Propagation of vortex beam around a Kerr black hole

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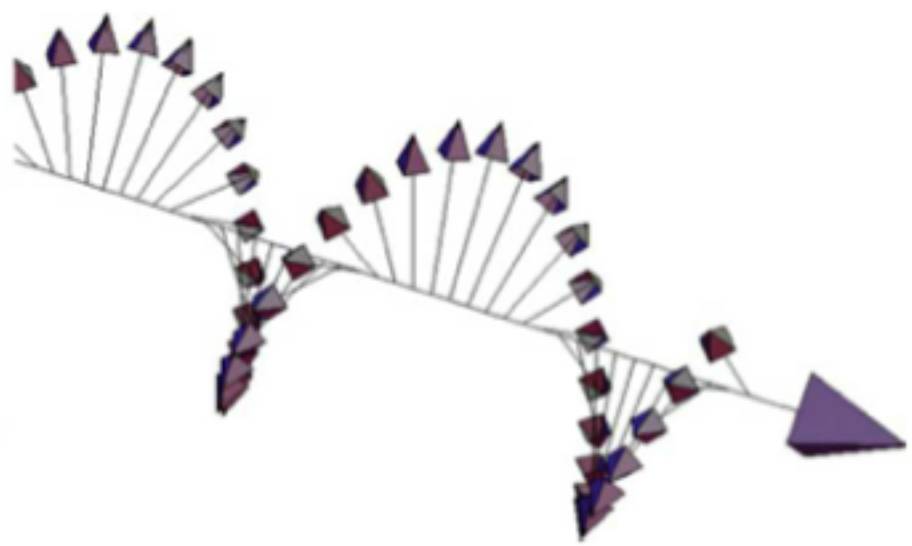
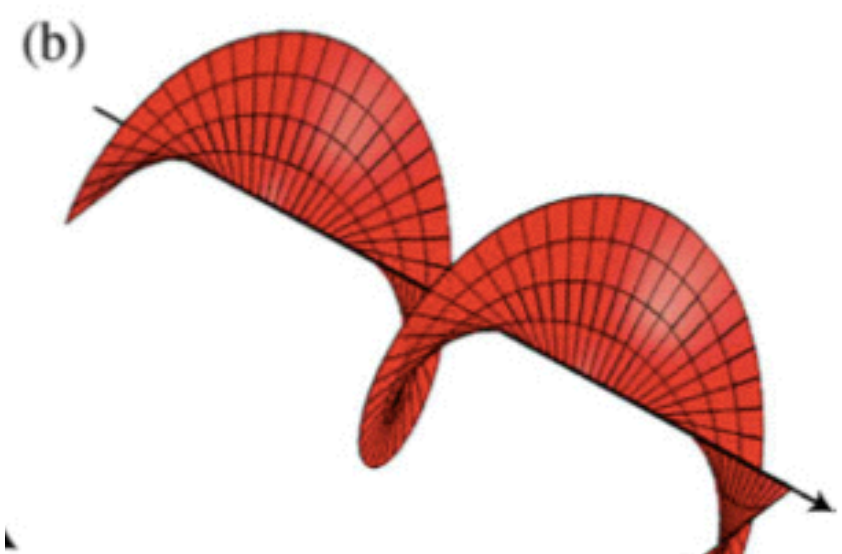
Angular momentum of light

- **Spin angular momentum**



→ circular polarization

- **Orbital angular momentum**



→ helical wavefront

Vortex beam

Contents

- What is a vortex beam

  Property, Production, Observation

- Propagation of plane wave

- Propagation of a vortex beam

- Results

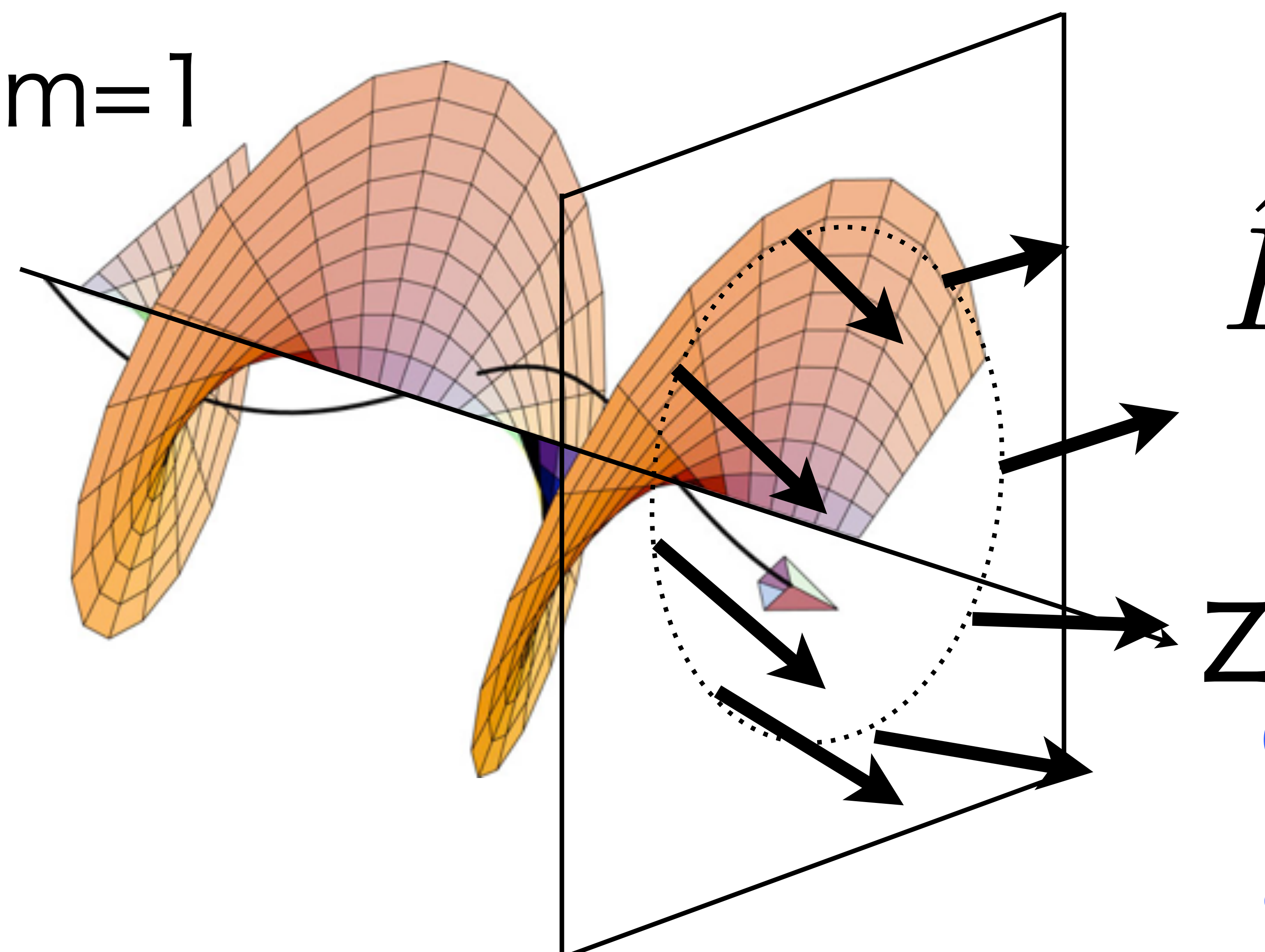
What is

Vortex beam?

What is vortex beam?

$$\psi \propto e^{i(kz+m\phi-\omega t)}$$

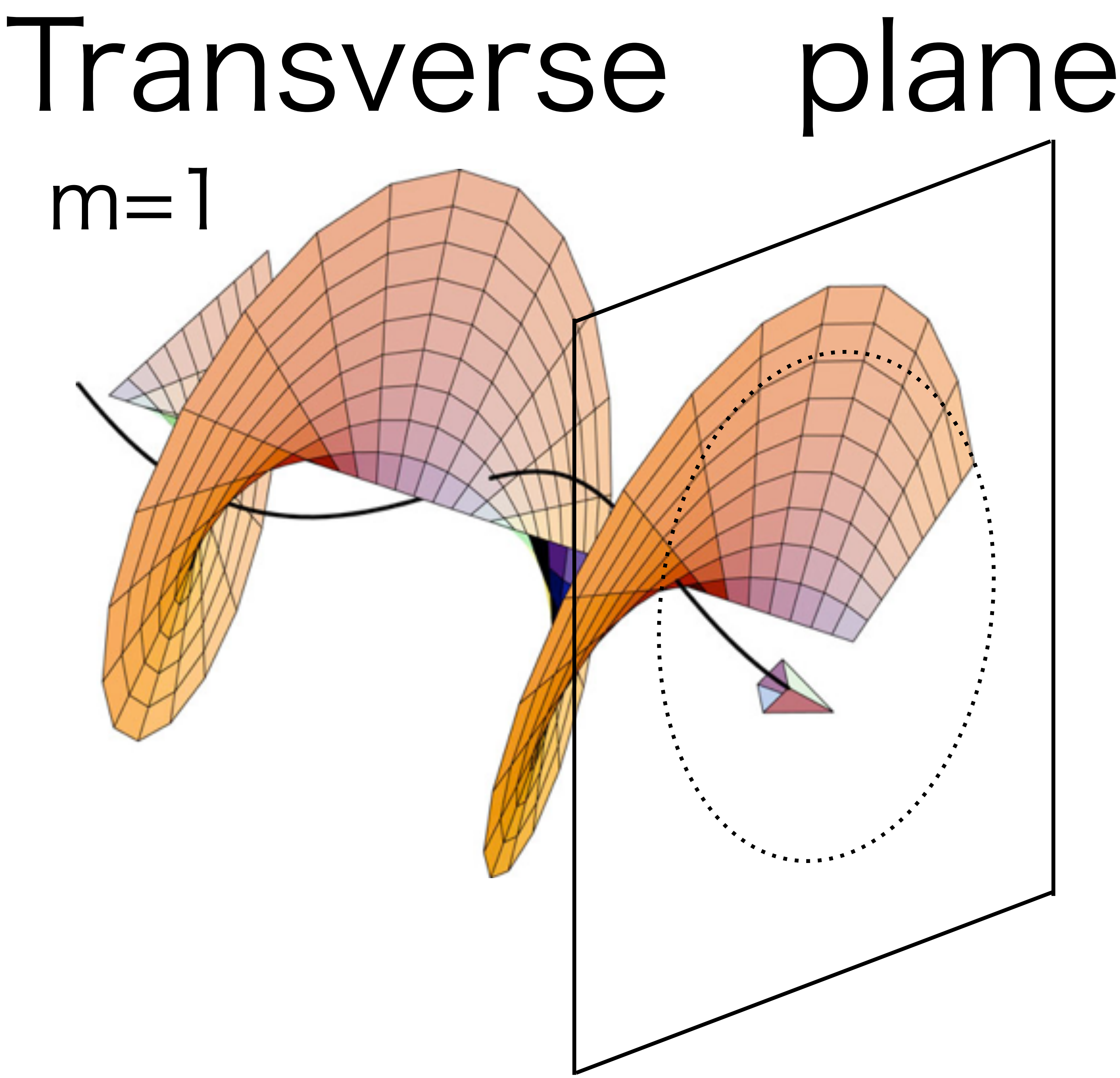
m :integer $\quad\quad$ ϕ :azimuthal angule

