Oscillations and propagation of Neutrinos through Magnetized GRB Fireball

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Questions of Magnetic Field in the GRBs:

- There is no way to get the magnetic field information directly from the fireball.
- It is strongly believed in the GRB community that the non-thermal $\gamma$-rays which we detect are mostly due to the synchrotron radiation of charged particles in the magnetic field although the strength of it is still unknown.
- Large magnetic field is expected if the progenitor is highly magnetized. Also amplification of small field due to turbulent dynamo mechanism, compression or shearing. Decreasing of magnetic field due to the expansion at large radii ($B(r) = B_0/r^2$)
- Here we take the weak field approximation and study the oscillation of neutrinos in the fireball environment.
- The fireball model explains the temporal structure of the bursts and nonthermal spectral behavior.
- Expanding fireball runs into the surrounding ISM to give afterglow.
Neutrino Oscillations in the Fireball:

Sources for the Neutrinos:

- Neutrinos of about 5-20 MeV are generated due to the stellar collapse or merger of compact binaries that trigger the burst.
- From nucleonic bremsstrahlung:
  \[ N + N \rightarrow N + N + \nu + \bar{\nu} \]

  From \( e^+e^- \) annihilation:
  \[ e^+ + e^- \rightarrow \nu + \bar{\nu} \]

- In the fireball:
  \[ p + e^- \rightarrow n + \nu_e \]

- All these neutrinos have the energy in the MeV range and will propagate through the fireball.

Effect of Heat Bath on Particle Properties:

- Massive Photons (Plasmon)
- Plasmon decay: \( \gamma_L \rightarrow \nu\bar{\nu} \)
- Massless neutrino acquires an effective mass
- MSW effect of neutrinos–flavor conversion
- Modification of dispersion relation in the medium with and without magnetic field:
  \[ p^2 - m^2 \neq 0 \]
Neutrino Oscillations in the Fireball:

Effective Potential for the Neutrinos:

\[ W(k - p) \]

\[ Z(k - p) \]

\[ f(q) \]

\[ Z \]

\[ \nu_e(k) \quad \nu_e(k) \]

\[ \nu_e(k) \quad \nu_e(k) \]

\[ \nu_e(k) \quad \nu_e(k) \]

Figure 1: One-loop diagrams for the neutrino self-energy in a medium.
Neutrino Self-Energy-I:

The total self-energy of neutrino in a magnetized medium:

\[ \Sigma(k) = \Sigma_W(k) + \Sigma_Z(k) + \Sigma_t(k). \tag{1} \]

with

\[ -i\Sigma_W(k) = \int \frac{d^4p}{(2\pi)^4} \left( \frac{-ig}{\sqrt{2}} \right) \gamma_\mu L iS_\ell(p) \left( \frac{-ig}{\sqrt{2}} \right) \gamma_\nu L iW^{\mu\nu}(q), \tag{2} \]

\[ -i\Sigma_Z(k) = \int \frac{d^4p}{(2\pi)^4} \left( \frac{-ig}{\sqrt{2} \cos \theta_W} \right) \gamma_\mu L iS_\nu(p) \left( \frac{-ig}{\sqrt{2} \cos \theta_W} \right) \gamma_\nu L iZ^{\mu\nu}(q), \tag{3} \]

and

\[ -i\Sigma_t(k) = -\left( \frac{g}{2 \cos \theta_W} \right)^2 R \gamma_\mu iZ^{\mu\nu}(0) \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ \gamma_\nu (C_V + C_A \gamma_5) iS_\ell(p) \right]. \tag{4} \]

