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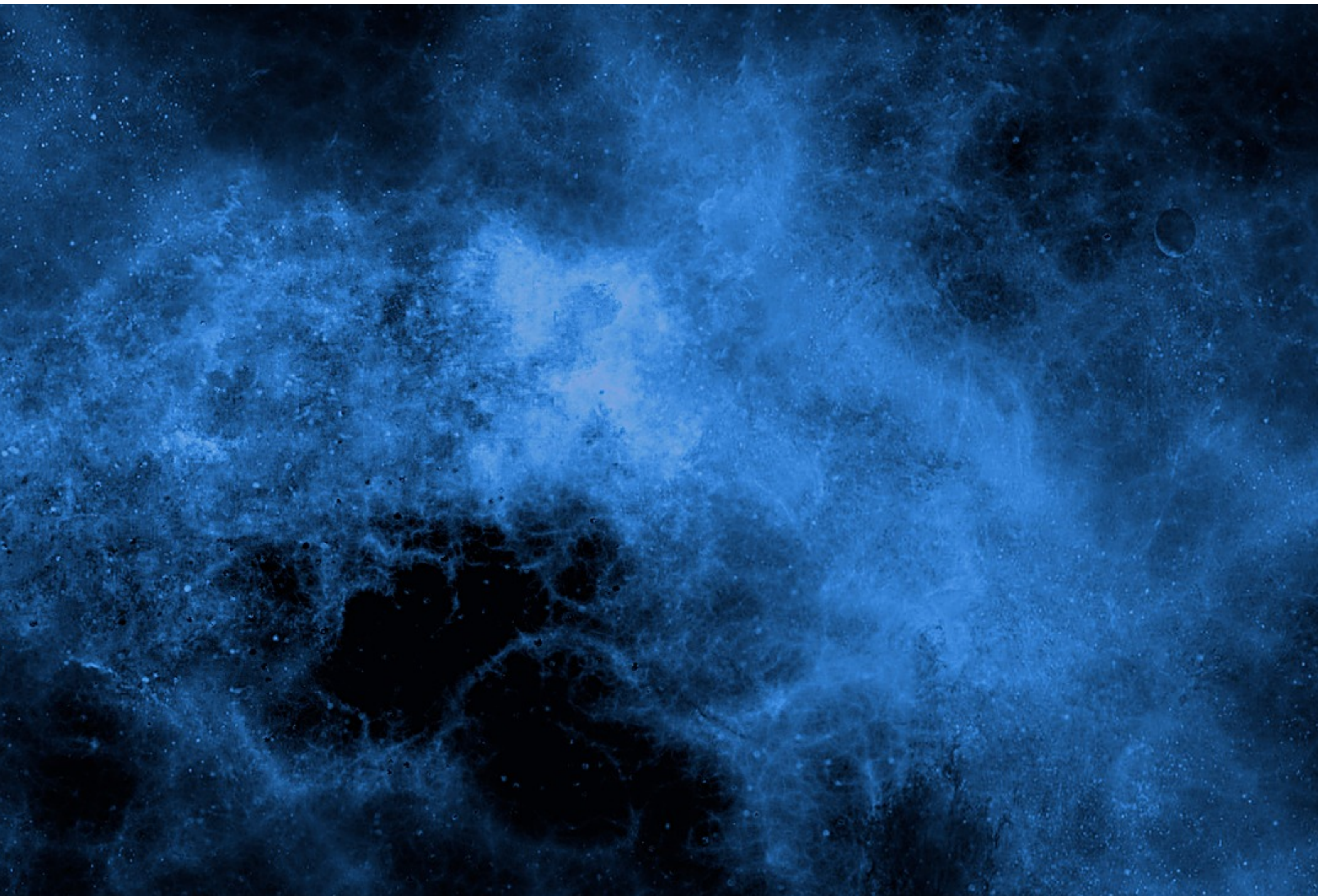

Session D2a 10:45–12:00

[Chair: Shuichiro Yokoyama]

Kazuharu Bamba

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“Generation of magnetic fields from the coupling of electromagnetic fields with a higher curvature term in inflationary cosmology”

(15 min.)

[JGRG30 (2021) 120911]

Generation of magnetic fields from the coupling of electromagnetic fields with a higher curvature term in inflationary cosmology

Reference: **JCAP 04 (2021) 009**

JGRG30, Waseda University, Tokyo, online

9th December, 2021

Fukushima University

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Tanmoy Paul（Chandernagore College）

I. Introduction

- Galactic magnetic fields

$$\sim \mu\mathrm{G}$$

[Sofue *et al.*, Annu. Rev. Astron. Astrophys. 24, 459 (1986)]

- Magnetic fields in clusters of galaxies

$$0.1 - 10\mu\mathrm{G}\,, \quad 10\mathrm{kpc} - \underline{1\mathrm{Mpc}}$$

[Clarke *et al.*, Astrophys. J. 547, L111 (2001)]

- Void region $B \geq 3 \times 10^{-16}\,\mathrm{G}\,, \quad 1\mathrm{Mpc}$

[Neronov and Vovk, Science 328, 73 (2010) [arXiv:1006.3504 [astro-ph.HE]]]

* Recent reviews (examples)

[Kandus *et al.*, Phys. Rep. 505, 1 (2011)] [Yamazaki *et al.*, Phys. Rep. 517, 141 (2012)]

[Maleknejad *et al.*, Phys. Rep. (2013)]

[Durrer and Neronov, Astron. Astrophys. Rev. 21, 62 (2013)]

Origin of cosmic magnetic fields

1. Astrophysical process: **Plasma instability**

(a) **Biermann battery mechanism**

[Biermann and Schlüter, Phys. Rev. 82, 863(1951)]

[Hanayama *et al*., Astrophys. J. 633, 941 (2005)]

(b) **Weibel instability**

[Weibel, Phys. Rev. Lett. 2, 83 (1959)]

[Fujita and Kato, Mon. Not. Roy. Astron. Soc. 364, 247 (2005)]

Origin of cosmic magnetic fields (2)

2. Cosmolosical processes:

- **(First-order) Cosmological Phase transitions**

(i) **Electroweak phase transition (EWPT)**

[Baym, Bödeker and McLerran, Phys. Rev. D 53, 662 (1996)]

(ii) **Quark-hadron phase transition (QCDPT)**

[Quashnock, Loeb and Spergel, Astrophys. J. 344, L49 (1989)]

- **Primordial density perturbations before the epoch of recombination**

[Ichiki *et al*., Science 311, 827 (2006)]

[Kobayashi, *et al*, Phys. Rev. D 75, 103501 (2007)]

Origin of cosmic magnetic fields (3)

- Coherence scale
- Strength

→ It is difficult that these processes generate the magnetic fields on megaparsec scales with sufficient field strength to account for the observed magnetic fields in galaxies and clusters of galaxies without requiring any dynamo amplification.

⇩

The most natural origin of large-scale magnetic fields:

Electromagnetic quantum fluctuation generated at the inflationary stage

Obstacle

- Friedmann-Lemaître-Robertson-Walker (FLRW) metric is conformally flat.

- The Maxwell theory is conformally invariat.

⇩

The conformal invariance of the electromagnetic fields has to be broken at the inflationary stage.

Breaking mechanisms (1)

1. Coupling of a scalar field to electromagnetic fields

[Ratra, Astrophys. J. 391, L1 (1992)]

Cf. [KB and Yokoyama, Phys. Rev. D 69, 043507 (2004)]

$$\mathcal{L} = -\frac{1}{4} \underline{f(\Phi) F_{\mu\nu} F^{\mu\nu}}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$: Electromagnetic field-strength tensor

A_μ : $U(1)$ gauge field

$$f(\Phi) = e^{-\lambda \kappa \Phi}$$

Φ : Dilaton field, $\kappa = \sqrt{8\pi G}$, G : Newton's constant λ : Dimensionless constant

2. Non-minimal coupling of electromagnetic fields to gravity

[Turner and Widrow, Phys. Rev. D 37, 2743 (1988)]

$$\underline{(R/m^2) F_{\mu\nu} F^{\mu\nu}}$$

$\longleftarrow$ Such a term is known to arise in curved spacetime due to one-loop vacuum-polarization effects.

R : Ricci scalar

m : Mass scale

[Drummond and Hathrell, Phys. Rev. D 22, 343 (1980)]

Breaking mechanisms (2)

3. The conformal anomaly in the trace of the energy-momentum tensor induced by quantum corrections to Maxwell electrodynamics

[Dolgov, Phys. Rev. D 48, 2499 (1993)]

Cf. Baryon isocurvature constraints on the primordial hypermagnetic fields

[K. Kamada, F. Uchida and J. Yokoyama, JCAP 04 (2021) 034]

II. Model

Action: $S = S_{grav} + S_{em}^{(can)} + S_{CB}$

(1) $S_{grav} = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi) - \xi(\Phi)\mathcal{G}\right]$

$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$: Gauss-Bonnet invariant

$V(\Phi)$: Potential of an inflaton Φ

$\xi(\Phi)$: Coupling between Φ and $\mathcal{G}$

R : Scalar curvature

(2) $S_{em}^{(can)} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$: Maxwell theory

(3) Non-minimal coupling between electromagnetic fields and gravity $S_{CB} = \int d^4x\sqrt{-g}\left[f(R,\mathcal{G})F_{\mu\nu}F^{\mu\nu}\right]$ ⇒ **Breaking of the conformal invariance of the electromagnetic fields.**

$\boxed{f(R,\mathcal{G}) = \kappa^{2q}\left(R^q + \mathcal{G}^{q/2}\right)}$

q : Model parameter

Spatially flat FLRW metric

$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right]$

$a(\eta)$: Scale factor

η : Conformal time

$(dt = a(\eta)d\eta)$

Field equations

$\partial_\alpha\left[\sqrt{-g}h^2(R,G)\, g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}\right] = 0$ $h^2(R,\mathcal{G}) = 1 - 4f(R,\mathcal{G})$

$A_i''(\eta,\vec{x}) + 2\frac{h'(R,G)}{h(R,G)}A_i' - \partial_l\partial^l A_i = 0$ ⟵ Coulomb gauge $A_0 = 0$

Quasi de Sitter Inflation

η_0 : Initial time of inflation

$N = \int^\eta aH\, d\eta$: e-folding number

$a(\eta) = \left(\frac{-\eta}{\eta_0}\right)^{\beta+1}$ $\beta = -2 - \epsilon = -3 + \frac{\mathcal{H}'}{\mathcal{H}^2}$

$\epsilon = -\frac{\mathcal{H}'}{\mathcal{H}^2} + 1$: Slow-roll parameter

$' = \frac{d}{d\eta}$ $\mathcal{H} = \frac{a'}{a}$: Conformal Hubble parameter

III. Energy density and spectrum of electric and magnetic fields

Energy-momentum tensor of electromagnetic fields

$$T_{\alpha\beta} = -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\alpha\beta}}\left[-\frac{1}{4}\sqrt{-g}\,\big(1-4f(R,\mathcal{G})\big)F_{\mu\nu}F^{\mu\nu}\right]$$

$$= -\frac{1}{4}\left\{g_{\alpha\beta}\big(1-4f(R,\mathcal{G})\big)F_{\mu\nu}F^{\mu\nu} - 4\big(1-4f(R,\mathcal{G})\big)g^{\mu\nu}F_{\mu\alpha}F_{\nu\beta} + 8F_{\mu\nu}F^{\mu\nu}\frac{\delta f(R,\mathcal{G})}{\delta g^{\alpha\beta}}\right\}$$

$$\Rightarrow \quad T_0^0 = -\frac{1}{2a^4}\left(A_i'\right)^2 P(\eta) - \frac{1}{4a^4}F_{ij}F_{ij}Q(\eta)$$

$$P(\eta) = 1 - 4\kappa^{2q}\left(R^q + \mathcal{G}^{q/2}\right) - \frac{24q\kappa^{2q}\mathcal{H}'}{a^2}\left(R^{q-1} + \frac{2\mathcal{H}^2}{a^2}\mathcal{G}^{\frac{q}{2}-1}\right)$$

$$Q(\eta) = 1 - 4\kappa^{2q}\left(R^q + \mathcal{G}^{q/2}\right) + \frac{24q\kappa^{2q}\mathcal{H}'}{a^2}\left(R^{q-1} + \frac{2\mathcal{H}^2}{a^2}\mathcal{G}^{\frac{q}{2}-1}\right)$$

<Quantization of U(1) gauge fields>

Mode expansion $$\hat{A}_i(\eta,\vec{x}) = \int \frac{d\vec{k}}{(2\pi)^3}\sum_{r=1,2}\epsilon_{ri}\left[\hat{b}_r(\vec{k})A_r(k,\eta)e^{i\vec{k}.\vec{x}} + \hat{b}_r^{+}(\vec{k})A_r^{*}(k,\eta)e^{-i\vec{k}.\vec{x}}\right]$$

$$\left[\hat{b}_p(\vec{k}), \hat{b}_r^{+}(\vec{k}')\right] = \delta_{pr}\delta\left(\vec{k}-\vec{k}'\right)$$

$\vec{k}$: Comoving wave number

ϵ_{ri} : Polarization vector

$\hat{b}_r(\vec{k})$: Annihilation operator $\hat{b}_r^{+}(\vec{k})$: Creation operator

Energy density

$$\rho(\vec{E}) = P(\eta)\sum_{r=1,2}\int\frac{dk}{2\pi^2}\frac{k^2}{a^4}\left|A_r'(k,\eta)\right|^2, \quad \rho(\vec{B}) = Q(\eta)\sum_{r=1,2}\int\frac{dk}{2\pi^2}\frac{k^4}{a^4}\left|A_r(k,\eta)\right|^2$$

Power spectrum

$$\frac{\partial\rho(\vec{E})}{\partial\ln k} = P(\eta)\sum_{r=1,2}\frac{k}{2\pi^2}\frac{k^2}{a^4}\left|A_r'(k,\eta)\right|^2, \quad \frac{\partial\rho(\vec{B})}{\partial\ln k} = Q(\eta)\sum_{r=1,2}\frac{k}{2\pi^2}\frac{k^4}{a^4}\left|A_r(k,\eta)\right|^2$$

Equations of motion of U(1) gauge fields

$$A_r''(k,\eta) + \frac{2h'}{h}A_r'(k,\eta) + k^2 A_r(k,\eta) = 0$$

$$\tilde{A}_r(k,\eta) = h(\eta)A_r(k,\eta)$$

$$\Rightarrow \quad \tilde{A}_r''(k,\eta) + \left(k^2 - \frac{h''}{h}\right)\tilde{A}_r(k,\eta) = 0$$

$$h(R,\mathcal{G}) = \left(1 - 4f(R,\mathcal{G})\right)^{1/2} = \left[1 - \frac{4B}{\eta_0^{2q}}\left(\frac{-\eta}{\eta_0}\right)^{2\epsilon q}\right]^{1/2}$$

$$B = \kappa^{2q}\left[\left[6\beta(\beta+1)\right]^q + \left[-24(\beta+1)^3\right]^{q/2}\right]$$

$$\longrightarrow \quad h(R,\mathcal{G}) = 1 - \frac{2B}{\eta_0^{2q}}\left(\frac{-\eta}{\eta_0}\right)^{2\epsilon q}$$

$$\tilde{A}_r''(k,\eta) + \left(k^2 - \frac{4B\epsilon q}{\eta_0^{2q}\left(1 - \frac{2B}{\eta_0^{2q}}\right)}\frac{1}{\eta^2}\right)\tilde{A}_r(k,\eta) = 0$$

J_ν : Bessel function

$$\tilde{A}_r(k,\eta) = \sqrt{-k\eta}\left[D_1\ J_\nu(-k\eta) + D_2\ J_{-\nu}(-k\eta)\right]$$

$$\nu^2 = \frac{1}{4} - \frac{4B\epsilon q}{\eta_0^{2q}\left(\frac{2B}{\eta_0^{2q}} - 1\right)}$$

- **Sub-horizon mode:** $|k\eta| \gg 1$

※Initial condition: Bunch-Davies vacuum

$\left(\tilde{A}_r(k,\eta) = \frac{1}{\sqrt{2k}}e^{-ik\eta} \right)$

$$\lim_{|k\eta|\gg 1} J_\nu(-k\eta) = \sqrt{\frac{2}{\pi(-k\eta)}}\ \cos\left[-k\eta - \frac{\pi}{2}(\nu + \frac{1}{2})\right]$$

$$\lim_{|k\eta|\gg 1} J_{-\nu}(-k\eta) = \sqrt{\frac{2}{\pi(-k\eta)}}\ \sin\left[-k\eta + \frac{\pi}{2}(\nu + \frac{1}{2})\right]$$

$$\begin{aligned}\lim_{|k\eta|\gg 1} \tilde{A}_r(k,\eta) &= \sqrt{\frac{2}{\pi}}\left[D_1\ \cos\left[-k\eta - \frac{\pi}{2}(\nu+\frac{1}{2})\right] + D_2\ \sin\left[-k\eta + \frac{\pi}{2}(\nu+\frac{1}{2})\right]\right] \\ &= \sqrt{\frac{2}{\pi}}\left[e^{-ik\eta}\left(\frac{D_1}{2}e^{-i\frac{\pi}{2}(\nu+\frac{1}{2})} + \frac{D_2}{2i}e^{i\frac{\pi}{2}(\nu+\frac{1}{2})}\right) + e^{ik\eta}\left(\frac{D_1}{2}e^{i\frac{\pi}{2}(\nu+\frac{1}{2})} - \frac{D_2}{2i}e^{-i\frac{\pi}{2}(\nu+\frac{1}{2})}\right)\right]\end{aligned}$$

$$D_1 = \frac{1}{2}\sqrt{\frac{\pi}{k}}\,\frac{e^{-i\frac{\pi}{2}(\nu+\frac{1}{2})}}{\cos\left[\pi(\nu+1/2)\right]}\,, \qquad D_2 = \frac{1}{2}\sqrt{\frac{\pi}{k}}\,\frac{e^{i\frac{\pi}{2}(\nu+\frac{3}{2})}}{\cos\left[\pi(\nu+1/2)\right]}$$

$$A_r(k,\eta) = \frac{1}{h(\eta)}\tilde{A}_r(k,\eta)$$

$$= \frac{\sqrt{-k\eta}}{2\left[1-\frac{4B}{\eta_0^{2q}}\left(\frac{-\eta}{\eta_0}\right)^{2\epsilon q}\right]^{1/2}}\left\{\sqrt{\frac{\pi}{k}}\,\frac{e^{-i\frac{\pi}{2}(\nu+\frac{1}{2})}}{\cos\left[\pi(\nu+1/2)\right]}\,J_\nu(-k\eta)+\sqrt{\frac{\pi}{k}}\,\frac{e^{i\frac{\pi}{2}(\nu+\frac{3}{2})}}{\cos\left[\pi(\nu+1/2)\right]}\,J_{-\nu}(-k\eta)\right\}$$

- **Super-horizon mode:** $k < \frac{1}{\mathcal{H}}$

$$\lim_{|k\eta|\ll 1} A_r(k,\eta) = \frac{1}{\left[1-\frac{4B}{\eta_0^{2q}}\left(\frac{-\eta}{\eta_0}\right)^{2\epsilon q}\right]^{1/2}}\left\{\frac{D_1}{2^\nu\Gamma(\nu+1)}\,(-k\eta)^{\nu+\frac{1}{2}}+\frac{D_2}{2^{-\nu}\Gamma(-\nu+1)}\,(-k\eta)^{-\nu+\frac{1}{2}}\right\}$$

$$\Rightarrow\quad \frac{\partial\rho(\vec{E})}{\partial\ln k} = \frac{P(\eta)\,(\nu-\frac{1}{2})^2\,H_0^4}{4\pi\left[1-\frac{4B}{\eta_0^{2q}}\left(\frac{-\eta}{\eta_0}\right)^{2\epsilon q}\right]\left\{2^{-\nu}\Gamma(-\nu+1)\cos\left[\pi(\nu+1/2)\right]\right\}^2}\underline{(-k\eta)^{3-2\nu}}$$

$$\frac{\partial\rho(\vec{B})}{\partial\ln k} = \frac{Q(\eta)\,H_0^4}{4\pi\left[1-\frac{4B}{\eta_0^{2q}}\left(\frac{-\eta}{\eta_0}\right)^{2\epsilon q}\right]\left\{2^{-\nu}\Gamma(-\nu+1)\cos\left[\pi(\nu+1/2)\right]\right\}^2}\underline{(-k\eta)^{5-2\nu}}$$

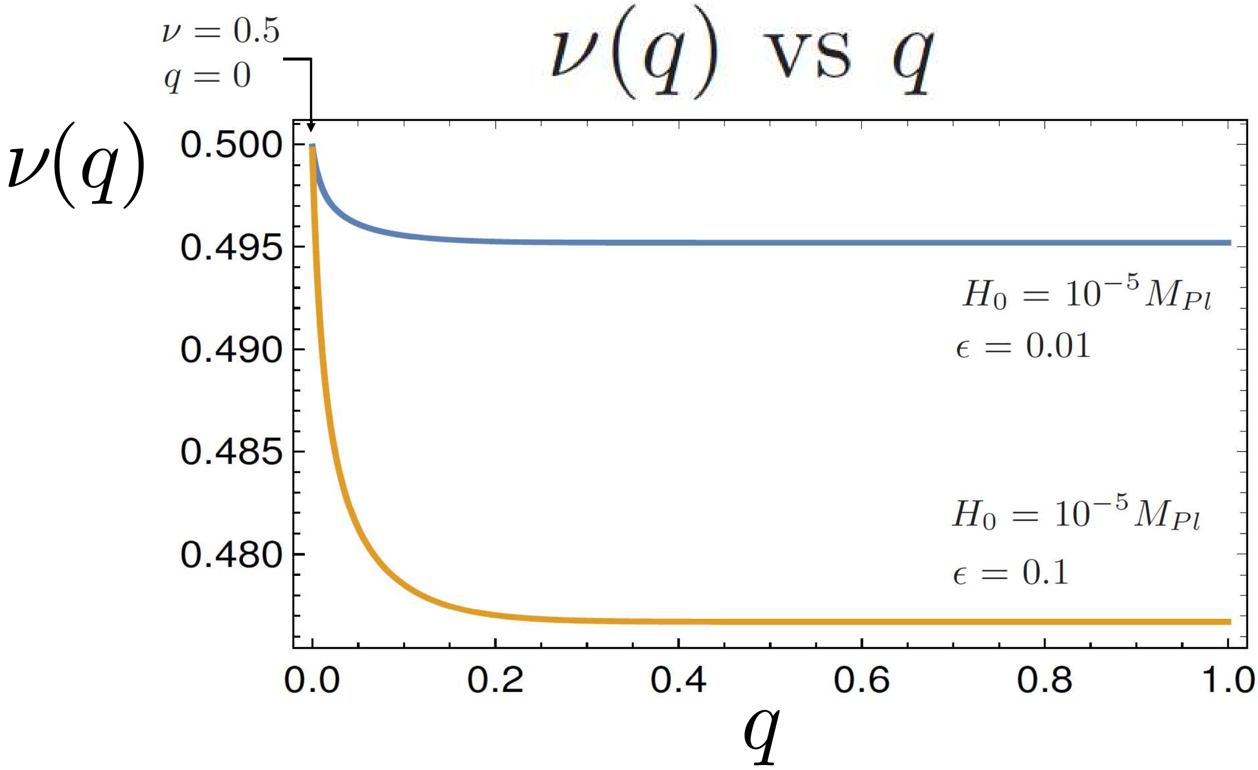
$\epsilon < 1$

※Energy density of electromagnetic fields during inflation ($\eta = \eta_c$) $\quad 0 < q < 1$

$\rho_{em}(\eta_c) \sim H_0^4 \qquad H_0 = 10^{-5} M_{Pl}$: Energy scale of inflation

$\longrightarrow \rho_{em} \ll M_{Pl}^2 H^2 \quad \Rightarrow$ Backreaction in the inflationary stage is negligible.

$\nu(q)$ vs q



$$\left.\frac{\partial \rho(\vec{B})}{\partial \ln k}\right|_0 = \left(\frac{a_f}{a_0}\right)^4 \left.\frac{\partial \rho(\vec{B})}{\partial \ln k}\right|_{\eta_f}$$

η_f : Final time of inflation

$$B_0 = \frac{1}{\sqrt{2\pi}} \left\{ \frac{2^\nu}{\Gamma(-\nu+1)\cos\left[\pi(\nu+1/2)\right] \left(1 - \frac{4B}{\eta_0^{2q}} \left(\frac{-\eta_f}{\eta_0}\right)^{2\epsilon q}\right)^{1/2}} \right\} \left(\frac{a_f}{a_0}\right)^2 H_0^2 (-k\eta_f)^{-\nu+5/2}$$

$$B = \kappa^{2q}\left\{\left[6\beta(\beta+1)\right]^q + \left[-24(\beta+1)^3\right]^{q/2}\right\} = \kappa^{2q}\left(12^q + 24^{q/2}\right)$$

⇨ **Present magnetic strength for the case of instantaneous reheating**

$$\left.B_0\right|_{CMB} \approx \underline{10^{-62}\mathrm{G}}$$

※Observation: $10^{-22}\mathrm{G} \lesssim B_0 \lesssim 10^{-10}\mathrm{G}$

$k_{CMB} \approx 10^{-40}\mathrm{GeV} \approx 0.02\mathrm{Mpc}^{-1}$

$q = 0.5$

$N_f = 58$, $1\mathrm{G} = 1.95 \times 10^{-20}\mathrm{GeV}^2$

$H_f = 5.1 \times 10^{-8} M_{Pl}$

$\frac{a_0}{a_f} \approx 10^{30}\left(H_f / 10^{-5} M_{Pl}\right)^{1/2}$

Electric fields are screened because the value of the electric conductivity becomes very large instantaneously.

IV. Present magnetic strength for the case of the reheating phase with a non-zero e-folding number

※Growth of magnetic strength at the reheating stage: [Kobayashi and Sloth, Phys.Rev.D 100, 023524 (2019)]

e-folds number at the reheating stage

[Dai, Kamionkowski and Wang, Phys. Rev. Lett 133, 041302 (2014)]
[Cook, Dimastrogiovanni, Easson and Krauss, JCAP 04(2015), 047]

$$N_{\mathrm{re}} = \frac{4}{(1-3\omega_{\mathrm{eff}})}\left[-\frac{1}{4}\ln\left(\frac{45}{\pi^2 g_{re}}\right)-\frac{1}{3}\ln\left(\frac{11 g_{s,re}}{43}\right)-\ln\left(\frac{k}{a_0 T_0}\right)-\ln\left(\frac{(3H_f^2 M_{\mathrm{Pl}}^2)^{1/4}}{H_0}\right)-N_f\right]$$

$$T_{\mathrm{re}} = H_0\left(\frac{43}{11 g_{s,re}}\right)^{\frac{1}{3}}\left(\frac{a_0 T_0}{k}\right)\exp\left[-(N_f+N_{\mathrm{re}})\right]$$: Reheating temperature

$T_0 = 2.725\mathrm{K}$: Present temperature of the cosmic microwave background (CMB) radiation

$\frac{k}{a_0} \approx 0.02\mathrm{Mpc}^{-1}$

a_0 : Present value of a

$g_{s,re} = g_{re} \approx 100$

ω_{eff} **: Effective equation of state**

Equations of motion of U(1) gauge fields

$A^{(re)}(k,\eta)$: Mode function of U(1) gauge fields at the reheating stage

$$A''^{(re)}(k,\eta) + k^2 A^{(re)}(k,\eta) = 0$$

$$\longrightarrow A^{(re)}(k,\eta) = \frac{1}{\sqrt{2k}}\left[c_k\, e^{-ik(\eta-\eta_f)} + d_k\, e^{ik(\eta-\eta_f)}\right]$$

$$c_k = \sqrt{\frac{k}{2}}\,A(k,\eta_f) + \frac{i}{\sqrt{2k}}\,A'(k,\eta_f), \quad d_k = \sqrt{\frac{k}{2}}\,A(k,\eta_f) - \frac{i}{\sqrt{2k}}\,A'(k,\eta_f)$$

※Connection condition at the end of inflation

$$A^{(re)}(k,\eta_f) = A(k,\eta_f)\,, \quad A'^{(re)}(k,\eta_f) = A'(k,\eta_f)$$

$$A(k,\eta_f) = \frac{\sqrt{-k\eta_f}}{\left[1-\frac{4B}{\eta_0^{2q}}\left(\frac{-\eta_f}{\eta_0}\right)^{2\epsilon q}\right]^{1/2}}\left\{D_1\, J_\nu(-k\eta_f) + D_2\, J_{-\nu}(-k\eta_f)\right\}$$

$$A'(k,\eta_f) = \frac{k}{\left[1-\frac{4B}{\eta_0^{2q}}\left(\frac{-\eta_f}{\eta_0}\right)^{2\epsilon q}\right]^{1/2}} \Bigg\{ \sqrt{-k\eta_f}\ \left[\ D_1\ J_{-1+\nu}(-k\eta_f) + D_2\ J_{-1-\nu}(-k\eta_f)\right]$$
$$+\frac{1}{\sqrt{-k\eta_f}}\ \left[\ D_1(\nu-\frac{1}{2})\ J_\nu(-k\eta_f) - D_2(\nu+\frac{1}{2})\ J_{-\nu}(-k\eta_f)\right] \Bigg\}$$

Bogoliubov coefficients at the reheating stage

$$\alpha_k(\eta) = \sqrt{\frac{k}{2}}\ A^{(re)}(k,\eta) + \frac{i}{\sqrt{2k}}\ A'^{(re)}(k,\eta)$$
$$\beta_k(\eta) = \sqrt{\frac{k}{2}}\ A^{(re)}(k,\eta) - \frac{i}{\sqrt{2k}}\ A'^{(re)}(k,\eta)$$

※ The vacuum of the electromagnetic fields change from the Bunch-Davies vacuum owing to the particle creation at the reheating stage.

$$\Rightarrow \quad \alpha_k(\eta) = c_k\ e^{-ik(\eta-\eta_f)}$$
$$\beta_k(\eta) = d_k\ e^{ik(\eta-\eta_f)}$$

$\longrightarrow$ Solution

$$A^{(re)}(k,\eta) = \frac{1}{\sqrt{2k}}\left[\alpha_k(\eta_f)\ e^{-ik(\eta-\eta_f)} + \beta_k(\eta_f)\ e^{ik(\eta-\eta_f)}\right]$$

- $$\alpha_k(\eta_f) = \frac{\sqrt{k/2}}{\left[1-\frac{4B}{\eta_0^{2q}}\left(\frac{-\eta_f}{\eta_0}\right)^{2\epsilon q}\right]^{1/2}} \Bigg\{ \sqrt{-k\eta_f}\left[D_1\ J_\nu(-k\eta_f) + D_2\ J_{-\nu}(-k\eta_f)\right]$$
$$+i\sqrt{-k\eta_f}\ \left[\ D_1\ J_{-1+\nu}(-k\eta_f) + D_2\ J_{-1-\nu}(-k\eta_f)\right] + \frac{i}{\sqrt{-k\eta_f}}\left[D_1(\nu-\frac{1}{2})\ J_\nu(-k\eta_f) - D_2(\nu+\frac{1}{2})\ J_{-\nu}(-k\eta_f)\right] \Bigg\}$$

- $$\beta_k(\eta_f) = \frac{\sqrt{k/2}}{\left[1-\frac{4B}{\eta_0^{2q}}\left(\frac{-\eta_f}{\eta_0}\right)^{2\epsilon q}\right]^{1/2}} \Bigg\{ \sqrt{-k\eta_f}\left[D_1\ J_\nu(-k\eta_f) + D_2\ J_{-\nu}(-k\eta_f)\right]$$
$$-i\sqrt{-k\eta_f}\ \left[\ D_1\ J_{-1+\nu}(-k\eta_f) + D_2\ J_{-1-\nu}(-k\eta_f)\right] - \frac{i}{\sqrt{-k\eta_f}}\left[D_1(\nu-\frac{1}{2})\ J_\nu(-k\eta_f) - D_2(\nu+\frac{1}{2})\ J_{-\nu}(-k\eta_f)\right] \Bigg\}$$

$$-k\eta_f = e^{-N_f}, \qquad \lim_{|k\eta|\ll 1} J_\nu(-k\eta) = \frac{1}{2^\nu\Gamma(\nu+1)}(-k\eta)^\nu$$

- $$k(\eta-\eta_f) = \frac{2k}{(3\omega_{\rm eff}+1)}\left[\frac{1}{aH} - \frac{1}{a_f H_f}\right], \qquad \eta-\eta_f = \int_{a_f}^{a}\frac{da}{a^2 H}$$

Spectrum of electric and magnetic fields at the reheating stage

$$\frac{\partial \rho(\vec{B})}{\partial \ln k} = \frac{1}{\pi^2}\left(\frac{k^4}{a^4}\right)|\beta_k(\eta_f)|^2 \left\{ \mathrm{Arg}\left[\alpha_k(\eta_f)\ \beta_k^*(\eta_f)\right] - \pi - \left(\frac{4k}{3\omega_{\mathrm{eff}}+1}\right)\left(\frac{1}{aH} - \frac{1}{a_f H_f}\right)\right\}^2$$

$$\propto\ (a^3 H)^{-2}$$

$$\frac{\partial \rho(\vec{E})}{\partial \ln k} = \frac{1}{\pi^2}\left(\frac{k^4}{a^4}\right)|\beta_k(\eta_f)|^2 \ \propto\ a^{-4}$$

Reheating stage: **Electric conductivity** $\sigma < H$	⇨	**Electric fields induce magnetic fields.**

Spectrum of magnetic fields at the present time

$$\left.\frac{\partial \rho(\vec{B})}{\partial \ln k}\right|_0 = \left(\frac{a_{re}}{a_0}\right)^4 \left.\frac{\partial \rho(\vec{B})}{\partial \ln k}\right|_{re}$$

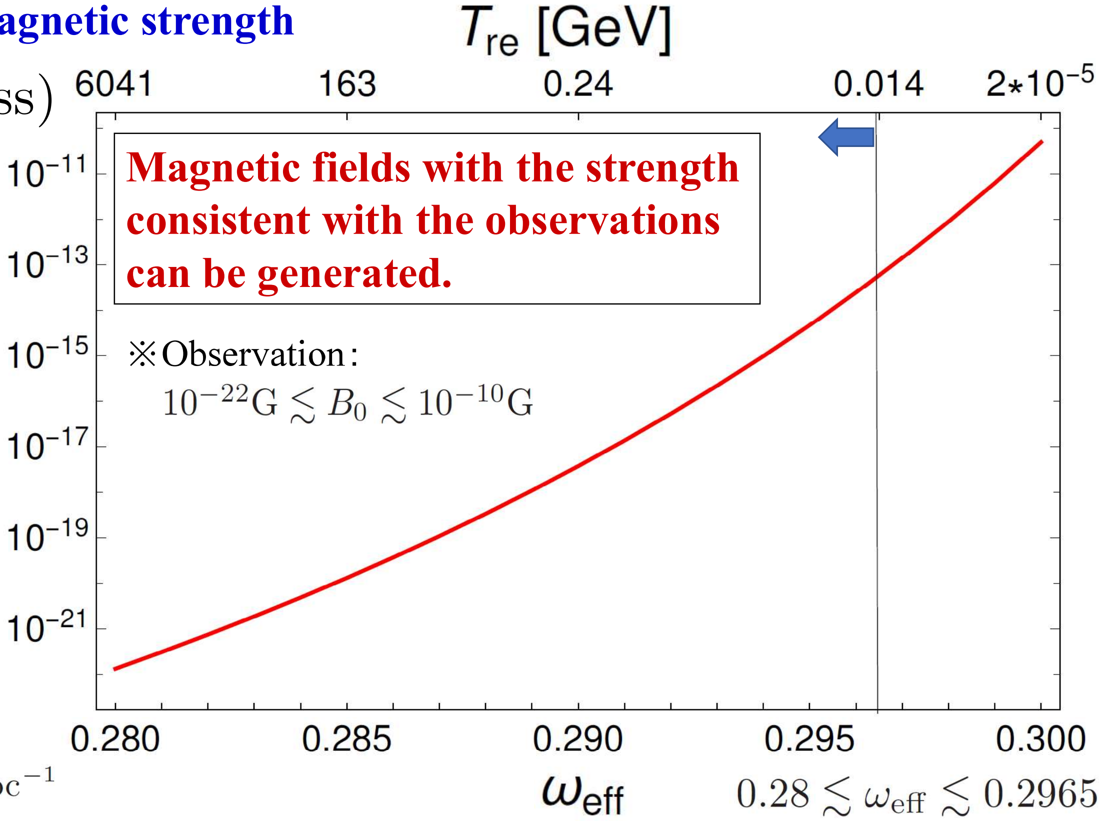
$$\left.\frac{\partial \rho(\vec{B})}{\partial \ln k}\right|_{re} = \frac{1}{\pi^2}\left(\frac{k^4}{a_{re}^4}\right)|\beta_k(\eta_f)|^2 \left\{ \mathrm{Arg}\left[\alpha_k(\eta_f)\ \beta_k^*(\eta_f)\right] - \pi - \left(\frac{4}{3\omega_{\mathrm{eff}}+1}\right)\left(\frac{k}{a_f H_f}\right)\left[\left(\frac{H_f}{H_{re}}\right)^{\frac{3\omega_{\mathrm{eff}}+1}{3\omega_{\mathrm{eff}}+3}} - 1\right]\right\}^2$$

$$H_{re} = H_f\left(\frac{a_{re}}{a_f}\right)^{-\frac{3}{2}(1+\omega_{eff})}, \qquad \frac{a_{re}}{a_f} = e^{N_{\mathrm{re}}}$$

⇨ **Magnetic fields at the present time**

$$B_0 = \frac{\sqrt{2}}{\pi}\left(\frac{k}{a_0}\right)^2 |\beta_k(\eta_f)| \left\{ \mathrm{Arg}\left[\alpha_k(\eta_f)\ \beta_k^*(\eta_f)\right] - \pi - \left(\frac{4}{3\omega_{\mathrm{eff}}+1}\right)\left(\frac{k}{a_f H_f}\right)\left[\left(\frac{H_f}{H_{re}}\right)^{\frac{1+3\omega_{\mathrm{eff}}}{3+3\omega_{\mathrm{eff}}}} - 1\right]\right\}$$

Present magnetic strength



$\frac{k}{a_0} = 0.02\mathrm{Mpc}^{-1}$

$0.28 \lesssim \omega_{\rm eff} \lesssim 0.2965$

V. Schwinger backreaction

[Kobayashi and Sloth, Phys. Rev. D 100, 023524 (2019)]

$$\frac{\mathcal{P}_B}{\mathcal{P}_E} = \left\{ \mathrm{Arg}\left[\alpha_k(\eta_f)\ \beta_k^*(\eta_f)\right] - \pi - \left(\frac{4k}{3\omega_{\rm eff}+1}\right)\left(\frac{1}{aH} - \frac{1}{a_f H_f}\right) \right\}^2$$

$$\mathcal{P}_B = \frac{\partial \rho(B)}{\partial \ln k},\ \mathcal{P}_E = \frac{\partial \rho(E)}{\partial \ln k}$$

$$\alpha_k(\eta_f) = \sqrt{\frac{k}{2}}\ A(k,\eta_f) + \frac{i}{\sqrt{2k}}\ A'(k,\eta_f) \qquad \Rightarrow \qquad \alpha_k(\eta_f) = \frac{i}{\sqrt{2k}}\ A'(k,\eta_f)$$

$$\beta_k(\eta_f) = \sqrt{\frac{k}{2}}\ A(k,\eta_f) - \frac{i}{\sqrt{2k}}\ A'(k,\eta_f) \qquad \qquad \beta_k(\eta_f) = -\frac{i}{\sqrt{2k}}\ A'(k,\eta_f)$$

$$\longrightarrow \quad \frac{\mathcal{P}_B}{\mathcal{P}_E} = \left\{ \left(\frac{4k}{3\omega_{\rm eff}+1}\right)\left(\frac{1}{aH} - \frac{1}{a_f H_f}\right) \right\}^2$$

$$\epsilon_{ijl}\partial_j B_l = E_i' + \left(a\sigma + \frac{a'}{a}\right)E_i$$

σ : Conductivity

ϵ_{ijl} : 3-Levi Civita symbol

$$\sigma = \frac{1}{12\pi^3}\frac{|e^3 E|}{H}\exp\left[-\frac{\pi m^2}{|eE|}\right]$$

$\boxed{|\sigma| < H}$ **:Condition that Schwinger backreaction is negligible**

$$\frac{|E|}{H^2} = \frac{12\pi^3}{e^3}\left(\frac{\sigma}{H}\right)\exp\left\{W\left(\frac{e^2}{12\pi^2}\frac{m^2}{H^2}\frac{H}{\sigma}\right)\right\}$$

Lambert W-function

$$We^W = x$$

$$W(0 < x \ll 1) \approx x$$

$$m^2 \ll H^2$$

$$\frac{|E|}{H^2} = \frac{12\pi^3}{e^3}\left(\frac{\sigma}{H}\right) \Rightarrow \frac{|E|}{H^2} < \frac{12\pi^3}{e^3} \approx 12\pi^3$$

$$|\beta_k(\eta_f)| = \left|\frac{2B\epsilon q}{\eta_0^{2q}\left(\frac{2B}{\eta_0^{2q}} - 1\right)}\right|(-k\eta_f)^{-\nu-\frac{1}{2}}, \qquad \nu^2 = \frac{1}{4} - \frac{4B\epsilon q}{\eta_0^{2q}\left(2B/\eta_0^{2q} - 1\right)}$$

$$B = \kappa^{2q}\left\{[6\beta(\beta+1)]^q + [-24(\beta+1)^3]^{q/2}\right\}$$

$$H^2 = H_f^2\left(\frac{a}{a_f}\right)^{-3(1+\omega_{\rm eff})}, \qquad H_f = H_0\exp\left[-\frac{\epsilon N_f}{1+\epsilon}\right]$$

$$\Rightarrow \frac{1}{12\pi^3}\left|\frac{2B\epsilon q}{\eta_0^{2q}\left(\frac{2B}{\eta_0^{2q}} - 1\right)}\right|(-k\eta_f)^{\frac{3}{2}-\nu}\left(\frac{a}{a_f}\right)^{1+3\omega_{\rm eff}} < 1$$

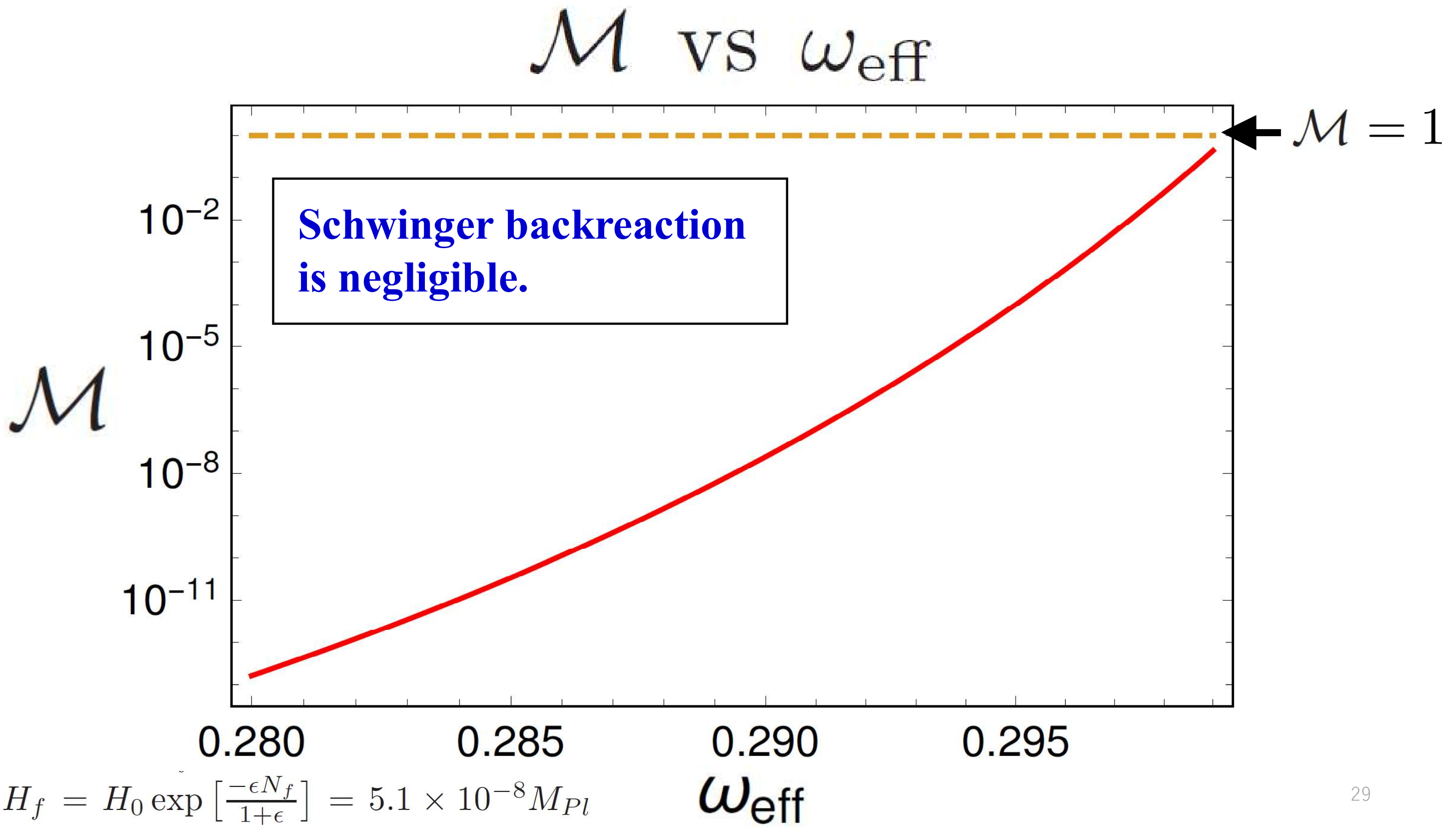
Condition that Schwinger backreaction is negligible

$$\mathcal{M} = \frac{1}{12\pi^3}\left|\frac{2B\epsilon q}{\eta_0^{2q}\left(\frac{2B}{\eta_0^{2q}} - 1\right)}\right|(-k\eta_f)^{\frac{3}{2}-\nu}\left(\frac{a_{\rm re}}{a_f}\right)^{1+3\omega_{\rm eff}}$$

The maximum of the left-hand side of the above relation

$$\longrightarrow \mathcal{M} = \left(\frac{\epsilon q(12^q + 24^{q/2})}{6\pi^3}\right)\left(\frac{H_0}{M_{Pl}}\right)^{2q}\exp\left\{-\left(\frac{3}{2}-\nu\right)N_f + (1+3\omega_{\rm eff})N_{\rm re}\right\}$$

$$-\left(\frac{3}{2}-\nu\right)N_f + (1+3\omega_{\rm eff})N_{\rm re} = -N_f\left[\left(\frac{3}{2}-\nu\right) + \frac{4(1+3\omega_{\rm eff})}{(1-3\omega_{\rm eff})}\right] - \frac{4(1+3\omega_{\rm eff})}{(1-3\omega_{\rm eff})}\left[\frac{1}{4}\ln\left(\frac{45}{\pi^2 g_{re}}\right) + \frac{1}{3}\ln\left(\frac{11 g_{s,re}}{43}\right) + \ln\left(\frac{k}{a_0 T_0}\right) + \ln\left(\frac{(3H_f^2 M_{\rm Pl}^2)^{1/4}}{H_0}\right)\right]$$



$H_f = H_0 \exp\left[\frac{-\epsilon N_f}{1+\epsilon}\right] = 5.1\times10^{-8} M_{Pl}$

V. Conclusions

- **We have investigated the generation of magnetic fields from inflation for the case that the electromagnetic fields couple with the Ricci scalar and the Gauss-Bonnet invariant.**

- **We have analyzed the evolution of the quantum fluctuations of the electromagnetic fields during inflation due to the breaking of the conformal invariance of the electromagnetic fields.**

- **It has been shown that for the case of the reheating phase with a non-zero e-folding number, the present strength of magnetic fields can be consistent with the observations.**

Session D2a 10:45–12:00

[Chair: Shuichiro Yokoyama]

Kanji Nishii

Kobe Univ.

"String Excitation by Initial Singularity of Inflation"

(15 min.)

[JGRG30 (2021) 120912]

String Excitation by Initial Singularity of Inflation

based on JHEP10(2021)025

This work is in collaboration with
Daisuke Yoshida (Kobe U. PD)

Kanji Nishii (Kobe U. M2)

Introduction (1)

・Inflation models are the most vigorously studied model of the early universe.

ex. Starobinsky model, hill-top model, etc...

・A singularity occurs at early stage of inflation due to a component of Ricci tensor diverges.

→Does the singularity due to the divergence of a component of Ricci tensor cause problems?

Introduction (2)

・From what point of view do you find out "whether the initial singularity causes problems"?

・Effects of quantum gravity become important on the early stage of inflation.

→ Supposing string theory to investigate the initial singularity of the inflationary universe.

・The singularity is expected to be resolved by the effects of quantum gravity if it causes a problem.

Short summary

・What is a breakdown of theory due to an initial singularity?

→ Calculating expectation values of mass of a string passing through the initial singularity.

・A divergence of expectation values of mass means a breakdown of theory.

・**Our results : Suggesting a method to distinguish inflation models whether they break down or not due to the initial singularity.**

	Is it OK?
Starobinsky model	×
Cosine type hill-top model	△

Contents

Initial singularity in FLRW space-time

・Considering flat FLRW space-time in light cone coordinates.

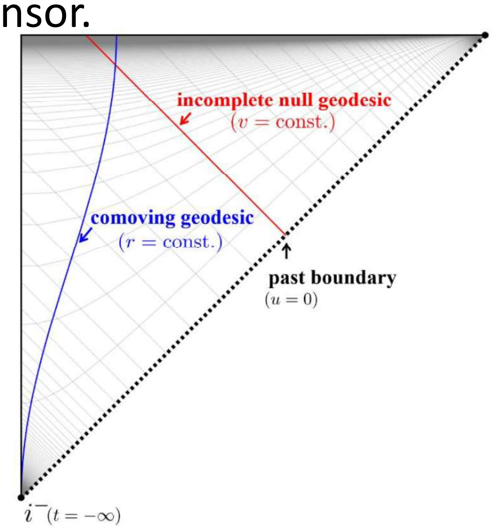
$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -2dudv + a^2(u)dv^2 + a^2(u)(v-\eta(u))^2 d\Omega^2$$

・Along a null geodesic, a singularity cause at the past $u=0$ by divergence of uu component of Ricci tensor.

$$R_{uu} = -2A(u)$$

$$A(u) := \frac{\dot{H}}{a^2}, \quad a(u) \overset{u\to 0}{\sim} 0$$

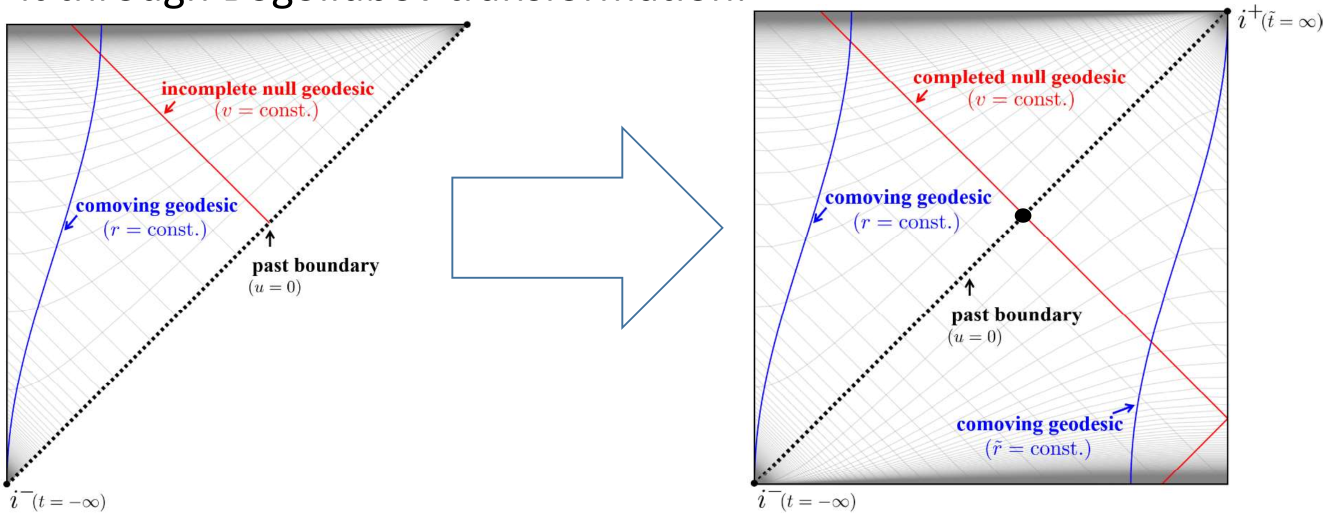
・What are expectation values of mass when strings pass through the singularity?



Contracting universe

・Constructing a contracting universe using a continuity of the metric.

・We can compare the mass before the singularity and after it through Bogoliubov transformation.



String theory

・String theory is strong candidate of quantum gravity.

・Thinking of the inflationary universe as a low-energy effective theory of string theory.

・Excitations of string correspond to expectation values of mass of particle.

→Calculating effects of the initial singularity to excitation of string.

String excitation by singularity (1)

・Considering a quantization of string and EOM in simplified previous FLRW space-time.

・A field $X_n^i(\tau)$ determine the motion of strings follow the following equation.

$$-\frac{d^2 X_n^i}{d\tau^2} + \mathcal{V}(\tau) X_n^i = E_n X_n^i, \quad \mathcal{V}(\tau) := \alpha'^2 p^2 A(\alpha' p \tau), \quad E_n := n^2$$

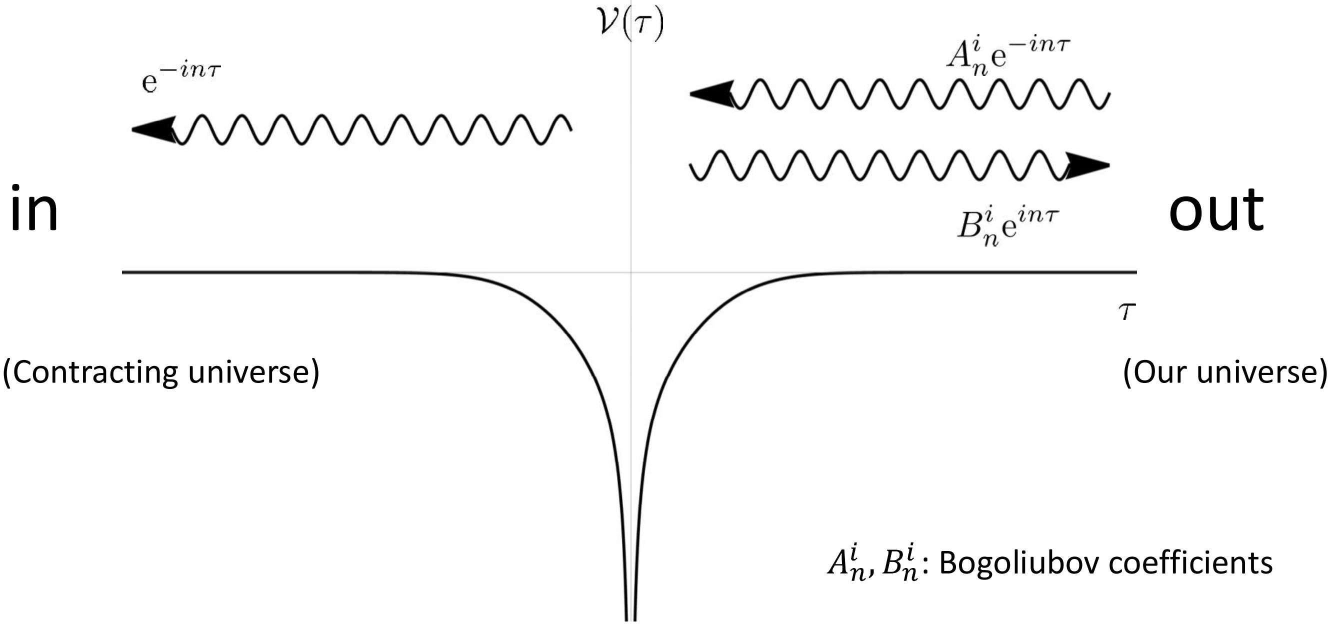
・This is Schrödinger-like equation has a potential determined by $A(u = \alpha' p \tau)$.

※ $R_{uu} = -2A(u)$

$$A(u) := \frac{\dot{H}}{a^2}, \quad a(u) \overset{u \to 0}{\sim} 0$$

String excitation by singularity (2)

・Considering Bogoliubov transformation from in-state to out-state.



A_n^i, B_n^i: Bogoliubov coefficients

String excitation by singularity (3)

・Calculating an expectation value of mass of out-state looked from in-state by using Bogoliubov coefficients.

$$\langle M_{\mathrm{out}}^2 \rangle = \frac{\langle 0_{\mathrm{in}} | \, M_{\mathrm{out}}^2 \, | 0_{\mathrm{in}} \rangle}{\langle 0_{\mathrm{in}} | 0_{\mathrm{in}} \rangle} \sim \frac{2}{\alpha'} \sum_{n=1}^{\infty} \sum_{i=1}^{2} n |B_n^i|^2$$

・Bogoliubov coefficients B_n^i is the following as,

$$B_n^i \simeq \frac{p^2 \alpha'^2}{2in} \int_{-\infty}^{\infty} d\tau \; e^{-2in\tau} A(\alpha' p \tau) e^{-Cu^2}, \quad C : \mathrm{const}$$

so we can calculate the mass when we obtain uu components of Ricci tensor $A(u = \alpha' p \tau)$.

String excitation by singularity (4)

・In general, $A(u)$ takes the following form in inflation models. (β varies from model to model)

$$A(u) = -\frac{\kappa}{|u|^\beta}, \quad \kappa > 0 : \mathrm{const}, \quad 0 < \beta < 2$$

・The expectation value of mass converges for $0 < \beta < 1$ and diverges for $1 \leq \beta < 2$.

$$\langle M_{\mathrm{out}}^2 \rangle = \begin{cases} \text{converge} & 0 < \beta < 1, \\ \text{diverge} & 1 \leq \beta < 2 \end{cases}$$

→We can distinguish models whether break down or not by reading the value of β.

Inflation model and Initial singularity

・Calculating $A(u)$ and reading β for inflation models.

	Is it OK?	Value of β
Starobinsky model	×	$1 \leq \beta < 2$
Cosine type hill-top model	△	$0 < \beta < 1$ or $1 \leq \beta < 2$

・The initial singularity in some inflation models causes the problem.

Preparing for the calculation

・$A(u)$ is calculated by potential $V(\phi)$.

$$V(\phi) = 3M_{pl}^2 \bar{H}^2 \left(1 + \delta V(\phi)\right)$$

$$A(u) = \frac{1}{a^2}\frac{dH}{dt} \simeq -\frac{M_{pl}^2}{2u^2} \times \delta V'(\phi(u))^2$$

・EOMs of inflaton ϕ are

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \qquad H^2 = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

Starobinsky model

・The potential of Starobinsky model is

$$V(\phi) = 3M_{pl}^2 \bar{H}^2 \left(1 - \mathrm{e}^{-\frac{\phi}{\mu}} + \cdots\right), \quad \mu : \mathrm{const}$$

・We can calculate $A(u)$ easily as

$$A(u) \sim -\frac{\kappa}{u^2 (\log u)^2}$$

This corresponds to $1 \leq \beta < 2$, so the mass diverges.

→The initial singularity in Starobinsky model should be removed by effects of quantum gravity.

Cosine type hill-top model

・The potential of Cosine type hill-top (axion) model is

$$V(\phi) = \frac{3M_{pl}^2 \bar{H}^2}{2} \left(1 + \cos\frac{\phi}{f}\right) = 3M_{pl}^2 \bar{H}^2 \left(1 - \frac{1}{2}\frac{M_{pl}^2}{2f^2}\frac{\phi^2}{M_{pl}^2} + \cdots\right)$$

Then, we obtain

$$A(u) = \frac{-\kappa}{u^{2(1-\gamma)}}, \quad \gamma = \frac{-3 + \sqrt{9 + \frac{6M_{pl}^2}{f^2}}}{2}$$

・If we ignore the slow-roll condition,

$$\langle M_{\mathrm{out}}^2 \rangle = \begin{cases} \text{converge} & f < \sqrt{\frac{6}{7}} M_{pl}, \\ \text{diverge} & f \geq \sqrt{\frac{6}{7}} M_{pl} \end{cases}$$

Summery

・The initial singularity causes at early stage of inflation.

→Do inflation models break down due to the singularity as EFT of string theory?

	Is it OK?	Value of β
Starobinsky model	×	$1 \leq \beta < 2$
Cosine type hill-top model	△	$0 < \beta < 1$ or $1 \leq \beta < 2$

・The calculation method can be easily used for other models as well.

・The strength of the initial singularity is important for quantum gravity.

Outlook

・Calculating the time-dependence of excitation without the contracting universe.

・Taking into account a back reaction of the excitation.

Session D2a 10:45–12:00

[Chair: Shuichiro Yokoyama]

Yuichiro Tada

Nagoya University

"Probability density functions of coarse-grained curvature and density perturbations in stochastic inflation"

(15 min.)

[JGRG30 (2021) 120913]

9th December 2021 @ JGRG30

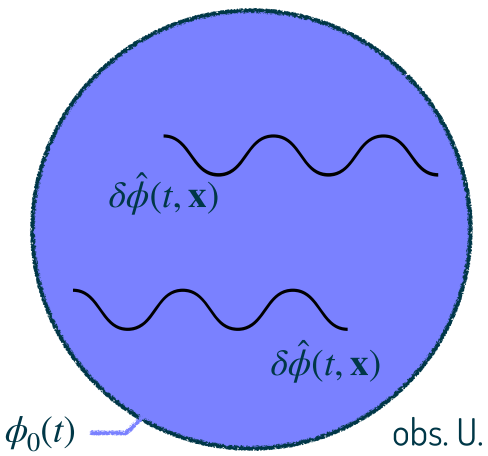
PDFs of coarse-grained ζ & δ in stochastic inflation

Yuichiro TADA Nagoya U.
w/ V. Vennin (APC) 2111.15280

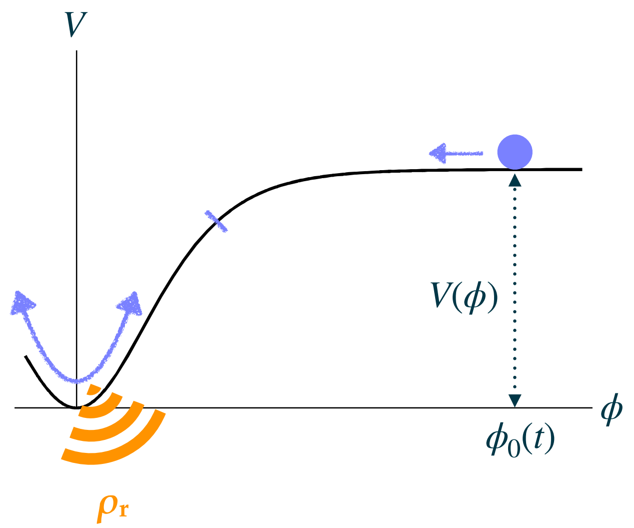
Stochastic Infl.

Starobinsky '86

- Standard QFT



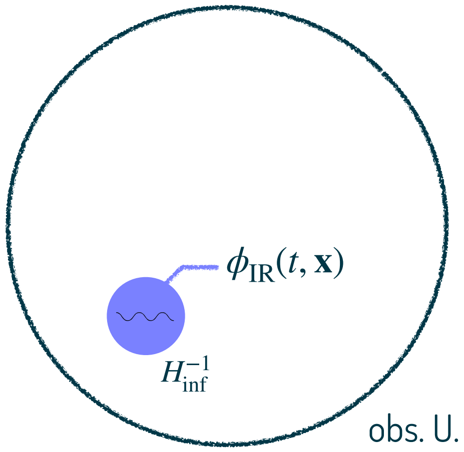
$$\frac{\mathrm{d}\phi_0}{\mathrm{d}N} = -\frac{V'}{3H^2}, \quad (\Box^2 - m^2)\delta\hat{\phi} = 0$$



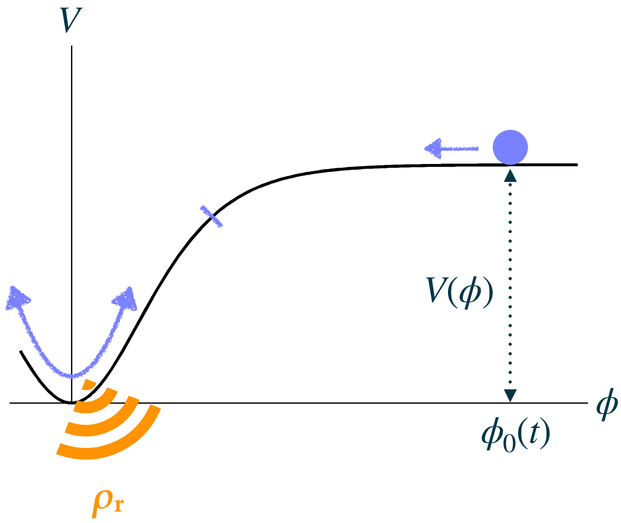
Stochastic Infl.

Starobinsky '86

- Stochastic approach : $\boldsymbol{H}$-EFT



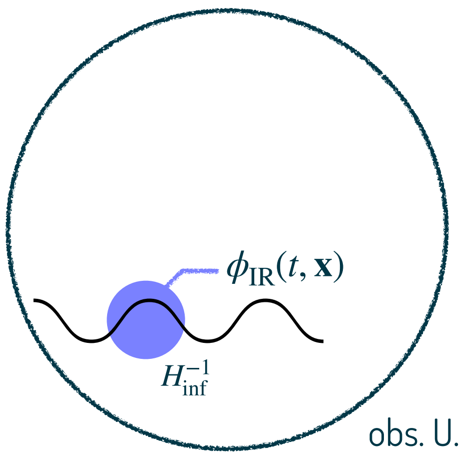
$$\frac{\mathrm{d}\phi_0}{\mathrm{d}N} = -\frac{V'}{3H^2}, \quad (\Box^2 - m^2)\delta\hat{\phi} = 0$$



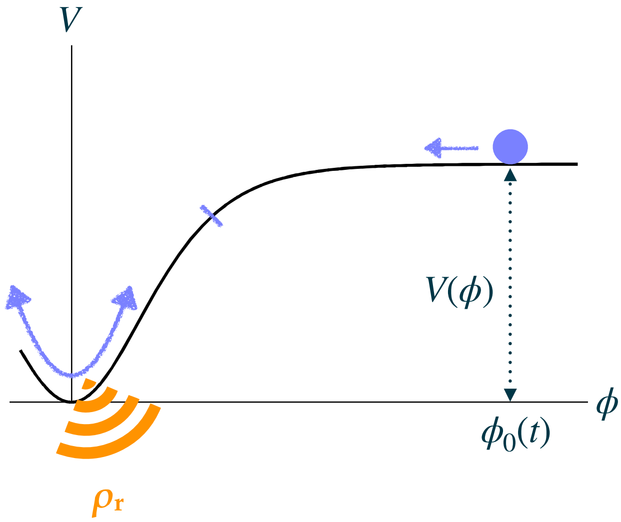
Stochastic Infl.

Starobinsky '86

- Stochastic approach : $\boldsymbol{H}$-EFT



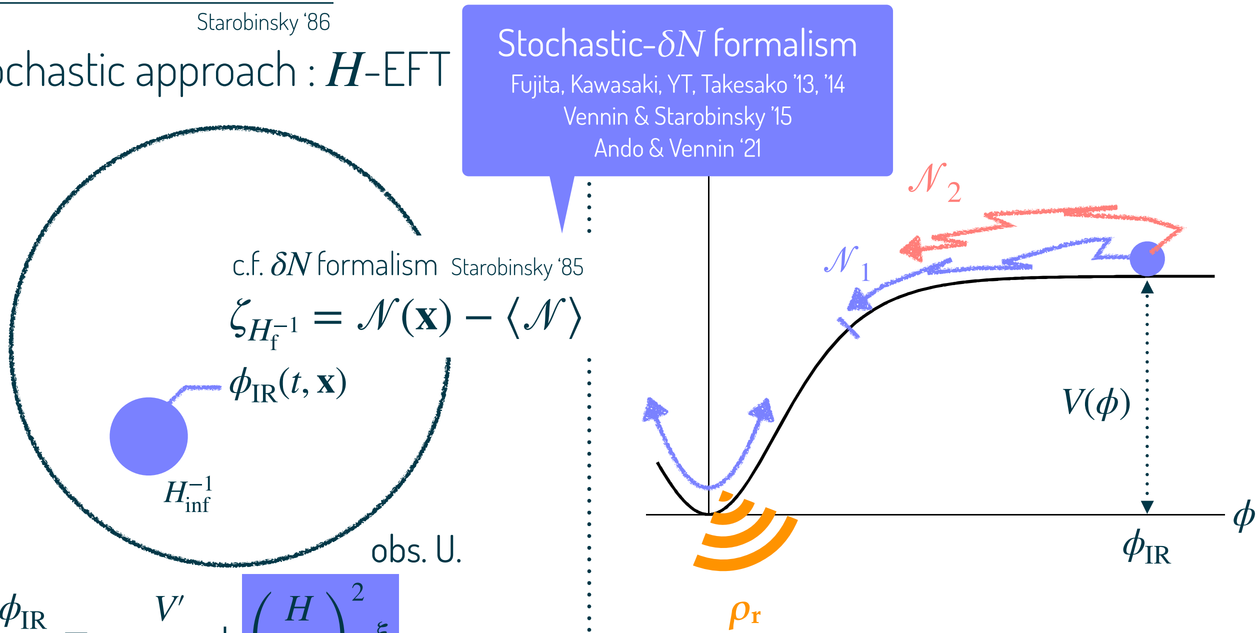
$$\frac{\mathrm{d}\phi_0}{\mathrm{d}N} = -\frac{V'}{3H^2}, \quad (\Box^2 - m^2)\delta\hat{\phi} = 0$$



Stochastic Infl.

Starobinsky '86

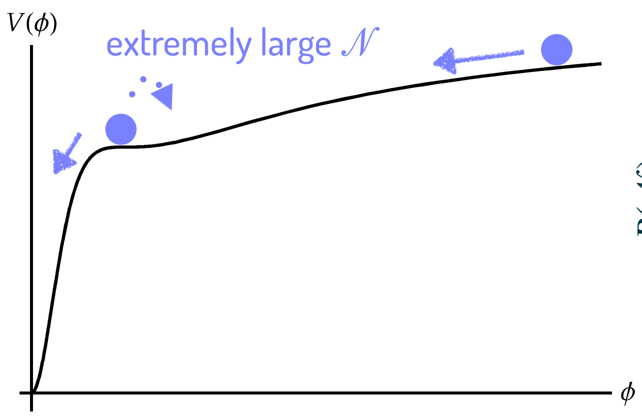
- Stochastic approach : H-EFT

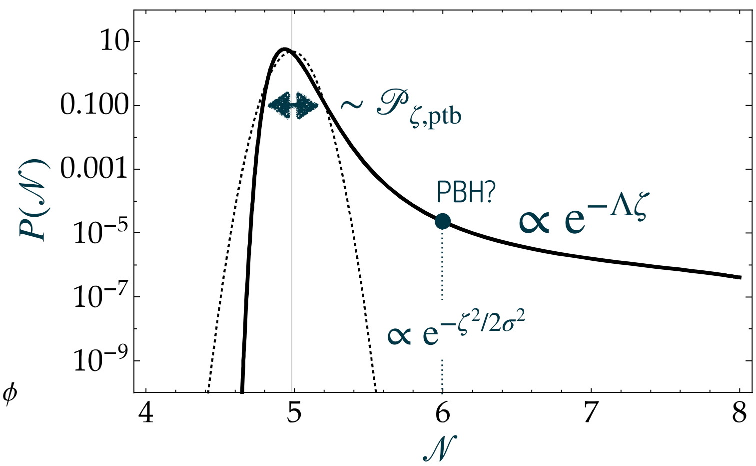


$$\frac{d\phi_{\rm IR}}{dN} = -\frac{V'}{3H^2} + \boxed{\left(\frac{H}{2\pi}\right)^2 \xi}$$

Exp-tail

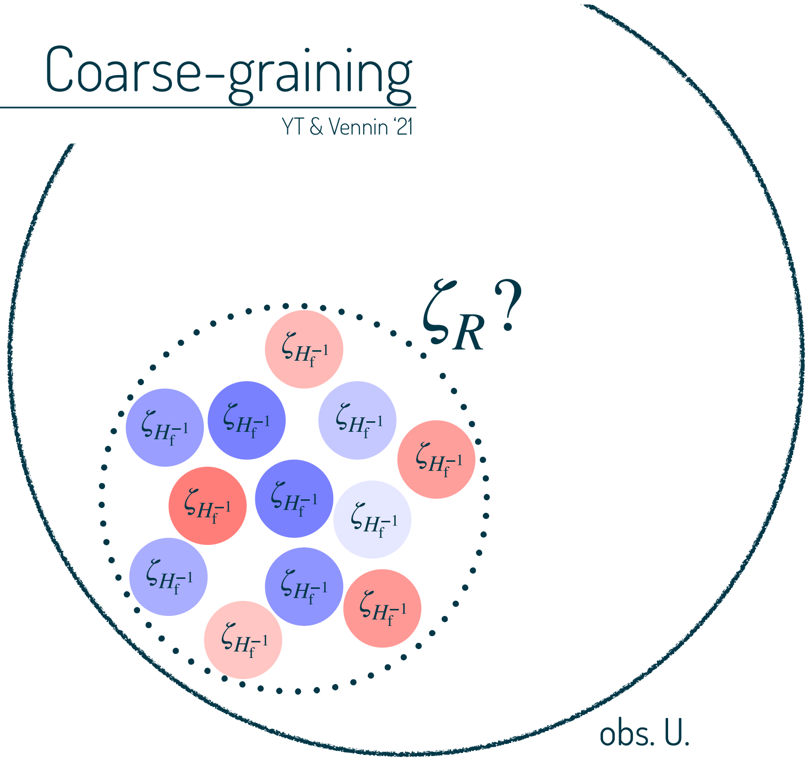
Pattison+ '17, '21, Ezquiaga+ '20, Figueroa+ '20





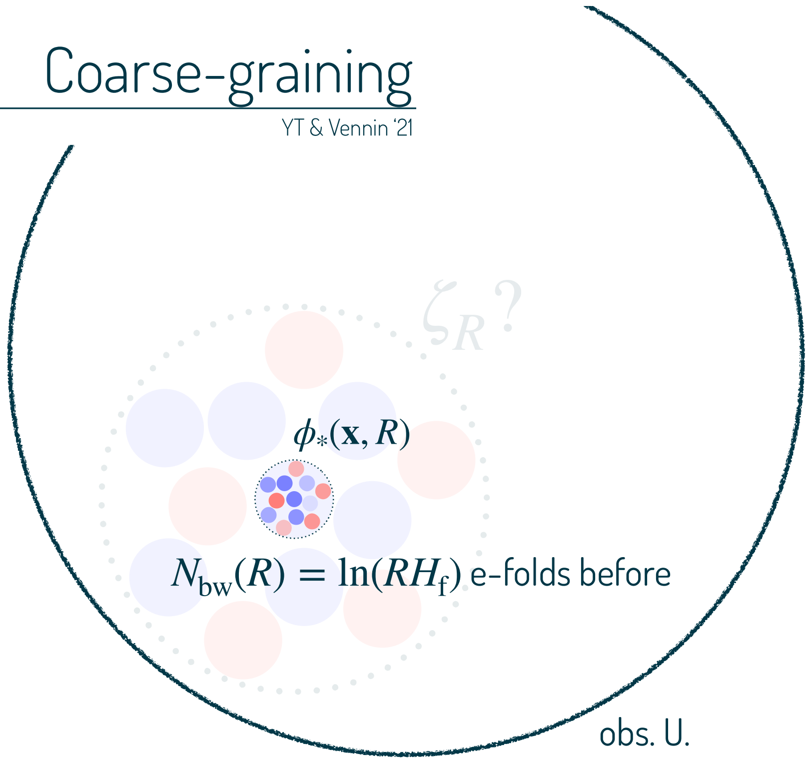
Coarse-graining

YT & Vennin '21



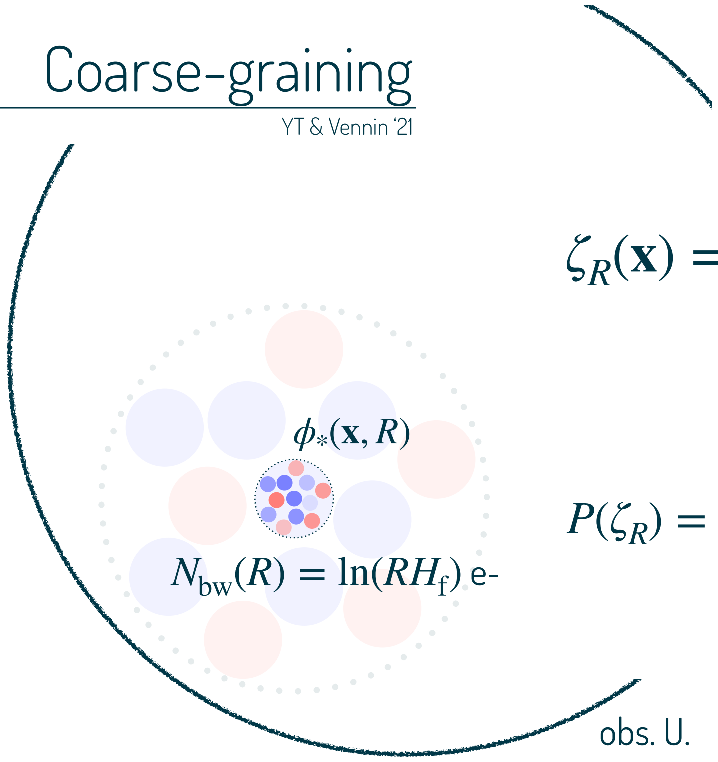
Coarse-graining

YT & Vennin '21



Coarse-graining

YT & Vennin '21



same dyn. until ϕ_*

$$\zeta_R(\mathbf{x}) = \mathcal{N}(\phi_{\rm i} \to \phi_*(\mathbf{x}, R)) + \langle \mathcal{N}(\phi_*(\mathbf{x}, R))\rangle - \langle \mathcal{N}(\phi_{\rm i})\rangle$$

independent after ϕ_*

$$P(\zeta_R) = \int \mathrm{d}\phi_* \, \mathbb{P}\left(\mathcal{N}_{\rm i\to *} = \langle \mathcal{N}_{\rm i}\rangle - \langle \mathcal{N}_*\rangle + \zeta_R\right) \times \mathbb{P}\left(\phi = \phi_* @ - N_{\rm bw}(R)\right)$$

δ as coarse-shelled ζ

- (linear) density contrast

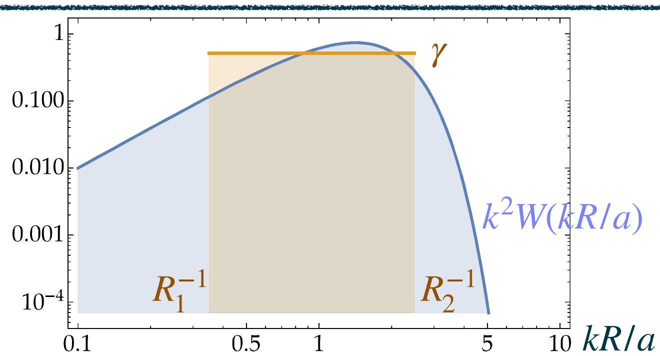
$$\delta = -\frac{9}{4}\frac{1}{a^2H^2}\nabla^2\zeta$$

- (nonlinear) compaction func.

$$\mathscr{C}(r) = \frac{2}{3}\left[1 - \left(1 + r\zeta'(r)\right)^2\right]$$

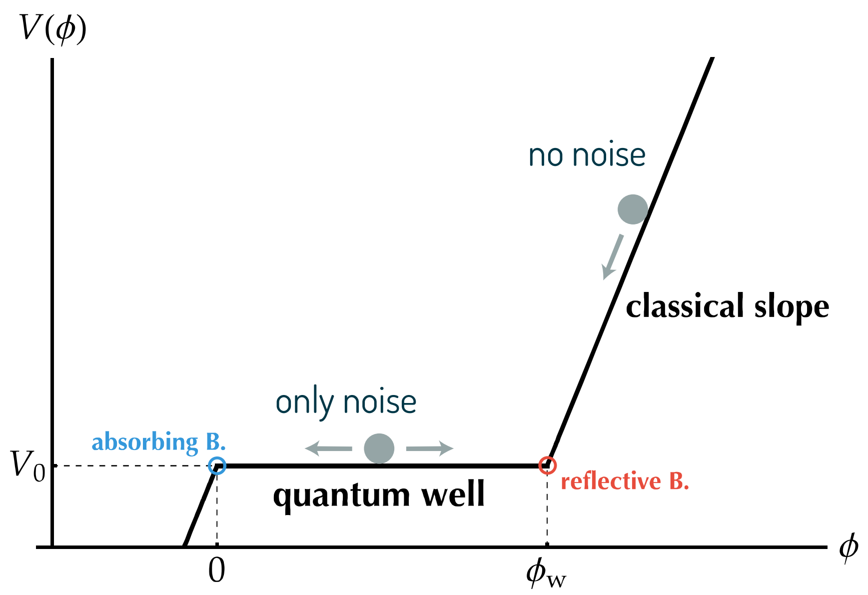
$$\boxed{(-\nabla^2\zeta_R)(\mathbf{k}) = k^2 W\left(\frac{kR}{a}\right)\zeta(\mathbf{k}) \approx \gamma\left(\zeta_{R_2} - \zeta_{R_1}\right) =: \Delta\zeta}$$

w/ appropriate R_1, R_2, γ



Quantum Well

Pattison+ '17 ...

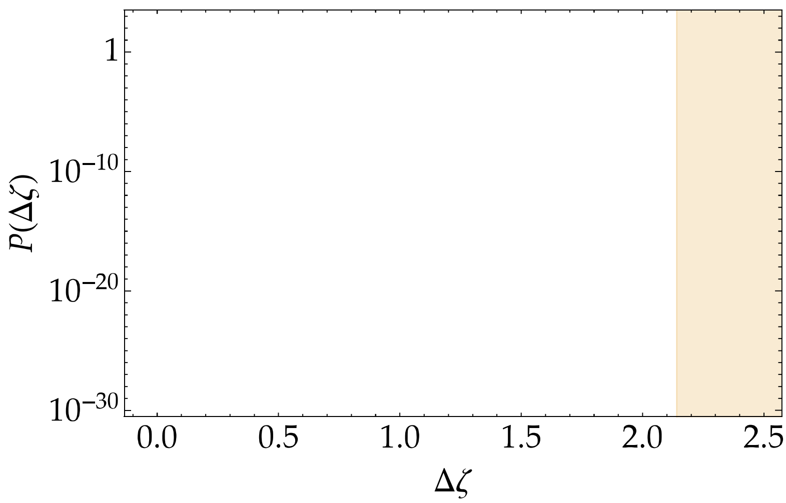


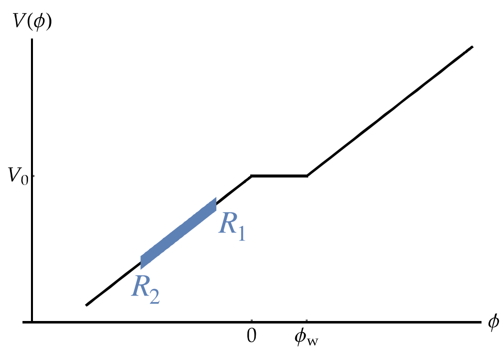
$$\mu = \sqrt{2}\frac{\phi_w}{H/2\pi}$$

Quantum Well

YT & Vennin '21

$$\mu = \frac{1}{\sqrt{6}}$$



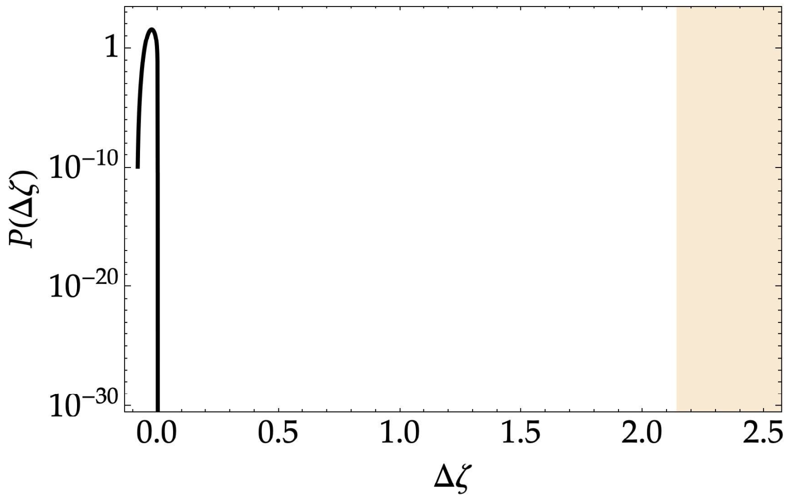


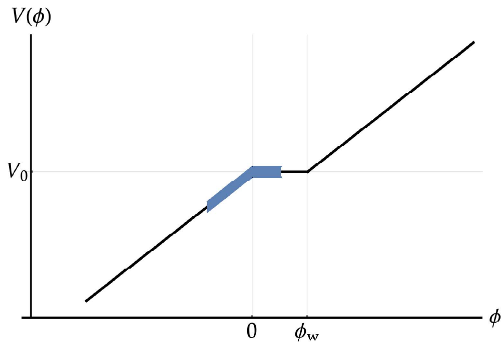
$$\mu = \sqrt{2}\frac{\phi_w}{H/2\pi}$$

Quantum Well

YT & Vennin '21

$$\mu = \frac{1}{\sqrt{6}}$$



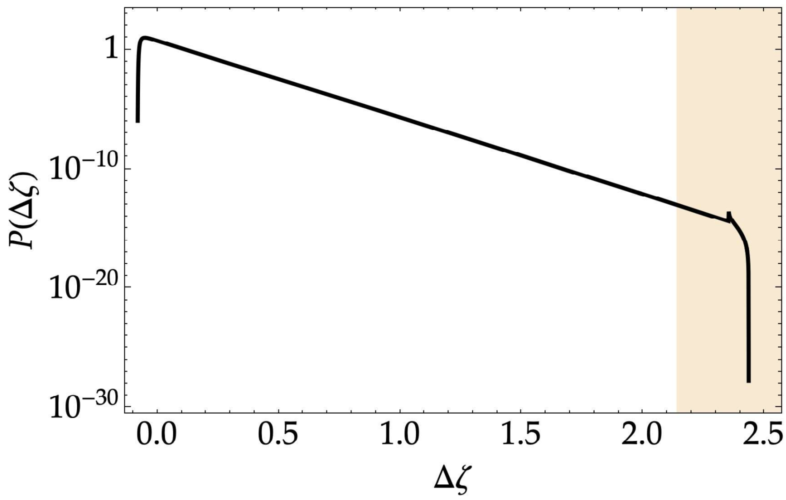


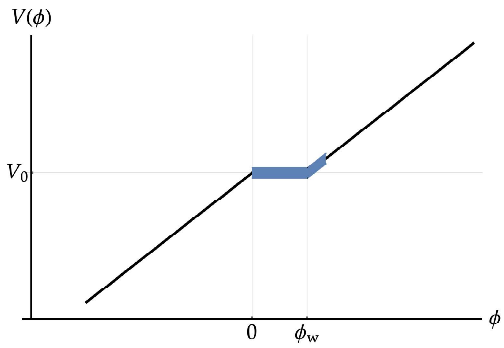
$$\mu = \sqrt{2}\frac{\phi_w}{H/2\pi}$$

Quantum Well

YT & Vennin '21

$$\mu = \frac{1}{\sqrt{6}}$$



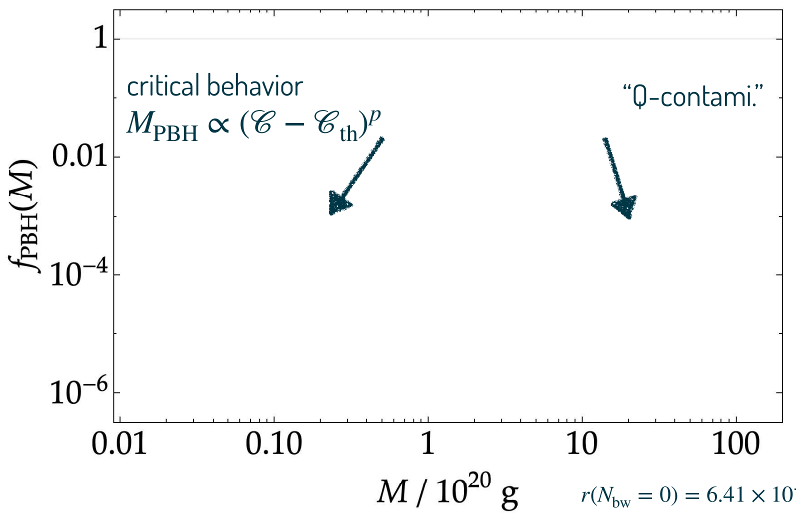


$$\mu = \sqrt{2}\frac{\phi_w}{H/2\pi}$$

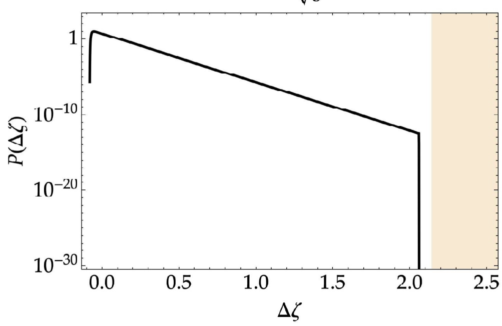
Quantum Well

YT & Vennin '21

$\mu = \frac{1}{\sqrt{6}}$



$r(N_{\mathrm{bw}} = 0) = 6.41 \times 10^{-14}\,\mathrm{Mpc}$



$$\mu = \sqrt{2}\frac{\phi_{\mathrm{w}}}{H/2\pi}$$

Summary

- Coarse-graining in stochastic infl. as partial average
- δ (or $\mathscr{C}$) as a coarse-shelled ζ
- High-mass tail in f_{PBH} due to "Q-contami."

Appendices

- $P(\phi \mid N, \phi_\mathrm{i})$: inflaton is ϕ @ the time N e-folds after ϕ_i

  Fokker–Planck eq. : $\partial_N P = \partial_\phi \left(\frac{V'}{3H^2}P\right) + \frac{1}{2}\partial_\phi^2\left(\left(\frac{H}{2\pi}\right)^2 P\right)$

- $P_\mathrm{FPT}(\mathcal{N} \mid \phi \to \phi_*)$: 1st passage time is $\mathcal{N}$ from ϕ to ϕ_*

  adjoint FP eq. : $\partial_N P_\mathrm{FPT} = -\frac{V'}{3H^2}\partial_\phi P_\mathrm{FPT} + \frac{1}{2}\left(\frac{H}{2\pi}\right)^2 \partial_\phi^2 P_\mathrm{FPT}$

- $P_\mathrm{bw}(\phi \mid N_\mathrm{bw})$: inflaton is ϕ @ the time N_bw e-folds before the end of infl.

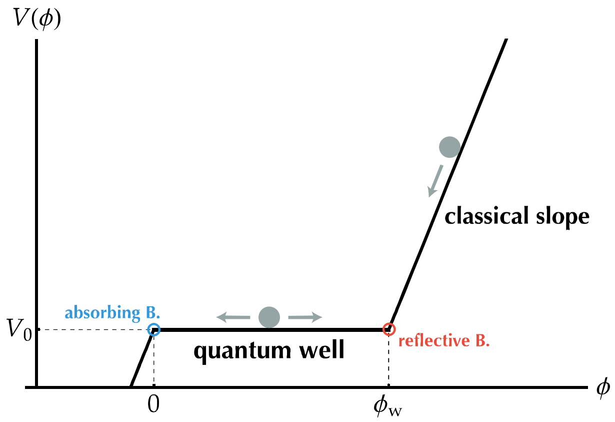
  Ando–Vennin eq. : $P_\mathrm{bw}(\phi \mid N_\mathrm{bw}) = \frac{P_\mathrm{FPT}(N_\mathrm{bw} \mid \phi)\int_0 \mathrm{d}N\, P(\phi \mid N)}{\int_{N_\mathrm{bw}} \mathrm{d}\mathcal{N}_\mathrm{tot} P_\mathrm{FPT}(\mathcal{N}_\mathrm{tot} \mid \phi_\mathrm{i})}$

Appendices

- $P(\phi \mid N, \phi_\mathrm{i})$: inflaton is ϕ @ the time N e-folds after ϕ_i
- $P_\mathrm{FPT}(\mathcal{N} \mid \phi \to \phi_*)$: 1st passage time is $\mathcal{N}$ from ϕ to ϕ_*
- $P_\mathrm{bw}(\phi \mid N_\mathrm{bw})$: inflaton is ϕ @ the time N_bw e-folds before the end of infl.

$$\boxed{\begin{aligned} P(\zeta_R) = \int \mathrm{d}\phi_*\, P_\mathrm{FPT}\left(\langle \mathcal{N}_\mathrm{i}\rangle - \langle \mathcal{N}_*\rangle + \zeta_R \mid \phi_\mathrm{i} \to \phi_*\right) \\ \times P_\mathrm{bw}(\phi_* \mid N_\mathrm{bw}(R)) \end{aligned}}$$

Appendices



$$x = \frac{\phi}{\phi_\mathrm{w}}, \quad \mu = \sqrt{2}\frac{\phi_\mathrm{w}}{H/2\pi}$$

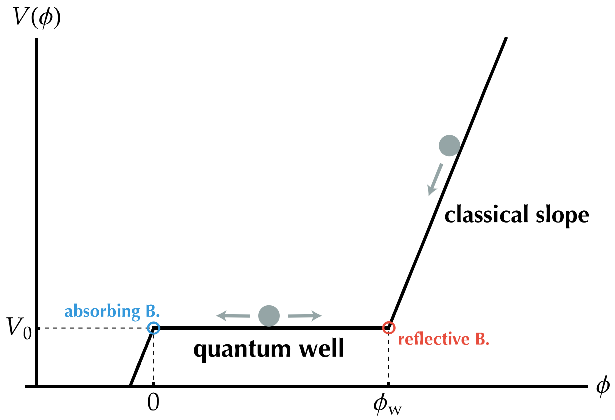
$$P(x \mid N) = \frac{1}{2}\vartheta_2\left[-\frac{\pi}{2}(x-1), \mathrm{e}^{-\pi^2 N/\mu^2}\right] - \frac{1}{2}\vartheta_2\left[-\frac{\pi}{2}(x+1), \mathrm{e}^{-\pi^2 N/\mu^2}\right]$$

$$P_\mathrm{FPT}(\mathcal{N} \mid x) = -\frac{\pi}{2\mu^2}\vartheta_2'\left(\frac{\pi}{2}x, \mathrm{e}^{-\pi^2\mathcal{N}/\mu^2}\right)$$

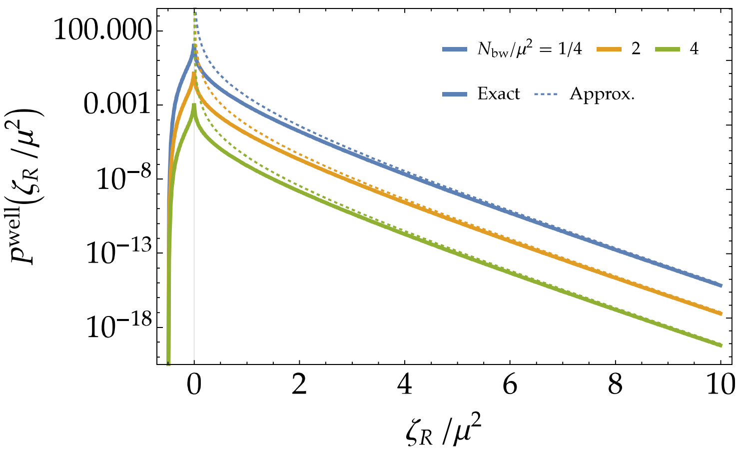
$$P_\mathrm{bw}(x \mid N_\mathrm{bw}) = -\frac{\pi x}{2}\vartheta_2'\left(\frac{\pi}{2}x, \mathrm{e}^{-\pi^2 N_\mathrm{bw}/\mu^2}\right)$$

$$P(\zeta_R) \underset{\zeta_R \gg 1}{\propto} \frac{1}{(\zeta_R/\mu^2)^3}\mathrm{e}^{-\frac{\pi^2}{4\mu^2}\zeta_R}$$

Appendices



$$x = \frac{\phi}{\phi_\mathrm{w}}, \quad \mu = \sqrt{2}\frac{\phi_\mathrm{w}}{H/2\pi}$$



Appendices

$$\Delta\zeta = \ln\left(\frac{R_1}{R_2}\right) + \langle \mathcal{N}(\phi_*(R_2))\rangle - \langle \mathcal{N}(\phi_*(R_1))\rangle$$

$$P(\Delta\zeta) = \int \mathrm{d}\phi_*^{(1)}\mathrm{d}\phi_*^{(2)}\, P_{\mathrm{bw}}\left(\phi_*^{(1)}, \phi_*^{(2)} \mid N_{\mathrm{bw}}^{(1)}(R_1), N_{\mathrm{bw}}^{(2)}(R_2)\right)$$
$$\times\, \delta\left(\Delta\zeta + \ln\left(\frac{R_1}{R_2}\right) + \langle \mathcal{N}(\phi_*^{(2)})\rangle - \langle \mathcal{N}(\phi_*^{(1)})\rangle\right)$$

Session D2a 10:45–12:00

[Chair: Shuichiro Yokoyama]

Jason Kristiano

RESCEU, The University of Tokyo

"Theoretical bound of primordial non-Gaussianity in single field inflation"

(15 min.)

[JGRG30 (2021) 120914]

Theoretical bound of primordial non-Gaussianity in single field inflation

Jason Kristiano (RESCEU, The University of Tokyo)
Based on arXiv:2104.01953 with Jun'ichi Yokoyama

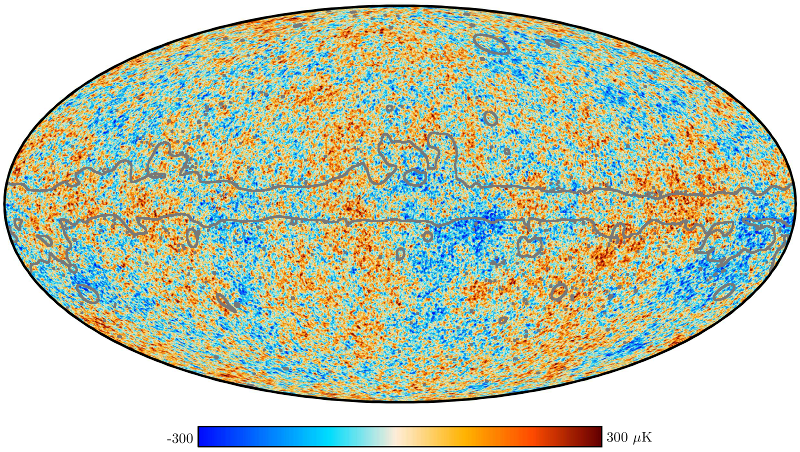
JGRG 2021

Introduction

Signature of inflation: small quantum fluctuations as the origin of CMB anisotropy and large-scale structures.

Observed by Planck 2018:

- Almost Gaussian.
- Almost scale-invariant.



Planck 2018

Introduction

An inflation model predicts:

- Deviation from scale-invariant (spectral tilt).
- Deviation from Gaussian distribution (non-Gaussianity).

Observation (Planck 2018) still allows quite large (equilateral) non-Gaussianity.

How large non-Gaussianity that still can be explained by cosmological perturbation theory?

k-inflation

Consider k-inflation, the simplest model which can generate large spectrum of non-Gaussianity, with action

$$\boxed{S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_{\mathrm{pl}}^2 R + 2P(X,\phi)\right],}$$

where $g = \det\, g_{\mu\nu}$, $g_{\mu\nu}$ is spacetime metric, R is Ricci scalar, ϕ is inflaton, and $X = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$.

Armendariz-Picon et. al., PLB 1998

Cosmological perturbations

Small perturbations:

$$\boxed{\begin{gathered}\phi(\mathbf{x},t) = \bar{\phi}(t) + \delta\phi(\mathbf{x},t),\\ \mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t).\end{gathered}}$$

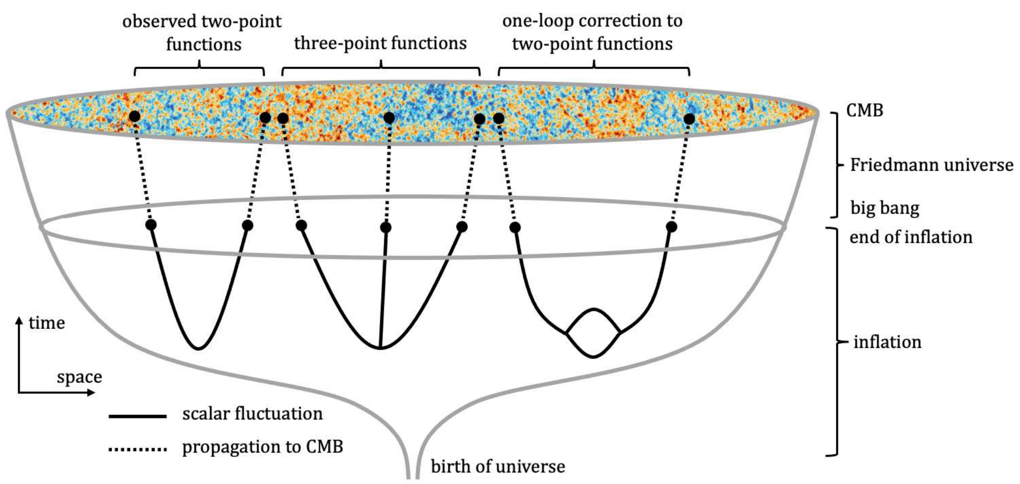
Gauge fixing condition (comoving gauge):

$$\boxed{\delta\phi = 0 \text{ and } \gamma_{ij} = a^2(1+2\zeta)\delta_{ij}.}$$

Some parameters ($\epsilon, \eta, s \ll 1$):

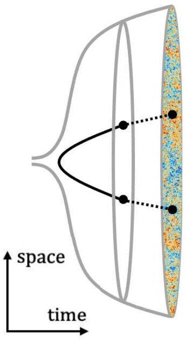
$$\boxed{\epsilon = -\frac{\dot{H}}{H^2}, \eta = \frac{\dot{\epsilon}}{\epsilon H}, s = \frac{\dot{c}_s}{c_s H}, \text{ and } c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}.}$$

Feynman-Witten diagram



Cosmological correlators

Two-point functions



Second-order action of ζ:

$$\boxed{S^{(2)} = M_{\rm pl}^2 \int \mathrm{d}t\ \mathrm{d}^3x \frac{\epsilon}{c_s^2} a^3 \left[\dot{\zeta}^2 - \frac{c_s^2}{a^2} (\partial_i \zeta)^2 \right]}$$

Inflationary power spectrum:

$$\boxed{\Delta_s^2(k) = \frac{k^3}{2\pi^2} \langle \zeta\zeta \rangle = \left(\frac{H^2}{8\pi^2 M_{\rm pl}^2 c_s \epsilon} \right)_H = \Delta_s^2(k_*) \left(\frac{k}{k_*} \right)^{n_s - 1},}$$

where subscript H denotes horizon crossing $c_s k = aH$ and $n_s - 1 = -2\epsilon - \eta - s$.

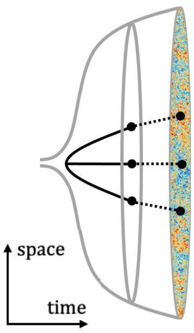
Cosmological correlators

Three-point functions

Cubic-order action of ζ (for $c_s \ll 1$ or $1 - c_s^2 \approx 1$):

$$\boxed{S_{\rm int} = \int \mathrm{d}t\ \mathrm{d}^3x \left[-\frac{2\lambda}{H^3} a^3 \dot{\zeta}^3 + \boxed{\frac{\epsilon}{H c_s^2} a \dot{\zeta} (\partial_i \zeta)^2} \right],}$$

where $\lambda = X^2 P_{,XX} + (2/3) X^3 P_{,XXX}$.



Primordial non-Gaussianity:

$$\boxed{f_{NL} = \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^2} \propto \frac{1}{c_s^2}}$$

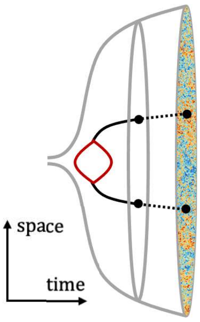
Chen et. al., JCAP 2009

Cheung et. al., JHEP 2008

Cosmological correlators

One-loop correction

One-loop correction generated by cubic-order action is computed using in-in perturbation theory:



$$\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle_{(1)} = \frac{1}{c_s^4}\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle_{(0)}\int_p^\infty \frac{\mathrm{d}^3k}{(2\pi)^3}\langle\zeta(\mathbf{k})\zeta(-\mathbf{k})\rangle_{(0)}$$

$$\Delta^2_{s(1)}(p) = \frac{1}{c_s^4}\Delta^2_{s(0)}(p)\int_p^\infty \frac{\mathrm{d}k}{k}\Delta^2_{s(0)}(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}$$

$$\Delta^2_{s(1)}(p) = \frac{1}{c_s^4}\Delta^2_{s(0)}(p)\frac{\Delta^2_{s(0)}(p)}{n_s-1}$$

converge for $n_s - 1 < 0$, based on observation $n_s = 0.97$

Renormalization?

Previous research did not consider spectral tilt to calculate the loop correction, so they found

$$\int_p^\infty \frac{\mathrm{d}^3k}{(2\pi)^3k^3} \to \int_p^\infty \frac{\mathrm{d}^{3+\delta}k}{(2\pi)^3k^3},$$

and introducing dimensional regularization to make the integral converge.

As a consequence, pole $1/\delta$ appears and it must be renormalized.

Our study shows that considering spectral tilt $n_s - 1$ naturally makes the integral converge and we do not need renormalization.

Senatore and Zaldarriaga, JHEP 2010

Bartolo et. al., JCAP 2010

Conclusion

- Requiring perturbativity of one-loop correction compared to the tree-level power spectrum yields

$$\boxed{\frac{\Delta^2_{s(0)}}{c_s^4(1-n_s)} \ll 1.}$$

- Substituting observation results $\Delta^2_{s(0)} = 2.1 \times 10^{-9}$ and $1 - n_s = 0.03$, theoretical bound of sound speed is $c_s \gg 0.02$.
- Such bound overlaps with observational constraint $c_s > 0.021$.
- Future observation will reveal the validity of cosmological perturbation theory.

Session D2a 10:45–12:00

[Chair: Shuichiro Yokoyama]

Taiga Hasegawa

Yamaguchi University

"Eisenhart-Duval lift for minisuperspace quantum cosmology"

(15 min.)

[JGRG30 (2021) 120915]

Eisenhart-Duval lift for minisuperspace quantum cosmology

Phys. Rev. **D104** (2021) 086001

Taiga Hasegawa

(Yamaguchi University)

Collaborators:

Nahomi Kan (National Institute of Technology, Gifu College)

Takuma Aoyama , Kiyoshi Shiraishi (Yamaguchi University)

1. Introduction

Wheeler-DeWitt (WDW) equation

$$\left[\underline{\frac{1}{a^{s+1}}\frac{\partial}{\partial a}a^s\frac{\partial}{\partial a}} - \frac{1}{a^3}\frac{\partial^2}{\partial\phi^2} + 2U(a,\phi)\right]\Psi(a,\phi) = 0$$

- There is a problem of factor ordering.
- It is difficult to define a positive-definite probability density.

➡ Dirac-square-root formulation

P. D. D'Eath, S. W. Hawking and O. Obregon (1993)

- probability density $||\Psi||^2$

Arbitrariness arises due to the presence of $U(a,\phi)$ in addition to s when replacing it with the Dirac equation.

Our purpose

- We apply the method in Eisenhart-Duval lift to the simple models and extend the minisuperspace .

➡ It is possible to describe a system by geometric treatment even in the presence of the potential term.

➡ There is no arbitrariness comes from s .

- We introduce Dirac type WDW equation in term of the covariance of the extended minisuperspace.

➡ There is no arbitrariness comes from $U(a, \phi)$.

- We derive a positive-definite probability density.

We obtain the fundamental solution to the Dirac type WDW equation in the extended minisuperspace of specific models.

2. WDW equation

Einstein-Hilbert action + scalar field action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

FLRW metric

$$ds^2 = -N^2dt^2 + a^2\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]$$

A scalar field depend only time : $\phi = \phi(t)$

$$\boxed{L = -\frac{1}{2N}a\dot{a}^2 + \frac{1}{2N}a^3\dot{\phi}^2 - NU(a,\phi)} \quad (\kappa^2 = 6)$$

$$U(a,\phi) = a^3V(\phi) - \frac{1}{2}Ka$$

2. WDW equation

Quantize a and ϕ using the canonical quantization.

Lagrangian

$$L = -\frac{1}{2N} a\dot{a}^2 + \frac{1}{2N} a^3 \dot{\phi}^2 - NU(a,\phi)$$

Canonical conjugate momenta

$$\Pi_a = \frac{\partial L}{\partial \dot{a}} = -\frac{a\dot{a}}{N} \qquad \Pi_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{a^3\dot{\phi}}{N}$$

Hamiltonian

$$H = -\frac{1}{2}\frac{\Pi_a^2}{a} + \frac{1}{2}\frac{\Pi_\phi^2}{a^3} + U(a,\phi)$$

2. WDW equation

Replace the momenta with differential operators

$$\Pi_a \to -i\frac{\partial}{\partial a} \qquad \Pi_\phi \to -i\frac{\partial}{\partial \phi}$$

Hamiltonian constraint condition : $H\Psi = 0$

WDW equation

$$\left[\frac{1}{a^{s+1}}\frac{\partial}{\partial a} a^s \frac{\partial}{\partial a} - \frac{1}{a^3}\frac{\partial^2}{\partial \phi^2} + 2U(a,\phi)\right]\Psi(a,\phi) = 0$$

S indicates the arbitrariness in factor ordering of $\frac{\Pi_a^2}{a}$.

Arbitrariness arises due to the presence of $U(a,\phi)$

in addition to S when replacing it with the Dirac equation.

3. Eisenhart-Duval lift for minisuperspace

L. P. Eisenhart (1928)
C. Duval et al. (1985)

Extension with a new degree of freedom χ .

$$L = -\frac{1}{2N}a\dot{a}^2 + \frac{1}{2N}a^3\dot{\phi}^2 - NU(a,\phi)$$

$$\rightarrow \tilde{L} = -\frac{1}{2}a\dot{a}^2 + \frac{1}{2}a^3\dot{\phi}^2 + \frac{1}{2}\frac{\dot{\chi}^2}{2U(a,\phi)} \quad (N=1)$$

$$= \frac{1}{2}\tilde{G}_{MN}\dot{X}^M\dot{X}^M$$

The introduction of χ allows us to treat it as a free system in a “ curved space”.

$X^M = (a,\phi,\chi)$: The component of the extended minisuperspace

$\tilde{G}_{MN} = \mathrm{diag.}(-a, a^3, [2U(a,\phi)]^{-1})$ The metric of the extended minisuperspace

3. Eisenhart-Duval lift for minisuperspace

Hamiltonian in the extended minisuperspace

$$\frac{1}{2}\tilde{G}^{MN}P_M P_N = 0 \quad \leftarrow \quad \tilde{L} = \frac{1}{2}\tilde{G}_{MN}\dot{X}^M\dot{X}^N$$

This Hamiltonian has conformal covariance.

$$G_{MN} = 2U(a,\phi)\tilde{G}_{MN} \quad \leftarrow \text{Conformal transformation}$$

$$= \mathrm{diag.}(-2U(a,\phi)a, 2U(a,\phi)a^3, 1)$$

Replace the Hamiltonian with differential operators in term of covariance

$$G^{MN}P_M P_N \rightarrow \frac{1}{\sqrt{-G}}\partial_M\sqrt{-G}G^{MN}\partial_N - \xi\mathcal{R}$$

3. Eisenhart-Duval lift for minisuperspace

WDW equation in the extended minisuperspace

$$\left[\frac{1}{\sqrt{-G}}\partial_M\sqrt{-G}G^{MN}\partial_N - \xi\mathcal{R}\right]\Psi = 0$$

Arbitrariness $\mathcal{S}$ does not exit.

$$\left[\frac{1}{a^2}\frac{\partial}{\partial a}a\frac{\partial}{\partial a} - \frac{1}{a^3}\frac{\partial^2}{\partial\phi^2} - (2a^3V - Ka)\frac{\partial^2}{\partial\chi^2} + 2\xi\mathcal{R}\right]\Psi(a,\phi,\chi) = 0$$

$$\mathcal{R} = \frac{2a^3[(V')^2 - V''V] + Ka[V'' - 4V]}{(2a^3V - Ka)^2}$$

: the scalar curvature of the extended minisuperspace

➡ We consider two models with $\mathcal{R} = 0$

Model 1 : $V(\phi) = 0$ C. Kiefer (1988)

Model 2 : $K = 0$ $V(\phi) = V_0 \exp\lambda\phi$ A. A. Andrianov et al.(2018)

3. Eisenhart-Duval lift for minisuperspace

Model 1 : $V(\phi) = 0$ C. Kiefer (1988)

$$\left[a\frac{\partial}{\partial a}a\frac{\partial}{\partial a} - \frac{\partial^2}{\partial\phi^2} + Ka^4\frac{\partial}{\partial\chi^2}\right]\Psi(a,\phi,\chi) = 0$$

Constraint condition : $-\frac{\partial^2}{\partial\chi^2}\Psi = p^2\Psi$

General solution : $\Psi = \int d\nu\mathcal{A}(\nu)\psi_{\nu p}e^{ip\chi}$

The fundamental solution :

$$\psi_{\nu p} = K_{i\nu/2}(\sqrt{K}|p|a^2/2)e^{i\nu(\phi-\phi_0)} \qquad (K>0)$$

$$\psi_{\nu p} = J_{\pm i\nu/2}(\sqrt{|K|}|p|a^2/2)e^{i\nu(\phi-\phi_0)} \qquad (K<0)$$

3. Eisenhart-Duval lift for minisuperspace

Model 2 : $K = 0$, $V(\phi) = V_0 \exp \lambda\phi$ A. A. Andrianov et al.(2018)

$$\left[a\frac{\partial}{\partial a}a\frac{\partial}{\partial a} - \frac{\partial^2}{\partial\phi^2} - 2a^6 V_0 \exp\lambda\phi \frac{\partial^2}{\partial\chi^2} \right] \Psi(a, \phi, \chi) = 0$$

Constraint condition : $-\dfrac{\partial^2}{\partial\chi^2}\Psi = p^2\Psi$

General solution : $\Psi = \int d\nu \mathcal{A}(\nu)\psi_{\nu p} e^{ip\chi}$

$$C = 2V_0 \left(1 - \frac{\lambda^2}{36}\right)^{-1}$$

The fundamental solution :

$$\psi_{\nu p} = J_{\pm i\nu/3}(i\sqrt{C}|p|e^{3x}/3)e^{i\nu(y-y_0)} \qquad (C > 0)$$

$$\psi_{\nu p} = K_{i\nu/3}(i\sqrt{|C|}|p|e^{3x}/3)e^{i\nu(y-y_0)} \qquad (C < 0)$$

4. Dirac type WDW equation

Dirac type equation in the extended minisuperspace

$$\gamma^A e_A^M D_M \Psi = 0$$

Since this equation is defined in term of covariance in extended minisuperspace, there is no arbitrariness comes from $U(a, \phi)$.

dreibein $e_A^M = \mathrm{diag.}((2U)^{-1/2}a^{-1/2}, (2U)^{-1/2}a^{-3/2}, 1)$

gamma matrices $\{\gamma^A, \gamma^B\} = -2\eta^{AB}$

covariant derivative $D_M \equiv \partial_M + \frac{1}{4}\omega_{MAB}\Sigma^{AB}$ $\Sigma^{AB} \equiv -\frac{1}{2}[\gamma^A, \gamma^B]$

spin connection $\omega_{MAB} = \frac{1}{2}e_A^N(\partial_M e_{NB} - \Gamma^L_{MN}e_{LB}) - (A \leftrightarrow B)$

4. Dirac type WDW equation   Model 1 : $V(\phi)=0$

$$\gamma^A e_A^M D_M \Psi = 0$$

$$\left[\sigma^1\left(a\frac{\partial}{\partial a}+1\right)+i\sigma^2\frac{\partial}{\partial\phi}+i\sigma^3\sqrt{-K}a^2\frac{\partial}{\partial\chi}\right]\Psi=0$$

By setting the wave function to $\Psi=\left(\begin{array}{c}\Psi_+\\ \Psi_-\end{array}\right)e^{ip\chi}$,

the equation is in matrix form:

$$\left(\begin{array}{cc} -p\sqrt{-K}a^2 & a\frac{\partial}{\partial a}+1+\frac{\partial}{\partial\phi} \\ a\frac{\partial}{\partial a}+1-\frac{\partial}{\partial\phi} & p\sqrt{-K}a^2 \end{array}\right)\left(\begin{array}{c}\Psi_+\\ \Psi_-\end{array}\right)=0$$

4. Dirac type WDW equation   Model 1 : $V(\phi)=0$

$K>0$

$$\Psi_{\pm,\nu p}=\frac{1}{\sqrt{2}}e^{\pm i\frac{\pi}{4}}K_{\frac{i\nu}{2}\mp\frac{1}{2}}(\sqrt{K}|p|a^2/2)e^{i\nu(\phi-\phi_0)}$$

$K<0$

$$\Psi_{\pm,\nu p}=\frac{1}{\sqrt{2}}J_{\frac{i\nu}{2}\mp\frac{1}{2}}(\sqrt{|K|}|p|a^2/2)e^{i\nu(\phi-\phi_0)}$$

$$\Psi_{\pm,\nu p}=\pm\frac{1}{\sqrt{2}}J_{-\frac{i\nu}{2}\pm\frac{1}{2}}(\sqrt{|K|}|p|a^2/2)e^{i\nu(\phi-\phi_0)}$$

4. Dirac type WDW equation Model 2 : $K = 0$, $V(\phi) = V_0 \exp \lambda\phi$

$$\gamma^A e_A^M D_M \Psi = 0$$

$$\left[\sigma^1 \left(a\frac{\partial}{\partial a} + \frac{3}{2}\right) + i\sigma^2 \left(\frac{\partial}{\partial \phi} + \frac{\lambda}{4}\right) + i\sigma^3 \sqrt{2V_0 e^{\lambda\phi}} a^3 \frac{\partial}{\partial \chi}\right] \Psi = 0$$

By setting the wave function to $\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} e^{ip\chi}$,

the equation is in matrix form.

$$\begin{pmatrix} -pa^3\sqrt{2V_0 e^{\lambda\phi}} & a\frac{\partial}{\partial a} + \frac{3}{2} + \frac{\partial}{\partial \phi} + \frac{\lambda}{4} \\ a\frac{\partial}{\partial a} + \frac{3}{2} - \frac{\partial}{\partial \phi} - \frac{\lambda}{4} & pa^3\sqrt{2V_0 e^{\lambda\phi}} \end{pmatrix} \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} = 0$$

4. Dirac type WDW equation Model 2 : $K = 0$, $V(\phi) = V_0 \exp \lambda\phi$

$C > 0$ $C = 2V_0 \left(1 - \frac{\lambda^2}{36}\right)^{-1}$

$$\Psi_{\pm,\nu p} = \frac{1}{\sqrt{2}} \sqrt{1 \pm \frac{\lambda}{6}} J_{\frac{i\nu}{3} \mp \frac{1}{2}} (\sqrt{C}|p|e^{3x}/3) e^{i\nu(y-y_0)}$$

$$\Psi_{\pm,\nu p} = \pm\frac{1}{\sqrt{2}} \sqrt{1 \pm \frac{\lambda}{6}} J_{-\frac{i\nu}{3} \pm \frac{1}{2}} (\sqrt{C}|p|e^{3x}/3) e^{i\nu(y-y_0)}$$

$C < 0$

$$\Psi_{\pm,\nu p} = \frac{1}{\sqrt{2}} e^{\pm i\frac{\pi}{4}} \sqrt{1 \pm \frac{\lambda}{6}} K_{\frac{i\nu}{3} \mp \frac{1}{2}} (\sqrt{|C|}|p|e^{3x}/3) e^{i\nu(y-y_0)}$$

4. Dirac type WDW equation

- the conservation law

$$\partial_M(\sqrt{-G}\bar{\Psi}\hat{\gamma}^M\Psi) = 0$$

- the probability density

$$\propto \sqrt{|2U|}a^{3/2}||\Psi||^2 = \sqrt{|2U|}a^{3/2}(|\Psi_+|^2 + |\Psi_-|^2)$$

$\Rightarrow$ positive-definite

4. Dirac type WDW equation

Comparing asymptotic behaviors of scale factor $y \to \infty$

The modified Bessel function of second kind in the Klein-Gordon type

$$K_{i\frac{\nu}{2}}\left(\frac{y}{2}\right) \sim \sqrt{4\pi}e^{-\frac{\nu\pi}{4}}(\nu^2 - y^2)^{-\frac{1}{4}} \sin\underline{\left(\frac{\pi}{4} - \frac{1}{2}\sqrt{\nu^2 - y^2} + \frac{\nu}{2}\cosh^{-1}\frac{\nu}{y}\right)}$$

The modified Bessel of second kind in the function in the Dirac type

$$K_{i\frac{\nu}{2}\pm\frac{1}{2}}\left(\frac{y}{2}\right) \sim \sqrt{4\pi}e^{-\frac{\nu\pi}{4}\pm i\frac{\pi}{4}}(\nu^2 - y^2)^{-\frac{1}{4}}\left[\sqrt{\frac{\nu+y}{2y}}\sin\underline{\left(\frac{\pi}{4} - \frac{1}{2}\sqrt{\nu^2 - y^2} + \frac{\nu}{2}\cosh^{-1}\frac{\nu}{y}\right)}\right.$$
$$\left. \mp i\sqrt{\frac{\nu-y}{2y}}\cos\underline{\left(\frac{\pi}{4} - \frac{1}{2}\sqrt{\nu^2 - y^2} + \frac{\nu}{2}\cosh^{-1}\frac{\nu}{y}\right)}\right]$$

$\Rightarrow$ These wave functions have a common wave packet solution.

5. Summary and Prospects

○ Summary

- We have applied the method in Eisenhart-Duval lift to a simple models and extend the minisuperspace.

  We have formulated Dirac type WDW equation in an extended minisuperspace.

  ⟹ There is no arbitrariness comes from S and $U(a,\phi)$.

  ⟹ The probability density is positive definite.

- For the case of simple models, Dirac type WDW equation can be solved exactly.

○ Prospects

- Applying the technique to the general cosmological models.
- Third quantization and global property of extended minisuperspace.

Session D2b 10:45–12:00

[Chair: Kunihito Uzawa]

Keiju Murata

Nihon University

"Superradiant instability of Myers-Perry black strings"

(15 min.)

[JGRG30 (2021) 120916]

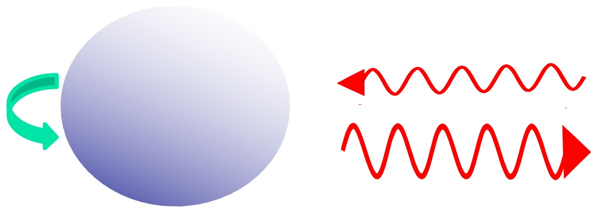
Superradiant instability of Myers-Perry black strings

Nihon University
Keiju Murata
with O.J.C.Dias, T.Ishii, J.E.Santos, B.Way

What is superradiant instability?

Superradiance

Wave amplification by the rotating black hole.

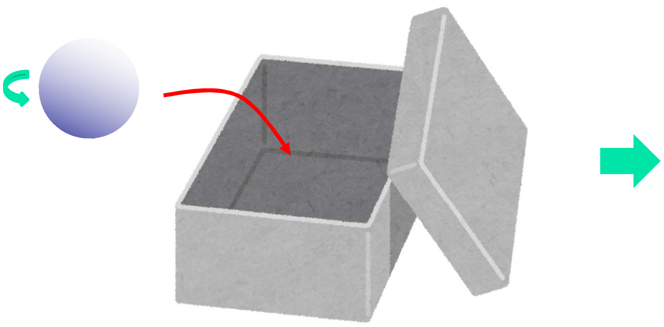


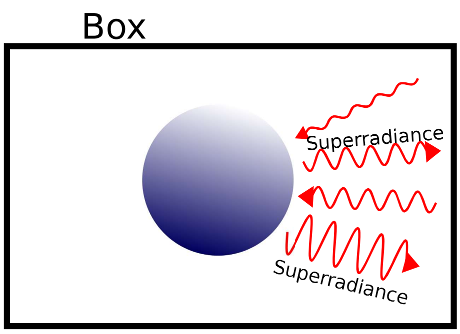
"Wave version of the Penrose process"

Black hole bomb

Press Teukolsky 72, Cardoso, Dias, Lemos, Yoshida 04

If you put a rotating black hole into a box





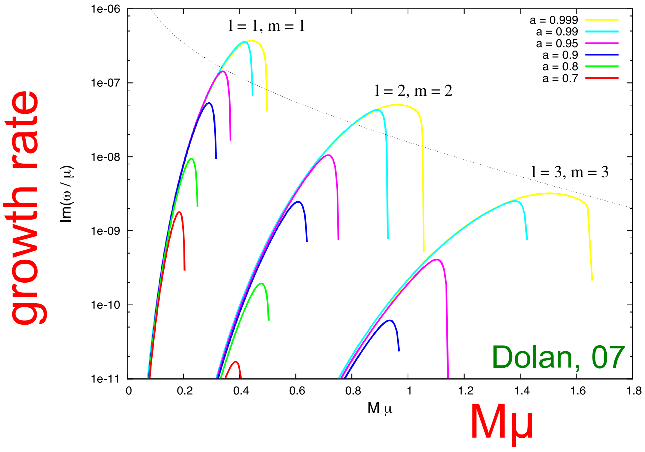
Instability caused by the repetition of superradiance
= Black hole bomb or Superradiant instability

Massive field in Kerr spacetime

$$\Box\Phi = \mu^2\Phi$$

The mass gives
an effective potential barrier at infinity.

Superradiant instability Detweiler, 80



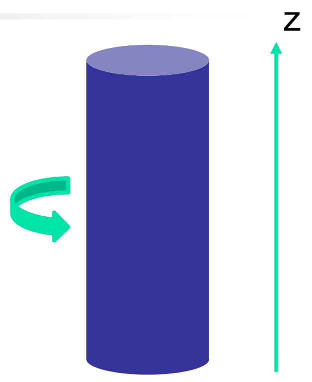
$$\mathrm{Im}(\omega/\mu) \approx 10^{-7} e^{-1.84 M\mu}/(M\mu),$$

M: Mass of Kerr

Perturbation of the rotating black string

Rotating black string

$$R_{\mu\nu} = 0$$



$$ds^2 = ds^2_{\text{Myers-Perry}} + dz^2$$

Superradiant instability of rotating black string

$$ds^2 = ds^2_{\text{Myers-Perry}} + dz^2$$

Perturbation in this geometry

Massive field in $ds^2_{\text{Myers-Perry}}$ from KK-mode.

Superradinat instability?

For the **scalar field** perturbation, $\Box\phi = 0$

4d Kerr + 1d: Unstable

Myers-Perry + 1d: Stable

Cardoso, Lemos 05, Cardoso, Yoshida 05

See also Marolf Palmer 04.

Gravitational perturbation of Myers-Perry black string

Myers-Perry-string with equal angular momenta

$$ds^2 = ds^2_{\text{Myers-Perry}} + dz^2$$

$$ds^2_{\text{Myers-Perry}} = -f(r)d\tau^2 + \frac{dr^2}{g(r)} + \frac{r^2}{4}\{\sigma_1^2 + \sigma_2^2 + \beta(r)(\sigma_3 + 2\Omega(r)d\tau)^2\}$$

Invariant 1-forms of SU(2)

$$\left\{\begin{aligned} \sigma_1 &= -\sin\chi \mathrm{d}\theta + \cos\chi \sin\theta \mathrm{d}\phi \ , \\ \sigma_2 &= \cos\chi \mathrm{d}\theta + \sin\chi \sin\theta \mathrm{d}\phi \ , \\ \sigma_3 &= \mathrm{d}\chi + \cos\theta \mathrm{d}\phi \ . \end{aligned}\right.$$

$$g(r) = 1 - \frac{2\mu}{r^2} + \frac{2a^2\mu}{r^4} \ , \quad \beta(r) = 1 + \frac{2a^2\mu}{r^4} \ ,$$
$$\Omega(r) = \Omega_H - \frac{2\mu a}{r^4 + 2a^2\mu} \ , \quad f(r) = \frac{g(r)}{\beta(r)} \ .$$

Symmetry of this spacetime:

$$SU(2) \times U(1) \times R_z \times R_t$$

χ-translation z-translation

$$\sigma_1 = -\sin\chi d\theta + \cos\chi \sin\theta d\phi \; ,$$
$$\sigma_2 = \cos\chi d\theta + \sin\chi \sin\theta d\phi \; ,$$
$$\sigma_3 = d\chi + \cos\theta d\phi \; .$$

Decoupled gravitational perturbation

$$h_{\mu\nu} dx^\mu dx^\nu = e^{-i\omega t + ikz} r^2 \delta\alpha(r) \sigma_+^2 \; .$$

where $\sigma_\pm = \frac{1}{2}(\sigma_1 \mp i\sigma_2)$

Under χ-translation,

$$\chi \to \chi + \lambda \quad \Rightarrow \quad \sigma_\pm \to e^{\mp i\lambda}\sigma_\pm$$

$\sigma_\pm$ has U(1)-charge m=±1.

Above is unique perturbation
which is SU(2)-symmetric and has U(1)-charge m=+2.

⇒ Decouple

KM&J.Soda 07

Master equation

$$h_{\mu\nu} dx^\mu dx^\nu = e^{-i\omega t + ikz} r^2 \delta\alpha(r) \sigma_+^2 \; .$$

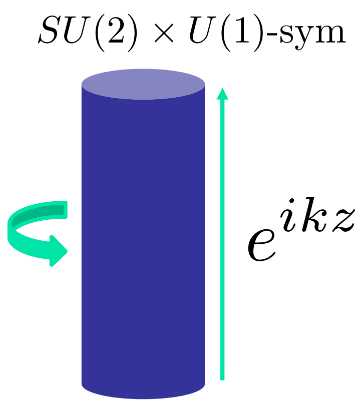
⇓

Single ODE

$$\delta\alpha'' + \left\{ \frac{(fg\beta)'}{2fg\beta} + \frac{3}{r} \right\} \delta\alpha' + \left\{ \frac{(fg\beta)'}{rfg\beta} + \frac{4}{r^2}\left(1 + \frac{\beta}{g} - \frac{4}{g\beta}\right) + \frac{(\omega - 4\Omega)^2 - fk^2}{fg} \right\} \delta\alpha = 0 \; .$$

Gregory-Laflamme instability

This spacetime also exhibits
Gregory-Laflamme instability.



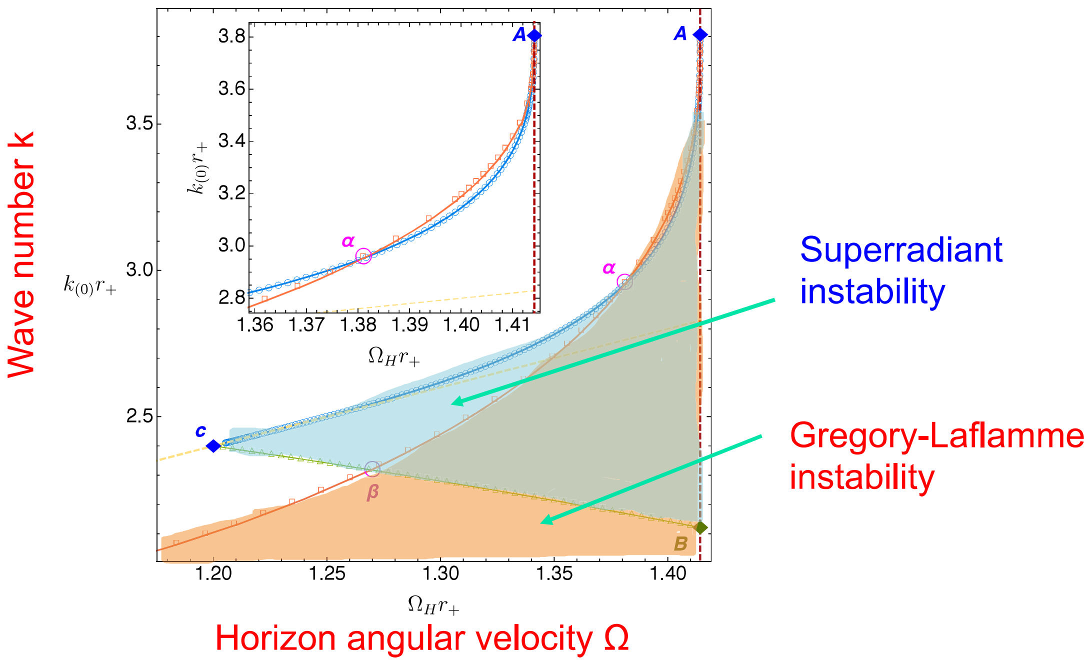
GL instability
= Instability of the mode with

$$SU(2)\times U(1)\times \cancel{R_z}$$

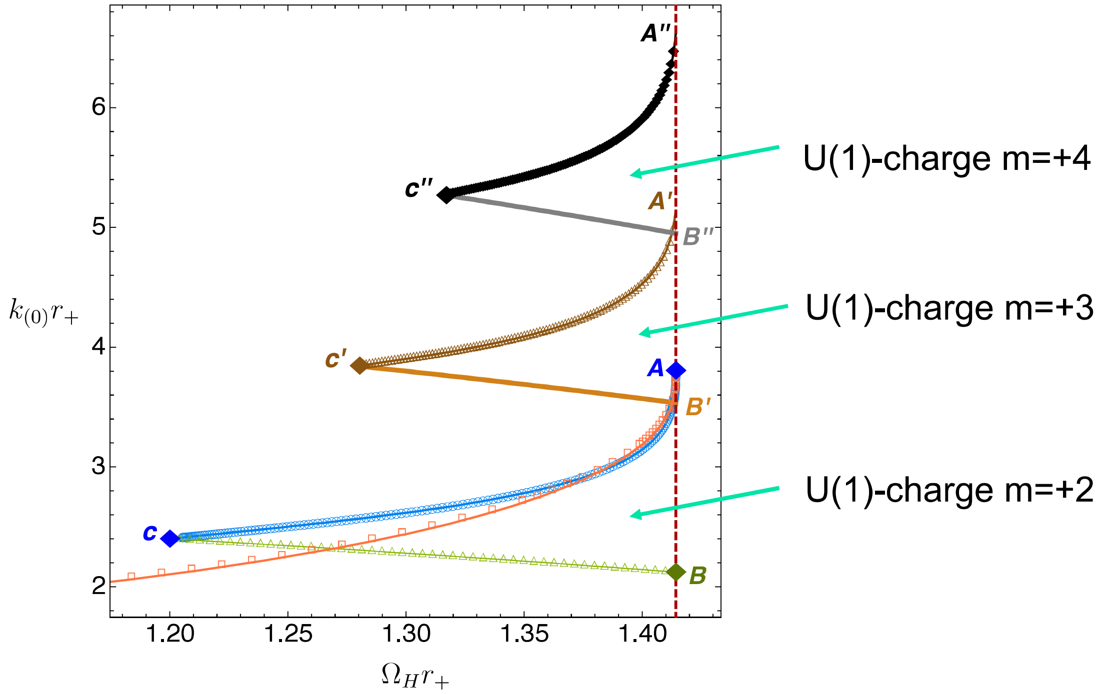
Dias et al 10

We will show the results
for both of GL and superradiant instabilities.

Unstable region

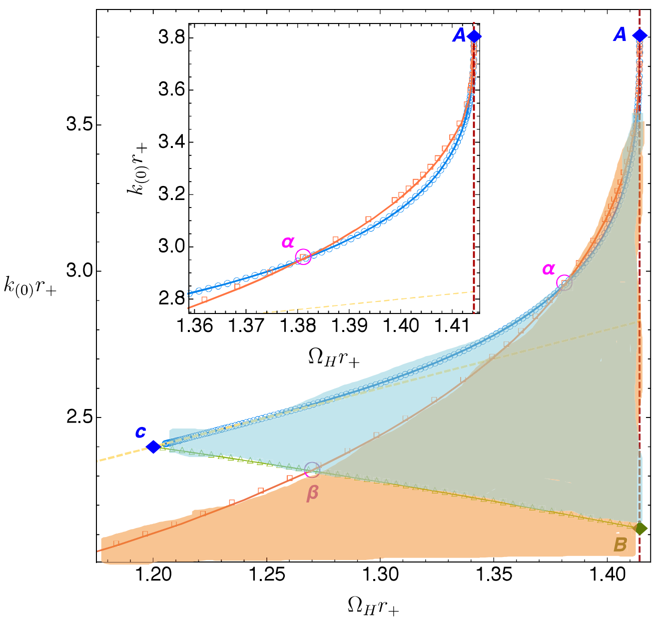


Results of higher modes



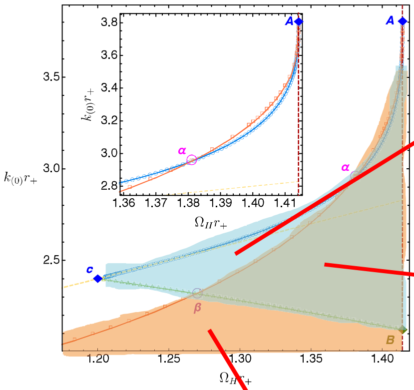
Summary

Superradiant instability of Myers-Perry black string was studied.



In a wider region in the parameter space,
the black string becomes unstable.

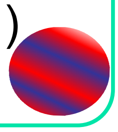
Possible end point?



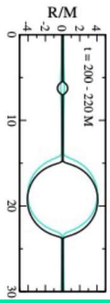
Helical black string?
(Both of U(1) and R_z broken.)

Ishii's Talk

"Helical" localized BH?
(Localized BH with broken U(1).)



"Localized" BH?



Taken from
Lehner & Pretorius
PRL 105, 101102 (2010)

Session D2b 10:45–12:00

[Chair: Kunihito Uzawa]

Takaaki Ishii

Rikkyo University

"Helical black strings from superradiant instability"

(15 min.)

[JGRG30 (2021) 120917]

Helical black strings from superradiant instability

Takaaki Ishii (Rikkyo U.)

To appear

w/ Oscar J. C. Dias, Keiju Murata, Jorge E. Santos, Benson Way

9 Dec 2021, JGRG30 (online)

Motivation

We have been interested in (gravitational) **superradiant instability and black resonators** in asymp AdS space.

Are there any examples in **non-AdS** space (i.e. without cosmological constant)?

YES: we consider the 6D equally spinning Myers-Perry black string (MPBS).

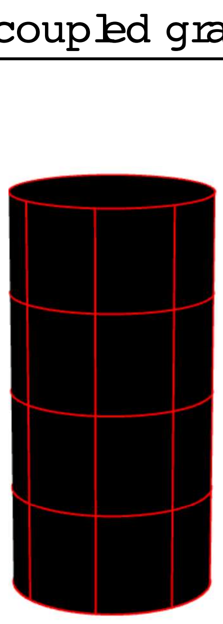
Setup

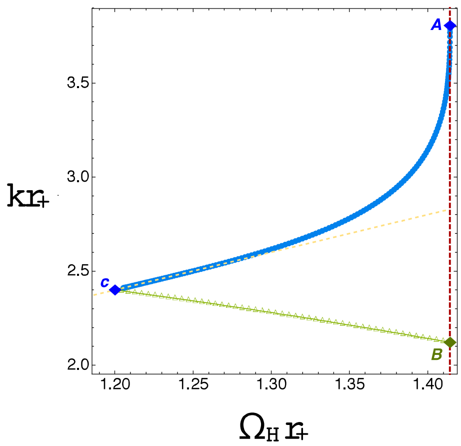
6D pure Einstein gravity

$$S = \frac{1}{16\pi G_6} \int \mathrm{d}^6 x \, R$$

Murata's talk

The 6D MPBS is superradiant (as well as GL) unstable to a decoupled gravitational perturbation.





Unstable for $\Omega_H / \Omega_{\mathrm{extreme}} \gtrsim 0.85$

6D equally spinning MPBS

For equal rotations, the MPBS can be given by a **cohomogeneity-1 metric.**

$$ds^2 = -f(r)\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{g(r)} + \frac{r^2}{4}\left[\underbrace{\sigma_1^2+\sigma_2^2}_{S^2} + \beta(r)\underbrace{\left(\sigma_3 + 2h(r)\mathrm{d}\tau\right)^2}_{S^1\ \text{fiber}}\right] + \mathrm{d}z^2$$

Isometries: **$R_\tau \times R_z \times SU(2) \times U(1)_\chi$**

SU(2)-invariant 1-forms:

$$\sigma_1 = \sin\chi \mathrm{d}\theta + \cos\chi \sin\theta \mathrm{d}\phi$$
$$\sigma_2 = -\cos\chi \mathrm{d}\theta + \sin\chi \sin\theta \mathrm{d}\phi$$
$$\sigma_3 = \mathrm{d}\chi + \cos\theta \mathrm{d}\phi$$

$U(1)_\chi$ symmetry: $\sigma_1^2+\sigma_2^2$ is invariant under $\chi \to \chi + \mathrm{const.}$

Coh-1 helical black string

We introduce a **cohomogeneity-1 metric ansatz** that realized a broken $U(1)_\chi$ isometry.

$$ds^2 = -f(r)\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{g(r)} + \frac{r^2}{4}\left[\alpha(r)\sigma_1^2 + \frac{1}{\alpha(r)}\sigma_2^2 + \beta(r)\left(\sigma_3 + 2h(r)\mathrm{d}\tau + \frac{k(r)}{2}\mathrm{d}z\right)^2\right] + \gamma(r)\left(\mathrm{d}z + q(r)\mathrm{d}\tau\right)^2$$

⇒ ODEs for 8 variables (f, g, h, k, q, α, β, γ)

<u>**Deformation**</u>

α(r)=1: MPBS with $R_\tau \times R_z \times SU(2) \times U(1)_\chi$

α(r)≠1: helical $R_\tau \times R_z \times SU(2)$

Helical spacetime

The generator of the horizon is made not only of ∂_t but also of $\partial_{\bar{\chi}}$ and ∂_y ⇒ **helical Killing vector**

Our ansatz is a rotating frame at infinity (r→∞) in general.

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}r^2 + \frac{r^2}{4}\left[\sigma_1^2 + \sigma_2^2 + \left(\sigma_3 + 2h_\infty \mathrm{d}\tau + \frac{1}{2}k_\infty \mathrm{d}z\right)^2\right] + \mathrm{d}z^2$$

Non-rotating frame at infinity: $(t, y, \bar{\chi}) = (\tau, z, \chi + 2h_\infty t + \frac{1}{2}k_\infty z)$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + \frac{r^2}{4}\left(\bar{\sigma}_1^2 + \bar{\sigma}_2^2 + \bar{\sigma}_3^2\right) + \mathrm{d}y^2$$

The generator of the horizon (v_H=0 for MPBS):

$$K = \partial_\tau + v_H \partial_z = \partial_t + v_H \partial_y + \Omega_H \partial_{\bar{\chi}/2}$$

Thermodynamics

We will evaluate thermodynamic quantities, which satisfy the 1st law of black hole mechanics.

E: energy
T: temperature
S: entropy
Ω_H: angular velocity
J: angular momentum

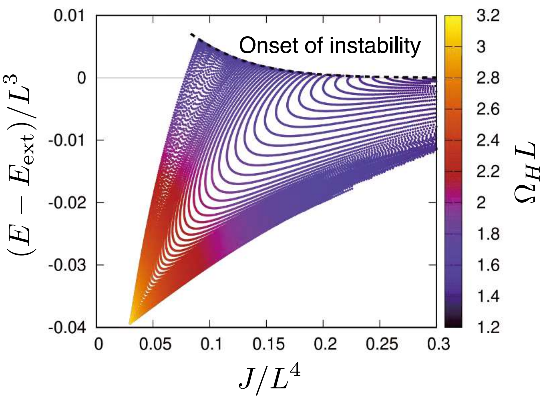
v_H: horizon velocity along the string
P: pressure along the string
$\mathcal{T}$: tension
L: length scale $L \equiv 2\pi/k_\infty$

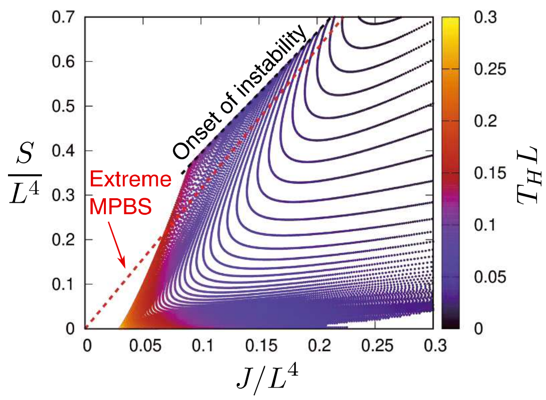
$$\mathrm{d}E = T\mathrm{d}S + \Omega_H \mathrm{d}J + v_H \mathrm{d}P + \mathcal{T}_{\mathrm{eff}}\mathrm{d}L \qquad \mathcal{T}_{\mathrm{eff}} \equiv \mathcal{T} + \frac{v_H P}{L}$$

We set **P=0 by Lorentz boost** for comparing solutions.

We use **L** to normalize dimensionful quantities.

Results 1



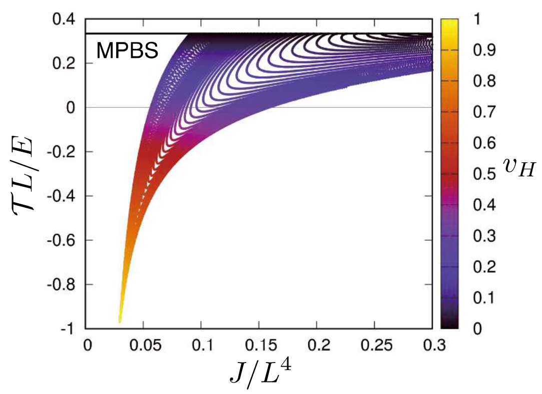


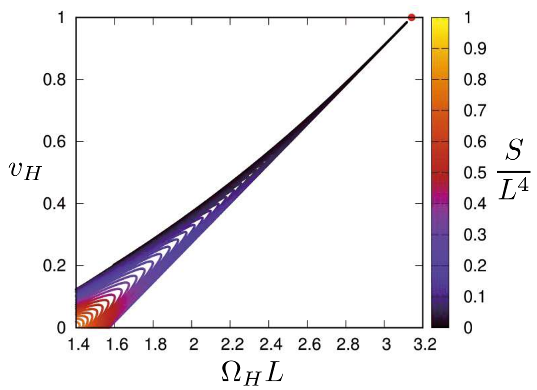
Solutions exist even in the region of super-extreme MPBS.

There seems to be an $S \to 0$ limit.

The maximal Ω_H appears to be $\Omega_H \to \pi$ as $S \to 0$.

Results 2





$v_H \neq 0$ even for P=0 and reaches the speed of light ($v_H = 1$) as $S \to 0$.

$S \to 0$ with $v_H \to 1$ is not a regular geon, likely a singular pp-wave.

Tension per energy seems to approach -1 as $v_H \to 1$.

This is a stationary spacetime

We can find an **asymptotically timeline Killing vector**, which means that the spacetime is **stationary**.

Asymp timelike Killing vector: $\zeta \equiv k_\infty \partial_\tau - 4h_\infty \partial_z$

$$\zeta^2 \to -(k_\infty^2 - 16h_\infty^2) \qquad (r \to \infty)$$

Meanwhile, by Lorentz boost, we could introduce a stationary "time" that the bulk metric does not depend on:

$$\tilde{t} = \frac{k_\infty \tau + 4h_\infty z}{\sqrt{k_\infty^2 - 16h_\infty^2}} \qquad \tilde{z} = \frac{k_\infty z + 4h_\infty \tau}{\sqrt{k_\infty^2 - 16h_\infty^2}}$$

But the Lorentz boost brings **P≠0.**

Summary

We constructed **cohomogeneity-1 helical black strings** branching off from the onset of a **gravitational superradiant instability of 6D equal-spinning MPBS**.

The helical strings are stationary with $\mathbf{R_t \times R_z \times SU(2)}$ isometries. $U(1)_\chi$ of the MPBS is broken by the instability.

The horizonless limit **S→0** appears to be accompanied by $\mathbf{v_H \to 1}$**,** and it is not likely a regular geon.

Session D2b 10:45–12:00

[Chair: Kunihito Uzawa]

Hidetoshi Omiya

Kyoto U

"Adiabatic evolution of the strongly self-interacting axion cloud"

(15 min.)

