New Model of massive spin-2 particle

Based on Phys.Rev. D90 (2014) 043006, Y.O, S. Akagi, S. Nojiri
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Introduction

Massive spin-2 = Massive graviton?

The free massive spin-2 field theory was formulated by Fierz and Pauli.

(They tried to construct field theories with arbitrary spin)

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

- ✓ No ghost (Consistent theory as QFT)
- ✓ Realization of 5 d.o.f in 4 dimensions (Massive spin-2 particle)

thanks to the Fierz-Pauli mass term.

$$\text{Massive spin-2 particle} \overset{?}{=} \text{Massive graviton}$$

Introduction

The 1st problem : vDVZ discontinuity

Massive spin-2 in the massless limit $\neq$ Linearized GR

<u>Does this mean the massive spin-2 particle can not be graviton?</u>

Vainshtein's argument $\Rightarrow$ Non-linearity screens the discontinuity!

Introduction

Einstein-Hilbert + Fierz-Pauli mass term

$$S = \frac{1}{2\kappa^2}\int d^4x\left[(\sqrt{-g}R) - \frac{1}{4}m^2\eta^{\mu\alpha}\eta^{\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\nu\beta}\right)\right]$$

Static and spherical solution of EH + FP

$m \to 0$

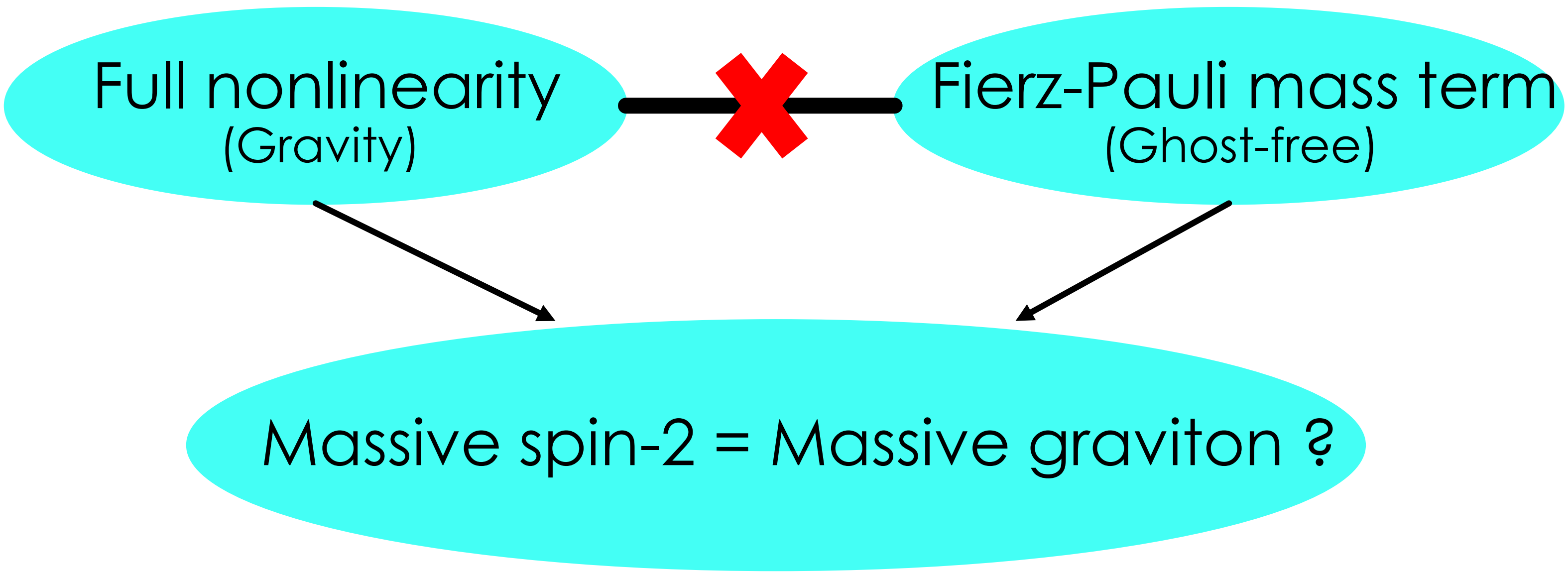
Schwarzschild Solution

Fully the non-linear massive spin-2

Fully the non-linear massless spin-2

No discrepancy!

Introduction



However…

Boulware and Deser suggested the nonlinearity and the ghost-free property are not compatible with each other.

Introduction

The 2nd problem : Boulware-Deser ghost

Nonlinearity $\Rightarrow$ ghost

e.g.) Einstein-Hilbert + Fierz-Pauli mass term

$$S=\frac{1}{2\kappa^2}\int d^4x\left[\left(\sqrt{-g}R\right)-\frac{1}{4}m^2\eta^{\mu\alpha}\eta^{\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta}-h_{\mu\alpha}h_{\nu\beta}\right)\right]$$

ADM variables (Lapse N, shift N_i, 3-metric γ_{ij}) $\left(h_{ij}\coloneqq g_{\mathrm{ij}}-\delta_{ij}\right)$

$$\begin{aligned}\eta^{\mu\alpha}\eta^{\mu\beta}\left(h_{\mu\nu}h_{\alpha\beta}-h_{\mu\alpha}h_{\mu\beta}\right)=&\delta^{ik}\delta^{jl}\left(h_{ij}h_{kl}-h_{ik}h_{jl}\right)+2\delta^{ij}h_{ij}\\&-\underline{2N}^2\delta^{ij}h_{ij}+\underline{2N_i\left(\gamma^{ij}-\delta^{ij}\right)N_i}\end{aligned}$$

Hamiltonian constraint Momentum constraints

Introduction

<u>Progress in 2000s</u>

1. DGP model Phys.Lett. B485 (2000) 208-214

Higher derivative scalar field theory without any ghost.

2. Effective field theoretical approach Annals Phys. 305 (2003) 96-118

Stuckelberg trick

Encoding the scalar mode into the lagrangian explicitly.
(Using the scalar field)

The origin of the Boulware-Deser ghost is the higher derivative of the scalar field.

Introduction

Progress in 2000s

Field theoretical approach
(Stuckelberg method)

The origin of BD ghost : Higher derivatives of the scalar field.

DGP model
(Ghost-free massive gravity)

Higher derivative scalar field theory without ghost.

dRGT massive gravity

Introduction

dRGT massive gravity

de Rham, Gabadadze, Tolley Phys.Rev.Lett. 106 (2011) 231101

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R + \frac{m^2}{4}\sum_{n=2}^{4}\alpha_n e_n(\mathcal{K})\right]$$