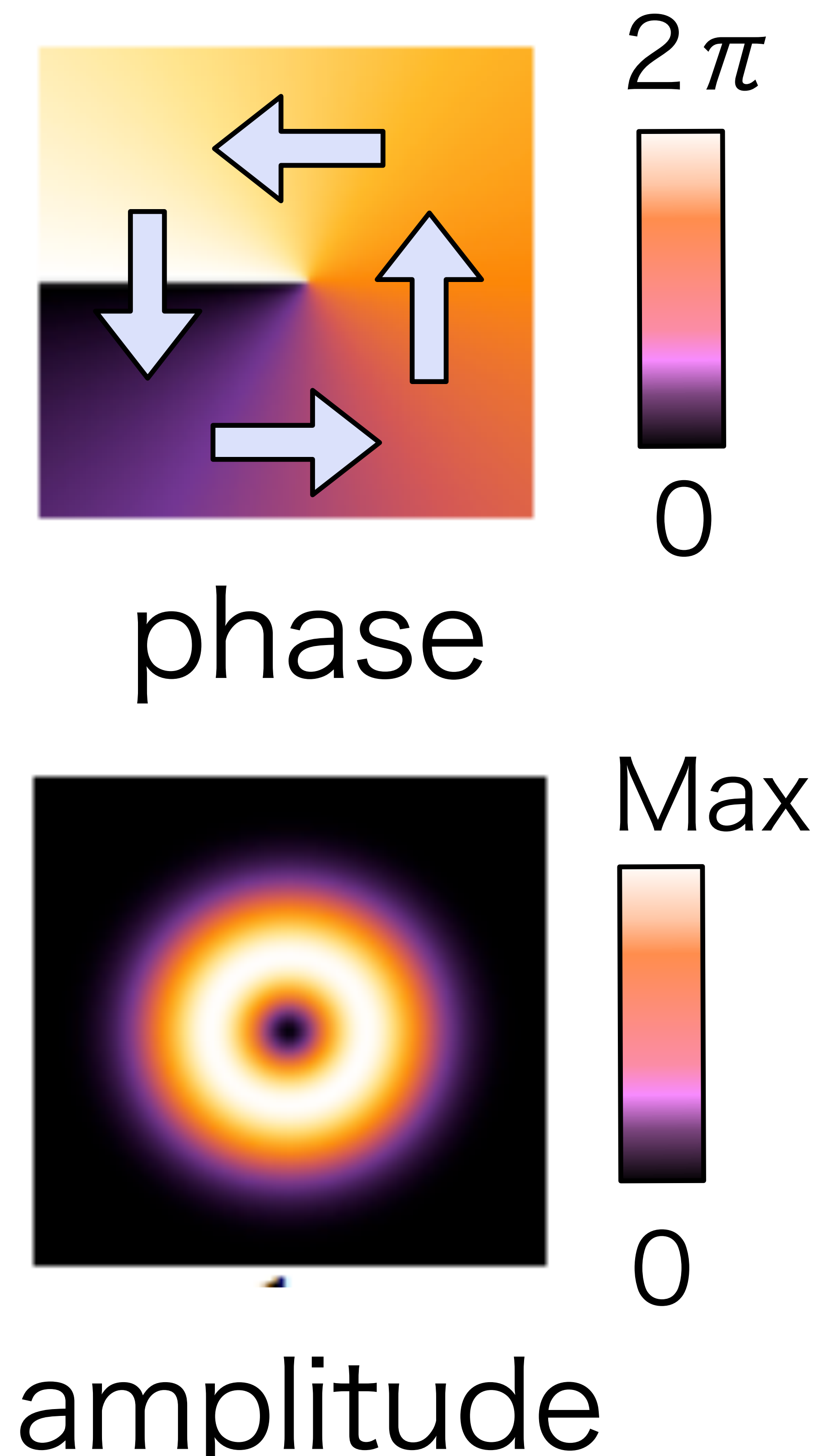


$$\hat{L}_z \psi = -i\frac{\partial}{\partial\phi}\psi$$
$$= m\psi$$

eigenstate of orbital angular momentum

vortex beam





Why vortex beam?

- **Allen et. al , Phvs. Rev. A, 45, 8185 (1992)**
- **J. Wangi et. al, Nature Photonics 6, 488-496(2012)**

