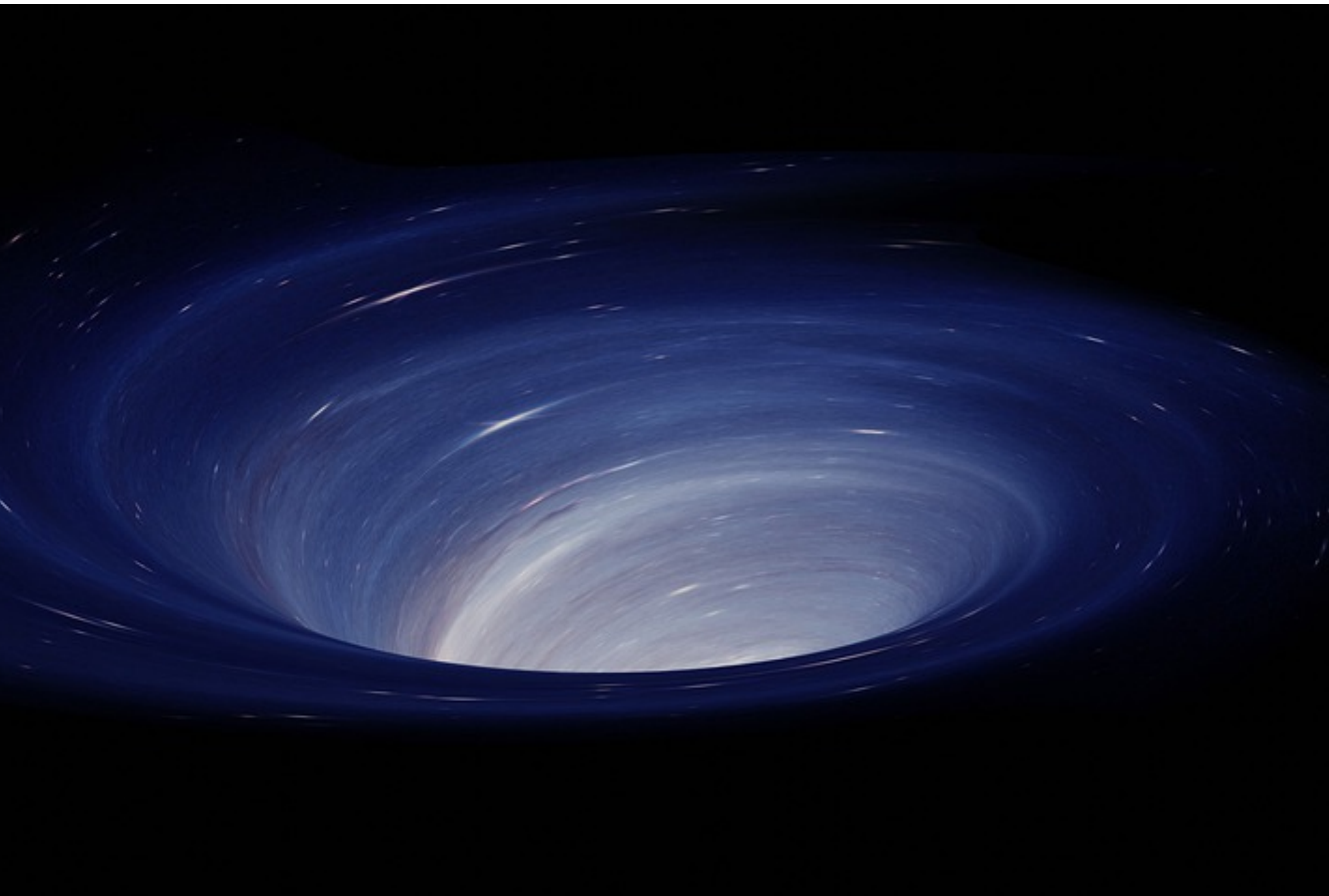
JGRG 30

Proceedings of The 30th Workshop on General Relativity and Gravitation in Japan

Online (https://www.tsujikawa.phys.waseda.ac.jp/jgrg30/)
Waseda University, Japan
6-10 December 2021

Volume II



Session B3a 14:30–16:00

[Chair: Ryo Namba]

Mian Zhu

The Hong Kong University of Science and Technology

“Alternative to inflation scenario from DHOST cosmology”

(15 min.)

[JGRG30 (2021) 120723]

Alternative-to-Inflation Scenarios from DHOST Cosmology

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December 6, 2021

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Content

Based on arXiv 2002.08269&2009.10351&2108.01339

1. Alternative-to-inflation scenarios: Motivation and challanges.
2. Evading the instabilities from DHOST cosmology
3. Phenomenological study of DHOST cosmology
4. Summary

Inflationary cosmology

A period of time before the hot big bang during which the universe undergoes exponential expansion, which is the leading paradigm of early universe cosmology.

Advantages

1. Solve the flatness, horizon and monopole problems
2. Provide a mechanism for the formation of Large Scale Structure through primordial fluctuations
3. The near de-Sitter expansion predicts a near scale invariant power spectrum of density perturbations
4. Predict small amounts of primordial non-Gaussianities

Why searching for alternative paradigm

- provides a possible solution to initial singularity problem and Trans-Planckian problem encountered in inflationary scenario.
- Alternative scenarios may provide distinctively phenomenology compared to inflation.
- We may look out of the paradigm of inflation to judge whether these solutions are economical or artificial, to measure the success of inflation.

Examples of early universe scenarios

We may classify the models of early universe cosmology through the evolution of the scale factor $a(t)$.

- Bounce cosmology: The universe starts with a contraction period ($H < 0$), followed by an expansion period ($H > 0$).
- Genesis cosmology: The universe starts in a quasi-Minkovskian configuration ($H \simeq 0$ but not 0), then transits to an expansion period ($H > 0$). Also named as "Emergent universe scenario" or "Slow expansion/contraction".

One exemplified Bounce cosmology model

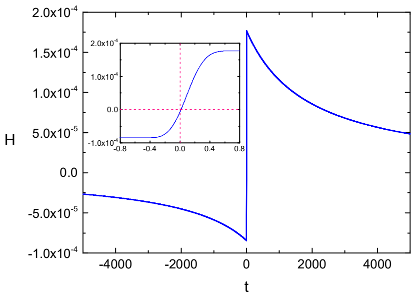

Figure 1: Cosmological evolution of a bounce cosmology model from [1].

One exemplified Galileon Genesis model

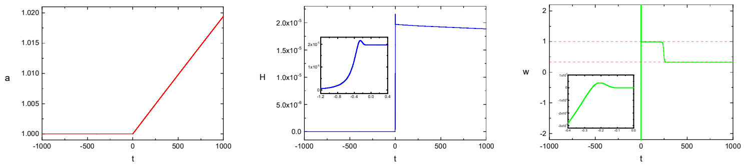

Figure 2: The background dynamics of an emergent universe model from [9].

Challange: NEC Violation

The equation-of-state parameter for the dominated matter content:

$$w = -1 - \frac{2\dot{H}}{3H^2} \ , \tag{1}$$

When the universe transit from a non-expanding state ($H < 0$ or $H \to 0$) to an expanding state $H > 0$, w will approach to $-\infty$, the Null Energy Condition(NEC) is violated.

NEC violation is a generic challange for alternative-to-inflation scenarios.

Constructing alternative scenarios in scalar tensor theory

- NEC violation commonly accompanied with ghost degree of freedom(DoF).

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- However, there always exists gradient instabilities (sound speed of scalar perturbations becomes negative) within the framework of ghost condensation.

- It is believed that within the framework Horndeski/Galileon theory [2, 3], which is the extension of ghost condensation theory, NEC can be stably violated, resulting in a healthy alternative model.

Horndeski/Galileon Theory

The most general scalar tensor theory with equation of motion up to second order, hence no ghost DoF.

$$L_H = \sum_{i=2}^{5} c_i L_i \ , \tag{2}$$

where $X \equiv 1/2(\partial\phi)^2$ and

$$L_2 = G_2(\phi, X) \ , \ L_3 = G_3(\phi, X)\Box\phi \ ,$$

$$L_4 = G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] \ ,$$

$$L_5 = G_5 G_{\mu\nu}\phi^{\mu\nu} + \frac{G_{5,X}}{3}\left[(\Box\phi)^3 - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi^{\mu\nu}\phi_{\mu\sigma}\phi^{\sigma}_{\nu}\right] \ .$$

No-go theorem for Horndeski theory

The Null Energy Condition cannot be stably violated in Horndeski theory, there will be either ghost degree of freedom or a gradient instability.

The theorem is proven to be true for bounce cosmology, genesis cosmology and Lorentzian wormhole [5, 6, 7, 8].

Solve the gradient instability: DHOST theory

- It is natural to guess if extending the Horndeski theory could evade the no-go theorem. However, modifying Horndeski action will lead to a higher order equation of motion, and generally it will result in the Ostrogradsky instability [11], where unexpectional ghost DoF appears.
- For scalar tensor theories with higher derivatives, the action should be in certain form to avoid the Ostrogradsky ghost. This kind of theory is dubbed as Degenerate Higher Order Scalar Tensor(DHOST) theory [12, 13, 14, 15].
- In our work [1, 9], we will show that specific DHOST theory can break the no-go theorem, and hence solve the gradient instability problem.

Quadratic DHOST theory

The most general action of scalar tensor theory with derivative up to second order is

$$S = \int d^4x\sqrt{-g}\left(f_2 R + C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma}\right) \equiv \int d^4x\sqrt{-g}\sum_{i=1}^{5} a_i L_i^{(2)} \ ,$$

with

$$L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu} \ , \ L_2^{(2)} = (\Box\phi)^2 \ , \ L_3^{(2)} = (\Box\phi)\phi^{\mu}\phi_{\mu\nu}\phi^{\nu} \ ,$$

$$L_4^{(2)} = \phi_{\mu}\phi^{\mu\rho}\phi_{\rho\nu}\phi^{\nu} \ , \ L_5^{(2)} = (\phi^{\mu}\phi_{\mu\nu}\phi^{\nu})^2 \ .$$

To evade the Ostrogradsky ghost, the form of $a_i = a_i(\phi, X)$ is constrained.

DHOST action in our model

In our work [1, 9], we use a Horndeski action

$$\mathcal{L}_H = K(\phi, X) + G(X)\Box\phi \ , \tag{3}$$

developed in [4, 10] to build up a bounce/Genesis cosmology, then merge it with the DHOST action, which is taken to be

$$\mathcal{L}_D = \frac{R}{2}f - \frac{f}{4X}\left(L_1^{(2)} - L_2^{(2)}\right) + \frac{f - 2Xf_X}{4X^2}\left(L_4^{(2)} - L_3^{(2)}\right) \ , \tag{4}$$

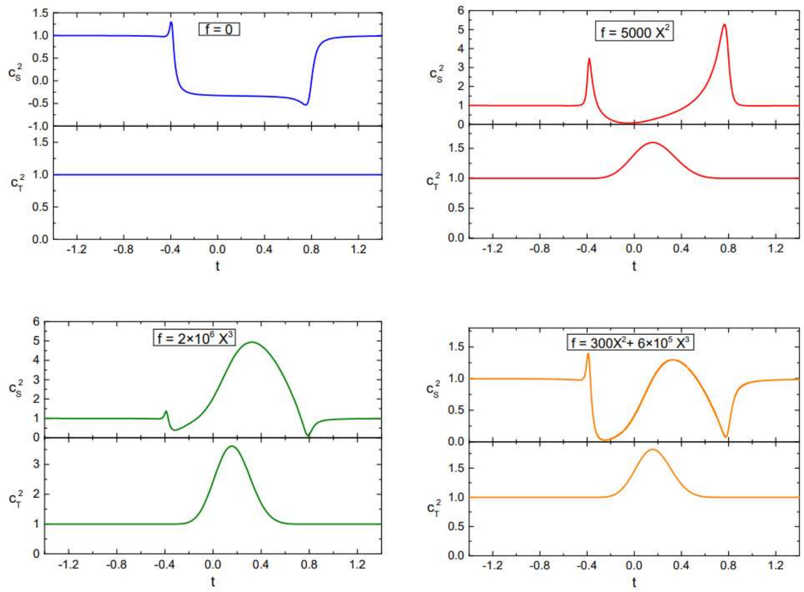
$f = f(\phi, X)$. The action (4) is of type ${}^{(2)}N - II$ DHOST theory, so the merge of $\mathcal{L}_D$ and $\mathcal{L}_H$ doesn't introduce ghost degree of freedom. Besides, when $f = f(X)$, the action (4) doesn't change the background dynamics.

EFT origin of our action

The action (4), when $f = f(X)$, corresponds to EFT operator $\delta K \delta g^{00}$ and $R^{(3)}\delta g^{00}$, which contribute only to the sound speed of scalar perturbation. Certain such operators are found to be able to remove the gradient instability in bounce cosmology [16, 17].

Hence, we believe the action (4) can help to build a healthy alternative-to-inflation model.

Solve gradient instability in bounce cosmology [1]



Solve gradient instability in emergent universe [9]

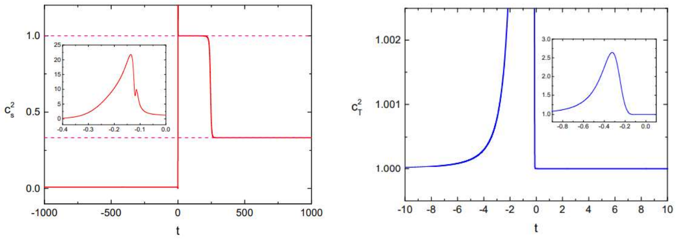


Figure 3: The dynamics of the sound speed of scalar and tensor perturbation c_s^2 and c_t^2 as a function of cosmic time t. The DHOST function is taken to be $f = 0.004X + 0.18X^2$.

Phenomenology of DHOST cosmology with $f = f(X)$

Main results: the DHOST action has negligible impact on the evolution of scalar/tensor perturbations. So the introduction of DHOST terms will not influence the predictions of observational signals [18].

Reason: the DHOST action has non-trivial contribution only during the transition period which is generically very quick, so its effect is restricted by the short duration of the transition period.

Conclusion: DHOST action (4) can serve as a mechanism to change the propogating speed of primordial perturbation, without altering the other physics.

Ongoing Project

The phenomenology of DHOST cosmology with $f = f(X)$ is limited: it only affect c_s^2 in limited region. We are searching for new phenomenons with a generic DHOST coupling $f(\phi, X)$.

We find in [1] that a DHOST coupling of the form $f(\phi) = e^{-a\phi^2}(1 - e^{-b\phi^2})$ can stably generate more than one bounce phase. It is interesting to study how to build up a multiple bounce cosmology with certain $f(\phi, X)$.

Besides, since conventional bounce and Genesis models give a blue scalar spectra, we wish to study whether certain $f(\phi)$ can help to acquire a scale invariant density spectrum.

Summary

- Alternative-to-inflation scenarios commonly require NEC violation, which generically leads to ghost and gradient instabilities.
- We introduce a type of DHOST action and construct stable cosmological models in bounce and Genesis scenarios.
- The DHOST coupling $f = f(X)$ provides negligible phenomenos. We expect new phenomenons for a more general DHOST coupling $f = f(\phi, X)$.

Thanks for your Attention

A. Ilyas, M. Zhu, Y. Zheng, Y. F. Cai and E. N. Saridakis, "DHOST Bounce," JCAP **09**, 002 (2020) doi:10.1088/1475-7516/2020/09/002 [arXiv:2002.08269 [gr-qc]].

G. W. Horndeski, "Second-order scalar-tensor field equations in a four-dimensional space," Int. J. Theor. Phys. **10**, 363 (1974). doi:10.1007/BF01807638

C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, "From k-essence to generalised Galileons," Phys. Rev. D **84**, 064039 (2011) doi:10.1103/PhysRevD.84.064039 [arXiv:1103.3260 [hep-th]].

Y. F. Cai, D. A. Easson and R. Brandenberger, "Towards a Nonsingular Bouncing Cosmology," JCAP **1208**, 020 (2012) doi:10.1088/1475-7516/2012/08/020 [arXiv:1206.2382 [hep-th]]

M. Libanov, S. Mironov and V. Rubakov, "Generalized Galileons: instabilities of bouncing and Genesis cosmologies and modified Genesis," JCAP **08**, 037 (2016) doi:10.1088/1475-7516/2016/08/037 [arXiv:1605.05992 [hep-th]].

T. Kobayashi, "Generic instabilities of nonsingular cosmologies in Horndeski theory: A no-go theorem," Phys. Rev. D **94**, no.4, 043511 (2016) doi:10.1103/PhysRevD.94.043511 [arXiv:1606.05831 [hep-th]].

R. Kolevatov and S. Mironov, "Cosmological bounces and Lorentzian wormholes in Galileon theories with an extra scalar field,” Phys. Rev. D **94**, no.12, 123516 (2016) doi:10.1103/PhysRevD.94.123516 [arXiv:1607.04099 [hep-th]].

- S. Akama and T. Kobayashi, “Generalized multi-Galileons, covariantized new terms, and the no-go theorem for nonsingular cosmologies,” Phys. Rev. D **95**, no.6, 064011 (2017) doi:10.1103/PhysRevD.95.064011 [arXiv:1701.02926 [hep-th]].
- A. Ilyas, M. Zhu, Y. Zheng and Y. F. Cai, “Emergent Universe and Genesis from the DHOST Cosmology,” JHEP **01**, 141 (2021) doi:10.1007/JHEP01(2021)141 [arXiv:2009.10351 [gr-qc]].
- P. Creminelli, A. Nicolis and E. Trincherini, “Galilean Genesis: An Alternative to inflation,” JCAP **11**, 021 (2010)

doi:10.1088/1475-7516/2010/11/021 [arXiv:1007.0027 [hep-th]].

- R. P. Woodard, “Ostrogradsky’s theorem on Hamiltonian instability,” Scholarpedia **10**, no.8, 32243 (2015) doi:10.4249/scholarpedia.32243 [arXiv:1506.02210 [hep-th]].
- J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, “Healthy theories beyond Horndeski,” Phys. Rev. Lett. **114**, no. 21, 211101 (2015)
- A. Nicolis, R. Rattazzi and E. Trincherini, “The Galileon as a local modification of gravity,” Phys. Rev. D **79**, 064036 (2009) doi:10.1103/PhysRevD.79.064036 [arXiv:0811.2197 [hep-th]].
- D. Langlois and K. Noui, “Degenerate higher derivative theories beyond Horndeski: evading the Ostrogradski instability,” JCAP **1602**, no. 02, 034 (2016)

doi:10.1088/1475-7516/2016/02/034 [arXiv:1510.06930 [gr-qc]].

- J. Ben Achour, M. Crisostomi, K. Koyama, D. Langlois, K. Noui and G. Tasinato, “Degenerate higher order scalar-tensor theories beyond Horndeski up to cubic order,” JHEP **1612**, 100 (2016) doi:10.1007/JHEP12(2016)100 [arXiv:1608.08135 [hep-th]].
- Y. Cai, Y. Wan, H. G. Li, T. Qiu and Y. S. Piao, “The Effective Field Theory of nonsingular cosmology,” JHEP **01**, 090 (2017) doi:10.1007/JHEP01(2017)090 [arXiv:1610.03400 [gr-qc]].
- P. Creminelli, D. Pirtskhalava, L. Santoni and E. Trincherini, “Stability of Geodesically Complete Cosmologies,” JCAP **11**,

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047 (2016) doi:10.1088/1475-7516/2016/11/047 [arXiv:1610.04207 [hep-th]].

- M. Zhu, A. Ilyas, Y. Zheng, Y. F. Cai and E. N. Saridakis, [arXiv:2108.01339 [gr-qc]].

Session B3a 14:30–16:00

[Chair: Ryo Namba]

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Tokyo Institute of Technology

“Conformal GUT inflation, Dark matter and Standard Model”

(15 min.)

[JGRG30 (2021) 120725]

Conformal GUT inflation, Dark matter and Standard Model

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Based on **arXiv:1806.09032 (Eur.Phys.J.C 79 (2019) 11, 945), arXiv: 2004.02921 (JHEP 07 (2020) 039)** in collaborations with Paulo Vargas Moniz, Simone Biondini

December 7, 2021

Outlines

Inflation (from particle physics)

- Inflationary cosmology after Planck
- Staorobinsky, Higgs inflation
- GUT inflation (Coleman-Weinberg inflation): Shafi-Vilenkin Model
- Scale invariance and Conformal invariance: Conformal Inflation ($\alpha-$ attractors etc..)
- Conformal GUT (CGUT) inflation
- Proton decay
- Reheating, Dark Matter and the Standard Model

Primordial seeds

Cosmic inflation 1980-201X

- The major problems of classical Big Bang cosmology: Horizon, flatness and monopole problems are resolved by the proposal of an accelerated (quasi de Sitter) expansion of the early Universe.
- Primordial quantum fluctuations can explain the LSS.