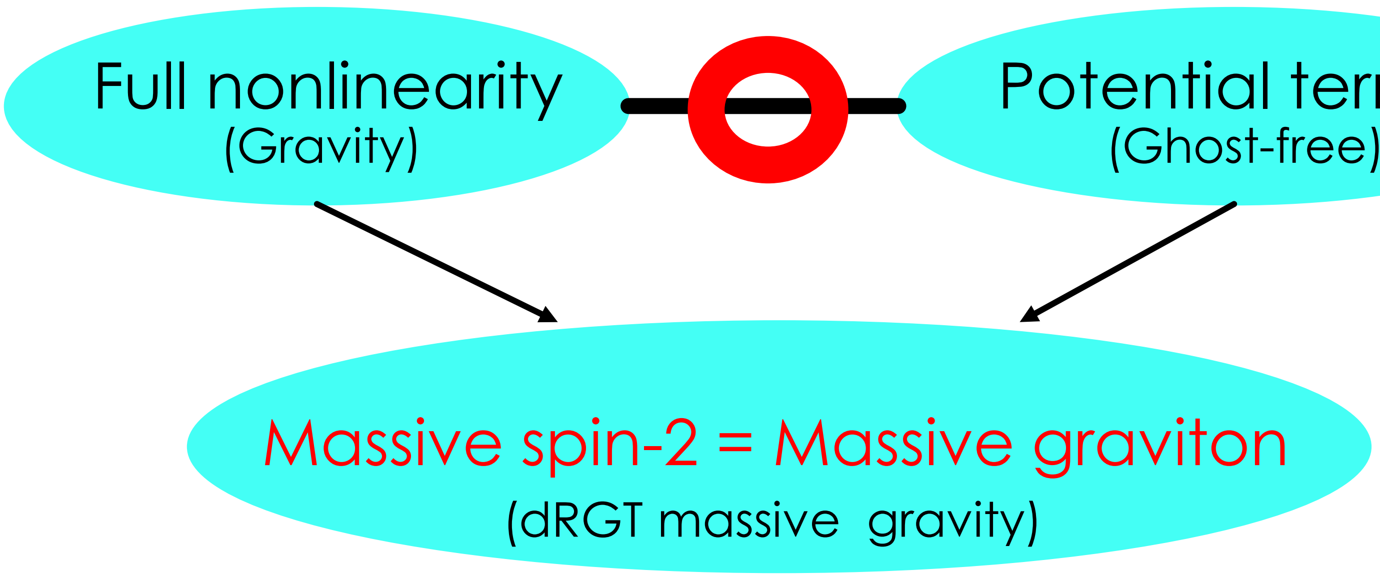
$$e_2(\mathbb{X}) = \left([\mathbb{X}]^2 - [\mathbb{X}^2]\right), \quad e_3(\mathbb{X}) = \left([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]\right),$$

$$e_4(\mathbb{X}) = \left([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4]\right)$$

$$\mathbb{X} = X^{\mu}{}_{\nu} \qquad [\mathbb{X}] := X^{\mu}{}_{\mu} \qquad \mathcal{K}^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \sqrt{g^{-1}\eta}^{\mu}{}_{\nu}$$

Nonlinearity and the ghost-free property are compatible now!

Introduction



Massive spin-2 particles can be identified with massive gravitons.

Should we identify the massive spin-2 with the massive graviton?

Motivation

Question 1

Is Massive spin-2 = Massive graviton necessary?

Massive spin-2 theory necessarily leads to modification of gravity?

As a fact,

There exist massive spin-2 particles in the hadron spectrum.

Motivation

Question 2

Which assumptions can we remove?

In the history of the massive spin-2 field….

vDVZ discontinuity Vainshten mechanism

Full non-linearity (EH term is introduced)

This is natural in some sense because….

- To avoid the vDVZ discontinuity.
- The spin-2 field $h_{\mu\nu}$ is naturally replaced by the metric $g_{\mu\nu}$

Motivation

Question 2

Which assumptions can we remove?

Einstein-Hilbert term

The massive spin-2 particle is not the graviton in this point of view.

Full nonlinearity is not necessary.

Construct the massive spin-2 theory.

Fierz-Pauli theory

Massless spin-2 field theory

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h$$

The phase space is spanned by h_{ij} and π^{ij}. (12 dimensions)

$$\text{8 second class constraints} \left\{ \begin{array}{c} \text{4 first class constraints} \\ + \\ \text{4 gauge fixing functions} \end{array} \right.$$

(12 dimensional phase space) – (8 constraints) = 4 independent comp.

Massless spin-2 particle has 2 degrees of freedom.

Fierz-Pauli theory

Massive spin-2 field theory

Possible quadratic terms $\qquad h_{\mu\nu}h^{\mu\nu} \qquad h^2$

Candidates for mass terms $\qquad \mathcal{L}_{mass} \sim h_{\mu\nu}h^{\mu\nu} - (1-a)h^2$

When $a \neq 0$, an extra d.o.f propagates with a negative kinetic energy.

Fierz-Pauli lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h \underbrace{- \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)}_{\text{Fierz-Pauli tuning}}$$

Fierz-Pauli theory

Hamiltonian analysis

Conjugate momenta $\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \dot{h}_{ij}} = \dot{h}_{ij} - \dot{h}_{kk}\delta_{ij} - 2\partial_{(i}h_{j)0} + 2\partial_k h_{0k}\delta_{ij}\,,$

Lagrangian density $\mathcal{L} = \pi_{ij}\dot{h}_{ij} - \mathcal{H} + h_{00}\left(\vec{\nabla}^2 h_{ii} - \partial_i\partial_j h_{ij} - m^2 h_{ii}\right)$

$$\begin{aligned}\mathcal{H} =& \frac{1}{2}\pi_{ij}^2 - \frac{1}{2}\frac{1}{D-2}\pi_{ii}^2 + \frac{1}{2}\partial_k h_{ij}\partial_k h_{ij} - \partial_i h_{jk}\partial_j h_{ik} \\ &+ \partial_i h_{ij}\partial_j h_{kk} - \frac{1}{2}\partial_i h_{jj}\partial_i h_{kk} + \frac{1}{2}m^2\left(h_{ij}h_{ij} - h_{ii}^2\right) + \frac{1}{m^2}\left(\partial_j\pi_{ij}\right)^2\end{aligned}$$

Fierz-Pauli theory

$$\mathcal{L} = \pi_{ij}\dot{h}_{ij} - \mathcal{H} + h_{00}\left(\vec{\nabla}^2 h_{ii} - \partial_i\partial_j h_{ij} - m^2 h_{ii}\right)$$

$$\begin{aligned}\mathcal{H} =& \frac{1}{2}\pi_{ij}^2 - \frac{1}{2}\frac{1}{D-2}\pi_{ii}^2 + \frac{1}{2}\partial_k h_{ij}\partial_k h_{ij} - \partial_i h_{jk}\partial_j h_{ik} \\ &+ \partial_i h_{ij}\partial_j h_{kk} - \frac{1}{2}\partial_i h_{jj}\partial_i h_{kk} + \frac{1}{2}m^2\left(h_{ij}h_{ij} - h_{ii}^2\right) + \frac{1}{m^2}\left(\partial_j \pi_{ij}\right)^2\end{aligned}$$

h_{00}: Lagrange multiplier (Linear) → Single constraint

$$\mathcal{C} = -\vec{\nabla}^2 h_{ii} + \partial_i\partial_j h_{ij} + m^2 h_{ii} = 0$$

Secondary constraint

$$\{H, \mathcal{C}\}_{\mathrm{PB}} = \frac{1}{2}m^2\pi_{ii} + \partial_i\partial_j\pi_{ij} = 0\,, \quad H = \int d^3x\,\mathcal{H}$$

Fierz-Pauli theory

In total, we have two second class constraints.

(12 dimensional phase space) − (2 constraints) = 10 independent comp.

(5 polarizations of the massive spin-2 particle)

h_{00}^2 does not appear thanks to the Fierz-Pauli tuning.

No ghost if h_{00} remains linear in general.