  Application information science,

  the vortex beam has more information

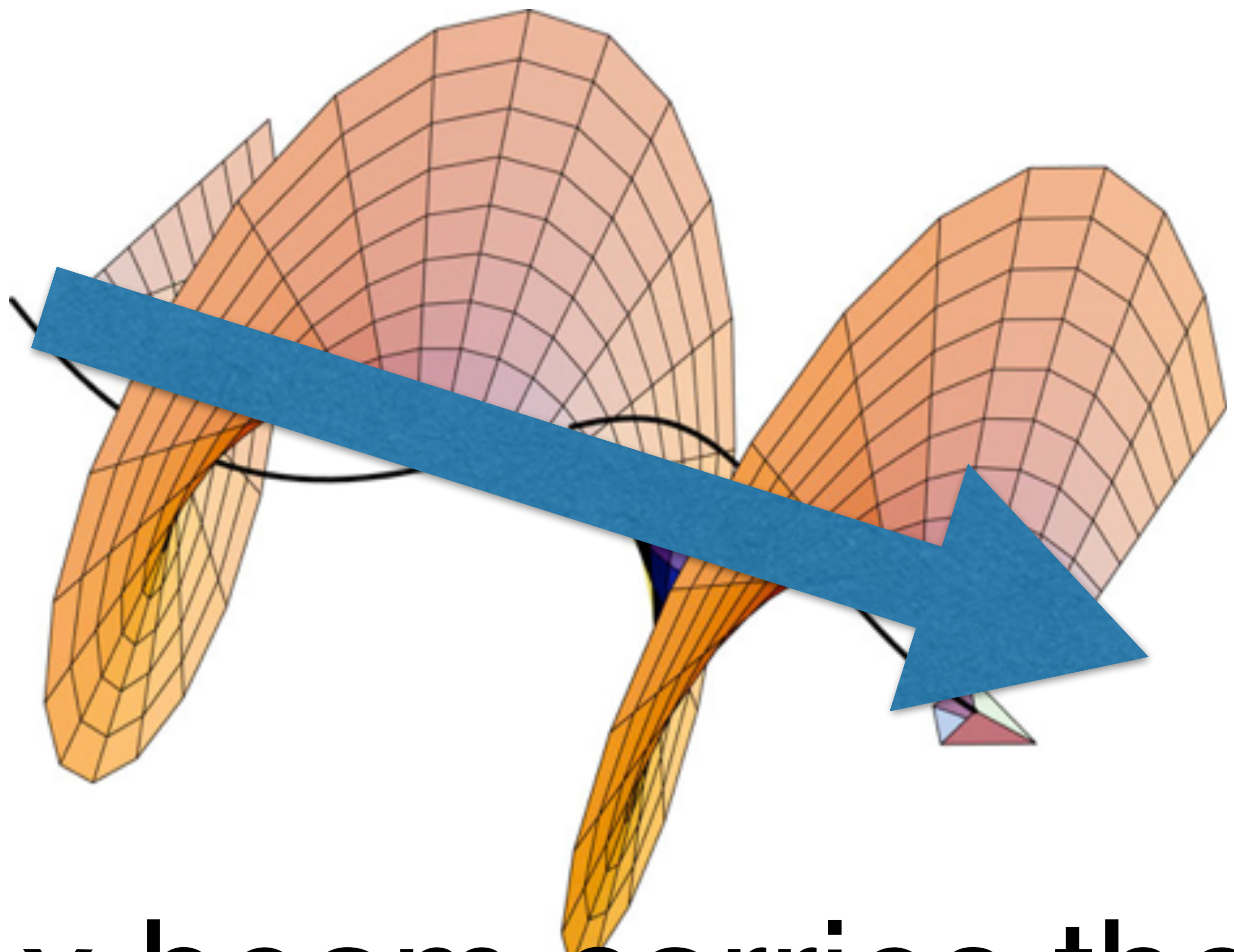
  than the plane wave

- **F. Tamburini et. al, Nature Physics 7, 195(2011)**

  Recently, some application of vortex beam to

  astrophysics have been considered

Vortex beam carries Orbital Angular Momentum

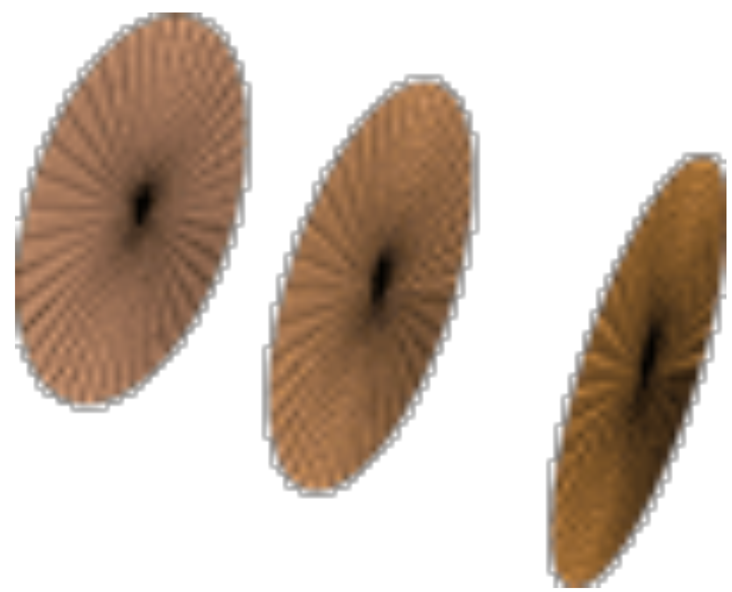


the vortex beam carries the orbital angular momentum
about the propagation axis.

wave fronts

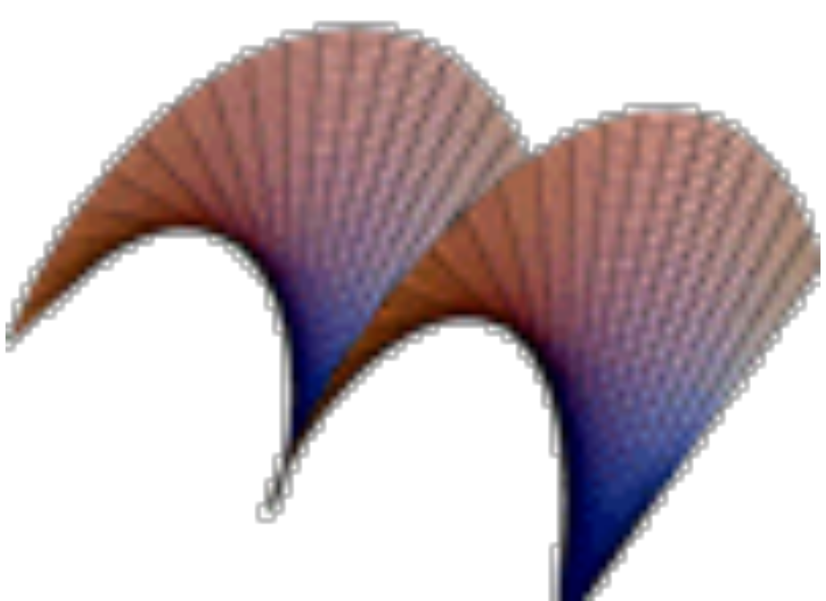
m=-1

m=0

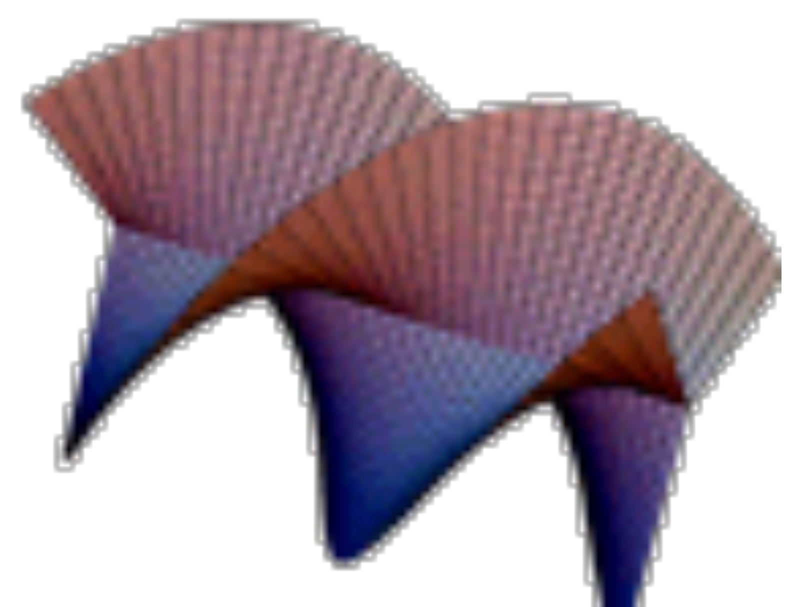


m=+1

m=+2

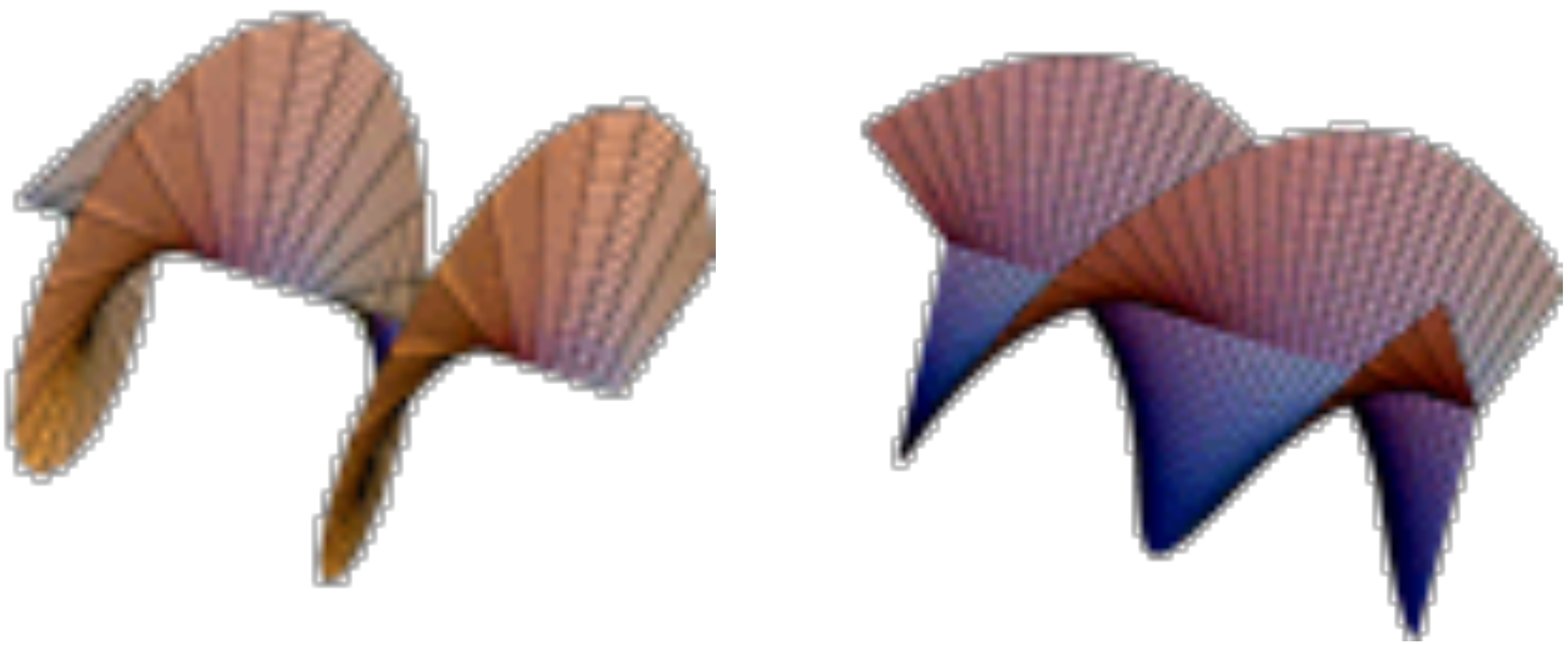


m=+3

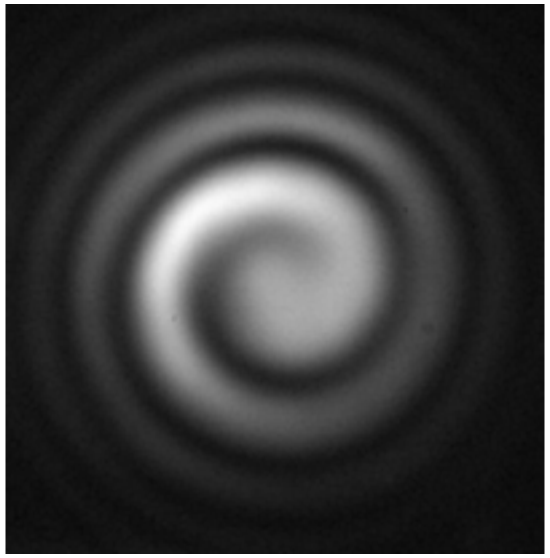


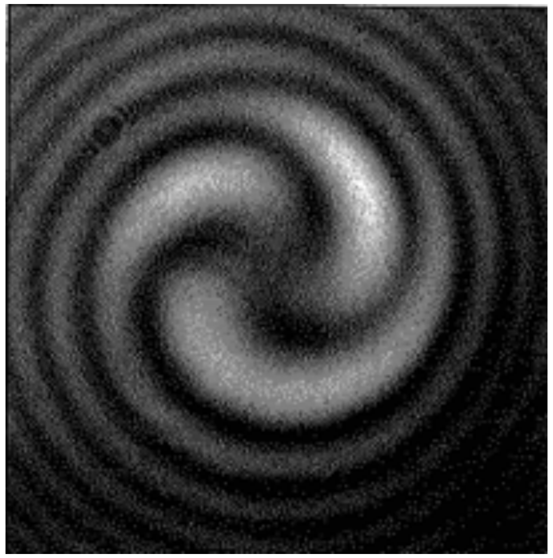
Production and Observation of the vortex beam