A. A. Starobinsky, A. H. Guth, A. D. Linde and F. Mukhanov etc.

Quantum gravity $\rightleftharpoons$ *Inflation* $\rightleftharpoons$ Standard Model of particle physics.

Planck CMB map

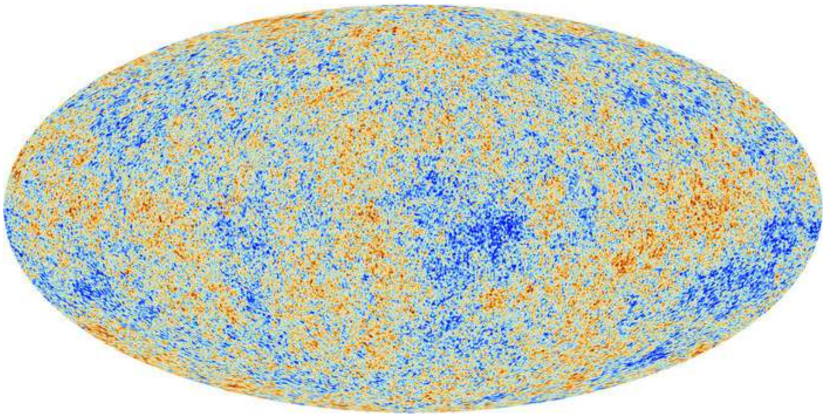


Figure: Planck 2015 CMB map

Inflation is not just about CMB but can get us BSM as well

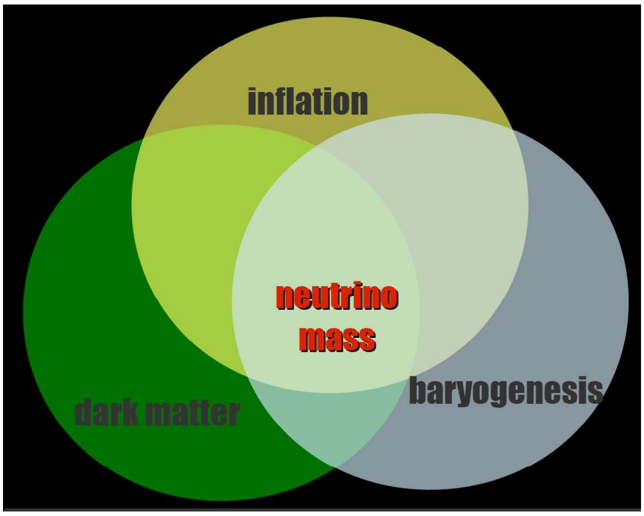


Figure: Taken from J. W. F. Valle arXiv: 1504.01913

Picture after Planck

- In the light of recent CMB data, Starobinsky ($R + R^2$) and Higgs inflation inflation stands out to be the best fit with ($N = 55 - 60$ $e-$ foldings)

$$n_s = 1 - \frac{2}{N}\,, \qquad r = \frac{12}{N^2}\,.$$

- (Quasi-) single field models are favored
- Canonical scalar that gives good fit is with data

$$\mathcal{L} = \sqrt{-g}\left[\frac{m_P^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \lambda\left(1-e^{-\sqrt{\frac{2}{3B}}\varphi}\right)^{2n}\right]$$

The above potential is what is known as Starobinsky like potential.

Starobinsky like potentials

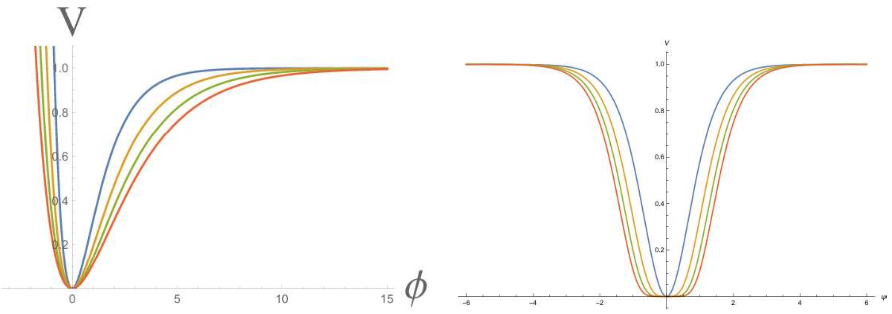


Figure: E-model and T-model

Local scale invariance

- A general action that is invariant under local scale transformations $g_{\mu\nu} \to \Omega^{-2}(x)\, g_{\mu\nu}\,,\ \phi \to \Omega(x)\phi\,,\ \chi \to \Omega(x)\chi$ can be written as

$$S_{local} = \int d^4x\,\sqrt{-g}\left[\frac{(\chi^2-\phi^2)}{12}R + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \phi^4 f\left(\frac{\phi}{\chi}\right)\right]$$

- Note that one of the field carries a wrong sign of kinetic term which must be gauge fixed to a constant $\chi = \sqrt{6}m_P$ (Spontaneous Breaking of Conformal Symmetry) A similar setup also proposed as conformal SM, bouncing Universe models in SUGRA by I. Bars, Steinhardt et al 2013-

Starobinsky like model from local scale invariance

- Considering $f\left(\frac{\phi}{\chi}\right) = \lambda\left(1-\frac{\phi^2}{\chi^2}\right)^2$, SBCS via gauge fixing $\chi = \sqrt{6}m_{\rm P}$ leads to the Einstein frame action in terms of a canonically normalized field $\phi = \sqrt{6}m_{\rm P}\tanh\left(\frac{\varphi}{\sqrt{6}m_{\rm P}}\right)$ and it is written as

$$S_{local} = \int d^4x\,\sqrt{-g}\left[\frac{m_{\rm P}^2}{2}R - \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \lambda m_{\rm P}^4\tanh^4\left(\frac{\varphi}{\sqrt{6}m_{\rm P}}\right)\right] .$$

Coleman-Weinberg inflation in GUT

- Shafi-Vilenkin (1983) model is one of the first realistic model of inflation which was based on SU(5) GUT
- inflation is a result of the spontaneous breaking of $\mathrm{SU}(5) \to \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ by a GUT field (**24-plet** adjoint Higgs) and an inflaton, which is a SU(5) singlet that rolls down to a vacuum expectation value (VEV).
- Interesting thing about the SV model is that it can lead to a successful reheating after inflation and predicts proton life time above the current lower bound Shafi et al (2008), Rehman (2008)
- Prediction of tensor-to-scalar ratio is $r \gtrsim 0.02$.

GUT symmetry breaking

- In this framework a new scalar field ϕ, a SU(5) singlet was considered and it weakly interacts with the GUT symmetry breaking field (adjoint) Σ and fundamental Higgs field H_5

$$\begin{aligned} V\left(\phi,\, \Sigma,\, H_5\right) =& \frac{1}{4}a\left(\mathrm{Tr}\Sigma^2\right)^2 + \frac{1}{2}b\mathrm{Tr}\Sigma^4 - \alpha\left(H_5^\dagger H_5\right)\mathrm{Tr}\Sigma^2 + \frac{\beta}{4}\left(H_5^\dagger H_5\right)^2 \\ & + \gamma H_5^\dagger \Sigma^2 H_5 + \frac{\lambda_1}{4}\phi^4 - \frac{\lambda_2}{2}\phi^2\mathrm{Tr}\Sigma^2 + \frac{\lambda_3}{2}\phi^2 H_5^\dagger H_5\,. \end{aligned}$$

- $\mathrm{SU}(5) \to \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ corresponds to

$$\langle\Sigma\rangle = \sqrt{\frac{1}{15}}\sigma.\mathrm{diag}\left(1,\, 1,\, 1, -\frac{3}{2}, -\frac{3}{2}\right)\,.$$

- The equations of motion for the σ field reads as

$$\Box\sigma + \frac{\lambda_c}{4}\sigma^3 - \frac{\lambda_2}{2}\sigma\phi^2 = 0$$

where $\lambda_c = a + \frac{7}{15}b$. Taking $\lambda_2 \ll \lambda_c$, the σ field quickly evolves to its local minimum of the potential given by

$$\sigma^2 = \frac{2\lambda_2}{\lambda_c}\phi^2$$

- Adding the radiative corrections due to the couplings $-\frac{\lambda_2}{2}\phi^2\mathrm{Tr}\Sigma^2$ and $\frac{\lambda_3}{2}\phi^2 H_5^\dagger H_5$, the effective potential of ϕ gets to the CW form given by (Linde (2005), Shafi (1983))

$$V_{eff} = A\phi^4\left[\ln\left(\frac{\phi}{\mu}\right) - \frac{1}{4}\right] + \frac{A\mu^4}{4}$$

GUT inflation with conformal symmetry

- Conformal symmetry is useful to generate flat potentials and the hierarchy of mass scales. Therefore, embedding conformal symmetry in GUT inflation is more realistic and helpful to generate simultaneously a Planck scale m_{P} along with the mass scale of X Bosons $M_X \sim 10^{15}$ GeV that sources proton decay.
- We extend the previously discussed CW inflation by means of introducing conformal symmetry in SU(5) GUT theory. We then obtain an interesting model of inflation by implementing spontaneous breaking of conformal symmetry together with GUT symmetry.
- We start with two complex singlet fields of SU(5) $(\Phi,\, \bar{X})$ where the real part of Φ $(\phi = \sqrt{2}\mathfrak{Re}\,[\Phi])$ is identified as inflaton. Gauge fixing the field $\bar{X}$ causes SBCS.

Conformal GUT setup

The conformally invariant action with complex SU(5) singlet fields $(\Phi,\, \bar{X})$ can be written as

$$S_G = \int d^4x\,\sqrt{-g}\Bigg[\left(|\bar{X}|^2 - |\Phi|^2 - \mathrm{Tr}\Sigma^2\right)\frac{R}{12} - \frac{1}{2}\left(\partial\Phi\right)^\dagger\left(\partial\Phi\right) + \frac{1}{2}\left(\partial\bar{X}\right)^\dagger\left(\partial\bar{X}\right)$$

$$- \frac{1}{2}\mathrm{Tr}\left[\left(D^\mu\Sigma\right)^\dagger\left(D_\mu\Sigma\right)\right] - \frac{1}{4}\mathrm{Tr}\left(\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu}\right) - V\left(\Phi,\, \bar{X},\, \Sigma\right)\Bigg],$$

where $D_\mu\Sigma = \partial_\mu\Sigma - ig\left[\boldsymbol{A}_\mu,\, \Sigma\right]$, $\boldsymbol{A}_\mu$ are the 24 massless Yang-Mills fields with Field strength defined by $\boldsymbol{F}_{\mu\nu} \equiv \boldsymbol{\nabla}_{[\mu}\boldsymbol{A}_{\nu]} - ig\left[\boldsymbol{A}_\mu,\, \boldsymbol{A}_\nu\right]$. Here we assume the Φ field coupling to the Higgs field H_5 is negligible and not very relevant during inflation.

Conformal GUT setup

We consider that the singlet field Φ is weakly coupled to the adjoint field Σ through the following tree level potential

$$V\left(\Phi,\,\bar{X},\,\Sigma\right)=\frac{1}{4}a\left(\mathrm{Tr}\Sigma^2\right)^2+\frac{1}{2}b\mathrm{Tr}\Sigma^4-\frac{\lambda_2}{2}|\Phi|^2\mathrm{Tr}\Sigma^2 f\left(\frac{\Phi}{\bar{X}}\right)+\frac{\lambda_1}{4}|\Phi|^4 f^2\left(\frac{\Phi}{\bar{X}}\right)\,.$$

the action (14) is conformally invariant under the following transformations

$$g_{\mu\nu}\to\Omega\left(x\right)^2 g_{\mu\nu}\quad,\quad\bar{X}\to\Omega^{-1}\left(x\right)\bar{X}\quad,\quad\Phi\to\Omega^{-1}\left(x\right)\Phi\quad,\quad\Sigma\to\Omega^{-1}\left(x\right)\Sigma\,.$$

The SBCS occurs with gauge fixing $\bar{X}=\bar{X}^*=\sqrt{3}M$, where $M\sim\mathcal{O}\left(m_{\mathrm{P}}\right)$. We assume inflation to happen in a direction $\mathrm{Im}\Phi=0$. We consider $\mathrm{SU}(5)\to\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ as a result we get

$$\sigma^2=\frac{2}{\lambda_c}\lambda_2\phi^2 f\left(\frac{\phi}{\sqrt{6}M}\right)\,.$$

where $f\left(\frac{\phi}{\sqrt{6}M}\right)=\left(1-\frac{\phi^2}{6M^2}\right)$.

Shape of effective potential in CGUT

After a conformal transformation the Einstein frame action look like $(m_P=1)$

$$S_G^E=\int d^4x\sqrt{-g}\left\{\frac{1}{2}R_E-\frac{1}{2M^2\left(1-\frac{\phi^2}{6M^2}\right)^2}\partial^\mu\phi\partial_\mu\phi-A\phi^4\left[\ln\left(\frac{\phi^2}{\mu^2}\right)-\frac{1}{4}\right]-\frac{A\mu^4}{4}\right.$$

Canonically normalizing the scalar field as $\phi=\sqrt{6}M\tanh\left(\frac{\varphi}{\sqrt{6}}\right)$ yields the Einstein frame potential

$$V_E\left(\varphi\right)=36AM^4\tanh^4\left(\frac{\varphi}{\sqrt{6}}\right)\left(\ln\left(\frac{6M^2\tanh^2\left(\frac{\varphi}{\sqrt{6}}\right)}{\mu^2}\right)-\frac{1}{4}\right)+\frac{A\mu^4}{4}\,. \qquad (1)$$

The corresponding VEV of the canonically normalized field is $\langle\varphi\rangle=\sqrt{6}\arctan\left(\frac{\mu}{\sqrt{6}M}\right)$.

Shape of the potential in CGUT

Inflationary predictions in CGUT inflation are exactly same as R^2 and Higgs inflation.

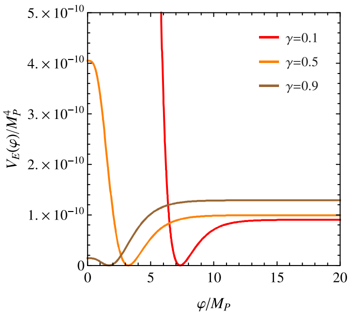


Figure: The potential $V_E(\varphi)$ for $A = 5 \times 10^{-12}$ and different values of $\gamma = \frac{m_P}{M}$.

Proton decay

From the VEV of the singlet field ϕ we can compute the masses of superheavy gauge bosons as

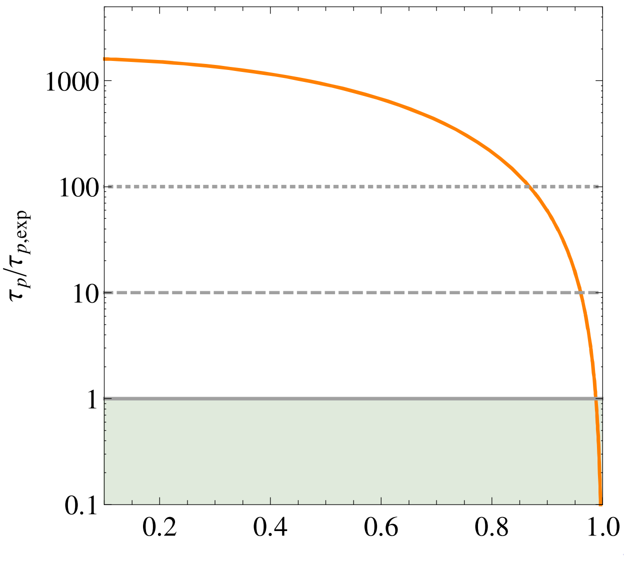
$$M_X = \sqrt{\frac{5}{3}}\frac{g v_\sigma}{2} \simeq \sqrt{5\lambda_2\left(1-\gamma^2\right)} M_{\mathrm{P}} \,. \tag{2}$$

The key prediction of GUT models is proton decay $\left(p \to \pi^0 + e^+\right)$ mediated by X, Y gauge bosons. The life time of proton can be computed using

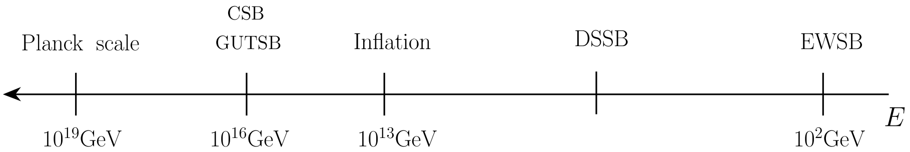
$$\tau_p = \frac{M_X^4}{\alpha_G^2 m_{pr}^5} \,, \tag{3}$$

where m_{pr} is proton mass and $\alpha_G \sim 1/40$ is the GUT coupling constant. The current lower bound on proton life time is given by $\tau_p > 1.6 \times 10^{34}$ years indicates $M_X \sim 4 \times 10^{15}$ GeV (Super-Kamiokande Collaboration, arXiv:1605:03597).

Proton Life time in CGUT



Dark matter and Standard Model from CGUT, arXiv: 2004.02921


Planck scale

CSB
GUTSB

Inflation

DSSB

EWSB

10^{19}GeV

10^{16}GeV

10^{13}GeV

10^{2}GeV

E

Figure: In this plot we depict the associated hierarchy of energy scales (from left to right) and symmetry breaking patterns in our model. We obtain Starobinsky-like inflation after Conformal Symmetry Breaking (CSB) and GUT Symmetry Breaking (GUTSB) respectively. Later on, a Dark Sector Symmetry Breaking (DSSB) occurs at some intermediate scale between the inflation scale and Electroweak Symmetry Breaking (EWSB) scale.

Dark matter and Standard Model from CGUT

$$
\begin{aligned}
V(\phi,\mathrm{SM},\mathrm{DS}) \equiv V(\phi,\chi,\psi,S,H) \\
&= Y_\psi f_\psi \left(\frac{\phi}{\chi}\right)\phi\,\bar{\psi}\psi - \lambda_{S1} f_{S1}\left(\frac{\phi}{\chi}\right)\phi^2 S^\dagger S + \lambda_{S2} f_{S2}\left(\frac{\phi}{\chi}\right)\left(S^\dagger S\right)^2 \\
&\quad - \lambda_{HS}\left(S^\dagger S\right)\left(H^\dagger H\right) + \lambda_H\left(H^\dagger H\right)^2 ,
\end{aligned}
$$

where

$$
f_\psi = \left(1-\frac{\phi^2}{\chi^2}\right)^{\alpha}, \quad f_{S1} = \left(1-\frac{\phi^2}{\chi^2}\right)^{\beta_1}, \quad f_{S2} = \left(1-\frac{\phi^2}{\chi^2}\right)^{\beta_2}.
$$

Therefore conformal symmetry is preserved

$$
S \to \Omega^{-1}(x)S\,, \quad H \to \Omega^{-1}(x)H\,, \quad \psi \to \Omega^{-3/2}(x)\psi\,.
$$

Conclusions

- In arXiv: 1806.09032, arXiv: 2004.02921 we have established conformal GUT inflation which gives same predictions as Starobinsky and Higgs inflation.
- In arXiv: 1806.09032 CGUT inflation studied with additional predictions such as proton life time, neutrino masses, non-thermal leptogenesis.
- In arXiv: 2004.02921 we addressed issue of dark matter and production of standard model particles.
- It would be interesting to combine the above studies in a coherent manner.
- Since there are heavy GUT fields during inflation, their signature can be probed with "cosmological collider physics".