[JGRG30 (2021) 120918]

Adiabatic evolution of the strongly self-interacting axion cloud

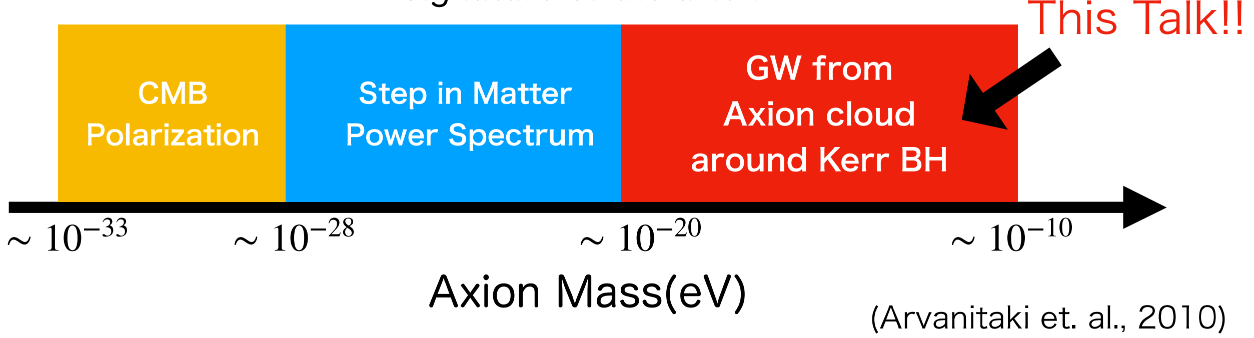
Hidetoshi Omiya (Kyoto U)
Work in progress with
Takuya Takahashi, Takahiro Tanaka

2021/12/9@JGRG

Introduction

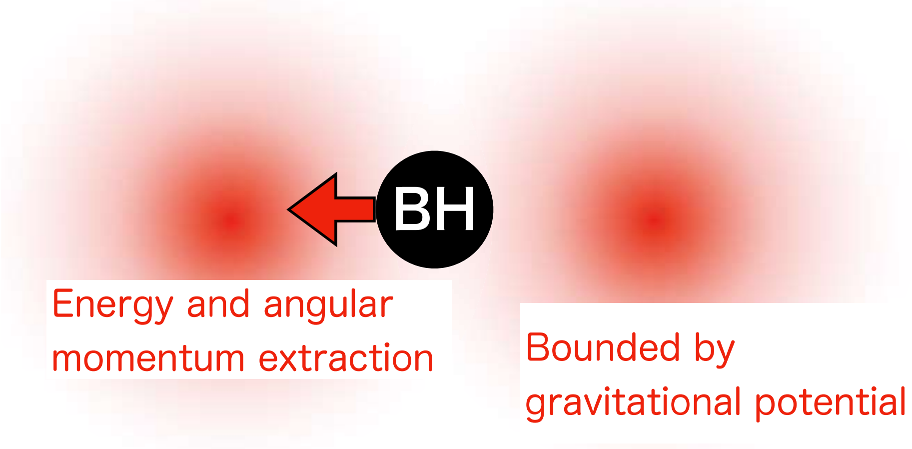
- Axion are attracting many interests!
  - Solve strong CP problem
  - Dark Matter candidates
  - Appears in low energy effective theory of string theory
  - Can be observed by cosmological/astrophysical phenomena

Signature of the axion



(Arvanitaki et. al., 2010)

Axion cloud



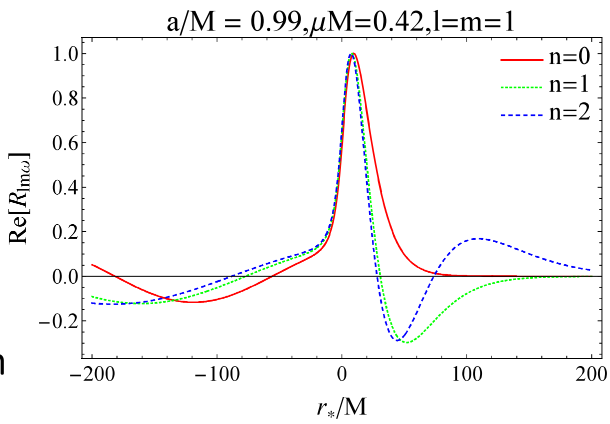
Observable Signal

- GW from the cloud
- Highly spinning BH is not allowed

$$\phi = R_{lm\omega}(r) S_{lm\omega}(\theta) e^{-i(\omega t - m\varphi)} + \mathrm{c.c.}$$

$$\omega = \omega_R + i\omega_I \qquad \tau = \omega_I^{-1} \sim 1\mathrm{min}(M = M_\odot)$$

$$\underbrace{\omega_I > 0}_{\text{Instability}},\ \omega_R \sim \mu \gg \omega_I,\ \underbrace{\omega_R - m\Omega_H < 0}_{\text{Superradiance condition}}$$



Self-interaction

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2 - \mu^2 F_a^2\left(1 - \boxed{\cos\frac{\phi}{F_a}}\right)\right]$$

After cloud grows enough, self-interaction works

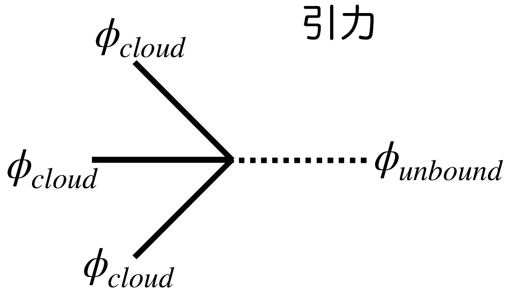
(Arvanitaki et. al.,2010)

$$\sim \frac{1}{2}\mu^2\phi^2 - \underbrace{\frac{\mu^2}{4!F_a^2}}_{\text{引力}}\phi^4 + \ldots$$

1. Energy dissipation to infinity by exciting unbounded mode
2. Bosenova, attractive force collapse the cloud and burst of gravitational wave
3. Energy dissipation to BH by mode coupling (Baryakhtar et. al. 2020)

*We neglect third effect in this talk for simplicity

1.



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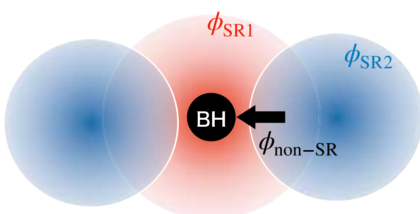
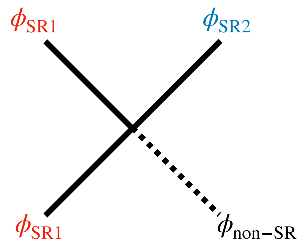
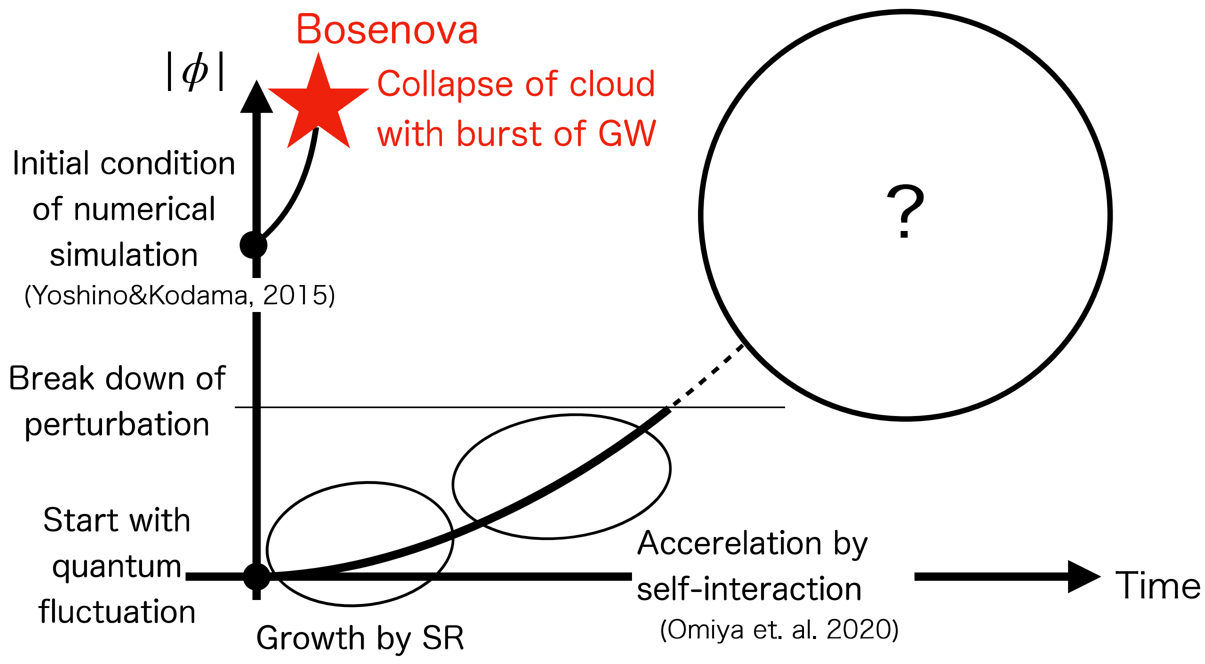




Evolution of self-interacting cloud



- Perturbation theory breaks down when growth is still slow.
- Long time numerical simulation is difficult($\omega_R \gg \omega_I$)

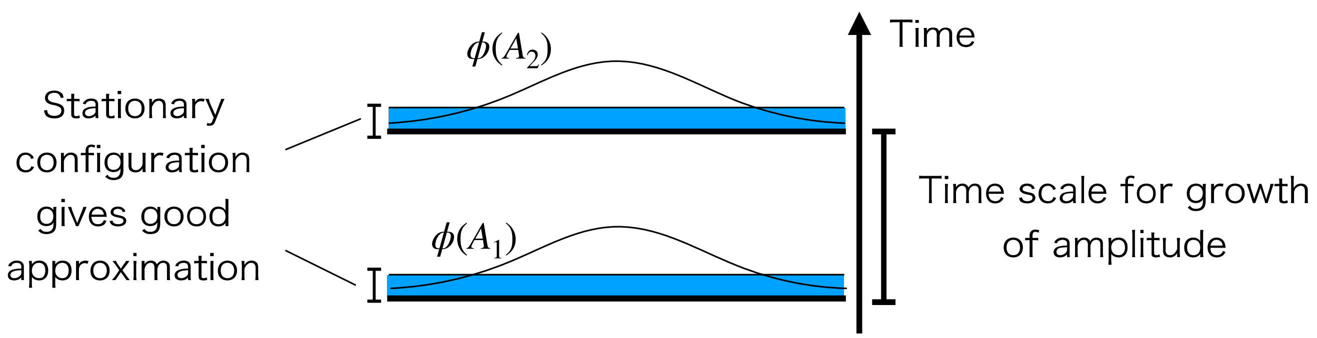
Non-Perturbative analysis of cloud

(Work in Progress)

- Because of adiabatic growth ($\omega_R \gg \omega_I$), locally ($\Delta t \ll \omega_I^{-1}$), cloud can be approximated to be stationary

Main Idea

Construct the one parameter family of stationary configuration $\{\phi(A_0)\}_{A_0}$ by varying amplitude A_0 and join them by energy conservation.



Impose $\dfrac{\mathrm{d}E(A_0)}{\mathrm{d}A_0}\dfrac{\mathrm{d}A_0}{\mathrm{d}t} = -F_{tot}(A_0)$

$E(A_0)$:Energy of $\phi(A_0)$

$F_{tot}(A_0)$:Total Energy flux

Ansatz of configuration

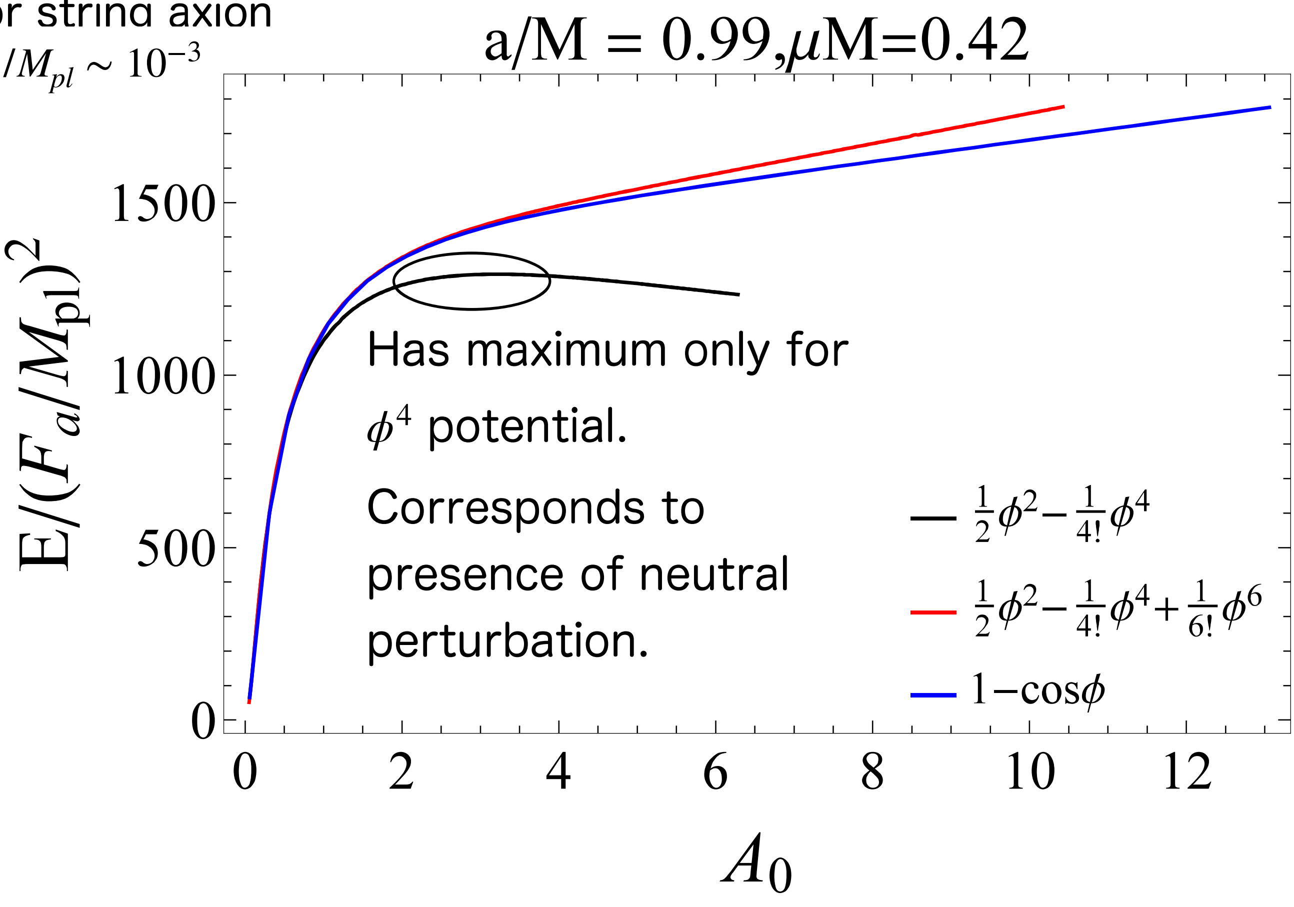
$$\begin{aligned}\phi(A_0) = &\left(\tilde{R}_{11}(r;A_0)Y_{11}(\theta) + \tilde{R}_{31}(r;A_0)Y_{31}(\theta) + \tilde{R}_{51}(r;A_0)Y_{51}(\theta)\right) e^{-i(\omega_0 t - \varphi)} \\ &+ \left(\tilde{R}_{33}(r;A_0)Y_{33}(\theta) + \tilde{R}_{53}(r;A_0)Y_{53}(\theta)\right) e^{-3i(\omega_0 t - \varphi)} \\ &+ \tilde{R}_{55}(r;A_0)Y_{55}(\theta) e^{-5i(\omega_0 t - \varphi)} + \mathrm{c.c.}\end{aligned}$$

- Helical symmetric
- A_0 is amplitude of $\tilde{R}_{11}$ at infinity

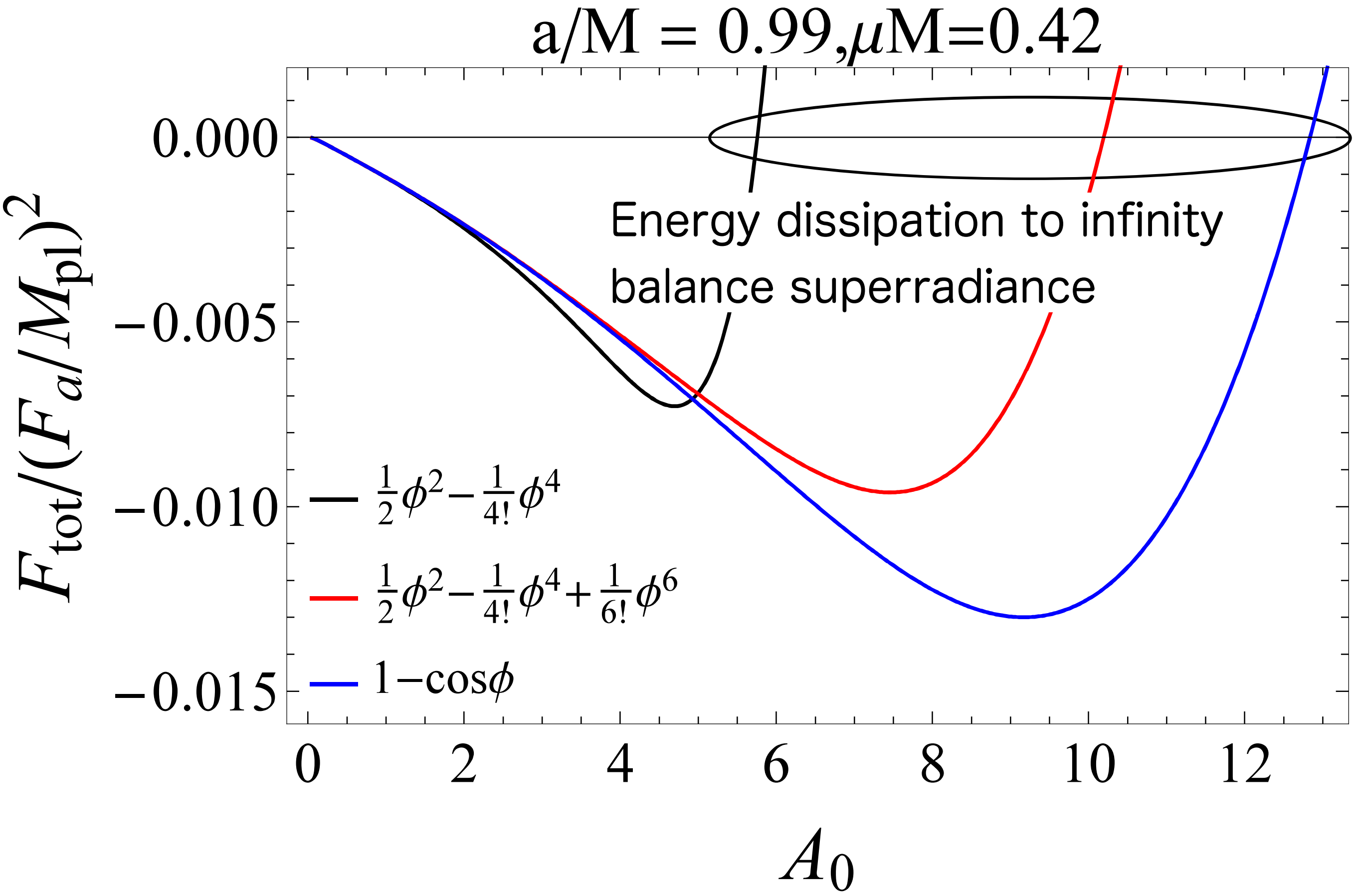
$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta \frac{\mathrm{d}\tilde{R}_{nl}}{\mathrm{d}r}\right) + \left[\frac{n^2(\omega_0(r^2+a^2) - am_0)^2}{\Delta} - \mu^2 r^2 + 2an^2\omega_0 m_0 - a^2 n^2 \omega_0^2 - l(l+1)\right. \\ &\left. + a^2(n^2\omega_0^2 - \mu^2)\frac{1 - 2l(l+1) + 2n^2 m_0^2}{3 - 4l(l+1)}\right]\tilde{R}_{nl} \\ &+ a^2(n^2\omega_0^2 - \mu^2)\left(\frac{(l-1-nm_0)(l-nm_0)}{(2l-3)(2l-1)}\frac{N^{nm_0}_{l-2}}{N^{nm_0}_{l}}\tilde{R}_{nl-2}\right. \\ &\left. + \frac{(l+2+nm_0)(l+1+nm_0)}{(2l+3)(2l+5)}\frac{N^{nm_0}_{l+2}}{N^{nm_0}_{l}}\tilde{R}_{nl+2}\right) \\ &+ \int_0^{2\pi} d\varphi \int_{-1}^{1} dx\, Y_{lnm_0}(x) e^{-inm_0\varphi}(r^2 + a^2 x^2) V'(\phi) = 0\,,\end{aligned}$$

Cloud energy

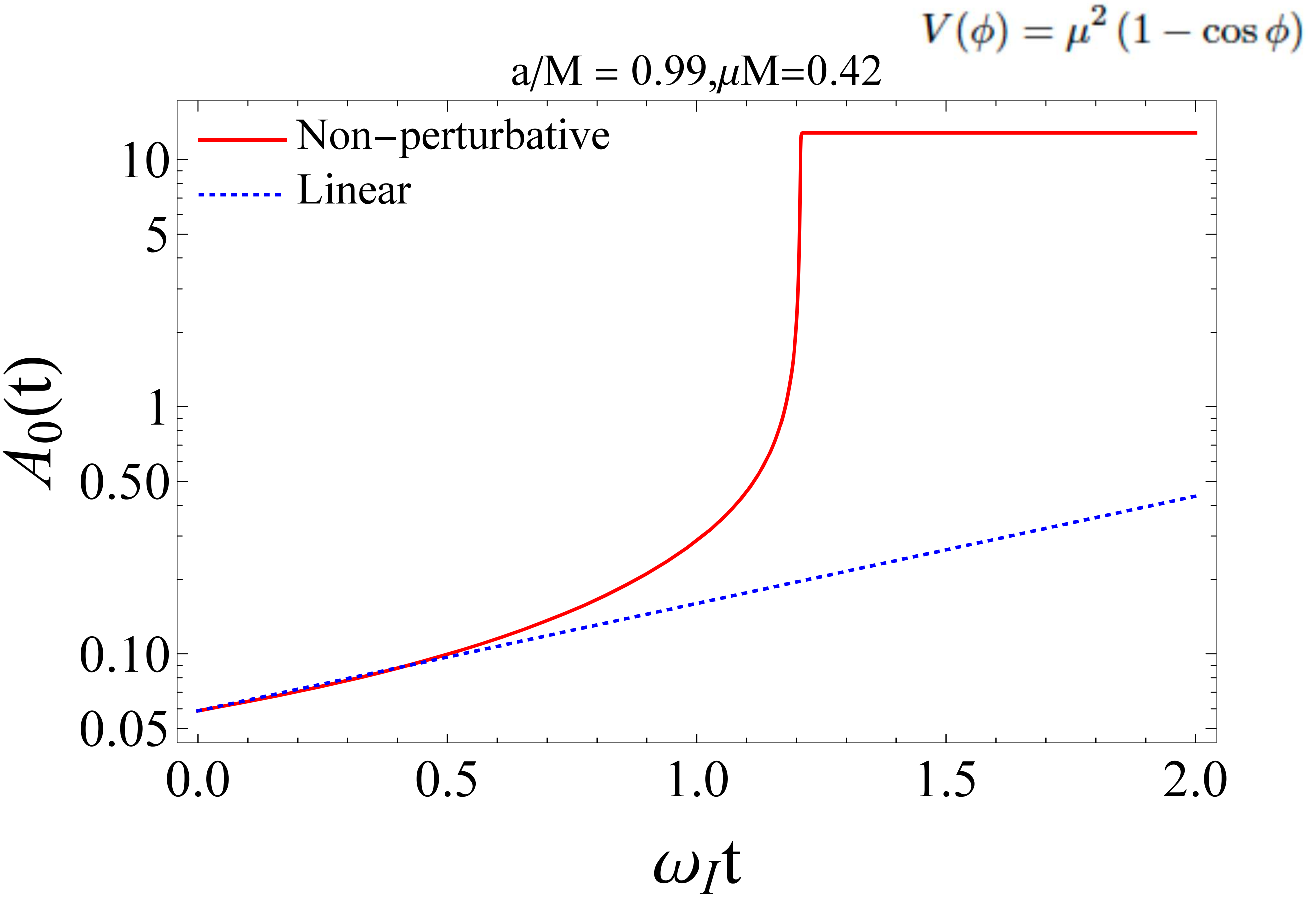
For string axion
$F_a/M_{pl} \sim 10^{-3}$



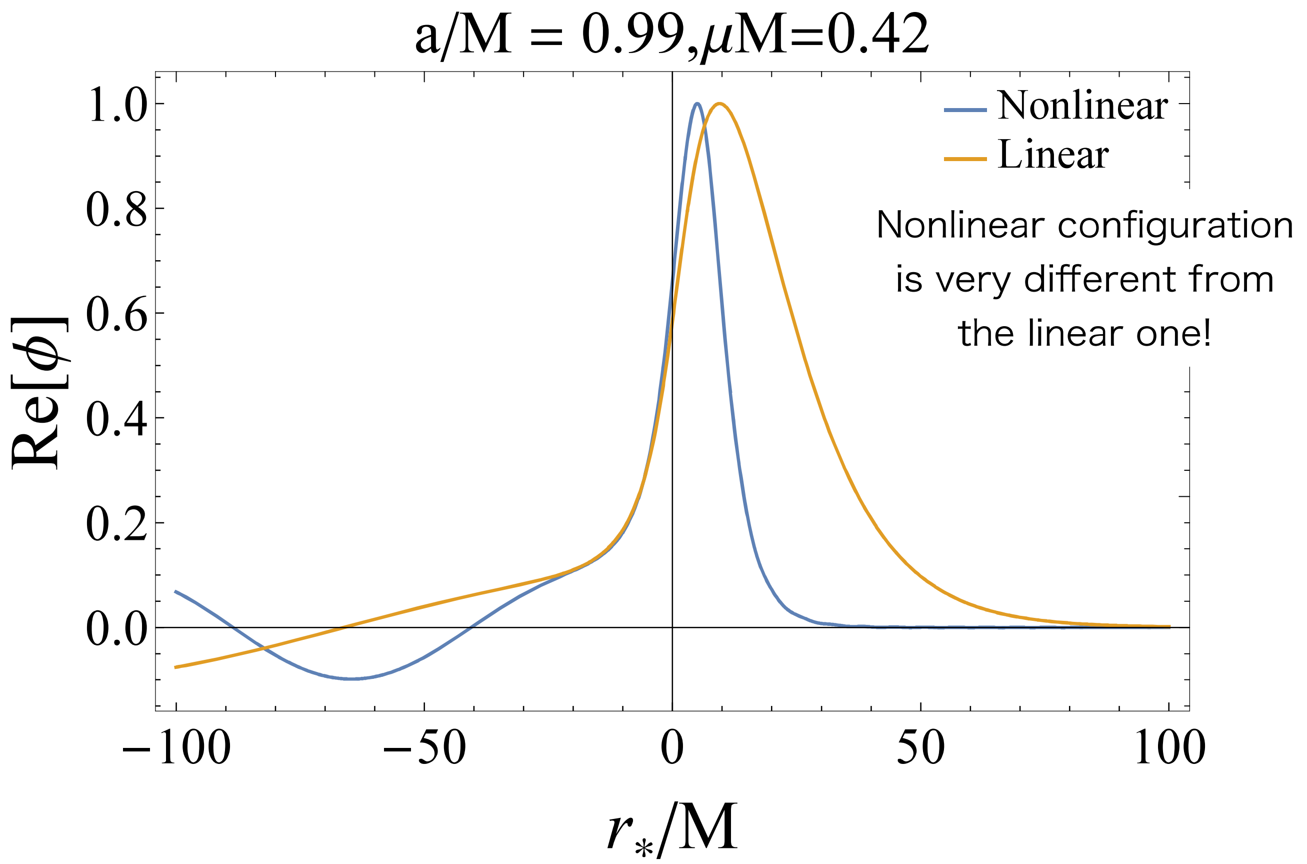
Total energy flux



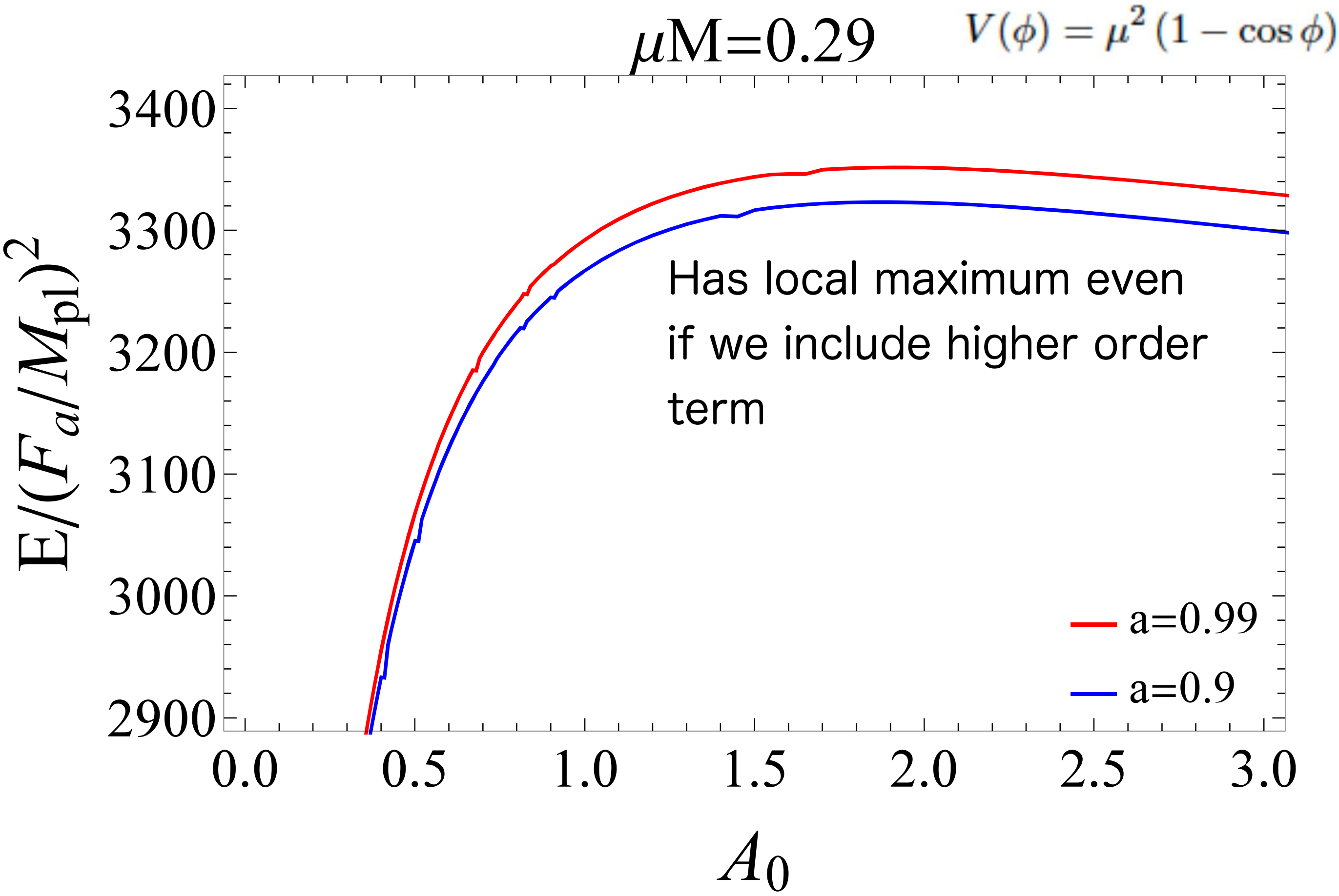
Time evolution of amplitude



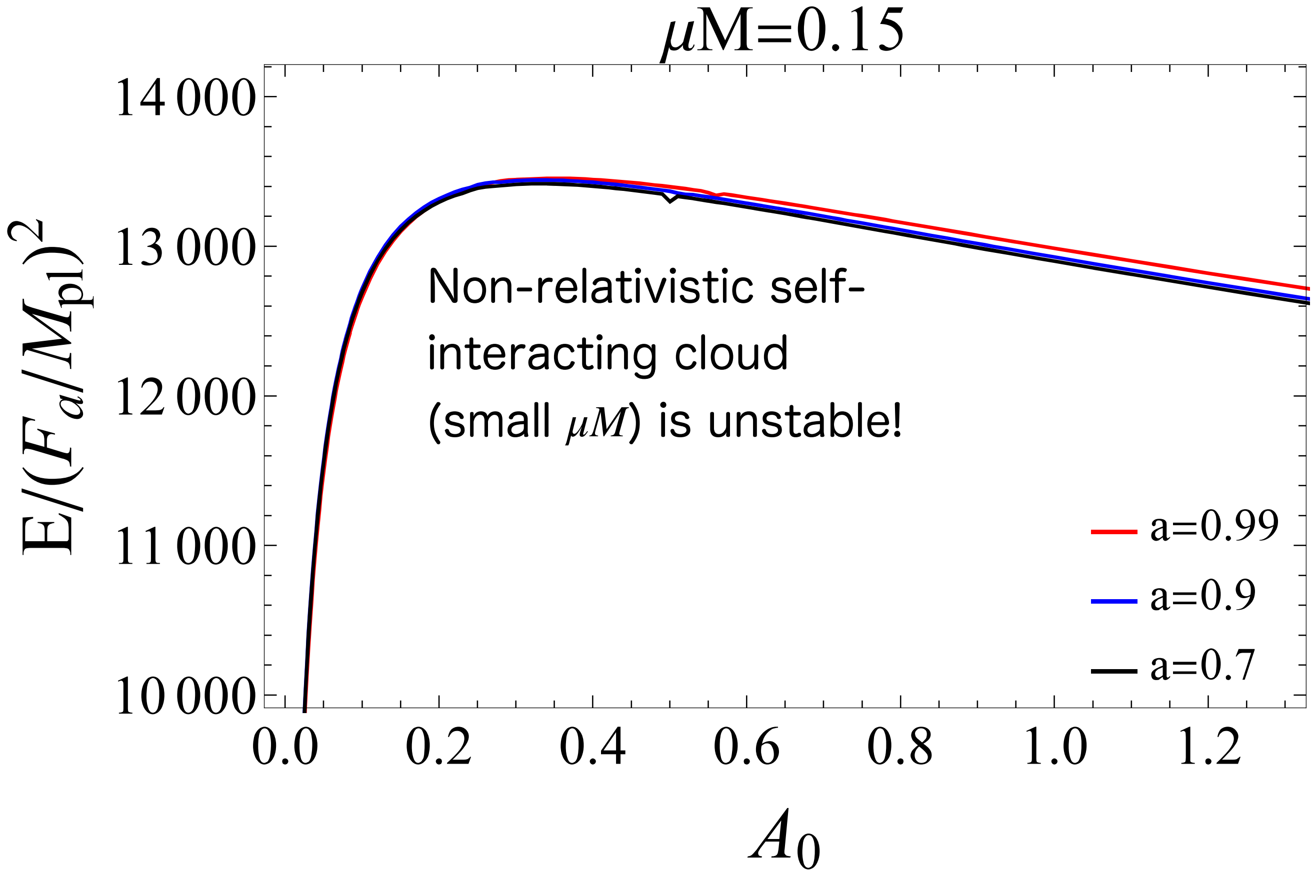
Field configuration at saturation



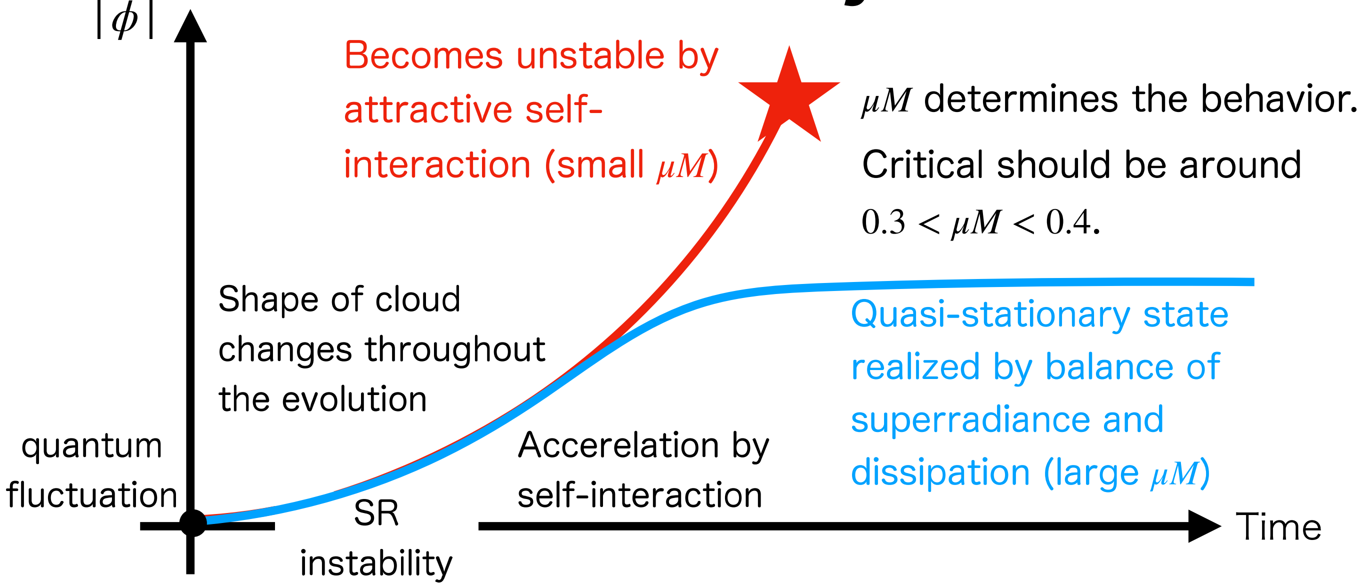
Energy($\mu M = 0.29$)



Energy($\mu M = 0.15$)



Summary

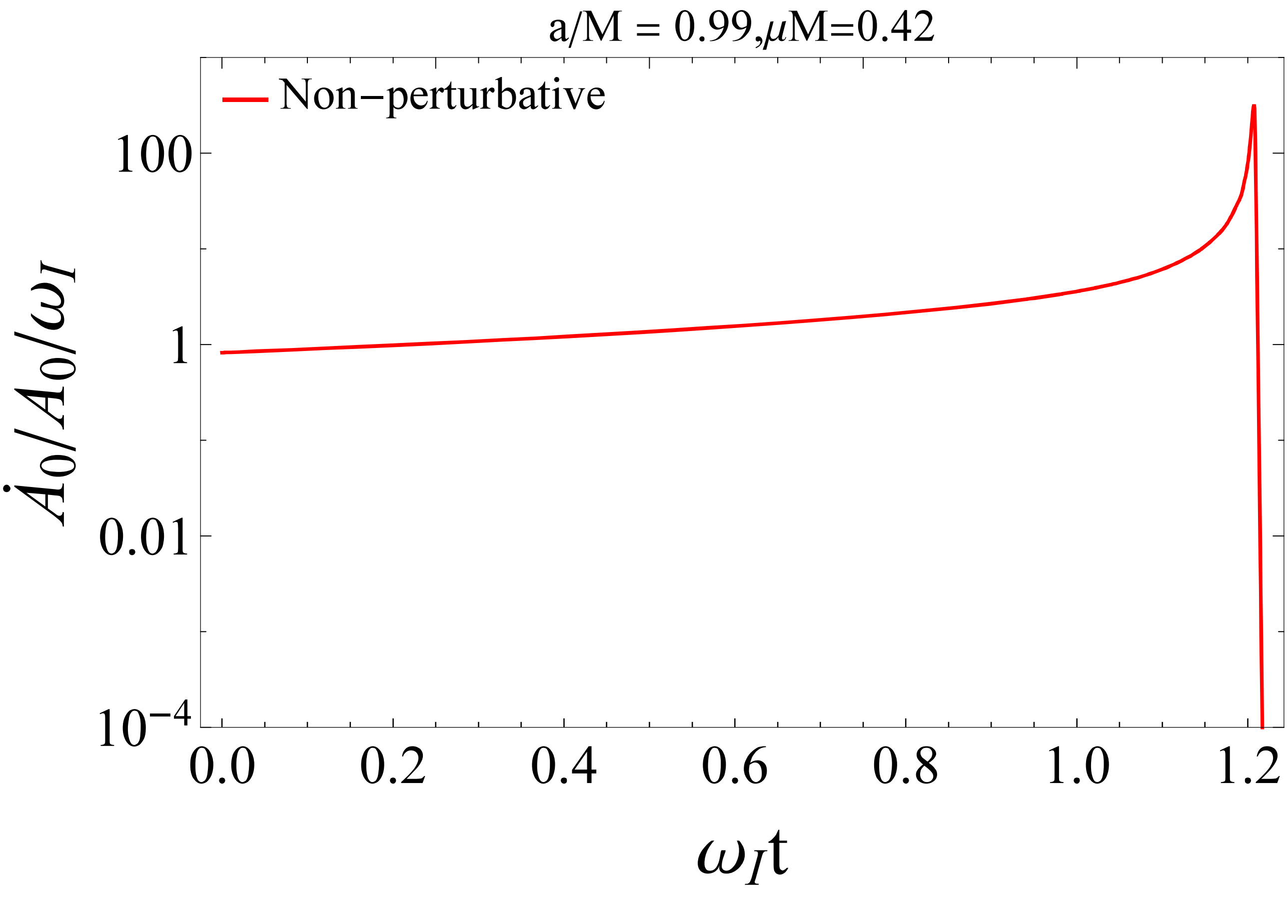


Future Work

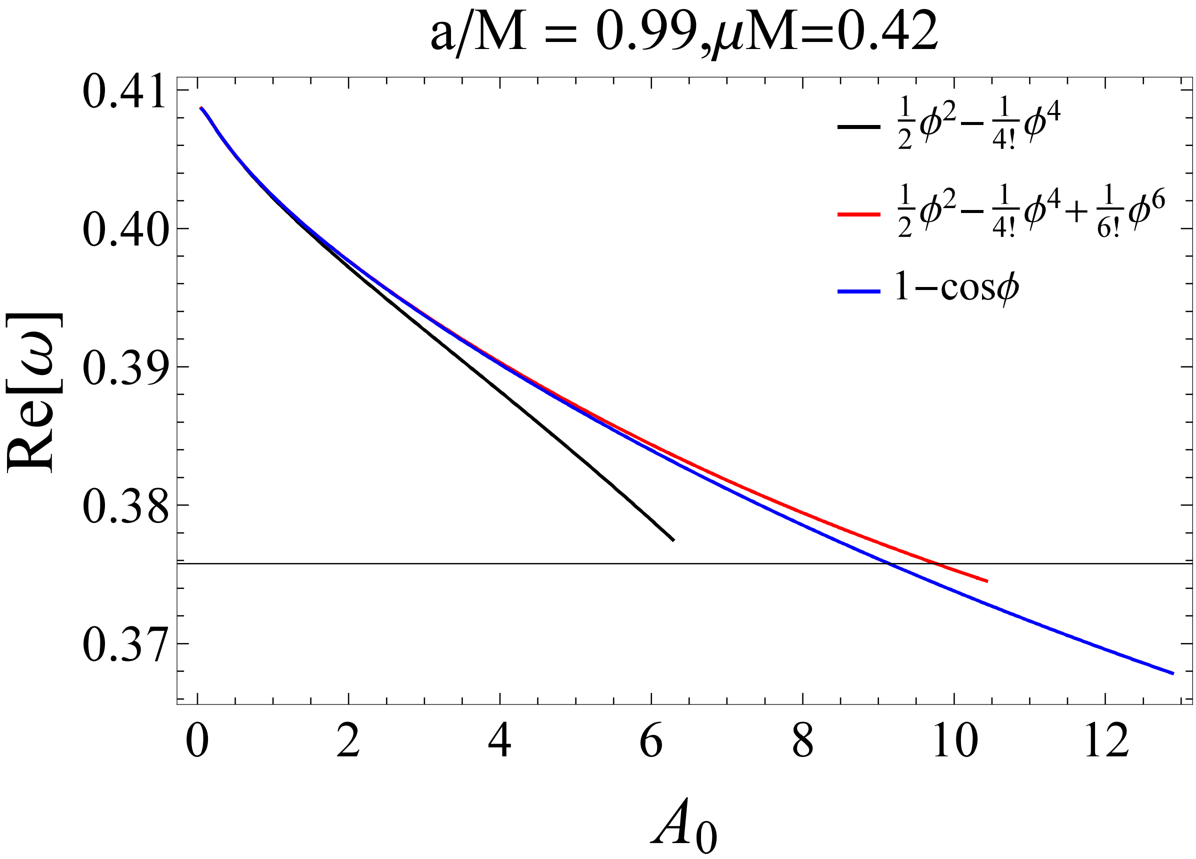
- (Large μM)Dynamical stability of quasi-stationary state？
- (Small μM)What happens after instability？　Bosenova?
- Effect of multiple mode?
- Implication on observability？

Back up

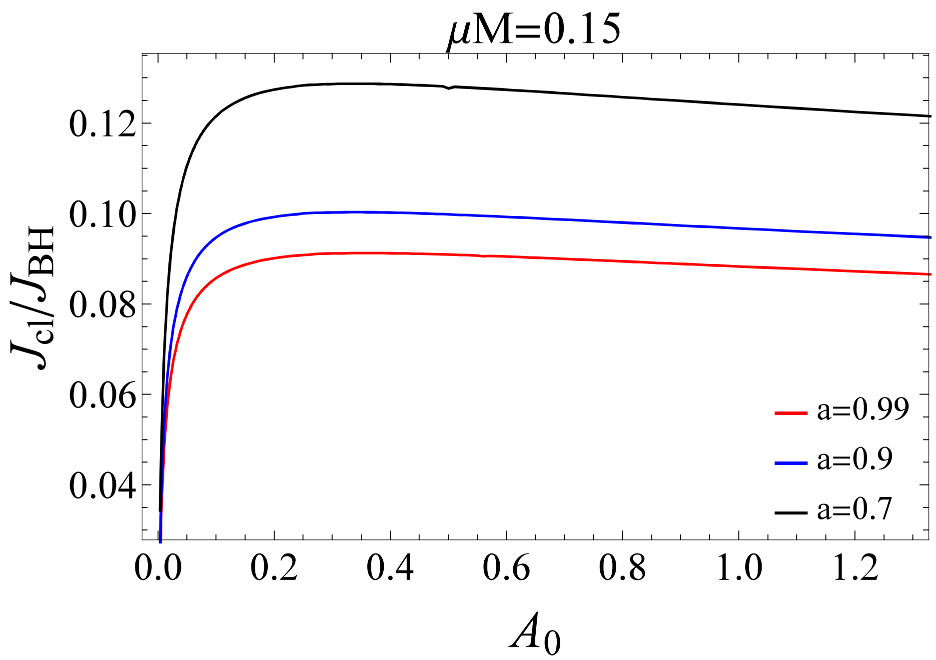
Growth rate



Frequency



Angular momentum($\mu M = 0.15$)

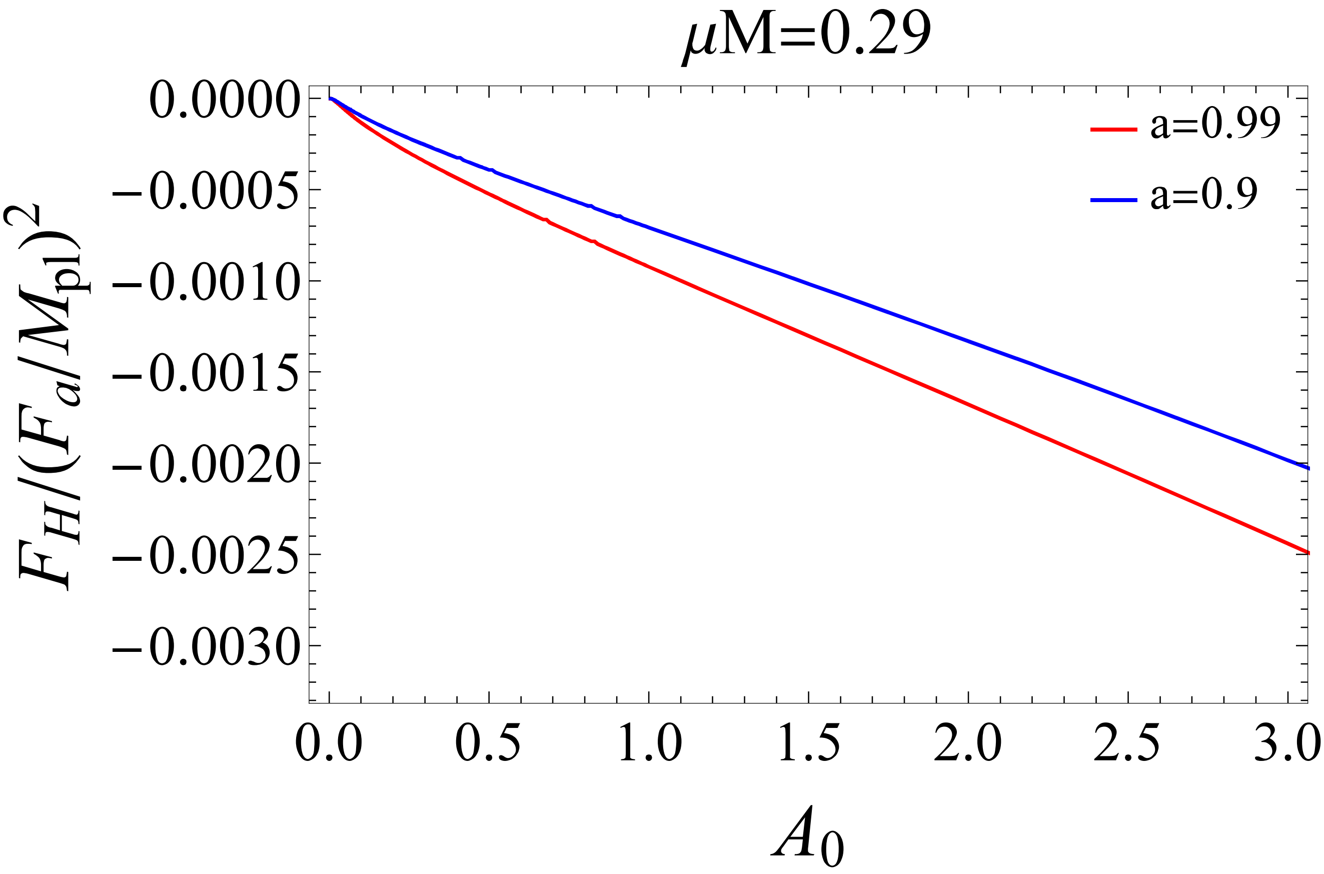


Change of BH spin:$\Delta(a/M_{BH}) \sim 0.1$

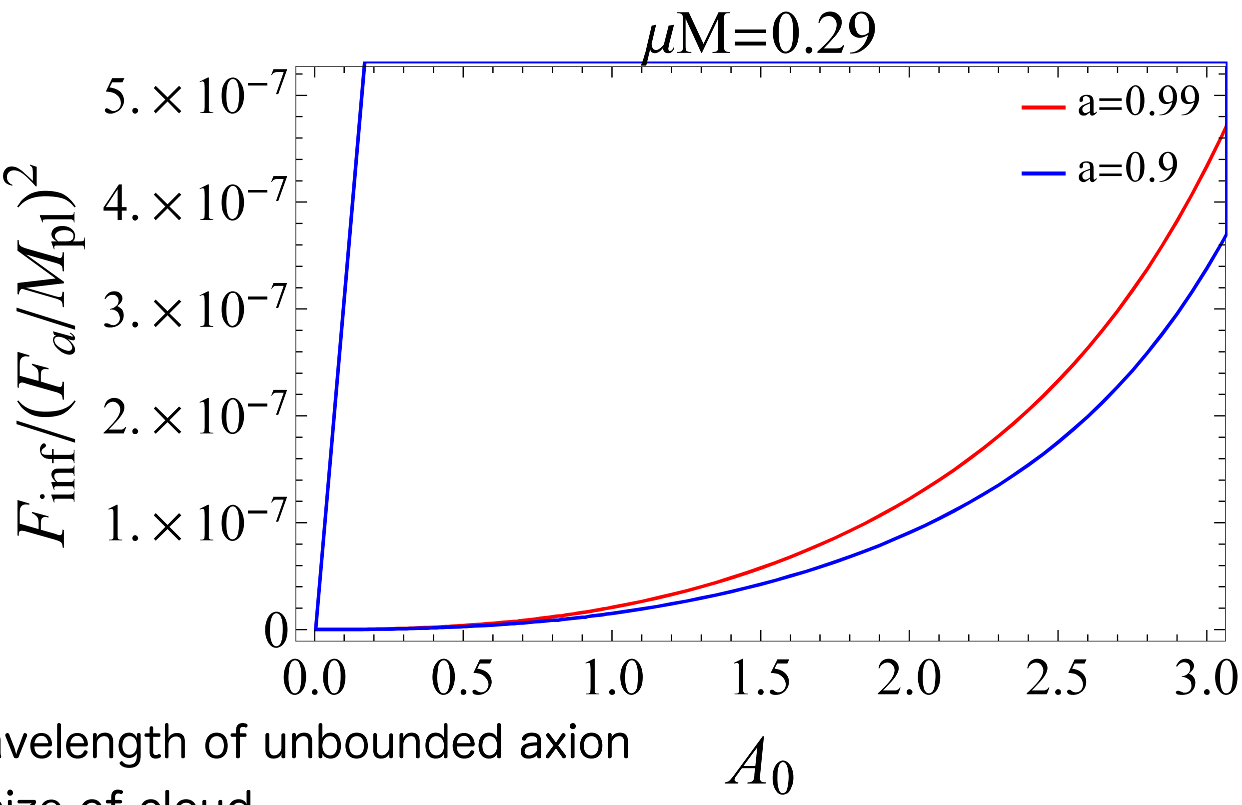
For small BH spin a/M_{BH}($a/M_{BH} \lesssim 0.65$ for this case),

saturation of SR condition occurs before the instability

Energy flux@horizon

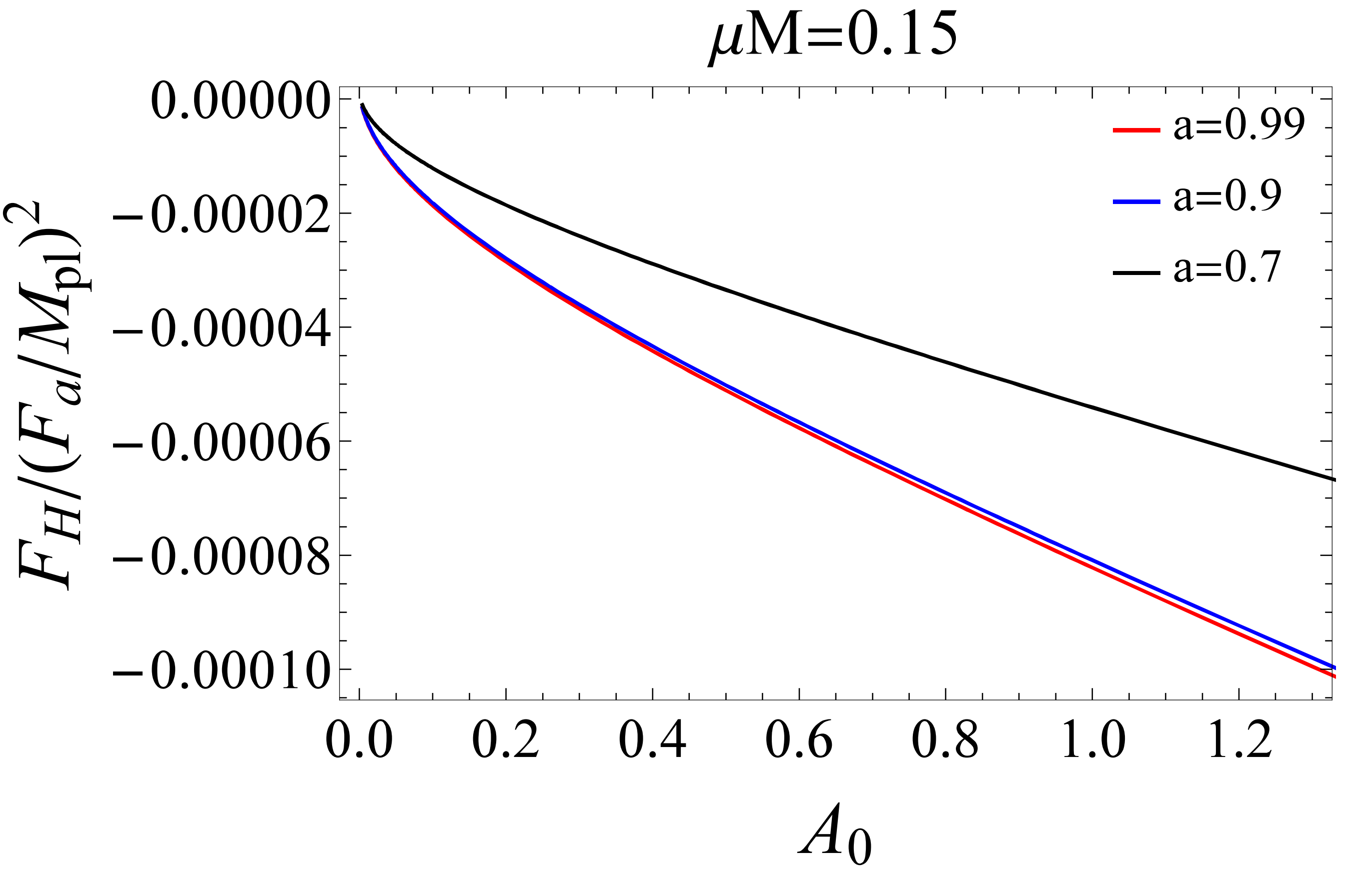


Energy flux@infinity

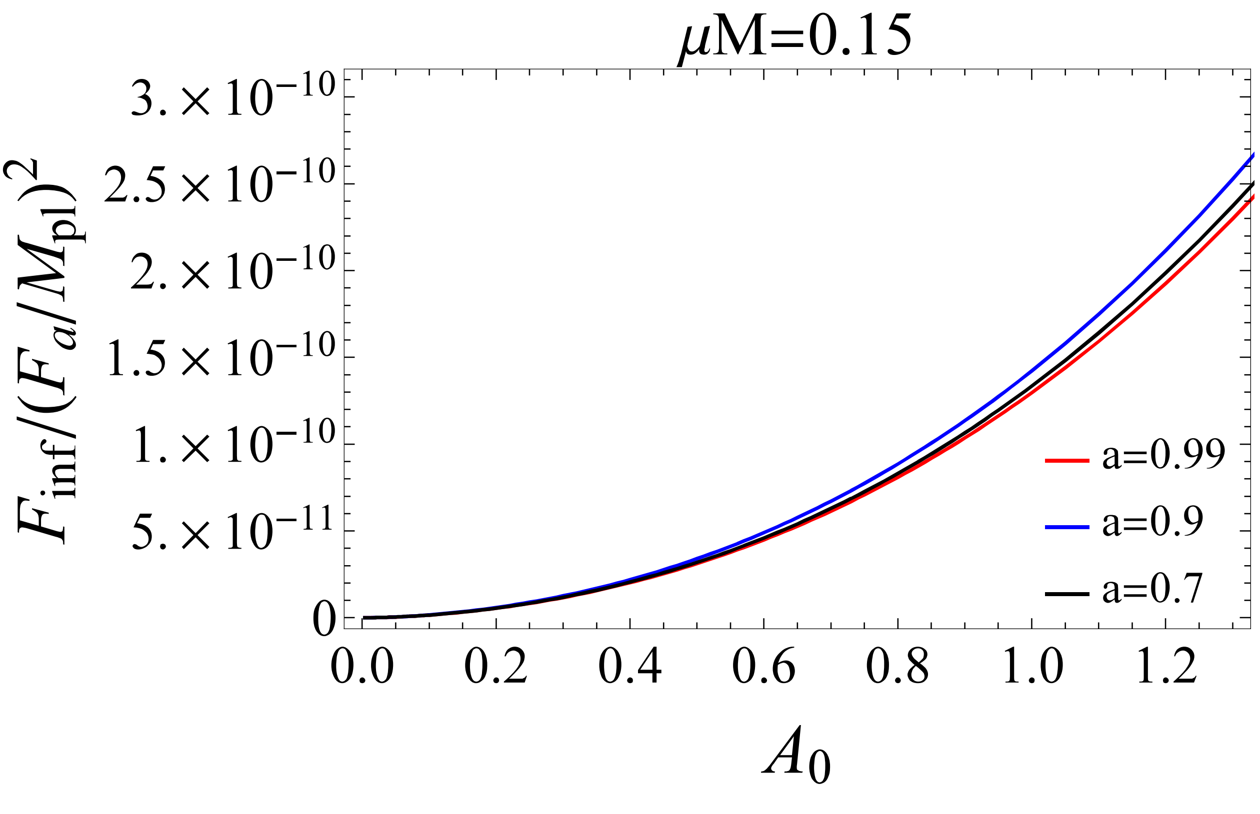


Wavelength of unbounded axion
>>size of cloud
Emission to infinity is surpressed

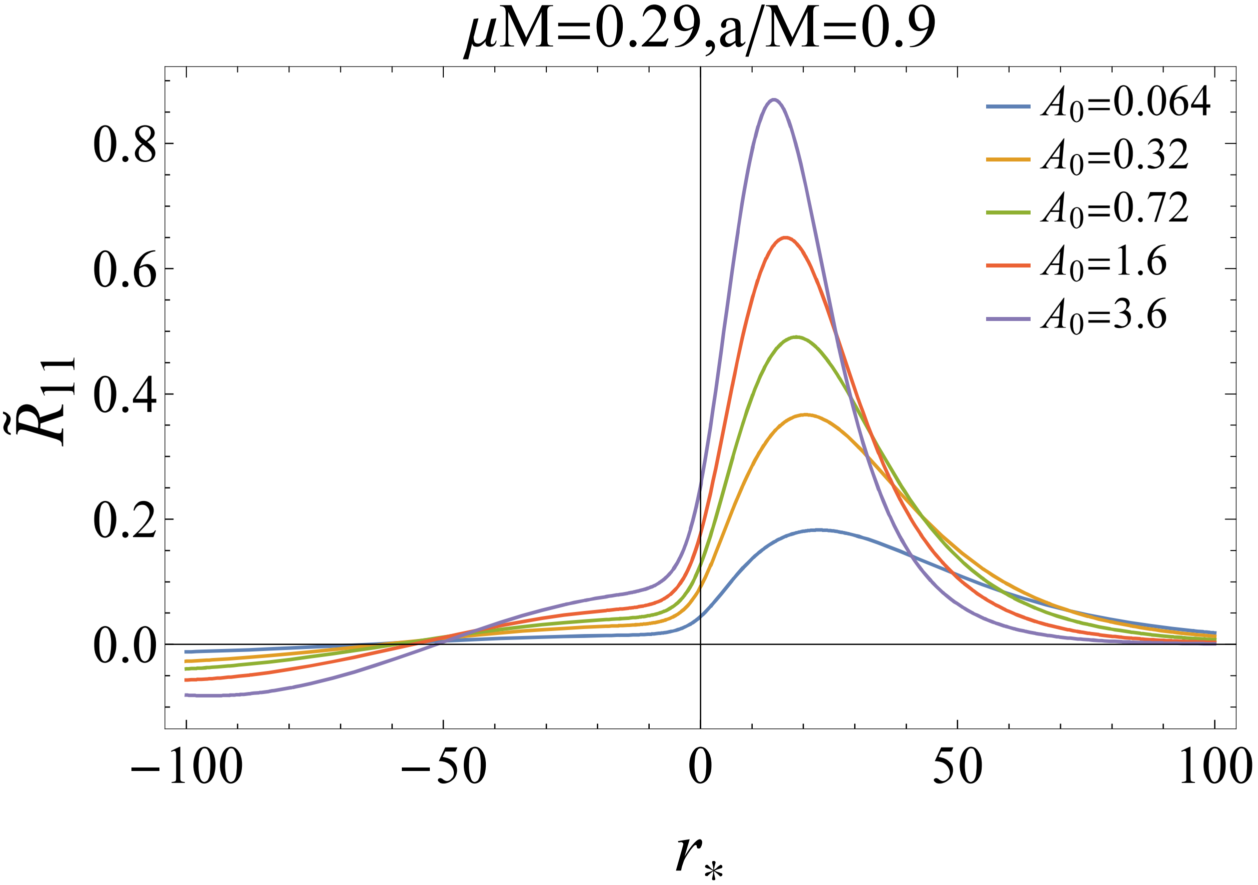
Result(energy flux@horizon)



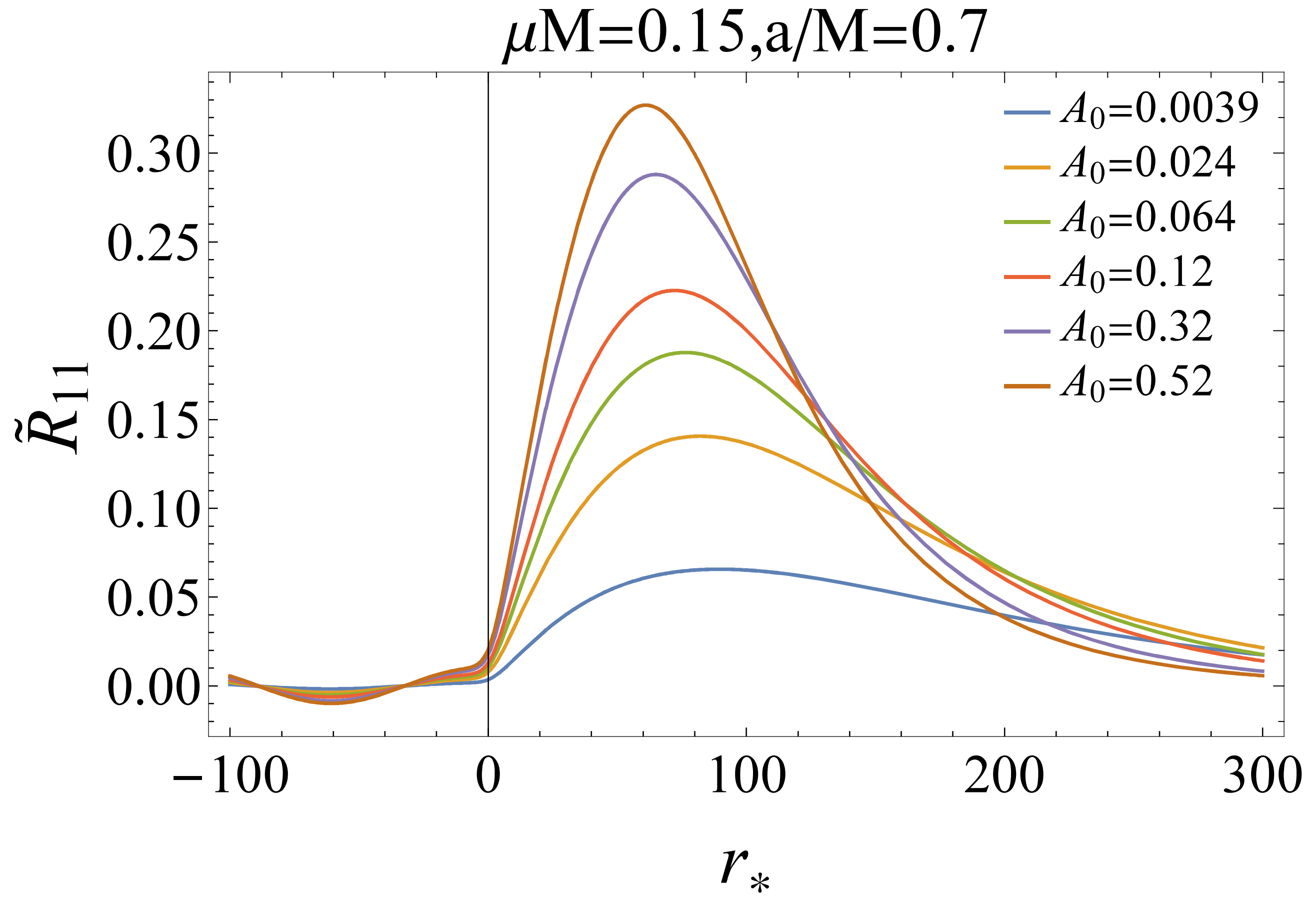
Result(energy flux@infinity)



Field configuration



Result(configuration)



Toy model

$$S_{\rm NR} = F_a^2 \int dt d^3x \left(\frac{i}{2}\left(\psi^* \dot{\psi} - \psi \dot{\psi}^*\right) - \frac{1}{\mu}(\partial_i \psi)^2 + \frac{\mu}{r}|\psi|^2 + \mu^2 \sum_{n=2} \frac{(-1/2)^n}{(n!)^2} \frac{|\psi|^{2n}}{\mu^n} \right)$$

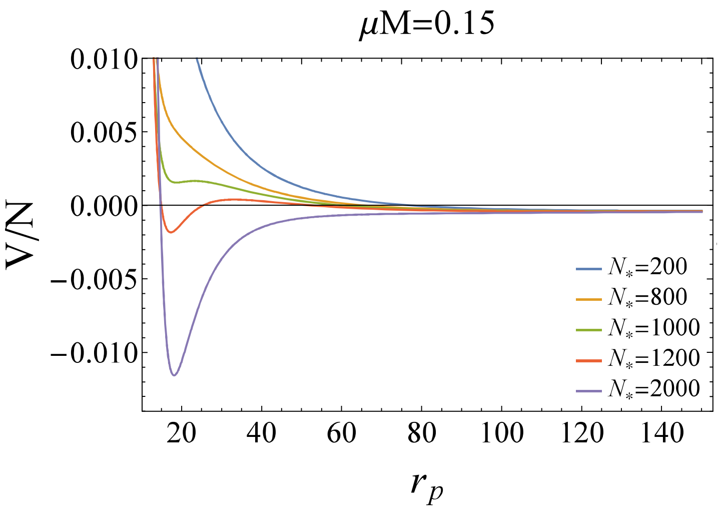
$$\phi = \frac{1}{\sqrt{2\mu}}\left(\psi e^{-i\mu t} + \psi^* e^{+i\mu t}\right)$$

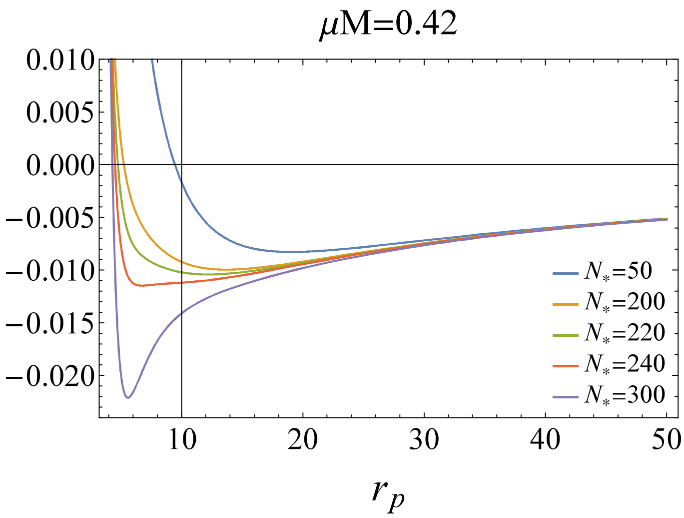
Potential energy of the configuration

$$\begin{aligned}\frac{V}{N} =& \frac{r_p^2 + 3\sigma^2}{8\mu\sigma^2(r_p^2+\sigma^2)} + \frac{1}{\mu(r_p^2+\sigma^2)} - \frac{\mu r_p}{r_p^2+\sigma^2} \\ &- \mu^2\left(\frac{N_*}{80\sqrt{2\pi}\mu^2(r_p^2+\sigma^2)} - \frac{3N_*^2}{4480\pi\mu^5\sigma^2(r_p^2+\sigma^2)^2} + \cdots\right)\end{aligned}$$

$$\psi = A_p e^{-\frac{(r-r_p)}{4\sigma^2}} Y_{l_0 m_0}(x) e^{+im_0\varphi} \qquad N = \int d^3x\, |\psi|^2 \sim 2\pi\sqrt{2\pi}\sigma(r_p^2+\sigma^2)A_p^2$$

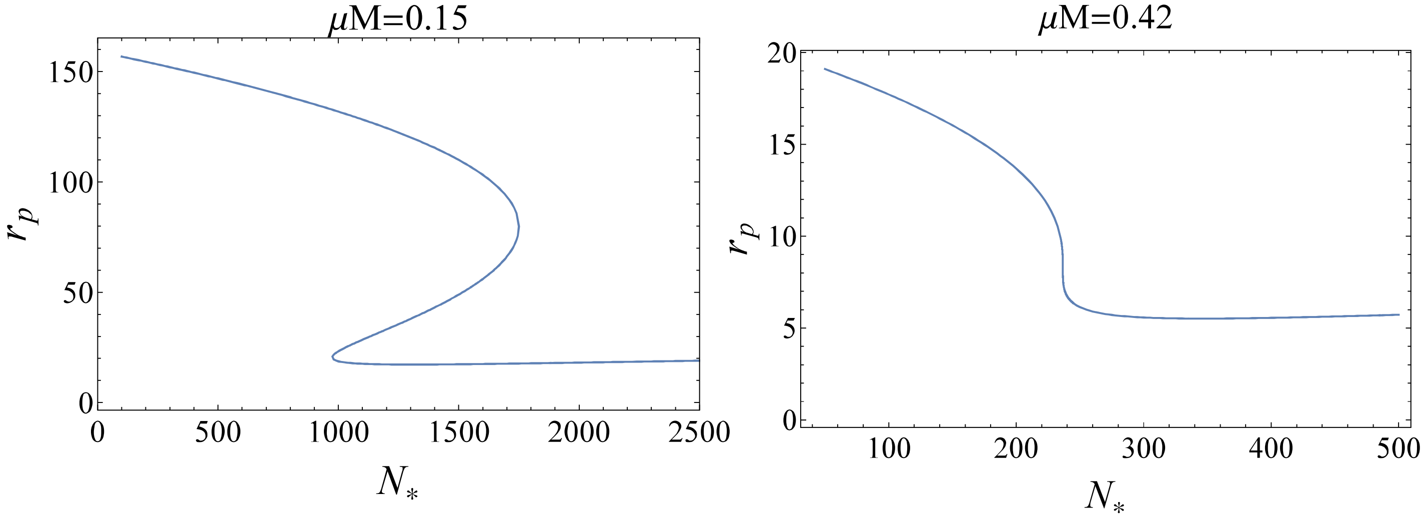
Potential





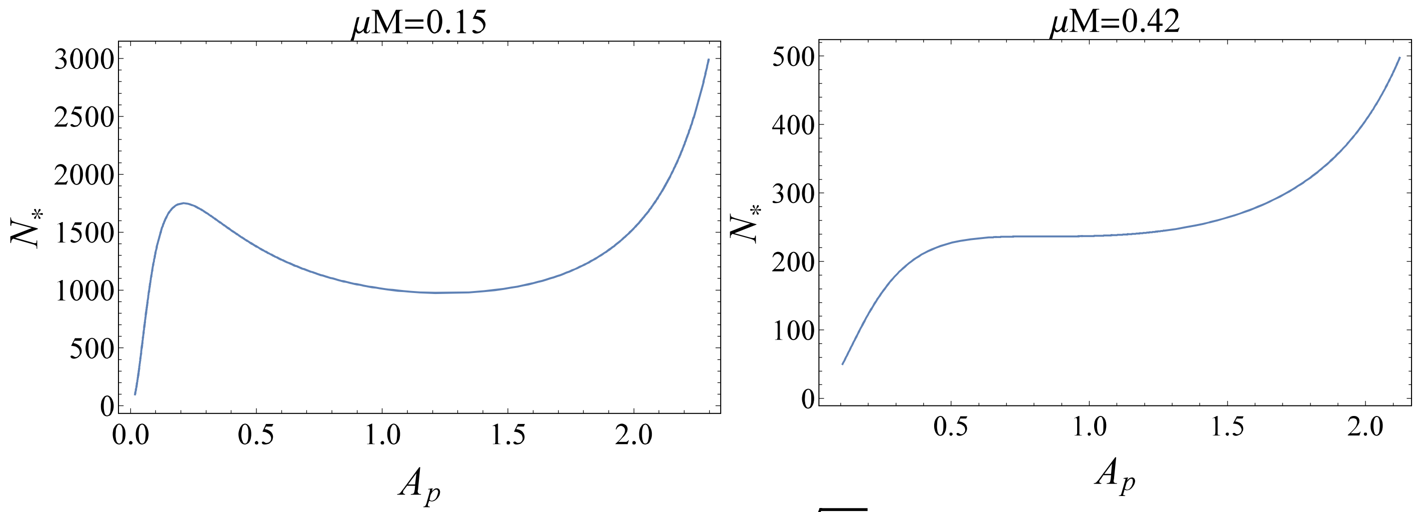
- Deep potential well is made for large N_*
- Local maximum appear for small μM

Equilibrium



- r_p at equilibrium
- Moves from left to right
- For small μM, r_p jumps at som N_*
- No jump for large μM

Equilibrium



$$A_p = A_0/\sqrt{2\mu}$$

- Converted previous figure to peak amplitude
- Same behavior as our numerical calculation

Session D2b 10:45–12:00

[Chair: Kunihito Uzawa]

Takuya Takahashi

Kyoto University

"Can we detect the signature of axion clouds in black hole binaries?"

(15 min.)

[JGRG30 (2021) 120919]

Can we detect the signature of axion clouds in black hole binaries?

Takuya Takahashi (Kyoto University, D1)

with Hidetoshi Omiya and Takahiro Tanaka

based on arXiv:2112.05774

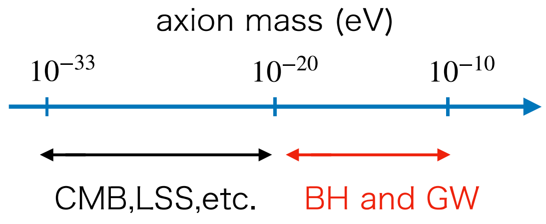
JGRG30 Dec 9th, 2021

- Introduction

Axion and Black Hole

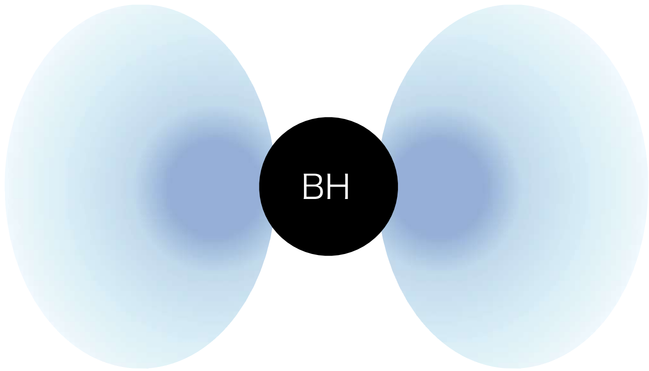
Axion-like particle

- Ultralight scalar field
- QCD axion/ string axion
- Dark matter candidate



BH superradiance

- energy extraction
- gravitationally bound

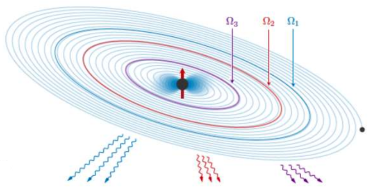


Axions form a cloud

Observation

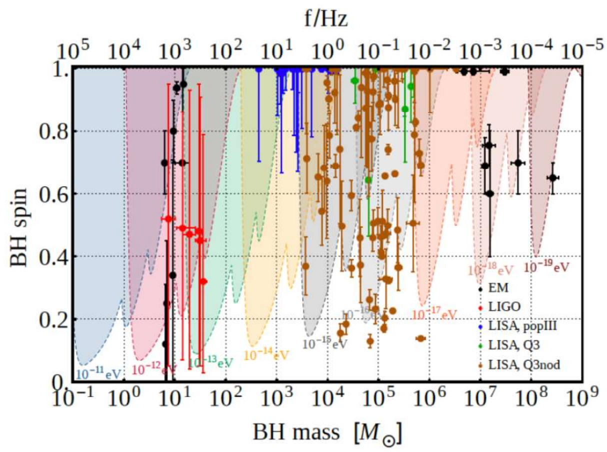
① Gravitational Wave

D.Baumann et al., 1912.04932



modulation of waveform
PN parameter, QNM, etc.

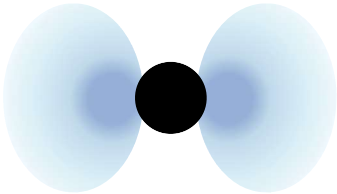
② BH parameter



R.Brito et al., arXiv:1706.06311

“forbidden region”
in mass and spin
distribution of BHs

Axion clouds in binary systems



hydrogen atom-like structure

$$i\frac{\partial}{\partial t}\psi = \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r}\right)\psi$$

$$\boxed{\alpha = M\mu}$$

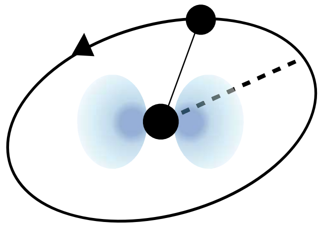
BH mass M

axion mass μ

$$\omega_{nlm} = (\omega_R)_{nlm} + i(\omega_I)_{nlm}$$

Axions initially occupy the fastest growing mode.

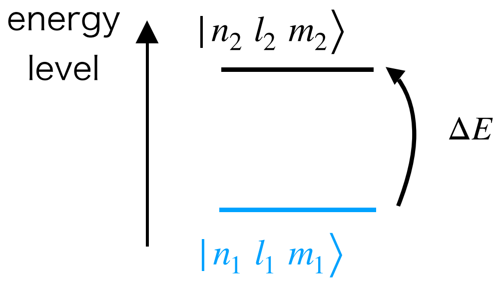
Orbital frequency Ω



tidal interaction

$$i\frac{\partial}{\partial t}\psi = \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r} + \underline{V_*(t)}\right)\psi$$

resonant level transition



Motivation

- To clarify the detectability with GWs.
- To make the constraints from BH distribution robust.

Our work

We extended the previous works and clarified the fate of axion clouds in binary systems.

① Higher-multipole moments
② Backreaction
③ Axion emission

- no self-interaction
- equal mass binaries

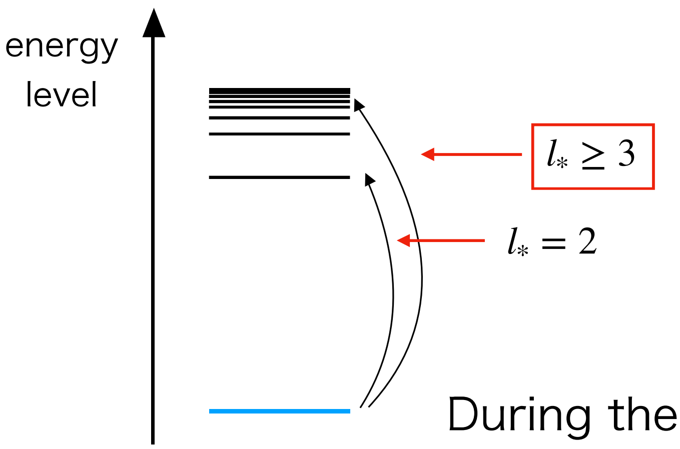
- Our work

① Higher-multipole moments

tidal potentail

$$V_*(t) = \sum_{l_* \geq 2} \sum_{|m_*| \leq l_*} \mathcal{E}_{l_* m_*} e^{-i m_* \Phi_*} \left(\frac{r^{l_*}}{R_*^{l_*+1}} \theta(R_* - r) + \frac{R_*^{l_*}}{r^{l_*}} \theta(r - R_*) \right)$$

In the previous works, the leading $l_* = 2$ has been focused.



resonance frequency

$$\Omega_0 = \frac{\Delta E}{\Delta m}$$

$|\Delta m| \leq l_*$, $\Delta E \simeq \mu \frac{\alpha^2}{2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

large l_* $\longrightarrow$ small Ω_0

During the binary inspiral,
higher-multipole moments work earlier.

Level transition

D.Baumann et al., arXiv:1912.04932

- Two level system $\psi = c_1(t)\varphi_1 + c_2(t)\varphi_2$
- Linear orbital evolution $\Omega(t) = \Omega_0 + \gamma t$

Transition is described by the Hamiltonian,

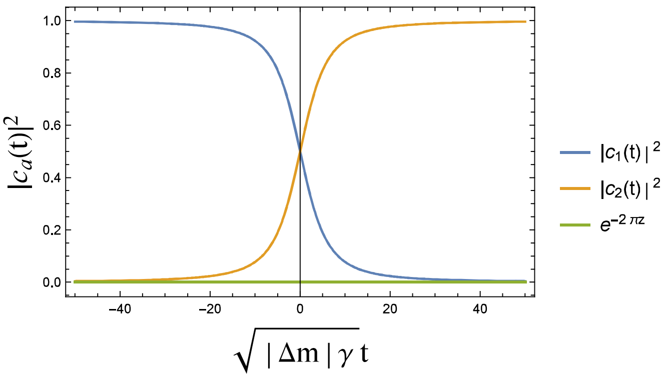
$$i\frac{\mathrm{d}c_i}{\mathrm{d}t} = \sum_{j=1,2} \mathcal{H}_{ij} c_j$$

$$\mathcal{H} = \begin{pmatrix} \pm\frac{\Delta m}{2}\gamma t & \eta \\ \eta & \mp\frac{\Delta m}{2}\gamma t \end{pmatrix}$$

transition rate

$$|c_2(+\infty)|^2 = 1 - \exp(-2\pi z)$$

normalized particle number



② Backreaction

angular momentum transfer to orbital motion

angular momentum of the cloud

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \boxed{\gamma} \underline{\pm 3R_J\Omega_0 \frac{\mathrm{d}}{\mathrm{d}t}\boxed{\left[m_1|c_1|^2 + m_2|c_2|^2\right]}}$$

radiation reaction backreaction

We describe transition including backreaction by non-linear model Hamiltonian.

$$\boxed{\mathcal{H} = \begin{pmatrix} \pm\frac{\Delta m}{2}\gamma t + \boxed{\frac{3|\Delta m|^2}{2}R_J\Omega_0|c_2|^2} & \eta \\ \eta & \mp\frac{\Delta m}{2}\gamma t - \boxed{\frac{3|\Delta m|^2}{2}R_J\Omega_0|c_2|^2} \end{pmatrix}}$$

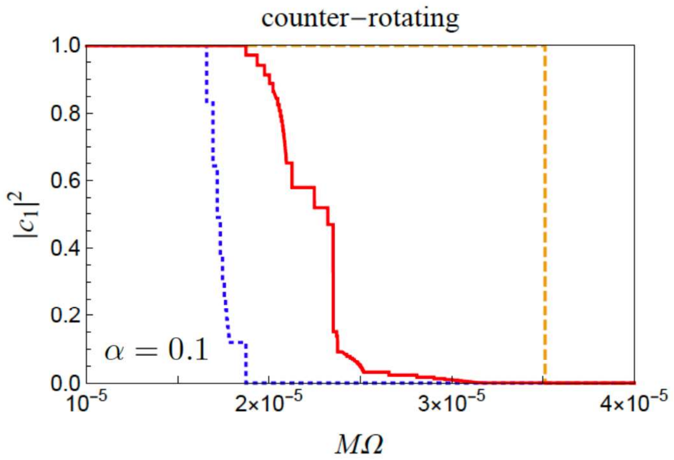
case (1) : orbit sinks ⟶ transition rate decrease

case (2) : orbit floats ⟶ transition rate increase

② Backreaction

1. Analytic derivation of the transition rate in a strongly non-linear regime
2. Backreaction to the hyperfine split thorough the change in geometry

Axions are transferred to higher levels with small decay rate gradually, irrespective of direction of orbital motion.

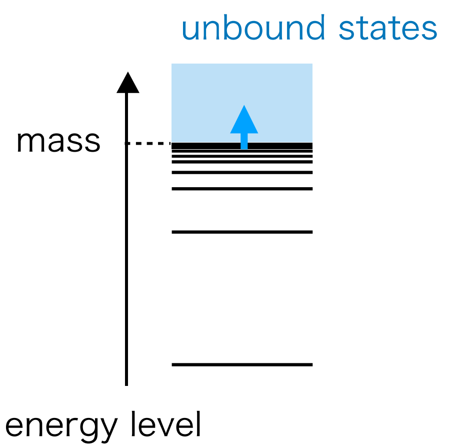


only quadrupole

+ higher-multirupole

+ higher-multiupole
+ backreaction

③ Axion emission



Axions transferred to higher levels can be further excited to unbound states by tidal interaction.

$$\left(i\partial_t + \frac{1}{2\mu}\nabla^2 + \frac{\alpha}{r}\right)\psi^r = V_*\psi^b$$

$$\psi^r(x) = \int d^4x' G(x,x') V_*\psi^b(x')$$

normalized particle number flux

$$F = \int d\theta \sin\theta d\varphi \frac{r^2}{2\mu i}\left(\psi^{r*}\partial_r\psi^r - \psi^r\partial_r\psi^{r*}\right)$$

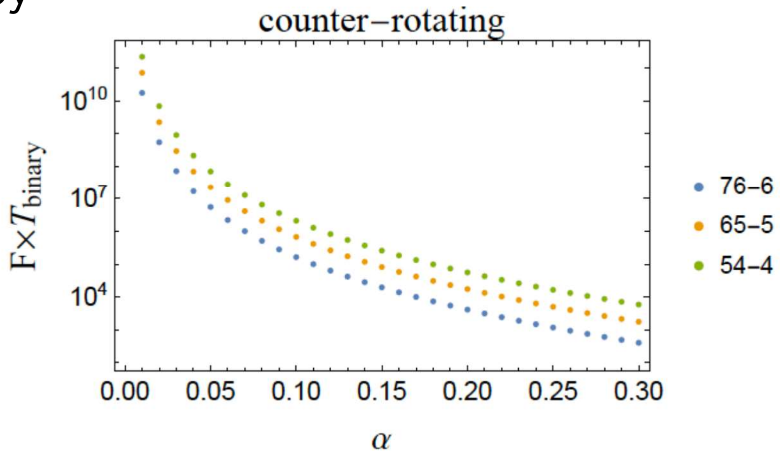
$$= \sum_{lm}\sum_{l_* m_*}\frac{4\mu}{k}\left|\int d^3x' R^0_{kl} Y^*_{lm} V_{*,l_* m_*} R_{n_0 l_0} Y_{l_0 m_0}\right|^2$$

③ Axion emission

Comparison with time scale of the evolution of orbital frequency

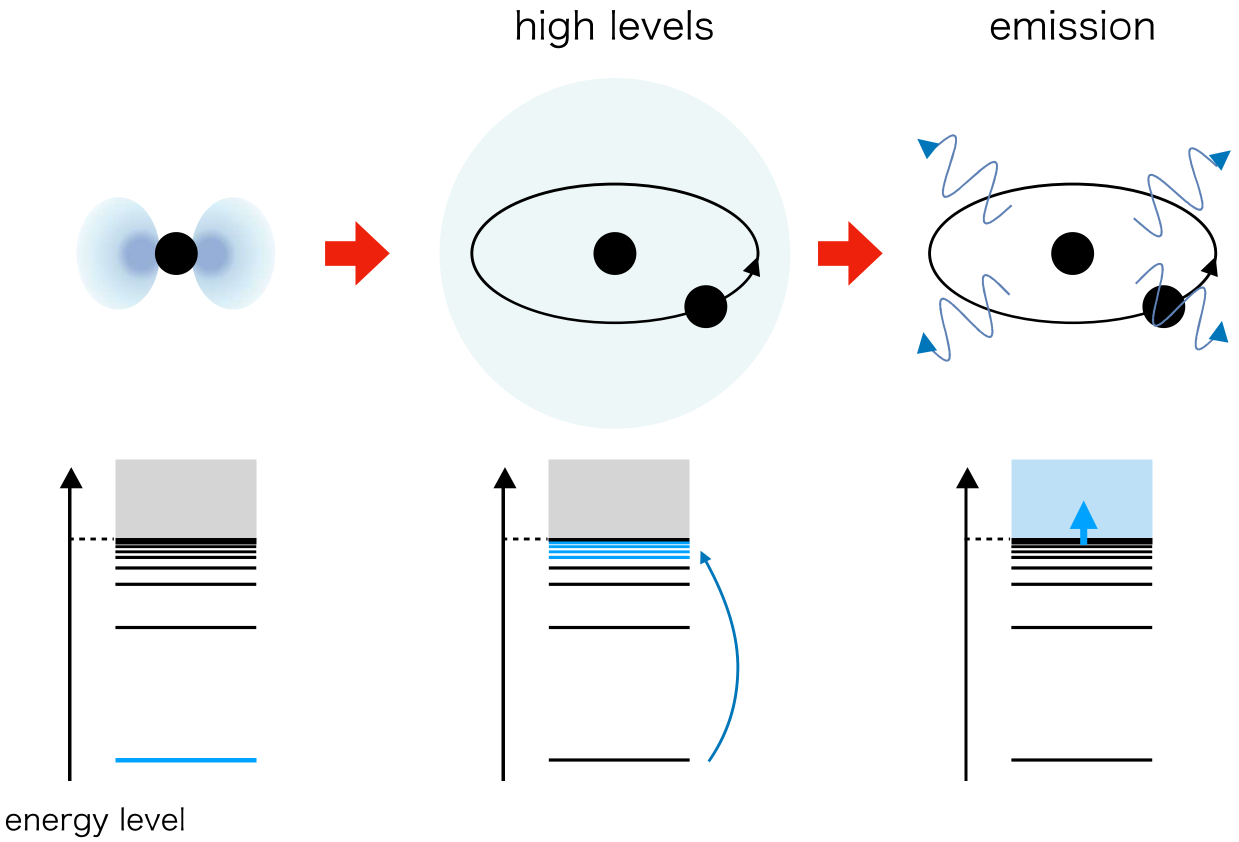
time scale

$$T_{\rm binary} \equiv \frac{\Omega}{\gamma}$$



Normalized particle number flux is sufficiently large to deplete the cloud.

Summary



Conclusion and Discussion

- We studied the dynamics of axion clouds in almost equal mass binary systems.

- Axion clouds disappear through the emission, without being reabsorbed by BHs.

- It may be difficult to detect the modulation of the gravitational waveform.
- The constraints from BH distribution will not be altered.

Session D2b 10:45–12:00

[Chair: Kunihito Uzawa]

Takeshi Shinohara

Nagoya Univ. Mathematics

"Divergence equations and uniqueness theorem of static spacetimes with conformal scalar hair"

(15 min.)

[JGRG30 (2021) 120920]

Divergence equations and uniqueness of static spacetime with conformal scalar hair

Takeshi Shinohara
Department of Mathematics, Nagoya University

Ref. Shinohara, Tomikawa, Izumi and Shiromizu
PTEP 2021 (2021) 9, 093E02, Prog Theor Exp Phys (2021)

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Introduction

Uniqueness theorem for black hole spacetime is one of important consequence in general relativity.

In Israel (1967) and Robinson (1977), divergence equations are used in the proof of uniqueness of vacuum static black hole solution.

However, the derivation of divergence equation is non-trivial.

Nozawa et al (2018) gave a systematic derivation of divergence equation.

Meanwhile, using divergence equations, Tomikawa et al (2017) discussed the uniqueness of static black hole spacetime in Einstein-conformal scalar system.

We will apply Nozawa et al's procedure for the proof of the uniqueness of static spacetime in Einstein-conformal scalar system.

	vacuum static spacetime	vacuum static spacetime with conformal scalar hair outside photon surface
Uniqueness proof based on divergence equation	Israel (1967) Robinson (1977)	Tomikawa et al (2017)
Systematic derivation	Nozawa et al (2018)	Our work

Contents

Einstein-conformal scalar system

- action $S = \int d^4x \sqrt{-g}\left[\left(-\frac{1}{2\kappa}+\frac{1}{12}\phi^2\right)R+\frac{1}{2}(\nabla\phi)^2\right]$

➡ field equations
$$\begin{cases}\left(1-\frac{\kappa}{6}\phi^2\right)R_{\mu\nu} = \kappa\left(\frac{2}{3}\nabla_\mu\phi\nabla_\nu\phi-\frac{1}{6}g_{\mu\nu}(\nabla\phi)^2-\frac{1}{3}\nabla_\mu\nabla_\nu\phi^2\right)\\ \nabla^2\phi = 0\end{cases}$$

Einstein equation is singular at $\phi = \pm\sqrt{6/\kappa} =: \phi_p$

- Bocharova-Bronnikov-Melnikov-Bekenstein (BBMB) solution

$$\begin{cases} g = -\left(1-\frac{m}{r}\right)^2 dt^2 + \left(1-\frac{m}{r}\right)^{-2} dr^2 + r^2 d\Omega_2^2 \\ \phi = \pm\sqrt{\frac{6}{\kappa}}\frac{m}{r-m}\end{cases}$$

m : mass
$r = m$: event horizon
$r = 2m$: photon sphere

Uniqueness of static spacetime in Einstein-conformal scalar

[Tomikawa et al, 2017]

Asymptotically flat, 4-dimension, vacuum static solution with conformal scalar field is unique outside photon sphere.

key divergence equation in proof:

EOM $\xrightarrow[\text{non trivial}]{}$

$$\begin{cases} D_i\left(\dfrac{(\rho k-2)n^i}{(2V-1)\rho^{3/2}}\right) = -\dfrac{1}{2V-1}\left(\rho^{-\frac{1}{2}}\tilde{k}_{ij}\tilde{k}^{ij} + \rho^{-\frac{3}{2}}\mathcal{D}^2\rho\right) \\ D_i\left((k\xi+\eta)n^i\right) = -\left(\tilde{k}_{ij}\tilde{k}^{ij} + \rho^{-1}\mathcal{D}^2\rho\right)\xi \end{cases}$$

$\xi := (2V-1)\rho^{-1/2}$

$\eta := 2(2V+1)\rho^{-3/2}$

$\Rightarrow$ systematic derivation

Contents

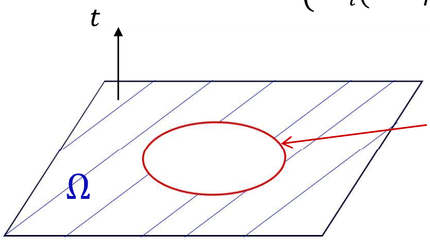
1. Introduction
2. Einstein-conformal scalar system
3. Setup
4. Systematic derivation of divergence equation
5. Summary

Static spacetime and field equations

- static metric $\quad ds^2 = -V^2(x^k)dt^2 + g_{ij}(x^k)dx^i dx^j$

- field equations $\Rightarrow$

$$\begin{cases} \left(1-\dfrac{\phi^2}{\phi_p^2}\right) VD^2V = \dfrac{\kappa}{6}\left[V^2(D\phi)^2 + 2\phi V D^i V D_i \phi\right] \\ \left(1-\dfrac{\phi^2}{\phi_p^2}\right)\left({}^{(3)}R_{\mu\nu} - V^{-1}D_iD_jV\right) = \kappa\left(\dfrac{2}{3}D_i\phi D_j\phi - \dfrac{1}{6}g_{ij}(D\phi)^2 - \dfrac{1}{3}\phi D_iD_j\phi\right) \\ D_i\left(VD^i\phi\right) = 0 \end{cases}$$



$S_p : \phi = \phi_p$

$\phi_p = \pm\sqrt{6/\kappa}$

D_i: covariant derivative on $t = const.surface$

- asymptotic condition at S_∞

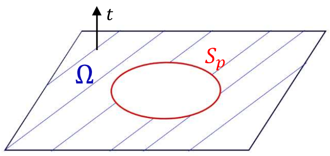
$$V = 1 - \frac{m}{r} + \mathcal{O}\left(\frac{1}{r^2}\right), \quad g_{ij} = \left(1+\frac{2m}{r}\right)\delta_{ij} + \mathcal{O}\left(\frac{1}{r^2}\right), \quad \phi = \mathcal{O}\left(\frac{1}{r}\right)$$

m : mass

Relation between V and ϕ

Set $\quad \Phi \coloneqq (1+\varphi)V, \quad \varphi \coloneqq \phi_p \quad (0 \le \varphi \le 1) \quad (\phi_p = \pm\sqrt{6/\kappa}\phi)$

[Tomikawa, Izumi, Shiromizu, 2017]



(0,0)-component of Einstein equation,
scalar field equation

$\Rightarrow \quad D_i\left[(1-\varphi)D^i\,\Phi\right] = 0 \quad \Rightarrow \quad \Phi = 1 \quad \Rightarrow \quad \varphi = V^{-1} - 1$

boundary conditions

$\varphi|_{S_p} = 1, \quad \varphi|_{S_\infty} = 0, \quad \Phi|_{S_\infty} = 1$

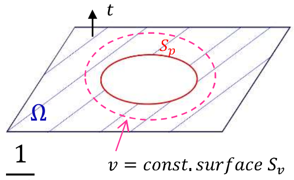
$S_p \leftrightarrow V = \dfrac{1}{2}$

Einstein equation $\xRightarrow[\varphi = V^{-1}-1]{}$

$$\begin{cases} D^2 v = 0 \qquad v \coloneqq lnV \\ (2V-1)\,{}^{(3)}R_{ij} = D_iD_jv + (4V+1)D_ivD_jv - g_{ij}(Dv)^2 \end{cases}$$

$\Rightarrow$ v is regarded as “radial” coordinate

Boundary condition at S_p



- curvature invariant

$$R_{\mu\nu}R^{\mu\nu} = \underbrace{\frac{1}{(2V-1)^2\rho^2}}_{\to\infty\ (V\to\frac{1}{2})}\left[\underbrace{\left(2(1-V)k_{ij}-\frac{1}{\rho}h_{ij}\right)^2}_{\to 0} + \underbrace{\left(-2(1-V)k+\frac{1+2V}{\rho}\right)^2}_{\to 0\ (V\to\frac{1}{2})} + \underbrace{\frac{8(1-V)^2}{\rho^2}(\mathcal{D}\rho)^2}_{\to 0}\right] + \frac{1}{\rho^4}$$

h_{ij} : metric on S_v, $\rho \coloneqq \left(D_i v D^i v\right)^{-1/2}$, $\mathcal{D}_i$: covariant derivative on S_v, k_{ij} : extrinsic curvature of S_v

- regularity at S_p

➡ boundary condition at S_p: $\mathcal{D}_i\rho\Big|_{S_p} = 0,\quad k_{ij}\Big|_{S_p} = \frac{1}{\rho_p}h_{ij}\Big|_{S_p}$

Contents

Generalization of the divergence equations

find divergence equation of following form :

$$\begin{cases} D_i\left(\dfrac{(\rho k-2)n^i}{(2V-1)\rho^{3/2}}\right) = -\dfrac{1}{2V-1}\left(\rho^{-\frac{1}{2}}\tilde{k}_{ij}\tilde{k}^{ij}+\cdots\right) \\ D_i\left((k\xi+\eta)n^i\right) = -\left(\tilde{k}_{ij}\tilde{k}^{ij}+\cdots\right)\xi \end{cases}$$

$$D_i J^i = [\underline{\text{terms with a definite sign}}]$$

a sum of tensors vanish if spacetime is BBMB solution

$+[(\ \ )^2+(\ \ )^2+\cdots+(\ \ )^2]$

set J_i as $\quad J_i = f_1(v)g_1(\rho)D_i\rho + f_2(v)g_2(\rho)D_iv$

$$\Rightarrow \quad D_iJ^i = -\rho^3 f_1 g_1\left[|H_{ij}|^2 - \left(\frac{g_1'}{\rho g_1}+\frac{3}{\rho}\right)|H_i|^2\right] + \rho f_1 g_1 D^i v H_i S_1 + \frac{1}{\rho^2} f_1 g_2 S_2$$

$S_1, S_2 \rightarrow$ 0 (control sign)

$$\begin{cases} H_{ij} := D_iD_jv + \dfrac{3V}{1-V}D_ivD_jv - \dfrac{V}{\rho^2(1-V)}g_{ij} \\ H_i := -\rho^2 H_{ij}D^jv \end{cases} \qquad \begin{cases} S_1 := \dfrac{f_1'}{f_1} + \dfrac{f_2}{f_1}\dfrac{g_2'}{g_1} + \dfrac{(4V-1)(3V-1)}{(2V-1)(1-V)} + \dfrac{4\rho V}{1-V}\dfrac{g_1'}{g_1} \\ S_2 := \dfrac{2\rho V}{1-V}\dfrac{g_1}{g_2}S_1 - \dfrac{4\rho V^2}{(1-V)^2}\dfrac{g_1}{g_2}\left[\rho\dfrac{g_1}{g_1} + \dfrac{8V^2-7V+2}{2V(2V-1)}\right] + \dfrac{f_2'}{f_1} \end{cases}$$

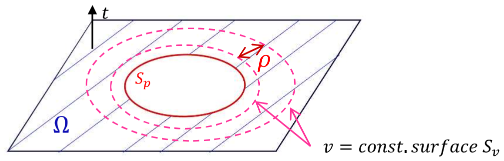
H_{ij} vanishes if spacetime is BBMB solution

Generalized divergence equation

set $g_1 = -c\rho^{-(c+1)}$, $g_2 = \rho^{-c}$ [Nozawa et al, (2018)]

$\Rightarrow$ ordinal differential equations for $f_1(V)$ and $f_2(V)$

$$\Rightarrow \begin{cases} f_1 = \dfrac{1}{4}(2V-1)^{-1}(1-V)^{1-2c}(a+b(2V-1)^2) \\ f_2 = \dfrac{1}{4}(2V-1)^{-1}(1-V)^{-2c}[(a+b)(2cV-2V+1)-8bcV^2(1-V)] \end{cases}$$



a, b:constant

$$\Rightarrow \quad \boxed{D_iJ^i = \frac{cf_1}{2\rho^c}[|2\rho^2 H_{i[j}D_{k]}v - g_{i[j}H_{k]}|^2 + (2c-1)|H_i|^2]}$$

If $\quad f_1 \geq 0\ ,\quad c \geq \dfrac{1}{2} \quad \Rightarrow \quad D_iJ^i \geq 0$ on Ω

divergence equations found by Tomikawa et al (2017), $\Leftrightarrow$ $b=0,\ c=\frac{1}{2}$, $\quad a=0,\ c=\frac{1}{2}$

Uniqueness of static solution with conformal scalar hair

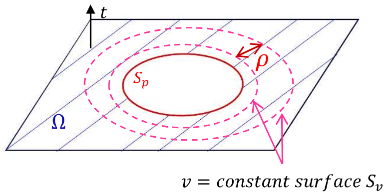
$$D_i J^i = \frac{cf_1}{2\rho^c}\left[\left|2\rho^2 H_{i[j}D_{k]}v - g_{i[j}H_{k]}\right|^2 + (2c-1)|H_i|^2\right] \geq 0$$

boundary condition of Ω

$$\pi\rho_p^2\left(\frac{4m}{\rho_p}\right)^{1-c} \leq \frac{1}{2}\pi\rho_p^2\chi$$

χ : Euler number

$\rho_p := \rho|_{S_p}$



Consider inequality in range of c ($c \geq 1/2$) :

$c = 1$ gives $\chi \geq 2$ ➡ topology of S_p is spherical ($\chi = 2$) ➡ equality should hold

➡ $D_i J^i = 0$ ➡ $H_{ij} = 0$ ➡ Ω : spherically symmetric ➡ BBMB solution

[Xanthopoulos and Zannias, 1991]

Contents

1. Introduction
2. Einstein-conformal scalar system
3. Setup
4. Systematic derivation of divergence equation
5. Summary

Summary

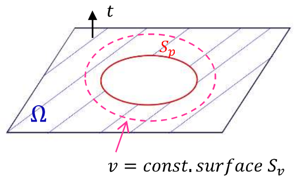
In Tomikawa et al (2017), for Einstein-conformal scalar system, the divergence equations were used to prove uniqueness.

We could derive the divergence equation systematically using Nozawa et al's procedure.

Using the divergence equation, we proved the uniqueness of the static solution with conformal scalar field outside photon sphere.

The deep physical/mathematical reason is expected to be hidden behind the presence of such a procedure.

	vacuum static spacetime	vacuum static spacetime with conformal scalar hair outside photon surface
Uniqueness proof using divergence equation	Israel (1967) Robinson (1977)	Tomikawa et al (2017)
Systematic derivation	Nozawa et al (2018)	Our work



- Kretschmann invariant

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{4}{\rho^2}\left[k_{ij}k^{ij} + \frac{2}{\rho^2}(\mathcal{D}\rho)^2 + \left(k - \frac{1}{\rho}\right)^2\right]$$
$$+ \frac{1}{(2V-1)^2\rho^2}\left[\left(k_{ij} - \frac{1}{\rho}h_{ij}\right)^2 + \frac{2}{\rho^2}(\mathcal{D}\rho)^2 + \left(k - \frac{4V}{\rho}\right)^2\right] - \frac{1}{\rho^4}$$

h_{ij} : metric on S_v, $\rho := \left(D_i v D^i v\right)^{-1/2}$, $\mathcal{D}_i$: covariant derivative on S_v, k_{ij} : extrinsic curvature of S_v

boundary condition at S_p: $\mathcal{D}_i\rho\Big|_{S_p} = 0,\quad k_{ij}\Big|_{S_p} = \frac{1}{\rho_p}h_{ij}\Big|_{S_p}$

Session D3a 14:30–16:00

[Chair: Tomo Takahashi]

Yo Toda

Hokkaido University

"Hubble tension with an extra radiation and neutrino degeneracy"

(15 min.)

[JGRG30 (2021) 120921]

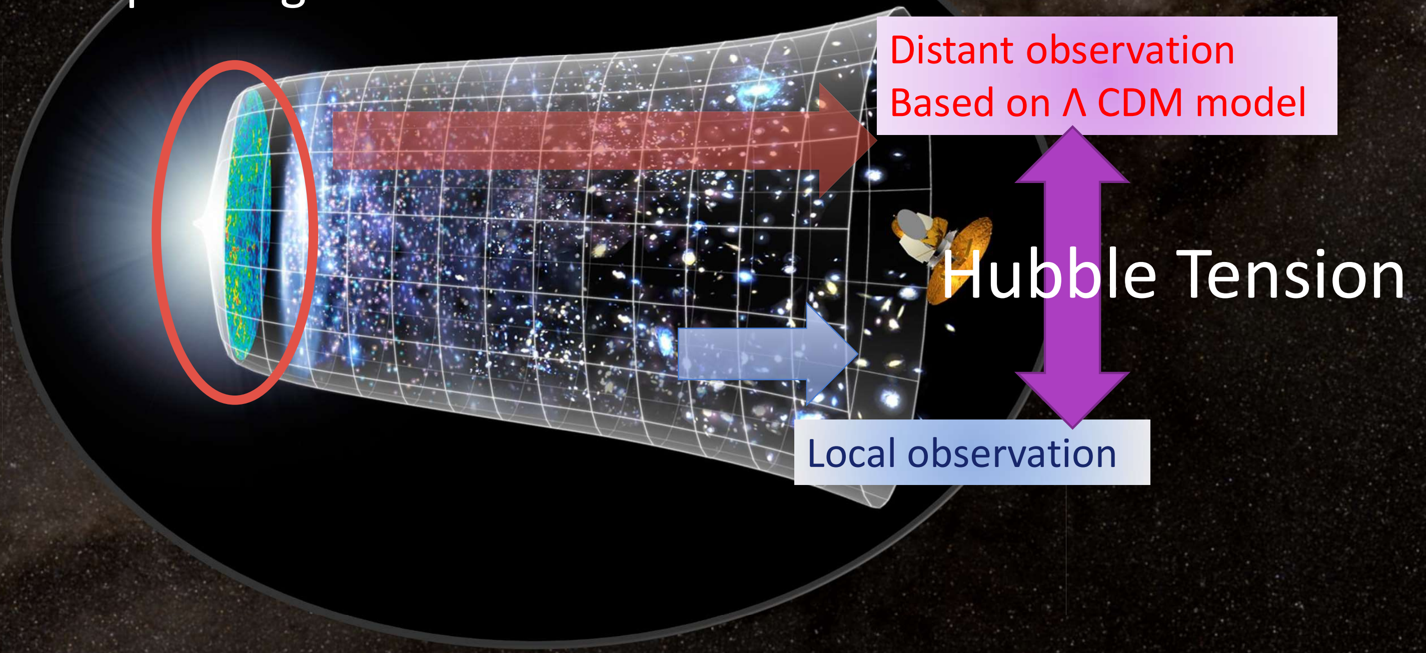
HUBBLE TENSION WITH EXTRA RADIATION AND NEUTRINO DEGENERACY

YO TODA

HOKKAIDO UNIVERSITY

Osamu Seto, Yo Toda *Phys.Rev.D* 104 (2021) 6, 063019

The Λ CDM model does well
in explaining the evolution of our Universe.



TODAY I WILL

Focus on the extra radiation and neutrino degeneracy

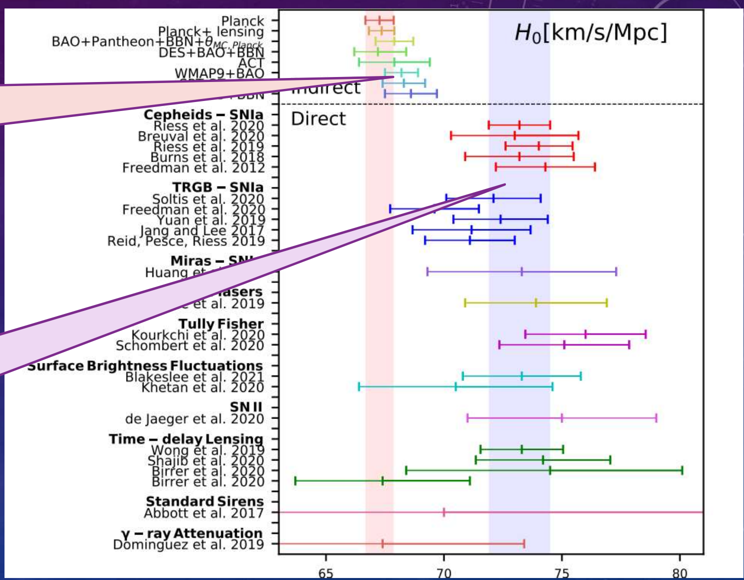
Treat the extra radiation parameter N_{eff} as independent of degeneracy ξ because we consider the sterile neutrinos or axions under the degeneracy

Conclude that
the combination of extra radiation and neutrino degeneracy
is a promising solution

WHAT IS HUBBLE TENSION?

Distant observations suggest
$H_0 \fallingdotseq 67$ km/s/Mpc

Local observations suggest
$H_0 \fallingdotseq 74$ km/s/Mpc



Di Valentino, arXiv:2011.00246 [astro-ph.CO]

Beyond ΛCDM Physics?

EXTRA RADIATION

The relativistic degrees of freedom N_{eff}

(increased by dark radiation, axion, sterile neutrino …)

$$\rho_{\mathrm{radiation}} = \Big(\underbrace{1}_{\text{photon}} + \underbrace{\frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} N_{\mathrm{eff}}}_{\text{Neutrino} + \alpha} \Big) * \rho_{\mathrm{photon}}$$

$$N_{\mathrm{eff}} = \underbrace{3}_{\text{neutrino}} + \underbrace{0.046}_{e^+e^- \text{ annihilation}} + (\text{Extra contribution})$$

How contribute to CMB?

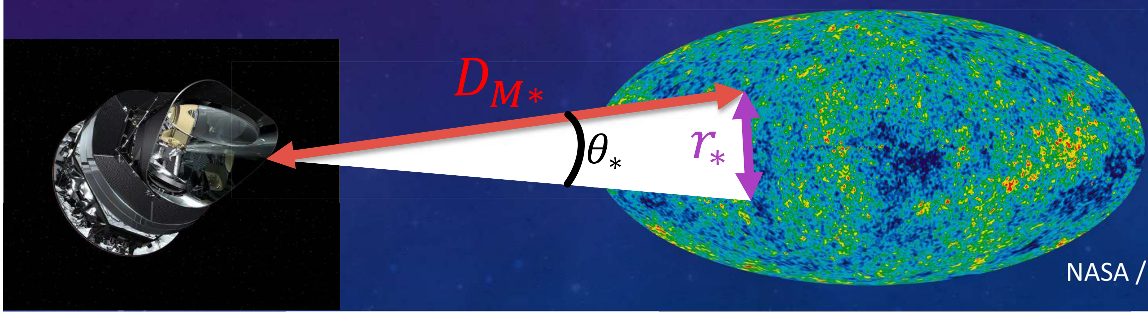
ANGULAR SIZE OF THE SOUND HORIZON

Directly Measured

Angular Size : $\theta_* = \dfrac{r_*}{D_{M*}} = (1.0411 \pm 0.0003) \times 10^{-2}$

$r_* = \int_0^{t_*} \dfrac{c_s d\tilde{t}}{a(\tilde{t})}$: comoving sound horizon at the recombination

$D_{M*} = \int_{t_*}^{t_0} \dfrac{d\tilde{t}}{a(\tilde{t})}$: comoving angular diameter distance



NASA / WMAP Science Team

ANGULAR SIZE OF THE SOUND HORIZON

Directly Measured

Angular Size : $\theta_* = \frac{r_*}{D_{M*}} = (1.0411 \pm 0.0003) \times 10^{-2}$

$\propto H_0 \frac{1}{\sqrt{\rho \text{ in the early universe}}}$

$\because \frac{dt}{a(t)} = \frac{dz}{H_0\sqrt{\rho(z)/\rho_0}}$

ρ : energy density

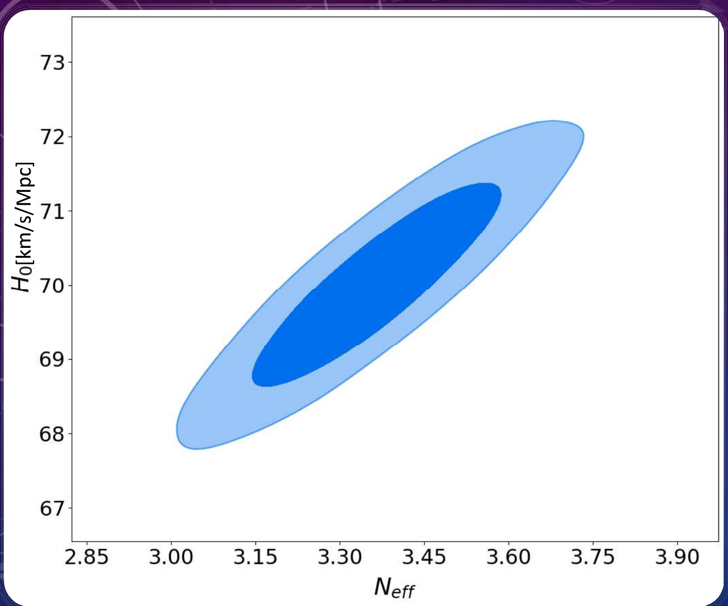
TO INCREASE H_0 …

Increase ρ in the early universe

Ex) **extra radiation**, early dark energy, …

Analysis results

EXTRA RADIATION RELIEVES THE HUBBLE TENSION

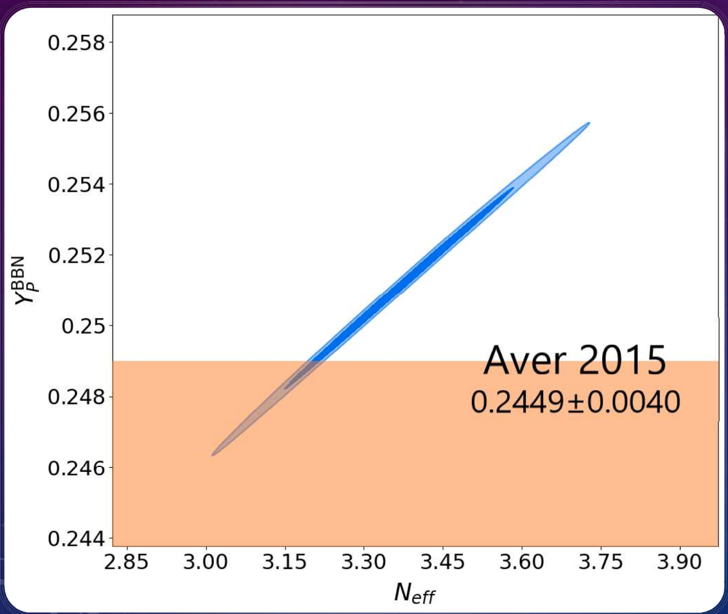


Higher N_{eff}
increase H_0

$N_{\text{eff}} = 3.046 + (\text{Extra contribution})$

Planck + Pantheon + BAO + R19

$N_{\rm eff}$ VS. HELIUM MASS FRACTION Y_P MEASUREMENT



Extra radiation increases :

- the expansion rate of the universe,
- the decoupling temperature of the weak interaction,
- the neutron-to-proton ratio

Larger $N_{\rm eff}$

increase Y_P

$$N_{\rm eff} = 3.046 + (\text{Extra contribution})$$

Planck + Pantheon + BAO + R19

Neutrino degeneracy

NEUTRINO DEGENERACY

The degeneracy parameter $\xi_i = \frac{\mu_{\nu i}}{T_\nu}$ $(i = e, \mu, \tau)$

μ_{ν_i} : chemical potential for neutrino ν_i

T_ν : temperature of neutrinos

Distribution functions

$$f_\nu = \frac{1}{\exp(p/T_\nu - \xi_i) + 1}$$

$$f_{\bar{\nu}} = \frac{1}{\exp(p/T_\nu + \xi_i) + 1}$$

Number Densities of neutrinos and antineutrinos

$$n_{\nu_i} + n_{\bar{\nu}_i} \propto T_{\nu_i}^3 (2(\xi_i/\pi)^2 + (\xi_i/\pi)^4) \rightarrow N_{\rm eff}$$

$$n_{\nu_i} - n_{\bar{\nu}_i} \propto T_{\nu_i}^3 (\pi^2 \xi_i + \xi_i^3) \rightarrow \text{BBN}$$

Positive electron neutrino degeneracy

$$n_{\nu_e} - n_{\bar{\nu}_e} \propto T_{\nu_e}^3(\pi^2\xi_e + \xi_e^3)$$

More neutrinos ν_e than antineutrinos $\bar{\nu}_e$

The process $p + \bar{\nu}_e \to n + e^+$ is more suppressed than $n + \nu_e \to p + e^-$

Neutron-to-proton ratio (n/p) decrease

Helium mass fraction Y_P decrease

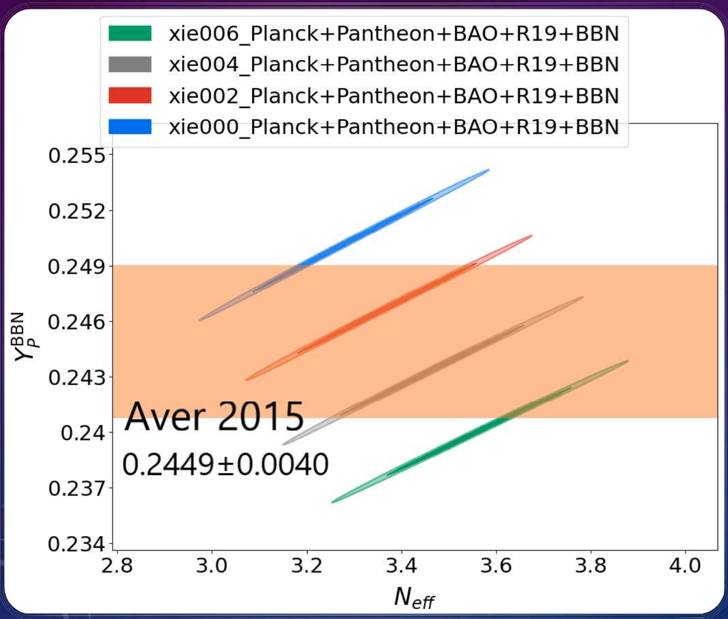
Next, I will show the results of analysis of

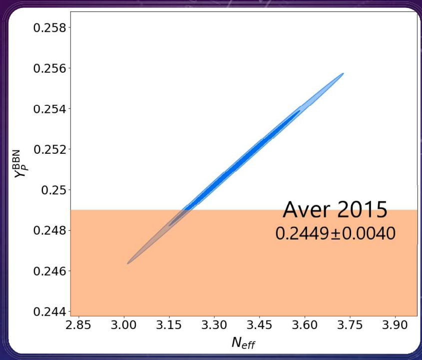
EXTRA RADIATION $\Delta N_{\rm eff}$

AND

ELECTRON NEUTRINO DEGENERACY ξ_e

Y_P VS. N_{eff} WITH THE DEGENERACY ξ_e





Larger N_{eff} increase Y_P

Larger ξ_e decrease Y_P

BEST-FIT

Parameter	ΛCDM	ξ_e=0	ξ_e=0.02	ξ_e=0.04	ξ_e=0.06
N_{eff}	3.046	3.243	3.31264	3.45451	3.63353
H_0	68.218	69.632	69.7162	70.2582	71.701
Y_P	0.2468	0.2497	0.2460	0.2432	0.2410
$\chi^2_{\mathrm{Cooke17}}$	0.2977	0.1036	0.0611	0.0015	6.62848e-06
χ^2_{Aver15}	0.2161	1.4540	0.0753	0.1725	0.9673
$\chi^2_{\mathrm{H074p03}}$	16.7501	9.5927	9.2288	7.0555	2.6900
χ^2_{JLA}	1034.77	1034.74	1034.74	1034.75	1034.81
χ^2_{prior}	4.5083	4.3142	2.3132	3.1993	7.2315
χ^2_{CMB}	2779.73	2781.6	2783.9	2782.84	2783.71
χ^2_{BAO}	5.2445	5.8010	5.4053	5.3761	6.5744
χ^2_{todal}	3841.52	3837.61	3835.72	3833.39	3836.22

Y_P measurement

Local (direct) H_0 measurement

TAKE-HOME MESSAGE

- The combination of extra radiation and neutrino degeneracy is a promising solution of the Hubble tension.

- Non-zero degeneracy $\xi_e = 0.04$ and extra radiation $N_{\mathrm{eff}} = 3.45$ is the best-fit at the combination of CMB, BBN, BAO and the local measurements.

- The model of particle physics which takes large neutrino degeneracy and extra radiation is worth constructing. (sterile neutrino + ξ_e …)

Thank you for your kind attention!

Osamu Seto, Yo Toda

Phys.Rev.D 104 (2021) 6, 063019

y-toda@particle.sci.hokudai.ac.jp

Session D3a 14:30–16:00

[Chair: Tomo Takahashi]

Shao-Jiang Wang

Institute of Theoretical Physics, Chinese Academy of Sciences

"Improved no-go argument from inverse distance ladder"

(15 min.)

[JGRG30 (2021) 120922]

Improved no-go argument from inverse distance ladder

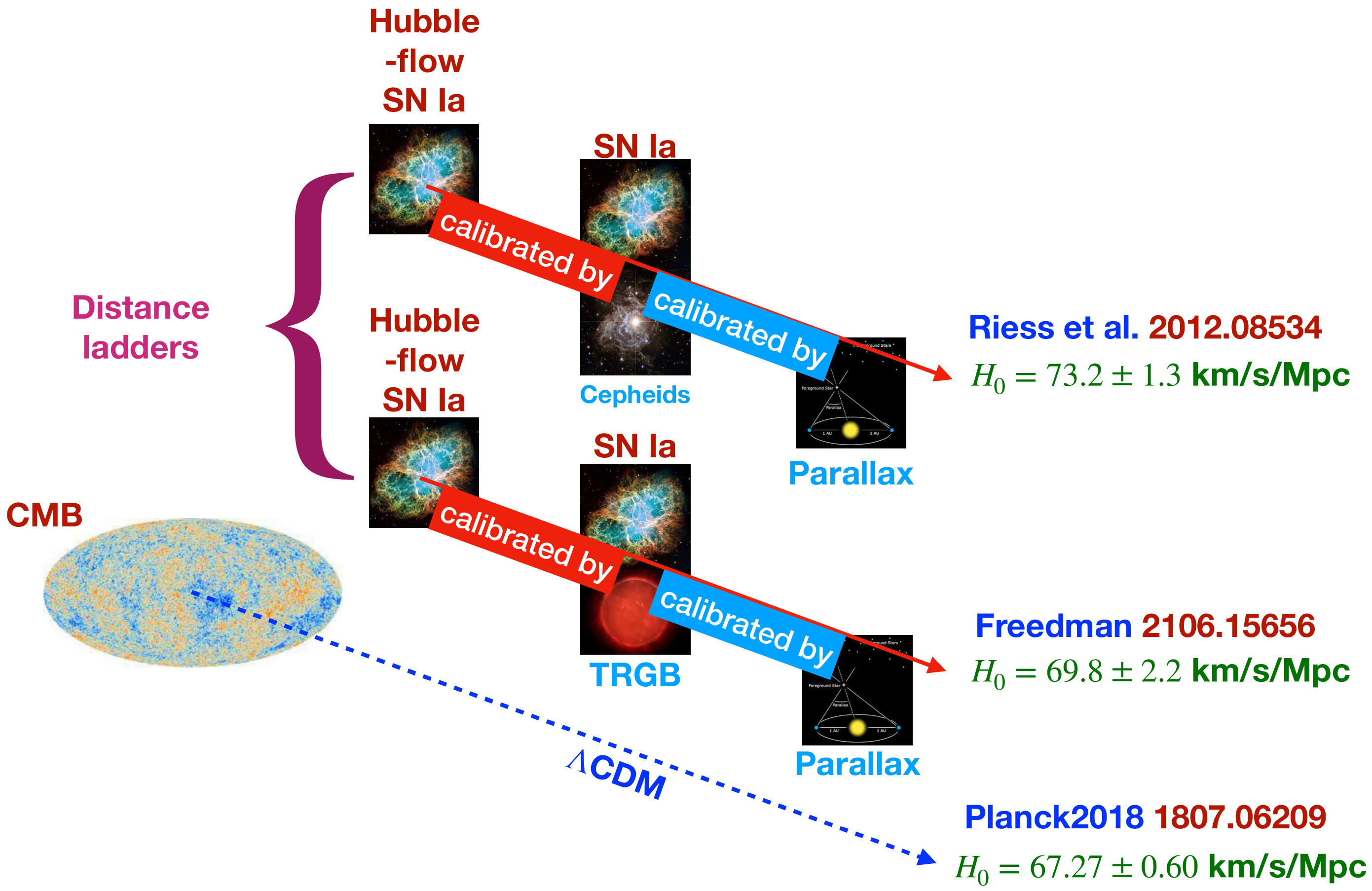
Shao-Jiang Wang

Institute of Theoretical Physics
Chinese Academy of Sciences

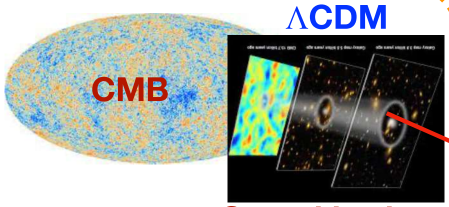
JGRG30-D3a2 @ 2021-12-209 @ 14:45-15:00(JST)

Based on 2107.13286 “A No-Go guide for the Hubble tension”
with Rong-Gen Cai, Zong-Kuan Guo, SJW, Wang-Wei Yu, Yong Zhou

Hubble tension



Inverse distance ladder

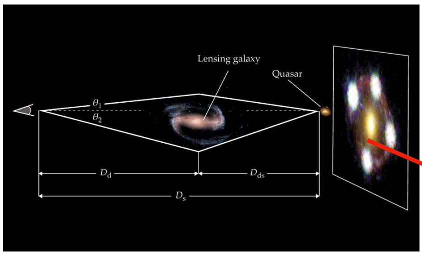


Λ**CDM**

Sound horizon

Early universe dependence

Gravitational lensing time delay



Lensing model dependence

H0LiCOW XIII 1907.04869

$H_0 = 73.3^{+1.7}_{-1.8}$ **km/s/Mpc**

TDCOSMO IV 2007.02941

$H_0 = 67.4^{+4.1}_{-3.2}$ **km/s/Mpc**

1. Cosmic chronometers (CC) data

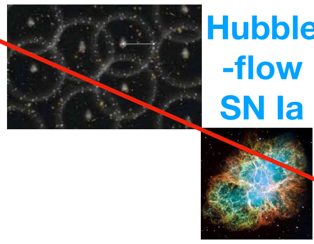
calibrated by

Late-time model parameterization

Taylor expansion of z or $y = 1 - a$

2. Parameterization based on the cosmic age: PAge model

BAO



Hubble -flow SN Ia

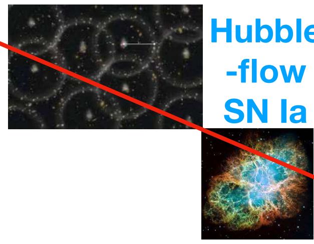
$H_0 \approx 68$ **km/s/Mpc**

3. Set SN Ia absolute magnitude as a free parameter in fitting

M_B

calibrated by

BAO



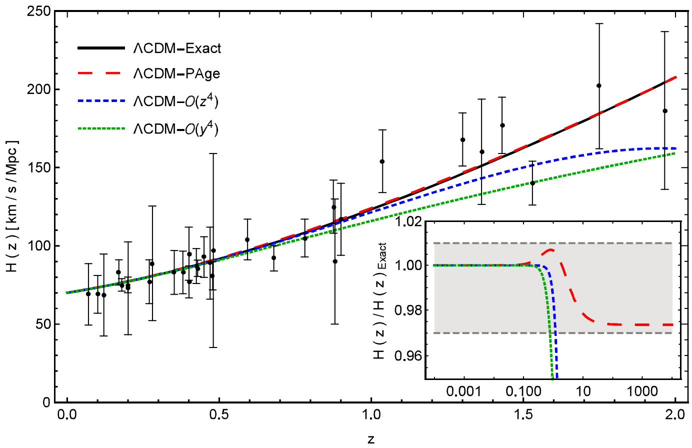
Hubble -flow SN Ia

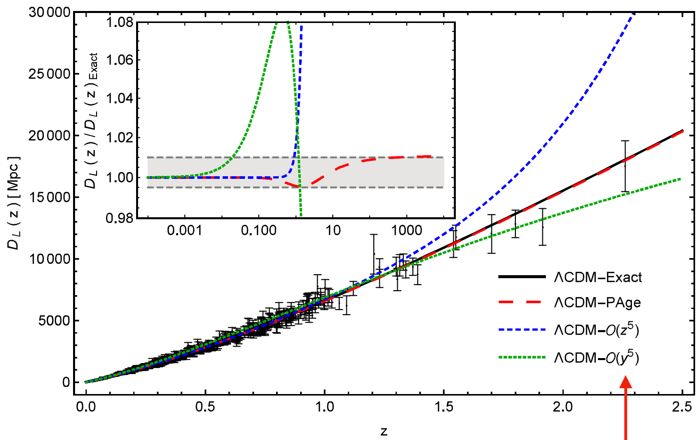
$H_0 \approx 68$ **km/s/Mpc**

$H_0 \approx 73$ **km/s/Mpc**

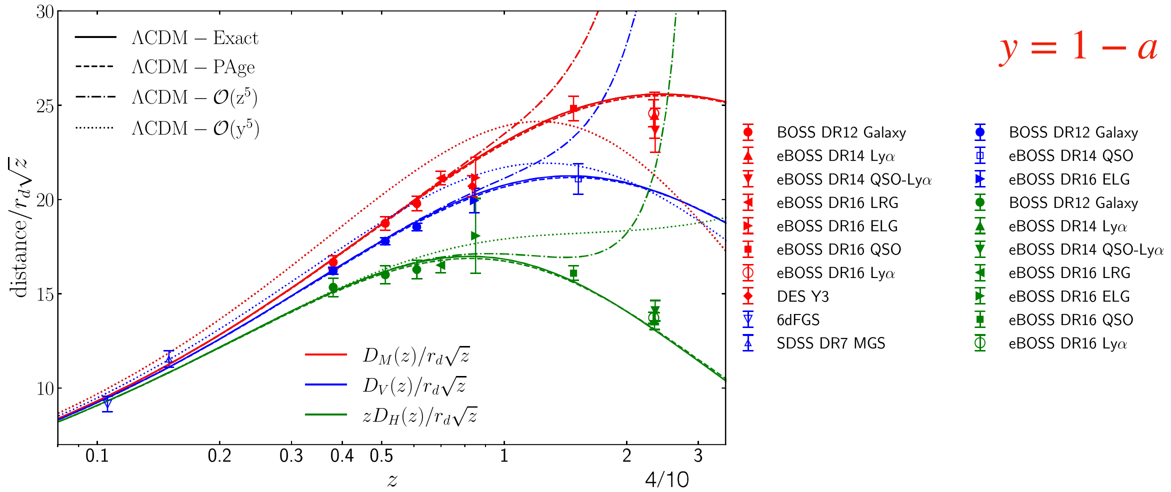
Nikki Arendse et al. 1909.07986

Poor Taylor expansion





$y = 1 - a = z/(1+z)$



PAge model

PAge : parameterization based on the cosmic age

$$\frac{H}{H_0} = 1 + \frac{2}{3}\left(1-\eta\frac{H_0 t}{P_{\rm age}}\right)\left(\frac{1}{H_0 t}-\frac{1}{P_{\rm age}}\right) \qquad P_{\rm age}\equiv H_0 t_0$$

(Zhiqi Huang 2020 ApJL 892:L28)

$$H(z) = -\frac{1}{1+z}\frac{\mathrm{d}z}{\mathrm{d}t} \qquad \Rightarrow E(z;\eta,P_{\rm age})$$

$$1+z = \left(\frac{p_{\rm age}}{H_0 t}\right)^{\frac{2}{3}} e^{\frac{1}{3}\left(1-\frac{H_0 t}{p_{\rm age}}\right)\left(3p_{\rm age}+\eta\frac{H_0 t}{p_{\rm age}}-\eta-2\right)} \qquad \Rightarrow (H_0 t)(z;\eta,P_{\rm age})$$

PAge model representation

$$\frac{H}{H_0} = 1 + \frac{2}{3}\left(1-\eta\frac{H_0 t}{P_{\rm age}}\right)\left(\frac{1}{H_0 t}-\frac{1}{P_{\rm age}}\right) \xrightarrow{q=-\frac{\ddot{a}a}{\dot{a}^2}} \eta = 1-\frac{3}{2}P_{\rm age}^2(1+q_0)$$

$\Omega_k w_{\rm CPL}$CDM

$$E(a)^2 = \Omega_m a^{-3} + \Omega_k a^{-2} + (1-\Omega_m-\Omega_k)a^{-3(1+w_0+w_a)}e^{-3w_a(1-a)}$$

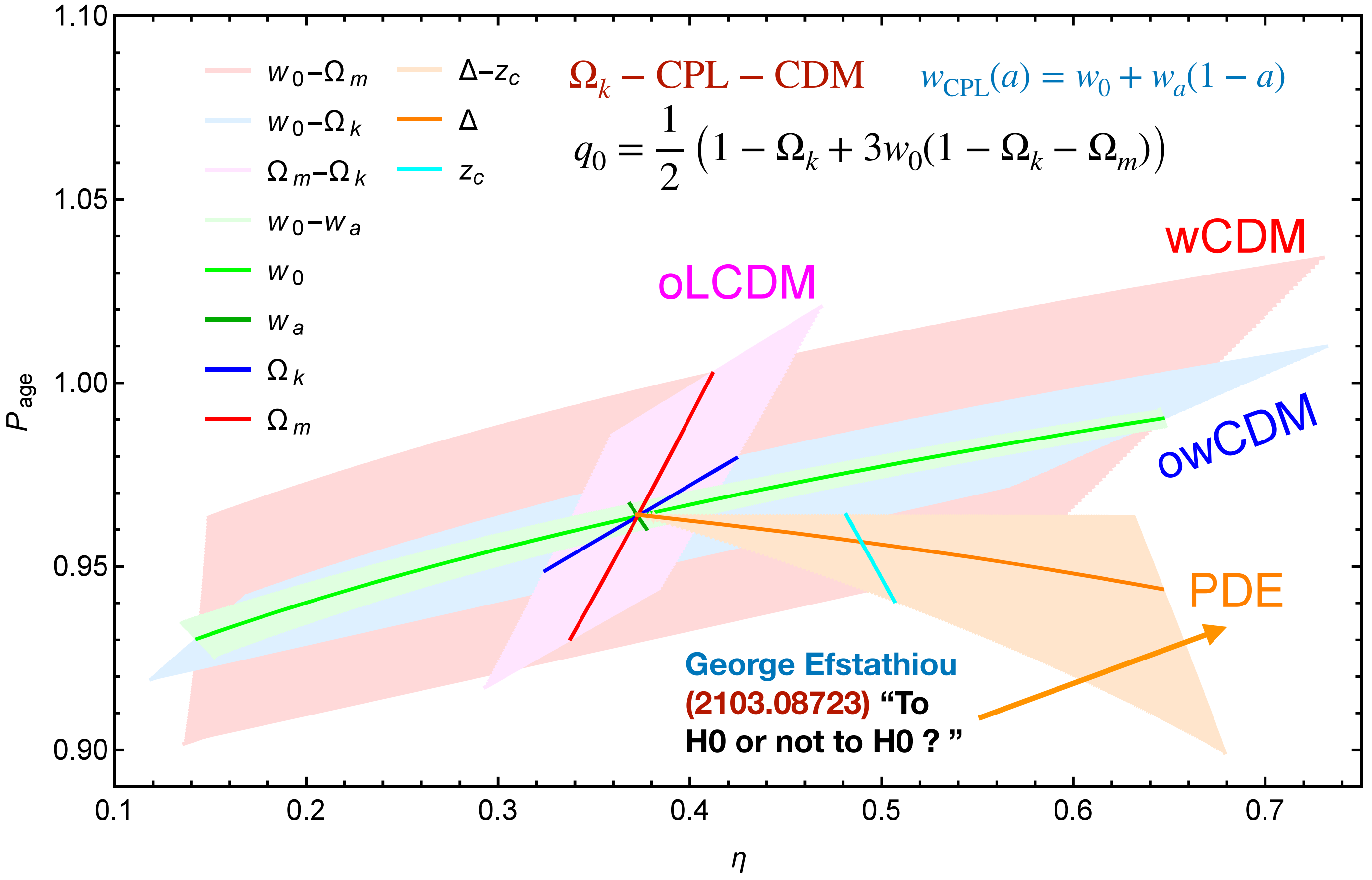
$$q_0 = \frac{1}{2}(1-\Omega_k) + \frac{3}{2}(1-\Omega_k-\Omega_m)w_0$$

Phantom DE transition model

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)\left(1+\Delta e^{-\left(\frac{z}{z_c}\right)^{\beta}}\right)$$

$$q_0 = -1 + \frac{3}{2}\frac{\Omega_m}{1+(1-\Omega_m)\Delta}$$

PAge model



PAge model

Table 1. PAge approximation: maximum relative errors in angular diameter distance ($0 < z \le 2.5$)

model	parameters	$p_{\rm age}$	η	max $\frac{\Delta D_A}{D_A}$
CDM	$\Omega_m = 1$	$\frac{2}{3}$	0	0
nonflat CDM	$\Omega_m = 0.3, \Omega_k = 0.7$	0.809	-0.128	0.011
flat ΛCDM	$\Omega_m = 0.3$	0.964	0.373	0.0045
nonflat ΛCDM	$\Omega_m = 0.5, \Omega_k = 0.2$	0.797	0.0955	0.0013
flat wCDM	$\Omega_m = 0.3, w = -1.2$	0.991	0.647	0.0060
nonflat wCDM	$\Omega_m = 0.33, \Omega_k = -0.25, w = -0.8$	0.967	0.269	0.014
flat w_0-w_aCDM	$\Omega_m = 0.3, w_0 = -1.0, w_a = 0.3$	0.953	0.387	0.0025
nonflat w_0-w_aCDM	$\Omega_m = 0.25, \Omega_k = 0.1, w_0 = -1.2, w_a = -0.2$	1.009	0.572	0.0050
GCG	$\Omega_b = 0.05, A = 0.75, \alpha = 0.1$	0.956	0.409	0.0041
DGP	$\Omega_m = 0.3$	0.907	0.146	0.0011
$R_h = ct$	-	1	$-\frac{1}{2}$	0.056

Advantages

Cosmic chronometers is cosmological-model-independent calibration of inverse distance ladder

Global parameterization of cosmic history, valid beyond the Taylor expansion domain

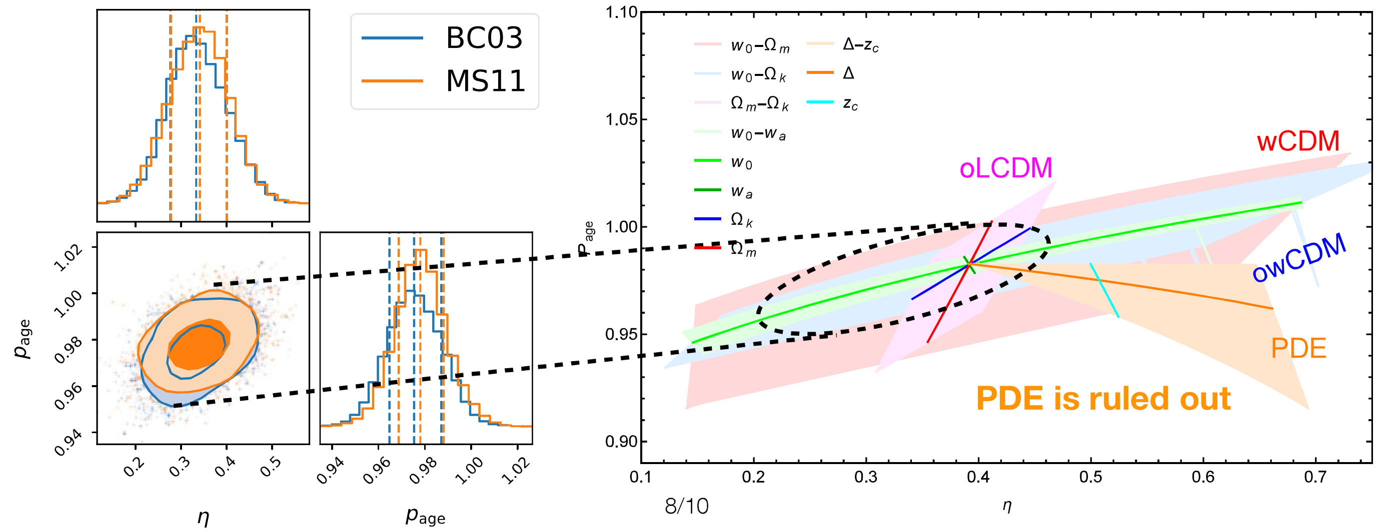
Covering a large class of late-time models to high accuracy over a large range of redshift

No need inputting rs, MB priors when fitting to standard candle+standard ruler+standard clock

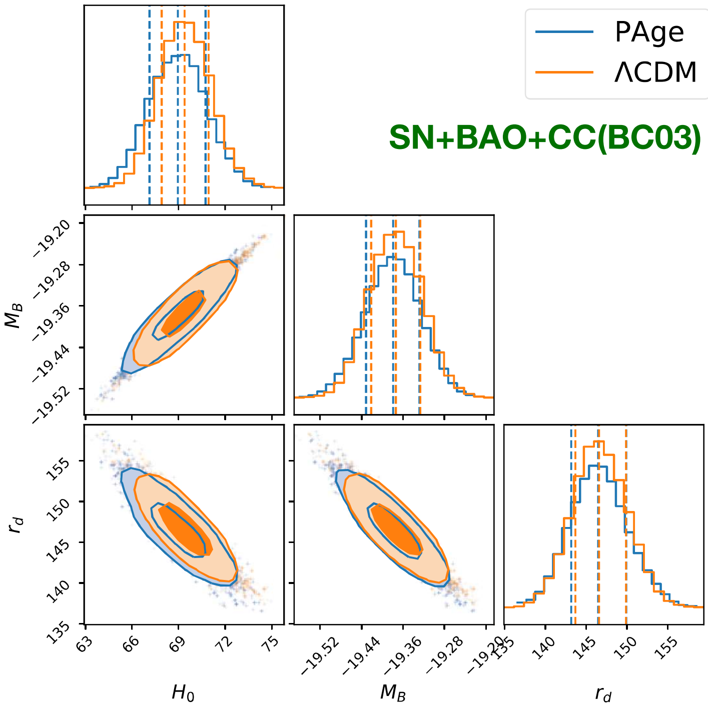
Results

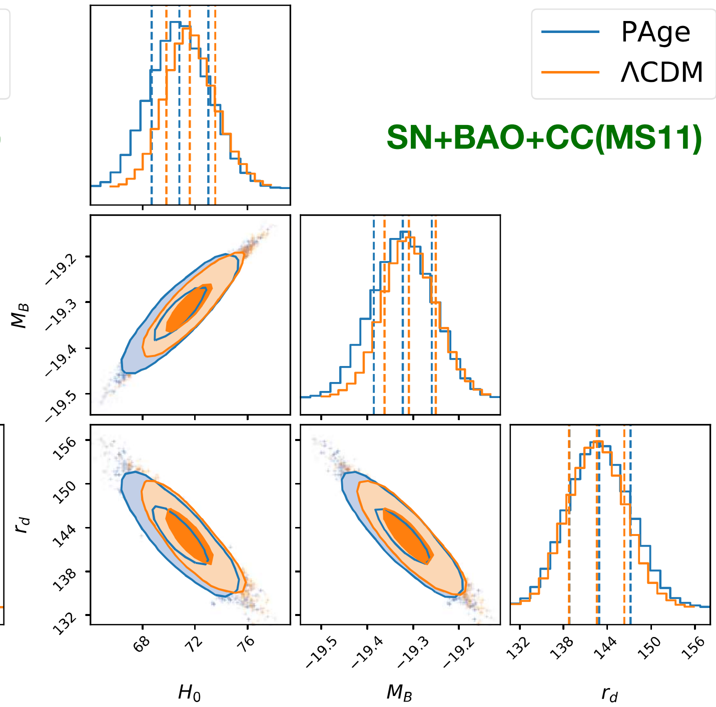
TABLE I. The cosmological constraints from fitting the datasets SNe+BAO+OHD(BC03) and SNe+BAO+OHD(MS11) to the ΛCDM and PAge models with free parameters $\{\Omega_m, M_B, H_0, r_d\}$ and $\{\eta, p_{\rm age}, M_B, H_0, r_d\}$, respectively.

Parameter	Prior range	BC03		MS11	
		ΛCDM	PAge model	ΛCDM	PAge model
Ω_m	$0.15 \sim 0.5$	0.288 ± 0.011	–	0.282 ± 0.011	–
η	$-2 \sim 2$	–	$0.334^{+0.067}_{-0.057}$	–	$0.341^{+0.059}_{-0.063}$
$p_{\rm age}$	$0.15 \sim 2.0$	–	$0.975^{+0.012}_{-0.011}$	–	$0.978^{+0.010}_{-0.009}$
M_B	$-20 \sim -19$	-19.374 ± 0.047	$-19.379^{+0.051}_{-0.052}$	$-19.309^{+0.059}_{-0.053}$	-19.322 ± 0.063
H_0	$60 \sim 80$	$69.389^{+1.547}_{-1.474}$	$68.958^{+1.779}_{-1.826}$	$71.591^{+1.950}_{-1.780}$	$70.799^{+2.220}_{-2.118}$
r_d	$120 \sim 160$	$146.563^{+3.293}_{-2.894}$	$146.466^{+3.448}_{-3.302}$	$142.573^{+3.754}_{-3.820}$	$142.885^{+4.287}_{-4.062}$
$\chi^2_{\rm min}$/d.o.f	–	0.9724	0.9708	0.9803	0.9789



Results





No evidence for going beyond ΛCDM model at late-time

Conclusion and discussions

Improved inverse distance ladders

- **Inverse distance ladder (SN+BAO) calibrated by sound horizon from CMB+LCDM is early-time model-dependent**
- **Inverse distance ladder (SN+BAO) calibrated by gravitational lensing time delay is cosmological model-dependent**
- **Inverse distance ladder (SN+BAO) calibrated by cosmic chronometers is cosmological model-independent**

Taylor expansion for late-time model is bad

A good late-time model representation is PAge : parameterization based on the cosmic age

Fitting PAge model to SN(standard candle)+BAO(standard ruler)+CC(standard clock) favors no evidence beyond LCDM model at late-time

Thank you

Session D3a 14:30–16:00

[Chair: Tomo Takahashi]

Jackson Levi Said

University of Malta

"Model-Independent Techniques to Reconstructing Late-Time Cosmological Data"

(15 min.)