W-boson diagram contributions:

\[ \text{Re } \Sigma_W(k) = R \left[ a_W k_\perp + b_W \ell + c_W \ell \right] L, \tag{5} \]

\[ a_W = -\frac{\sqrt{2} G_F}{M_W^2} \left\{ E_{\nu_e}(N_e - \bar{N}_e) + k_3(N_e^0 - \bar{N}_e^0) \right\} \]

\[ + \frac{\epsilon B}{2\pi^2} \int_0^\infty dp_3 \sum_{n=0}^\infty (2 - \delta_{n,0}) \left( \frac{m_e^2}{E_n} - \frac{H}{E_n} \right) (f_{e,n} + \bar{f}_{e,n}), \tag{6} \]
Neutrino Self-Energy-II:

\[ b_W = b_{W0} + \tilde{b}_W \]
\[ = \sqrt{2}G_F \left[ \left(1 + \frac{3}{2} \frac{m_e^2}{M_W^2} + \frac{E_{\nu e}^2}{M^2_W} \right)(N_e - \bar{N}_e) + \left( \frac{eB}{M^2_W} + \frac{E_{\nu e}k_3}{M^2_W} \right)(N_e^0 - \bar{N}_e^0) \right] \]
\[ - \frac{eB}{2\pi^2M^2_W} \int_0^\infty dp_3 \sum_{n=0}^\infty (2 - \delta_{n,0}) \left\{ 2k_3E_n \delta_{n,0} + 2E_{\nu e} \left( E_n + \frac{m_e^2}{2E_n} \right) \right\} (f_{e,n} + \bar{f}_{e,n}) \]

\[ c_W = c_{W0} + \tilde{c}_W \]
\[ = \sqrt{2}G_F \left[ \left(1 + \frac{1}{2} \frac{m_e^2}{M_W^2} - \frac{k_3^2}{M^2_W} \right)(N^0_e - \bar{N}^0_e) + \left( \frac{eB}{M^2_W} - \frac{E_{\nu e}k_3}{M^2_W} \right)(N_e - \bar{N}_e) \right] \]
\[ - \frac{eB}{2\pi^2M^2_W} \int_0^\infty dp_3 \sum_{n=0}^\infty (2 - \delta_{n,0}) \left\{ 2E_{\nu e} \left( E_n - \frac{m_e^2}{2E_n} \right) \delta_{n,0} \right\} + 2k_3 \left( E_n - \frac{3}{2} \frac{m_e^2}{E_n} - \frac{H}{E_n} \right) \right\} (f_{e,n} + \bar{f}_{e,n}) \].
Neutrino Self-Energy-III:

**Z-boson diagram contributions:**

\[\text{Re} \Sigma_Z(k) = R(a_Z k + b_Z \bar{\phi}) L,\] (8)

where

\[a_Z = \sqrt{2} G_F \left[ \frac{E_{\nu e}}{M_Z^2} (N_{\nu e} - \bar{\bar{N}}_{\nu e}) + \frac{2}{3} \frac{1}{M_Z^2} \left( \langle E_{\nu e} \rangle N_{\nu e} + \langle \bar{E}_{\nu e} \rangle \bar{N}_{\nu e} \right) \right],\] (9)

and

\[b_Z = \sqrt{2} G_F \left[ (N_{\nu e} - \bar{\bar{N}}_{\nu e}) - \frac{8 E_{\nu}}{3 M_Z^2} \left( \langle E_{\nu e} \rangle N_{\nu e} + \langle \bar{E}_{\nu e} \rangle \bar{N}_{\nu e} \right) \right].\] (10)

**Tadpole diagram contributions:**

\[\text{Re} \Sigma_t(k) = \sqrt{2} G_F R \left\{ C_{\nu e} (N_{\nu e} - \bar{\bar{N}}_{\nu e}) + C_{\nu p} (N_p - \bar{\bar{N}}_p) + C_{\nu n} (N_n - \bar{\bar{N}}_n) + (N_{\nu e} - \bar{\bar{N}}_{\nu e}) \right.\]

\[\left. + (N_{\nu \mu} - \bar{\bar{N}}_{\nu \mu}) + (N_{\nu \tau} - \bar{\bar{N}}_{\nu \tau}) \right\} \bar{\phi} - C_{A_{e}} (N_{e}^0 - \bar{\bar{N}}_{e}^0) \bar{\phi} \right] L.\] (11)
Neutrino Oscillations in the Fireball:

Effective Potential for the Neutrinos:
In a relativistic and non-degenerate $e^+ - e^-$ plasma,