The pattern of interference with plane wave



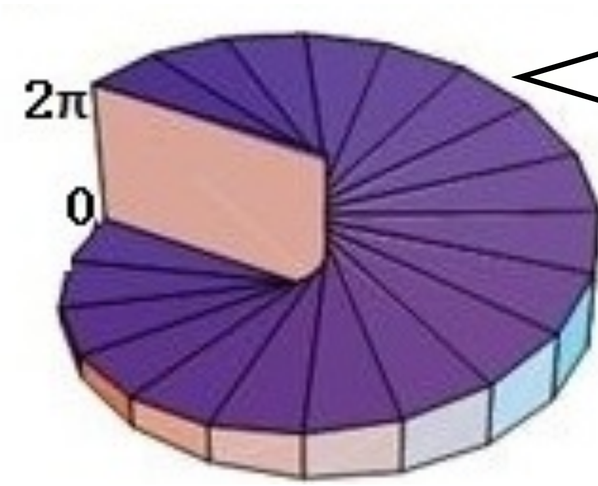
m=1





m=3

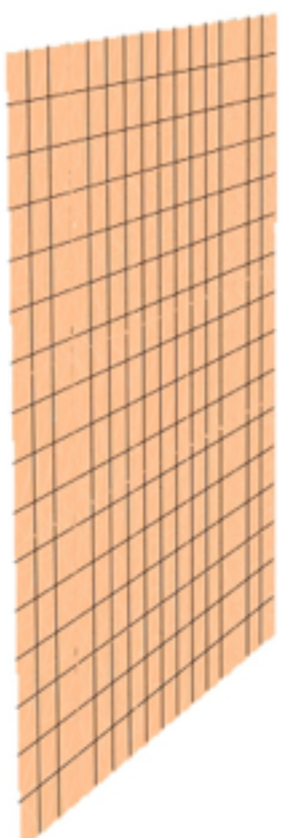
Production of vortex beam



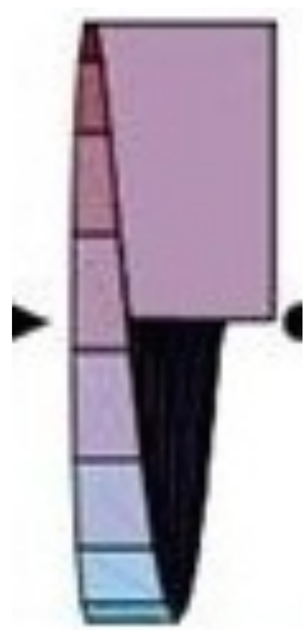
This plate is made of glass ,called spiral phase plate

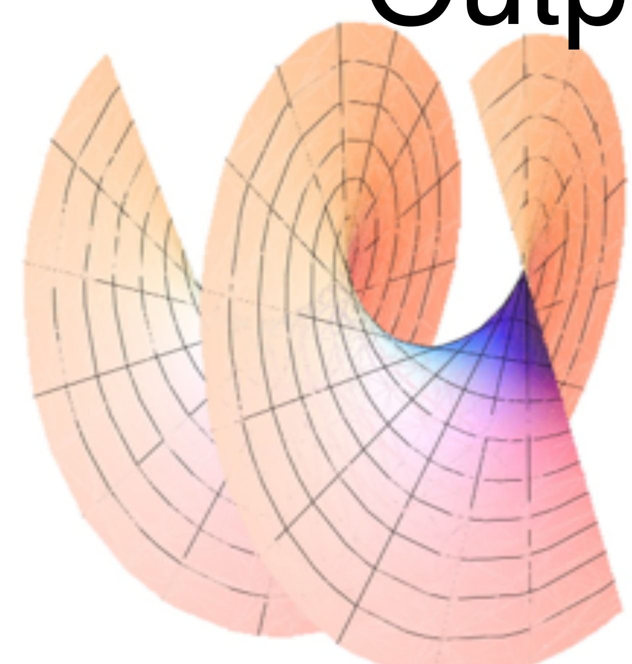
input

Output



Plane wave





vortex beam

Solutions of vortex beam

Bessel function

Bessel beam $\psi = J_m(q\rho)\exp[i(-\omega t + kz)]\boxed{\exp(im\phi)}$

dispersion relation $q^2 = \omega^2 - k^2$

Laguerre function

Laguerre Gaussian beam

$$\psi = \left(\sqrt{2}\rho/w\right)^m L_0^m(-2\rho^2/w^2)\boxed{\exp(im\phi)}(w_0/w)$$
$$\exp[-\rho^2(1/w^2 - ik/2R) - i\Phi)]$$

$$w^2 = w_0\left[1 + \left(2z/kw_0^2\right)^2\right], R = z\left[1 + \left(kw_0^2/2z\right)^2\right], \Phi = (m+1)\arctan\left(2z/kw_0^2\right)$$

Propagation of plane waves in a Gravitational field

Eikonal approximation

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\psi = 0 \qquad \psi \equiv A e^{i\frac{S}{\epsilon}}$$

$$\sim \frac{1}{\epsilon^2} g^{\mu\nu}(\nabla_\mu S)(\nabla_\nu S) A e^{i\frac{S}{\epsilon}} = 0$$

Hamilton equation of massless particle

wave vector $k_\mu \equiv \nabla_\mu S$

$$\Downarrow \quad \dot{x}^\alpha = \frac{\partial H}{\partial k_\alpha}, \quad \dot{k}^\alpha = -\frac{\partial H}{\partial x^\alpha}$$

$$\boxed{\frac{D\dot{x}^\mu}{D\tau} = 0, \quad g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0}$$

Propagation of wave

plane wave

Eikonal approximation

Geodesic equation

vortex beam

Eikonal approximation

?

Propagation of vortex beam

~flat spacetime~

Orbit of Bessel beam in flat spacetime

ψ_B :Bessel beam solution

(exact solution of wave equation in flat spacetime)