[JGRG30 (2021) 120924]

Model-Independent Techniques to Reconstructing Late-Time Cosmological Data

Jackson Levi Said, Jurgen Mifsud, Reginald Christian Bernardo

ISSA

Based on: arXiv:2106.08688, arXiv:2103.05021, arXiv:2009.14582, arXiv:2105.14332

L-Università ta' Malta

Outline

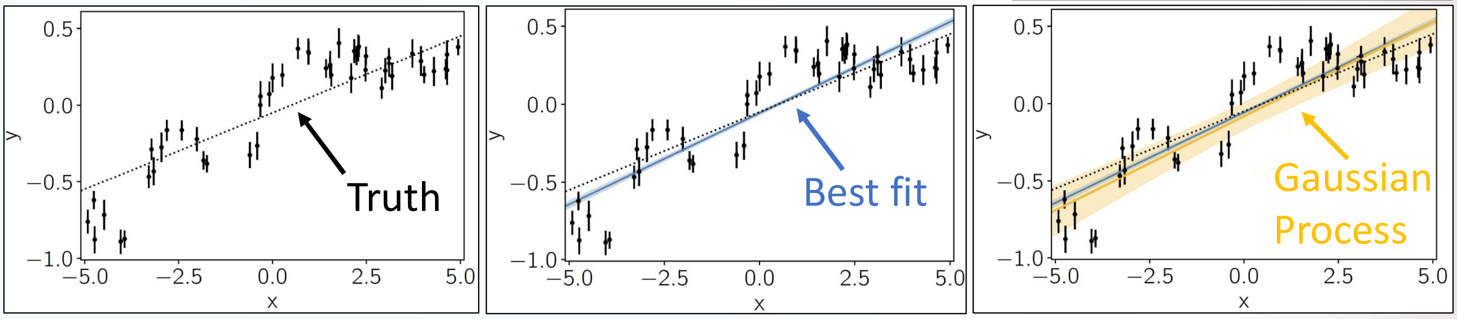
- Using **Gaussian Processes** (GP) for cosmology
- Estimating $\boldsymbol{H_0}$ **value** using GP
- Improving GP with **genetic algorithms**
- Utilizing **artificial neural networks (ANNs)** to approximate H_0

What are Gaussian Processes?

<u>Definition</u>: A GP is a stochastic (random) process where any finite subset is a **multivariant Gaussian distribution** with <u>mean</u> $\mu(x)$ and <u>covariance</u> $k(x, x')$

Setting the each $\mu(x)$ to zero, the **covariance function** can be used to **learn the behavior** that produced the data points

$$y(x_i) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{Exp}\left[-\frac{1}{2}\left(\frac{x_i - \mu}{\sigma}\right)^2\right]$$



Gaussian Processes Regression

- The covariance function contains **non-physical hyperparameters** θ which define the distribution $k(\theta, x, x')$
- Iterating over these values using **Bayesian inference** (or others) can produce better hyperparameters
- The result is a (physics) **model-independent reconstruction** of the behavior of some parameter
- This is superior to regular fitting because it is nonparametric and so **assumes no physical model** whatsoever

The Covariance Functions

- **Squared Exponential** (Radial basis function - RBF)

$$k(x,x') = \sigma_f^2 \ \mathrm{Exp}\left[-\frac{1}{2}\left(\frac{x-x'}{l_f}\right)^2\right]$$

- **Rational Quadratic** (RQ)

$$k(x,x') = \sigma_f^2\left(1+\frac{(x-x')^2}{2\alpha l_f^2}\right)^{-\alpha}$$

Cauchy (CHY) occurs for $\alpha = 1$

- **Matérn** (M)

$$k(x,x') = \sigma_f^2 \frac{2^{1-\nu}}{\Gamma(\nu)}\left(\frac{\sqrt{2\nu(x-x')^2}}{l_f}\right)^{\nu} K_\nu\left(\frac{\sqrt{2\nu(x-x')^2}}{l_f}\right)$$

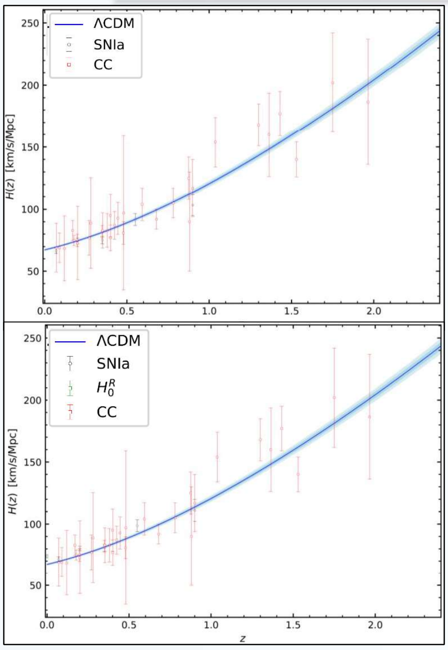
Hubble Data ($H(z)$)

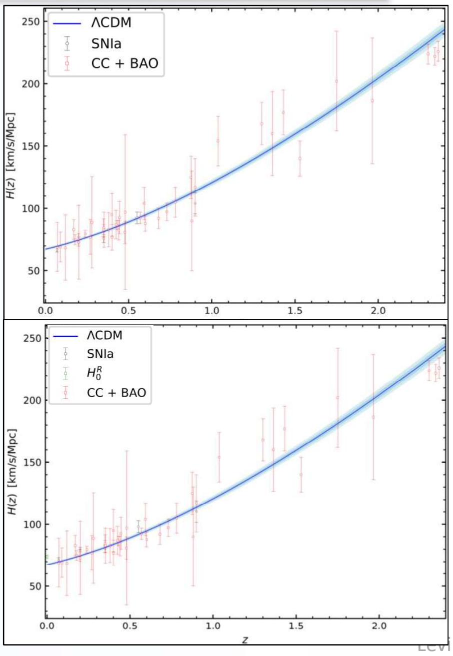
- Cosmic Chronometers (CC): Spectroscopic dating that depends on **stellar evolution** and **differential aging** but **independent of cosmological models**
- SnIa: 5 compressed redshift gaussian points $(z < 2)$ based on 1048 SnIa at $z < 1.5$ from **Pantheon data** + 15 SnIa at $1 < z < 2.3$ from **CANDELS and CLASH programs** obtained by the Hubble Space Telescope
- Baryonic Acoustic Oscillations (BAO): 10 model dependent points from **SDSS** for $z < 2.4$

Priors

- Planck Collaboration (18): **ΛCDM Model dependent** → $H_0 = 67.4 \pm 0.5 \ \mathrm{km\ s^{-1} Mpc^{-1}}$
- SH0ES Survey [H_0^R]: Riess et al. (2019) mainly using **Cepheid variables** → $H_0 = 74.03 \pm 1.42 \ \mathrm{km\ s^{-1} Mpc^{-1}}$

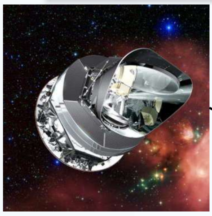
Hubble Data ($H(z)$)





ΛCDM using Planck parameters

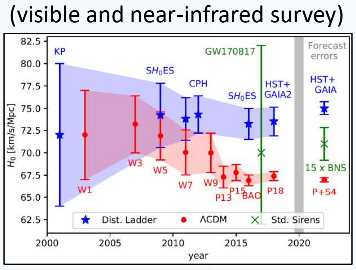
The H_0 Tension

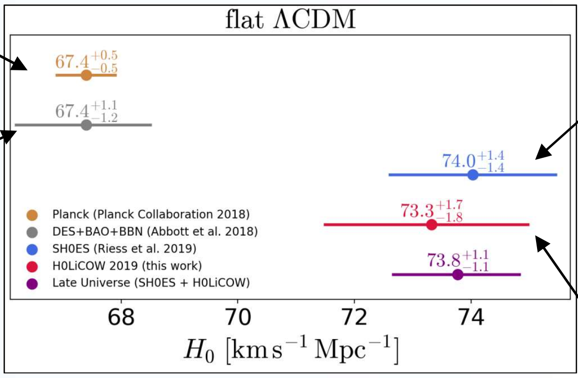


Planck Mission (predictions from the CMB)

THE DARK ENERGY SURVEY

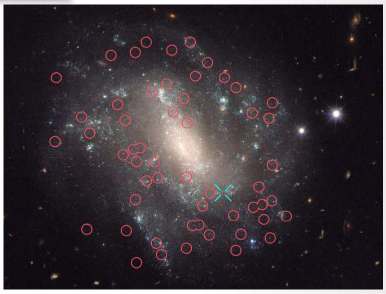
(visible and near-infrared survey)



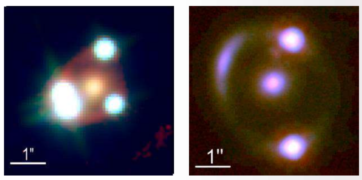


Wong et al. (2019)

5.3σ tension



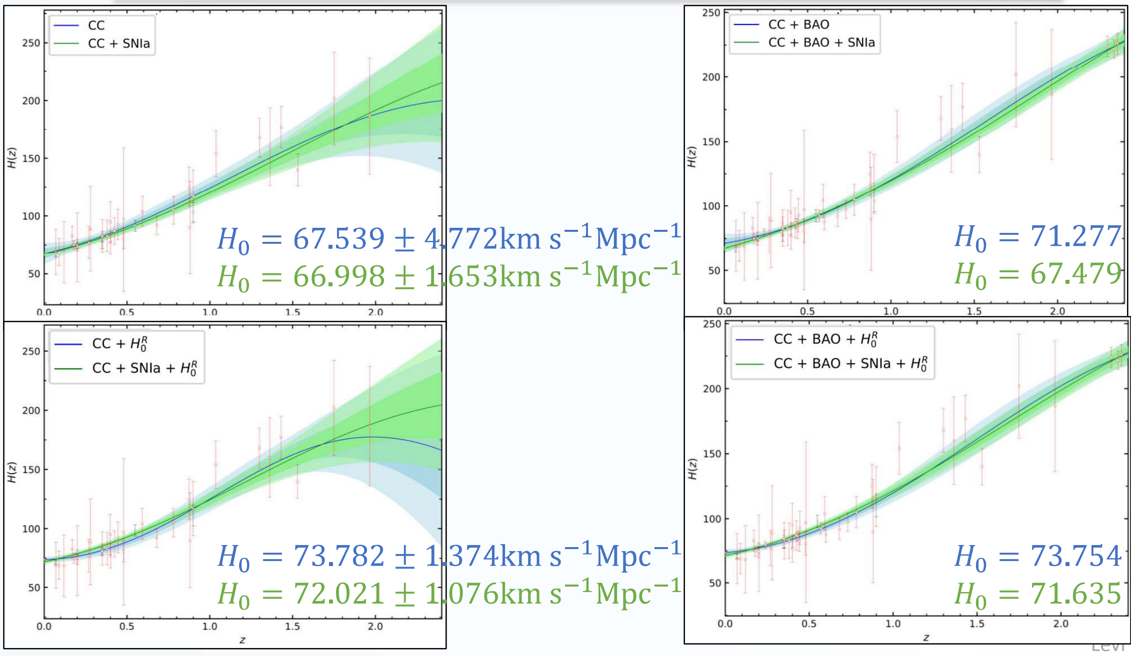
The SH0ES Project (mainly Cepheid variables)



H0LICOW (strong lensing - cosmography)

Square Exponential H_0 GP

GaPP code from Seikel et al. (2012)



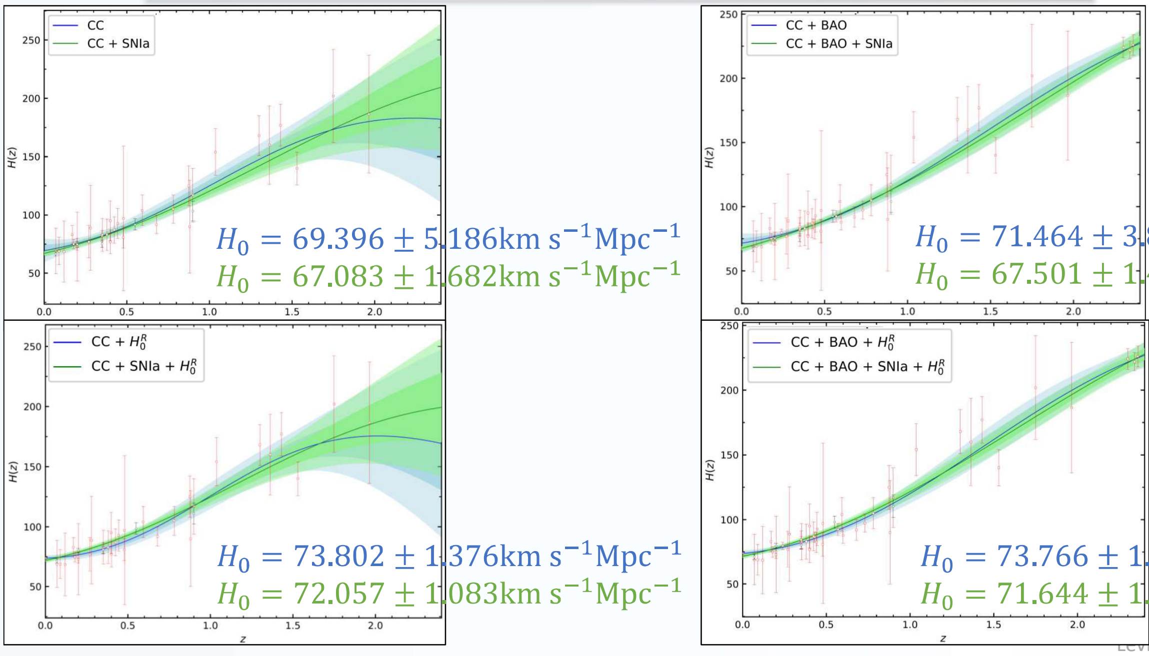
$H_0 = 67.539 \pm 4.772 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 66.998 \pm 1.653 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 71.277 \pm 3.734 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 67.479 \pm 1.455 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 73.782 \pm 1.374 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 72.021 \pm 1.076 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 73.754 \pm 1.333 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 71.635 \pm 1.030 \mathrm{km\ s^{-1} Mpc^{-1}}$

Cauchy H_0 GP



$H_0 = 69.396 \pm 5.186 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 67.083 \pm 1.682 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 71.464 \pm 3.873 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 67.501 \pm 1.479 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 73.802 \pm 1.376 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 72.057 \pm 1.083 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 73.766 \pm 1.337 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 71.644 \pm 1.037 \mathrm{km\ s^{-1} Mpc^{-1}}$

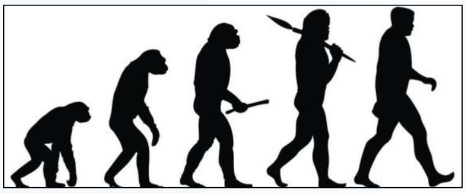
Testing the H_0 Fit

$$d\left(H_{0,i}, H_{0,j}\right) = \frac{H_{0,i} - H_{0,j}}{\sqrt{\sigma_i^2 + \sigma_j^2}}$$

Distance (in σ units) between the H_0 arguments:

	Square Exponential kernel			Cauchy kernel		
Data set(s)	H_0	$d(H_0, H_0^{\mathrm{R}})$	$d(H_0, H_0^{\mathrm{Planck18}})$	H_0	$d(H_0, H_0^{\mathrm{R}})$	$d(H_0, H_0^{\mathrm{Planck18}})$
CC	$\mathbf{67.539 \pm 4.772}$	**-1.304**	**0.029**	69.396 ± 5.186	-0.862	0.383
CC+H_0^{R}	73.782 ± 1.374	-0.126	4.364	73.802 ± 1.376	-0.115	4.374
CC+SNIa	66.998 ± 1.653	-3.227	-0.233	67.083 ± 1.682	-3.156	-0.180
CC+SNIa+H_0^{R}	72.021 ± 1.076	-1.128	3.896	72.057 ± 1.083	-1.105	3.905
CC+BAO	71.277 ± 3.734	-0.689	1.029	71.464 ± 3.873	-0.622	1.041
CC+BAO+$\boldsymbol{H_0^{\mathrm{R}}}$	$\mathbf{73.754 \pm 1.333}$	**-0.142**	**4.463**	$\mathbf{73.766 \pm 1.337}$	**-0.135**	**4.460**
CC+BAO+SNIa	67.479 ± 1.455	-3.222	0.051	$\mathbf{67.501 \pm 1.479}$	**-3.184**	**0.065**
CC+BAO+SNIa +H_0^{R}	71.635 ± 1.030	-1.366	3.700	71.644 ± 1.037	-1.357	3.688

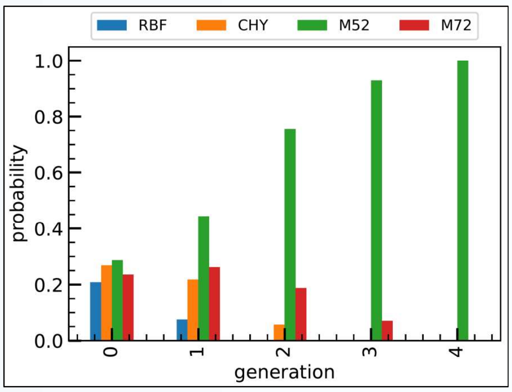
Genetic Algorithms (GAs)



Fitness function: Score to characterize the performance of each generation (BIC inspire)

$$\mathcal{F} = \ln \mathcal{L} - \frac{k_{\mathrm{eff}} \ln N}{2}$$

Selection: Population that will survive
Crossover: Inheritance of kernels
Mutation: Changes in addition to crossover



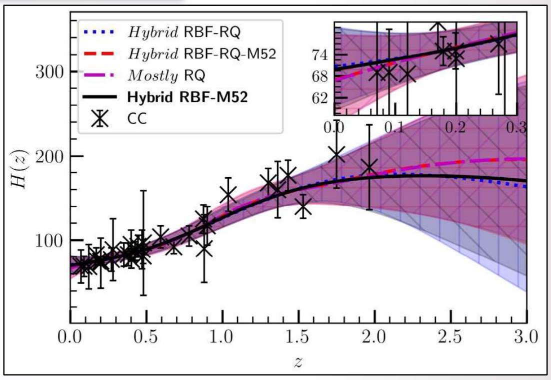
CC dataset

PyGAD code from Ahmed Fawzy Gad et al. (2021)

Trials for GAs

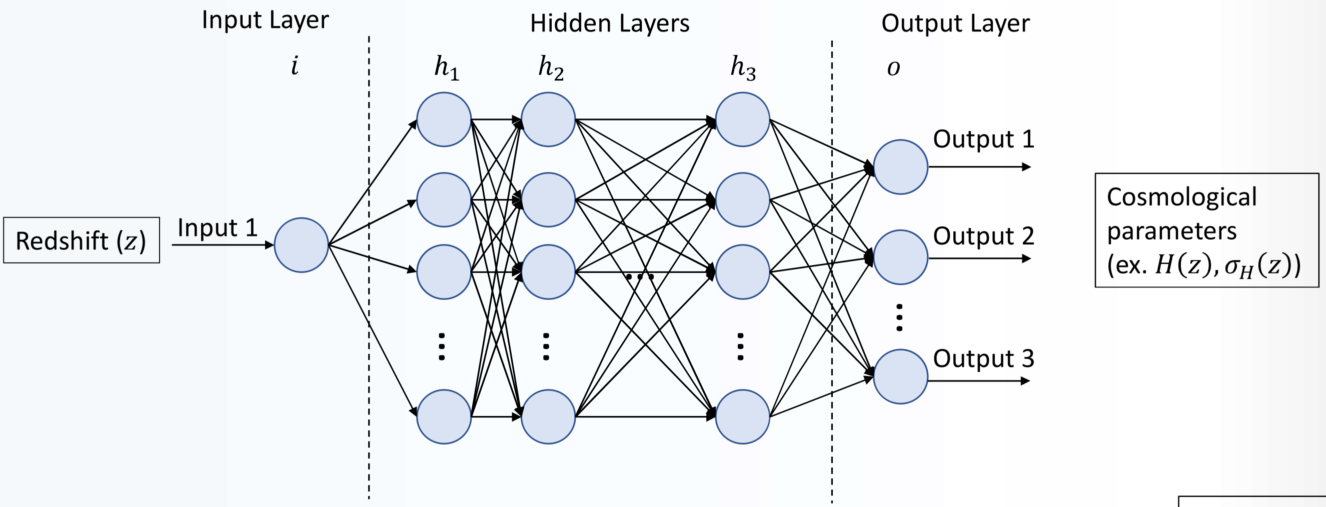
Trial	Population size	Selection rate	Mutation rate	No. of generations	Best fitness
1	10^4	0.5	0.15	10^1	-143.5
2	10^4	0.3	0.30	10^1	-148.5
3	10^3	0.1	0.10	10^2	-143.4
4	$\mathbf{10^3}$	**0.3**	**0.50**	$\mathbf{10^2}$	$\mathbf{-141.8}$

Kernel	H_0	$\ln \mathcal{L}$	χ	fitness	Penalty
Hybrid RBF-RQ	70.6 ± 5.5	-131.49	13.1	-143.5	12.0
Hybrid RBF-RQ-M52	66.9 ± 6.3	-131.38	12.0	-148.5	17.2
Mostly RQ	66.7 ± 6.4	-131.36	11.7	-143.4	12.0
***Hybrid* RBF-M52**	$\mathbf{69.8 \pm 5.8}$	$\mathbf{-131.48}$	**12.7**	$\mathbf{-141.8}$	**10.3**



$$\text{Penalty} = \frac{k_{\text{eff}} \ln N}{2}$$

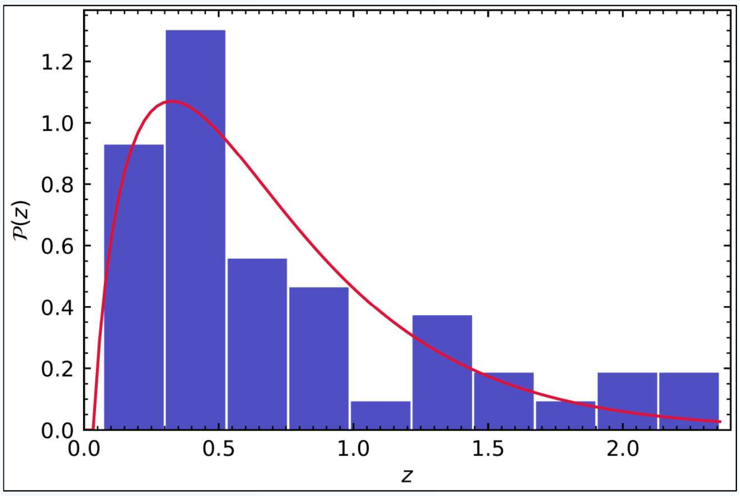
Artificial Neural Networks (ANNs)

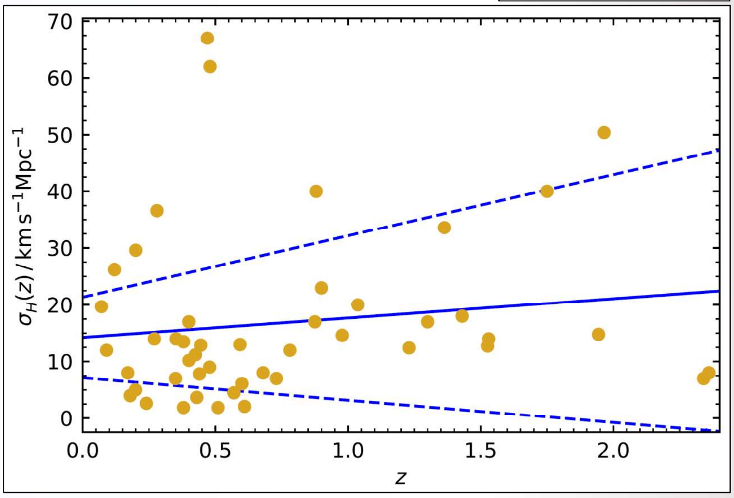


RefANN code from Wang et al. (2020)

Training Data for the ANN

CC+BAO dataset





This observes the gamma distribution:

$$\mathcal{P}(z,\alpha,\lambda) = \frac{\lambda^\alpha}{\Gamma(\alpha)} z^{\alpha-1} e^{-\lambda z}$$

Mean: $\sigma_H = 14.25 + 3.42z$
Upper error: $\sigma_H = 21.37 + 10.79z$
Lower error: $\sigma_H = 7.14 - 3.95z$

Designing the ANN

- **<u>Risk</u>** – Optimizes the **number of hidden layers and neurons** in an ANN

$$\text{risk} = \sum_{i=1}^{N} (\text{Bias}_i^2 + \text{Variance}_i) = \sum_{i=1}^{N} \left(\left[H_{Obs}(z_i) - H_{pred}(z_i) \right]^2 + \sigma_H^2(z_i) \right)$$

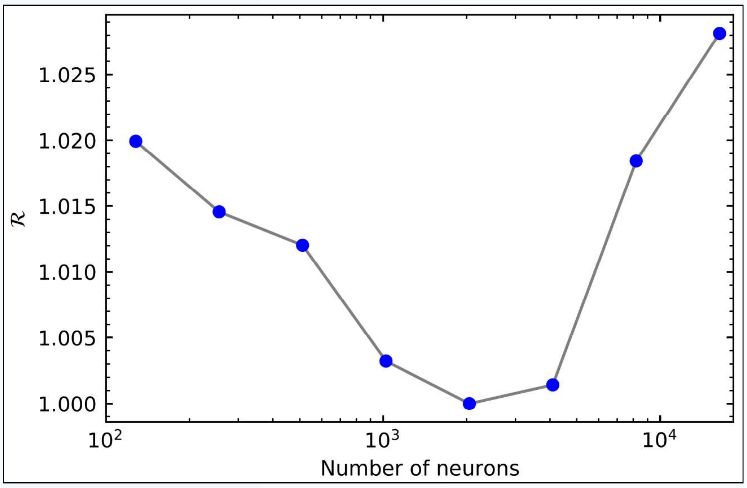
- **<u>Loss</u>** – Balances the **number of iterations** a system needs
  1. **L1** (Least absolute deviation)

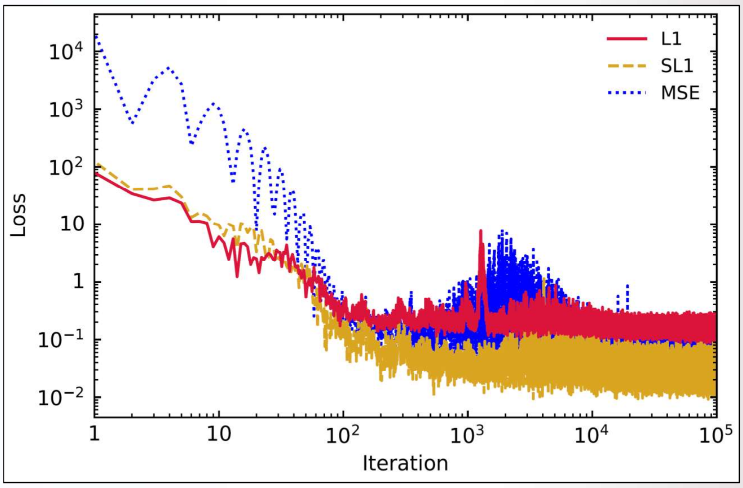
$$\text{L1} = \sum_{i=1}^{N} \left| H_{Obs}(z_i) - H_{pred}(z_i) \right|$$

  2. Smoothed L1 (**SL1**)
  3. Mean Square Error (**MSE**)

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} \left(H_{Obs}(z_i) - H_{pred}(z_i) \right)^2$$

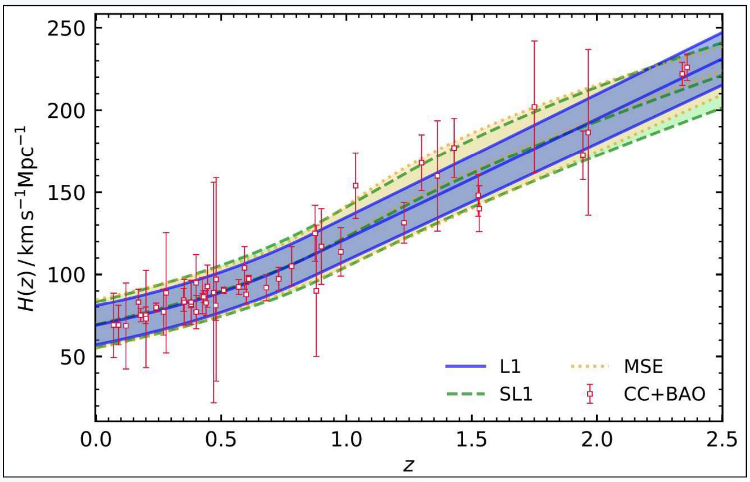
Building the ANN

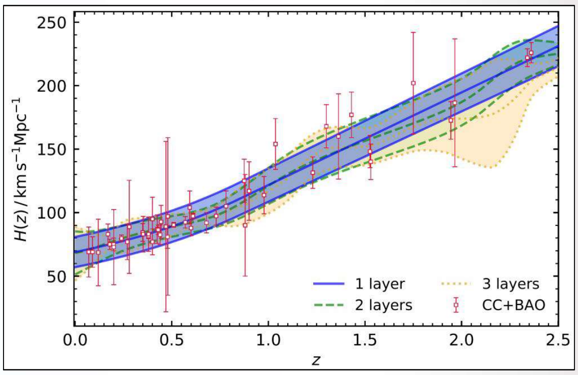




Risk function for **one layer** ($number\ of\ neurons = 2^n$ $n \in \{7, .. 14\}$)

Using the ANN

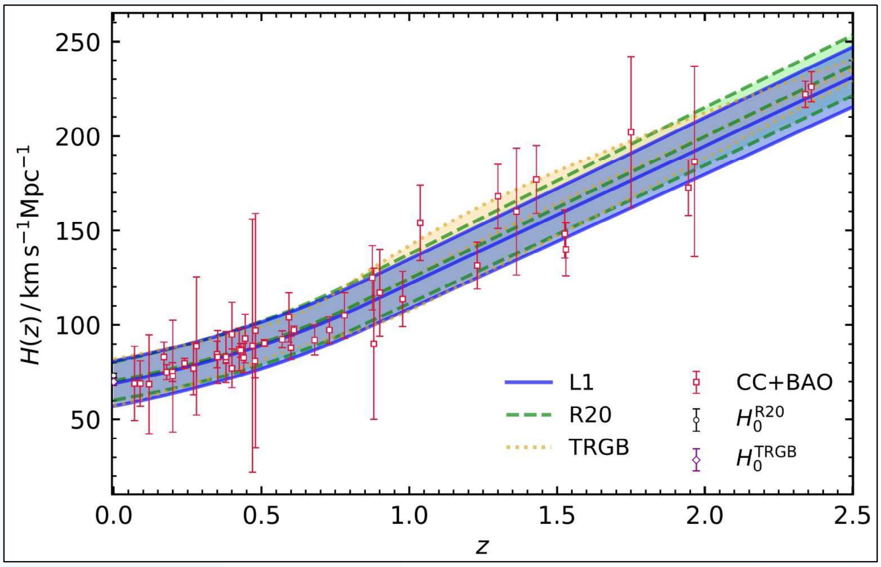




One layer is preferred

MSE: $H_0 = 69.76 \pm 14.82 \text{ km s}^{-1}\text{Mpc}^{-1}$
L1: $H_0 = 68.93 \pm 11.90 \text{ km s}^{-1}\text{Mpc}^{-1}$
SL1: $H_0 = 69.18 \pm 13.92 \text{ km s}^{-1}\text{Mpc}^{-1}$

What about priors?



$\boldsymbol{H_0^{\mathrm{R}}}$: $H_0 = 70.24 \pm 10.08 \text{ km s}^{-1}\text{Mpc}^{-1}$
$\boldsymbol{H_0^{\mathrm{TRGB}}}$: $H_0 = 69.47 \pm 12.37 \text{ km s}^{-1}\text{Mpc}^{-1}$

$H_0^{\mathrm{TRGB}} = 69.8 \pm 1.9 \text{ km s}^{-1}\text{Mpc}^{-1}$

Conclusion and Prospects

- GPs offers an interesting approach to tackling **tension in H_0**
- GAs provide a **model independent approach to resolving the kernel selection problem**
- Using ANNs, we can determine a completely **nonparametric reconstruction of the Hubble diagram**

Thank You

Session D3a 14:30–16:00

[Chair: Tomo Takahashi]

Takashi Hiramatsu

Rikkyo University

"CMB constraints on a subclass of DHOST theories"

(15 min.)

[JGRG30 (2021) 120926]

CMB constraints on a subclass of DHOST theories

Takashi Hiramatsu

Rikkyo University

Motivation / Goal

(single) scalar-tensor theory

generalisation

- Quadratic DHOST (Type-I) → This work
- Quartic GLPV
- Quartic Horndeski
- Quartic Galileon
- (GR)

qDHOST Langlois, Noui, JCAP 1602 (2016) 034

CMB2nd TH, Yamauchi, PRD 102 (2020) 083525

Quartic GLPV, Quartic Horndeski, Quartic Galileon, (GR): Many works on the observational constrains (CMB, LSS, stellar objects, ...)

How well can the Planck results put constraint on the model parameters in DHOST theory with MCMC sampler ?

DHOST theory

(Shift-symmetric) quadratic DHOST theory

Langlois, Noui, JCAP 1602 (2016) 034

$$S = \int d^4x \sqrt{-g} \left[P(X) + Q(X)\Box\phi + f_2(X)^{(4)}R + \sum_{A=1}^{5} a_A(X) L_A \right],$$

$$L_1 := \phi_{\mu\nu}\phi^{\mu\nu} \qquad L_2 := (\Box\phi)^2 \qquad L_3 := (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu \qquad X = \partial_\mu\phi\partial^\mu\phi$$

$$L_4 := \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu \qquad L_5 := (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2$$

arbitrary functions : $P, Q, f_2, a_1, a_2, a_3, a_4, a_5$

$c_{\rm GW}^2 = 1 \quad a_1 = 0$

LIGO/Virgo/Fermi/INTEGRAL, ApJ 848 (2017) L13

degeneracy condition
$$\begin{cases} a_2 = -a_1 \\ a_4 = a_4(f_2, a_2, a_3; X) \\ a_5 = a_5(f_2, a_2, a_3; X) \end{cases}$$

$\Downarrow$

P, Q, f_2, a_3

Crisostomi & Koyama model

Parametrisation proposed by Crisostomi & Koyama

Crisostomi, Koyama, PRD 97 (2018) 084004

(simple extension from cubic Galileon, realising self-acceleration)

$$P = c_2 X \qquad Q = \frac{c_3}{\Lambda^3}X \qquad f_2 = \frac{M_{\rm pl}^2}{2} + c_4\frac{X^2}{\Lambda^6} \qquad a_3 = -\frac{\beta + 8c_4}{\Lambda^6}$$

arbitrary constants (dimensionless) : c_2, c_3, c_4, β

normalisation $\quad c_2 = 1$

avoid graviton decay $\quad \beta = -8c_4 \quad (a_3 = 0)$

Creminelli, Lawandowski, Tambalo, Vernizzi, JCAP 12 (2018) 025

$\Downarrow$

$c_3, \ c_4$

DHOST

(Only $a_4 L_4 = \dfrac{6 f_{2X}^2}{f_2}\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu$ survives even if c1=c3=0.)

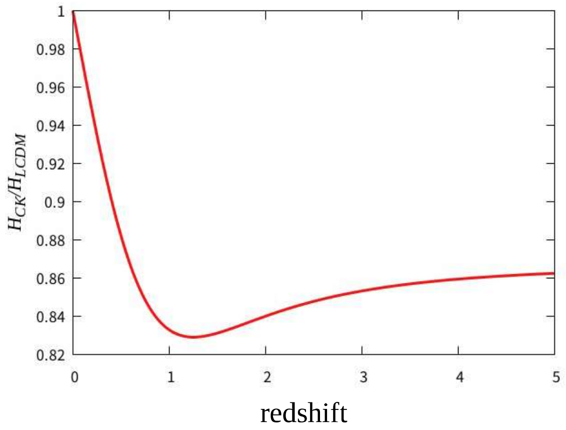
Compare with LCDM

Background equations

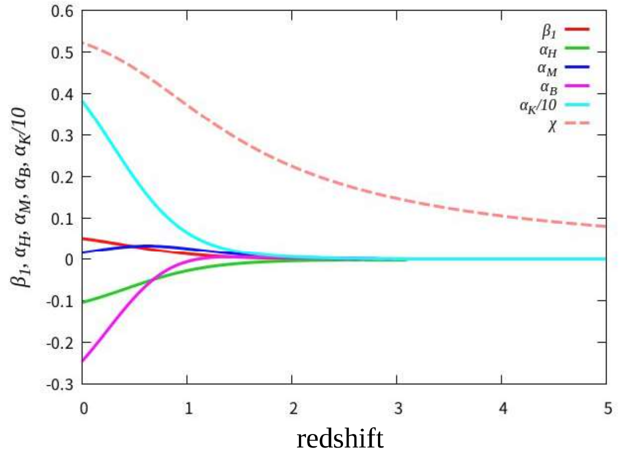
$$\frac{\delta S_0}{\delta N} = \rho_s \qquad -\frac{a}{3}\frac{\delta S_0}{\delta a} = p_s \qquad \frac{\delta S_0}{\delta \phi_0} = 0$$

$$\left(\alpha_M = \frac{1}{H}\frac{d\ln M^2}{dt}\right)$$

$H_{\rm CK}/H_{\Lambda{\rm CDM}}$



$\beta_1(t), \alpha_H(t), \alpha_M(t), \alpha_B(t), \alpha_K(t), \dot\phi_0(t)$



$$(c_2, c_3, c_4, \beta) = (1.00, 1.68, 0.179, -1.43)$$

TH, Yamauchi, PRD 102 (2020) 083525

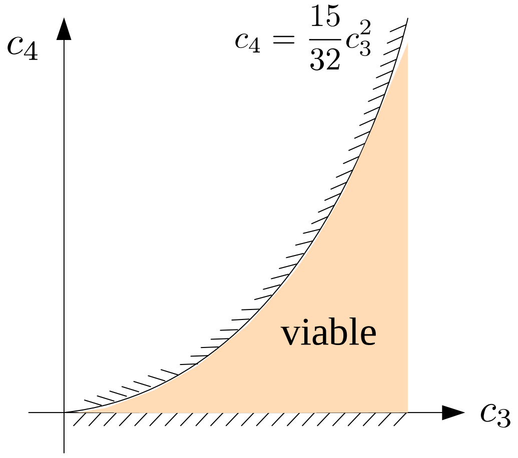
Viable parameter region in CK

- Existence of tracker solution $c_3 > 0 \quad c_4 > 0$ Crisostomi, Koyama, PRD 97 (2018) 084004

Action for curvature perturbations : $\mathcal{L}_2 = a^3 \frac{M^2}{2}\left[A\dot\zeta^2 + \frac{B}{a^2}(\partial_i\zeta)^2\right]$

- Avoidance of gradient instability $B_{\rm MD} < 0, B_{\rm DS} < 0$
- Avoidance of ghost instability $A_{\rm MD} > 0, A_{\rm DS} > 0$

$$0 < c_3, 0 < c_4 < \frac{15}{32}c_3^2$$



Markov-Chain Monte-Carlo simulation

Method : Metropolis sampling

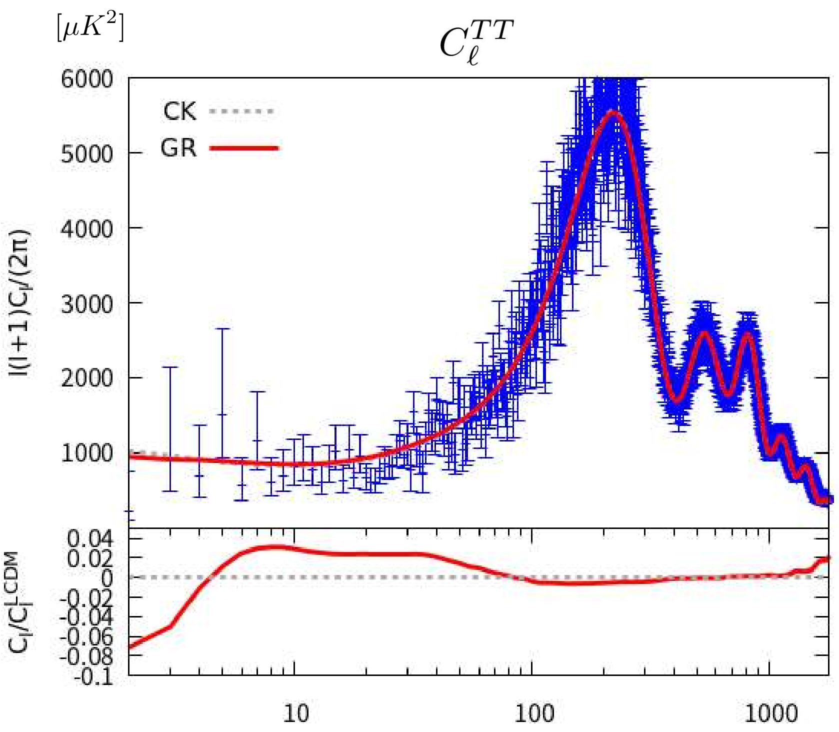
Likelihood : Planck 2018 likelihood code
* `commander` $(\ell \leq 29)$,
* `Plik_lite_TTTEEE` $(30 \leq \ell \leq 2508)$

# of chains : 6

# of params : 8 (CK), 6 (LCDM)

$e^{-2\tau}A_s$	amplitude of curvature perturbation
n_s	spectral index
h	reduced Hubble parameter
$h^2\Omega_\mathrm{c}$	CDM density parameter
$h^2\Omega_\mathrm{b}$	baryon density parameter
τ	optical depth (prior : $\tau = 0.067 \pm 0.023$)
$\ln c_3$	cubic Galileon term (CK)
$\ln c_4$	Einstein-Hilbert term (CK)

Best-fit

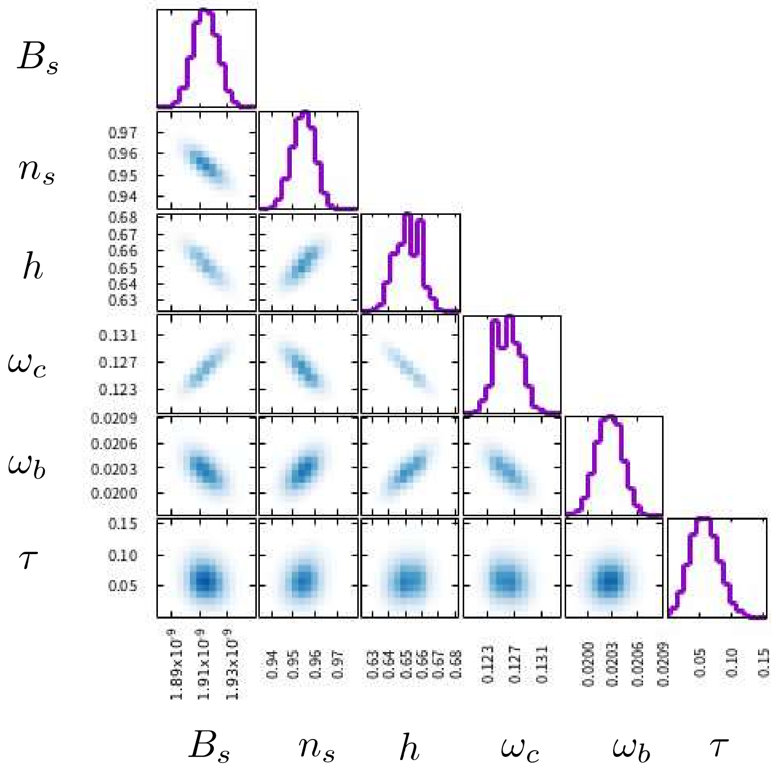


best-fit parameters

	CK	ΛCDM
c_3	1.661	-
c_4	0.003294	-
$e^{2\tau}10^9 A_s$	1.918	1.910
n_s	0.9633	0.9595
h	0.7524	0.6601
$h^2\Omega_\mathrm{c}$	0.1262	0.1251
$h^2\Omega_\mathrm{b}$	0.02031	0.02041
τ	0.1360	0.05311
$\ln\mathcal{L}$	-697.6	-686.0

Large-scale anisotropies are a little bit suppressed. The Hubble parameter is enhanced, which is observed in the cubic Galileon theory.

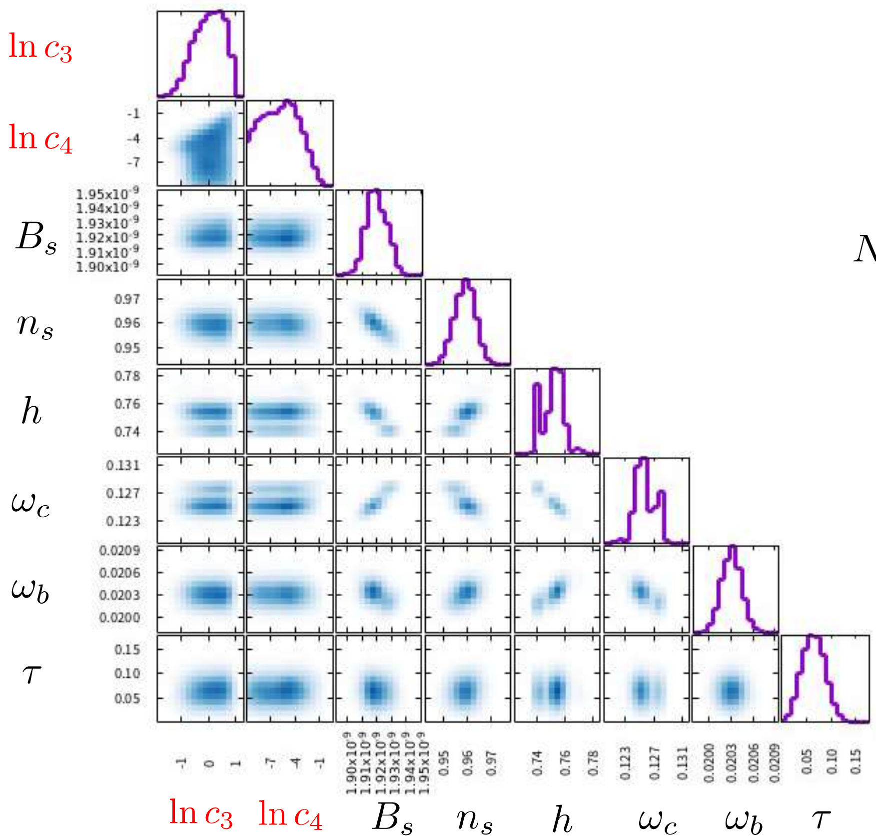
Parameter distribution in LCDM



$$B_s \equiv e^{-2\tau} A_s$$
$$\omega_c \equiv h^2 \Omega_c$$
$$\omega_b \equiv h^2 \Omega_b$$

$$N_{\rm chain} = 184803$$
$$N_{\rm bin} = 20$$

Parameter distribution in CK



$$B_s \equiv e^{-2\tau} A_s$$
$$\omega_c \equiv h^2 \Omega_c$$
$$\omega_b \equiv h^2 \Omega_b$$

$$N_{\rm chain} = 740448$$
$$N_{\rm bin} = 20$$

Constraints

Rikkyo University 立教大学
Takashi Hiramatsu

Crisostomi-Koyama (qDHOST)		Lambda-CDM	
Gal3 → c_3	$1.53^{+0.46}_{-0.91}$		
DHOST → c_4	< 0.045		
$10^9 A_s e^{-2\tau}$	$1.9193^{+0.010}_{-0.0044}$	$10^9 A_s e^{-2\tau}$	$1.9124^{+0.013}_{-0.0045}$
n_s	$0.9592^{+0.0053}_{-0.0034}$	n_s	$0.9553^{+0.0060}_{-0.0048}$
h	$0.7524^{+0.0075}_{-0.0092}$	h	$0.6510^{+0.011}_{-0.0056}$
ω_C	$0.12556^{+0.0025}_{-0.00069}$	ω_C	$0.1263^{+0.0023}_{-0.0016}$
ω_B	$0.02033^{+0.00015}_{-0.00012}$	ω_B	$0.02029^{+0.00017}_{-0.00014}$
τ	$0.059^{+0.034}_{-0.014}$	τ	$0.052^{+0.035}_{-0.011}$

- It is the first time to put constraints on c_4 from CMB.
- Basically, the standard 6 parameters are not changed even in CK.
- Cubic Galileon model indicates $h \sim 0.75$ — Barreira, Li, Baugh, Pascoli, JCAP 08 (2014) 059; Renk, Zumalacárregui, Montanari, Barreira, JCAP 10 (2017) 020

  … is consistent to this study.
- The orbital decay of the Hulse-Taylor pulser gives stronger constraint, implying $|Xf_{2X}/f_2| \sim c_4 < \mathcal{O}(10^{-3})$ — Hirano, Kobayashi, Yamauchi, PRD 99 (2019) 104073; Crisostomi, Lewandowski, Vernizzi, PRD 100 (2019) 024025

Summary

Rikkyo University 立教大学
Takashi Hiramatsu

- MCMC simulation : constraints on c_4 in Crisostomi-Koyama model

  (a concrete model of a subclass of DHOST theory with $c_{\rm GW}^2 = 1$)

$$(0 <)\ c_4 < 0.045$$

- Need to take other stuffs like the matter power (or σ_8) into account to improve the constraints from the cosmological observations

Session D3b 14:30–16:00

[Chair: Keisuke Izumi]

Sofia Di Gennaro

Yangzhou University

"Maximum Force and Black Hole Thermodynamic Instability"

(15 min.)

[JGRG30 (2021) 120927]

Maximum Force and Black Hole Thermodynamic Instability

Based on the article arXiv:2108.13435 [gr-qc] by Sofia Di Gennaro, Michael R. R. Good, Yen Chin Ong

The maximum force conjecture

$$F = \frac{1}{4} F_{\rm pl}$$

- Initially proposed by Gibbons and Schiller:

C. Schiller (1997-2004), "Maximum force a simple principle encompassing general relativity", http://www.motionmountain.net, Sec. 36.

C. Schiller (2005), “General Relativity and Cosmology Derived From Principle of Maximum Power or Force”, https://doi.org/10.1007/s10773-005-4835-2, Int J Theor Phys 44, 1629–1647.

C. Schiller (2006), “Simple derivation of minimum length, minimum dipole moment and lack of space-time continuity”, doi:10.1007/s10773-005-9018-7, Int. J. Theor. Phys. 45, 221-235

G.W. Gibbons (2002), “The Maximum tension principle in general relativity”, doi:10.1023/A:1022370717626 [arXiv:hep-th/0210109[hep-th]], Found. Phys. 32, 1891-1901

- Challenged by Jowsey and Visser:

A.Jowsey, M.Visser (2021), "Counterexamples to the maximum force conjecture", arXiv:2102.01831 [gr-qc].

- Restricted to black holes by Faraoni, who proposes a connection with cosmic censorship:

V. Faraoni (2021), "Maximum force and cosmic censorship", arXiv:2105.07929 [gr-qc].

The Hookean Force

In four dimensions, for Kerr black holes:

$$T = \frac{1}{2\pi}(g - k), \qquad g := \frac{1}{4M}$$

$$k := M\Omega_+^2 \qquad \Omega_+ = \frac{a}{r_+^2 + a^2}$$

$$\lim_{J \to M^2} F_1 = \frac{1}{4M}\left(\lim_{J \to M^2} r_+\right) = \frac{1}{4M} \cdot M = \frac{1}{4}.$$

Based on the article "Are Black Holes Springy?" arXiv:1412.5432 [gr-qc] by Michael R. R. Good, Yen Chin Ong

In five dimensions, for Myers-Perry black holes:

$$T = \frac{r_+}{2\pi\mu}, \qquad T_s = \frac{1}{2\pi r_{+s}} = \frac{1}{2\pi\sqrt{\mu}} \qquad r_+ = \sqrt{\mu - a^2}$$

$$T = \frac{1}{2\pi}\left[\frac{1}{\sqrt{\mu}} - \left(\frac{\sqrt{\mu} - \sqrt{\mu - a^2}}{\mu}\right)\right] = \frac{1}{2\pi}(g - k)$$

$$F_1 = kr_h = \sqrt{1 - \frac{a^2}{\mu}}\left(1 - \sqrt{1 - \frac{a^2}{\mu}}\right) \leq \frac{1}{4}$$

For singly-spinning higher-dimensional Myers-Perry black holes in the ultra-spinning limit:

Temperature and Hookean force:

$$T = \frac{1}{2\pi}\left\{\frac{d-3}{2\mu^{\frac{1}{d-3}}} - \underbrace{\left[\frac{d-3}{2r_+}\left(\frac{r_+}{a}\right)^{\frac{2}{d-3}} - \frac{d-5}{2r_+} - \frac{r_+}{a^2}\right]}_{=:k}\right\}$$

$$F_1 = \frac{d-3}{2}\left(\frac{r_+}{a}\right)^{\frac{2}{d-3}} - \frac{d-5}{2} - \frac{r_+^2}{a^2} \leq 0.21$$

(result obtained numerically).

Emparan-Myers fragmentation limit:

$$\frac{a}{r_+} \leq 1.36 \qquad \Longrightarrow \qquad \frac{a^3}{\mu} \leq 0.88$$

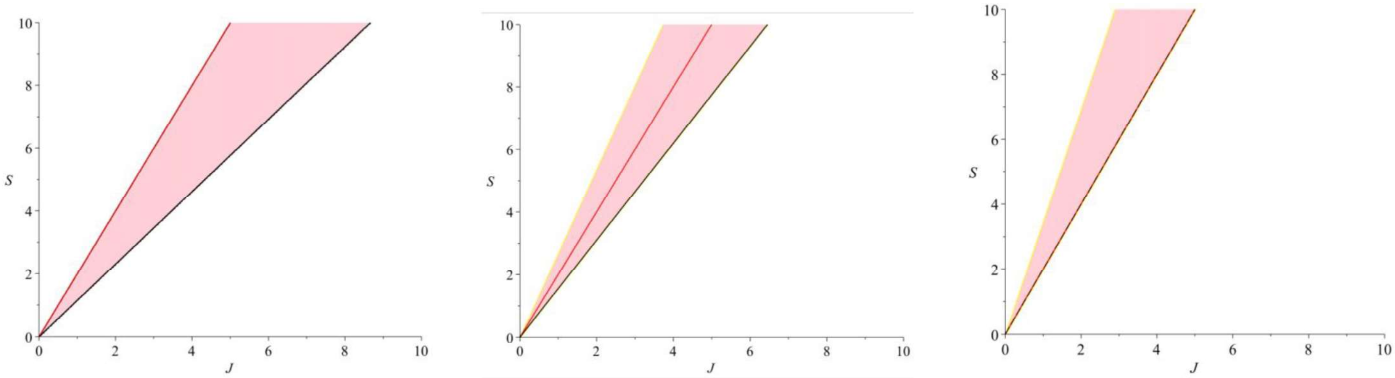
results in the following bound in d=6:

$$F_1 = \frac{3}{2}\frac{r_+}{\mu^{\frac{1}{3}}} - \frac{r_+^3}{\mu} - \frac{1}{2} = \frac{3}{2}\frac{r_+}{a}\frac{a}{\mu^{\frac{1}{3}}} - \frac{r_+^3}{a^3}\frac{a^3}{\mu} - \frac{1}{2} \leq 0.21$$

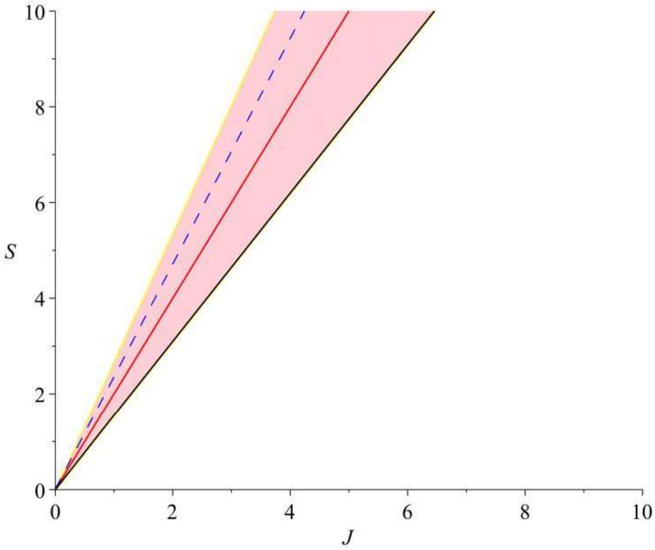
Thermodynamic Geometry

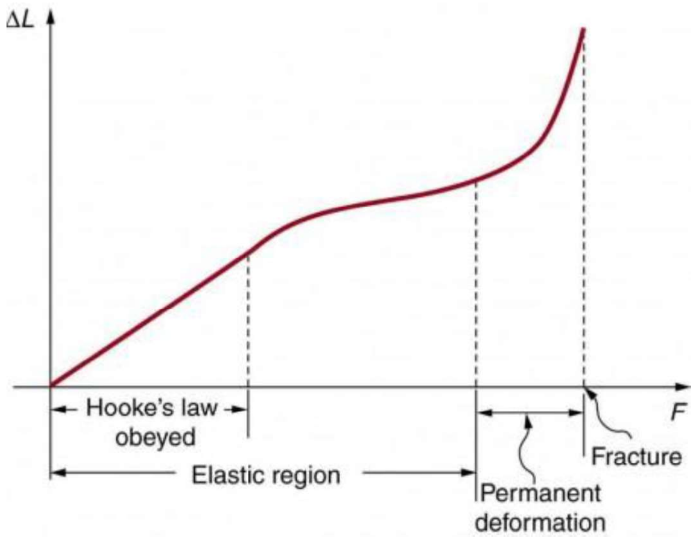
We introduce the Ruppeiner metric: $g_{ij}^R = -\partial_i\partial_j S(M, N^a)$

$$\frac{1}{12}\left(\frac{d-3}{d-5}\right) \leq F_2 := \frac{J^2}{S^2} < \frac{1}{4}\left(\frac{d-3}{d-5}\right)$$



Interpretation





Conclusions

- The maximum force conjecture always holds in all the cases analysed;
- it is not related to cosmic censorship in general.
- The bound on the Hookean force corresponds to the fragmentation limit.
- The value ¼ holds some significance in the thermodynamic properties of the black hole.

Session D3b 14:30–16:00

[Chair: Keisuke Izumi]

Che-Yu Chen

Institute of Physics, Academia Sinica

"Testing equatorial reflection symmetry of rotating black holes"

(15 min.)

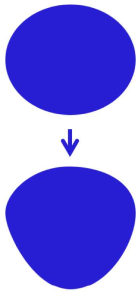
[JGRG30 (2021) 120928]

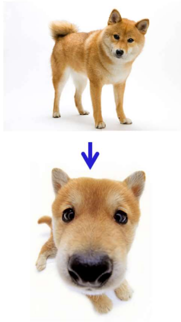
Testing *equatorial reflection symmetry* of rotating black holes

Che-Yu Chen

Institute of Physics, Academia Sinica, Taiwan

- **CYC,** *JCAP* 05 (2020) 040, arXiv:2004.01440
- **CYC, HYY,** arXiv:2109.00564





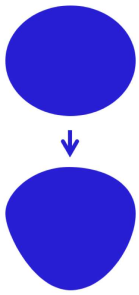
JGRG30

Testing *equatorial reflection symmetry* of rotating black holes

Che-Yu Chen

Institute of Physics, Academia Sinica, Taiwan

- **CYC,** *JCAP* 05 (2020) 040, arXiv:2004.01440
- **CYC, HYY,** arXiv:2109.00564



JGRG30

Black holes without $\mathbb{Z}_2$ symmetry

(Pontryagin scalar)

$$S_{eff} = \int d^4x\sqrt{-g}2M_{pl}^2[R + \alpha C\tilde{C} + \phi(\beta_1 R_{GB} + \beta_2\tilde{C}) + \cdots], \quad C \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \quad \tilde{C} \equiv R_{\alpha\beta\gamma\delta}\tilde{R}^{\alpha\beta\gamma\delta}$$

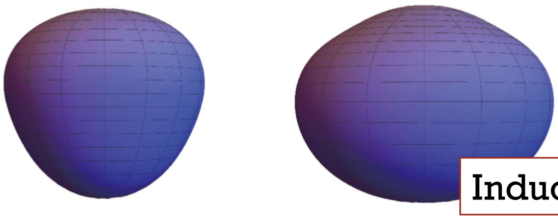
- Parity violating EFTs
- ECOs inspired in string theory

Endlich, Gorbenko, Huang, Senatore (2017)

Sennett, Brito, Buonanno, Gorbenko, Senatore (2020)

Cano, Ruipérez (2017)

Cardoso, Kimura, Maselli, Senatore (2018)



Induced by spin

- Theory-agnostic metric with $\mathbb{Z}_2$ asymmetry
  - Non-Kerr metric designed to probe this asymmetry
  (Principled-Parameterized approach)

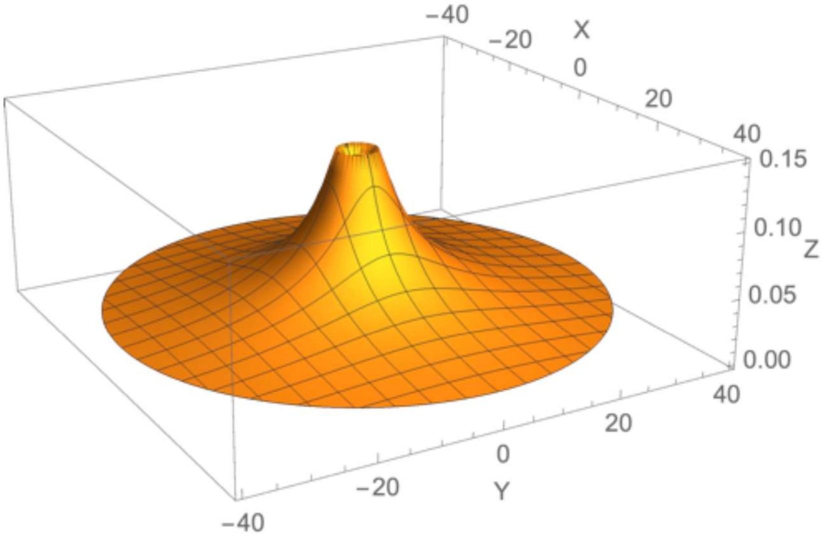
The Kerr-like metric

We want…

- Asymptotically reduces to Kerr
- Single deviation parameter ϵ: Equatorial reflection asymmetry
  - possibility induced by spin
- Separable geodesic equations (Carter constant)

Chen (2020)

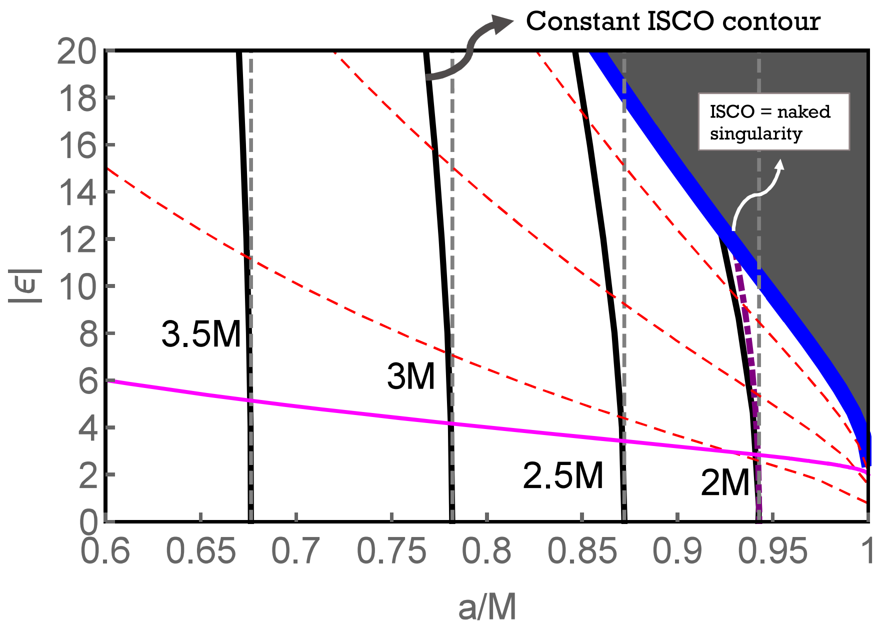
Curved accretion disk



- When $r \to \infty$, the orbits approach $y \to 0$ (asymptotic flatness)

Chen, Yang (2021)

What happen to ISCO?

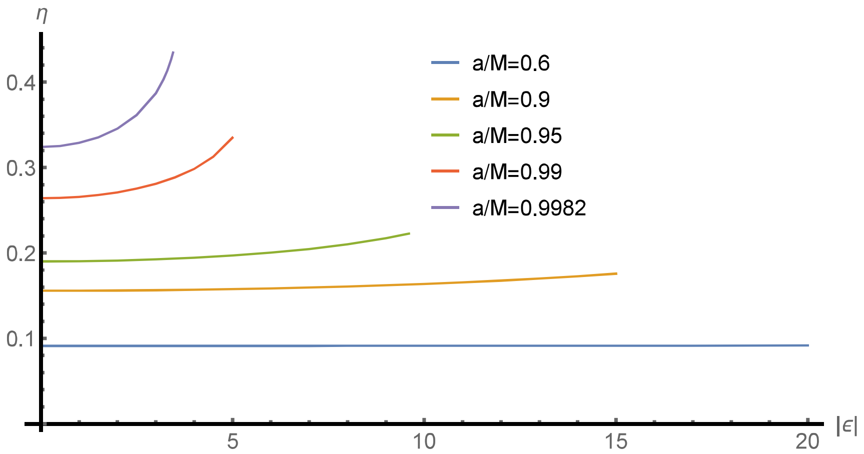


- <u>Prograde ISCOs are shrunk by ϵ</u>
  - A generic feature when deviations from Kerr are small
- Shaded region: Part of the disk becomes singular

Chen, Yang (2021)

Radiative process is enhanced

- The radiative efficiency: $\eta = 1 - E_{ISCO}$

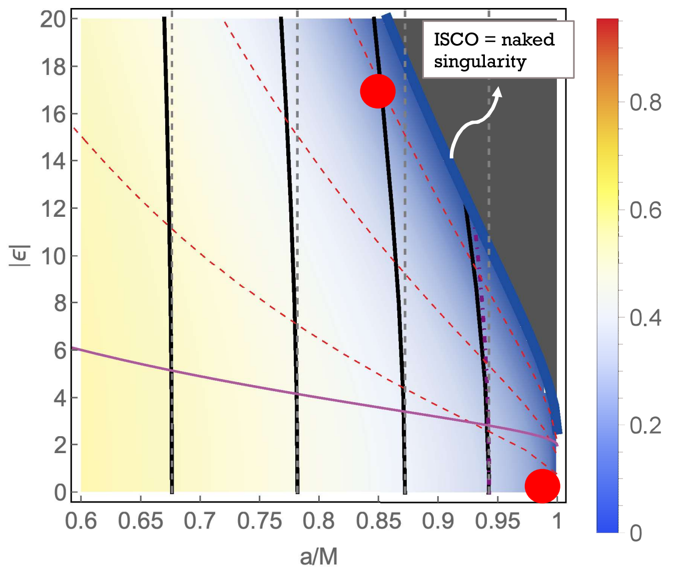


- We neglect the photon capture effect
  - One needs a reliable thin disk model for "curved" disk to consider that

Thorne (1974)

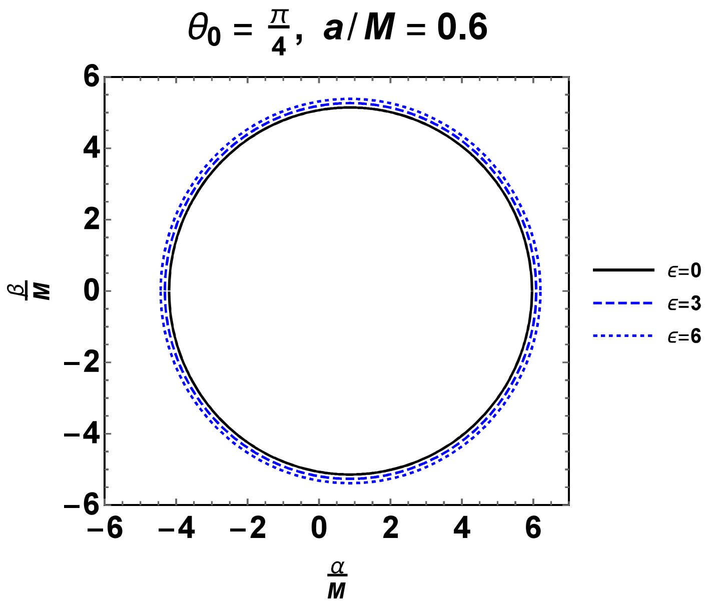
Gravitational redshifts

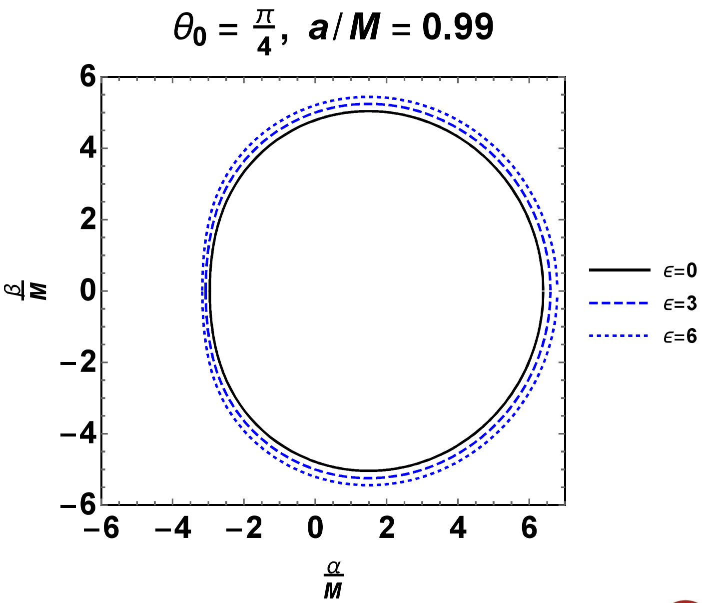
- Redshift g-factor: $g = \frac{1}{1+z} = \frac{E_o}{E_e} = \frac{1}{\dot{t}(1-\lambda_o\Omega)}$



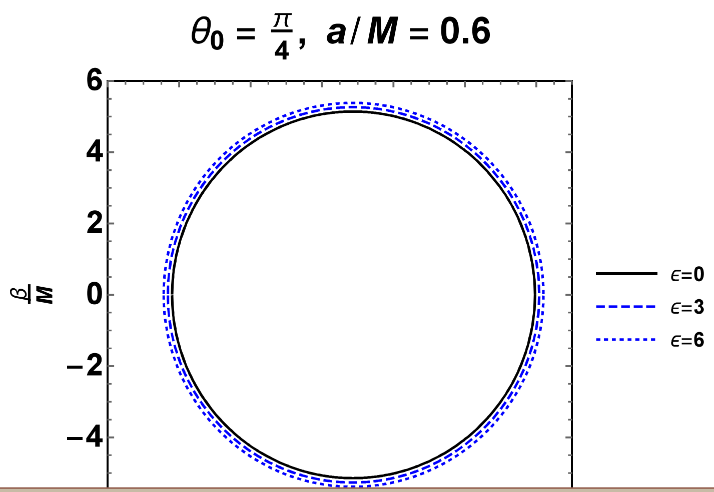
- Assuming Kerr, all spin measurements based on redshifts would overestimate the true spin of Kerr-like BH
- For example: $g = 0.164$ when
  - $a = 0.99M$ for Kerr BH
  - $a = 0.85M$ and $|\epsilon| = 17.37$ for Kerr-like BH
  - This is $\sim 16.4\%$ difference!

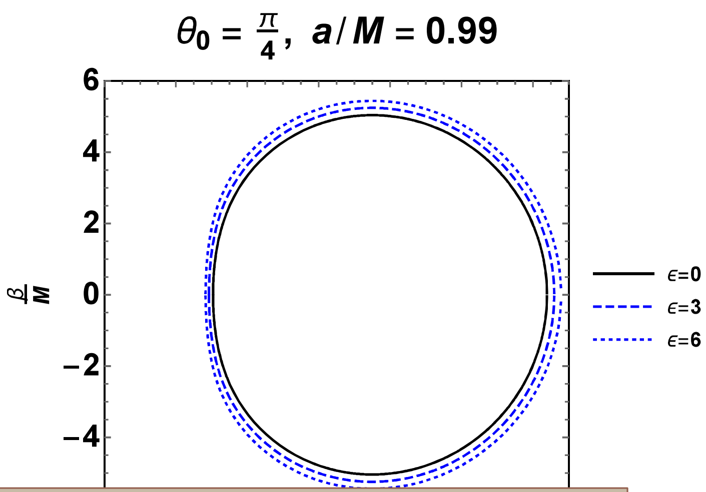
Shadow critical curve





Shadow critical curve





Remark:

- The critical curve is symmetric w.r.t. the horizontal-axis, no matter what θ_0 is!
- ϵ changes the shadow size, but does not change the shape much

Symmetry w.r.t the horizontal axis

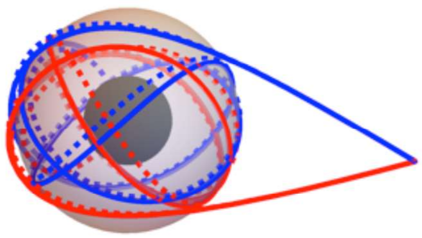
- Mathematically…

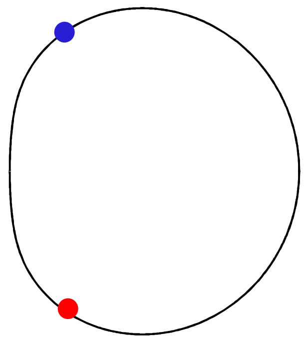
$$(A_1 + B_1)\dot{\theta}/E = \pm\sqrt{\Theta(\theta,\xi,\eta)}, \qquad \beta = \lim_{r_0\to\infty}\left(r_0^2\frac{d\theta}{dr}\right)\Big|_{r_0,\theta_0}$$

  - It is related to the separability of the geodesic equations (Carter symmetry)

Grenzebac, Perlick, Lämmerzahl (2014), Cunha, Herdeiro, Radu (2018)

- Physically…

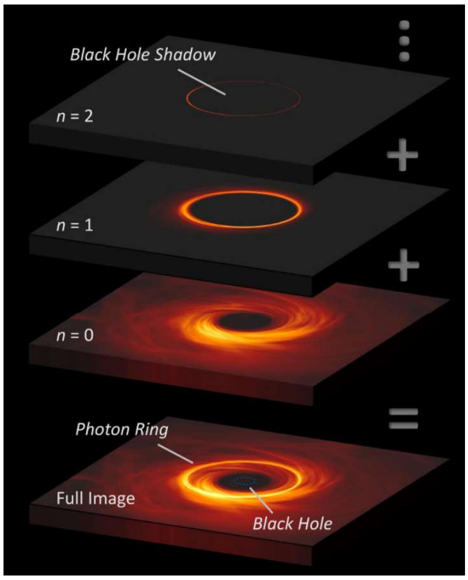




Chen (2020)

Conclusions

- Kerr-like metric: testing $\mathbb{Z}_2$ symmetry
- Astrophysics implications:
  - Curved accretion disk
  - Smaller prograde ISCO radius
- The shadow critical curve:
  - Symmetric w.r.t the horizontal axis (regardless of the inclination)
  - The deviation parameter is more sensitive to the apparent size
- Future:
  - Curved thin disk model, luminosity, temperature…
  - plunging processes
  - How the $\mathbb{Z}_2$ asymmetry manifests in sub-ring images ($n < \infty$)



Thank you for your attention!

Session D3b 14:30–16:00

[Chair: Keisuke Izumi]

Hsu-Wen Chiang

LeCosPA, National Taiwan University

"Would gravitational wave implosion around a central black hole generate horizon deformation akin to supertranslation?"

(15 min.)

[JGRG30 (2021) 120929]

“Modification to the Hawking temperature of a dynamical black hole by a time-dependent supertranslation,” JHEP 20 (2020) 089 arXiv:2004.05045
“Implanting soft hairs on the black hole horizon through gravitational wave implosion,” will be online this year (?)
“Probing the structure of dynamical soft hairs through scalar field quasinormal modes,” will be online this year (?)
“Extracting energy from a dynamical-soft-hair-induced ergosphere via the Blandford–Znajek process,” will be online this year (?)

Would GW Implosion around BH Generate Horizon Deformation akin to Super-translation?

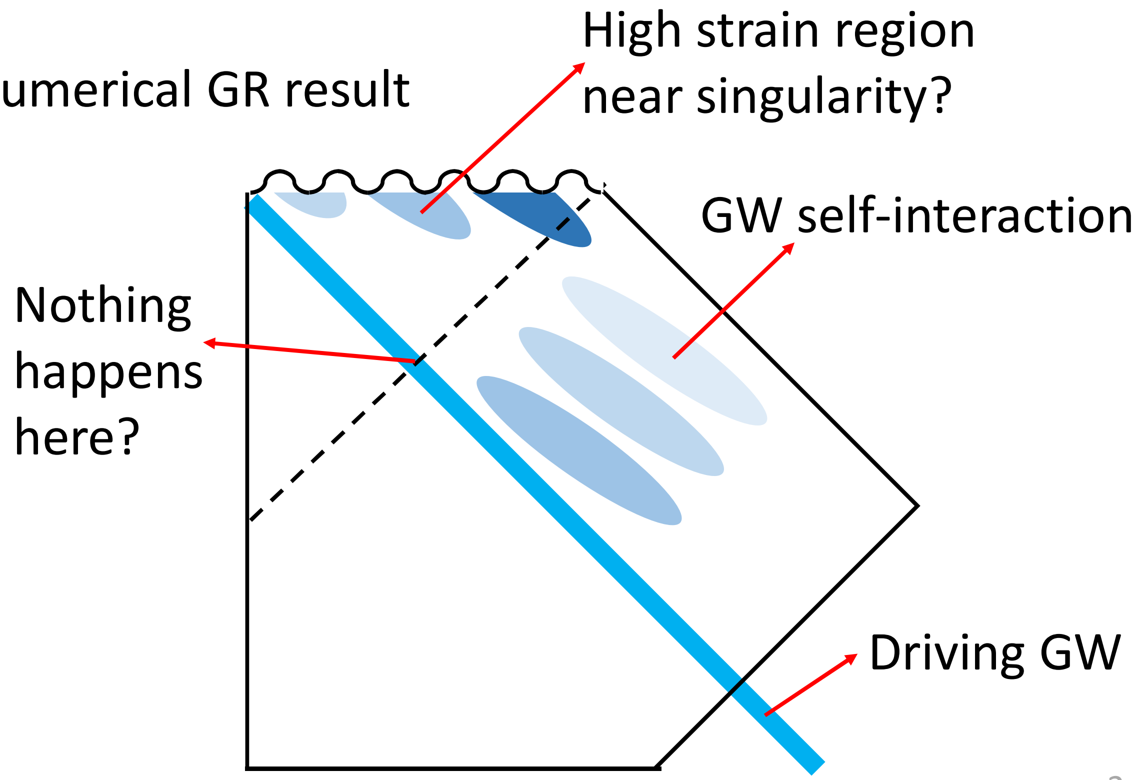
Hsu-Wen Chiang
based on works in collaboration with
Yu-Hsein Kung, Che-Yu Chen, Feng-Li Lin and Pisin Chen

Leung Center for Cosmology and Particle Astrophysics (LeCosPA)
National Taiwan University (NTU)

“How to kick a black hole,” J. Frauendiener, plenary talk at AAPPS-DACG 2021.

How to Kick a Black Hole?

- Numerical GR result

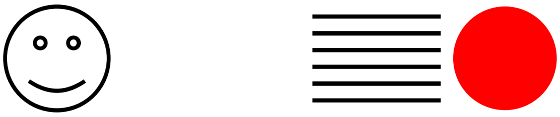


"Gravitational-wave bursts with memory: The Christodoulou effect," Kip S. Thorne. Phys. Rev. D 45, 520 (1992)

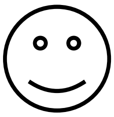
Intuitive Derivation of Soft Hair

- Doppler effect on Gravitational attraction.
- Moving toward us → Blueshift → More attraction.

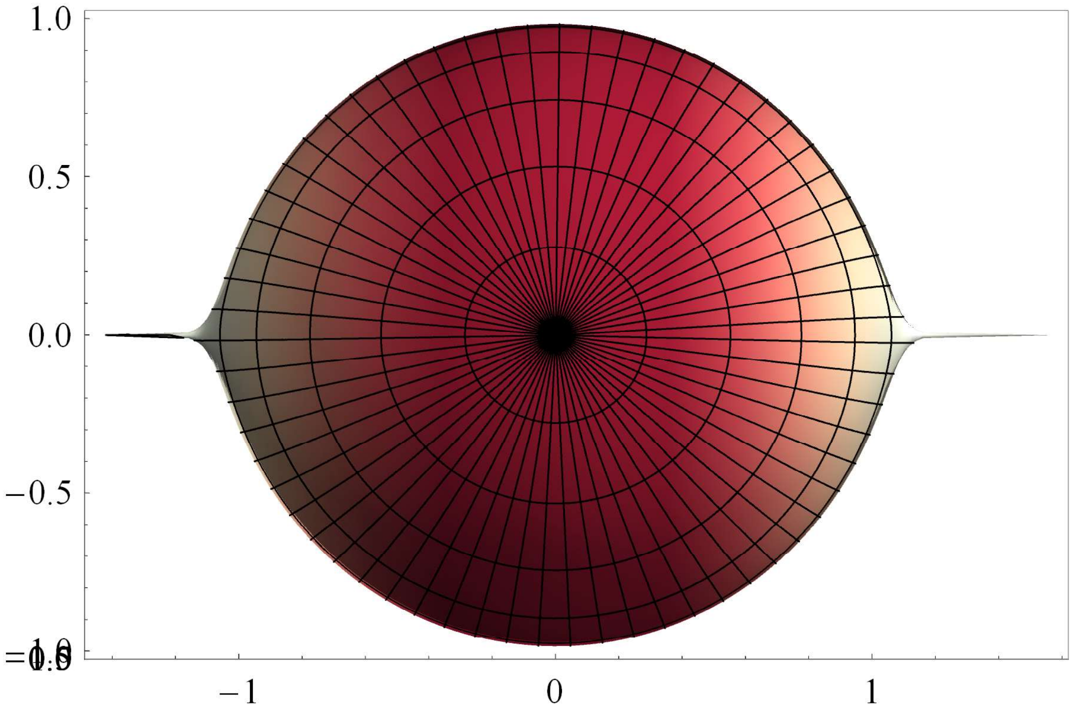
- Moving away from us → Redshift → Less attraction.



- Net result → Permanent displacement: soft hair.



Constant r Surface after 2 Particles toward and Staying at the Center



“Gravitational waves in general relativity VII. Waves from isolated axisymmetric systems,” H. Bondi, M. G. J. van der Burg & A. W. K. Metzner, Proc. Roy. Soc. Lond. A269, 21 (1962)
“Superrotation Charge and Supertranslation Hair on Black Holes,” by Hawking, Perry, Strominger (1611.09175)

Soft Hair…or Soft Wig?

- Most general large r coor. transform w/ asymptotic flat condition w/o rigid boost, rotation &translation is supertranslation (static anisotropic translation)

(Abelian) $\delta v = f(\theta^A), \delta r = -\frac{1}{2}\mathfrak{D}^2 f, \delta\theta^A = \frac{1}{r}\mathfrak{D}^A f$ Lensing Pot.

- Supertranslation can be generated by matter flow.

$\mathfrak{D}_A \equiv r P_{S^2} \nabla_A P_{S^2}$ Bondi news aspect $|g_{AB}|^2_{TT}$

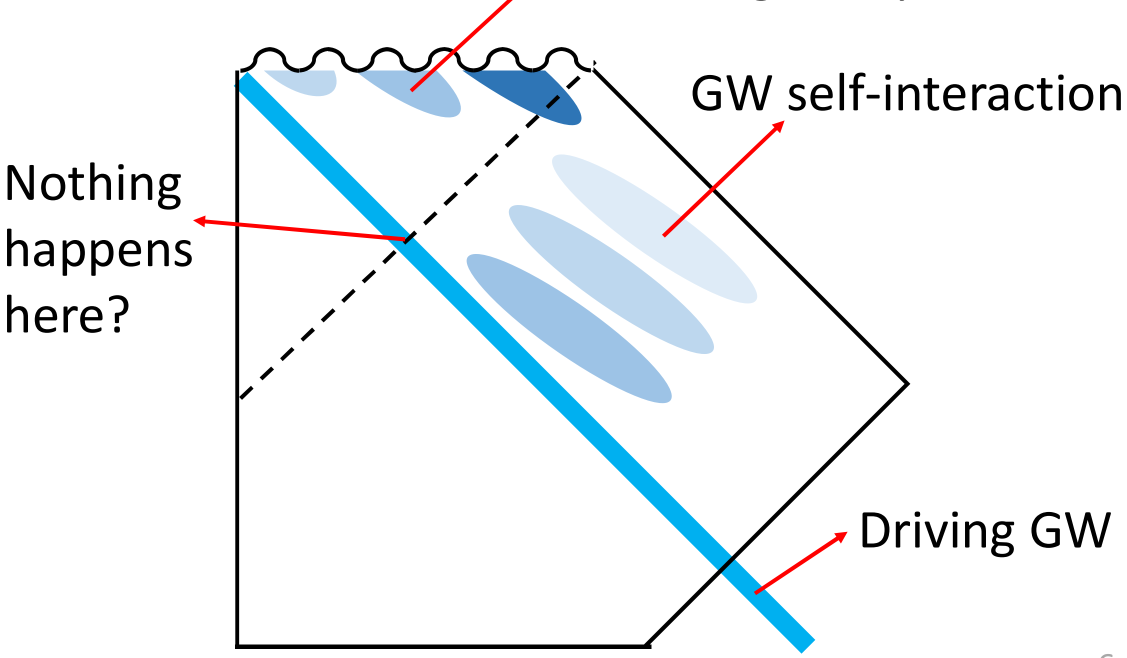
$$\partial_v M = 4\pi r^2 \left(T^M_{vv} + T^G_{vv}\right)\Big|_{I^-} + \mathfrak{D}^2(\mathfrak{D}^2 + 2)\partial_v f/4$$

- Hawking, Perry, Strominger (HPS) extended it to near horizon region (no GW).

“How to kick a black hole,” J. Frauendiener, plenary talk at AAPPS-DACG 2021.

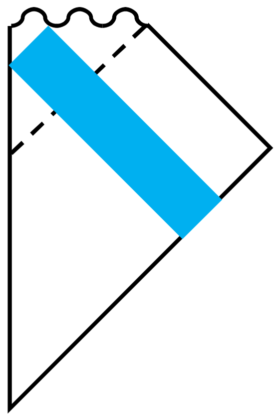
How to Kick a Black Hole?

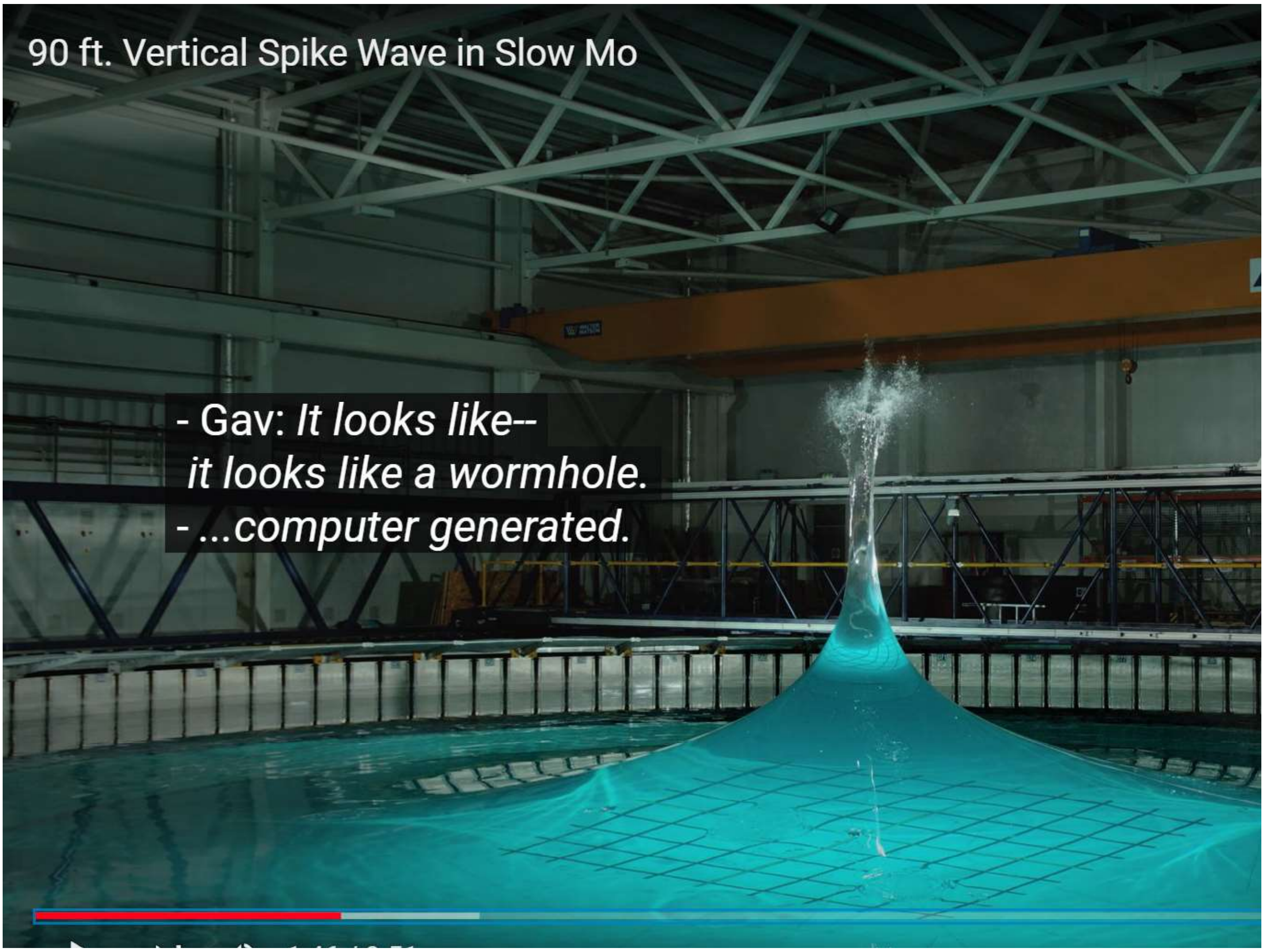
- Numerical GR result



Gravitational Wave Implosion around Black Hole

- Gravitational wave? → Solution of linearized EFE
  $R^{(1)}_{\mu\nu}\left[g^{(1)}_{\mu\nu}\right] = 0$ (polar/axial perturbation).
- Soft hair? → 2nd order DC effect!
  $\mathrm{T}^{(1)}_{\mu\nu}\left[g^{(2)}_{\mu\nu}\right] + \mathrm{T}^{(2)}_{\mu\nu}\left[g^{(1)}_{\mu\nu}\right] = 0.$
- $\mathrm{T}^{(2)}_{\mu\nu}\left[g^{(1)}_{\mu\nu}\right]$ is our target of interest.
- Could it appear as HPS envisioned?
- What should be the i.b.c. of GW?

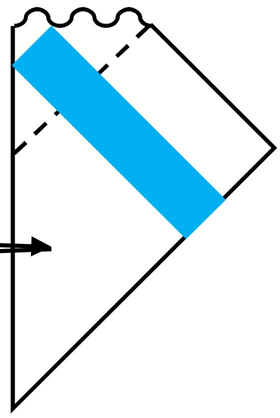




“The formation of black holes in general relativity,” D. Christodoulou [0805.3880]

UV & Coarse-graining Limit

- Implosion radius ~ inverse of GW freq. $\omega^{-1} \ll 2M$. Metric perturbation is small even inside horizon.
- 2nd order term bilinear in GW form → Complicated except for special cases.
- We choose squeezed triangle limit. I.e., coarse-graining: $l, s \gg j \sim 0$.

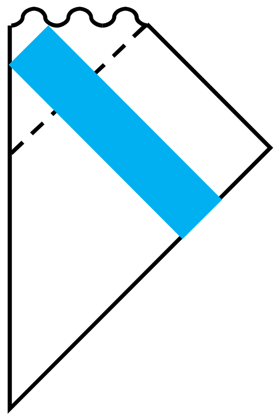


- Only free parameter is the scaling relation between l and ωM^{-1}.

Scaling Relation and the Emergence of HPS Soft Hair

Parametrizing ωM^{-1} as l^n.

- $n > 2$: Soft hair, i.e., focused beam $O(r^{-2})$.
- $n = 2$: GW self-interaction between polar and axial mode dominates. Breakdown of the perturbation series.
- $1 > n > 2$: Polarized GW only. Inter-mode self-interaction diverges.
- $n = 1$: Axial mode generates soft hair but the polar mode breaks down.
- $n < 1$: $T^{(2)}_{\mu\nu}\left[g^{(1)}_{\mu\nu}\right]$ diverges inside $r = O(M\, l^{1-n})$



Conclusion and Future Work

- GW implosion leads to either soft hair or strong gravity region outside BH, validating HPS.
- Near-singularity behavior? → Implosion core or supertranslation singularity perhaps.
- Nonstationary setup such as shockwaves?
- AC (time-domain) 2nd order effect?
- Fine-grain horizon deformation?

Session D3b 14:30–16:00

[Chair: Keisuke Izumi]

Timothy Anson

ITMP, Moscow

"Disforming the Kerr metric"

(15 min.)

[JGRG30 (2021) 120930]

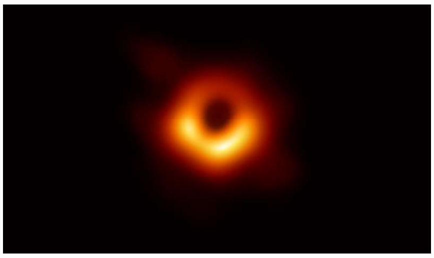
Disforming the Kerr metric

December 9th 2021,
30th Workshop on General Relativity and Gravitation in Japan

Timothy Anson
Institute for Theoretical and Mathematical Physics, Moscow

- TA, E. Babichev, C. Charmousis and M. Hassaine, *Disforming the Kerr metric*, *JHEP* **01** (2021) 018
- TA, E. Babichev and C. Charmousis, *Deformed black hole in Sagittarius A*, *Phys. Rev. D* **103** (2021) 124035

Introduction



- There is growing evidence that black holes exist in Nature: observation of stars around Sgr A* **[GRAVITY]**; detection of gravitational waves **[LIGO/VIRGO, 2015,...]**; imaging of M87* **[EHT, 2019]**
- In general relativity, rotating black holes are described by the Kerr metric
- It is interesting to construct deformations of the Kerr spacetime, in order to test general relativity and find signatures of modified gravity
- Usually, *ad hoc* deformations of the Kerr spacetime are constructed **[Psaltis+, 2011; Johannsen, 2013; Papadopoulos+, 2018; ...]**
- Using the disformal map, one can construct deformed versions of the Kerr spacetime which are solutions to higher-order scalar-tensor theories

Stealth-Kerr solution in higher-order gravity

$$\mathcal{L} = f(\phi, X)R + K(\phi, X) - G_3(\phi, X)\Box\phi + A_1(\phi, X)\,\phi_{\mu\nu}\phi^{\mu\nu} + A_2(\phi, X)\,(\Box\phi)^2 \\ + A_3(\phi, X)\,\phi_{\mu\nu}\phi^{\mu}\phi^{\nu}\Box\phi + A_4(\phi, X)\,\phi_{\mu\alpha}\phi^{\alpha\nu}\phi^{\mu}\phi_{\nu} + A_5(\phi, X)\,(\phi_{\mu\nu}\phi^{\mu}\phi^{\nu})^2$$

+ degeneracy conditions on the A_i, $X = (\partial\phi)^2$

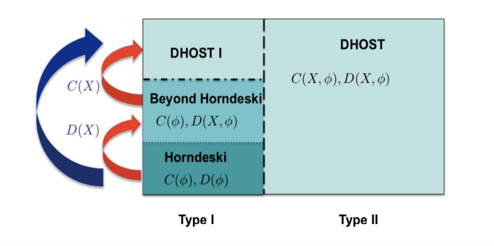
- DHOST theories **[Langlois+,Crisostomi+,2015]** contain higher derivatives in the action but are free of the Ostrogradsky ghost
- A stealth-Kerr solution was constructed **[Charmousis+, 2019]**, where the scalar field is the Hamilton-Jacobi potential of the Kerr spacetime

$$g = g_{\mathrm{Kerr}}$$
$$\phi = -Et + L_z\varphi \pm \int \frac{\sqrt{\mathcal{R}(r)}}{\Delta}\mathrm{d}r \pm \int \sqrt{\Theta(\theta)}\mathrm{d}\theta$$

- The scalar defines a geodesic direction because

$$\nabla_\nu\left(\nabla^\mu\phi\nabla_\mu\phi\right) = 0 \Rightarrow \nabla^\mu\phi\nabla_\mu\nabla_\nu\phi = 0$$

Stability of the DHOST Ia class under the disformal map



[Langlois, 2018]

- The Ia subclass can be obtained from Horndeski theories by a disformal transformation of the metric **[Ben Achour+; Crisostomi+, 2016; ...]**:

$$\tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

- The theories are different because of the matter coupling:

$$\tilde{S}\left[\tilde{g}_{\mu\nu}, \phi\right] + S_{\mathrm{m}}\left[\tilde{g}_{\mu\nu}, \Psi_{\mathrm{m}}\right] \xrightarrow{\text{DISFORMAL}} S\left[g_{\mu\nu}, \phi\right] + S_{\mathrm{m}}\left[g_{\mu\nu}, \Psi_{\mathrm{m}}\right]$$

Disformed Kerr metric

$$\tilde{g}_{\mu\nu} = g^{\mathrm{K}}_{\mu\nu} - \frac{D}{q^2}\,\partial_\mu\phi\,\partial_\nu\phi\,,$$
$$\phi = q\left[t + \int \frac{\sqrt{2Mr(a^2+r^2)}}{\Delta}\mathrm{d}r\right]\,.$$

- We start from the Kerr solution g^{K}, and perform a disformal transformation

$$\mathrm{d}\tilde{s}^2 = -\left(1-\frac{2\tilde{M}r}{\rho^2}\right)\mathrm{d}t^2 + \frac{\rho^2\Delta - 2\tilde{M}rD(1+D)(a^2+r^2)}{\Delta^2}\mathrm{d}r^2 - \frac{4\sqrt{1+D}\tilde{M}ar\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi$$
$$+\frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta\right]\mathrm{d}\varphi^2 + \rho^2\mathrm{d}\theta^2 - 2D\frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}\mathrm{d}t\mathrm{d}r$$

with $\tilde{M} = M/(1+D)$ and the rescaling $t \to \sqrt{1+D}t$

- a is the black hole spin, $\Delta = r^2 + a^2 - 2Mr$, $\rho^2 = r^2 + a^2\cos^2\theta$
- If $a = 0$, we recover the Schwarzschild metric with mass $\tilde{M}$ **[Babichev+, 2017]**

Regular solution

- The disformed metric has the following curvature scalars

$$\tilde{R} = -\frac{Da^2Mr[1+3\cos(2\theta)]}{(1+D)\rho^6}\,,\quad \tilde{R}_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta} = \frac{M^2Q_2(r,\theta)}{\rho^{12}(r^2+a^2)(1+D)^2}\,,\ldots$$

- The solution is not Ricci-flat, but the only singularity is at $\rho = 0$, like Kerr. To verify this, one changes coordinates to

$$t \to v - r - \int\frac{2Mr}{\Delta}\mathrm{d}r\,,\qquad \varphi \to -\Phi - a\int\frac{\mathrm{d}r}{\Delta}$$

- The metric components are regular in these coordinates, and the scalar field reads

$$\phi = q_0\left(v - r + \int\frac{\mathrm{d}r}{1+\sqrt{\frac{r^2+a^2}{2Mr}}}\right)$$

- The spacetime is stably causal (ϕ is a cosmic time function)

Properties of the disformed metric

- We still have axisymmetry (two commuting Killing vectors ∂_t and ∂_φ)
- However, defining $\xi_{(t)} = \tilde{g}_{t\mu}\mathrm{d}x^\mu$ and $\xi_{(\varphi)} = \tilde{g}_{\varphi\mu}\mathrm{d}x^\mu$, we now have

$$\xi_{(t)} \wedge \xi_{(\varphi)} \wedge \mathrm{d}\xi_{(t)} = -D\frac{4a^2\tilde{M}r\sqrt{2\tilde{M}r(a^2+r^2)}\cos\theta\sin^3\theta}{\rho^4}\mathrm{d}t\wedge\mathrm{d}r\wedge\mathrm{d}\theta\wedge\mathrm{d}\varphi$$

- This noncircularity means we cannot write the metric in a form that is invariant under the reflection $(t,\varphi)\to(-t,-\varphi)$
- It also has an impact on the separability structure of the spacetime, and we no longer have a nontrivial Killing tensor **[Benenti+, 1979,1980]**
- Interesting because noncircular spacetimes can exist even in GR (for instance in the presence of toroidal magnetic fields), and the circular ansatz fails in certain situations **[Van Aelst+, 2019]**

Asymptotically similar to Kerr

- Asymptotically, the Kerr metric can be written

$$\mathrm{d}s^2_{\text{Kerr}} = -\left[1-\frac{2\tilde{M}}{r}+\mathcal{O}\left(\frac{1}{r^3}\right)\right]\mathrm{d}T^2 - \left[\frac{4\tilde{a}\tilde{M}}{r^3}+\mathcal{O}\left(\frac{1}{r^5}\right)\right][x\mathrm{d}y-y\mathrm{d}x]\,\mathrm{d}T + \left[1+\mathcal{O}\left(\frac{1}{r}\right)\right]\left[\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\right]$$

- The physical parameters determined from the asymptotic expansion are

$$\tilde{M} = \frac{M}{1+D}\,, \qquad \tilde{a} = a\sqrt{1+D}$$

- After a coordinate transformation, one can write the disformal metric as

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}s^2_{\text{Kerr}} + \frac{D}{1+D}\left[\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)\mathrm{d}T^2 + \left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i\mathrm{d}T\mathrm{d}x^i + \left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$$

with $\alpha_i, \beta_{ij} \sim \mathcal{O}(1)$.

Stationary observers

- Consider constant (r, θ) observers, with a 4-velocity

$$\boxed{u = \partial_t + \omega \partial_\varphi}$$

- The condition $u^2 \leq 0$ implies $\omega \in [\omega_-, \omega_+]$, where

$$\boxed{\omega_\pm = \frac{1}{\tilde{g}_{\varphi\varphi}} \left(-\tilde{g}_{t\varphi} \pm \sqrt{\tilde{g}_{t\varphi}^2 - \tilde{g}_{tt}\tilde{g}_{\varphi\varphi}} \right)}$$

- Inside the static limit defined by $\tilde{g}_{tt} = 0$, one necessarily has $\omega_- > 0$
- These observers no longer exist when $\tilde{g}_{t\varphi}^2 - \tilde{g}_{tt}\tilde{g}_{\varphi\varphi} = 0$, which happens when

$$\boxed{P(r, \theta) \equiv r^2 + a^2 - 2\tilde{M}r + \frac{2\tilde{M}Da^2 r \sin^2\theta}{\rho^2(r, \theta)} = 0}$$

- The outermost surface $r = R_0(\theta)$ which satisfies $P(R_0(\theta), \theta) = 0$ is called the stationary limit

Nature of the stationary limit

$$\boxed{P(r, \theta) \equiv r^2 + a^2 - 2\tilde{M}r + \frac{2\tilde{M}Da^2 r \sin^2\theta}{\rho^2(r, \theta)} = 0}$$

- When $D = 0$, the stationary limit coincides with the event horizon
- In the general case, the normal vector N to this surface is

$$N_\mu = \left(0, 1, -R_0'(\theta), 0\right)$$

- One can check that

$$\boxed{N^2|_{r=R_0} = \tilde{g}^{rr} + \tilde{g}^{\theta\theta} R_0'^2 > 0}$$

- Hence the surface is timelike and cannot be the event horizon in the general case
- All Killing vectors of the form $\partial_t + \omega \partial_\varphi$ are spacelike inside this surface, so if there is an event horizon, it cannot be a Killing horizon

Event horizon ?

- For Kerr, the horizons are found by solving $g^{rr} = 0 \implies \Delta = 0$ which admits constant r solutions. In our case, we have $\tilde{g}^{rr} = 0 \implies P = 0$, which doesn't admit constant r solutions when $D \neq 0$
- We look for more general null hypersurface of the form $r = R(\theta)$. The normal has components

$$\boxed{n_\mu = \left(0, 1, -R'(\theta), 0\right)}$$

- The condition $n^2 = 0$ yields

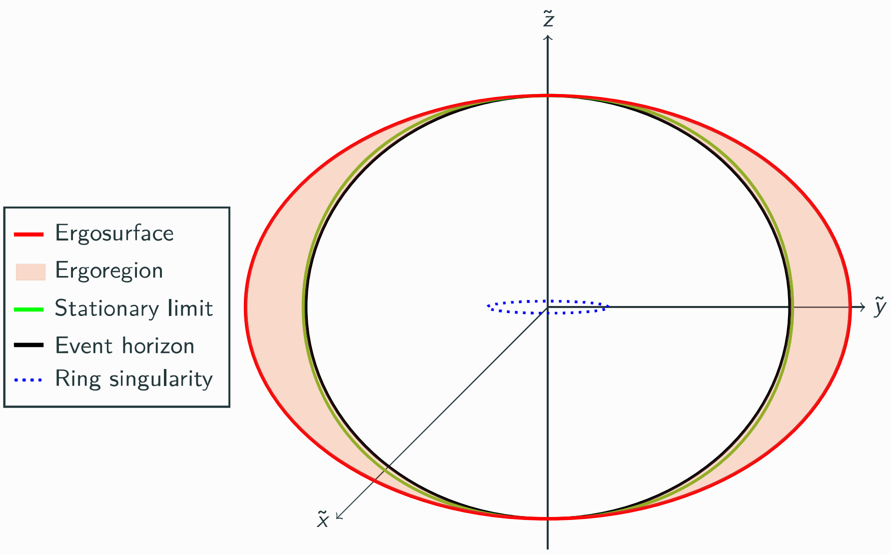
$$\boxed{R'(\theta)^2 + P(R,\theta) = R'(\theta)^2 + R^2 + a^2 - 2\tilde{M}R + \frac{2\tilde{M}Da^2 R\sin^2\theta}{\rho^2(R,\theta)} = 0}$$

- To have a smooth solution, we must have

$$\boxed{R'(0) = R'(\tfrac{\pi}{2}) = 0}$$

- This puts bounds on the rotation parameter $a \leq a_c(D)$

Hypersurfaces in the disformed Kerr spacetime



Limit $D \to \infty$ and $D = -1$, simpler noncircular spacetimes

- In the limit $D \to \infty$, we obtain with $\tilde{\chi} = \tilde{a}/\tilde{M}$ (noncircular Schwarzschild)

$$\mathrm{d}\tilde{s}^2_{\mathrm{NCS}} = -\left(1 - \frac{2\tilde{M}}{r}\right)\left(\mathrm{d}T + \frac{2\tilde{\chi}\tilde{M}^2\sin^2\theta}{r - 2\tilde{M}}\mathrm{d}\varphi\right)^2 + \left(1 - \frac{2\tilde{M}}{r}\right)^{-1}\left(\mathrm{d}r - \sqrt{\frac{2\tilde{M}^3}{r}}\tilde{\chi}\sin^2\theta\mathrm{d}\varphi\right)^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)$$

- In the limit $D \to -1$ (quasi-Weyl)

$$\mathrm{d}\tilde{s}^2_{\mathrm{QW}} = -\left(1 - \frac{2\tilde{M}r}{r^2 + a^2\cos^2\theta}\right)\mathrm{d}t^2 + \frac{r^2 + a^2\cos^2\theta}{r^2 + a^2}\mathrm{d}r^2 + 2\sqrt{\frac{2\tilde{M}r}{r^2 + a^2}}\mathrm{d}t\mathrm{d}r + \left(r^2 + a^2\cos^2\theta\right)\mathrm{d}\theta^2 + \left(r^2 + a^2\right)\sin^2\theta\mathrm{d}\varphi^2$$

- Examples of noncircular spacetimes which may be useful to understand some properties of the disformed Kerr metric

Conclusion

- We constructed deformations of the Kerr spacetime using the disformal transformation. While asymptotically very similar to Kerr, the solution presents many interesting properties: noncircularity, horizon not located at constant r and not a Killing horizon, the stationary limit is distinct from the event horizon
- In another paper, we have calculated the secular variation of orbital parameters for stars around a deformed black hole, and shown that the no-hair theorem of GR is violated in general for these spacetimes. The 1PN corrections are modified if $D \sim -1$
- Other papers have studied some aspects of these solutions: the particular DHOST theories that these objects are a solution of **[Achour+, 2020]**; shadows of this black hole **[Long+, 2020]**; testing noncircularity with pulsars orbiting Sgr A* **[Takamori+, 2021]**; ...

Thank you for your attention.

Session D3b 14:30–16:00

[Chair: Keisuke Izumi]

Masato Minamitsuji

CENTRA, IST, U-Lisboa

"Black holes in the extended vector-tensor theories"

(15 min.)

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Black holes in extended vector-tensor theories (D3b5)

Masato Minamitsuji
(CENTRA, IST, Lisbon)

Classical and Quantum Gravity **38**, 105011 (2021)

JGRG30
(Waseda University, Tokyo)

December 9, 2021

Introduction

- Higher-derivative theories have attracted revived interests in theoretical physics.
- Degeneracy among higher-derivative EOMs can avoid Ostrogradsky instabilities

  Motohashi, Noui, Suyama, Yamaguchi, and Langlois (2016)
- Degenerate higher-order scalar-tensor (DHOST) theories have been constructed and applied to cosmology and black hole (BH) physics

  Langlois and Noui (2015)
- Extended vector-tensor theories are degenerate vector-tensor theories

  Kimura, Naruko and Yoshida (2016)
- We investigate static and spherically symmetric vacuum solutions in extended vector-tensor theories.

Extended vector-tensor theories

$$S = \int d^4x\sqrt{-g}\,[f_0(\mathcal{Y}) + f_2(\mathcal{Y})\,R + C^{\mu\nu\rho\sigma}\nabla_\mu A_\nu \nabla_\rho A_\sigma]\,, \quad \mathcal{Y} = g^{\mu\nu}A_\mu A_\nu$$

Kimura, Naruko, and Yoshida (2016)

$$\begin{aligned} C^{\mu\nu\rho\sigma} \quad & \alpha_1\ \mathcal{Y}\ g^{\mu(\rho}g^{\sigma)\nu} \quad \alpha_2\ \mathcal{Y}\ g^{\mu\nu}g^{\rho\sigma} \quad \frac{\alpha_3\ \mathcal{Y}}{\quad}\ A^\mu A^\nu g^{\rho\sigma} \quad A^\rho A^\sigma g^{\mu\nu} \\ & \frac{\alpha_4\ \mathcal{Y}}{\quad}\left(A^\mu A^{(\rho}g^{\sigma)\nu} \quad A^\nu A^{(\rho}g^{\sigma)\mu}\right) \quad \alpha_5\ \mathcal{Y}\ A^\mu A^\nu A^\rho A^\sigma \quad \alpha_6\ \mathcal{Y}\ g^{\mu[\rho}g^{\sigma]\nu} \\ & \frac{\alpha_7\ \mathcal{Y}}{\quad}\left(A^\mu A^{[\rho}g^{\sigma]\nu} - A^\nu A^{[\rho}g^{\sigma]\mu}\right) \quad \frac{\alpha_8\ \mathcal{Y}}{\quad}\ A^\mu A^\rho g^{\sigma\nu} - A^\nu A^\sigma g^{\mu\rho}\ . \end{aligned}$$

- symmetric contractions $\alpha_1 - \alpha_5$
  $\Longrightarrow$ quadratic DHOST theories in the limit of $A_\mu \to \partial_\mu\phi$
- antisymmetric contractions $\alpha_6 - \alpha_8$
  $\Longrightarrow$ interactions with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- Maxwell term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ from $\alpha_6 = -1 \Longrightarrow$ only $U(1)$-invariant

Degeneracy conditions

- Class A $(\alpha_1 + \alpha_2 = 0)$;
  disformally mapped from generalized Proca theories
  - A1: $\alpha_1 = -\alpha_2 = \frac{f_2}{\mathcal{Y}}$, $\alpha_3 = \frac{2(f_2 - 2f_{2,\mathcal{Y}}\mathcal{Y})}{\mathcal{Y}^2}$
  - A2; $\alpha_1 = -\alpha_2 = \frac{f_2}{\mathcal{Y}}$, $\alpha_4 = \frac{6f_2 + \mathcal{Y}\beta}{\mathcal{Y}^2} - \alpha_8$, $\beta := -2\alpha_6 - \alpha_7$
  - A3; $\alpha_1 \quad -\alpha_2 \quad -\frac{(\alpha_4+\alpha_8)\mathcal{Y}-\beta}{2}$
    $\alpha_3 \quad \frac{1}{4f_2} \quad \alpha_4 \quad \alpha_8\ \mathcal{Y} - \beta \quad - \ \beta \quad f_{2,\mathcal{Y}} \quad \alpha_8\mathcal{Y} - \quad \alpha_4 \quad \alpha_8\ f_2$
  - A4; $\alpha_2 \quad -\alpha_1$
    $\alpha_4 \quad \frac{-2f_2^2\Xi_3 + 2\mathcal{Y}\alpha_1\Xi_4 + f_2\Xi_5}{\Xi_1}$

$$\alpha_3 \quad \alpha_4 \quad \alpha_5\mathcal{Y} - \frac{\alpha_2 \quad \mathcal{Y}\alpha_3 \quad f_{2\mathcal{Y}} \ ^2}{f_2 \quad \mathcal{Y}\alpha_2}\ ,$$

- Class-B $(\alpha_1 + \alpha_2 \neq 0)$

Static and spherically symmetric solutions

$$
\begin{aligned}
g_{\mu\nu}dx^{\mu}dx^{\nu} &= -f(r)dt^2 + \frac{dr^2}{h(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \\
A_{\mu}dx^{\mu} &= A_t(r)dt + A_r(r)dr
\end{aligned}
$$

1. Reissner-Nordström [-(anti-)de Sitter] solutions with $\mathcal{Y} = \mathcal{Y}_0 := \mathrm{const}$

$$
f(r) = h(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 + \frac{\mathcal{Q}^2}{r^2}, \quad A_t(r) = q + \frac{Q}{r}
$$

$$
\Longrightarrow \mathcal{Q} = \mathcal{F}(\alpha_1, \alpha_2, \cdots)Q
$$

charged stealth Schwarzschild solutions if $\mathcal{F} = 0$.

2. Black hole solutions with non-GR metric and $\mathcal{Y}(r) \neq \mathrm{const}$ obtained via disformal mapping from generalized Proca theories

RN [-(anti-) de Sitter] solutions ($Q \neq 0$)

- $\mathcal{Y}_0 = -q^2$

$$
\begin{aligned}
&f_0 \quad -2\Lambda\left(f_2 \quad q^2\alpha_1\right), \quad f_{0,\mathcal{Y}} \quad \frac{\Lambda}{}\left[\ \alpha_1 - \quad f_{2,\mathcal{Y}} \quad q^2 \quad \alpha_3 \quad \alpha_{1,\mathcal{Y}}\ \right], \\
&\alpha_2 \quad -\alpha_1, \quad \alpha_{2,\mathcal{Y}} \quad -\alpha_{1,\mathcal{Y}}, \\
&\alpha_8 \quad -\left[\ \alpha_3 - \quad \alpha_4 - \alpha_7 \quad \alpha_{1,\mathcal{Y}} - \quad \alpha_{6,\mathcal{Y}} \quad q^2\left(\alpha_{4,\mathcal{Y}} \quad \alpha_{7,\mathcal{Y}} \quad \alpha_{8,\mathcal{Y}}\right)\right], \\
&\mathcal{Q} \quad \sqrt{\frac{\alpha_1 \quad q^2\ \alpha_4 \quad \alpha_7 \quad \alpha_8 \ - \ \alpha_6}{f_2 \quad q^2\alpha_1 \quad q^4\ \alpha_3 \quad \alpha_{1,\mathcal{Y}}}}\,Q
\end{aligned}
$$

- $\mathcal{Y}_0 \neq -q^2$

$$
\begin{aligned}
&f_0 \quad -2\Lambda f_2, \quad f_{0,\mathcal{Y}} = \Lambda(- \quad f_{2,\mathcal{Y}} \quad \mathcal{Y}_0\alpha_{1,\mathcal{Y}} \quad , \\
&\alpha_1 \quad \alpha_2 \quad \prime, \quad \alpha_{2,\mathcal{Y}} \quad -\alpha_{1,\mathcal{Y}}, \quad \alpha_3 \quad - \ \alpha_{1,\mathcal{Y}}, \\
&\alpha_8 \quad -- \quad \alpha_4 \quad \alpha_7 - \quad \alpha_{1,\mathcal{Y}} \quad \alpha_{6,\mathcal{Y}} \quad \mathcal{Y}_0\ \alpha_{4,\mathcal{Y}} \quad \alpha_{7,\mathcal{Y}} \quad \alpha_{8,\mathcal{Y}} \quad , \\
&\mathcal{Q} \quad \sqrt{-\frac{\alpha_4 \quad \alpha_7 \quad \alpha_8\ \mathcal{Y}_0 \quad \alpha_6}{f_2 \quad \mathcal{Y}_0^2\alpha_{1,\mathcal{Y}}}}\,Q
\end{aligned}
$$

- compatible only with Class A ($\alpha_1 + \alpha_2 = 0$)

Disformal mapping from generalized Proca theories

generalized Proca theories

$$
\begin{aligned}
\mathcal{L}_{\rm GP} &= \sqrt{-\bar{g}}\Big[\bar{G}_2(\bar{\mathcal{Y}}) + \bar{G}_4(\bar{\mathcal{Y}})\bar{R} - \frac{1}{4}\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} \\
&- 2\bar{G}_{4\bar{\mathcal{Y}}}(\bar{\mathcal{Y}})\left\{\left(\bar{g}^{\mu\nu}\bar{\nabla}_\mu A_\nu\right)^2 - \bar{g}^{\mu\nu}\bar{g}^{\alpha\beta}\left(\bar{\nabla}_\mu A_\alpha\right)\left(\bar{\nabla}_\beta A_\nu\right)\right\}\Big]
\end{aligned}
$$

$\Updownarrow$ disformal transformation

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + B(\mathcal{Y})A_\mu A_\nu$$

extended vector tensor theories

$$
\begin{aligned}
f_0(\mathcal{Y}) &= \bar{G}_2(\bar{\mathcal{Y}})\sqrt{B(\mathcal{Y})\mathcal{Y}+1}, \quad f_2(\mathcal{Y}) = \bar{G}_4(\bar{\mathcal{Y}})\sqrt{B(\mathcal{Y})\mathcal{Y}+1}, \\
\alpha_1(\mathcal{Y}) &= -\alpha_2(\mathcal{Y}) = \frac{2\bar{G}_{4\bar{\mathcal{Y}}}(\bar{\mathcal{Y}}) + B(\mathcal{Y})\bar{G}_4(\bar{\mathcal{Y}})(B(\mathcal{Y})\mathcal{Y}+1)}{(B(\mathcal{Y})\mathcal{Y}+1)^{3/2}}, \\
\alpha_3(\mathcal{Y}) &= -\alpha_4(\mathcal{Y}) = -\frac{2B_{\mathcal{Y}}(\mathcal{Y})\left(\bar{G}_4(\bar{\mathcal{Y}})(B(\mathcal{Y})\mathcal{Y}+1) - 2\mathcal{Y}\bar{G}_{4\bar{\mathcal{Y}}}(\bar{\mathcal{Y}})\right)}{(B(\mathcal{Y})\mathcal{Y}+1)^{3/2}}, \quad \alpha_5(\mathcal{Y}) = \alpha_8(\mathcal{Y}) = 0, \\
\alpha_6(\mathcal{Y}) &= -(1+2\bar{G}_{4,\bar{\mathcal{Y}}}(\bar{\mathcal{Y}}))\sqrt{B(\mathcal{Y})\mathcal{Y}+1} - B(\mathcal{Y})\bar{G}_4(\bar{\mathcal{Y}})\sqrt{B(\mathcal{Y})\mathcal{Y}+1}, \\
\alpha_7(\mathcal{Y}) &= \frac{2\left(2(B(\mathcal{Y})+\mathcal{Y}B_{\mathcal{Y}}(\mathcal{Y}))\bar{G}_{4\bar{\mathcal{Y}}}(\bar{\mathcal{Y}}) + \bar{G}_4(\bar{\mathcal{Y}})(B\mathcal{Y}+1)(B(\mathcal{Y})^2 - B_{\mathcal{Y}}(\mathcal{Y}))\right)}{(B(\mathcal{Y})\mathcal{Y}+1)^{3/2}} + \frac{2B(\mathcal{Y})\left(1+2\bar{G}_{4\mathcal{Y}}(\bar{\mathcal{Y}})\right)}{\sqrt{B(\mathcal{Y})\mathcal{Y}+1}}
\end{aligned}
$$

Seed model $\bar{G}_2(\bar{\mathcal{Y}}) = -m^2\bar{\mathcal{Y}} - 2\zeta\Lambda$, $\bar{G}_4(\bar{\mathcal{Y}}) = \zeta - \frac{\beta}{2}\bar{\mathcal{Y}}$

(1) Charged asymptotically AdS BHs ($\beta = \frac{1}{4}$)

$$A_t(r) = \frac{Q}{r}, \quad \mathcal{Y}(r) = -\frac{4r^2\zeta(m^2+\beta\Lambda)}{m^2r^2(1+4B_0\zeta) + 4\beta(\beta + r^2B_0\zeta\Lambda)}$$

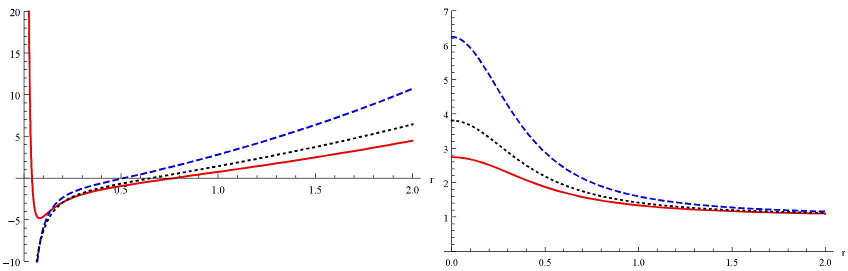


Figure: $h\ r$ $f\ r\ /h\ r$ B_0 . , − . , β . m ζ M .0, $\Lambda = -$.

Vector disformal transformation modifies the inner structure of BHs.

(2) Charged asymptotically flat BHs $\beta \neq \frac{1}{4}$ and $m = \Lambda = 0$

$$
\begin{aligned}
f(r) &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2}\frac{2\beta(-2+q^2B_0) + (1-q^2B_0)(1+4B_0\zeta)}{2\left[2\zeta + q^2(-3\beta+8\beta^2-2B_0\zeta)\right]} + \mathcal{O}\left(\frac{1}{r^3}\right),\\
h(r) &= 1 - \frac{2M}{r} - \frac{Q^2}{r^2}\frac{16\beta^2 - 2B_0\left(-1+q^2B_0\right)\zeta - 4\beta(1+4B_0\zeta) + q^2\beta B_0(1+16B_0\zeta)}{4\beta\left[2\zeta + q^2(-3\beta+8\beta^2-2B_0\zeta)\right]} + \mathcal{O}\left(\frac{1}{r^3}\right),\\
A_t(r) &= q + \frac{Q}{r} + \frac{Q^2}{r^2}\frac{q(1-4\beta)\beta}{2\zeta + q^2(-3\beta+8\beta^2-2B_0\zeta)} + \mathcal{O}\left(\frac{1}{r^3}\right),\\
\mathcal{Y}(r) &= -q^2 - \frac{Q^2}{r^2}\frac{(1-4\beta)(1-q^2B_0)\left(2\zeta - q^2(\beta+2B_0\zeta)\right)}{4\beta\left[2\zeta + q^2(-3\beta+8\beta^2-2B_0\zeta)\right]} + \mathcal{O}\left(\frac{1}{r^3}\right).
\end{aligned}
$$

distinguished with future near-horizon observations

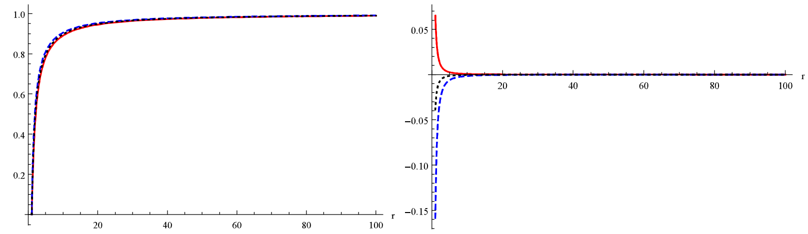


Figure: β . h r $\frac{f(r)}{h(r)}$ − B_0 . , − . , r_h ζ f_1 .
a_1 − .

Seed model $\bar{G}_2(\bar{\mathcal{Y}}) = \frac{\gamma_1}{\bar{\mathcal{Y}}}$ and $\bar{G}_4(\bar{\mathcal{Y}}) = \zeta + \frac{\beta}{\bar{\mathcal{Y}}}$

$\bar{h}(r) \to -\frac{\gamma_1}{18\beta}r^2$; cosmological horizon ($\frac{\beta}{\gamma_1} > 0$)
$h(r) \to \frac{-\gamma_1\zeta}{18\beta(\zeta+6\beta B_0)}r^2$; BH event horizon ($\zeta + 6\beta B_0 < 0$)

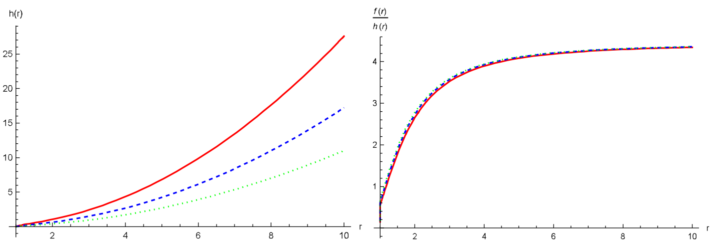


Figure: h r f r / h r B_0 − . , − . , − . ζ r_h γ_1 . β .

Vector disformal transformation maps a cosmological horizon to a black hole horizon.

Summary

Summary

1. Static and spherically symmetric BHs in extended vector-tensor theories
2. BHs with constant spacetime norm are compatible only with Class-A.
3. Vector disformal transformation could modify the internal structure of black holes and the charachter of the horizon.

Future direction

1. stability?
2. rotating solutions?
3. other compact objects; neutron stars, warmholes, and Proca stars ?

Thank you.

Session E 10:00–12:00

[Chair: Ken-ichi Nakao]

Yoshimune Tomikawa

Matsuyama University

“On uniqueness of static spacetime with conformal scalar in higher dimensions”

(15 min.)

[JGRG30 (2021) 121002]

JGRG30　12/6-10（2021）

Uniqueness of static spacetime with conformal scalar in higher dimensions

Yoshimune Tomikawa
Faculty of Economics, Matsuyama University

based on
K. Izumi, Y. Tomikawa, T. Shiromizu, PRD 104, 104025 (2021)
(arXiv: 2108.02588 [gr-qc])

Contents

1. Introduction

BBMB black hole solution

(Bocharova-Bronnikov-Melnikov-Bekenstein)

Bocharova, Bronnikov, Melnikov (1970),
Bekenstein (1974, 1975)

n=4 static and spherically symmetric

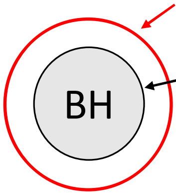
metric : $ds^2 = -\left(1 - \frac{M}{r}\right)^2 dt^2 + \left(1 - \frac{M}{r}\right)^{-2} dr^2 + r^2 d\Omega_2^2$

scalar field : $\phi = \pm\sqrt{\frac{6}{\kappa}}\frac{M}{r-M}$

event horizon : $r = M$ scalar field diverges at event horizon, but curvature does not

BBMB black hole has photon surface at $r = 2M$

photon surface $r = 2M \rightarrow 1 - \frac{\kappa}{6}\phi^2 = 0 \rightarrow S_p$



event horizon $r = M$

S_p : surface specified by $\phi = \phi_p := \pm\sqrt{6/\kappa}$

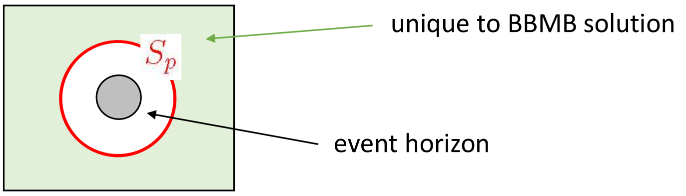
Uniqueness of BBMB solution

Tomikawa, Shiromizu, Izumi, PTEP (2017), CQG (2017),
Shinohara, Tomikawa, Izumi, Shiromizu (2021)

n=4

The outside region of S_p for the static and asymptotically flat spacetime in the Einstein-conformal scalar field system is unique to the BBMB solution.

(dose not mean uniqueness of black hole!)



S_p : surface specified by $\phi = \phi_p := \pm\sqrt{6/\kappa}$
corresponding to photon sphere

Einstein-conformal scalar system in higher dim.

Xanthopoulos, Dialynas (1992), Klimcik (1993),
Martinez, Nozawa (2021)

For n>4, static and spherically symmetric solution exist, but not black hole

No singularity outside S_p $\sim$ photon sphere

⇩

Uniqueness outside S_p such as 4 dim.?

2. Setup

Einstein-conformal scalar system in n-dim.

$g := \det(g_{\mu\nu})$

R : Ricci scalar

ϕ : scalar field

$\xi := \frac{1}{4}\frac{n-2}{n-1}$

Einstein gravity + conformal scalar field

action : $S = \frac{1}{2\kappa}\int d^n x\sqrt{-g}R - \int d^n x\sqrt{-g}\left(\frac{1}{2}(\nabla\phi)^2 + \frac{\xi}{2}R\phi^2\right)$

Einstein eq. is singular when this part vanishes

field equations :
$$\begin{cases} (1-\kappa\xi\phi^2)R_{\mu\nu} = \kappa\left[\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2 + \xi(g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu)\phi^2\right] \\ \nabla^2\phi = 0 \qquad (R = 0) \end{cases}$$

$$\phi = \phi_p := \pm\frac{1}{\sqrt{\kappa\xi}}$$

Static and Asymptotically flat spacetime (n-dim.)

metric : $ds^2 = -V^2(x^k)dt^2 + g_{ij}(x^k)dx^i dx^j$

event horizon : $V = 0$

asymptotic boundary conditions : $\begin{cases} V = 1 - \dfrac{m}{r^{n-3}} + O(1/r^{n-2}) \\ g_{ij} = \left(1 + \dfrac{2}{n-3}\dfrac{m}{r^{n-3}}\right)\delta_{ij} + O(1/r^{n-2}) \\ \phi = O(1/r^{n-3}) \end{cases}$

m : mass

$r := |\delta_{ij}x^i x^j|^{1/2}$

Field equations

static, n-dim.

D_i : covariant derivative on Ω

${}^{(n-1)}R_{ij}$: Ricci tensor on Ω

- Einstein equation

(0,0) $$(1-\kappa\xi\phi^2)D^2V = \kappa\left[\left(\frac{1}{2}-2\xi\right)V(D\phi)^2 + 2\xi\phi D^iVD_i\phi\right]$$

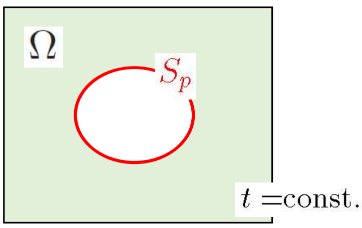
=0 when $\phi = \phi_p := \pm\dfrac{1}{\sqrt{\kappa\xi}}$

(i, j) $$(1-\kappa\xi\phi^2)\left({}^{(n-1)}R_{ij} - \frac{1}{V}D_iD_jV\right) = \kappa\left[(1-2\xi)D_i\phi D_j\phi - \left(\frac{1}{2}-2\xi\right)(D\phi)^2 g_{ij} - 2\xi\phi D_iD_j\phi\right]$$

- scalar field equation

$$D_i(VD^i\phi) = 0$$

S_p : surface specified by $\phi = \phi_p := \pm\dfrac{1}{\sqrt{\kappa\xi}}$



3. Uniqueness of static spacetime with conformal scalar in n-dimensions

n>4

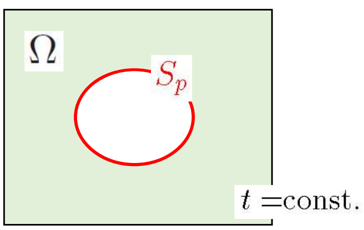
Relation between ϕ and V n>4

$$(1-\kappa\xi\phi^2)D^2V=\kappa\left[\left(\frac{1}{2}-2\xi\right)V(D\phi)^2+2\xi\phi D^iVD_i\phi\right] \qquad D_i(VD^i\phi)=0$$

$$D^i\left[(1-\varphi)(1+\varphi)^{\frac{n-4}{n-2}}D_i\left\{(1+\varphi)^{\frac{2}{n-2}}V\right\}\right]=0$$

$$\varphi:=\sqrt{\kappa\xi}\phi$$

integration over Ω , boundary conditions



$$\boxed{\varphi=V^{-\frac{n-2}{2}}-1}$$

S_p : surface specified by $\phi=\phi_p:=\pm\frac{1}{\sqrt{\kappa\xi}}$

$\varphi|_{S_p}=1$ $\quad V|_{S_p}=2^{-\frac{2}{n-2}}=:V_p$

Curvature invariant & Regularity at S_p

n>4

$\rho := (D^i v D_i v)^{-1/2}$ $\quad$ h_{ij} : induced metric of S_p

$v := V^{-\frac{n-4}{2}}$ $\quad$ k_{ij} : extrinsic curvature of S_p

$V|_{S_p} = 2^{-\frac{2}{n-2}} =: V_p$ $\quad$ $\mathcal{D}_i$: covariant derivative w.r.t. h_{ij}

$$
\begin{aligned}
R_{\mu\nu}R^{\mu\nu} &= \frac{4(n-2)^2}{(n-4)^4}\frac{1}{v^4\rho^4} \\
&+ \frac{4(n-2)^2}{(n-4)^4}\frac{1}{v^4\rho^4}\frac{1}{\left(2v^{-\frac{n-2}{n-4}}-1\right)^2}\left[\left\{(n-3)+2v^{-\frac{n-2}{n-4}}-(n-4)v\left(1-v^{-\frac{n-2}{n-4}}\right)\rho k\right\}^2\right. \\
&\left.+\left\{(n-4)v\left(1-v^{-\frac{n-2}{n-4}}\right)\rho k_{ij}-h_{ij}\right\}^2+2(n-4)^2v^2\left(1-v^{-\frac{n-2}{n-4}}\right)^2(\mathcal{D}\rho)^2\right]
\end{aligned}
$$

$\xrightarrow{S_p} 0$ $\quad$ $\xrightarrow{S_p} 0$

From regularity at S_p,

$$
\boxed{k_{ij}|_{S_p} = \frac{2^{\frac{2}{n-2}}}{n-4}\frac{1}{\rho_p}h_{ij}|_{S_p} \quad , \quad \mathcal{D}_i\rho|_{S_p} = 0}
$$

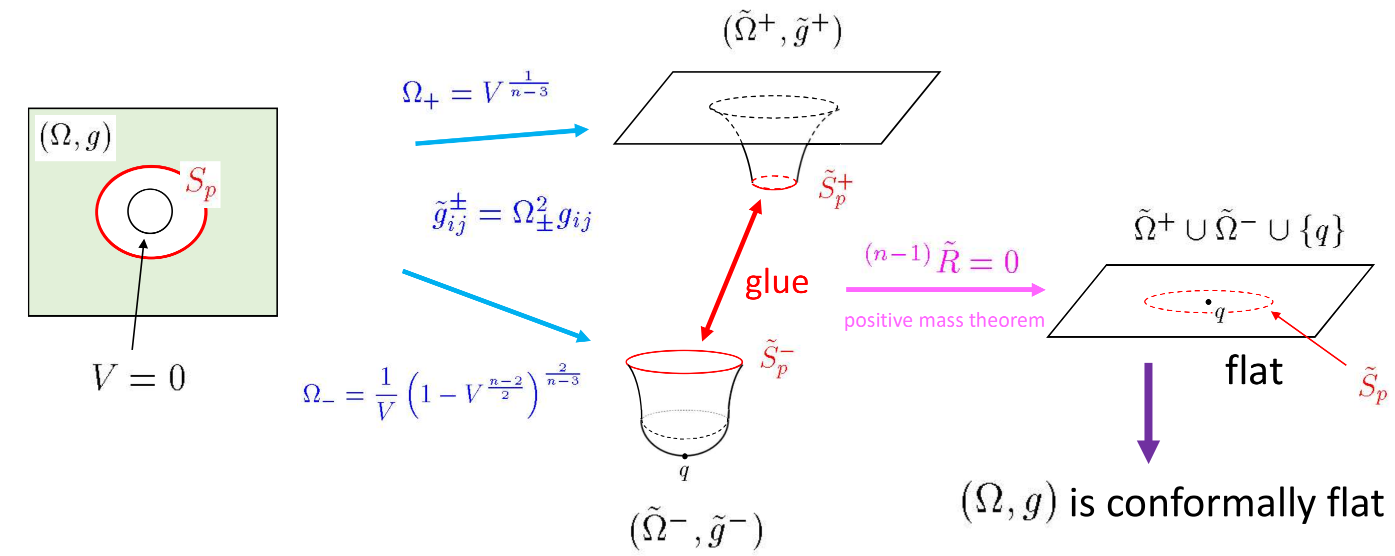
Uniqueness theorem

n>4

> The outside region of S_p for static and asymptotically flat spacetime in the Einstein-conformal scalar field system is spherically symmetric spacetime.

S_p : surface specified by $\phi = \phi_p := \pm\frac{1}{\sqrt{\kappa\xi}}$

Proof (1/2)

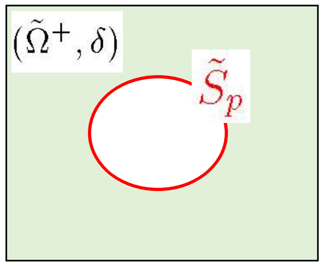


$\tilde{S}_p$ is totally umbilic in flat space

q : spatial infinity

S_p : surface specified by $\phi = \phi_p := \pm\frac{1}{\sqrt{\kappa\xi}}$

Proof (2/2)



$$\Delta_\delta V^{-\frac{n-2}{2}} = 0$$ in flat space $(\tilde{\Omega}^+, \delta)$

Δ_δ : flat Laplacian

$\tilde{S}_p$ is totally umbilic in the flat space.

$\tilde{S}_p$ is spherically symmetric in the Euclid space.

The electrostatic potential problem tells us that (Ω, g) is spherically symmetric.

Spacetime outside S_p is unique!

4. Summary & Comment

We proved that the region outside S_p of static and asymptotically flat spacetime in the n-dim. Einstein-conformal scalar field system is unique.

S_p : surface specified by $\phi = \phi_p := \pm \frac{1}{\sqrt{\kappa \xi}}$

future work

However, the uniqueness inside of S_p in the 4/higher dim. spacetime has not been proven because the analyticity at S_p might not hold.

Session E 10:00–12:00

[Chair: Ken-ichi Nakao]

Kohei Kamada

RESCEU, U-Tokyo

"wash-in leptogenesis and its application"

(15 min.)

[JGRG30 (2021) 121003]

Wash-in Leptogenesis and its application

Based on : V. Domcke (CERN), KK, K. Mukaida (KEK), K, Schmitz (CERN), M. Yamada (Tohoku),
Phys. Rev. Lett 126 (2021) 201802 (arXiv: 2011.09347[hep-ph])

Kohei Kamada
(RESCEU, U Tokyo)

JGRG30
10/12/2021 @ on-line / Waseda University

Courtesy: H. Oide

Sakharov's condition ('67 Sakharov) ··· necessary condition for the BAU.

1. B-violation
2. C & CP-violation
3. Deviation from thermal equilibrium

Heavy particle decay in B/C&CP-violating way easily satisfies this condition.

e.g.) GUT gauge boson/Higgs boson decay

('78 Yoshimura, '78 Dimopoulous & Suskind, '79 Toussaint+, '79 Weinberg, '79 Barr+, ···)

Well-motivated model.
We have understood the origin of the matter-antimatter asymmetry of the Universe, though difficult to prove it experimentally···?

Courtesy: H. Oide

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Well-motivated model.
We have understood the origin of the matter-antimatter asymmetry of the Universe, though difficult to prove it experimentally⋯?

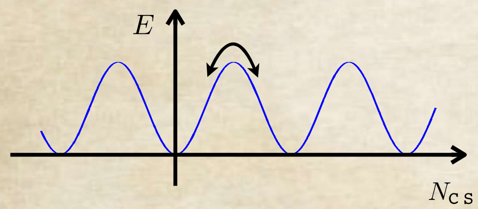
The story is not so simple.

Courtesy: H. Oide

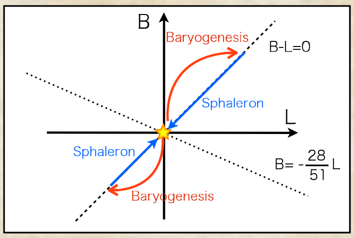
Electroweak sphaleron ('84 Klinkhamer & Manton)

⋯ washes out B+L asymmetry ('85 Kuzmin, Rubakov & Shaposhnikov).

killed SU(5) GUT baryogenesis.



$$\Delta N_{\rm CS} \propto \Delta B = \Delta L$$



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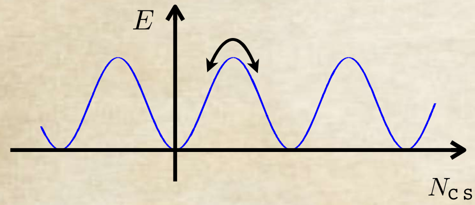

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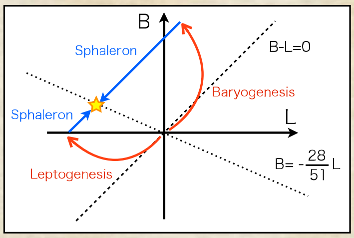

Popular option thus far:

- Generate B-L asymmetry (much) before EWSB.

This option···

needs to violate B-L symmetry,

exact (non-anomalous) symmetry of the SM.

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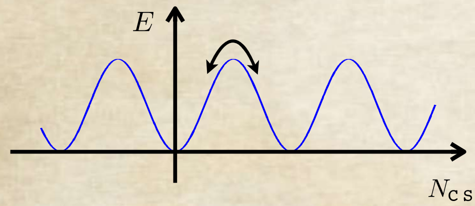

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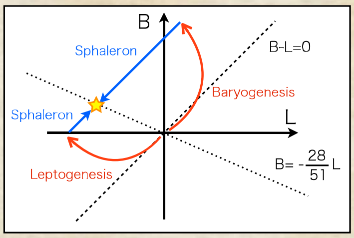

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Right-handed neutrinos can be naturally introduced.

(Leptogenesis; '86 Fukugita & Yanagida)

But BAU from their decay needs severe conditions on the CP-violation and right-handed neutrino mass.

(See e.g., '05 Buchmüller, Di Bari, & Plumacher)

Courtesy: H. Oide

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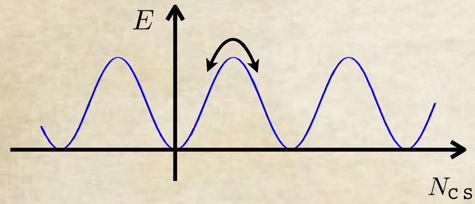

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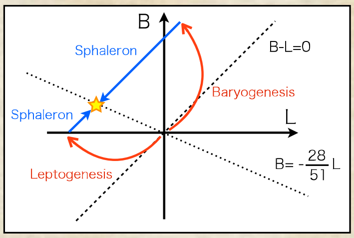

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Other ways?

Courtesy: H. Oide

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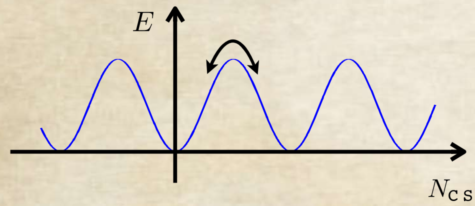

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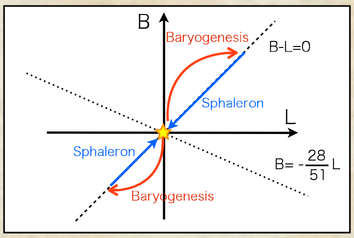

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Other ways?

Courtesy: H. Oide

An issue in the B+L washout by EW sphalerons.

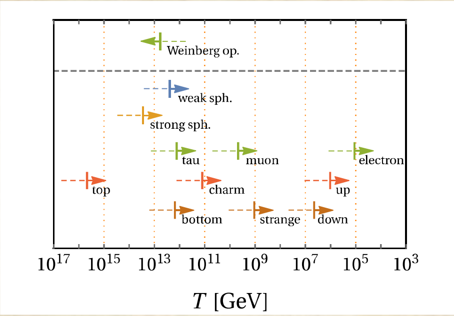
It completes when the electron Yukawa gets in equilibrium @ $T \lesssim 100\mathrm{TeV}$

('92 Campbell+)

Approx. conserved quantities prevents from the completion of washout.

@lower temp., the SM has only B/3-Li as a global conserved quantities

@higher temp., many other global conserved quantities appear, depending on temp.



Equilibrium temperature of Yukawa/sphalerons

(Figure from '20 Domcke+)

	T[GeV]	y_e	y_{ds}	y_d	y_s	y_{sb}	y_μ	y_c	y_τ	y_b	WS	SS	y_t
(v)	$(10^5, 10^6)$	q_e	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
(iv)	$(10^6, 10^9)$	q_e	$q_{2B_1-B_2-B_3}$	q_{u-d}	✓	✓	✓	✓	✓	✓	✓	✓	✓
(iii)	$(10^9, 10^{11-12})$	q_e	$q_{2B_1-B_2-B_3}$	q_{u-d}	q_{d-s}	$q_{B_1-B_2}$	q_μ	✓	✓	✓	✓	✓	✓
(ii)	$(10^{11-12}, 10^{13})$	q_e	$q_{2B_1-B_2-B_3}$	q_{u-d}	q_{d-s}	$q_{B_1-B_2}$	q_μ	q_{u-c}	q_τ	q_{d-b}	q_B	✓	✓
(i)	$(10^{13}, 10^{15})$	q_e	$q_{2B_1-B_2-B_3}$	q_{u-d}	q_{d-s}	$q_{B_1-B_2}$	q_μ	q_{u-c}	q_τ	q_{d-b}	q_B	q_u	✓

List of conserved quantities at several temperature regime

('21 Domcke, KK+)

Courtesy: H. Oide

An issue in the B+L washout by EW sphalerons.

It completes when the electron Yukawa gets in equilibrium @ $T \lesssim 100\mathrm{TeV}$

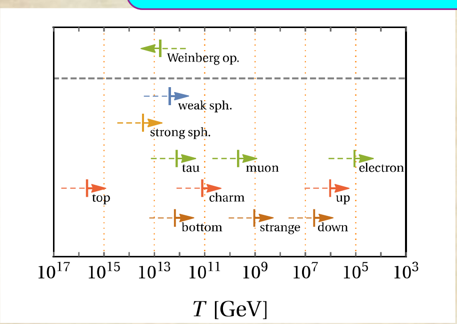
('92 Campbell+)

Approx. conserved quantities prevents from the completion of washout.

@lov

@hig ... g on temp.

Something can be implemented after the generation of asymmetry with B-L=0, before the completion of washout for net B asymmetry.



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(v)	$(10^5, 10^6)$	q_e	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
(iv)	$(10^6, 10^9)$	q_e	$q_{2B_1-B_2-B_3}$	q_{u-d}	✓	✓	✓	✓	✓	✓	✓	✓	✓
(iii)	$(10^9, 10^{11-12})$	q_e	$q_{2B_1-B_2-B_3}$	q_{u-d}	q_{d-s}	$q_{B_1-B_2}$	q_μ	✓	✓	✓	✓	✓	✓
(ii)	$(10^{11-12}, 10^{13})$	q_e	$q_{2B_1-B_2-B_3}$	q_{u-d}	q_{d-s}	$q_{B_1-B_2}$	q_μ	q_{u-c}	q_τ	q_{d-b}	q_B	✓	✓
(i)	$(10^{13}, 10^{15})$	q_e	$q_{2B_1-B_2-B_3}$	q_{u-d}	q_{d-s}	$q_{B_1-B_2}$	q_μ	q_{u-c}	q_τ	q_{d-b}	q_B	q_u	✓

List of conserved quantities at several temperature regime

('21 Domcke, KK+)

Courtesy: H. Oide

For concrete calculation…

Once initial asymmetry is generated by a "genesis" mechanism,
one can determine the conserved charges and evolution of each particle asymmetries redistributed in a la Turner & Harvey ('90)'s way.

Yukawa: $\mu_{u_R} = \mu_{H_0} + \mu_{u_L}$

Sphaleron: $3N_f \sum_q \mu_q + \sum_l \mu_{=0}$

$$\mu_i = \sum_C \tilde{S}_{iC} \mu_C$$

or

$$\mu_\alpha = \sum_C S_{\alpha C} \mu_C$$

μ_i : chemical potential for each particle

μ_C : chemical potential for conserved charge

μ_α : chemical potential for approx. conserved charge, such as B or B+L

From this calculation we obtain the well-known formula $\mu_B = \frac{28}{79}\mu_{B-L}^{\text{ini}}$
after the electron Yukawa equilibration.

Courtesy: H. Oide

Once we introduce the right-handed neutrinos,
the way how asymmetries are redistributed changes.

We might expect the L asymmetry is induced.

(also see '02 Fukugita & Yanagida)

New equilibrium condition:

$$\mu_{L_\alpha} + \mu_H = \mu_{N_R^i}$$

When the right-handed neutrinos are almost massless,
the L-violating effect should not be relevant.

Courtesy: H. Oide

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the way how asymmetries are redistributed changes.

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(also see '02 Fukugita & Yanagida)

New equilibrium condition:

$$\mu_{L_\alpha} + \mu_H = \mu_{N_R^i} \longrightarrow 0 \qquad \text{when } T \simeq m_{N_R}$$

When the right-handed neutrinos become massive,
the L-violating effects should be effective.