- the effective potential for the (anti-)electron neutrino is: (if the fireball is charge neutral: $L_e = L_p$):

$$V_{\nu_e,\bar{\nu}_e} = \sqrt{2} G_F N_\gamma \left[ \pm L_e \mp \frac{1}{2} L_n - \left( \frac{7 \xi(4)}{\xi(3)} \right)^2 \frac{T^2}{M_W^2} \right].$$  \hspace{1cm} (12)

- For the muon (tau)-neutrinos:

$$V_{\nu_{\mu,\tau},\bar{\nu}_{\mu,\tau}} \simeq \sqrt{2} G_F N_\gamma L_{\mu,\tau}.$$ \hspace{1cm} (13)

where the particle asymmetry is defined as

$$L_i \equiv \frac{N_i - \bar{N}_i}{N_\gamma}$$
Neutrino Oscillations in the Fireball:

Effective Potential for the Neutrinos:
Including the (weak) magnetic field contributions (\(B \ll m^2/e = B_c \sim 10^{13} \ G\)), the effective potentials is given:

\[
V = V_{\nu_e} - V_{\nu_\mu} \simeq \sqrt{2}G_F \frac{m^3}{\pi^2} \left[ \Phi_1 - \Phi_2 - \frac{4}{\pi^2} \left( \frac{m}{M_W} \right)^2 \frac{E_{\nu_e}}{m} (\Phi_3 - \Phi_4) \right].
\] (14)

\[
N_e^0 - \bar{N}_e^0 = \frac{m^3}{\pi^2} \frac{B}{B_c} \sum_{l=0}^{\infty} (-1)^l \sinh \alpha K_1(\sigma) = \frac{m^3}{\pi^2} \Phi_1,
\] (15)

\[
N_e - \bar{N}_e = \frac{m^3}{\pi^2} \sum_{l=0}^{\infty} (-1)^l \sinh \alpha \left[ \frac{2}{\sigma} K_2(\sigma) - \frac{B}{B_c} K_1(\sigma) \right] = \frac{m^3}{\pi^2} \Phi_2,
\] (16)

\[
\Phi_3 = \sum_{l=0}^{\infty} (-1)^l \cosh \alpha \left[ \left( \frac{3}{\sigma^2} - \frac{1}{4} \frac{B}{B_c} \right) K_0(\sigma) + \left( 1 + \frac{6}{\sigma^2} \right) \frac{K_1(\sigma)}{\sigma} \right],
\] (17)

\[
\Phi_4 = \sum_{l=0}^{\infty} (-1)^l \cosh \alpha \frac{1}{\sigma^2} \left[ K_0(\sigma) + \frac{2}{\sigma} K_1(\sigma) \right],
\] (18)
**Neutrino Oscillations $\nu_e \leftrightarrow \nu_{\mu,\tau}$:**

Neutrino Oscillation $\nu_e \leftrightarrow \nu_{\mu,\tau}$ (I):

The evolution equation for the propagation of neutrinos is:

$$i \left( \begin{array}{c} \dot{\nu}_e \\ \dot{\nu}_\mu \end{array} \right) = \left( \begin{array}{cc} V - \Delta \cos 2\theta & \frac{\Delta}{2} \sin 2\theta \\ \frac{\Delta}{2} \sin 2\theta & 0 \end{array} \right) \left( \begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right),$$

$$\Delta = \delta m^2 / 2 E_\nu, \quad V = V_{\nu_e} - V_{\nu_\mu},$$

$E_\nu$ is the neutrino energy (5-20 MeV) and $\theta$ is the neutrino mixing angle.

The conversion probability at a given time $t$ is given by

$$P_{\nu_e \rightarrow \nu_\mu(\nu_\tau)}(t) = \frac{\Delta^2 \sin^2 2\theta}{\omega^2} \sin^2 \left( \frac{\omega t}{2} \right), \quad (19)$$

with

$$\omega = \sqrt{(V - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}. \quad (20)$$

the effective potential for the (anti-)electron neutrino is: The oscillation length for the neutrino is given by

$$L_{osc} = \frac{L_v}{\sqrt{\cos^2 2\theta (1 - \frac{V}{\Delta \cos 2\theta})^2 + \sin^2 2\theta}}, \quad (21)$$

where $L_v = 2\pi / \Delta$ is the vacuum oscillation length.
**Neutrino Oscillations** $\nu_e \leftrightarrow \nu_{\mu,\tau}$:

**Neutrino Oscillation** $\nu_e \leftrightarrow \nu_{\mu,\tau}$ (II):

At Resonance,

$$V = \Delta \cos 2\theta.$$  \hspace{1cm} (22)

The resonance condition is

$$\Phi_1 - \Phi_2 - 3.196 \times 10^{-11} E_{MeV} (\Phi_3 - \Phi_4) = 2.26 \frac{\delta m^2}{E_{MeV}} \cos 2\theta,$$  \hspace{1cm} (23)

The left hand side depends on the chemical potential $\mu$ of the background electrons and positrons, temperature $T$ of the plasma and the neutrino energy. On the other hand the right hand side depends only on the neutrino energy (for a given set of neutrino mass square difference and the mixing angle).

The resonance condition can be written as

$$L_e T^{3}_{MeV} = 0.124 \frac{\delta m^2 \cos 2\theta}{E_{MeV}}$$  \hspace{1cm} (24)

**Baryon Load:**

$$M_b \sim 2.23 \times 10^{-4} R^3_7 T^3_{MeV} L_e M_\odot.$$  \hspace{1cm} (25)
Our Analysis:

Our Analysis of Neutrino Oscillations:

- We take into account the neutrino oscillation parameters from solar, atmospheric (SNO and SuperKamiokande), and the Liquid Scintillator Neutrino Detector (LSND) reactor neutrinos to study the resonance conditions in the fireball.

- The resonance oscillation of neutrinos can constraint the fireball parameters.

- For the best fit neutrino oscillation parameter sets $\delta m^2$ and $\sin^2 2\theta$ of the above three different state of the art experiments (SNO, Super Kamiokande and LSND), we have shown what should be the values of $\mu$ and $T$ to satisfy the resonance condition for different neutrino energies in the fireball plasma.

- Afterward these values of $\mu$ and $T$ are used to calculate the lepton asymmetry $L_e$, baryon load $M_b$ and the resonance length $L_{res}$ of the propagating neutrinos.
Solar Neutrinos: SNO + KamLAND

\[ 6 \times 10^{-5} \text{eV}^2 < \delta m^2 < 10^{-4} \text{eV}^2 \] and \[ 0.64 < \sin^2 2\theta < 0.96. \]

Probably very few or No resonant oscillations take place within the fireball and most of the neutrinos will come out.

Table 1: SNO: The best fit values of the neutrino oscillation parameters \( \delta m^2 \sim 7.1 \times 10^{-5} \text{eV}^2 \) and \( \sin^2 2\theta \sim 0.69 \) from the combined analysis of the salt phase data of SNO and KamLAND are used in the resonance condition for different neutrino energies in this table. The magnetic field used here is \( B/B_c = 0.1 \).

<table>
<thead>
<tr>
<th>( E_{MeV} )</th>
<th>( T(\text{MeV}) )</th>
<th>( L_e )</th>
<th>( L_{res}(\text{cm}) )</th>
<th>( M_b(R_{J}^3 M_{\odot}) )</th>
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<td>5</td>
<td>3</td>
<td>4.93 \times 10^{-8}</td>
<td>2.10 \times 10^7</td>
<td>2.97 \times 10^{-10}</td>
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<td>3.28 \times 10^{-8}</td>
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<td>9.14 \times 10^{-10}</td>
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<tr>
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<td>5.07 \times 10^{-8}</td>
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<td>1.49 \times 10^{-9}</td>
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<tr>
<td></td>
<td>10</td>
<td>9.99 \times 10^{-8}</td>
<td></td>
<td>2.23 \times 10^{-8}</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
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<td>8.42 \times 10^7</td>
<td>4.11 \times 10^{-10}</td>
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<td></td>
<td>2.85 \times 10^{-9}</td>
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<td></td>
<td>10</td>
<td>1.99 \times 10^{-7}</td>
<td></td>
<td>4.44 \times 10^{-8}</td>
</tr>
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</table>
Atmospheric Neutrinos: Super-Kamiokande

1.9 \times 10^{-3} \text{eV}^2 < \delta m^2 < 3.0 \times 10^{-3} \text{eV}^2 \text{ and } 0.9 \leq \sin^2 2\theta \leq 1.0

Neutrinos can have many resonant oscillations within the fireball.