Thank you for your attention

Session B3a 14:30–16:00

[Chair: Ryo Namba]

Beatriz Elizaga Navascués

Waseda University

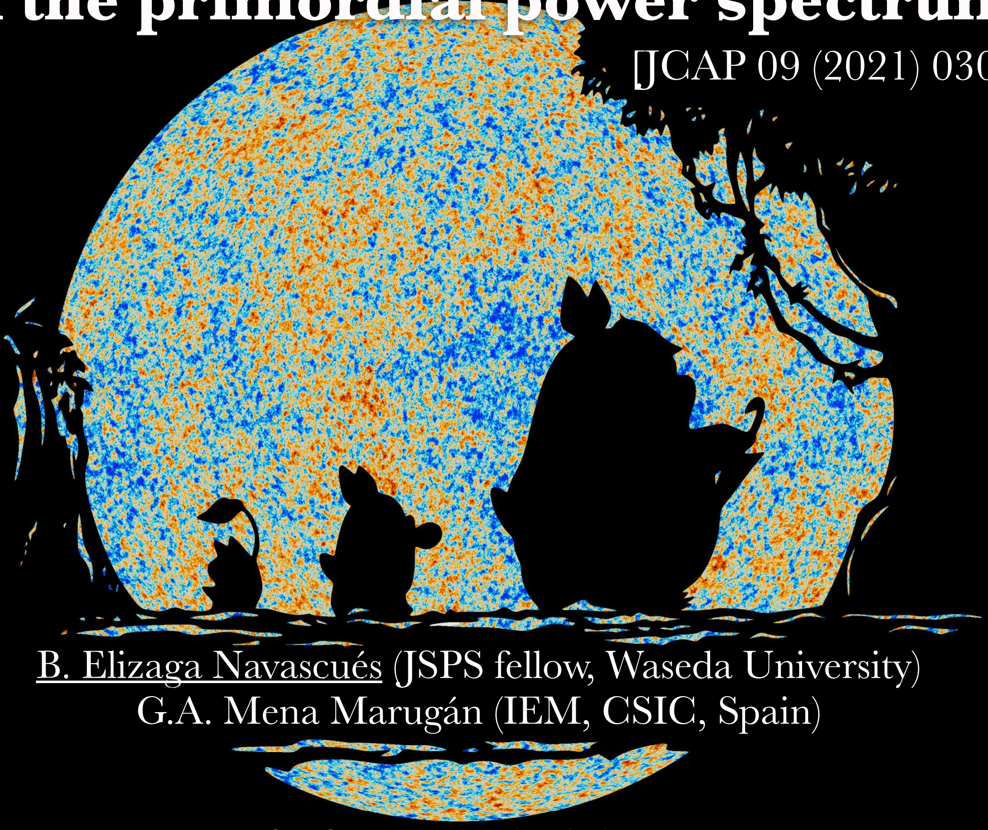
“Relativistic vs. loop quantum effects in the primordial power spectrum”

(15 min.)

[JGRG30 (2021) 120726]

Relativistic vs. loop quantum effects in the primordial power spectrum

[JCAP 09 (2021) 030]



B. Elizaga Navascués (JSPS fellow, Waseda University)
G.A. Mena Marugán (IEM, CSIC, Spain)

JGRG30, December 7th, 2021

Motivation

- Standard cosmological model with inflation is extremely successful describing primordial inhomogeneities.

- Is it completely satisfactory, considering the primeval stages of the Universe with increasingly high curvature?

Motivation

- Standard cosmological model with inflation is extremely successful describing primordial inhomogeneities.
- Is it completely satisfactory, considering the primeval stages of the Universe with increasingly high curvature?
- Theoretically:
    - Big-Bang singularity: loss of predictability.
    - Quantum gravity phenomena?
    - Non-inflationary epoch: State for the perturbations?

Motivation

- Standard cosmological model with inflation is extremely successful describing primordial inhomogeneities.
- Is it completely satisfactory, considering the primeval stages of the Universe with increasingly high curvature?

- Observationally:
    - Angular power spectrum in CMB: Anomalies.
    - Power suppression $\ell \lesssim 30$, lensing amplitude > 1, …
    - Strongly affected by cosmic variance, but could point to new physics → Planck regime of the Universe?

Motivation

- Standard cosmological model with inflation is extremely successful describing primordial inhomogeneities.
- Is it completely satisfactory, considering the primeval stages of the Universe with increasingly high curvature?
- Theoretical and observational concerns.
- Promising candidate: Loop Quantum Cosmology (LQC).
- Typically includes a classical pre-inflationary epoch.
- Robust predictions **require** disentangling LQC from GR effects on the evolution of the perturbations.

Loop Quantum Cosmology: Mukhanov-Sasaki equations

Why LQC?

- Canonical quantization program for spacetimes with high degree of symmetry: e.g. cosmological spacetimes.
- Techniques from the non-perturbative theory of LQG.
- Widely studied in e.g. FLRW-type cosmologies.
- Provides robust quantum mechanisms to resolve the cosmological singularity ⟶ Big Bounce.
- Effective bouncing regimes with modified Friedmann eqs.
- Can be combined with standard quantum field theory techniques to include inhomogeneities.

Perturbations in LQC

- Hybrid quantization of perturbed cosmology with inflaton:
  - ★ Background cosmology: LQC techniques.
  - ★ Gauge-invariant perturbations: Fock representation.

Perturbations in LQC

- Hybrid quantization of perturbed cosmology with inflaton:
  - ★ Background cosmology: LQC techniques.
  - ★ Gauge-invariant perturbations: Fock representation.
- Mean-field approximation on quantum constraint equation

  → Effective constraint for the perturbations, depends on background geometry via expectation values.
- Effective Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + [k^2 + s_{\text{eff}}]v_{\vec{k}} = 0, \qquad s_{\text{eff}} = s_{\text{eff}}(\eta)$$

Mass codifies LQC effects on the background.

Mukhanov-Sasaki equations

- Effective Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + [k^2 + s_{\text{eff}}]v_{\vec{k}} = 0$$

- Restrict to background states with effective LQC behavior.
- Phenomenologically interesting solutions: Large observable scales a/k today were ~ order of curvature at the bounce.
- They are all such that the kinetic energy of the inflaton greatly dominate over its potential at the bounce.

Mukhanov-Sasaki equations

- Effective Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + [k^2 + s_{\text{eff}}]v_{\vec{k}} = 0$$

- Restrict to background states with effective LQC behavior.
- Phenomenologically interesting solutions: Large observable scales a/k today were $\sim$ order of curvature at the bounce.
- They are all kinetically dominated at the bounce.
- Quantum effects tightly narrowed around the bounce.
- They imply a short-lived inflation ($\gtrsim$ 65 e-folds), and a classical deccelerated preinflationary expansion.

GR with KD and LQC: Approximations

Inflation with KD epoch in GR

- Deep in the pre-inflationary epoch, the potential is completely negligible compared with kinetic energy.

- Approximate this classical epoch (η_0, η_i) as a Friedmann universe with a massless scalar field.

- The evolution of the Universe during slow-roll inflation is of quasi-de Sitter type.

- For our purposes here, we ignore transition effects and deviations from an exact de Sitter phase.

Inflation with KD epoch in GR

- Deep in the pre-inflationary epoch, the potential is completely negligible compared with kinetic energy.

- Approximate this classical epoch (η_0, η_i) as a Friedmann universe with a massless scalar field.

- Ignore slow-roll corrections: Approximate the inflationary period $[\eta_i, \eta_{\text{end}}]$ as de Sitter.

- Ignore transition effects: Make an instantaneous matching between both periods.

Inflation with KD epoch in LQC

- Approximate pre-inflationary epoch (η_0, η_i) as a Friedmann universe with a massless scalar field.
- Approximate the inflationary period $[\eta_i, \eta_{\text{end}}]$ as de Sitter.
- For the interval $[\eta_B, \eta_0]$ with strong loop quantum effects, we approximate the mass by a Pöschl–Teller potential.
- The potential is fixed to match the exact values of the (KD) LQC and GR masses at, respectively, the bounce and η_0.
- The goodness of the approximation depends on the choice of η_0. Relative error can be made to grow at most to 15%, and quickly become negligible afterwards.

Vacuum state and power spectra

Power spectrum in de Sitter

- In de Sitter, solutions to Mukhanov-Sasaki equations:

$$\mu_k = A_k \frac{e^{ik(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}}{\sqrt{2k}}\left[1+\frac{i}{k(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}\right]$$

$$+B_k \frac{e^{-ik(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}}{\sqrt{2k}}\left[1-\frac{i}{k(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}\right]$$

- Primordial power spectrum is well-approximated by:

$$\mathscr{P}(k) = \frac{H_\Lambda^2}{4\pi^2}|B_k - A_k|^2, \qquad |B_k|^2 - |A_k|^2 = 1$$

- Dephasing between constants typically leads to oscillations.

Power spectrum in de Sitter

- In de Sitter, solutions to Mukhanov-Sasaki equations:

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- Dephasing between constants: Oscillations.

→ If no interference in previous epoch(s), origin can be traced to instantaneous changes of the mass function.

Power spectrum in de Sitter

- In de Sitter, solutions to Mukhanov-Sasaki equations:

$$\mu_k = A_k \frac{e^{ik(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}}{\sqrt{2k}}\left[1+\frac{i}{k(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}\right]$$

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- Primordial power spectrum:

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- Dephasing between constants: Oscillations.
- For well-behaved initial state, we remove it in the end.

Choice of vacuum state

- Vacuum state: Initial conditions for the perturbations.
- In de Sitter, natural choice is Bunch-Davies: $A_k = 0$, $B_k = 1$.
- What if there are observable scales k that are sensitive to the spacetime curvature in the pre-inflationary epoch?

Choice of vacuum becomes an open question

Choice of vacuum state

- Vacuum state: Initial conditions for the perturbations.

- In de Sitter, natural choice is Bunch-Davies: $A_k = 0$, $B_k = 1$.

- What if there are observable scales k that are sensitive to the spacetime curvature in the pre-inflationary epoch?

- For a robust comparative study: Criterion of choice should be applicable to different types of cosmological dynamics.

- Ideally, it should also be motivated by fundamental considerations, and lead to positive-frequency solutions that do not present rapid oscillations in time and/or k.

Choice of vacuum state

- Vacuum state: Initial conditions for the perturbations.

- Here, criterion is fixed based on previous investigations:

    - ★ Originates from an ultraviolet diagonalization of the Hamiltonian in quantum cosmology.

    - ★ In the ultraviolet regime, it is the unique one that does not display rapid time oscillations of frequency k.

    - ★ Applied to Minkowski and de Sitter spacetimes, leads to Poincaré and Bunch-Davies vacua.

Power spectrum in GR

- In the case of GR with KD, our criterion fixes the following positive-frequency solutions in the epoch (η_0, η_i):

$$\mu_k = \sqrt{\frac{\pi y}{4}} H_0^{(2)}(ky), \qquad y = \eta - \eta_0 + \frac{1}{2H_0 a_0}$$

- By continuity, fixes positive-frequency solutions in de Sitter.

- Resulting power spectrum displays artificial oscillations around $k_I = a_i H_\Lambda$, which we remove with the transformation:

$$A_k \to A_k^{\text{kin}} = |A_k|, \qquad B_k \to B_k^{\text{kin}} = |B_k|$$

Power spectrum in (hybrid) LQC

- In the case of hybrid LQC, our criterion fixes the following positive-frequency solutions in the epoch $[\eta_B, \eta_0]$:

$$\mu_k = \sqrt{-\frac{1}{2\text{Im}(h_k)}} e^{i \int_{\eta_0}^{\eta} \text{Im}(h_k)}, \qquad x = \left[1 + e^{-2\alpha(\eta - \eta_B)}\right]^{-1},$$

$$h_k = -i\alpha\tilde{k} - 2\alpha x(1-x)\frac{cd}{1+i\tilde{k}}\frac{{}_2F_1\left(c+1, d+1; 2+i\tilde{k}; x\right)}{{}_2F_1\left(c, d; 1+i\tilde{k}; x\right)}, \quad \tilde{k} = k/\alpha$$

Power spectrum in (hybrid) LQC

- In the case of hybrid LQC, our criterion fixes the following positive frequencies $-\mathrm{Im}(h_k)$ in the epoch $[\eta_B, \eta_0]$:

$$h_k = -i\alpha\tilde{k} - 2\alpha x(1-x)\frac{cd}{1+i\tilde{k}}\frac{{}_2F_1\left(c+1, d+1; 2+i\tilde{k}; x\right)}{{}_2F_1\left(c, d; 1+i\tilde{k}; x\right)}, \quad \tilde{k} = k/\alpha$$

- By continuity, fixes positive-frequency solutions in the KD classical epoch and these, in turn, in the de Sitter regime.

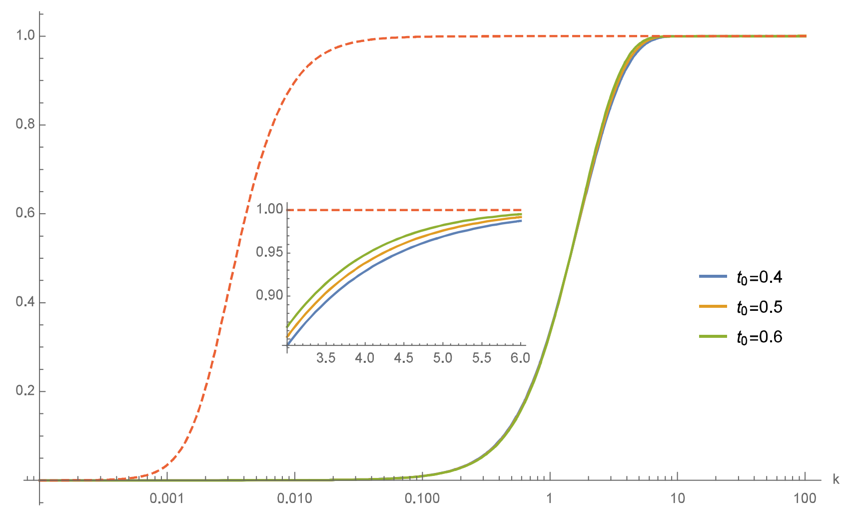
- Resulting power spectrum displays artificial oscillations for $k \lesssim k_{\mathrm{LQC}} = \alpha\,(\sim 3)$, which we remove in analogous way:

$$A_k \to A_k^{\mathrm{LQC}} = |A_k|, \qquad B_k \to B_k^{\mathrm{LQC}} = |B_k|$$

Power spectrum in (hybrid) LQC

- Resulting power spectrum displays artificial oscillations for $k \lesssim k_{\mathrm{LQC}} = \alpha\,(\sim 3)$, which we remove.

- We compare it with the one in the GR with KD model for which inflation starts at the same scale as in LQC: $k_I \sim 10^{-3}$.



┄┄ GR

── LQC

Conclusions

- Approximative methods to understand **analytically** the main differences between (classical) KD preinflationary and LQC effects leading to suppression in power spectra.
- Differences traceable to existence of two distinct scales:
  - ★ Curvature at onset of inflation (both models).
  - ★ Curvature around the bounce (only in LQC).
- They always differ in 3 orders of magnitude for interesting LQC solutions (phenomenologically speaking).
- Study can be used to compare other preinflationary models.
- Approximations yet rough: Call for further developing the studies about the dynamical behavior of the chosen vacua.

Session B3a 14:30–16:00

[Chair: Ryo Namba]

Lucas Pinol

IFT Madrid (UAM-CSIC)

“The non-linear Universe as a particle detector”

(15 min.)

[JGRG30 (2021) 120727]

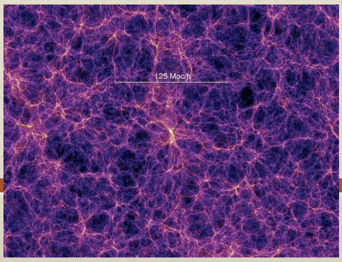
Based on: **[Fumagalli *et al.*, Lucas Pinol, 2019]**
Phys. Rev. Lett. 123, 201302
[Garcia-Saenz, Lucas Pinol, Renaux-Petel 2020]
J. High Energ. Phys. **2020,** 73 (2020)
[Lucas Pinol 2020]
J. Cosm. & Astro. Phys. 04(**2021**)048
[Aoki *et al.*, Lucas Pinol, 2021 soon]
ArXiv:2112.xxxxx

JGRG30
December 2021, 7th, from Madrid, Spain

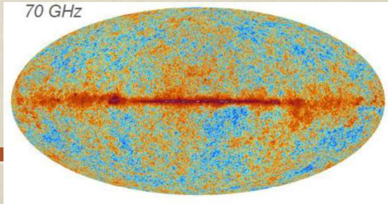
Lucas Pinol
Instituto de Física Teórica (IFT) UAM-CSIC

PRIMORDIAL NON-GAUSSIANITIES: THE NON-LINEAR UNIVERSE AS A PARTICLE DETECTOR

Clouds: a non-linear process



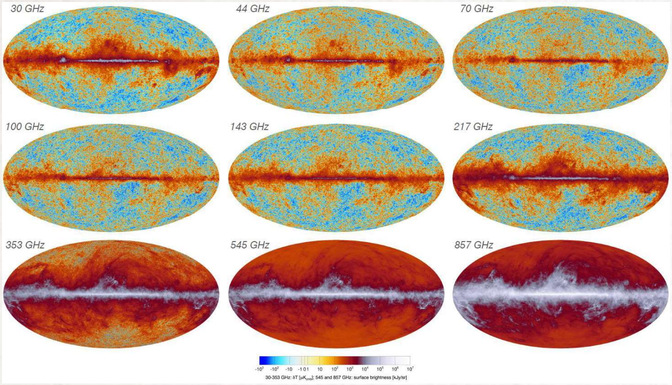
Millenium simulation



Planck 2018

Non-linearities in the sky

(but no diamonds)



Planck CMB intensity maps

Sources of non-Gaussianity:

- Foreground
- Late-time evolution: lensing, etc.
- Early-time evolution: gravity, interactions, etc.
- **Initial conditions:**

Primordial non-Gaussianities from inflation

$$\boldsymbol{T}_{\text{ini}}(\boldsymbol{\theta},\boldsymbol{\varphi}) = \boldsymbol{T}^{\boldsymbol{G}}_{\text{ini}}(\boldsymbol{\theta},\boldsymbol{\varphi}) + \boldsymbol{f}^{\text{local}}_{\text{NL}} \times \left[\boldsymbol{T}^{\boldsymbol{G}}_{\text{ini}}(\boldsymbol{\theta},\boldsymbol{\varphi})\right]^2$$

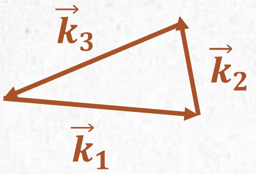
Gaussian　　*Non-Gaussian if* $f^{\text{local}}_{\text{NL}} \neq 0$

PRIMORDIAL BISPECTRUM

Power spectrum $= 2.10 \times 10^{-9}$

ζ the primordial curvature perturbation

$$< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} >= (2\pi)^7\,\delta^{(3)}\left(\overrightarrow{k_1}+\overrightarrow{k_2}+\overrightarrow{k_3}\right)\frac{A_s^2}{(k_1k_2k_3)^2}\times S(k_1,k_2,k_3)$$



Shape function

PRIMORDIAL BISPECTRUM

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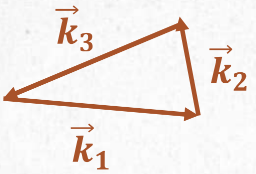


Shape function

< 0.0035 (from $r < 0.056$)

(attractor)

Ex: Single-field inflation

[Maldacena 2003]

$$S = \frac{5}{12}(1-n_s)S_{\mathrm{loc}} + \frac{\epsilon}{8}S_{\mathrm{eq}} + \cdots =$$ **VERY SMALL**

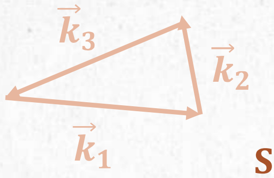
0.0351

PRIMORDIAL BISPECTRUM

ζ The curvature perturbation

Power spectrum $= 2.10 \times 10^{-9}$

$$< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} >= (2\pi)^7\,\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{A_s^2}{(k_1k_2k_3)^2}\times S(k_1,k_2,k_3)$$