$$\psi_B = J_m(q\rho)e^{iS} \quad S = -\omega t + kz + m\phi$$

Jm:Bessel function

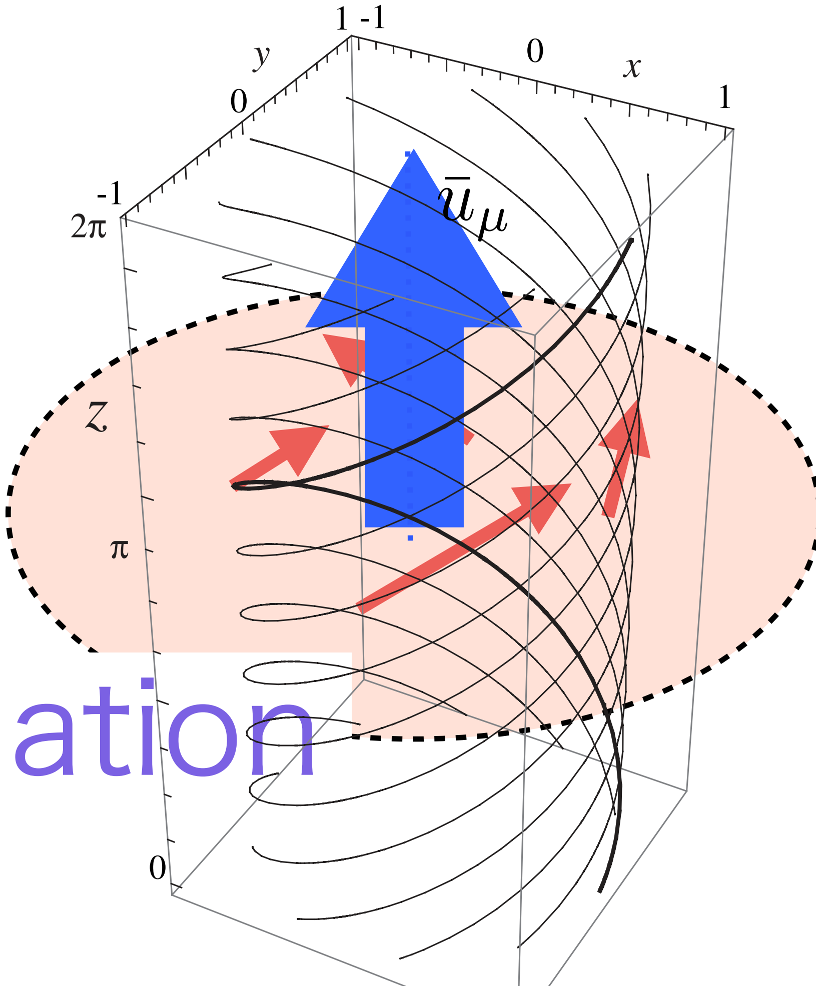
q,k,ω:constant

$$u_\mu = \nabla_\mu S$$
$$= (-\omega, 0, m, k)$$

$$\bar{u}_\mu = \frac{1}{\int dS}\int u_\mu dS$$

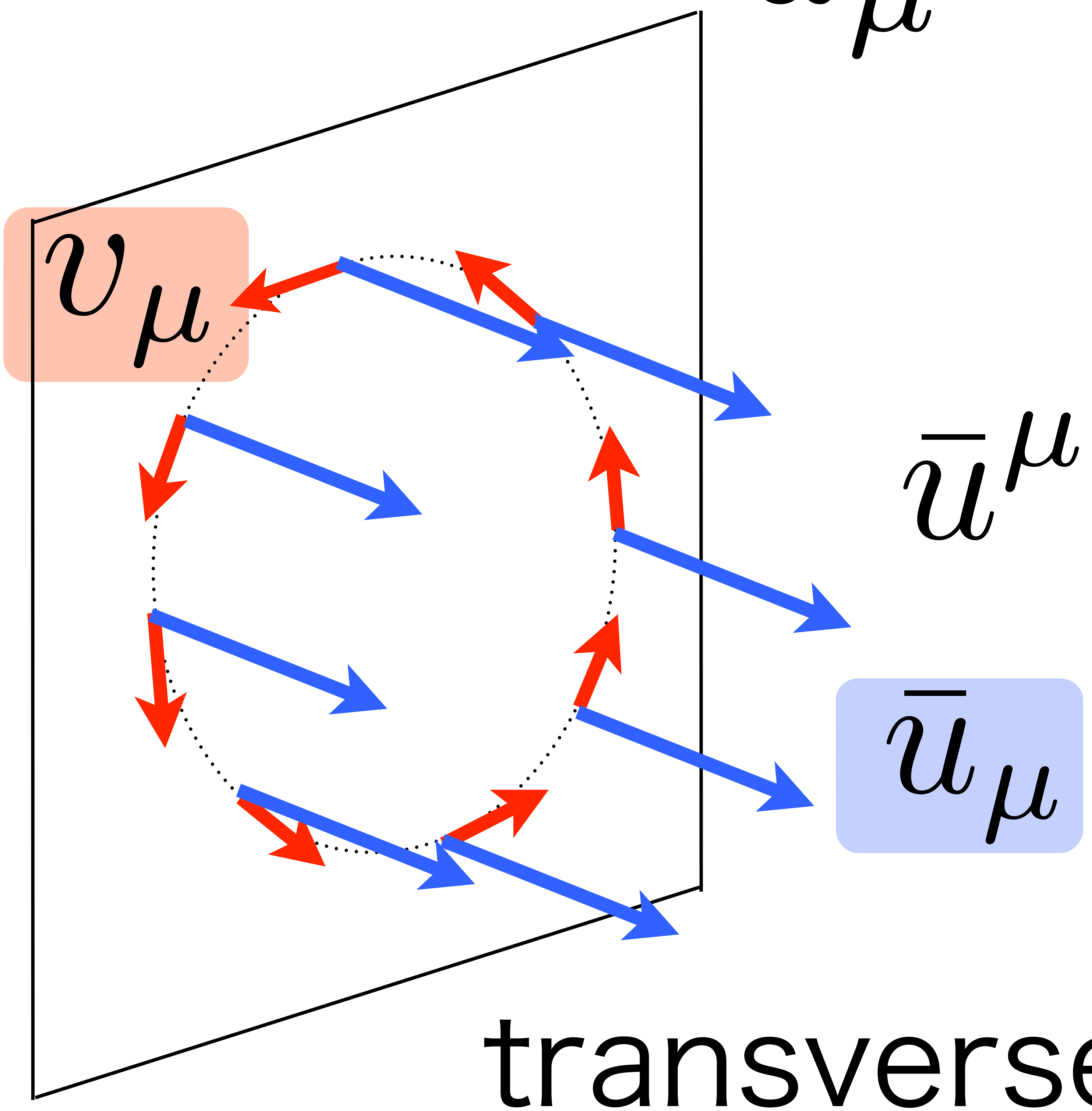
satisfying geodesic equation

$$= (-\omega, 0, 0, k)$$



Decomposition of wave vectors

$$u_\mu = \bar{u}_\mu + v_\mu$$



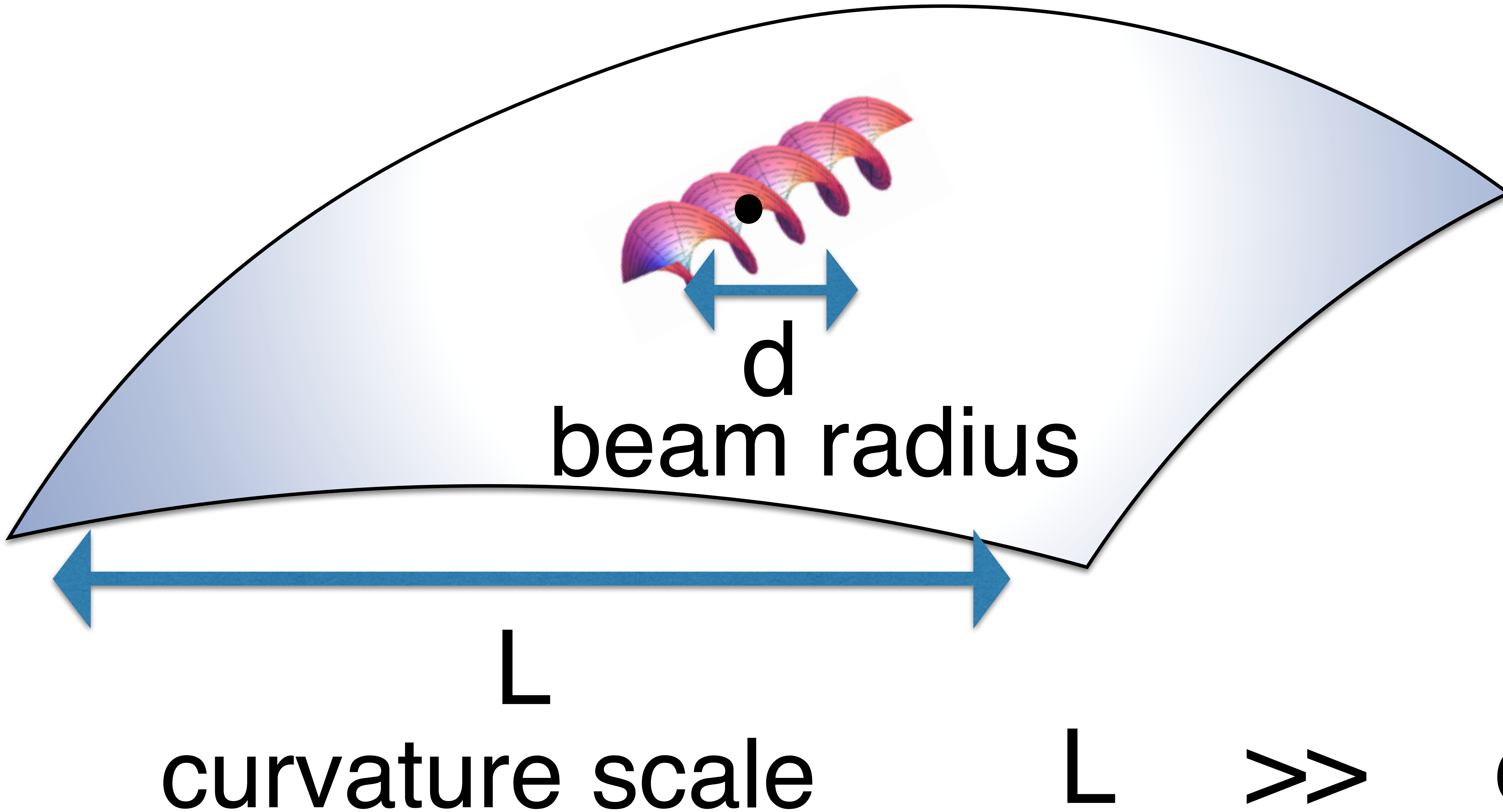
$$\bar{u}^\mu \partial_\mu \bar{u}_\nu = 0$$

transverse plane

Propagation of vortex beam

~curved spacetime~

Scale of beam radius



L >> d

Orbit of Bessel beam in a curved spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} \boxed{+h_{\mu\nu}}$$

metric perturbation

$$\psi = J_m(q\rho)e^{i\frac{S+\boxed{\delta S}}{\epsilon}}$$

correction term

Orbit

$$\delta u_\mu \equiv \partial_\mu \delta S$$

$$\bar{k}_\mu := \bar{u}_\mu + \boxed{\delta\bar{u}_\mu}$$

Perturbed eikonal equation

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\psi = 0$$

Ansatz

$$g = \eta + h$$

$$\psi = \psi_B e^{i\frac{\delta S}{\epsilon}}$$

$$k_\mu = \overline{u}_\mu + \overline{\delta u_\mu}$$

Averaging

$$H := \frac{1}{2}g^{\mu\nu}k_\mu k_\nu - k_\mu \overline{h^{\mu\nu}v_\nu} + \frac{1}{2}q^2 = 0$$