Ghost-free interaction

Ghost-free interactions for Fierz-Pauli theory

Ghost-free term

Folkerts *et al.* arXiv:1107.3157 [hep-th]
Hinterbichler, JHEP 10 (2013) 102

$$\mathcal{L}_{d,n} \sim \eta^{\mu_1\nu_1\cdots\mu_n\nu_n}\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\cdots\partial_{\mu_{d-1}}\partial_{\nu_{d-1}}h_{\mu_d\nu_d}h_{\mu_{d+1}\nu_{d+1}}\cdots h_{\mu_{d/2+n}\nu_{d/2+n}}$$
$$(d/2 \leq n \leq D - d/2)$$

d : The number of derivatives, n : The number of the fields, D : Spacetime dim

$$\begin{aligned}
\eta^{\mu_1\nu_1\mu_2\nu_2} \equiv& \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2} - \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}\,,\\
\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \equiv& \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3} - \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_2} + \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_1}\\
&- \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_3} + \eta^{\mu_1\nu_3}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_2} - \eta^{\mu_1\nu_3}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_1}
\end{aligned}$$

- The kinetic term and the mass term are included.
- We use this term to construct the massive spin-2 model.

Ghost-free interaction

- Linear with respect to h_{00} in the Hamiltonian.

$$\eta^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \sim h_{00}\,(h_{11} + h_{22} + h_{33}) \\ + \text{terms not including } h_{00}$$

- The terms which include both of h_{00} and h_{0i} never appear.

Variation of h_{00}

$\Rightarrow$ a constraint for h_{ij} and their conjugate momenta π_{ij}
+ secondary constraint

(12 dimensional phase space) − (2 constraints) = 10 independent comp.

$\Rightarrow$ No ghost

Ghost-free interaction

The Fierz-Pauli lagrangian

The kinetic term : $\mathcal{L}_{2,2} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\right)h_{\mu_3\nu_3}$

The mass term : $\mathcal{L}_{0,2} \sim \eta^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}$

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\lambda h^\lambda{}_\mu\partial_\nu h^{\mu\nu} - \partial^\mu h_{\mu\nu}\partial^\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) \\ &= -\frac{1}{2}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\right)h_{\mu_3\nu_3} + \frac{m^2}{2}\eta^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}\end{aligned}$$

Ghost-free interaction

In 4 dimensions, the allowed interaction is following:

Non-derivative int.

$$\begin{cases} \mathcal{L}_{0,3} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} \\ \mathcal{L}_{0,4} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4} \end{cases}$$

Derivative int.

$$\mathcal{L}_{2,3} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \partial_{\mu_1} \partial_{\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4}$$

Other possibilities are excluded due to the antisymmetric properties.

New model of massive spin-2

New model of massive spin-2

$$\mathcal{L} = -\frac{1}{2}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\right)h_{\mu_3\nu_3} + \frac{m^2}{2}\eta^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}$$
$$-\frac{\mu}{3!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{\zeta}{4!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$
$$-\frac{\lambda}{4!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

μ, ζ, λ : constants

New model of massive spin-2

Possible application (Additional motivation)

- Supersymmetry breaking mechanism?

$$\mathcal{L} = \mathcal{L}_{FP} - \frac{\mu}{3!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{\lambda}{4!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

$\Rightarrow$ Can this model be used to realize SUSY breaking?

- BH physics and cosmology?

The new spin-2 model on curved spacetime.

FP theory in curved space

The simplest model (Minimal coupling)

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\nabla_\mu h \nabla^\mu h - \frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho} - \nabla^\mu h_{\mu\nu}\nabla^\nu h + \nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu} - \frac{m^2}{2}(h_{\mu\nu}h^{\mu\nu} - h^2)\right\}$$

We don't regard the massive spin-2 as the perturbation of metric.

Unfortunately, this model does not have 5 degrees of freedom.

↓

To see this reason, let us see the FP theory in flat spacetime.

FP theory in curved space

Taking the variation gives e.o.m

$$E_{\mu\nu} = \Box h_{\mu\nu} - \partial_\lambda \partial_\mu h^\lambda{}_\nu - \partial_\lambda \partial_\nu h^\lambda{}_\mu + \eta_{\mu\nu} \partial_\lambda \partial_\sigma h^{\lambda\sigma} \\ + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \Box h - m^2 (h_{\mu\nu} - \eta_{\mu\nu} h) = 0$$

Two constrains obtained from $E_{\mu\nu}$

$$\begin{cases} \partial^\mu E_{\mu\nu} = m^2 (\partial_\nu h - \partial^\mu h_{\mu\nu}) = 0 \\ \dfrac{m^2}{2} \eta^{\mu\nu} E_{\mu\nu} + \partial^\mu \partial^\nu E_{\mu\nu} = \dfrac{3}{2} h m^4 = 0 \end{cases}$$

$$\Rightarrow \quad (\Box - m^2) h_{\mu\nu} = 0 \,, \quad \partial^\mu h_{\mu\nu} = 0 \,, \quad h = 0$$

FP theory in curved space

Key point

$$\begin{cases} \partial^\mu E_{\mu\nu} = m^2(\partial_\nu h - \partial^\mu h_{\mu\nu}) = 0 \\ \dfrac{m^2}{2}\eta^{\mu\nu}E_{\mu\nu} + \partial^\mu\partial^\nu E_{\mu\nu} = \dfrac{3}{2}hm^4 = 0 \end{cases}$$

Existence of the second equation $\Leftrightarrow$ Commutativity of ∂_μ

On the other hand……

Covariant derivatives ∇_μ do not commute with each other.

$\Rightarrow$ $R\nabla\nabla h$ type terms appear and the constraint is lost.

FP theory in curved space

FP theory in curved spacetime was considered by Buchbinder *et al.*

I. L. Buchbinder, D. M. Gitman, V. A. Krykhtin and V. D. Pershin, Nucl. Phys. B **584** (2000) 615 [hep-th/9910188]
I. L. Buchbinder, V. A. Krykhtin and V. D. Pershin, Phys. Lett. B **466** (1999) 216 [hep-th/9908028].

They constructed the theory having 5 d.o.f in curved spacetime.

Problem : $R\nabla\nabla h$ type terms appear and the constraint is lost.

↑

Prepare non-minimal coupling terms like Rhh
(quadratic in derivatives)

FP theory in curved space

Prepare non-minimal coupling terms like Rhh

(quadratic in derivatives)

$$S=\int d^4x\sqrt{-g}\Big\{\frac{1}{4}\nabla_\mu H\nabla^\mu H-\frac{1}{4}\nabla_\mu H_{\nu\rho}\nabla^\mu H^{\nu\rho}-\frac{1}{2}\nabla^\mu H_{\mu\nu}\nabla^\nu H+\frac{1}{2}\nabla_\mu H_{\nu\rho}\nabla^\rho H^{\nu\mu}$$
$$+\frac{a_1}{2}RH_{\alpha\beta}H^{\alpha\beta}+\frac{a_2}{2}RH^2+\frac{a_3}{2}R^{\mu\alpha\nu\beta}H_{\mu\nu}H_{\alpha\beta}+\frac{a_4}{2}R^{\alpha\beta}H_{\alpha\sigma}H_\beta{}^\sigma+\frac{a_5}{2}R^{\alpha\beta}H_{\alpha\beta}H$$
$$-\frac{m^2}{4}H_{\mu\nu}H^{\mu\nu}+\frac{m^2}{4}H^2\Big\}$$

⇓

They determined a_i and found that the theory can be ghost-free on Einstein manifold.