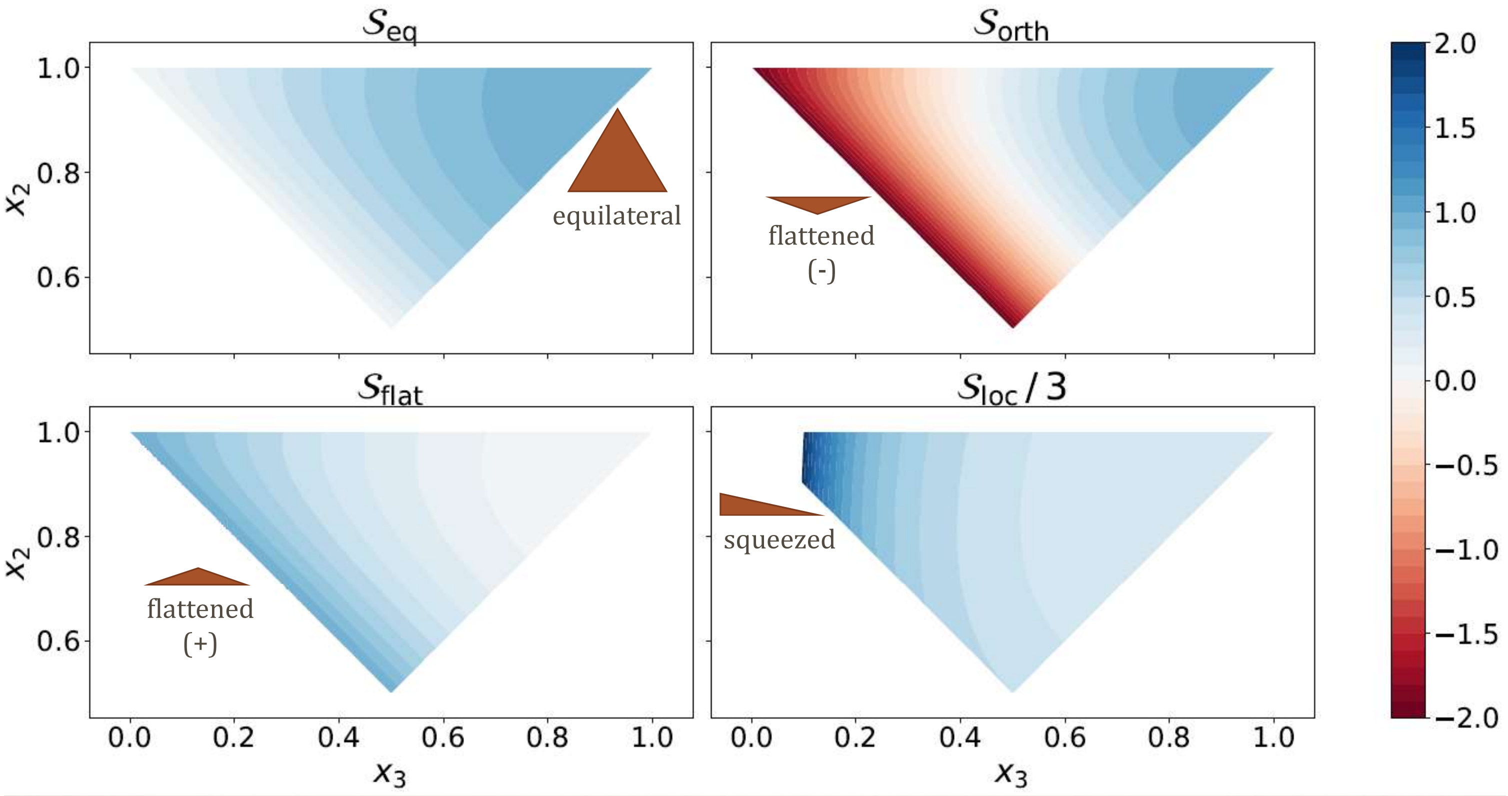


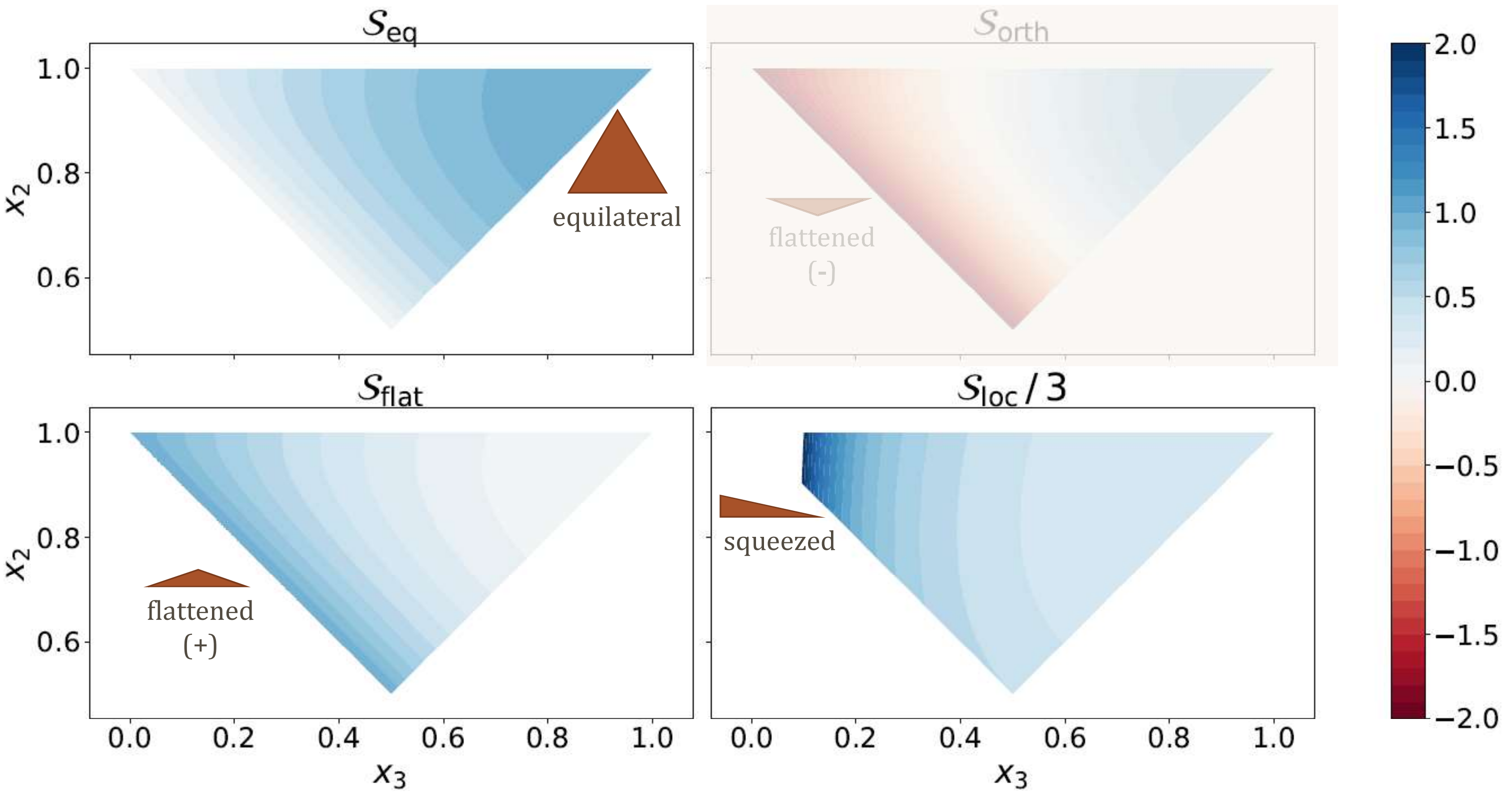
Shape function

Shape templates

(attractor)

Ex: Single-field inflation $S=\frac{5}{12}(1-n_s)\boldsymbol{S}_{\mathbf{loc}}+\frac{\epsilon}{8}\boldsymbol{S}_{\mathbf{eq}}+\cdots=$ **VERY SMALL**





BISPECTRUM IN MULTIFIELD INFLATION

More model-dependent but typically $f_{\rm NL} \gtrsim O(1)$

<u>Example</u>: super-Hubble interactions of light fields → $f_{\rm NL}^{\rm local} \gtrsim O(1)$

Detection of $f_{\rm NL}$ in the near future ($f_{\rm NL} \gtrsim 1$)

⇓

Multiple primordial degrees of freedom

But can we do better?

BISPECTRUM IN MULTIFIELD INFLATION

The squeezed limit as a cosmological collider

Remember the single-field result: $\boxed{\underset{\textit{consistency relation}}{f_{\text{NL}}^{\text{squeezed}} \propto n_s - 1 \ll 1}}$

Two-field result:

[Chen, Wang 2009]

[Noumi, Yamaguchi, Yokoyama 2013] (one extra heavy field $m_s > 3H/2$, perturbatively coupled)

[Arkani-Hamed, Maldacena 2015]

[Arkani-Hamed, Baumann, Lee, Pimentel 2018]

BISPECTRUM IN MULTIFIELD INFLATION

The squeezed limit as a cosmological collider

[Noumi, Yamaguchi, Yokoyama 2013]

Small coupling between the two fields : $\eta_\perp \ll 1$

Two-field result: $f_{\text{NL}}^{\text{squeezed}} \underset{k_l \ll k_s}{\simeq} \eta_\perp^2 e^{-\pi\mu} \cos\left[\mu \log\left(\frac{k_l}{k_s}\right)\right]$

Oscillatory pattern: massive particle

$\mu = \sqrt{\frac{m_s^2}{H^2} - \frac{9}{4}}$ **the reduced mass**

Boltzmann suppression for heavy particles

$H/2\pi \sim T$ **so** $e^{-\pi\mu} \sim e^{-\frac{m_s}{T}}$

BISPECTRUM IN MULTIFIELD INFLATION

The squeezed limit as a cosmological collider

[Noumi, Yamaguchi, Yokoyama 2013]

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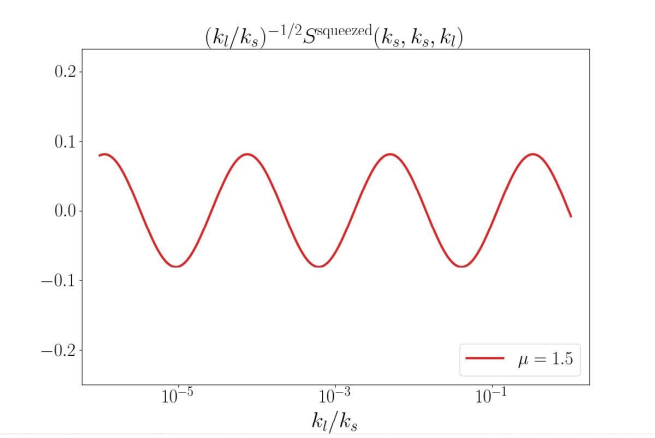

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BISPECTRUM IN MULTIFIELD INFLATION

Probing other regimes

- Large coupling, $\eta_\perp \gg 1$ → Multifield instability → Large flattened NGs:

[Fumagalli, Garcia-Saenz, Lucas Pinol, Renaux-Petel, Ronayne 2019]
Phys. Rev. Lett. 123, 201302

$$f_{\text{NL}}^{\text{flat}} = \mathcal{O}(50)$$

BISPECTRUM IN MULTIFIELD INFLATION

Probing other regimes

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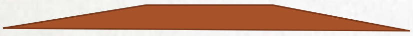
Phys. Rev. Lett. 123, 201302

$f_{\rm NL}^{\rm flat} = \mathcal{O}(50)$

Higher-order correlation functions are boosted in similar configurations



$g_{\rm NL}^{\rm flat} = \mathcal{O}(10^5)$ etc.

BISPECTRUM IN MULTIFIELD INFLATION

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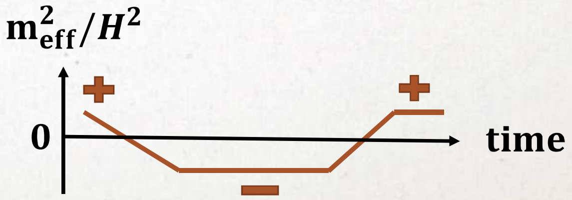
[Fumagalli, Garcia-Saenz, Lucas Pinol, Renaux-Petel, Ronayne 2019]
Phys. Rev. Lett. 123, 201302

$f_{\rm NL}^{\rm flat} = \mathcal{O}(50)$

Higher-order correlation functions are boosted in similar configurations

$g_{\rm NL}^{\rm flat} = \mathcal{O}(10^5)$ etc.

Clear sign of transiently unstable degrees of freedom:



BISPECTRUM IN MULTIFIELD INFLATION

Probing other regimes

- Large mass, $|m_s^2| \gg H^2$ → Single-field effective theory for ζ

  (including the instability with $m_s^2 < 0$)

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  (including the instability with $m_s^2 < 0$)

$$\hookrightarrow \quad f_{\mathrm{nl}}^{\mathrm{eq}} \simeq \left(\frac{1}{c_s^2} - 1\right)\left(-\frac{85}{324} + \frac{15}{243}\boldsymbol{A}\right)$$

Speed of sound:
Dictated by the bilinear coupling $\eta_\perp$

[Achucarro, Gong, Hardeman, Palma, Patil 2012]

Single-field effective interactions
Dictated by the multifield cubic interactions

[Garcia-Saenz, Lucas Pinol, Renaux-Petel 2019]
J. High Energ. Phys. **2020,** 73 (2020)

Later extended to any number of heavy fields: **[Lucas Pinol 2020]** *J. Cosm. & Astro. Phys.* 04(**2021**)048

THE EFT OF INFLATION

REVISITED…

Bottom-up approach: unknown natural values of the coefficients

[Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2009]

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2}-1\right)\left(\zeta'(\partial_i\zeta)^2+\frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

with $\boldsymbol{A} = \mathcal{O}(1)$ but **undetermined**

THE EFT OF INFLATION

REVISITED…

In our top-down approach we DERIVE those coefficients

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2}-1\right)\left(\zeta'(\partial_i\zeta)^2+\frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = \underbrace{-\frac{1}{2}(1+c_s^2)}_{\text{Previously known}} + \underbrace{\frac{2}{3}(1+2c_s^2)\frac{\epsilon R_{\mathrm{fs}} H^2 M_p^2}{m_s^2}}_{\text{Scalar curvature of the field space}} - \frac{1}{6}(1-c_s^2)\left(\underbrace{\frac{\kappa V_{;sss}}{m_s^2}}_{3^{\text{rd}}\text{ derivative of the potential}} + \underbrace{\frac{\kappa\epsilon H^2 M_p^2 R_{\mathrm{fs},s}}{m_s^2}}_{\text{Derivative of the scalar curvature}}\right)$$

[Garcia-Saenz, Lucas Pinol, Renaux-Petel 2019]
J. High Energ. Phys. **2020,** 73 (2020)

Bending radius of the trajectory: $\kappa = \sqrt{2\epsilon}M_p/\eta_\perp$

BISPECTRUM IN MULTIFIELD INFLATION

Probing more than one extra field

[Lucas Pinol 2020]
J. Cosm. & Astro. Phys. 04(**2021**)048

- I extended previous works for any number N_{field} of kinetically coupled scalars:
  - Most generic cubic action for perturbations at lowest order in derivatives

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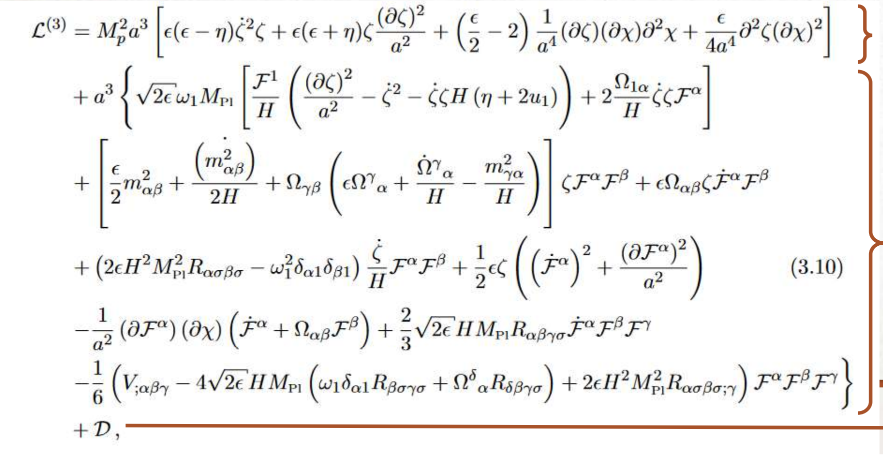

$$\mathcal{L}^{(3)} = M_p^2 a^3 \left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]$$

single-field (recovering Maldacena)

$$\begin{aligned}
&+ a^3 \Bigg\{ \sqrt{2\epsilon}\,\omega_1 M_{\text{Pl}} \left[\frac{\mathcal{F}^1}{H}\left(\frac{(\partial\zeta)^2}{a^2} - \dot{\zeta}^2 - \dot{\zeta}\zeta H\left(\eta + 2u_1\right)\right) + 2\frac{\Omega_{1\alpha}}{H}\dot{\zeta}\zeta\mathcal{F}^\alpha\right] \\
&+ \left[\frac{\epsilon}{2}m^2_{\alpha\beta} + \frac{\dot{\left(m^2_{\alpha\beta}\right)}}{2H} + \Omega_{\gamma\beta}\left(\epsilon\Omega^\gamma{}_\alpha + \frac{\dot{\Omega}^\gamma{}_\alpha}{H} - \frac{m^2_{\gamma\alpha}}{H}\right)\right]\zeta\mathcal{F}^\alpha\mathcal{F}^\beta + \epsilon\Omega_{\alpha\beta}\zeta\dot{\mathcal{F}}^\alpha\mathcal{F}^\beta \\
&+ \left(2\epsilon H^2 M_{\text{Pl}}^2 R_{\alpha\sigma\beta\sigma} - \omega_1^2\delta_{\alpha 1}\delta_{\beta 1}\right)\frac{\dot{\zeta}}{H}\mathcal{F}^\alpha\mathcal{F}^\beta + \frac{1}{2}\epsilon\zeta\left(\left(\dot{\mathcal{F}}^\alpha\right)^2 + \frac{(\partial\mathcal{F}^\alpha)^2}{a^2}\right) \qquad (3.10) \\
&- \frac{1}{a^2}\left(\partial\mathcal{F}^\alpha\right)\left(\partial\chi\right)\left(\dot{\mathcal{F}}^\alpha + \Omega_{\alpha\beta}\mathcal{F}^\beta\right) + \frac{2}{3}\sqrt{2\epsilon}\,H M_{\text{Pl}} R_{\alpha\beta\gamma\sigma}\dot{\mathcal{F}}^\alpha\mathcal{F}^\beta\mathcal{F}^\gamma \\
&- \frac{1}{6}\left(V_{;\alpha\beta\gamma} - 4\sqrt{2\epsilon}\,H M_{\text{Pl}}\left(\omega_1\delta_{\alpha 1}R_{\beta\sigma\gamma\sigma} + \Omega^\delta{}_\alpha R_{\delta\beta\gamma\sigma}\right) + 2\epsilon H^2 M_{\text{Pl}}^2 R_{\alpha\sigma\beta\sigma;\gamma}\right)\mathcal{F}^\alpha\mathcal{F}^\beta\mathcal{F}^\gamma \Bigg\}
\end{aligned}$$

≥ two fields

$$+ \mathcal{D}\,, \longrightarrow$$

≥ boundary terms (they contribute!)

BISPECTRUM IN MULTIFIELD INFLATION

Probing more than one extra field **[Lucas Pinol 2020]**
J. Cosm. & Astro. Phys. 04(**2021**)048

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- In the case of heavy fields, integrating out procedure still possible:

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$$\begin{aligned} A = & -\frac{1}{2}(1+c_s^2) + \frac{4}{3}(1+2c_s^2)\epsilon H^2 M_p^2 \left(m^{-2}\right)_{11} R_{m\sigma m\sigma} \\ & -\frac{\kappa}{6}(1-c_s^2)\left(m^{-2}\right)_{11}\Big[V_{;mmm} + 2\epsilon H^2 M_p^2 R_{m\sigma m\sigma;m} \\ & +4\sqrt{2\epsilon}HM_p\left({\Omega^\alpha}_m + \frac{1}{\left(m^{-2}\right)_{11}}\frac{\mathrm{d}\left(m^{-2}\right)^\alpha}{\mathrm{d}t}\right) R_{m\alpha m\sigma}\Big], \end{aligned}$$

Depends on:

❖ The whole geometry of the target space (**Riemann tensor**)

❖ Third derivative of the **potential**

❖ New many-field (≥ 3) **bilinear interactions**

$f_{\text{nl}}^{\text{eq}} \simeq \left(\frac{1}{c_s^2} - 1\right)\left(-\frac{85}{324} + \frac{15}{243}\boldsymbol{A}\right)$ → Primordial NGs sensitive to the whole geometry and interactions!