Perturbed eikonal equation

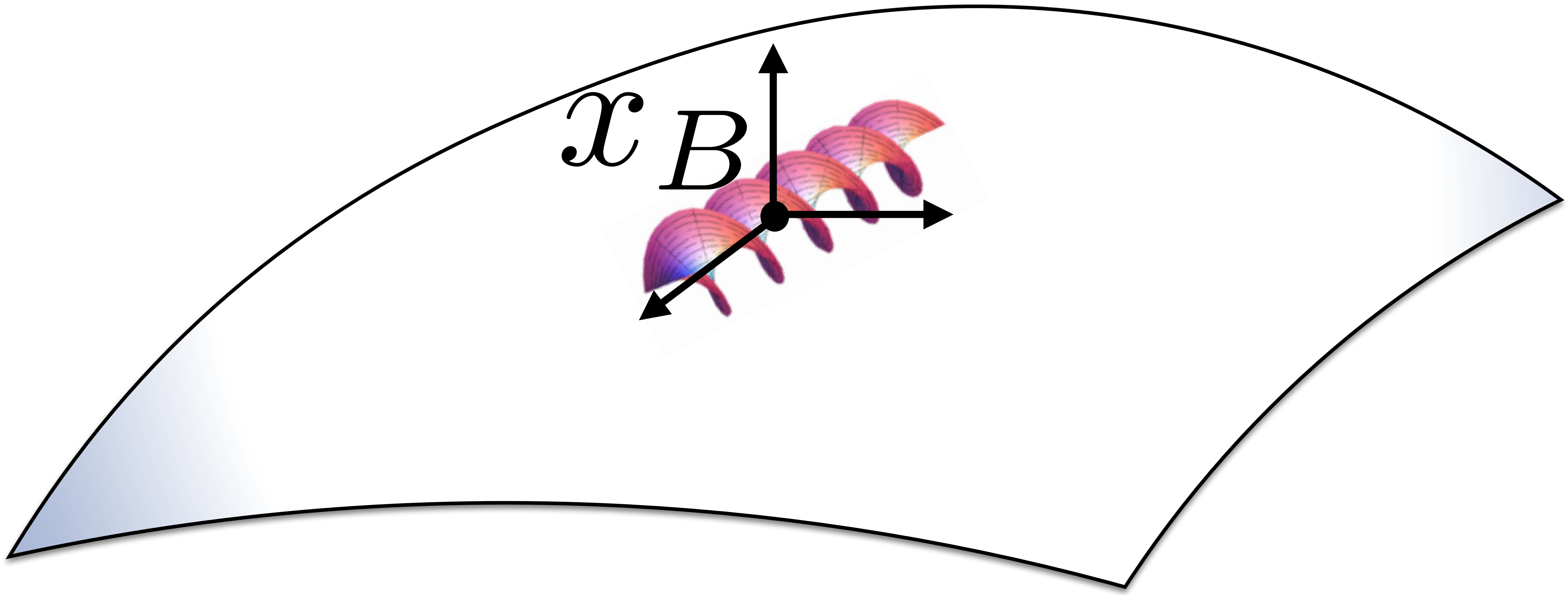
$$H := \frac{1}{2} g^{\mu\nu} k_\mu k_\nu - k_\mu \overline{h^{\mu\nu} v_\nu} + \frac{1}{2} q^2 = 0$$

$$\dot{x}^\alpha = \frac{\partial H}{\partial k_\alpha} \quad \dot{k}^\alpha = -\frac{\partial H}{\partial x^\alpha}$$

$$\frac{D\dot{x}^\mu}{D\tau} = \dot{x}^\nu g_{\nu\alpha} \nabla^\mu \overline{h^{\alpha\beta} v_\beta}$$

the extra force between angular momentum of the vortex beam and curved space-time.

Riemann normal coordinate



$$h_{\mu\nu} = -\frac{1}{3} R_{\mu\alpha\nu\beta}(x^\alpha - x_B^\alpha)(x^\beta - x_B^\beta)$$

$$\frac{D\dot{x}^\mu}{D\tau} = \dot{x}^\alpha g_{\nu\alpha} \nabla^\nu \overline{h^{\alpha\beta} v_\beta}$$

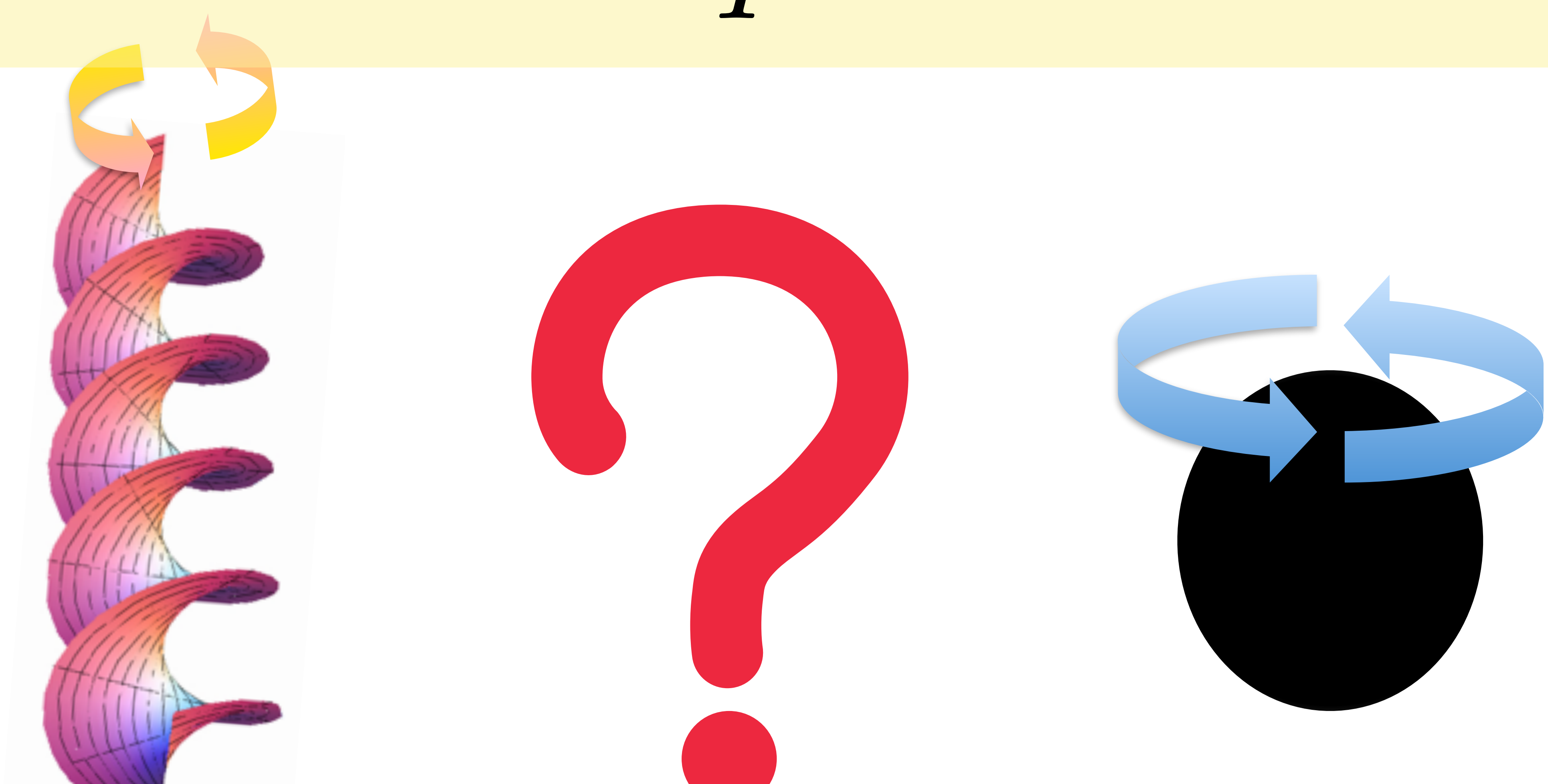
$$\frac{D\dot{x}^\mu}{D\tau} = -\frac{1}{2q} R_{\mu\nu\alpha\beta} u^\nu S^{\alpha\beta}$$

where $$S^{\nu\beta} = \frac{1}{2}\overline{(X_B^\nu v^\beta - X_B^\beta v^\nu)}$$

$$X_B^\mu = x^\mu - x_B^\mu$$

How does vortex beam propagate around Kerr B.H?

$$\frac{D\dot{x}^{\mu}}{D\tau} = -\frac{1}{2q} R_{\mu\nu\alpha\beta} u^{\nu} S^{\alpha\beta}$$



Orbit of vortex beam
on the equatorial plane
of a Kerr Black hole

perturbative form of Kerr metric

$$ds^2 = -(1-2\Phi)dt^2 + 2h_{ti}dx^i dt + (1+2\Phi)\delta_{ij}dx^i dx$$

$$\Phi = \frac{M}{r} \qquad h_{ti} = \frac{2Ma}{r^3}(-y, x, 0)$$

M:mass of black hole

a : Kerr parameter

Expanding metric around Beam

$$-\frac{1}{2q}R_{\mu\nu\alpha\beta}u^{\nu}S^{\alpha\beta}=\frac{1}{2q}\partial_{\mu}(\partial_{l}h_{tk}-\partial_{k}h_{tl})u^{t}S^{kl}$$