FP theory in curved space

Ghost-free FP theory on curved space

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \nabla_\mu h \nabla^\mu h - \frac{1}{2} \nabla_\mu h_{\nu\rho} \nabla^\mu h^{\nu\rho} - \nabla^\mu h_{\mu\nu} \nabla^\nu h \right.$$
$$\left. + \nabla_\mu h_{\nu\rho} \nabla^\rho h^{\nu\mu} + \frac{\xi}{4} R h_{\mu\nu} h^{\mu\nu} + \frac{1-2\xi}{8} R h^2 - \frac{m^2}{2} (h_{\mu\nu} h^{\mu\nu} - h^2) \right\}$$

ξ : Real parameter

- The background is restricted to Einstein manifold $R_{\mu\nu} = \frac{1}{4} g_{\mu\nu} R$

New model of massive spin-2 in a curved spacetime

Interaction on the Einstein manifold

Int. in a flat spacetime

$$\begin{cases} \mathcal{L}_{0,3} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} \\ \mathcal{L}_{0,4} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4} \\ \mathcal{L}_{2,3} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \partial_{\mu_1} \partial_{\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4} \end{cases}$$

Int. on Einstein manifold

$$\begin{cases} \mathcal{L}_{0,3} \sim g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} \\ \mathcal{L}_{0,4} \sim g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4} \\ \mathcal{L}_{2,3} \sim g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \nabla_{\mu_1} \nabla_{\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4} \end{cases}$$

New model of massive spin-2 in a curved spacetime

New model of massive spin-2 on the Einstein manifold

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\,\Big\{ & \frac{1}{2}\nabla_\mu h \nabla^\mu h - \frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho} - \nabla^\mu h_{\mu\nu}\nabla^\nu h \\
& + \nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu} + \frac{\xi}{4} R h_{\mu\nu}h^{\mu\nu} + \frac{1-2\xi}{8} R h^2 - \frac{m^2}{2}(h_{\mu\nu}h^{\mu\nu} - h^2) \\
& - \frac{\mu}{3!} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{\zeta}{4!} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}\nabla_{\mu_1}\nabla_{\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4} \\
& - \frac{\lambda}{4!} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4} \Big\}
\end{aligned}
$$

Is this model ghost-free on Einstein manifold?

$\Downarrow$

Counting the degrees of freedom using Lagrangian analysis.

New model of massive spin-2 in a curved spacetime

Lagrangian analysis

1. The system containing some set of fields $\phi^A(\mathrm{x}), \quad \mathrm{A} = 1, 2, \cdots N$

2. The second time derivatives are defined only for $r < N$ fields in e.o.m.

3. N-r primary constraints are constructed from e.o.m.

4. Requirement of conservation in time of the primary constraints defines the second time derivatives for remaining fields or new secondary constraints.

5. This procedure continues until the second time derivatives are defined for all fields ϕ^A

New model of massive spin-2 in a curved spacetime

Example : FP theory in a flat spacetime

$$\mathcal{L} = -\frac{1}{2}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\right)h_{\mu_3\nu_3} + \frac{m^2}{2}\eta^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}$$

1. The system containing some set of fields $\phi^A(\mathrm{x})$, $\mathrm{A} = 1, 2, \cdots N$

$\Rightarrow$ $h_{\mu\nu}$: 10 components

Equations of motion

$$0 = E_{\mu\nu} = -\eta_{(\mu\nu)\mu_1\nu_1\mu_2\nu_2}\partial^{\mu_1}\partial^{\nu_1}h^{\mu_2\nu_2} + m^2\eta_{\mu\nu\mu_1\nu_1}h^{\mu_1\nu_1}$$

$$\begin{cases} E_{ij} = (\ddot{h}_{ij}\,\text{part}) + (\text{terms without}\,\ddot{h}) \\ E_{0\mu} = (\text{terms without}\,\ddot{h}) \end{cases}$$

New model of massive spin-2 in a curved spacetime

2. The second time derivatives are defined only for $r < N$ fields in e.o.m.

$$E_{ij} = (\ddot{h}_{ij}\,\text{part}) + (\text{terms without}\,\ddot{h})$$

⇨ h_{ij}: 6 components

3. N-r primary constraints are constructed from e.o.m.

$$\phi^{(1)}{}_{\mu} := E_{0\mu} = (\text{terms without}\,\ddot{h})$$

⇨ 4 constraints (@ some time t)

$h_{0\mu}$: Undetermined (4 components)

New model of massive spin-2 in a curved spacetime

4-1. Requirement of conservation in time of the primary constraints .

$$\dot{\phi}^{(1)\mu} = \partial_0 E^{0\mu} = 0$$

This equations do not contain $\ddot{h}_{0\mu}$ and $\ddot{h}_{ij}$ are eliminated using e.o.m.

Secondary constraint-1 $\phi^{(2)\mu} = \dot{\phi}^{(1)\mu} = \partial_0 E^{0\mu} = 0$

$\Rightarrow$ $\underline{\phi^{(1)\,\mu} = 0,}$ (all time) $\underline{\phi^{(2)\mu} = 0}$ (some time)

Continue the same procedure.

New model of massive spin-2 in a curved spacetime

4-2. Requirement of conservation in time of the primary constraints .

$$\dot{\phi}^{(2)i} = 0$$

This equations do contain $\ddot{h}_{0i}$ and determine the dynamics of $\ddot{h}_{0i}$

On the other hand,

$$\dot{\phi}^{(2)0} = 0$$

This equations do not contain any time derivative of h.

Secondary constraint-2 $\quad \phi^{(3)} = \dot{\phi}^{(2)0} = 0$

$$\Rightarrow \underbrace{\underbrace{\phi^{(1)\mu} = 0, \quad \phi^{(2)\mu} = 0,}_{\text{(all time)}} \quad \underbrace{\phi^{(3)} = 0,}_{\text{(some time)}}}_{\text{Constraints}} \quad \underbrace{\dot{\phi}^{(2)i} = 0}_{\ddot{h}_{0i}}$$

New model of massive spin-2 in a curved spacetime

4-3. Requirement of conservation in time of the primary constraints .

$$\dot{\phi}^{(3)} = 0$$

This equations do not contain $\ddot{h}_{00}$ $(\ddot{h})$

Secondary constraint-3 $\qquad \phi^{(4)} = \dot{\phi}^{(3)} = 0$

$$\phi^{(1)\mu} = 0, \quad \phi^{(2)\mu} = 0, \quad \phi^{(3)} = 0, \quad \phi^{(4)} = 0, \quad \dot{\phi}^{(2)i} = 0$$

(all time)

(some time)

Constraints

New model of massive spin-2 in a curved spacetime

4-4. Requirement of conservation in time of the primary constraints .

$$\dot{\phi}^{(4)} = 0$$

This equations do contain $\ddot{h}_{00}$

The dynamics of all components of $h_{\mu\nu}$ is determined.

$$\underbrace{\underline{\phi^{(1)\mu} = 0, \quad \phi^{(2)\mu} = 0, \quad \phi^{(3)} = 0, \quad \phi^{(4)} = 0,}_{\text{(all time)}}}_{\text{Constraints}} \quad \underbrace{\dot{\phi}^{(2)i} = 0, \quad \dot{\phi}^{(4)} = 0}_{\ddot{h}_{0i}\ ,\ \ddot{h}_{00}}$$

$\Updownarrow$

$$\phi^{(1)\mu} \approx 0, \quad \phi^{(2)\mu} \approx 0, \quad \phi^{(3)} \approx 0, \quad \phi^{(4)} \approx 0$$

Constraints for initial values

New model of massive spin-2 in a curved spacetime

The space spanned by $h_{\mu\nu}$ and $\dot{h}_{\mu\nu}$ has 20 degrees of freedom.