Table 2: SK: The best fit values of the atmospheric neutrino oscillation parameters $\delta m^2 \sim 2.5 \times 10^{-3} \text{eV}^2$ and $\sin^2 2\theta \sim 0.9$ from Super-Kamiokande Collaboration are used in the resonance condition for different neutrino energies in this table.

<table>
<thead>
<tr>
<th>$E_{\text{MeV}}$</th>
<th>$T$ (MeV)</th>
<th>$L_e$</th>
<th>$L_{res}$ (cm)</th>
<th>$M_b (R^3_7 M_\odot)$</th>
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<tr>
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<td>$2.30 \times 10^{-9}$</td>
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<td>$1.30 \times 10^{-7}$</td>
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<td>$3.62 \times 10^{-9}$</td>
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<td>$1.09 \times 10^{-7}$</td>
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<td>$2.44 \times 10^{-8}$</td>
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<tr>
<td>20</td>
<td>3</td>
<td>$2.38 \times 10^{-7}$</td>
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<td>$1.44 \times 10^{-9}$</td>
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<tr>
<td></td>
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<td>$1.37 \times 10^{-7}$</td>
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<td>$3.81 \times 10^{-9}$</td>
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<tr>
<td></td>
<td>10</td>
<td>$2.04 \times 10^{-7}$</td>
<td></td>
<td>$4.54 \times 10^{-8}$</td>
</tr>
</tbody>
</table>
The effective Hamiltonian is

\[ H = U \cdot H_0^d \cdot U^\dagger + \text{diag}(V_e, 0, 0), \]  

(26)

with

\[ H_0^d = \frac{1}{2E_\nu} \text{diag}(-\Delta m_{21}^2, 0, \Delta m_{32}^2). \]  

(27)

Here \( V_e \) is the charge current (CC) matter potential and \( U \) is the three neutrino mixing matrix.

The different neutrino probabilities are given as

\[
\begin{align*}
P_{ee} & = 1 - 4s_{13,m}^2 c_{13,m}^2 s_{31}, \\
P_{\mu\mu} & = 1 - 4s_{13,m}^2 c_{13,m}^2 s_{23}^4 S_{31} - 4s_{13,m}^2 s_{23}^2 c_{23} S_{21} - 4c_{13,m}^2 s_{23}^2 c_{23} S_{32}, \\
P_{\tau\tau} & = 1 - 4s_{13,m}^2 c_{13,m}^2 c_{23}^4 S_{31} - 4s_{13,m}^2 s_{23}^2 c_{23} S_{21} - 4c_{13,m}^2 s_{23}^2 c_{23} S_{32}, \\
P_{e\mu} & = 4s_{13,m}^2 c_{13,m}^2 s_{23}^2 S_{31}, \\
P_{e\tau} & = 4s_{13,m}^2 c_{13,m}^2 c_{23}^2 S_{31}, \\
P_{\mu\tau} & = -4s_{13,m}^2 c_{13,m}^2 s_{23}^2 c_{23}^2 S_{31} + 4s_{13,m}^2 s_{23}^2 c_{23}^2 S_{21} + 4c_{13,m}^2 s_{23}^2 c_{23}^2 S_{32},
\end{align*}
\]