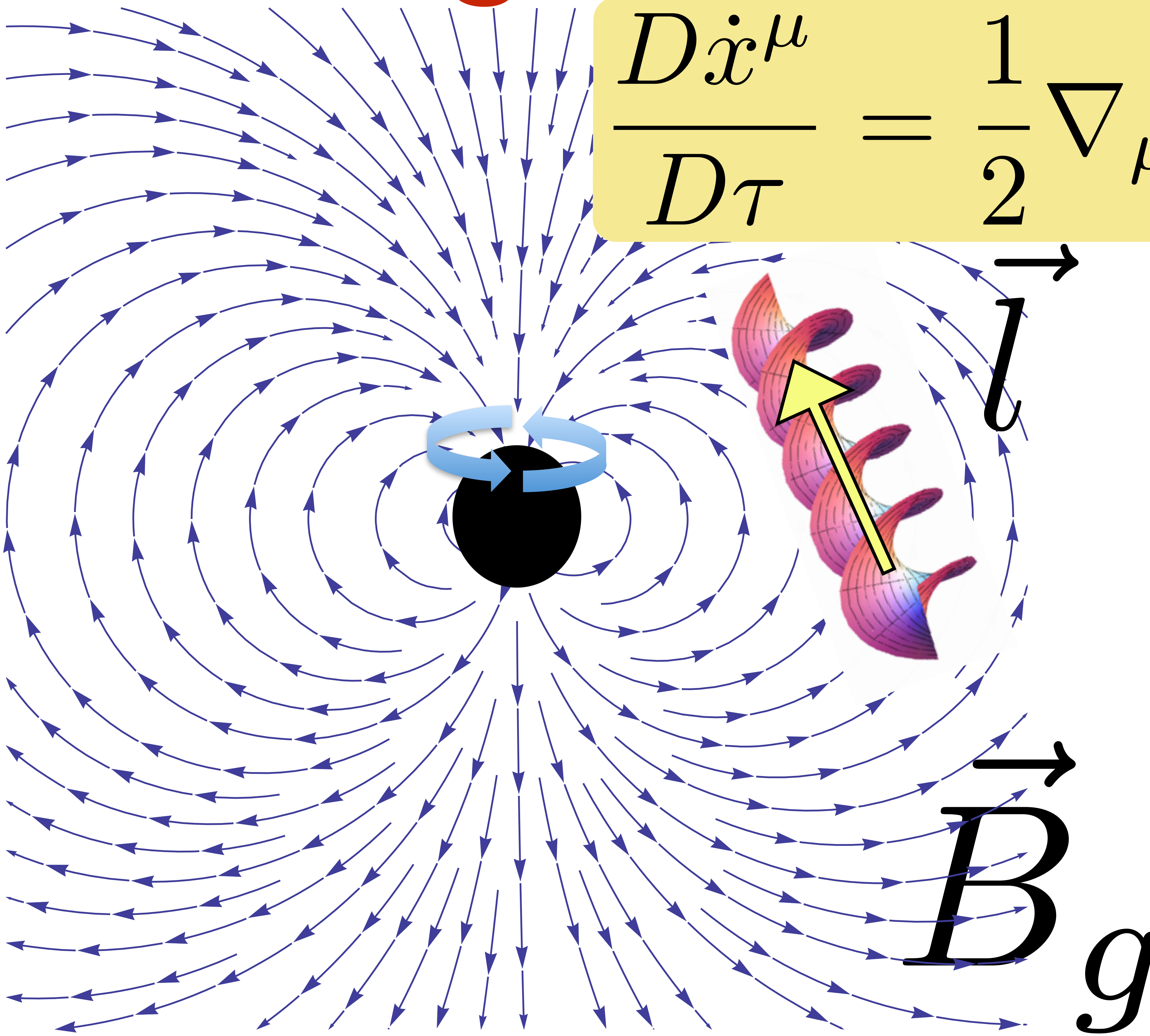
$$=\frac{1}{2}\nabla_{\mu}(\vec{B}_{g}\cdot\vec{l})$$

where $B_{ij}=\frac{1}{4}\left(\frac{\partial h_{ti}}{\partial x^{j}}-\frac{\partial h_{tj}}{\partial x^{i}}\right),\quad l^{i}=\frac{u^{t}}{2q}\epsilon^{ijk}S_{jk}$

$$\boxed{\frac{D\dot{x}^{\mu}}{D\tau}=\frac{1}{2}\nabla_{\mu}(\vec{B}_{g}\cdot\vec{l})}$$

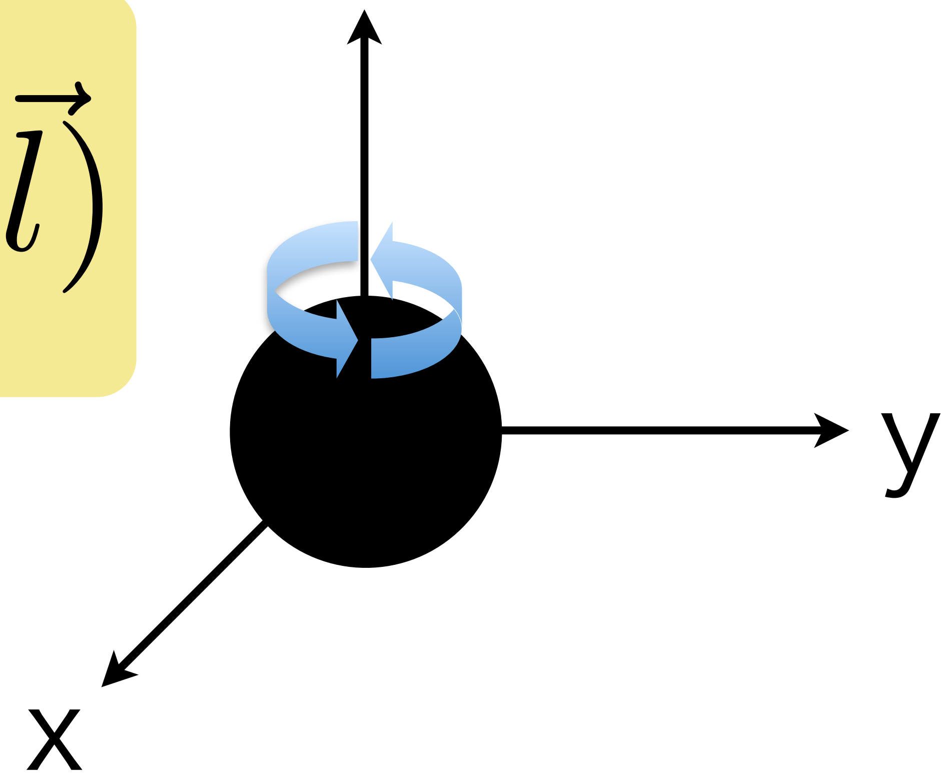
Configuration of Bg

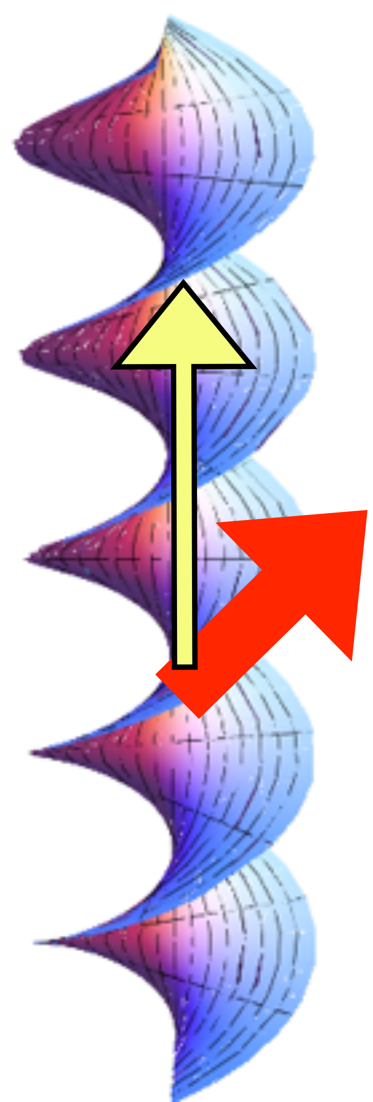
$$\frac{D\dot{x}^{\mu}}{D\tau} = \frac{1}{2}\nabla_{\mu}(\vec{B}_g \cdot \vec{l})$$



Propagation of parallel to axis of black hole

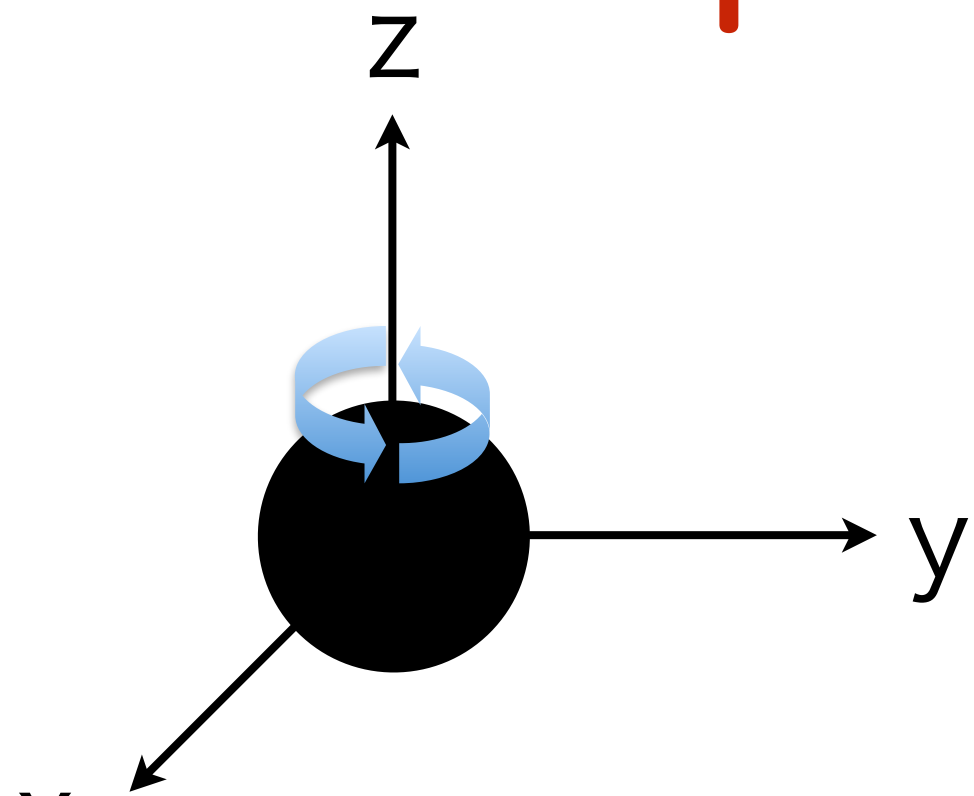
$$\frac{D\dot{x}^{\mu}}{D\tau} = \frac{1}{2}\nabla_{\mu}(\vec{B}_g \cdot \vec{l})$$