At this point, $\Delta_i \equiv B/3 - L_i$ are not conserved charge.

$$\mu_\alpha = \sum_C S_{\alpha C}\mu_C \quad \Rightarrow \quad 0 = \sum_{C \neq \Delta_i} S_{\alpha C}\mu_C + \sum_i S_{\alpha \Delta_i}\mu_{\Delta_i} \quad (\alpha = L_\alpha + H) \quad \Rightarrow \quad \mu_{\Delta_i} = \sum_C S^{-1}_{\alpha\Delta_i} S_{\alpha C}\mu_C$$

B-L asymmetry is induced in the system (and fixed quickly)!

We name it "wash-in" process.

Courtesy: H. Oide

Once we introduce the right-handed neutrinos,
the w

	T_{B-L} [GeV]	Index α	μ_e	$\mu_{2B_1-B_2-B_3}$	μ_{u-d}	μ_{d-s}	$\mu_{B_1-B_2}$	μ_μ	μ_{u-c}	μ_τ	μ_{d-b}	μ_B	μ_u	$\mu_{\Delta_\perp}$
(v)	$(10^5, 10^6)$	e, μ, τ	$-\frac{3}{10}$	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
(iv)	$(10^6, 10^9)$	e, μ, τ	$-\frac{3}{17}$	0	$-\frac{7}{17}$	✗	✗	✗	✗	✗	✗	✗	✗	✗
(iii)	$(10^9, 10^{11-12})$	$\parallel_\tau, \tau$	$\frac{142-225P_\tau}{247}$	0	$-\frac{123}{247}$	$-\frac{82}{247}$	$\frac{123}{494}$	$\frac{142-225P_\tau}{247}$	✗	✗	✗	✗	✗	$\frac{225}{247}$
(ii)	$(10^{11-12}, 10^{13})$	$\parallel$	$\frac{-23P+7}{30}$	$\frac{1}{5}$	$-\frac{3}{5}$	$-\frac{1}{6}$	$-\frac{3}{10}$	$\frac{-23P+7}{30}$	$\frac{3}{10}$	$\frac{-23P+7}{30}$	$-\frac{4}{15}$	$\frac{23}{90}$	✗	$\frac{23}{30}$
(i)	$(10^{13}, 10^{15})$	$\parallel$	$\frac{-3P+1}{4}$	$\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{-3P+1}{4}$	$\frac{1}{4}$	$\frac{-3P+1}{4}$	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{3}{4}$

uced.

ugita & Yanagida)

New e

$$\mu_{L_\alpha} + \mu_H = \mu_{N_R^i} \longrightarrow 0 \qquad \text{when } T \simeq m_{N_R}$$

When the right-handed neutrinos become massive,
the L-violating effects should be effective.

At this point, $\Delta_i \equiv B/3 - L_i$ are not conserved charge.

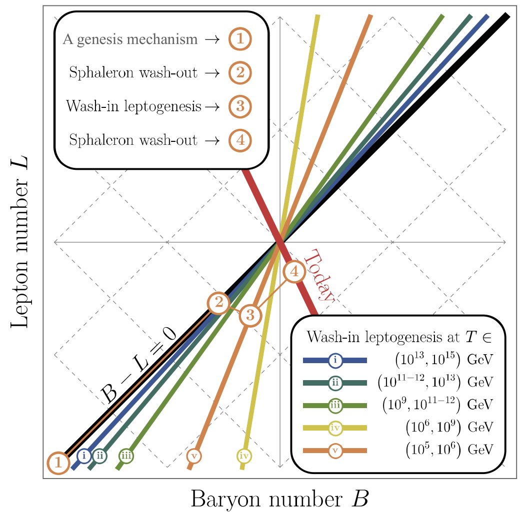
$$\mu_\alpha = \sum_C S_{\alpha C}\mu_C \quad \Rightarrow \quad 0 = \sum_{C \neq \Delta_i} S_{\alpha C}\mu_C + \sum_i S_{\alpha \Delta_i}\mu_{\Delta_i} \quad (\alpha = L_\alpha + H) \quad \Rightarrow \quad \mu_{\Delta_i} = \sum_C S^{-1}_{\alpha\Delta_i} S_{\alpha C}\mu_C$$

B-L asymmetry is induced in the system (and fixed quickly)!

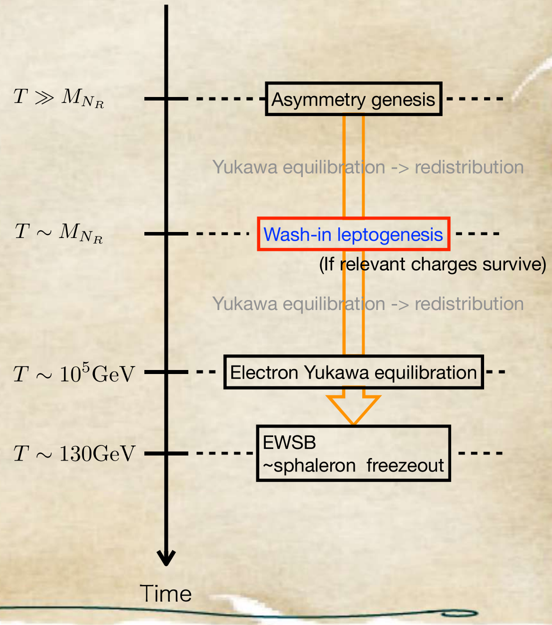
We name it "wash-in" process.

Courtesy: H. Oide

The way how B and L asymmetry is redistributed



('21 Domcke, KK+)



Courtesy: H. Oide

Advantages of wash-in leptogenesis

- Sakharov's condition does not have to be satisfied simultaneously.

  C&CP-violation/Deviation from equilibrium: asymmetry genesis @high temp.

  (not have to be B-L)

  B-L violation: Right-handed neutrino decoupling @ $T \simeq m_{N_R}$

  # RHN sector does not have to violate C & CP.

- Depending on the initial asymmetries, wash-in works with relatively light RHNs.

  can be consistent with naturalness $m_{N_R} \lesssim 10^7\mathrm{GeV}$ (Vissani bound ('98))

  and also the neutrino option ('17 Brivio & Trott)

  c.f.) The vanilla leptogenesis requires $m_{N_R} \gtrsim 10^9\mathrm{GeV}$ (Davidson-Ibarra bound ('02))

Courtesy: H. Oide

Applications/Realizations

- SU(5) GUT baryogenesis from the GUT Higgs decay

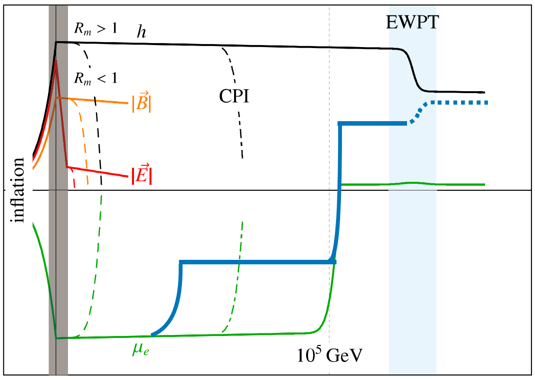
B+L but not B-L asymmetry is generated.
Wash-in helps to avoid the sphaleron washout.
Lowest RHN mass for the wash-in depends on the decay ratio of the GUT Higgs.

Courtesy: H. Oide

Applications/Realiza

- SU(5) GUT baryogene

B+L but not B-L asy
Wash-in helps to av
Lowest RHN mass of the GUT Higgs.



[plot by E. Morgante]

- Axion inflation

$\phi Y_{\mu\nu}\tilde{Y}^{\mu\nu}$ coupling generate both B+L asym. and hypermagnetic helicity through chiral anomaly.
They can annihilate each other at a later time.
Wash-in prevents from the complete cancellation of B+L and helicity.

Final baryon asymmetry is the summation of the B+L with wash-in
and hypermagnetic helicity decay at EWSB. (Domcke, KK+, in prep.)

Courtesy: H. Oide

Summary

- Wash-in leptogenesis is a new framework for baryogenesis.

- It uses the redistribution of the asymmetries with the RHN mass term, between the asymmetry generation and completion of would-be sphaleron washout.

- The idea is based on the fact that the Sakharov's condition does not have to be satisfied simultaneously.

- Relatively light RHN can be useful, consistent with naturalness problem and neutrino option.

- SU(5) GUT baryogenesis and baryogenesis from axion inflation is rescued, but the idea does not have to be limited to these examples.

Courtesy: H. Oide

Session E 10:00–12:00

[Chair: Ken-ichi Nakao]

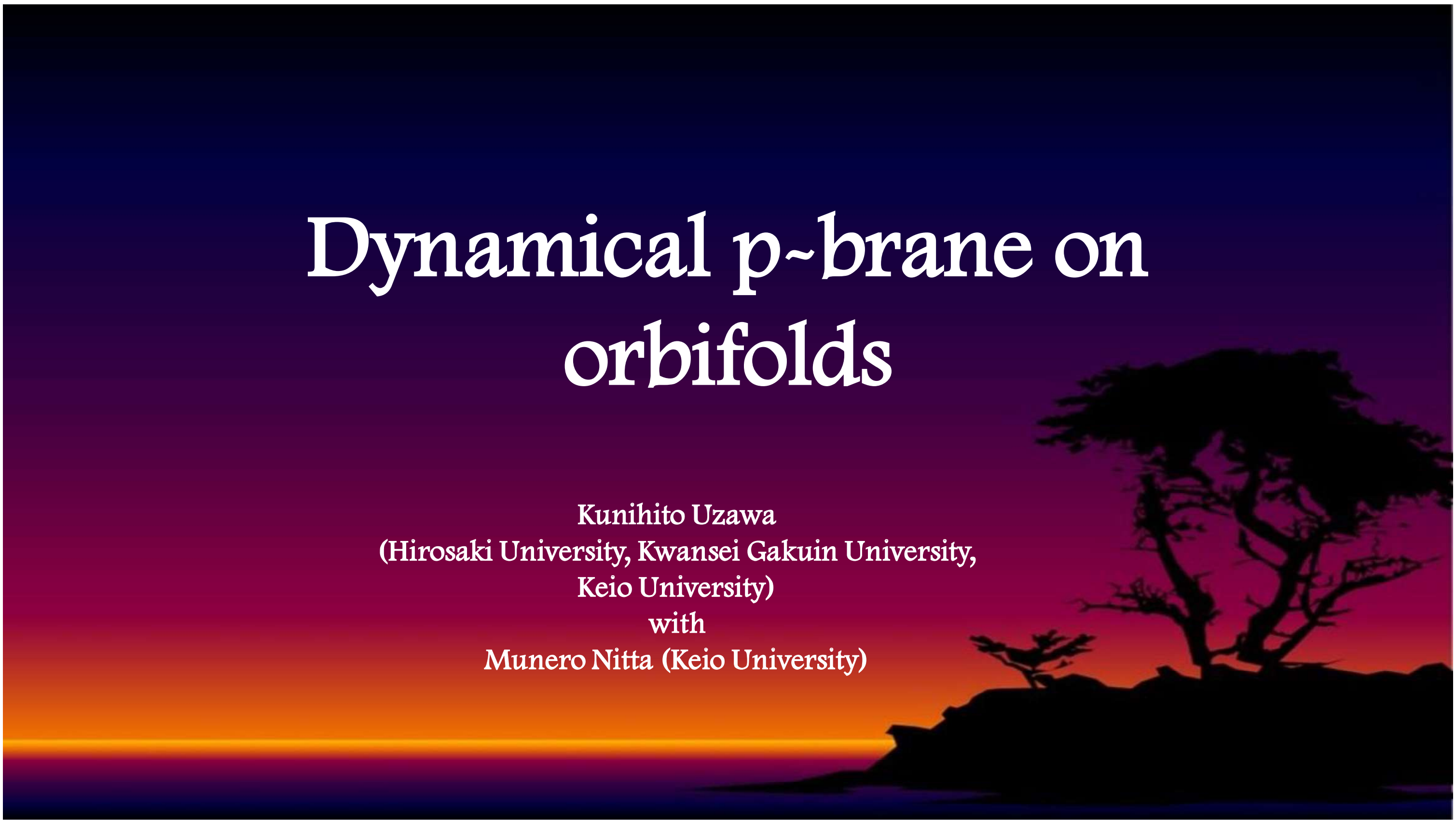
Kunihito Uzawa

Hirosaki University

“Dynamical brane on orbifold”

(15 min.)

[JGRG30 (2021) 121004]



[1] Introduction and our results

- We construct a dynamical p-brane solution on orbifold (the complex line bundle over CP^n space) as a solution.

- There are interesting properties when branes are located at an orbifold.
  (M.R. Douglas and G.W. Moore, hep-th/9603167)
  (M.R. Douglas, B.R. Greene and D.R. Morrison, Nucl. Phys.B 506 (1997) 84 [hep-th/9704151])

- Orbifold singularities are resolved in the D-brane world volume theory.

- One can expect that the spacetime itself becomes regular without any naked singularity once p-branes are placed at the orbifold singularities.

- This is important when one finds hints that the gravity theory giving a p-brane on the orbifold is known and well-defined supergravity, thus giving a handle on the strong coupling dynamics of string theory.

- Black hole on the Eguchi-Hanson space

(H. Ishihara, M. Kimura, K. Matsuno and S. Tomizawa,Phys. Rev. D 74 (2006) 047501 [hep-th/0607035])
(Hideki Ishihara, Masashi Kimura, Shinya Tomizawa, Class.Quant.Grav.23 (2006) L89 [hep-th/0609165])
(Chul-Moon Yoo, Hideki Ishihara, Masashi Kimura, Ken Matsuno, Shinya Tomizawa, Class.Quant.Grav. 25 (2008) 095017 [0708.0708 [gr-qc])

- Black holes on the complex line bundle over CP^n space

(M. Nitta, K. Uzawa, Eur.Phys.J.C 81 (2021) 6, 513, [arXiv:2011.13316 [hep-th]])

- (Static) Black p-brane on the orbifolds

(M. Nitta, K. Uzawa, JHEP 03 (2021) 018, [arXiv:2012.13285 [hep-th]])

★ Plan of my talk

- Black holes on Eguchi-Hanson space in 5D Einstein-Maxwell theory
- Black p-brane
- Extension to dynamical p-brane
- Summary and discussions

[2] Black holes on Eguchi-Hanson space in 5D Einstein-Maxwell theory

(H. Ishihara, et al., Phys. Rev. D 74 (2006) 047501 [hep-th/0607035])

- 5D-metric

$$ds^2 = -\left(1+\frac{m_1}{r^2-a^2\cos^2\theta}+\frac{m_2}{r^2+a^2\cos^2\theta}\right)^{-2}dt^2+\left(1+\frac{m_1}{r^2-a^2\cos^2\theta}+\frac{m_2}{r^2+a^2\cos^2\theta}\right)$$
$$\times\underbrace{\left[\left(1-\frac{a^4}{r^4}\right)^{-1}dr^2+\frac{r^2}{4}\left\{\left(1-\frac{a^4}{r^4}\right)(d\psi+\cos\theta\,d\phi)^2+d\theta^2+\sin^2\theta\,d\phi^2\right\}\right]}$$

Eguchi-Hanson space (4D)
Complex line bundle over CP^1

- 1-form gauge potential

$$A_{(1)} = \pm\frac{\sqrt{3}}{2}\left(1+\frac{m_1}{r^2-a^2\cos^2\theta}+\frac{m_2}{r^2+a^2\cos^2\theta}\right)^{-1}dt$$

(i) Extension to p-brane

$$ds^2 = -h^{-2}(y)\boxed{dt^2}+h(y)\boxed{u_{ij}(Y)dy^i dy^j}$$

(2) Extension to orbifold

[3] Black p-brane

Gravity and antisymmetric tensor fields of arbitrary rank in D-dim.

- Action in the Einstein frame
  (K. S. Stelle, [arXiv:hep-th/9701088 [hep-th]])

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left[R - \frac{1}{2\cdot(p+2)!}F^2_{(p+2)}\right]$$

- Einstein eq. : $R_{MN} = \frac{1}{2\cdot(p+2)!}\left[(p+2)F_{MA\cdots B}{F_N}^{A\cdots B} - \frac{p+1}{D-2}g_{MN}F^2_{(p+2)}\right]$
- gauge field eq. : $d\left[*F_{(p+2)}\right] = 0$
- Bianchi identity : $dF_{(p+2)} = 0$

♦ Solution

(M. Nitta & K. Uzawa, JHEP 2103 (2021) 018, arXiv:2012.13285 [hep-th])
(G.W. Gibbons, G.T. Horowitz and P.K. Townsend, Class. Quant. Grav. 12 (1995) 297 [hep-th/9410073])

- metric

$$ds^2 = \left(c_0 + \frac{c_1}{r^{D-p-3}}\right)^{-\frac{2}{p+1}} q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + \left(c_0 + \frac{c_1}{r^{D-p-3}}\right)^{\frac{2}{D-p-3}} u_{ij}(\mathrm{Y})dy^i\, dy^j$$

- p-form field strength

$$F_{(p+2)} = \sqrt{\frac{2\,(D-2)}{(p+1)\,(D-p-3)}}\, d\left(c_0 + \frac{c_1}{r^{D-p-3}}\right)^{-1} \wedge \Omega\,(\mathrm{X})$$

[3] Extension to dynamical p-brane on orbifolds

- Metric of dynamical p-brane in D dimensions

(Pierre Binetruy, Misao Sasaki, Kunihito Uzawa, Phys.Rev.D 80 (2009) 026001 0712.3615 [hep-th])
(Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa, JHEP 06 (2009) 051, arXiv: 0903.5483 [hep-th])

$$ds^2 = h^a(x,\ y)\, q_{\mu\nu}(\mathrm{X})\, dx^\mu dx^\nu + h^b(x,\ y) \boxed{u_{ij}(\mathrm{Y})\, dy^i dy^j},$$

orbifold

$$a = -\frac{D-p-3}{D-2}, \qquad b = \frac{p+1}{D-2},$$

- (p+2)-form field strength

$$F_{(p+2)} = d\left(h^{-1}\right) \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p$$

- It is straightforward to check that with such an ansatz the Bianchi identity is trivially satisfied.

- The field equation for the antisymmetric tensor becomes

(Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa, JHEP 06 (2009) 051, arXiv: 0903.5483 [hep-th])

$$\partial_\mu \partial_i h = 0, \qquad \triangle_{\mathrm{Y}} h = 0,$$

$$\to \quad h(x,y) = h_0(x) + h_1(y), \qquad \triangle_{\mathrm{Y}} h_1 = 0,$$

- Einstein equations

$$R_{\mu\nu}(\mathrm{X}) - h^{-1} D_\mu D_\nu h_0 - \frac{a}{2} h^{-1} q_{\mu\nu} \left(\triangle_{\mathrm{X}} h_0 + h^{-1} \triangle_{\mathrm{Y}} h_1 \right) = 0 \,,$$

$$R_{ij}(\mathrm{Y}) - \frac{b}{2} q_{\mu\nu} \left(\triangle_{\mathrm{X}} h_0 + h^{-1} \triangle_{\mathrm{Y}} h_0 \right) = 0 \,,$$

$$\Rightarrow \quad R_{\mu\nu}(\mathrm{X}) = 0 \,, \quad R_{ij}(\mathrm{Y}) = \frac{1}{2} b \, (p+1) \, \lambda \, u_{ij}(\mathrm{Y}) \,, \quad D_\mu D_\nu h_0 = \lambda \, q_{\mu\nu}(\mathrm{X}) \,,$$

- The space Y is not Ricci flat, but the Einstein space such as CP^n if $\lambda \neq 0$, and the function h can be more non-trivial.

(Pierre Binetruy, Misao Sasaki, Kunihito Uzawa, Phys.Rev.D 80 (2009) 026001 0712.3615 [hep-th])
(Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa, JHEP 06 (2009) 051, arXiv: 0903.5483 [hep-th])

If we set

$$q_{\mu\nu}(\mathrm{X}) = \eta_{\mu\nu}(\mathrm{X}) \,, \qquad D_\mu h_0 \neq 0 \,, \qquad (D_\mu h_0)\,(D^\mu h_0) \neq 0 \,,$$

the solution for h_0 is given by

$$h_0(x) = \frac{\lambda}{2} x_\mu x^\mu + \bar{a}_\mu x^\mu + \bar{a}$$

- When the space Y is Ricci flat like orbifold, the function h_0 is linear in the coordinates x^μ because of λ=0.

♦ Dynamical p-brane on the orbifold

cf) Black holes on Eguchi-Hanson space :

(H. Ishihara, M. Kimura and S. Tomizawa, Class.Quant.Grav. 23 (2006) L89 [hep-th/0609165])

Eguchi-Hanson space – complex line bundle over CP^1 (2-sphere)

- p-brane on orbifold

$$u_{ij}(\mathrm{Y})dy^i dy^j = dr^2 + r^2\left[\left\{d\rho + \sin^2\xi_{n-1}\left(d\psi_{n-1} + \frac{1}{2(n-1)}\omega_n\right)\right\}^2 + ds^2_{\mathbb{C}\mathrm{P}^{n-1}}\right]$$

If we impose $h_1 = h_1(r)$, the field equations become for

$$\triangle_{\mathrm{Y}} h_1(r) = 0\,, \quad \Rightarrow \quad h_1(r) = b_1 + \frac{b_2}{r^{D-p-3}}$$

the solution for $h(x,r)$ is given by

$$h(x,r) = \bar{a}_\mu x^\mu + b_1 + \frac{b_2}{r^{D-p-3}}$$

- There is a naked singularity at h=0.

(1) $r \to 0$: static p-brane solution.

(2) $r \to \infty$: asymptotically Kasner geometry

♦ Dynamical solution on the CP^n space

- Metric

CP^n space

$$ds^2 = h^a(x,\ y)\, q_{\mu\nu}\,(\mathrm{X})\, dx^\mu dx^\nu + h^b(x,\ y) \boxed{u_{ij}\,(\mathrm{Y})\, dy^i dy^j},$$

$$a = -\frac{D-p-3}{D-2}, \qquad b = \frac{p+1}{D-2},$$

- For the CP^1 space

$$ds^2_{\mathbb{C}\mathrm{P}^1} = u_{ij}(\mathrm{Y})dy^i dy^j = \left(1+r^2\right)^{-2}\left(dr^2 + r^2 d\theta^2\right)$$

- Solution

$$h(x, r, \theta) = \frac{\lambda}{2} x_\mu x^\mu + \bar{a}_\mu x^\mu + \bar{a}$$
$$-\tilde{c}\, r + \tilde{c}_1 \ln r + \frac{\tilde{c}}{2}\,\theta^2 + \tilde{c}_2\,\theta$$

For the CP^2 space

$$\begin{aligned} ds^2_{\mathbb{C}\mathrm{P}^2} &= u_{ij}(\mathrm{Y})dy^i dy^j \\ &= \left(1+\rho^2\right)^{-2} d\rho^2 + \frac{\rho^2}{4}\left(1+\rho^2\right)^{-2}\left(d\psi + \cos\theta d\phi\right)^2 \\ &\quad + \frac{\rho^2}{4}\left(1+\rho^2\right)^{-1}\left(d\theta^2 + \sin^2\theta d\phi^2\right) \end{aligned}$$

• Solution

$$h(x,\rho,\theta) = \frac{\lambda}{2}x_\mu x^\mu + \bar{a}_\mu x^\mu + \bar{a} + \ln\left(\frac{\rho}{\sqrt{1+\rho^2}}\right) + \bar{c}\ln\sin\theta + \bar{c}_2\ln\tan\frac{\theta}{2}$$

[4] Discussion and remarks

(1) Extension to dynamical p-brane

• Dynamical p-brane on the complex line bundle over CP^n space.

• We will be able to describe p-brane collision.

(2) Black hole

• Black hole on orbifolds (complex line bundle over CP^n space).

• Time dependent black hole.

For Kei-ichi Maeda san,

Wishing you a new journey of success and happiness in the new page of your life!

- Kei-ichi Maeda, Kunihito Uzawa, Phys.Rev.D 68 (2003) 084017, arXiv: hep-th/0308137.
- Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa Journal of High Energy Physics, JHEP 06 (2009) 051, arXiv: 0903.5483 [hep-th]
- Kei-ichi Maeda, Masato Minamitsuji, Nobuyoshi Ohta, Kunihito Uzawa, Phys.Rev.D 82 (2010) 046007, arXiv: 1006.2306 [hep-th]
- Kei-ichi Maeda, Kunihito Uzawa Physical Review D, Phys.Rev.D 85 (2012) 086004, arXiv: 1201.3213 [hep-th]
- Kei-ichi Maeda, Kunihito Uzawa Physical Review D, Phys.Rev.D 94 (2016) 12, 126016, arXiv:1603.01948 [hep-th]

Session E 10:00–12:00

[Chair: Ken-ichi Nakao]

Sebastian Garcia-Saenz

SUSTech

"A new class of vector-tensor theories and its cosmology"

(15 min.)

[JGRG30 (2021) 121005]

A new class of vector-tensor theories and its cosmology

Based on **2110.14327** with C. de Rham, L. Heisenberg and V. Pozsgay

Sebastian Garcia-Saenz
SUSTech

Why vector-tensor theories?

- Massive vector condensate as candidate for **dark energy**

De Felice, Heisenberg, Kase, Mukohyama, Tsujikawa, Zhang (2016)

- Massive vector particles as candidate (light) **dark matter**

Arkani-Hamed, Weiner (2008)

- Natural alternative to scalar-tensor theories
- Interesting astrophysical solutions with **vector hair**
- Important differences with scalar-tensor theories, e.g. no tachyonic growth of hair

SGS, Held, Zhang (2021), Silva et al. (2021)

Catalogue of theories

We define a **consistent** massive vector-tensor theory by an action $S[g, A]$ with **2+3 degrees of freedom**

- Generalized Proca — Tasinato (2014), Heisenberg (2014)
- Beyond Generalized Proca — Heisenberg, Kase, Tsujikawa (2016)
- Extended vector-tensor theories — Kimura, Naruko, Yoshida (2016)
- Proca-Nuevo — de Rham, Pozsgay (2020)
- **Extended Proca-Nuevo** — de Rham, SGS, Heisenberg, Pozsgay (2021)

Generalized Proca vs. Proca-Nuevo

- **Generalized Proca theory**

$$\mathscr{L}_{\text{GP}} = \sum_{n=1}^{4} \beta_n(X)\mathscr{L}_n[\partial A] + \ldots$$

$$X \propto A^\mu A_\mu$$

$$\mathscr{L}_n[M] \propto \epsilon^{\cdots}\epsilon^{\cdots}M_{\cdot\cdot}\cdots M_{\cdot\cdot}$$

Properties

— Polynomial in $\partial_\mu A_\nu$

— Constraint structure: A_0 non-dynamical

— Covariant theory includes non-minimal curvature terms

Generalized Proca vs. Proca-Nuevo

- **Proca-Nuevo theory**

$$X \propto A^\mu A_\mu$$

$$\mathcal{L}_n[M] \propto \epsilon^{\cdots}\epsilon^{\cdots} M_{..} \cdots M_{..}$$

$$\mathcal{L}_{\rm PN} = \sum_{n=1}^{4} \alpha_n(X)\mathcal{L}_n[\mathcal{K}]$$

$$\boxed{\begin{gathered}\mathcal{K} = \sqrt{\eta^{-1}f} - \mathbf{1} \\ f_{\mu\nu} = \eta_{\mu\nu} + 2\partial_{(\mu}A_{\nu)} + \partial_\mu A^\rho \partial_\nu A_\rho\end{gathered}}$$

Properties

— Non-polynomial in $\partial_\mu A_\nu$

— Constraint structure: non-trivial functional of A_μ

— Covariant theory **not completely known**

Extended Proca-Nuevo

$$X \propto A^\mu A_\mu$$

$$\mathcal{L}_n[M] \propto \epsilon^{\cdots}\epsilon^{\cdots} M_{..} \cdots M_{..}$$

$$\boxed{\mathcal{L}_{\rm EPN} = \sum_{n=1}^{4} \alpha_n(X)\mathcal{L}_n[\mathcal{K}] + \beta_n(X)\mathcal{L}_n[\partial A]}$$

$$\mathcal{K} = \sqrt{\eta^{-1}f} - \mathbf{1}$$

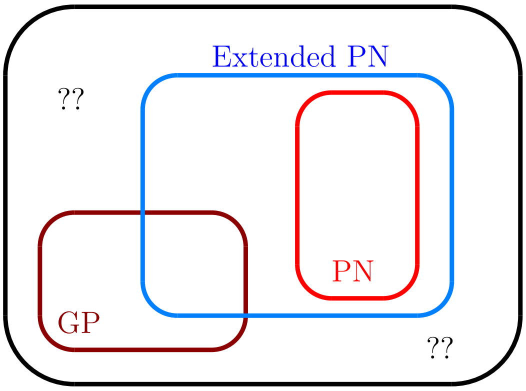
$$f_{\mu\nu} = \eta_{\mu\nu} + 2\partial_{(\mu}A_{\nu)} + \partial_\mu A^\rho \partial_\nu A_\rho$$

Remarks

— Non-trivial because of constraint structure

— Interesting link between GP and PN

— Caveat: not all GP class included



Extended Proca-Nuevo

Minimal coupling prescription does **not** work in general

$$\mathscr{L}_{\mathrm{EPN}} = \sum_{n=1}^{4} \alpha_n(X)\mathscr{L}_n[\mathscr{K}] + \beta_n(X)\mathscr{L}_n[\nabla A] + \boxed{\text{non-minimal curvature terms ?}}$$

Remark: OK if background is curved but non-dynamical

Two options

- Restrict the theory by (partially) fixing the free functions α_n , β_n ⟶ **"special model"**
- Add non-minimal couplings ⟶ **"general model"**

Extended Proca-Nuevo

- Our two covariantization schemes are only **partial** ones
- They are consistent for **linear** perturbations about any background such that

$$\bar{\nabla}_\mu \bar{A}_\nu = \bar{\nabla}_\nu \bar{A}_\mu$$

- This is sufficient for **cosmological solutions** with background

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \ , \qquad \bar{A}_\mu dx^\mu = -\phi(t)dt$$

- For more general backgrounds one typically has $m_{\mathrm{ghost}} \sim M_{\mathrm{Pl}}$

Cosmology of the special model

$$\mathscr{L} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \alpha_0(X) + \alpha_1(X)\mathscr{L}_1[\mathscr{K}] + \beta_1(X)\mathscr{L}_1[\nabla A]$$
$$+\alpha_2(X)\big(\mathscr{L}_2[\mathscr{K}] - \mathscr{L}_2[\nabla A]\big) + \alpha_3(X)\big(\mathscr{L}_3[\mathscr{K}] - \mathscr{L}_3[\nabla A]\big)$$

$$X \propto A^\mu A_\mu$$
$$\mathscr{L}_n[M] \propto \epsilon^{\cdots}\epsilon^{\cdots}M_{\cdot\cdot}\cdots M_{\cdot\cdot}$$
$$\mathscr{K} = \sqrt{\eta^{-1}f} - \mathbf{1}$$
$$f_{\mu\nu} = \eta_{\mu\nu} + 2\partial_{(\mu}A_{\nu)} + \partial_\mu A^\rho \partial_\nu A_\rho$$

Remarks

- Relatively simple model
- First instance of a generalized vector-tensor theory with **only minimal couplings**

Cosmology of the special model

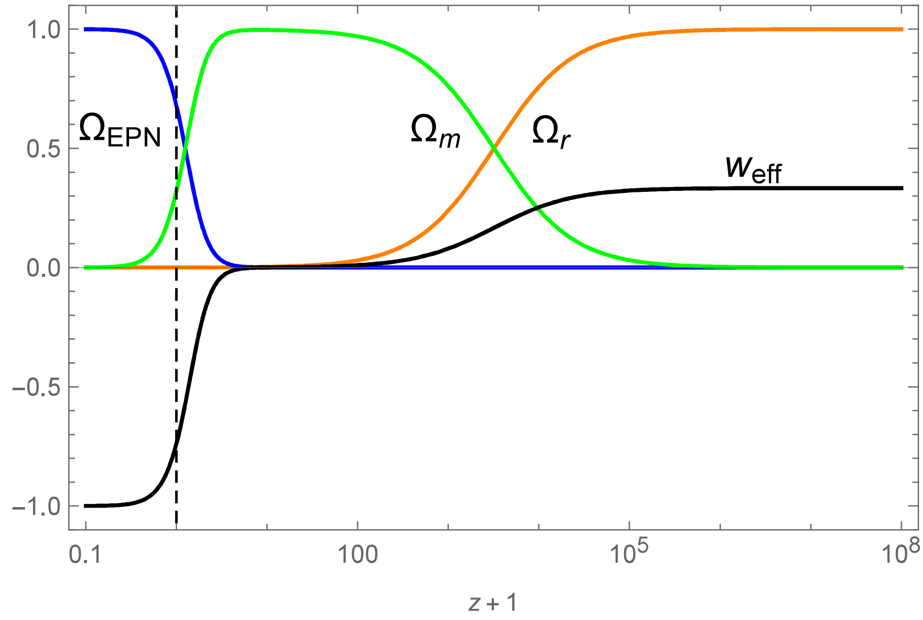
Set-up

- Flat FLRW background, general linear perturbations (scalar, vector, tensor)
- Perfect fluid matter with general equation of state

Results

- Correct number of degrees of freedom (2 tensor, 2+2 vector, 1+1 scalar)
- Exactly **luminal GWs**
- Window of parameters with no instabilities (ghost/gradient) or superluminality
- Consistent Big Bang solutions with late-time **dark energy**

Cosmology of the special model



Ω_r → radiation

Ω_m → matter

$\Omega_{\rm EPN}$ → dark energy

$w_{\rm eff}$ → effective EoS parameter

Cosmology of the general model

$$\begin{aligned}\mathscr{L} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \alpha_0(X) + \alpha_1(X)\mathscr{L}_1[\mathscr{K}] + \beta_1(X)\mathscr{L}_1[\nabla A] \\ +(\alpha_2(X)+\beta_2(X))R + \alpha_{2,X}(X)\mathscr{L}_2[\mathscr{K}] + \beta_{2,X}(X)\mathscr{L}_2[\nabla A] \\ +(\alpha_3(X)\mathscr{K}^{\mu\nu} + \beta_3(X)\nabla^\mu A^\nu)G_{\mu\nu} + \alpha_{3,X}(X)\mathscr{L}_3[\mathscr{K}] + \beta_{3,X}(X)\mathscr{L}_3[\nabla A]\end{aligned}$$

$$X \propto A^\mu A_\mu$$

$$\mathscr{L}_n[M] \propto \epsilon^{\cdots}\epsilon^{\cdots}M_{\cdot\cdot}\cdots M_{\cdot\cdot}$$

$$\mathscr{K} = \sqrt{\eta^{-1}f} - \mathbf{1}$$

$$f_{\mu\nu} = \eta_{\mu\nu} + 2\partial_{(\mu}A_{\nu)} + \partial_\mu A^\rho \partial_\nu A_\rho$$

Remarks

- Non-minimal couplings to curvature
- Includes Generalized Proca as particular case

Cosmology of the general model

De Felice, Heisenberg, Kase, Mukohyama, Tsujikawa, Zhang (2016)

Results (like Generalized Proca)

- Correct number of degrees of freedom (2 tensor, 2+2 vector, 1+1 scalar)
- Window of parameters with no instabilities (ghost/gradient) or superluminality
- Consistent Big Bang solutions with late-time **dark energy**

Results (unlike Generalized Proca)

- Non-linear dispersion relation for vector mode
- Modified speed of sound for perfect fluid

Take-home message

- **Extended Proca-Nuevo**: new class of vector-tensor theories
- Interesting link between structurally dissimilar theories (Generalized Proca vs. Proca-Nuevo)
- Two partial covariantization schemes, both allowing for consistent cosmological solutions and linear perturbations
- Interesting differences with Generalized Proca

Thank you

Session E 10:00–12:00

[Chair: Ken-ichi Nakao]

Hayato Motohashi

Kogakuin University

"Exact solution for wave scattering from black holes"

(15 min.)

[JGRG30 (2021) 121006]

Exact solution for wave scattering from black holes

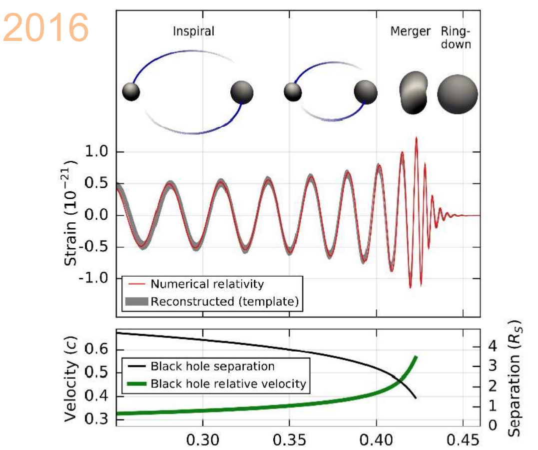
Hayato Motohashi
(Kogakuin University)

HM, Sousuke Noda,
PTEP 2021 083, [arXiv:2103.10802]

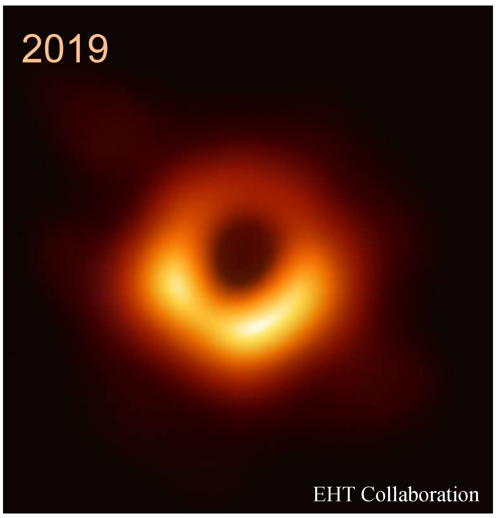
2021.12.10 JGRG30

KOGAKUIN UNIVERSITY

科研費 KAKENHI



LIGO Collaboration



EHT Collaboration

The Nobel Prize in Physics 2017

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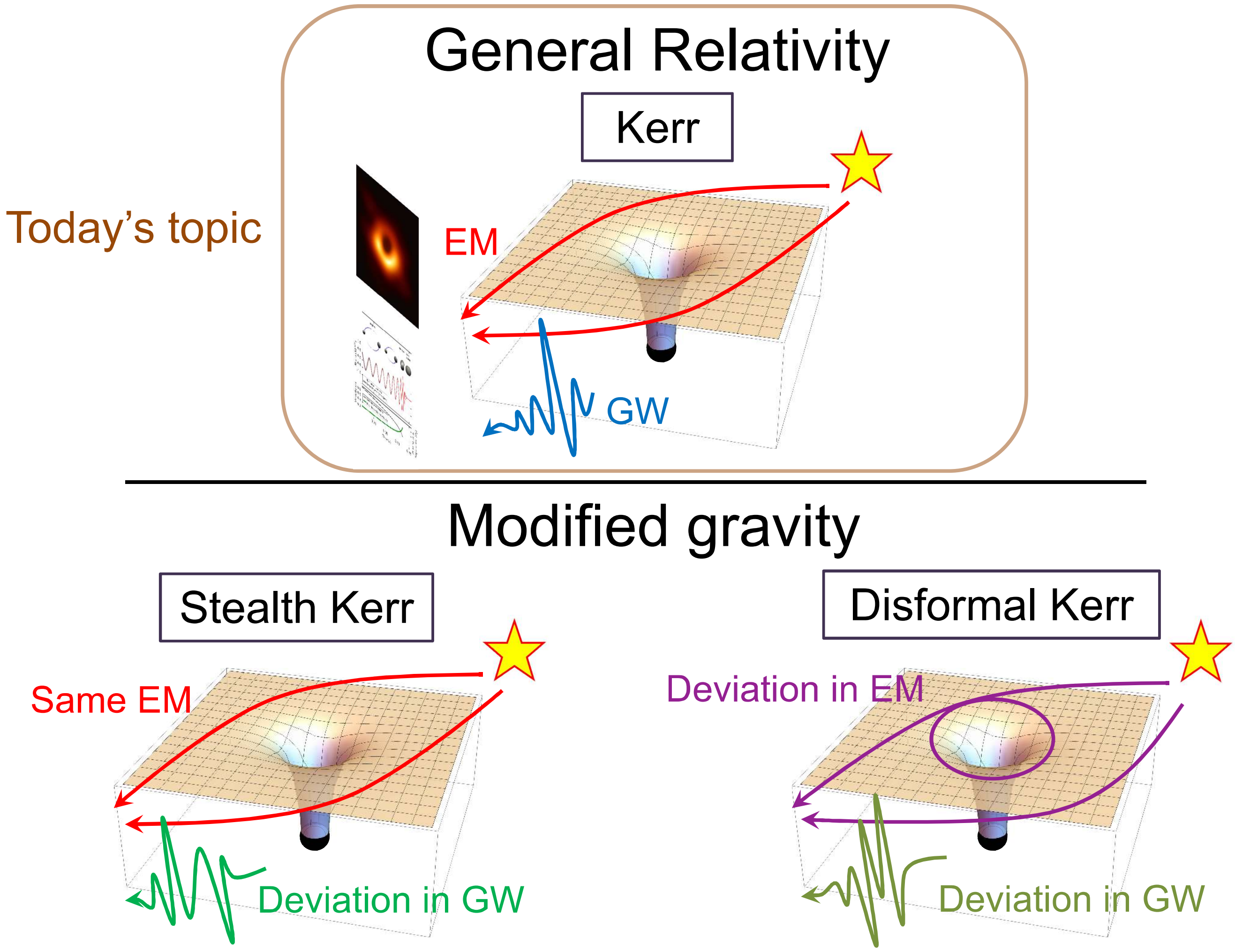
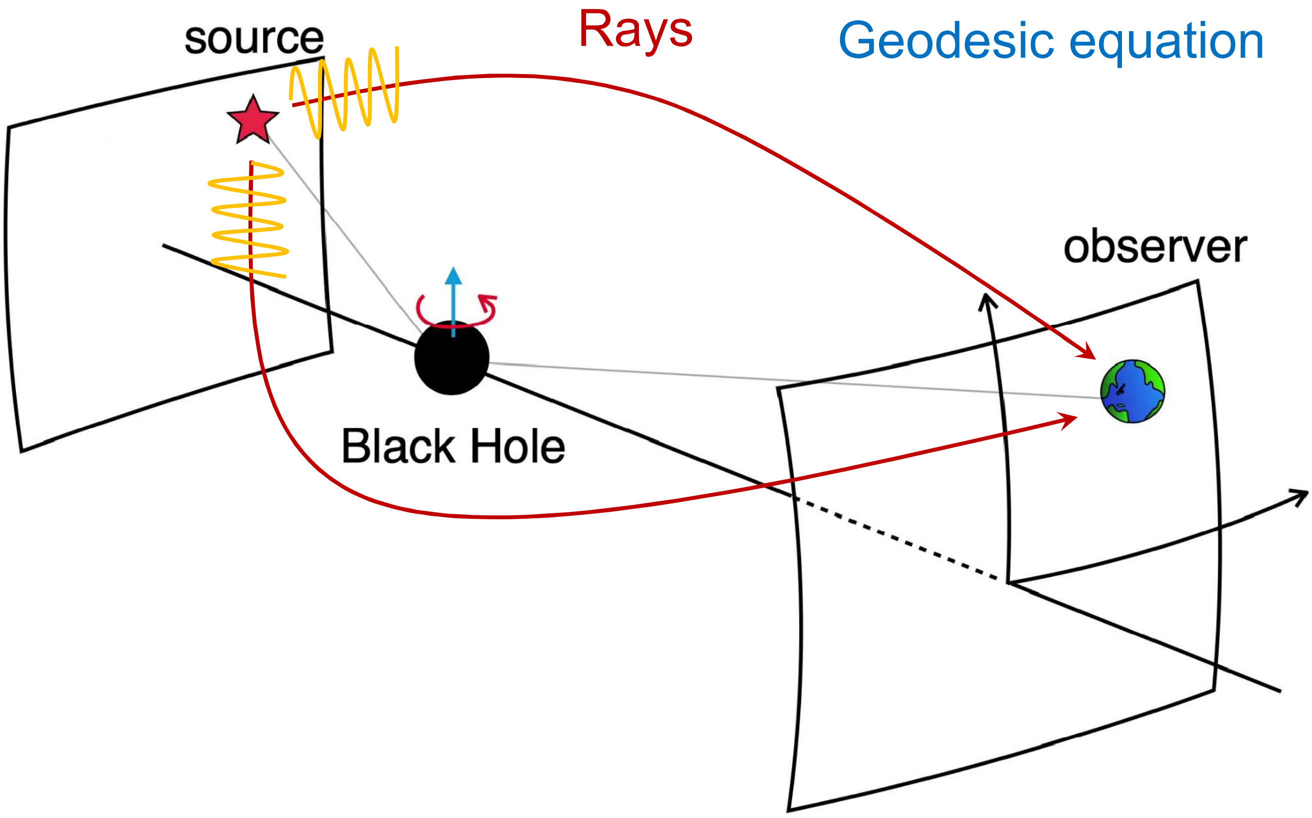
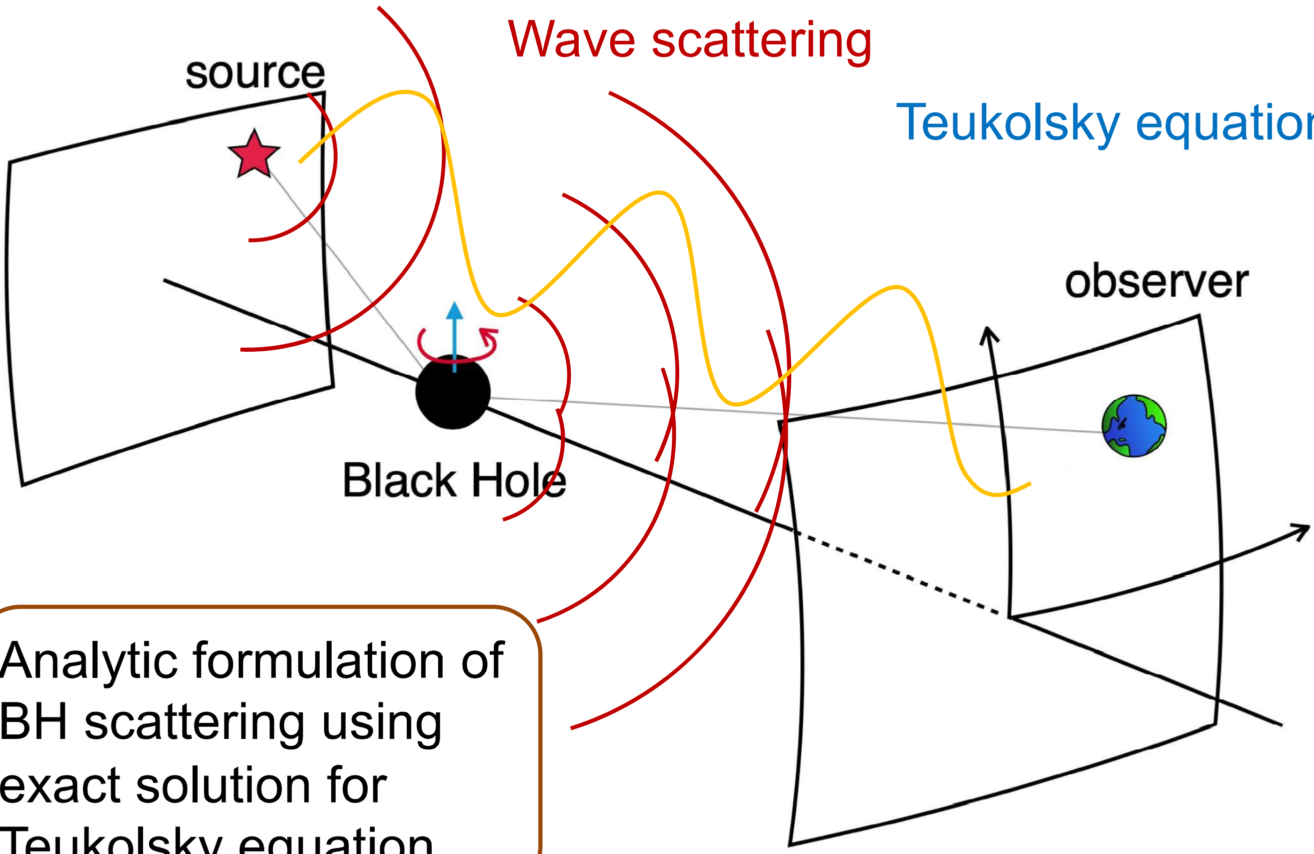


Geometrical optics



Wave optics



Analytic formulation of BH scattering using exact solution for Teukolsky equation

Kerr-Newman-de Sitter spacetime

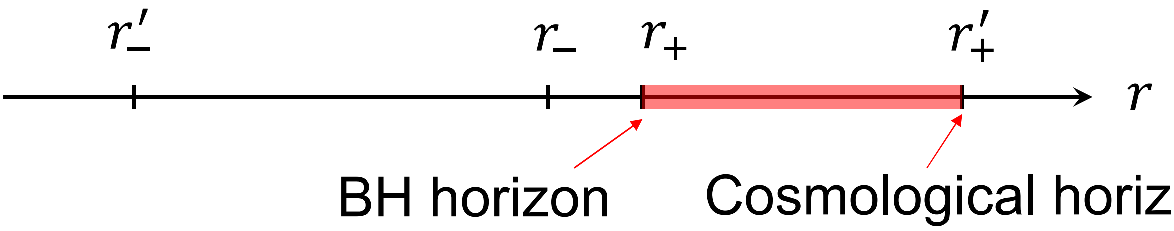
$$ds^2 = -\frac{\Delta}{(1+\alpha)^2\rho^2}(dt - a\sin^2\theta d\varphi)^2 + \rho^2\left(\frac{dr^2}{\Delta} + \frac{d\theta^2}{1+\alpha\cos^2\theta}\right)$$
$$+\frac{(1+\alpha\cos^2\theta)\sin^2\theta}{(1+\alpha)^2\rho^2}[adt - (r^2+a^2)d\varphi]^2$$

$$\Delta(r) = (r^2 + a^2)\left(1 - \frac{\Lambda r^2}{3}\right) - 2Mr + Q^2,$$
$$\alpha = \frac{\Lambda a^2}{3}, \quad \rho^2 = r^2 + a^2\cos^2\theta$$

Assumption: $\Lambda > 0$.

$\Delta(r) = 0$ has four roots: $r_\pm$, $r'_\pm$

$$\Delta(r) = -\frac{\Lambda}{3}(r - r_-)(r - r_+)(r - r'_-)(r - r'_+)$$



Teukolsky equation

For a massless test field on KNdS background, the Teukolsky eq is separable: $\psi_s = R_s(r) S_s(\theta) e^{-i\omega t + im\varphi}$

Spin 0, 1/2, 1, 3/2, 2 fields on the Kerr-de Sitter
Spin 0, 1/2 fields on the KNdS

Angular part

$$\left[\frac{d}{dx}(1+\alpha x^2)(1-x^2)\frac{d}{dx} + \lambda - s(1-\alpha) - 2\alpha x^2 + \frac{4sx(1+\alpha)[m\alpha - c(1+\alpha)]}{1+\alpha x^2} - \frac{(1+\alpha)^2[m+sx-(1-x^2)c]^2}{(1+\alpha x^2)(1-x^2)}\right] S_s = 0$$

λ: separation constant
$x = \cos\theta$, $c = a\omega$
$J(r) = (1+\alpha)[\omega(r^2+a^2) - am] - eQr$

- $a = 0$: ${}_sY_{\ell m}(\theta,\varphi)$, $\lambda = \ell(\ell+1) - s(s-1)$
- $\Lambda = 0$: Spin-weighted spheroidal harmonics

Radial part

$$\left[\Delta^{-s}\frac{d}{dr}\Delta^{s+1}\frac{d}{dr} + \frac{J^2 - isJ\Delta'}{\Delta} + 2isJ' - \frac{2}{3}\Lambda r^2(s+1)(2s+1) + 2s(1-\alpha) - \lambda\right] R_s = 0$$

Both eqs can be transformed to Heun equation.

Suzuki, Takasugi, Umetsu, PTEP 100 (1998) 491

- Teukolsky

$$\left[\Delta^{-s}\frac{d}{dr}\Delta^{s+1}\frac{d}{dr} + \frac{J^2 - isJ\Delta'}{\Delta} + 2isJ' - \frac{2}{3}\Lambda r^2(s+1)(2s+1) + 2s(1-\alpha) - \lambda\right] R_s = 0$$

- Schrödinger

$$\left(\frac{d^2}{dr_*^2} + V_s\right)\mathcal{Y}_s = 0$$

- Heun

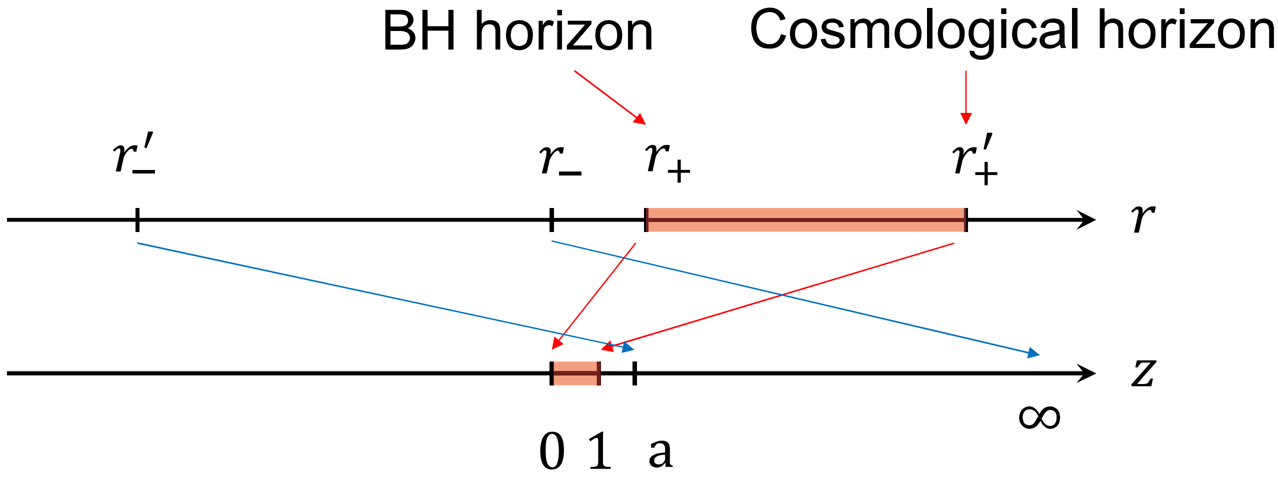
$$y'' + \left(\frac{\gamma}{z} + \frac{\delta}{z-1} + \frac{\varepsilon}{z-a}\right) y' + \frac{\alpha\beta z - q}{z(z-1)(z-a)} y = 0$$

Teukolsky → Heun

Transformation: $R_s(r) \to y_s(z)$

$$z = \frac{r'_+ - r_-}{r'_+ - r_+}\frac{r - r_+}{r - r_-}$$

$$R_s(r) = z^{B_1}(z-1)^{B_2}(z-z_r)^{B_3}(z-z_\infty)^{2s+1}y_s^{(\mathrm{r})}(z)$$



Heun equation

$$y'' + \left(\frac{\gamma}{z} + \frac{\delta}{z-1} + \frac{\varepsilon}{z-\mathrm{a}}\right)y' + \frac{\alpha\beta z - \mathrm{q}}{z(z-1)(z-\mathrm{a})}y = 0$$

$$\varepsilon = \alpha + \beta - \gamma - \delta + 1, \qquad \mathrm{a} \neq 0, 1$$

Two indep. solutions around $z = 0$ are

$y_{01}(z) = Hl(\mathrm{a}, \mathrm{q}; \alpha, \beta, \gamma, \delta; z)$

$y_{02}(z) = z^{1-\gamma}Hl(\mathrm{a}, (\mathrm{a}\delta + \varepsilon)(1-\gamma) + \mathrm{q}; \alpha + 1 - \gamma, \beta + 1 - \gamma, 2 - \gamma, \delta; z)$

Local Heun function

Three-term recurrence relation

$$Hl(\mathrm{a}, \mathrm{q}; \alpha, \beta, \gamma, \delta; z) = \sum_{k=0}^{\infty} c_k z^k$$

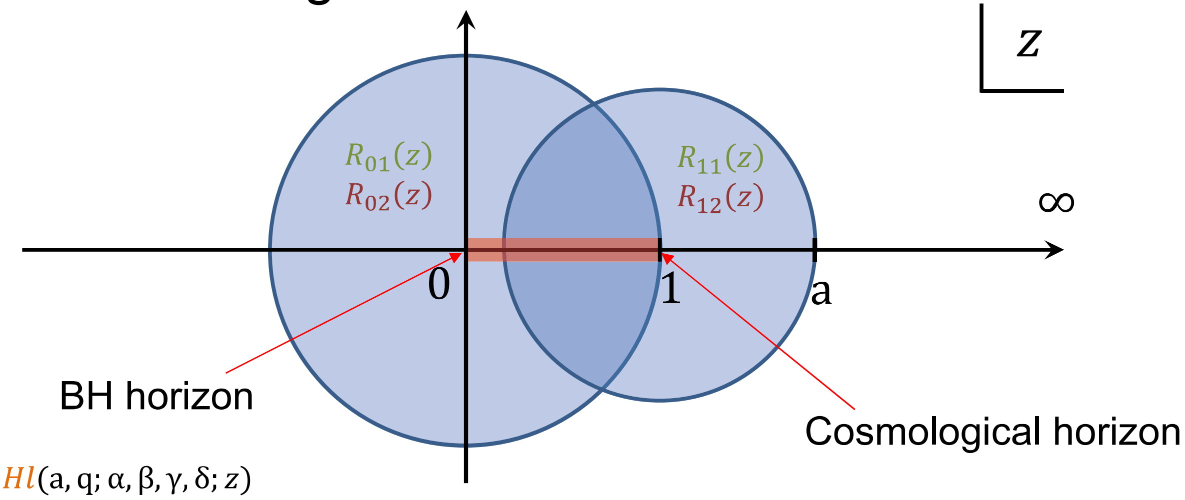
- Hl = "**HeunG**" in Mathematica 12.1 or later.
- Python implementation

Birkandan et al, 2106.13729

Scattering problem in terms of Hl

Two indep. solutions around $z = 0,1$

$\Leftrightarrow$ Ingoing and outgoing waves at BH/cosmological horizons.

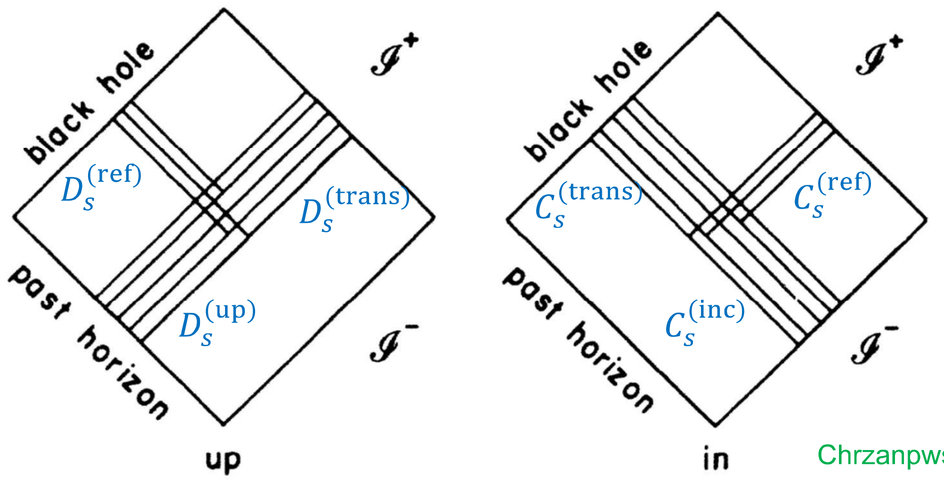


$y_{01}(z) = Hl(\mathrm{a}, \mathrm{q}; \alpha, \beta, \gamma, \delta; z)$

$y_{02}(z) = z^{1-\gamma} Hl(\mathrm{a}, (\mathrm{a}\delta + \varepsilon)(1-\gamma) + \mathrm{q}; \alpha + 1 - \gamma, \beta + 1 - \gamma, 2 - \gamma, \delta; z)$

$y_{11}(z) = Hl(1 - \mathrm{a}, \alpha\beta - \mathrm{q}; \alpha, \beta, \delta, \gamma; 1 - z)$

$y_{12}(z) = (1-z)^{1-\delta} Hl\big(1 - \mathrm{a}, \big((1-\mathrm{a})\gamma + \varepsilon\big)(1-\delta) + \alpha\beta - \mathrm{q}; \alpha + 1 - \delta, \beta + 1 - \delta, 2 - \delta, \gamma; 1 - z\big)$

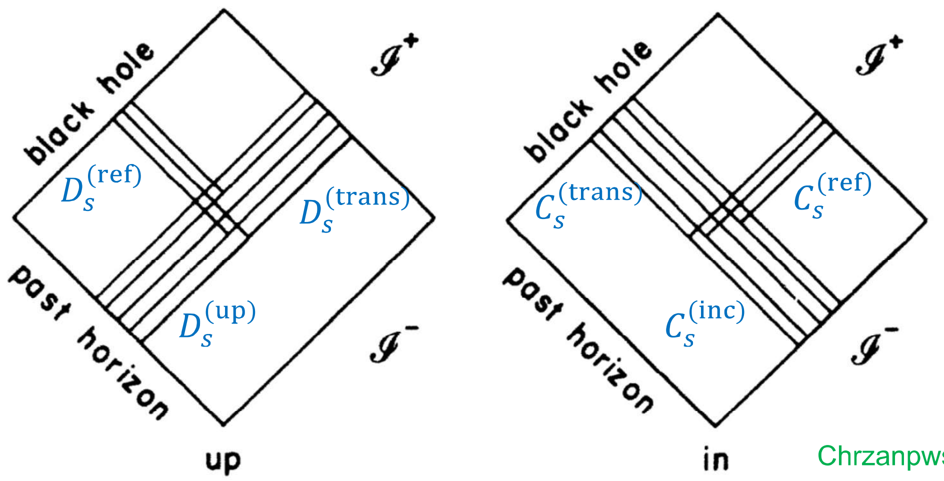


Chrzanpwski, Misner (1974)

Boundary condition

$$R_{\mathrm{in},s} \to \begin{cases} C_s^{(\mathrm{trans})} \Delta^{-B_1 - s}, & (r \to r_+) \\ C_s^{(\mathrm{ref})} \Delta^{B_2} + C_s^{(\mathrm{inc})} \Delta^{-B_2 - s}, & (r \to r'_+) \end{cases}$$

$$R_{\mathrm{up},s} \to \begin{cases} D_s^{(\mathrm{up})} \Delta^{B_1} + D_s^{(\mathrm{ref})} \Delta^{-B_1 - s}, & (r \to r_+) \\ D_s^{(\mathrm{trans})} \Delta^{B_2}, & (r \to r'_+) \end{cases}$$



Chrzanpwski, Misner (1974)

Exact solution

$$R_{\mathrm{in},s} = \begin{cases} R_{02,s}, & (r \to r_+) \\ C_{21} R_{11,s} + C_{22} R_{12,s}, & (r \to r'_+) \end{cases}$$

$$R_{\mathrm{up},s} = \begin{cases} D_{11} R_{01,s} + D_{12} R_{02,s}, & (r \to r_+) \\ R_{11,s}, & (r \to r'_+) \end{cases}$$

where $C_{22} = \frac{W_z[y_{02}, y_{11}]}{W_z[y_{12}, y_{11}]}$ etc.

$$W_z[u, v] = u\frac{dv}{dz} - \frac{du}{dz}v$$

QNM frequencies

Hatsuda, 2006.08957

- QNM boundary condition on radial solution:

$$W_z\left[y_{02}^{(\mathrm{r})}, y_{11}^{(\mathrm{r})}\right] = 0$$

- Regularity condition on angular solution:

$$W_z\left[y_{0i}^{(\mathrm{a})}, y_{1j}^{(\mathrm{a})}\right] = 0$$

with $i = \begin{cases} 1 \ (m - s \geq 0) \\ 2 \ (m - s < 0) \end{cases}$ and $j = \begin{cases} 1 \ (m + s \leq 0) \\ 2 \ (m + s > 0) \end{cases}$.

Finding root of two equations determines $(\omega_{\mathrm{QNM}}, \lambda)$ simultaneously ($\sim O(1)$ sec for 20 digits of precision).

Reflection / absorption rate

Reflection rate $\mathcal{R}_s = \dfrac{W_z\left[y^{(\mathrm{r})}_{12,s}, y^{(\mathrm{r})}_{02,s}\right]}{W_z\left[y^{(\mathrm{r})}_{02,s}, y^{(\mathrm{r})}_{11,s}\right]}\left(\dfrac{W_z\left[y^{(\mathrm{r})}_{12,-s}, y^{(\mathrm{r})}_{02,-s}\right]}{W_z\left[y^{(\mathrm{r})}_{02,-s}, y^{(\mathrm{r})}_{11,-s}\right]}\right)^*$

Absorption rate (=greybody factor) $\mathcal{T}_s = 1 - \mathcal{R}_s$.

Simple and fast.

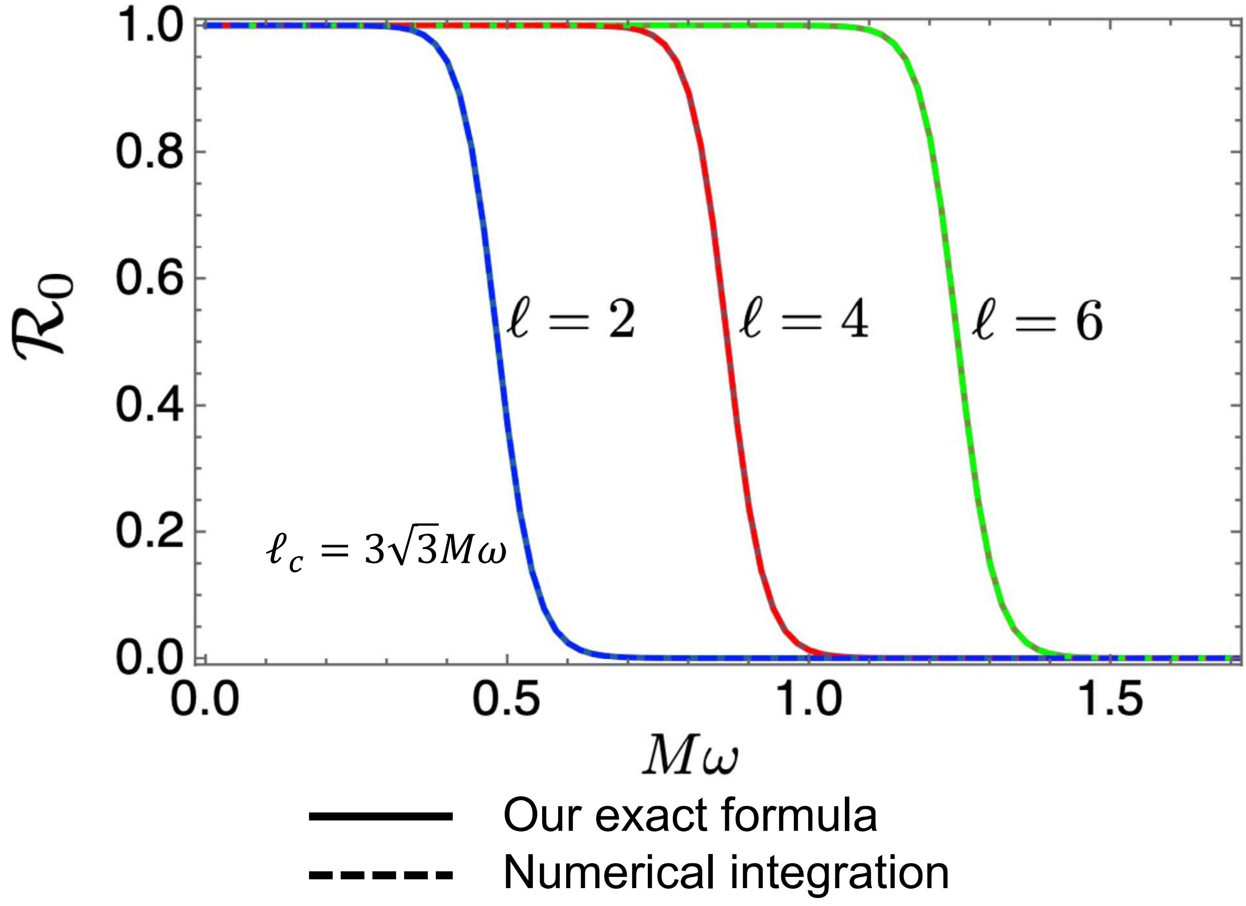
No approximation.

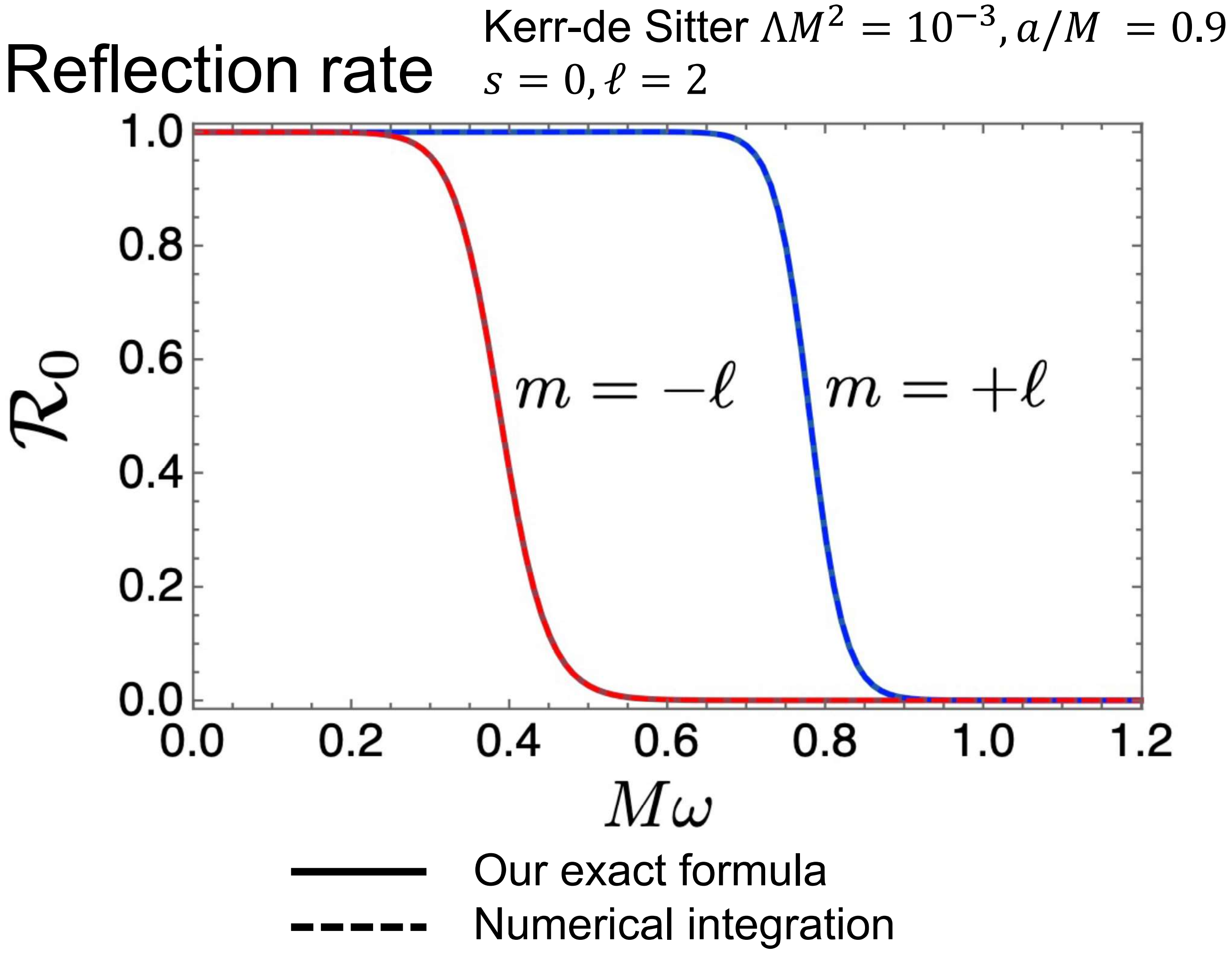
Arbitrary-precision arithmetic.

cf.
Gregory, Moss, Oshita, Patrick, 2103.09862

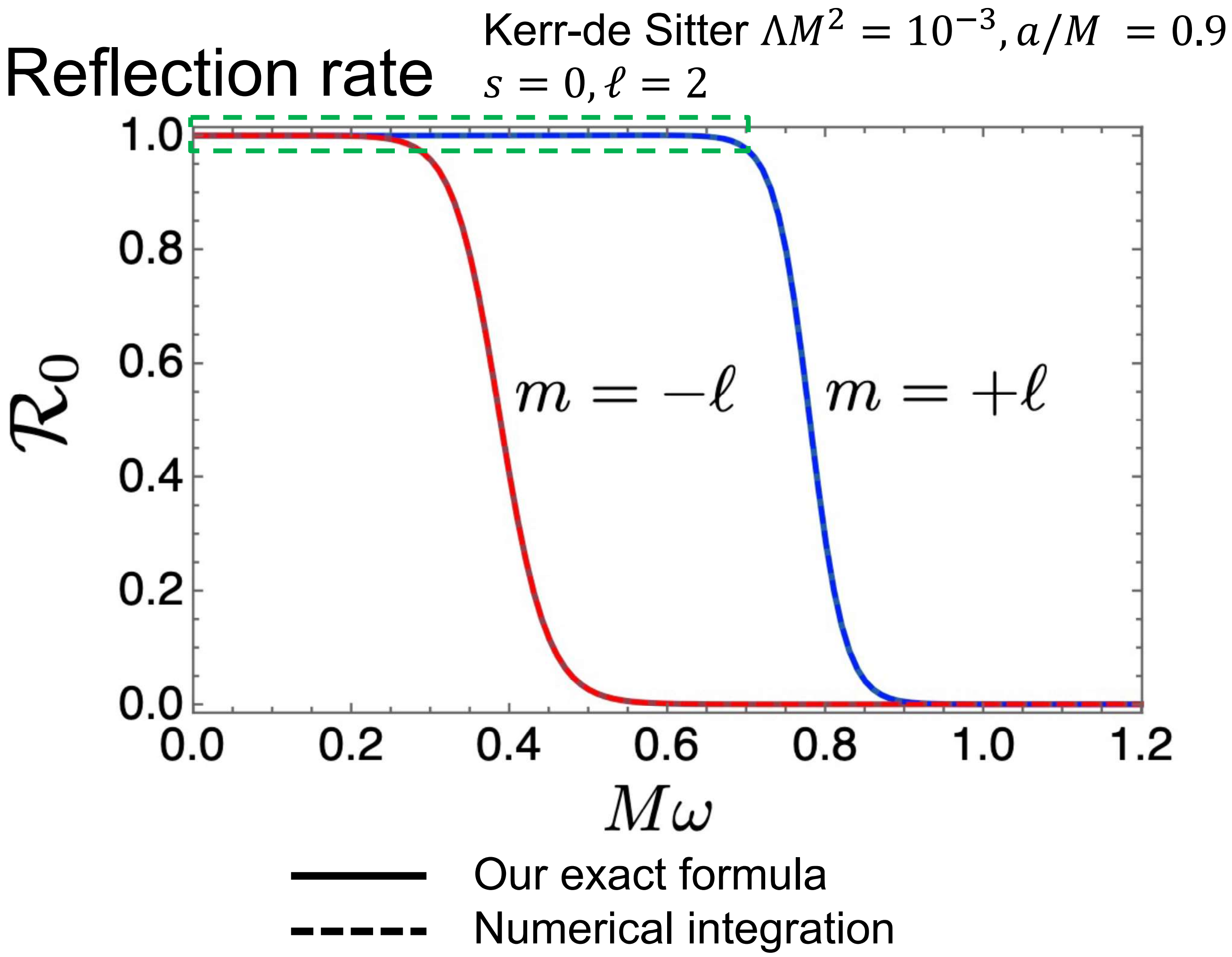
Schwarzschild-de Sitter $\Lambda M^2 = 10^{-3}$
$s = 0, \ell = 2, 4, 6$

Reflection rate

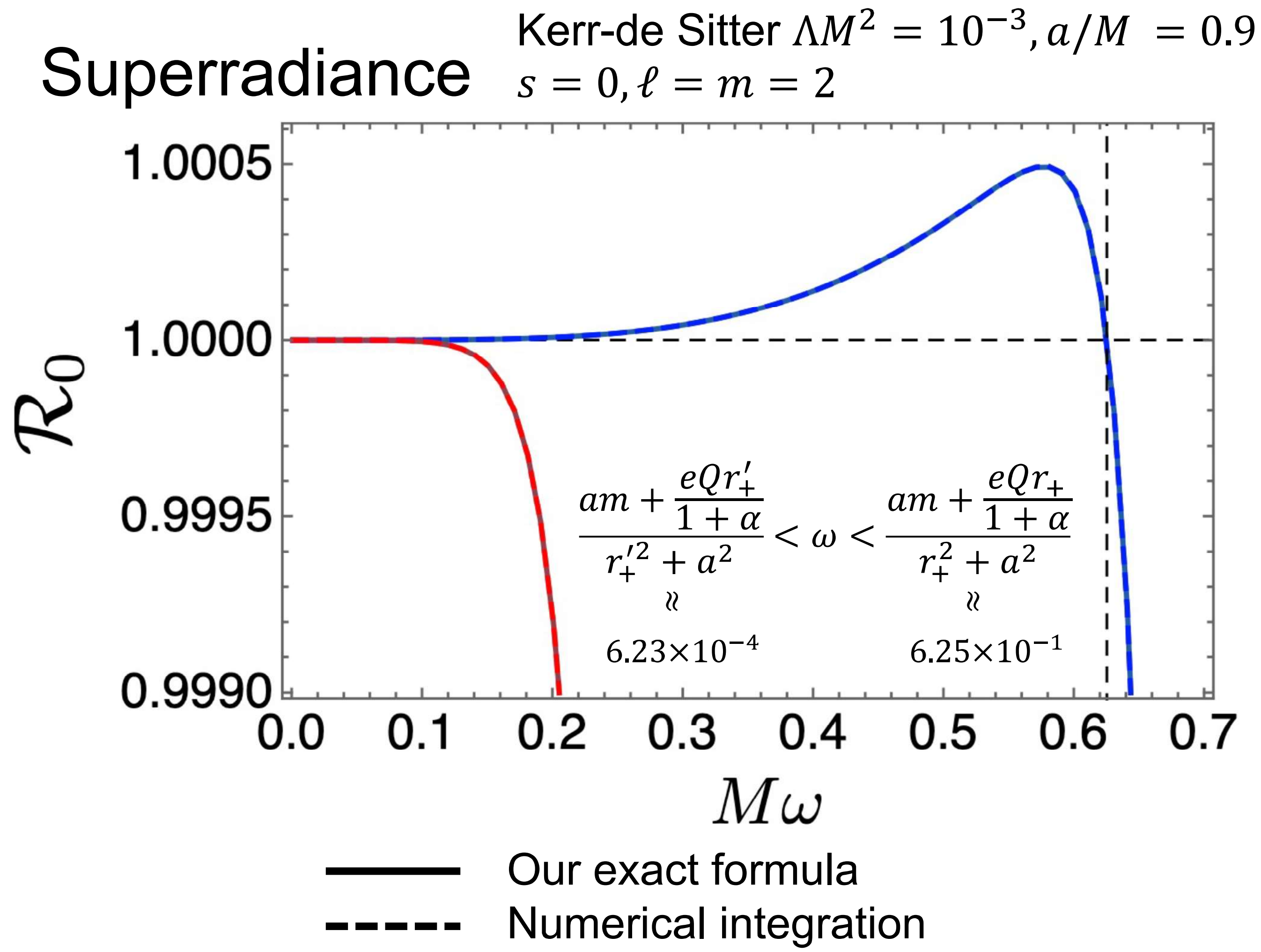




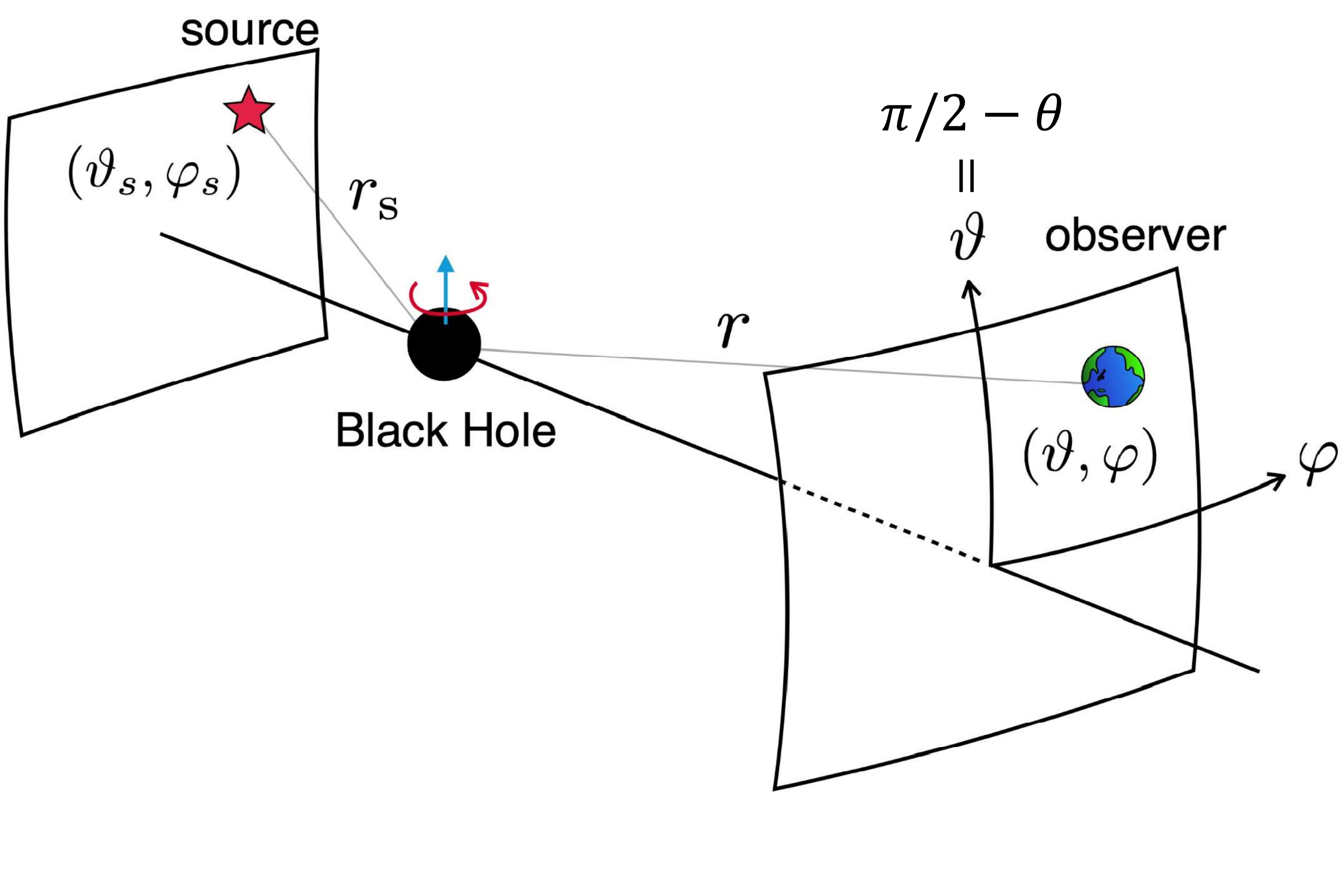
Reflection rate
Kerr-de Sitter $\Lambda M^2 = 10^{-3}, a/M = 0.9$
$s = 0, \ell = 2$
$\mathcal{R}_0$
$m = -\ell$
$m = +\ell$
$M\omega$
Our exact formula
Numerical integration



Reflection rate
Kerr-de Sitter $\Lambda M^2 = 10^{-3}, a/M = 0.9$
$s = 0, \ell = 2$
$\mathcal{R}_0$
$m = -\ell$
$m = +\ell$
$M\omega$
Our exact formula
Numerical integration



Superradiance
Kerr-de Sitter $\Lambda M^2 = 10^{-3}, a/M = 0.9$
$s = 0, \ell = m = 2$
1.0005
1.0000
0.9995
0.9990
$\mathcal{R}_0$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$M\omega$
$\frac{am + \frac{eQr'_+}{1+\alpha}}{r'^2_+ + a^2} < \omega < \frac{am + \frac{eQr_+}{1+\alpha}}{r^2_+ + a^2}$
$\approx$
6.23×10^{-4}
$\approx$
6.25×10^{-1}
Our exact formula
Numerical integration



source
(ϑ_s, φ_s)
$r_{\rm s}$
Black Hole
r
$\pi/2 - \theta$
ϑ
observer
(ϑ, φ)
φ

Green function

$$G(\boldsymbol{x}, \boldsymbol{x}_{\rm s}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{-\Delta^s(r_{\rm s})\left[R_{\rm in}(r_{\rm s}) R_{\rm up}(r_{\rm s}) \Theta(r - r_{\rm s}) + (r \leftrightarrow r_{\rm s})\right]}{\Delta^{s+1} W_r\left[R_{\rm in}, R_{\rm up}\right]} \times {}_sS_{\ell \rm m}(\theta, \varphi)\, {}_sS^*_{\ell \rm m}(\theta_{\rm s}, \varphi_{\rm s})$$

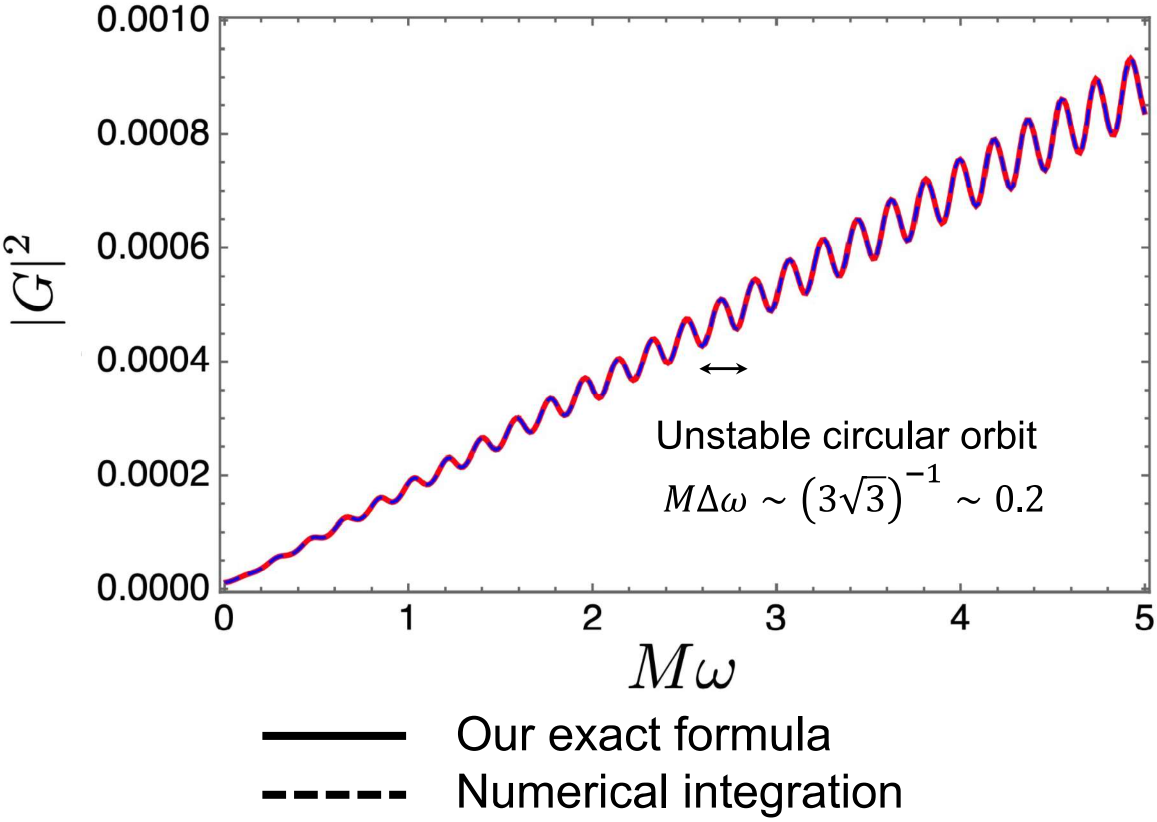
No approximation such as

- $r, r_{\rm s} \gg 1$
- $\vartheta, \varphi \ll 1$
- $M\omega \gg 1$ or $M\omega \ll 1$

cf.
Nambu, Noda, 1502.05468
Nambu, Noda, Sakai, 1905.01793

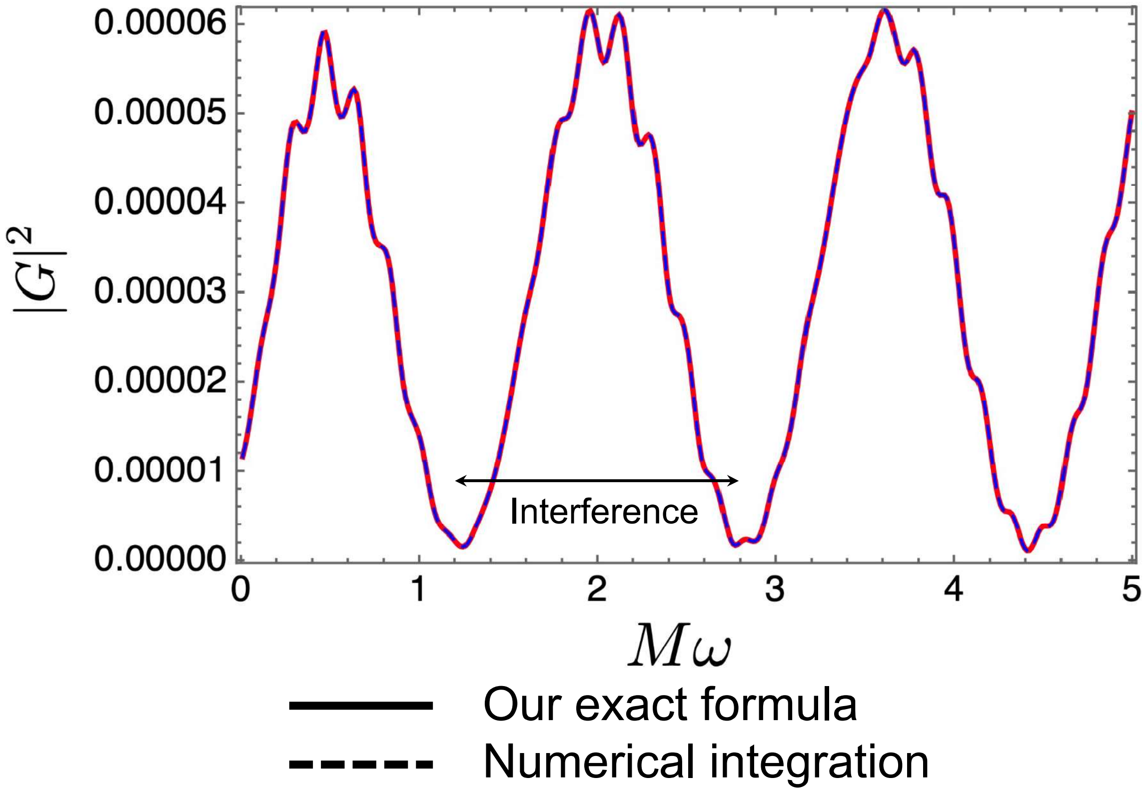
Forward scattering

$s = 0$ wave on SdS $\Lambda M^2 = 10^{-3}$
$(r, \vartheta, \varphi) = (20M, 0, 0)$
$(r_{\rm s}, \vartheta_{\rm s}, \varphi_{\rm s}) = (6M, 0, \pi)$

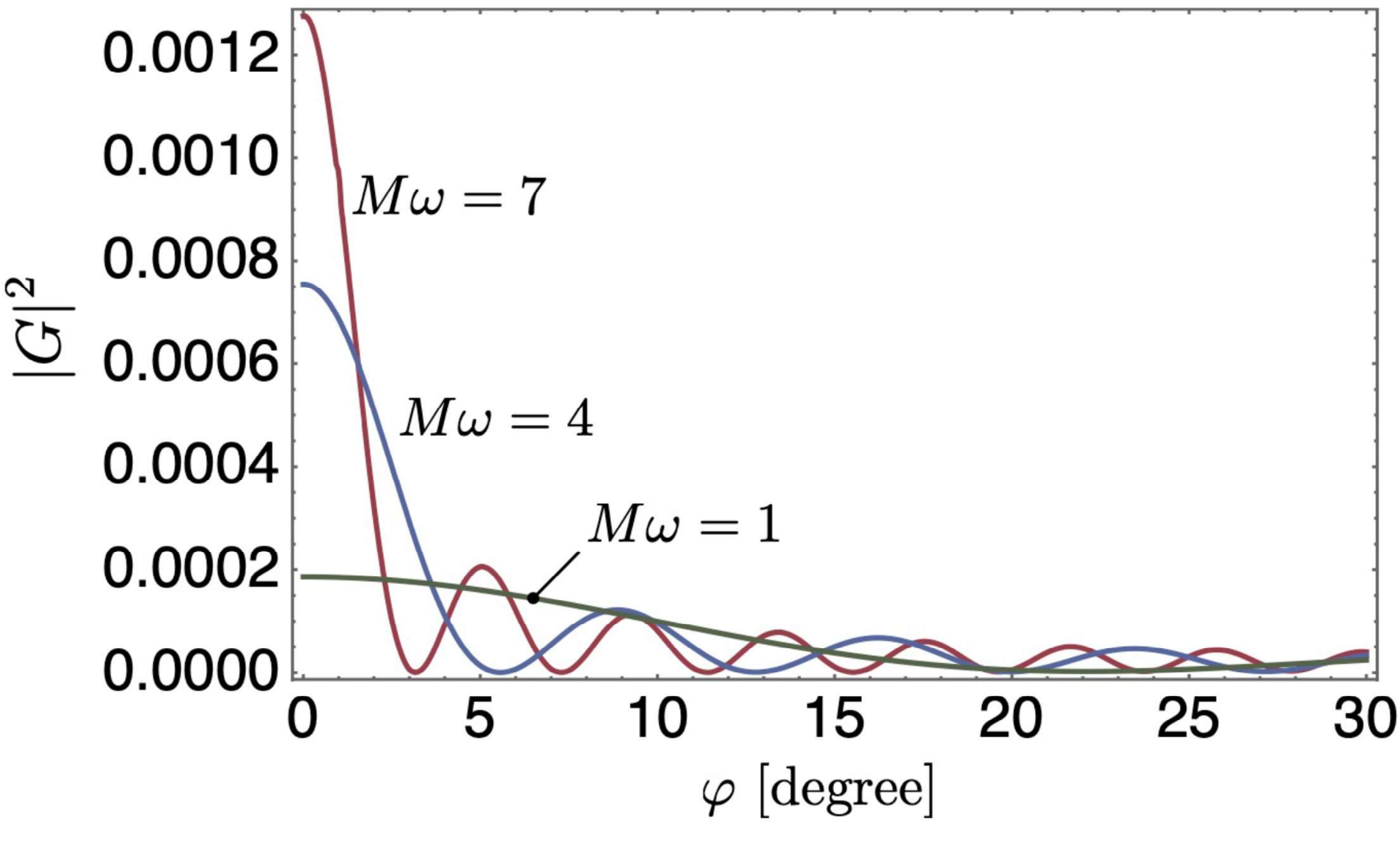


Slightly off forward scattering

$$(r, \vartheta, \varphi) = (20M, 0, \pi/10)$$
$$(r_s, \vartheta_s, \varphi_s) = (6M, 0, \pi)$$



Angular dependence



Summary

- We have established the exact formulation of the scattering problem of massless field on KNdS background by using the exact solution of Teukolsky eq. in terms of local Heun function Hl.
- Simple and fast formulae with arbitrary high precision w/o any approximations.
- Applications include:
  - QNM
  - Reflection / absorption rates, greybody factor
  - Green function
  - S-matrix, cross section, BH image, …and more!

Session E 10:00–12:00

[Chair: Ken-ichi Nakao]

Hideki Maeda

Hokkai-Gakuen University

"Quest for realistic non-singular black-hole geometries:"

(15 min.)

[JGRG30 (2021) 121007]

Hokkai-Gakuen University

@JGRG30

10th December 2021, Waseda University (online), Tokyo, Japan

Quest for realistic non-singular black-hole geometries

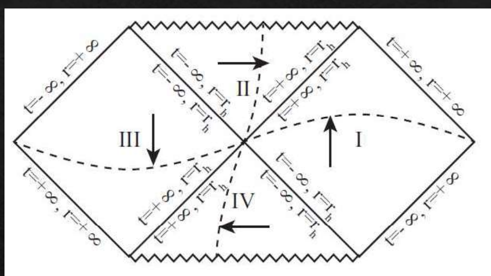
Hideki Maeda

(Hokkai-Gakuen University, Japan)

This talk is based on

e-Print:2107.04791 [gr-qc]: HM “Quest for realistic non-singular black-hole geometries: Regular-center type”

Black Holes in General Relativity



- GR is successful
- Exact BH solutions in GR
  - Vacuum: Schwarzschild, Kerr
  - Einstein-Maxwell system: Reissner-Nordstrom, Kerr-Newman
- Curvature singularity inside a BH is generic
  - Quantum Gravity effect should dominate around there
- Belief: A regular description of spacetime must be possible in QG
  - But we still don’t know a complete theory of QG

Singularity resolution: A belief in Physics

- We assume: Non-singular BH is realized in the low-energy CLASSICAL theory of QG
  - A bold assumption
  - Such a theory should be a modified gravity: Higher-curvature terms, non-minimal coupling, etc
  - Deviation from GR should be small in asymptotically flat regions
- Central problem in this talk: What is a physically reasonable model of non-singular BHs?
  - A bottom-up approach
  - Actually (too) many models have been proposed since the pioneering work by Bardeen in 1968
- We propose 7 criteria for model construction

7 criteria for realistic non-singular BHs

$$\tilde{T}_{\mu\nu} := G_{\mu\nu}$$

Effective energy-momentum tensor

- C1: Any kind of non-coordinate singularity is absent
  - Weak version of C1 (W-C1): Curvature singularities are absent
- C2: Closed causal curves are absent
- C3: $\tilde{T}_{\mu\nu}$ satisfies the standard energy conditions in asymptotically flat regions
- C4: $\tilde{T}_{\mu\nu}$ satisfies the standard energy conditions on the event horizon of a large BH
- C5: The limiting curvature condition (LCC) is respected
- C6: Realized for a set of non-zero measure in the parameter space of the BH solution
- C7: Dynamically stable

C1-C5 are purely geometric and can be examined without specifying a theory
We seek non-singular BH geometries satisfying C1-C5

Limiting Curvature Condition (LCC)

- Curvature invariants are bounded in the parameter space of the solution
  (Frolov `16 based on works in 80s by Markov, Polchinski, & Morgan)
- LCC ensures that singularity resolution is completed within the theory under consideration
  - LCC violated → Even higher-order corrections needed or resolution is possible only in full QG
- Suppose that a spacetime around a regular center (r=0) is given by

$$\mathrm{d}s^2 \simeq -(1-\lambda(m,l)r^2)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\lambda(m,l)r^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)$$

  - Ricci scalar near r=0 is $R \cong 12\lambda(m,l)$
  - According to LCC, the function λ should not blow up in the parameter space of l and m

Spherical non-singular BH with a regular center

- Metric ansatz:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2),$$
$$f(r) := 1 - \frac{2\boxed{M(r)}}{r},$$

functon of r

- Effective energy-momentum tensor: A particular Hawking-Ellis type I matter

$$\tilde{T}^{(a)(b)} := \tilde{T}^{\mu\nu} E^{(a)}_{\mu} E^{(b)}_{\nu} = \mathrm{diag}(\rho, p_1, p_2, p_3),$$

$$\rho = -p_1 = \frac{2M'}{r^2}, \qquad p_2 = p_3 = -\frac{M''}{r}$$

- Energy conditions

$$\begin{aligned} \mathrm{NEC}: &\quad \rho + p_i \geq 0 \ (i=1,2), \\ \mathrm{WEC}: &\quad \rho \geq 0 \text{ in addition to NEC}, \\ \mathrm{DEC}: &\quad \rho - p_i \geq 0 \ (i=1,2) \text{ in addition to WEC}, \\ \mathrm{SEC}: &\quad \rho + p_1 + 2p_2 \geq 0 \text{ in addition to NEC}. \end{aligned}$$

- DEC ⊂ WEC ⊂ NEC and SEC ⊂ NEC
- NEC violation → All energy conditions are violated
- DEC holds → WEC & NEC are respected as well

4 non-singular BHs with a regular center

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$
$$f(r) := 1 - \frac{2M(r)}{r},$$

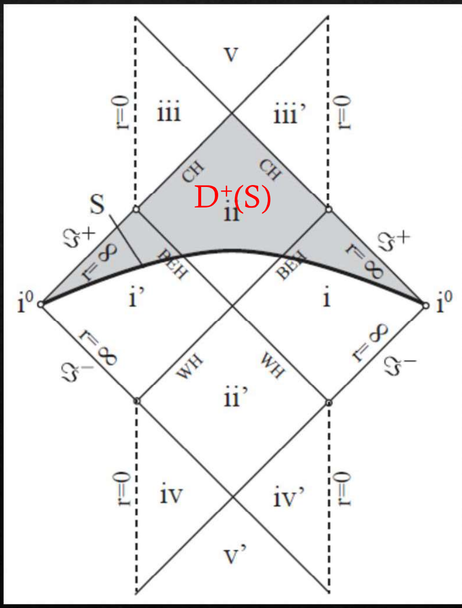
- Parameters: m & *l*
- f(r)=0 determines Killing horizons
  - Two horizons in general
- M(r)→m as r→ ∞ (Asymptotically flat)
- $M(r) \propto r^3$ as r→0 (Regular center)
  - de Sitter core

- Bardeen (1968): $M(r) = \frac{mr^3}{(r^2+l^2)^{3/2}}$
- Hayward (2006): $M(r) = \frac{mr^3}{r^3+2ml^2}$
- Dymnikova (2004): $M(r) = \frac{2m}{\pi}\left\{\arctan\left(\frac{r}{l}\right) - \frac{lr}{r^2+l^2}\right\}$
- Fan & Wang (2016): $M(r) = \frac{mr^3}{(r+l)^3}$

Only Hayward BH satisfies LCC
(the metric reduces to de Sitter as m → ∞)

How to avoid singularity theorem

- Penrose's singularity theorem (`64):
  A spacetime is null geodesically incomplete if
  1. The spacetime is globally hyperbolic and admits a non-compact Cauchy surface S,
  2. $\tilde{T}_{\mu\nu}$ (:=$G_{\mu\nu}$) satisfies the NEC,
  3. S contains a trapped surface
- These non-singular BHs are not globally hyperbolic
  - Assumption 1 does not hold
  - S in the figure is just a partial Cauchy surface



Where are Energy Conditions respected?

	NEC	WEC	DEC	SEC	$r=0$	Black Hole
Bardeen ($m>0$)	everywhere	everywhere	$0\le r\le 2l$	$r\ge\sqrt{2/3}l$	Analytic	$m\ge 3\sqrt{3}l/4$
Bardeen ($m<0$)	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	Analytic	n/a
Hayward ($m>0$)	everywhere	everywhere	$0\le r\le(4ml^2)^{1/3}$	$r\ge(ml^2)^{1/3}$	Regular	$m\ge 3\sqrt{3}l/4$
Hayward ($m_s<m<0$)	everywhere	everywhere	$\emptyset$	everywhere	n/a	n/a
Dymnikova ($m>0$)	everywhere	everywhere	everywhere	$r\ge l$	Analytic	$m\gtrsim 2.21l$
Dymnikova ($m<0$)	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	Analytic	n/a
Fan-Wang ($m>0$)	everywhere	everywhere	everywhere	$r\ge l$	Regular	$m\ge 27l/8$
Fan-Wang ($m<0$)	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	Regular	n/a

- Bardeen & Hayward BHs don't respect DEC at infinity: Discarded
- Dymnikova & Fan-Wang BHs respect DEC everywhere
  - We will focus on the rotating counterparts of these two BHs

Rotating counterparts: Metric ansatz

- Gurses-Gursey (GG) metric (`74): M(r) is a function and Δ(r)=0 is a Killing horizon

$$
\begin{aligned}
\mathrm{d}s^2 = &-\left(1-\frac{2M(r)r}{\Sigma(r,\theta)}\right)\mathrm{d}t^2 - \frac{4aM(r)r\sin^2\theta}{\Sigma(r,\theta)}\mathrm{d}t\mathrm{d}\phi \\
&+\frac{\Sigma(r,\theta)}{\Delta(r)}\mathrm{d}r^2 + \Sigma(r,\theta)\mathrm{d}\theta^2 + \left(r^2+a^2+\frac{2a^2M(r)r\sin^2\theta}{\Sigma(r,\theta)}\right)\sin^2\theta\mathrm{d}\phi^2, \\
&\Sigma(r,\theta) := r^2+a^2\cos^2\theta, \qquad \Delta(r) := r^2+a^2-2r\boxed{M(r)}
\end{aligned}
$$

- Effective energy-momentum tensor: Hawking-Ellis type I (Gurses & Gursey `74)

$$\tilde{T}^{(a)(b)} := \tilde{T}^{\mu\nu}E^{(a)}_{\mu}E^{(b)}_{\nu} = \mathrm{diag}(\rho, p_1, p_2, p_3),$$

$$\rho = -p_1 = \frac{2r^2M'}{\Sigma^2},$$

$$p_2 = p_3 = -\frac{rM''\Sigma + 2M'a^2\cos^2\theta}{\Sigma^2}$$

Stable causality with rM(r)≥0

- Doran coordinates in a region with rM(r)≥0 in the GG spacetime:

$$\mathrm{d}s^2 = -\,\mathrm{d}\bar{t}^2 + \Sigma(r,\theta)\mathrm{d}\theta^2 + (r^2+a^2)\sin^2\theta\mathrm{d}\varphi^2 + \frac{\Sigma(r,\theta)}{r^2+a^2}\left\{\mathrm{d}r + \frac{\sqrt{2M(r)r(r^2+a^2)}}{\Sigma(r,\theta)}(\mathrm{d}\bar{t} - a\sin^2\theta\mathrm{d}\varphi)\right\}^2.$$

- If rM(r)≥0 holds everywhere, the maximally extended spacetime is stably causal
- Due to $g^{00}=-1$, $\nabla_\mu \bar{t}$ is everywhere timelike and hypersurface-orthogonal one-form
  - $T=\bar{t}$ is a time function
  - The spacetime is stably causal by Prop. 6.4.9 in Hawking-Ellis textbook
    (No closed causal curves appear even under small perturbations)

Rotating counterparts with GG metric

- Rotating Dymnikova BH:

$$M(r) = \frac{2m}{\pi}\left\{\arctan\left(\frac{r}{l}\right) - \frac{lr}{r^2+l^2}\right\}$$

  - Singularity-free in -∞<r<∞
    except at the ring (r,θ)=(0,π/2)
  - All ECs are respected at spatial infinity
  - DEC is respected on & outside the event horizon
  - Stably causal

- Rotating Fan-Wang BH:

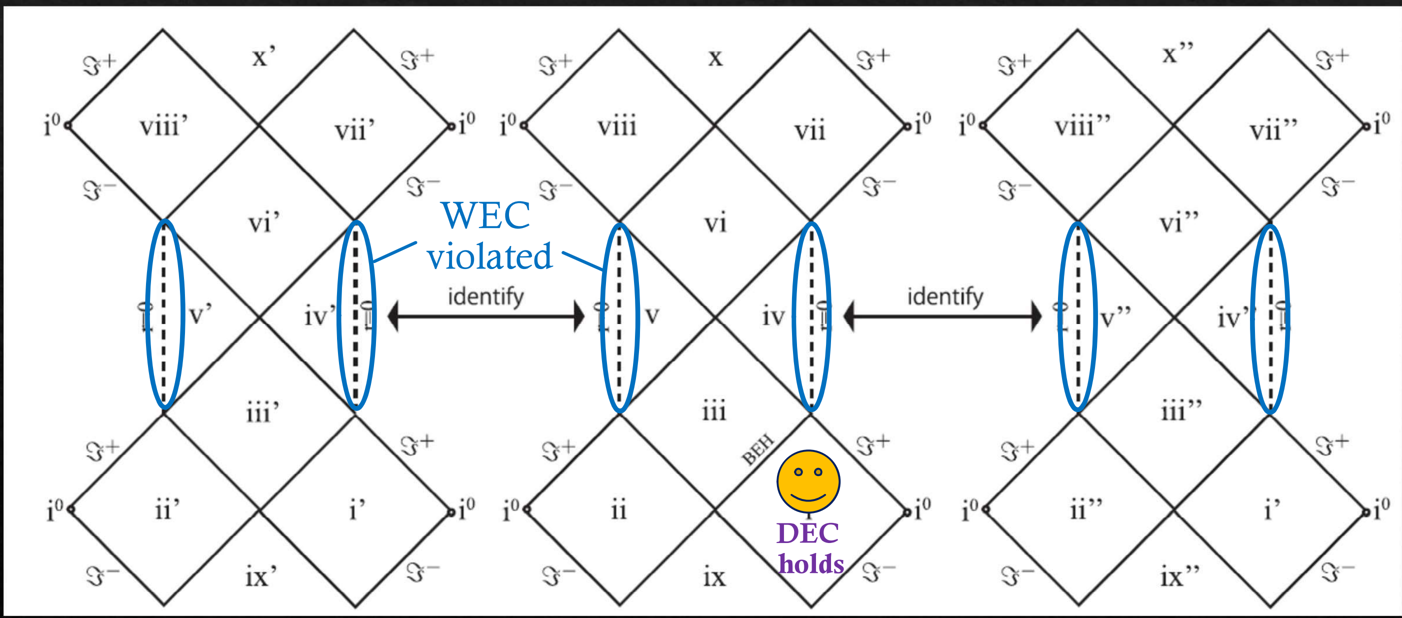
$$M(r) = \frac{mr^3}{(r+l)^3}$$

  - Curvature singularity at r=-*l*(<0)
  - Domain of r is r_s<r<∞
  - All ECs are respected at spatial infinity
  - Stably causal
  - Discarded by its singular nature

Note:

- (r,θ)=(0,π/2) is NOT a scalar polynomial curvature singularity (Torres & Fayos`17), but still a conical singularity as Kerr
- Still not clear whether the ring (r,θ)=(0,π/2) is a p.p. scalar curvature singularity or not

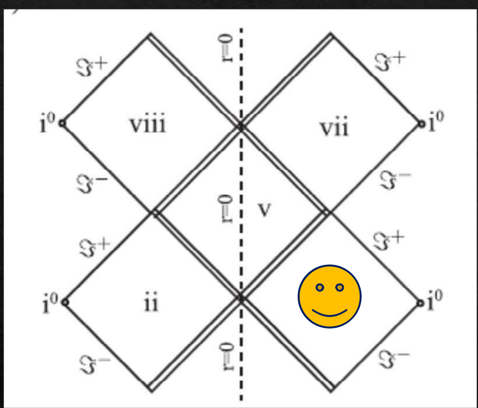
Rotating Dymnikova BH for $m>m_{ex}$



4 non-degenerate Killing horizons in $-\infty<r<\infty$

The ring $(r,\theta)=(0,\pi/2)$ could be a p.p. singularity

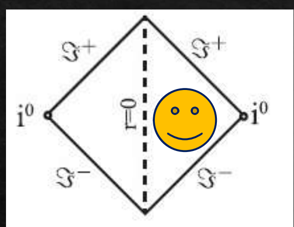
$m=m_{ex}$



2 degenerate Killing horizons in $-\infty<r<\infty$

(Extreme BH)

$m<m_{ex}$



No Killing horizon in $-\infty<r<\infty$

(Wormhole)

Summary: Which is better?

- C1: Singularity-free in any sense
  - Weak-C1: Curvature singularity-free
- C2: No closed causal curves
- C3: ECs in asymp. flat regions
- C4: ECs on EH of large BH
- C5: LCC is respected
- C6: Without fine-tuning in the solution
- C7: Dynamically stable

	C1	C2	C3	C4	C5	C6	C7
Bardeen	Ring?	OK	×	?	×	Theory dependent	
Hayward	×	OK	×	?	OK		
Dymnikova	Ring?	OK	OK	OK	×		
Fan-Wang	×	OK	OK	?	×		

Dymnikova BH is preferable but LCC is not satisfied
Still not clear if the ring $(r,\theta)=(0,\pi/2)$ is a p.p. scalar curvature singularity or not

Remaining problems

- Q1: Clarify whether $(r,\theta)=(0,\pi/2)$ is a p.p. singularity or not
- Q2: Find better models
- Q3: Find well-motivated theories which admit such non-singular BHs solutions
- Q4: Clarify stability of inner Cauchy horizon
  - Mass inflation instability of Reissner-Nordstrom BH in GR (Poisson & Israel `89)
  - Different groups have obtained opposite results for spherically symmetric non-singular BHs (Bonanno et al. `21, Carballo-Rubio et al. `21)

Final take-home message

- If you use a model in astrophysics, check energy conditions at least at spacelike infinity
- If LCC is violated, you need even higher-order corrections or full QG

Session E 10:00–12:00

[Chair: Ken-ichi Nakao]

Sousuke Noda

National Institute of Technology, Miyakonojo College

“Alfv\'enic superradiance in a force-free magnetosphere of a Kerr black hole”

(15 min.)

[JGRG30 (2021) 121008]

Alfvénic superradiance
in a force-free magnetosphere of a Kerr black hole

Sousuke Noda [National Institute of Technology, Miyakonojo College]

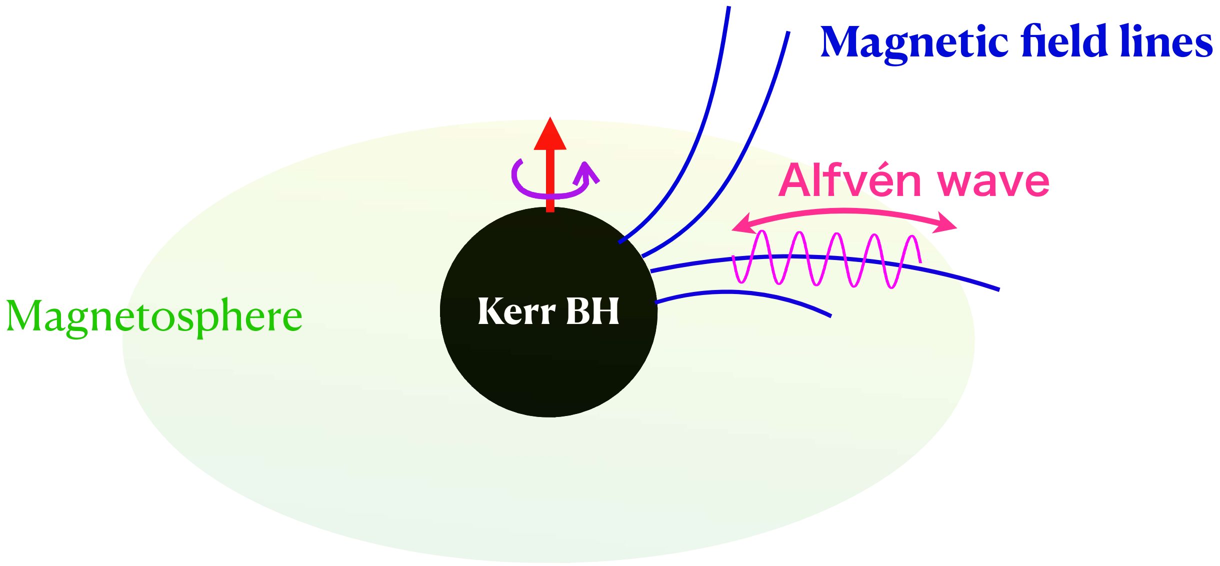
Collaborators
Yasusada Nambu (Nagoya University)
Masaaki Takahashi (Aichi University of Education)
Shinji Koide (Kumamoto University)

References

1. S.N *etal.,* Phys. Rev. D **101** 023003 (2020) ⇒ BTZ string (each z-const = BTZ)
2. S. Koide, S.N, M. Takahashi, and Y. Nambu, arXiv: 2109.05703
3. S.N, Y. Nambu, M. Takahashi, and T. Tsukamoto, arXiv: 2111.01149 ⇒ Kerr case (this talk)
   accepted for publication in PRD

Alfvénic superradiance
in a force-free magnetosphere of a Kerr black hole

Sousuke Noda [National Institute of Technology, Miyakonojo College]



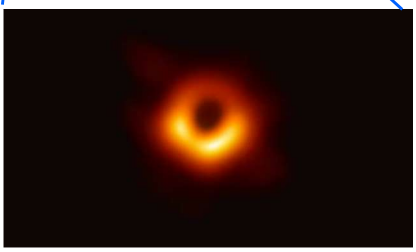
Relativistic jet & BH

M87

Credits: NASA and the Hubble Heritage Team (STScI/AURA)

Collimated stream of plasma

speed ~ c

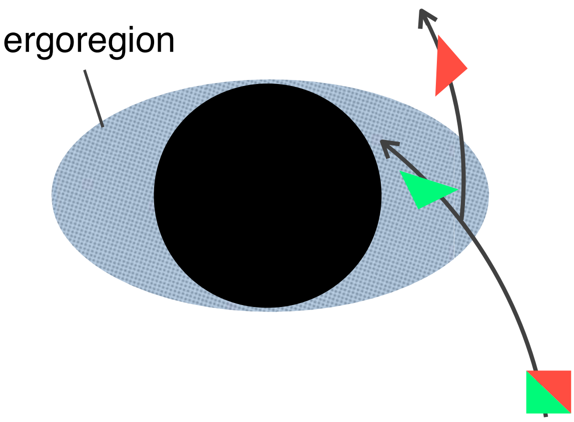


BH Shadow (EHTC)

Energy source of jets/gamma ray bursts $\Rightarrow$ **BH's rotational energy**

$\longrightarrow$ **Energy extraction processes**

Penrose Process



Energy (Killing energy)

$$E := -p_\mu \xi^\mu_{(t)}$$

Energy conservation

$$\boxed{E = E + E}$$

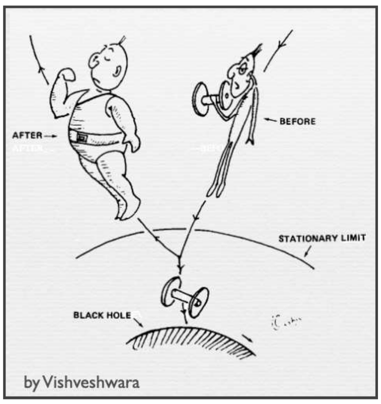
Timelike Killing vector is spacelike in the ergoregion

$\longrightarrow$ E can be negative. (It depends on p_μ)

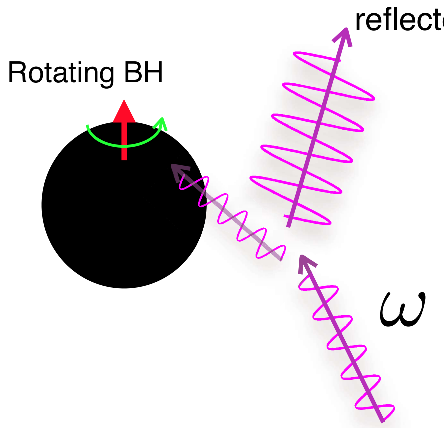
From the energy conservation law,

$$E = E - \underbrace{E}_{<0} \longrightarrow \boxed{E > E}$$

energy gain



Superradiant wave scattering (superradiance)



$$\Phi = \int d\omega \; e^{-i\omega t} e^{im\varphi} f(r, \theta)$$

For wave modes satisfying $\omega < m\Omega_{\rm H}$

will be amplified by scattering.

angular velocity of the horizon

Superradiant wave scattering is possible for

scalar wave	Y. B. Zel'dovich	Zh. Eksp. Teor. Fiz. Pis'ma Red. **14**, 270, (1971)
EM, gravitational waves	A.A. Starobinsky	Zh. Eksp. Teor. Phyz. **64**, 48 (1973)
	A.A. Starobinsky and S.M. Churilov	Zh. Eksp. Teor. Phyz. **65**, 3 (1973)
acoustic wave in fluids	M. Visser ,	Class.Quant. Grav, **15** 1767 (1998)
magnetosonic fast wave	S.N *etal.,*	Phys. Rev. D **95** 104055 (2017)
Alfvén wave	S.N *etal.,*	Phys. Rev. D **101** 023003 (2020)
	S.N, *etal.*	arXiv: 2111.01149, press in PRD (2021? 2022?)

Blandford-Znajek process

BH's rotational energy ⇒ jet ?

Blandford & Znajek (1977)

1. Kerr BH
2. Stationary axisymmetric rotating magnetosphere (plasma + electromagnetic field)
3. Force-free magnetosphere (magnetically dominated)

$$\Omega_{\rm H} := -\left.\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right|_{r=r_{\rm H}}$$

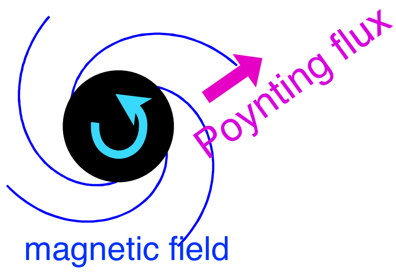
Poynting flux

$$P^r \propto \Omega_F(\Omega_{\rm H} - \Omega_F)$$ If $0 < \Omega_F < \Omega_{\rm H}$, nonzero outward flux!!

Rotational energy is extracted by magnetosphere

BZ power

$$P_{\rm BZ} \approx 10^{45} {\rm erg/s} \left(\frac{a}{M}\right)^2 \left(\frac{B_0}{10^4 {\rm G}}\right)^2 \left(\frac{M}{10^9 M_\odot}\right)^2$$



Questions

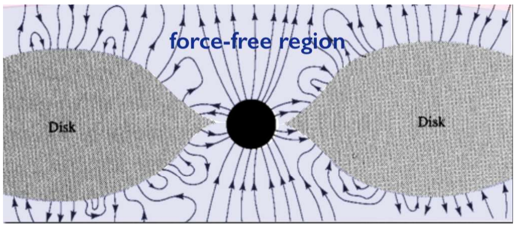
- Non-stationary magnetosphere case ??
- Wave propagation ?? Fast & Alfvén
- Essence of the BZ process ??

Force-Free Electromagnetic Field

Maxwell eq.

$$\nabla_{[a}F_{bc]} = 0 \qquad \nabla_b T^{ab}_{(\mathrm{EM})} = -F^{ab} j_b$$

$$\nabla_b F^{ab} = 4\pi j^a \quad \nabla_b\left(T^{ab}_{(\mathrm{EM})} + \cancel{T^{ab}_{\mathrm{plasma}}}\right) = 0$$



If the EM fields are dominant , $\nabla_b T^{ab}_{(\mathrm{EM})} \approx 0 \;\; \rightarrow \;\; F^{ab} j_b \approx 0.$ (Force-free approximation)

$$\nabla_{[a}F_{bc]} = 0 \;, \quad F_{ab}\nabla_c F^{bc} = 0 \;, \quad j^a \propto \nabla_b F^{ab} \neq 0$$

$\Rightarrow$ $\boxed{F = d\phi_1 \wedge d\phi_2}$ $\quad \phi_1 \,, \phi_2$: Euler potential

Carter (1979)
Uchida (1997)
Gralla & Jacobson (2014)

Force-Free EM field eq.

$$\boxed{\partial_a \underline{\phi_1} \partial_b \left[\sqrt{-g}\left(\partial^a\phi_1\partial^b\phi_2 - \partial^b\phi_1\partial^a\phi_2\right)\right] = 0}$$

$$\boxed{\partial_a \underline{\phi_2} \partial_b \left[\sqrt{-g}\left(\partial^a\phi_1\partial^b\phi_2 - \partial^b\phi_1\partial^a\phi_2\right)\right] = 0}$$

Euler potential & FF-Magnetosphere

For stationary and axisymmetric (**rotating**) force-free magnetosphere

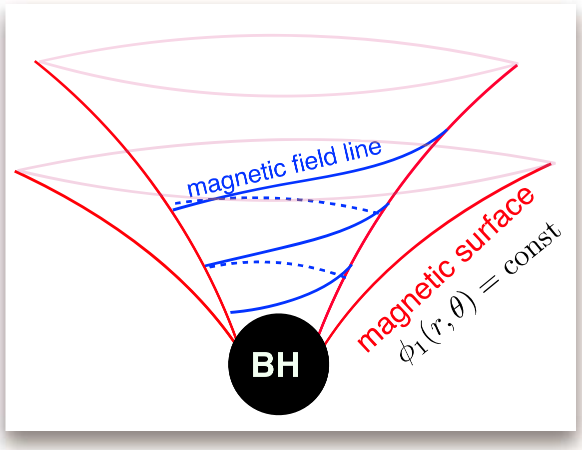
$$\boxed{\begin{aligned} \phi_1 &= \Psi_1(r,\theta) \\ \phi_2 &= \varphi - \Omega_F(\Psi_1)t + \Psi_2(r,\theta) \end{aligned}}$$

This can be derived with Killing vector [e.g. Uchida (1997)]

ϕ_1= const. defines magnetic surface

Axisymmetric magnetosphere
is **foliated** by magnetic surfaces

ϕ_2 = const. magnetic field line on a magnetic surface



wave modes

Alfvén waves propagate on the magnetic surface

Fast magnetosonic wave can across the surfaces

To obtain global magnetosphere in the
Kerr spacetime is very difficult (Grad-Shafranov eq)

Background magnetosphere

Magnetosphere near the equatorial plane.

Structure of mag field = Monopole-type

Ansatz $\phi_1 = \phi_1(\underline{\theta})\ ,\ \ \phi_2 = \varphi - \Omega_F t + \Psi(\underline{r})$

Maxwell equation $\Rightarrow \quad \partial_\theta\left(\phi_1 \sin\theta\right) = 0\ \ ,\ \ \partial_r\left(\partial^r \phi_2\right) = 0$

$$\Rightarrow \quad \boxed{\begin{aligned} \phi_1 &= q\cos\theta \\ \phi_2 &= \varphi - \Omega_F t + C\int \frac{r^2}{\Delta} \end{aligned}}$$

Regularity at horizon

$$C = \frac{r_{\mathrm{H}}^2 + a^2}{r_{\mathrm{H}}^2}(\Omega_{\mathrm{H}} - \Omega_F)$$

The BZ process for this solution

energy flux vector: $P^\mu = -T^\mu{}_\nu \xi^\nu_{(t)}$

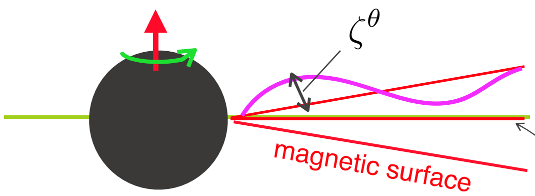
timelike Killing vector $\xi^\nu_{(t)} := (\partial_t)^\nu$

Poynting flux

$$P^r = \Omega_F(\Omega_{\mathrm{H}} - \Omega_F)\frac{r_{\mathrm{H}}^2 + a^2}{r_{\mathrm{H}}^2}\frac{B_0^2}{r^2}$$

$0 < \Omega_F < \Omega_{\mathrm{H}}$ $\Rightarrow$ outward flux

Perturbation & Mode decoupling



Definition of the perturbation

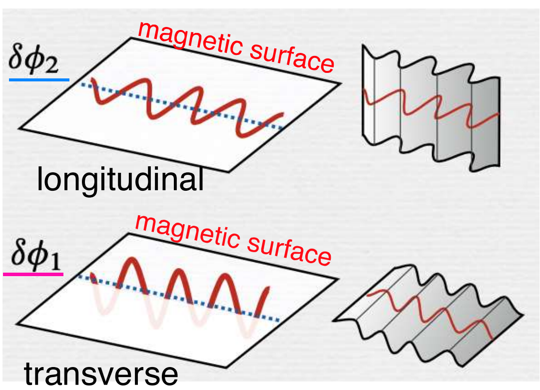
$$\delta\phi_i = \zeta^\mu \partial_\mu \phi_i$$

displacement vector

$\phi_1 = q\cos\theta$

$\phi_2 = \varphi - \Omega_F t + C\int \frac{r^2}{\Delta}$

On the equatorial plane magnetic surface…



Fast magnetosonic wave

$$\partial_j\left(\sqrt{-g}\,\partial^j \underline{\delta\phi_2}\right) = 0$$

Alfvén wave

$$\partial_\nu\phi_2\partial_\mu\left(\sqrt{-g}\,\partial^{[\mu}\underline{\delta\phi_1}\partial^{\nu]}\phi_2\right) = 0$$

propagate along magnetic surface

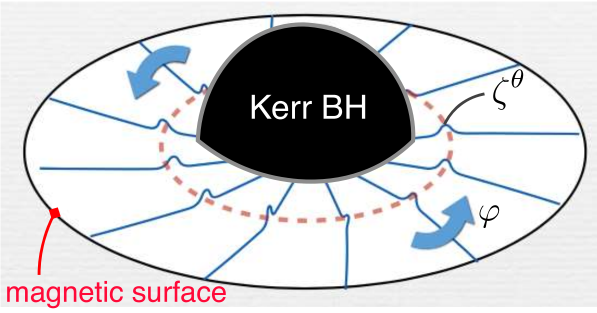
Wave equation for Alfvén wave

$$\boxed{\partial_\nu \phi_2 \partial_\mu \left(\sqrt{-g}\partial^{[\mu}\delta\phi_1 \partial^{\nu]}\phi_2\right) = 0}$$

separation of variables

$$\delta\phi_1 = A(\rho)\psi(t,r) \qquad \rho = \varphi - \Omega_F t + \int dr \frac{r^2}{\Delta}$$

Choice of A(ρ)



$\rho = \text{const}$ gives a magnetic field line.

$A(\rho)$: configuration of the perturbation in φ direction

$A(\rho) = 1 \quad \Rightarrow \quad$ axisymmetric

$A(\rho) = \delta(\rho - \rho_0) \quad \Rightarrow \quad$ only $\rho = \rho_0$ is perturbed

Wave equation

$$\left[\Gamma\left(\partial_r - \frac{Cr^2 g_{\varphi\varphi}}{\Gamma\Delta}(\Omega - \Omega_F)\partial_t\right)\psi\right]_r - \frac{Cr^2 g_{\varphi\varphi}}{\Delta}(\Omega - \Omega_F)\psi_{tr} + \frac{C^2 r^2 - \Gamma}{\Delta}\psi$$
$$+ \frac{r^2 g_{\varphi\varphi}}{\Delta^2}\left[C^2 r^2 - \Gamma + g_{\varphi\varphi}(\Omega - \Omega_F)^2\right]\psi_{tt} = 0,$$

Coordinate transformation

Transformation reflecting the structure of magnetic field line

$$r = X, \quad t = T - C\int dX \frac{X^2 g_{\varphi\varphi}}{\Gamma\Delta}(\Omega - \Omega_F),$$

to eliminate of the cross term (tr-term) in the wave eq.

$$\boxed{\partial_X(\Gamma\partial_X\psi) + \frac{X^2 g_{\varphi\varphi}}{\Delta^2}(c^2X^2 - \Gamma)\left[1 - \frac{g_{\varphi\varphi}(\Omega - \Omega_F)^2}{\Gamma}\right]\psi_{TT} + \frac{c^2X^2 - \Gamma}{\Delta}\psi = 0.}$$

(※) Γ=0: Light surfaces (inner LS and outer LS) $\quad \Gamma := g_{\mu\nu}\chi^\mu\chi^\nu, \quad \chi^\mu = \xi^\mu_{(t)} + \Omega_F \xi^\mu_{(\varphi)}$

- Light surfaces are one way boundaries for Alfvén waves (← shown by ray motion)
- The inner LS is an effective horizon for Alfvén waves (purely ingoing / coordinate singularity

Wave eq in tortoise coordinate

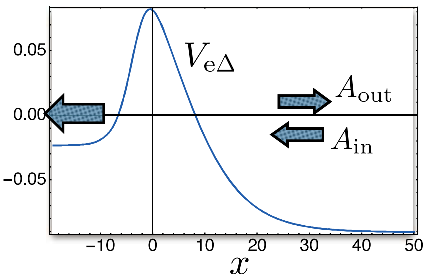
$$\frac{dx}{dX} = -\frac{1}{\Gamma}$$

r	$r_{\rm in}$	·····	$r_{\rm out}$
x	$-\infty$	·····	$+\infty$

$\psi = e^{-i\omega T}R$ stationary scattering

$$\boxed{\frac{d^2R}{dx^2} - V_{\rm eff}R = 0,} \qquad V_{\rm eff} = -\frac{C^2X^2 - \Gamma}{\Delta}\left[\Gamma + \frac{\omega^2 X^2 g_{\varphi\varphi}}{\Delta}\left[g_{\varphi\varphi}(\Omega - \Omega_F)^2 - \Gamma\right]\right].$$

Alfvénic superradiance



Asymptotic solutions

$$\frac{d^2R}{dx^2} - V_{\rm eff}R = 0,$$

$$R = \begin{cases} \exp\left[-iC\omega\int \frac{dx}{\Delta}X^2 g_{\varphi\varphi}(\Omega-\Omega_F)\right]. & \text{for} \quad x \to -\infty. \\ A_{\rm in}\exp\left[-iC\omega\int \frac{dx}{\Delta}X^2 g_{\varphi\varphi}(\Omega-\Omega_F)\right] + A_{\rm out}\exp\left[i c\omega\int \frac{dx}{\Delta}X^2 g_{\varphi\varphi}(\Omega-\Omega_F)\right] & \text{for} \quad x \to +\infty. \end{cases}$$

purely ingoing at inner LS

Boundary Condition should be given at the inner LS for energy extraction

not BH horizon !!

Superradiant condition

Wronskian = const.

$$\left|\frac{A_{\rm out}}{A_{\rm in}}\right|^2 = 1 - \frac{f_{\rm in}}{f_{\rm out}}\frac{\Omega|_{r_{\rm in}} - \Omega_F}{\Omega|_{r_{\rm out}} - \Omega_F}\frac{1}{|A_{\rm in}|^2} \qquad f_{\rm in/out} = r^2 g_{\varphi\varphi}/\Delta|_{r_{\rm in/out}}$$

reflection rate

$$\boxed{\Omega|_{r_{\rm out}} < \Omega_F < \Omega|_{r_{\rm in}}}$$ $\Rightarrow$ reflection rate exceeds 1

angular velocity of magnetic field line

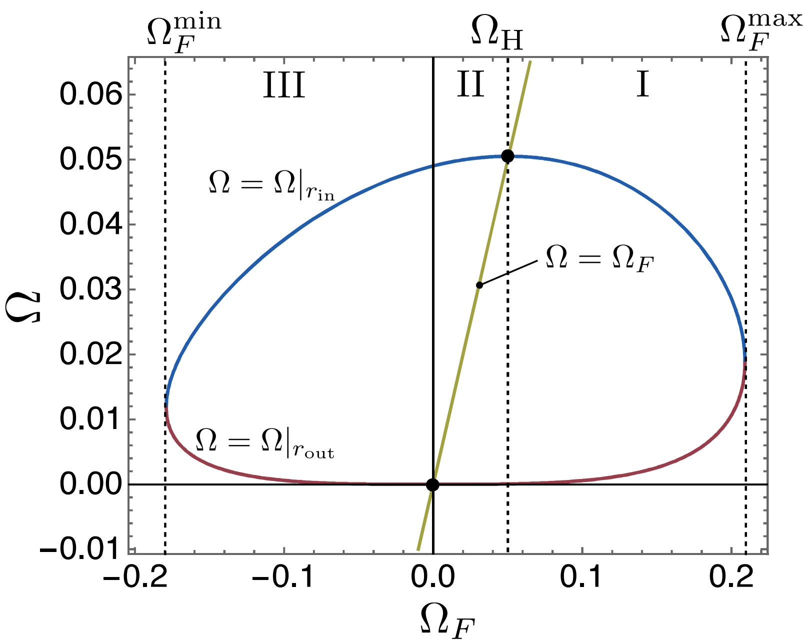
$$\Omega|_{r_{\rm in/out}} = -\left.\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right|_{r=r_{\rm in/out}}$$

Dragging effect at light surfaces

vs

Angular velocity of magnetic field line

Superradiant condition and the BZ process

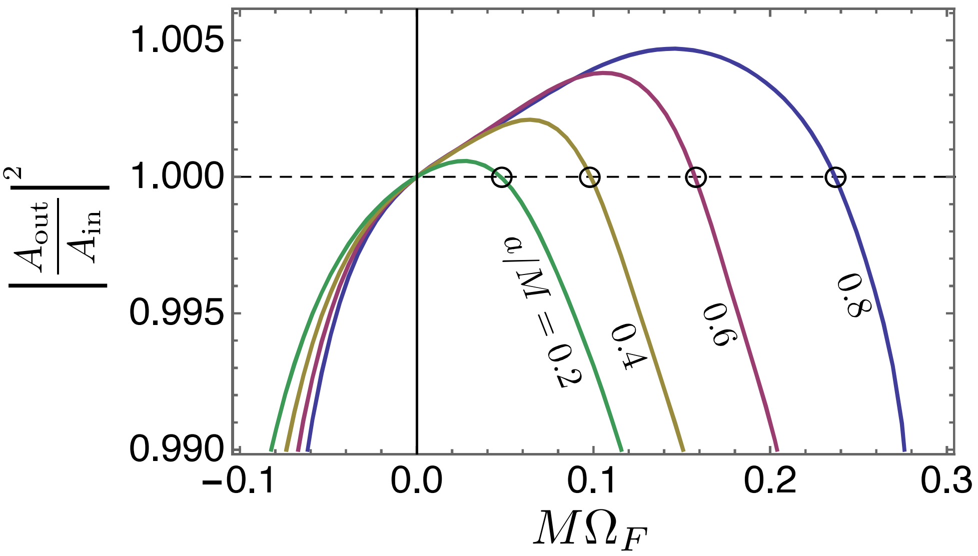


Superradiant condition

$$\Omega|_{r_{\rm out}(\Omega_F)} < \Omega_F < \Omega|_{r_{\rm in}(\Omega_F)} \longrightarrow 0 < \Omega_F < \Omega_{\rm H}$$

Condition for the BZ process!!!!

Reflection rate (Numerical calculation)



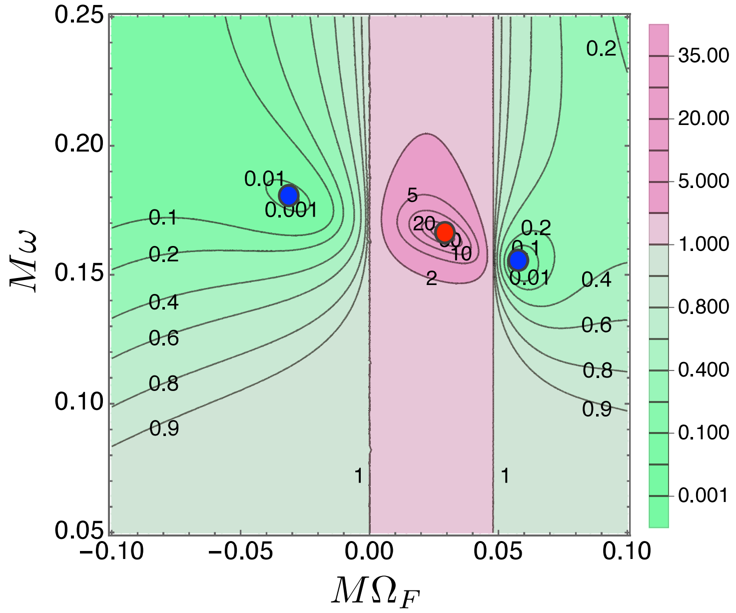
$\circ\ \Omega_F = \Omega_{\rm H}$

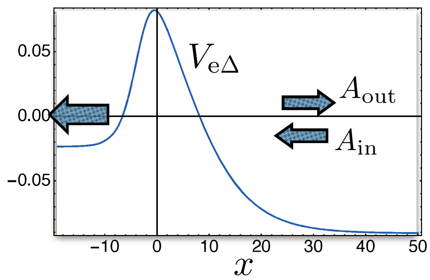
$$\boxed{0 < \Omega_F < \Omega_{\rm H}}$$

Reflection rates exceed unity when the superradiant condition (condition for the BZ process) is satisfied.

Resonance of Alfvén waves

Contours of $|A_{\rm out}/A_{\rm in}|^2$





● $A_{\rm in} \sim 0$ resonant reflection

● $A_{\rm out} \sim 0$ resonant absorption

Searching $A_{\rm in}$ in Re[ω]-Im[ω] plane,

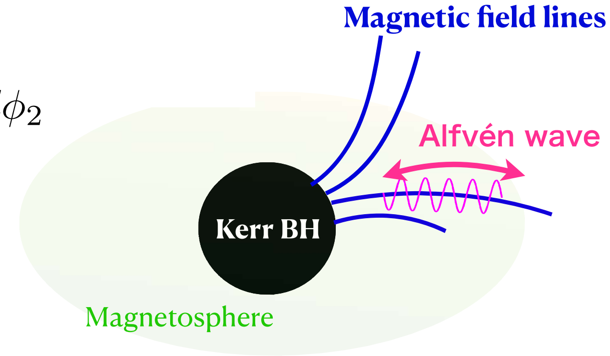
$A_{\rm in} = 0$ at $M\omega = 0.167 - 0.002\,i$.

"QNM of magnetosphere"

Resonances of Alfvén waves makes some observable phenomena?

(burst-like emission of EM wave?)

Summary



Force-free magnetosphere & Alfvén wave

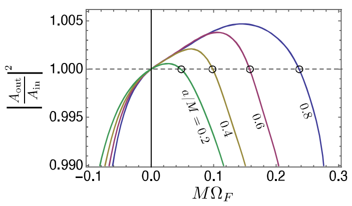
$$\nabla_b \left(T^{ab}_{(\mathrm{EM})} + \cancel{T^{ab}_{\mathrm{plasma}}} \right) = 0 \ , \ F = d\phi_1 \wedge d\phi_2$$

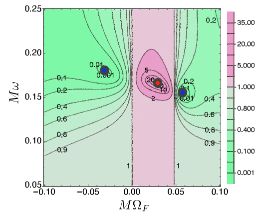
$$\longrightarrow \ \partial_\nu \phi_2 \partial_\mu \left(\sqrt{-g}\, \partial^{[\mu} \delta\phi_1 \partial^{\nu]} \phi_2 \right) = 0$$

Alfvén wave eq

$$\longrightarrow \ \frac{d^2 R}{dx^2} - V_{\mathrm{eff}} R = 0$$

Alfvénic superradiance & Resonance





Superradiant condition & BZ process

$$\Omega|_{r_{\mathrm{out}}} < \Omega_F < \Omega|_{r_{\mathrm{in}}} \ \Leftrightarrow \ 0 < \Omega_F < \Omega_{\mathrm{H}}$$

What's next ?

Plasma effect (Magnetohydrodynamics) $\nabla_b \left(T^{ab}_{(\mathrm{EM})} + T^{ab}_{\mathrm{plasma}} \right) = 0$

Extracted energy by Alfvén waves $\longrightarrow$ Kinetic energy of plasma $\Rightarrow$ Jet

Poster session

Shingo Akama

Rikkyo University

"Non-Bunch-Davies effect on primordial tensor non-Gaussianities in beyond GLPV theories"

[JGRG30 (2021) PA1]

P1

Non-Bunch-Davies effect on primordial tensor non-Gaussianities in beyond GLPV theories

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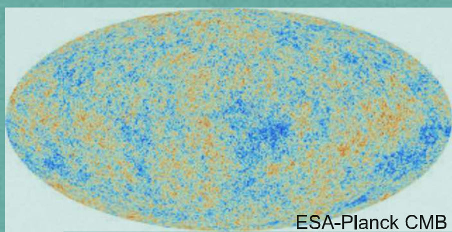
Introduction

Inflation

・A lot of models consistent with current observations

・Convenient to use general framework of inflation



Primordial Tensor non-Gaussianities

- Time evolution of perturbations
- Inflaton- and graviton-interactions
- Initial states of perturbation modes **: Our work**

⋮

Non-Gaussianities from inflation

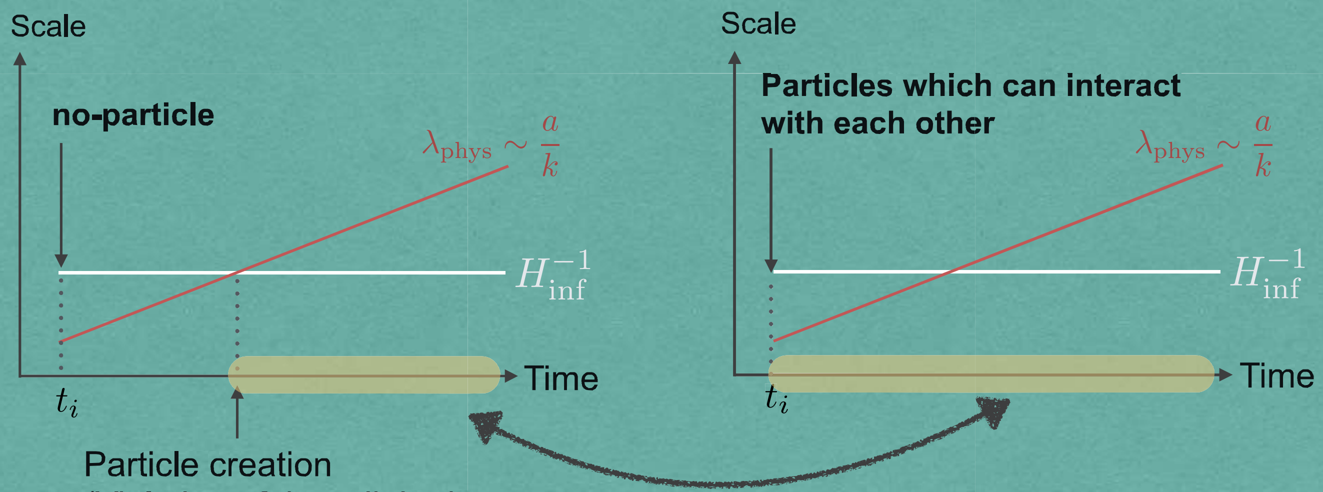
・Have been mainly studied in the context of the BD-state

・Sensitive to choice of initial state

Introduction

Generation mechanism of the Non-Gaussianity
(from the viewpoint of particle-interaction)

1. Bunch-Davies (BD) state | **2. Non-BD state**



Particle creation
(Violation of the adiabatic
(WKB) approximation)

Additional non-Gaussianity generated
by the interactions due to the excited modes

Introduction

Non-Gaussianities with non-BD states

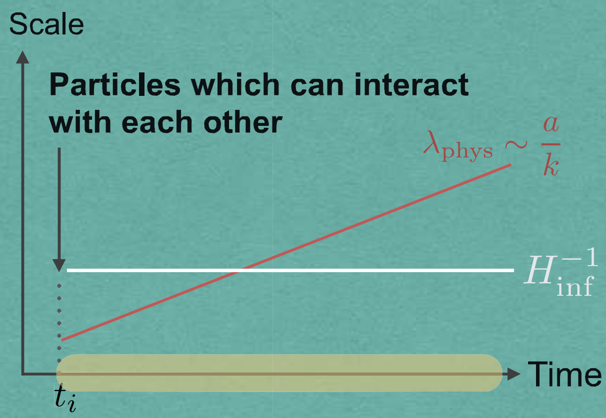
Enhanced non-Gaussianities at flattened and squeezed configurations

X. Chen et al, (2007), R. Holman and A. J. Tolley (2008), S. Kundu (2014), S. Akama et al, (2020), …

Previous research on tensor non-Gaussianities (Non-BD) : **within GLPV**

- Higher-derivative cubic interactions have more impacts on the non-Gaussianities
- Generalizing the theory yields the higher-derivative interactions

$\longrightarrow$ **beyond GLPV**



Setup : Lagrangian

A general Lagrangian written in terms of
the extrinsic curvature tensor K_{ij} and the intrinsic curvature tensor $R^{(3)}_{ij}$
of uniform inflation hypersurfaces as constant time hypersurfaces

$$\mathcal{L} = \mathcal{L}_{\rm GR} + \mathcal{L}_{\rm ex1} + \mathcal{L}_{\rm ex2} + \mathcal{L}_{\rm ex3} + \mathcal{L}_{\rm ex4}$$ Gao (2014)

where

$$\mathcal{L}_{\rm GR} = a_1 R^{(3)} + a_2 K_{ij}^2 + a_3 K^2,$$
$$\mathcal{L}_{\rm ex1} = b_1 K R^{(3)} + b_2 K^{ij} R^{(3)}_{ij} + b_3 K^3_{ij} + b_4 K K^2_{ij} + b_5 K^3,$$
$$\mathcal{L}_{\rm ex2} = c_1 (R^{(3)}_{ij})^2 + c_2 K^2 R^{(3)} + c_3 K^2_{ij} R^{(3)} + c_4 K K^{ij} R^{(3)}_{ij} + c_5 K^k_i K^j_k R^{(3)}_{ij},$$
$$\mathcal{L}_{\rm ex3} = d_1 K (R^{(3)}_{ij})^2 + d_2 K^i_j R^{(3)}_{ki} R^{(3)}_{kj},$$
$$\mathcal{L}_{\rm ex4} = e_1 (R^{(3)}_{ij})^3.$$

Setup : Lagrangian

A general Lagrangian written in terms of
the extrinsic curvature tensor K_{ij} and the intrinsic curvature tensor $R^{(3)}_{ij}$
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$$\mathcal{L} = \mathcal{L}_{\rm GR} + \mathcal{L}_{\rm ex1} + \mathcal{L}_{\rm ex2} + \mathcal{L}_{\rm ex3} + \mathcal{L}_{\rm ex4}$$ Gao (2014)

where

$$\mathcal{L}_{\rm GR} = a_1 R^{(3)} + a_2 K_{ij}^2 + a_3 K^2,$$
$$\mathcal{L}_{\rm ex1} = b_1 K R^{(3)} + b_2 K^{ij} R^{(3)}_{ij} + b_3 K^3_{ij} + b_4 K K^2_{ij} + b_5 K^3,$$

GLPV theory

Gleyzes et al.(2014)

$$a_2 = -a_3, \; -2b_1 = b_2, \; b_3 = -2b_4/3 = 2b_5$$

Setup

- **Tensor perturbations** $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h^k_j + \cdots\right)\mathrm{d}x^i\mathrm{d}x^j$

Beyond GLPV terms modify the dispersion relation in general:

$$S_h^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x a^3\left[\mathcal{G}_T\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial_k h_{ij})^2 + \frac{(\cdots)}{a^4}(\partial^2 h_{ij})^2\right]$$

In the perturbed action

$$K^2R^{(3)},\ KK^{ij}R^{(3)}_{ij},\ K^2_{ij}R^{(3)},\ K^k_iK^j_kR^{(3)}_{ij} \sim (\partial_i h_{jk})^2 + \mathcal{O}(h^3),$$
$$c_1(R^{(3)}_{ij})^2 \sim c_1(\partial^2 h_{ij})^2 + \mathcal{O}(h^3),$$
$$d_1K(R^{(3)}_{ij})^2,\ d_2K^i_jR^{(3)}_{ki}R^{(3)}_{kj} \sim (3d_1 + d_2)H(\partial^2 h_{ij})^2 + \mathcal{O}(h^3),$$
$$(R^{(3)}_{ij})^3 = \mathcal{O}(h^3).$$

Setup

- **Tensor perturbations** $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h^k_j + \cdots\right)\mathrm{d}x^i\mathrm{d}x^j$

The modification of the dispersion relation prevents us
from calculating the primordial non-Gaussianities analytically.