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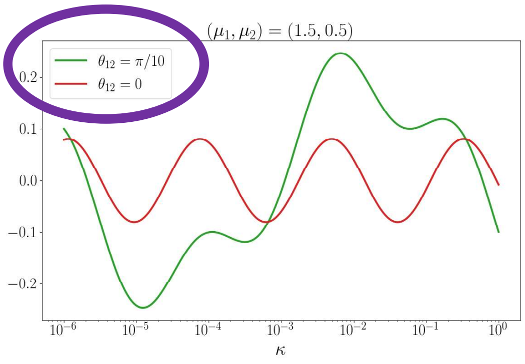


[Aoki, Lucas Pinol, Renaux-Petel, Yamaguchi 2021] *ArXiv:2112.xxxx*

Stay tuned ☺

Interesting features:

❖ Several extra massive fields lead to modulated oscillations

❖ Light fields or light and heavy also lead to characteristic signals

❖ Theory described with **mixing angles** for flavour and mass eigenstates

CONCLUSION

- Primordial NGs contain much more information than a single number $f_{\text{NL}}^{\text{local}}$
- We expect the Early Universe to be much richer than single-clock inflation
- Depending on the mass spectrum and interactions of primordial field content, NGs are of different **amplitudes** and **shapes**
- It is crucial to think about experiments to constrain better the squeezed bispectrum *(e.g. 21-cm radio-astronomy from the far side of the Moon?)*

Formidable opportunity to use the non-linear Universe as a particle detector

---

OBSERVATIONAL CONSTRAINTS

$$f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1 \qquad g_{\text{NL}}^{\text{local}} = (-5.8 \pm 6.5)10^4$$

$$f_{\text{NL}}^{\text{eq}} = -26 \pm 47$$

$$f_{\text{NL}}^{\text{ortho}} = -38 \pm 24$$

Planck 2018

GENERALISATION TO N FIELDS

$$\begin{aligned} A = & -\frac{1}{2}(1+c_s^2) + \frac{4}{3}(1+2c_s^2)\epsilon H^2 M_p^2 \left(m^{-2}\right)_{11} R_{m\sigma m\sigma} \\ & -\frac{\kappa}{6}(1-c_s^2)\left(m^{-2}\right)_{11} \Bigg[V_{;mmm} + 2\epsilon H^2 M_p^2 R_{m\sigma m\sigma;m} \\ & +4\sqrt{2\epsilon} H M_p \left(\Omega^\alpha{}_m + \frac{1}{\left(m^{-2}\right)_{11}} \frac{\mathrm{d}\left(m^{-2}\right)^\alpha{}_1}{\mathrm{d}t} \right) R_{m\alpha m\sigma} \Bigg], \end{aligned}$$

INTEGRATING OUT HEAVY ENTROPIC FLUCTUATIONS

AN EFFECTIVE THEORY FOR THE OBSERVABLE CURVATURE PERTURBATION

$$S[\zeta,\mathcal{F}] \xrightarrow{\mathcal{F}_{\text{heavy}}(\zeta)} S_{\text{EFT}}[\zeta] = S[\zeta,\mathcal{F}_{\text{heavy}}(\zeta)]$$

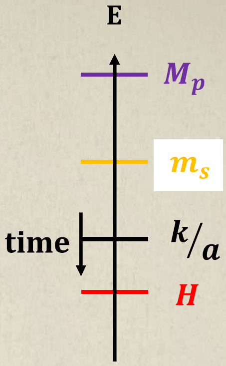
A HIERARCHY OF SCALES

WHEN ENTROPIC FLUCTUATIONS ARE HEAVY

- Equation of motion for $\mathcal{F}$:

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \left(m_s^2 + \frac{k^2}{a^2}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

Hierarchy of scales



Energy of the "experiment"

$H \ll m_s$

Integrate out the heavy perturbation

Like in the Fermi theory:
Integrate out the heavy W, Z bosons and consider contact interactions for fermions

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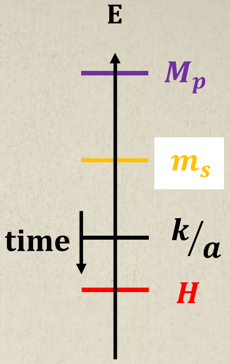
When $\mathcal{F}$ is heavy $\boxed{\mathcal{F}^{\mathrm{LO}}_{\mathrm{heavy}} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}}$ $\boxed{\omega^2, \omega H, \frac{k^2}{a^2} \ll m_s^2}$

Hierarchy of scales



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A HIERARCHY OF SCALES

THE QUADRATIC EFFECTIVE ACTION

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When $\mathcal{F}$ is heavy $\boxed{\mathcal{F}^{\mathrm{LO}}_{\mathrm{heavy}} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}}$

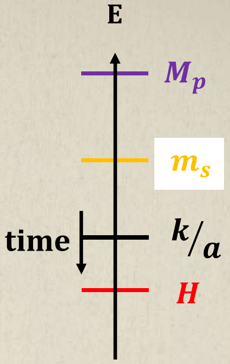
$\Downarrow$

Effective single-field action for the curvature perturbation

$$S_2^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 \epsilon \left(\frac{\zeta'^2}{c_s^2} - (\partial_i \zeta)^2\right)$$

With a speed of sound c_s: $\boxed{\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}}$

Hierarchy of scales



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$s = \dot{c}_s / H c_s$

THE CUBIC EFFECTIVE ACTION

FULL RESULT

P(X) cubic lagrangian:

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2} \left(\begin{array}{c} \frac{g_1}{\mathcal{H}}\zeta'^3 + \\ g_2 \zeta'^2 \zeta + \\ g_3 c_s^2 \zeta (\partial_i \zeta)^2 + \\ \frac{\tilde{g}_3 c_s^2}{\mathcal{H}} \zeta' (\partial_i \zeta)^2 + \\ g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta + \\ g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2 \end{array} \right) \text{with} \left\{ \begin{array}{l} g_1 = \left(\frac{1}{c_s^2} - 1 \right) \boldsymbol{A} \\ g_2 = \epsilon - \eta + 2s \\ g_3 = \epsilon + \eta \\ \tilde{g}_3 = \frac{1}{c_s^2} - 1 \\ g_4 = \frac{-2\epsilon}{c_s^2} \left(1 - \frac{\epsilon}{4} \right) \\ g_5 = \frac{\epsilon^2}{4 c_s^2} \end{array} \right.$$

The only new parameter is **A**, and depends on the UV physics

---

THE CUBIC EFFECTIVE ACTION

RECOVERING THE EFT OF INFLATION

$\epsilon, \eta, s \to 0$

Slow-varying result:

Non-Gaussianities $\sim \frac{1}{c_s^2} - 1$

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2} \left(\begin{array}{c} \frac{g_1}{\mathcal{H}}\zeta'^3 + \\ \cancel{g_2 \zeta'^2 \zeta} + \\ \cancel{g_3 c_s^2 \zeta (\partial_i \zeta)^2} + \\ \frac{\tilde{g}_3 c_s^2}{\mathcal{H}} \zeta' (\partial_i \zeta)^2 + \\ \cancel{g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta} + \\ \cancel{g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2} \end{array} \right) \text{with} \left\{ \begin{array}{l} g_1 = \left(\frac{1}{c_s^2} - 1 \right) \boldsymbol{A} \\ \\ \tilde{g}_3 = \frac{1}{c_s^2} - 1 \\ \\ \end{array} \right.$$

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$$\boldsymbol{h^2 = \frac{V'}{MH^2} - 9 \gg 1}$$

HYPERINFLATION

LINEAR PERTURBATIONS

$$\text{We compute} \begin{cases} \eta_\perp^2 \approx h^2 \\ \epsilon R_{fs} M_p^2 \approx -h^2 \\ V_{;ss}/H^2 \ll 1 \end{cases} \Rightarrow \begin{cases} \dfrac{m_s^2}{H^2} \approx -2h^2 < 0 \\ \dfrac{m_{s,\text{eff}}^2}{H^2} \approx 2h^2 > 0 \end{cases}$$

→ Unstable, growing sub-Hubble perturbations

→ Stable, decaying super-Hubble perturbations

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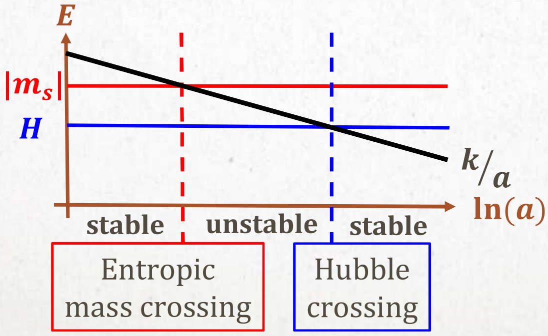
⚠ **This tachyonic instability is only transient for each k-mode**

Remember in the e.o.m. for Q_s, the mass term is $\left(\frac{k^2}{a^2} + m_s^2\right) > 0$ *deep under the horizon*

$m_{s,\text{eff}}^2 > 0$ *on super-horizon scales*



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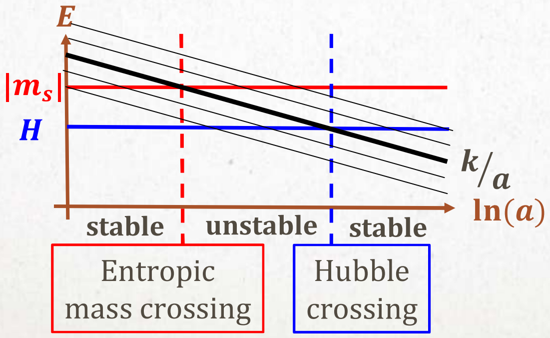

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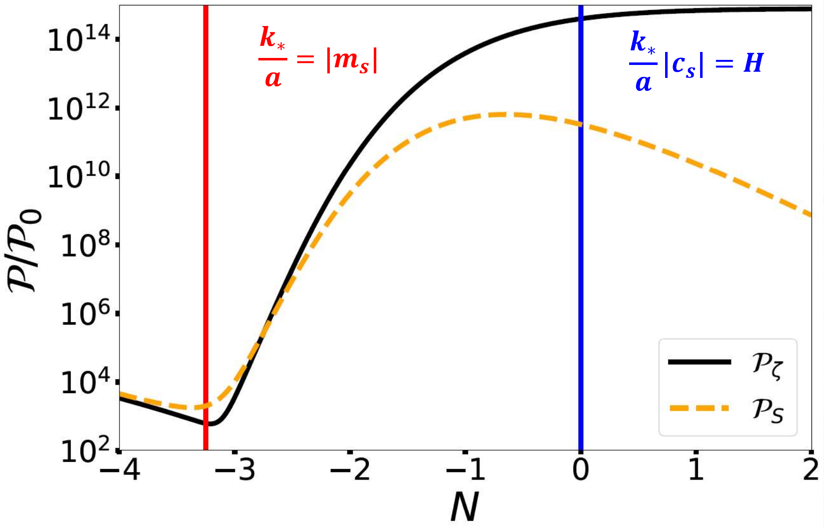

$r \ll 0.01$,
$n_s > 1$:

Excluded by CMB

$V = \frac{1}{2}m^2\phi^2$ with $m = M = 10^{-2}M_p$

Session B3a 14:30–16:00

[Chair: Ryo Namba]

P. Jishnu Sai

Indian Institute of Science

“On the primordial correlation of gravitons with gauge fields”

(15 min.)

[JGRG30 (2021) 120728]

On the primordial correlation of gravitons with gauge fields

Based on arXiv:2108.10887 [hep-th]
Collaborators: Rajeev Kumar Jain, Martin S. Sloth

P. Jishnu Sai

Department of Physics
Indian Institute of Science, Bangalore, India

The 30th Workshop on General Relativity and Gravitation in Japan (JGRG30)

Outline

1. Introduction
   - Inflationary magnetogenesis
   - Quantum fluctuations of metric perturbations
   - Quantum fluctuations of gauge field
2. Cross-correlation of inflationary tensor perturbation with primordial gauge fields
   - The in-in formalism
   - Consistency relations for cosmic magnetic field
   - Semi-classical derivation of the consistency relations
   - A direct correlation of tensor and curvature perturbations
3. Summary

Dynamics of primordial tensors and gauge field during inflation

---

Inflationary magnetogenesis

Typical model

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$$S_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}\,\lambda(\phi)F_{\mu\nu}F^{\mu\nu} \text{ with } \quad \lambda(\phi(a)) = \lambda_I\left(\frac{a}{a_I}\right)^{2n}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

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where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

- The perturbed metric: $ds^2 = -dt^2 + a^2(t)\, e^{2\zeta(t,\mathrm{x})}[e^{\gamma(t,\mathrm{x})}]_{ij}dx^i dx^j$

Quantum Fluctuations

Metric perturbations and gauge field

- The power spectra associated with metric perturbations

$$\langle\zeta(\mathbf{k},\tau)\zeta(\mathbf{k}',\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\zeta(k,\tau)$$

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$$\big\langle A_i(\tau,\mathbf{k})A_j(\tau,\mathbf{k}')\big\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)|A_k(\tau)|^2$$

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$$\big\langle A_i(\tau,\mathbf{k})A_j(\tau,\mathbf{k}')\big\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)|A_k(\tau)|^2$$

  with the mode function $A_k(\tau)$ is given by

$$A_k(\tau) = \frac{1}{\sqrt{\lambda_I}}\frac{\sqrt{\pi}}{2}e^{i\pi(n+1)/2}\sqrt{-\tau}\left(\frac{\tau}{\tau_I}\right)^n H^{(1)}_{n+\frac{1}{2}}(-k\tau)$$

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Thus,

$$\begin{aligned}P_B(k, \tau) &= 2\frac{k^2}{a^4} |A_k(\tau)|^2 \\ P_E(k, \tau) &= \frac{2}{a^4} |A_k'(\tau)|^2\end{aligned}$$

Cross-correlation of inflationary tensor perturbation with primordial gauge fields

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- Using in-in formalism, we have calculated $\langle \gamma A_\mu A^\mu \rangle$,$\langle \gamma B_\mu B^\mu \rangle$ and $\langle \gamma E_\mu E^\mu \rangle$ perturbatively

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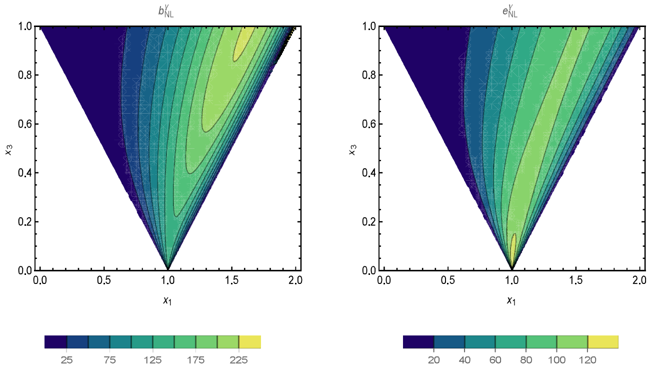
The in-in results



The extent of the non-linearity parameters b^γ_{NL} (left) and e^γ_{NL} (right) corresponding to different triangular configuration are plotted for the case of $n = 2$. Here, we defined $x_1 = \frac{k_1}{k_2}$ and $x_3 = \frac{k_3}{k_2}$ while k_2 is set at an arbitrary scale. The color legends representing the magnitude are also shown below each plot.

Squeezed/Soft limit and new consistency relations

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In this limit, we have $\mathbf{k}_1 \to 0$ and $\mathbf{k}_2 \to -\mathbf{k}_3 \equiv \mathbf{k}$. The primed correlator $\langle ... \rangle'$ indicate that we have suppressed the factor $(2\pi)^3 \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$.

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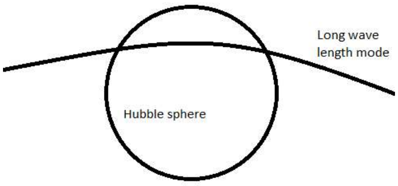

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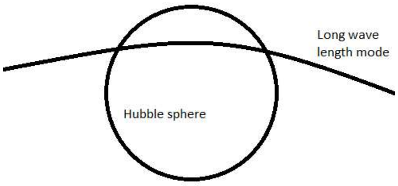

- Then the rescaled background will be: $ds^2 = -dt^2 + a^2(t)d\tilde{x}^2$ with $d\tilde{x}^2 \to dx^2 + \gamma_{ij}^B dx^i dx^j$.

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with $Y_\mu = \{A_\mu, B_\mu, E_\mu\}$

- The two point function in the modified background:

$$\langle Y_\mu(\mathrm{x}) Y^\mu(\mathrm{x}) \rangle_B = \langle Y_\mu(\mathrm{x}) Y^\mu(\mathrm{x}) \rangle_0 + \gamma^B_{ij} \frac{\partial}{\partial \gamma^B_{ij}} \langle Y_\mu(\tilde{\mathbf{x}}) Y^\mu(\tilde{\mathbf{x}}) \rangle |_{\gamma^B = 0} + \dots$$

Semi-classical derivation of consistency relations

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$$\lim_{k_1\to 0}\langle\gamma(\tau_I,\mathbf{k}_1)B_\mu(\tau_I,\mathbf{k}_2)B^\mu(\tau_I,\mathbf{k}_3)\rangle' = \left(n-\frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2}P_\gamma(k_1)P_B(k_2),$$

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$$\lim_{k_1\to 0} \langle \gamma(\tau_I, \mathbf{k}_1) B_\mu(\tau_I, \mathbf{k}_2) B^\mu(\tau_I, \mathbf{k}_3)\rangle' = \left(n - \frac{1}{2}\right) \epsilon_{ij} \frac{k_{2i}k_{2j}}{k_2^2} P_\gamma(k_1) P_B(k_2)$$

- Similarly for graviton electric fields cross-correlator can only be trusted for $n < -1/2$

$$\lim_{k_1\to 0} \langle \gamma(\tau_I, \mathbf{k}_1) E_\mu(\tau_I, \mathbf{k}_2) E^\mu(\tau_I, \mathbf{k}_3)\rangle' = -\left(n + \frac{1}{2}\right) \epsilon_{ij} \frac{k_{2i}k_{2j}}{k_2^2} P_\gamma(k_1) P_E(k_2)$$

A direct correlation of tensor and curvature perturbations

- The curvature perturbation induced by any magnetic field is

$$\zeta_B(\tau) = \int_{\tau_0}^{\tau} d\ln\tau' \lambda(\tau') \frac{B_i B^i}{3H^2\epsilon}$$

A direct correlation of tensor and curvature perturbations

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$$\langle\gamma\zeta\rangle \simeq \langle\gamma\zeta_B\rangle \propto \langle\gamma\mathbf{B}\cdot\mathbf{B}\rangle \neq 0$$

A direct correlation of tensor and curvature perturbations

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- But, we explicitly showed that such a correlator in this scenario actually vanishes due to the statistical isotropy.

$$\langle\gamma\zeta\rangle = 0$$

Summary

- In a particular model of inflationary magnetogenesis, we defined and calculated the non-Gaussian cross correlation of gauge fields with the tensor perturbations.
- We showed that there exist a leading order correction to these non-Gaussian cross correlations.
- We studied the shape function associated with these non-Gaussian correlators.
- We have derived new set of consistency relations analogous to known consistency relations in the literature.
- We have calculated a direct correlation between one graviton mode and a curvature perturbation mode.

Thank You

Session B3b 14:30–16:00

[Chair: Norihiro Tanahashi]

Masashi Kimura

Rikkyo University

“Metric Backreaction of the Blandford-Znajek Process”

(15 min.)

[JGRG30 (2021) 120729]

Metric Backreaction of the Blandford-Znajek Process

arXiv:2105.05581 (PTEP **2021**, 093E03)

Masashi Kimura
(Rikkyo University)
w/ T.Harada, A.Naruko, K.Toma

7th Dec 2021 JGRG30

Summary

In this talk

- We discuss the metric backreaction of the mass and angular momentum accretion on the Schwarzschild BH (monopole and dipole linear gravitational perturbation against generic $T_{\mu\nu}$)

- We apply our formalism to the Blandford-Znajek process

Introduction

We usually consider
fixed Kerr black hole + test fields

What happens beyond test field approximation?

If there are mass and angular momentum accretion on BH, we expect M and a slowly change.

We want to discuss this issue by solving the Einstein eqs.