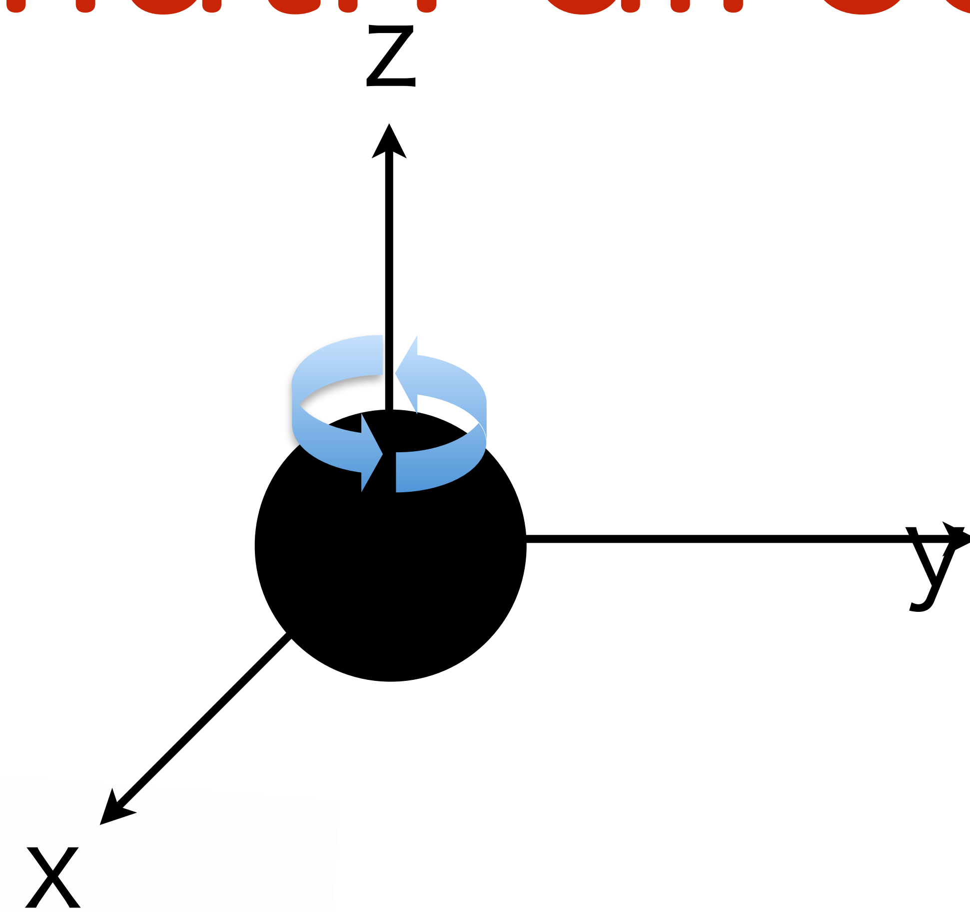
attracting force !

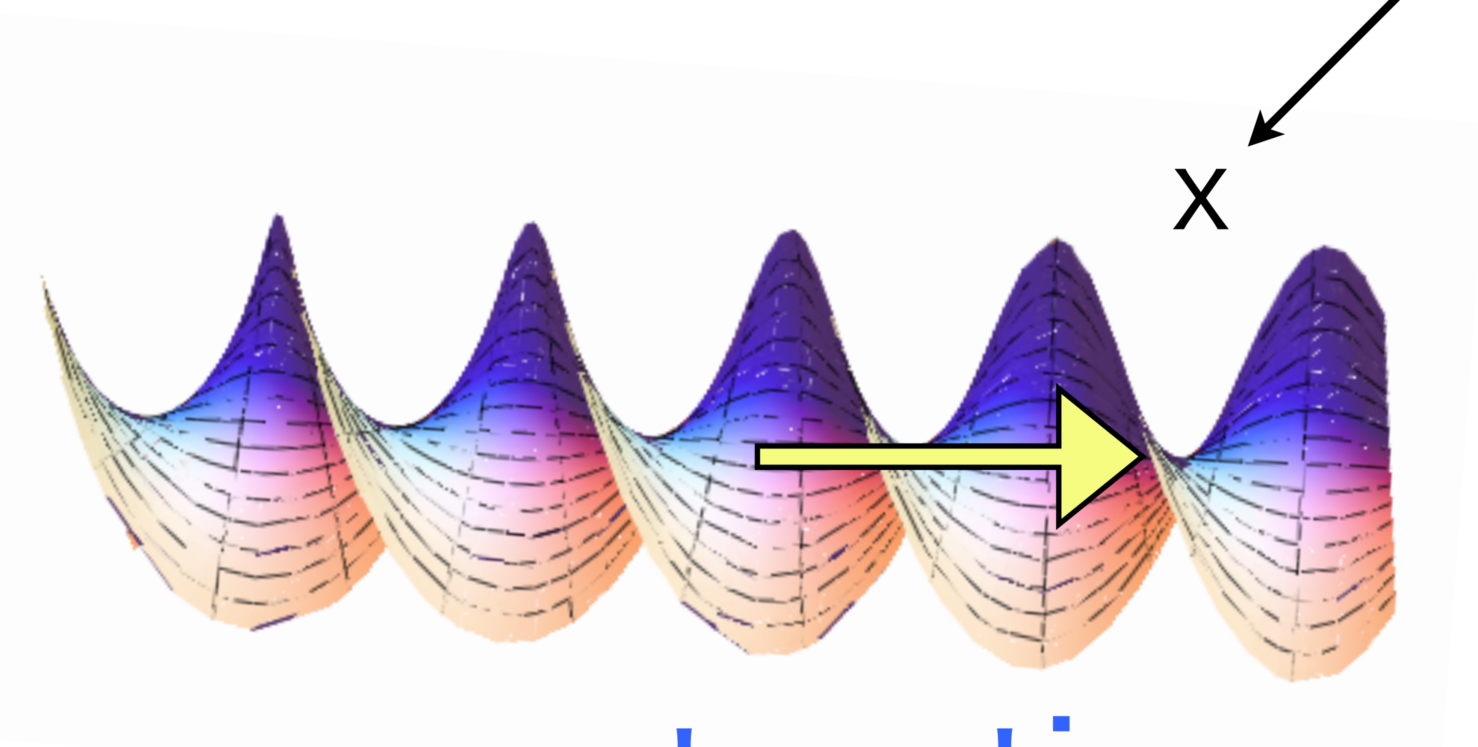
Toward black hole on equatorial plane



the force acts in the z direction

Propagation to the azimuth direction





not acting extra force

Summary

- We obtained the equation for orbit of the vortex beam in the Kerr spacetime.

$$\frac{D\dot{x}^{\mu}}{D\tau} = \frac{1}{2}\nabla_{\mu}(\vec{B}_g \cdot \vec{l})$$

$$\vec{B}_g = \vec{\nabla} \times \vec{h} \qquad l^i = \frac{u^t}{2q}\epsilon^{ijk}S_{jk}$$

- Extra force depend on $\dfrac{m}{q}$.

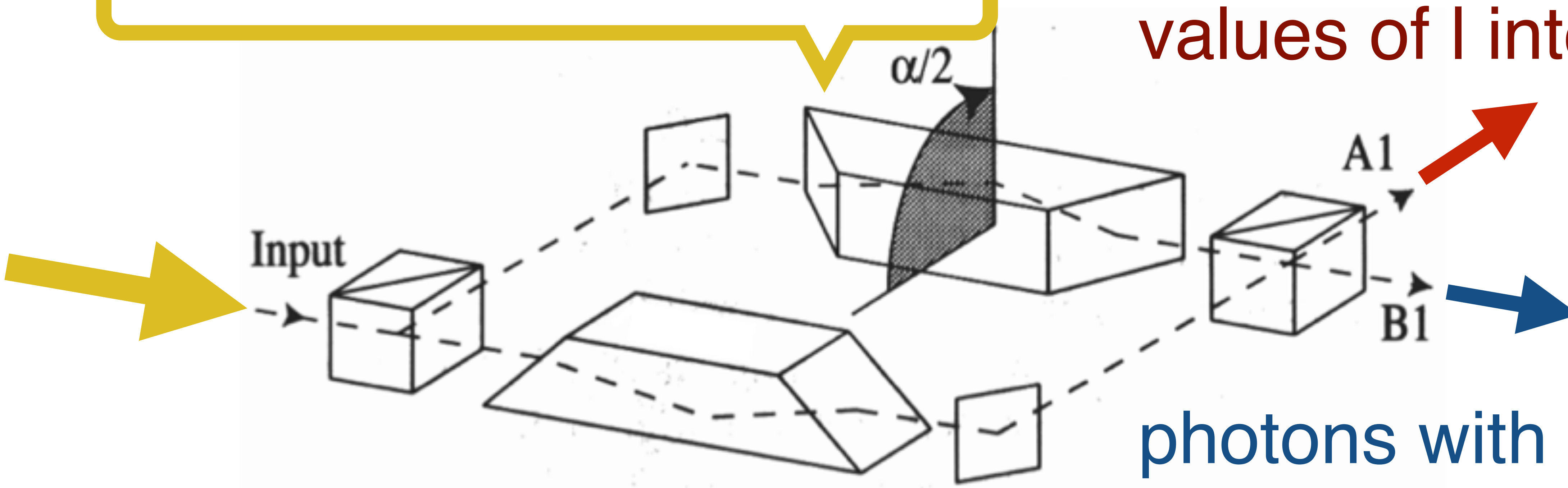
Future Work

- By using vortex beam,
  we determine spin parameter
  of Black hole
  - observing distribution of m of light emitted by a same source in the Kerr space-time

Observation of vortex photon

Phys. Rev. Lett. 88, 257901(2002)

$$e^{im\phi} \longrightarrow e^{im\phi + im\alpha}$$



photons with even values of l into Port A1

photons with odd values of l into Port B1