(The solution of the mode function is written by the Whittaker function.)

A. Ashoorioon et al, (2011), Y. Akita and T. Kobayashi (2016)

To perform the calculation analytically, we impose

$$c_1 = 0 = 3d_1 + d_2,\ (d_2 \neq 0).$$

In the cubic action,

$$a_4K(R^{(3)}_{ij})^2,\ a_7K^i_jR^{(3)}_{ki}R^{(3)}_{kj} \sim (3d_1 + d_2)\partial^2 h(\partial h)^2,\ d_2\dot{h}(\partial^2 h)^2.$$

Setup

Mode function

$$\tilde{h}_{ij} = \sum_s \left[\psi_k^{(s)} e_{ij}^{(s)}(\mathbf{k}) b_{\mathbf{k}}^{(s)} + \psi_k^{(s)*} e_{ij}^{(s)*}(-\mathbf{k}) b_{-\mathbf{k}}^{(s)\dagger}\right]$$

where $\psi_k^{(s)} = \underline{\alpha_k^{(s)} u_k} + \underline{\beta_k^{(s)} u_k^*}$ with $u_k = \frac{\sqrt{\pi}}{a}\frac{c_h}{M_T}\sqrt{-\eta}H_{3/2}^{(1)}(-c_h k\eta)$

Positive- and Negative-frequency modes

Bogoliubov coefficients

- BD-state $\beta_k^{(s)} = 0,\ \alpha_k^{(s)} = 1$

  Small corrections

- Non-BD-state $\beta_k^{(s)} = \delta_1 + i\delta_2,\ \alpha_k^{(s)} = 1 + i\delta_3$ with $\delta_1 \sim \delta_2 \sim \delta_3 \ll 1$

Setup

Backreaction from the excited modes SA et al, (2020)

$$\rho_{\text{excited}} \sim \mathcal{O}\left(\frac{c_h}{a_i^4}\int^{M_* a(\eta_0)} \mathrm{d}^3 k k|\beta_k^{(s)}|^2\right) \lesssim \rho_{\text{inf}} \sim \mathcal{O}(M_{\text{pl}}^2 H_{\text{inf}}^2)$$

$$\beta_k^{(s)} \sim \beta \exp\left[-\frac{k^2}{M_*^2 a^2(\eta_0)}\right]$$

$|\beta|^2 \lesssim \mathcal{P}_h \frac{M_{\text{pl}}^2}{M_*^2}\frac{\mathcal{G}_T}{M_*^2} = r\mathcal{P}_\zeta \frac{M_{\text{pl}}^2}{M_*^2}\frac{\mathcal{G}_T}{M_*^2}$ where $\mathcal{P}_h \sim \frac{H_{\text{inf}}^2}{c_h \mathcal{G}_T}$ and $r := \frac{\mathcal{P}_h}{\mathcal{P}_\zeta}$

CMB observations $r \lesssim \mathcal{O}(10^{-2}),\ \mathcal{P}_\zeta = \mathcal{O}(10^{-9})$ Planck 2018

Cutoff scale $M_* \sim M_{\text{pl}} \sim \mathcal{G}_T^{1/2}$

$|\beta| \lesssim 10^{-6}$

Calculation of non-Gaussianities

Primordial bispectrum

$$\langle 0_b|\xi^{(s_1)}(0,\mathbf{k}_1)\xi^{(s_2)}(0,\mathbf{k}_2)\xi^{(s_3)}(0,\mathbf{k}_3)|0_b\rangle \qquad \xi^{(s)} := \tilde{h}_{ij}(t,\mathbf{k})e^{(s)*}_{ij}(\mathbf{k})$$

$$= -i\int_{\eta_0}^{0} \mathrm{d}\eta a(\eta)\langle[\xi^{(s_1)}(0,\mathbf{k}_1)\xi^{(s_2)}(0,\mathbf{k}_2)\xi^{(s_3)}(0,\mathbf{k}_3), H_{\rm int}(\eta')]\rangle$$

$$=: (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\left[\mathcal{B}^{s_1s_2s_3}_{(\mathcal{F}_T)} + \mathcal{B}^{s_1s_2s_3}_{(\mathrm{b}_3)} + +\mathcal{B}^{s_1s_2s_3}_{(\mathrm{c}_5)} + \mathcal{B}^{s_1s_2s_3}_{(\mathrm{d}_2)} + \mathcal{B}^{s_1s_2s_3}_{(\mathrm{e}_1)}\right]$$

where

$$H_{\rm int} := -\int \mathrm{d}^3x a^3\left\{\frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl}-\frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl} + \frac{b_3}{8}\dot{h}^3_{ij} - \frac{c_5}{8a^2}\dot{h}^i_k\dot{h}^k_j\partial^2 h^j_i + \frac{d_2}{8a^4}\dot{h}^i_k\partial^2 h^k_j\partial^2 h^j_i - \frac{e_1}{8a^6}(\partial^2 h_{ij})^3\right\}$$

η_0

- Perturbations are deep inside the horizon, $|c_h k_i \eta_0| \gg 1$
- η_0 is associated with cutoff scale $M_* = \dfrac{k}{a(\eta_0)} \simeq (-k\eta_0)H_{\rm inf}$

Calculation of non-Gaussianities

Auto-bispectrum includes two types of integrals:

- $I_1 := \displaystyle\int_{\eta_0}^{0} \mathrm{d}\eta(-\eta)^n e^{-ic_h(k_1+k_2+k_3)\eta}$

- $I_2 := \displaystyle\int_{\eta_0}^{0} (-\eta)^n e^{-ic_h(k_j-k_{j+1}-k_{j+2})\eta}$ with j being defined modulo 3

Mixing between the positive and negative frequency modes

The value of n depends on the interaction terms:

$$H_{\rm int} := -\int \mathrm{d}^3x a^3\left\{\underbrace{\frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl}-\frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl}}_{n=1} + \underbrace{\frac{b_3}{8}\dot{h}^3_{ij}}_{n=2} - \underbrace{\frac{c_5}{8a^2}\dot{h}^i_k\dot{h}^k_j\partial^2 h^j_i}_{n=2} + \underbrace{\frac{d_2}{8a^4}\dot{h}^i_k\partial^2 h^k_j\partial^2 h^j_i}_{n=4} - \underbrace{\frac{e_1}{8a^6}(\partial^2 h_{ij})^3}_{n=5}\right\}$$

Calculation of non-Gaussianities

Auto-bispectrum includes two types of integrals:

■ $I_1 := \int_{\eta_0}^{0} \mathrm{d}\eta (-\eta)^n e^{-ic_h(k_1+k_2+k_3)\eta}$

$\propto (k_1+k_2+k_3)^{-(n+1)}$ for all configurations

■ $I_2 := \int_{\eta_0}^{0} (-\eta)^n e^{-ic_h(k_j-k_{j+1}-k_{j+2})\eta}$

$$\propto \begin{cases} (-\eta_0)^{(n+1)} & \text{for } |(k_j-k_{j+1}-k_{j+2})\eta_0| \ll 1 \\ & \text{(nearly-)flattened configuration} \\ \underline{(k_j-k_{j+1}-k_{j+2})}^{-(n+1)} & \text{for other configurations} \end{cases}$$

enhanced at the squeezed limit $k_j = k_{j+1} \gg k_{j+2}$

Results

Squeezed configuration

$(k_3 := k_L \ll k_1 = k_2 =: k_S)$

$$\mathcal{B}^{s_1s_2s_3}_{(\mathcal{F}_T)} = \mathcal{B}^{s_1s_2s_3}_{(\mathcal{F}_T),\mathrm{BD}}\left[1+\mathcal{O}\left((\beta^{(s_1)}_{k_S}+\beta^{(s_2)}_{k_S})(k_S/k_L)\right)\right],$$
$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{b}_3)} = \mathcal{B}^{s_1s_2s_3}_{(\mathrm{b}_3),\mathrm{BD}}\left[1+\mathcal{O}\left((\beta^{(s_1)}_{k_S}+\beta^{(s_2)}_{k_S})(k_S/k_L)^3\right)\right],$$
$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{c}_5)} = \mathcal{B}^{s_1s_2s_3}_{(\mathrm{c}_5),\mathrm{BD}}\left[1+\mathcal{O}\left((\beta^{(s_1)}_{k_S}+\beta^{(s_2)}_{k_S})(k_S/k_L)^3\right)\right],$$
$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{d}_2)} = \mathcal{B}^{s_1s_2s_3}_{(\mathrm{d}_2),\mathrm{BD}}\left[1+\mathcal{O}\left((\beta^{(s_1)}_{k_S}+\beta^{(s_2)}_{k_S})(k_S/k_L)^5\right)\right],$$
$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{e}_1)} = \mathcal{B}^{s_1s_2s_3}_{(\mathrm{e}_1),\mathrm{BD}}\left[1+\mathcal{O}\left((\beta^{(s_1)}_{k_S}+\beta^{(s_2)}_{k_S})(k_S/k_L)^5\right)\right].$$

$\mathcal{O}(\beta^{(s)}_k(k_S/k_L)^n) \lesssim 10^{-6}(k_S/k_L)^n$ (Backreaction)

If we take $k_S/k_L \sim 10^2$,

the non-Bunch-Davies contribution can be larger than $\mathcal{O}(1)$ for $n \geq 3$.

Nearly-flattened configuration

$(|(k_1-k_2-k_3)\eta_0| \ll 1)$

$$\mathcal{B}^{s_1s_2s_3}_{(\mathcal{F}_T)} = \mathcal{B}^{s_1s_2s_3}_{(\mathcal{F}_T),\mathrm{BD}}\left[1+\mathcal{O}(\beta^{(s_1)}_{k_1}|k_i\eta_0|^2)\right],$$
$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{b}_3)} = \mathcal{B}^{s_1s_2s_3}_{(\mathrm{b}_3),\mathrm{BD}}\left[1+\mathcal{O}(\beta^{(s_1)}_{k_1}|k_i\eta_0|^3)\right],$$
$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{c}_5)} = \mathcal{B}^{s_1s_2s_3}_{(\mathrm{c}_5),\mathrm{BD}}\left[1+\mathcal{O}(\beta^{(s_1)}_{k_1}|k_i\eta_0|^3)\right],$$
$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{d}_2)} = \mathcal{B}^{s_1s_2s_3}_{(\mathrm{d}_2),\mathrm{BD}}\left[1+\mathcal{O}(\beta^{(s_1)}_{k_1}|k_i\eta_0|^5)\right],$$
$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{e}_1)} = \mathcal{B}^{s_1s_2s_3}_{(\mathrm{e}_1),\mathrm{BD}}\left[1+\mathcal{O}(\beta^{(s_1)}_{k_1}|k_i\eta_0|^6)\right].$$

$\mathcal{O}(\beta^{(s)}_k|c_hk_i\eta_0|^n) \lesssim 10^{-6}|c_hk_i\eta_0|^n \lesssim 10^{-6}c_h^n\left(\frac{M_*}{H_{\mathrm{inf}}}\right)^n$

If we assume $c_h = \mathcal{O}(1)$,

the non-Bunch-Davies contribution can be larger than $\mathcal{O}(1)$ for $n \geq 2$.

Results

Nearly-flattened configuration

- Non-Bunch-Davies effects $\mathcal{O}(\beta_{k_1}^{(s_1)}|c_h k_i \eta_0|^{n})$

- Resultant bispectra $\mathcal{B}^{(\bullet)} = (\cdots)(k_1 - k_2 - k_3)$

$$\mathcal{B}^{(\bullet)} \propto |(k_1 - k_2 - k_3)\eta_0|\mathcal{O}(\beta_{k_1}^{(s_1)})|c_h k_i \eta_0|^{n-1}$$

- Reduction of the non-Bunch-Davies effects due to an angular average

CMB bispectrum (temperature fluctuations)

$$B := \sum_{m_1 m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle \quad \text{where} \quad \frac{\Delta T}{T}(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

$$\mathcal{B}_\zeta \propto \mathcal{O}(\beta_k)|k_i \eta_0|^{\alpha} \quad \underset{\text{(angular average)}}{\Rightarrow} \mathcal{B} \propto |\eta_0|^{\alpha - 1}$$

R. Holman and A. J. Tolley (2008)

Summary

- **We investigated the tensor non-Gaussianities from inflation with non-BD initial states in beyond GLPV theories**

$$S_h^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x a^3\left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial_k h_{ij})^2 + \frac{(\cdots)}{a^4}(\partial^2 h_{ij})^2\right], \quad (\cdots) = 0$$

- **Squeezed and flattened non-Gaussianities can potentially be enhanced.**

Future works

- CMB bispectrum (B-mode auto-bispectrum,..)
- Strong coupling

Poster session

ZhenYuan Wu

Yamaguchi University

“Antipodal correlation of the inflationary primordial gravitational waves”

[JGRG30 (2021) PA2]

P2

Antipodal correlations of the inflationary primordial gravitational waves

Zhen-Yuan Wu

Yamaguchi University

Collaborators:

Ryo Saito and Nobuyuki Sakai

Introduction

- Motivation: Distinguish Primordial Gravitational-Waves (PGWs) with the other types of Stochastic Gravitational-Wave Background (SGWB).
- How? By the special property--the antipodal correlation--of PGWs.

PGWs are generated on super-horizon scales → They form **standing waves** after horizon re-entry → PGWs possess a special angular correlation--the **antipodal correlations**

- The problems:.
  - Due to the finite frequency resolution, the angular correlation is undetectable *(Allen et al., 00).*
  - Phase decoherence problem due to the gravitational time delay *(Margalit et al., 20; Bartolo et al., 19).*

- The problem seems to be the ***phase***, so might the ***intensity map*** method *(Bartolo et al., 19; Contaldi, 17)* be work?

- We show that even in the homogeneous universe, the intensity map method will *not* work in revealing the antipodal correlation of PGWs.

Antipodal correlations of PGWs

① Characterizing SGWB:

$$h_{ij}(t,\vec{x}) = \sum_A \int_{-\infty}^{\infty} df \int d^2\hat{n}\, h_A(f,\hat{n})\, e_{ij}^A(\hat{n})\, e^{-i2\pi f(t-\vec{x}\cdot\hat{n})}$$

- $h_{ij}(t,\vec{x})$ and $h_A(f,\vec{n})$ are random variables
- $A = \pm$ is chosen to be the circular polarization basis

  i.e. $e_{ij}^{+} = e_{ij}^{+} + ie_{ij}^{\times}$

  $e_{ij}^{-} = e_{ij}^{+} - ie_{ij}^{\times}$
- $\vec{n}$ represents the propagation direction of each mode

Antipodal correlations of PGWs

② Assumptions on SGWB:

Usual assumption on SGWB:

$$\langle h_A^{\dagger}(f_1,\hat{n}_1)h_B(f_2,\hat{n}_2)\rangle = \frac{1}{4\pi}\,\delta_{AB}\,\delta(f_1-f_2)\,\boldsymbol{S_h^{(D)}(f_1)\delta^2(\hat{n}_1,\hat{n}_2)}$$

No angular correlation

PGWs:

$$\langle h_A^{\dagger}(f_1,\hat{n}_1)h_B(f_2,\hat{n}_2)\rangle = \frac{1}{4\pi}S_h^{(D)}(f_1)\,\delta_{AB}\,\delta(f_1-f_2)\,\delta^2(\hat{n}_1,\hat{n}_2) + \frac{1}{4\pi}\,\delta_{A(-B)}\,\delta(f_1+f_2)\,\boldsymbol{A_h^{(D)}(f_1)\delta^2(\hat{n}_1,-\hat{n}_2)}$$

Antipodal correlation

The upper index D means the quantity is defined as double-sided.

Antipodal correlations of PGWs

③ Where does the antipodal term $A_h^{(D)}(f)$ come from?

1. Standing wave nature of PGWs:

$$h''_{B,\vec{k}} + 2\frac{a'}{a}h'_{B,\vec{k}} + k^2 h_{B,\vec{k}} = 0$$

- $B = +, \times$
- $a = a(\eta)$: the scale factor
- $' = {}^{d}/_{d\eta}$: derivative with respect to the conformal time

Point:

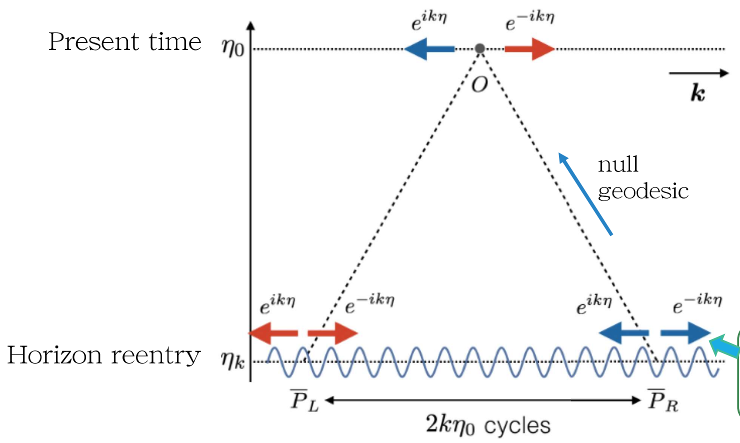
➢ $h_{B,\vec{k}}$ is non-zero on super-horizon scales, $\eta \to 0, h_{B,\vec{k}} \to$constant

$\Rightarrow h_{B,\vec{k}} = \frac{1}{a}(\alpha_B e^{ik\eta} + \alpha_B^* e^{-ik\eta}) \Rightarrow$ $\boxed{\text{Standing wave!}}$

Antipodal correlations of PGWs

③ Where does the antipodal term $A_h^{(D)}(f)$ come from?

2. Propagation to the observer



$\Rightarrow$ $\boxed{h_\pm(f,\hat{n}) = -e^{i4\pi f\eta_0} h_\mp(-f,-\hat{n})}$

$\Downarrow$

One-to-one correspondence between GWs from opposite directions

standing wave
(two traveling waves with the same amplitude and opposite directions)

(Un)Detectability of the antipodal correlations

① The $A_h^{(D)}(f)$ term

- The $A_h^{(D)}(f)$ term has the same amplitude with the $S_h^{(D)}(f)$ term :

$$\left\langle h_A^\dagger(f,\hat{n})\, h_{-B}(-f,-\hat{n})\right\rangle = -e^{i4\pi f\eta_0}\,\langle h_A^\dagger(f_1,\hat{n}_1)\, h_B(f_2,\hat{n}_2)\rangle$$

$$\Rightarrow A_h^{(D)}(f) = \underline{-e^{i4\pi f\eta_0}} S_h^{(D)}(f)$$

Highly oscillate!

"period" $(\Delta f) \sim {}^{1}/_{\eta_0} \sim {}^{1}/_{T_{age}} \sim 10^{-18}\mathrm{Hz}$

(Un)Detectability of the antipodal correlations

② Un-observability of $\boldsymbol{h_A(f,\hat{n})}$

Due to the finite observational time, T, the *actual observable quantity* is $h_{A,T}(f,\hat{n})$:

$$h_{A,T}(f,\hat{n}) \cong \int df'\, h_A(f',\hat{n})\, W_T(f-f'),$$

where $W_T(x)$ is a window function of the width $\sim {}^{1}/_{T}$,

e.g. $W_T(x) = \frac{\sin(\pi T x)}{\pi x}\, e^{i\pi T x}$.

$h_{A,T}(f,\hat{n})$ is the superposition of modes within $|f'-f| < 1/T$.

(Un)Detectability of the antipodal correlations

② The $A_h^{(D)}(f)$ term is un-observational

The antipodal correlations in the observational quantity $h_{A,T}(f,\hat{n})$ is

$$A_{h,T}^{(D)}(f) \equiv \langle h_{A,T}^{\dagger}(f,\hat{n})\, h_{B,T}(-f,-\hat{n})\rangle = \int_{-\infty}^{\infty} df'\, |W_T(f-f')|^2 A_h^{(D)}(f')$$

$$\cong \int_{|f-f'|\leq 1/T} df' |W_T(f-f')|^2 S_h^{(D)}(f') \cos(4\pi f' \eta_0)$$

$\boxed{{}^{1}/_{\eta_0} \ll {}^{1}/_{T}}$ ⇨ the integrand is rapidly oscillating over the integral range

$$\approx 0$$

The antipodal correlation disappears in observation!

(Allen et al., 00; Bartolo et al., 19)

Antipodal correlations in the intensity map

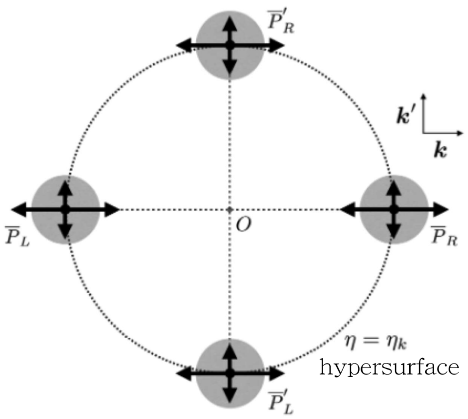
- The definition of the intensity $I_{A,T}(f,\hat{n})$:

$$I_{A,T}(f,\hat{n}) \equiv |h_{A,T}(f,\hat{n})|^2$$

- A note on the definition:

Realization: $I_{A,T}(f,\hat{n}) \equiv |h_{A,T}(f,\hat{n})|^2$ vs. Expectation value: $I_{A,T}(f) \equiv \langle |h_{A,T}(f,\hat{n})|^2 \rangle$

➢ Frequently used *(Bartolo et al., 19)* but *not good* to consider angular correlations!



➢ Arrows represent the realization of intensity of PGWs. They have directional dependence.
➢ Radius of the shaded circles represents the expectation value. ***The directional dependence is lost due to the statistical isotropy.***

Antipodal correlations in the intensity map

- The antipodal correlation in the intensity map can be calculated from:

$$\underbrace{\langle I_T(f,\hat{n}) I_T(-f,-\hat{n})\rangle}_{\langle h_T^\dagger(f,\hat{n})\, h_T(f,\hat{n})\, h_T^\dagger(-f,-\hat{n})\, h_T(-f,-\hat{n})\rangle} - \langle I_T(f,\hat{n})\rangle \langle I_T(-f,-\hat{n})\rangle = \langle h_T^\dagger(-f,\hat{n}) h_T(-f,-\hat{n})\rangle \langle h_T^\dagger(-f,-\hat{n}) h_T(-f,\hat{n})\rangle$$

- Under the assumption of Gaussian distribution, it can be written as the sum of each pairs of 2-point function.

Both **equal 0**, because there is no antipodal correlation for the same frequencies.

$$+ \langle h_T^\dagger(f,\hat{n}) h_T(-f,-\hat{n})\rangle \langle h_T^\dagger(-f,-\hat{n}) h_T(f,\hat{n})\rangle$$

Both **equal** $A_{h,T}^{(D)} \approx \mathbf{0}$ due to the finite frequency resolution.

The antipodal correlation also disappears in the intensity map!

Conclusion

- PGWs form standing waves after its re-entry of the horizon, this property gives PGWs a special angular correlation--the antipodal correlation.
- However, as shown by Allen et al. (2000), the antipodal correlation term we see today in the power spectrum is highly oscillating, and since the observational time is finite, the antipodal correlation is undetectable in the correlation of GW strains.
- In order to get rid of the highly oscillating phase factor in the antipodal correlation, we tried another method: the intensity correlations. Unfortunately, it turns out that the intensity correlation also will not reveal the antipodal correlations in PGWs.

Acknowledgment

- This work was supported by JST SPRING, Gant Number JPMJSP2111.

Poster session

Akira Dohi

Kyushu University

"Neutron Star Cooling in Scalar-Tensor Theories"

[JGRG30 (2021) PA3]

Neutron Star Cooling in Scalar-Tensor Theories

Akira Dohi[1] (Based on PTEP, **2021**, 093E01)
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Introduction

Motivation to modify general relativity (GR)

- Early inflationary universe
  ⇒ Dark Energy or Possibility of Modified Gravity in Large Scale
  - Cassini experiment (Bertotti+03)
    ⇒ Screening mechanism of fifth force mediating scalar field
  - GW observation (Abbott+16,17)
    ⇒ Constant light speed $c_{\rm GW} = c_{\rm EM}$

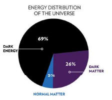

Fig. 1: Proportion of Matter in the Universe (taken from NASA)

- As the *healthy* scalar-tensor (ST) theories to satisfy the above observations, we adopt $f(R)$-gravity theory as

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}f(R) + S_{\rm m}[\tilde{g}_{\mu\nu}, \Psi_{\rm m}].$$

- For the scalar potential to cause the screening (*Chameleon*) mechanism, we adopt $V(\phi) = \frac{1}{4}\tilde{\lambda}\phi^4$ with a parameter $\tilde{\lambda}$.

Cooling Scenario of Isolated Neutron Stars (NSs)

- Most isolated NSs: Slow cooling processes + nucleon superfluidity (*minimal cooling scenario*, Page+04)
- Some cold isolated NSs: Rapid ν cooling process, above all the Direct Urca (DU) process ($Y_{p,{\rm center}} \gtrsim 1/9$)
  - However, Prohibiting the DU process is highly possible (AD+19)
    ⇒ Exotic States or Modification of TOV Equations due to fifth force

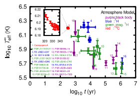

Fig. 2: Age and redshifted surface temperature observations of isolated NSs (Lim+17). At least, 17 and 19 require an enhanced cooling process (e.g., Page+04)

Purpose of this study

- In ST theories which could explain the early inflationary universe, we investigate how the fifth force affects the cooling curves.

Numerical Setup and Method

- 1D spherically metric: $ds^2 = -f(r)dt^2 + h(r)^{-1}dr^2 + r^2d\Omega^2$
- BSk21 EOS (Goriety+10): *Extended* Skyrme func. with AME2010

1. Modified TOV Equations in ST theories

$$\frac{f'}{f} = -\frac{2M_{\rm pl}^2(h-1)s - 2F^{-1}r^2(P-V) + hr\phi'[(6Q^2-1)r\phi' - 8QM_{\rm pl}]}{2hrM_{\rm pl}(M_{\rm pl} - Qr\phi')},$$

$$\mathcal{M}' = 4\pi F^{-1}r^2\left[(1-2Q^2)\rho + 6Q^2P + (1-8Q^2)V - 2QM_{\rm pl}V_{,\phi}\right] + \phi'\frac{2QM_{\rm pl}\mathcal{M} + 8QM_{\rm pl}\pi r^3F^{-1}(P-V) + r\phi'(4\pi rM_{\rm pl}^2 - \mathcal{M})(1+2Q^2)}{2M_{\rm pl}(M_{\rm pl} - Qr\phi')},$$

$$\phi'' = -\frac{\phi'}{2M_{\rm pl}^2rh}\left[2(h+1)M_{\rm pl}^2 + r^2F^{-1}\{P - \rho + 2QM_{\rm pl}V_{,\phi} - 2V + 2(\rho - 3P + 4V)Q^2\}\right] + \frac{1}{M_{\rm pl}hF}\left[4QV + V_{,\phi}M_{\rm pl} + Q(\rho - 3P)\right].$$

Here, $h(r) = 1 - \frac{2G\mathcal{M}(r)}{r}$, $F(\phi) = e^{-2Q\phi/M_{\rm pl}}$. $Q = -1/\sqrt{6}$ for $f(R)$ gravity theory

>>> Boundary Conditions <<<

$$f'(r=0) = h'(r=0) = \phi'(r=0) = \rho'(r=0) = 0\,,$$
$$f(r\to\infty) = h(r\to\infty) = 1\,, \quad \phi'(r\to\infty) = V(\phi(r\to\infty)) = 0.$$

⇒ Runge-Kutta+Shooting Method (Kase & Tsujikawa 19)

2. Cooling Transport Equations (without heating source)

$$C_V\frac{dT}{dt} = -\frac{1}{f^{1/2}}\left\{\frac{h^{1/2}}{4\pi r\rho}\frac{d(L_\gamma f)}{dr} + f\epsilon_\nu\right\}$$

Here, L_γ is the photon luminosity, ϵ_ν is neutrino emissivity, and C_V is the specific heat.

>>> Boundary Conditions <<<

$$T(t=0) = 10^{10}\ {\rm K}\,, \quad L_\gamma(r=0) = 0\,, \quad L_\gamma(r={\rm R}_s) = 7\times10^{35}\ {\rm erg\ s^{-1}}\,(R_s/10\ {\rm km})\,aT_{{\rm eff},6}^4\,.$$

(${\rm R}_s$: NS Radius in Jordan frame, a: Stefan–Boltzmann constant, $T_{{\rm eff},6}$: Surface temperature in units of 10^6 K)

⇒ To obtain the cooling curves, we use the public code `NSCool` (Page 89,16) as the post-process calculation of NS structure.

NS Structure in ST Theories

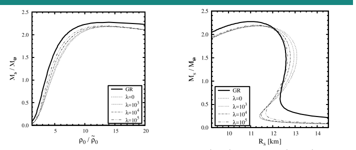

Fig. 3: ADM mass as a function of central density (Left) and radius (Right). $\tilde{\rho}_0 = 1.675\times10^{14}\ {\rm g\cdot cm^{-3}}$ indicates the approximate density of the NS core surface.

- Higher central density, but larger radius compared with GR
- Chameleon screening does work from GR-like behavior with higher $\tilde{\lambda}$, but clear difference between GR and $\tilde{\lambda} = 10^5$.

Cooling Curves in ST Theories

For simplicity, we assume ^{56}Fe envelope and no nucleon superfluidity in this poster. (see our paper for the superfluidity effect on cooling curves in ST theories)

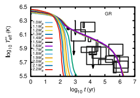

Fig. 4: Cooling curves in GR

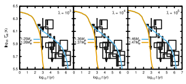

Fig. 6: Similar to Fig. 5, but for the ADM mass around DU threshold ($M_{\rm DU}$)

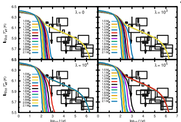

Fig. 5: Cooling curves in $f(R)$ gravity theory, i.e., difference of screening effect

- Smaller $M_{\rm DU}$ from GR
  ⇐ Due to higher central density in a same ADM mass
- If $\tilde{\lambda}$ is higher, $M_{\rm DU}$ is also higher and the cooling curve approaches to that with GR ($1.6 < M_{\rm DU}/M_\odot \le 1.7$) (but $M_{\rm DU}$ is different by $\gtrsim 0.1M_\odot$ from that with $\tilde{\lambda} = 10^5$)

The effect of scalar force on NS structure and cooling

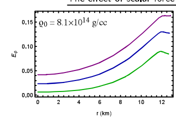

Fig. 7: Characteristic physical amount of fifth force $E_\phi = \left(\frac{dP}{dr}\right)^{-1}\frac{Q}{M_{\rm pl}}(\rho - 3P)\frac{d\phi}{dr}$ as a function of radius with $\tilde{\lambda} = 10^3$ (purple), 10^4 (blue), and 10^5 (green).

- Since fifth force is more screened with higher $\tilde{\lambda}$, NS structure and cooling curve approach to GR.
- The fifth force becomes more effective near the surface, they do not match with GR even with higher $\tilde{\lambda}$.
  ⇒Breaking of Chameleon Mechanism in Cooling Curves

Conclusion

- We investigated NS cooling curves in ST theories.
  1. Compared with GR, $M_{\rm DU}$ is lower at most $0.4M_\odot$.
     ⇒ Possibility to solve the "No Cooling" Problem (AD+19)
  2. If $\tilde{\lambda}$ is higher, cooling curves approach to those with GR.
  3. Due to the fifth force near the NS surface, Chameleon mechanism is broken. This implies that NS temperature observations are useful for probing modified-gravity theories.

Poster session

Kouji Nakamura

NAOJ, GWSP

"Proposal of a gauge-invariant treatment of l=0,1-mode perturbations on the Schwarzschild background spacetime"

[JGRG30 (2021) PA4]

Poster # P4

Proposal of a gauge-invariant treatment of l=0,1-mode perturbations on Schwarzschild Background Spacetime

@JGRG30 (online conference)
[6th Dec. – 10th Dec. (2021).]

Kouji Nakamura (NAOJ)

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K.N. PTP **121** (2009), 1321; K.N. Adv. in Astron. **2010** (2010), 576273.
A.J. Christopherson, et al, CQG **28** (2011), 225024.
K.N. CQG **28** (2011), 122001; K.N. PTEP **2013** (2013), 043E02;
K.N. IJMPD **21** (2012), 1242004;
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K.N. LHEP **2021** (2021), 215; [arXiv:2102.10650 [gr-qc]];
K.N. arXiv:2110.13508 [gr-qc]; Full paper Part I : --- Formulation and odd-mode perturbations ---
K.N. arXiv:2110.13512 [gr-qc]; Full paper Part II : --- Even-mode perturbations ---
K.N. arXiv:2110.13519 [gr-qc]; Full paper Part III: --- Realization of exact solutions ---

I. Introduction

- **The higher order perturbation theory in general relativity has very wide physical motivation.**
  - Cosmological perturbation theory
    - Expansion law of universe : "Back-reaction?" (ΛCDM ?, inhomogeneous cosmology ?, or modified gravity ?)
    - Non-Gaussianity in CMB (beyond Planck)
  - Gravitational-wave physics
    - Black hole perturbations
      - Radiation reaction effects due to the gravitational wave emission.
        - GW from SgA* EMRI. --> LISA target.
    - Binary coalescence through the post-Minkowski expansion
      - LIGO-Virgo detected GW from BH-BH binary coalescence !!!
  - Perturbation of a star (Neutron star)
    - Rotation – pulsation coupling (Kojima 1997)

Gravitational physics is now toward a precise science.

There are many physical situations to which higher order perturbation theory should be applied.

However, general relativistic perturbation theory requires very delicate treatments of “gauges”.

It is worthwhile to formulate the higher-order gauge-invariant perturbation theory from general point of view.

- According to this motivation, from 2003, we have been formulating a general-relativistic higher-order perturbation theory in a gauge-invariant manner.
  - **General formulation :**
    - General framework of higher-order gauge-invariant perturbation theory : K.N. PTP**110** (2003),723; *ibid.* **113** (2005), 413.
    - Construction of gauge-invariant variables for the linear-order metric perturbation and the proposal of the “zero-mode problem” : K.N. CQG**28** (2011),122001; PTEP**2013** (2013),043E02; IJMPD**21** (2012), 1242004.
    - The nth-order extension of the definitions of gauge-invariant variables : K.N. CQG **31** (2014), 135013.
  - **Application to cosmological perturbation theory** :
    - Einstein equations : K.N. PRD**74** (2006), 101301R; PTP**117** (2007), 17.
    - Equations of motion for matter fields : K.N. PRD**80** (2009), 124021.
    - Consistency of the 2nd order Einstein equations : K.N. PTP**121** (2009), 1321.
    - Summary of current status of this formulation : K.N. Adv. in Astron. **2010** (2010), 576273.
    - Comparison with a different formulation : A.J. Christopherson, et al., CQG**28** (2011), 225024.
    - Summary of current status updated 2019 : K.N. Book *“Theory and Applications of Physical Science Vol.3”*, Chapter I, (2020); arXiv:1912.1280v2 [gr-qc].
  - **Application to Black Hole perturbation theory** :
    - **Proposal of a gauge-invariant treatment of l=0,1-mode perturbations on Schwarzschild background spacetime:** K.N. CQG **38** (2021), 145010. [arXiv:2102.00830v3[gr-qc]].
    - Formal solutions of the any-order mass, angular-momentum, dipole perturbations on the Schwarzschild background spacetime: K.N., LHEP **2021** (2021), 215. [arXiv:2102.10650[gr-qc]].
    - Full Paper-Series of these short papers: Part I : --- Formulation and odd-mode perturbation --- K.N., arXiv:2110.13508 [gr-qc];
      Part II : --- Even-mode perturbations --- K.N., arXiv:2110.13512 [gr-qc];
      Part III : --- Realization of exact solutions -- K.N. arXiv:2110.13519 [gr-qc].

Our general formulations for the higher-order gauge-invariant perturbation theory is based on the following conjecture:

Conjecture (Decomposition conjecture):

If the gauge-transformation rule for a tensor field h_{ab} is given by ${}_{\mathscr{Y}}h_{ab} - {}_{\mathscr{X}}h_{ab} = \pounds_{\sigma} g_{ab}$ with the background metric g_{ab}, then, there exist a tensor field $\mathscr{F}_{ab}$ and a vector field Y^a such that h_{ab} is decomposed as $h_{ab} = \mathscr{F}_{ab} + \pounds_{Y} g_{ab}$, where $\mathscr{F}_{ab}$ and Y^a are transformed as ${}_{\mathscr{Y}}\mathscr{F}_{ab} - {}_{\mathscr{X}}\mathscr{F}_{ab} = 0$, ${}_{\mathscr{Y}}Y^a - {}_{\mathscr{X}}Y^a = \sigma^a$ under the gauge-transformation $\Phi_{\varepsilon} = \mathscr{X}_{\varepsilon}^{-1} \circ \mathscr{Y}_{\varepsilon}$, respectively.

In [K.N.(2011); K.N.(2013).], a proof of this conjecture was discussed, but the existence of Green functions for some elliptic differential operators was assumed. For this reason, the kernel modes (**zero modes**) of these elliptic differential operators were ignored (**zero-mode problem**).

In the perturbations on the Schwarzschild background spacetime, l=0,1 modes correspond to these kernel modes!!

⇒ **If we found a gauge-invariant treatment of l=0,1 modes, our general framework of the higher-order gauge-invariant perturbation theory becomes applicable to any order.**

In this poster, ...

I propose a gauge-invariant treatment of l=0,1-mode perturbations on the Schwarzschild background spacetime.

[K.N. arXiv:2102.00830v3 [gr-qc].-->CQG38 (2021), 145010.]

II. "Gauge" in general relativity

(R.K. Sachs (1964).)

- There are two kinds of "gauge" in general relativity.
  - The concepts of these two "gauges" are closely related to the general covariance.
  - "General covariance" :

    There is no preferred coordinate system in nature.

- **The first kind "gauge"** is a coordinate system on a single spacetime manifold.

- **The second kind "gauge"** appears in the perturbation theory.

  This is a point identification between the physical spacetime and the background spacetime.
  - **Our gauge-invariant formulation exclude this second kind "gauge".**

III. Linear perturbations on spherically symmetric background

- The background spacetime has the spherically symmetric.
  - Spacetime topology : $\mathscr{M} = \mathscr{M}_1 \times S^2$.
  - Metric : $g_{ab} = y_{ab} + r^2\gamma_{ab}, \quad y_{ab} = y_{AB}(dx^A)_a(dx^B)_b, \quad \gamma_{ab} = \gamma_{pq}(dx^p)_a(dx^q)_b.$

$$y_{ab} = -f(dt)_a(dt)_b + f^{-1}(dr)_a(dr)_b, \quad f = 1 - \frac{2M}{r}.$$

$$\gamma_{ab} = (d\theta)_a(d\theta)_b + \sin^2\theta(d\phi)_a(d\phi)_b.$$

  - Metric perturbation : $\mathscr{X}^*\bar{g}_{ab} = g_{ab} + \varepsilon_{\mathscr{X}} h_{ab} + O(\varepsilon^2)$.

$$h_{ab} = h_{AB}(dx^A)_a(dx^B)_b + 2h_{Ap}(dx^A)_{(a}(dx^p)_{b)} + h_{pq}(dx^p)_a(dx^q)_b.$$

  - Conventional decomposition of the metric perturbation : $S = Y_{lm}$,

$$h_{AB} = \sum_{l,m} \tilde{h}_{AB}S, \quad h_{Ap} = r\sum_{l,m}\left[\tilde{h}_{(e1)A}\hat{D}_pS + \tilde{h}_{(o1)A}\varepsilon_{pq}\hat{D}^qS\right],$$

$$h_{pq} = r^2\sum_{l,m}\left[\frac{1}{2}\gamma_{pq}\tilde{h}_{(e0)}S + \tilde{h}_{(e2)}\left(\hat{D}_p\hat{D}_q - \frac{1}{2}\gamma_{pq}\Delta\right)S + \tilde{h}_{(o2)}2\varepsilon_{r(p}\hat{D}_{q)}\hat{D}^rS\right],$$

  where $\varepsilon_{pq} = \varepsilon_{[pq]}$, $\hat{D}_p\gamma_{qr} = 0$, and $\hat{D}^p := \gamma^{pq}\hat{D}_q$.
  - **This conventional decomposition requires the existence of the Green functions of the operators $\hat{\Delta} := \hat{D}^r\hat{D}_r$ and $\hat{\Delta} + 2$.**

The conventional decomposition requires the existence of the Green functions of the operators $\hat{\Delta} := \hat{D}^r\hat{D}_r$ and $\hat{\Delta}+2$.

$$h_{AB} = \sum_{l,m}\tilde{h}_{AB}S, \quad h_{Ap} = r\hat{D}_p\sum_{l,m}\tilde{h}_{(e1)A}S + r\varepsilon_{pq}\hat{D}^q\sum_{l,m}\tilde{h}_{(o1)A}S,$$
$$h_{pq} = \frac{1}{2}\gamma_{pq}r^2\sum_{l,m}\tilde{h}_{(e0)}S + r^2\left(\hat{D}_p\hat{D}_q - \frac{1}{2}\gamma_{pq}\Delta\right)\sum_{l,m}\tilde{h}_{(e2)}S + r^2 2\varepsilon_{r(p}\hat{D}_{q)}\hat{D}^r\sum_{l,m}\tilde{h}_{(o2)}S.$$

Inverse relation

$$\begin{aligned}
\sum_{l,m}\tilde{h}_{AB}S &= h_{AB},\\
\sum_{l,m,(l\neq 0)}\tilde{h}_{(e1)A}S &= \frac{1}{r}\hat{\Delta}^{-1}\hat{D}^p h_{Ap}, \quad \sum_{l,m,(l\neq 0)}\tilde{h}_{(o1)A}S = -\frac{1}{r}\hat{\Delta}^{-1}\hat{D}_p\varepsilon^{pq}h_{Aq},\\
\sum_{l,m}\tilde{h}_{(e0)}S &= \frac{1}{r^2}\gamma^{pq}h_{pq}, \quad \mathbb{H}_{pq} := h_{pq} - \frac{1}{2}\gamma_{pq}\gamma^{rs}h_{rs},\\
\sum_{l,m,(l\neq 0,1)}\tilde{h}_{(e2)}S &= \frac{2}{r^2}\hat{\Delta}^{-1}\left[\hat{\Delta}+2\right]^{-1}\hat{D}^q\hat{D}^p\mathbb{H}_{pq},\\
\sum_{l,m,(l\neq 0,1)}\tilde{h}_{(o2)}S &= -\frac{1}{r^2}\hat{\Delta}^{-1}\left[\hat{\Delta}+2\right]^{-1}\varepsilon^{qs}\hat{D}_s\hat{D}^p\mathbb{H}_{pq}.
\end{aligned}$$

- This situation is also seen from the spherical harmonics $S = Y_{lm}$:

$$\hat{D}_pY_{00} = \varepsilon_{pq}\hat{D}^qY_{00} = 0, \quad \left(\hat{D}_p\hat{D}_q - \frac{1}{2}\gamma_{pq}\hat{\Delta}\right)Y_{00} = 2\varepsilon_{r(p}\hat{D}_{q)}\hat{D}^rY_{00} = 0, \quad \left(\hat{D}_p\hat{D}_q - \frac{1}{2}\gamma_{pq}\hat{\Delta}\right)Y_{1m} = 2\varepsilon_{r(p}\hat{D}_{q)}\hat{D}^rY_{1m} = 0.$$

- Zero-modes are kernel modes of the operators $\hat{\Delta}$ or $\hat{\Delta}+2$.

  ---> **l=0,1 mode ($\hat{\Delta}Y_{lm} = -l(l+1)Y_{lm}$) are zero-modes.**

One-to-one correspondence between the variable $\{h_{Ap}, h_{pq}\}$ and $\{h_{(e1)A}, h_{(o1)A}, h_{(e0)}, h_{(e2)}, h_{(o2)}\}$ is not guaranteed in l=0,1 mode.

Proposal to solve this l=0,1-mode problem (1)

- To resolve the problem of l=0,1-mode perturbations, **we use the decomposition of the perturbations by $S = S_\delta$ instead of by $S = Y_{lm}$.**

$$S_\delta = \begin{cases} Y_{lm} & \text{for} \quad l \geq 2; \\ k_{(\hat{\Delta}+2)m} & \text{for} \quad l = 1; \quad k_{(\hat{\Delta}+2)m} \in \mathscr{K}_{(\hat{\Delta}+2)} := \{f \in \mathscr{F}(S^2) | [\hat{\Delta}+2]f = 0\}. \\ k_{(\hat{\Delta})} & \text{for} \quad l = 0. \quad k_{(\hat{\Delta})} \in \mathscr{K}_{(\hat{\Delta})} := \{f \in \mathscr{F}(S^2) | \hat{\Delta}f = 0\}. \end{cases} \quad \text{– (I)}$$

$$h_{AB} = \sum_{l,m}\tilde{h}_{AB}S_\delta, \quad h_{Ap} = r\hat{D}_p\sum_{l,m}\tilde{h}_{(e1)A}S_\delta + r\varepsilon_{pq}\hat{D}^q\sum_{l,m}\tilde{h}_{(o1)A}S_\delta,$$
$$h_{pq} = \frac{1}{2}\gamma_{pq}r^2\sum_{l,m}\tilde{h}_{(e0)}S_\delta + r^2\left(\hat{D}_p\hat{D}_q - \frac{1}{2}\gamma_{pq}\Delta\right)\sum_{l,m}\tilde{h}_{(e2)}S_\delta + r^2 2\varepsilon_{r(p}\hat{D}_{q)}\hat{D}^r\sum_{l,m}\tilde{h}_{(o2)}S_\delta, \quad \text{– (II)}$$

- This decomposition is invertible if the following conditions are satisfied.

$$\left(\hat{D}^pk_{(\hat{\Delta})}\right)\left(\hat{D}_pk_{(\hat{\Delta})}\right) \neq 0, \quad \left(\hat{D}^p\hat{D}^q\right)k_{(\hat{\Delta})}\left(\hat{D}_p\hat{D}_q\right)k_{(\hat{\Delta})} \neq 0,$$
$$k_{(\Delta+2)m} = \Theta(\theta)e^{im\phi}, \quad \left(\hat{D}_p\hat{D}_qk_{(\hat{\Delta}+2)}\right)\left(\hat{D}^p\hat{D}^qk_{(\hat{\Delta}+2)}\right) - 2\left(k_{(\hat{D}+2)m}\right)^2 \neq 0.$$

$$\because \quad \hat{\Delta}^{-1}\hat{\Delta}: \quad \text{Projection operator} \quad L^2(S^2) \mapsto L^2(S^2)\backslash\mathscr{K}_{(\hat{\Delta})}$$
$$\left[\hat{\Delta}+2\right]^{-1}\left[\hat{\Delta}+2\right]: \quad \text{Projection operator} \quad L^2(S^2) \mapsto L^2(S^2)\backslash\mathscr{K}_{(\hat{\Delta}+2)}$$

Proposal to solve this l=0,1-mode problem (2)

- Explicitly, we use the following functions.

$$k_{(\hat{\Delta})} = 1+\delta\ln\left(\frac{1-z}{1+z}\right), \quad \delta\in\mathbb{R}, \quad z=\cos\theta$$
$$k_{(\hat{\Delta}+2)m=0} = z\left\{1+\delta\left(\frac{1}{2}\ln\frac{1+z}{1-z}-\frac{1}{z}\right)\right\}, \qquad \text{– (III)}$$
$$k_{(\hat{\Delta}+2)m=\pm1} = (1-z^2)^{1/2}\left\{1+\delta\left(\frac{1}{2}\ln\frac{1+z}{1-z}+\frac{z}{1-z^2}\right)\right\}e^{\pm i\phi}.$$

- These functions satisfy the previous conditions when $\delta\neq 0$.
- When $\delta=0$, we have $k_{(\hat{\Delta})}\propto Y_{00}$, $k_{(\hat{\Delta}+2)m}\propto Y_{1m}$.
- These functions are singular except for the case $\delta=0$.

Proposal :

We decompose perturbations h_{ab} on a spherically symmetric background spacetime through Eqs. (I), (II), and (III). Then, the decomposition (II) becomes invertible including $l=0,1$ modes. After deriving the field equations such as linearized Einstein equations by using the harmonic function S_δ, we choose $\delta=0$ when we solve these field equations as regularity of solutions.

- Since the decomposition (II) is invertible, we can construct gauge-invariant variables and evaluate the field equations through the mode-by-mode analyses including l=0,1 modes.

IV. Construction of gauge-invariant variables for linear perturbations

The gauge-transformation rule of linear-metric perturbation h_{ab} :

$${}_{\mathcal{Y}}h_{ab} - {}_{\mathcal{X}}h_{ab} = \pounds_\xi g_{ab} = 2\nabla_{(a}\xi_{b)}. \qquad \nabla_a g_{ab}=0.$$

- Mode decomposition of the generator ξ_a.

$$\xi_a = \xi_A(dx^A)_a + \xi_p(dx^p)_a, \quad \xi_A = \sum_{l,m}\zeta_A S_\delta, \quad \xi_p = r\sum_{l,m}\left(\zeta_{(e1)}\hat{D}_p S_\delta + \zeta_{(o1)}\varepsilon_{pr}\hat{D}^r S_\delta\right).$$

- Mode by mode gauge-transformation of metric perturbations :
  - Odd modes : $${}_{\mathcal{Y}}\tilde{h}_{(o1)A} - {}_{\mathcal{X}}\tilde{h}_{(o1)A} = r\bar{D}_A\left(\frac{1}{r}\zeta_{(o1)}\right), \quad {}_{\mathcal{Y}}\tilde{h}_{(o2)} - {}_{\mathcal{X}}\tilde{h}_{(o2)} = -\frac{1}{r}\zeta_{(o1)}; \qquad \bar{D}_A y_{BC}=0$$

$$\underbrace{\tilde{F}_A := \tilde{h}_{(o1)A} + r\bar{D}_A\tilde{h}_{(o2)},}_{\text{gauge-invariant variable}} \quad \underbrace{\tilde{Y}_{(o2)} := -r^2\tilde{h}_{(o2)},}_{\text{gauge-variant variable}} \quad \underbrace{{}_{\mathcal{Y}}\tilde{Y}_{(o2)} - {}_{\mathcal{X}}\tilde{Y}_{(o2)} = r\zeta_{(o1)}.}_{\text{gauge-transformation rule}}$$

  - Even modes :
$$\begin{aligned} {}_{\mathcal{Y}}\tilde{h}_{AB} - {}_{\mathcal{X}}\tilde{h}_{AB} &= 2\bar{D}_{(A}\zeta_{B)}, \\ {}_{\mathcal{Y}}\tilde{h}_{(e1)A} - {}_{\mathcal{X}}\tilde{h}_{(e1)A} &= \frac{1}{r}\zeta_A + r\bar{D}_A\left(\frac{1}{r}\zeta_{(e1)}\right), \quad {}_{\mathcal{Y}}\tilde{h}_{(e2)} - {}_{\mathcal{X}}\tilde{h}_{(e2)} = +\frac{2}{r}\zeta_{(e1)}; \\ {}_{\mathcal{Y}}\tilde{h}_{(e0)} - {}_{\mathcal{X}}\tilde{h}_{(e0)} &= -\frac{2}{r}l(l+1)\zeta_{(e1)} + \frac{4}{r}(\bar{D}^A r)\zeta_A,, \end{aligned}$$

$$\underbrace{\tilde{Y}_{(e2)} := \frac{r^2}{2}\tilde{h}_{(e2)}, \quad \tilde{Y}_A := r\tilde{h}_{(e1)A} - \frac{r^2}{2}\bar{D}_A\tilde{h}_{(e2)},}_{\text{gauge-variant variables}} \quad \underbrace{{}_{\mathcal{Y}}\tilde{Y}_{(e2)} - {}_{\mathcal{X}}\tilde{Y}_{(e2)} = r\zeta_{(e1)}, \quad {}_{\mathcal{Y}}\tilde{Y}_A - {}_{\mathcal{X}}\tilde{Y}_A = \zeta_A.}_{\text{gauge-transformation rules}}$$

$$\underbrace{\tilde{F}_{AB} := \tilde{h}_{AB} - 2\bar{D}_{(A}\tilde{Y}_{B)}, \quad \tilde{F} := \tilde{h}_{(e0)} - \frac{4}{r}(\bar{D}^A r)\tilde{Y}_A + \frac{2}{r^2}l(l+1)\tilde{Y}_{(e2)}.}_{\text{gauge-invariant variable}}$$

The expression of the original metric perturbation h_{ab} :

- Using the gauge-variant variables $\tilde{Y}_{(o2)}$, $\tilde{Y}_{(e2)}$, $\tilde{Y}_{A}$, we define the vector field Y_a as

$$Y_a := Y_A(dx^A)_a + Y_p(dx^p)_a \Longrightarrow \boxed{{}_{\mathcal{Y}}Y_a - {}_{\mathcal{X}}Y_a = \xi_a.}$$

$$Y_A := \sum_{l,m} \tilde{Y}_A S_\delta, \quad Y_p := \sum_{l,m} \left(\tilde{Y}_{(e2)} \hat{D}_p S_\delta + \tilde{Y}_{(o2)} \varepsilon_{pq} \hat{D}^q S_\delta \right).$$

- Using the gauge-invariant variables $\tilde{F}_{AB}$, $\tilde{F}_{A}$, $\tilde{F}$, we define the variables as

$$F_{AB} := \sum_{l,m} \tilde{F}_{AB} S_\delta, \quad F_{Ap} := \sum_{l,m} \tilde{F}_{A} \varepsilon_{pq} \hat{D}^q S_\delta, \quad F := \sum_{l,m} \tilde{F} S_\delta.$$

- The original components of the metric perturbation h_{ab} as

$$\begin{aligned} h_{AB} &= \sum_{l,m} \tilde{h}_{AB} S_\delta, \quad h_{Ap} = r\hat{D}_p \sum_{l,m} \tilde{h}_{(e1)A} S_\delta + r\varepsilon_{pq} \hat{D}^q \sum_{l,m} \tilde{h}_{(o1)A} S_\delta, \\ h_{pq} &= \frac{1}{2}\gamma_{pq} r^2 \sum_{l,m} \tilde{h}_{(e0)} S_\delta + r^2 \left(\hat{D}_p \hat{D}_q - \frac{1}{2}\gamma_{pq}\Delta \right) \sum_{l,m} \tilde{h}_{(e2)} S_\delta + r^2 2\varepsilon_{r(p}\hat{D}_{q)}\hat{D}^r \sum_{l,m} \tilde{h}_{(o2)} S_\delta. \end{aligned}$$

$$\Longrightarrow \begin{aligned} h_{AB} &= F_{AB} + 2\bar{D}_{(A}Y_{B)},, \quad h_{Ap} = rF_{Ap} + \hat{D}_p Y_A + \bar{D}_A Y_p - \frac{2}{r}(\bar{D}_A r) Y_p. \\ h_{pq} &= \frac{1}{2}\gamma_{pq} r^2 F + 2r(\bar{D}^A r)\gamma_{pq} Y_A + 2\hat{D}_{(p}Y_{q)}. \end{aligned}$$

- Identifying the gauge-invariant part $\mathcal{F}_{ab}$ of the metric perturbation h_{ab} as

$$\mathcal{F}_{AB} := F_{AB}, \quad \mathcal{F}_{Ap} := rF_{Ap}, \quad \mathcal{F}_{pq} := \frac{1}{2}\gamma_{pq} r^2 F.$$

the original metric perturbation is given by

$$\boxed{h_{ab} = \mathcal{F}_{ab} + \pounds_Y g_{ab}.}$$

- This expression includes not only $l \geq 2$ modes but also $l = 0, 1$ modes.
- This is the assertion of the above Decomposition Conjecture!!!

The decomposition conjecture is proved in the perturbations on the Schwarzschild background spacetime including l=0,1 modes.

V. l=0,1 solutions to the linearized Einstein equations

The linearized Einstein equations are given in the form

$${}^{(1)}\mathcal{G}_a{}^b = 8\pi {}^{(1)}\mathcal{T}_a{}^b.$$

- Here, ${}^{(1)}G_a{}^b = {}^{(1)}\mathcal{G}_a{}^b + \pounds_X G_a{}^b = {}^{(1)}\mathcal{G}_a{}^b$ is the linearized Einstein tensor with the vacuum background Einstein equation $G_a{}^b = 0$.
- Similarly, the linear perturbation of the energy-momentum tensor is given by ${}^{(1)}T_a{}^b = {}^{(1)}\mathcal{T}_a{}^b + \pounds_X T_a{}^b = {}^{(1)}\mathcal{T}_a{}^b$ with the vacuum background condition $T_a{}^b = 0$.
  - Here, we consider the conventional Einstein equation. Therefore, the linear metric perturbation h_{ab} is not included in ${}^{(1)}\mathcal{T}_a{}^b$.
- The linear perturbation of the energy-momentum tensor should satisfy the equation

$$\nabla^a {}^{(1)}\mathcal{T}_a{}^b = 0.$$

  - c.f. Identity:

$${}^{(1)}\left(\bar{\nabla}_a \bar{T}_b{}^a\right) = \nabla_a {}^{(1)}\mathcal{T}_b{}^a + H_{ca}{}^a[\mathcal{F}] T_b{}^c - H_{ba}{}^c[\mathcal{F}] T_c{}^a + \pounds_Y \nabla_a T_b{}^a, \quad H_{abc}[\mathcal{F}] := \nabla_{(a}\mathcal{F}_{b)c} - \frac{1}{2}\nabla_c \mathcal{F}_{ab}.$$

- Here, we decompose the components of ${}^{(1)}\mathcal{T}_{ac}$ as

$$\begin{aligned} {}^{(1)}\mathcal{T}_{ac} &= \sum_{l,m} \tilde{T}_{AC} S_\delta (dx^A)_a (dx^C)_c + 2r \sum_{l,m} \left\{ \tilde{T}_{(e1)A} \hat{D}_p S_\delta + \tilde{T}_{(o1)A} \varepsilon_{pq} \hat{D}^q S_\delta \right\} (dx^A)_{(a} (dx^p)_{c)} \\ &\quad + r^2 \sum_{l,m} \left\{ \tilde{T}_{(e0)} \frac{1}{2}\gamma_{pq} S_\delta + \tilde{T}_{(e2)} \left(\hat{D}_p \hat{D}_q - \frac{1}{2}\hat{\Delta} \right) S_\delta + \tilde{T}_{(o2)} 2\varepsilon_{s(p}\hat{D}_{q)}\hat{D}^s S_\delta \right\} (dx^p)_a (dx^q)_c. \end{aligned}$$

- The components of $\nabla^a {}^{(1)}\mathcal{T}_a{}^b = 0$ are summarized as

$$\bar{D}^C \tilde{T}_C{}^B + \frac{2}{r}(\bar{D}^D r)\tilde{T}_D{}^B - \frac{l(l+1)}{r}\tilde{T}^B_{(e1)} - \frac{1}{r}(\bar{D}^B r)\tilde{T}_{(e0)} = 0,$$

$$\bar{D}^C \tilde{T}_{(e1)C} + \frac{3}{r}(\bar{D}^C r)\tilde{T}_{(e1)C} + \frac{1}{2r}\tilde{T}_{(e0)} - \frac{(l-1)(l+2)}{2r}\tilde{T}_{(e2)} = 0.$$

$$\bar{D}^C \tilde{T}_{(o1)C} + \frac{3}{r}(\bar{D}^C r)\tilde{T}_{(o1)C} + \frac{(l-1)(l+2)}{r}\tilde{T}_{(o2)} = 0.$$

- Notes on components of ${}^{(1)}\mathscr{T}_{ac}$ when $\delta=0$:
  - For l=0,1 modes, we may choose $\tilde{T}_{(e2)}=\tilde{T}_{(o2)}=0$.
  - For l=0 mode, we may also choose $\tilde{T}_{(e1)A}=\tilde{T}_{(o1)A}=0$.
    - Because of $\nabla^{a(1)}\mathscr{T}_a{}^b=0$, these conditions for l=0 modes yields $\tilde{T}_{(e0)}=0$ for l=0 mode.

4-1. l=1 odd mode perturbations

- If we impose the regularity $\delta=0$, there is no l=0 mode in odd-mode perturbation.
- l=1 mode (we concentrate only on m=0 mode):
  - Components of the odd-mode gauge-invariant variable:

$$r\tilde{F}_A(dx^A)_a =: X_{(o)}(dt)_a + r^2\partial_r W_{(o)}(dr)_a, \quad \frac{6M}{r^4}a_1(t,r) := \partial_r\left(\frac{1}{r^2}X_{(o)}\right) - \partial_t\partial_r W_{(o)}.$$

  - Einstein equations:

$$\partial_t^2\left(rf\partial_r W_{(o)}\right) - f\partial_r\left(f\partial_r\left(rf\partial_r W_{(o)}\right)\right) + \frac{f}{r^2}\left[2-3(1-f)\right]\left(rf\partial_r W_{(o)}\right) = 16\pi f^2\tilde{T}_{(o1)r}.$$

[l=1 version of Regge-Wheeler eq.]

$$\partial_t a_1(t,r) = -\frac{16\pi}{3M}r^3 f\tilde{T}_{(o1)r}, \quad \partial_r a_1(t,r) = -\frac{16\pi}{3M}\frac{r^3}{f}\tilde{T}_{(o1)t}.$$

Constraints

$$\bar{D}^C\tilde{T}_{(o1)C} + \frac{3}{r}(\bar{D}^C r)\tilde{T}_{(o1)C} = 0.$$

Odd-mode of $\nabla^{a(1)}\mathscr{T}_a{}^b=0$. **Integrability condition**

l=1 Odd-mode sol.

$$a_1(t,r) = -\frac{16\pi}{3M}r^3 f\int dt\tilde{T}_{(o1)r} + a_{10} = -\frac{16\pi}{3M}\int dr\frac{r^3}{f}\tilde{T}_{(o1)t} + a_{10}.$$

Kerr parameter perturbation (a_{10})

$$\boxed{2\mathscr{F}_{Ap}(dx^A)_{(a}(dx^p)_{b)} = \left(6Mr^2\int dr\frac{a_1(t,r)}{r^4}\right)\sin^2\theta(dt)_{(a}(d\phi)_{b)} + \pounds_{V_{(o1)}}g_{ab}}$$

$$V_{(o1)a} := \left(\beta(t) + W_{(o)}(t,r)\right)r^2\sin^2\theta(d\phi)_a, \quad \beta(t)$$ **: arbitrary function of t.**

4-2. l=0,1 even mode perturbations

- For l=0,1 even mode, $\tilde{T}_{(e2)}=0$. $\xrightarrow{\text{Einstein eq.}}$ $\tilde{F}_D{}^D=0$.
  - Components of the gauge-invariant tensor $\tilde{F}_{AB}$:

$$\tilde{F}_{AB}(dx^A)_a(dx^B)_b =: X_{(e)}\left\{-(dt)_a(dt)_b - f^{-2}(dr)_a(dr)_b\right\} + 2Y_{(e)}(dt)_{(a}(dr)_{b)}$$

  - Einstein equations:

$$\begin{cases} \partial_t X_{(e)} + f\partial_r Y_{(e)} + \dfrac{1-f}{f}Y_{(e)} - \dfrac{1}{2}\partial_t\tilde{F} = 16\pi r\tilde{T}_{(e1)t}, \\ \partial_t Y_{(e)} + f\partial_r X_{(e)} + \dfrac{1-f}{f}X_{(e)} + \dfrac{1}{2}f\partial_r\tilde{F} = -16\pi rf\tilde{T}_{(e1)r}. \end{cases}$$

    - Moncrief variable $\Phi_{(e)} := \dfrac{r}{\Lambda}\left[fX_{(e)} - \dfrac{\Lambda}{4}\tilde{F} + \dfrac{rf}{2}\partial_r\tilde{F}\right], \quad \Lambda := (l-1)(l+2) + 3(1-f).$

$$\begin{cases} l(l+1)Y_{(e)} = \dfrac{2\Lambda}{f}\partial_t\Phi_{(e)} + \dfrac{\Lambda+3f-1}{2f}r\partial_t\tilde{F} + 16\pi r^2\tilde{T}_{tr}, \\ l(l+1)\Lambda\tilde{F} = -8f\Lambda\partial_r\Phi_{(e)} + \dfrac{4}{r}\left[6f(1-f) - l(l+1)\Lambda\right]\Phi_{(e)} - 64\pi r^2\tilde{T}_{tt}. \end{cases}$$

    - Zerilli equations:

$$\begin{cases} -\dfrac{1}{f}\partial_t^2\Phi_{(e)} + \partial_r\left[f\partial_r\Phi_{(e)}\right] - V_{\rm even}\Phi_{(e)} = 16\pi\dfrac{r}{\Lambda}S_{(\Phi_{(e)})}, \quad V_{\rm even} = \dfrac{\Lambda^2[\Lambda-2(2-3f)] + 6(1-f)[(1-3f)\Lambda + 3f(1-f)]}{r^2\Lambda^2} \\ S_{(\Phi_{(e)})} = \left(\dfrac{\Lambda}{4f} - \dfrac{1}{2}\right)\tilde{T}_{tt} - \dfrac{1}{2}r\partial_r\tilde{T}_{tt} - \dfrac{3(1-f)}{\Lambda}\tilde{T}_{tt} + \left(\dfrac{2-f}{2} - \dfrac{\Lambda}{4}\right)f\tilde{T}_{rr} + \dfrac{1}{2}f^2 r\partial_r\tilde{T}_{rr} - \dfrac{f}{2}\tilde{T}_{(e0)} - l(l+1)f\tilde{T}_{(e1)r}. \end{cases}$$

    - Evolution equation for $\tilde{F}$ (l=0):

$$-\frac{1}{f}\partial_t^2\tilde{F} + \partial_r\left(f\partial_r\tilde{F}\right) + \frac{3(1-f)}{r^2}\tilde{F} + \frac{4\Lambda}{r^3}\Phi_{(e)} = 16\pi\left[-\frac{1}{f}\tilde{T}_{tt} + f\tilde{T}_{rr} + 4f\tilde{T}_{(e1)r}\right].$$

l=0 version of Regge-Wheeler eq.

  - Even-mode of $\nabla^{a(1)}\mathscr{T}_a{}^b=0$.

$$\begin{cases} \bar{D}^C\tilde{T}_C{}^B + \dfrac{2}{r}(\bar{D}^D r)\tilde{T}_D{}^B - \dfrac{l(l+1)}{r}\tilde{T}^B_{(e1)} - \dfrac{1}{r}(\bar{D}^B r)\tilde{T}_{(e0)} = 0, \\ \bar{D}^C\tilde{T}_{(e1)C} + \dfrac{3}{r}(\bar{D}^C r)\tilde{T}_{(e1)C} + \dfrac{1}{2r}\tilde{T}_{(e0)} = 0. \end{cases}$$

  - The consistency of these equations can be check due to the tedious calculations.

4-2-1. l=0 even-mode solutions (1)

- For l=0 even mode, it is convenient to introduce the variables $m(t,r)$:
  - Mass perturbation definition: $m(t,r) = -\frac{1}{2}(1-3f)\Phi_{(e)}$.
  - Einstein equations for $m(t,r)$:

$$l(l+1)Y_{(e)} = \frac{2\Lambda}{f}\partial_t\Phi_{(e)} + \frac{\Lambda+3f-1}{2f}r\partial_t\tilde{F} + 16\pi r^2\tilde{T}_{tr},$$

$$l(l+1)\Lambda\tilde{F} = -8f\Lambda\partial_r\Phi_{(e)} + \frac{4}{r}\left[6f(1-f)-l(l+1)\Lambda\right]\Phi_{(e)} - 64\pi r^2\tilde{T}_{tt}.$$

$$\partial_t m_1(t,r) = 4\pi r^2 f\tilde{T}_{tr}, \quad \partial_r m_1(t,r) = 4\pi r^2\frac{1}{f}\tilde{T}_{tt}.$$

  - Even-mode of $\nabla^{a(1)}\mathscr{T}_a{}^b = 0.$

$$\bar{D}^C\tilde{T}_C{}^B + \frac{2}{r}(\bar{D}^D r)\tilde{T}_D{}^B - \frac{l(l+1)}{r}\tilde{T}^B_{(e1)} - \frac{1}{r}(\bar{D}^B r)\tilde{T}_{(e0)} = 0.$$

Integrability condition

$$\bar{D}^C\tilde{T}_{(e1)C} + \frac{3}{r}(\bar{D}^C r)\tilde{T}_{(e1)C} + \frac{1}{2r}\tilde{T}_{(e0)} = 0. \Longrightarrow T_{(e0)} = 0.$$

- l=0 mode $\Longrightarrow$ $\tilde{T}_{(e1)C} = 0$
- Solution of the mass perturbation $m(t,r)$:

$$\boxed{m_1(t,r) = 4\pi\int dr\left[\frac{r^2}{f}\tilde{T}_{tt}\right] + M_1 = 4\pi\int dt\left[r^2 f\tilde{T}_{tt}\right] + M_1}$$

Schwarzschild mass perturbation

- Zerilli equations becomes trivial.
- Moncrief variable definition and $\tilde{F} =: \partial_t\Upsilon$ $\Longrightarrow$

$$\boxed{fX_{(e)} = -\frac{2m(t,r)}{r} + \frac{1}{4}(1-3f)\partial_t\Upsilon - \frac{1}{2}rf\partial_t\partial_r\Upsilon.}$$

- Integrability of Eqs. $f\partial_r Y_{(e)} + \partial_t X_{(e)} + \frac{1-f}{f}Y_{(e)} - \frac{1}{2}\partial_t\tilde{F} = 0, \quad \partial_t Y_{(e)} + f\partial_r X_{(e)} + \frac{1-f}{f}X_{(e)} + \frac{1}{2}f\partial_r\tilde{F} = 0$ with respect to $Y_{(e)}$ is guaranteed by the Zerilli equations and evolution equation for $\tilde{F}$:

$$-\frac{1}{f}\partial_t^2\tilde{F} + \partial_r\left(f\partial_r\tilde{F}\right) + \frac{3(1-f)}{r^2}\tilde{F} + \frac{4\Lambda}{r^3}\Phi_{(e)} = 16\pi\left[-\frac{1}{f}\tilde{T}_{tt} + f\tilde{T}_{rr}\right].$$

$$\boxed{Y_{(e)} = -\frac{2}{r^2}\int dt\, m_1(t,r) + \frac{3}{4r}(1-f)\Upsilon - \frac{1}{4}(1-3f)\partial_r\Upsilon + \frac{1}{2}r\partial_r(f\partial_r\Upsilon) + 8\pi\frac{r}{f}\int dt\tilde{T}_{tt} + \frac{1}{2}r\zeta(r) - \frac{1}{4f}\int dr(1-3f)\zeta(r) + \frac{\xi}{f},}$$

where $\zeta(r)$ is an arbitrary function of r, and ξ is an arbitrary constant.

4-2-1. l=0 even-mode solutions (2)

- Then, the metric perturbation for l=0 even mode is given by

$$\boxed{\mathscr{F}_{ab} = \frac{2}{r}\left(M_1 + 4\pi\int dr\left[\frac{r^2}{f}\tilde{T}_{tt}\right]\right)\left((dt)_a(dt)_b + \frac{1}{f^2}(dr)_a(dr)_b\right) + 2\left[4\pi r\int dt\left(\frac{1}{f}\tilde{T}_{tt} + f\tilde{T}_{rr}\right)\right](dt)_{(a}(dr)_{b)} + \pounds_{V_{e0}}g_{ab},}$$

where

$$V_{(e0)a} := \left(\frac{1}{4}f\Upsilon + \frac{1}{4}rf\partial_r\Upsilon + \gamma(r)\right)(dt)_a + \frac{r}{4f}\partial_t\Upsilon(dr)_a, \quad \gamma(r) := f\int dr\left[\frac{1}{4f}r\zeta(r) - \frac{1}{4f^2}\int dr(1-3f)\zeta(r) + \frac{\xi}{f^2}\right].$$

4-2-2. l=1 even-mode solutions

- Solution to the Einstein equations for l=1 even mode:

$$\begin{aligned}\mathscr{F}_{ab} \; = \; & -\frac{16\pi r^2 f^2}{3(1-f)}\left[\frac{1+f}{2}\tilde{T}_{rr} + rf\partial_r\tilde{T}_{rr} - \tilde{T}_{(e0)} - 4\tilde{T}_{(e1)r}\right]\cos\theta(dt)_a(dt)_b \\ & +16\pi r^2\left[\tilde{T}_{tr} - \frac{2r}{3f(1-f)}\partial_t\tilde{T}_{tt}\right]\cos\theta(dt)_{(a}(dr)_{b)} \\ & +\frac{8\pi r^2(1-3f)}{f^2(1-f)}\left[\tilde{T}_{rr} - \frac{2rf}{3(1-3f)}\partial_r\tilde{T}_{tt}\right]\cos\theta(dr)_a(dr)_b \\ & -\frac{16\pi r^4}{3(1-f)}\tilde{T}_{tt}\cos\theta\gamma_{ab} \\ & +\pounds_{V_{(e1)}}g_{ab},\end{aligned}$$

where

$$V_{(e1)a} \; := \; -r\partial_t\Phi_{(e)}\cos\theta(dt)_a + \left(\Phi_{(e)} - r\partial_r\Phi_{(e)}\right)\cos\theta(dr)_a - r\Phi_{(e)}\sin\theta(d\theta)_a.$$

VI. Summary and Discussions on our proposal

- We proposed a gauge-invariant treatment of the l=0,1 mode perturbations on the Schwarzschild background spacetime.
  - Harmonics:

(I)

$$S_\delta = \begin{cases} Y_{lm} & \text{for} \quad l \geq 2; \\ k_{(\hat{\Delta}+2)m} & \text{for} \quad l = 1; \quad k_{(\hat{\Delta}+2)m} \in \mathscr{K}_{(\hat{\Delta}+2)} := \{ f \in \mathscr{F}(S^2) | \, [\hat{\Delta}+2] f = 0 \}. \\ k_{(\hat{\Delta})} & \text{for} \quad l = 0. \quad k_{(\hat{\Delta})} \in \mathscr{K}_{(\hat{\Delta})} := \{ f \in \mathscr{F}(S^2) | \hat{\Delta} f = 0 \}. \end{cases}$$

(III)

$$k_{(\hat{\Delta})} = 1 + \delta \ln\left(\frac{1-z}{1+z}\right), \quad \delta \in \mathbb{R}, \quad z = \cos\theta$$

$$k_{(\hat{\Delta}+2)m=0} = z\left\{1 + \delta\left(\frac{1}{2}\ln\frac{1+z}{1-z} - \frac{1}{z}\right)\right\},$$

$$k_{(\hat{\Delta}+2)m=\pm 1} = (1-z^2)^{1/2}\left\{1 + \delta\left(\frac{1}{2}\ln\frac{1+z}{1-z} + \frac{z}{1-z^2}\right)\right\} e^{\pm i\phi}$$

  - Decomposition:

(II)

$$h_{AB} = \sum_{l,m} \tilde{h}_{AB} \underline{S_\delta}, \quad h_{Ap} = r\hat{D}_p \sum_{l,m} \tilde{h}_{(e1)A} \underline{S_\delta} + r\varepsilon_{pq}\hat{D}^q \sum_{l,m} \tilde{h}_{(o1)A} \underline{S_\delta},$$

$$h_{pq} = \frac{1}{2}\gamma_{pq} r^2 \sum_{l,m} \tilde{h}_{(e0)} \underline{S_\delta} + r^2 \left(\hat{D}_p\hat{D}_q - \frac{1}{2}\gamma_{pq}\Delta\right) \sum_{l,m} \tilde{h}_{(e2)} \underline{S_\delta} + r^2 2\varepsilon_{r(p}\hat{D}_{q)}\hat{D}^r \sum_{l,m} \tilde{h}_{(o2)} \underline{S_\delta}.$$

  - **Proposal :**

    We decompose perturbations h_{ab} on a spherically symmetric background spacetime through Eqs. (I), (II), and (III). Then, the decomposition (II) becomes invertible including $l = 0, 1$ modes. After deriving the field equations such as linearized Einstein equations by using the harmonic function S_δ, we choose $\delta = 0$ when we solve these field equations as regularity of solutions.

  - Following our proposal, we derive the l=0,1 mode perturbation solution to the linearized Einstein equation with a generic linearized energy-momentum tensor for the matter field.
  - Derived solutions include Kerr parameter perturbations and the additional mass parameter perturbations in the vacuum case, except for the terms which has the form of the Lie derivative of the background spacetime. --> **Our proposal is reasonable!!!**

- **Thus, we have resolved the zero-mode problem in the perturbation theory on the Schwarzschild background spacetime.**
- **Then, we can apply our general formulation of higher-order perturbation theory to this background spacetime and can develop it to the any-order perturbations.**

Poster session

Junsei Tokuda

Kobe U.

“Gravitational positivity bounds on scalar potentials and applications to the Higgs sector”

[JGRG30 (2021) PA5]

Gravitational positivity bounds on scalar potentials
(I apologize for changing the title slightly…)

Junsei Tokuda (Kobe university)

Mainly based on [PRD104,066022(2021), T. Noumi, **JT**]([arXiv: 2105.01436])

See also:
[PRL127,091602(2021), K. Aoki, T.Q. Loc. T. Noumi, **JT**]
[JHEP11(2020)054 **JT**, K. Aoki, S. Hirano]

A. Motivation & Abstract

- Motivation: We want to clarify **which low-energy effective field theories (EFT) are consistent with UV complete quantum gravity.**
- Background: Recently, it has been noticed that "positivity bounds" which are derived from S-matric unitarity, analyticity, etc, have been useful in this context.
- Method: We discuss **positivity bounds on 4D scalar QFT + GR:** $\mathcal{L} \sim M_{\rm pl}^2 R - (\partial\phi)^2 - V(\phi) + \cdots$, to derive **swampland conditions on** $V(\phi)$.
- Result: It is found that **scalar potentials $V(\phi)$ cannot be arbitrarily flat.**
  e.g.) $V(\phi) = m^2\phi^2 + \lambda\phi^4$ ➡ λ: fixed, $m^2 \to 0$ limit is prohibited: Lower bound on the mass
  * See [D. Results] for details.

B-1. background: (1) Positivity bound *without* gravity

[Lee–Quigg–Thacker ('77)]

- S-matrix Unitarity・・・useful for finding new physics   e.g.) W-boson scat. → (perturbative) unitarity requires (Higgs mass) $\lesssim$ 1 TeV
- S-matrix **analyticity**, **unitarity etc** + **unitarity** → More info. of UV theory (new physics)

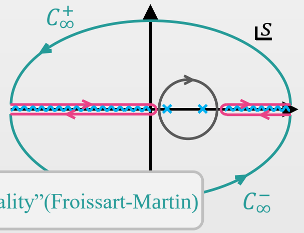
$\phi\phi \to \phi\phi$ scat. amplitude $\mathcal{M}(s,t)$
* ϕ: a massive scalar

Low-energy expansion
$\mathcal{M}(s,t) \sim (s,t,u\text{ poles}) + O(s^0) + c_2 s^2 + \cdots$

$$c_2 \sim \underbrace{\int_{4m^2}^{\infty} ds \frac{\mathrm{Im}\,\mathcal{M}(s,0)}{s^3}}_{\text{Positive! (unitarity)}} + \underbrace{\oint_{C_\infty} \frac{ds}{\pi i}\frac{\mathcal{M}(s,0)}{s^3}}_{0} > 0$$

$\lim_{|s|\to\infty} |\mathcal{M}(s,0)/s^2| = 0$. "Locality"(Froissart-Martin)

Analytic structure of $\mathcal{M}(s,0)$ & Integration contour



- Separating EFT pieces ($4m^2 < s < \Lambda^2$ integral, Λ: EFT cutoff) and high-energy pieces,

$$c_2(\Lambda) \coloneqq c_2 - \int_{4m^2}^{\Lambda^2} ds \frac{\mathrm{Im}\,\mathcal{M}(s,0)}{s^3} \qquad \boldsymbol{c_2(\Lambda) \sim \int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}\,\mathcal{M}(s,0)}{s^3} > 0}$$

"**Positivity bounds (*without* gravity)**" [Adams *et al.* ('06), Bellazzini ('16), de Rham *et al.* ('17)]

B-2. background: (2) Positivity bound *with* gravity

- Under clear assumptions, we can derive positivity bounds **with gravity.** [JT, K. Aoki, S. Hirano ('20)]
  (*see also [Hamada *et al.* ('18), Herrero-Valea *et al.* ('20), Bellazzini *et al.* ('19), Alberte *et al.* ('20, '21), Caron-Huot *et al.* ('21)])

*Additional assumptions (motivated by perturbative string amplitude)

1. Mild high-energy behavior $\lim_{|s|\to\infty} |\mathcal{M}(s,t<0)/s^2| = 0$
2. Regge behavior with single scaling $\mathrm{Im}\,\mathcal{M}(s,t\sim 0)|_{s\gg M_s^2} \sim f(t)\left(\frac{s}{M_s^2}\right)^{2+\alpha' t+\alpha'' t^2+\cdots}$, $\left|\frac{\partial_t f}{f}\right|, \left|\frac{\alpha''}{\alpha'}\right| \lesssim \alpha' \sim M_s^{-2}$

M_s: scale of Reggeization ~ mass of the lightest higher-spin state

$$c_2(\Lambda) = \lim_{t\to 0^-}\left\{\int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}\,\mathcal{M}(s,t)}{s^3} + \frac{1}{M_{\rm pl}^2 t}\right\} > \frac{-O(1)}{M_{\rm pl}^2 M_s^2}$$

(approximate) ***gravitational* positivity bounds**

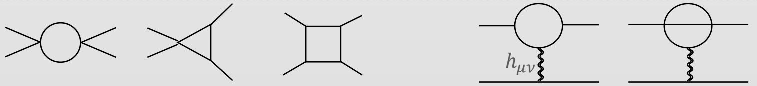
C. Method・Setup

- Consider **4D scalar QFT+GR** and compute $c_2(\Lambda)$. Discuss implications of $\boldsymbol{c_2(\Lambda) > \frac{-O(1)}{M_{pl}^2 M_s^2}}$.

$$\mathcal{L} = \frac{M_{\rm pl}^2 R}{2} - \frac{1}{2}(\partial\phi)^2 - V(\phi) + (\text{counterterms}) \qquad V(\phi) = \frac{m^2\phi^2}{2} + \frac{g\phi^3}{3!} + \frac{\lambda\phi^4}{4!}$$

*In this poster, we neglect higher-order terms such as ϕ^6 and $(\partial\phi)^4$ just for simplicity.

D. Result

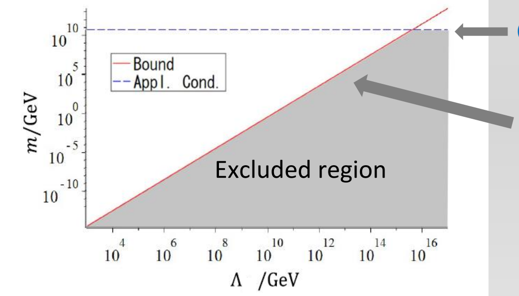


- Results :

$$c_2(\Lambda) \simeq \underbrace{\frac{\lambda^2}{16\pi^2\Lambda^4} - \frac{\lambda g^2}{6\pi^2\Lambda^6}\left[\ln\left(\frac{\Lambda^2}{m^2}\right) - \frac{1}{6}\right] + \frac{g^4}{12\pi^2 m^2\Lambda^6}}_{\text{(Nongravitational terms)} > 0} \underbrace{- \frac{1}{M_{\rm pl}^2}\left(\frac{45-8\pi\sqrt{3}}{1296\pi^2}\frac{g^2}{m^4} + \frac{10-\pi^2}{4608\pi^4}\frac{\lambda^2}{m^2}\right)}_{\text{(Gravitational terms)} < 0} > -O\left(M_{\rm pl}^{-2}M_s^{-2}\right).$$

✓ In the limit $(\lambda, g/m)$: fixed, $m \to 0$, we have **(Gravitational terms)** $\to -\infty$, leading to the violation of an inequality. This implies that **$V(\phi)$ cannot be arbitrarily flat.**

- E.g.) $\lambda\phi^4$ theory: $m \geq 0.0068\,\Lambda^2/M_{\rm pl}$



(Grav. terms) $= M_{\rm pl}^{-2}M_s^{-2}$
$\Leftrightarrow m = 5.4\times 10^{-4}|\lambda| M_s$

$m = 0.0068\,\Lambda^2/M_{\rm pl}$
$\approx 2.8\times 10^{9}\left(\frac{\Lambda}{10^{15}\,\text{GeV}}\right)^2$ GeV

*$\lambda = 10^{-2}$, $M_s = 10^{16}$ GeV is used in this plot.

Summary & Prospects

- Under some clear assumptions, we derived **swampland conditions on $V(\phi)$.**
- **$V(\phi)$** cannot be arbitrarily flat.
- Bounds on $V(\phi)$ around $\langle\phi\rangle \neq 0$?
- Justification of assumptions? Physical meaning of $c_2(\Lambda)$?
- Applications to other models.

Poster session

Keitaro Tomikawa

Rikkyo University

“Propagation of gravitational waves in an inhomogeneous universe in modified gravity”

[JGRG30 (2021) PA6]

Propagation of gravitational waves in an inhomogeneous universe in modified gravity

Keitaro Tomikawa
Rikkyo University
Collaborator: Tsutomu Kobayashi

JGRG 30, Online, Dec, 2021

Introduction

In this work, we investigate 3-dim covariant theory.

Lovelock's theorem
- 4-dim
- 4-dim covariance
- The theory contains the metric only
- EoMs are 2nd-order

→ In these assumptions, GR is unique.

➡

We consider following assumptions
- 4-dim
- 3-dim covariance
- The theory contains the metric only
- EoMs are 2nd-order (w.r.t time)

Motivations
- Universe has 3-dim covariance, not 4-dim covariance.
- Broken symmetry can be recovered by introducing a new field.(Stückelberg trick)
  e.g. "3-dim covariant theory" is equivalent to the "4-dim covariant theory + scalar field"

Spatially covariant theory

Action (Gao 2014)

$$S = \int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h}\ \mathscr{L}(t, N, h_{ij}, K_{ij}, R_{ij}, \varepsilon_{ijk}, \nabla_i)$$

N : Lapse function

h_{ij} : Metric of 3D space

K_{ij} : Extrinsic curvature

R_{ij} : Intrinsic curvature

∇_i : Covariant derivative

$\varepsilon_{ijk} = \sqrt{h}\epsilon_{ijk}, \epsilon_{123} = 1$

The property of this theory.

- D.o.fs are at most 3.
- Include Ghost condensate, EFT of inflation, Horndeski, etc..
- Contains higher order spatial derivatives.
- Include parity violating terms.

In [Gao and Hong 2020], they found the conditions under which the propagation speed of gravitational waves is equal to the speed of light in a homogeneous and isotropic spacetime.

GWs in homogeneous and isotropic s.p.

Brief review of [Gao and Hong 2020]

I)Classify the terms in the action by the order of differentiation.

$$S = \int \mathrm{d}t\mathrm{d}x^3 N\sqrt{h}(L^{(0)} + L^{(1)} + L^{(2)} + L^{(3)} + L^{(4)})$$

$$L^{(0)} = c_1^{(0,0)}(t, N)$$

$$L^{(1)} = c_1^{(1,0)}K$$

$$L^{(2)} = c_1^{(2,0)}K_{ij}K^{ij} + c_2^{(2,0)}K^2 + c_1^{(0,2)}R$$

d	(d_t, d_s)	Operators
0	(0,0)	1
1	(1,0)	K
	(0,1)	$\cdots$
2	(2,0)	$K_{ij}K^{ij},\ K^2$
	(1,1)	$\cdots$
	(0,2)	R
3	(3,0)	$K_{ij}K^{jk}K^i_k,\ K_{ij}K^{ij}K,\ K^3$
	(2,1)	$\varepsilon_{ijk}K^i_l\nabla^jK^{kl}$
	(1,2)	$\nabla^i\nabla^jK_{ij},\ \nabla^2K,\ R^{ij}K_{ij},\ RK$
	(0,3)	$\cdots$
4	(4,0)	$K_{ij}K^{jk}K^i_kK,\ (K_{ij}K^{ij})^2,\ K_{ij}K^{ij}K^2,\ K^4$
	(3,1)	$\varepsilon_{ijk}\nabla_mK^i_nK^{jm}K^{kn},\ \varepsilon_{ijk}\nabla^iK^j_mK^k_nK^{mn},\ \varepsilon_{ijk}\nabla^iK^j_lK^{kl}K$
	(2,2)	$\nabla_kK_{ij}\nabla^kK^{ij},\ \nabla_iK_{jk}\nabla^kK^{ij},\ \nabla_iK^{ij}\nabla_kK^k_j,\ \nabla_iK^{ij}\nabla_jK,$ $\nabla_iK\nabla^iK,\ R_{ij}K^i_kK^{jk},\ RK_{ij}K^{ij},\ R_{ij}K^{ij}K,\ RK^2$
	(1,3)	$\varepsilon_{ijk}R^{il}\nabla^jK^k_l,\ \varepsilon_{ijk}\nabla^iR^j_lK^{kl}$
	(0,4)	$\nabla^i\nabla^jR_{ij},\ \nabla^2R,\ R_{ij}R^{ij},\ R^2$

II)Consider the tensor perturbation around FLRW spacetime

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(\delta_{ij} + \gamma_{ij})\mathrm{d}x^i\mathrm{d}x^j$$

and expand the action up to 2nd-order.

III)Determine the conditions under which the propagation speed of gravitational waves equals the speed of light.

GWs in homogeneous and isotropic s.p.

The result of [Gao and Hong 2020].

$$S=\int \mathrm{d}t\mathrm{d}x^3 N\sqrt{h}(L^{(0)}+L^{(1)}+\tilde{L}^{(2)}+\tilde{L}^{(3)}+\tilde{L}^{(4)})$$

$$\tilde{L}^{(2)}=c_1^{(2,0)}(K_{ij}K^{ij}+R)+c_2^{(2,0)}K^2$$

$$\tilde{L}^{(3)}=c_1^{(3,0)}(K_{ij}K^{jk}K^i_k+RK)+c_2^{(3,0)}(K_{ij}K^{ij}+R)K+c_3^{(3,0)}K^3+c_1^{(1,2)}\nabla^i\nabla^j K_{ij}+c_2^{(1,2)}\nabla^2 K+c_3^{(1,2)}G^{ij}K_{ij}-\frac{1}{2N}\partial_t c_3^{(1,2)}R,$$

d	(d_t, d_s)	Operators
0	(0,0)	1
1	(1,0)	K
	(0,1)	$\cdots$
2	(2,0)	$K_{ij}K^{ij}$, K^2
	(1,1)	$\cdots$
	(0,2)	R
3	(3,0)	$K_{ij}K^{jk}K^i_k$, $K_{ij}K^{ij}K$, K^3
	(2,1)	$\varepsilon_{ijk}K^i_l\nabla^j K^{kl}$
	(1,2)	$\nabla^i\nabla^j K_{ij}$, $\nabla^2 K$, $R^{ij}K_{ij}$, RK
	(0,3)	$\cdots$
4	(4,0)	$K_{ij}K^{jk}K^i_k K$, $(K_{ij}K^{ij})^2$, $K_{ij}K^{ij}K^2$, K^4
	(3,1)	$\varepsilon_{ijk}\nabla_m K^i_n K^{jm}K^{kn}$, $\varepsilon_{ijk}\nabla^i K^j_m K^k_n K^{mn}$, $\varepsilon_{ijk}\nabla^i K^j_l K^{kl}K$
	(2,2)	$\nabla_k K_{ij}\nabla^k K^{ij}$, $\nabla_i K_{jk}\nabla^k K^{ij}$, $\nabla_i K^{ij}\nabla_k K^k_j$, $\nabla_i K^{ij}\nabla_j K$, $\nabla_i K\nabla^i K$, $R_{ij}K^i_k K^{jk}$, $RK_{ij}K^{ij}$, $R_{ij}K^{ij}K$, RK^2
	(1,3)	$\varepsilon_{ijk}R^{il}\nabla^j K^k_l$, $\varepsilon_{ijk}\nabla^i R^j_l K^{kl}$
	(0,4)	$\nabla^i\nabla^j R_{ij}$, $\nabla^2 R$, $R_{ij}R^{ij}$, R^2

:The conditions restrict the relation of some terms.

In particular, they found a more general theory in which both the odd and even modes of gravitational waves propagate at the speed of light even if the theory has parity violating terms.

Inhomogeneous universe

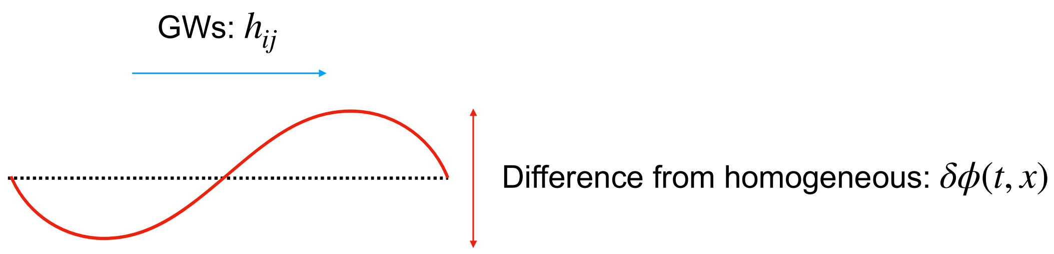
The result of [Gao and Hong 2020] is based on the symmetry of the “homogeneous” and “isotropic”.
→ How GWs propagate in less symmetric spacetime?

In particular, how GWs propagate in an inhomogeneous universe.

GWs $\longrightarrow$

$\longrightarrow$?

GWs in an inhomogeneous universe



We investigate the propagation of GWs in slightly inhomogeneous universe.
→ The propagation of GWs can be found in 3-point interactions $\delta\phi hh$.

We consider the metric such as

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + 2N_i\mathrm{d}t\mathrm{d}x^i + \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

$$N = 1+\alpha, \quad N_i = \partial_i\chi, \quad \gamma_{ij} = a^2 e^{2\zeta}\left(e^h\right)_{ij}$$

α, χ, ζ : scalar perturbations

and expand the action up to 3rd-order.

GWs in an inhomogeneous universe

For example, we consider the following term.

$$\mathscr{L} = c(t,N)(K_{ij}K^{ij} + R)$$

After the expansion, we obtain

$$S_{hh} = \frac{1}{4}\int \mathrm{d}t\mathrm{d}^3x a^3 c\left[\dot{h}_{ij}^2 - \frac{1}{a^2}(\partial_k h_{ij})^2\right],$$

$$S_{shh} = \frac{1}{4}\int \mathrm{d}t\mathrm{d}^3x a^3 c\left[(-\alpha+3\zeta)\dot{h}_{ij}^2 - \frac{2}{a^2}\partial_k\chi\dot{h}_{ij}\partial_k h_{ij} - (\alpha+\zeta)\frac{1}{a^2}(\partial_k h_{ij})^2\right] + \frac{c_{,N}}{c}L_{hh}.$$

We write this action as $S_{hh} + S_{shh} = \int \mathrm{d}t\mathrm{d}^3x\sqrt{-Z}Z^{\mu\nu}\partial_\mu h_{ij}\partial_\nu h_{ij}.$

$Z_{\mu\nu}$: Effective metric for GWs

If the effective metric is conformal to the background metric, i.e., $Z_{\mu\nu} = \Omega^2(x)g_{\mu\nu}$,
the speed of GWs are identical to the speed of light.

In our case, the background metric satisfies,

$$\sqrt{-g}g^{00} = -a^3(1-\alpha+3\zeta), \quad \sqrt{-g}g^{0i} = a\partial_i\chi, \quad \sqrt{-g}g^{ij} = a(1+\alpha+\zeta)\delta_{ij} \quad \Rightarrow \quad Z_{\mu\nu} \propto g_{\mu\nu}$$

GWs in an inhomogeneous universe

Next, we consider the terms

$$L = N\sqrt{\gamma}c_1^{(3,0)}(t,N)(K_{ij}K^{jk}K_k^i + RK)$$

In this case, the action for GWs are

$$L_{hh} = \frac{3}{4}c_1^{(3,0)}(t,1)a^3H\left[\dot{h}_{ij}^2 - \frac{1}{a^2}(\partial_k h_{ij})^2\right]$$

$$L_{shh} = a^3c_1^{(3,0)} \cdot \frac{3H}{4}\left[(-\alpha+3\zeta)\dot{h}_{ij}^2 - 2a^{-2}\partial_k\chi\dot{h}_{ij}\partial_k h_{ij} - (\alpha+\zeta)a^{-2}\left(\partial_k h_{ij}\right)^2\right] + \delta\Omega L^{(2)}$$

$$-\frac{3a}{4}c_1^{(3,0)}\tilde{\chi}_{ij}\dot{h}_{ik}\dot{h}_{kj}$$

This term changes the speed of GWs.

$$\delta\Omega(t,\vec{x}) := -\alpha + \frac{c_{1,N}^{(3,0)}}{c_1^{(3,0)}}\alpha + \frac{1}{H}\left(\dot{\zeta} - \frac{\partial^2\chi}{3a^2}\right)$$

$$\tilde{\chi}_{ij} := \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\partial^2\right)\chi$$

In FLRW spacetime, the speed of GWs is equal to the speed of light.
But if spacetime is slightly inhomogeneous, the speed of GWs is not equal to the speed of light.

Summary

- We studied the propagation of GWs in an inhomogeneous universe in spatially covariant theory.
- We have shown that the propagation speed of gravitational waves changes in an inhomogeneous universe.
- We can test the theories of gravity and inhomogeneity of Universe from the observations of the speed of GWs.

Poster session

Kota Ogasawara

Kyoto University

"Photon escape in the extremal Kerr black hole spacetime"

[JGRG30 (2021) PA7]

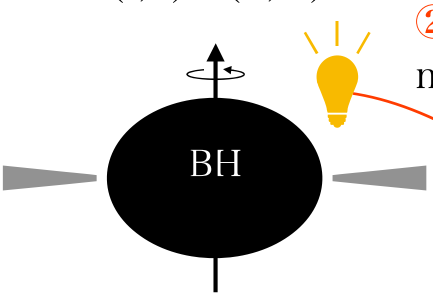
Photon escape in the extremal Kerr black hole spacetime

[P7] Kota Ogasawara (Kyoto Univ.)

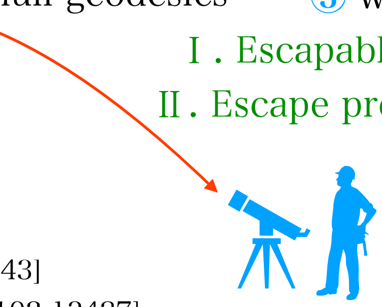
collaborator [B1b4] Takahisa Igata (KEK)

① emission point

$(r, \theta) = (r_*, \theta_*)$



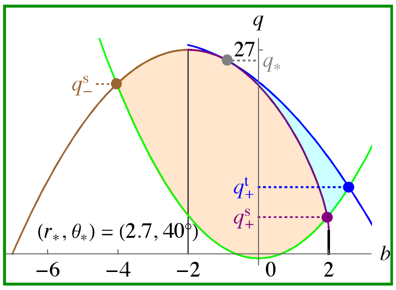
② investigate null geodesics

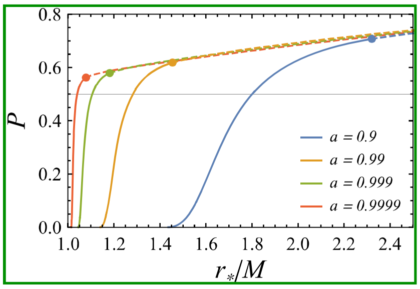


③ we obtain

I. Escapable region

II. Escape probability





Ref.

[1] KO and Igata [2111.03243]

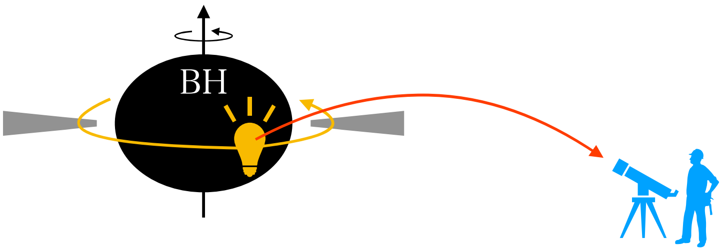
[2] Igata, Kohri, and KO [2102.13427]

Introduction

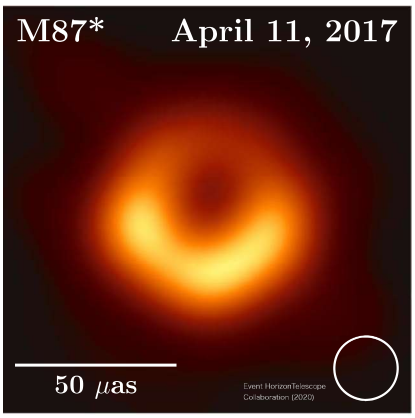
Photon escape & Escape probability ⇔ Observability

Black hole (BH) observation

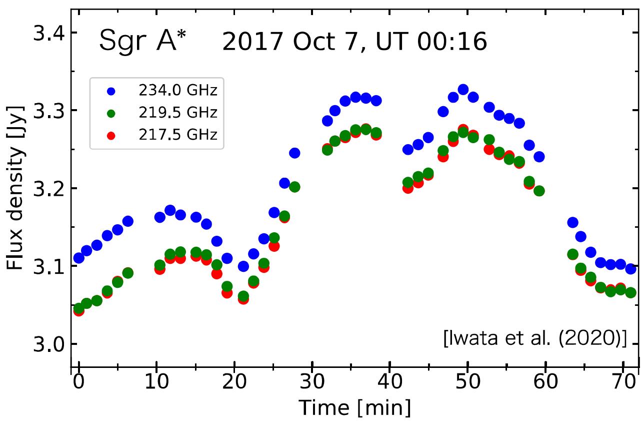
it must receive signals from sources near the BH



BH shadow

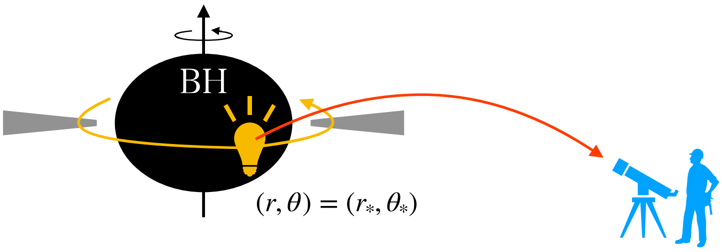


Flux variations



Introduction

① isotropic emitter (source)
- ✤ circular orbit
- ✤ plunge orbit
- ✤ rest at LNRF

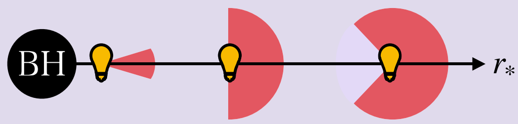


② photon escape conditions

→ investigate the null geodesics

③ observability
- ✤ photon escape probability $P(\gamma_*; a)$
- ✤ frequency shift $z(\gamma_*; a)$

generally : as source approach BH..
- P decreases
- strongly redshifted



photon escape cone shrinks

- source orbits γ_* { position (r_*, θ_*) ; proper motion }
- BH spin a

Ⅰ. Photon escape in the extremal Kerr black hole spacetime

[1] KO and Takahisa Igata
arXiv: 2111.03243

Photon Motion in Kerr BH

we use units in which $c = G = M = 1$

Kerr metric (mass M, spin a)

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\frac{\Sigma\Delta}{A}\mathrm{d}t^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma \mathrm{d}\theta^2 + \frac{A}{\Sigma}\sin^2\theta\left(\mathrm{d}\varphi - \frac{2Mar}{A}\mathrm{d}t\right)^2$$

$$\Sigma \equiv r^2 + a^2\cos^2\theta,\ \Delta \equiv r^2 - 2Mr + a^2,\ A \equiv \left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta$$

Constants of motion

$$E \equiv -\xi^a k_a = -k_t,\ L \equiv \psi^a k_a = k_\varphi$$

$$\mathcal{Q} \equiv K_{ab}k^a k^b - (L - aE)^2$$

Killing vectors: ξ^a and ψ^a, Killing tensor: K_{ab}

$\Rightarrow$ **I**mpact parameters

$$b \equiv \frac{L}{E},\ q \equiv \frac{\mathcal{Q}}{E^2}$$

Equation of motion

$$k^r = \dot{r} = \frac{\sigma_r}{\Sigma}\sqrt{R},\ k^\theta = \dot{\theta} = \frac{\sigma_\theta}{\Sigma}\sqrt{\Theta}$$

$$R(r) \equiv \left(r^2 + a^2 - ab\right)^2 - \Delta\left[q + (b-a)^2\right]$$

$$\Theta(\theta) \equiv q - \cos^2\theta\left(\frac{b^2}{\sin^2\theta} - a^2\right)$$

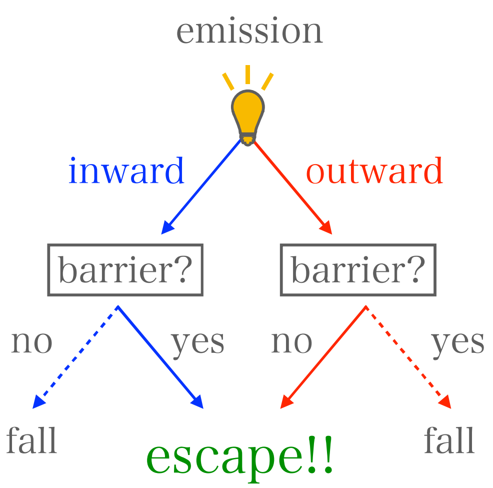
$\Rightarrow$ **A**llowed regions

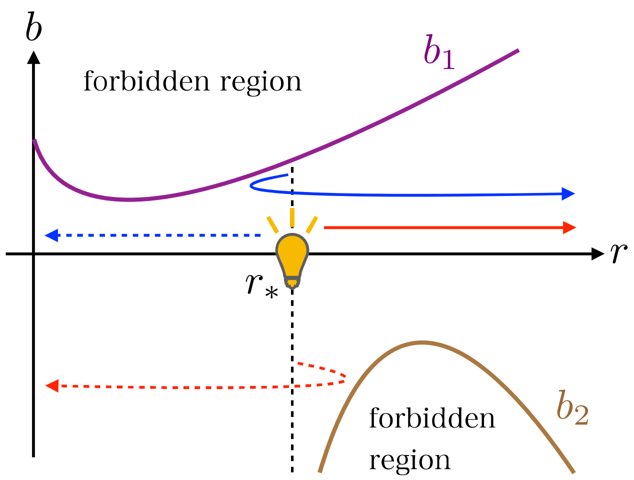
$$R(r) \geq 0,\ \Theta(\theta) \geq 0$$

Photon Dynamics

Radial eq: $\dot{r}^2 = \dfrac{r(2-r)}{\Sigma^2}\left[b - b_1(r;q)\right]\left[b - b_2(r;q)\right]$

$$b_{1,2}(r;q) = \frac{-2ar \pm \sqrt{r\Delta\left[r^3 - q(r-2)\right]}}{r(r-2)}$$

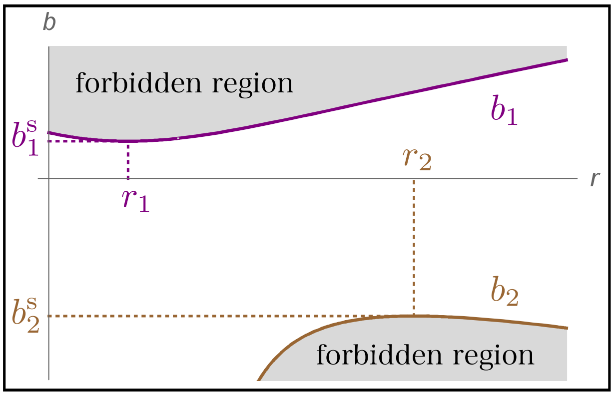




Allowed Parameter Region of b

Turning points: $R = 0$

$\rightarrow b = b_1(r; q)$ and $b = b_2(r; q)$



Typical shape of $b_1(r;q)$ and $b_2(r;q)$
$b_2(r;q)$ in the range $r_{\rm H} \leq r < 2$ is not plotted

Allowed region of b derived from $R \geq 0$

$b \leq b_1, \; b_2 \leq b \quad$ for $\quad r_{\rm H} \leq r < 2$

$b_2 \leq b \leq b_1 \quad$ for $\quad r \geq 2$

r-motion: necessary conditions for photons to reach infinity

Allowed region of b derived from $\Theta \geq 0$

$|b| \leq B(\theta; q) \equiv \tan\theta \sqrt{q + a^2 \cos^2\theta}$

θ-motion: further restricts the allowed region of photon motion

necessary & sufficient conditions "escapable region"

!! If $\theta = \pi/2$, $\Theta \geq 0 \Rightarrow q \geq 0$

$\Rightarrow$ no restriction of b

Spherical Photon Orbits (SPO)

Spherical photon orbits: $R = R' = 0$

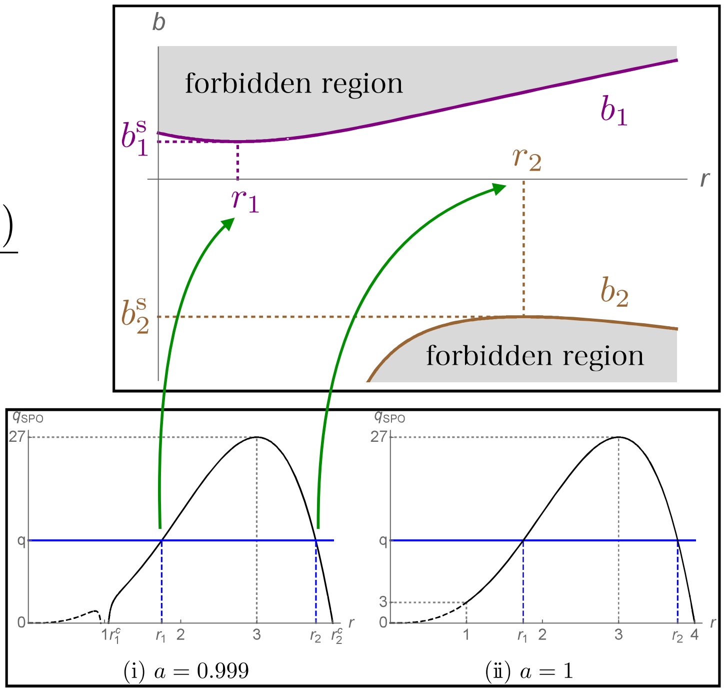
$$b = b_{\rm SPO}(r) \equiv -\frac{r^3 - 3r^2 + a^2 r + a^2}{a(r-1)}$$

$$q = q_{\rm SPO}(r) \equiv -\frac{r^3\left(r^3 - 6r^2 + 9r - 4a^2\right)}{a^2\left(r-1\right)^2}$$

Radii of SPO: solving $q = q_{\rm SPO}(r)$ for r

$r = r_1(q)$: extremal point of b_1

$r = r_2(q)$: extremal point of b_2



Relation between q and the radii r_i $(i = 1,2)$
of the extremum points of b_i

Impact parameter for SPO

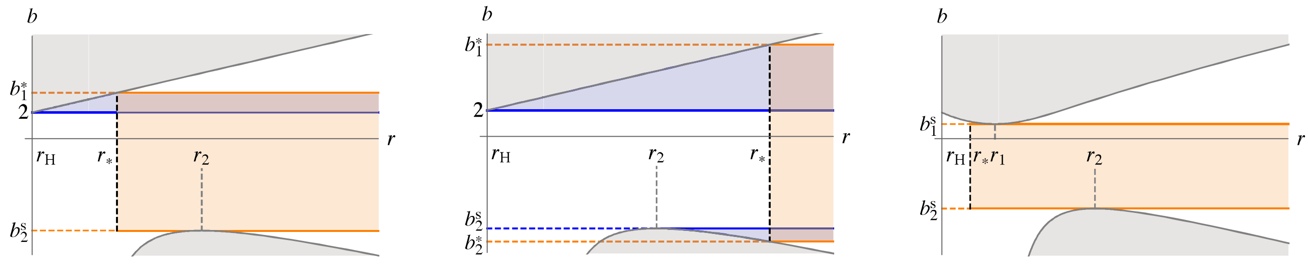
$b_1^{\rm s}(q) \equiv b_{\rm SPO}(r_1)$: extremal value of b_1

$b_2^{\rm s}(q) \equiv b_{\rm SPO}(r_2)$: extremal value of b_2

circular photon orbits: $r_1^{\rm c} \equiv r_1(0) = 2 + 2\cos\left[\frac{2}{3}\arccos(a) - \frac{2\pi}{3}\right]$

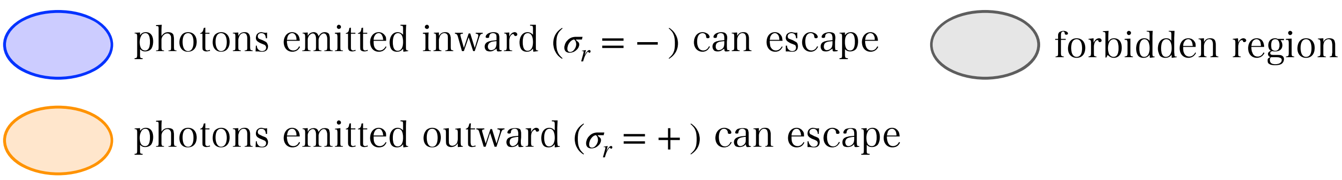
$r_2^{\rm c} \equiv r_2(0) = 2 + 2\cos\left[\frac{2}{3}\arccos(a)\right]$

Necessary Conditions for Escape



(a) $r_1 < r_H < r_* \leq r_2$ (b) $r_1 < r_H < r_2 < r_*$ (c) $r_H < r_* \leq r_1$

(d) $r_H \leq r_1 < r_* \leq r_2$ (e) $r_H \leq r_1 < r_2 < r_*$ (f) $q \geq 27$



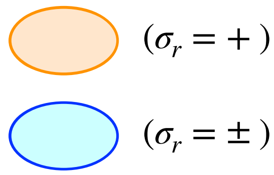
Necessary Conditions for Escape

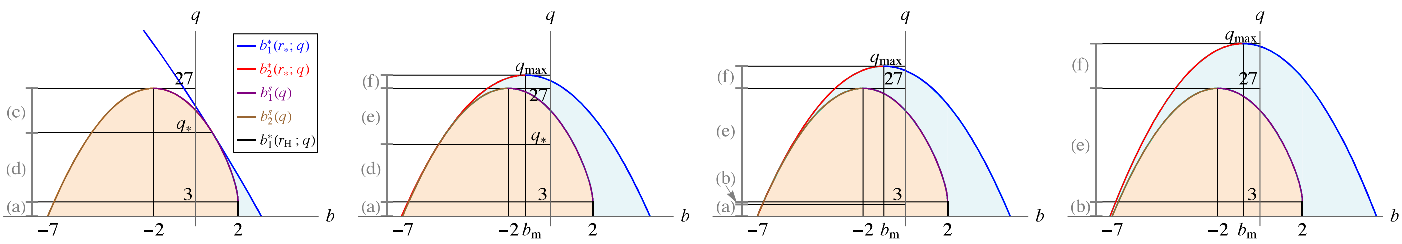
Fixing r_* determines the necessary conditions for escapable region (b, q)

TABLE I: Necessary conditions for photon escape from $r = r_*$ to infinity. Here "n/a" means not applicable.

Radial position of an emitter	q	$b\ (\sigma_r = +)$	$b\ (\sigma_r = -)$	Shape of b_i
(i) $r_H < r_* < 3$	$0 \leq q < 3$	$b_2^s < b \leq b_1^*$	$2 < b < b_1^*$	Fig. 2(a)
	$3 \leq q < q_*$	$b_2^s < b \leq b_1^*$	$b_1^s < b < b_1^*$	Fig. 2(d)
	$q_* \leq q \leq 27$	$b_2^s < b < b_1^s$	n/a	Fig. 2(c)
(ii) $3 \leq r_* < \tilde{r}$	$0 \leq q < 3$	$b_2^s < b \leq b_1^*$	$2 < b < b_1^*$	Fig. 2(a)
	$3 \leq q < q_*$	$b_2^s < b \leq b_1^*$	$b_1^s < b < b_1^*$	Fig. 2(d)
	$q_* \leq q < 27$	$b_2^* \leq b \leq b_1^*$	$b_2^* < b < b_2^s,\ b_1^s < b < b_1^*$	Fig. 2(e)
	$27 \leq q \leq q_{\max}$	$b_2^* \leq b \leq b_1^*$	$b_2^* < b < b_1^*$	Fig. 2(f)
(iii) $\tilde{r} \leq r_* < 4$	$0 \leq q < q_*$	$b_2^s < b \leq b_1^*$	$2 < b < b_1^*$	Fig. 2(a)
	$q_* \leq q < 3$	$b_2^* \leq b \leq b_1^*$	$b_2^* < b < b_2^s,\ 2 < b < b_1^*$	Fig. 2(b)
	$3 \leq q < 27$	$b_2^* \leq b \leq b_1^*$	$b_2^* < b < b_2^s,\ b_1^s < b < b_1^*$	Fig. 2(e)
	$27 \leq q \leq q_{\max}$	$b_2^* \leq b \leq b_1^*$	$b_2^* < b < b_1^*$	Fig. 2(f)
(iv) $r_* \geq 4$	$0 \leq q < 3$	$b_2^* \leq b \leq b_1^*$	$b_2^* < b < b_2^s,\ 2 < b < b_1^*$	Fig. 2(b)
	$3 \leq q < 27$	$b_2^* \leq b \leq b_1^*$	$b_2^* < b < b_2^s,\ b_1^s < b < b_1^*$	Fig. 2(e)
	$27 \leq q \leq q_{\max}$	$b_2^* \leq b \leq b_1^*$	$b_2^* < b < b_1^*$	Fig. 2(f)

$$q_* = q_{\rm SPO}(r_*)$$



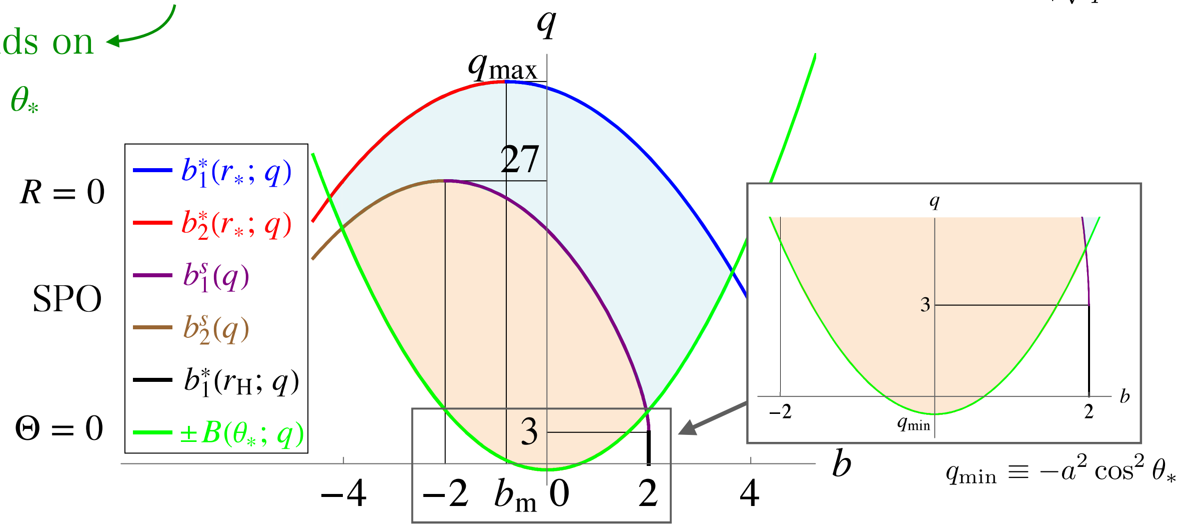


(i) $r_H < r_* < 3$ (ii) $3 \leq r_* < \tilde{r}$ (iii) $\tilde{r} \leq r_* < 4$ (iv) $r_* \geq 4$

Necessary & Sufficient Conditions

Escapable region: "necessary conditions" + "$\Theta \geq 0$" $\leftarrow$ $|b| \leq B(\theta_*; q) \equiv \tan\theta_* \sqrt{q + a^2\cos^2\theta_*}$

depends on r_* and θ_*



$q_{\min} \equiv -a^2\cos^2\theta_*$

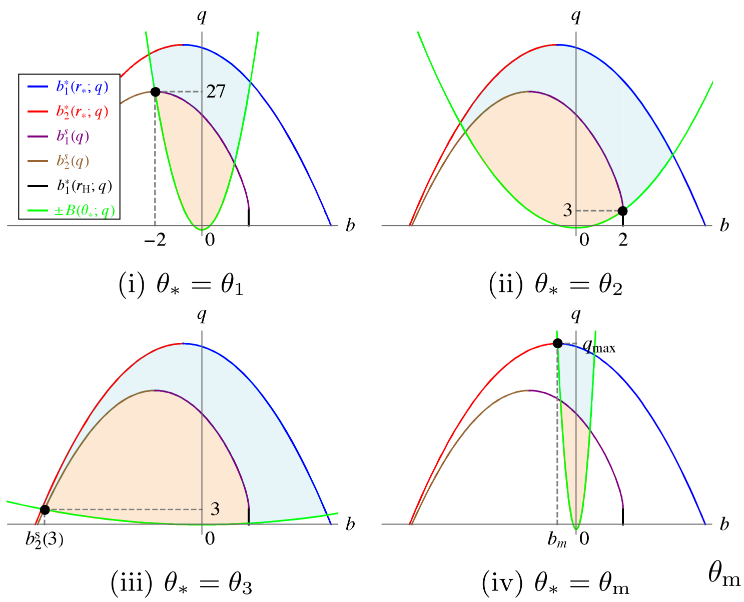
photons emitted only radially outward ($\sigma_r = +$) can escape to infinity

photons emitted both radially outward & inward ($\sigma_r = \pm$) can escape to infinity

Critical Angles

Critical angles: special intersections of $b = b_i^s(q)$, $b = b_i(r_*; q)$ and $b = \pm B(\theta_*; q)$

$\Rightarrow$ the classification of the escapable region varies qualitatively



(i) $\theta_* = \theta_1$ (ii) $\theta_* = \theta_2$

(iii) $\theta_* = \theta_3$ (iv) $\theta_* = \theta_{\rm m}$

Solving $\Theta = 0$ for θ, we obtain

$$\theta = \tilde{\theta}(b, q) = \arcsin\left[\frac{\left|\sqrt{(b+1)^2 + q} - \sqrt{(b-1)^2 + q}\right|}{2}\right]$$

$$\theta_1 = \tilde{\theta}(-2, 27) = \arcsin\left(3 - \sqrt{7}\right) \simeq 20.7^\circ$$

$$\theta_2 = \tilde{\theta}(2, 3) = \arcsin\left(\sqrt{3} - 1\right) \simeq 47.1^\circ$$

$$\theta_3 = \tilde{\theta}(b_2^s(3), 3) \simeq 75.4^\circ$$

$$\theta_{\rm m}(r_*) = \tilde{\theta}(b_{\rm m}(r_*), q_{\max}(r_*)) = \arcsin\left[\frac{r(r-1) - \sqrt{r^2(r-1)^2 - 8(r-2)}}{2(r-2)}\right]\Bigg|_{r=r_*}$$

Critical Values of q

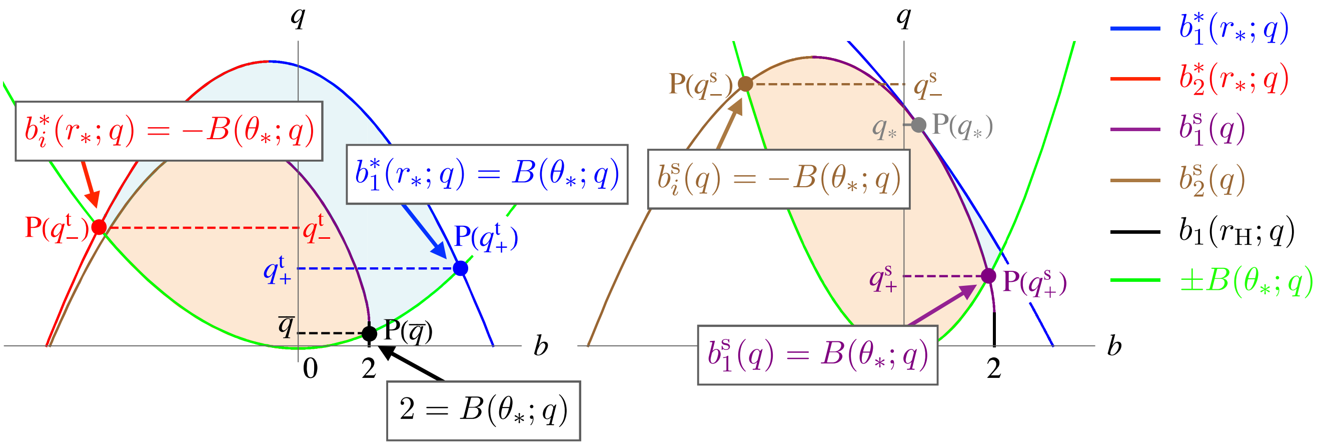
Critical values of q:

special intersections of $b = b_1(r_{\mathrm{H}}; q)$, $b_i(r_*; q)$, $b_i^{\mathrm{s}}(q)$ and $\pm B(\theta_*; q)$

$\Rightarrow$ the classification of the parameter ranges varies qualitatively

TABLE II: Definition of special points P on the b-q plane and the values of q and b at these points.

P	Intersection	q	b
$\mathrm{P}(\bar{q})$	$b = 2$ and $b = B$	$\bar{q}(\theta_*)$	$2 = B(\theta_*; \bar{q})$
$\mathrm{P}(q_+^{\mathrm{t}})$	$b = b_1^*$ and $b = B$	$q_+^{\mathrm{t}}(r_*, \theta_*)$	$b_1^*(r_*; q_+^{\mathrm{t}}) = B(\theta_*; q_+^{\mathrm{t}})$
$\mathrm{P}(q_-^{\mathrm{t}})$	$b = b_i^*$ and $b = -B$	$q_-^{\mathrm{t}}(r_*, \theta_*)$	$b_i^*(r_*; q_-^{\mathrm{t}}) = -B(\theta_*; q_-^{\mathrm{t}})$
$\mathrm{P}(q_+^{\mathrm{s}})$	$b = b_1^{\mathrm{s}}$ and $b = B$	$q_+^{\mathrm{s}}(\theta_*)$	$b_1^{\mathrm{s}}(q_+^{\mathrm{s}}) = B(\theta_*; q_+^{\mathrm{s}})$
$\mathrm{P}(q_-^{\mathrm{s}})$	$b = b_i^{\mathrm{s}}$ and $b = -B$	$q_-^{\mathrm{s}}(\theta_*)$	$b_i^{\mathrm{s}}(q_-^{\mathrm{s}}) = -B(\theta_*; q_-^{\mathrm{s}})$
$\mathrm{P}(q_*)$	–	$q_*(r_*)$	$b_1^*(r_*; q_*)$ for $r_* < 3$
			$b_2^*(r_*; q_*)$ for $3 \le r_* \le 4$



Classification of Photon Escape

Relationship of characteristic q's

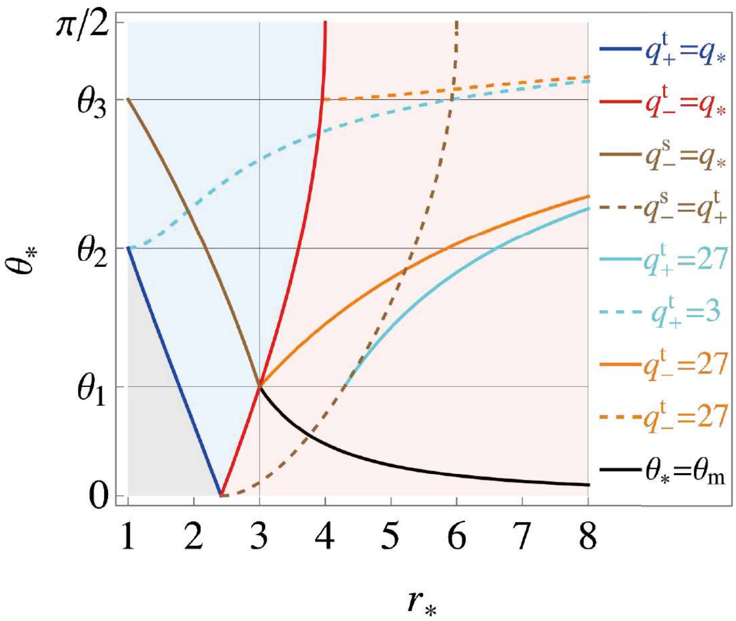


TABLE III: Definition of each Class and the characteristic values of q that appear in the classification in each class.

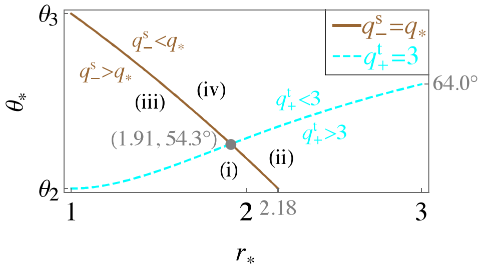
Class	Range of (r_*, θ_*)	characteristic q's
I	$r_* < 3$ and $\theta_* \in (0, \theta_1)$	q_*, $q_\pm^{\mathrm{t}}$, and $q_\pm^{\mathrm{s}}$
II	$r_* < 3$ and $\theta_* \in [\theta_1, \theta_2)$	q_*, q_+^{t}, and $q_\pm^{\mathrm{s}}$
III	$r_* < 3$ and $\theta_* \in [\theta_2, \theta_3)$	q_*, $\bar{q}$, q_+^{t}, q_-^{s}, and 3
IV	$r_* < 3$ and $\theta_* \in [\theta_3, \pi/2)$	q_*, $\bar{q}$, q_+^{t}, q_-^{s}, and 3
V	$r_* \ge 3$ and $\theta_* \in (0, \theta_1)$	$q_\pm^{\mathrm{t}}$ and $q_\pm^{\mathrm{s}}$
VI	$r_* \ge 3$ and $\theta_* \in [\theta_1, \theta_2)$	q_*, $q_\pm^{\mathrm{t}}$, $q_\pm^{\mathrm{s}}$, and 27
VII	$r_* \ge 3$ and $\theta_* \in [\theta_2, \theta_3)$	q_*, $\bar{q}$, $q_\pm^{\mathrm{t}}$, q_-^{s}, 3, and 27
VIII	$r_* \ge 3$ and $\theta_* \in [\theta_3, \pi/2)$	q_*, $\bar{q}$, $q_\pm^{\mathrm{t}}$, q_-^{s}, 3, and 27

Escapable parameters are different for each region separated by each curve, i.e., the shape of the escapable region is different. $\to$ 37 cases in total.

Escapable Region: Class III ($r_* < 3$ and $\theta_2 \le \theta_* < \theta_3$)

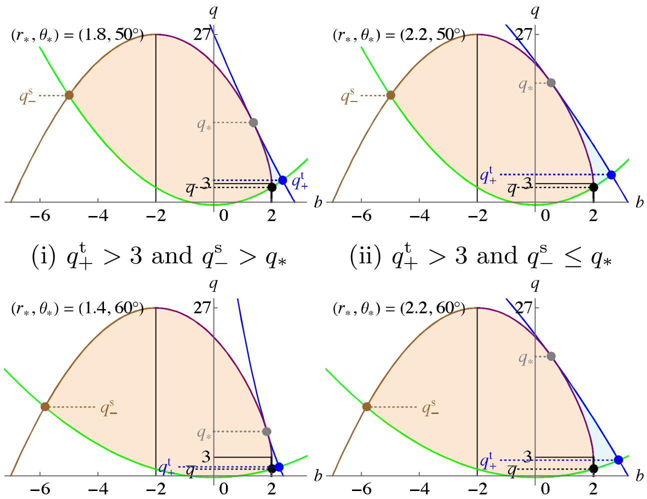
see [1] for other classes

Relationship of characteristic q's



There are four cases according to the relative values of q_+^t and 3, and q_-^s and q_*.

Case	q	b ($\sigma_r=+$)	b ($\sigma_r=-$)
(i)–(iv)	$q_{\min}\le q<\bar{q}$	$-B\le b\le B$	n/a
(i) and (ii)	$\bar{q}\le q<3$	$-B\le b\le B$	$2<b\le B$
	$3\le q<q_+^t$	$-B\le b\le B$	$b_1^s<b\le B$
(iii) and (iv)	$\bar{q}\le q<q_+^t$	$-B\le b\le B$	$2<b\le B$
	$q_+^t\le q<3$	$-B\le b\le b_1^*$	$2<b<b_1^*$
(i)	$q_+^t\le q<q_*$	$-B\le b\le b_1^*$	$b_1^s<b<b_1^*$
(ii)	$q_+^t\le q<q_-^s$		
(iii)	$3\le q<q_*$		
(iv)	$3\le q<q_-^s$		
(i) and (iii)	$q_*\le q<q_-^s$	$-B\le b<b_1^s$	n/a
	$q_-^s\le q\le 27$	$b_2^s<b<b_1^s$	n/a
(ii) and (iv)	$q_-^s\le q<q_*$	$b_2^s<b\le b_1^*$	$b_1^s<b<b_1^*$
	$q_*\le q\le 27$	$b_2^s<b<b_1^s$	n/a



(i) $q_+^t>3$ and $q_-^s>q_*$ (ii) $q_+^t>3$ and $q_-^s\le q_*$

(iii) $q_+^t\le 3$ and $q_-^s>q_*$ (iv) $q_+^t\le 3$ and $q_-^s\le q_*$

II. Photon emission from inside the innermost stable circular orbit

[2] Takahisa Igata, Kazunori Kohri, and KO
Phys. Rev. D **103**, 104028 (2021). [2102.13427]

Source Dynamics (isotropic emitter, $\theta_* = \pi/2$)

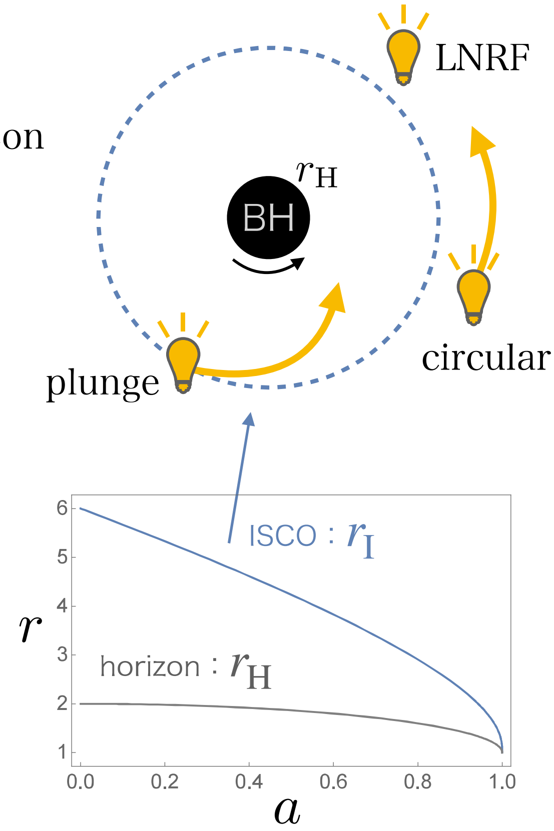
Plunge orbits: $r_*(\tau) < r_{\rm I}$

innermost stable circular orbit (ISCO) → horizon
gently leaves the ISCO and falls into BH

$$r_*(\tau_i) = r_{\rm I} - \varepsilon \quad (0 < \varepsilon \ll 1)$$

$$E_{\rm I} = \frac{r_{\rm I}^{3/2} - 2r_{\rm I}^{1/2} + a}{r_{\rm I}^{3/4}(r_{\rm I}^{3/2} - 3r_{\rm I}^{1/2} + 2a)^{1/2}}$$

$$L_{\rm I} = \frac{r_{\rm I}^2 - 2ar_{\rm I}^{1/2} + a^2}{r_{\rm I}^{3/4}(r_{\rm I}^{3/2} - 3r_{\rm I}^{1/2} + 2a)^{1/2}}$$



Circular orbits: $r_* \geq r_{\rm I}$

LNRF: $r_* \geq r_{\rm H}$

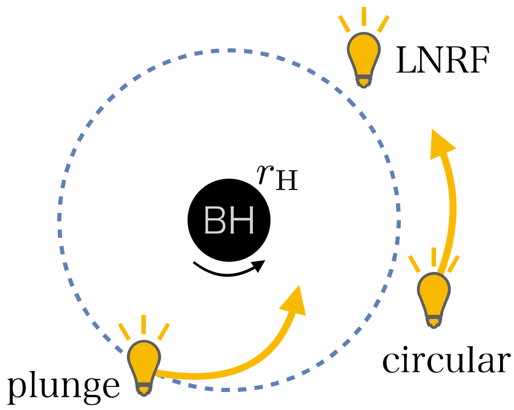
Locally Non Rotating Frame
→ no proper motion

Escape Probability

Step 1. construct comoving frame $\{\zeta^\mu\}$

$$\zeta^{(0)}_{\rm plunge} = -E{\rm d}t - \frac{\sqrt{R}}{\Delta}{\rm d}r + L{\rm d}\varphi$$

$$\zeta^{(0)}_{\rm circular} = -E{\rm d}t + L{\rm d}\varphi, \quad \zeta^{(0)}_{\rm LNRF} = \sqrt{\frac{\Delta\Sigma}{A}}{\rm d}t$$

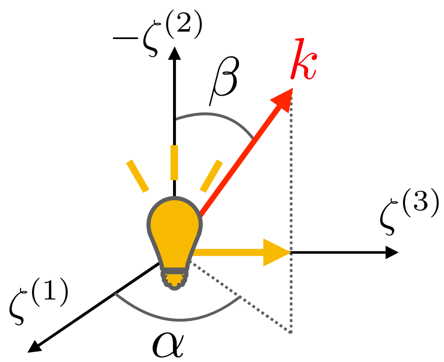


Step 2. specify tetrad components of k

$$k^{(\mu)} = \zeta^{(\mu)}_a k^a$$

Step 3. define the emission angles (α, β)

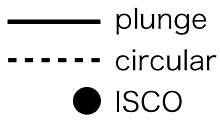
$$\cos\alpha \sin\beta = \frac{k^{(1)}}{k^{(0)}}, \quad \cos\beta = \frac{k^{(2)}}{k^{(0)}}, \quad \sin\alpha\sin\beta = \frac{k^{(3)}}{k^{(0)}}$$

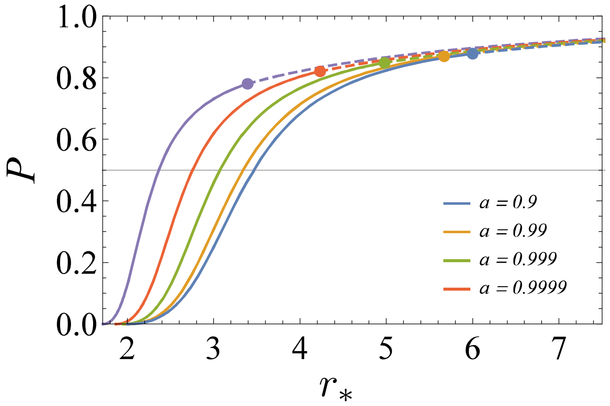


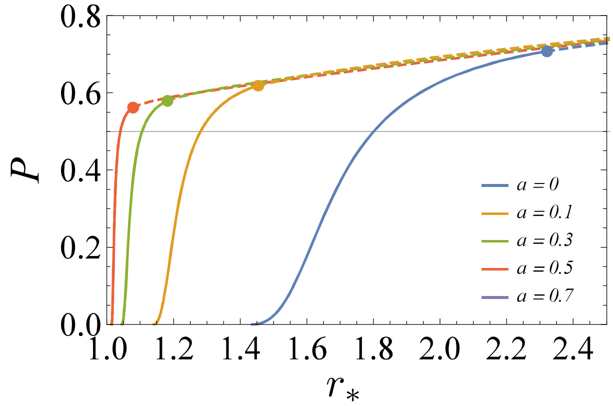
Step 4. integrate over the escapable region

$$P(r_*; a) = \frac{1}{4\pi}\iint_{\mathcal{E}} {\rm d}\alpha {\rm d}\beta \sin\beta$$

Results: Photon Escape Probability

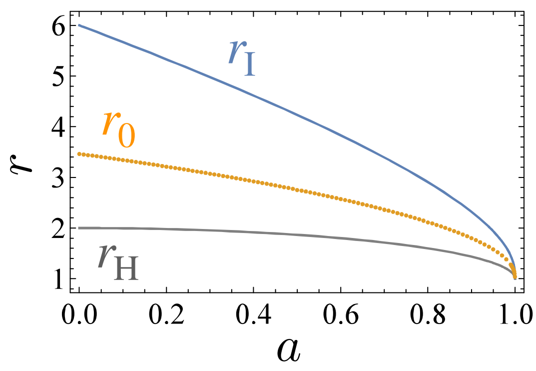




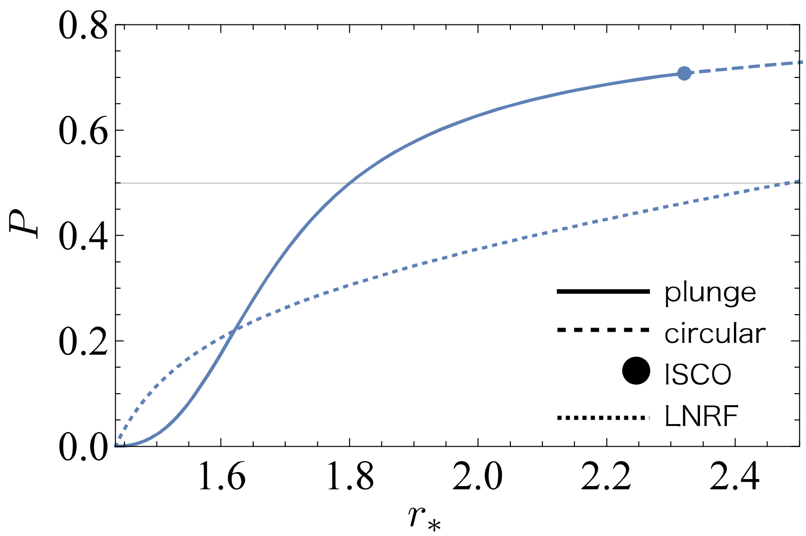


- P monotonically decreases as r_* decreases
- $P > 50\%$ for any stable circular orbits
- $P \to 0$ in the limit $r_* \to r_{\rm H}$
- Radial position where $P = 50\%$

$$r_0 \simeq \frac{r_{\rm I} + r_{\rm H}}{2}$$



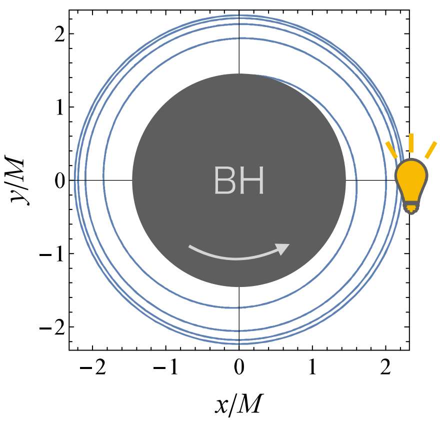
Results: Proper Motion Effect (P vs $P_{\rm LNRF}$)



Early stage: $P > P_{\rm LNRF}$

proper φ-motion is dominant

→ outward relativistic beaming is significant

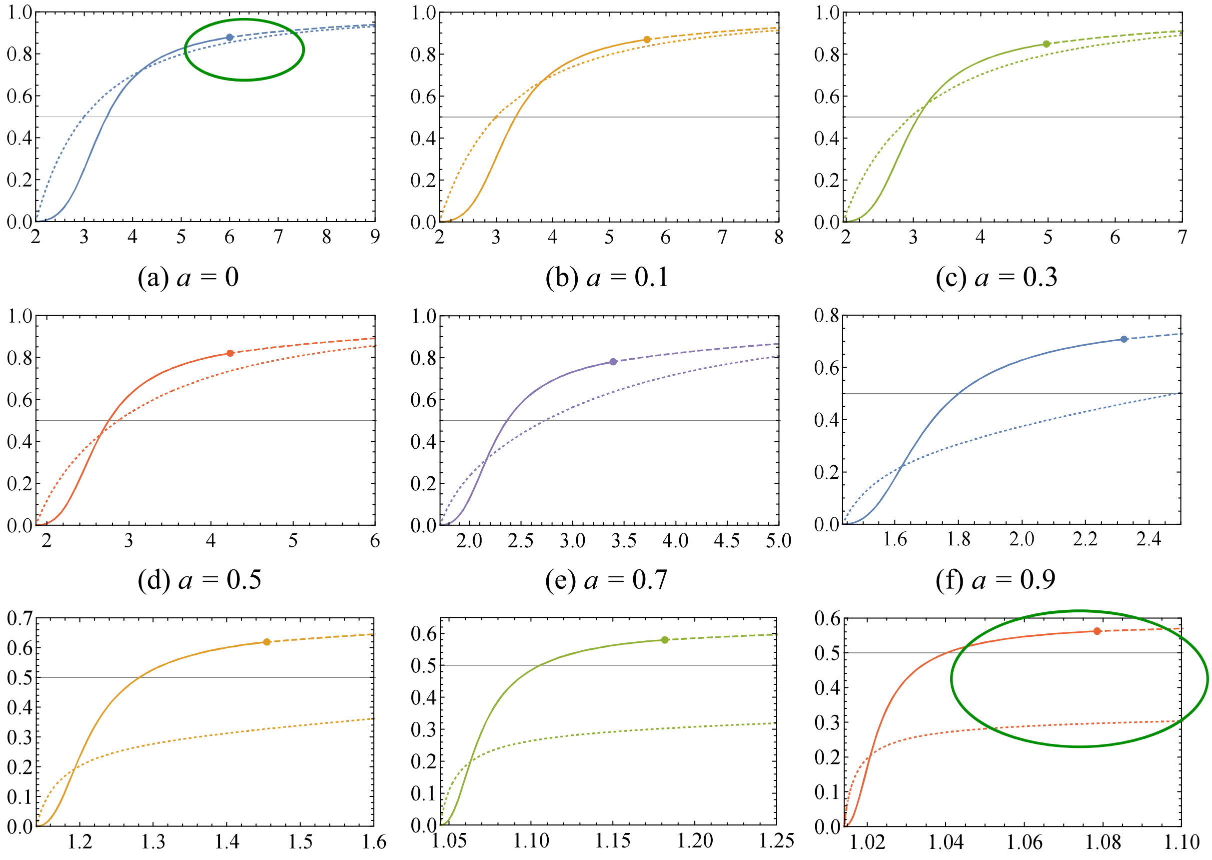


Final stage: $P < P_{\rm LNRF}$

proper r-motion is dominant

→ inward relativistic beaming is significant

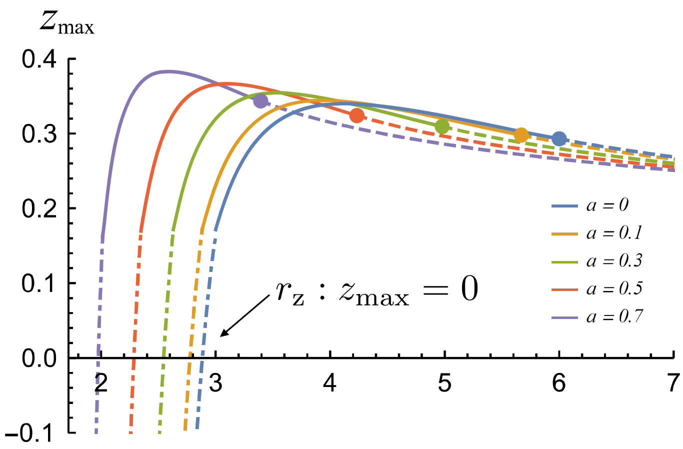
Results: Proper Motion Effect (P vs P_{LNRF})



(a) $a = 0$ (b) $a = 0.1$ (c) $a = 0.3$ (d) $a = 0.5$ (e) $a = 0.7$ (f) $a = 0.9$ (g) $a = 0.99$ (h) $a = 0.999$ (i) $a = 0.9999$

Results: Maximum Blue Shift

$$z_{\max} = \max\left(1 + k^{(0)}\right)$$



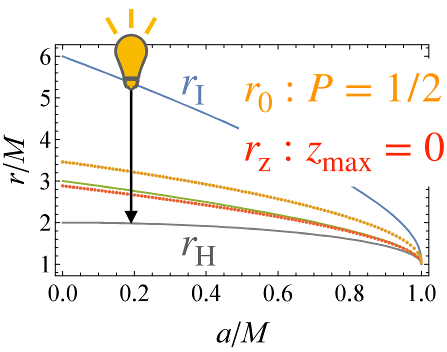
Early stage: $z_{\max} > 0$

Doppler effect > Gravitational redshift
→ outward relativistic beaming

Final stage: $z_{\max} < 0$

Doppler effect < Gravitational redshift
→ inward relativistic beaming

- There are always blue shifted photons in the early stage and $P > 50\%$.
- $P = 50\%$ in the middle stage, but $z > 0$ still exist.
- At the final stage, there are no blueshifted photons and $P \to 0$.



SUMMARY Ⅰ. [1] KO and Takahisa Igata, arXiv: 2111.03243

We have completely classified the range of impact parameters (b, q) necessary and sufficient for photons emitted from an arbitrary spacetime position of the extremal Kerr black hole to escape to infinity, i.e., “escapable region”.