Einstein eqs

We want to solve $G_{\mu\nu} = 8\pi\underline{\epsilon}T_{\mu\nu}$

small parameter

$\mathcal{O}(\epsilon^0)$:vacuum sols

We choose $\mathcal{O}(\epsilon^0)$ sol as Schwarzschild metric

$$g^{\rm Sch}_{\mu\nu}dx^\mu dx^\nu = -f dt^2 + f^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$f = 1 - \frac{r_0}{r} \quad (r_0 = 2M)$$

Einstein eqs

We consider $\mathcal{O}(\epsilon)$ effect

$$g_{\mu\nu} = g^{\mathrm{Sch}}_{\mu\nu} + \epsilon h_{\mu\nu} \qquad G_{\mu\nu} = 8\pi\epsilon T_{\mu\nu}$$

$$\begin{aligned}
G_{\mu\nu} &= \epsilon\delta G_{\mu\nu} \\
&= \epsilon\underline{\Big[-\frac{1}{2}\nabla_\mu\nabla_\nu h^\alpha{}_\alpha - \frac{1}{2}\nabla^\alpha\nabla_\alpha h_{\mu\nu} + \nabla^\alpha\nabla_{(\mu}h_{\nu)\alpha}} \\
&\qquad \underline{+\frac{1}{2}g_{\mu\nu}(\nabla^\alpha\nabla_\alpha h^\beta{}_\beta - \nabla^\alpha\nabla^\beta h_{\alpha\beta})\Big]} + \mathcal{O}(\epsilon^2) \\
&= \epsilon\mathcal{L}^{\mathrm{Sch}}[h_{\alpha\beta}]_{\mu\nu}
\end{aligned}$$

Einstein eqs

We need to solve

$$\epsilon\mathcal{L}^{\mathrm{Sch}}[h_{\alpha\beta}]_{\mu\nu} = 8\pi\epsilon T_{\mu\nu}$$

The energy momentum tensor satisfies $\nabla^\mu T_{\mu\nu} = 0$

$Y_{\ell m}$ decomposition

Due to the spherical symmetry of $g^{\rm Sch}_{\mu\nu}$, we can discuss different (ℓ, m) separately (Regge,Wheeler 1957)

$\ell \geq 2$ modes: GWs

We focus on $\ell = 0, 1$ modes: mass and angular momentum perturbations

---

Eddington-Finkelstein coords

We work in the Eddington-Finkelstein coordinates (V, r, θ, Φ)

$$dV = dt + f^{-1} dr \qquad (d\Phi = d\phi)$$

$$g^{\rm Sch}_{\mu\nu} dx^\mu dx^\nu = -f dV^2 + 2 dV dr + r^2 (d\theta^2 + \sin^2\theta d\Phi^2)$$

- EF coords cover BH horizon
- regularity condition is trivial

Monopole perturbation $(\ell = 0)$

Perturbed metric:

$$h^{(+)}_{\mu\nu}|_{\ell=0}dx^{\mu}dx^{\nu} = H_0(V,r)dV^2 + 2H_1(V,r)dVdr$$

Energy momentum tensor:

$$\begin{aligned} T^{(+)}_{\mu\nu}|_{\ell=0}dx^{\mu}dx^{\nu} &= T_{VV}(V,r)dV^2 + 2T_{Vr}(V,r)dVdr \\ &\quad + T_{rr}(V,r)dr^2 + T_{\Omega}(V,r)r^2(d\theta^2 + \sin^2\theta d\Phi^2) \end{aligned}$$

We want to solve Einstein eqs

$$\epsilon\mathcal{L}^{\mathrm{Sch}}[h_{\alpha\beta}]_{\mu\nu} = 8\pi\epsilon T_{\mu\nu}$$

Monopole perturbation $(\ell = 0)$

We introduce new variables (cf: Babichev et al 2012)

$$H_0(V,r) = \frac{2\delta M(V,r)}{r} + 2f\lambda(V,r)$$

$$H_1(V,r) = -\lambda(V,r)$$

$$\left[\ ds^2 = -\left(1 + \frac{2M + 2\epsilon\delta M}{r}\right)e^{2\epsilon\lambda}dV^2 + 2e^{\epsilon\lambda}dVdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)\ \right]$$

Einstein eqs reduce to

$$\partial_V \delta M = \mathcal{A}$$

$$\partial_r \delta M = -4\pi r^2 T_{Vr}$$

$$\partial_r \lambda = -4\pi r T_{rr}$$

$$\mathcal{A} := \int_0^{2\pi}\int_0^{\pi} T_V{}^r r^2 \sin\theta d\theta d\Phi$$

: accretion rate of the energy

Monopole perturbation $(\ell = 0)$

General sols are

$$\delta M = \delta m + \int_{V_0}^{V} \mathcal{A}(\bar{V}, r) d\bar{V} - 4\pi \int_{r_0}^{r} \bar{r}^2 T_{Vr}(V_0, \bar{r}) d\bar{r}$$

$$\lambda = -4\pi \int_{r_0}^{r} \bar{r} T_{rr}(V, \bar{r}) d\bar{r} + \chi(V)$$

$$h^{(+)}_{\mu\nu}|_{\ell=0} dx^\mu dx^\nu = \left(\frac{2\delta M}{r} + 2f\lambda\right) dV^2 - 2\lambda dV dr$$

Monopole perturbation $(\ell = 0)$

Misner-Sharp mass at (V, r) becomes

$$M_{\mathrm{MS}} = M + \epsilon \delta M$$

$$\delta M = \delta m + \int_{V_0}^{V} \mathcal{A}(\bar{V}, r) d\bar{V} - 4\pi \int_{r_0}^{r} \bar{r}^2 T_{Vr}(V_0, \bar{r}) d\bar{r}$$

$\mathcal{A}$ determines the time dependence

$$\partial_V M_{\mathrm{MS}} = \epsilon \mathcal{A}$$

dipole perturbation ($\ell = 1$)

Perturbed metric (odd parity):

$$h^{(-)}_{\mu\nu}|_{\ell=1}dx^\mu dx^\nu = -2h_0(V,r)\sin^2\theta dV d\Phi$$

Energy momentum tensor:

$$\begin{aligned}T^{(-)}_{\mu\nu}|_{\ell=1}dx^\mu dx^\nu = -2\sin^2\theta d\Phi[&t_{V\Phi}(V,r)dV \\ &+t_{r\Phi}(V,r)dr]\end{aligned}$$

Equations: $\epsilon\mathcal{L}^{\rm Sch}[h_{\alpha\beta}]_{\mu\nu} = 8\pi\epsilon T_{\mu\nu}$

We also can solve these eqs!

---

dipole perturbation ($\ell = 1$)

$$\epsilon h^{(-)}_{\mu\nu}|_{\ell=1}dx^\mu dx^\nu = -\epsilon\frac{2r_0\sin^2\theta}{r}d\Phi dV$$

$$\times\left[\delta a + \int_{V_0}^{V}\mathcal{B}(\bar V, r_0)d\bar V + \frac{r}{r_0}(h_0^{\rm IH} + r^2C_2(V))\right]$$

$$\mathcal{B} := -\frac{1}{M}\int_0^{2\pi}\int_0^{\pi}T_\Phi{}^r r^2\sin d\theta d\Phi$$

$$= \frac{16\pi r^2}{3r_0}(t_{V\Phi} + f t_{r\Phi})$$: accretion rate of the angular momentum

$$h_0^{\rm IH}(V,r) = 16\pi r^2\int_{r_0}^{r}\frac{1}{\bar r^4}\left[\int_{r_0}^{\bar r}\tilde r^2 t_{r\Phi}(V,\tilde r)d\tilde r\right]d\bar r$$

dipole perturbation ($\ell = 1$)

Komar angular momentum

$$J_{\mathrm{Komar}} = \epsilon M \left[\delta a + \int_{V_0}^{V} \mathcal{B}(\bar{V}, r_0) d\bar{V} + \frac{r}{6M}\left(2h_0^{\mathrm{IH}} - r\partial_r h_0^{\mathrm{IH}} \right) \right]$$

$\mathcal{B}$ determines the time dependence

$$\partial_V J_{\mathrm{Komar}} = \epsilon M \mathcal{B}$$

Application to the Blandford-Znajek process

$T_{\mu\nu}$ for BZ process

Split monopole sol around slow rot Kerr

$$T^{\rm BZ}_{\mu\nu} = \left(F_{\mu\alpha} F_{\nu}{}^{\alpha} - \frac{1}{4} g^{\rm Kerr}_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$$

Blandford and Znajek (1977)

$F_{\mu\nu} \propto C$: strength of magnetic field

(explicit form can be seen in McKinney and Gammie (2004))

Energy and angular momentum extraction rate:

$$\dot{E}_{\rm BZ} := -\int_0^{2\pi} \int_0^{\pi} \sqrt{|g|}\, T_T{}^r d\theta d\Phi = \frac{\pi}{24} \frac{a^2 C^2}{M^4} + \mathcal{O}(a^4)$$

$$\dot{J}_{\rm BZ} := \int_0^{2\pi} \int_0^{\pi} \sqrt{|g|}\, T_\Phi{}^r d\theta d\Phi = \frac{\pi}{3} \frac{a C^2}{M^2} + \mathcal{O}(a^3)$$

We expect that the mass and angular momentum of BH $M_{\rm BH}, J_{\rm BH}$ decrease according to these rates

$$\dot{M}_{\rm BH} = -\dot{E}_{\rm BZ}, \quad \dot{J}_{\rm BH} = -\dot{J}_{\rm BZ}$$

How to discuss backreaction

We introduce two small parameters:

$$\alpha := \frac{a}{M}, \quad \beta := \frac{C^2}{M^2}$$

energy-momentum tensor:

$$T^{\rm BZ}_{\mu\nu} = \beta T^{(0,1)}_{\mu\nu} + \alpha\beta T^{(1,1)}_{\mu\nu} + \alpha^2 \beta T^{(2,1)}_{\mu\nu} + \mathcal{O}(\alpha^3 \beta)$$

metric: $g_{\mu\nu} = g^{\rm Kerr}_{\mu\nu} + g^{\rm BZ}_{\mu\nu}$

$$g^{\rm Kerr}_{\mu\nu} = g^{\rm Sch}_{\mu\nu} + \alpha h^{(1,0)}_{\mu\nu} + \alpha^2 h^{(2,0)}_{\mu\nu} + \mathcal{O}(\alpha^3)$$

$$g^{\rm BZ}_{\mu\nu} = \beta h^{(0,1)}_{\mu\nu} + \alpha\beta h^{(1,1)}_{\mu\nu} + \alpha^2 \beta h^{(2,1)}_{\mu\nu} + \mathcal{O}(\alpha^3 \beta)$$

$$G_{\mu\nu} = \beta G^{(0,1)}_{\mu\nu} + \alpha\beta G^{(1,1)}_{\mu\nu} + \alpha^2 \beta G^{(2,1)}_{\mu\nu} + \mathcal{O}(\alpha^3 \beta)$$

How to discuss backreaction

$$G_{\mu\nu} = 8\pi T^{\rm BZ}_{\mu\nu}$$

$$G_{\mu\nu} = \beta G^{(0,1)}_{\mu\nu} + \alpha\beta G^{(1,1)}_{\mu\nu} + \alpha^2\beta G^{(2,1)}_{\mu\nu} + \mathcal{O}(\alpha^3\beta)$$

$$T^{\rm BZ}_{\mu\nu} = \beta T^{(0,1)}_{\mu\nu} + \alpha\beta T^{(1,1)}_{\mu\nu} + \alpha^2\beta T^{(2,1)}_{\mu\nu} + \mathcal{O}(\alpha^3\beta)$$

We can discuss order by order

$$\beta G^{(0,1)}_{\mu\nu} = 8\pi\beta T^{(0,1)}_{\mu\nu}$$

$$\alpha\beta G^{(1,1)}_{\mu\nu} = 8\pi\alpha\beta T^{(1,1)}_{\mu\nu}$$

$$\alpha^2\beta G^{(2,1)}_{\mu\nu} = 8\pi\alpha^2\beta T^{(2,1)}_{\mu\nu}$$

How to discuss backreaction

At each order, Eqs can be written as

$$\beta\mathcal{L}^{\rm Sch}[h^{(0,1)}_{\alpha\beta}]_{\mu\nu} = 8\pi\beta T^{(0,1)}_{\mu\nu}$$

$$\alpha\beta\mathcal{L}^{\rm Sch}[h^{(1,1)}_{\alpha\beta}]_{\mu\nu} = 8\pi\alpha\beta T^{{\rm eff}\,(1,1)}_{\mu\nu}$$

$$\alpha^2\beta\mathcal{L}^{\rm Sch}[h^{(2,1)}_{\alpha\beta}]_{\mu\nu} = 8\pi\alpha^2\beta T^{{\rm eff}\,(2,1)}_{\mu\nu}$$

$$\left(\begin{aligned}\epsilon\mathcal{L}^{\rm Sch}[h_{\alpha\beta}]_{\mu\nu} = \epsilon\Big[& -\frac{1}{2}\nabla_\mu\nabla_\nu h^\alpha{}_\alpha - \frac{1}{2}\nabla^\alpha\nabla_\alpha h_{\mu\nu} + \nabla^\alpha\nabla_{(\mu}h_{\nu)\alpha} \\ & + \frac{1}{2}g_{\mu\nu}(\nabla^\alpha\nabla_\alpha h^\beta{}_\beta - \nabla^\alpha\nabla^\beta h_{\alpha\beta})\Big]\end{aligned}\right)$$

At each order, eqs are same forms as the linear perturbation around $g^{\rm Sch}_{\mu\nu}$ against effective energy momentum tensor

Results

[MK+ arXiv:2105.05581]

$\mathcal{O}(\beta)$:magnetized RN metric

$\mathcal{O}(\beta\alpha)$:time dependence of angular momentum

$$\partial_V J_{\mathrm{Komar}} = -\frac{\alpha\beta\pi M}{3} = -\dot{J}_{\mathrm{BZ}}$$

$\mathcal{O}(\beta\alpha^2)$:time dependence of mass??

We want to discuss whether our result can be fit by the Kerr metric with time dependent parameters or not.

Comparison with Kerr metric

Our perturbative sol can be written as

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{Kerr}} + g_{\mu\nu}^{\mathrm{BZ}}$$

$=$ [time dependent part] + [time independent part]

coincides with Kerr in EF coords with

$$M \to M - \dot{E}_{\mathrm{BZ}}(V - V_0)$$

$$a \to a - \dot{J}_{\mathrm{BZ}}(V - V_0)/M$$

BH mechanics

If we regard $-\dot{E}_{\mathrm{BZ}}$ as time dependence of BH mass, 1st law of BH mechanics holds at the location of the apparent horizon

$$\partial_T M = \frac{\kappa}{8\pi}\underline{\partial_T A} + \Omega_H \partial_T J$$

growth rate of
apparent horizon area

Summary and Discussions

- We discussed the metric backreaction of the mass and angular momentum accretion on the Schwarzschild BH
- We applied our formalism to the Blandford-Znajek process
  We determined the time dependence of the metric due to the back reaction of BZ process

- extension to higher order or consistent BZ sols.
- Penrose process, superradiance, etc
  [ongoing with Ogasawara et al]
- appropriate definition of mass

How to discuss backreaction

At each order, Einstein tensors become

$$\beta G^{(0,1)}_{\mu\nu} = \beta \mathcal{L}^{\rm Sch}[h^{(0,1)}_{\alpha\beta}]_{\mu\nu}$$

$$\alpha\beta G^{(1,1)}_{\mu\nu} =: \alpha\beta \mathcal{L}^{\rm Sch}[h^{(1,1)}_{\alpha\beta}]_{\mu\nu} - 8\pi\alpha\beta \tilde{T}^{(1,1)}_{\mu\nu}$$

$$\alpha^2\beta G^{(2,1)}_{\mu\nu} =: \alpha^2\beta \mathcal{L}^{\rm Sch}[h^{(2,1)}_{\alpha\beta}]_{\mu\nu} - 8\pi\alpha^2\beta \tilde{T}^{(2,1)}_{\mu\nu}$$

$$\epsilon\mathcal{L}^{\rm Sch}[h_{\alpha\beta}]_{\mu\nu} = \epsilon\Big[-\frac{1}{2}\nabla_\mu\nabla_\nu h^\alpha{}_\alpha - \frac{1}{2}\nabla^\alpha\nabla_\alpha h_{\mu\nu} + \nabla^\alpha\nabla_{(\mu}h_{\nu)\alpha} + \frac{1}{2}g_{\mu\nu}(\nabla^\alpha\nabla_\alpha h^\beta{}_\beta - \nabla^\alpha\nabla^\beta h_{\alpha\beta})\Big]$$

$$g_{\mu\nu} = g^{\rm Kerr}_{\mu\nu} + g^{\rm BZ}_{\mu\nu}$$

$$g^{\rm Kerr}_{\mu\nu} = g^{\rm Sch}_{\mu\nu} + \alpha h^{(1,0)}_{\mu\nu} + \alpha^2 h^{(2,0)}_{\mu\nu} + \mathcal{O}(\alpha^3)$$

$$g^{\rm BZ}_{\mu\nu} = \beta h^{(0,1)}_{\mu\nu} + \alpha\beta h^{(1,1)}_{\mu\nu} + \alpha^2\beta h^{(2,1)}_{\mu\nu} + \mathcal{O}(\alpha^3\beta)$$

Monopole perturbation $(\ell = 0)$

$\nabla^\mu T_{\mu V} = 0$: $\partial_r \mathcal{A} = -4\pi r^2 \partial_V T_{Vr}$

$$\mathcal{A} := \int_0^{2\pi}\int_0^{\pi} T_V{}^r r^2 \sin\theta d\theta d\Phi$$

$$= 4\pi r^2 (f T_{Vr} + T_{VV})$$

$\mathcal{A}$: accretion rate of the energy

$\mathcal{E} := \int_0^{2\pi}\int_0^{\pi} T_{VV} r^2 \sin\theta d\theta d\Phi$:energy (density)

Eq becomes $(f\partial_r + \partial_V)\mathcal{A} = \partial_V \mathcal{E}$

In static coords $f\partial_r \mathcal{A} = \partial_t \mathcal{E}$

Monopole perturbation $(\ell = 0)$

$\nabla^\mu T_{\mu V} = 0$: $\partial_r \mathcal{A} = -4\pi r^2 \partial_V T_{Vr}$

$$\mathcal{A} := \int_0^{2\pi}\int_0^{\pi} T_\mu{}^\nu (\partial_V)^\mu (dr)_\nu r^2 \sin\theta d\theta d\Phi$$

$$= 4\pi r^2 (f T_{Vr} + T_{VV})$$

$\mathcal{A}$: accretion rate of the energy

$\nabla^\mu T_{\mu r} = 0$:

$$4 r T_\Omega - 2\partial_r (r^2 (T_{Vr} + f T_{rr})) - r^2 T_{rr} \partial_r f$$
$$-2r^2 \partial_V T_{rr} = 0$$

Session B3b 14:30–16:00

[Chair: Norihiro Tanahashi]

Daniele Gregoris

Jiangsu University of Science and Technology

“Understanding Gravitational Entropy of Black Holes: A New Proposal via Curvature Invariants”

(15 min.)

[JGRG30 (2021) 120730]

UNDERSTANDING GRAVITATIONAL ENTROPY OF BLACK HOLES: A NEW PROPOSAL VIA CURVATURE INVARIANTS

danielegregoris@libero.it

Daniele Gregoris
(Jiangsu University of Science and Technology)

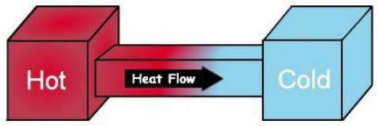
Based on: arXiv:2109.11968 [gr-qc]
with Yen Chin Ong

GRG IN JAPAN 30 – 2021 online workshop

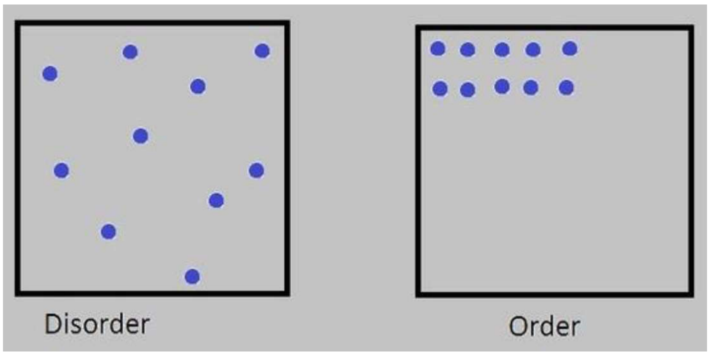
MANY DIFFERENT APPROACHES TO THE CONCEPT OF ENTROPY

- From thermodynamics: entropy as the arrow of time, entropy cannot decrease in time (Clausius);

Second law of Thermodynamics

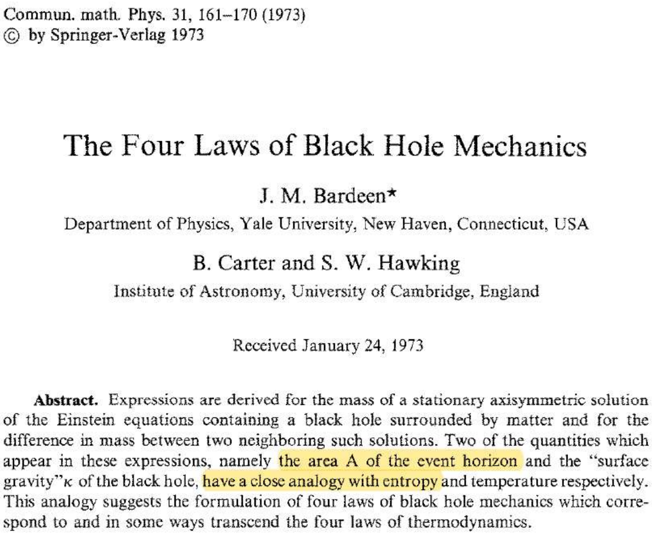


- From statistical mechanics: as a measure of disgregation and as a quantification of the number of different possible microscopic realizations of the same macroscopic system (Maxwell, Botzmann, Gibbs);
- From information theory: from a probabilistic perspective (von Neumann, Shannon);



- Can we assign a notion of entropy to the gravitational field?