10 second-order differential equations

$$\underbrace{E_{ij} = 0,}_{h_{ij}} \quad \underbrace{\dot{\phi}^{(2)i} = 0, \quad \dot{\phi}^{(4)} = 0}_{h_{0\mu}}$$

10 Constraints for initial values

$$\phi^{(1)\mu} \approx 0, \quad \phi^{(2)\mu} \approx 0, \quad \phi^{(3)} \approx 0, \quad \phi^{(4)} \approx 0$$

As a result, we have 5 degrees of freedom

New model of massive spin-2 in a curved spacetime

Apply the Lagrangian analysis to the model in a curved spacetime.

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2}\nabla_\mu h\nabla^\mu h-\frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho}-\nabla^\mu h_{\mu\nu}\nabla^\nu h\right.$$
$$+\nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu}+\frac{\xi}{4}Rh_{\mu\nu}h^{\mu\nu}+\frac{1-2\xi}{8}Rh^2-\frac{m^2}{2}(h_{\mu\nu}h^{\mu\nu}-h^2)$$
$$-\frac{\mu}{3!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}-\frac{\zeta}{4!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}\nabla_{\mu_1}\nabla_{\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$
$$\left.-\frac{\lambda}{4!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}\right\}$$

The model consists of two types of interaction.

$$\left\{\begin{array}{l}\bullet\ \text{Non-derivative interaction}\\ \bullet\ \text{Derivative interaction}\end{array}\right.$$

New model of massive spin-2 in a curved spacetime

- Non-derivative interaction

For simplicity, consider the cubic interaction only ($\zeta = 0$, $\lambda = 0$).

Equations of motion

$$\begin{aligned} E_{\mu\nu} &= -\, g_{(\mu\nu)}{}^{\mu_1\nu_1\mu_2\nu_2} \nabla_{\mu_1} \nabla_{\nu_1} h_{\mu_2\nu_2} + (\text{terms without } \nabla\nabla h) \\ &= -\, g_{i(\mu} g_{\nu)j} g^{ij00\mu_2\nu_2} \nabla_0 \nabla_0 h_{\mu_2\nu_2} + (\text{terms without } \nabla_0 \nabla_0 h) \end{aligned}$$

Again, the equations of motion contain $\ddot{h}_{ij}$, but not $\ddot{h}_{0\mu}$.

Primary constraints

In this case, not $E_{0\mu}$. Instead,

$$\phi^{(1)}_\nu := E^0{}_\nu = -g_{\nu\sigma} g^{(0\sigma)\mu_1\nu_1\mu_2\nu_2} \nabla_{\mu_1} \nabla_{\nu_1} h_{\mu_2\nu_2} + (\text{terms without } \nabla\nabla h)$$

4 constraints (@ some time)

New model of massive spin-2 in a curved spacetime

1.Requirement of conservation in time of the primary constraints .

$$\dot{\phi}_\nu^{(1)} = \partial_0 E^0{}_\nu = 0$$

Now we have

$$\phi_\mu^{(1)} = 0, \quad \dot{\phi}_\mu^{(1)} = 0$$

It is unclear whether $\dot{\phi}^{(1)\mu} = 0$ are secondary constraints or not.

By using the e.o.m $E_{\mu\nu}$ and $\phi^{(1)\mu} = 0$, we find

$$\dot{\phi}_\nu^{(1)} = \nabla^\mu E_{\mu\nu} = 0$$

(Up to constraints and e.o.m)

New model of massive spin-2 in a curved spacetime

The explicit form of $\nabla^\mu E_{\mu\nu} = 0$ is given by

$$\nabla^\mu E_{\mu\nu} = \left(\frac{1-\xi}{2}R + m^2\right) g_{\nu\nu_1} g^{\mu_1\nu_1\mu_2\nu_2} \nabla_{\mu_1} h_{\mu_2\nu_2}$$
$$- \mu g_{\nu\nu_1} g^{(\mu_1\nu_1)\mu_2\nu_2\mu_3\nu_3} \left(\nabla_{\mu_1} h_{\mu_2\nu_2}\right) h_{\mu_3\nu_3} = 0$$

$\ddot{h}$ never appears.

$\dot{\phi}^{(1)\mu} = 0$ are (secondary) constraints-1.

At this stage, we have 8 constraints.

$\underline{\phi^{(1)}_\nu = 0,}$ $\underline{\phi^{(2)}_\nu := \dot{\phi}^{(1)}_\nu = \nabla^\mu E_{\mu\nu} = 0}$

(all time) (some time)

New model of massive spin-2 in a curved spacetime

Continue the procedure but before that…

$$\phi^{(2)}_\nu = 0$$

(Linear combination)

$$\phi^{(2)0} \equiv g^{00}\phi^{(2)}_0 + g^{0i}\phi^{(2)}_i = 0$$

$$\phi^{(2)}_i = 0$$

2-(a).Requirement of conservation in time of the primary constraints .

$$\dot{\phi}^{(2)}_i = 0$$

This equations do contain $\ddot{h}_{0i}$ and determine the dynamics of $\ddot{h}_{0i}$

New model of massive spin-2 in a curved spacetime

2-(b).Requirement of conservation in time of the primary constraints .

$$\dot{\phi}^{(2)0} = 0$$

Using $\phi_0^{(2)} = 0$, $\phi_i^{(2)} = 0$ and e.o.m. , we have

$$\begin{aligned}
\dot{\phi}^{(2)0} &= \nabla^\mu \nabla^\nu E_{\mu\nu} + \frac{m^2}{2} g^{\mu\nu} E_{\mu\nu} - \mu h^{\mu\nu} E_{\mu\nu} + \frac{1-\xi}{4} R g^{\mu\nu} E_{\mu\nu} \\
&= h \left(\frac{3m^4}{2} + \frac{5-6\xi}{4} m^2 R + \frac{(1-\xi)(2-3\xi)}{8} R^2 \right) \\
&- \frac{3\mu m^2}{2} g^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} - \mu g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \left(\nabla_{\mu_1} h_{\mu_2\nu_2}\right) \nabla_{\nu_1} h_{\mu_3\nu_3} \\
&+ \frac{\mu^2}{2} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} - \frac{7-9\xi}{12} \mu R g^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \\
&- \mu C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha\beta}
\end{aligned}$$

New model of massive spin-2 in a curved spacetime

$$
\begin{aligned}
\dot{\phi}^{(2)0} = &\, h\left(\frac{3m^4}{2} + \frac{5-6\xi}{4}m^2R + \frac{(1-\xi)(2-3\xi)}{8}R^2\right) \\
&- \frac{3\mu m^2}{2} g^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} - \mu g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(\nabla_{\mu_1} h_{\mu_2\nu_2}\right)\nabla_{\nu_1} h_{\mu_3\nu_3} \\
&+ \frac{\mu^2}{2} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} - \frac{7-9\xi}{12}\mu R g^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \\
&- \mu C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha\beta}
\end{aligned}
$$

There are no $\ddot{h}$ and $\dot{\phi}^{(2)0}$ can be identified with a constraint.