Main calculation

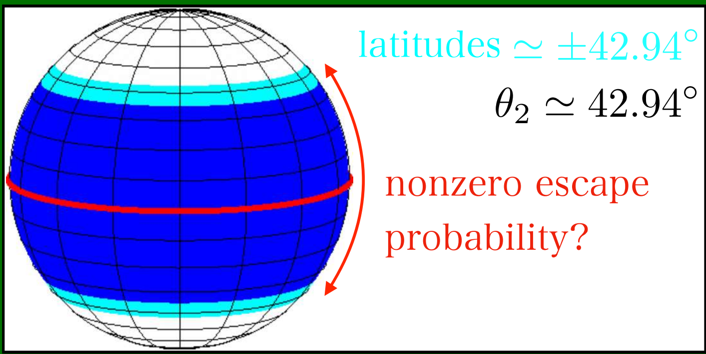
- $R(r) \geq 0$
- SPO $(R = R' = 0)$

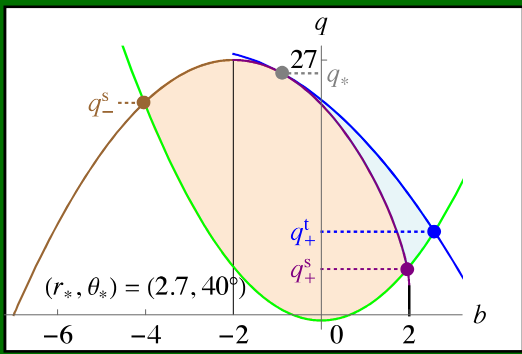
→ necessary conditions for photon escape

- $\Theta(\theta) \geq 0$

→ necessary and sufficient conditions for photon escape “escapable region”

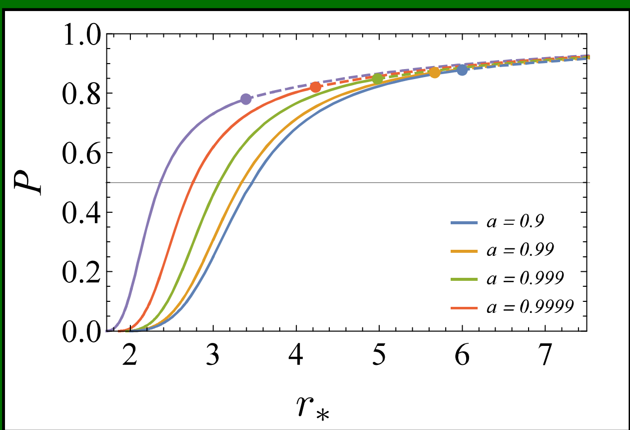
Collision belt, Harada & Kimura (2011)

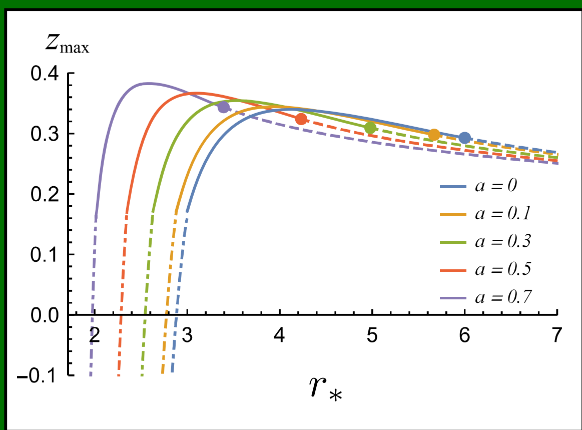




SUMMARY Ⅱ. [2] Takahisa Igata, Kazunori Kohri, and KO Phys. Rev. D **103**, 104028 (2021). [2102.13427]

- Evaluated the escape probability and maximum blueshift of photons along the orbits of a source falling “gently” from ISCO.
- $P(r_*)$ gradually decreases as the source falls into BH.
- $P = 50\%$ is halfway between the ISCO radius and the horizon radius.
- Due to the effect of proper motion,
  - ✓ $P(r_*)$ tends to be larger and $z_{\max} > 0$ in the former half of falling
  - ✓ $P(r_*)$ tends to be smaller and $z_{\max} < 0$ in the latter half of falling





Poster session

Masato Nozawa

Osaka Institute of Technology

“Gauged supergravities in six dimensions from orientifold compactifications”

[JGRG30 (2021) PA8]

Gauged supergravities in six dimensions from orientifold compactifications

Masato Nozawa

Based on JHEP 05 (2020) 015
w/ G. Dibitetto (Padua) & J-F. Melgarejo (Murcia)

OSAKA INSTITUTE OF TECHNOLOGY

SYNOPSIS

theoretical challenges in lower dimensional *extended supergravities*

- non-geometric flux
- double/exceptional field theory
- de Sitter vacua & moduli stabilization

Q. Is the rareness of dS vacua intrinsic to D=4 supergravity?

we explore the vacuum hunting in $D = 6$ & $\mathcal{N} = (1,1)$ supergravity

IIA/B string/M-theory $\xrightarrow{\text{orientifold compactifications}}$ 6D N=(1,1) SUGRA

- embedding tensor/flux dictionary
- general form of scalar potential
- new dS/AdS/Mink vacua

$\mathcal{N} = (1,1)$ gauged SUGRA in 6D

$\mathcal{N} = (1,1)$ SUGRA in 6D

- exhibits a novel duality $E_8 \times E_8$ hetero on $T^4 \Leftrightarrow$ IIA on K3 *Sen 1995*
- can be "deformed" in such a way that the scalar fields have a potential
- general Lagrangian yet unknown *Romans 1985, D'Auria, Ferrara, Valua 2000*

bosonic Lagrangian $\mathscr{L}_6 = R + 2\mathscr{L}_{\text{kin}} - V$

kinetic term $\underbrace{\mathbf{R}\times SO(4,4)}_{\text{T-duality}}/SO(4)^2$: $\mathcal{L}_{\text{kin}} = -2\Sigma^{-2}(\partial\Sigma)^2 + \frac{1}{16}\partial\mathcal{H}_{MN}\partial\mathcal{H}^{MN}$.

$\mathscr{H}_{MN}$: SO(4,4)/SO(4)^2

scalar potential V

- spontaneous SUSY breaking
- non-abelian gauge field
- fermion mass term
- expressed in quadratic form of *embedding tensor*

Embedding tensor

embedding tensor formalism: duality covariant gauging *deWit, Samtleben, Trigiante 2003*

gauge group G_{gauge} must a subgroup of duality group

$$\boxed{X_M = \Theta_M{}^\alpha t_\alpha}$$

X_M: gauge generator of G_{gauge}

$(t_\alpha)_M{}^N$: generator of duality group

embedding tensor $\Theta_M{}^\alpha$ specifies which subgroup is used to gaugings

gauging: gauge covariant derivative: $\partial_\mu \to D_\mu = \partial_\mu - gA_\mu{}^M X_M$.

Consistency requires

- linear constraint

$$\Theta = \{\underbrace{\zeta_M \in \mathbf{8}_c^{(+3)}}_{\text{massive def.}}, \underbrace{\xi_M \in \mathbf{8}_c^{(-1)}, f_{MNP} \in \mathbf{56}_c^{(-1)}}_{\text{gauging}}\} \quad \text{irrep. of } \mathbf{R}\times SO(4,4)$$

Bergshoeff, Gomis, Nutma, Roest 2008

- quadratic constraint

$$[X_M, X_N] = -X_{MN}{}^P X_P$$

$$\begin{aligned} 3f_{R[MN}f_{PQ]}{}^R - 2f_{[MNP}\xi_{Q]} &= 0 & \zeta_{(M}\xi_{N)} &= 0 \\ f_{MNP}\zeta^P - \xi_{[M}\zeta_{N]} &= 0 & \xi_M\xi^M &= 0 \\ f_{MNP}\xi^P &= 0 & \zeta_M\xi^M &= 0 \end{aligned}$$

Scalar potential

General form of scalar potential

$$V=\frac{g^2}{4}\left[f_{MNP}f_{QRS}\Sigma^{-2}\left(\tfrac{1}{12}\mathcal{H}^{MQ}\mathcal{H}^{NR}\mathcal{H}^{PS}-\tfrac{1}{4}\mathcal{H}^{MQ}\eta^{NR}\eta^{PS}+\tfrac{1}{6}\eta^{MQ}\eta^{NR}\eta^{PS}\right)\right.$$
$$\left.+\tfrac{1}{2}\zeta_M\zeta_N\Sigma^6\mathcal{H}^{MN}+\tfrac{2}{3}f_{MNP}\zeta_Q\Sigma^2\mathcal{H}^{MNPQ}+\tfrac{5}{4}\xi_M\xi_N\mathcal{H}^{MN}\Sigma^{-2}\right]\ ,$$

$\eta_{MN}=\eta^{MN}=\begin{pmatrix}0_4 & I_4\\ I_4 & 0_4\end{pmatrix}$: SO(4,4) invariant metric $\quad$ g: gauge coupling constant

procedure for derivation:

1) truncation of 6D $\mathcal{N}=(2,2)$ scalar potential *Bergshoeff, Samtleben, Sezgin 2007*

2) dimensional reduction of 5D $\mathcal{N}=4$ scalar potential *Schoen, Weidner 2006*

vacuum hunting:

1) solve *quadractic constraints* of embedding tensor

$\Rightarrow$ too complicated to solve exhaustively

2) extremize the scalar potential (wrt 1+16 scalar fields)

3) look for critical points & study stability

Our strategy: focus on the case/ orientifold compactification from IIA/IIB/M

Example: Type II string down to 6D

$$S^{\rm II}=\int \mathrm{d}^{10}x\sqrt{-g^{(10)}}\left(e^{-2\Phi}\left(\mathcal{R}^{(10)}+4(\partial\Phi)^2-\frac{1}{12}|H_{(3)}|^2\right)-\sum_p\frac{1}{2p!}|F_{(p)}|^2\right)+S_{\rm CS}\ ,$$

w/local sources: $$S^{(\mathrm{O}p/\mathrm{D}p)}=-T_p\int_{\mathcal{C}_{p+1}}\mathrm{d}^{p+1}x\sqrt{-\tilde{g}^{(p+1)}}e^{-\Phi}\ ,$$

Scherk-Schwarz reduction *Scherk, Schwarz 1979; de Witt 1964*

$$g^{(10)}_{MN}\mathrm{d}x^M\otimes\mathrm{d}x^N=\tau^{-2}g^{(6)}_{\mu\nu}\mathrm{d}x^\mu\otimes\mathrm{d}x^\nu+\rho\,\mathrm{d}s_4^2\ ,\qquad \rho^2\stackrel{!}{=}e^{2\Phi}\tau^4$$

group mfd

6D Lagrangian $\quad \mathcal{L}_6=\sqrt{-g^{(6)}}\left(\mathcal{R}^{(6)}+2\mathcal{L}_{\rm kin}-V\right)\ ,\quad$ 1+1+15 scalars

$$V=V_H+V_\omega+\sum_p V_{F_p}+V_{\mathrm{O}p/\mathrm{D}p}\ .\qquad \mathcal{L}_{\rm kin}=-\frac{(\partial\rho)^2}{2\rho^2}-\frac{2(\partial\tau)^2}{\tau^2}+\frac{1}{8}\mathrm{Tr}(\partial\mathcal{M}\partial\mathcal{M}^{-1})\ .$$

internal flux $\quad V_\omega\equiv-\rho^{-1}\tau^{-2}\,\mathcal{R}^{(4)}.\qquad \mathcal{M}_{mn}$: SL(4,$\mathbb{R}$)/SO(4)

$$\mathcal{R}^{(4)}=-\frac{1}{4}\mathcal{M}_{mq}\mathcal{M}^{nr}\mathcal{M}^{ps}\omega_{np}{}^q\omega_{rs}{}^m-\frac{1}{2}\mathcal{M}^{np}\omega_{mn}{}^q\omega_{qp}{}^m$$

Example: massive IIA w/O6/D6

O6 configurations $\underbrace{\overset{t}{\times}| \overset{x^\alpha}{\times \times \times \times \times}}_{6D} \underbrace{\overset{y^m}{\times - - -}}_{4D}$,

note: 6D part should preserve maximal Killing symmetry

orientifold involution: $\sigma_{O6} : \begin{cases} y^0 & \mapsto & y^0 \quad , \\ y^i & \mapsto & -y^i \quad , \quad i = 1,\, 2,\, 3 \ . \end{cases}$

SL(4,$\mathbb{R}$) → $\mathbb{R}^+ \times$ SL(3,$\mathbb{R}$)

retain only the even part for orientifold involution/fermion #/worldsheet parity

adj. of SL(4) under R×SL(3)

c.f Graña 2006

$$\mathbf{15} \longrightarrow \mathbf{1}_{(0)} \oplus \cancel{\mathbf{3}_{(-4)}} \oplus \cancel{\mathbf{3}'_{(+4)}} \oplus \mathbf{8}_{(0)} \ . \qquad \mathcal{M}_{mn} = \left(\begin{array}{c|c} \sigma^3 & \\ \hline & \sigma^{-1} M_{ij} \end{array} \right) ,$$

SL(3,$\mathbb{R}$)/SO(3)

$$H_{mnp} \in \mathbf{4}' \longrightarrow \mathbf{1}_{(-3)} \oplus \cancel{\mathbf{3}'_{(+1)}}$$

$$\omega_{mn}{}^p \in \mathbf{20} \longrightarrow \mathbf{3}_{(-1)} \oplus \cancel{\mathbf{3}'_{(-5)}} \oplus \mathbf{8}_{(+3)} \oplus \cancel{\mathbf{6}'_{(-1)}}$$

$$F_{(0)} \in \mathbf{1} \longrightarrow \mathbf{1}_{(0)}$$

$$F_{mn} \in \mathbf{6} \longrightarrow \cancel{\mathbf{3}_{(+2)}} \oplus \mathbf{3}'_{(-2)}$$

$$F_{mnpq} \in \mathbf{1} \longrightarrow \cancel{\mathbf{1}_{(0)}}$$

$H_{ijk} \equiv h\epsilon_{ijk}$

$\omega_{ij}{}^0 \equiv \theta_{ij} \qquad \omega_{0i}{}^j \equiv \kappa_i{}^j$

$F_{(0)} \equiv f_0$

$F_{0i} \equiv f_i$

Embedding tensor/flux dictionary

IIA Flux type	Flux parameters	σ_{O6}	$(-1)^{F_L}\Omega_p$	Θ components
$F_{(0)}$	$F_{(0)} = f_0$	$+$	$+$	$f_{\bar{i}\bar{j}\bar{k}} = f_0\ \epsilon_{\bar{i}\bar{j}\bar{k}}$
$F_{(2)}$	$F_{0i} = f_i$	$-$	$-$	$f_{\bar{1}\bar{j}\bar{k}} = f_i\ \epsilon^{ijk}$
$H_{(3)}$	$H_{ijk} = h\ \epsilon_{ijk}$	$-$	$-$	$\zeta_{\bar{1}} = h$
ω	$\omega_{ij}{}^0 = \theta_{ij} \equiv \epsilon_{ijk}\ \theta^k$	$+$	$+$	$\zeta_{\bar{i}} = \theta^i$
	$\omega_{0i}{}^j = \kappa_i{}^j$	$+$	$+$	$f_{\bar{1}i\bar{j}} = \kappa_i{}^j$

embedding of R×SL(3) into R×SO(4,4)

$$\Lambda = \tau^{-2} \qquad \Gamma = \rho^{1/2}\sigma^{1/2} \qquad \Sigma = \rho^{-1/4}\sigma^{3/4}$$

$$\mathcal{H}_{MN} = \left(\begin{array}{c|c} \begin{array}{c|c} \Lambda\Gamma^{-3} & 0 \\ \hline 0 & \Lambda\Gamma M_{ij} \end{array} & 0 \\ \hline 0 & \begin{array}{c|c} \Lambda^{-1}\Gamma^{3} & 0 \\ \hline 0 & \Lambda^{-1}\Gamma^{-1}M^{ij} \end{array} \end{array} \right)$$

quadratic constraints: $\boxed{\theta^i \kappa_i{}^j = 0}$

3×3 zero eigenvalue problem

⇒ *exhaustive classification is possible*

tadpole cancellation $\quad T_6 \equiv N_{D6} - 2N_{O6} \overset{!}{=} f_0 h - \theta^i f_i$

description of maximal SUGRA is possible only iff T_6=0

Go to the origin approach

Scalar target space R×SO(4,4)/SO(4)² is *homogeneous*

= every point can be connected by isometry (=T-duality)

Dibitetto, Guarino, Roest 2011; Dall'Agata-Inverso 2011

V is invariant by *simultaneous* tr. of coset rep. & embedding tensor

At origin, extrema condition and mass matrix reduce to quadratic eqn. on Θ

3 new stable Minkowski vacua

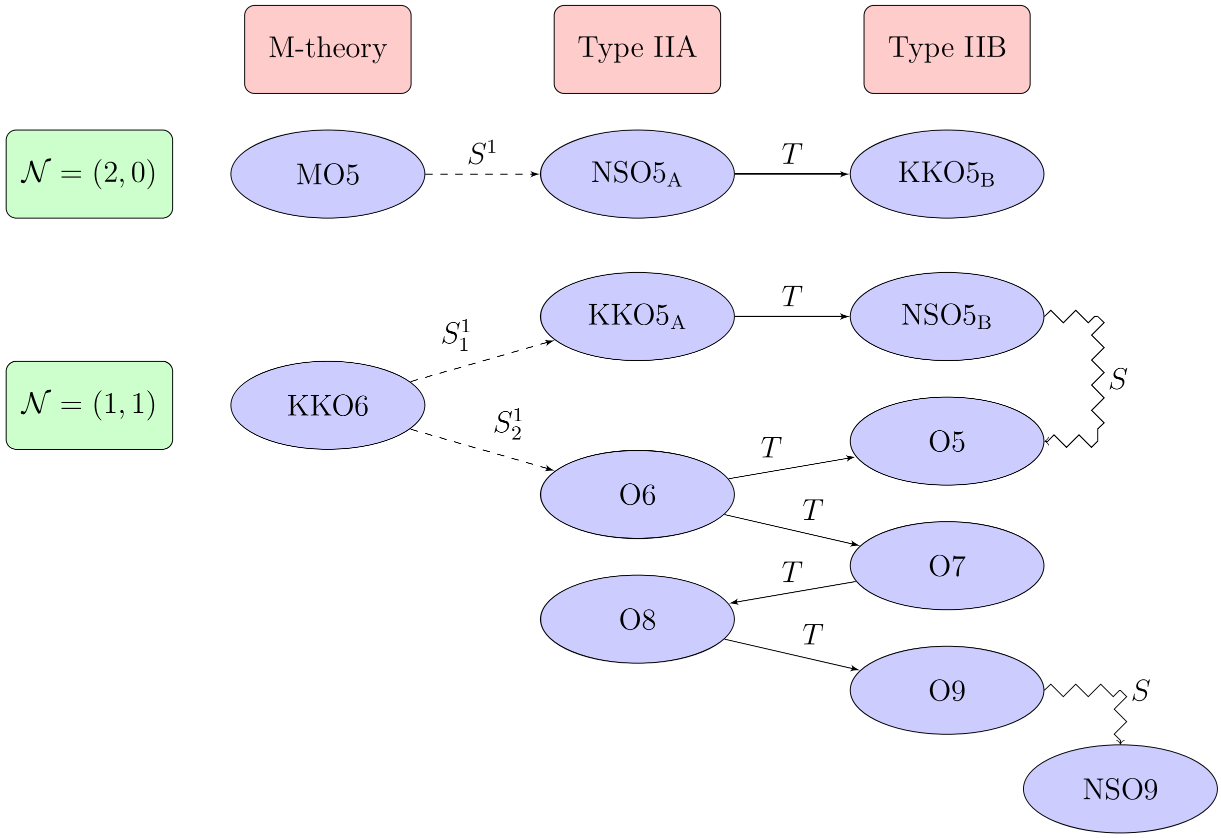
Sol #	f_0	f_i	h	${\kappa_i}^j$	θ^i	T_6	m^2
1	α	β_i	α	$\mathbb{O}_3$	β_i	$\alpha^2-\lvert\vec{\beta}\rvert^2$	$0_{(\times 13)},\ (\alpha^2+\lvert\vec{\beta}\rvert^2)_{(\times 3)},\ 4(\alpha^2+\lvert\vec{\beta}\rvert^2)_{(\times 1)}$
2	α	0	α	$\begin{pmatrix} 0 & \beta_1 & \beta_2 \\ -\beta_1 & 0 & \beta_3 \\ -\beta_2 & -\beta_3 & 0 \end{pmatrix}$	0	α^2	$0_{(\times 9)},\ \alpha^2_{(\times 1)},\ 4\alpha^2_{(\times 1)},\ \lvert\vec{\beta}\rvert^2_{(\times 2)},\ 4\lvert\vec{\beta}\rvert^2_{(\times 2)},\ (\alpha^2+\lvert\vec{\beta}\rvert^2)_{(\times 2)}$
3	0	0	0	$\begin{pmatrix} 0 & \alpha & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -\beta \\ \alpha \end{pmatrix}$	0	$0_{(\times 13)},\ (\alpha-\beta)^2_{(\times 3)},\ 4(\alpha-\beta)^2_{(\times 1)}$

internal mfd for sol.1: Nil geometry $\sigma^1 = \mathrm{d}\tau - \beta_3 x\mathrm{d}y - \beta_1 y\mathrm{d}z - \beta_2 z\mathrm{d}x\ ,\quad \sigma^2 = \mathrm{d}x\ ,\quad \sigma^3 = \mathrm{d}y \quad \sigma^4 = \mathrm{d}z$

Critical points from orientifold compactification

Theory	Λ	stability	SUSY
massive IIA w/O6/D6	Mink ×3	✔	×
massive IIA w/O8/D8	Mink	✔	×
massive IIA w/KKO5/KK5	Mink ×3 AdS AdS×2 dS (A)dS	✔ ✔ × × ×	× ✔ × × ×
IIB w/O5/D5	Mink	✔	×
IIB w/O7/D7	Mink×3	✔	×
IIB w/O9/D9	Mink	✔	×
M-theory w/KKO6/KK6	Mink×3	✔	×

Web of duality/reduction



massive IIA w/KKO5/KK5

Sol #	$\Lambda = \frac{1}{2}V_0$	m^2	
1	$-\frac{5}{2}g^2\alpha^2$	$\frac{12}{5}{}_{(\times 1)}, \frac{7}{5}{}_{(\times 3)}, -\frac{3}{5}{}_{(\times 4)}, \frac{3}{5}{}_{(\times 5)}, -\frac{2}{5}{}_{(\times 1)}, 0_{(\times 3)}$	AdS
2	$-\frac{1}{2}g^2\alpha^2$	$2_{(\times 1)}, -1_{(\times 8)}, 1_{(\times 4)}, 0_{(\times 4)}$	AdS
3 (3′)	0	$4\alpha^2_{(\times 1)}, \alpha^2_{(\times 3)}, 0_{(\times 13)}$	Mink
4	$\frac{1}{2}\left(8\sqrt{7}-21\right)g^2\alpha^2$	$\frac{1}{14}(77+36\sqrt{7}\pm\sqrt{22057+7896\sqrt{7}})_{(\times 1)}, 5+\frac{12}{\sqrt{7}}{}_{(\times 3)}, 1+\frac{8}{\sqrt{7}}{}_{(\times 5)}, 2+\frac{4}{\sqrt{7}}{}_{(\times 1)}, -1+\frac{4}{\sqrt{7}}{}_{(\times 3)}, 0_{(\times 3)}$	dS
5	$-\frac{1}{2}\left(8\sqrt{7}+21\right)g^2\alpha^2$	$\frac{1}{14}(-77+36\sqrt{7}\pm\sqrt{22057-7896\sqrt{7}})_{(\times 1)}, -5+\frac{12}{\sqrt{7}}{}_{(\times 3)}, -1+\frac{8}{\sqrt{7}}{}_{(\times 5)}, -2+\frac{4}{\sqrt{7}}{}_{(\times 1)}, 1+\frac{4}{\sqrt{7}}{}_{(\times 3)}, 0_{(\times 3)}$	AdS
6	$\frac{1}{2}g^2\alpha\beta$	$0_{(\times 3)}, 1_{(\times 5)}, \frac{(\alpha+\beta)^2\pm\sqrt{(\alpha+\beta)^4+16\alpha^2\beta^2}}{4\lvert\alpha\beta\rvert}{}_{(\times 3)}, \frac{\lambda_{(i)}}{2\lvert\alpha\beta\rvert(\alpha\beta)^3}\ (i=1,2,3)$	(A)dS
7	0	$32(7\pm 4\sqrt{3})\alpha^2_{(\times 9)}, 0_{(\times 8)}$	Mink

$$f(\lambda) \equiv \lambda^3 - 2(2\alpha^2+\alpha\beta+2\beta^2)\lambda^2 + 2\alpha\beta(\alpha^2+6\alpha\beta+\beta^2)\lambda + 12\alpha^2\beta^2(\alpha+\beta)^2 = 0\,.$$

this (A)dS vacuum is always unstable (η-parameter is -ve)

Bottom line

We have

- studied dimensional reductions M/II theories down to 6D in the presence of branes/orientifolds

- presented an embedding tensor/flux dictionary

- provided a concrete form of scalar potential & performed an exhaustive vacuum hunting

Outlooks:

- similar exhaustive classification of vacua in diverse dimensions

- explore the complete classification of vacua in D=6 theory

Poster session

Ken Matsuno

Osaka City University

“Hawking radiation from squashed Kaluza-Klein black holes with quantum gravity effects”

[JGRG30 (2021) PA9]

Hawking radiation from squashed Kaluza-Klein black holes with quantum gravity effects

Ken Matsuno

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arXiv: 2104.00891

Introduction

Hawking radiation is interesting phenomenon where both of general relativity and quantum theory play a role.

➢ During its final stages, semiclassical approach would be expected to break down due to dominance of quantum gravity effects.

✓ Some quantum gravity models suggest that there exists a minimal measurable length which would be of order Planck length.

✓ From string theories, such minimal length would be obtained by generalized uncertainty principle, which is quantum gravity inspired modification to Heisenberg uncertainty principle.

Introduction

- ✓Some generalized uncertainty principles, derived from thought experiments, have been applied to some different systems and played an important role to consider its corrections by supposed quantum theories of gravity.

- We investigate **Hawking radiation** from 5D charged static squashed Kaluza-Klein black holes by **tunneling of charged fermions and charged scalar particles**, including **quantum gravity effects predicted by quadratic generalized uncertainty principle with minimal measurable length**.

- We consider evaporation process of 5D black holes with quantum gravity effects by tunneling of particles.

5D charged squashed Kaluza-Klein black holes

(H. Ishihara, K.M.)

$$\begin{cases} ds^2 = -F dt^2 + \dfrac{K^2}{F} d\rho^2 + \rho^2 K^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) + \dfrac{r_\infty^2}{4K^2}\left(d\psi + \cos\theta d\phi\right)^2 \\ A_\mu dx^\mu = \pm \dfrac{\sqrt{3\rho_+\rho_-}}{2\rho} dt, \quad F = \dfrac{(\rho-\rho_+)(\rho-\rho_-)}{\rho^2}, \qquad K^2 = \dfrac{\rho+\rho_0}{\rho} \end{cases}$$

$$\begin{cases} r_\infty^2 = 4(\rho_+ + \rho_0)(\rho_- + \rho_0) & \text{: compact extra dimension size at infinity} \\ \rho_+ \geq \rho_- \geq 0, \ \rho_- + \rho_0 > 0 & \text{: parameter regions } (\rho_\pm\text{: horizons}) \end{cases}$$

- Mass, Charge, Hawking temperature, Entropy :

$$\begin{cases} M = \dfrac{\pi r_\infty}{G_5}(\rho_+ + \rho_-) = \dfrac{\rho_+ + \rho_-}{2G_4}, \qquad |Q| = \dfrac{2\pi r_\infty}{G_5}\sqrt{\rho_+\rho_-} = \dfrac{\sqrt{\rho_+\rho_-}}{G_4} \\ T_{\rm KK} = \dfrac{\kappa}{2\pi} = \dfrac{\rho_+ - \rho_-}{4\pi\rho_+\sqrt{\rho_+(\rho_+ + \rho_0)}}, \quad S_{\rm KK} = \dfrac{A}{4G_5} = \dfrac{\pi\rho_+\sqrt{\rho_+(\rho_+ + \rho_0)}}{G_4} \end{cases}$$

✓ Smarr-type formula :

$$M - 2QA_+/\sqrt{3} = 2T_{\rm KK}S_{\rm KK}, \quad |A_+| = \sqrt{3\rho_-/\rho_+}\,/\,2$$

Horizon and asymptotic structures

- Squashed S^3 horizons in the form of Hopf bundle :

$$ds_3^2 = \rho_\pm^2 K^2(\rho_\pm)\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \frac{r_\infty^2}{4K^2(\rho_\pm)}(d\psi + \cos\theta d\phi)^2$$

$$\begin{cases} \rho_+ = M + \sqrt{M^2 - Q^2} & : \text{ outer horizon} \\ \rho_- = M - \sqrt{M^2 - Q^2} & : \text{ inner horizon} \end{cases}$$

- Surface gravity on outer horizon of black hole :

$$\kappa = \frac{\rho_+ - \rho_-}{2\rho_+\sqrt{\rho_+(\rho_+ + \rho_0)}}$$

- At infinity ($\rho \to \infty$) :

$$ds^2 \simeq -dt^2 + d\rho^2 + \rho^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \frac{r_\infty^2}{4}(d\psi + \cos\theta d\phi)^2$$

A twisted S^1 fiber bundle over 4D Minkowski spacetime

(r_∞ : size of compactified extra dimension at infinity)

Physical meanings of parameter ρ_0

✓ For observer $\rho_0 \ll \rho_\pm \lesssim \rho$,

4D Reissner-Nordstrom black hole with a twisted constant S^1 fiber :

$$ds^2 \simeq -\frac{(\rho - \rho_+)(\rho - \rho_-)}{\rho^2}dt^2 + \frac{\rho^2}{(\rho - \rho_+)(\rho - \rho_-)}d\rho^2 + \rho^2 d\Omega_{S^2}^2 + \frac{r_\infty^2}{4}(d\psi + \cos\theta d\phi)^2$$

✓ For observer $\rho_\pm \lesssim \rho \ll \rho_0$,

5D Reissner-Nordstrom black hole with round S^3 horizons :

$$ds^2 \simeq -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^4}dt^2 + \frac{r^4}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2 + r^2 d\Omega_{S^3}^2$$

➢ Parameter ρ_0 gives typical scale of transition from five dimensions to effective four dimensions.

Heisenberg Uncertainty Principle

- For operator *A*, $\Delta A = A - \langle A \rangle$, $(\Delta A)^2 = \left\langle A^2 \right\rangle - \langle A \rangle^2$

➢ Robertson inequality: $(\Delta A)^2 (\Delta B)^2 \geq \frac{1}{4} |\langle [A, B] \rangle|^2$

✓ Canonical commutation relation: $[x, p] = i\hbar$

➢ Heisenberg uncertainty relation: $\Delta x \Delta p \geq \frac{\hbar}{2}$

⇒ Δx can be made arbitrarily small by letting Δp grow correspondingly.

✓ Resolution of small distances requires test particles of short wavelengths and thus of high energies.

➢ At such small scales, gravitational effects by high energies of test particles would significantly disturb spacetime structure which was tried.

Generalized Uncertainty Principle (GUP)

(Kempf, Mangano, Mann)

✓ Some quantum gravity theories, including string theory and loop quantum gravity, suggest that there would exist a finite limit to possible resolution of distances, which would be of order Planck length and obtained by modified Heisenberg uncertainty principle.

➢ Modified commutation relation with correction term:

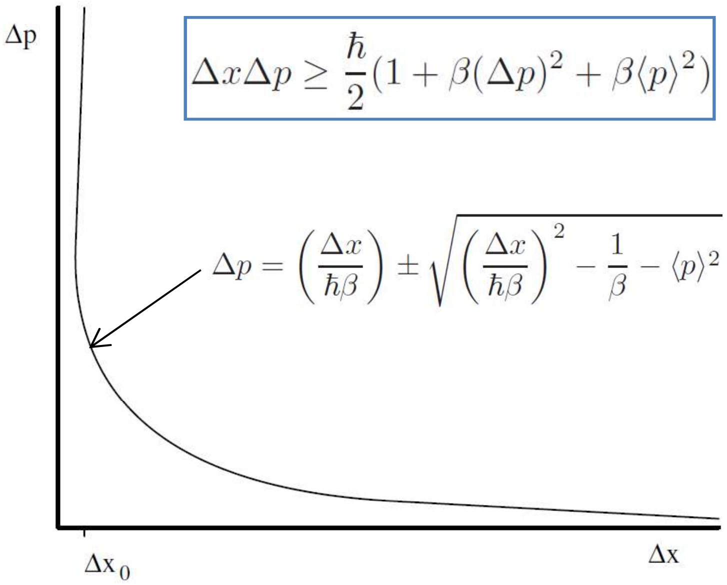
$$[x, p] = i\hbar \left(1 + \beta p^2\right) \quad \left[\begin{array}{l} \beta = \beta_0 l_p^2 = \beta_0 / m_p^2, \\ l_p = \sqrt{\hbar G_4}, \ m_p = \sqrt{\hbar / G_4} \end{array} \right.$$

➢ Modified uncertainty relation from Robertson inequality:

$$\boxed{\Delta x \Delta p \geq \frac{\hbar}{2} (1 + \beta (\Delta p)^2 + \beta \langle p \rangle^2)}$$

Generalized Uncertainty Principle (GUP)

(Kempf, Mangano, Mann)



✓ Minimal position uncertainty:

$$\Delta x_{\min}(\langle p\rangle) = \hbar\sqrt{\beta}\sqrt{1+\beta\langle p\rangle^2}$$

➢ Absolutely smallest uncertainty in position with $\langle p\rangle = 0$:

$$\Delta x_0 = \hbar\sqrt{\beta}$$

➢ Generalized uncertainty principle with minimal measurable length:

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+\beta\left(\Delta p\right)^2\right), \quad \beta = \beta_0 l_p^2 = \frac{\beta_0}{m_p^2}$$

(β_0 : quantum gravity parameter)

Generalized Uncertainty Principle (GUP)

(Kempf, Mangano, Mann)

$$\begin{cases} \Delta x \Delta p \geq \frac{1}{2}\left[1+\beta\left(\Delta p\right)^2\right] \\ \beta = \beta_0 l_p^2 = \beta_0 / m_p^2, \quad l_p = \sqrt{\hbar G_4}, \quad m_p = \sqrt{\hbar / G_4} \end{cases}$$

(β_0 : quantum gravity parameter)

✓ Minimal measurable length : $\Delta x_0 = l_p\sqrt{\beta_0}$

➢ Modified commutation relation : $\left[x_i, p_j\right] = i\left(1+\beta g^{kl}p_k p_l\right)\delta_{ij}$

➢ Modified position and momentum operators :

$$x_i = x_{0i}, \;\; p_i = p_{0i}\left(1+\beta g^{jk}p_{0j}p_{0k}\right), \quad \left[x_{0i}, p_{0j}\right] = i\delta_{ij}$$

$$\Rightarrow \; p^2 = g^{ij}p_i p_j = -g^{ij}\partial_i\partial_j\left(1-2\beta g^{kl}\partial_k\partial_l\right) + O\left(\beta^2\right)$$

➢ Using energy mass shell condition $E^2=p^2+m^2$ (m : particle mass), modified energy operator up to first order in β :

$$\tilde{E} = E\left(1-\beta m^2-\beta g^{ij}p_i p_j\right), \quad E = i\left(\partial/\partial t\right)$$

Modified Dirac equation with Maxwell field

(Hossenfelder *et al*, Chen *et al.*)

- According to GUP, modified Dirac equation up to first order in β:

$$\left[i\hbar\gamma^t\partial_t + \left(i\hbar\left(\gamma^i\partial_i + \gamma^\mu\Omega_\mu\right) - e\gamma^\mu A_\mu + m\right)\left(1 - \beta m^2 + \beta\hbar^2 g^{jk}\partial_j\partial_k\right)\right]\Psi = 0$$

$$\begin{cases} \Omega_\mu = i\,\omega_\mu^{ab}\Sigma_{ab}/2, \quad \omega_{\mu\ b}^{\ a} = e_\nu^a e_b^\lambda \Gamma_{\mu\lambda}^\nu - e_b^\nu \partial_\mu e_\nu^a, \quad \Sigma^{ab} = i\left[\gamma^a, \gamma^b\right]/4 \\ \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad g^{\mu\nu} e_\mu^a e_\nu^b = \eta^{ab} = \mathrm{diag}\left(-1, 1, 1, 1, 1\right) \end{cases}$$

- ✓ Assuming spinor field for spin-up state: $\Psi = \begin{pmatrix} U(x^\mu) \\ 0 \\ V(x^\mu) \\ 0 \end{pmatrix} \exp\left(\frac{i}{\hbar} I(x^\mu)\right)$

- ✓ Gamma matrices of squashed KK BH:

$$\begin{cases} \gamma^t = \frac{1}{\sqrt{F}}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \gamma^\rho = \frac{\sqrt{F}}{K}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \gamma^\theta = \frac{1}{\rho K}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \\ \gamma^\phi = \frac{1}{\rho K \sin\theta}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^\psi = \begin{pmatrix} -2K/r_\infty & 0 & 0 & i\tilde{K} \\ 0 & -2K/r_\infty & -i\tilde{K} & 0 \\ 0 & i\tilde{K} & 2K/r_\infty & 0 \\ -i\tilde{K} & 0 & 0 & 2K/r_\infty \end{pmatrix}, \\ \tilde{K} = \cos\theta/(\rho K \sin\theta) \end{cases}$$

Equation of motion for charged fermions

- WKB approximation to leading order in $\hbar$:

$$\begin{cases} U\left(\frac{1}{\sqrt{F}}\left(\partial_t I + eA_t\sigma\right) - \sigma\frac{\sqrt{F}}{K}\partial_\rho I\right) - \sigma V\left(\frac{2K}{r_\infty}\partial_\psi I - m\right) = 0, \\ V\left(\frac{1}{\sqrt{F}}\left(\partial_t I + eA_t\sigma\right) + \sigma\frac{\sqrt{F}}{K}\partial_\rho I\right) - \sigma U\left(\frac{2K}{r_\infty}\partial_\psi I + m\right) = 0, \\ \sigma\left(\partial_\theta I + \frac{i}{\sin\theta}\left(\partial_\phi I - \cos\theta\partial_\psi I\right)\right) = 0, \\ \sigma := 1 - \beta m^2 - \frac{\beta F}{K^2}\left(\partial_\rho I\right)^2 - \frac{\beta}{\rho^2 K^2}\left(\partial_\theta I\right)^2 - \frac{\beta\left(\partial_\phi I - \cos\theta\partial_\psi I\right)^2}{\rho^2 K^2 \sin^2\theta} - \frac{4\beta K^2}{r_\infty^2}\left(\partial_\psi I\right)^2 \end{cases}$$

- ✓ According to Killing vector fields $\partial/\partial t,\ \partial/\partial\phi$ and $\partial/\partial\psi$, the action ansatz:

$$I = -\omega t + R(\rho) + \Theta(\theta) + J\phi + L\psi$$

(ω : fermion energy, J and L : fermion angular momenta)

- From 3rd EOM: $\frac{d\Theta}{d\theta} + \frac{i}{\sin\theta}\left(J - L\cos\theta\right) = 0 \quad \Rightarrow$ complex function Θ

Tunneling of fermions

- From 1st and 2nd EOM:

$$
\begin{aligned}
&\left(\frac{4K^2L^2}{r_\infty^2}-\frac{e^2A_t^2}{F}-m^2\right)\left(\frac{4\beta K^2L^2}{r_\infty^2}+\beta m^2-1\right)^2-\frac{2\omega eA_t}{F}\left(\frac{4\beta K^2L^2}{r_\infty^2}+\beta m^2-1\right)-\frac{\omega^2}{F}\\
&+\left(\frac{F}{K^2}\left(\frac{4\beta K^2L^2}{r_\infty^2}+\beta m^2-1\right)\left(\frac{12\beta K^2L^2}{r_\infty^2}-\frac{2\beta e^2A_t^2}{F}-\beta m^2-1\right)-\frac{2\omega\beta eA_t}{K^2}\right)\left(\frac{dR}{d\rho}\right)^2\\
&+\frac{\beta F^2}{K^4}\left(\frac{12\beta K^2L^2}{r_\infty^2}-\frac{\beta e^2A_t^2}{F}+\beta m^2-2\right)\left(\frac{dR}{d\rho}\right)^4+\frac{\beta^2F^3}{K^6}\left(\frac{dR}{d\rho}\right)^6=0.
\end{aligned}
$$

- Action for outgoing and ingoing modes for classically forbidden trajectory:

$$
\begin{cases}
\mathrm{Im}R_{\text{out}}=-\mathrm{Im}R_{\text{in}}=\dfrac{\pi\rho_+\sqrt{\rho_+(\rho_++\rho_0)}\,(\omega-eA_+)}{\rho_+-\rho_-}\left(1+\beta\Xi_f\right)+O\left(\beta^2\right)\\
\Xi_f=\dfrac{3\omega m^2}{2(\omega-eA_+)}+\dfrac{\omega L^2}{2\rho_+(\rho_-+\rho_0)(\omega-eA_+)}\\
\qquad+\dfrac{\omega^2\rho_+\left(4\rho_+(\rho_+-2\rho_-)+\rho_0(3\rho_+-7\rho_-)\right)+e\omega\rho_+A_+\left(4\rho_+\rho_-+\rho_0(\rho_++3\rho_-)\right)}{2(\rho_++\rho_0)(\rho_+-\rho_-)^2}
\end{cases}
$$

- Tunneling probability amplitude of charged fermions:

$$
\Gamma\simeq\frac{\exp\left(-2\mathrm{Im}R_{\text{out}}\right)}{\exp\left(-2\mathrm{Im}R_{\text{in}}\right)}\simeq\exp\left(-\frac{4\pi\rho_+\sqrt{\rho_+(\rho_++\rho_0)}\,(1+\beta\Xi_f)}{\rho_+-\rho_-}(\omega-eA_+)\right)
$$

Modified Hawking temperature

- Comparing tunneling probability amplitude with Boltzmann factor $\Gamma=\exp\left(-(\omega-eA_+)/T\right)$ at temperature *T*, we obtain **modified Hawking temperature of squashed Kaluza-Klein black hole**:

$$
T=T_{\text{KK}}\left(1-\beta\Xi_f\right)+O\left(\beta^2\right)
$$

✓ Hawking temperature depends upon energy ω, mass m, charge e and angular momentum L of emitted fermion, and is modified by squashed Kaluza-Klein geometry, Maxwell field and generalized uncertainty principle through parameters ρ_0, ρ_- and β.

✓ Taking limits, we obtain some known quantum-corrected Hawking temperatures of 5D and 4D black holes:

- $\rho_-=0$, $L=0$, $r_\infty\to\infty$: 5D Schwarzschild-Tangherlini black hole
- $\rho_0=0$, $e=0$, $L=0$: 4D Reissner-Nordstrom black hole
- $\rho_-=0$, $\rho_0=0$, $e=0$, $L=0$: 4D Schwarzschild black hole

Modified thermodynamics

- ✓ We consider uncharged massless fermion radiation without momentum in extra dimensional direction from uncharged Kaluza-Klein black hole, i.e., $\rho_-=0$, $m=0$, $L=0$.

- ➢ Using saturated form of uncertainty principle $\omega \gtrsim 1/\Delta x$ and uncertainty in position x for events near black hole horizon $\Delta x \simeq 2\rho_+$,
Hawking temperature and entropy with quantum corrections:$(\mu = M/m_p,\ \nu = r_\infty/l_p)$

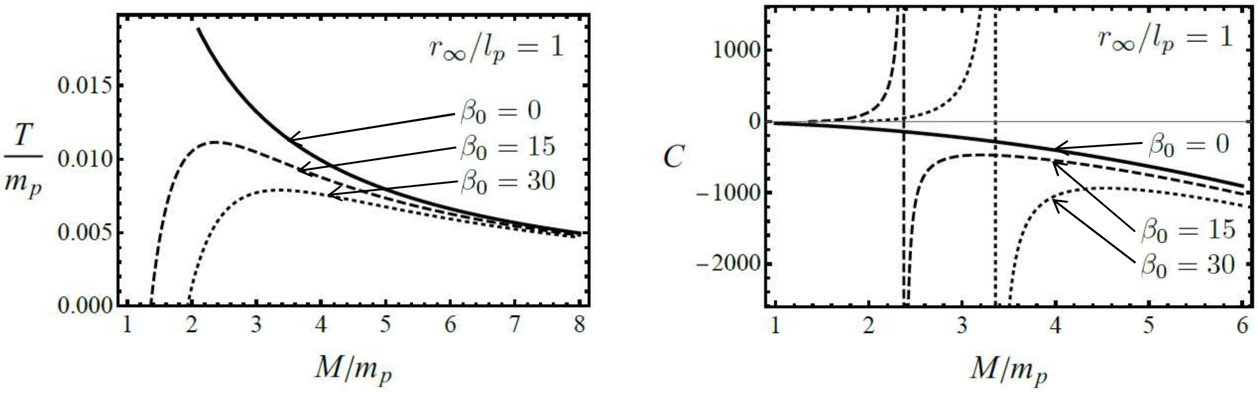
$$\begin{cases} T = \dfrac{m_p}{4\pi\sqrt{\mu\left(2\mu+\sqrt{4\mu^2+\nu^2}\right)}}\left(1-\beta_0\dfrac{10\mu+3\sqrt{4\mu^2+\nu^2}}{32\mu^2\left(2\mu+\sqrt{4\mu^2+\nu^2}\right)}\right)+O\left(\beta_0^2\right) \\ S = 2\pi\mu\sqrt{\mu\left(2\mu+\sqrt{4\mu^2+\nu^2}\right)}\left(1+\beta_0\dfrac{10\mu+3\sqrt{4\mu^2+\nu^2}}{32\mu^2\left(2\mu+\sqrt{4\mu^2+\nu^2}\right)}\right)+O\left(\beta_0^2\right) \end{cases}$$

- ➢ Heat capacity $C = T\left(\partial S/\partial T\right)$ with quantum correction:

$$\begin{aligned} C = \pi & \left(2048\mu^5+448\mu^3\nu^2-2\beta_0\mu\nu^2+\left(1024\mu^4+96\mu^2\nu^2-3\beta_0\nu^2\right)\sqrt{4\mu^2+\nu^2}\right) \\ & \times\left(64\mu^3-10\beta_0\mu+\left(32\mu^2-3\beta_0\right)\sqrt{4\mu^2+\nu^2}\right)\sqrt{\mu\left(2\mu+\sqrt{4\mu^2+\nu^2}\right)} \\ & \times\Big[48\beta_0\mu\left(512\mu^4+136\mu^2\nu^2+5\nu^4+\left(256\mu^3+36\mu\nu^2\right)\sqrt{4\mu^2+\nu^2}\right) \\ & -512\mu^3\left(128\mu^4+32\mu^2\nu^2+\nu^4+\left(64\mu^3+8\mu\nu^2\right)\sqrt{4\mu^2+\nu^2}\right)\Big]^{-1}, \end{aligned}$$

  - ✓ BH evaporation modified by GUP: quantum gravity effect in Hawking radiation

Modified evaporation process of black hole



- ✓ As black hole mass decreases, quantum-corrected temperature with $\beta_0 \neq 0$ reaches local maximum value and then decreases to zero at minimum value of mass M_{rm}.
- ✓ At local maximum temperature, the system undergoes transition from unstable negative heat capacity phase to stable positive heat capacity cooling down towards cold extremal configuration with mass M_{rm}.
- ✓ At minimum mass M_{rm}, since both Hawking temperature and heat capacity vanish, black hole may not exchange its energy with surrounding environment.

- ➢ **Generalized uncertainty principle prevents squashed Kaluza-Klein black hole to completely evaporate and results in thermodynamic stable remnant.**

- ✓ If quantum gravity parameter is $\beta_0 \fallingdotseq 1$ and extra dimension size is $r_\infty \fallingdotseq 0.1$ mm, mass of black hole remnant is $M_{rm} \fallingdotseq 10^{-8}$ kg, which is of order Planck mass.

Tunneling of scalar particles

- According to GUP, modified Klein-Gordon equation with Maxwell field up to first order in β (Feng, Li, Zu, Yang):

$$\left[g^{t\mu}\left(\hbar\partial_t + ieA_t\right)\left(\hbar\partial_\mu + ieA_\mu\right)\right.$$
$$\left. + \left(g^{ij}\left(\hbar\partial_i + ieA_i\right)\left(\hbar\partial_j + ieA_j\right) - m^2\right)\left(1 - 2\beta m^2 + 2\beta\hbar^2 g^{kl}\partial_k\partial_l\right)\right]\tilde{\Psi} = 0$$

- Tunneling probability amplitude of charged s-wave scalar particles:

$$\begin{cases} \tilde{\Gamma} \simeq \dfrac{\exp\left(-2\mathrm{Im}\tilde{R}_{\mathrm{out}}\right)}{\exp\left(-2\mathrm{Im}\tilde{R}_{\mathrm{in}}\right)} \simeq \exp\left(-\dfrac{4\pi\rho_+\sqrt{\rho_+\left(\rho_+ + \rho_0\right)}\left(1+\beta\Xi_s\right)}{\rho_+ - \rho_-}\left(\tilde{\omega} - eA_+\right)\right) \\ \Xi_s = \dfrac{m^2}{2} + \dfrac{\rho_+\left(\tilde{\omega} - eA_+\right)}{2\left(\rho_+ + \rho_0\right)\left(\rho_+ - \rho_-\right)^2}\left[\tilde{\omega}\left(4\rho_+\left(\rho_+ - 2\rho_-\right) + \rho_0\left(3\rho_+ - 7\rho_-\right)\right)\right. \\ \qquad \left. + eA_+\left(2\rho_+\left(\rho_+ + \rho_-\right) + \rho_0\left(3\rho_+ + \rho_-\right)\right)\right], \end{cases}$$

- **Modified Hawking temperature of squashed Kaluza-Klein black hole**:

$$\tilde{T} = T_{\mathrm{KK}}\left(1 - \beta\Xi_s\right) + O\left(\beta^2\right)$$

⇒ Thermodynamic stable Planck mass remnant after evaporation

Summary

We study Hawking radiation from 5D charged static squashed Kaluza-Klein black hole by tunneling of charged particles based on GUP with minimal measurable length.

- We derive corrections of Hawking temperature to general relativity, which are related to energy of emitted particle, size of compact extra dimension, charge of black hole, quantum gravity effect coming from the existence of minimal length.

- We show that GUP may slow down increase of Hawking temperature due to radiation, which may lead to thermodynamic stable remnant of the order of Planck mass after evaporation of squashed Kaluza-Klein BH.

Poster session

Takashi Mishima

Nihon University

"Nonlinear behavior of new cylindrically symmetric waves of Einstein-Maxwell system"

[JGRG30 (2021) PA10]

2021 Dec. / Poster
(JGRG30 Waseda Univ. online)

P10

Nonlinear behavior of new cylindrically symmetric waves of Einstein-Maxwell system

T. Mishima (Nihon Univ.)
S. Tomizawa (Toyota Tech. Inst.)

We construct exact solutions of gravitational and electromagnetic waves that interact nonlinearly, applying a simple harmonic mapping method to the cylindrically symmetric Einstein-Maxwell system. As an interesting feature of the solutions, we show that the large conversion phenomena occur in the intense area of fields with no background.

I. Purpose and Introduction

I.1 purpose

Construction of exact solutions of Einstein-Maxwell system(E-M system) with full modes:4modes

⇩

Full nonlinear analysis of conversion between gravitational waves and electro-magnetic waves by using the exact solutions

＜ some previous studies＞

(i) Gravit-elemag conversion in the background electromagnetic field: perturbative approach
Olson & Unruh[1974], … Crispino,Dolan,Higuchi & Oliveira[2014], Saito, Soda & Yoshino[2021], Hadj & Dolan[2021]

(ii) Colliding plane waves in E-M system: many works, not focused on the conversion
Chandrasekhar & Xanthopoulos[1985〜], Halilsoy[1989]… ⇒Griffiths「Colliding Plane Waves」for more other treatments

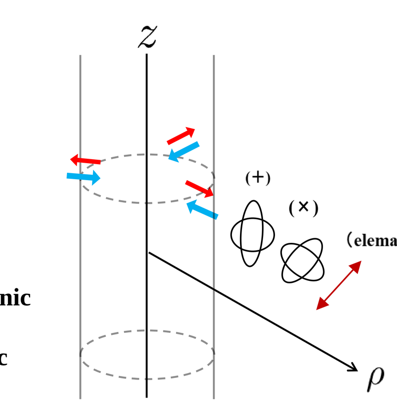
(iii) Cosmological behavior of E-M system : not focused on the conversion
Charach & Malin[1980], … Narita, Torii & Maeda(Ken)[2000], Yazadjiev[2003]

(iv) Behavior of cylindrical waves of E-M system: not so many
Halilsoy[1983], Yazadjiev[2005], Alekseev[2015], Barreto,Oliveira & Rodrigues[2017]
(cosmologically conversion treated) (conversion treated, numerical study)

Following (iv): analysis of the non-linear conversion done using cylindrical exact solutions in E-M system

＜features or assumptions＞

- No conical singularity on the axis
- The wave fields become strong near the axis and weak near infinity generally.
- Some useful quantities like Thorn's C-energy or Bondi news are available[Thorne '65],[Stachel '66]



I.2 storyline

II. Settings and solution-generating method using simple harmonic map method
III. Construction of interacting gravitational and electromagnetic waves
IV. Study of conversion phenomena in the Einstein-Maxwell system
V. Summary and discussion

II. Settings and solution-generating method

II.1 Settings and basic equations

＜Kompaneets - Jordan - Ehlers metric and elemag. gauge field＞*** $(f,\ w,\ A,\ B)$

$$(1)\quad \begin{cases} ds^2 = f(dz - w d\phi)^2 + f^{-1}\rho^2 d\phi^2 + f^{-1}e^{2\gamma}(-dt^2 + d\rho^2), & [f := e^{2\psi}] \\ \mathbf{A} = A d\phi + B dz & \text{(gauge field)} \end{cases}$$

＜ Ernst equations for E-M system[Ernst '68] ＞

$$(2)\quad \begin{cases} \text{(i)} & (\Re[\mathbf{E}] - |\mathbf{F}|^2)\nabla^2\mathbf{F} = (\nabla\mathbf{E} - 2\bar{\mathbf{F}}\nabla\mathbf{F})\cdot\nabla\mathbf{F} \\ \text{(ii)} & (\Re[\mathbf{E}] - |\mathbf{F}|^2)\nabla^2\mathbf{E} = (\nabla\mathbf{E} - 2\bar{\mathbf{F}}\nabla\mathbf{F})\cdot\nabla\mathbf{E} \end{cases}$$

(Ernst potentials)

$$(3)\quad \begin{cases} \mathbf{F} := B + i\chi \\ \mathbf{E} := f + |\mathbf{F}|^2 - i\Phi \end{cases}$$

- Here twist potentials are introduced: $(\chi,\ \Phi) \leftarrow (A,\ w)$
- These are defined on *a hypothetical 3-dim. Minkowski space* $\mathbf{M}^{2,1}$ with cylindrical symmetry. (so-called *target space*)

II.2 Alternative basic equations and target space/potential space

Introducing another set of complex potentials (ξ,η) and the corresponding basic equations

$$\left[\mathbf{E} = \frac{1+\xi}{1-\xi},\quad \mathbf{F} = \frac{\eta}{1-\xi}\right] \quad \text{Griffiths[1991]}$$

$$(4)\quad \begin{cases} (\Re[\mathbf{E}] - |\mathbf{F}|^2)\nabla^2\mathbf{F} = (\nabla\mathbf{E} - 2\bar{\mathbf{F}}\nabla\mathbf{F})\cdot\nabla\mathbf{F} \\ (\Re[\mathbf{E}] - |\mathbf{F}|^2)\nabla^2\mathbf{E} = (\nabla\mathbf{E} - 2\bar{\mathbf{F}}\nabla\mathbf{F})\cdot\nabla\mathbf{E} \end{cases}$$

$$(5)\quad \begin{cases} (\xi\bar\xi + \eta\bar\eta - 1)\,\nabla^2\xi = 2(\bar\xi\nabla\xi + \bar\eta\nabla\eta)\cdot\nabla\xi \\ (\xi\bar\xi + \eta\bar\eta - 1)\,\nabla^2\eta = 2(\bar\xi\nabla\xi + \bar\eta\nabla\eta)\cdot\nabla\eta \end{cases}$$

Kinnersley[1973]

⇩

Solutions of Equation set (5) are given as the harmonic map/wave map from $\mathbf{M}^{2,1}$ (base space) to a ball model of complex 2-dim. Hyperbolic space $\mathrm{H}^2_{\mathrm{C}}$(target space).

Metric of $\mathrm{H}^2_{\mathrm{C}}$

$$(6)\quad d\ell^2 = \frac{(1 - \xi\bar\xi - \eta\bar\eta)(d\xi d\bar\xi + d\eta d\bar\eta) + (\bar\xi d\xi + \bar\eta d\eta)(\xi d\bar\xi + \eta d\bar\eta)}{(\xi\bar\xi + \eta\bar\eta - 1)^2}$$

Parker['03] , Goldman['99]

II.3 Simple and convenient solution-generation method by composit harmonic mappinig

As a simple method, we adopt here the composite harmonic method[Eells and Sampson '63].

＜ Procedure ＞

i. construct an harmonic mapping from the base space M to a intermediate space K
ii. find out a totally geodesic embedding map of K into the target space N
iii. combine finally these maps

$$\varphi(v(x)) : \underset{(x^i)}{M} \longrightarrow \underset{(v^a)}{K} \longrightarrow \underset{(\varphi^A)}{N}$$

$$\nabla^2\varphi^A + \Gamma^A_{BC}\,h^{ij}\nabla_i\varphi^B\nabla_j\varphi^C = 0 \quad \text{(harmonic map from } M \text{ to } N\text{)} \quad \overset{\varphi(v(x))}{\Longleftarrow} \quad \begin{cases} \nabla^2 v^a + \Gamma^a_{bc}\,h^{ij}\nabla_i v^b\nabla_j v^c = 0 & \text{(harmonic map from } M \text{ to } K\text{)} \\ \partial_a\partial_\beta\varphi^A - \Gamma^c_{ab}\partial_c\varphi^A + \Gamma^A_{BC}\,\partial_a\varphi^B\,\partial_\beta\varphi^C = 0 & \text{(zero extrinsic curvature)} \end{cases}$$

III. Construction of interacting gravitational and electromagnetic waves

III.1 Choice of totally geodesic subspaces in $\mathrm{H}^2_{\mathrm{C}}$: $(z_1,\ z_2) = (\xi,\ \eta)$

Isometrically only *two different types of totally geodesic surfaces*:
(a) Poincare disc model $\mathrm{H}^1_{\mathrm{C}}$ (complex line L)
(b) Klein-Beltrami disk model (totally real and totally geodesic plane R)

Case (a)

Poincare disc model
↓
$$\varphi(v(x)) : \underset{(t,\rho)}{M^{2,1}} \longrightarrow \underset{(\xi_v)}{L} \longrightarrow \underset{(z_1,z_2)}{\mathrm{H}^2_{\mathrm{C}}}$$
M K N

⇩

- Totally geodesic embedding of L in $\mathrm{H}^2_{\mathrm{C}}$:

$$(7)\quad (\xi,\eta) = (\cos 2\theta\,\xi_v, \sin 2\theta\,\xi_v)$$

: simple example for (a)

Case (b)

Klein disc model
↓
$$\varphi(v(x)) : \underset{(t,\rho)}{M^{2,1}} \longrightarrow \underset{(v_1,\,v_2)}{R} \longrightarrow \underset{(z_1,z_2)}{\mathrm{H}^2_{\mathrm{C}}}$$
M K N

⇩

- Totally geodesic embedding of R in $\mathrm{H}^2_{\mathrm{C}}$:

$$(8)\quad (\xi,\eta) = (v_1 e^{i\alpha},\ v_2 e^{i\beta})$$

: simple example for (b)

III.2 Formal solutions of Ernst potentials

(i) ξ_v is replaced with the solutions $\xi_v(t,\rho)$ of vacuum Ernst equation. $\left[(\xi\bar\xi - 1)\nabla^2\xi = 2\bar\xi\nabla\xi\cdot\nabla\xi\right]$

⇩

the solutions $(\xi,\ \eta)$ of Ernst equation (5) $\left[(5)\ \begin{cases} (\xi\bar\xi + \eta\bar\eta - 1)\,\nabla^2\xi = 2(\bar\xi\nabla\xi + \bar\eta\nabla\eta)\cdot\nabla\xi \\ (\xi\bar\xi + \eta\bar\eta - 1)\,\nabla^2\eta = 2(\bar\xi\nabla\xi + \bar\eta\nabla\eta)\cdot\nabla\eta \end{cases}\right]$

case (a)

$$(7)'\quad (\xi,\eta) = (\cos\theta\,\xi_v(t,\rho),\ \sin\theta\,\xi_v(t,\rho))$$

Halilsoy[1989], Griffith['91]

case (b)

$$(8)'\quad (\xi,\eta) = \left(e^{i\alpha}v_1(t,\rho),\ e^{i\beta}v_2(t,\rho)\right)$$

$$\left[v_1 + iv_2 = \frac{2\xi_v}{1+|\xi_v|^2}\right]$$

$$\left[\mathbf{E} = \frac{1+\xi}{1-\xi},\quad \mathbf{F} = \frac{\eta}{1-\xi}\right]$$

(ii) Fields and twist potentials: $f,\ B,\ \Phi,\ \chi$ ⇦ $\left[(3)\quad E = f + |F|^2 - i\Phi, \qquad F = B + i\chi\right]$

(iii) Using the previous solution[MT '17] as a simple harmonic map from $\mathbf{M}^{2,1}$ to $\mathrm{H}^1_{\mathrm{C}}$ to construct $(f,\ B,\ \Phi,\ \chi)$:

$$(9)\quad \xi_v = \frac{1 - A^2 - s^2}{(1+s)^2 + A^2} + i\frac{2A}{(1+s)^2 + A^2} \qquad (s = e^{-2\tau}\ :\ \nabla^2\tau = 0) \qquad \text{MT ['17]}$$

τ: seed function

case (a) : $(\xi,\eta) = (\cos 2\theta\,\xi_v, \sin 2\theta\,\xi_v)$

$$(10)\quad \begin{aligned} f &= \frac{s}{A^2\cos^4\theta + (s\cos^2\theta + \sin^2\theta)^2} \\ \Phi &= -\frac{A\cos 2\theta}{A^2\cos^4\theta + (s\cos^2\theta + \sin^2\theta)^2} \\ B &= -\frac{1}{2}\frac{[A^2\cos^2\theta + (s-1)(s\cos^2\theta + \sin^2\theta)]\sin 2\theta}{A^2\cos^4\theta + (s\cos^2\theta + \sin^2\theta)^2} \\ \chi &= \frac{1}{2}\frac{A\sin 2\theta}{A^2\cos^4\theta + (s\cos^2\theta + \sin^2\theta)^2} \end{aligned}$$

- two parameters: (A,θ)

case (b) : $(\xi,\eta) = (v_1 e^{i\alpha},\ v_2 e^{i\beta})$

$$(11)\quad \begin{aligned} f &= \frac{2s^2}{1 - \cos\alpha + (A^2 + s^2)^2(1+\cos\alpha)} \\ \Phi &= -\frac{[(A^2+s^2)^2 - 1]\sin\alpha}{1 - \cos\alpha + (A^2+s^2)^2(1+\cos\alpha)} \\ B &= A\frac{(A^2 + s^2 - 1)\cos(\alpha - \beta) + (A^2 + s^2 + 1)\cos\beta}{1 - \cos\alpha + (A^2+s^2)^2(1+\cos\alpha)} \\ \chi &= -A\frac{(A^2 + s^2 - 1)\sin(\alpha - \beta) - (A^2 + s^2 + 1)\sin\beta}{1 - \cos\alpha + (A^2+s^2)^2(1+\cos\alpha)} \end{aligned}$$

- Three parameters: (A,α,β)

(iv) metric component w and gauge field component A are derived by integrations. Here we neglect their explicit form.

IV. Study of the conversion phenomena in the Einstein-Maxwell system:

- To what extent such phenomena occur ?

IV.1 Preparation: useful quantities related to C-energy γ

Thorne[1965] Piran, Safier and Stark [1985]

The C-energy density is generalized to treat the conversion and divided into gravitational part(+-mode and ×-mode) and electromagnetic part(A_z and A_φ).

(Amplitudes) $\left[u = \frac{1}{2}(t - r) \text{ and } v = \frac{1}{2}(t + r)\right]$

$$(12)\quad \begin{aligned} &\mathcal{A}_+ := 2\,\partial_v\psi, \quad \mathcal{A}_\times := \frac{1}{\rho}e^{2\psi}\partial_v w = \frac{1}{2}e^{-2\psi}[\partial_v\Phi + 2(B\partial_v\chi - \chi\partial_v B)], \quad \mathcal{A}_z := e^{-\psi}\partial_v B, \quad \mathcal{A}_\phi := \frac{1}{\rho}e^{\psi}(\partial_v A + w\partial_v A) = e^{-\psi}\partial_v\chi, \\ &\mathcal{B}_+ := 2\,\partial_u\psi, \quad \mathcal{B}_\times := \frac{1}{\rho}e^{2\psi}\partial_u w = \frac{1}{2}e^{-2\psi}[\partial_u\Phi + 2(B\partial_u\chi - \chi\partial_u B)], \quad \mathcal{B}_z := e^{-\psi}\partial_u B, \quad \mathcal{B}_\phi := \frac{1}{\rho}e^{\psi}(\partial_u A + w\partial_u A) = e^{-\psi}\partial_u\chi. \end{aligned}$$

Generalization of Piran, Safier and Stark [1985]

(Generalized C-energy density)

$$(13)\quad \begin{aligned} \partial_\rho\gamma &= \frac{\rho}{8}\big(\underbrace{\mathcal{A}_+^2 + \mathcal{B}_+^2 + \mathcal{A}_\times^2 + \mathcal{B}_\times^2}_{\mathcal{E}_g} + \underbrace{\mathcal{A}_z^2 + \mathcal{B}_z^2 + \mathcal{A}_\phi^2 + \mathcal{B}_\phi^2}_{\mathcal{E}_{em}}\big) \\ \mathcal{E} &= \qquad \mathcal{E}_g \qquad + \qquad \mathcal{E}_{em} \end{aligned}$$

(gravitational part) (electromagnetic part)

(Generalized C-energy γ)

$$(14)\quad \gamma_\bullet(t,r) := \int_0^r \mathcal{E}_\bullet(t,\rho)\,d\rho$$

IV.2 Occupancy ratios of gravitational and electromagnetic parts for the C-energy density

(Occupancy: definition and results)

$$(15)\quad \begin{aligned} \mathcal{E}_{\mathrm{g}} &= R_{\mathrm{g}}\mathcal{E}, \quad R_{\mathrm{g}} := \frac{A^2e^{4\tau}\cos^4\theta + (\cos^2\theta - e^{2\tau}\sin^2\theta)^2}{A^2e^{4\tau}\cos^4\theta + (\cos^2\theta + e^{2\tau}\sin^2\theta)^2} \\ \mathcal{E}_{\mathrm{em}} &= R_{\mathrm{em}}\mathcal{E}, \quad R_{\mathrm{em}} := \frac{e^{2\tau}\sin^2(2\theta)}{A^2e^{4\tau}\cos^4\theta + (\cos^2\theta + e^{2\tau}\sin^2\theta)^2} \end{aligned} \qquad \left[R_{\mathrm{g}} + R_{\mathrm{em}} = 1\right]$$

⇩

Once the parameters (A,θ) given, the spacetime dependence of the occupancy of each part is controlled by the seed function $\tau(t,\rho)$!

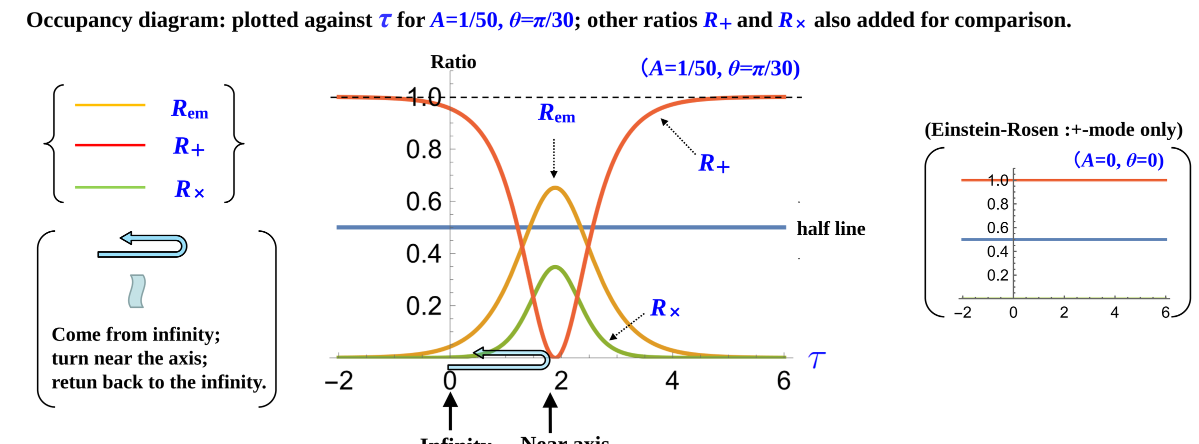
- Total C-energy density $\mathcal{E}$ has no dependence on (A,θ) as below; that is, the C-energy has the same dependence as the solution for $(A,\theta) = (0, 0)$, which corresponds to so called Einstein-Rosen waves.

$\mathcal{E} = \rho\left[(\partial_t\tau)^2 + (\partial_\rho\tau)^2\right]$ ⇨ All solutions have the same overall shape, but with different distribution ratios of contents !

IV.3 An example of conversion phenomena: based on the case (a): $(\xi,\eta) = (\cos 2\theta\,\xi_v, \sin 2\theta\,\xi_v)$

＜Occupancy R_{em} ＞

Occupancy diagram: plotted against τ for A=1/50, θ=π/30; other ratios R_+ and $R_\times$ also added for comparison.



- By using such diagrams, the approximate behavior of the conversion can be known through the behavior of seed wave τ !

＜An interesting example: large amplification of electromagnetic waves ＞

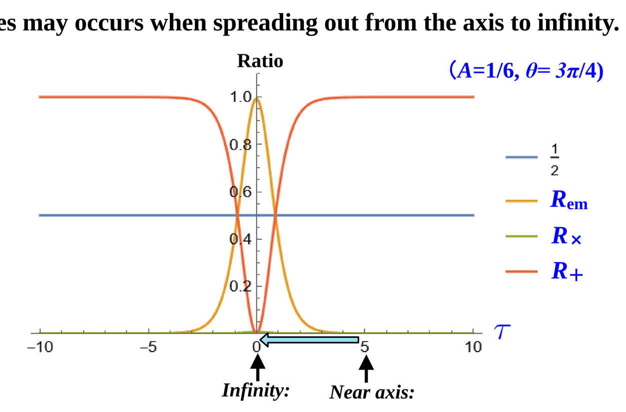
(a) diagram for A=1/6, θ= 3π/4

The large amplification of electromagnetic waves may occurs when spreading out from the axis to infinity.

R_{em} = 0.0001767
(near axis: τ = 5, gravitational wave dominance)
R_{em} = 0.9931
(near infinity: τ = 0, elemag. wave dominance)

⇩

Amplification rate = 0.9931/0.0001767
= 5.620×10^3



- It should be noticed that the seed function τ cannot always take appropriate values for the optimal conversion, so that the electromagnetic contribution to the C-energy really becomes smaller.

⇩

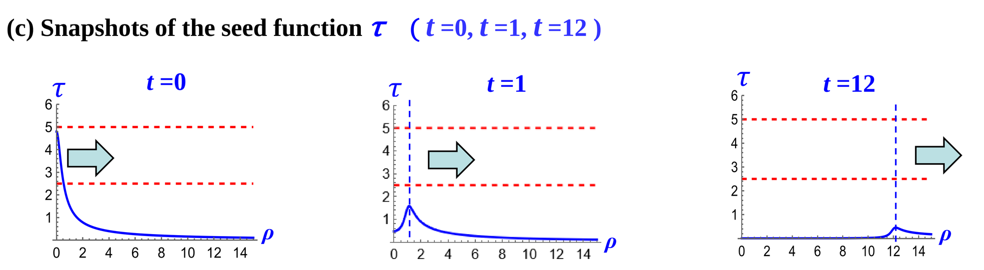
Analysis using a specific seed function

(b) WWB solution as a seed wave function τ : Weber-Wheeler['57]・Bonnor['57]

- Regular and packet-like wave solution 〜 one of the Einstein-Rosen solutions

$$\begin{aligned} \tau(t,\rho) &= c\int_0^\infty e^{-ak}J_0(k\rho)\cos(kt)\,dk \\ &= \frac{c}{\sqrt{2}}\left[\frac{\sqrt{4a^2t^2 + (a^2 + \rho^2 - t^2)^2} + a^2 + \rho^2 - t^2}{4a^2t^2 + (a^2 + \rho^2 - t^2)^2}\right]^{1/2} \end{aligned}$$

(c) Snapshots of the seed function τ (t =0, t =1, t =12)



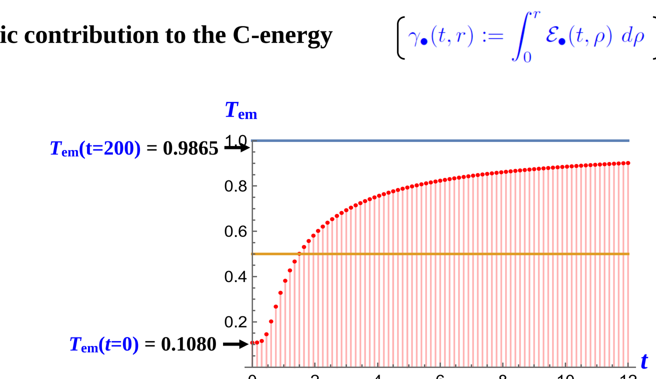
(d) Time dependence of total electromagnetic contribution to the C-energy $\left[\gamma_\bullet(t,r) := \int_0^r \mathcal{E}_\bullet(t,\rho)\,d\rho\right]$

$$(15)\quad T_{\mathrm{em}}(t) := \frac{\gamma_{\mathrm{em}}(t, r = 2000)}{\gamma(t, r = 2000)}$$

$T_{em}(t=0)$ = 0.1080, $T_{em}(t=200)$ = 0.9865

⇩

$T_{em}(t=200)/ T_{em}(t=0)$ = 9.14



- The contribution of Electromagnetic waves are amplified by about 10 times against the initial electromagnetic waves.

V. Summary and discussion

- We have shown a convenient procedure to generate new solutions that represent non-linearly interacting gravitational and electromagnetic waves, by using the composite harmonic mapping method.
- Observing the behavior of the generalized C-energy density, we have clarified the conversion phenomena between gravitational waves and electro-magnetic waves.
- Depending on the parameters, the solutions show interesting aspects of conversion: especially, it is found, as conjectured in the previous paper[TM,'17], that if gravitational waves sufficiently concentrate near the axis and very weak electromagnetic waves exist together, large amplification of electromagnetic waves occurs as the waves spread out from the axis to infinity.

- Analysis of spacetime structure
- more systematic analysis and introducing more general totally geodesic embeddings
- Including soliton solutions
- Behavior of test charges
- Extension to other systems (K-K gravity, higher dimension…)
- ・・・・・

Poster session

Tatsuki Kodama

Saga University

“Relaxing inflation models with non-minimal coupling: A general study”

[JGRG30 (2021) PA11]

Relaxing inflation models with non-minimal coupling: A general study

Tatsuki Kodama
Saga University
in collaboration with **Tomo Takahashi** (Saga U.)
TK, T.Takahashi, in preparation

10 MINITES VERSION

JGRG30 Dec.6 - 10@online

Definitions and Notations (1)

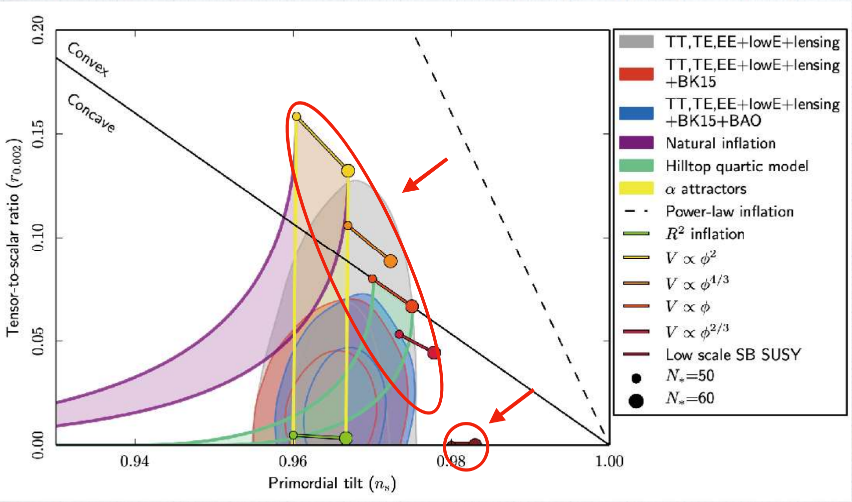
* We choose the natural unit, i.e., $c = \hbar = k_B = 1$.
* Slow-roll parameters: $\epsilon = (M_{\rm pl}^2/2)(V'/V)^2\,, \quad \eta = M_{\rm pl}^2 V''/V\,.$
* The spectral index: $n_s = 1 - 6\epsilon + 2\eta\,.$
* The tensor-to-scalar ratio: $r = 16\epsilon$

Definitions and Notations (2)

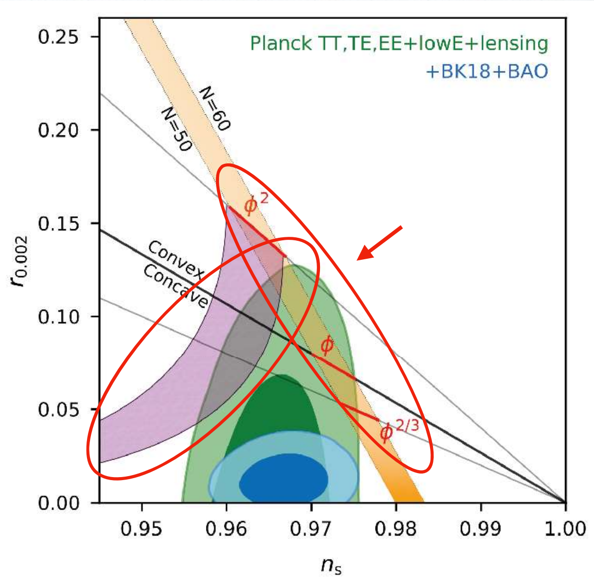
* The values in the Jordan frame **always** have a subscript J .
* For example, $S_J, V_J, \epsilon_J, \eta_J$.
* The values in the Einstein frame have a subscript E, though **not always.**
* For example, S_E and V_E have a subscript, but other parameters don't.

Motivation

Y. Akrami et al. 1807.06211



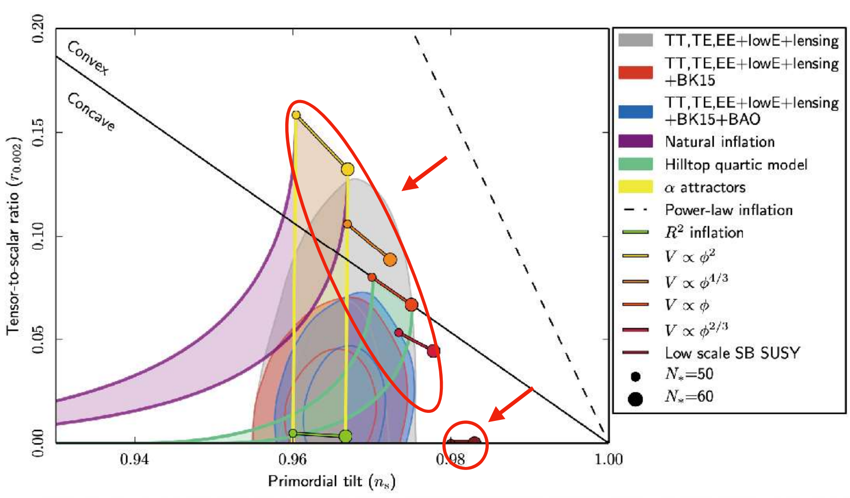
P. A. R. Ade et al. 2110.00483



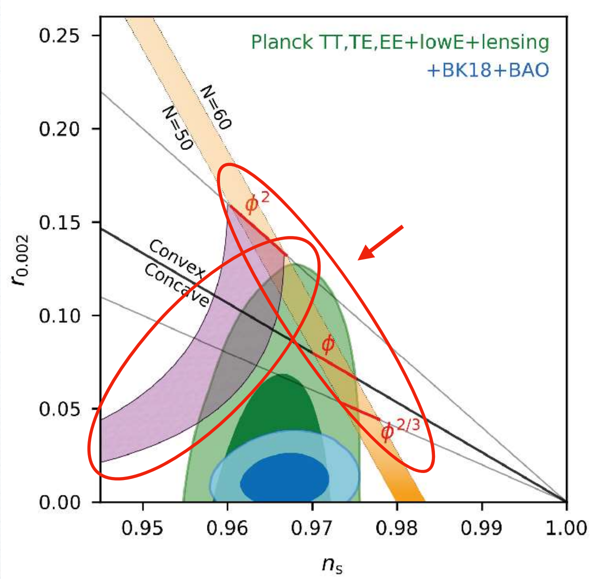
These models are excluded by observational constraints.

Motivation

Y. Akrami et al. 1807.06211



P. A. R. Ade et al. 2110.00483



How n_s and r parameters are modified by introducing a non-minimal coupling?

Set up

Action in the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_{pl}^2 A(\phi) g^{\mu\nu}R_{\mu\nu}(\Gamma) - \frac{1}{2}B(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V_J(\phi)\right]$$

$A(\phi), B(\phi)$: arbitrary scalar function

$V_J(\phi)$: potential in the Jordan frame

Weyl transformation

$\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu}\,,\qquad \Omega^2(\phi) = A(\phi) = 1 + \xi F(\phi)$

$$S_E = \int d^4x\sqrt{-\hat{g}}\left[-\frac{1}{2}M_{pl}^2 \hat{g}^{\mu\nu}\hat{R}_{\mu\nu}(\hat{\Gamma}) - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + V_E(\chi(\phi))\right]$$

$$\frac{d\phi}{d\chi} = \sqrt{\frac{2A(\phi)^2}{2A(\phi) + 3\kappa M_{pl}^2(A'(\phi))^2}}$$

$\kappa = 1$: metric formalism
$\kappa = 0$: Palatini formalism

V_E : potential in the Einstein frame

$$V_E(\phi(\chi)) = \frac{V_J(\phi)}{A^2(\phi)} = \frac{V_J(\phi)}{\Omega^4(\phi)}$$

Slow-roll parameters in the Einstein frame

- We need slow-roll parameter to get inflationary observables such as n_s and r

$$\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{V_{E\chi}}{V_E}\right)^2, \qquad \eta = M_{Pl}^2 \frac{V_{E\chi\chi}}{V_E} \quad \Rightarrow \quad n_s = 1 - 6\epsilon + 2\eta\,, \qquad r = 16\epsilon$$

Expressing by ϕ

$$\epsilon = P^2(\phi)\left[\epsilon_J - 2M_{Pl}^2 \frac{A'}{A}\frac{V_J'}{V_J} + 2M_{Pl}^2\left(\frac{A'}{A}\right)^2\right]$$

$$\eta = P^2(\phi)\left[\eta_J - 4M_{Pl}^2 \frac{A'}{A}\frac{V_J'}{V_J} + 6M_{Pl}^2\left(\frac{A'}{A}\right)^2 - 2M_{Pl}^2\frac{A''}{A} + M_{Pl}^2\frac{P'}{P}\left(-2\frac{A'}{A} + \frac{V_J'}{V_J}\right)\right]$$

where $P(\phi) \equiv \dfrac{d\phi}{d\chi}$.

- e-folding number

$$N = \frac{1}{M_{Pl}^2}\int_{\phi_{\rm end}}^{\phi_*} \frac{1}{P^2}\left(\frac{V_J'}{V_J} - 2\frac{A'}{A}\right)^{-1} d\phi \qquad A(\phi) = 1 + \xi F(\phi)$$

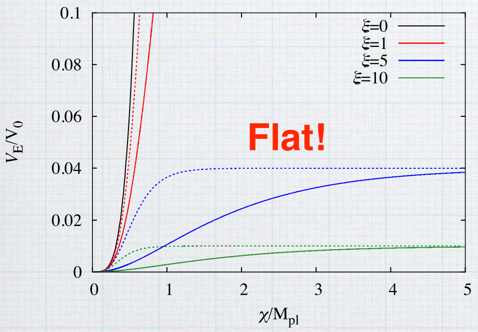
Model classification

- The derivative of the Einstein potential with respect to χ is given by

$$\frac{dV_E}{d\chi} = \frac{d\phi}{d\chi}\left(\frac{V_J}{A^2}\right)\left(\frac{V_J'}{V_J} - 2\frac{A'}{A}\right) \quad \Rightarrow \quad \frac{dV_E}{d\chi} = P(\phi)\left(\frac{V_J}{F^2}\right)\boxed{\left(\frac{V_J'}{V_J} - 2\frac{F'}{F}\right)}\frac{1}{\xi^2} + \mathcal{O}\left(\frac{1}{\xi^3}\right)$$

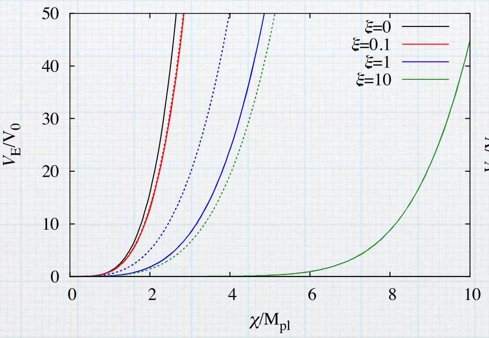
Chaotic inflation

TK, T.Takahashi, in preparation



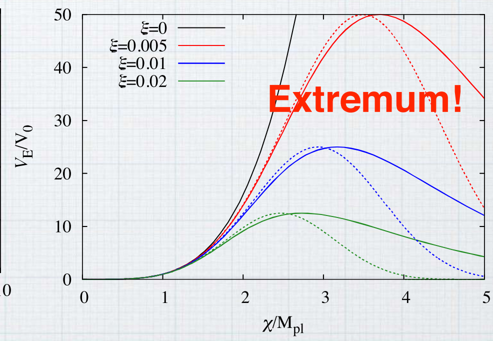
$\dfrac{V_J'}{V_J} = 2\dfrac{F'}{F}$

Attractor type



$\dfrac{V_J'}{V_J} > 2\dfrac{F'}{F}$

V_J-dominant type



$\dfrac{V_J'}{V_J} < 2\dfrac{F'}{F}$

F-dominant type

Example models

- Chaotic inflation with power-law $F(\phi)$
  - $p = 2n$ (attractor)
  - $p > 2n$ (V_J-dominant)
  - $p < 2n$ (F-dominant)

$$V_J(\phi) = V_0\left(\frac{\phi}{M_{Pl}}\right)^p, \qquad F(\phi) = \left(\frac{\phi}{M_{Pl}}\right)^n$$

$n = 2, p = 4$

$$V_J(\phi) = V_0\left(\frac{\phi}{M_{Pl}}\right)^4, \quad F(\phi) = \left(\frac{\phi}{M_{Pl}}\right)^2$$

F. Bezrukov, M. Shaposhinkov
0710.3755

- Natural inflation with cosine-type $F(\phi)$
  - $n = 1/2$ (attractor)
  - $n > 1/2$ (V_J-dominant)
  - $n < 1/2$ (F-dominant)

$$V_J(\phi) = V_0\left[1 - \cos\frac{\phi}{f}\right], \qquad F(\phi) = \left[1 - \cos\frac{\phi}{f}\right]^n$$

$n = 1$

$$V_J(\phi) = \Lambda^4\left[1 + \cos\frac{\phi}{f}\right], \quad A(\phi) = 1 + \alpha\left[1 + \cos\frac{\phi}{f}\right]$$

R. Z. Ferreira et at. 1806.0511

- Loop inflation with log-type $F(\phi)$
  - $n = 1/2$ (attractor)
  - $n > 1/2$ (V_J-dominant)
  - $n < 1/2$ (F-dominant)

$$V_J(\phi) = V_0\left[1 + a_h \log\frac{\phi}{M_{Pl}}\right], \qquad F(\phi) = \left[1 + a_h \log\frac{\phi}{M_{Pl}}\right]^n$$

Attractor type (metric formalism)

$$\boxed{\frac{V_J'}{V_J} = 2\frac{F'}{F}} \longrightarrow \boxed{V_J = CF^2 \quad (C : const.)}$$

- S. C. Park, S. Yamaguchi 0801.1722
- R. Kallosh, A. Linde, D. Roset 1310.3950

slow-roll parameters expanded around $\xi = \infty$

$$\epsilon = \frac{4}{3F^2\xi^2} + \mathcal{O}\left(\frac{1}{\xi^3}\right), \quad \eta = -\frac{4}{3F\xi} + \mathcal{O}\left(\frac{1}{\xi^2}\right)$$

e-folding number expanded around $\xi = \infty$

$$N \simeq \frac{3}{4}\xi F(\phi_*)$$

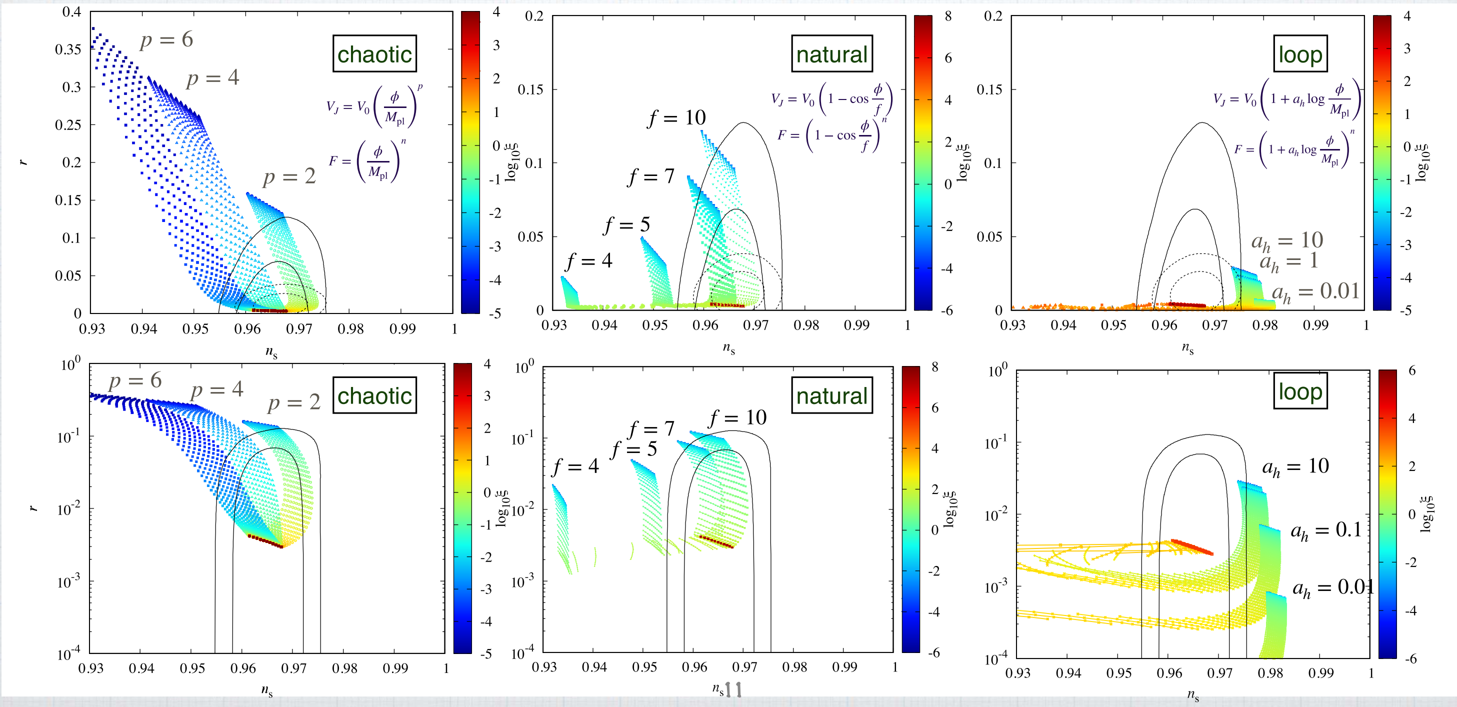
$$n_s - 1 \simeq -\frac{2}{N} \qquad r = \frac{12}{N^2}$$

n_s and r approach to some attractor point **model independently!!**

$n_s - 1 = -\frac{9}{2N^2} - \frac{2}{N} \simeq -\frac{2}{N}$ (typical value of N is $50 - 60$)

Attractor type (metric formalism: $\kappa = 1$)

$$n_s - 1 \simeq -\frac{2}{N} \begin{cases} = 0.960 \; (N = 50) \\ \simeq 0.967 \; (N = 60) \end{cases} \qquad r = \frac{12}{N^2} \begin{cases} = 4.8 \times 10^{-3} \; (N = 50) \\ \simeq 3.3 \times 10^{-3} \; (N = 60) \end{cases}$$



Attractor type (Palatini formalism)

$$\boxed{\frac{V_J'}{V_J} = 2\frac{F'}{F}} \longrightarrow \boxed{V_J = CF^2 \quad (C : const.)}$$

slow-roll parameters expanded around $\xi = \infty$

$$\epsilon = \frac{2(F')^2 M_{Pl}^2}{F^3 \xi} + \mathcal{O}\left(\frac{1}{\xi^3}\right) \xrightarrow{\xi \to \infty} 0$$

$$\eta = M_{Pl}^2 \left(-3\left(\frac{F'}{F}\right)^2 + 2\frac{F''}{F}\right) + \cancel{\frac{5(F')^2 M_{Pl}^2}{F^3 \xi}} + \mathcal{O}\left(\frac{1}{\xi^2}\right)$$

e-folding number expanded around $\xi = \infty$

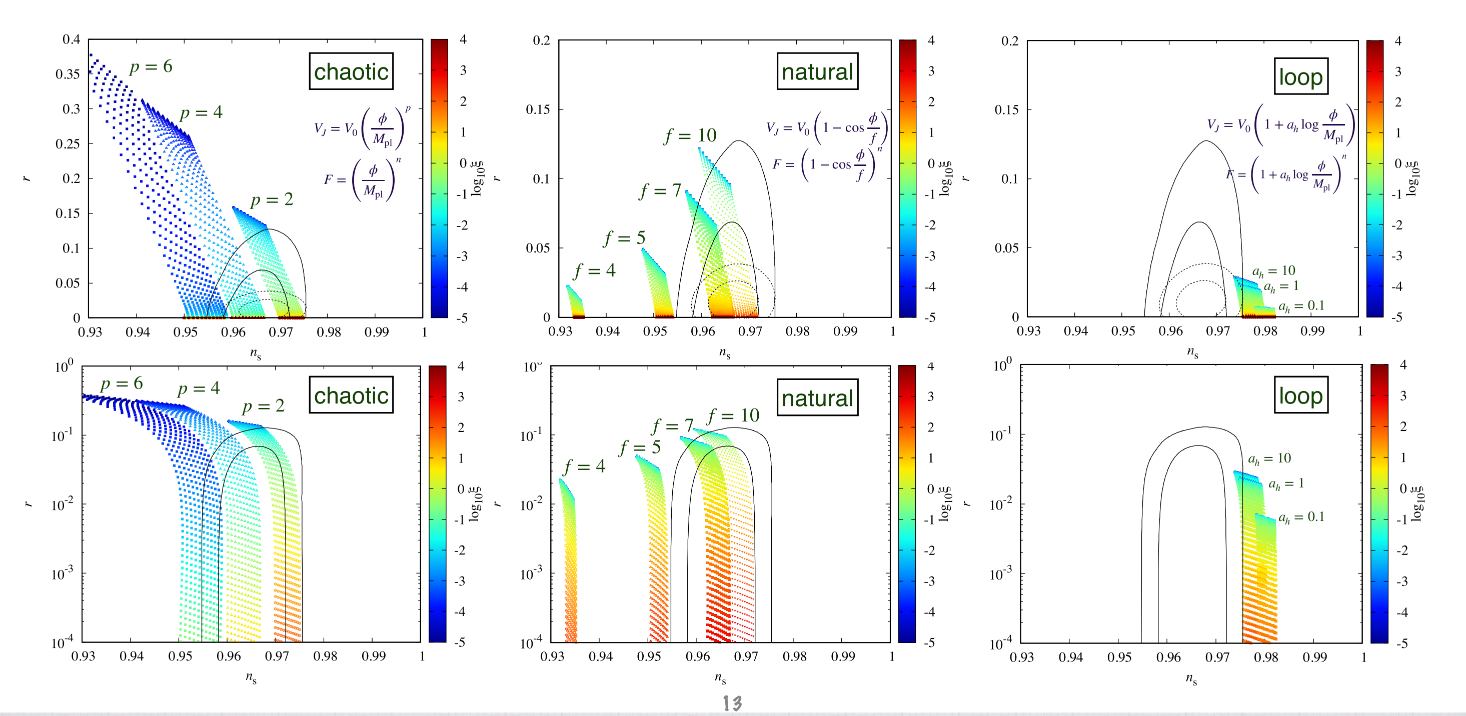
$$N = \frac{1}{M_{\rm pl}^2} \int_{\phi_{\rm end}}^{\phi_*} \frac{V_J}{V_J'} d\phi$$

(Same as the **minimal** coupling case!!)

r is suppressed to 0. $(r = 16\epsilon)$

$$n_s - 1 \simeq 2\eta = -5\epsilon_J + 2\eta_J$$

Attractor type (Palatini formalism: $\kappa = 1$)



V_J dominant type

We assume the relation between V_J and F to be

$$\boxed{\alpha\frac{V_J'}{V_J} = 2\frac{F'}{F}} \qquad (\alpha > 1)$$

where α is constant.

When $\xi \gg 1$, slow-roll parameters are

$$\epsilon = P^2\epsilon_J(1-\alpha)^2$$

$$\eta = P^2\left[(1-\alpha)\eta_J - 2\alpha(1-\alpha)\epsilon_J + \frac{1}{2}\alpha(1-\alpha)P^2\epsilon_J\left\{\frac{1}{\xi F} + 3\kappa\left(\alpha\epsilon_J - \frac{\alpha}{2}\eta_J\right)\right\}\right]$$

e-folding number in the large ξ limit

$$N = \frac{1}{M_{Pl}^2}\int_{\phi_{\rm end}}^{\phi_*}\frac{1}{P^2}\frac{F}{F'}$$

V_J dominant type

e-folding number in the large ξ limit

$$N = \frac{1}{M_{Pl}^2}\int_{\phi_{\rm end}}^{\phi_*} \frac{1}{P^2}\frac{F}{F'}$$

$$P^2(\phi) = \frac{1}{\frac{1}{\xi F} + \frac{3}{2}\kappa M_{Pl}^2\left(\frac{F'}{F}\right)^2}$$

We define two parameters such as

$$a(\phi) \equiv \frac{1}{\xi F} \qquad b(\phi) \equiv \frac{3}{2}\kappa M_{Pl}^2\left(\frac{F'}{F}\right)^2$$

This type can be further divided into two cases

$$R \equiv \frac{b(\phi_*)}{a(\phi_*)} \begin{cases} > 1 & \text{(case 1)} \\ < 1 & \text{(case 2)} \end{cases}$$

V_J dominant type (case 1)

- **case 1**

The slow-roll parameters are reduced to

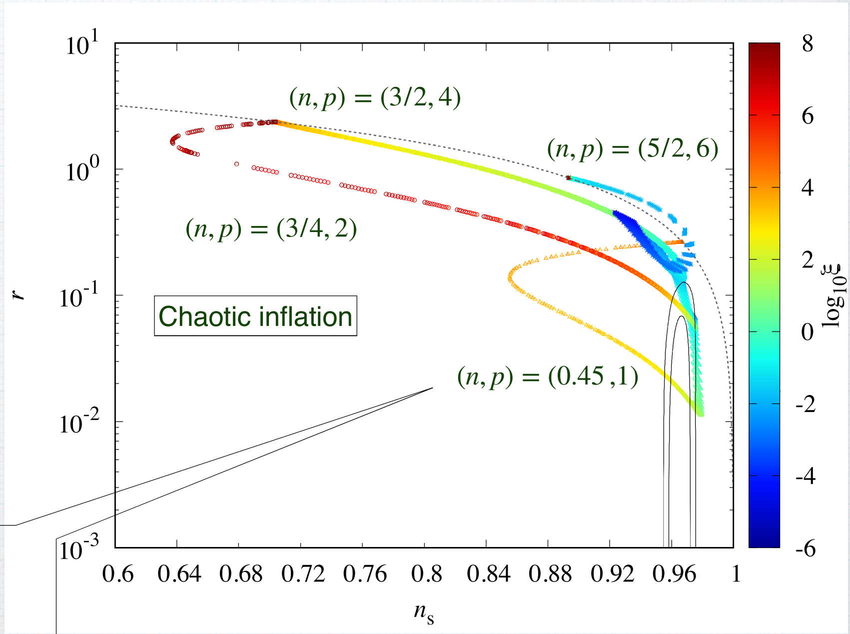
$$\epsilon = \frac{4}{3}\left(\frac{\alpha-1}{\alpha}\right)^2, \quad \eta = \frac{8}{3}\left(\frac{\alpha-1}{\alpha}\right)^2$$

$$(\epsilon = 2\eta)$$

<u>Consistency relation</u>

$$\boxed{r = 8(1-n_s)}$$

The **Case 1** is an attractor independent on the e-folding number (however, n_s and r are model dependent)



V_J dominant type (case 2)

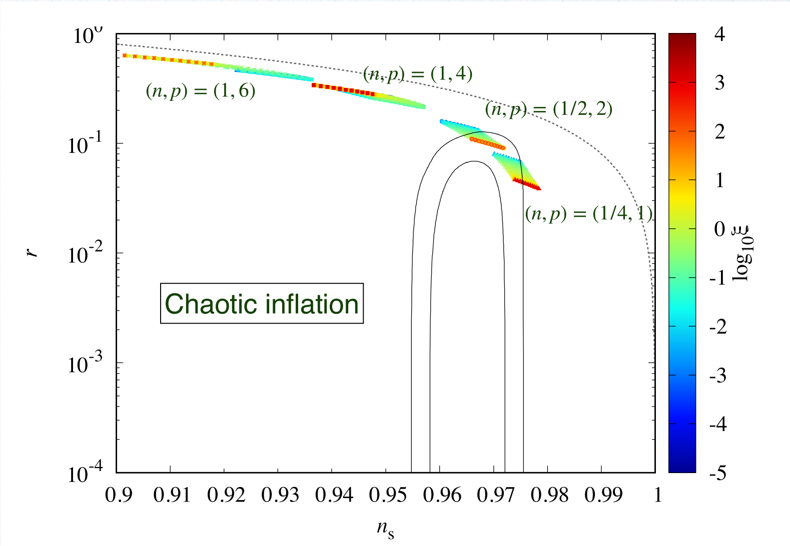
- **case 2**

The slow-roll parameters are reduced to

$$\epsilon = (1-\alpha)^2 \epsilon_J F \xi$$
$$\eta = F\xi \left[(1-\alpha)\eta_J - \frac{3}{2}\alpha(1-\alpha)\epsilon_J \right]$$

The e-folding number is

$$N_e = \frac{1}{2(1-\alpha)\xi M_{Pl}^2} \int_{\phi_{\rm end}}^{\phi_*} \frac{d\phi}{F'}$$



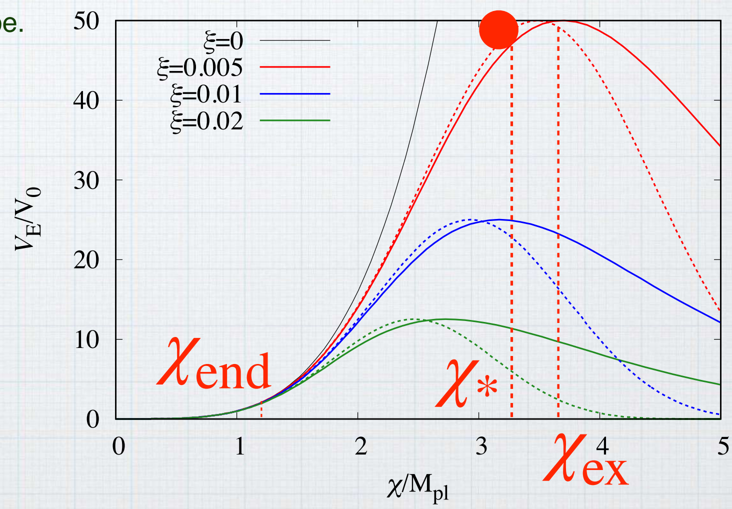
$$n_s - 1 = \frac{-p+3n-2}{(2-n)N_e} \qquad r = \frac{8(p-2n)}{2-n}\frac{1}{N_e} \qquad N_e \simeq \frac{1}{(p-2n)(2-n)\xi}\left(\frac{\phi_*}{M_{Pl}}\right)^{2-n}$$

The Palatini case is automatically is **case 2**.

F dominant type

- Even if V_J doesn't have an extremum, V_E always has an extremum in F-dominant type.
- The more ξ increases, the more $\chi_{\rm ex}$ decreases.
- We should fine-tune the value of χ_* to obtain a sufficient amount of e-folding number N_e
- So χ_* very close to $\chi_{\rm ex}$

We expand χ_* around $\chi_{\rm ex}$!



F dominant type

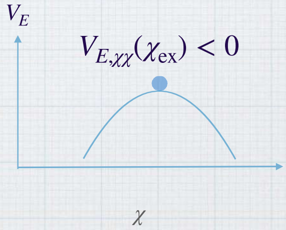
- Expanding χ_* around χ_{ex},

$$\epsilon(\chi_*) = \underbrace{\cancel{\epsilon(\chi_{ex})} + \cancel{\left.\frac{d\epsilon}{d\chi}\right|_{\chi=\chi_{ex}}}}_{\because\ V_{E,\chi}=0}(\chi_*-\chi_{ex}) + \frac{1}{2}\left.\frac{d^2\epsilon}{d\chi^2}\right|_{\chi=\chi_{ex}}(\chi_*-\chi_{ex})^2 + \mathcal{O}\left((\chi_*-\chi_{ex})^3\right)$$

- The second derivative of $\epsilon(\chi)$ with respect to χ is $\left.\frac{d^2\epsilon}{d\chi^2}\right|_{\chi=\chi_{ex}} = \left.\left(M_{Pl}^2\frac{V_{E,\chi\chi}}{V_E}\right)^2\right|_{\chi=\chi_{ex}} = \eta^2(\chi_{ex}),.$

$$\Rightarrow\quad \epsilon(\chi_*) = \frac{1}{2}\eta(\chi_{ex})(\chi_*-\chi_{ex})^2 + \mathcal{O}\left((\chi_*-\chi_{ex})^3\right)$$ ← Very suppressed !!

- Thus, the spectral index and tensor-to-scalar ratio are given by

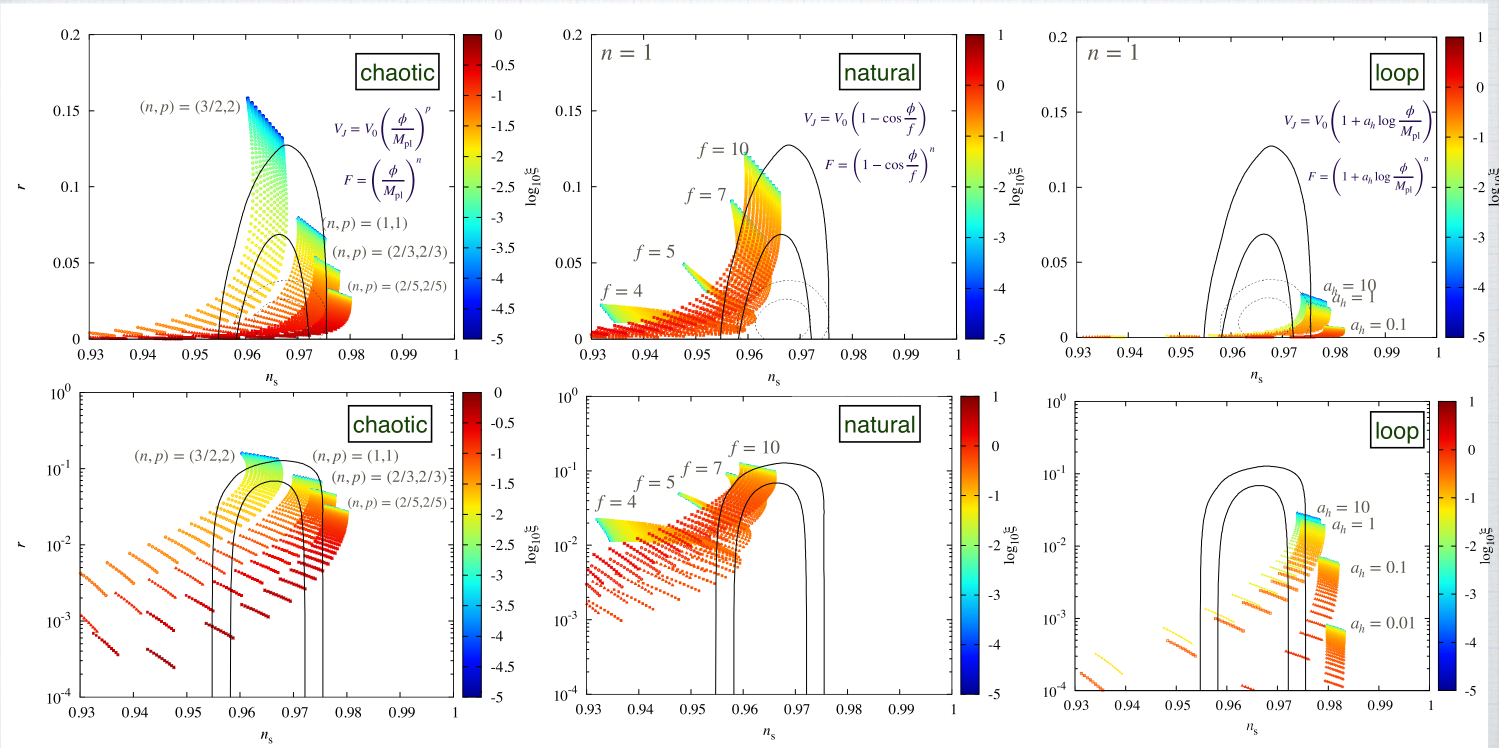


$$\boxed{\begin{cases} r \simeq 8\eta^2(\chi_{ex})(\chi_*-\chi_{ex})^2 \\ n_s \simeq 1 - 2|\eta(\chi_{ex})| \end{cases}}$$

← Very suppressed !!

← Red tilted !!

F dominant type



Summary

* We have investigated how inflation models with non-minimal coupling relax in the general setting.
* Inflation models can be classified into three types.
* Attractor type (metric formalism) is always consistent with current observational constraints, but it isn't necessarily in Palatini formalism.
* V_J dominant type is not consistent with observations in most cases.
* n_s is generically red-tilted and r is suppressed in F dominant type. Therefore, when n_s and r are larger than the allowed region, they would enter the allowed region as increasing ξ.

Poster session

Seiga Sato

Waseda University

“Predicting complex trajectories around Kerr black hole via deep learning”

[JGRG30 (2021) PA12]

[P12]

Predicting complex **trajectories** around **Kerr** black hole via **deep learning**

Seiga SATO

Waseda University

@JGRG30, online

- **Motivation & Summary**

➢ Some previous works claim that the Deep Learning(DL) can not predict the chaotic system.

➢ I would like to know that whether chaos is critical for the deep learning.

➢ I investigate whether the DL can predict the test particle trajectories around the Kerr Black hole without EoM.

➢ The test particle dynamics is quite complicated, and the dynamics becomes chaos when it has spin.

➢ I employed 2 DL models, LSTM(previous work) and Lagrangian Neural Network.

Previous Work : Y.Misonoh (2017)

"Imitating Equation of Motion via Deep Learning"

$$\omega^1 = -\sin x^3\, dx^1 + \sin x^1 \cos x^3\, dx^2$$
$$\omega^2 = \cos x^3\, dx^1 + \sin x^1 \sin x^3\, dx^2$$
$$\omega^3 = \cos x^1\, dx^2 + dx^3$$

Set-up: Bianchi type-IX (a : const)

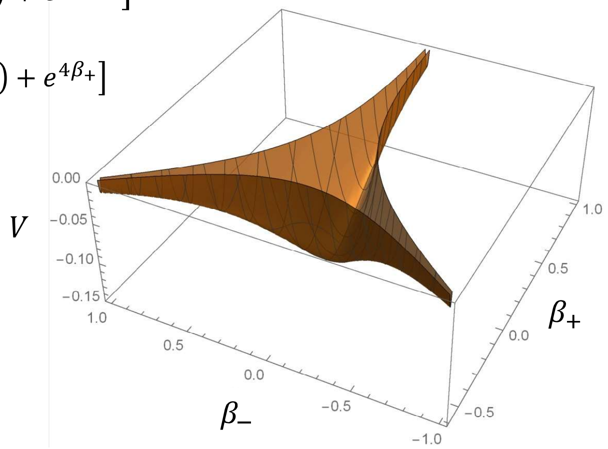
$$ds^2 = -dt^2 + \frac{a^2}{4} e^{\beta_{ij}} \omega_i \omega_j \quad , \quad d\omega^i = \frac{1}{2}\varepsilon^i{}_{jk}\omega^j \wedge \omega^k \quad , \quad \beta_{ij} = diag\left(\beta_+ + \sqrt{3}\beta_-, \beta_+ - \sqrt{3}\beta_-, -2\beta_+\right)$$

EoM

$$\dot{\beta}_\pm = p_\pm \qquad V(\beta_+, \beta_-) = \frac{1}{24}\left[2e^{4\beta_+}\cosh\left(4\sqrt{3}\beta_-\right) + e^{-8\beta_+}\right]$$
$$\dot{p}_\pm = -\frac{dV}{d\beta_\pm} \qquad -\frac{1}{12}\left[2e^{-2\beta_+}\cosh\left(2\sqrt{3}\beta_-\right) + e^{4\beta_+}\right]$$

Dynamic is same as the ball on the right potential.

- High energy : Chaos
- Low energy : Almost periodic

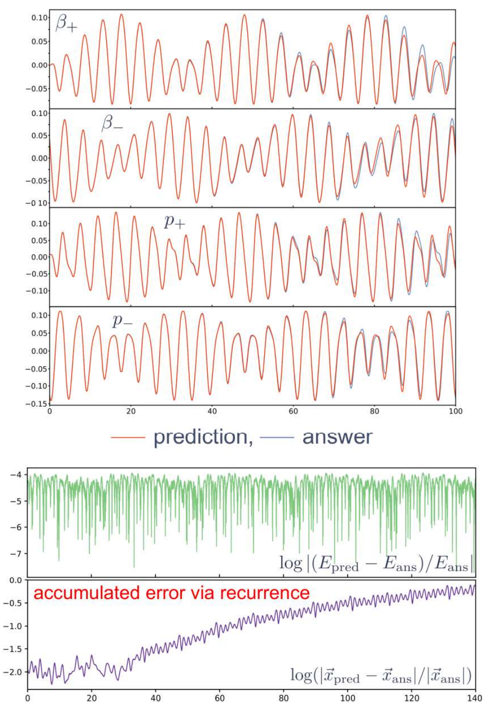


Prediction and Error

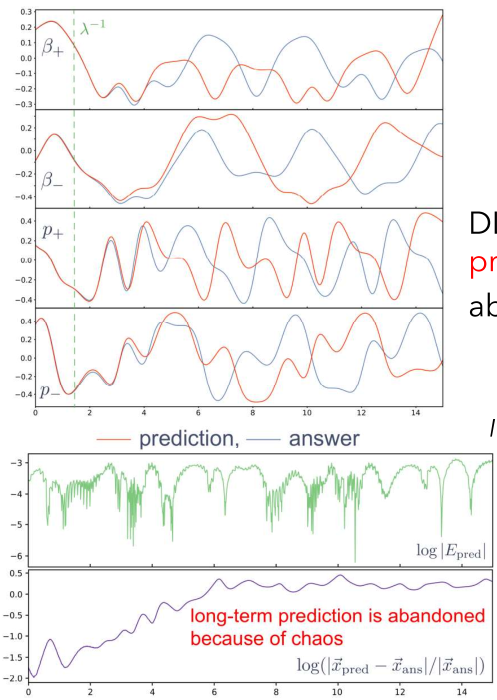
cf. Y.Misonoh, JGRG27 proceeding Vol.3

λ: Lyapunov exponent

➢ Low energy



➢ High energy



DL(LSTM) can not predict longer than about $\frac{1}{\lambda}$

I will explain LSTM later…

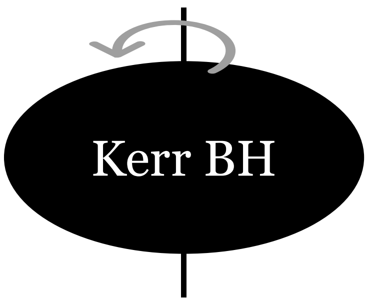
- Motivation : *Chaos is so important for DL?*

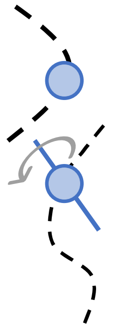
At low energy in the previous work, the exact EoMs are non-integrable but the solution is almost periodic. DL could predict it at low energy.

How about the integrable but complicated trajectories?

⇨ test particle around the Kerr BH

S.Suzuki&K.Maeda(1996),
K.Kiuchi & K.Maeda(2018)





predictable ?

unpredictable ?

It is a more realistic situation.
Spin interactions between the test particle and BH make the dynamics chaos.
Unlike the previous study, the EoMs itself are changed.
Since I suppose the observations, I provide the information of position and speed only without EoMs.

- Basic equations

S.Suzuki&K.Maeda(1996), Dixon(1970),
K.Kiuchi & K.Maeda(2018)

➢ Kerr Metric

$$\boxed{\begin{aligned}\Sigma &= r^2 + a^2\cos^2\theta\\ \Delta &= r^2 - 2Mr + a^2\end{aligned}}$$

$$ds^2 = -\left(1-\frac{2Mr}{\Sigma}\right)dt^2 - \frac{4Mar\sin^2\theta}{\Sigma}dtd\phi + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \sin^2\theta\left(r^2 + a^2 + \frac{2Ma^2r\sin^2\theta}{\Sigma}\right)$$

➢ Non-Spinning

Integrable

$$\Sigma\frac{d\theta}{d\tau} = \pm\sqrt{\Theta},$$
$$\Sigma\frac{dr}{d\tau} = \pm\sqrt{R},$$
$$\Sigma\frac{d\phi}{d\tau} = -\left(aE - \frac{L_z}{\sin^2\theta}\right) + \frac{a}{\Delta}P,$$
$$\Sigma\frac{dt}{d\tau} = -a(aE\sin^2\theta - L_z) + \frac{r^2+a^2}{\Delta}P,$$

$$\boxed{\begin{aligned}&\Theta(\theta) = C - \cos^2\theta\left\{a^2(1-E^2) + \frac{L_z^2}{\sin^2\theta}\right\},\\ &P(r) = E(r^2+a^2) - aL_z,\\ &R(r) = P^2 - \Delta\{r^2 + (L_z - aE)^2 + C\},\end{aligned}}$$

➢ Spinning Dixon(1970)

Non-integrable

$$\boxed{M = m = 1}$$

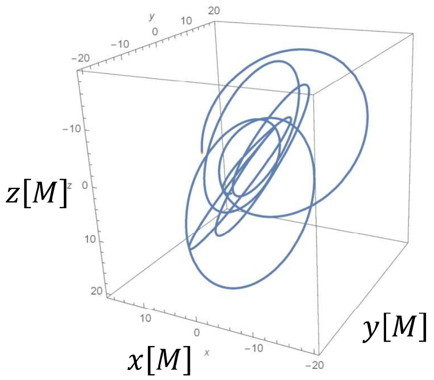
$$\frac{dx^\mu}{d\tau} = v^\mu,$$
$$\frac{Dp^\mu}{d\tau} = -\frac{1}{2}R^\mu{}_{\nu\rho\sigma}v^\nu S^{\rho\sigma},$$
$$\frac{DS^{\mu\nu}}{d\tau} = 2p^{[\mu}v^{\nu]},$$

L_z: Angular momentum
E : test particle energy
C : Carter Constant

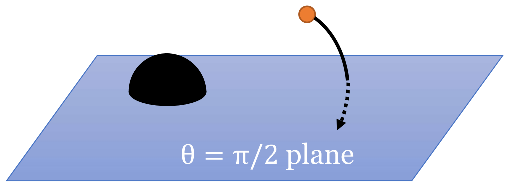
$$\boxed{S_{ab} = 2\int dx^3\,\rho x_{[a}u_{b]}}$$

- Trajectories and Poincare map

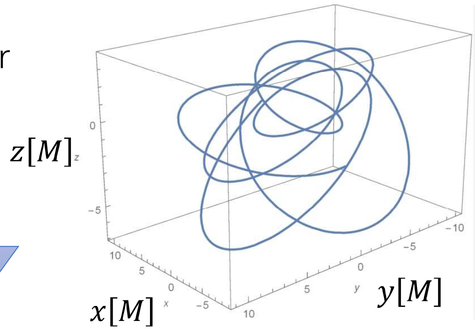
  - Spinless *Not chaos*

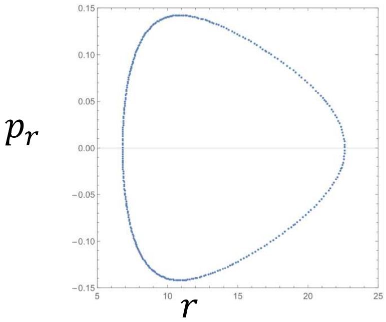


It is difficult to judge whether it is chaotic or not just from its appearance.

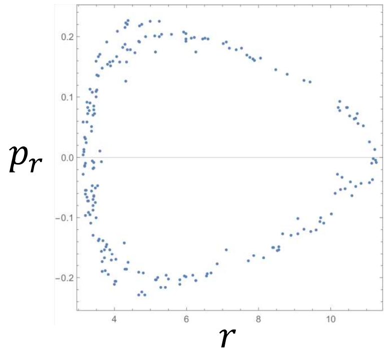


  - Spinning *Chaos*



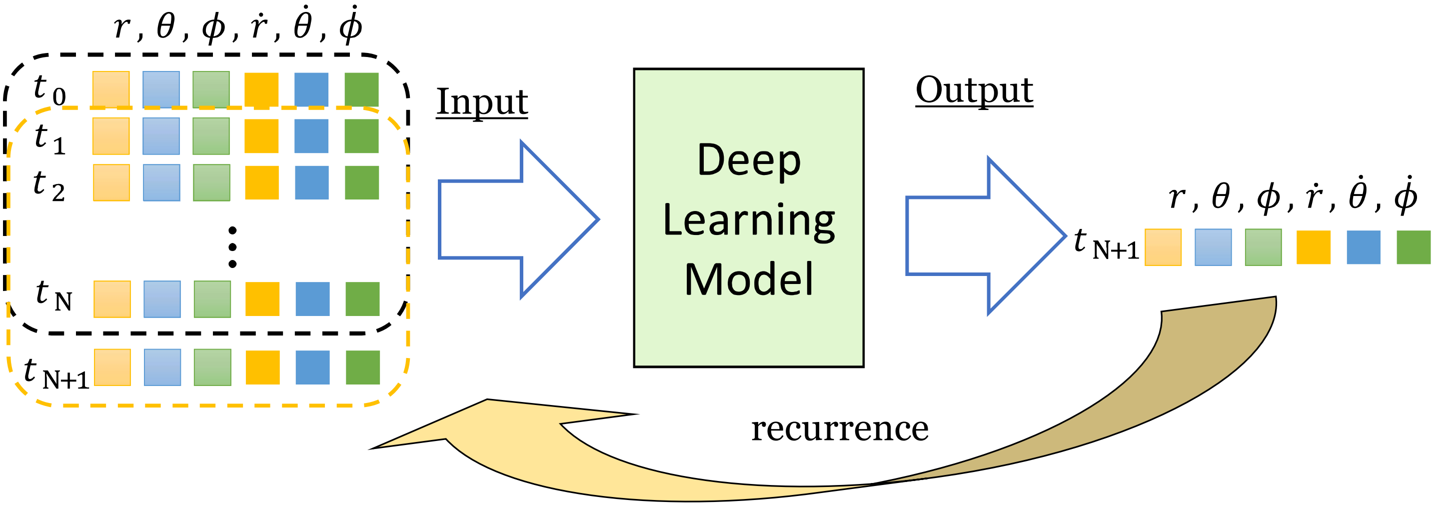


In the case of chaos, the Poincare map is not closed.



- How to predict deep learning : LSTM model

Training data are obtained by numerical calculation.
We give the information of position and speed for training.
The input form is sequence of them, which is about 1/27 cycle of r dynamics.
The dynamics is predicted by repeatedly reinserting the obtained output into the input.



Can the DL predict the trajectories without EoMs?

● How to predict deep learning : LSTM model

➢ Standardized data set

Input data value should be 0~1.
I "Standardized" the values for each values.

All values
$r\,,\,\theta\,,\,\phi\,,\,\dot{r}\,,\,\dot{\theta}\,,\,\dot{\phi}$ ⇒ $0 \le \tilde{r}\,,\,\tilde{\theta}\,,\,\tilde{\phi}\,,\,\tilde{\dot{r}}\,,\,\tilde{\dot{\theta}}\,,\,\tilde{\dot{\phi}} \le 1$

- r

  Since r is always positive, it is easy.

$$\tilde{r} = \frac{r}{r_{max}}$$

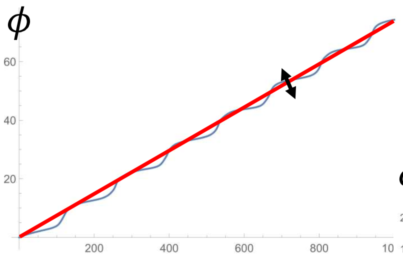
- $\theta\,,\,\phi\,,\,\dot{r}\,,\,\dot{\theta}\,,\,\dot{\phi}$

  After the scaling, the values are shifted

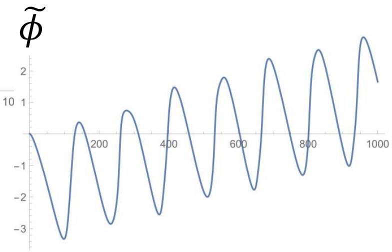
  $ex)$

$$\tilde{\dot{r}} = \frac{\dot{r}}{2\,|\dot{r}_{max}|} + \frac{1}{2}$$

- Unbouded values: ϕ



ϕ is unbounded.
I define the $\tilde{\phi}$ as the deviation from the linear evolution.

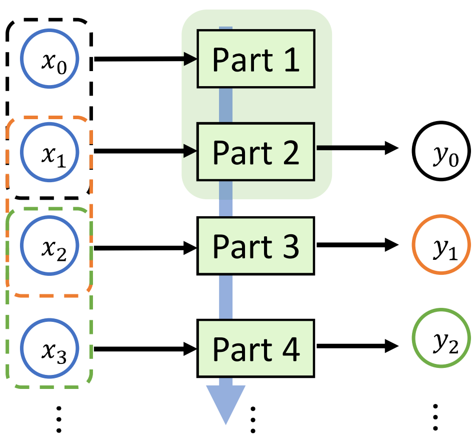


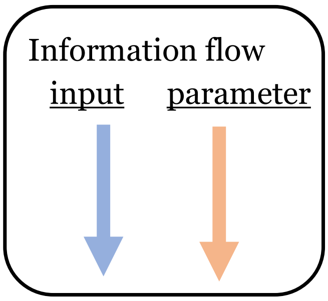
● How to predict deep learning : LSTM model

S.Hochreiter & J. Schmidhuber (1997)
F.Gers, J.Schmidhuber & F. Cummins (1999,2000)

I adopted Long Short Term Memory (LSTM) model.
It is one of the Recurrent Neural Network (RNN), which treats the sequence data.
RNN is good at predicting the future from sequence of the previous information, like translations .
LSTM is an advanced model of RNN, which has an additional flow of the parameters in the model.

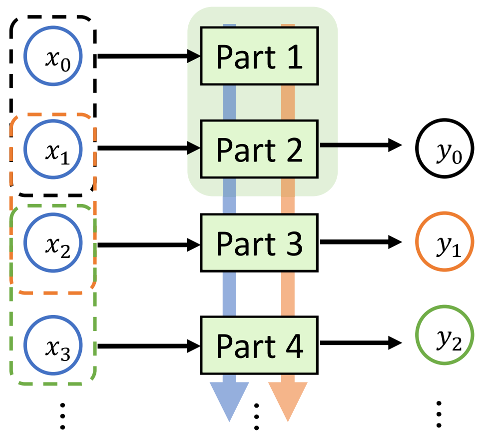
➢ Previous RNN





In this study,
#para $\simeq 4 \times 10^4$

➢ LSTM

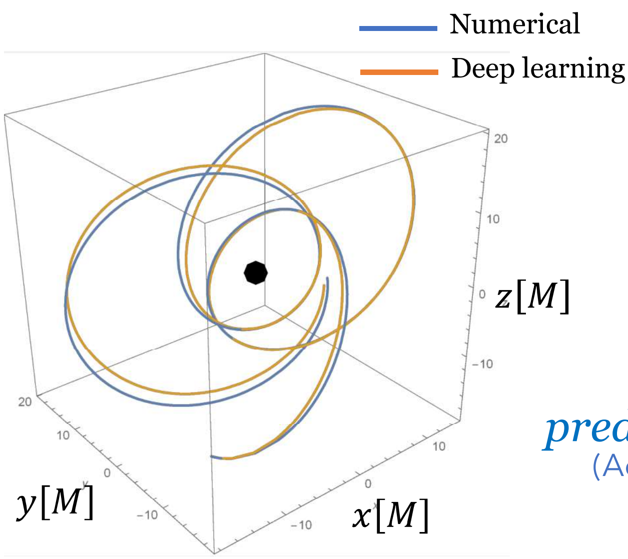


● Result (spinless)

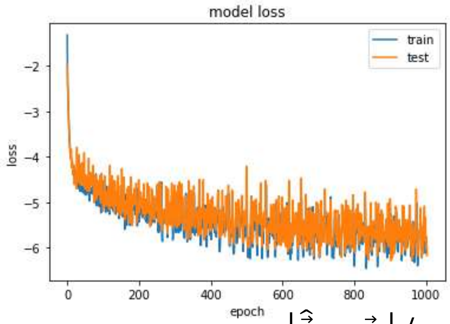
Setup

$\hat{\vec{x}}_i$: Answer values
$\vec{x}_i$: Predicted values

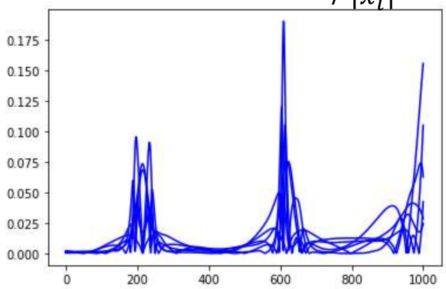
$$L_z = 2, a = \frac{1}{\sqrt{2}}, E = \frac{968}{1000}, C = 10$$



loss function: MSE $: \frac{1}{n}\sum\left|\hat{\vec{x}}_i - \vec{x}_i\right|^2$

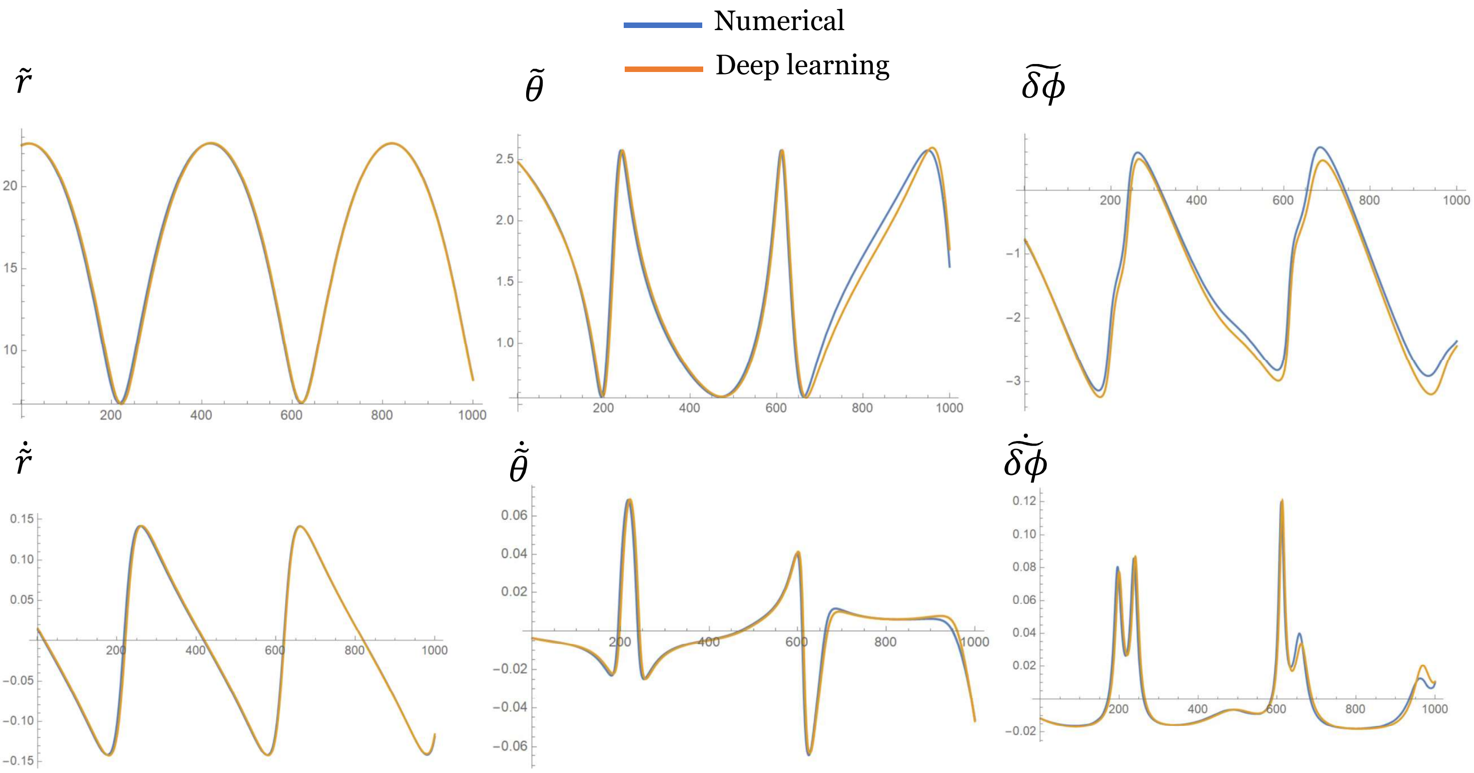


Error rate (Prediction): $\left|\hat{\vec{x}}_i - \vec{x}_i\right| / \left|\vec{x}_i\right|$



predictable
(Accracy≥ 95 %)

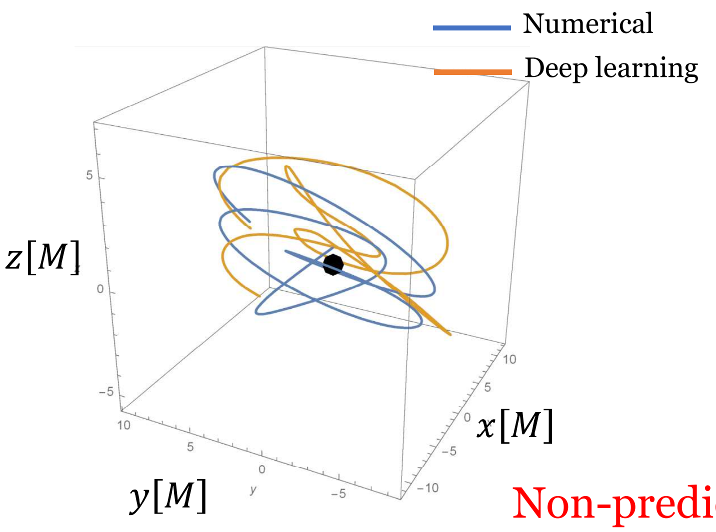
$$Accuracy: 100\left(1 - \frac{1}{n}\sum\frac{|\hat{\vec{x}}_i - \vec{x}_i|}{|\vec{x}_i|}\right) = 98.8\%$$



● Result (spinning)

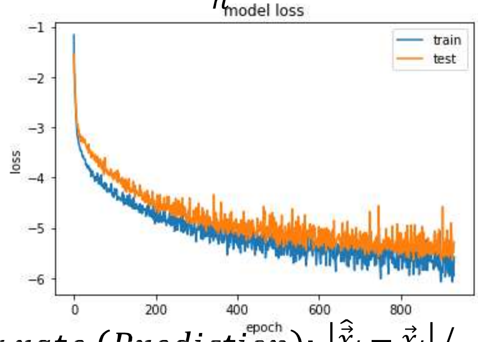
Setup

$$L_z = \frac{28}{10}, a = \frac{8}{10}, E = \frac{9328}{10000}, \frac{1}{2} S^{\mu\nu} S_{\mu\nu} = 1$$

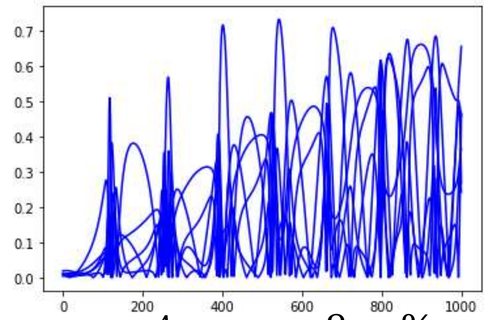


Non-predictable
(Accracy ≤ 95 %)

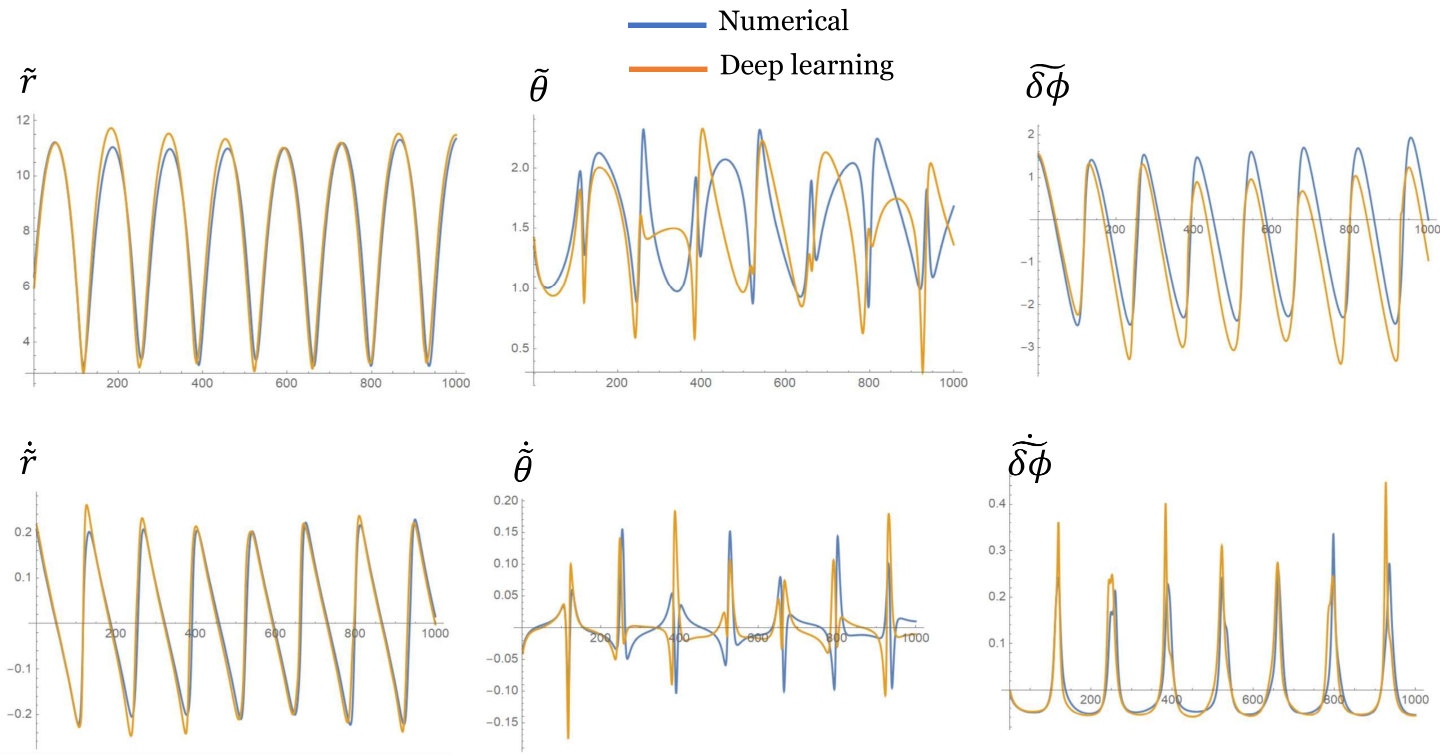
loss function: MSE $: \frac{1}{n}\sum\left|\hat{\vec{x}}_i - \vec{x}_i\right|^2$



Error rate (Prediction): $\left|\hat{\vec{x}}_i - \vec{x}_i\right| / \left|\vec{x}_i\right|$



Accuracy: 80.5%

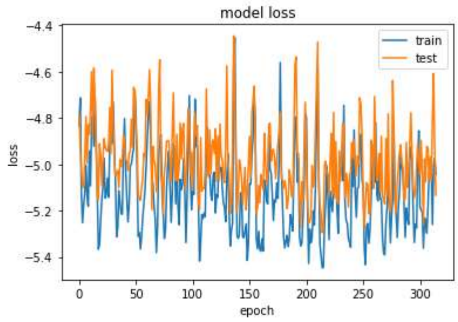


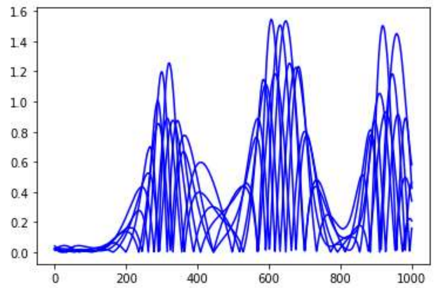
- **Learning coordinate dependence : Cartesian**

I used the Cartesian coordinate as the input data. $r, \theta, \phi, \dot{r}, \dot{\theta}, \dot{\phi} \Rightarrow x, y, z, \dot{x}, \dot{y}, \dot{z}$

➢ **Result (spinless)** $\boxed{L_z = 2, a = \frac{1}{\sqrt{2}}, E = \frac{968}{1000}, C = 10}$

loss function: MSE $: \frac{1}{n}\sum\left|\hat{\vec{x}}_i - \vec{x}_i\right|^2$ *Error rate (Prediction)*: $\left|\hat{\vec{x}}_i - \vec{x}_i\right| / \left|\vec{x}_i\right|$





Accuracy: 62.4%

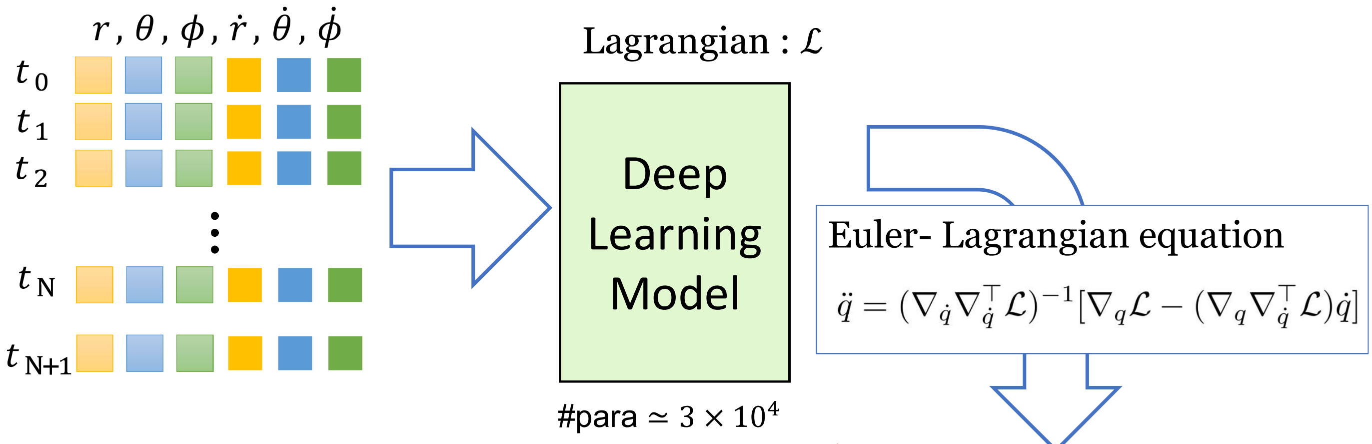
The learning becomes much worse. The accuracy is worse than chaos case.
DL did not learn the covariant information. The learning seems depends on the coordinate.

Coordinate independent model ⇒ *How about learning the Lagrangian?*

- **How to predict deep learning : LNN model** C.Miles et al (2020)

I also adopted another model, Lagrangian Neural Network(LNN) model.
In this model, the DL model learn the Lagrangian functional by the unsupervised method.
This model is expected effective when the system contains conserved quantities.

<u>Input</u> : not sequence



I gave additionally the information of $\ddot{q}$, and estimated the Mean Squared Error as the criteria for learning.
I did not give the information of EoMs.

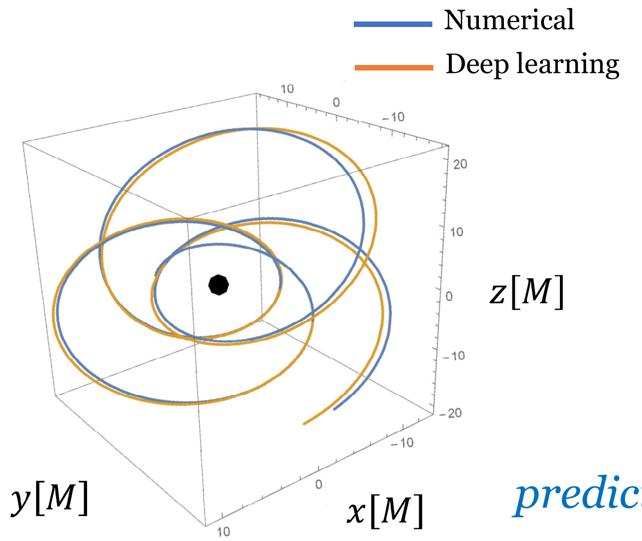
Loss fun. = $\sum\left|\hat{\ddot{q}}_i - \ddot{q}_i\right|^2$

- Result (spinless)
  - Dynamics

Setup : same as LSTM

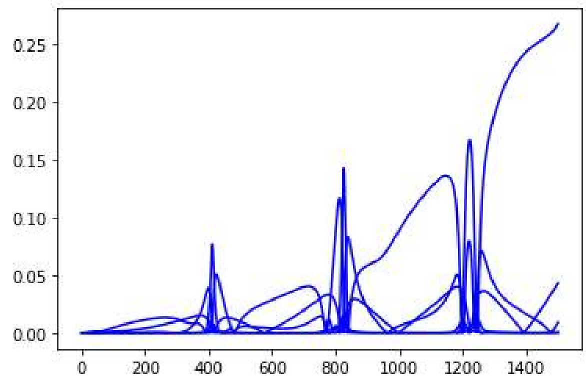
$$L_z = 2, a = \frac{1}{\sqrt{2}}, E = \frac{968}{1000}, C = 10$$

DL can predict with less parameters than LSTM.
The accuracy is as good as LSTM model.



predictable

$$\text{Loss fun.} = \sum \left| \ddot{\hat{q}}_i - \ddot{q}_i \right|^2 \sim 10^{-7}$$



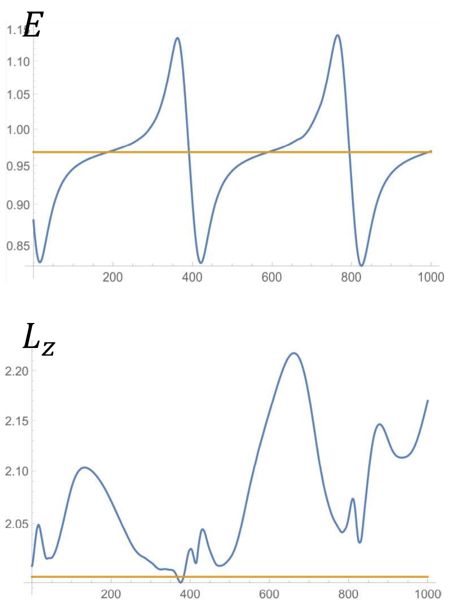
$Accuracy$: 98.3%

- Result (spinless)
  - Cnserved Quantities

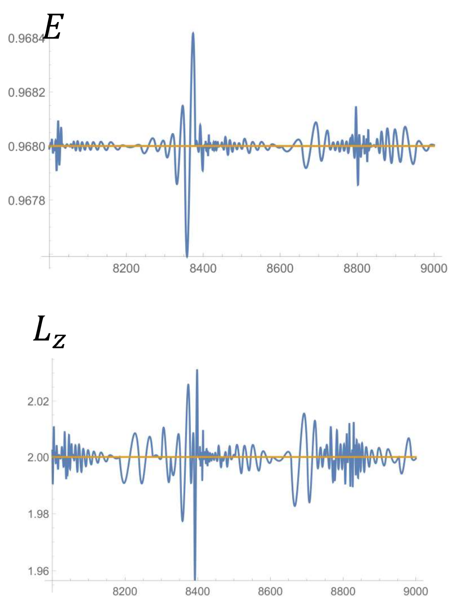
There does not seem to be much difference between LNN and LSTM in the evaluation of conserved quantities.
The longer time scale may cause difference.

$$\boxed{L_z = 2, E = \frac{968}{1000}}$$

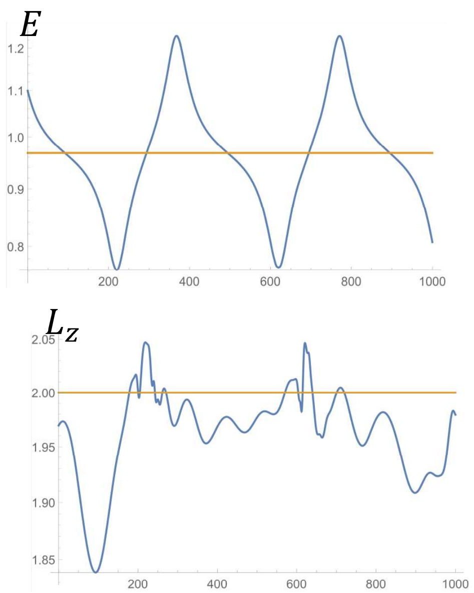
- LNN



- Numerical Cal.