(28)
where

\[
\sin 2\theta_{13,m} = \frac{\sin 2\theta_{13}}{\sqrt{(\cos 2\theta_{13} - 2E_\nu V_e/\Delta m_{32}^2)^2 + (\sin 2\theta_{13})^2}}, \tag{29}
\]

and

\[
S_{ij} = \sin^2 \left( \frac{\Delta \mu_{ij}^2}{4E_\nu} L \right). \tag{30}
\]

\[
\Delta \mu_{21}^2 = \frac{\Delta m_{32}^2}{2} \left( \frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} - 1 \right) - E_\nu V_e
\]

\[
\Delta \mu_{32}^2 = \frac{\Delta m_{32}^2}{2} \left( \frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} + 1 \right) + E_\nu V_e
\]

\[
\Delta \mu_{31}^2 = \frac{\Delta m_{32}^2}{\sin 2\theta_{13,m}} \sin 2\theta_{13} \tag{31}
\]

where

\[
\sin^2 \theta_{13,m} = \frac{1}{2} \left( 1 - \sqrt{1 - \sin^2 2\theta_{13,m}} \right)
\]

\[
\cos^2 \theta_{13,m} = \frac{1}{2} \left( 1 + \sqrt{1 - \sin^2 2\theta_{13,m}} \right) \tag{32}
\]
The oscillation length for the neutrino is given by

$$L_{osc} = \frac{L_v}{\sqrt{\cos^2 2\theta_{13} \left(1 - \frac{2E_\nu V_e}{\Delta m_{32}^2 \cos 2\theta_{13}}\right)^2 + \sin^2 2\theta_{13}}}$$

(33)

where $L_v = \frac{4\pi E_\nu}{\Delta m_{32}^2}$ is the vacuum oscillation length. For resonance to occur, we should have $V_{eff,B} = V_e > 0$ and

$$\cos 2\theta_{13} = \frac{2E_\nu V_e}{\Delta m_{32}^2}.$$  

(34)

**Comparision between $B = 0.1B_c$ and $B=0$**

$$\Phi_A - 1.58027 \times 10^{-10} E_{\nu,MeV} \Phi_B \simeq 2.24208 \frac{\Delta m_{32}^2}{E_{\nu,MeV}} \cos 2\theta_{13},$$

(35)
Comparison between $B = 0.1B_c$ and $B=0$ (I)

The contour plot of the resonance condition as functions of $T/m_e$ and $\mu = 10^p m_e$ is shown for different neutrino energies and $B = 0.1 B_c$ where (a) is for $\Delta m_{32}^2 = 10^{-2.9} \text{eV}^2$ and $\Delta m_{32}^2 = 10^{-2.2} \text{eV}^2$. 
Comparision between $B = 0.1B_c$ and $B=0$ (II)

Comparision between $B = 0.1B_c$ and $B=0$ in $P_{\mu\mu}$:

The survival probability of muon neutrinos $P_{\mu\mu}$ is plotted as a function of $\Delta m_{32}^2 eV^2 = 10^X eV^2$, for the fireball radius $L = 100 \, km$ (a) and $L = 1000 \, km$ (b). The neutrino energy and magnetic field are shown in it.
Comparision between $B = 0.1B_c$ and $B=0$ (III)

Comparision between $B = 0.1B_c$ and $B=0$ in $P_{e\mu}$:

The probability $P_{e\mu}$ is plotted as a function of $\Delta m_{32}^2$, for the fireball radius $L = 100 \, km$ with $E_\nu = 5.0 \, MeV$ (a) and $E_\nu = 30 \, MeV$ (b).

Comparision between $B = 0.1B_c$ and $B=0$ in $P_{\mu\tau}$:

The probability $P_{\mu\tau}$ is plotted as a function of $\Delta m_{32}^2$, for the fireball radius $L = 100 \, km$ (a) and $L = 1000 \, km$ (b). The neutrino energy and magnetic field are shown in it.
Summary and Discussions:

- From the Collapsar/Hypernova model of GRBs, lots of neutrinos will be produced and fractions of these neutrinos will propagate through the fireball. We studied the resonant oscillation of these neutrinos in the fireball.

- We assume
  - Spherical Fireball R 100 - 1000 Km
  - Charge Neutral
  - No. of Protons = No. of Neutrons
  - \( L_e > 6.14 \times 10^{-9} T_{MeV}^2 \)

  Neutrino Oscillation can be occurred for the Neutrino mass square difference mixing angle are in the Atmospheric and Reactor expt. ranges, so that the average conversion probability of neutrinos will be 0.5. however for the solar neutrino range, a few or no resonant oscillation will take place.