Secondary constraint-2 $\quad \phi^{(3)} := \dot{\phi}^{(2)0} = 0$

$$
\underbrace{\underbrace{\phi^{(1)\mu} = 0, \quad \phi^{(2)\mu} = 0,}_{\text{(all time)}} \quad \underbrace{\phi^{(3)} = 0,}_{\text{(some time)}}}_{\text{Constraints}} \quad \underbrace{\dot{\phi}^{(2)}_i = 0}_{\ddot{h}_{0i}}
$$

New model of massive spin-2 in a curved spacetime

3.Requirement of conservation in time of the primary constraints .

$$\dot{\phi}^{(3)} = 0$$

The equation does not contain $\ddot{h}_{00}$. ➡ Constraint.

($\ddot{h}_{0i}$ are eliminated with $\dot{\phi}_i^{(2)}$)

Secondary constraint-3 $\phi^{(4)} := \dot{\phi}^{(3)} = 0$

The structure of $\phi^{(3)}$

$$\phi^{(3)} = \cdots - \mu g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \left(\nabla_{\mu_1} h_{\mu_2\nu_2}\right) \nabla_{\nu_1} h_{\mu_3\nu_3} \cdots$$

$g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}$ is antisymmetric w.r.t μ

➡ No $\dot{h}_{00}$.

New model of massive spin-2 in a curved spacetime

4.Requirement of conservation in time of the primary constraints .

$$\dot{\phi}^{(4)} = 0$$

As $\phi^{(4)} = \dot{\phi}^{(3)}$ includes $\dot{h}_{00}$, this requirement defines $\ddot{h}_{00}$.

$$\underbrace{\phi^{(1)\mu} = 0, \quad \phi^{(2)\mu} = 0, \quad \phi^{(3)} = 0, \quad \phi^{(4)} = 0,}_{\text{(all time)}} \qquad \underbrace{\dot{\phi}_i^{(2)} = 0, \quad \dot{\phi}^{(4)} = 0}_{\ddot{h}_{0i}\ ,\ \ddot{h}_{00}}$$

Constraints

$\Updownarrow$

$$\phi^{(1)\mu} \approx 0, \quad \phi^{(2)\mu} \approx 0, \quad \phi^{(3)} \approx 0, \quad \phi^{(4)} \approx 0$$

Constraints for initial values

New model of massive spin-2 in a curved spacetime

We can extend this analysis in the case $\lambda \neq 0$ case and obtain

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\nabla_\mu h \nabla^\mu h - \frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho} - \nabla^\mu h_{\mu\nu}\nabla^\nu h\right.$$
$$+\nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu} + \frac{\xi}{4}R h_{\mu\nu}h^{\mu\nu} + \frac{1-2\xi}{8}Rh^2 - \frac{m^2}{2}(h_{\mu\nu}h^{\mu\nu} - h^2)$$
$$\left.-\frac{\mu}{3!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{\lambda}{4!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}\right\}$$

Here the back ground metric satisfy the relation $R_{\mu\nu} = \frac{1}{4}g_{\mu\nu}R$

This system has 5 degrees of freedom.

What about the derivative interaction? ($\zeta \neq 0$)

New model of massive spin-2 in a curved spacetime

Derivative interactions ($\mu = \lambda = 0,\ \zeta \neq 0$)

$$\zeta g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}\nabla_{\mu_1}\nabla_{\nu_1}h_{\mu_2\nu_2}\cdot h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

The same analysis is also applied to this case.

$$\phi^{(2)\mu} = \nabla_\nu E^{\mu\nu} \supset g^{\mu\nu\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(-\frac{1}{2}\nabla_{\nu_2}h_{\mu_2\sigma} + \nabla_\sigma h_{\mu_2\nu_2} - \nabla_{\mu_2}h_{\sigma\nu_2}\right)R_{\mu_1}{}^{\sigma}{}_{\nu\nu_1}h_{\mu_3\nu_3}$$

$$\Downarrow \qquad R_{\mu\alpha\nu\beta} = C_{\mu\alpha\nu\beta} + \frac{R}{12}g_{\mu\nu\alpha\beta}$$

$$\phi^{(2)\mu} = \nabla_\nu E^{\mu\nu} \supset \left\{-C^{\mu\alpha0\beta}g^{00} + C^{\alpha0\beta0}g^{\mu0} + C^{\mu00\alpha}g^{\beta0}\right\}h_{\alpha\beta}\nabla_0 h_{00}$$

At this stage, the constraint contains any time derivative of h_{00}.

Otherwise, we can not have 10 constraints.

New model of massive spin-2 in a curved spacetime

$$\phi^{(2)\mu} = \nabla_\nu E^{\mu\nu} \supset \left\{ -C^{\mu\alpha 0\beta} g^{00} + C^{\alpha 0 \beta 0} g^{\mu 0} + C^{\mu 0 0 \alpha} g^{\beta 0} \right\} h_{\alpha\beta} \nabla_0 h_{00}$$

To eliminate time derivative of h_{00}, non-minimal terms are required.

General form

$$c_1 C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha\beta} h + c_2 C^{\mu\alpha\nu\beta} h_{\mu\nu} h_\alpha{}^\lambda h_{\lambda\beta}$$

Contribution from this term to the constraint

$$\phi^{(2)\nu} = \nabla_\mu E^{\mu\nu} \supset \left\{ (2c_1 + c_2) C^{\mu\alpha 0\beta} g^{00} + (2c_1 + c_2)\, C^{0\alpha 0\beta} g^{\mu 0} \right\} h_{\alpha\beta} \nabla_0 h_{00}$$
$$+ \left(\text{terms not including } \nabla_0 h_{00}\right)$$

Thus, the time derivative of h_{00} can not be eliminated unless the background is conformally flat.

New model of massive spin-2 in a curved spacetime

New non-derivative interactions

$$c_1 C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha\beta} h + c_2 C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha}^{\ \lambda} h_{\lambda\beta}$$

The contribution to the constraint

$$\begin{aligned}\nabla_\mu E^{\mu\nu} \supset & \left\{(2c_1 + c_2) C^{\mu\alpha 0\beta} g^{00} + (2c_1 + c_2)\, C^{0\alpha 0\beta} g^{\mu 0}\right\} h_{\alpha\beta} \nabla_0 h_{00} \\ & + (\text{terms not including } \nabla_0 h_{00})\end{aligned}$$

Thus, we have the new interactions by tuning the coefficients.

$$C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha\beta} h - 2 C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha\lambda} h^{\lambda}_{\ \beta}$$

Similar terms can be constructed.

New model of massive spin-2 in a curved spacetime

New non-minimal coupling term

$$C^{\mu_1\mu_2\nu_1\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}\ ,$$

$$\delta^{\mu_1}{}_{\rho_1}{}^{\mu_2}{}_{\rho_2}{}^{\mu_3}{}_{\rho_3}\delta^{\nu_1}{}_{\sigma_1}{}^{\nu_2}{}_{\sigma_2}{}^{\nu_3}{}_{\sigma_3}C^{\rho_1\rho_2\sigma_1\sigma_2}g^{\rho_3\sigma_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}$$

$$\delta^{\mu_1}{}_{\rho_1}{}^{\mu_2}{}_{\rho_2}{}^{\mu_3}{}_{\rho_3}{}^{\mu_4}{}_{\rho_4}\delta^{\nu_1}{}_{\sigma_1}{}^{\nu_2}{}_{\sigma_2}{}^{\nu_3}{}_{\sigma_3}{}^{\nu_4}{}_{\sigma_4}C^{\rho_1\rho_2\sigma_1\sigma_2}g^{\rho_3\sigma_3\rho_4\sigma_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

- Derivative interaction induce an extra degree of freedom and can not be eliminated.
- Instead, the non-minimal coupling terms with Weyl tensor are found.