THE PIONEERING WORKS OF HAWKING AND …



Commun. math. Phys. 31, 161–170 (1973)
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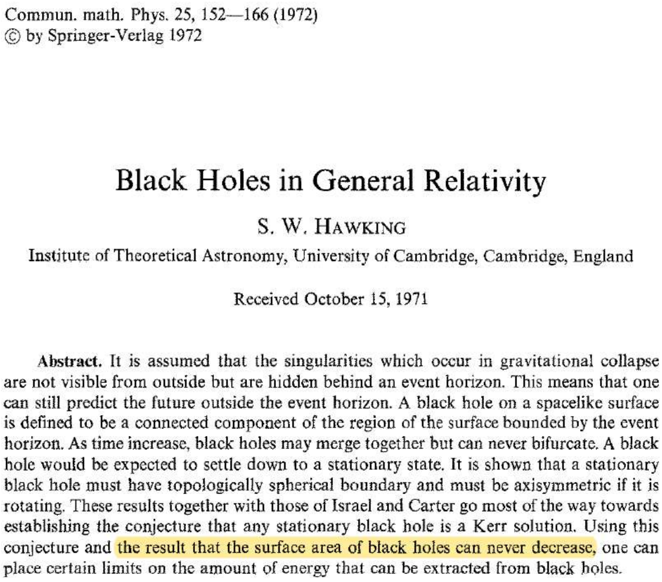
The Four Laws of Black Hole Mechanics

J. M. Bardeen*
Department of Physics, Yale University, New Haven, Connecticut, USA

B. Carter and S. W. Hawking
Institute of Astronomy, University of Cambridge, England

Received January 24, 1973

Abstract. Expressions are derived for the mass of a stationary axisymmetric solution of the Einstein equations containing a black hole surrounded by matter and for the difference in mass between two neighboring such solutions. Two of the quantities which appear in these expressions, namely the area A of the event horizon and the "surface gravity" κ of the black hole, have a close analogy with entropy and temperature respectively. This analogy suggests the formulation of four laws of black hole mechanics which correspond to and in some ways transcend the four laws of thermodynamics.



Commun. math. Phys. 25, 152—166 (1972)
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Black Holes in General Relativity

S. W. Hawking
Institute of Theoretical Astronomy, University of Cambridge, Cambridge, England

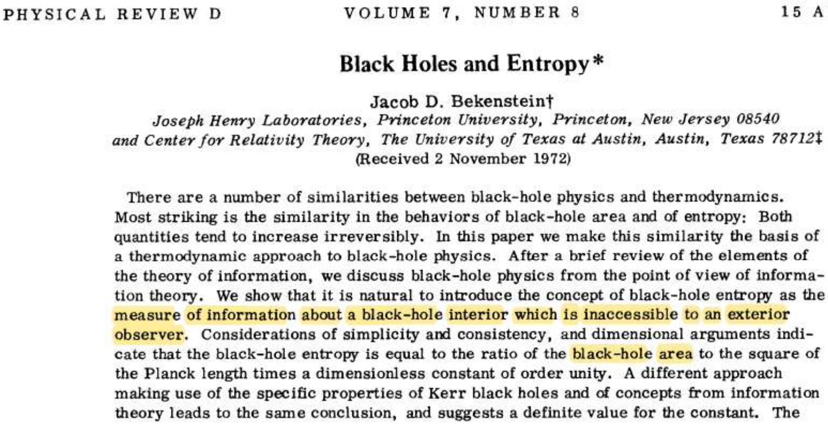
Received October 15, 1971

Abstract. It is assumed that the singularities which occur in gravitational collapse are not visible from outside but are hidden behind an event horizon. This means that one can still predict the future outside the event horizon. A black hole on a spacelike surface is defined to be a connected component of the region of the surface bounded by the event horizon. As time increase, black holes may merge together but can never bifurcate. A black hole would be expected to settle down to a stationary state. It is shown that a stationary black hole must have topologically spherical boundary and must be axisymmetric if it is rotating. These results together with those of Israel and Carter go most of the way towards establishing the conjecture that any stationary black hole is a Kerr solution. Using this conjecture and the result that the surface area of black holes can never decrease, one can place certain limits on the amount of energy that can be extracted from black holes.

HAWKING: BLACK HOLE ENTROPY IS GIVEN BY THE HORIZON AREA +
NEVER DECREASE AREA THEOREMS

THIS IS THE THERMODYNAMICAL
APPROACH TO ENTROPY

… AND BEKENSTEIN



PHYSICAL REVIEW D VOLUME 7, NUMBER 8 15 APRIL 1973

Black Holes and Entropy*

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(Received 2 November 1972)

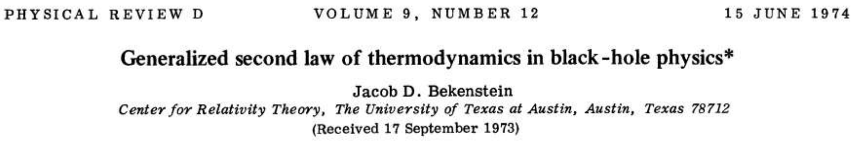
There are a number of similarities between black-hole physics and thermodynamics. Most striking is the similarity in the behaviors of black-hole area and of entropy: Both quantities tend to increase irreversibly. In this paper we make this similarity the basis of a thermodynamic approach to black-hole physics. After a brief review of the elements of the theory of information, we discuss black-hole physics from the point of view of information theory. We show that it is natural to introduce the concept of black-hole entropy as the measure of information about a black-hole interior which is inaccessible to an exterior observer. Considerations of simplicity and consistency, and dimensional arguments indicate that the black-hole entropy is equal to the ratio of the black-hole area to the square of the Planck length times a dimensionless constant of order unity. A different approach making use of the specific properties of Kerr black holes and of concepts from information theory leads to the same conclusion, and suggests a definite value for the constant. The

BEKENSTEIN: BLACK HOLE ENTROPY AS (SHANNON) INFORMATION
ENTROPY

REMARKABLY THE SAME RESULT AS HAWKING WAS OBTAINED: BLACK
HOLE ENTROPY IS HORIZON AREA

COMPLETELY DIFFERENT PHYSICAL
ARGUMENTS WERE USED

… BUT WHAT IS THIS ENTROPY ACTUALLY REFERRING TO?



PHYSICAL REVIEW D VOLUME 9, NUMBER 12 15 JUNE 1974

Generalized second law of thermodynamics in black-hole physics*

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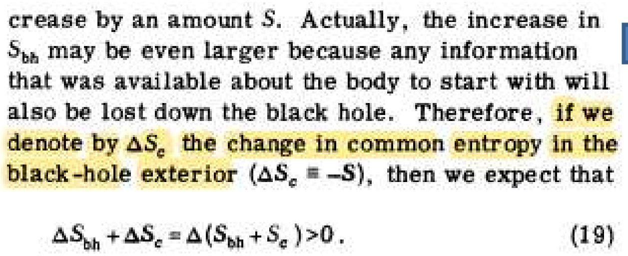


crease by an amount S. Actually, the increase in S_{bh} may be even larger because any information that was available about the body to start with will also be lost down the black hole. Therefore, if we denote by ΔS_c the change in common entropy in the black-hole exterior ($\Delta S_c \equiv -S$), then we expect that

$$\Delta S_{bh} + \Delta S_c = \Delta(S_{bh} + S_c) > 0. \quad (19)$$

BLACK HOLE ENTROPY IS THE ENTROPY OF THE PURE GRAVITATIONAL FIELD, AND IT SHOULD NOT BE CONFUSED WITH THE ENTROPY OF A MATTER FIELD OUTSIDE THE EVENT HORIZON

SCHWARZSCHILD IS AN EMPTY SPACETIME, BUT NEVERTHELESS IT COMES WITH A NONZERO ENTROPY

WHEELER: *IN GENERAL RELATIVITY WE CAN HAVE MASS WITHOUT HAVING MATTER*

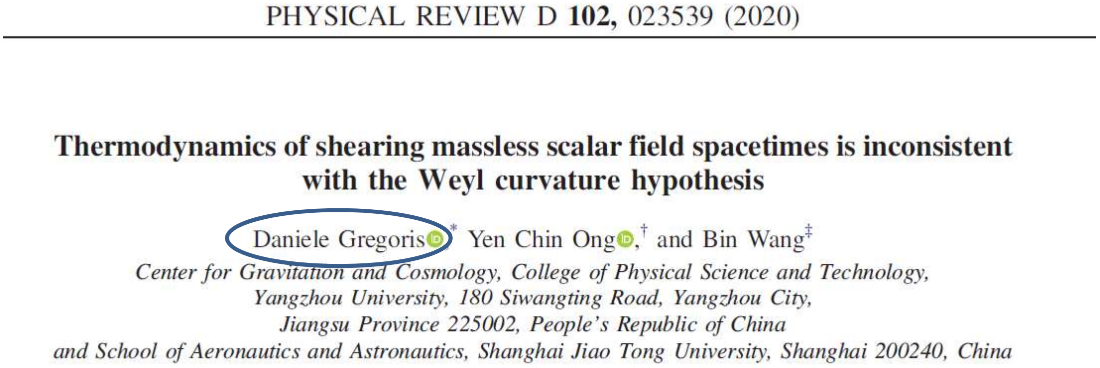
ADDING COSMOLOGICAL MOTIVATIONS: THE WEYL CURVATURE HYPOTHESIS BY ROGER PENROSE

- IT CONJECTURES THAT THE WEYL TENSOR SHOULD BE A GOOD MEASURE OF GRAVITATIONAL ENTROPY;
- IT IS EXPECTED THAT THE BIG BANG SINGULARITY SHOULD COME WITH ZERO WEYL CURVATURE, WHEREAS BIG CRUNCHES AND BLACK HOLE SINGULARITIES DUE TO GRAVITATIONAL COLLAPSE SHOULD HAVE LARGE WEYL CURVATURE;
- DURING THE COLLAPSE OF A STAR OF MASS M, ENTROPY INCREASES BY A FACTOR OF $10^{20}\,(M/M_o)^{1/2}$
- SINCE WEYL CURVATURE QUANTIFIES TIDAL DEFORMATIONS, THIS IS JUST THE STATEMENT THAT WE EXPECT BLACK HOLE AND BIG CRUNCH SINGULARITIES TO EXHIBIT VERY MESSY AND CHAOTIC CURVATURE BEHAVIOR, PERHAPS LIKE THOSE IN THE BKL DESCRIPTION.
- RIEMANN CURVATURE CAN BE DECOMPOSED INTO WEYL AND RICCI CURVATURE. RICCI CURVATURE IS GIVEN BY EINSTEIN EQUATIONS ONCE THE MATTER CONTENT IS KNOWN, WHILE WEYL CURVATURE CAN BE NONZERO ALSO IN VACUUM.

IMPLEMENTING THE WEYL CURVATURE HYPOTHESIS IS NOT A SIMPLE TASK

- Clifton-Ellis-Tavakol, Class. Quant. Grav. 30 (2013) 125009.
- It has been adopted by several authors for describing the formation of astrophysical structures (galaxies, filaments, voids, overdensities,...) in late-time cosmology (assuming dust) both using exact and approximate formalisms.
- Density of the gravitational entropy: $T_{\rm grav}\dot{s}_{\rm grav} = -dV\sigma_{ab}\left(\pi^{ab}_{\rm grav} + \frac{(\rho c^2+p)}{3\rho_{\rm grav}}E^{ab}\right)$

It is not a measure of the "pure" gravitational field because it depends directly also on ρ and p (e.g. on the matter content).



PHYSICAL REVIEW D **102,** 023539 (2020)

Thermodynamics of shearing massless scalar field spacetimes is inconsistent with the Weyl curvature hypothesis

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IMPLEMENTING THE WEYL CURVATURE HYPOTHESIS IS NOT A SIMPLE TASK

- The proposal of considering an entropy density proportional to the square of the Weyl curvature works for 5-dimensional Schwarzschild and Schwarzschild-anti-de Sitter black holes, but not for the Reissner-Nordström spacetime
- Li-Li-Song, EPJC 76 (2016) 111
- $S = \int_V C_{abcd}C^{abcd}dV$ <u>does not admit</u> a general applicability in black hole physics
- It was proposed to consider $S = \int_V \frac{C_{abcd}C^{abcd}}{R_{ab}R^{ab}}dV$ when studying isotropic cosmological singularities, but this proposal is directly sensitive to the matter content of the spacetime via the Ricci tensor
- Pelavas-Coley, Int. Jour. Theor. Phys. 45 (2006) 1258

FORMULATION OF THE QUESTION WE WANT TO ANSWER:

- Does an appropriate quantity χ function only of the Weyl curvature such that
$$S = \int_V \chi dV = \frac{A_H}{4}$$
exist for static and spherically-symmetric (possibly distorted) black holes
$$ds^2 = -f(r)[1+h(r)]dt^2 + \frac{[1+h(r)]dr^2}{f(r)} + r^2 d\Omega^2$$
$$h(r) = \sum_{k=0}^{\infty} \epsilon_k \left(\frac{M}{r}\right)^k ,$$
in 4 and 5 dimensions?

[For this matric ansatz see Yunes-Stein, PRD 83 (2011) 104002, Johannsen-Psaltis, PRD 83 (2011) 124015.]

OUR ANSWER: YES

Working with the Newman-Penrose formalism we can compute

$$\Psi_2 = \frac{r^2(1+h)^2 f'' + r^2 f(1+h)h'' + r(1+h)(rh'-2h-2)f' - f(h')^2 r^2 + 2(1+h)^2(f-1-h)}{12(1+h)^3 r^2},$$

$$D\Psi_2 = \frac{[r^2(1+h)^2 f'' + r^2 f(1+h)h'' + r(1+h)(rh'-2h-2)f' - (h')^2 f r^2 + 2(1+h)^2(f-1-h)]\sqrt{2f}}{8(1+h)^{7/2} r^3},$$

Therefore

$$S = \frac{1}{3\sqrt{2}} \int_0^{r_H} \int_\Omega \left|\frac{D\Psi_2}{\Psi_2}\right| r^2 \sqrt{\frac{1+h}{f}} dr d\Omega = \frac{A_H}{4}$$

Spatial hypersurface volume element

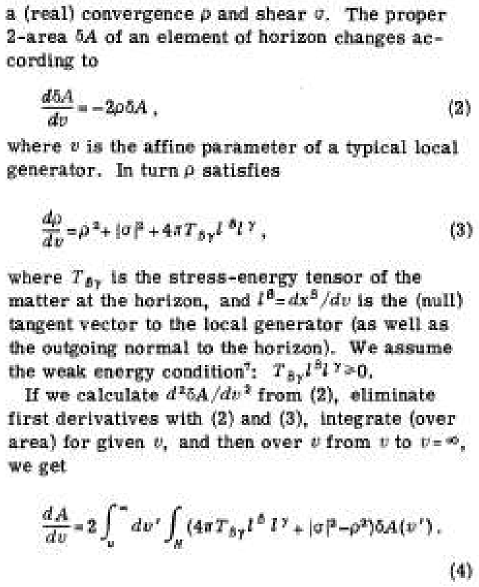
Remarks:

- Our formalism is fully based on the Weyl curvature: it is an appropriate result for a density of gravitational entropy;
- We have not made assumptions on $f(r)$: our formalism comes with a general applicability to all black hole spacetimes regardless of whether they are empty space solutions or not;
- Our formalism can be applied also to Bardeen regular black holes for which $f(r) = 1 - \frac{2Mr^2}{(r^2+Q^2)^{3/2}} + \frac{Q^2 r^2}{(r^2+Q^2)^2}$.

PHYSICAL CONSIDERATIONS

What we learnt about black hole entropy in general relativity:

- Black hole entropy is a related to tidal effects;
- Black hole entropy is a property of the focusing of light rays because we can use the expression for the Newman-Penrose spin coefficient $\rho \propto \frac{D\Psi_2}{\Psi_2}$.



a (real) convergence ρ and shear σ. The proper 2-area δA of an element of horizon changes according to

$$\frac{d\delta A}{dv} = -2\rho\delta A\,, \tag{2}$$

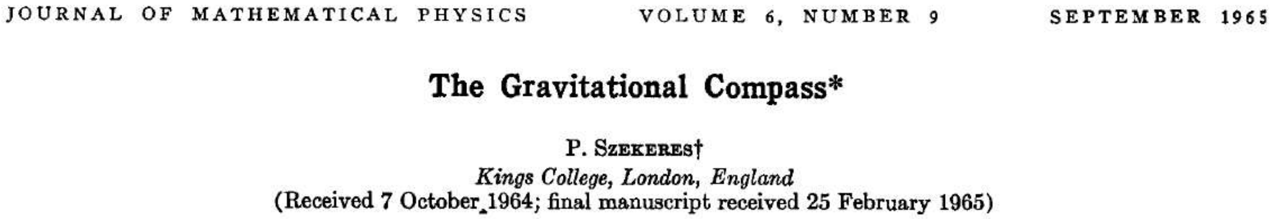
where v is the affine parameter of a typical local generator. In turn ρ satisfies

$$\frac{d\rho}{dv} = \rho^2 + |\sigma|^2 + 4\pi T_{\delta\gamma} l^{\delta} l^{\gamma}\,, \tag{3}$$

where $T_{\delta\gamma}$ is the stress-energy tensor of the matter at the horizon, and $l^{\delta} = dx^{\delta}/dv$ is the (null) tangent vector to the local generator (as well as the outgoing normal to the horizon). We assume the weak energy condition[7]: $T_{\delta\gamma} l^{\delta} l^{\gamma} \geq 0$.

If we calculate $d^2\delta A/dv^2$ from (2), eliminate first derivatives with (2) and (3), integrate (over area) for given v, and then over v from v to $v=\infty$, we get

$$\frac{dA}{dv} = 2\int_v^{\infty} dv' \int_H (4\pi T_{\delta\gamma} l^{\delta} l^{\gamma} + |\sigma|^2 - \rho^2)\delta A(v')\,. \tag{4}$$



JOURNAL OF MATHEMATICAL PHYSICS VOLUME 6, NUMBER 9 SEPTEMBER 1965

The Gravitational Compass*

P. Szekeres†
Kings College, London, England
(Received 7 October 1964; final manuscript received 25 February 1965)

OPEN PROBLEMS

Open question about gravitational entropy in general relativity:

- If we try to compute gravitational entropy according to our recipe in some inhomogeneous universe , do we obtain a function which is increasing in time in the same intervals in which spatial shear effects are? If yes, ours would be a good tool for investigating the formation of astrophysical structures.

Open question about black hole entropy beyond general relativity:

- Ours is a purely geometrical result because we have never used that $f(r)$ should arise as a solution of the Einstein field equations. Thus, if we apply our formula to some black hole which possesses the same symmetries but it is a solution is some modified gravity theory we still get a result which is an area.
- However, it has been argued that in modified gravitational theories, the entropy does not obey anylonger to an area law;
- So, in principle, a different combinations of curvature quantities should be adopted as a density of gravitational entropy;
- So: what is the physical foundation of entropy in modified gravity?

Session B3b 14:30–16:00

[Chair: Norihiro Tanahashi]

Chun-Hung Chen

The Institute for Fundamental Study, Naresuan University

“ On the Dolan-Ottewill method for solving quasinormal modes”

(15 min.)

[JGRG30 (2021) 120731]

On the Dolan-Ottewll method for solving quasinormal modes (QNMs).

$*$ Chun-Hung Chen,

Collaboration with:

† Hing-Tong Cho, ‡ Anna Chrysostomou, ‡ Alan S. Cornell.

$*$ The Institute for Fundamental Study, Naresuan University, Thailand.
† Department of Physics, Tamkang University, Taiwan.
‡ Department of Physics, University of Johannesburg, South Africa.