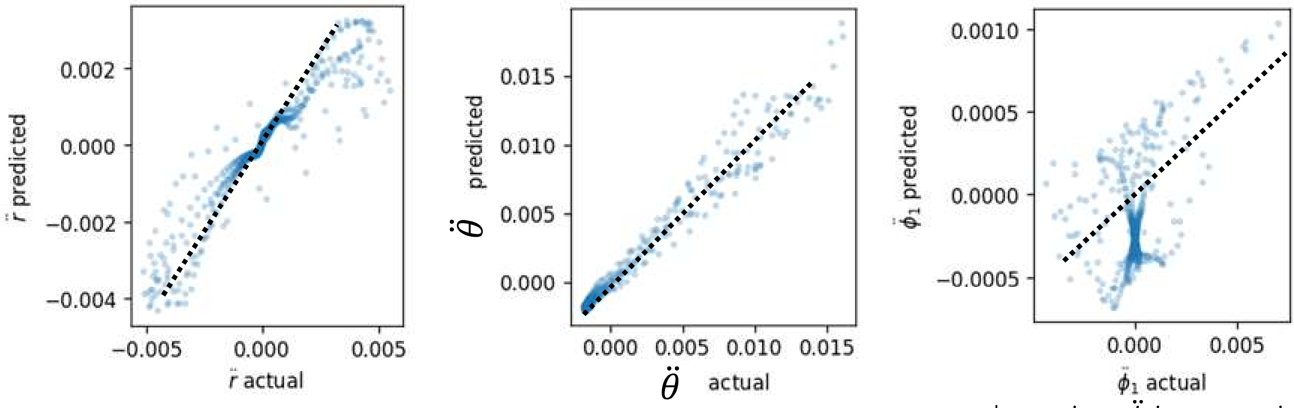


- LSTM



● Result (spinning/chaos)

➢ Learning



Learning $\ddot{\phi}$ is not going well. Standardization spoiled the physical meaning?

If learning is successful, the line will be aligned on the diagonal (dashed line).
Continuing the learning any longer causes the over learning.
This figure shows the learning are not going well In the chaos case, as long as the number of parameters are same as LSTM.

● Summary

➢ If the system is not chaos, the trajectories can be predicted without the EoMs by DL.

➢ Though I employed the new DL model for the chaos, Neural Lagrangian Network, they could not predict the chaos dynamics either.

➢ The chaos is critical for the deep learning.

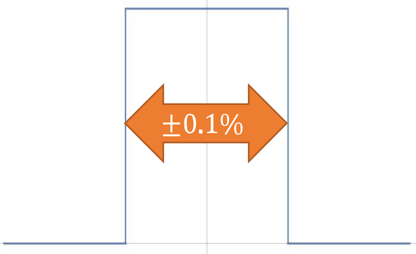
● Reference

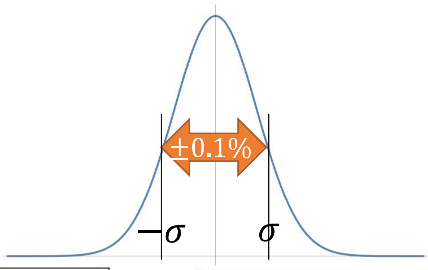
- K.Kiuchi & K.Maeda, Phys Rev D.70.064036, gr-qc/0404124
- S.Suzuki & K.Maeda, Phys Rev D.55.4848
- S.Suzuki & K.Maeda, Phys Rev D.61.024005
- Y.Misono, JGRG27 proceeding Volume.3 P26
- M.Cranmer et al, ICLR 2020 Deep Differential Equations Workshop, arXiv:2003.04630
  code: github.com/MilesCranmer/lagrangian_nns]
- http://colah.github.io/posts/2015-08-Understanding-LSTMs/

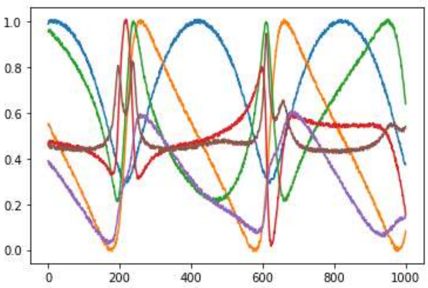
- Appendix : Noise effect on the learning
  - Eccentric dynamics with noise (1%)

I estimate the noise effect on the learning.
I adopt two types of noise, random noise and Gaussian noise

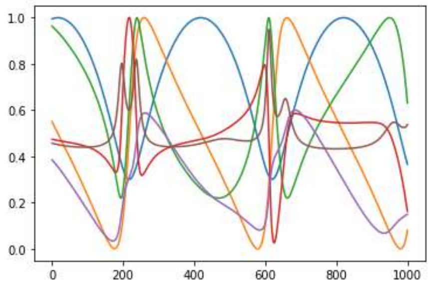
- Random noise distribution
- Gaussian noise distribution



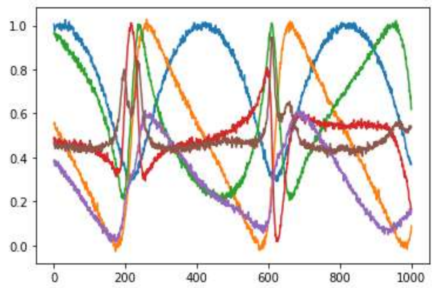




Random noise



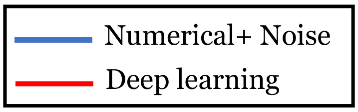
raw data



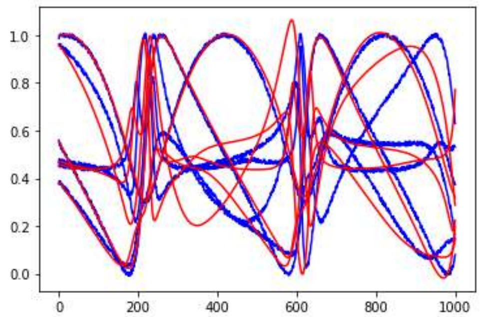
Gaussian noise

- Appendix : effect on the learning
  - Result (spinless)

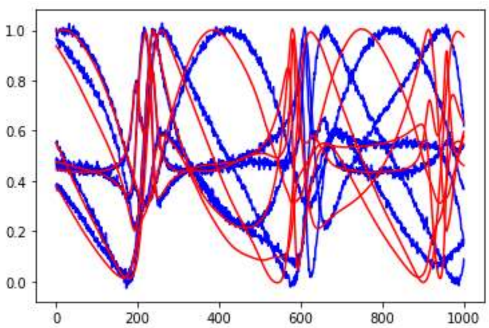
Dynamics



- Random

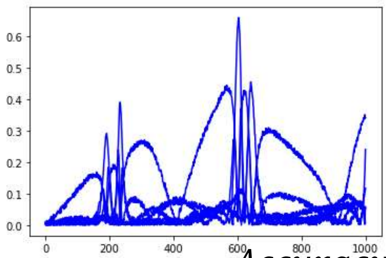


- Gaussian

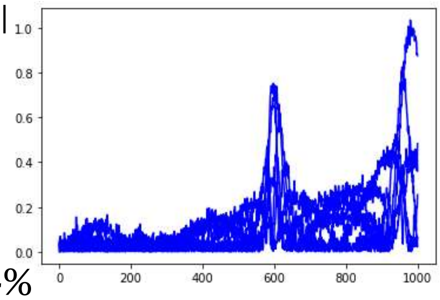


Noise effect is not negligible.

$Error\ rate\ (Prediction)$: $|\hat{\vec{x}}_i - \vec{x}_i| / |\vec{x}_i|$



$Accuracy$: 93.4%



Should I remove it before training by another deep learning?

$Accuracy$: 89.0%

Poster session

Yuuki Sugiyama

Kyushu University

“The effect of dynamical electromagnetic fields on entanglement”

[JGRG30 (2021) PA13]

The effect of dynamical electromagnetic fields on entanglement

Yuuki Sugiyama

work in progress

Collaborator : A. Matsumura, K. Yamamoto

JGRG @ online 2021/12/06 ~ 10

Poster: P13

Gravity and Quantum mechanics are consistent？

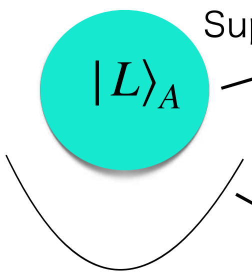
||

Spacetime curvature and Superposition principle are consistent ??

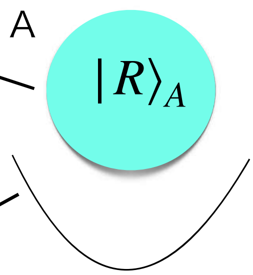
||

Superposition state of curvature realize (Quantumness of gravity)??

Superposition state of massive particle A



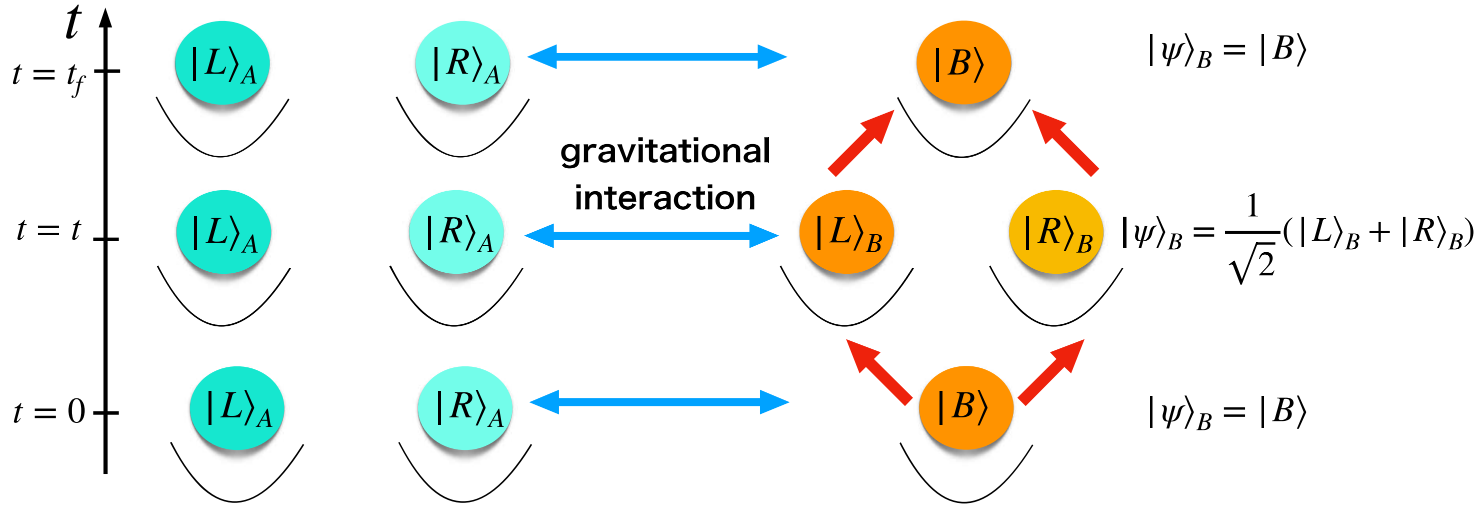
$$|\psi\rangle_A = \frac{1}{\sqrt{2}}(|L\rangle_A + |R\rangle_A)$$



Superposition state of gravitational potential induced by particle A ?

Feynman's thought experiment

- Consider a particle A in superposition state and a test particle B.



Quantum superposition of gravitational fields may
induce test particles to be superposed

Is it possible to test the quantum superposition of gravitational field ?

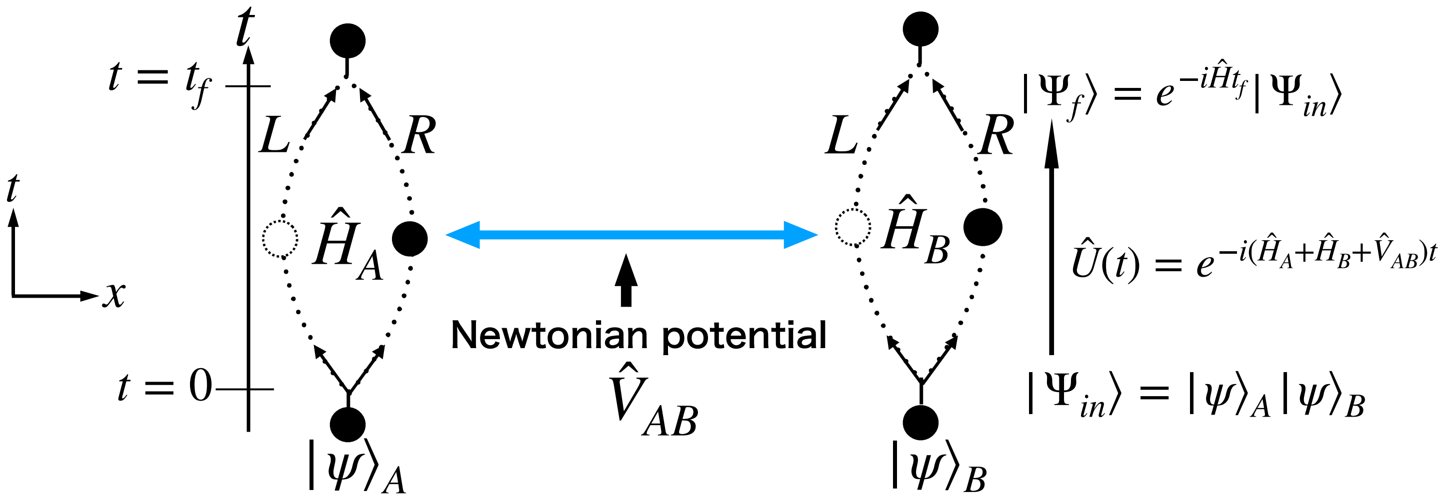
BMV experiment

[S. Bose, et al, (2017)]
[C. Marletto & V. Vedral, (2017)]

Is it possible to test the quantum superposition of gravitational field ?

➡

Entanglement may use the test of quantum superposition of gravitational field ! ?



What is entangled state ?

For two quantum system $|\Psi\rangle_\alpha, |\Psi\rangle_\beta$ and the product state $|\Psi\rangle$;

$$|\Psi\rangle \begin{cases} = |\psi\rangle_\alpha |\psi\rangle_\beta & \text{separable state} \\ \neq |\psi\rangle_\alpha |\psi\rangle_\beta & \text{entangled state} \end{cases}$$

In the setup of BMV;

Newtonian potential

$$|\Psi_f\rangle = e^{-i\hat{H}t_f}|\Psi_{in}\rangle$$

$$\approx \frac{1}{2} e^{-it_{\mathrm{f}}\left(\hat{H}_{\mathrm{A}} + \hat{H}_{\mathrm{B}}\right)} \sum_{\mathrm{P,Q=R,L}} \exp\left[i \int_0^{t_{\mathrm{f}}} dt \frac{G m_{\mathrm{A}} m_{\mathrm{B}}}{|\boldsymbol{x}_{\mathrm{A_P}}(t) - \boldsymbol{x}_{\mathrm{B}_Q}(t)|} \right] |P\rangle_{\mathrm{A}} |Q\rangle_{\mathrm{B}}$$

entangled state (can't factorize)

Two quantum system can be entangled state through the gravitational interaction.

Entanglement use the test of quantum superposition of gravitational field !

Questions about BMV experiment

- **We want to understand the effect of dynamical d.o.f. of gravity on entanglement generation.** ($\because$ Newtonian potential is non-dynamical
- **We want to understand the quantumness of gravity from QFT.**
  - It is nontrivial to affect the quantum phenomenon by graviton.
- **We consider the interaction of electromagnetic field and charged particle to compare with gravity in the future.**

Newtonian potential $\longleftrightarrow$ Coulomb potential

Graviton $\longleftrightarrow$ Photon

Interferometer of charged particle [L. H. Ford, (1993)]

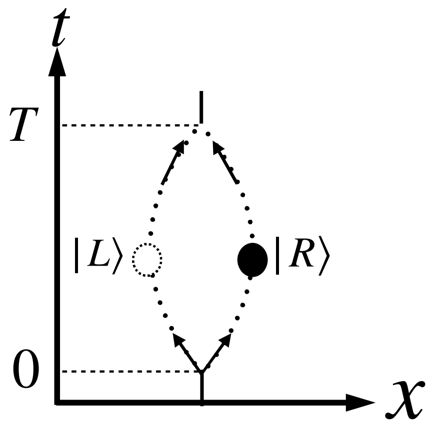
Consider the interaction of a charged particle and photon (QED).

- Total Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{V} \qquad \hat{V} = \int d^3x \hat{J}_\mu(\mathbf{x})\hat{A}^\mu(\mathbf{x})$$

$$\hat{H}_0 = \underbrace{\hat{H}_p}_{\text{Charged particle}} + \underbrace{\hat{H}_{ph}}_{\text{Photon}}$$

interaction term



- Initial state

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}\underbrace{\left(|R\rangle + |L\rangle\right)}_{\text{Charged particle}} \otimes \underbrace{|0\rangle_{ph}}_{\text{Photon}}$$

Reduced density matrix of charged particle

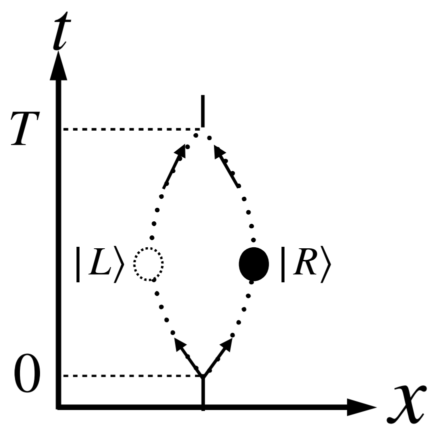
- Quantum state evolves in the total Hamiltonian

$$|\Psi(T)\rangle = e^{-i\hat{H}T}|\Psi(0)\rangle \approx \frac{e^{-i\hat{H}_0T}}{\sqrt{2}}\sum_{P=R,L}|P\rangle \otimes \hat{U}_P|0\rangle_{ph}$$

- Density matrix with reduced d.o.f. of photon

$$\rho_{\rm p} = {\rm Tr}_{\rm ph}[|\Psi(T)\rangle\langle\Psi(T)|]$$

$$= \frac{1}{2}\begin{pmatrix} |R\rangle\langle R| & e^{-\Gamma_{\rm RL}+i\Phi_{\rm RL}}|R\rangle\langle L| \\ e^{-\Gamma_{\rm RL}-i\Phi_{\rm RL}}|L\rangle\langle R| & |L\rangle\langle L| \end{pmatrix}$$



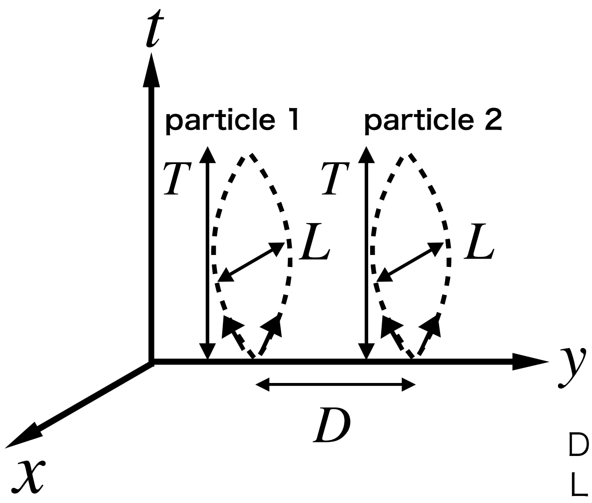
: interference term

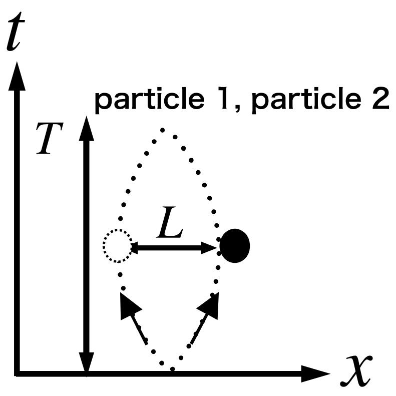
$\Gamma_{RL} \geqq 0$: decrease interference term = decoherence induced by photon

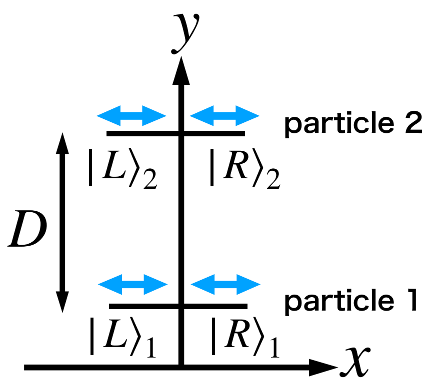
Φ_{RL} : induce the phase shift of interference pattern

Parallel configuration

We extend the previous model as two charged particles model and investigate the entanglement between them.







D : length of two trajectories
L : length of superposition
T : time scale of superposition

We assume non-relativistic particle (cT>>L) and evaluate the entanglement.

Evaluation of entanglement

- Total Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{V} \qquad \hat{V} = \int d^3x \left(\hat{J}^\mu_1(\mathbf{x}) + \hat{J}^\mu_2(\mathbf{x}) \right) \hat{A}_\mu(\mathbf{x})$$

$$\hat{H}_0 = \hat{H}_1 + \hat{H}_2 + \hat{H}_{ph}$$

- reduced density matrix and its negativity

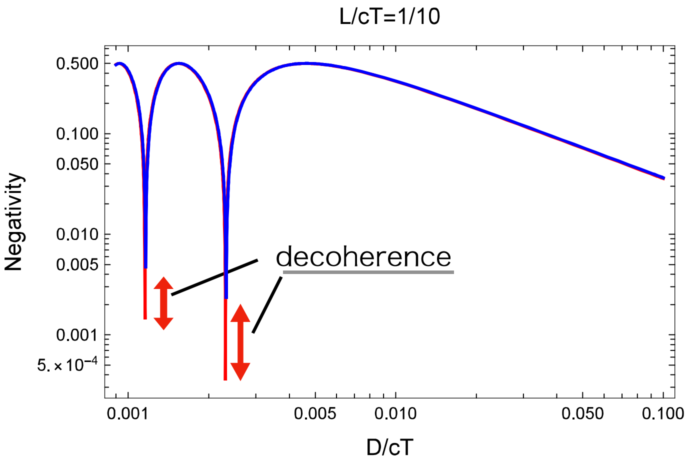
$$\rho_{12} = \mathrm{Tr}_{\mathrm{ph}}[\,|\Psi(T)\rangle\langle\Psi(T)|\,]$$
$$= \frac{1}{4} \sum_{P,Q=R,L} \sum_{P',Q'=R,L} e^{-\Gamma_{P'Q'PQ} + i\Phi_{P'Q'PQ}} |P\rangle\langle P'| \otimes |Q\rangle\langle Q'|$$

$\mathcal{N} = max\left[-\lambda_{min}, 0\right], \quad \mathcal{N} > 0 \Rightarrow$ **entangled**

- minimum eigenvalue of partial transposed reduced density matrix

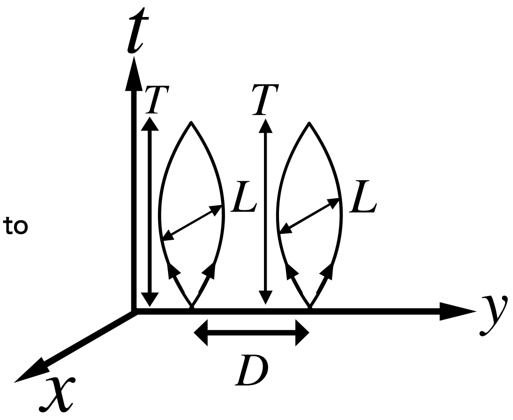
$$\lambda_{min}[\rho_{12}^{T_1}] = \frac{1}{4}\left[1 - e^{-\Gamma_1 - \Gamma_2}\cosh[\Gamma_c] - \left\{ \left(e^{-\Gamma_1} - e^{-\Gamma_2}\right)^2 + 4e^{-\Gamma_1-\Gamma_2}\sin^2(\Phi/2) + e^{-2\Gamma_1 - 2\Gamma_2}\sinh^2[\Gamma_c] \right\}^{\frac{1}{2}}\right]$$

cT >> L >> D regime



Blue : Our result

Red : Corespond to BMV result

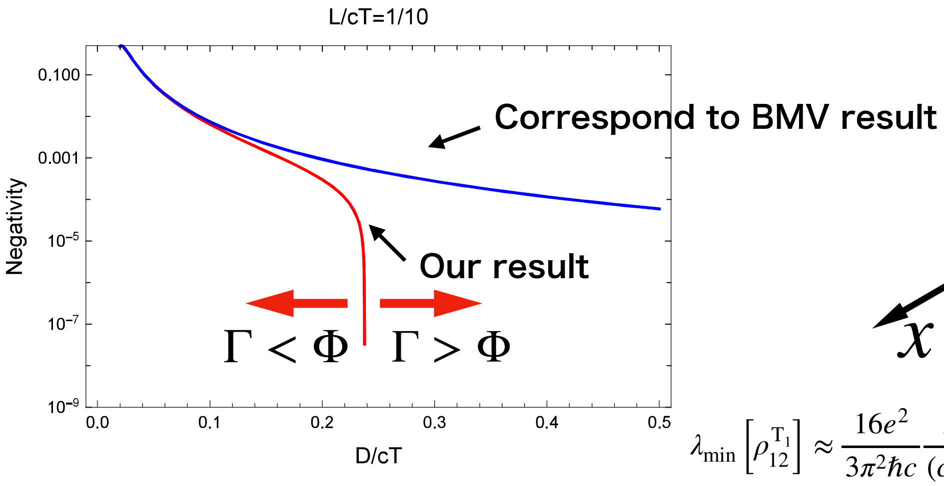


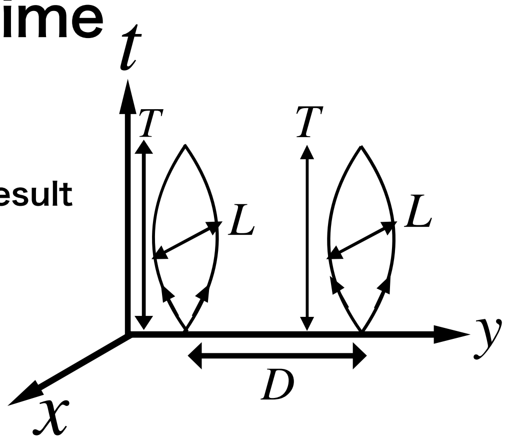
$$\lambda_{\min}\left[\rho_{12}^{\mathrm{T}_1}\right] \approx \frac{16e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2} - \frac{1}{2}\left|\sin\left[\frac{e^2}{4\pi\hbar c}\frac{cT}{D}\left(1-\frac{64L^2}{105(cT)^2}\right)\right]\right|$$

There exist parameter region where is consistent with BMV result.

Quantum decoherence decrease the negativity.

cT >> D >> L regime





$$\lambda_{\min}\left[\rho_{12}^{\mathrm{T}_1}\right] \approx \frac{16e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2} - \left(\frac{8e^2}{315\pi\hbar c}\frac{cTL^2}{D^3} - \frac{16e^2}{105\pi\hbar c}\frac{L^2}{(cT)D}\right)$$

Non-relativistic ; Coulomb potential only

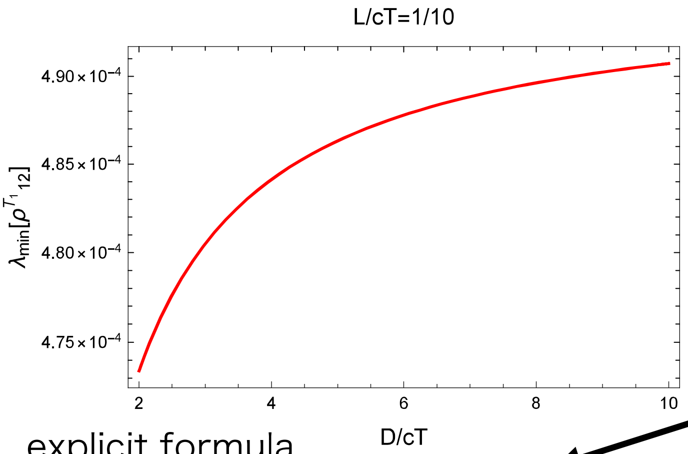
Relativistic correction ; decoherence + Coulomb potential

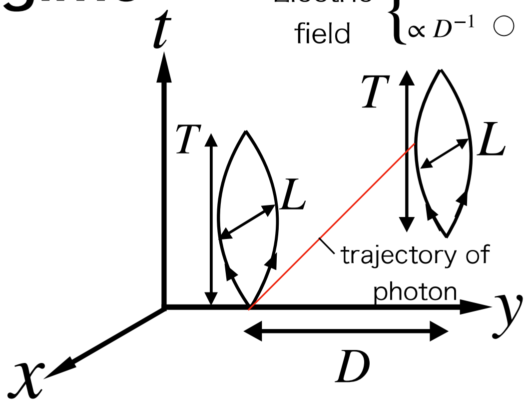
There exist parameter region where the negativity vanishes by decoherence.

D >> cT >> L regime

Wave zone;
Electric field $\begin{cases} \propto D^{-2} & \times \\ \propto D^{-1} & \bigcirc \end{cases}$

The minimum eigenvalue behaves as





decoherence
= decrease entanglement

explicit formula

$$\lambda_{\min}\left[\rho_{12}^{T_1}\right] \approx \frac{16e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2} - \frac{16e^2}{105\pi\hbar c}\frac{L^2}{D(cT)}$$

phase shift (induced by acceleration)
= increase entanglement

decoherence >> phase shift ⟶ Entanglement doesn't occur $(\mathcal{N} > 0 \Leftrightarrow \lambda_{min} < 0)$

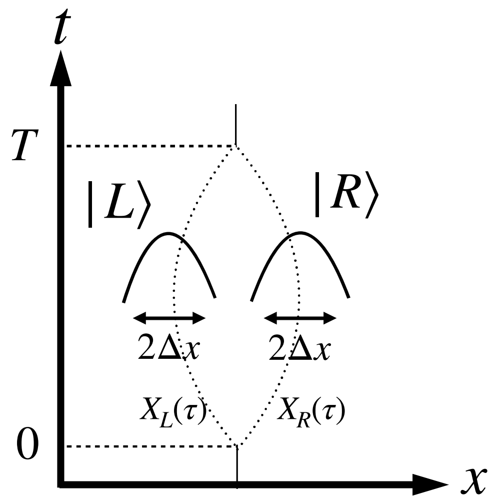
Summary and future works

- We evaluate the negativity analytically between two charged particles with dynamical d.o.f. of photon. ⟷ BMV : non-dynamical
- Quantum decoherence which comes from vacuum fluctuation of photon and phase shift induced by the acceleration affect the entanglement generation
- The negativity of non-relativistic particles showed …
  - In cT >> L >> D, a parameter region where is consistent with BMV result exist.
  - In cT >> D >> L, there exist a parameter region where the negativity vanishes by decoherence.
  - In D >> cT >> L, decoherence >> phase shift and entanglement doesn't occur.
- Evaluate the negativity for relativistic particles
- Gravitational field case …

Thank you

Backup

Approximation



① $\lambda_{dB} \ll \Delta x$

λ_{dB} : de Broglie length of charged particle

Δx : wave packet of charged particle

➡ $\hat{J}^{\mathrm{I}\mu}(x)\left|P\right\rangle \approx J^{\mu}_{\mathrm{P}}(x)\left|P\right\rangle$ **(Classical current)**

$$J^{\mu}_{\mathrm{P}}(x) = e\int d\tau \frac{dX^{\mu}_{\mathrm{P}}}{d\tau}\delta^{(4)}\left(x - X_{\mathrm{P}}(\tau)\right)$$

② $\lambda_C \ll \lambda$

$\lambda_C = \hbar/mc$: Compton length of charged particle

$\lambda = c/\omega$: wave length of photon coming from the classical current

➡ $\hbar\omega \ll mc^2$

(neglect the pair creation or annihilation)

D >> T >> L regime

$$\Gamma_1 = \Gamma_2 \approx \frac{32e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2} \qquad \Gamma_{\mathrm{c}} \approx -\frac{e^2}{225\pi^2}\frac{L^2T^2}{D^4} \qquad \Phi \approx -\frac{64e^2}{105\pi\hbar c}\frac{L^2}{DT}\left(1 - \underbrace{\frac{(cT)^2}{12D^2}}_{\ll 1}\right)$$

$$\lambda_{\min}\left[\rho^{\mathrm{T}_1}_{12}\right] \approx \frac{16e^2}{3\pi^2}\frac{L^2}{T^2} - \frac{16e^2}{105\pi}\frac{L^2}{DT}$$

T >> D >> L regime

$$\Gamma_1 = \Gamma_2 \approx \frac{32e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2} \quad \Gamma_{\mathrm{c}} \approx \frac{64e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2}\left(1 + \frac{4D^2}{(cT)^2}\ln\left[\frac{D}{cT}\right]\right) \quad \Phi \approx -\frac{32e^2}{315\pi\hbar c}\frac{cTL^2}{D^3}\left(1 - \frac{6D^2}{(cT)^2}\right)$$

$$\lambda_{\min}\left[\rho^{\mathrm{T}_1}_{12}\right] \approx \frac{16e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2} - \left(\frac{8e^2}{315\pi\hbar c}\frac{cTL^2}{D^3} - \frac{16e^2}{105\pi\hbar c}\frac{L^2}{(cT)D}\right)$$

T >> L >> D regime

$$\Gamma_1 = \Gamma_2 \approx \frac{32e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2} \qquad \Gamma_{\mathrm{c}} \approx \frac{64e^2}{3\pi^2\hbar c}\frac{L^2}{(cT)^2} \qquad \Phi \approx -\frac{e^2}{2\pi\hbar c}\frac{cT}{D}\left(1 - \frac{64L^2}{105(cT)^2}\right)$$

$$\lambda_{\min}\left[\rho^{\mathrm{T}_1}_{12}\right] \approx \frac{1}{4}\left[\frac{64e^2}{3\pi^2}\frac{L^2}{T^2} - \sqrt{\underbrace{\left(4\sin\left[\frac{e^2}{4\pi}\frac{T}{D}\left(1 - \frac{64L^2}{105T^2}\right)\right]\right)^2}_{\leqq 1} + \underbrace{\left(\frac{64e^2}{3\pi^2}\frac{L^2}{T^2}\right)^2}_{\ll 1}}\right] \approx \frac{1}{2}\left[\frac{32e^2}{3\pi^2}\frac{L^2}{T^2} - \left|\sin\left[\frac{e^2}{4\pi}\frac{T}{D}\left(1 - \frac{64L^2}{105T^2}\right)\right]\right|\right]$$

D >> T >> L regime

$$X^0_{1R}(t) = t, \quad \mathbf{X}_{1R}(t) = \left[8L\left(1-\frac{t}{T}\right)^2\left(\frac{t}{T}\right)^2, 0, 0\right]^T$$
$$X^0_{1L}(t) = t, \quad \mathbf{X}_{1L}(t) = \left[-8L\left(1-\frac{t}{T}\right)^2\left(\frac{t}{T}\right)^2, 0, 0\right]^T$$
$$X^0_{2R}(t) = t, \quad \mathbf{X}_{2R}(t) = \left[8L\left(1-\frac{t-D}{T}\right)^2\left(\frac{t-D}{T}\right)^2, D, 0\right]^T$$
$$X^0_{2L}(t) = t, \quad \mathbf{X}_{2L}(t) = \left[-8L\left(1-\frac{t-D}{T}\right)^2\left(\frac{t-D}{T}\right)^2, D, 0\right]^T$$

$$\Gamma_{RL} = \frac{e^2}{16}\int_S d\sigma^{\mu\nu}\int_S d\sigma'^{\alpha\beta}\langle\{\hat{F}^I_{\mu\nu}(x), \hat{F}^I_{\alpha\beta}(x')\}\rangle$$

$$\Phi_{RL} = -\frac{e}{2}\int_S d\sigma_{\mu\nu}(F^{\mu\nu}_R(x) + F^{\mu\nu}_L(x)) + e\int_S d\sigma^{\mu\nu}F^c_{\mu\nu}(x)$$

T >> L >> D, T >> D >> L regime

$$X^0_{1R}(t) = t, \quad \mathbf{X}_{1R}(t) = \left[8L\left(1-\frac{t}{T}\right)^2\left(\frac{t}{T}\right)^2, 0, 0\right]^T$$
$$X^0_{1L}(t) = t, \quad \mathbf{X}_{1L}(t) = \left[-8L\left(1-\frac{t}{T}\right)^2\left(\frac{t}{T}\right)^2, 0, 0\right]^T$$
$$X^0_{2R}(t) = t, \quad \mathbf{X}_{2R}(t) = \left[8L\left(1-\frac{t}{T}\right)^2\left(\frac{t}{T}\right)^2, D, 0\right]^T$$
$$X^0_{2L}(t) = t, \quad \mathbf{X}_{2L}(t) = \left[-8L\left(1-\frac{t}{T}\right)^2\left(\frac{t}{T}\right)^2, D, 0\right]^T$$

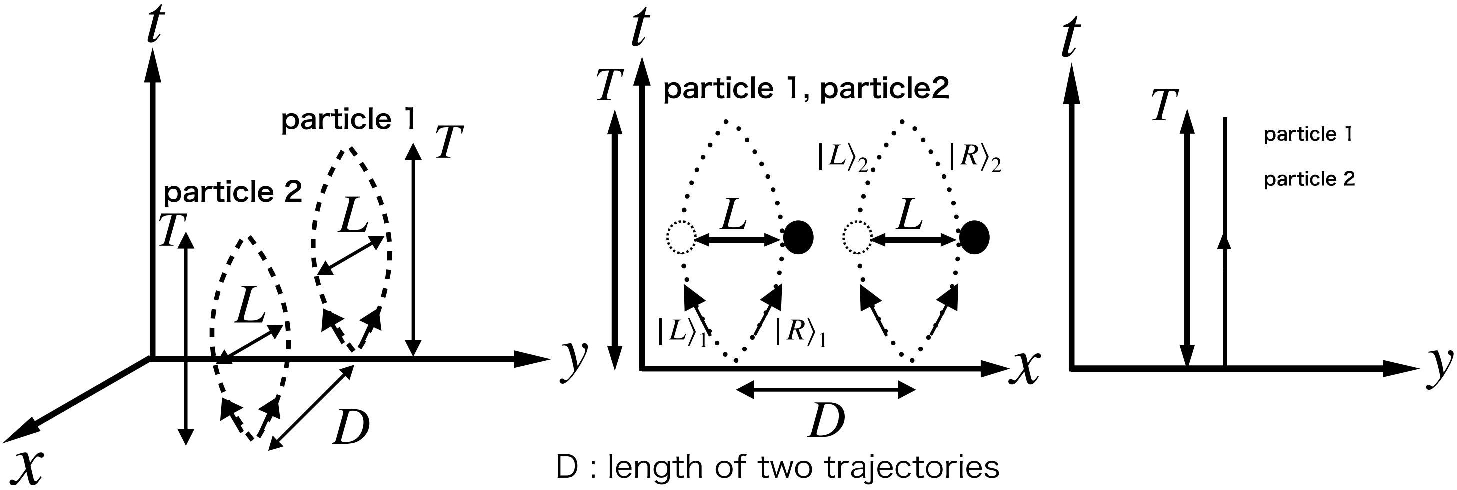
$$\Gamma_i = \frac{1}{4}\int d^4x\int d^4y \Delta J^\mu_i(x)\Delta J^\nu_i(y)\langle\{\hat{A}^I_\mu(x), \hat{A}^I_\nu(y)\}\rangle$$

$$\Gamma_c = \frac{1}{2}\int d^4x\int d^4y\, \Delta J^\mu_1(x)\Delta J^\nu_2(y)\langle\{\hat{A}^I_\mu(x), \hat{A}^I_\nu(y)\}\rangle$$

$$\Phi = \frac{1}{2}\int d^4x\int d^4y\left\{\Delta J^\mu_1(x)\Delta J^\nu_2(y) + \Delta J^\mu_2(x)\Delta J^\nu_1(y)\right\}G^r_{\mu\nu}(x,y)$$

Linear configuration

We extend the previous model as two charged particles model and investigate the entanglement between them.



D : length of two trajectories
L : length of superposition
T : time scale of superposition

Poster session

Yong Song

University of Science and Technology of China

“The evolutions of the innermost stable circular orbits in dynamical spacetimes”

[JGRG30 (2021) PA14]

The evolutions of the innermost stable circular orbits in dynamical spacetimes

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Introduction

In 2019, the Event Horizon Telescope Published the first image of a black hole at the center of the M87 galaxy [1]. In the image, one can see a shadow region which is called the black hole shadow, and the black hole lies in the shadow. One can also see a ring-like structure that corresponds to the accretion disk, and the study of the innermost stable circular orbit (ISCO) plays a vital role in analyzing this image.
Up to date, there are many studies based on the effective potential to study ISCO in spacetime. On the one hand, ISCO has many important properties. For example, it is the inner edge of an accretion disk; it is also the boundary between the stable orbits and the unstable orbits, and the accretion flow changes dramatically across the ISCO in a thin disk. On the other hand, ISCO has many applications. Such as, for a rotating black hole, the radius of ISCO is a key fit parameter to measure the spin of the black hole, and there are many other studies about the ISCOs in Kerr-like spacetimes. In the modified gravitational theories, ISCOs may also exist. Also, ISCO may have some applications in AdS/CFT. In recent years, some studies suggest that ISCO should describe field theory long-lived excitations that do not thermalize like typical excitations.
Through the effective potential method, one can efficiently study the ISCOs in static and stationary spacetimes. But, this method is not suitable for dynamical spacetimes because the effective potential cannot be defined in dynamical spacetimes.
In this paper, we get a method to study the evolutions of the ISCOs in dynamical spacetimes. As examples, we studied the ISCOs in Vaidya spacetime, Vaidya-AdS spacetime and the slow rotation limit of Kerr-Vaidya spacetime. The results given by these examples are reasonable and have similar evolution curves to photon spheres in dynamical spacetimes [3].

Lessons from Schwarzschild spacetime

we get two critical properties of the ISCO in Schwarzschild spacetime:

(1). For a general circular orbit, it does not evolve in time, i.e.,

$$dr_o/d\tau = d^2r_o/d\tau^2 = 0 \, . \tag{1}$$

(2). For a family of circular orbits, ISCO has a minimal orbital angular momentum, i.e.,

$$\delta l_o/\delta r_o = \delta l_o^2/\delta r_o = 0 \, , \tag{2}$$

where l_o should be regarded as a function of r_o

If the solution of eq.(2) is single-valued, it is an ISCO. If the solution of eq.(2) is double-valued, such as Schwarzschild-dS spacetime, Kerr-dS spacetime and so on, the one with $\delta^2 l_o/\delta r_o^2 > 0$ is ISCO, and the one with $\delta^2 l_o/\delta r_o^2 < 0$ is OSCO (outermost stable circular orbit). Below, we only treat single-valued cases.

ISCOs in static and stationary spacetimes

In the general static and stationary spacetimes, eq.(1) is obviously valid. Below we will demonstrate that eq.(2) is also valid in some conditions.
In the general static and stationary spacetimes, suppose one can define the effective potential as $V_l(r)$, where l is the conserved orbital angular momentum. Consider a free point particle, and for a given circular orbit, one always has the following relation

$$V'_{l_o}(r_o) = 0 \, , \tag{3}$$

and for this circular orbit, l_o is a constant. Considering a family of circular orbits and varying eq.(3), one can get the following equation,

$$0 = \frac{\delta V'_{l_o}(r_o)}{\delta r_o} = V''_{l_o}(r_o) + \frac{\partial V'_{l_o}(r_o)}{\partial l_o}\frac{\delta l_o}{\delta r_o} \, . \tag{4}$$

Here, l_o should regard as a function of r_o. Then, one have the following relation

$$V''_{l_o}(r_o) = -\frac{\partial V'_{l_o}(r_o)}{\partial l_o}\frac{\delta l_o}{\delta r_o} \, , \tag{5}$$

where we have assumed that $\partial V'_{l_o}(r_o)/\partial l_o\big|_{r_{isco},l_{isco}} \neq 0$. In general, this assumption can be satisfied. So, for an ISCO, the condition $V''_{l_{isco}}(r_{isco}) = 0$ is equivalent to $\delta l_o/\delta r_o = 0$ [2], and the stable circular orbits should satisfy the condition that $\delta l_o/\delta r_o \geq 0$.

The evolutions of the ISCOs in dynamical spacetimes

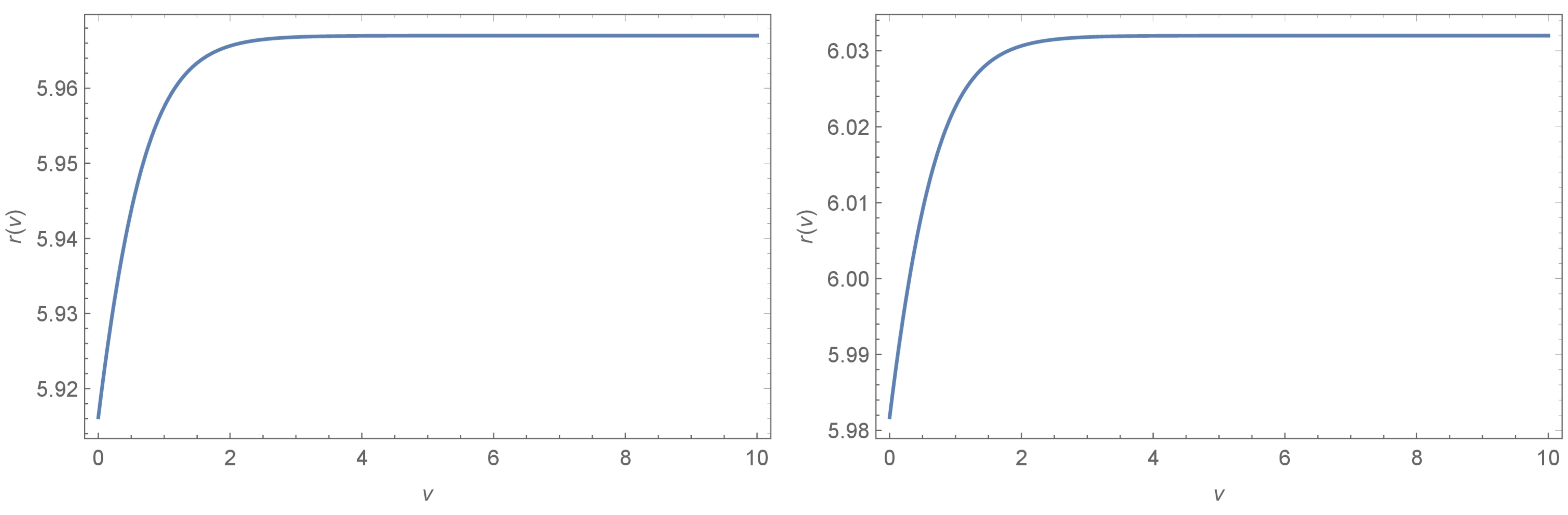
In general dynamical spacetimes, condition (1) does not hold anymore. Enlightened by [3], we assume

$$dr_o(t) = \frac{\partial r_o(t)}{\partial t}dt = \dot{r}_o(t)dt \, , \tag{6}$$

where t is the coordinate time and a dot stands for the derivative with respect to this coordinate time. As for condition (2), we generalize it to the following equation

$$\frac{\delta l_o}{\delta r_o(t)} = \frac{\delta l_o^2}{\delta r_o(t)} = 0 \, . \tag{7}$$

Here, l_o should regard as a function of $r_o(t)$. The evolution of the ISCO of Kerr-Vaidya spacetime in the slow rotation limit is shown below.



The left figure is correspond to the "direct" rotation and The right figure is correspond to the "retrograde" rotation. Thses results are similar to the evolutions of the photon sphere in Kerr-Vaidya spacetime under the slow rotation limit [3].

Conclusion

In this paper, we reviewed the two methods to get the ISCO in Schwarzschild spacetime. We domenstrated the extremum method is equivalent to the effective potential method in static and stationary spacetimes. We verify this equivalence in general spherically symmetric spacetimes and Kerr spacetime. We then generalized the extremum method into dynamical spacetimes. From this generalization, we studied the evolutions of the ISCOs in Vaidya spacetime, Vaidya-AdS spacetime, and Kerr-Vaidya spacetime under the limit of slow rotation. These examples are all giving reasonable resluts.

Reference

[1] K. Akiyama *et al.* [Event Horizon Telescope Collaboration], Astrophys. J. **875**, no. 1, L1 (2019) [arXiv:1906.11238 [astro-ph.GA]].
[2] T. Berry, A. Simpson and M. Visser, Universe **7**, no.1, 2 (2020) doi:10.3390/universe7010002 [arXiv:2008.13308 [gr-qc]].
[3] A. K. Mishra, S. Chakraborty and S. Sarkar, Phys. Rev. D **99**, no.10, 104080 (2019) doi:10.1103/PhysRevD.99.104080 [arXiv:1903.06376 [gr-qc]].

Poster session

Takahiro Tanaka

Kyoto University

“Simple justification of \delta N formalism and its generalization”

[JGRG30 (2021) PA15]

Simple justification of δN formalism and its generalization

Takahiro Tanaka and Yuko Urakawa

JCAP 07 (2021) 051 arXiv:2101.05707

Locality condition

In higher order cosmological perturbation theory, we often meet with Δ^{-1}.

Even if the original Lagrangian looks local, it may become non-local after solving the constraints.

- Constraints related to gauge symmetry including diffeo.
- Constraints to simply eliminate auxiliary fields

We can leave the first constraints because they are guaranteed to be satisfied once the initial conditions are properly chosen.

Then, the locality condition that we have to impose is the locality after solving the constraints of the second kind.

Relaxed equations

Def.) Relaxed equations

≡ set of evolution eqs. excluding spatial derivative constraint like MC(momentum constraint), which vanishes at the leading order of spatial grad. expansion.

The locality condition: The relaxed equations give local time evolution equations to all the remaining variables.

Locality ⇒ Kinetic terms of relaxed eqs. are not degenerate.

⇔ No elliptic type equation

⇔ One can uniquely determine the background homogeneous universe model once the initial conditions are provided.

Non-trivial issue about MC

MC necessarily contains spatial derivatives, and so finding a solution of MC to specify the initial conditions becomes non-local.

Form of MC

$$x^i \to x'^i = x^i + \xi^i \qquad (\partial_t)_x \to (\partial_t)_{x'} + \dot{\xi}^i \partial_i$$

$$N^i \to N^i - \dot{\xi}^i$$

Time derivative and the shift should appear in the combination.

$$\partial_t \phi^I + £_{N^i} \phi^I \qquad \phi^I : \text{fields with an arbitrary spin}$$

$$\int d^3x \sqrt{-g} H_i \delta N^i = \int d^3x \frac{\delta S}{\delta N^i} \delta N^i = \int d^3x \frac{\delta S}{\delta \dot{\phi}^I} £_{\delta N^i} \phi^I = \int d^3x \sqrt{-g}\, \pi_I \, £_{\delta N^i} \phi^I$$

$$\Rightarrow \{\pi_I, H_i\} = {}_{\delta N^i}\phi^I$$ MC is the generator of 3d diffeo. transform.

However, we do not have to solve MC in general at a sufficiently late time for the reason explained in the next slide.

Noether charge

The locality condition

⇒ "Lagrangian in the long-wavelength limit ($\nabla \to 0$)"

="Lagrangian for the homogeneous universe model"

Large gauge transformation: $M^j_{\ j} = 0$

$$x \to x'^i = x^i + M^i_{\ j} x^j \qquad A \to A'^i = A^i + M^i_{\ j} A^j$$

$$\delta \int L(\phi^I) dt = \int \left(\frac{\partial L}{\partial \dot{\phi}^I} \delta \dot{\phi}^I + \frac{\partial L}{\partial \phi^I} \delta \phi^I \right) dt = \int \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\phi}^I} \delta \phi^I \right) dt = 0$$

$$\Rightarrow \text{const.} = \frac{\partial L}{\partial \dot{\phi}^I} \delta \phi^I = \sqrt{-g}\, \pi_I \delta \phi^I$$

For the model with a vector field:

$$\delta S = \sqrt{-g} \left[\frac{1}{2} \left(\pi_\gamma^{ij} M^k_{\ j} \gamma_{ki} + \pi_\gamma^{ij} M^k_{\ i} \gamma_{jk} \right) + \pi^i_{\ A} M^j_{\ i} A_j \right]$$

$$\Rightarrow \left[\pi^{\ i}_{\gamma\ j} + \pi^i_{\ A} A_j \right]^{TF} = Q^i_j \Big/ \sqrt{-g} \qquad Q^i_j = \text{const.}$$

At a late time, right hand side vanishes, and hence shear $[\pi^{\ i}_{\gamma j}]^{TF}$ can be determined by the matter field.

$$\left[\pi^i_{\gamma,j} + \pi^i_{\ A} A_j \right]^{TF} = Q^i_j \Big/ \sqrt{-g}$$

If we substitute this equation into MC

⇒ We have an equation that determines $Q^i_{\ j}$.

$$\partial_i Q^i_j = (\text{matter contribution})$$

symmetric part of $Q^i_{\ j}$:

5 components = 3 components to be determined by MC

+ 2 decaying modes of GWs

Approximate initial data for the relaxed eqs. can be obtained by simply setting $Q^i_{\ j}$=0.

In most cases, the decay of $\pi^{\ i}_{\gamma j}$ means the decay of $\partial_t \gamma_{ij}$.

Similar problem for gauge constraint

Gauge field constraint also contains spatial derivatives, and so finding its solution to specify the initial conditions can be non-local.

$$0=\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta A_0}\approx\frac{1}{2}\partial_i F^{0i}+\frac{\partial L_{matt}}{\partial A_0}$$

1) When charged fields are non-vanishing

- The second term, which is not associated with spatial differentiation, can be used to solve the constraint equation locally.
- So, the gauge field constraint is not a spatial derivative constraint in this case.

2) When there is no non-vanishing charged field

- Solving gauge constraint introduces non-locality.

The same issue for gauge constraint

Our focus is on the case 2).

When there is no non-vanishing charged field

$$A_\mu \to A_\mu - \partial_\mu \Lambda(x) \qquad \Lambda(x) = M_i x^i$$

$$\Rightarrow A_i \to A_i - M_i$$

$$\delta\int L(\phi^I)dt=\int\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\phi^I}\delta\phi^I\right)dt=0 \Longrightarrow \text{const.}=\frac{\partial L}{\partial\dot\phi^I}\delta\phi^I=\sqrt{-g}\,\pi_I\delta\phi^I$$

$$\Longrightarrow \pi^i_A = Q^i/\sqrt{-g}$$

If we are allowed to adjust Q^i simply as decaying modes, one can freely cure the violation of the gauge constraint. However, $\delta\pi^i_A=\delta Q^i/\sqrt{-g}$ guarantees the decay of $\delta\pi^i{}_A$ but not the decay of $\partial_t A_i$.

$$L=\sqrt{-g}\frac{f(\phi)}{4}F_{\mu\nu}F^{\mu\nu} \Longrightarrow \pi^i{}_A \approx f(\phi)\partial_t A_i$$

Existence of adiabatic modes

- LGT generates a solutions of relaxed eqs.

$$\phi^I \to \phi^I + \delta\phi^I$$

$\delta\phi^I(\boldsymbol{x}, s)$ is promoted to $\delta\phi^I\left(\boldsymbol{x}, s(\boldsymbol{x})\right)$

⇨ MC is not satisfied after this promotion.

- But $Q^i{}_j$=0 is maintained by the LGT, since $Q^i{}_j$ transforms as a 3-d tensor

- When we have some gauge field in the background, the violation of gauge constraint must be cured, which is slightly non-trivial. It depends on the relation between $\delta\pi^i{}_A$ and $\partial_t A_i$

When δN-formalism works?

- δN-formalism=Separate universe approach

Main message) Even if we are not allowed to set $Q^i{}_j$=0 and Q^i=0, the super-horizon dynamics can be described by a set of background homogeneous universes as long as the locality condition is satisfied.

- Given a set of gauge invariant variables obtained by solving the sub-horizon dynamics

⇨ To give the initial data for the relaxed eqs., we need to solve the MC, which makes the problem non-local.

- At a late time, $Q^i{}_j$=0 can be a substitute of MC. Then, the late time super-horizon evolution is described by the solutions of relaxed eqs. = anisotropic separate universes.

⇨ The formalism can be extended to include anisotropic inflation, If we can use the approximation Q^i=0 for gauge field.

Summary

When the action is local after solving all the constraints except for gauge constraints, related to the symmetry of the system, like momentum constraint, we refer to such a model as local.

Relaxed equations
 ≡ set of evolution eqs. excluding spatial derivative constraint like momentum constraint, which vanishes at the leading order of spatial grad. expansion.

Main claim) If the model is local, the super-horizon dynamics can be described by a set of background homogeneous universe solutions, because relaxed equations at the leading order of gradient expansion are identical to the equations governing the background homogeneous (but not isotropic) universe.

Important remark) Gauge invariant variables are not enough to give the initial conditions for the super-horizon dynamics in general. To give a complete initial data, we need to solve the momentum constraint, which introduces singular behavior in the IR limit. However, we can use an approximation to set the generically existing decaying modes to zero. Then, the initial data can be locally obtained from gauge invariant perturbations.

Poster session

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“Angular momenta in microstate geometries and black holes in five dimensions”

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Lower bound for angular momenta of microstate geometries in five dimensions

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We study the Bogomol'nyi-Prasad-Sommerfield (BPS) solutions of the asymptotically flat, stationary microstate geometries with bi-axisymmetry and reflection symmetry in the five-dimensional ungauged minimal supergravity. We show that the angular momenta of the microstate geometry with a small number of centers (at least, five centers) have lower bounds, which are slightly smaller than those of the maximally spinning BMPV black hole. Therefore, there exists a certain narrow parameter region such that the microstate geometry with a small number of the centers admits the same angular momenta as the BMPV black hole. Moreover, we investigate the dependence of the topological structure of the evanescent ergosurfaces on the magnetic fluxes through the 2-circles between two centers.

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I. INTRODUCTION

The microstate geometries [1–14] are smooth horizonless solutions in the bosonic sector of supergravity which have the same asymptotic structure as a given black hole or a black ring. So far, these solutions have been constructed and thought of as one of ways to resolve the problem of black hole information loss. This idea to describe black hole microstates by horizonless geometries originated from the works on fuzzballs of Mathur [15–17]. The existence of such solutions itself should be surprising because of the earlier results [18–21] on "No-Go" which exclude completely smooth soliton solutions which are regular in four spacetime dimensions. In five dimensional supergravity, the conclusion of the no-go theorem can be evaded because the spacetime admits the spatial cross sections with non-trivial second homology and the Chern-Simons interactions.

Therefore, despite the absence of horizons, the microstate geometries should closely approximate the geometries of black holes and need to describe all phenomenon which could occur in black hole spacetimes, such as the gravitational lens effect and gravitational wave radiation. However, the analysis is not still sufficient to say that such microstate geometries well describes black hole

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physics. From this point of view, it is an important issue to probe what extent asymptotically flat microstate geometries possess the classical features of stationary black holes with the same asymptotic structure. There are many ways to probe physical aspects of such microstate geometries. A natural and simple way is to investigate whether these spacetimes can carry the same asymptotic charges, the mass and angular momenta, as rotating black holes [22–24], rotating black rings [25–28] and rotating black lenses [29–33] in the same theory. If not so, such spacetimes cannot be regarded as the description of these black objects. For instance, as proved mathematically in [34], there are no asymptotically static microstate geometries in higher dimensional Einstein-Maxwell theory, which implies that any static black hole cannot be described by the soliton solutions. In particular, it is well known that there exist the microstate geometries corresponding to maximally-spinning black holes and maximally-spinning black rings that have zero horizon area, which are referred to " zero-entropy microstate geometries" [37]. Moreover, using the merge of such zero-entropy microstate geometries, Refs. [35, 36] constructed the first microstate geometries with the same charges as black holes and black rings which have nonzero horizon area. In general, it is, however, not known how to construct the microstate geometries that correspond to black holes and black rings with non-zero horizon area without introducing the merger of zero-entropy microstate geometries.

The main purpose of this paper is to investigate whether there exist the microstate geometries in five dimension having the same asymptotic charges (mass and angular momenta) as the black hole, without using zero-entropy microstate geometries and by merely imposing a simple symmetry. In this paper, based on the work developed by Gauntlett *et al.* [38] in the framework of the five-dimensional minimal ungauged supergravity, we consider asymptotically flat, stationary and bi-axisymmetric BPS microstate geometries with n centers on the z-axis of the Gibbons-Hawking space. In addition, we impose reflection symmetry, which means the invariance under the transformation $z \to -z$, on the solution since such an assumption dramatically simplifies the constraint equations for the parameters included in the solutions, a so-called " bubble equations", and this enables us to solve the constraint equations for the parameters. It can be shown that under the symmetry assumptions, the geometry has equal angular momenta. It is of physical interest to compare the mass and angular momenta of the Breckenridge-Myers-Peet-Vafa (BMPV) solution [24] since it describes a spinning black hole with equal angular momenta in the same theory. We will show that asymptotically flat, stationary, bi-axisymmetric and reflection-symmetric microstate geometries (at least, for five centers) can have the same mass and angular momenta as the BMPV black hole.

The rest of the paper is organized as follows: In the following Sec. II, we review the BPS solutions of the microstate geometries in the five-dimensional minimal supergravity. In Sec. III, we compute the mass, angular momenta and magnetic fluxes through the bubbles, and show the existence of evanescent ergosurfaces. In Sec. IV, imposing reflection symmetry, we simplify the solution and the bubble equations and thereafter show numerically that the microstate geometries have the same angular momentum as the BMPV black hole. In Sec. V, we summarize our results and discuss possible generalizations of our analysis.

II. MICROSTATE GEOMETRY

A. Solutions

First, we begin with supersymmetric solutions in the five-dimensional minimal ungauged supergravity [38], whose bosonic Lagrangian consists of the Einstein-Maxwell theory with a Chern-Simons term. In this theory, the metric and the gauge potential of Maxwell field for the supersymmetric solutions take the form:

$$ds^2 = -f^2(dt+\omega)^2 + f^{-1}ds_M^2, \tag{1}$$

$$A = \frac{\sqrt{3}}{2}\left[f(dt+\omega) - \frac{K}{H}(d\psi+\chi) - \xi\right]. \tag{2}$$

Here, the four-dimensional metric ds_M^2 is the metric of an arbitrary hyper-Kähler space, where we use the Gibbons-Hawking space metric [39] which is written as

$$ds_M^2 = H^{-1}(d\psi+\chi)^2 + H ds_{\mathbb{E}^3}^2, \tag{3}$$

$$ds_{\mathbb{E}^3}^2 = dx^2+dy^2+dz^2, \tag{4}$$

$$H = \sum_{i=1}^{n} \frac{h_i}{r_i}, \tag{5}$$

with

$$r_i := |\boldsymbol{r}-\boldsymbol{r}_i| = \sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}, \tag{6}$$

$$\boldsymbol{r} := (x,y,z), \tag{7}$$

$$\boldsymbol{r}_i := (x_i,y_i,z_i), \tag{8}$$

The function H in Eq.(5) is a harmonic function with n point sources (n centers) on three-dimensional Euclid space $\mathbb{E}^3$, and the 1-form χ on $\mathbb{E}^3$ is determined by

$$*d\chi = dH, \tag{9}$$

where the Hodge dual $*$ is associated with $\mathbb{E}^3$. χ can be written as

$$\chi \;=\; \sum_{i=1}^{n} h_i \tilde{\omega}_i, \tag{10}$$

where the 1-form $\tilde{\omega}_i$ on $\mathbb{E}^3$, which is defined by

$$*d\tilde{\omega}_i = d(1/r_i), \tag{11}$$

can be written as

$$\tilde{\omega}_i = \frac{z - z_i}{r_i} \frac{(x - x_i)dy - (y - y_i)dx}{(x - x_i)^2 + (y - y_i)^2}. \tag{12}$$

The vectors $\partial/\partial t$ and $\partial/\partial\psi$ are commuting Killing vector fields. The Gibbons-Hawking metric (3) is preserved under the scaling transformation $H \to \lambda^2 H$, $\chi \to \lambda^2\chi$, $\psi \to \lambda\psi$ and $x^i \to \lambda^{-1}x_i$, which enables us to fix the period of the coordinate ψ as $0 \le \psi < 4\pi$. These Gibbons-Hawking spaces are nontrivial $U(1)$ fibration over a flat space $\mathbb{E}^3$ and the unique class of four-dimensional hyper-Kähler metric with tri-holomorphic isometry.

The function f^{-1} and the 1-forms (ω, ξ) are given by

$$f^{-1} \;=\; H^{-1}K^2 + L, \tag{13}$$

$$\omega \;=\; \omega_\psi(d\psi + \chi) + \hat{\omega}, \tag{14}$$

$$\omega_\psi \;=\; H^{-2}K^3 + \frac{3}{2}H^{-1}KL + M, \tag{15}$$

where the functions K, L and M are harmonic functions on $\mathbb{E}^3$,

$$K \;=\; \sum_{i=1}^{n} \frac{k_i}{r_i}, \tag{16}$$

$$L \;=\; l_0 + \sum_{i=1}^{n} \frac{l_i}{r_i}, \tag{17}$$

$$M \;=\; m_0 + \sum_{i=1}^{n} \frac{m_i}{r_i}, \tag{18}$$

The 1-forms $\hat{\omega}$ are ξ are determined by

$$*d\hat{\omega} \;=\; HdM - MdH + \frac{3}{2}(KdL - LdK), \tag{19}$$

$$*d\xi \;=\; -dK, \tag{20}$$

and take the forms

$$\hat{\omega} \;=\; \sum_{i,j=1(i\neq j)}^{n} \left(h_i m_j + \frac{3}{2}k_i l_j\right)\hat{\omega}_{ij} - \sum_{i=1}^{n}\left(m_0 h_i + \frac{3}{2}l_0 k_i\right)\tilde{\omega}_i, \tag{21}$$

$$\xi \;=\; -\sum_{i=1}^{n} k_i \tilde{\omega}_i, \tag{22}$$

where the 1-form $\hat{\omega}_{ij}$ $(i \neq j)$ on $\mathbb{E}^3$, which is determined by

$$*d\hat{\omega}_{ij} = (1/r_i)d(1/r_j) - (1/r_j)d(1/r_i), \tag{23}$$

can be written as

$$\hat{\omega}_{ij} = -\frac{(\boldsymbol{r}-\boldsymbol{r}_i)\cdot(\boldsymbol{r}-\boldsymbol{r}_j)}{r_i r_j} \frac{\left[(\boldsymbol{r}_i-\boldsymbol{r}_j)\times(\boldsymbol{r}-\frac{\boldsymbol{r}_i+\boldsymbol{r}_j}{2})\right]_k dx^k}{\left|(\boldsymbol{r}_i-\boldsymbol{r}_j)\times(\boldsymbol{r}-\frac{\boldsymbol{r}_i+\boldsymbol{r}_j}{2})\right|^2}. \tag{24}$$

In this paper, we set $\boldsymbol{r}_i = (0,0,z_i)$ $(i = 1,\dots,n)$, by which $x\partial/\partial y - y\partial/\partial x$ becomes another $U(1)$ Killing vector field, and assume $z_i < z_j$ for $i < j$ $(i,j = 1,\dots,n)$ without loss of generality. In terms of standard spherical coordinates (r,θ,ϕ) such that $(x,y,z) = (r\sin\theta\cos\phi, r\sin\theta\sin\phi, r\cos\theta)$, the 1-forms $\tilde{\omega}_i$ and $\hat{\omega}_{ij}$ are simplified as

$$\tilde{\omega}_i = \frac{r\cos\theta - z_i}{r_i} d\phi, \tag{25}$$

$$\hat{\omega}_{ij} = \frac{r^2 - (z_i+z_j)r\cos\theta + z_i z_j}{z_{ji} r_i r_j} d\phi, \quad z_{ji} := z_j - z_i. \tag{26}$$

and so the 1-form $\hat{\omega}$ can be written as

$$\begin{aligned}\hat{\omega} = &\left[\sum_{i,j=1}^{n}\left(h_i m_j + \frac{3}{2}k_i l_j\right)\frac{r^2 - (z_i+z_j)r\cos\theta + z_i z_j}{z_{ji} r_i r_j}\right.\\ &\left. - \sum_{i=1}^{n}\left(m_0 h_i + \frac{3}{2}l_0 k_i\right)\frac{r\cos\theta - z_i}{r_i} + c\right]d\phi,\end{aligned} \tag{27}$$

where we have added the integration constant c since $\hat{\omega}$ is determined by only the derivatives in Eq. (19).

B. Boundary conditions

As the detail is reviewed in [10, 40], in order that the supersymmetric solution describes the BPS microstate geometry solution of physical interest, we must impose suitable boundary conditions (i) at infinity, (ii) at the Gibbon-Hawking centers $\boldsymbol{r} = \boldsymbol{r}_i$ $(i = 1, ..., n)$ and (iii) on the z-axis $x = y = 0$ of $\mathbb{E}^3$ in the Gibbons-Hawking space. More precisely, we consider the following boundary conditions:

- **(i)** at infinity $r \to \infty$, the spacetime is asymptotically Minkowski spacetime.
- **(ii)** at the n centers $\boldsymbol{r} = \boldsymbol{r}_i$ $(i = 1, ..., n)$ such that each harmonic function diverges, the spacetime is regular, and behaves as the coordinate singularities like the origin of the Minkowski spacetime. The spacetime admits no causal pathology such as closed timelike curve (CTCs) around these points.

(iii) on the z-axis $I = \{(x, y, z) \mid x = y = 0\}$ of $\mathbb{E}^3$ in the Gibbons-Hawking space, there appear no Dirac-Misner strings, no orbifold singularities and no conical singularities.

1. Infinity

The asymptotic flatness demands that at infinity $r \to \infty$, the functions $(f^{-1},\, H)$, the 1-forms $(\chi,\, \omega)$ behave as, respectively,

$$f^{-1} \simeq 1, \tag{28}$$

$$H \simeq \frac{1}{r}, \tag{29}$$

$$\omega \simeq 0, \tag{30}$$

$$\chi \simeq \pm\cos\theta d\phi, \tag{31}$$

which ensure that in terms of the radial coordinate $\rho = 2\sqrt{r}$, and at $r \to \infty$ $(\rho \to \infty)$ the metric is indeed approximated as

$$ds^2 \simeq -dt^2 + d\rho^2 + \frac{\rho^2}{4}\left[(d\psi + \cos\theta d\phi)^2 + d\theta^2 + \sin^2\theta d\phi^2\right]. \tag{32}$$

This is the metric of five-dimensional Minkowski spacetime where the metric on S^3 is written in terms of Euler angles (ψ, ϕ, θ), whose ranges must be $0 \le \theta \le \pi$, $0 \le \phi < 2\pi$ and $0 \le \psi < 4\pi$ with the identification $\phi \sim \phi + 2\pi$ and $\psi \sim \psi + 4\pi$.

At infinity $r \to \infty$, the metric functions f and H behave, respectively, as

$$f^{-1} \simeq l_0 + \left[\left(\sum_i k_i\right)^2 + \sum_i l_i\right]\left(\sum_i h_i\right)^{-1} r^{-1}, \tag{33}$$

$$H \simeq \left(\sum_i h_i\right) r^{-1}. \tag{34}$$

Since for $r \to \infty$, the metric function ω_ψ and the 1-forms $(\tilde{\omega}_i,\, \hat{\omega}_{ij})$ behave as, respectively,

$$\omega_\psi \simeq m_0 + \frac{3}{2} l_0 \left(\sum_i h_i\right)^{-1} \sum_i k_i, \tag{35}$$

$$\tilde{\omega}_i \simeq \cos\theta d\phi, \tag{36}$$

$$\hat{\omega}_{ij} \simeq \frac{d\phi}{z_{ji}}, \tag{37}$$

the 1-forms χ and ω are approximated by

$$\chi = \sum_i h_i \hat{\omega}_i \simeq \sum_i h_i \cos\theta d\phi, \tag{38}$$

$$\begin{aligned}\omega \simeq& \left(m_0 + \frac{3}{2} l_0 \sum_i k_i\right)(d\psi + \cos\theta d\phi) - \sum_i \left(m_0 h_i + \frac{3}{2} l_0 k_i\right) \cos\theta d\phi \\ &+ \left(\sum_{i,j(i\neq j)} \frac{h_i m_j + \frac{3}{2} k_i l_j}{z_{ji}} + c\right) d\phi.\end{aligned} \tag{39}$$

Thus, in comparison with Eqs. (28)-(31) and Eqs. (33), (34),(38), (39), the parameters must satisfy the following constraints

$$l_0 = 1, \tag{40}$$

$$\sum_{i=1}^{n} h_i = 1, \tag{41}$$

$$m_0 = -\frac{3}{2} l_0 \sum_{i=1}^{n} k_i, \tag{42}$$

$$c = -\sum_{i,j=1(i\neq j)}^{n} \frac{h_i m_j + \frac{3}{2} k_i l_j}{z_{ji}}. \tag{43}$$

2. Gibbons-Hawking centers

The metric obviously has divergence at the points $\boldsymbol{r} = \boldsymbol{r}_i$ $(n = 1, ..., n)$ on the Gibbons-Hawking space. We hence impose the boundary conditions at the points $\boldsymbol{r} = \boldsymbol{r}_i$ $(n = 1, ..., n)$ so that these become regular points like the origin of Minkowski spacetime:

$$ds^2 \simeq -dt'^2 + \left[d\rho^2 + \frac{\rho^2}{4}\left\{(d\psi' \pm \cos\theta d\phi')^2 + d\theta^2 + \sin^2\theta d\phi'^2\right\}\right]. \tag{44}$$

Let us choose the coordinates (x, y, z) on $\mathbb{E}^3$ of the Gibbons-Hawking space so that the ith point $\boldsymbol{r} = \boldsymbol{r}_i$ is an origin of $\mathbb{E}^3$. Near the origin $\boldsymbol{r} = 0$, the four harmonic functions H, K, L and M behave as, respectively,

$$H \simeq \frac{h_i}{r} + \sum_{j(\neq i)} \frac{h_j}{|z_{ji}|}, \quad K \simeq \frac{k_i}{r} + \sum_{j(\neq i)} \frac{k_j}{|z_{ji}|}, \tag{45}$$

$$L \simeq \frac{l_i}{r} + 1 + \sum_{j(\neq i)} \frac{l_j}{|z_{ji}|}, \quad M \simeq \frac{m_i}{r} + m_0 + \sum_{j(\neq i)} \frac{m_j}{|z_{ji}|}, \tag{46}$$

which lead to

$$f^{-1} \simeq \frac{k_i^2 h_i^{-1} + l_i}{r} + c_{1(i)}, \tag{47}$$

$$\omega_\psi \simeq \frac{k_i^3 h_i^{-2} + \frac{3}{2} k_i l_i h_i^{-1} + m_i}{r} + c_{2(i)}, \tag{48}$$

where the constants $c_{1(i)}$ and $c_{2(i)}$ are defined by

$$\begin{aligned}
h_i c_{1(i)} &:= h_i l_0 + \sum_{j=1(j\neq i)}^{n} \frac{2h_i^2 k_i k_j - h_i k_i^2 h_j + h_i^3 l_j}{|z_{ij}| h_i^2} \\
&= h_i + \sum_{j=1(j\neq i)}^{n} \frac{2k_i k_j - h_i k_i^2 h_j - h_i h_j k_j^2}{|z_{ij}|}, \qquad (49)\\
h_i c_{2(i)} &:= h_i m_0 + \frac{3}{2} k_i l_0 + \sum_{j=1(j\neq i)}^{n} \frac{-2k_i^3 h_j + 3h_i k_i^2 k_j + 3h_i^2 k_i l_j + 2h_i^3 m_j}{2|z_{ij}| h_i^2} \\
&= h_i m_0 + \frac{3}{2} k_i + \sum_{j=1(j\neq i)}^{n} \frac{-2m_i h_j - 3l_i k_j + 3k_i l_j + 2h_i m_j}{2|z_{ij}|}, \qquad (50)
\end{aligned}$$

where we have used $h_i^2 = 1$ ($h_i = \pm 1$ will be imposed below. See Eq. (65)) in the second equalities of Eqs.(49) and (50). The 1-forms $\tilde{\omega}_j$ and $\hat{\omega}_{kj}$ are approximated by

$$\tilde{\omega}_i \simeq \cos\theta d\phi, \qquad \tilde{\omega}_j \simeq -\frac{z_{ji}}{|z_{ji}|} d\phi \ (j \neq i), \qquad (51)$$

$$\hat{\omega}_{ij} \simeq -\frac{\cos\theta}{|z_{ji}|} d\phi \ (i \neq j), \qquad \hat{\omega}_{kj} \simeq \frac{z_{ji} z_{ki}}{|z_{ji} z_{ki}| z_{jk}} d\phi \ (k \neq j, k, j \neq i), \qquad (52)$$

and hence, 1-forms χ and $\hat{\omega}$ behave as

$$\chi \simeq \left(h_i \cos\theta + \chi_{0(i)}\right) d\phi\,, \qquad \hat{\omega} \simeq (\hat{\omega}_{1(i)} \cos\theta + \hat{\omega}_{0(i)}) d\phi\,, \qquad (53)$$

where

$$\chi_{0(i)} := -\sum_{j(\neq i)} \frac{h_j z_{ji}}{|z_{ji}|}\,, \qquad (54)$$

$$\hat{\omega}_{0(i)} := \sum_{k,j(\neq i, k\neq j)} \left(h_k m_j + \frac{3}{2} k_k l_j\right) \frac{z_{ji} z_{ki}}{|z_{ji} z_{ki}| z_{jk}} + \sum_{j(\neq i)} \left(m_0 h_j + \frac{3}{2} k_j\right) \frac{z_{ji}}{|z_{ji}|} + c\,, \qquad (55)$$

$$\hat{\omega}_{1(i)} := -\sum_{j(\neq i)} \left(h_i m_j - h_j m_i + \frac{3}{2}(k_i l_j - k_j l_i)\right) \frac{1}{|z_{ji}|} - \left(m_0 h_i + \frac{3}{2} k_i\right). \qquad (56)$$

One therefore obtains the asymptotic behavior of the metric around the ith point as

$$\begin{aligned}
ds^2 \simeq & -\left(\frac{k_i^2 h_i^{-1} + l_i}{r} + c_{1(i)}\right)^{-2} \left[dt + \left(\frac{k_i^3 h_i^{-2} + \frac{3}{2} k_i l_i h_i^{-1} + m_i}{r} + c_{2(i)}\right) \{d\psi + (h_i \cos\theta + \chi_{0(i)}) d\phi\}\right. \\
& \left. + (\hat{\omega}_{1(i)} \cos\theta + \hat{\omega}_{0(i)}) d\phi\right]^2 + \left(\frac{k_i^2 h_i^{-1} + l_i}{r} + c_{1(i)}\right) \frac{r}{h_i} \\
& \times \left[\{d\psi + (h_i \cos\theta + \chi_{0(i)}) d\phi\}^2 + h_i^2 \left(\frac{dr^2}{r^2} + d\theta^2 + \sin^2\theta d\phi^2\right)\right]. \qquad (57)
\end{aligned}$$

To remove the divergence of the metric, it is sufficient to impose the following conditions on the parameters (k_i, l_i, m_i) $(i = 1, ..., n)$:

$$k_i^2 + h_i l_i = 0, \qquad (58)$$

$$k_i^3 h_i^{-2} + \frac{3}{2} k_i l_i h_i^{-1} + m_i = 0, \qquad (59)$$

which are equivalent to the condition for the parameters (l_i, m_i),

$$l_i = -\frac{k_i^2}{h_i}, \tag{60}$$

$$m_i = \frac{k_i^3}{2h_i^2}, \tag{61}$$

and these yield the equation

$$h_i c_{2(i)} = -\hat{\omega}_{1(i)}. \tag{62}$$

Introducing the new coordinates (ρ, ψ', ϕ') by

$$\rho = 2\sqrt{h_i^{-1} c_{1(i)} r}, \qquad \psi' = \psi + \chi_{0(i)}\phi, \qquad \phi' = \phi\,, \tag{63}$$

we can write the metric near $\boldsymbol{r} = \boldsymbol{r}_i$ as

$$ds^2 \simeq -c_{1(i)}^{-2} d[t + c_{2(i)}\psi' + \hat{\omega}_{0(i)}\phi']^2 + \left[d\rho^2 + \frac{\rho^2}{4}\left\{(d\psi' + h_i \cos\theta d\phi')^2 + d\theta^2 + \sin^2\theta d\phi'^2\right\}\right]. \tag{64}$$

Comparing the (ϕ', ψ')-part of the above metric (64) with the boundary condition (44), we must impose for each h_i $(i = 1, \ldots, n)$

$$h_i = \pm 1. \tag{65}$$

To ensure the five-dimensional metric with Lorentzian signature, the following inequities must be satisfied

$$h_i^{-1} c_{1(i)} = h_i + \sum_{j=1(j\neq i)}^{n} \frac{2k_i k_j + l_i h_j + h_i l_j}{|z_{ij}|} > 0 \ (i = 1, \ldots, n). \tag{66}$$

The above metric (64) is locally isometric to the flat metric, but CTCs necessarily appear near $\rho \simeq 0$ because the Killing vector $\partial/\partial\psi' = \partial/\partial\psi$ becomes timelike. To avoid the existence of CTCs around $\boldsymbol{r}_i$ $(i = 1, \ldots, n)$, $c_{2(i)} = 0$ and $\omega_{0(i)} = 0$ must be simultaneously satisfied at $\boldsymbol{r} = \boldsymbol{r}_i$ $(i = 1, ..., n)$ but it is sufficient to impose only $c_{2(i)} = 0$, which can be written as

$$\begin{aligned} 0 &= h_i c_{2(i)} \\ &= h_i m_0 + \frac{3}{2}k_i + \sum_{j=1(j\neq i)}^{n} \frac{h_i m_j - m_i h_j - \frac{3}{2}(l_i k_j - k_i l_j)}{|z_{ij}|} \\ &= h_i m_0 + \frac{3}{2}k_i + \sum_{j=1(j\neq i)}^{n} \frac{(h_i k_j - h_j k_i)^3}{2|z_{ij}|}. \end{aligned} \tag{67}$$

These equations are so-called "bubble equations" in Refs. [9, 41], which physically means the balance between the gravitational attraction and the repulsion by the magnetic fluxes over the

2-cycles. Moreover, let us note that $\hat{\omega}_{0(i)} = 0$ automatically hold for all $i = 1, ..., n$, if we impose (67) since from Eqs. (60) and (61), $\hat{\omega}_{0(i)}$ can be shown to vanish,

$$\begin{aligned}
\hat{\omega}_{0(i)} &= \sum_{k,j(k,j\neq i,k\neq j)} \left(h_k m_j + \frac{3}{2} k_k l_j\right) \frac{z_{ji} z_{ki}}{|z_{ji} z_{ki}| z_{jk}} + \sum_{j(\neq i)} \left(m_0 h_j + \frac{3}{2} k_j\right) \frac{z_{ji}}{|z_{ji}|} \\
&\quad - \sum_{k,j(j\neq k)} \frac{h_k m_j + \frac{3}{2} k_k l_j}{z_{jk}} \\
&= \sum_{k,j(k,j\neq i,k\neq j)} \frac{(h_k k_j - h_j k_k)^3}{4 z_{jk}} \frac{z_{ji} z_{ki}}{|z_{ji} z_{ki}|} + \sum_{k,j(j\neq i,k\neq j)} \frac{(h_k k_j - h_j k_k)^3}{2|z_{jk}|} \frac{z_{ji}}{|z_{ji}|} \\
&\quad - \sum_{k,j(k\neq j)} \frac{(h_k k_j - h_j k_k)^3}{4 z_{jk}} \\
&= 0,
\end{aligned} \tag{68}$$

where we have used Eq. (67) for the 2nd term in the right-hand side of the first equality, and the last equality can be shown by long but simple computations.

Furthermore, the n bubble equations $c_{2(i)} = 0$ $(i = 1, \ldots, n)$ are not independent because the summation of $h_i c_{2(i)}$ $(i = 1, \ldots, n)$ automatically vanishes, regardless of the bubble equations, as

$$\begin{aligned}
\sum_{i=1}^{n} h_i c_{2(i)} &= \sum_{i=1}^{n} h_i m_0 + \frac{3}{2} \sum_{i=1}^{n} k_i \\
&\quad + \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n} \frac{h_i m_j - m_i h_j - \frac{3}{2}(l_i k_j - k_i l_j)}{|z_{ij}|} \\
&= \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n} \frac{(h_i k_j - h_j k_i)^3}{2|z_{ij}|} \\
&= 0,
\end{aligned} \tag{69}$$

where we have used Eqs. (40) and (42) in the second equality and the antisymmetry for i and j in the last summation. Thus, the bubble equations $h_i c_{2(i)} = 0$ $(i = 1, \ldots, n)$ give $(n-1)$ independent constraint equations for the parameters (k_i, z_i) $(i = 1, \ldots, n)$.

3. Axis

The z-axis of $\mathbb{E}^3$ (i.e., $x = y = 0$) in the Gibbons-Hawking space consists of the $(n+1)$ intervals: $I_- = \{(x, y, z)|x = y = 0, z < z_1\}$, $I_i = \{(x, y, z)|x = y = 0, z_i < z < z_{i+1}\}$ $(i = 1, ..., n-1)$ and $I_+ = \{(x, y, z)|x = y = 0, z > z_n\}$. On the z-axis, the 1-forms $\hat{\omega}_{ij}$ and $\tilde{\omega}_i$ are, respectively, simplified to

$$\hat{\omega}_{ij} = \frac{(z - z_i)(z - z_j)}{z_{ji}|z - z_i||z - z_j|} d\phi, \qquad \tilde{\omega}_i = \frac{z - z_i}{|z - z_i|} d\phi. \tag{70}$$

In particular, on $I_\pm$, $\hat{\omega}_{ij}$ and $\tilde{\omega}_i$ become, respectively,

$$\hat{\omega}_{ij} = \frac{1}{z_{ji}} d\phi, \qquad \tilde{\omega}_i = \pm d\phi. \tag{71}$$

Hence, on $I_\pm$, $\hat{\omega} = \hat{\omega}_\phi d\phi$ vanishes since

$$\begin{aligned}
\hat{\omega} &= \sum_{k,j(k\neq j)} \left(h_k m_j + \frac{3}{2} k_k l_j \right) \hat{\omega}_{kj} - \sum_j \left(m_0 h_j + \frac{3}{2} k_j \right) \hat{\omega}_j + c d\phi \\
&= \sum_{k,j(k\neq j)} \left(h_k m_j + \frac{3}{2} k_k l_j \right) \frac{d\phi}{z_{jk}} \mp \sum_j \left(m_0 h_j + \frac{3}{2} k_j \right) d\phi - \sum_{k,j(k\neq j)} \left(h_k m_j + \frac{3}{2} k_k l_j \right) \frac{d\phi}{z_{jk}} \\
&= \mp \sum_j \left(m_0 h_j + \frac{3}{2} k_j \right) d\phi \\
&= \mp \left(m_0 + \frac{3}{2} \sum_j k_j \right) d\phi \\
&= 0,
\end{aligned} \tag{72}$$

where we have used Eq. (42) in the last equality.

On $z \in I_i$ $(i = 1, ..., n-1)$, the 1-forms $\hat{\omega}_{ij}$ and $\hat{\omega}_j$ are written as

$$\hat{\omega}_{kj} = \frac{z_{ik} z_{ij}}{z_{jk} |z_{ik} z_{ij}|} d\phi \ (k, j \neq i), \qquad \hat{\omega}_{ij} = -\frac{1}{|z_{ij}|} d\phi \ (j \neq i) \tag{73}$$

$$\tilde{\omega}_j = \frac{z_{ij}}{|z_{ij}|} d\phi \ (j \neq i), \qquad \tilde{\omega}_i = d\phi, \tag{74}$$

and therefore,

$$\begin{aligned}
\hat{\omega}_\phi - \hat{\omega}_{0(i)} &= -\sum_{j(j\neq i)} \frac{h_i m_j - h_j m_i + \frac{3}{2}(k_i l_j - k_j l_i)}{z_{ji}} - \left(m_0 h_i + \frac{3}{2} k_i \right) \\
&= h_i c_{2(i)} \\
&= 0,
\end{aligned} \tag{75}$$

where we have used Eq. (67) and (68). Thus, we can show that $\hat{\omega} = 0$ also holds on I_i for $i = 1, ..., n-1$. We therefore conclude that $\hat{\omega} = 0$ holds at each interval $I_\pm$ and I_i $(i = 1, \ldots, n-1)$. This means that there are no Dirac-Misner strings in the spacetime, which can be obtained as the result of the bubble equations (67) (see [10, 41]).

Next, we show the absence of orbifold singularities. On the intervals $I_\pm$, the 1-form χ becomes

$$\chi = \pm d\phi, \tag{76}$$

and on the intervals I_i $(i = 1, \ldots, n-1)$, it takes the form

$$\begin{aligned}\chi &= \left(\sum_{j=1}^{i} h_j \frac{z-z_j}{|z-z_j|} + \sum_{j=i+1}^{n} h_j \frac{z-z_j}{|z-z_j|}\right) d\phi \\ &= \left(\sum_{j=1}^{i} h_j - \sum_{j=i+1}^{n} h_j\right) d\phi. \qquad (77)\end{aligned}$$

The two-dimensional (ϕ, ψ)-part of the metric on the intervals $I_\pm$ and I_i can be written in the form

$$ds_2^2 = (-f^2\omega_\psi^2 + f^{-1}H^{-1})(d\psi + \chi_\phi d\phi)^2. \qquad (78)$$

Here let us use the coordinate basis vectors $(\partial_{\phi_1}, \partial_{\phi_2})$ with 2π periodicity, instead of $(\partial_\phi, \partial_\psi)$, where the coordinates ϕ_1 and ϕ_2 are defined by $\phi_1 := (\psi + \phi)/2$ and $\phi_2 := (\psi - \phi)/2$. It can be shown from (78) that the Killing vector $v := \partial_\phi - \chi_\phi \partial_\psi$ vanishes on each interval. Indeed we can show

1. on the interval I_-, the Killing vector $v_- := \partial_\phi + \partial_\psi = \partial_{\phi_1}$ vanishes,

2. on each interval I_i $(i = 1, ..., n-1)$, the Killing vector

$$\begin{aligned}v_i &:= \partial_\phi - \chi_\phi\big|_{I_i}\partial_\psi \qquad (79)\\ &= \frac{1-\chi_\phi\big|_{I_i}}{2}\partial_{\phi_1} - \frac{1+\chi_\phi\big|_{I_i}}{2}\partial_{\phi_2} \qquad (80)\\ &= \frac{1}{2}\left(1 - \sum_{j=1}^{i} h_j + \sum_{j=i+1}^{n} h_j\right)\partial_{\phi_1} - \frac{1}{2}\left(1 + \sum_{j=1}^{i} h_j - \sum_{j=i+1}^{n} h_j\right)\partial_{\phi_2} \qquad (81)\\ &= \left(\sum_{j=i+1}^{n} h_j\right)\partial_{\phi_1} - \left(\sum_{j=1}^{i} h_j\right)\partial_{\phi_2} \qquad (82)\end{aligned}$$

vanishes, where we have used $\sum_i h_j = 1$ in the last equation.

3. on the interval I_+, the Killing vector $v_+ := \partial_\phi - \partial_\psi = -\partial_{\phi_2}$ vanishes.

From these, we can observe that the Killing vectors $v_\pm,\ v_i$ on the intervals satisfy with

$$\det\,(v_-^T, v_1^T) = h_1, \quad \det\,(v_{n-1}^T, v_+^T) = -h_n, \qquad (83)$$

$$
\begin{aligned}
\det\,(v_{i-1}^T, v_i^T) &= -\left(\sum_{j=i}^{n} h_j\right)\left(\sum_{j=1}^{i} h_j\right) + \left(\sum_{j=i+1}^{n} h_j\right)\left(\sum_{j=1}^{i-1} h_j\right) \\
&= -\left(\sum_{j=i}^{n} h_j\right)\left(\sum_{j=1}^{i} h_j\right) + \left(\sum_{j=i}^{n} h_j - h_i\right)\left(\sum_{j=1}^{i} h_j - h_i\right) \\
&= h_i^2 - \left(\sum_{j=i}^{n} h_j + \sum_{j=1}^{i} h_j\right) h_i \\
&= h_i^2 - \left(\sum_{j=i}^{n} h_j + h_i\right) h_i \\
&= -\left(\sum_{j=i}^{n} h_j\right) h_i \\
&= -h_i. \qquad (84)
\end{aligned}
$$

Therefore, it turns out that $|\det\,(v_-^T, v_1^T)| = |\det\,(v_{n-1}^T, v_+^T)| = |\det\,(v_{i-1}^T, v_i^T)| = 1$ hold, which means that there exist no orbifold singularities at adjacent intervals, as proved in Ref. [42].

C. Gauge freedom

As discussed in Ref. [43], the supersymmetric solutions have a gauge freedom, which means that for the linear transformation for the harmonic functions H, K, L and M,

$$K \to K + \bar{\lambda} H, \qquad L \to L - 2\bar{\lambda} K - \bar{\lambda}^2 H, \qquad M \to M - \frac{3}{2}\bar{\lambda} L + \frac{3}{2}\bar{\lambda}^2 K + \frac{1}{2}\bar{\lambda}^3 H, \qquad (85)$$

the metric and Maxwell field are invariant, where $\bar{\lambda}$ is a constant. Indeed, it is easy to show that under the transformation (85), (f, ω_ψ, χ) remain invariant, and the 1-form ξ changes as $\xi \to \xi - \bar{\lambda}\chi$, which merely corresponds to the gauge shift of A, $A \to A + \bar{\lambda} d\psi$. Using this gauge transformation and the appropriate choice of $\bar{\lambda}$, one can set

$$k_m = 0, \qquad (86)$$

for a certain m $(m = 1, \dots, n)$ because the coefficient of $1/r_m$ in K changes $k_m \to k_m + \bar{\lambda} h_m$. Moreover, using the shift of $z \to z+$const., one can set

$$z_m = 0 \qquad (87)$$

for a certain m $(m = 1, \dots, n)$.

D. Parameter counting

The solution (1) and (2) includes the $4n+3$ continuous parameters $(k_i, l_0, l_i, m_0, m_i, z_i, c)$ and the n discrete parameters $h_i = \pm 1$ $(i = 1, \dots n)$. The conditions (40), (42), (43) (60), (61), (67) and the gauge conditions (86), (87) reduce the number of independent continuous parameters from $4n+3$ to $n-1$, where the bubble equations (67) give not n but rather $(n-1)$ independent equations due to the constraint equation (69), and the condition (41) reduces the number of independent discrete parameters from n to $n-1$. Moreover, these parameters must be subject to the n inequalities (66).

It follows from the constraint equation (41) and the conditions (65) that the number n of centers $\boldsymbol{r} = \boldsymbol{r}_i$ must be odd, and so in Sec. IV, we consider three centers and five centers as the simplest nontrivial examples of the microstate geometries (the case $n = 1$ corresponds to Minkowski spacetime).

III. PHYSICAL PROPERTIES

Under the appropriate boundary conditions mentioned in the previous section, let us investigate some physical properties of the solutions.

A. Conserved quantities

To begin with, we consider conserved quantities of the microstate geometries. From the boundary conditions at infinity (40)-(43), the ADM mass and two ADM angular momenta can be computed as

$$M = \frac{\sqrt{3}}{2}Q = 3\pi\left[\left(\sum_i k_i\right)^2\left(\sum_i h_i\right)^{-1} + \sum_i l_i\right], \tag{88}$$

$$J_\psi = \pi\left[\left(\sum_i k_i\right)^3 + \sum_i m_i + \frac{3}{2}\left(\sum_i h_i\right)^{-1}\left(\sum_i k_i\right)\left(\sum_i l_i\right)\right], \tag{89}$$

$$J_\phi = \frac{3\pi}{2}\left(\sum_i h_i\right)^{-1}\left[-\left(\sum_i k_i\right)\left(\sum_i h_i z_i\right) + \left(\sum_i k_i z_i\right)\right], \tag{90}$$

where Q is the electric charge, which saturates the BPS bound [44].

Each interval I_i $(i = 1, ..., n-1)$, which is introduced in Sec. II B 3, denotes the bubble which is topologically a two-dimensional sphere since the ψ-fiber of the Gibbons-Hawking space (3) collapses to zero at the centers $z = z_i$ and $z = z_{i+1}$, and so along the interval, the fiber sweeps out two-dimensional sphere. Since the Maxwell gauge field A_μ is obviously smooth on the bubbles, the magnetic fluxes through I_i $(i = 1, ..., n-1)$ can be defined as

$$q[I_i] := \frac{1}{4\pi}\int_{I_i} F, \tag{91}$$

which are computed as

$$q[I_i] = [-A_\psi]_{z=z_i}^{z=z_{i+1}} = \frac{\sqrt{3}}{2}\left(\frac{k_i}{h_i} - \frac{k_{i+1}}{h_{i+1}}\right) \quad (i = 1, ..., n-1). \tag{92}$$

B. Evanescent ergosurface

The existence of ergoregions gives rise to strong instability due to a superradiant-triggered mechanism in spite of the existence of the horizon [45, 46]. It was demonstrated that a certain class of non-supersymmetric microstate geometries with ergoregion in type IIB supergravity are unstable, which is a general feature of horizonless geometries with ergoregion [47]. The BPS microstate geometries does not admit the presence of ergoregions but evanescent ergosurfaces [10, 48], which are defined as timelike hypersurfaces such that a stationary Killing vector field becomes null there and timelike everywhere except there. Reference [49] proved that on such surfaces, massless particles with zero energy $(E = 0)$ relative to infinity move along stable trapped null geodesics. Since this stably trapping leads to a classical non-linear instability of the spacetime [45, 49, 50], it is of physical importance to investigate the existence of evanescent ergosurfaces, which exist at $f = 0$ which corresponds to

$$H = \sum_{i=1}^{n} \frac{h_i}{r_i} = 0. \tag{93}$$

For simplicity, let us consider the microstate geometries with reflection symmetry $z_m = -z_{n-m+1}$ and $k_m = k_{n-m+1}$ $(m = 1, \ldots n)$. For the microstate geometries with three centers $(n = 3)$ and $(h_1, h_2, h_3) = (1, -1, 1)$, they intersect the z-axis at the points

$$F_3(z) := |z||z - z_3| - |z - z_1||z - z_3| + |z||z - z_1| = 0. \tag{94}$$

It turns out from simple computations that $F_3(z) = 0$ has no root on $I_\pm$ and a single root I_i $(i = 1, 2)$. As seen FIG. 1, the evanescent ergosurfaces on the timeslice $t =$const. is the closed surface surrounding the center $\boldsymbol{r}_2 = (0, 0, 0)$, where we have introduced the radial coordinate by $\rho = \sqrt{x^2 + y^2}$.

For the microstate geometry with five centers ($n = 5$) and $(h_1, h_2, h_3, h_4, h_5) = (1, -1, 1, -1, 1)$, they intersect the z-axis at the points z satisfying $F_5(z) = 0$, where $F_5(z)$ is written as

$$
\begin{aligned}
F_5(z) :=\ & |z + z_2||z||z - z_2||z - z_1| - |z + z_1||z||z - z_2||z - z_1| + |z + z_1||z + z_2||z - z_2||z - z_1| \\
& -|z + z_1||z + z_2||z||z - z_1| + |z + z_1||z + z_2||z||z - z_2|.
\end{aligned} \tag{95}
$$

The roots of the equation $F_5(z) = 0$ are determined by the ratio k_2/k_1 through the bubble equations (67). As seen in FIG. 2, for the small ratio $0 < k_2/k_1 \ll 1$, the intervals $z_{21}(z_{54})$ of I_1 (I_4) are much larger the intervals $z_{32}(z_{43})$ of I_2 (I_3), whereas for the comparable ratio $1 \lesssim k_2/k_1 \lesssim 2$, the intervals $z_{21}(z_{54})$ also become comparable with $z_{32}(z_{43})$. This reason can be physically interpreted as the result that the magnetic fluxes need to support the bubbles. More precisely, this is caused by the force balance between a gravitational force that tend to contract the bubbles and a repulsive force by the magnetic fluxes that tend to expand the bubbles. For $k_2/k_1 \ll 1$, the magnetic flux through I_1 (I_4) is much larger than one through I_2 (I_3) $[|q[I_1]| \gg |q[I_2]|$ $(|q[I_4]| \gg |q[I_3]|)]$, and so the size of the bubble on I_1 (I_4) is larger than I_2 (I_3), whereas for $k_2/k_1 \simeq 2$, two magnetic fluxes are comparable $[|q[I_1]| \simeq |q[I_2]|$ $(|q[I_4]| \simeq |q[I_3]|)]$, and hence the size of the bubbles also becomes comparable. For $k_2/k_1 \ll 1$, the evanescent ergosurface exists as a common surface surrounding three centers $\boldsymbol{r} = \boldsymbol{r}_i = (0, 0, z_i)$ ($i = 2, 3, 4$), for $k_2/k_1 \simeq 1$, another ergosurface appears as the surface surrounding the center $\boldsymbol{r} = \boldsymbol{r}_3 = (0, 0, z_3)$, whereas for $k_2/k_1 \simeq 2$, two ergosurfaces combine into one, and thereafter separates into two parts.

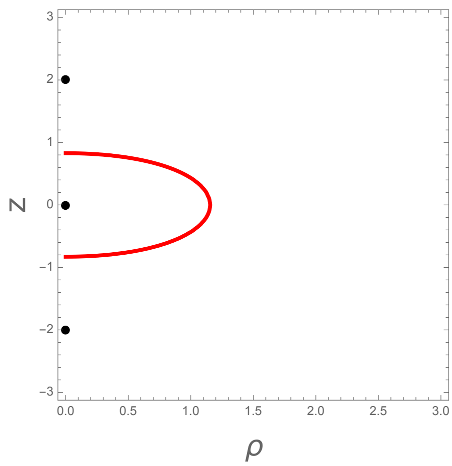

FIG. 1: Evanescent ergosurface in the microstate geometry for $n = 3$ in the (ρ, z)-plane: The black points corresponds to three centers that are located at $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ and $\boldsymbol{r}_3$ on the z-axis, and the red curve denotes an evanescent ergosurface, which surrounds a center at $\boldsymbol{r}_2 = (0, 0)$ but does not other two centers $\boldsymbol{r}_1 = (0, -2)$ and $\boldsymbol{r}_3 = (0, 2)$.

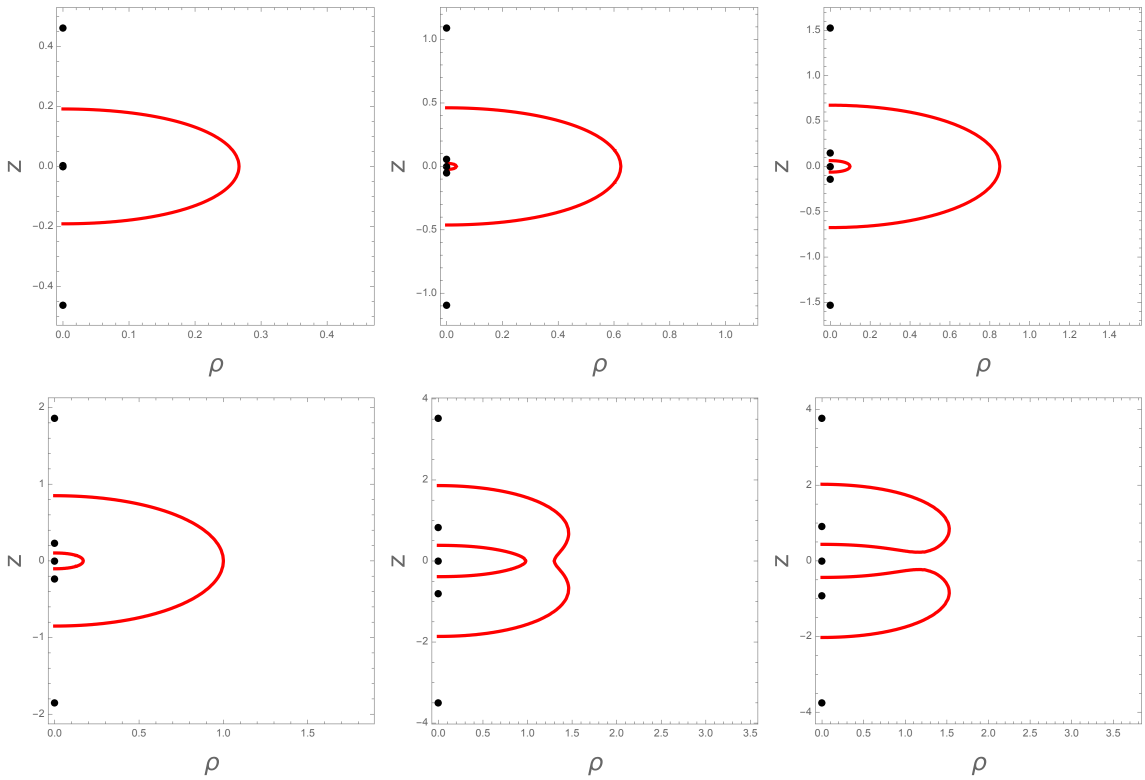

FIG. 2: Evanescent ergosurfaces in the microstate geometry with five centers in the (ρ, z)-plane for $k_3 = 0,\ k_4 = k_2,\ k_5 = k_1,\ z_3 = 0,\ z_4 = -z_2,\ z_5 = -z_1$: The upper and lower figures correspond to the ratios $k_2/k_1 = 0.1,\ 0.6,\ 0.9$, and $k_2/k_1 = 1.1,\ 1.9,\ 2.0$, respectively, from left to right. The black points correspond to the five centers that are located at $\boldsymbol{r}_i$ $(i = 1, \ldots, 5)$ on the z-axis, and the red curves denote the evanescent ergosurfaces, whose shapes depend on k_1 and k_2.

IV. MICROSTATE GEOMETRIES WITH REFLECTION SYMMETRY

In Sec. II, we have considered the stationary and bi-axisymmetric microstate geometries with n centers on the z-axis of the Gibbons-Hawking space which satisfy the bubble equations (67). The nasty constraint equations (for the parameters included in the solutions) make it difficult for us to understand the physical properties. In this section, in addition to such symmetry assumptions, we impose a further reflection symmetry on the solutions:

$$z_m = -z_{n-m+1}, \qquad k_m = k_{n-m+1}\ (m = 1, \ldots n), \tag{96}$$

which means the invariance of the solutions under the transformation $z \to -z$. This additional assumption extremely simplifies the bubble equations so that one can solve them and express z_i $(1, \ldots, n)$ in terms of k_i $(i = 1, \ldots, n)$, at least, for small n. In particular, it is easy to show from Eq. (90) that the angular momentum J_ϕ always vanishes under the additional symmetry assumption. In this section, for simplicity, let us consider only two cases of $n = 3$ and $n = 5$.

A. Three-center solution

First, let us consider the solution with three centers ($n = 3$) and $(h_1, h_2, h_3) = (1, -1, 1)$ that describes the simplest asymptotically flat, stationary and bi-axisymmetric microstate geometry, which has the four parameters (k_1, k_3, z_1, z_3), where we have set $k_2 = 0$ and $z_2 = 0$ from the two gauge conditions (86) and (87). Moreover, under the assumption of the reflection symmetry

$$z_3 = -z_1 =: a \ (> 0), \qquad k_3 = k_1, \tag{97}$$

the bubble equations (67) are simply written as

$$c_{2(1)} = -\frac{1}{2} c_{2(2)} = c_{2(3)} = \frac{k_1 [k_1^2 - 3a]}{2a} = 0, \tag{98}$$

which imply

$$k_1 = 0, \tag{99}$$

$$a = \frac{k_1^2}{3}. \tag{100}$$

It is obvious that in the former case $h_i c_{1(i)} = 0$ $(i = 1, 2, 3)$, and so the inequalities (66) cannot be satisfied. In the meanwhile, in the latter case, the inequalities (66) can be automatically satisfied because $h_i c_{1(i)}$ $(i = 1, 2, 3)$ can be directly computed as

$$h_1 c_{1(1)} = h_3 c_{1(3)} = 4, \quad h_2 c_{1(2)} = 5. \tag{101}$$

Therefore, for arbitrary nonzero k_1, this describes a regular and causal solution of an asymptotically flat, stationary microstate geometry with the bi-axisymmetry and reflection symmetry. This solution was previously analyzed in Ref. [10].

The z-axis of $\mathbb{E}^3$ in the Gibbons-Hawking space consists of the four intervals: $I_- = \{(x, y, z)|x = y = 0, z < z_1\}$, $I_i = \{(x, y, z)|x = y = 0, z_i < z < z_{i+1}\}$ $(i = 1, 2)$ and $I_+ = \{(x, y, z)|x = y = 0, z > z_3\}$. From the result in Sec. II B 3, one can see

1. on I_-, the Killing vector $v_- = \partial_{\phi_1}$ vanishes,
2. on I_1, the Killing vector $v_1 = \left(\sum_{j=2}^{3} h_j\right) \partial_{\phi_1} - h_1 \partial_{\phi_2} = -\partial_{\phi_2}$ vanishes,
3. on I_2, the Killing vector $v_2 = h_3 \partial_{\phi_1} - \left(\sum_{j=1}^{2} h_j\right) \partial_{\phi_2} = \partial_{\phi_1}$ vanishes, and

4. on I_+, the Killing vector $v_+ = -\partial_{\phi_2}$ vanishes.

Thus the rod structure of this three-center microstate geometry is displayed in Fig. 3.

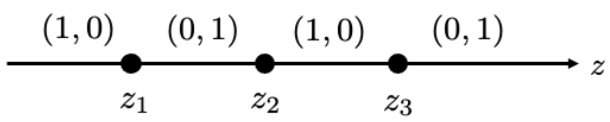

FIG. 3: Rod structure for the microstate geometry with three centers and $(h_1, h_2, h_3) = (1, -1, 1)$.

Under the symmetric conditions (97) and gauge conditions $k_2 = 0$, $z_2 = 0$, the ADM mass and two ADM angular momenta in Eqs. (88)-(90) are reduced to

$$M = \frac{\sqrt{3}}{2}Q = 6\pi k_1^2, \tag{102}$$

$$J_\psi = 3\pi k_1^3, \tag{103}$$

$$J_\phi = 0, \tag{104}$$

and the magnetic fluxes in Eq. (92) are written as

$$q[I_1] = -q[I_2] = \frac{\sqrt{3}}{2}k_1. \tag{105}$$

B. Five-center solution

Next, let us consider the stationary, bi-axisymmetric microstate geometry with five centers ($n = 5$), which has the four parameters (k_1, k_2, z_1, z_2) under the reflection-symmetric conditions

$$k_5 = k_1, \quad k_4 = k_2, \quad z_5 = -z_1 =: a + b, \quad z_4 = -z_2 =: b \tag{106}$$

and the gauge conditions $k_3 = 0$, $z_3 = 0$. Here, let us notice that for the five-center solutions, there are two possible types of reflection-symmetric solutions, one with $(h_1, h_2, h_3, h_4, h_5) = (1, -1, 1, -1, 1)$ and one with $(h_1, h_2, h_3, h_4, h_5) = (-1, 1, 1, 1, -1)$, but the latter numerically seems not to satisfy the conditions (66). Thus, we here concentrate on only the former, in which case the conditions (67) are simplified to give

$$2h_1c_{2(1)} = 2h_5c_{2(5)} = -3(k_1 + 2k_2) - \frac{k_1^3}{a+b} + \frac{(k_1+k_2)^3}{a} + \frac{(k_1+k_2)^3}{a+2b} = 0, \tag{107}$$

$$2h_2c_{2(2)} = 2h_4c_{2(4)} = 3(2k_1 + 3k_2) - \frac{k_2^3}{b} - \frac{(k_1+k_2)^3}{a} - \frac{(k_1+k_2)^3}{a+2b} = 0, \tag{108}$$

$$h_3c_{2(3)} = -3(k_1 + k_2) + \frac{k_1^3}{a+b} + \frac{k_2^3}{b} = 0, \tag{109}$$

where we note that Eqs. (107)-(109) are not independent due to the constraint equation $\sum_{i=1}^{5} h_i c_{2(1)} = 2h_1 c_{2(1)} + 2h_2 c_{2(2)} + h_3 c_{2(3)} = 0$. Therefore, this solution has only two independent parameters. If we regard a and b as the functions of k_1 and k_2 from Eqs. (107), (109), this solution is a two-parameter family for (k_1, k_2).

Furthermore, the parameters k_1 and k_2 must satisfy the inequalities (66), which are reduced to

$$h_1 c_{1(1)} = h_5 c_{1(5)} = 1 - \frac{k_1^2}{a+b} + \frac{(k_1+k_2)^2}{a} + \frac{(k_1+k_2)^2}{a+2b} > 0, \tag{110}$$

$$h_2 c_{1(2)} = h_4 c_{1(4)} = -1 + \frac{k_2^2}{b} + \frac{(k_1+k_2)^2}{a} + \frac{(k_1+k_2)^2}{a+2b} > 0, \tag{111}$$

$$h_3 c_{1(3)} = 1 - \frac{2k_1^2}{a+b} + \frac{2k_2^2}{b} > 0, \tag{112}$$

together with the inequalities

$$a > 0, \qquad b > 0. \tag{113}$$

In the below, we assume $k_1 \neq 0$ and $k_2 \neq 0$ because from Eqs. (107) and (109), the case $k_1 = 0$ leads to

$$(a, b) = \left(\frac{-1 \pm \sqrt{5}}{6} k_2^2, \frac{1}{3} k_2^2 \right), \tag{114}$$

where only the solution with the positive sign can satisfy (110)-(113) and has $j^2 = 25/24$, and from Eqs. (107) -(109), the case $k_2 = 0$ yields $(a, b) = (k_1^2/3, 0)$, which cannot satisfy one of the inequalities (113). In what follows, we remove both cases of $k_1 = 0$ and $k_2 = 0$.

As shown in Fig.4, these inequalities are equivalent with

$$k_2/k_1 < -1, \quad -0.2063... < k_2/k_1 < 0, \quad k_2/k_1 > 0. \tag{115}$$

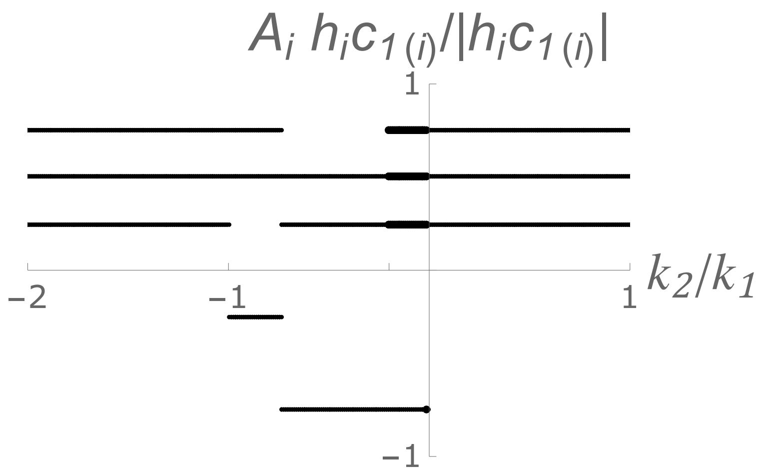

FIG. 4: The plots of $A_i h_i c_{1(i)}/|h_i c_{1(i)}|$ $[i = 1, 2, 3,\ (A_1, A_2, A_3) = (0.25, 0.5, 0.75)]$ for the microstate geometry with five centers and $(h_1, h_2, h_3, h_4, h_5) = (1, -1, 1, -1, 1)$, where we set $k_1 = 1$. The inequalities (110)-(113) are simultaneously satisfied in the range $k_2/k_1 < -1,\ -0.2063... < k_2/k_1 < 0,\ k_2/k_1 > 0$, where all graphs are positive. In particular, in the range $-0.2063... < k_2/k_1 < 0$, the solution to Eqs. (107)-(109) has the two branches which have the same nonzero pair of (k_1, k_2) but two different positive pairs of (a, b). One of two branches cannot satisfy the inequality (110).

The z-axis of $\mathbb{E}^3$ in the Gibbons-Hawking space consists of the six intervals: $I_- = \{(x, y, z)|x = y = 0, z < z_1\}$, $I_i = \{(x, y, z)|x = y = 0, z_i < z < z_{i+1}\}$ $(i = 1, ..., 4)$ and $I_+ = \{(x, y, z)|x = y = 0, z > z_5\}$. Applying the result in Sec. II B 3 to this solution, one can see

1. on I_-, the Killing vector $v_- = \partial_{\phi_1}$ vanishes,
2. on I_1, the Killing vector $v_1 = \left(\sum_{j=2}^{5} h_j\right) \partial_{\phi_1} - h_1 \partial_{\phi_2} = -\partial_{\phi_2}$ vanishes,
3. on I_2, the Killing vector $v_2 = \left(\sum_{j=3}^{5} h_j\right) \partial_{\phi_1} - \left(\sum_{j=1}^{2} h_j\right) \partial_{\phi_2} = \partial_{\phi_1}$ vanishes,
4. on I_3, the Killing vector $v_3 = \left(\sum_{j=4}^{5} h_j\right) \partial_{\phi_1} - \left(\sum_{j=1}^{3} h_j\right) \partial_{\phi_2} = -\partial_{\phi_2}$ vanishes,
5. on I_4, the Killing vector $v_4 = h_5 \partial_{\phi_1} - \left(\sum_{j=1}^{4} h_j\right) \partial_{\phi_2} = \partial_{\phi_1}$ vanishes, and
6. on I_+, the Killing vector $v_+ = -\partial_{\phi_2}$ vanishes,

Thus, it turns out that this five-center microstate geometry has the rod structure displayed in Fig. 5.

For this solution, the ADM mass and two ADM angular momenta in Eqs. (88)-(90) are reduced to

$$M = \frac{\sqrt{3}}{2}Q = 6\pi(k_1^2 + 4k_1k_2 + 3k_2^2), \tag{116}$$

$$J_\psi = 3\pi(k_1^3 + 6k_1^2k_2 + 10k_1k_2^2 + 5k_2^3), \tag{117}$$

$$J_\phi = 0, \tag{118}$$

and the magnetic fluxes in Eq. (92) are written as

$$q[I_1] = -q[I_4] = \frac{\sqrt{3}}{2}(k_1 + k_2), \quad q[I_2] = -q[I_3] = -\frac{\sqrt{3}}{2}k_2. \tag{119}$$

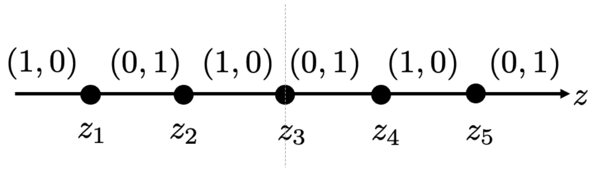

FIG. 5: Rod structure for the microstate geometry with five centers and $(h_1, h_2, h_3, h_4, h_5) = (1, -1, 1, -1, 1)$.

C. Comparison with BMPV black hole

Finally, we compare the BPS microstate geometries for $n = 3$ and $n = 5$ described in the previous section with the rotating BPS black hole in the five-dimensional minimal supergravity, i.e., the BMPV black hole [24], which carries mass (saturated the BPS bound) and equal angular momenta ($J_\phi = 0$). For this purpose, let us define a dimensionless angular momentum by

$$j := \frac{3\sqrt{3\pi}|J_\psi|}{M^{3/2}}. \tag{120}$$

For the BMPV black hole, the dimensionless angular momentum j has the range of

$$0 \le j < 1, \tag{121}$$

where $j = 0$ corresponds to the extremal Reissner-Nordstrom black hole. The absence of CTCs around the horizon requires the upper bound, $j = 1$.

It is shown from Eqs. (102) and (103) that for $n = 3$, the squared angular momentum j^2 takes only the value of

$$j^2 = \frac{9}{8}\ (> 1), \tag{122}$$

which is a larger value than the upper bound for the BMPV black hole.

Similarly, for $n = 5$, we evaluate the value of the squared angular momentum j^2 from Eqs. (116) and (117), where the ratio k_2/k_1 lies in the range (115). As seen in Fig. 6, The squared angular momentum j^2 asymptotically approaches $25/24$ at $k_2/k_1 \to -\infty$. For $k_2/k_1 < -1$, j^2 monotonically increases and diverges at $k_2/k_1 \to -1$, whereas for $k_2/k_1 > -1$, it has the lower bound 0.841... at $k_2/k_1 \to -0.206...$, where Eqs. (107)-(109) cannot be satisfied. Thereafter, it increases and approaches $9/8$ at $k_2/k_1 \to 0$, for $k_2/k_1 > 0$ monotonically decreases and asymptotically approaches $25/24$ at $k_2/k_1 \to \infty$. Thus, because the squared angular momentum does not have an upper bound but have the lower bound $j^2 = 0.841...$, we find that it must run the range

$$j^2 > 0.841... \,. \tag{123}$$

From this analysis, we can conclude that the bi-axisymmetric and reflection-symmetric microstate geometry with five centers can have the angular momentum of the range $0.841... < j^2 < 1$ as the BMPV black hole, while the microstate geometry with three centers cannot have.

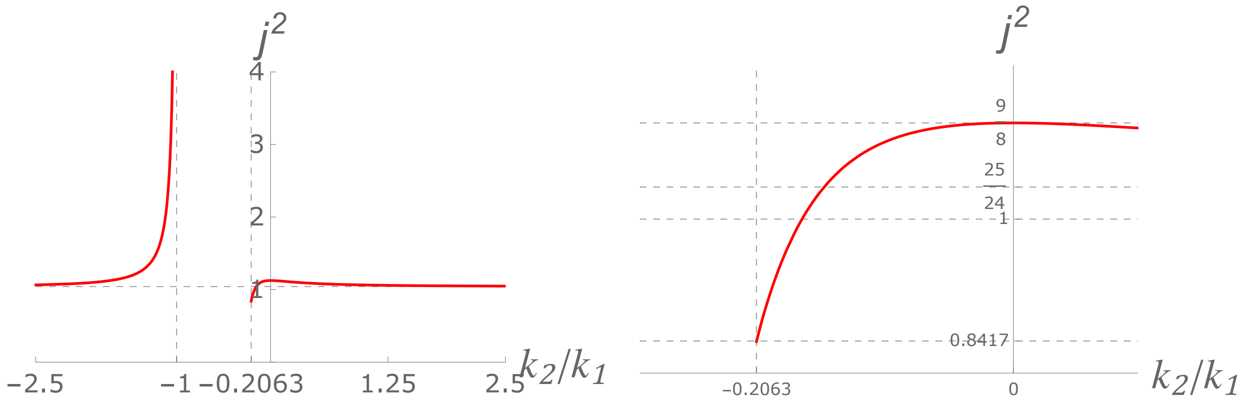

FIG. 6: The range of j^2 for the asymptotically flat, stationary, bi-axisymemtric and reflection-symmemtic microstate geometry with five centers ($n = 5$). The left figure shows the plots of j^2, and the right figure the close-up region of $-0.206.... < k_2/k_1 < 0$ in the left figure.

V. SUMMARY AND DISCUSSIONS

In this paper, we have analyzed the solutions of the asymptotically flat, stationary, BPS microstate geometries with bi-axisymmetry in the five-dimensional minimal supergravity. Moreover, we have imposed additional reflection symmetry since this symmetry assumption extremely simplifies the expression of the solutions and enables us to solve the bubble equations. We have also computed the conserved charges, the ADM mass, two ADM angular momenta, and $(n-1)$ magnetic fluxes through the bubbles between two centers. In particular, we have compared the mass and angular momenta for the three-center solution and the five-center solution of microstate geometries with those of the BMPV black hole. We have shown that the dimensionless angular momentum of the five-center microstate geometry does not have the upper bound but has the lower bound which is smaller than the angular momentum for the maximally spinning BMPV black hole, and hence there are the parameter region such that the microstate geometry has the same angular momentum as the BMPV black hole.

In our present analysis, we have restricted ourselves to the reflection-symmetric microstate geometries for $n = 3$ and $n = 5$, but it is not trivial whether there exist the reflection-symmetric solutions with a larger number of centers ($n = 7, 9, \ldots$) which admit the same mass and angular momentum as the BMPV black hole or the microstate geometries for $n = 3, 5$. The bi-axisymmetric and reflection-symmetric microstate geometry with n centers seems to have $(n+3)/2$ independent physical charges or fluxes [the mass M, the angular momentum J_ψ or the $(n-1)/2$ magnetic fluxes $q[I_i]$ $(i = 1, \ldots, (n-1)/2)$], among which only $(n-1)/2$ are independent since the number of the parameters reduces to half due to reflection symmetry. The analysis for such microstate geometries with $n \geq 7$ deserves future works. Moreover, it may be interesting to compare the five-center solution dealt with in this paper with the spherical black holes having a topologically nontrivial domain of outer communication in Refs. [51, 52], which can have not only same asymptotic charges as the BMPV black hole but also different ones. The solution without the reflection symmetry should be compared with the supersymmetric black ring [27] and supersymmetric black lenses [29, 31, 32] which does not admit the limit to equal angular momenta. This may be an interest issue as our future study. Finally, we comment that the solutions of the five-dimensional minimal supergravity can be uplifted to the solutions of both type IIB supergravity and eleven supergravity [53, 54], and as discussed in Ref. [55], such solutions are relevant for the most general four-dimensional superconformal field theories (SCFTs) with holographic duals. This enables one to study some aspects of the dual strongly coupled thermal plasma with a non-zero R-charge chemical potential. Therefore, it might be physically interesting to study the fluid-dynamics of the thermal plasma of the SCFTs corresponding to the microstate geometries.

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Poster session

Shinpei Tonosaki

Hirosaki University

"A master hydrostatic equation for Newtonian stars in generic higher-curvature gravity"

[JGRG30 (2021) PA17]

A master hydrostatic equation for Newtonian stars in generic higher-curvature gravity

JGRG30 at Waseda (online), Dec. 6-10, 2021

Shinpei Tonosaki (Hirosaki University)

with Tomoya Tachinami and Yuuiti Sendouda

HIROSAKI UNIVERSITY

Abstract: We derive a sixth-order master equation for spherically symmetric hydrostatic configurations in the Newtonian limit of higher-curvature gravity that admits generic non-linear curvature corrections to the Lagrangian. We also demonstrate how the solutions to the master equation look like assuming constant density.

1. Introduction

Generic higher-curvature gravity

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(\text{Riemann}) + S_{\text{matter}}$$

$$= \frac{1}{2\kappa}\int d^4x\sqrt{-g}(R - \alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \beta R^2 + \cdots) + S_{\text{matter}} \quad (1)$$

R : Ricci scalar, $C_{\mu\nu\rho\sigma}$: Weyl tensor,

$\kappa = 8\pi G$: gravitational constant,

α and β : positive constants $[M^{-2}]$

- 8 gravitational dofs (2 tensor (massless) + 2 tensor, 2 vector, 2 scalar (all massive)) on flat background [1]

Questions to answer

- Is this theory astrophysically viable? What roles do the extra dofs play in stellar structure?

Strategy

- Derive and solve a **Lane–Emden-like master equation** for hydrostatic equilibrium in the Newtonian limit

2. Scalar perturbations

Variables

Consider metric perturbations about Minkowski background sourced by minimally coupled stress-energy tensor $T_{\mu\nu}$. Scalar variables to use are (in the Newtonian gauge)

$$g^{(S)}_{\mu\nu}dx^\mu dx^\nu = -(1+2\Psi)\,dt^2 + (1+2\Phi)\,\delta_{ij}\,dx^i dx^j\,,$$

$$T^{(S)}_{\mu\nu}\,dx^\mu dx^\nu = \rho\,dt^2 + 2\partial_i S\,dt dx^i + \left[P\delta_{ij} + \left(\partial_{ij} - \frac{1}{3}\delta_{ij}\Delta\right)\sigma\right]\,dx^i dx^j$$

with constraints $\dot\rho - \Delta S = 0$ and $\dot S - P - \frac{2}{3}\Delta\sigma = 0$.

4th-order equations for gravitational potentials

Hereafter, we will restrict ourselves to the **static spherically symmetric configuration** and **Newtonian** source with negligible pressure. The scalar linear perturbation equations from action (1) reduce to a set of *decoupled* 4th-order equations

$$(\Delta - m_2^2)\Delta Y = -\frac{\kappa}{2}m_2^2\rho\,,$$
$$(\Delta - m_0^2)\Delta Z = -\frac{\kappa}{2}m_0^2\rho\,, \quad (2)$$

where $Y \equiv \frac{1}{2}\Psi - \frac{1}{2}\Phi$, $Z \equiv -\Psi - 2\Phi$, $m_2^2 = \frac{1}{2\alpha}$ and $m_0^2 = \frac{1}{6\beta}$. By solving (2) for Y and Z , one can obtain Ψ and Φ .

Einsteinian modes and higher-curvature modes

The 4th-order equations (2) admit 4 independent solutions for Y and Z. By analogy with the findings in the study of gravitational waves [1], we can expect 2 of these are the massless solutions in GR (**Einstein modes**), while the remaining 2 are massive solutions arising genuinely from the higher-curvature corrections in (1) (**high-curvature modes**).

In fact, it is confirmed that the linear combination $Y = Y^E + Y^H$, where Y^E and Y^H satisfy the Einsteinian (Poisson) and higher-curvature (Helmholtz) equations

$$\Delta Y^E = \frac{\kappa}{2}\rho\,, \quad (\Delta - m_2^2)Y^H = -\frac{\kappa}{2}\rho\,,$$

respectively, satisfies the Y equation in (2). Likewise, it is obvious that Z can be decomposed into $Z = Z^E + Z^H$.

The Einstein modes Y^E and Z^E are identical. Interestingly and importantly, the sum of these Einstein modes recovers the correct Newton potential $\Psi_N = \frac{4Y^E - Z^E}{3} = -\frac{GM}{r}$ (outside a star). (3)

3. Hydrostatic equilibrium

We then formulate a Lane–Emden-like equation that determines the stellar structure. The hydrostatic equilibrium equation is

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{\mathrm{d}\Psi}{\mathrm{d}r}\,.$$

We adopt the **polytropic equations of state**, $P = K\rho^{1+\frac{1}{n}}$. Differentiating both sides with respect to r and using (3) for the Einstein modes, one obtains

$$(1+n)K\Delta\rho^{\frac{1}{n}} = -\frac{4}{3}m_2^2Y^H + \frac{1}{3}m_0^2Z^H\,.$$

Here, Y^H and Z^H involve formal integrals of the matter density ρ. Applying $(\Delta - m_0^2)(\Delta - m_2^2)$ on both sides, performing change of variables $\rho = \rho_c\theta^n(\xi)$ with $\xi \equiv \gamma r$ and $\gamma \equiv \sqrt{\frac{\kappa\rho_c^2}{2(n+1)P_c}}$, and re-defining $\mu_0 \equiv m_0/\gamma$ and $\mu_2 \equiv m_2/\gamma$, we obtain

$$\left(\Delta - \mu_0^2\right)\left(\Delta - \mu_2^2\right)\Delta\theta - \frac{4\mu_2^2 - \mu_0^2}{3}\Delta\theta^n + \mu_0^2\mu_2^2\theta^n = 0\,. \quad (4)$$

This is our **master equation for a Newtonian star in hydrostatic equilibrium** in the generic higher-curvature gravity.

4. Solving the master equation

As a preliminary analysis, we show the solutions for $n = 0$.

We have noticed there are uncertainties in the boundary conditions at the stellar center. The boundary condition here is

$$\theta(0) = 1\,, \frac{d^k\theta}{d\xi^k}(0) = 0\,(1 \le k \le 5)\,.$$

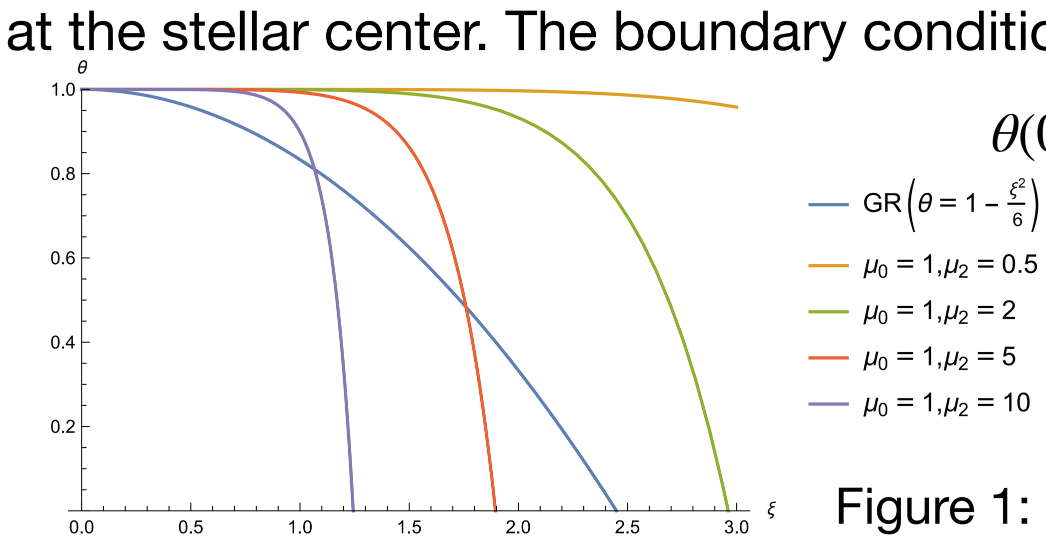


Figure 1: Solutions for $n = 0$.

5. Conclusion

- A hydrostatic equilibrium equation (4) has been formulated as a 6th-order differential equation using the same variables as in the Lane–Emden equation in GR.

Future work

- Imposing boundary conditions at the center of the star seems more complicated than GR due to the higher-derivative nature of the equation (4). It cannot be found by just requiring the first derivative of the density to vanish.
- Extension to more realistic equations of state.

References

[1] T. Tachinami, S. Tonosaki and Y. Sendouda, Phys. Rev. D 103, 104037 (2021) [arXiv:2102.05540 [gr-qc]]

Poster session

Tsutomu Kobayashi

Rikkyo University

“Black holes in spatially covariant gravity with two tensorial degrees of freedom”

[JGRG30 (2021) PA18]

JGRG30@Waseda 2021/12

Black holes in spatially covariant gravity with two tensorial degrees of freedom

Tsutomu Kobayashi

Rikkyo University

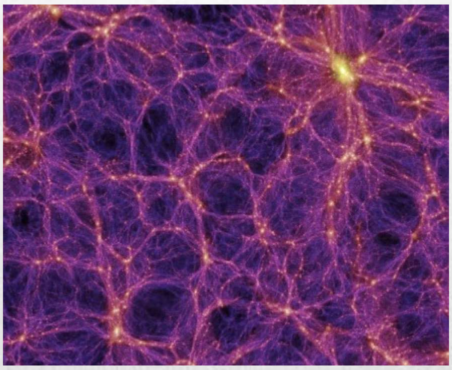
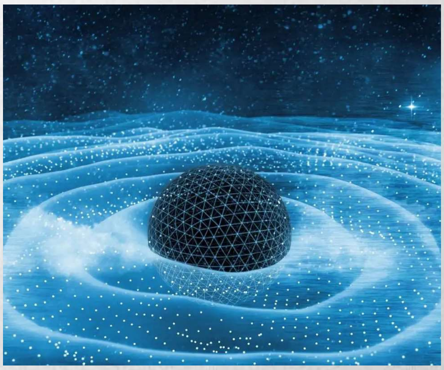
Based on work with **Aya Iyonaga** (Rikkyo)

2109.10615, Phys. Rev. D, accepted

Extreme Universe
A New Paradigm for Spacetime and Matter from Quantum Information
Grant-in-Aid for Transformative Research Areas (A)

Modified gravity: Why?

- Playing with **modified gravity**, one can…
  - tackle the mystery of dark energy.
  - test gravity with cosmological/astrophysical/gravitational-wave observations.
  - deep understand the nature of gravity.

Modified gravity: How?

- Lovelock theorem Lovelock (1971, 1972)
  - ☑ Second-order EOMs
  - ☑ Diff. invariance
  - ☑ Metric tensor alone
  - ☑ 4D

⇨ $G_{\mu\nu} + \Lambda g_{\mu\nu} =$

2 tensorial DOFs
(gravitational waves)

- To modify gravity, relax one of the postulates:
  - ☐ Higher-order EOMs? ⟶ Ostrogradsky instability
  - ☐ Less invariance than full diff.?
  - ☐ Metric tensor + extra DOF(s)?
  - ☐ Higher D?

Contents

- Modified gravity with
  - ☑ spatial diff. (no time diff.)
  - ☑ 2 tensorial DOFs (no scalar mode)
- *Can we distinguish such a theory from GR through observations?*
- Contents
  - ☐ Modified gravity with two tensorial DOFs
  - ☐ Solar-system tests, speed of gravitational waves
  - ☐ Black holes

For cosmology, see Aya Iyonaga's talk!
Thursday morning, Parallel D1(a) ⟶

Gravity with less invariance

- Spatially covariant theory:

  cf. EFT of inflation, Horava gravity

  $$x^i \to \tilde{x}^i(t, x^j), \qquad \text{no time diff.}$$

  Cheung *et al.* (2008), Horava (2009)

- Action in terms of ADM variables:

  3D Ricci　3D covariant D

  $$S = \int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{\gamma}\mathcal{L}(N, \gamma_{ij}, K_{ij}, R_{ij}, D_i; t)$$

  Gao (2014)

  Lapse function　3D metric　Extrinsic curvature　Explicit *t* dependence

- In general, this is a scalar-tensor theory (with full 4D diff.)

4D covariant expression $\mathcal{L}(g_{\mu\nu}, \phi)$

Unitary gauge: $\phi = \phi(t)$ $\longrightarrow$

$\longleftarrow$ Stuckelberg trick: $n_\mu \propto \nabla_\mu \phi$

ADM expression $\mathcal{L}(N, \gamma_{ij}, \cdots)$

"Scalarless" scalar-tensor theories

- So, basically,
  spatially covariant gravity = gravity with a scalar DOF.

- But, it is possible to construct spatially covariant gravity with just 2 tensorial DOFs and no propagating scalar DOF.

  - Example: *cuscuton*　Afshordi *et al.* (2007)

ADM expression

$$\boxed{\mathcal{L} = \frac{1}{2}\left(K_{ij}K^{ij} - K^2 + R\right) + c_1(t) + \frac{c_2(t)}{N}}$$

Unitary gauge　Stuckelberg trick

4D covariant expression

$$\boxed{\mathcal{L} = \frac{1}{2}\mathcal{R}^{(4)} - V(\phi) + \mu(\phi)\sqrt{-(\partial\phi)^2}}$$

Stuckelberg scalar ϕ does not propagate.

Spatially covariant gravity with two tensorial DOFs

- Gao & Yao derived the necessary and sufficient conditions for

$$S = \int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{\gamma}\mathcal{L}(N, \gamma_{ij}, K_{ij}, R_{ij}, D_i; t)$$

to propagate no scalar mode through a Hamiltonian analysis.

- For $\mathcal{L} = (1/2)[b_1(t,N)K^2 + b_2(t,N)K_{ij}K^{ij} + d_1(t,N) + d_2(t,N)R]$, the conditions fix the coefficients as

$$b_1 = -\frac{\beta_0(t)}{3}\left[\frac{2N}{\beta_1(t)+N} + \frac{N}{\beta_2(t)+N}\right], \quad b_2 = \frac{\beta_0(t)N}{\beta_2(t)+N},$$

$$d_1 = \alpha_1(t) + \frac{\alpha_3(t)}{N}, \quad d_2 = \alpha_2(t) + \frac{\alpha_4(t)}{N}.$$

Gao & Yao (2020)

  - ~ The kinetic matrix for $g_{\mu\nu}$ and ϕ is degenerate.
  - This includes the *cuscuton* theory as a specific case.

From ADM to 4D covariant expression

$$\mathcal{L} = \frac{1}{2}\left[\left(\frac{\beta_0 N}{\beta_2 + N}\right)K_{ij}K^{ij} - \frac{\beta_0}{3}\left(\frac{2N}{\beta_1 + N} + \frac{N}{\beta_2 + N}\right)K^2 + \alpha_1 + \alpha_2 R + \frac{1}{N}(\alpha_3 + \alpha_4 R)\right]$$

Stuckelberg trick ↓ 🙄 ADM expression is simpler… ↑ Unitary gauge

$$\mathcal{L} = \frac{\tilde{\alpha}_1}{2} - \tilde{\alpha}_2''(2X - X\ln X) + \frac{\tilde{\alpha}_3}{2}\sqrt{2X} + \tilde{\alpha}_4''(2X)^{3/2} - \left(\frac{\tilde{\alpha}_2'}{2}\ln X + 2\tilde{\alpha}_4'\sqrt{2X}\right)\Box\phi$$
$$+ \tilde{f}\mathcal{R}^{(4)} + A_1\phi_{\mu\nu}\phi^{\mu\nu} + A_2(\Box\phi)^2 + A_3\Box\phi\phi^\mu\phi_{\mu\nu}\phi^\nu + A_4\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu + A_5(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2,$$

where $X := -(\partial\phi)^2/2$,

$$A_1 = \frac{\tilde{B}}{4X} - \frac{\tilde{f}}{2X}, \quad A_2 = -A_1 + 4X^2A_5, \quad A_3 = -A_4 + 4XA_5, \quad A_4 = \frac{\tilde{f}_{,X} + A_1}{X}, \quad A_5 = \frac{\tilde{C}}{8X^3},$$

$$\tilde{f} = \frac{1}{2}\left[\tilde{\alpha}_2(\phi) + \tilde{\alpha}_4(\phi)\sqrt{2X}\right], \quad \tilde{B} = \frac{\tilde{\beta}_0(\phi)}{\tilde{\beta}_2(\phi)\sqrt{2X}+1}, \quad \tilde{C} = \frac{\tilde{\beta}_0(\phi)}{3}\cdot\frac{[\tilde{\beta}_1(\phi) - \tilde{\beta}_2(\phi)]\sqrt{2X}}{[\tilde{\beta}_1(\phi)\sqrt{2X}+1][\tilde{\beta}_2(\phi)\sqrt{2X}+1]}.$$

Particular case of *U-DHOST* theories De Felice *et al.* (2018)

Solar-system tests & $c_{\rm GW}$

$$\mathcal{L}=\frac{1}{2}\left[\left(\frac{\beta_0 N}{\beta_2+N}\right)K_{ij}K^{ij}-\frac{\beta_0}{3}\left(\frac{2N}{\beta_1+N}+\frac{N}{\beta_2+N}\right)K^2+\alpha_1+\alpha_2 R+\frac{1}{N}\left(\alpha_3+\alpha_4 R\right)\right]$$

Solar-system constraints and the limit on the speed of GWs are so stringent that we focus on the subset of the theory evading them trivially…

7 free functions of *t* $\longrightarrow$ **3**

Solar-system tests & $c_{\rm GW}$

These terms modify weak gravitational fields and GW propagation.

$$\mathcal{L}=\frac{1}{2}\left[\left(\frac{\beta_0 N}{\beta_2+N}\right)K_{ij}K^{ij}-\frac{\beta_0}{3}\left(\frac{2N}{\beta_1+N}+\frac{N}{\beta_2+N}\right)K^2+\alpha_1+\alpha_2 R+\frac{1}{N}\left(\alpha_3+\alpha_4 R\right)\right]$$

$\longrightarrow \boxed{\beta_0=1,\ \beta_2=0}$ $\qquad \longrightarrow \boxed{\alpha_2=1}$ $\qquad \longrightarrow \boxed{\alpha_4=0}$

- Solar-system tests

$$N=1+\Phi(\vec{x}),\quad N_i=\partial_i\chi(\vec{x}),\quad \gamma_{ij}=[1-2\Psi(\vec{x})]\delta_{ij},\quad \mathcal{L}_{\rm m}=-\Phi\rho(\vec{x})$$

$$\longrightarrow\quad \Delta\Phi=\Delta\Psi=\frac{\rho}{2},\quad \Delta\chi=0$$

(Other PPN parameters?)

- Gravitational waves

$$\gamma_{ij}=\delta_{ij}+h_{ij}(t,\vec{x})\quad\longrightarrow\quad c_{\rm GW}=1$$

(Units: $8\pi G=1,\ c=1$)

Phenomenologically interesting subclass

$$\mathcal{L} = \frac{1}{2}\left[K_{ij}K^{ij} - \frac{1}{3}\left(\frac{2N}{\beta + N} + 1\right)K^2 + R + \underbrace{\alpha_1 + \frac{\alpha_3}{N}}_{\text{Cuscuton terms}}\right]$$

$\downarrow$

New!

- 3 functions of *t*.
- The same predictions as in GR for weak fields and GW propagation.
- *Black holes?*
- *Cosmology?*

Black holes

$$\mathcal{L} = \frac{1}{2}\left[K_{ij}K^{ij} - \frac{1}{3}\left(\frac{2N}{\beta + N} + 1\right)K^2 + R + \underbrace{\alpha_1 + \frac{\alpha_3}{N}}_{\sim H^2}\right]$$

Assumptions: $\dot{\beta} \sim H\beta$

$$\longrightarrow \qquad \beta = \text{const.}, \quad \alpha_1 = \alpha_3 = 0.$$

- Look for static, spherically symmetric, asymptotically flat solutions

$$N = N(r), \quad N_i \mathrm{d}x^i = B(r)F(r)\mathrm{d}r, \quad \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = F^2(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

The shift vector cannot be removed (no time diff.!)

- Field equations:

$$N' = \cdots, \quad F' = \cdots, \quad \beta B'' = \cdots.$$

$$\alpha_1 + \frac{2\alpha_2}{r^2}\left[\left(\frac{F^2-1}{F^2}\right)r\right]' + \frac{2(rB^2)'}{r^2(\beta+N)^2F^2} - \frac{2r^2}{3}\frac{\beta(\beta+2N)}{N^2(\beta+N)^2F^2}\left[(B/r)'\right]^2 = 0,$$

$$\frac{r^2B}{(\beta+N)^2F^2}\left[(\beta+N)F\right]' + \frac{\beta}{3}\left[\frac{r^4(B/r)'}{N(\beta+N)F}\right]' = 0,$$

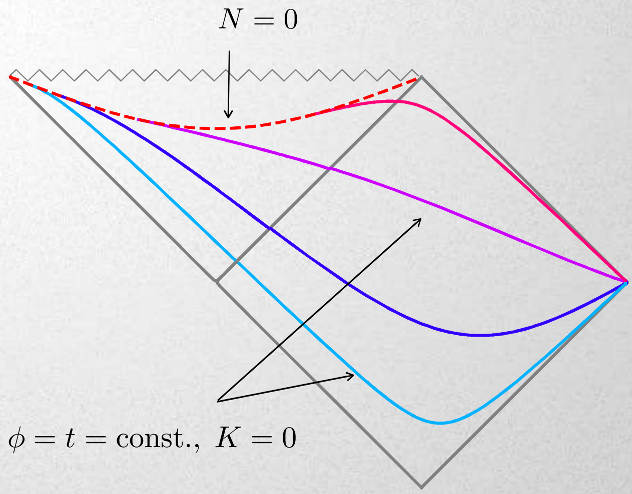
$$\alpha_1\frac{N}{F} + \frac{2\alpha_2}{r^2}\frac{N}{F}\left(\frac{F^2-1}{F^2} - \frac{2rN'}{NF^2}\right) + \frac{2(rB^2)'}{r^2NF^3} - \frac{2\beta}{3r^4N(\beta+N)F^3}\left[(r^2B)'\right]^2 = 0.$$

Schwarzschild solution

Schwarzschild foliated with maximal slices ($K = 0$) *is* a solution:

$$N = N_0\sqrt{f(r)}, \quad F = \frac{1}{\sqrt{f(r)}}, \quad B = \frac{N_0 b_0}{r^2},$$

where $\quad f(r) := 1 - \frac{\mu_0}{r} + \frac{b_0^2}{r^4}. \qquad N_0, b_0, \mu_0 :$ integration constants



✓ β does not appear.

✓ The same solution found in Einstein-aether/IR limit of Horava gravity.

Barausse *et al.* (2011)
Blas & Sibiryakov (2011)

Schwarzschild solution

Schwarzschild foliated with maximal slices ($K = 0$) *is* a solution:

$$N = N_0\sqrt{f(r)}, \quad F = \frac{1}{\sqrt{f(r)}}, \quad B = \frac{N_0 b_0}{r^2},$$

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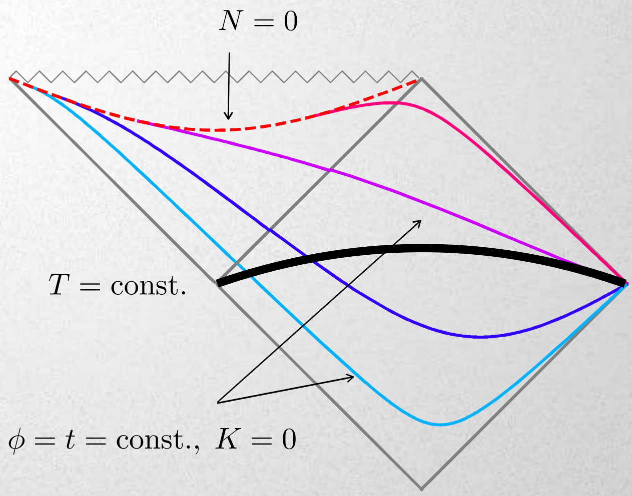


<u>In the non-unitary gauge…</u>

$$ds^2 = -\left(1 - \frac{\mu_0}{r}\right) dT^2 + \frac{dr^2}{1 - \mu_0/r} + r^2 d\Omega^2,$$

$$\phi(T, r) = \frac{1}{N_0}\left(T + \int^r \frac{b_0/r^2}{(1 - \mu_0/r)\sqrt{f(r)}} dr\right).$$

Static perturbations

Unique solution? May have another integration constant?

Static and spherically symmetric perturbations:

$$N = N_0\sqrt{f + h_0(r)}, \quad F = \frac{1}{\sqrt{f + h_1(r)}}, \quad B = N_0\left[\frac{b_0}{r^2} + h_2(r)\right].$$

Solution:

$$h_0 = -\frac{\mu_1}{r} + C_1\left(f - \frac{b_0^2}{r^4}\right) + \frac{2b_0h_2}{r^2},$$

$$h_1 = -\frac{\mu_1}{r} - \frac{C_1 b_0^2}{r^4} + \frac{2b_0h_2}{r^2},$$

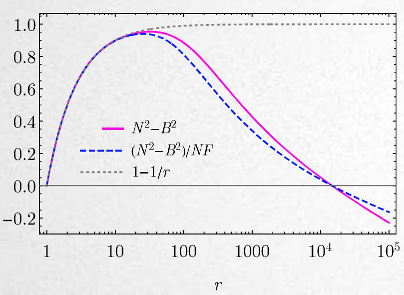
$$h_2 = \frac{b_1}{r^2} + \frac{\epsilon_1}{r^2}\int^r r^2\left(\beta + N_0\sqrt{f}\right)\mathrm{d}r. \quad \boxed{\to \infty \quad \text{for} \quad r \to \infty} \longrightarrow \epsilon_1 = 0$$

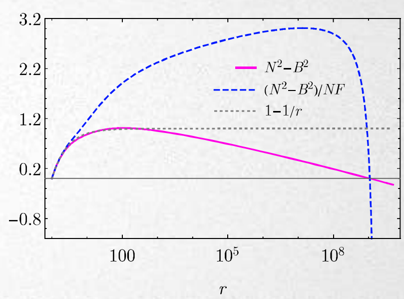
No asymptotically flat solution can be obtained by a small deformation of Schwarzschild.