Summary

- We have proposed the new model of massive spin-2 particle in the Minkowski space-time and a curved spacetime.

- Couple the model with gravity by adding non-minimal coupling term and prove the system is ghost-free on the Einstein manifold.

- The derivative interaction can be added without a ghost in the Minkowski space-time. On the other hand, such a interaction induces a ghost on the Einstein manifold unless $C^{\mu\nu\rho\sigma}$=0

- New non-minimal coupling terms are obtained thanks to the lagrangian analysis.

Partially massless gauge theory

Fierz-Pauli action in D dimensions

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2}\nabla_\mu h\nabla^\mu h-\frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho}-\nabla^\mu h_{\mu\nu}\nabla^\nu h\right.$$
$$\left.+\nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu}+\frac{R}{D}\left(h_{\mu\nu}h^{\mu\nu}-\frac{1}{2}h^2\right)-\frac{m^2}{2}(h_{\mu\nu}h^{\mu\nu}-h^2)\right\}$$

- The background is restricted to Einstein manifold $R_{\mu\nu}=\frac{1}{D}g_{\mu\nu}R$
- The model does not have the symmetry under $\delta h_{\mu\nu}=2\nabla_{(\mu}\xi_{\nu)}$ and recover the sym. In the massless limit.

The model does have the symmetry under

$$\delta h_{\mu\nu}=\nabla_\mu\nabla_\nu\lambda+\frac{R}{D(D-1)}\lambda g_{\mu\nu}$$ provided that $$R=\frac{D(D-1)}{D-2}m^2$$

Partially massless gauge theory

Toward Partially massless gauge theory with non-linear terms

$$S=\int d^Dx\sqrt{-g}\left\{\frac{1}{2}\nabla_\mu h\nabla^\mu h-\frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho}-\nabla^\mu h_{\mu\nu}\nabla^\nu h+\nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu}\right.$$
$$\left.+\frac{m^2}{2}g^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}+\frac{R}{D}h_{\alpha\beta}h^{\alpha\beta}-\frac{R}{2D}h^2\right\}$$
$$=\int d^Dx\sqrt{-g}\left[\frac{1}{2}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\nabla_{\mu_1}h_{\mu_2\nu_2}\nabla_{\nu_1}h_{\mu_3\nu_3}+\frac{m^2}{2}g^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}\right.$$
$$\left.-\frac{D-2}{2D(D-1)}Rg^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}+\frac{1}{2}C^{\mu\alpha\nu\beta}h_{\mu\nu}h_{\alpha\beta}\right]$$

The Weyl tensor appears through the non-commutativity of the covariant derivatives.

These two expressions are equivalent.

Partially massless gauge theory

Toward Partially massless gauge theory with non-linear terms

$$S = \int d^D x \sqrt{-g} \left\{ \frac{1}{2} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \nabla_{\mu_1} h_{\mu_2\nu_2} \nabla_{\nu_1} h_{\mu_3\nu_3} + \frac{m^2}{2} g^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \right.$$
$$\left. - \frac{D-2}{2D(D-1)} R g^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} + \frac{1}{2} C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha\beta} \right\}$$

This is equivalent to the FP action in a curved space-time.

Invariant under

$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \lambda + \frac{R}{D(D-1)} \lambda g_{\mu\nu}$ provided that $R = \frac{D(D-1)}{D-2} m^2$

Partially massless gauge theory

Substituting $R = \frac{D(D-1)}{D-2} m^2$ gives

$$S = \int d^D x \sqrt{-g} \left\{ \frac{1}{2} g^{\mu_1 \nu_1 \mu_2 \nu_2 \mu_3 \nu_3} \nabla_{\mu_1} h_{\mu_2 \nu_2} \nabla_{\nu_1} h_{\mu_3 \nu_3} + \frac{1}{2} C^{\mu\alpha\nu\beta} h_{\mu\nu} h_{\alpha\beta} \right\}$$

Thus, assuming $C^{\mu\nu\rho\sigma}$=0, the partially gauge invariance can be translated into

$$g^{\mu_1 \nu_1 \mu_2 \nu_2 \mu_3 \nu_3} \nabla_{\mu_1} \delta h_{\mu_2 \nu_2} = 0$$

Generalization

$$\Longrightarrow \qquad g^{\mu_1 \nu_1 \mu_2 \nu_2 \cdots \mu_n \nu_n} \nabla_{\mu_1} \delta h_{\mu_2 \nu_2} = 0$$

This fact suggests the possibility of constructing partially massless theory.

Partially massless gauge theory

Gauss-Bonnet type $\xi g^{\mu_1\nu_1\cdots\mu_5\nu_5}\nabla_{\mu_1}h_{\mu_2\nu_2}\nabla_{\nu_1}h_{\mu_3\nu_3}\nabla_{\mu_4}\nabla_{\nu_4}h_{\mu_5\nu_5}$

This term is invariant under $\delta h_{\mu\nu}=\nabla_\mu\nabla_\nu\lambda+\frac{R}{D(D-1)}\lambda g_{\mu\nu}$

$$(g^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}\nabla_{\mu_1}\delta h_{\mu_2\nu_2}=0)$$

However, the cubic interaction does not have the linearized diffeo.
(Some extra terms appear in the trans.)

We expect that some non-minimal coupling terms recover the linearized diffeomorphism and realize partially massless gauge sym.

Back up

Back up

Massive spin-2 fields

Why is the negative kinetic term undesirable?

- We can not define the vacuum.

  In the healthy QFT, “particle” is defined as the fluctuation from the vacuum.

- If we quantize the theory neglecting the fact, we are faced with the negative norm (Ghost).

If the ghost sate is not in the physical subspace, the theory remains consistent.

Back up

<u>Calculation of secondary constraint</u>

$$\partial^{\mu} E_{\mu\nu} = m^2 \eta_{(\mu\nu)\mu_1\nu_1} \partial^{\mu} h^{\mu_1\nu_1}$$

$$\Downarrow$$

$$-E_{i\nu,i} + m^2 \eta_{(\mu\nu)\mu_1\nu_1} \partial^{\mu} h^{\mu_1\nu_1} = -\dot{E}_{0\nu} = -\dot{\phi}_{\nu}^{(1)}$$

$$\Downarrow$$

e.o.m $E_{ij} = 0$ and $\phi^{(1)} = 0$

$$\dot{\phi}^{(1)} = \partial^{\mu} E_{\mu\nu} = 0$$

(Up to constraints and e.o.m)

Back up

Using the constraint obtained before ($\phi_\nu^{(1)} = 0$, $\phi_i^{(2)} = 0$).

$$\dot{\phi}_i^{(2)} = \nabla_0 \nabla^\mu E_{\mu i} = B_i{}^j \nabla_0 \nabla_0 h_{0j} + \underline{C_i{}^{kl} \nabla_0 \nabla_0 h_{kl}} + (\text{terms without } \nabla_0 \nabla_0 h) = 0$$

eliminated by e.o.m.