(μ_1, C_1, b_1) can be absorbed into redefinitions of (μ_0, N_0, b_0).

Some remarks

- Numerical solutions:
  - Field equations are solved numerically from the horizon outward.
  - No solution other than Schwarzschild can be found.





- Slowly rotating solutions:
  - To first order in black hole spin, the geometry is identical to Kerr.

- Black hole perturbations and quasi-normal modes:
  - work in progress with **Jin Saito** (M1 student@Rikkyo).

Poster session

Chulmoon Yoo

Nagoya Univ.

"Simulation of PBH formation from iso-curvature perturbations"

[JGRG30 (2021) PA19]

P19

Simulation of PBH formation from iso-curvature perturbations

Chulmoon Yoo

With Tomohiro Harada, Shin'ichi Hirano, Hirotada Okawa and Misao Sasaki

Introduction:Primordial BHs [Zeldovich and Novikov(1967), Hawking(1971), Carr and Hawking(1974)]

◎Remnant of primordial non-linear inhomogeneity

◎Trace the inhomogeneity in the early universe

◎May provide a fraction of dark matter and BH binaries

◎Several aspects

- Inflationary models which provide a number of PBHs
- Theoretical estimation and observational constraints on PBH abundance
- Threshold of PBH formation
- Mass and spin distribution of PBHs

◎Standard scenario: Primordial curvature perturbation → PBH

What about iso-curvature perturbation?

First example of PBH simulation from an iso-curvature initial condition

◎Setting

- Spherically symmetric
- Full GR Geometry + fluid with $p=\rho/3$ + massless scalar field
- Asymptotically FLRW
- iso-curvature:
  the metric form is identical to the background one at the lowest order of the gradient expansion

◎Simple questions

- How to construct appropriate iso-curvature initial data?
- Is it possible to produce a PBH from iso-curvature of a massless scalar field?
- If possible,
  How large amplitude?
  What about critical scaling?
  What about the accretion after the formation?

Cosmological long-wavelength solutions of the iso-curvature mode

Cosmological 3+1 decomposition-1

◎Metric

$$ds^2 = -\alpha^2 dt^2 + a^2\psi^4\tilde{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

◎Extrinsic curvature

$$K_{ij} = \psi^4 a^2 (\tilde{A}_{ij} + \tfrac{1}{3}\tilde{\gamma}_{ij}K)$$

◎Constraint equations

- Hamiltonian constraint

$$\tilde{\Delta}\psi = \frac{\tilde{\mathcal{R}}^k_k}{8}\psi - 2\pi\psi^5 a^2 E - \frac{\psi^5 a^2}{8}\left(\tilde{A}_{ij}\tilde{A}^{ij} - \tfrac{2}{3}K^2\right) \qquad E := n_\mu n_\nu T^{\mu\nu}$$

- Momentum constraint

$$\tilde{\mathcal{D}}^j(\psi^6\tilde{A}_{ij}) - \tfrac{2}{3}\psi^6\tilde{\mathcal{D}}_i K = 8\pi J_i\psi^6 \qquad J_i := -\gamma_{i\mu}n_\nu T^{\mu\nu}$$

$\tilde{\mathcal{D}}_i$: covariant derivative w. r. t. $\tilde{\gamma}_{ij}$

$\tilde{\mathcal{R}}_{ij}$: Ricci tensor w. r. t. $\tilde{\gamma}_{ij}$

Cosmological 3+1 decomposition-2

◎Evolution equations for geometrical variables

$$(\partial_t - \mathcal{L}_\beta)\psi = -\frac{\dot{a}}{2a}\psi + \frac{\psi}{6}(-\alpha K + \mathcal{D}_k\beta^k),$$

$$(\partial_t - \mathcal{L}_\beta)\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} - \tfrac{2}{3}\tilde{\gamma}_{ij}\mathcal{D}_k\beta^k$$

$$(\partial_t - \mathcal{L}_\beta)K = \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \tfrac{1}{3}K^2\right) - D_k D^k\alpha + 4\pi\alpha(E + S^k_k)$$

$$(\partial_t - \mathcal{L}_\beta)\tilde{A}_{ij} = \frac{1}{a^2\psi^4}\left[\alpha\left(\mathcal{R}_{ij} - \frac{\gamma_{ij}}{3}\mathcal{R}\right) - \left(D_i D_j\alpha - \frac{\gamma_{ij}}{3}D_k D^k\alpha\right)\right]$$

$$+\alpha(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k_j) - \tfrac{2}{3}(\mathcal{D}_k\beta^k)\tilde{A}_{ij} - \frac{8\pi\alpha}{a^2\psi^4}\left(S_{ij} - \frac{\gamma_{ij}}{3}S^k_k\right) \qquad S_{ij} := \gamma_{i\mu}\gamma_{j\nu}T^{\mu\nu}$$

D_i : covariant derivative w. r. t. γ_{ij}

$\mathcal{D}_i$: covariant derivative w. r. t. the flat metric f_{ij}

$\mathcal{R}_{ij}$: Ricci tensor w. r. t. γ_{ij}

Cosmological 3+1 decomposition-3

◎Fluid stress-energy tensor

$$T^{\mathrm{f}}_{\mu\nu} = T_{\mu\nu} - T^{sc}_{\mu\nu} = (\rho+p)u_\mu u_\nu + p g_{\mu\nu}$$

◎Fluid EoM

$$\begin{aligned}&\left[\psi^6 a^3 \left\{(\rho+p)\Gamma^2 - p\right\}\right]_{,t} + \frac{1}{\sqrt{\eta}}\left[\sqrt{\eta}\psi^6 a^3 \left\{(\rho+p)\Gamma^2 - p\right\} v^l\right]_{,l} \\ &\quad = -\frac{1}{\sqrt{\eta}}\left[\sqrt{\eta}\psi^6 a^3 p(v^l+\beta^l)\right]_{,l} + \alpha\psi^6 a^3 pK - \alpha^{-1}\alpha_{,l}\psi^6 a^3\Gamma^2(\rho+p)(v^l+\beta^l) \\ &\quad + \alpha^{-1}\psi^{10}a^5\Gamma^2(\rho+p)(v^l+\beta^l)(v^m+\beta^m)\left(\tilde{A}_{lm} + \frac{\tilde{\gamma}_{lm}}{3}K\right)\end{aligned}$$

$$\begin{aligned}&(\Gamma\psi^6 a^3(\rho+p)u_j)_{,t} + \frac{1}{\sqrt{\eta}}(\sqrt{\eta}\Gamma\psi^6 a^3(\rho+p)v^k u_j)_{,k} \\ &\quad = -\alpha\psi^6 a^3 p_{,j} + \Gamma\psi^6 a^3(\rho+p)\left(-\Gamma\alpha_{,j} + u_k\beta^k_{,j} - \frac{u_k u_l}{2u^t}\gamma^{kl}_{,j}\right)\end{aligned}$$

$$(\Gamma\psi^6 a^3 n)_{,t} + \frac{1}{\sqrt{\eta}}(\sqrt{\eta}\Gamma\psi^6 a^3 n v^k)_{,k} = 0$$

Cosmological 3+1 decomposition-4

◎Scalar field EoM

$$(\partial_t - \beta^i\partial_i)\phi = -\alpha\Pi$$

$$(\partial_t - \beta^i\partial_i)\Pi = -\alpha\triangle\phi - \gamma^{\mu\nu}\partial_\mu\alpha\partial_\nu\phi + \alpha K\Pi$$

◎Stress-energy tensor of the massless scalar field

$$E^{\mathrm{sc}} = n^\mu n^\nu T^{\mathrm{sc}}_{\mu\nu} = \frac{1}{2}\Pi^2 + \frac{1}{2}\psi^{-4}a^{-2}\tilde{\gamma}^{ij}\partial_i\phi\partial_j\phi,$$

$$J^{\mathrm{sc}}_i = -\gamma_i{}^\mu n^\nu T^{\mathrm{sc}}_{\mu\nu} = \Pi\partial_i\phi,$$

$$S^{\mathrm{sc}}_{ij} = \gamma_i{}^\mu\gamma_j{}^\nu T^{\mathrm{sc}}_{\mu\nu} = \partial_i\phi\partial_j\phi - \frac{1}{2}\tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_k\phi\partial_l\phi + \frac{1}{2}\psi^4 a^2\tilde{\gamma}_{ij}\Pi^2$$

Gradient expansion and solutions of the iso-curvature mode

Gradient expansion

◎Expansion parameter $\varepsilon \sim k/(aH_b)$

◎Assumption: the scalar field does not contribute to the background

$$\Rightarrow \phi = \Phi(\vec{x}) + \lambda(t,\vec{x}) \;\; \text{with } \Phi(\vec{x}) = \mathcal{O}(1) \text{ and } \lambda = \mathcal{O}(\epsilon)$$

◎Scalar field equation and the solution

$$\partial_t \lambda = -\Pi + \mathcal{O}(\epsilon^2)$$
$$\partial_t \Pi = K_{\rm b}\Pi + \mathcal{O}(\epsilon^2)$$

$$\Rightarrow \lambda = \underbrace{\lambda_1(\vec{x})}_{\text{absorbed into O(1)}} + \underbrace{\lambda_2(\vec{x}) \int a^{-3} dt}_{\text{decaying mode}} + \mathcal{O}(\epsilon^2)$$

$$\Rightarrow \lambda = \mathcal{O}(\epsilon^2)$$

All variables up to O(ε^2) and EoM

◎Expansion parameter $\varepsilon \sim k/(aH_\mathrm{b})$

=1 for iso-curvature perturbation

$$\psi = \Psi(\vec{x})(1+\xi(t,\vec{x})) + \mathcal{O}(\epsilon^3)$$
$$\tilde{\gamma}_{ij} = \eta_{ij} + h_{ij} + \mathcal{O}(\epsilon^3)$$
$$K = K_\mathrm{b}(1+\kappa) + \mathcal{O}(\epsilon^3)$$
$$\alpha = 1 + \chi + \mathcal{O}(\epsilon^3)$$
$$\beta^i = \mathcal{O}(\epsilon)$$
$$\rho = \rho_\mathrm{b}(1+\delta) + \mathcal{O}(\epsilon^3),$$
$$v^i = \mathcal{O}(\epsilon)$$

seed function for iso-curvature

$$\phi = \Phi(\vec{x}) + \lambda(t,\vec{x}) + \mathcal{O}(\epsilon^3),$$
$$\Pi = \mathcal{O}(\epsilon^2)$$

$$\dot{\partial}_t a + 6\dot{\xi} + 3H_\mathrm{b} w\chi = \mathcal{O}(\epsilon^4)$$
$$\frac{1}{1+w}\dot{\partial}_t a + 6\dot{\xi} = \mathcal{O}(\epsilon^4)$$
$$\partial_t[a^3(1+w)\rho_\mathrm{b} u_j] = -a^3\rho_\mathrm{b}[w\partial_j\partial_t a + (1+w)\partial_j\chi] + \mathcal{O}(\epsilon^5)$$
$$6\dot{\xi} - 3H_\mathrm{b}\chi = \mathcal{O}(\epsilon^4)$$
$$\rho_\mathrm{b}\partial_t a + \frac{1}{2}a^{-2}(\mathcal{D}\Phi)^2 = \mathcal{O}(\epsilon^4), 3(1+w)\chi + (1+3w)\partial_t a = \mathcal{O}(\epsilon^4)$$
$$\partial_t h_{ij} = -2\tilde{A}_{ij} + \mathcal{O}(\epsilon^4)$$
$$\partial_t \tilde{A}_{ij} + \frac{3\dot{a}}{a}\tilde{A}_{ij} = -\frac{8\pi}{a^2}\left(\mathcal{D}_i\Phi\mathcal{D}_j\Phi - \frac{1}{3}\eta_{ij}(\mathcal{D}\Phi)^2\right) + \mathcal{O}(\epsilon^4)$$
$$\bar{\mathcal{D}}_i(\Psi\tilde{A}^i_j) = 8\pi\Psi^6(1+w)\rho_\mathrm{b} u_j + 8\pi\Pi\mathcal{D}_i\Phi + \mathcal{O}(\epsilon^5)$$
$$\partial_t\lambda = -\Pi + \mathcal{O}(\epsilon^3)$$
$$\partial_t\Pi = -a^{-2}\triangle\Phi - 3H_\mathrm{b}\Pi$$

$$(\mathcal{D}\Phi)^2 = f^{ij}\mathcal{D}_i\Phi\mathcal{D}_j\Phi,\ \triangle = f^{ij}\mathcal{D}_i\mathcal{D}_j$$

Solutions of the iso-curvature up to O(ε^2)

◎Gauge: constant-mean-curvature(κ=0), normal coordinate(β^i=0)

◎Growing mode solutions

$$\delta = -\frac{4\pi}{3}\left(\frac{1}{aH_\mathrm{b}}\right)^2(\mathcal{D}\Phi)^2$$
$$\chi = -\frac{1+3w}{3(1+w)}\delta$$
$$\xi = \frac{2\pi}{9}\frac{1}{1+w}\left(\frac{1}{aH_\mathrm{b}}\right)^2(\mathcal{D}\Phi)^2$$
$$u_i = -\frac{8\pi}{9}\frac{a}{(1+w)(5+3w)}\left(\frac{1}{aH_\mathrm{b}}\right)^3\mathcal{D}_i(\mathcal{D}\Phi)^2$$
$$\tilde{A}_{ij} = -\frac{16\pi}{5+3w}\frac{1}{a}\frac{1}{aH_\mathrm{b}}\left[\mathcal{D}_i\Phi\mathcal{D}_j\Phi - \frac{1}{3}f_{ij}(\mathcal{D}\Phi)^2\right]$$
$$h_{ij} = \frac{32\pi}{(5+3w)(1+3w)}\left(\frac{1}{aH_\mathrm{b}}\right)^2\left[\mathcal{D}_i\Phi\mathcal{D}_j\Phi - \frac{1}{3}f_{ij}(\mathcal{D}\Phi)^2\right]$$
$$\Pi = -\frac{2}{5+3w}\frac{1}{a}\frac{1}{aH_\mathrm{b}}\triangle\Phi$$
$$\lambda = \frac{2}{(5+3w)(1+3w)}\left(\frac{1}{aH_\mathrm{b}}\right)^2\triangle\Phi$$

◎Seed function Φ
⇒corresponding growing mode sol.

◎Metric form is not identical to the flat one at the order ε^2 of the gradient expansion

Simulation of PBH formation from iso-curvature perturbation

1. initial configuration configuration and the numerical code

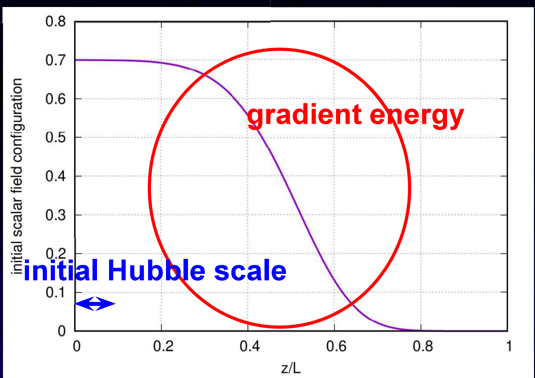
Functional form of Φ

◎Gaussian form of Φ

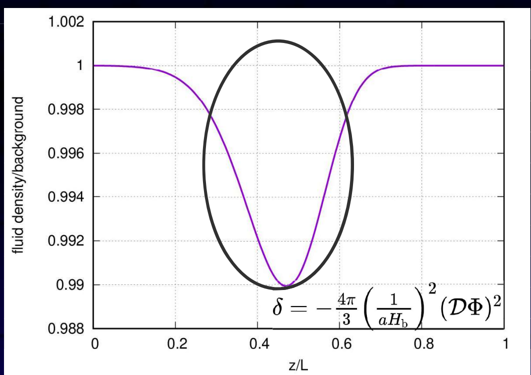
$$\Phi(R) \simeq \mu \exp\left(-\frac{1}{6}k^2 R^2\right)$$

$$E^{\rm sc} = \frac{1}{2}a^{-2}(\mathcal{D}\Phi)^2 + \mathcal{O}(\epsilon^4)$$

◎Scalar field



◎fluid density / background density



the scalar field gradient energy density is compensated by the fluid energy density

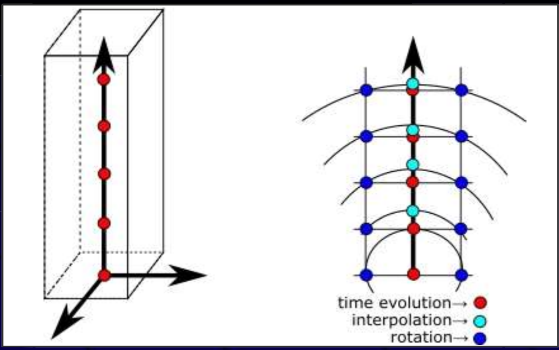
Numerical simulation of PBH formation

◎Numerical code

- Spherically symmetric
- Full GR Geometry + fluid with $p=w\rho$ (w=⅓ in the trial) + real scalar field
- Asymptotically FLRW with $p=w\rho$
- inhomogeneous grid spacing (finer resolution in the center)
- based on BSSN formalism with CARTOON method

◎What is CARTOON?

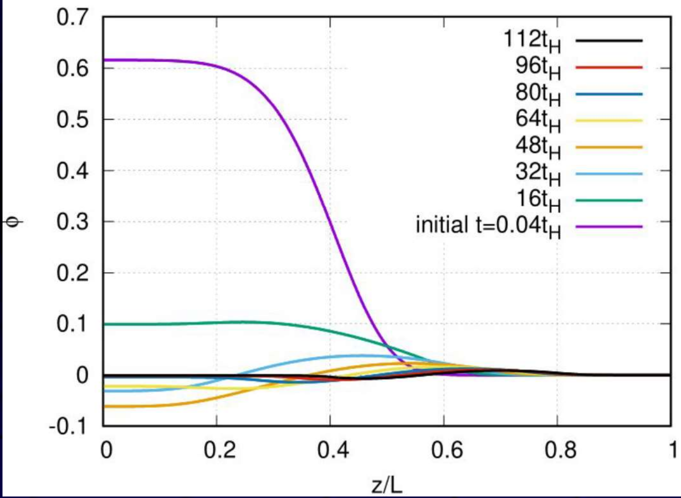
- We solve 3+1 dimensional equations on z-axis
  → We need values around z-axis
- No coord. sing. at the center (∵Cartesian coord)

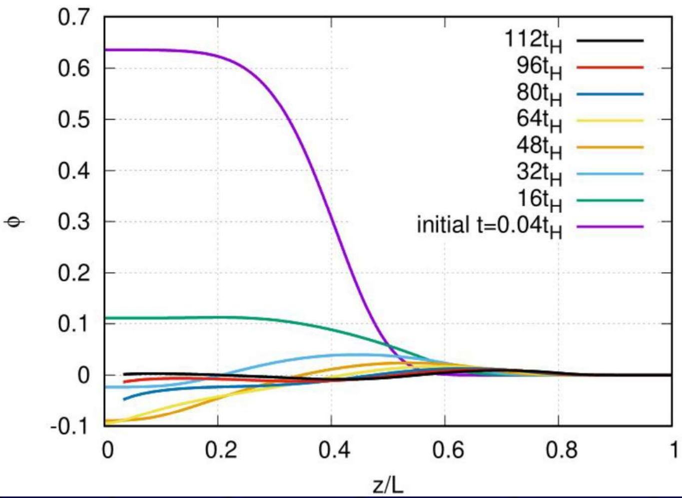


2. Time evolution of two typical cases

Scalar field behavior

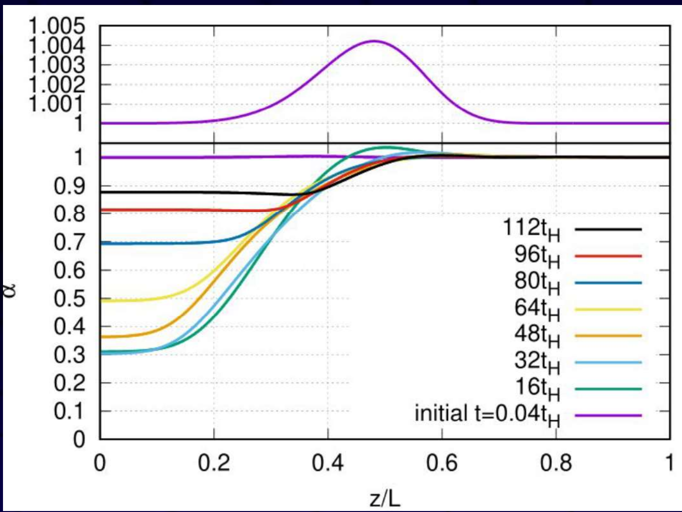
◎Initial amplitude μ=0.62: no PBH formation ◎Initial amplitude μ=0.64: PBH formation

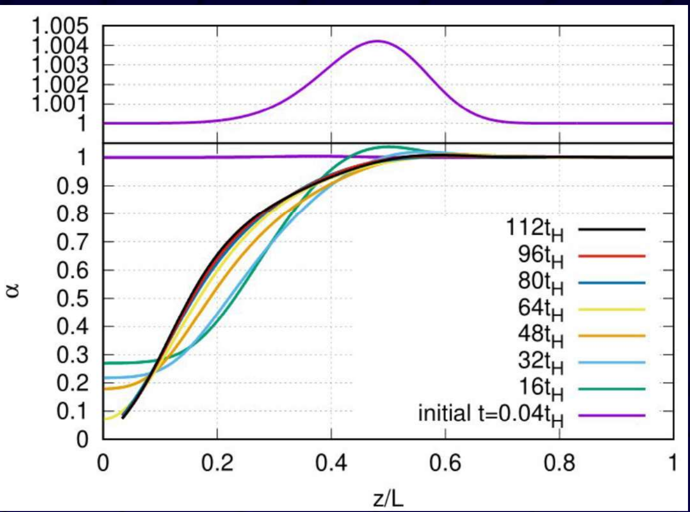




Lapse function

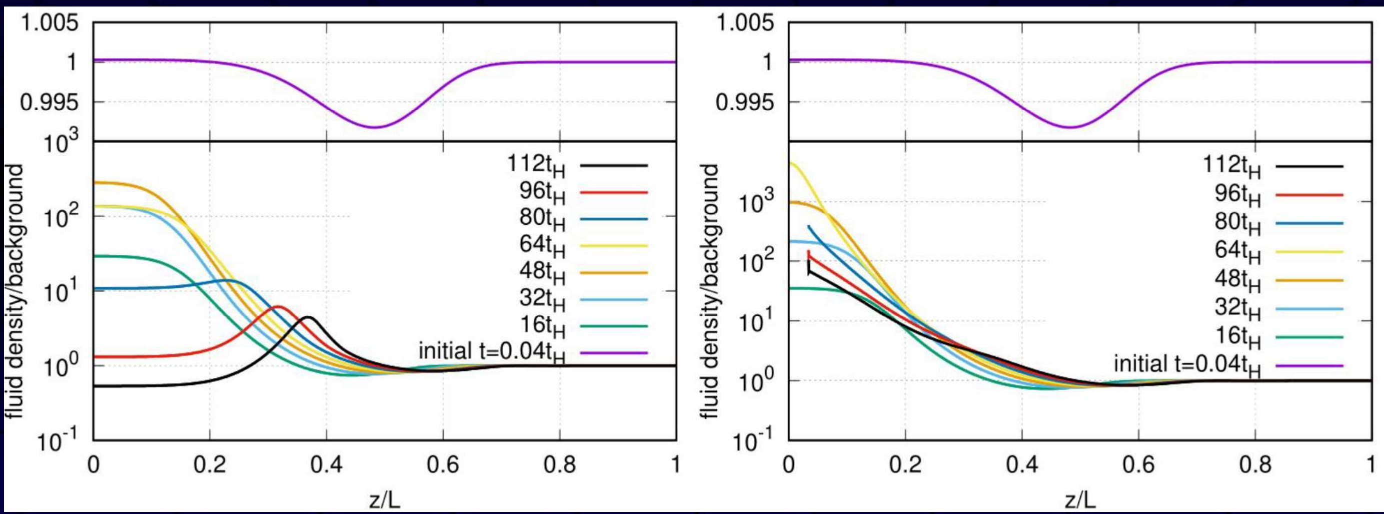
◎Initial amplitude μ=0.62: no PBH formation ◎Initial amplitude μ=0.64: PBH formation





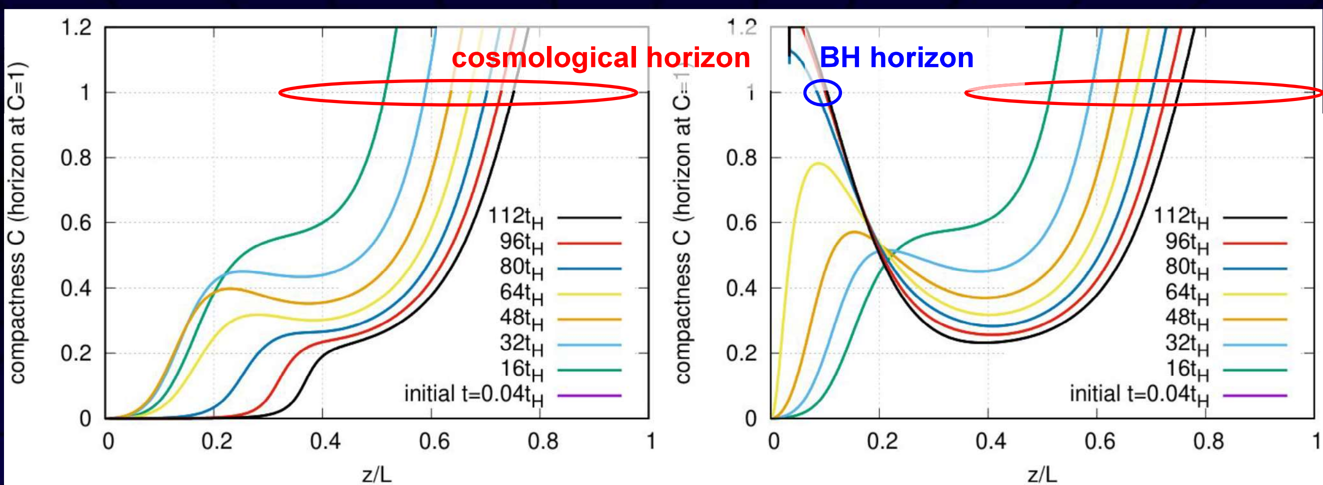
Fluid energy density

◎Initial amplitude μ=0.62: no PBH formation ◎Initial amplitude μ=0.64: PBH formation



Compactness(Misner-Sharp mass/2×Areal radius)

◎Initial amplitude μ=0.62: no PBH formation ◎Initial amplitude μ=0.64: PBH formation



3. Time evolution of the PBH mass

Fitting formula

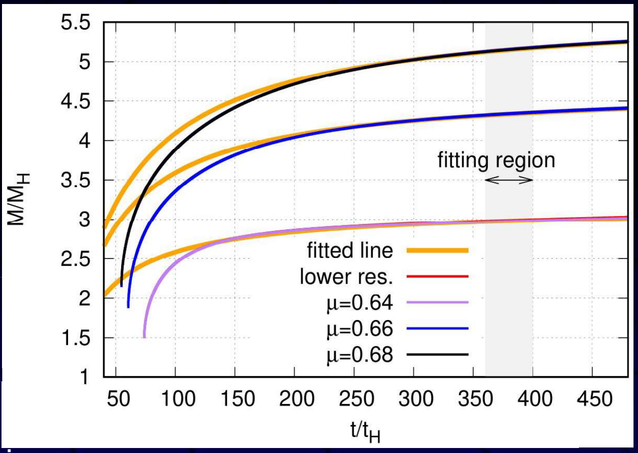
◎A simple model: the mass accretion rate is proportional to
the horizon area and the background energy density

$$\frac{dM}{dt} = 16\pi F M^2 \rho_{\rm b}$$

coefficient of O(1)

$$\Rightarrow M(t) = 1/\left(\frac{1}{M_{\rm f}} + \frac{3F}{2t}\right)$$

two fitting parameters

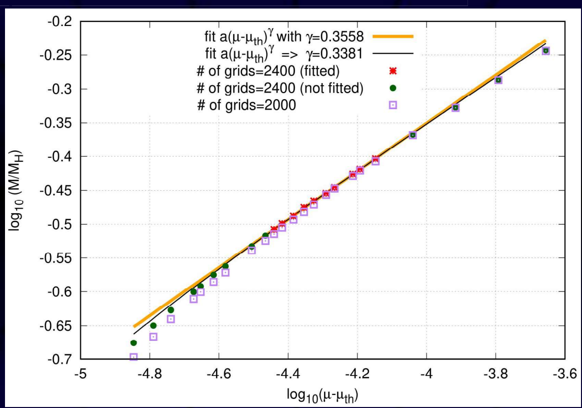


◎Well described by the model of
the time evolution in the radiation dominated universe

4. Behavior near the threshold

Mass scaling behavior

◎Critical behavior for the radiation fluid $M \propto (\mu - \mu_{\rm th})^{\gamma}$ with $\gamma \simeq 0.3558$



◎Unfortunately, the resolution of the simulation is not sufficient for finding the clear scaling.
But the result seems roughly consistent with the critical exponent ϒ=0.3558
within the resolution of the simulation

Summary

◎Simulation of PBH formation from iso-curvature perturbations

◎PBH can be formed from the isocurvature of a massless scalar field

◎The formation and accretion process are similar to those in the RD dominated universe

◎Constraints on the isocurvature modes of a massless scalar field would be given by observational constraints of the PBH abundance similarly to those for the curvature perturbation.

Thank you for your attention

Poster session

Yosuke Mishima

RIkkyo University

“Preferred frame effects for nonluminal GWs in pulsar timing arrays”

[JGRG30 (2021) PA20]

Preferred frame effects for nonluminal GWs in pulsar timing arrays

Work in progress

Yosuke Mishima (Rikkyo University)

Collaborator: Hiroaki Tahara (Rikkyo University)

Introduction - Modified gravity

Extension of General Relativity

- General Relativity has almost succeeded as a theory of gravity
- In the context of accelerated expansion, there are various extensions of General Relativity, such as
  1. Massive gravity — 5 degrees of freedom with mass m_{graviton}
  2. Scalar-Tensor theories — 2+1 degrees of freedom with massless
- The speed of GWs doesn't coincide with the speed of light in general

Introduction - Speed of GWs

GW170817 & GRB170817A

$$-3 \times 10^{-15} \leq \frac{c_T}{c} - 1 \leq 7 \times 10^{-16}$$

c_T : phase speed of gravitational waves

[Virgo, Fermi-GBM, INTEGRAL, *and* LIGO Scientific Collaborations (2017)]

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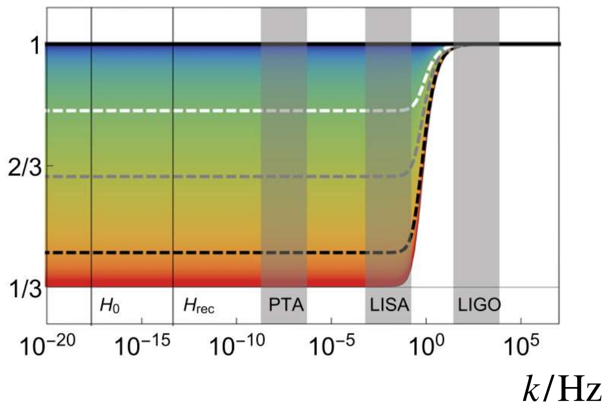
Gravitational Rainbows

[C. de Rham *and* S. Melville (2018)]

- the limited speed of GWs
  $c_T = c \ (k \sim 10 - 100 \text{ Hz})$
- In the others frequency band,
  c_T isn't necessarily the speed of light



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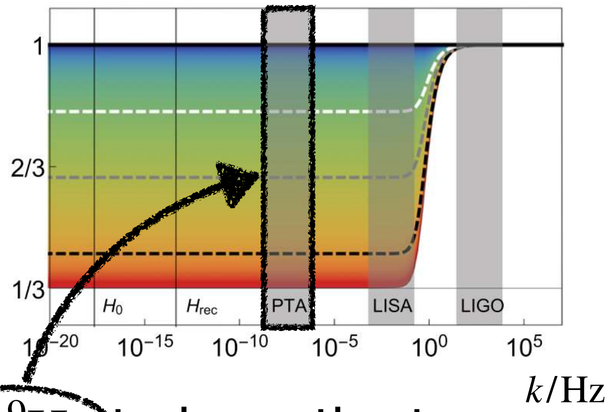

We focus on the range of 10^{-9}Hz to investigate c_T

Introduction - Nonluminal propagation

Massive gravity

as for the pulsar timing arrays, discussed in [Q. Liang *and* M. Trodden (2021)]

- Polarization: Tensor mode ×2, Vector mode ×2, Scalar mode ×1

- Phase velocity is always superluminal propagation

$$c > v_{\rm group} = \frac{d\omega}{dk} = \frac{1}{v_{\rm phase}}$$

Dispersion relation

$$\omega^2 = k^2 + m_{\rm graviton}^2$$

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Scalar-Tensor theories

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- If $c_T \neq c$, the background scalar field identifies the preferred direction

→ $\partial_\mu \bar{\phi}$ alongside with cosmological evolution

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- Polarization: Tensor mode ×2, Scalar mode ×1
- If $c_T \neq c$, the background scalar field identifies the preferred direction

→ $\partial_\mu \bar{\phi}$ alongside with cosmological evolution

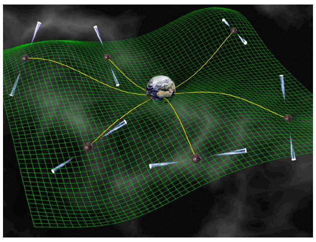
even though $c_T > c$

Preferred frame effects are characteristic in Scalar-Tensor theories

Introduction - Pulsar Timing Arrays

Overview

- Observe pulses from an array of millisecond pulsars
- The GWs affects the time of arrival of the pulses



[David J. Champion]

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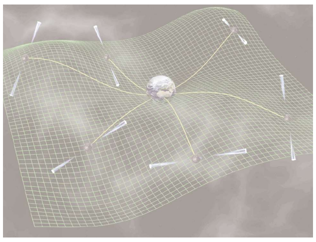

[David J. Champion]

- This redshift is evaluated by correlation of several pulses
  - Two point correlation function in General Relativity
    [R. W. Hellings *and* G. S. Downs (1983)]
  - Two point correlation function in Massive gravity
    [Q. Liang *and* M. Trodden (2021)]

Introduction - Pulsar Timing Arrays

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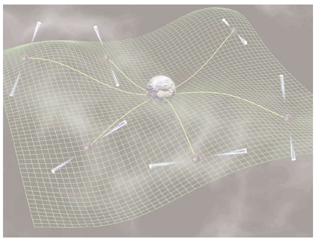

[David J. Champion]

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    - Two point correlation function in General Relativity
      [R. W. Hellings *and* G. S. Downs (1983)]
    - Two point correlation function in Massive gravity
      [Q. Liang *and* M. Trodden (2021)]

Our work

Evaluate the two point correlation function in Scalar-Tensor theories. Especially c_T is superluminal/subluminal propagation, incorporating the effect of moving with respect to the preferred frame.

Redshift of pulses (at rest)

The redshift of pulses crossed by GWs at rest

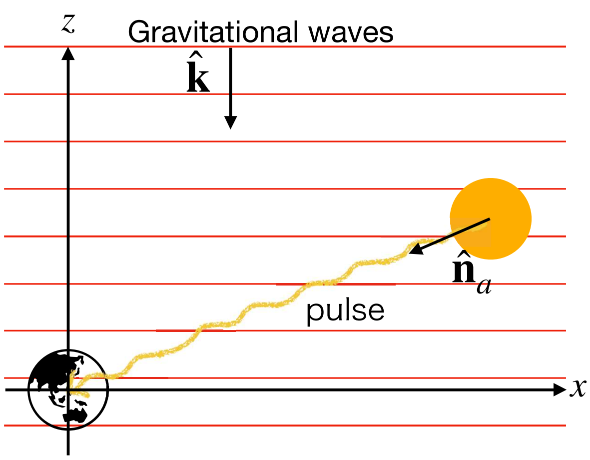
$$z_a(t) := \left(\frac{\Delta T_a}{T_a}\right) \simeq \frac{c_T}{c_T + \hat{\mathbf{k}}\cdot\hat{\mathbf{n}}_a}\frac{n_a^i n_a^j}{2}\left[h_{ij}^{\mathrm{TT}}(t, \mathbf{x}=0) - h_{ij}^{\mathrm{TT}}(t-\tau_a, \mathbf{x}_a)\right]$$

$$\tau_a := t_{\mathrm{obs}} - t_{\mathrm{em}}$$

Sketch:

1. Evaluate the arrival time of photon

Integrate the null geodesic along the path from a pulsar $(\mathbf{x} = \mathbf{x}_a)$ to the Earth $(\mathbf{x} = \mathbf{0})$



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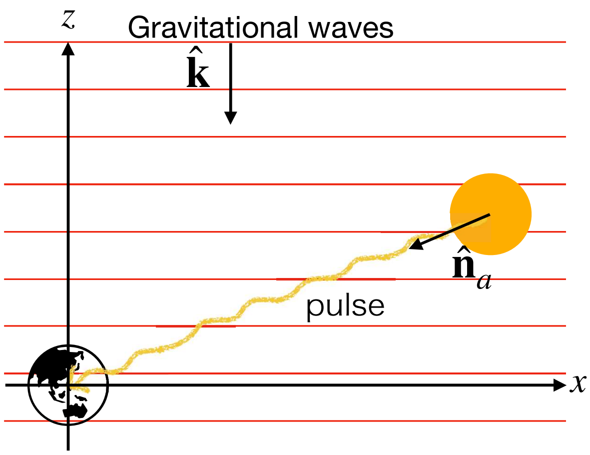
Sketch:

1. Evaluate the arrival time of photon

Integrate the null geodesic along the path from a pulsar ($\mathbf{x} = \mathbf{x}_a$) to the Earth ($\mathbf{x} = \mathbf{0}$)

2. Calculate the period of the pulse

If GWs crosses the path of light, the accurate period of pulses changes



Redshift of pulses (in motion)

The redshift of pulses crossed by GWs in motion

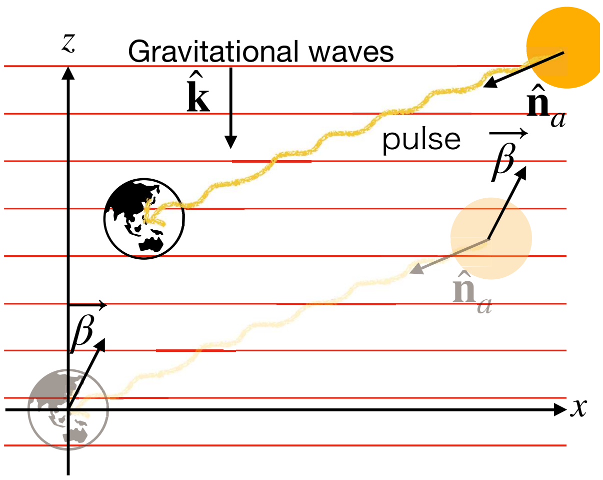
$$z_a(t) := \left(\frac{\Delta T_a}{T_a}\right) \simeq \frac{k_\mu u^\mu}{k_\nu n_a^\nu}\frac{n_a^i n_a^j}{2}\left[h_{ij}^{\mathrm{TT}}(t, \mathbf{x}=0) - h_{ij}^{\mathrm{TT}}(t-\tau_a, \mathbf{x}_a)\right]$$

u^μ : 4-velocity
k^μ : wave vector of GWs
n_a^μ : null vector (pulse a)
$\tau_a := t_{\mathrm{obs}} - t_{\mathrm{em}}$

Sketch:

1. Move against the GWs background

The galaxy, including not only the Earth but also an array of pulsar, has the same velocity $\vec{\beta}$



Redshift of pulses (in motion)

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u^μ : 4-velocity
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$\tau_a := t_{\rm obs} - t_{\rm em}$

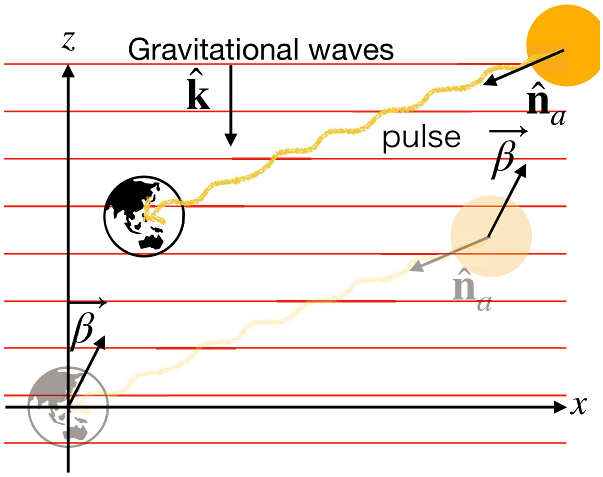
Sketch:

1. Move against the GWs background

The galaxy, including not only the Earth but also an array of pulsar, has the same velocity $\vec{\beta}$

2. Change in frequency of the pulse

Doppler shift occurs until observe the pulses



Configuration

Ansatz 1. Stochastic GWs background

$$h_{ij}(t, \mathbf{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty} df \int d(\cos\theta)d\phi\, \tilde{h}_A(f, \hat{\mathbf{n}})e_{ij}^A(\hat{\mathbf{n}})e^{k^\mu x_\mu}$$

e_{ij}^A : polarization
$\tilde{h}_A$: stochastic amplitude

Ansatz 2. stationary, Gaussian, isotropic, unpolarized

$$\langle \tilde{h}_A^*(f, \hat{\mathbf{n}})\tilde{h}_B(f', \hat{\mathbf{n}}')\rangle = \delta(f-f')\frac{\delta(\phi-\phi')\delta(\cos\theta-\cos\theta')}{4\pi}\delta_{AB}\frac{1}{2}S_h(f)$$

$S_h(f)$: spectral density

Configuration

Ansatz 1. Stochastic GWs background

$$h_{ij}(t,\mathbf{x}) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d(\cos\theta)d\phi\ \tilde{h}_A(f,\hat{\mathbf{n}})e^A_{ij}(\hat{\mathbf{n}})e^{k^\mu x_\mu}$$

e^A_{ij} : polarization
$\tilde{h}_A$: stochastic amplitude

Ansatz 2. stationary, Gaussian, isotropic, unpolarized

$$\langle \tilde{h}^*_A(f,\hat{\mathbf{n}})\tilde{h}_B(f',\hat{\mathbf{n}}')\rangle = \delta(f-f')\frac{\delta(\phi-\phi')\delta(\cos\theta-\cos\theta')}{4\pi}\delta_{AB}\frac{1}{2}S_h(f)$$

$S_h(f)$: spectral density

Finally, evaluate two point correlation function $\langle z_a(t)z_b(t)\rangle$ ...

Notice!
The GWs background rest frame doesn't necessary coincide with the CMB rest frame.
If these frames are identical, the velocity(rapidity) $|\vec{\beta}| = 2\times 10^{-3}$

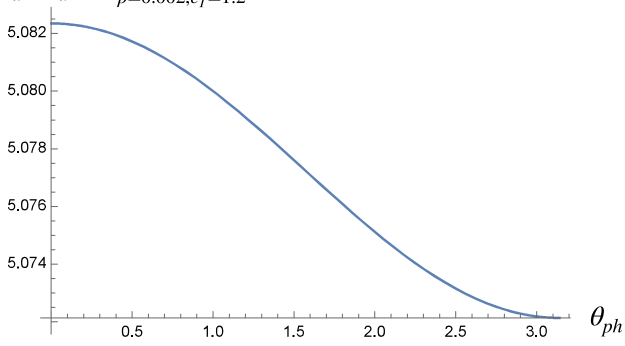
Result (superluminal propagation case)

Autocorrelation of pulses

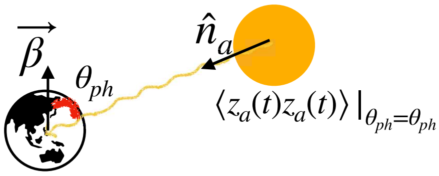
$$\begin{aligned}\langle z_a(t)z_a(t)\rangle = \frac{\pi}{15\,(\beta^2-1)}\Bigg[&40c_T^2(2-3c_T^2)+6(8-25c_T^2+15c_T^4)\beta^2+(60c_T^3(c_T^2-1)-45c_T(c_T^2-1)^2\beta^2)\ln\left[\frac{c_T+1}{c_T-1}\right]\\ &+10c_T\beta\cos\theta_{\rm ph}\left(26c_T-30c_T^3+3(1-6c_T^2+5c_T^4)\ln\left[\frac{c_T+1}{c_T-1}\right]\right)\\ &+\beta^2\cos^2\theta_{\rm ph}\left(-64+330c_T^2-270c_T^4+15c_T(5-14c_T^2+9c_T^4)\ln\left[\frac{c_T+1}{c_T-1}\right]\right)\Bigg]\end{aligned}$$

for $c_T > c$

$\langle z_a(t)z_a(t)\rangle|_{\beta=0.002,c_T=1.2}$



$\vec{\beta}\cdot(-\hat{n}_a) := \beta\cos\theta_{ph}$



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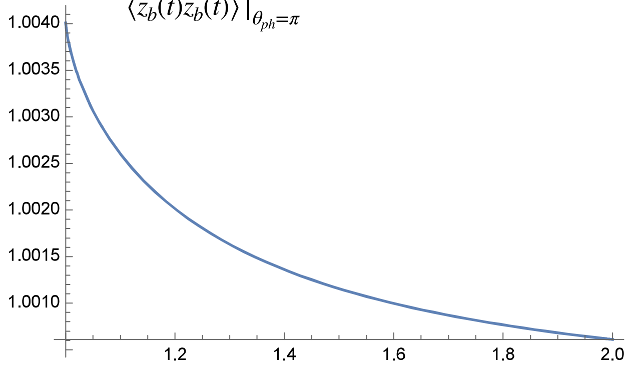
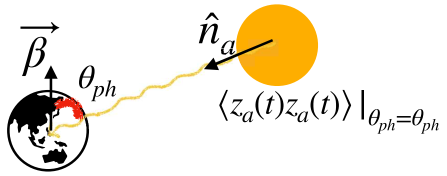
$$\text{ratio} := \frac{\langle z_a(t) z_a(t)\rangle|_{\theta_{ph}=0}}{\langle z_b(t) z_b(t)\rangle|_{\theta_{ph}=\pi}}$$

$$\vec{\beta}\cdot(-\hat{n}_a) := \beta\cos\theta_{ph}$$



c_T



Be careful of the difference of c_T when evaluate $\vec{\beta}$

The ratio of autocorrelation depends on the propagation speed

Discussion

Summary

- In Scalar-Tensor theories, if the phase velocity of GWs $c_T \neq c$, the background scalar field identifies a preferred direction.
- Taking into account the Doppler shift, autocorrelation of pulses depends on the velocity of motion $\vec{\beta}$ and the phase velocity of GWs c_T
    - When evaluating $\vec{\beta}$ relative to the GWs background rest frame, be careful of the difference in c_T
    - If the GWs background rest frame is same as the CMB rest frame, measure c_T

Discussion

Summary

- In Scalar-Tensor theories, if the phase velocity of GWs $c_T \neq c$,
  the background scalar field identifies a preferred direction.

- Taking into account the Doppler shift, autocorrelation of pulses depends on the velocity of motion $\vec{\beta}$ and the phase velocity of GWs c_T

  - When evaluating $\vec{\beta}$ relative to the GWs background rest frame, be careful of the difference in c_T

  - If the GWs background rest frame is same as the CMB rest frame, measure c_T

In preparation

- In the $c_T < c$ case, evaluate the effect of resonance contribution.

Poster session

Taishi Ikeda

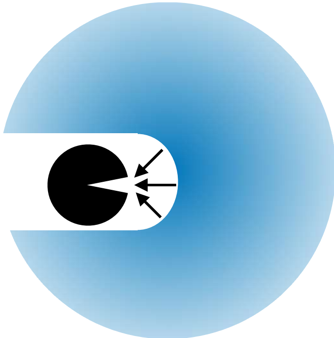
Sapienza University of Rome

“Black hole eating boson stars”

[JGRG30 (2021) PA21]

P21

Black hole eating boson star



Taishi Ikeda
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Outline

- Introduction
  - Motivation
  - Set up
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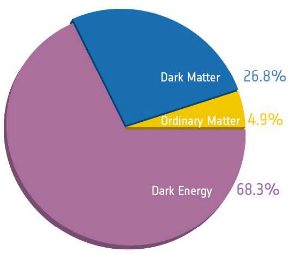
Scalar field beyond GR and SM



- Mystery in our Universe
  - Dark matter ??
  - Dark energy ??
  - Quantum theory of gravity ??
- Light scalar fields are smoking guns for new physics beyond SM.
- Boson stars of the complex scalar field may be dark matter.

Sang-Jin (1994)

  - Boson stars interact with other astrophysical objects.
- The simplest model of boson stars

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi} - g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi^* - \mu^2|\psi|^2\right)$$

ψ : complex scalar field

Boson star

- Non-rotating Boson star profile

$$\begin{cases} ds^2 = -\alpha_{\rm BS}^2 dt^2 + a_{\rm BS}^2 dr^2 + r^2 d^2\Omega \\ \psi_{\rm BS}(t,r) = e^{i\omega t}\psi_{0,\rm BS}(r) \end{cases}$$

ω : boson star frequency

  - Einstein eq. and KG eq.

$$\begin{cases} \partial_r \alpha_{\rm BS} = \cdots \\ \partial_r a_{\rm BS} = \cdots \\ \partial_r^2 \psi_{0,\rm BS} = \cdots \end{cases}$$

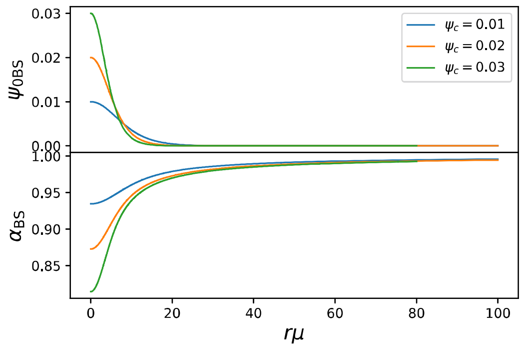
  - Boundary conditions

$$\begin{cases} a_{\rm BS}(\infty) = \alpha_{\rm BS}(\infty)^{-1} \\ \psi_{0,\rm BS}(\infty) = 0 \end{cases}$$

  - Eigenfrequency : $\omega_{\rm BS}(\psi_c, n)$

$\psi_{0,\rm BS}(0) = \psi_c$ $\quad$ n : # node

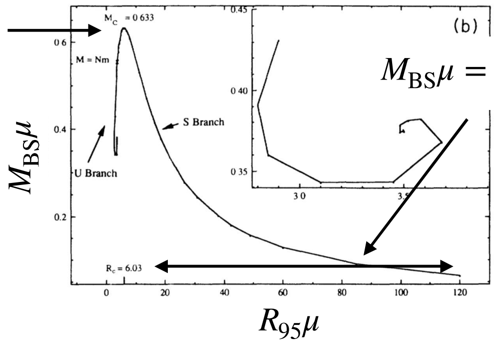
solutions with $n = 0$



Mass-radius relation

$$M_{\rm max} \simeq 0.6\frac{M_{\rm p}^2}{\mu}$$

$$M_{\rm BS}\mu = \frac{9.1}{R_{\rm BS}\mu}$$



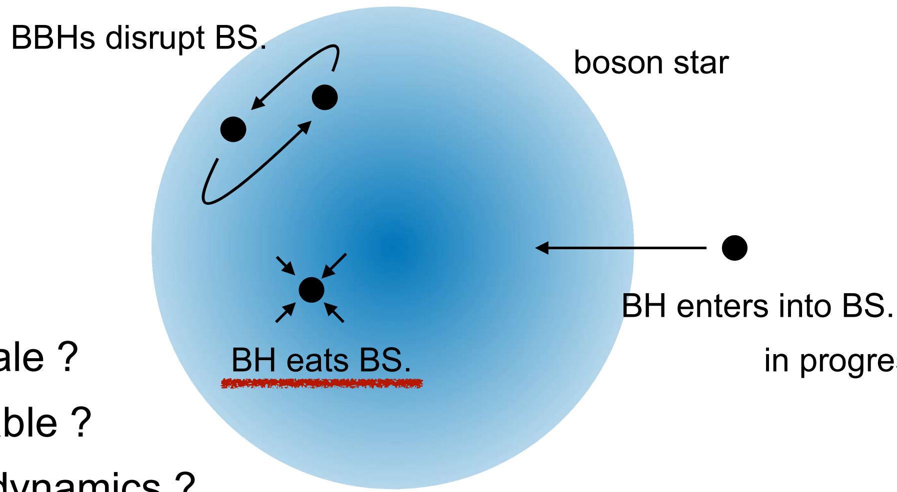
Seidel et.al 1990

Possible interactions with BHs

- Boson stars with light fields

$$\frac{M_{\rm BS}}{M_\odot} = 9\times 10^9 \frac{100{\rm pc}}{R_{\rm BS}}\left(\frac{10^{-22}\ {\rm eV}}{\mu}\right)^2$$

- Boson stars can interact with BHs.

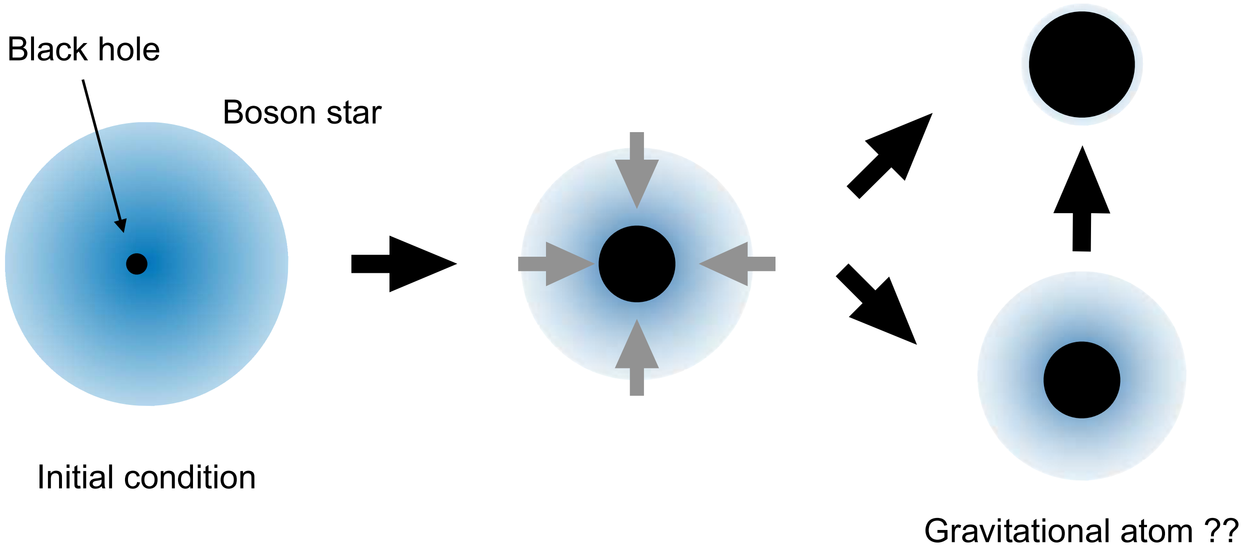


in progress with Zhen

- Time scale ?
- Observable ?
- Typical dynamics ?

Set up

- Set up : BS-BH system
  - Spherical symmetry (for simplicity)
  - Initial profile of the scalar field is same as boson star profile
  - We solve the evolution of metric and the complex scalar field.



Gravitational atom

- Gravitational atom is long-lived state of the scalar field around BH.

$$\left(\Box_{\mathrm{Kerr\ BH}} - \mu^2\right)\psi = 0 \qquad \psi = e^{-i\omega t + im\phi}R(r)S(\theta)$$

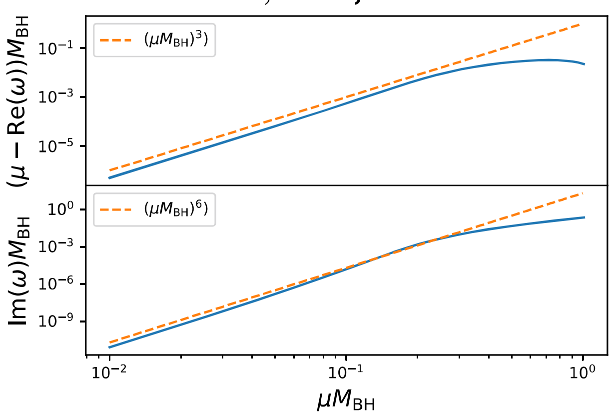
$$\Delta\frac{d}{dr}\left(\Delta\frac{dR}{dr}\right) + \left(\omega^2(r^2+a^2)^2 - 4aMrm\omega + a^2m^2 - \Delta(\mu^2r^2 + a^2\omega^2 + \Lambda)\right)R = 0$$

- Boundary condition
  - Ingoing on BH horizon
  - decaying at infinity
- Leaver method

$\rightarrow$ $\omega = \omega_R + i\omega_I$

  - We can estimate typical life time of the gravitational atom.

$a = 0, l = 0$, nodeless



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Numerical formulation

- We use (generalized-)BSSN formulation.

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

  - conformal decomposition

$$\begin{cases} \gamma_{ij} = e^{4\phi}\tilde{\gamma}_{ij} \\ K_{ij} = e^{4\phi}\tilde{A}_{ij} + \dfrac{1}{3}\gamma_{ij}K \end{cases}$$

  - auxiliary field

$$\tilde{\Lambda}^k = \tilde{\gamma}^{ij}(\tilde{\Gamma}^k_{ij} - \bar{\Gamma}^k_{ij})$$

$\bar{\gamma}_{ij}$: reference metric

  - spherical symmetry : (t, r, θ, ϕ)

$$\begin{cases} \tilde{\gamma}_{ij} = \mathrm{diag}(\tilde{a}, \tilde{b}r^2, \tilde{b}r^2 \sin^2\theta) \\ \tilde{A}_{ij} = \mathrm{diag}(A, Br^2, Br^2 \sin^2\theta) \\ \tilde{\Lambda}^k = (\tilde{\Lambda}, 0, 0) \end{cases}$$

  - constraint eq.

$$\mathscr{H} \equiv \left(\frac{\phi''}{a} + \frac{\phi'^2}{a} - (\frac{a'}{2a^2} - \frac{b'}{ab} - \frac{2}{ar})\phi'\right)e^{\phi} - \frac{e^{\phi}}{8}\tilde{R} + \frac{e^{5\phi}}{8}\left(\frac{A^2}{a^2} + 2\frac{B^2}{b^2}\right) - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}E$$

$$\mathscr{M} \equiv 6\phi'\frac{A}{a} + \frac{A'}{a} - \frac{a'A}{a^2} + \frac{b'}{b}\left(\frac{A}{a} - \frac{B}{b}\right) + \frac{2}{r}\left(\frac{A}{a} - \frac{B}{b}\right) - \frac{2}{3}K' - 8\pi p = 0$$

Numerical formulation

- evolution eq.

$$
\begin{cases}
\partial_t \phi = \beta \phi' - \frac{1}{6}\alpha K + \sigma \frac{1}{6}\mathscr{B} \\
\partial_t a = \beta a' + 2a\beta' - 2\alpha A - \sigma \frac{2}{3} a \mathscr{B} \\
\partial_t b = \beta b' + 2\beta \frac{b}{r} - 2\alpha B - \sigma \frac{2}{3} b \mathscr{B} \\
\partial_t K = \beta K' - \mathscr{D} + \alpha (\frac{1}{3}K^2 + \frac{A^2}{a^2} + 2\frac{B^2}{b^2}) + 4\pi\alpha(E+S) \\
\partial_t A = \beta A' + 2A\beta' + e^{-4\phi}(-\mathscr{D}_{rr}^{TF} + \alpha(R_{rr}^{TF} - 8\pi S_{rr}^{TF})) + \alpha(KA - 2\frac{A^2}{a}) - \sigma\frac{2}{3}A\mathscr{B} \\
\partial_t B = \beta B' + \frac{e^{-4\phi}}{r^2}(-\mathscr{D}_{\theta\theta}^{TF} + \alpha(R_{\theta\theta}^{TF} - 8\pi S_{\theta\theta}^{TF})) + \alpha(KB - 2\frac{B^2}{b}) + 2\frac{\beta}{r}B - \sigma\frac{2}{3}B\mathscr{B} \\
\partial_t \tilde{\Lambda} = \beta\tilde{\Lambda}' - \beta'\tilde{\Lambda} + \frac{2\alpha}{a}(\frac{6A\phi'}{a} - \frac{2}{3}K' - 8\pi p) + \frac{\alpha}{a}(\frac{a'A}{a^2} - \frac{2b'B}{b^2} + 4B\frac{a-b}{rb^2}) + \sigma(\frac{2}{3}\tilde{\Lambda}\mathscr{B} + \frac{\mathscr{B}'}{3a}) + \frac{2}{rb}(\beta' - \frac{\beta}{r}) - 2\frac{\alpha' A}{a^2} + \frac{1}{a}\beta''
\end{cases}
$$

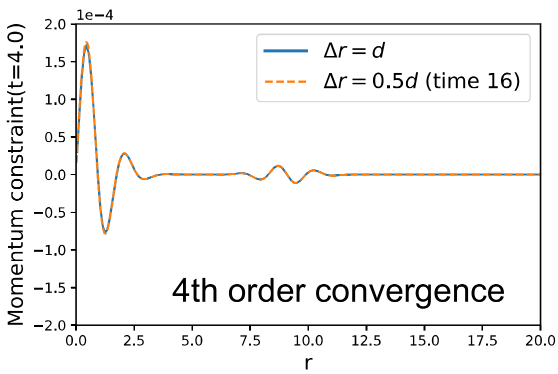
skip the details…..

- Our numerical code
  - Time integration : 4th order Runge-Kutta method
  - Radial derivative: 4th order accurate centered finite difference
  - Open MP, KO dissipation, excision procedure ….

Test simulations

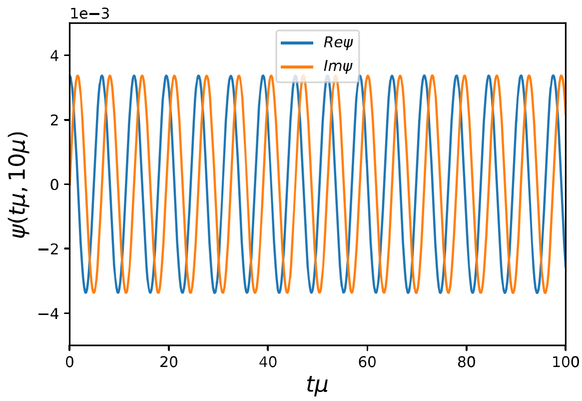
- Numerical convergence, and boson star evolution
  - Pure gauge evolution

$$\alpha(r,0) = 1 + \frac{\alpha_0 r^2}{1+r^2}\left(e^{-(r-r_0)^2} + e^{-(r+r_0)^2}\right)$$



  - Boson star evolution

$$
\begin{cases}
\phi(0,r) = \phi_{\rm BS}(r) \\
\psi(0,r) = \psi_{\rm BS}(r) \\
\alpha(0,r) = e^{-4\phi_{\rm BS}(r)}
\end{cases}
\qquad \psi_c = 0.01
$$



Evolution is consistent with stable boson star configuration.

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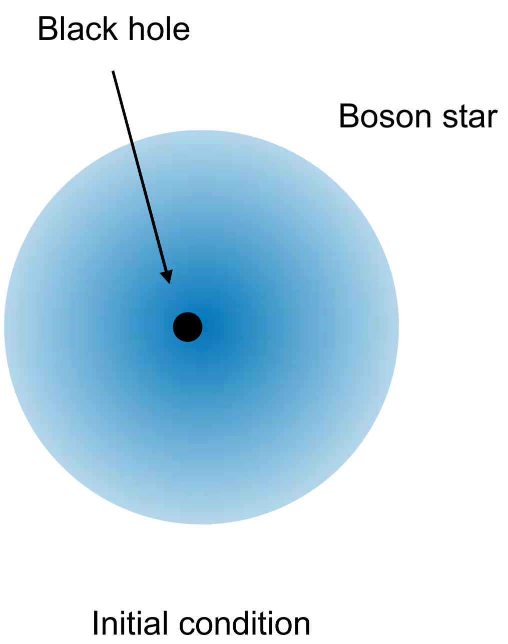
Construction of ID



Construction of ID

- We construct BS-BH initial data by solving constraint equation.
  - assumptions for initial data
    - momentarily static : $K = A = B = 0$ $\Rightarrow$ $\mathscr{M} = 0$
    - conformally flat : $a = b = 1$
    - Profile of the scalar field is same as boson star profile.
    - precollapse lapse, zero shift : $\alpha(0,r) = e^{-4\phi(0,r)}$, $\beta(0,r) = 0$
  - Parameters: ψ_c , $M_{0,\mathrm{BH}}$
  - 1. Construct the BS star profile in isotropic coordinate.

$$\begin{cases} ds^2 = -\alpha_{\mathrm{BS}}^2(r)dt^2 + \Phi_{\mathrm{BS}}(r)^4(dr^2 + r^2 d^2\Omega) \\ \psi(t,r) = \psi_{0,\mathrm{BS}}(r)e^{i\omega t} \end{cases}$$

  - 2. Apply the window function to scalar field

$$\psi_{\mathrm{BS}}^{\mathrm{W}}(r) = W(r)\psi_{0,\mathrm{BS}}(r)$$

$$W(r) = \begin{cases} 0 & (r < M_{0,\mathrm{BH}} + \epsilon_1) \\ f(r) & (M_{0,\mathrm{BH}} + \epsilon_1 < M_{0,\mathrm{BH}} + \epsilon_1 + \epsilon_2) \\ 1 & (M_{0,\mathrm{BH}} + \epsilon_1 + \epsilon_2 < r) \end{cases}$$

Construction of ID

- 3.Sum of conformal factor

$$\Phi = \Phi_{\mathrm{BS}} + \Phi_{\mathrm{BH}} - 1 + \delta\Phi$$

$$\Phi = e^{\phi}$$

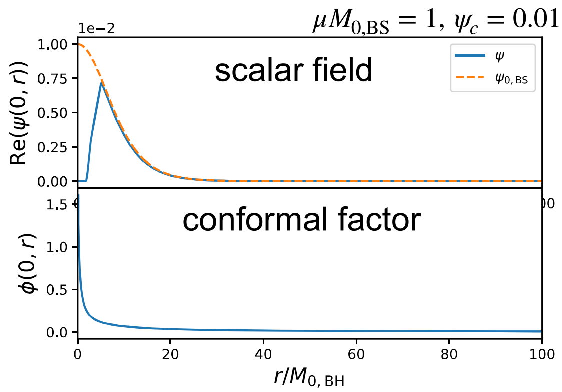
$$\Phi_{\mathrm{BH}} = 1 + \frac{M_{0,\mathrm{BH}}}{2r}$$

- 4.Solve Hamiltonian constraint for $\delta\Phi$

$$\mathscr{H} = \Phi'' + \frac{2}{r}\Phi' + 2\pi\Phi^5 E_{\mathrm{BS}}^{\mathrm{W}} = 0$$

$$\Rightarrow \delta\Phi'' + \frac{2}{r}\delta\Phi' + 2\pi\left(\Phi^5 E_{\mathrm{BS}}^{\mathrm{W}} - \Phi_{\mathrm{BS}}^5 E_{\mathrm{BS}}\right) = 0$$

Integrate from infinity with $\delta\Phi(\infty) = \partial_r\delta\Phi(\infty) = 0$



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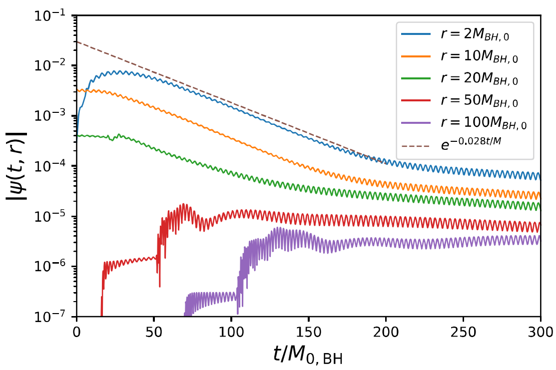
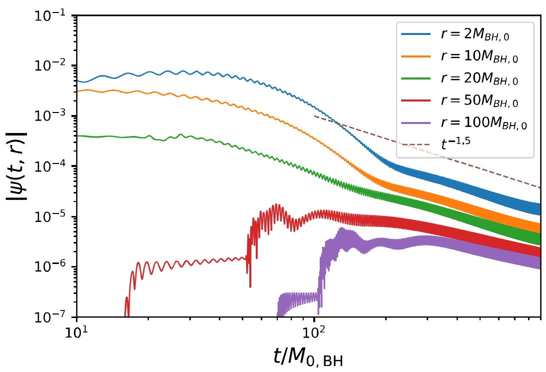
Preliminary results

- Here, we show one preliminary result. $\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01$

The scalar field decays exponentially around BH in early phase.



In late time, we observe power-law tail.



In general, we can expect power low tail

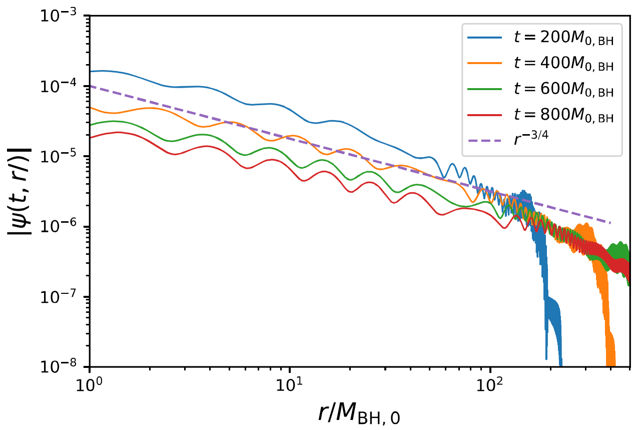
$$\psi \sim t^p \sin(\mu t)$$

$$\begin{cases} p = -(l+3/2) & \text{at late time} \\ p = -5/6 & \text{at very late time} \end{cases}$$

Preliminary results

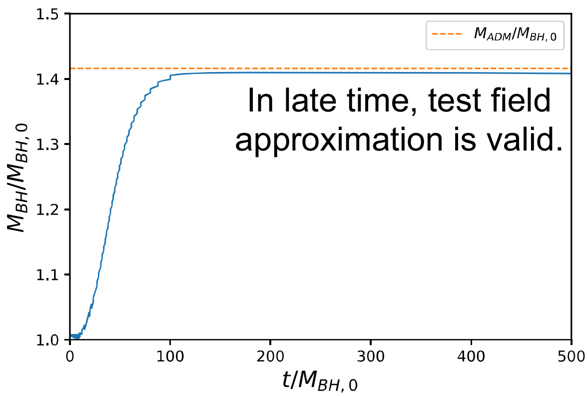
$$\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01$$

Late time radial profile of scalar field is $\sim r^{-3/4}$



cf: Lam (2019)

BH eat almost boson star energy in early phase.



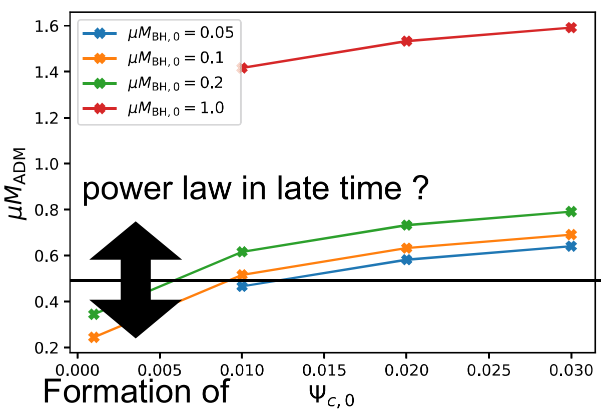
In late time, $\mu M_{BH} \simeq 1.4$

The life time of the corresponding gravitational atom is very short.

power low behavior dominates.

Preliminary results

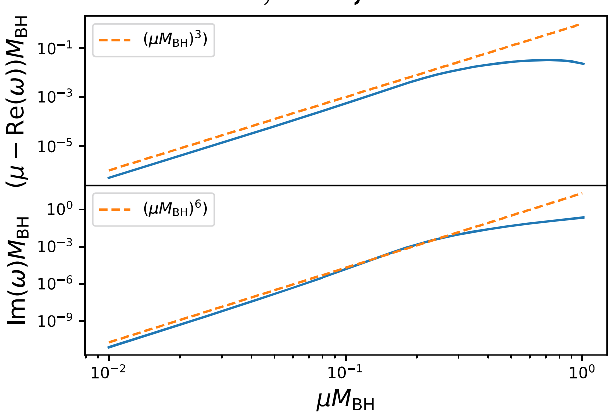
- We guess late time behavior from ADM mass and mass of the scalar field.



Formation of gravitational atom ??

We need further simulations to check the expectations.

$a = 0, l = 0$, nodeless



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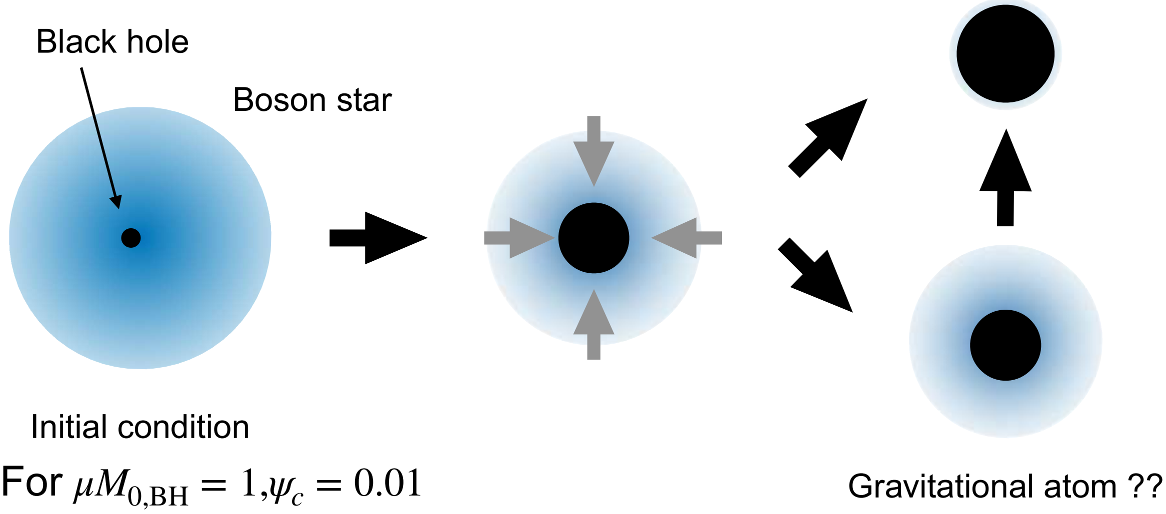
Summary

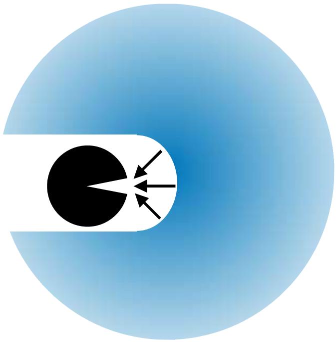
- We considered the accretion process of boson star into black hole.



  - For $\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01$

    we observed late time power low decay, and power low profile.
  - We can guess late time behavior from ADM mass and mass of the scalar field.
- We need further simulations….

Finish



Poster session

Koki Tokeshi

The University of Tokyo

“PBH abundance revised from joint formation criteria”

[JGRG30 (2021) PA22]

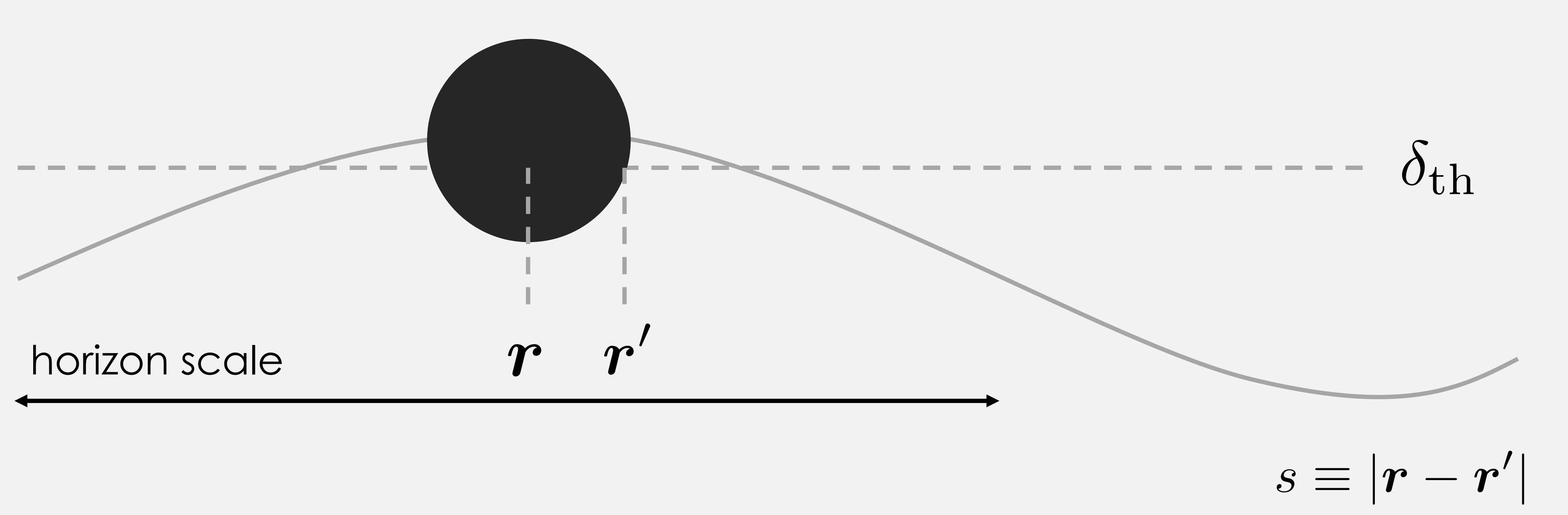
PBH abundance from joint formation criteria

Koki Tokeshi and Jun'ichi Yokoyama (RESCEU, The University of Tokyo)

Abstract: Several formation conditions have been proposed for realizing primordial black holes from the curvature and/or density perturbations in the early Universe. A joint formulation in which both conditions are combined under the conditional multivariate Gaussian distribution is presented here. The modified PBH abundance becomes smaller by some orders compared to the conventional prediction, as well as smaller thresholds are obtained.

1. INTRODUCTION

- Primordial black hole (PBH) from curvature/density perturbations, in the radiation-dominated Universe [1], is considered here.



- Formation condition(s)

(1) Compaction function [2] must exceed its threshold:

$$C(s) = \frac{1}{3}\left[1 - (1+\xi)^2\right] > C_{\rm th} \quad \text{at} \quad \boldsymbol{r}' \qquad (\xi \equiv s\zeta')$$

where, according to numerical simulation, $\xi \in (0.554, 1.446)$

(2) Density perturbation must be "peaked" at formation point [3]:

$$\mathcal{P} = \{\delta(\boldsymbol{r}) \mid \delta > \delta_{\rm th},\ \boldsymbol{\eta} \equiv \boldsymbol{\nabla}\delta = \boldsymbol{0},\ \chi \equiv \boldsymbol{\nabla} \otimes \boldsymbol{\nabla}\delta \prec 0\}$$

We expect that by **combining these conditions** the PBH abundance today would be evaluated more precisely.

2. JOINT FORMALISM

- Prepare the variables assumed to be Gaussian

$$\boldsymbol{x} \equiv \boldsymbol{\nabla}\zeta(\boldsymbol{r}'), \quad \boldsymbol{y} \equiv (\delta, \boldsymbol{\eta}, \chi)(\boldsymbol{r})$$

- Two-point correlators: $(10+3)\times(10+3)$ matrix

auto-correlations of density perturbations (given in [3])

correlations between curvature/density perturbations

$$\Sigma = \begin{pmatrix} \Sigma_{\boldsymbol{yy}} & \boxed{\Sigma_{\boldsymbol{yx}}} \\ \boxed{\Sigma_{\boldsymbol{xy}}} & \Sigma_{\boldsymbol{xx}} \end{pmatrix} \Longrightarrow \Sigma_{\boldsymbol{x}|\boldsymbol{y}} = \Sigma_{\boldsymbol{xx}} - \Sigma_{\boldsymbol{xy}}\Sigma_{\boldsymbol{yy}}^{-1}\Sigma_{\boldsymbol{xy}}^{\rm T}$$

auto-correlations of curvature perturbations

- Conditional probability of multivariate Gaussian

$$f(\boldsymbol{x} \mid \boldsymbol{y}) = \frac{1}{\sqrt{(2\pi)^3 \det \Sigma_{\boldsymbol{x}|\boldsymbol{y}}}} \exp\left[-\frac{1}{2}{}^t(\Delta\boldsymbol{x})\Sigma_{\boldsymbol{x}|\boldsymbol{y}}^{-1}(\Delta\boldsymbol{x})\right]$$

Required that density perturbation is "peaked" at formation point

Under that, statistical behavior of the compaction function is considered

- Correlators with spherical symmetry assumed

$$\langle\delta(\boldsymbol{r}) \otimes \boldsymbol{\nabla}\zeta(\boldsymbol{r}')\rangle = O_{1\times3}$$

$$\langle\boldsymbol{\eta}(\boldsymbol{r}) \otimes \boldsymbol{\nabla}\zeta(\boldsymbol{r}')\rangle = \frac{4}{9}R^2 \cdot \frac{1}{3}\left[\sigma_\zeta^{(4,0)}\right]^2 \boldsymbol{1}_{3\times3}$$

$$\langle\chi(\boldsymbol{r}) \otimes \boldsymbol{\nabla}\zeta(\boldsymbol{r}')\rangle = O_{6\times3}$$

Under the spherical symmetry, **the threshold and negative-definite condition do not affect the behavior of the curvature perturbation** (, which might be carefully re-considered.)

3. MODIFIED PBH ABUNDANCE

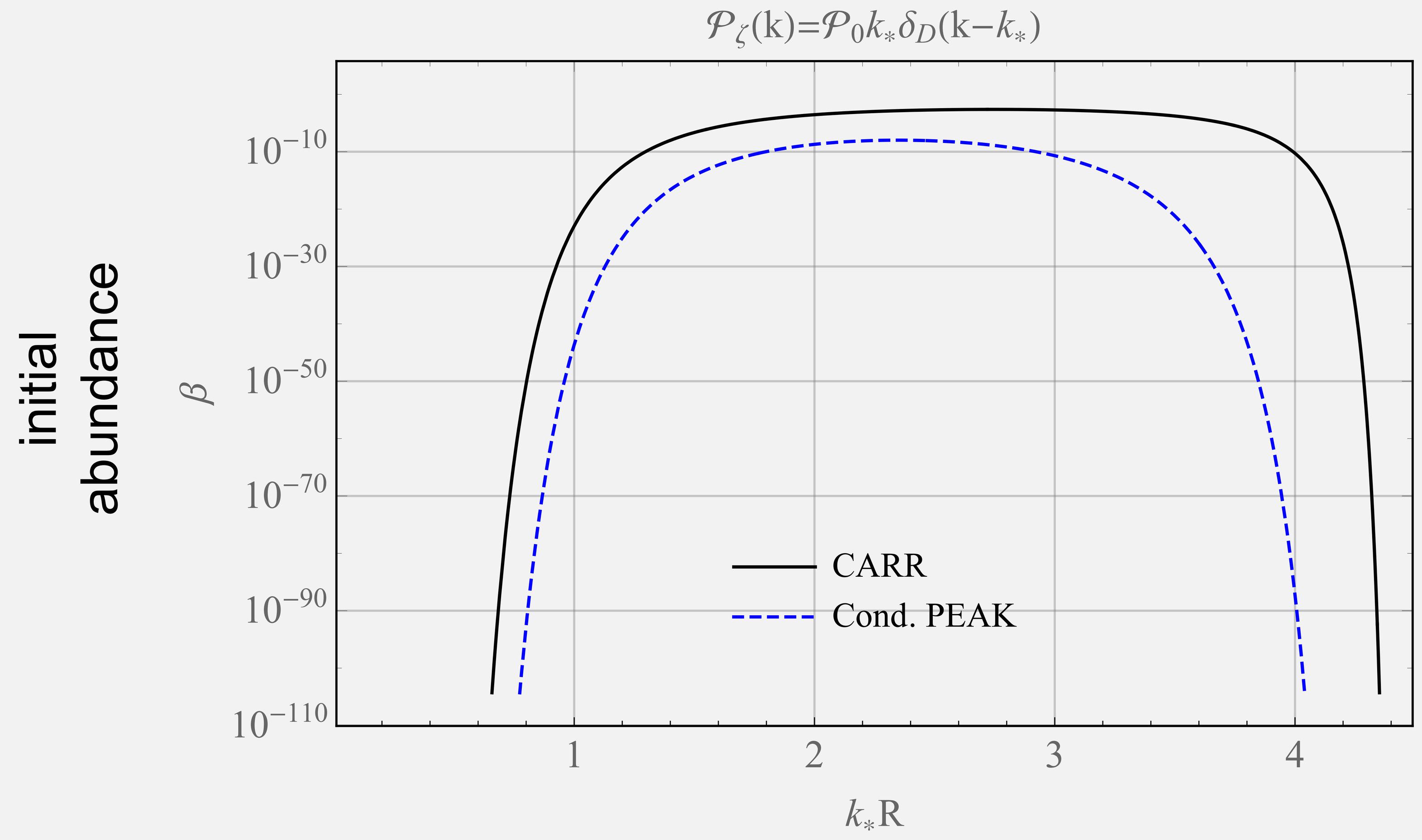
- PBH abundance at formation time

Integration of the conditional PDF over the region in which the condition on the compaction function is satisfied gives the initial PBH abundance.

$$\beta = \frac{1}{\sqrt{2\pi}}\int_{X_-}^{X_+} {\rm d}t \, \exp\left(-t^2\right)$$

$$= \frac{1}{2}\left[\operatorname{erf}\left(\frac{\xi_+}{\sqrt{2}s}\sqrt{\frac{b}{-\alpha^2 + b\beta}}\right) - \operatorname{erf}\left(\frac{\xi_-}{\sqrt{2}s}\sqrt{\frac{b}{-\alpha^2 + b\beta}}\right)\right]$$

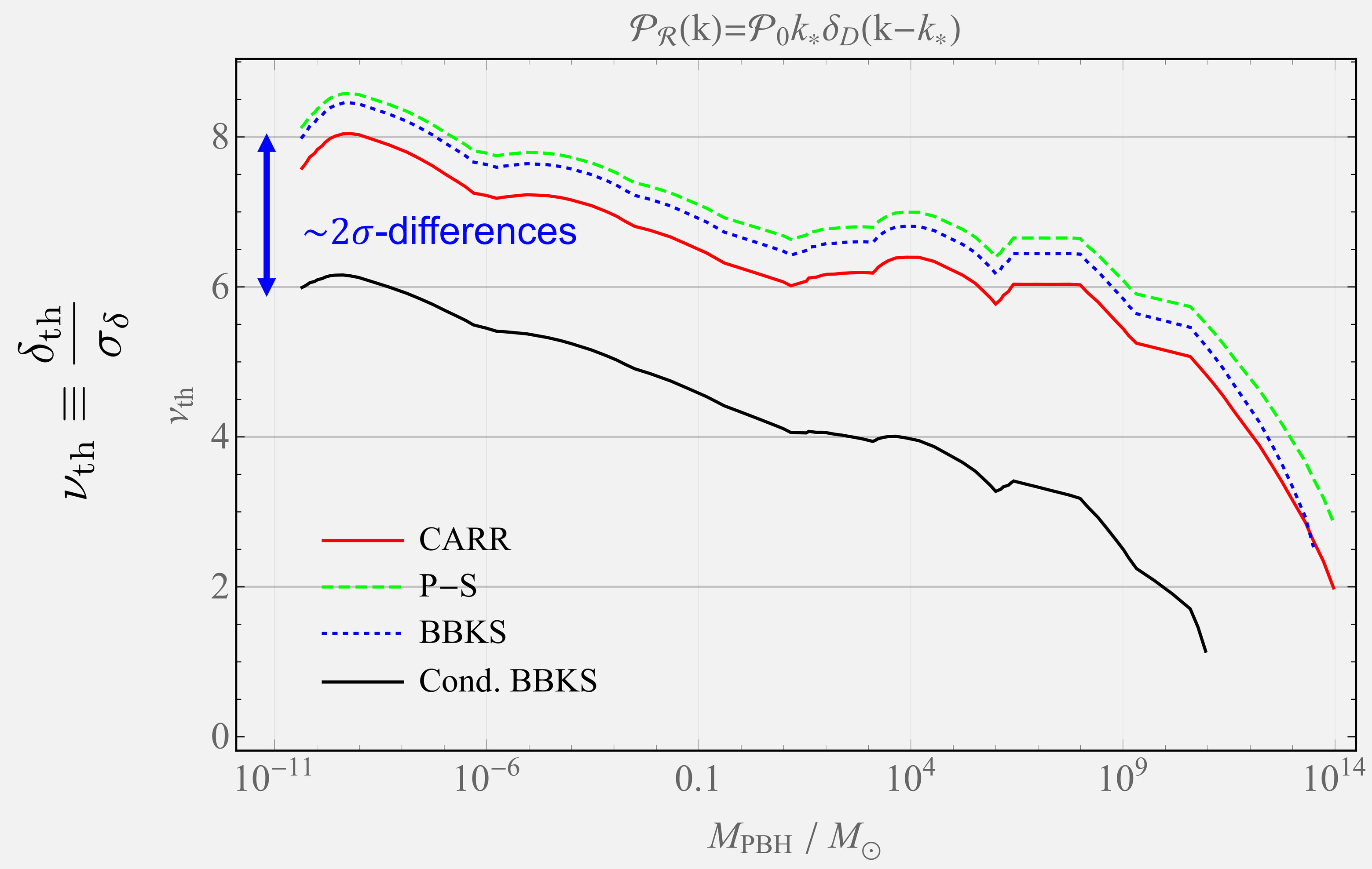
uniquely determined for each perturbations' profile



The joint analysis here predicts smaller PBH abundance by some orders compared to that from previous formulations.

4. REQUIRED THRESHOLD

Lower threshold limits for each mass (and upper bounds of the amplitudes of perturbations) can be obtained by converting the observational upper bounds on the PBH abundance today.



Various estimation formulas are compared in the graph: Carr's original, Press-Schechter's, peaks counting formulas, and the conditional method (based on the last one) presented here.

5. CONCLUSION

- A **joint formalism**, in which the compaction function & density perturbation conditions are both required, is proposed.
- Consequently, **smaller PBH abundance** is predicted as well as **smaller threshold** bounds.

References: [1] S. W. Hawking, Mon. Not. R. Astron. Soc. 1971, 152, 75-78; B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. 1974, 168, 399-415, [2] M. Shibata and S. Sasaki, PRD 1999, 60, 084002, [3] J. M. Bardeen, et al., Astrophys J., 1986, 304, 15.

JGRG30, 2021

Poster session

J. Fernando Abalos

CIEM-CONICET, Universidad Nacional de Córdoba (UNC)

"On constraint preservation for quasi-linear first-order PDEs"

[JGRG30 (2021) PA23]

On constraint preservation for quasi-linear first-order PDEs

Fernando Abalos

FaMAF-UNC, CIEM-CONICET, Córdoba, Argentina.

Abstract

We use partial differential equations (PDEs) to describe physical systems. In general, these equations include evolution and constraint equations. One method used to find solutions to these equations is the Free-Evolution approach, which consists in obtaining the solutions of the entire system by solving only the evolution equations. Certainly, this is valid only when the chosen initial data satisfies the constraints and the constraints are preserved in the evolution. We establish sufficient conditions that the PDEs have to satisfy to guarantee the constraint preservation. This is achieved by considering quasi-linear first-order PDEs, assuming the sufficient conditions and deriving strongly hyperbolic first-order partial differential evolution equations for the constraints. We show that, in general, these constraint evolution equations correspond to a family of equations parametrized by a set of free parameters.

Geometric formalism [1, 2, 3]

- **Quasi-Linear First-Order systems**:

$$E^A := \mathfrak{N}^{Aa}_{\alpha}(x,\phi)\nabla_a\phi^{\alpha} - J^A(x,\phi) = 0.$$

over a manifold M with $\dim M = n+1$

$\boldsymbol{a} \to$ index space-time, $\boldsymbol{A} \to$ index equations, $\boldsymbol{\alpha} \to$ index fields.

- **Foliation:** Let $t : M \to \mathbb{R}$ and the hypersurfaces $\Sigma_{t_0} = \{p \in M \mid t(p) = t_0\}$.

Locally $M = (-T,T) \times \underset{t\in(-T,T)}{\cup} \Sigma_t$.

- Adapted coordinates $x^a := (t, x^i)$, $i = 1,..,n$, with x^i adapted to the Σ_t's.
- Definitions $\quad t^a := (\partial_t)^a, \quad n_a := \nabla_a t \quad \Longrightarrow \quad t^a n_a = 1$
- Projector $\quad \eta^a_b := \delta^a_b - t^a n_b \quad \Longrightarrow \quad \eta^a_b t^b = 0 = \eta^a_b n_a$
- Initial value problem: To solve

$$E^A = \left(\mathfrak{N}^{Ac}_{\alpha} n_c\right) t^b \nabla_b\phi^{\alpha} + \mathfrak{N}^{Ac}_{\alpha}\eta^b_c\nabla_b\phi^{\alpha} - J^A = \mathfrak{N}^{A0}_{\alpha}\nabla_0\phi^{\alpha} + \mathfrak{N}^{Ai}_{\alpha}\nabla_i\phi^{\alpha} - J^A = 0 \text{ with initial data } \phi|_{\Sigma_0}.$$

Where $t^a\nabla_a = \nabla_0$, and $\eta^b_c\nabla_b = \nabla_i$ has no derivatives in the t^c direction since $t^c\eta^b_c\nabla_b = 0$

- **Condition 1:** the system has not gauge freedoms or the gauge has been fixed,

$$\mathfrak{N}^{A0}_{\alpha} := \mathfrak{N}^{Ac}_{\alpha} n_c \text{ has maximal rank}$$

- **Geroch fields** $C^{\Gamma a}_A$: $\quad C^{\Gamma(a}_A \mathfrak{N}^{|A|b)}_{\alpha} = 0$

$\Gamma \to$ index constraints.

- **Condition 2**: $\quad \dim \Gamma + \dim \alpha = \dim A$
- **Condition 3**: $\quad C^{\Gamma 0}_A := C^{\Gamma a}_A n_c$ has maximal rank
- **Constraints**

$$\psi^{\Gamma} := n_a C^{\Gamma a}_A E^A = C^{\Gamma 0}_A \mathfrak{N}^{A0}_{\alpha}\nabla_t\phi^{\alpha} + C^{\Gamma 0}_A\mathfrak{N}^{Ai}_{\alpha}\nabla_i\phi^{\alpha} - C^{\Gamma 0}_A J^A = C^{\Gamma 0}_A\mathfrak{N}^{Ai}_{\alpha}\nabla_i\phi^{\alpha} - C^{\Gamma 0}_A J^A = 0$$

ψ^{Γ} are called constraints equations since they do not have time derivatives

- **Evolution equations**:

$$e^{\alpha} := h^{\alpha}_A E^A = \nabla_t\phi^{\alpha} + h^{\alpha}_A\mathfrak{N}^{Ai}_{\beta}\nabla_i\phi^{\beta} - h^{\alpha}_A J^A = 0, \tag{1}$$

where the tensor h^{α}_A is called reduction and it satisfies $\quad h^{\alpha}_A\mathfrak{N}^{A0}_{\beta} = \delta^{\alpha}_{\beta}$

- Evolution and Constraint equations: $\begin{bmatrix} e^{\alpha} \\ \psi^{\Delta} \end{bmatrix} := \begin{bmatrix} h^{\alpha}_A \\ C^{\Delta 0}_A \end{bmatrix} E^A$

$$\begin{bmatrix} h^{\alpha}_A \\ C^{\Delta 0}_A \end{bmatrix} \text{ is invertible} \qquad \begin{bmatrix} \mathfrak{N}^{A0}_{\alpha} & h^A_{\Delta} \end{bmatrix}\begin{bmatrix} h^{\alpha}_B \\ C^{\Delta 0}_B \end{bmatrix} = \mathfrak{N}^{A0}_{\alpha} h^{\alpha}_B + h^A_{\Delta} C^{\Delta 0}_B = \delta^A_B$$

- **Geroch fields** $M^{\tilde{\Delta} a}_A$:

$$M^{\tilde{\Delta}(a}_A\mathfrak{N}^{|A|b)}_{\alpha} = 0, \qquad M^{\tilde{\Delta} a}_A \mathfrak{N}^{A0}_{\alpha} = 0 \qquad \Longrightarrow \qquad M^{\tilde{\Delta}0}_A = 0$$

$\tilde{\Delta} \to$ numbers the $M^{\tilde{\Delta} a}_A$ fields

- **Non unicity on Geroch fields**

$$\tilde{C}^{\Gamma a}_A = C^{\Gamma a}_A + N^{\Gamma}_{\tilde{\Delta}} M^{\tilde{\Delta} a}_A, \text{ with } N_{\tilde{\Delta}} \text{ free, produces the same constraints } \psi^{\Delta}$$

- **Integrability conditions**: Off-shell identities (for any ϕ^{α})

$$\nabla_d\left(C^{\Gamma d}_A E^A\right) = L^{\Gamma}_{1A}(x,\phi,\nabla\phi)\, E^A(x,\phi,\nabla\phi), \qquad \nabla_d\left(M^{\tilde{\Delta} a}_A h^A_{\Delta} C^{\Gamma 0}_A E^A\right) = L^{\tilde{\Delta}}_{2A}(x,\phi,\nabla\phi)\, E^A(x,\phi,\nabla\phi).$$

Results [3]

- **Theorem 1** : Assuming condition 1 to 4 and the integrability conditions, the followings off-shell identity are satisfied

$$\nabla_0\psi^{\Gamma} + \left(C^{\Gamma i}_A h^A_{\Delta} + N^{\Gamma}_{\tilde{\Delta}} M^{\tilde{\Delta} i}_{\Delta}\right)\nabla_i\psi^{\Delta} = +\left(L^{\Gamma}_{1A}h^A_{\Delta} - \nabla_d\left(C^{\Gamma d}_A h^A_{\Delta}\right) + N^{\Gamma}_{\tilde{\Delta}}\left(L^{\tilde{\Delta}}_{2A}h^A_{\Delta} - \nabla_i\left(M^{\tilde{\Delta} i}_{\Delta}\right)\right)\right)\psi^{\Delta}$$
$$+\left(L^{\Gamma}_{1A}\mathfrak{N}^{A0}_{\alpha} - \nabla_d\left(C^{\Gamma d}_A\mathfrak{N}^{A0}_{\alpha}\right)\right)e^{\alpha} - C^{\Gamma d}_A\mathfrak{N}^{A0}_{\alpha}\nabla_d e^{\alpha}, \tag{2}$$

where $N^{\Gamma}_{\tilde{\Delta}}$ can be freely chosen. Here $M^{\tilde{\Delta} a}_{\Gamma} := M^{\tilde{\Delta} a}_A h^A_{\Gamma}$

- The Subsidiary family systems are obtained in the on-shell case $e^{\alpha}(\phi) = 0$.
- If the system does not admit Geroch fields $M^{\tilde{\Delta} a}_A$, then $N^{\Gamma}_{\tilde{\Delta}} = 0$, $L^{\tilde{\Delta}}_{2A} = 0$. In these cases, the Subsidiary System is unique.

Strong hyperbolicity

- **Existence, Uniqueness and Continuity** of solutions with respect to the initial data
- **Principal symbol**: In the constant coefficient case, the evolution equations

$$e^{\alpha} = \partial_t\phi^{\alpha} + h^{\alpha}_A\mathfrak{N}^{Ai}_{\beta}\partial_i\phi^{\beta} - h^{\alpha}_A\tilde{J}^A = 0$$

are strongly hyperbolics if $h^{\alpha}_A(k_i)\,\mathfrak{N}^{Ai}_{\beta}k_i$ is diagonalizable with real eingenvalues. Here the reduction $h^{\alpha}_A(k)$ can depend on the wave vector k_i.

- We are looking for conditions under which the evolution equations (1) and (2) (with freezzing coefficients) are strongly hyperbolic. Namely,

$$\text{When are } h^{\alpha}_A\mathfrak{N}^{Ai}_{\beta}k_i \text{ and } \left(C^{\Gamma i}_A h^A_{\Delta} + N^{\Gamma}_{\tilde{\Delta}} M^{\tilde{\Delta} i}_{\Delta}\right) k_i \text{ diagonalizable with real eingenvalues?} \tag{3}$$

Referencias

[1] Geroch, R.. "Partial differential equations of physics." General Relativity, Aberdeen, Scotland (1996): 19-60.
[2] Abalos, F. and Reula, O. "On necessary and sufficient conditions for strong hyperbolicity in systems with constraints." Classical and Quantum Gravity 37.18 (2020): 185012.
[3] Abalos, F. "On constraint preservation and strong hyperbolicity." arXiv preprint arXiv:2111.06295 (2021).

Kronecker decomposition of pencil matrices

- **Principal symbol**: Since $\mathfrak{N}^{A0}_{\eta}$ has not right kernel, there exist $Y^A_B(k)$, $W^{\alpha}_{\eta}(k)$ invertible matrices, such that

$$\mathfrak{N}^{Ab}_{\eta} l(\lambda)_b = \lambda\left(-\mathfrak{N}^{A0}_{\eta}\right) + \left(\mathfrak{N}^{Ai}_{\eta}k_i\right) = Y^A_B(k)\, K^B_{\alpha}(\lambda,k)\, W^{\alpha}_{\eta}(k) \tag{4}$$

where $l(\lambda)_b = -\lambda n_b + k_b$ and $K^B_{\alpha}(\lambda,k)$ **is a diagonal block matrix** with the following three types of blocks

$$J_m(\lambda_i) = \begin{pmatrix} \lambda-\lambda_i & 1 & 0 & 0 \\ 0 & \lambda-\lambda_i & ... & 0 \\ 0 & 0 & ... & 1 \\ 0 & 0 & 0 & \lambda-\lambda_i \end{pmatrix} \in \mathbb{C}^{m\times m}, \quad L^T_m = \begin{pmatrix} \overbrace{\begin{matrix}\lambda & 0 & 0 & 0\end{matrix}}^{m} \\ \begin{matrix}1 & \lambda & 0 & 0\end{matrix} \\ \begin{matrix}0 & 1 & ... & 0\end{matrix} \\ \begin{matrix}0 & 0 & ... & \lambda\end{matrix} \\ \begin{matrix}0 & 0 & 0 & 1\end{matrix} \end{pmatrix} \in \mathbb{C}^{m+1\times m} \text{ and vanishing rows}$$

Results [3]

- **Definitions**: For each k_i, we call

$$d(k) := \dim\left(right\_\ker\left(C^{\Gamma 0}_A\mathfrak{N}^{Ai}_{\alpha}k_i\right)\right) \qquad r(k) := rank\left(C^{\Gamma 0}_A\mathfrak{N}^{Ai}_{\alpha}k_i\right) \qquad s(k) =: \dim\left(left\_\ker\left(C^{\Gamma 0}_A\mathfrak{N}^{Ai}_{\alpha}k_i\right)\right)$$

- **Lemma** : Assuming that conditions 1 to 3 are satisfied, then the **Kronecker decomposition of the pencil** $-\lambda\mathfrak{N}^{A0}_{\alpha} + \mathfrak{N}^{Ai}_{\alpha}k_i$ has the following blocks

$$J, r(k) \times L^T_1, s(k) \times \text{vanishing rows}$$

where J is a $d(k)\times d(k)$ matrix that includes all the Jordan matrices of the pencil.

- **Theorem 2** :There exist $h^{\alpha}_A(k)$, $h^A_{\Delta}(k)$ and $N^{\Gamma}_{\tilde{\Delta}}(k)$ such that **condition (3) is satisfied** if and only if $J = d(k)\times J_1(\lambda_i)$.

Wave Equation

- **Setting**: Let $g^{ab}\nabla_b\nabla_a\phi = 0$ over a space-time M with a Lorentzian metric g_{ab}.
  - Definitions: $\quad n_a = \nabla_a t, \quad \tilde{n}_b := -N n_a, \quad N := \frac{1}{\sqrt{-\nabla t.\nabla t}} \Longrightarrow \tilde{n}^a\tilde{n}_a = -1$

$$p^a := (\partial_t)^a - \beta^a, \quad (\partial_t)^a n_a = 1, \quad \beta^a n_a = 0, \quad N \text{ lapse and } \beta^a \text{ shift}$$

  - Defining $u_b := \nabla_b\phi$, we reduce the equation to first order in derivatives

$$E := g^{ab}\nabla_a u_b = 0 \qquad E_b := \nabla_b\phi - u_b = 0 \qquad E_{ab} := \nabla_{[a}u_{b]} = 0$$

  - Projector $\tilde{\eta}^a_b$, over Σ_t: $\quad \tilde{\eta}^a_b := \delta^a_b - p^a n_b = \delta^a_b + \tilde{n}^a\tilde{n}_b$
  - **Projected variables**: $\quad \tilde{u}^0 := \tilde{n}_b u^b \quad \tilde{u}_d := \tilde{\eta}_{db}u^b$
- **Evolution ($\tilde{e}$) and constraint (ψ) equations**

$$\begin{aligned} E &= -\tfrac{1}{N}\tilde{e}_1, \\ E_c &= -\tfrac{1}{N}\tilde{n}_c\tilde{e}_2 + \psi_{1c} \\ E_{ac} &= \tfrac{1}{N}\tilde{e}_{3[a}\tilde{n}_{c]} + \psi_{2ac} \end{aligned} \qquad \begin{aligned} \tilde{e}_1 &:= \mathcal{L}_p\tilde{u}^0 - N D_d\tilde{u}^d - N(\tilde{u}^w S_w) - N\tilde{u}^0 K, \\ \tilde{e}_2 &:= \mathcal{L}_p\phi - N\tilde{u}^0, \\ \tilde{e}_{3a} &:= \mathcal{L}_p\tilde{u}_a - N D_a\tilde{u}^0 - N\left(\tilde{u}^r K_{ar} + \tilde{u}^0 S_a\right) - N\tilde{u}^f K_{fa}, \\ \psi_{1c} &:= D_f\phi - \tilde{u}_f, \\ \psi_{2ac} &:= D_{[a}\tilde{u}_{c]}. \end{aligned}$$

- **Principal symbol**:

$$\begin{bmatrix} h^{\alpha}_A \\ C^{\Gamma 0}_A \end{bmatrix}\mathfrak{N}^{Aq}_{\alpha}\nabla_q\begin{bmatrix} \tilde{u}^0 \\ \phi \\ \tilde{u}_w \end{bmatrix} = \begin{bmatrix} N\tilde{n}^q & 0 & -N\tilde{\eta}^{qw} \\ 0 & N\tilde{n}^q & 0 \\ -N\tilde{\eta}^q_s & 0 & N\tilde{\eta}^w_s\tilde{n}^q \\ 0 & \tilde{\eta}^q_s & 0 \\ 0 & 0 & \tilde{\eta}^{[q}_s\tilde{\eta}^{w]}_y \end{bmatrix}\nabla_q\begin{bmatrix} \tilde{u}^0 \\ \phi \\ \tilde{u}_w \end{bmatrix}$$

  - **Kronecker decomposition of** $\begin{bmatrix} h^{\alpha}_A \\ C^{\Gamma 0}_A \end{bmatrix}\mathfrak{N}^{Aq}_{\alpha} l(\lambda)_q$:

$$J_1\left(N\sqrt{k.k} - \beta.k\right), J_1\left(-N\sqrt{k.k} - \beta.k\right), 3\times L^T_1, 3\times L^T_0$$

It satisfies the conditions of theorem 2.

  - $h^{\alpha}_A\mathfrak{N}^{Ai}_{\alpha}k_i$ is diagonalizable with real eingenvalues
- **Off-shell identities**: $0 = \nabla_f\left(\delta^{[f}_a\delta^{g]}_b E_g\right) + E_{ab}, \quad 0 = \nabla_c\left(\delta^{[c}_f\delta^a_g\delta^{b]}_h E_{ab}\right)$. Namely,

$$\nabla_d\left(C^{\Gamma d}_A E^A\right) = -L^{\Gamma}_{1A}E^A \qquad \nabla_d\left(\begin{bmatrix} 0 & -\tilde{\eta}^d_r & 0 & N\tilde{n}^d\tilde{\eta}^w_r & 0 \\ 0 & 0 & \tilde{\eta}^{[q}_g\tilde{\eta}^{d]}_h & 0 & N\tilde{n}^d\tilde{\eta}^{[w}_g\tilde{\eta}^{y]}_h \\ 0 & 0 & 0 & \tilde{\eta}^d_{[s}\tilde{\eta}^w_{r]} & 0 \\ 0 & 0 & 0 & 0 & \tilde{n}_f\varepsilon^{fdab}\tilde{\eta}^w_a\tilde{\eta}^y_b \end{bmatrix}\begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \tilde{e}_{3q} \\ \psi_{1w} \\ \psi_{2wy} \end{bmatrix}\right) \propto \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \tilde{e}_{3q} \\ \psi_{1w} \\ \psi_{2wy} \end{bmatrix},$$

  - **Subsidiary system** (on-shell case $\tilde{e}_1 = \tilde{e}_2 = \tilde{e}_{3q} = 0$):

$$0 = \begin{bmatrix} \tilde{\eta}^q_c & 0 \\ 0 & \tilde{\eta}^g_s\tilde{\eta}^h_r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\mathcal{L}_p\begin{bmatrix} \psi_{1q} \\ \psi_{2gh} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \tilde{\eta}^f_{[s}\tilde{\eta}^q_{r]} & 0 \\ 0 & \tilde{n}_d\varepsilon^{dfgh} \end{bmatrix} D_f\begin{bmatrix} \psi_{1q} \\ \psi_{2gh} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \tilde{\eta}^g_{[s}\tilde{\eta}^h_{r]} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \psi_{1q} \\ \psi_{2gh} \end{bmatrix}.$$

$\left(C^{\Gamma i}_A h^A_{\Delta} + N^{\Gamma}_{\tilde{\Delta}} M^{\tilde{\Delta} i}_{\Delta}\right) k_i$ is diagonalizable with real eingenvalues for $N^{\Gamma}_{\tilde{\Delta}} = 0 \Rightarrow$ constraint preservation

Conclusions

- In this work, we continue with the formalism proposed by Geroch for dealing with constraints. We give explicit expressions for the evolution of these constraints and find conditions that guarantee their strong hyperbolicity, and consequently, their preservation.

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Poster session

Minxi He

KEK

"Reheating Process in Mixed Higgs-R^2 Model"

[JGRG30 (2021) PA24]

Reheating Process in Mixed Higgs-R^2 Model

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Institute of Particle and Nuclear Studies, KEK

MH, R. Jinno, K. Kamada, S. C. Park, A. A. Starobinsky, J. Yokoyama, PLB791 (2019) 36-42;
MH, R. Jinno, K. Kamada, A. A. Starobinsky, J. Yokoyama, JCAP 01 (2021) 066;
MH, JCAP 05 (2021) 021

Introduction

Mixed Higgs-R^2 Model

Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R + \frac{M_{pl}^2}{12M^2}R^2 + \xi\left|\mathscr{H}\right|^2 R - \left|\partial\mathscr{H}\right|^2 - \lambda\left|\mathscr{H}\right|^4\right]$$

Einstein frame

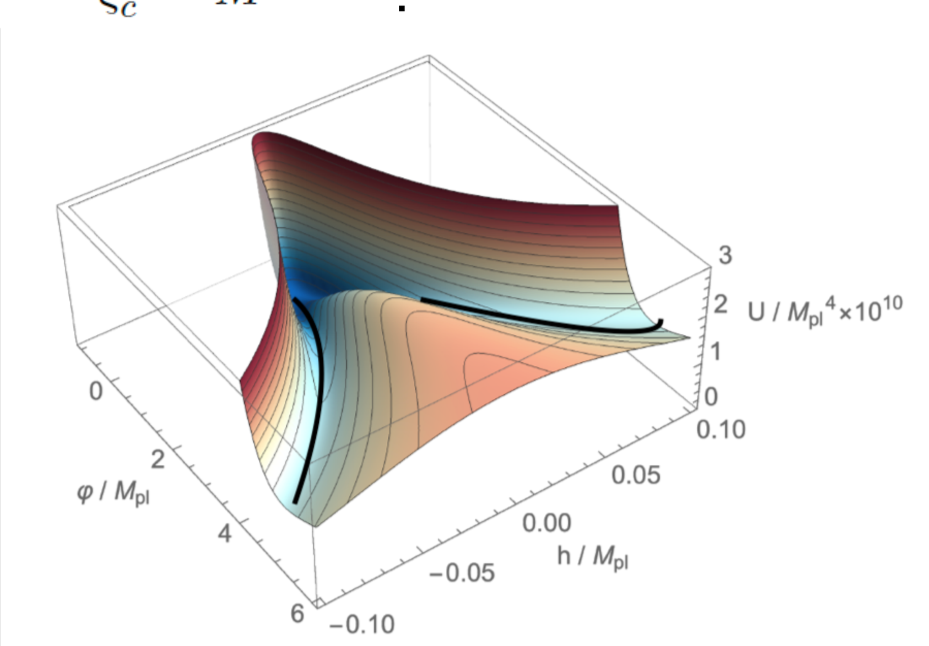
$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}(\partial\varphi)^2 - e^{-\alpha\varphi}\left|\partial\mathscr{H}\right|^2 - U(\varphi,\mathscr{H})\right]$$

$$U(\varphi,\mathscr{H}) = \lambda\left|\mathscr{H}\right|^4 e^{-2\alpha\varphi} + \frac{3}{4}M_{pl}^2M^2e^{-2\alpha\varphi}\left(e^{\alpha\varphi} - 1 - \frac{2}{M_{pl}^2}\xi\left|\mathscr{H}\right|^2\right)^2$$

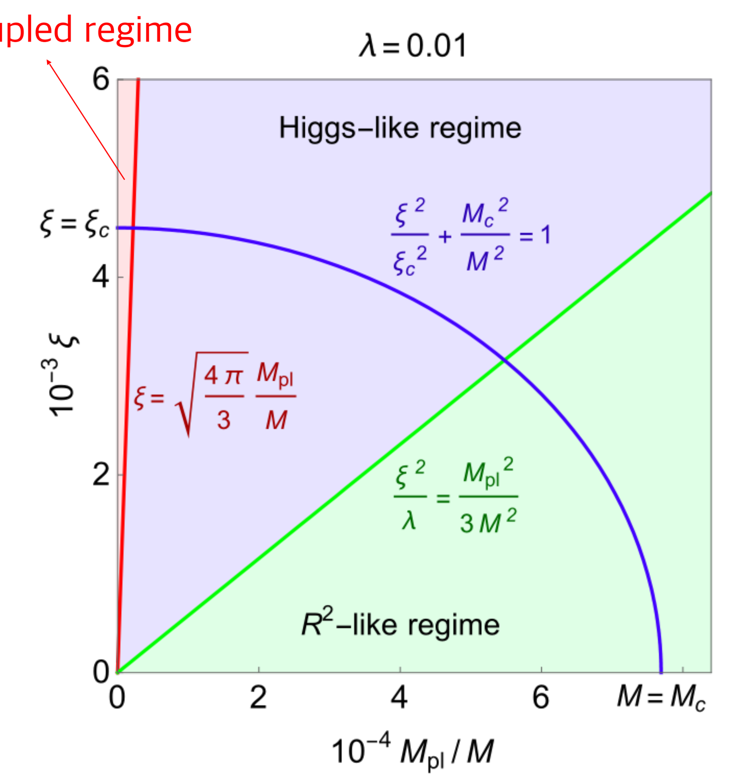
Motivation:

- Inevitable emergence of R^2 due to quantum correction [1]
- Combination of General Relativity and Standard Model of particle physics
- Natural UV-extension of Higgs inflation: cutoff scale can be pushed up to Planck scale [2]
- Effective single-field inflation (effective R^2-inflation) and observationally favored $n_s - r$ predictions with condition

$$\frac{\xi^2}{\xi_c^2} + \frac{M_c^2}{M^2} = 1 .$$



Potential and trajectory



Parameter space

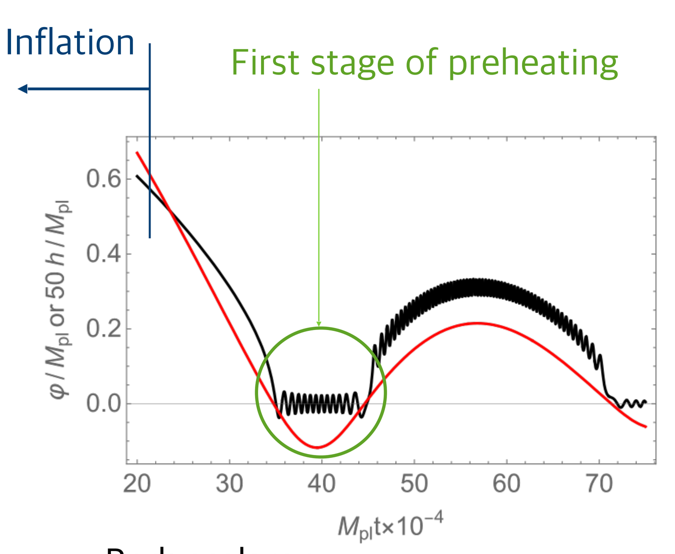
- Higgs-limit: $\xi \to \xi_c$ and $M \to \infty$
- R^2-limit: $\xi \to 0$ and $M \to M_c$

First Stage of Preheating

Production of Nambu-Goldstone (NG) mode by its spiky effective mass is analytically proven to be insignificant to reheat the Universe. The spike scale is much lower than the cutoff, unlike the Higgs inflation case.

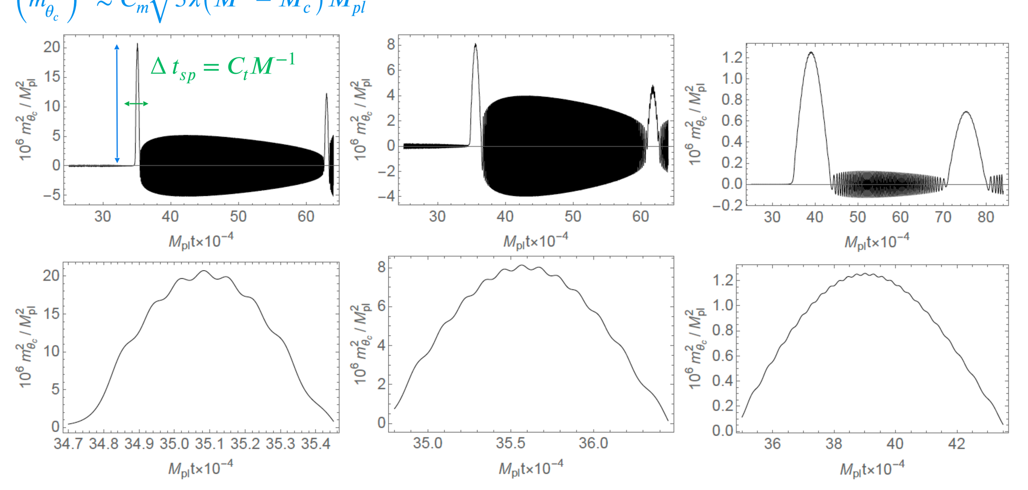
$$\mathcal{H}_{\rm SM}(t,\mathbf{x}) = h(t)e^{i\boxed{\theta_h(t,\mathbf{x})}}/\sqrt{2}$$

$$m_{\theta_c}^2 = -\frac{\alpha}{2}\frac{\partial U}{\partial\varphi} + \frac{e^{\alpha\varphi}}{h}\frac{\partial U}{\partial h} - \frac{3}{4}\frac{U}{M_{pl}^2} + \frac{5}{24}\frac{1}{M_{pl}^2}\left(\dot\varphi^2 + e^{-\alpha\varphi}\dot h^2\right)$$



Red: scalaron
Black: Higgs
Time evolution of Higgs and scalaron

$$\left(m_{\theta_c}^{sp}\right)^2 \approx C_m\sqrt{3\lambda\left(M^2 - M_c^2\right)}M_{pl}$$



Numerical results of the effective mass squared of NG mode

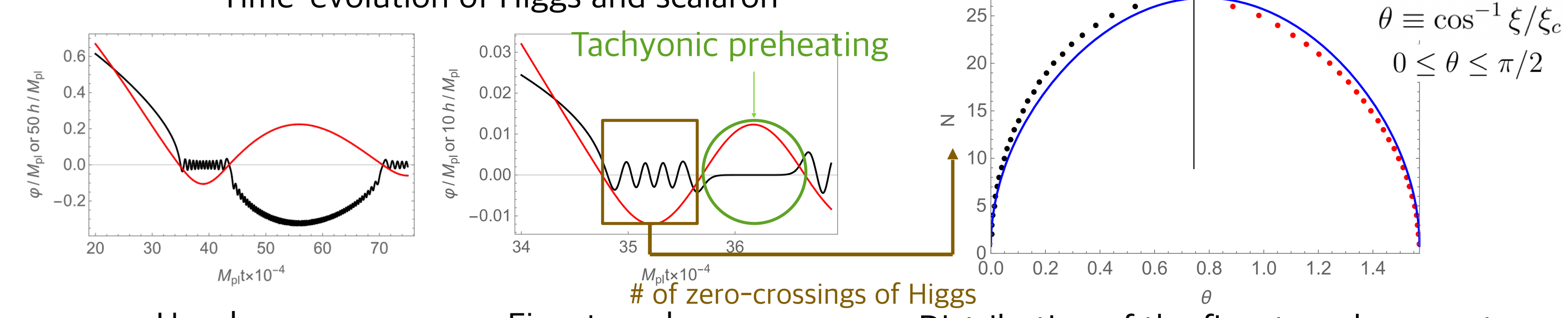
$$\rho_{\theta_c} \simeq 10^{-7}\left(\frac{\lambda}{0.01}\right)^{-2}\left(\frac{C_t}{2}\right)^{-4}U_{inf} \ll U_{inf}$$

Particle production by the spike is insufficient to reheat the Universe.

Tachyonic Preheating

The Higgs field and the NG mode can experience tachyonic instability with fine-tuned parameters, which can reheat the Universe within 1 e-fold.

Time-evolution of Higgs and scalaron



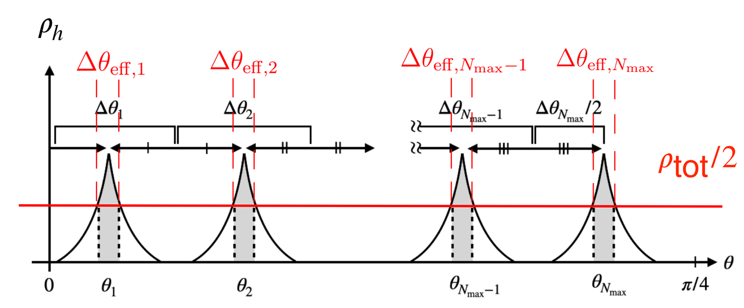
Usual case | Fine-tuned case | Distribution of the fine-tuned parameters

Number density and energy density of produced particles (e.g. Higgs) by tachyonic instability are given as

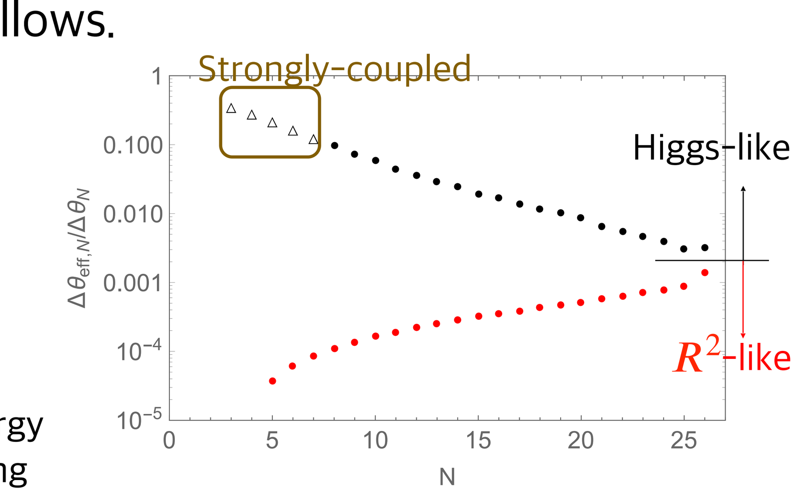
$$\Omega_k \equiv \int_{t_{\rm enter}(k)}^{t_{\rm exit}(k)}|\omega_{h,k}(t)|dt , \quad \omega_{h,k}^2 = k_p^2 + m_h^2(t)$$

$$n_k(t_{\rm exit,0}) \simeq \left|e^{\Omega_k} - e^{-\Omega_k}/4\right|^2 \simeq e^{2\Omega_k} , \quad \rho_{\delta h}(t_{\rm exit,0}) = \int\frac{d^3k}{(2\pi)^3}\omega_{h,k}(t)n_k(t)\Big|_{t=t_{\rm exit,0}}$$

Necessary degree of fine-tuning is defined as $\Delta\theta_{{\rm eff},N}/\Delta\theta_N$ where the two quantities are shown schematically as follows.



Schematic picture of parameter-dependence of energy density of produced particles by tachyonic preheating

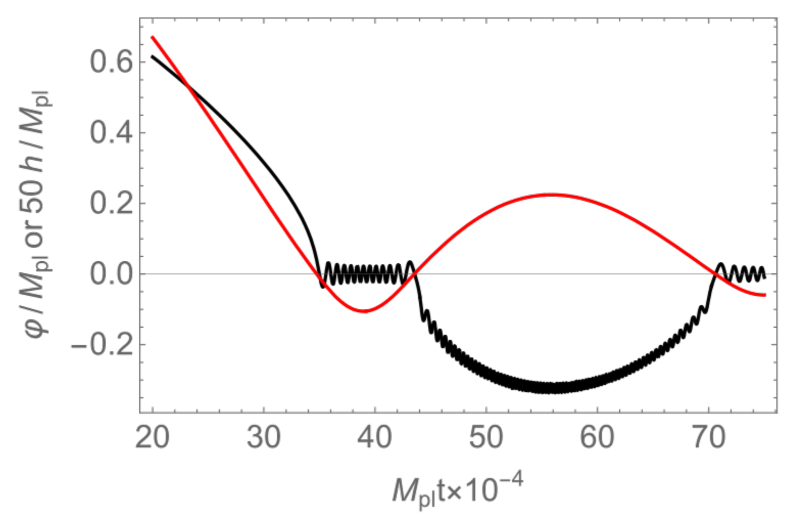


Necessary degree of fine-tuning

Perturbative Reheating

Small coherent oscillations of Higgs and scalaron induce perturbative decay of both field.

Time-evolution of Higgs and scalaron



Dominant decay channels

- Scalaron decays into Higgs

$$\Gamma_{\varphi\to\delta\tilde h\delta\tilde h} = \begin{cases} \frac{3}{16\pi}\frac{M^3}{M_{\rm pl}^2}\left(\frac{1}{6}+\xi\right)^2\left(1-12\xi\alpha|\varphi|\right)^{1/2} , & \varphi<0 , \\ \frac{3}{16\pi}\frac{\tilde M^3}{M_{\rm pl}^2}\left(\frac{1}{6}-2\xi\frac{M^2}{\tilde M^2}\right)^2\left(1-24\xi\frac{M^2}{\tilde M^2}\alpha\varphi\right)^{1/2} , & \varphi>0 , \end{cases}$$

- Higgs decays into top quarks

$$\Gamma_{h\to\tilde t\bar{\tilde t}} = \begin{cases} \frac{3y_t^2}{16\pi}\left(3\xi\alpha|\varphi|\right)^{1/2}M , & \varphi<0 , \\ \frac{3y_t^2}{16\pi}\left(6\xi\alpha\varphi\right)^{1/2}M\left(1-\frac{y_t^2}{\lambda}\frac{\tilde M^2}{M^2}\right)^{3/2} , & \varphi>0 , \end{cases}$$

The following equations are solved numerically.

$$\ddot\varphi + \left(3H + \Gamma_{\varphi\to\delta\tilde h\delta\tilde h}\right)\dot\varphi + \frac{\partial U}{\partial\varphi} + \frac{\alpha}{2}e^{-\alpha\varphi}\dot h^2 = 0 ,$$

$$\ddot h + \left(3H + \Gamma_{h_0\to\tilde t\bar{\tilde t}} + \Gamma_{h_0\to\tilde b\bar{\tilde b}} + \Gamma_{h_0\to\tilde W\tilde W} + \Gamma_{h_0\to\tilde Z\tilde Z}\right)\dot h + e^{\alpha\varphi}\frac{\partial U}{\partial h} - \alpha\dot\varphi\dot h = 0 ,$$

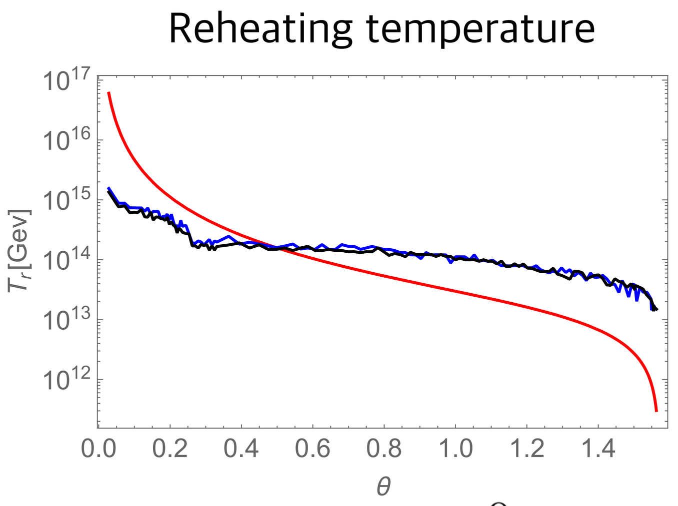
$$\frac{d\rho_{\rm rad}}{dt} + 4H\rho_{\rm rad} = \Gamma_{\varphi\to\delta\tilde h\delta\tilde h}\dot\varphi^2 + \left(\Gamma_{h_0\to\tilde t\bar{\tilde t}} + \Gamma_{h_0\to\tilde b\bar{\tilde b}} + \Gamma_{h_0\to\tilde W\tilde W} + \Gamma_{h_0\to\tilde Z\tilde Z}\right)\dot h^2 ,$$

$$3M_{\rm pl}^2H^2 = \rho_{\rm bg} + \rho_{\rm rad}$$

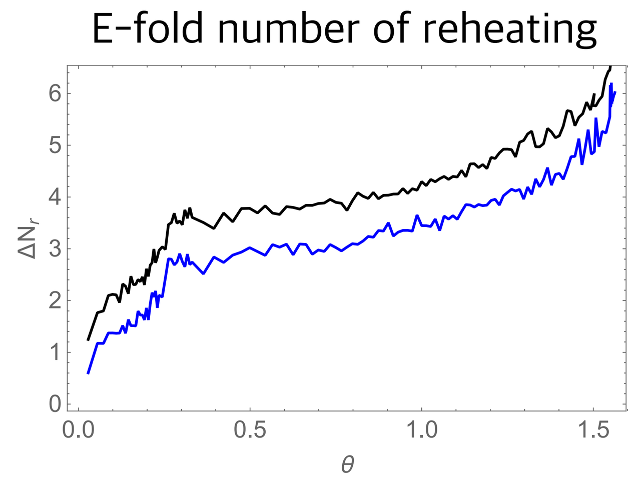
Numerical calculation continues until $\rho_{\rm rad}/\rho_{\rm tot} \gtrsim 20$ from which the Hubble parameter H_r is fixed.

Reheating temperature $\quad T_r = \left(\frac{90}{g_r\pi^2}\right)^{1/4}\sqrt{H_rM_{\rm pl}}$

Reheating temperature



E-fold number of reheating



Initial conditions: $\rho_{\rm rad} = 0,\ \rho_{\rm bg} = \rho_{\rm tot}$
Initial conditions: $\rho_{\rm rad} = 90\%\rho_{\rm tot},\ \rho_{\rm bg} = 10\%\rho_{\rm tot}$

The reheating temperature is independent of the detail of tachyonic preheating if such preheating cannot completely reheat the Universe. The number of e-fold of reheating depends on whether there exists preheating process.

References

[1] Salvi et al (2015), Netto et al (2016), Calmet, Kuntz (2016), Liu et al (2018), Ghilencea (2018), Ema (2019), Ema et al (2021).
[2] Ema (2017), Gorbunov, Tokareva (2019).

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Poster session

Takumi Hayashi

The University of Tokyo

"Vacuum decay with the Lorentzian path integral"

[JGRG30 (2021) PA25]

Vacuum decay with the Lorentzian path integral

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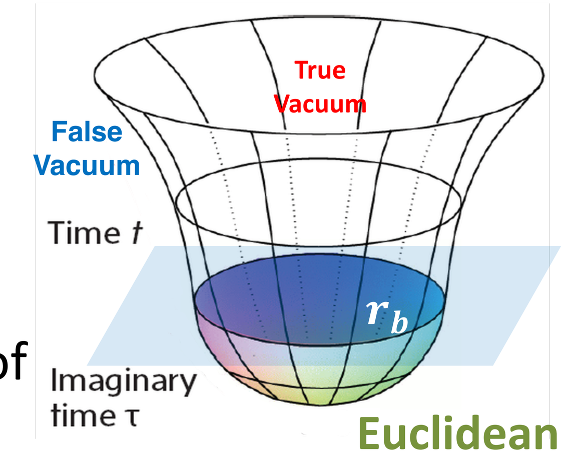
Introduction: Vacuum decay in Euclidean analysis

- Quantum tunneling of scalar field with metastable vacuum was analyzed in Euclidean spacetime.

Decay rate(=imaginary part of energy) is estimated by the action of bounce solution[1]

$$\Gamma_{\rm decay} \propto \int D\phi e^{-S_E[\phi]} \sim e^{-B}$$



After tunneling, bubble with specific size is nucleated and expands.

Q1. Is there any direct interpretation? (Notion of energy is ambiguous in curved spacetime)

Q2. How to analyze the bubble nucleation with different size?

✓ **Evaluate bubble nucleation probability in Lorentzian path integral**

$$P \propto \int_{\phi_{\rm false\ vac}}^{\phi_{\rm bubble}} D\phi e^{iS[\phi]}$$

Method: Bubble wall nucleation by Lorentzian integral

- Analyze bubble wall nucleation with thin-wall approximation

Polyakov-type quadratic action for bubble wall is

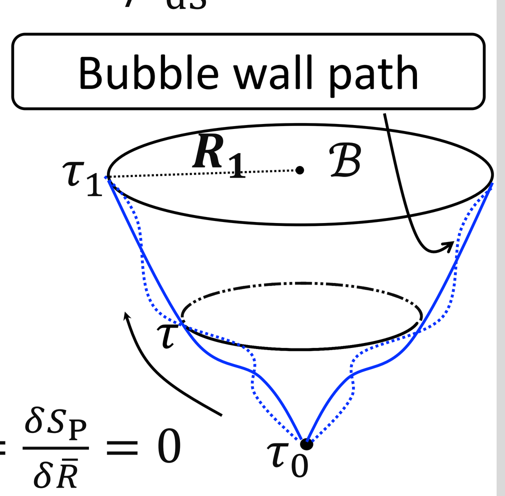
$$S_P[X^\mu,\gamma^{ab}] = -\sigma\int_{\partial\mathcal{B}} d^3x\sqrt{-\gamma}\,\frac{1}{2}[\gamma^{ab}\partial_a X^\mu \partial_b X_\nu - 1] + \Delta V\int_{\mathcal{B}} d^4\tilde{x}\sqrt{-g}$$

γ_{ab}: metric on wall,

and the bulk $\mathcal{B}$ is de-Sitter space (including Minkowski spacetime)

$$ds^2 = -f(R)dT^2 + f(R)^{-1}dR^2 + R^2 d\Omega_2^2,\quad f = 1 - R^2/l_{\rm ds}^2$$

Spherical symmetry: $X^\mu \to T(\tau), R(\tau)$ and $\gamma_{ab} \to N(\tau)$



✓ **Transition amplitude for bubble nucleation**

$$G(R_1;0) = \int_0^\infty dN \int_{R(\tau_0)=0}^{R(\tau_1)=R_1} \mathcal{D}T\mathcal{D}R \exp(iS_P[T,R,N])$$
$$\sim \int_0^\infty dN \exp[iS_{\rm eff}(N)],$$

with $S_{\rm eff}(N) = S_P[\bar{T},\bar{R},N]$, $\bar{T},\bar{R}$: classical sol. of $\frac{\delta S_P}{\delta T} = \frac{\delta S_P}{\delta R} = 0$

Path integral is reduced to the single **oscillatory** (phase) integral.[2]

✓ **Evaluate oscillatory integral by Picard-Lefshetz theory[3]**

$$\int_0^\infty dN \exp[iS_{\rm eff}(N)] \to \int_{\mathcal{C}} dN \exp[iS_{\rm eff}(N)],$$

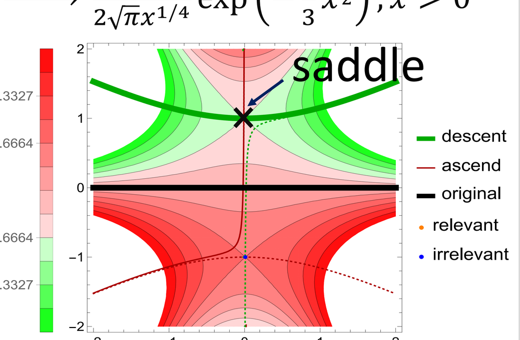
$\mathcal{C}$: steepest contours of $\mathbf{Re}[iS_{\rm eff}(N)]$

On steepest contours $\mathcal{C}$, $\mathbf{Im}[iS_{\rm eff}(N)]$ is stationary.

→ integration over $\mathcal{C}$ is **no longer oscillatory**!

approximate integration at the saddle point.

ex) $\mathrm{Ai}(x) = \int_{\mathbb{R}} dN \exp\left[i\left(\frac{N^3}{3}+xN\right)\right] \xrightarrow{N\sim i\sqrt{x}} \frac{1}{2\sqrt{\pi}x^{1/4}}\exp\left(-\frac{2}{3}x^{\frac{3}{2}}\right), x>0$



Result 1: Transition amplitude of critical size bubble

✓ **The transition amplitude is consistent with Euclidean decay rate**

Bubble radius predicted by Euclidean analysis: $\boldsymbol{r_b}$

boundary condition: $\bar{R}(\tau_0) = 0, \bar{R}(\tau_1) = r_b$

$$\to S_{\rm eff}(N) = \frac{2\pi\sigma r_b^3}{(1+r_b/r_0)^2}\left[\coth\frac{N}{r_b} - \frac{N}{r_b}\right]$$

Transition amplitude for **critical size** bubble

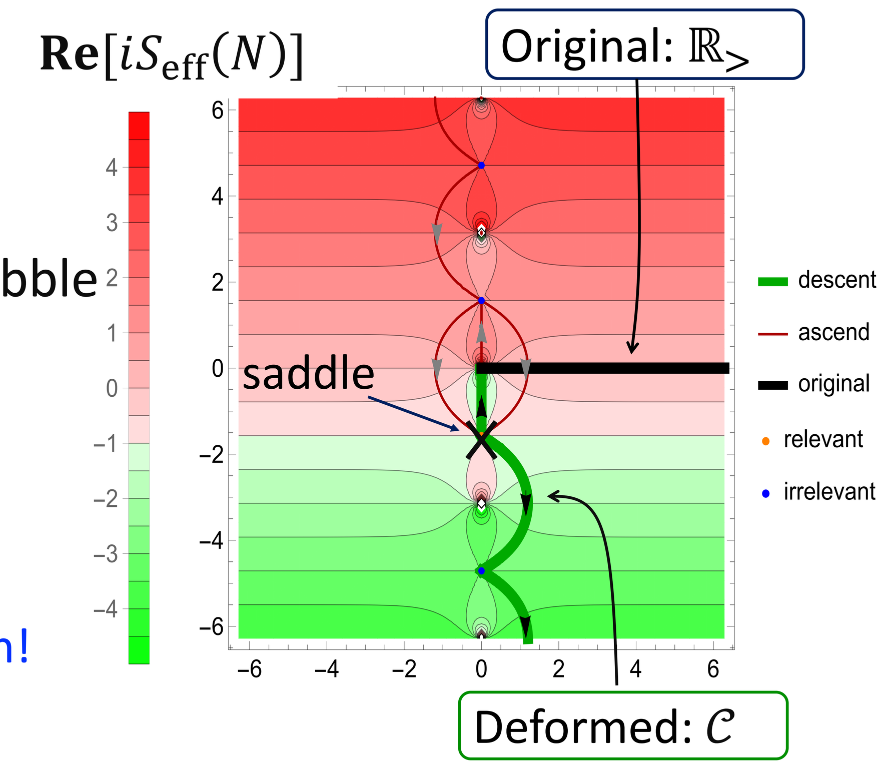
$$G(r_b;0) \sim \int_0^\infty dN \exp[iS_{\rm eff}(N)]$$

$$\overset{\rm saddle}{\sim} \exp\left[-\frac{\pi^2\sigma r_b^3}{(1+r_b/r_0)^2}\right]$$

$= B/2$, bounce action!

$$\to \boxed{P \sim |G(r_b;0)|^2 \sim \exp(-B)}$$

cf. $\Gamma_{\rm decay} \sim e^{-B}$



Result 2: Nucleation of larger bubble size

✓ **Probability of larger bubble nucleation is same**

boundary condition: $\bar{R}(\tau_0) = 0, \bar{R}(\tau_1) = R_1 > \boldsymbol{r_b}$

$$\to S_{\rm eff}(N) = \frac{2\pi\sigma l_{ds}^4}{r_b}\left\{\left(1+\frac{r_b^2}{r_0^2}\right)(\coth z - z) - \left(\frac{R_1^2}{r_b^2}-1\right)\frac{r_b^2}{l_{ds}^2}\coth z\right.$$
$$\left. - 2\left[\sqrt{\cosh^2 z - \left(\frac{R_1^2}{r_b^2}-1\right)\frac{r_0^2}{l_{ds}^2}} - \frac{r_b}{r_0}\mathrm{Arccoth}\left(\mathrm{csch}\, z\sqrt{\cosh^2 z - \left(\frac{R_1^2}{r_b^2}-1\right)\frac{r_0^2}{l_{ds}^2}}\right)\right]\right\} \quad (z = N/r_b)$$

Transition amplitude for **large size** bubble:

$$G(R_1;0) \sim \int_0^\infty dN \exp[iS_{\rm eff}(N)]$$

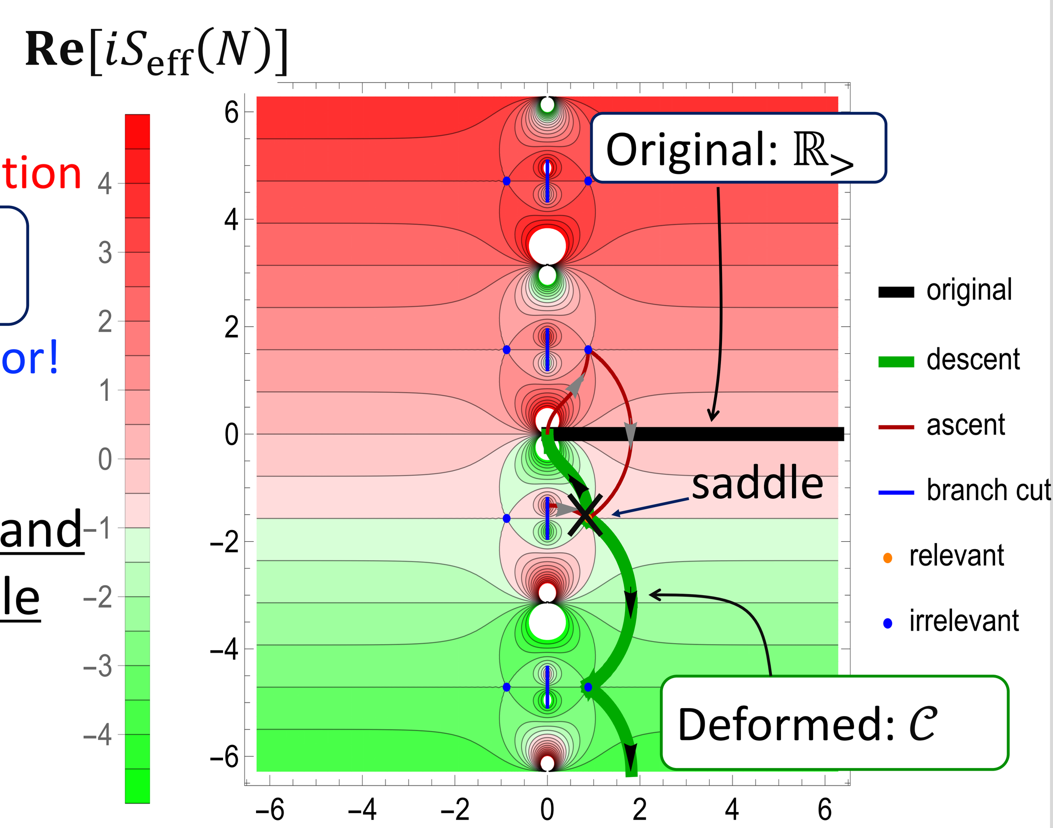
$$\overset{\rm saddle}{\sim} \exp\left[-B/2 + iS_{\rm cl}(r_b \to R_1)\right]$$

classical phase rotation

$$\to \boxed{P \sim |G(r_b;0)|^2 \sim \exp(-B)}$$

same factor!

Critical size bubble is nucleated and **classically** expand to large bubble



Result 3: Nucleation of larger bubble size

✓ **Nucleation probability of smaller bubble is larger**

boundary condition: $\bar{R}(\tau_0) = 0, \bar{R}(\tau_1) = R_1 < \boldsymbol{r_b}$

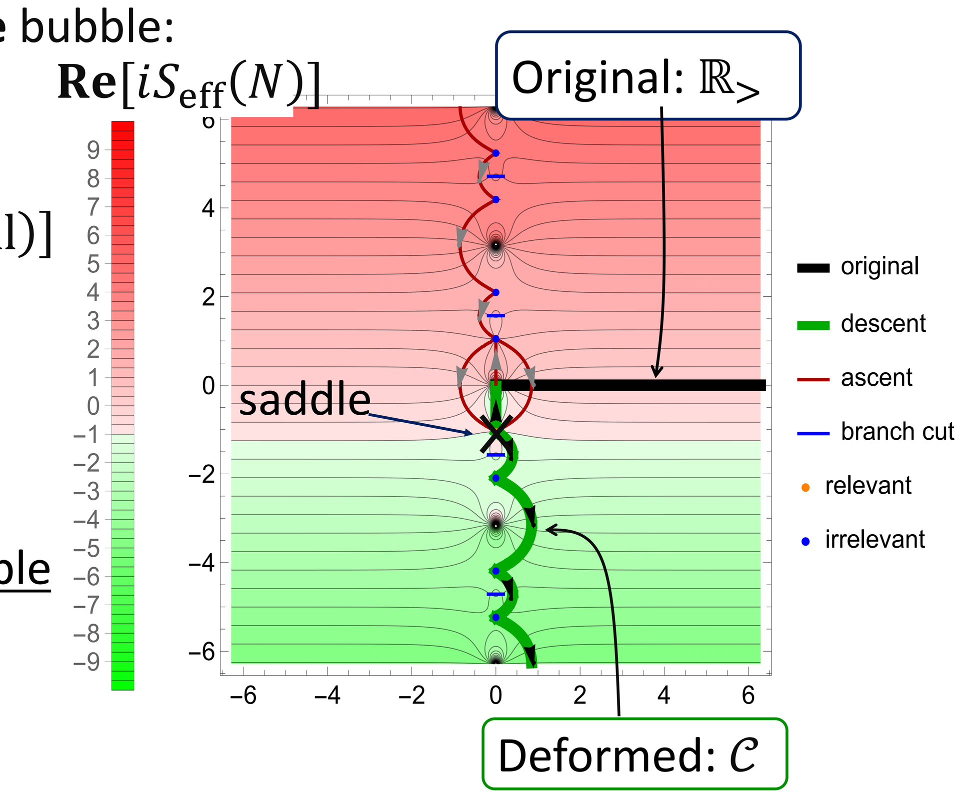
Transition amplitude for **small size** bubble:

$$G(R_1;0) \sim \int_0^\infty dN \exp[iS_{\rm eff}(N)]$$
$$\sim \exp[-B/2 + (\text{positive real})]$$

$$\boxed{P \sim |G(r_b;0)|^2 > \exp(-B)}$$

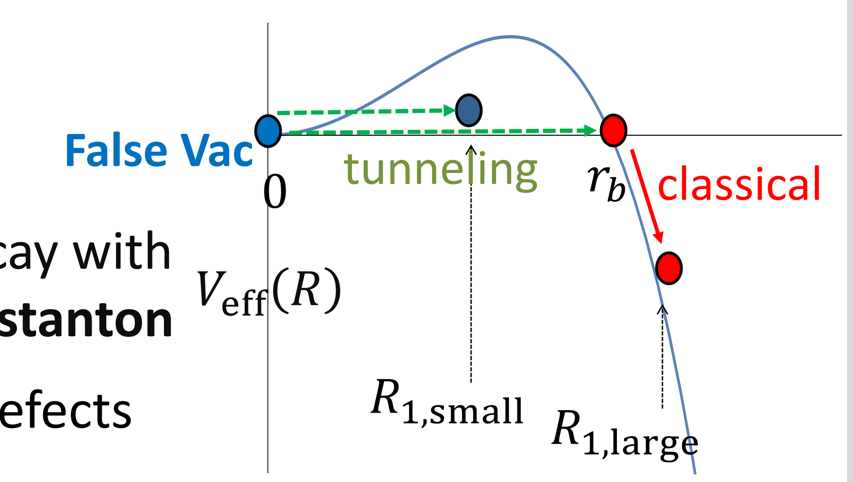
Higher probablity to find the bubble

But small bubble is not classical, due to energy conservation.



Summary and Discussion

- We formulate vacuum decay in Lorentzian path integral, providing the **direct interpretation as transition of bubble wall**.
- We find (1)**consistent probability** for bubble nucleation with Euclidean analysis, (2)large bubble nucleation is understood as bubble nucleation + classical expansion, and (3)higher probability to find small bubble.



Future work:

- generalization to the vacuum decay with dynamical gravity, known as **CDL instanton**
- nucleation of other topological defects

References

1. C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D 16, 1762 (1977).
2. Feldbrugge, J.-L. Lehners, and N. Turok, Phys. Rev. D 95, 103508 (2017)
3. S. Lefschetz, The Picard-Lefschetz Theory and Feynman Integrals (Springer-Verlag New York, 1975)

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