Here, B and C are defined as follows.

$$\begin{cases} B_i{}^j \equiv \dfrac{1}{N^2} \left[\left(\dfrac{1-\xi}{2} R + m^2 \right) \delta_i{}^j - \mu e^j{}_i{}^{mn} h_{mn} \right] \\ C_i{}^{kl} \equiv -\dfrac{1}{N^2} \left[N^k \delta_i{}^l - \mu \left\{ e^{kl}{}_i{}^j h_{0j} - N^k e_i{}^{lmn} h_{mn} - N^m e_i{}^{nkl} h_{mn} \right\} \right] \end{cases}$$

e_{ij} : 3-metric, N_i : Shift, N : Lapse

Back up

<u>Derivative int.</u>

$$\zeta g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}\nabla_{\mu_1}\nabla_{\nu_1}h_{\mu_2\nu_2}\cdot h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

Contribution to the equations of motion $E_D^{\mu\nu}$

$$\begin{aligned} E_D^{\mu\nu} \propto\ & 2\zeta g^{(\mu\nu)\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\nabla_{\mu_1}\nabla_{\nu_1}h_{\mu_2\nu_2}\cdot h_{\mu_3\nu_3} \\ & + \zeta g^{(\mu\nu)\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\nabla_{\mu_1}h_{\mu_2\nu_2}\cdot\nabla_{\nu_1}h_{\mu_3\nu_3} \end{aligned}$$

$\Rightarrow$ $E_D^{\mu\nu}$ do not include $\dot{h}_{00}$ and $\dot{h}_{0i}$

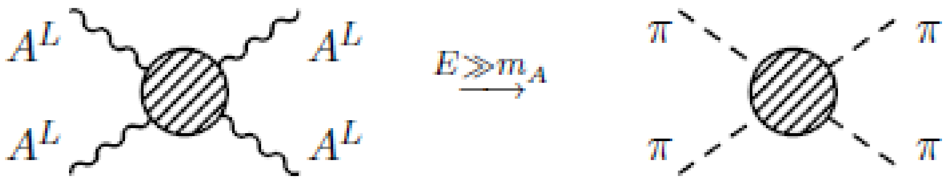
Back up

<u>What is the equivalence theorem?</u>

Equivalence theorem states that the relation between massive spin particles and Stuckelberg fields (Nambu-Goldstone bosons).

Equivalence theorem

$\boxed{\text{(Amplitude of Longitudinal mode)} \sim \text{(Amplitude of NG boson)} + O(m/E)}$



E : Energy scale m : particle mass

Back up

- The scattering amplitude involving massive gauge bosons.

  After the discovery of Higgs particle, the detail behavior of Electroweak sector @ high energy scale is now investigated in the context of BSM.

  Transparent description of the model

  - Power counting
  - Massless limit in the internal line

- Construction of massive gauge theories

  e.g.) dRGT massive spin-2

  N. Arkani-Hamed, H. Georgi, and M. D. Schwartz, Annals Phys. 305 (2003) 96-118 hep-th/0210184 HUTP-02-A051

  de Rham, Gabadadze, Tolley Phys.Rev.Lett. 106 (2011) 231101

Back up

Boulware-Deser ghost problem D. G. Boulware and S. Deser," Annals Phys.**89** (1975) 193.

Non-linear terms for massive spin-2 particles lead to a ghost in general.

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}V(g,h)\right]$$

$$V(g,h) = V_2(g,h) + V_3(g,h) + \cdots$$

$$V_2 = \langle h^2\rangle - \langle h\rangle^2$$

$$V_3 = c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3$$

$$V_4 = d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4$$

$$\vdots$$

Back up

Cut-of scale and the origin of BD ghost

Stuckelberg trick : Restoration of the gauge inv.

$$H_{\mu\nu} = h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\partial_\mu\partial_\nu\phi - \partial_\mu A^\alpha \partial_\nu A_\alpha - \partial_\mu A^\alpha \partial_\nu \partial_\alpha \phi - \partial_\mu \partial^\alpha \phi \partial_\nu A_\alpha - \partial_\mu \partial^\alpha \phi \partial_\nu \partial_\alpha \phi.$$

Equivalence theorem : Amp of Longitudinal mode~ Amp of NG boson +O(m/E)

- Higher derivative terms give us the cut-off scale of the theory and suggest the existence of a ghost.
- Thanks to Stuckelberg field and ET theorem, it becomes easier to treat the problematic helicity 0 mode.

Review of massive spin-2

NG boson description of massive gravity

$$S=\frac{1}{2\kappa^2}\int d^Dx\Big[(\sqrt{-g}R)-\frac{1}{4}m^2\eta^{\mu\alpha}\eta^{\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta}-h_{\mu\alpha}h_{\nu\beta}\right)\Big]$$

$h_{\mu\nu}\to h_{\mu\nu}+\partial_\mu A_\nu+\partial_\nu A_\mu+2\partial_\mu\partial_\nu\phi+$ (Quadratic terms in A ,ϕ)

This replacement leads to higher derivative terms in terms of NG boson.

Interaction terms : $\sim\Lambda_\lambda^{4-n_h-2n_A-3n_\phi}h^{n_h}(A)^{n_A}(\partial^2\phi)^{n_\phi},\quad\Lambda_\lambda=(M_pm^{\lambda-1})^{1/\lambda}$ $\lambda\leq 5$

In this theory, the lowest scale is Λ_5 accompanied with $(\partial^2\phi)^3$

At $E\sim\Lambda_5$, tree level amplitude of the scalar NG boson amplitude ~ 1

This theory breaks down at $E\sim\Lambda_5$

Review of massive spin-2

Strategy: Adding non-derivative interactions of h and tuning coefficients

Terms suppressed with factors below Λ_3 are all eliminated !

➡ Effective theory with Λ_3 (Λ_3 theory), dRGT model

$$S = M_p^2 \int d^4x\sqrt{-g}\left[R + 2m^2 \sum_{n=2}^{4} \bar{\alpha}_n e_n(\mathcal{K})\right]$$

$$\bar{\alpha}_2 = 1, \quad \bar{\alpha}_3,\ \bar{\alpha}_4 : \text{ free parameters}$$

This means $g^{\mu_1\nu_1}\cdots g^{\mu_d\nu_d}$ anti-symmetrized over ν.

$$e_n(X) = g^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_1\nu_n} X_{\mu_1\nu_1} X_{\mu_2\nu_2}\cdots X_{\mu_n\nu_n} \text{ antisymmetrized in } \nu$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{g^{-1}f}^\mu{}_\nu, \quad \mathcal{K}_{\mu\nu} \sim h_{\mu\nu} \quad \text{around flat spacetime}$$

$$f_{\mu\nu}(x) = \partial_\mu Y^\alpha(x)\partial_\nu Y^\beta(x)\eta_{\alpha\beta}$$

corresponding to NG bosons ($Y^\alpha = x^\alpha$: Unitary gauge)

Back up

$$
\begin{aligned}
g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\nabla_{\mu_1}\delta h_{\mu_2\nu_2} &= g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\nabla_{\mu_1}\left\{\nabla_{\mu_2}\nabla_{\nu_2}\lambda + \frac{R}{D(D-1)}\lambda g_{\mu_2\nu_2}\right\} \\
&= \frac{R}{2D(D-1)}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}g_{\nu_2\mu_1}{}^{\sigma}{}_{\nu_2}\nabla_\sigma\lambda + \frac{R}{D(D-1)}g^{\mu_1\nu_1\mu_2}{}_{\mu_2}{}^{\mu_3\nu_3}\nabla_{\mu_1}\lambda \\
&= -\frac{R(D-2)}{D(D-1)}\lambda g^{\sigma\nu_1\mu_3\nu_3}\nabla_\sigma\lambda + \frac{R(D-2)}{D(D-1)}\lambda g^{\mu_1\nu_1\mu_3\nu_3}\nabla_{\mu_1}\lambda \\
&= 0
\end{aligned}
$$