7th Dec. 2021, JGRG30.

Outline

Motivations - Methods

Having the studies for the fermionic QNMs and comparing with the bosonic QNMs of the spherically symmetric black hole spacetimes within the following limits

Large angular momentum limit.
Phys. Rev. D **104**, 024009 (2021)

Asymptotic QNMs (large overtone “n” limit).
Preprint version as arXiv:2107.00939.

In today’s presentation, we will focus on the large angular momentum limit one.

Motivations - Methods

The Dolan and Ottewill’s method (DO) based on

Expanding all the variables, including the wave function and the frequency, with finite positive and infinite negative power of the angular momentum parameter L.

Imposing an ansatz function on the wave function which related to the null geodesic for massless perturbations.

With the ansatz function on the wave function, the QNMs boundary conditions: purely ingoing and purely outgoing waves to the event horizon and spatial infinite, respectively, shall satisfy.

S. R. Dolan and A. C. Ottewill, Class. Quant. Grav. 26, 225003 (2009), arXiv:0908.0329[gr-qc].

The effective potentials

The metric elements for spherically symmetric black holes

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r) + r^2 d\Omega_{D-2},$$

$$f(r) = 1 - \frac{2\mu}{r^{d-3}} + \frac{\theta^2}{r^{2(d-3)}} - \lambda r^2 \ .$$

Here, μ, θ, and λ parametrise M, Q, and the cosmological constant Λ:

$$\mu \quad = \quad \frac{8\pi G_d}{(d-2)\,\Omega_{d-2}}\,M \quad , \quad \theta^2 = \frac{8\pi G_d}{(d-2)(d-3)}\,Q^2 \quad , \quad \lambda = \frac{2\Lambda}{(d-1)(d-2)}.$$

And the Schrödinger-like radial equation is (usually) given by

$$\frac{d^2}{dr_*^2}\Psi_s + \left(\omega^2 - V_s\right)\Psi_s = 0,$$

where r_* represent the tortoise coordinate and V_s represent the effective potential dominated the behavior of propagation wave.

The effective potentials

The bosonic effective potentials (Regge-Wheeler type)

$$V_{s=0,1,2}(r) = \frac{f(r)}{r^2}\left[\ell(\ell+d-3) + \frac{(d-2)(d-4)}{4} - \frac{K}{4}\,\lambda r^2 + \frac{P}{2r^{d-3}}\,\mu\right] ,$$

where the forms of K and P are associated with the perturbation of interest, as summarised in the Table.

perturbation type	K	P
scalar	$d(d-2)$	$(d-2)^2$
electromagnetic - scalar	$(d-2)(d-4)$	$d(d-4)$
electromagnetic - vector	$(d-4)(d-6)$	$-(d-4)(3d-8)$
gravitational - vector	$(d-2)(d-4)$	$-3(d-2)^2$
gravitational - tensor	$d(d-2)$	$(d-2)^2$

As a review article in E. Berti, V. Cardoso, and A. O Starinets, Class. Quant. Grav. **26**, No. 16. (2009)

The bosonic effective potentials (Zerilli type)

- gravitational scalar perturbation.

H. Kodama, and A. Ishibashi, Prog. Theor. Phys. 110 (2003) 701-722.

The effective potentials

The effective potentials - The Dirac perturbation

$$V_{s=1/2} = \pm f\,(r)\,\frac{dW}{dr} + W^2 \;\; ; \;\; W = \frac{\sqrt{f}}{r}\left(l + \frac{d-2}{2}\right),$$

where the sign $\pm$ represent a pair of supersymmetric partner potential, and $l = 0,\ 1,\ 2,\ ....$ corresponding to the eigenspinor on sphere. One can further simplify $V_{s=1/2}$ as,

$$V_{s=1/2} = W\left(\frac{1}{2}\frac{df\,(r)}{dr} - \frac{f\,(r)}{r} + W\right).$$

H. T. Cho, A. S. Cornell, J. Doukas, and W. Naylor, Phys. Rev. D **75**, 104005 (2007)

The effective potentials - The Rarita-Schwinger perturbation

non-TT (non transverse and traceless) related potentials.

TT related potentials.

C.-H. Chen, H. T. Cho, A. S. Cornell, and G. Harmsen, Phys. Rev. D **100**, 104018 (2019)

The Dolan and Ottewill's method (DO)

Starting with the Lagrangian in the equatorial plane ($\theta = \pi/2$) for spherically symmetric black holes is written as

$$\mathcal{L} = \frac{1}{2}g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = \frac{1}{2}\left(-f(r)\dot{t}^2 + f(r)^{-1}\dot{r}^2 + r^2\dot{\phi}^2\right) .$$

The corresponding orbital equations for null geodesic can be written as

$$\dot{r}^2 = L^2\left[\frac{E^2}{L^2} - \frac{f(r)}{r^2}\right] \equiv L^2\left[\frac{1}{b^2} - \frac{f(r)}{r^2}\right] .$$

Define a new function as

$$k^2(r,b) = \frac{1}{b^2} - \frac{f(r)}{r^2} .$$

The critical impact parameter may given by

$$k^2(r_c, b_c) = \partial_r k^2(r_c, b_c) = 0 .$$

One may further define the function

$$k_c(r) = \text{sgn}(r - r_c)\sqrt{k^2(r, b_c)} .$$

and take the wave function for the radial equation as

$$\psi(r) = \exp\left\{i\omega\int^{r_*} b_c k_c(r)dr_*\right\}\, v(r) ,$$

which shall be the ansatz function for the DO method.

The Dolan and Ottewill's method (DO)

Putting in the ansatz function to the radial equation,

$$\frac{d^2}{dr_*^2}\psi(r) + \left(\omega^2 - V_{eff}(r)\right)\psi(r) = 0,$$

and cancel out the exponential terms after obtaining the partial derivative coefficients with respect to each terms, we have

$$f(r)\frac{d}{dr}\left(f(r)\frac{dv}{dr}\right) + 2i\omega\rho(r)f(r)\frac{dv}{dr} + \left[i\omega f(r)\frac{d\rho(r)}{dr} + \left(1-\rho(r)^2\right)\omega^2 - V_{eff}(r)\right]v(r) = 0,$$

where $\rho(r) = b_c k_c(r)$, replace l in the effective potential with $L = l + \frac{1}{2}$ for bosonic cases, and the $\bar{L} = \kappa$(spinor eigenvalue on sphere) for fermionic cases, and expand ω and $v(r)$ as the polynomials of L as

$$\begin{aligned} \omega &= \sum_{k=-1}^{\infty} \omega_k L^{-k}, \\ v(r) &= \exp\left\{\sum_{k=0}^{\infty} S_k(r) L^{-k}\right\}. \end{aligned}$$

and solving the coefficient order by order, we may find the QNM solutions.

The Dolan and Ottewill's method (DO)

As an example for Regge-Wheeler type potentials in Schwarzschild cases, we have

$$r_c = 3\,, \quad b_c = \sqrt{27} \quad \Rightarrow \; \rho(r) = \left(1 - \frac{3}{r}\right)\sqrt{1 + \frac{6}{r}}\ .$$

The radial equations transform to

$$\begin{aligned} \frac{d}{dr}\left(f(r)\frac{dv(r)}{dr}\right) &+ 2i\omega\left(1-\frac{3}{r}\right)\left(1+\frac{6}{r}\right)^{\frac{1}{2}}\frac{dv(r)}{dr} \\ &+ \left[\frac{27i\omega}{r^3}\left(1+\frac{6}{r}\right)^{-\frac{1}{2}} + \left(\frac{27\omega^2 - L^2}{r^2}\right) + \frac{1}{4r^2} - \frac{2(1-s^2)}{r^3}\right]v(r) = 0. \end{aligned}$$

Putting in

$$\begin{aligned} \omega &= \sum_{k=-1}^{\infty} \omega_k L^{-k}, \\ v(r) &= \exp\left\{\sum_{k=0}^{\infty} S_k(r) L^{-k}\right\}, \end{aligned}$$

and comparing the order of L, we shall have the analysis in next page.

The Dolan and Ottewill's method (DO)

For L^2 order:

$$\frac{27}{r^2}\omega_{-1}^2 - \frac{1}{r^2} = 0 \quad \Rightarrow \quad \omega_{-1} = \frac{1}{\sqrt{27}}.$$

For L^1 order:

$$2i\omega_{-1}\rho(r)\partial_r S_0(r) + 2\frac{27}{r^2}\omega_{-1}\omega_0 + \frac{27}{r^3}\left(1+\frac{6}{r}\right)^{-\frac{1}{2}}\omega_{-1} = 0.$$

Taking $r = r_c = 3$, we have $\rho(r_c) = 0$, and putting ω_{-1} one may solve

$$\omega_0 = \frac{-i}{2\sqrt{27}} \quad \Rightarrow \quad b_c\omega_0 = \frac{-i}{2}.$$

Putting back to the coefficient of L^1 order, and consider r in general, one may solve

$$\partial_r S_0(r) = \frac{\sqrt{27}\left((1+\frac{6}{r})^{\frac{1}{2}} - \frac{\sqrt{27}}{r}\right)}{2(r+6)(r-3)}.$$

Consider the next L^0 order, together with ω_{-1}, ω_0, and $\partial_r S_0(r)$, we may find ω_1 and $\partial_r S_1(r)$. As well as continue to the higher orders.

The Dolan and Ottewill's method (DO)

As a remark that the setting of r_c, b_c, and $\rho(r)$ shall depends on the black hole spacetimes, here are the setting for RN and Schwarzschild dS black holes.

For the metric function for RN black hole shall be $f(r) = 1 - 2/r + \theta^2/r^2$,

$$r_c = \frac{3 \pm \alpha}{2}\,, \qquad b_c = \sqrt{\frac{(\alpha+3)^3}{2(\alpha+1)}}$$

$$\Rightarrow \qquad \rho(r) = \left(1 - \frac{r_c}{r}\right)\sqrt{1 + \frac{(\alpha-3)}{(\alpha+1)}\left(\frac{r_c}{r}\right)^2 + \frac{(\alpha+3)}{r}}\,.$$

for $\alpha = \sqrt{9 - 8\theta^2}$ and using the outer orbit.

The SdS BH spacetime is markedly similar to its flat-space counterpart. From the metric function $f(r) = 1 - 2/r - \eta r^2/27$, where $\eta = 27\lambda = 9\Lambda$, we obtain

$$r_c = 3\,, \qquad b_c = \sqrt{\frac{27}{1-\eta}}\,, \qquad \rho(r) = \left(1 - \frac{3}{r}\right)\sqrt{\frac{r+6}{(1-\eta)\,r}}\,.$$

Results

Table: *The inverse multipolar expansions for the effective QNFs of spin s in Schwarzschild black hole.*

s	$b_c \sum_{k=-1}^{6} \omega_k L^{-k}$ and $b_c = \sqrt{27}$
	perturbations of integer spin
0	$L - \frac{i}{2} + \frac{7}{216L} - \frac{137}{7776L^2} i + \frac{2615}{1259712L^3} + \frac{590983}{362797056L^4} i - \frac{42573661}{39182082048L^5} + \frac{11084613257}{8463329722368L^6} i$
1	$L - \frac{i}{2} - \frac{65}{216L} + \frac{295}{7776L^2} i - \frac{35617}{1259712L^3} + \frac{3374791}{362797056L^4} i - \frac{342889693}{39182082048L^5} + \frac{74076561065}{8463329722368L^6} i$
2	$L - \frac{i}{2} - \frac{281}{216L} + \frac{1591}{7776L^2} i - \frac{710185}{1259712L^3} + \frac{92347783}{362797056L^4} i - \frac{7827932509}{39182082048L^5} - \frac{481407154423}{8463329722368L^6} i$
	perturbations of half-integer spin
1/2	$\bar{L} - \frac{i}{2} - \frac{11}{216\bar{L}} - \frac{29}{7776\bar{L}^2} i + \frac{1805}{1259712\bar{L}^3} + \frac{27223}{362797056\bar{L}^4} i + \frac{23015171}{39182082048\bar{L}^5} - \frac{6431354863}{8463329722368\bar{L}^6} i$
3/2	$\bar{L} - \frac{i}{2} - \frac{155}{216\bar{L}} + \frac{835}{7776\bar{L}^2} i - \frac{214627}{1259712\bar{L}^3} + \frac{25750231}{362797056\bar{L}^4} i - \frac{2525971453}{39182082048\bar{L}^5} + \frac{292606736465}{8463329722368\bar{L}^6} i$

Note that $L = l + \frac{1}{2}$(half integer) for bosonic cases, and the $\bar{L} = l + 1$(integer) for fermionic cases in four dimensional cases. For the RN and Schwarzschild dS results are presented in detail in our current work.

We studied the DO methods until $\sim O(L^{-6})$ order for large angular momentum limit with various kind of perturbations, including fermionic and bosonic perturbations, in Schwarzschild, RN, and Schwarzschild dS black holes.

Results

Compare the low-lying results for WKB, DO, and references.

Table: *Spin-1/2 QNFs of Schwarzschild BHs from references and computed with the 6th-order WKB, Posch-Teller (PT) approximation, and DO expansion.*

ℓ	ω (WKB)	ω (PT)	ω (DO)
1	0.3801-0.0964i	0.3855-0.0991i	0.3800-0.0964i
2	0.5741-0.0963i	0.5779-0.0975i	0.5741-0.0963i
3	0.7674-0.0963i	0.7702-0.0969i	0.7674-0.0963i
4	0.9603-0.0963i	0.9625-0.0963i	0.9603 - 0.0963i

Table: *Spin-0 QNFs for 4D RN BHs calculated using the DO method and compared with the 6th-order WKB PT results.*

ℓ	ω $(\theta = 0.2)$	ω $(\theta = 0.4)$	ω $(\theta = 0.6)$	ω $(\theta = 0.8)$
2 (DO)	0.4876 - 0.0971i	0.5001 - 0.0978i	0.5245 - 0.0989i	0.6078 - 0.0973i
2 (WKB)	0.4869-0.09697i	0.4974-0.09756i	0.5174-0.09833i	0.5531-0.09834i
2 (PT)	0.4913-0.0982 i	0.5041-0.0989 i	0.5288-0.0998 i	0.5747-0.1001 i
3 (DO)	0.6804 - 0.0967i	0.6970 - 0.0974i	0.7277 - 0.0984i	0.8318 - 0.0967i
3 (WKB)	0.6805 - 0.0967i	0.6967 - 0.0974i	0.7281 - 0.0983i	0.7853 - 0.0985i
3 (PT)	0.6832 - 0.0973i	0.6994 - 0.0980i	0.7306 - 0.0989i	0.7876 - 0.0990i

The QNM results evaluated by DO methods are not only sufficient in large angular momentum limit but also the low-lying modes in an approximate consistency with WKB and the other methods..

The further applications for DO method

The higher dimensional fermionic perturbation cases are not yet done.

Especially for the spin-3/2 non-TT related potentials in the higher dimensional Schwarzschild dS black hole spacetimes since the irregular properties of the radial equation and seems not able to solve by the WKB methods.

Is it possible to generalize DO method to the other boundary conditions?

For example, the Dirichlet or vanishing energy flux boundary conditions for the AdS black holes, or the boundary conditions for solving the greybody factors.

The extension to the perturbations in Kerr-like black holes.

The extension for $l = m$ modes were done by Dolan after the first paper of this method, the $l \neq m$ modes seems not yet been done.
S. R. Dolan, Phys. Rev. D **82**, 104003 (2010).

The excitation factors and sub-dominated modes.

The QNMs, the corresponding Green function, and the excitation factors were known as the ways to understand the sub-dominated modes for ring-down behavior. The following papers presented interesting results and warrant further studies.
H. Yang, F. Zhang, A. Zimmerman, and Y. Chen, Phys. Rev. D **89**, 064014 (2014).
Z. Zhang, E. Berti, and V. Cardoso Phys. Rev. D **88**, 044018 (2013).

Thank you for your attention.

Session B3b 14:30–16:00

[Chair: Norihiro Tanahashi]

Ratchaphat Nakarachinda

Naresuan university

“Effective thermodynamical system of Schwarzschild–de Sitter black holes from Renyi statistics”

(15 min.)

[JGRG30 (2021) 120732]

Effective thermodynamical system of Schwarzschild–de Sitter black holes from Rényi statistics

Ratchaphat Nakarachinda

The Institute for Fundamental Study (IF), Naresuan University, Thailand

Present in
The 30th Workshop on General Relativity
and Gravitation in Japan (JGRG30)

Outline

1. Introduction
    - Non-extensivity of BH entropy
    - Effective approach
2. Effective BH system with Rényi statistics [2106.02838]
3. Conclusion

Outline

1. Introduction
   - Non-extensivity of BH entropy
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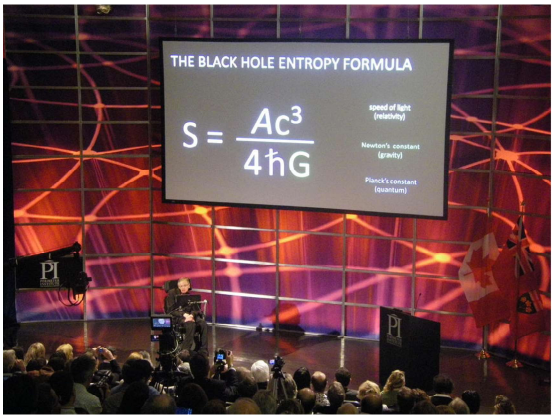
Introduction

- It is well known that the entropy of the black hole (BH) is actually the surface area at its horizon. [Bekenstein, 1973] & [Hawking, 1975]

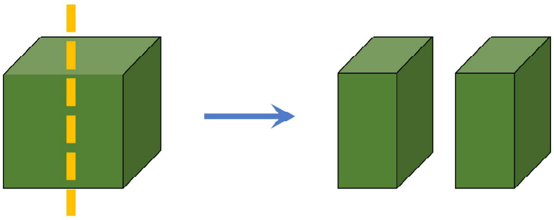


Hawking discusses the black hole entropy formula at the Perimeter Institute for Theoretical Physics (June 2010).

From www.570news.com

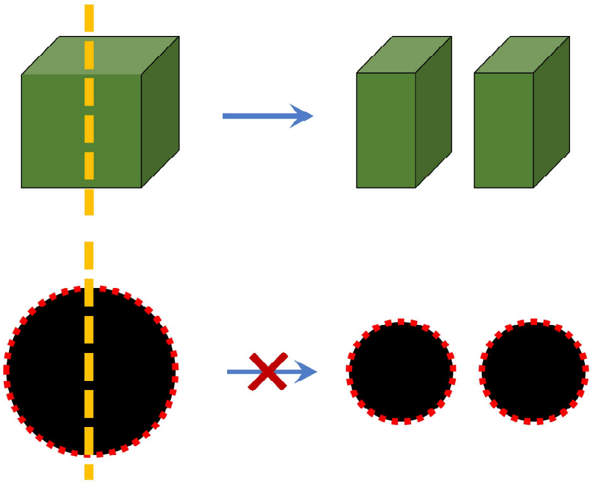
Non-extensivity of BH entropy

- The entropy of the normal system is an extensive quantitiy.



Non-extensivity of BH entropy

- However, it is non-extensive for BH.



BH thermodynamics with Rényi entropy

- The BH entropy is regarded as the Tsallis entropy and then study in the 0th law compatible form (i.e. the Rényi entropy).

$$S_{\mathrm{R}} = \frac{1}{\lambda} \ln \left(1 + \lambda S_{\mathrm{BH}}\right), \qquad S_{\mathrm{BH}} = \pi r_h^2. \tag{1}$$

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- Studies of BH thermo with Rényi entropy
  - → Spherical sym BH [1511.06963, 2106.02406]
  - → Rotating BH [1702.05341]
  - → Charged BH [2003.12986]
  - → Spherical sym BH with Λ [2002.00377, 2106.02838]

  etc.

BH thermodynamics with Rényi entropy

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- Studies of BH thermo with Rényi entropy
  - → Spherical sym BH [1511.06963, 2106.02406]
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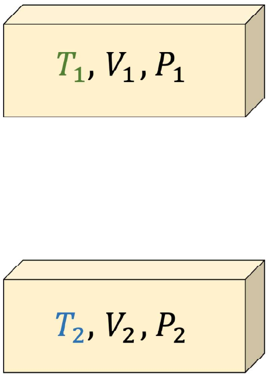
  - → Charged BH [2003.12986]
  - → Spherical sym BH with Λ [2002.00377, 2106.02838]

    etc.
- The effect of the Rényi entropy gives the similar thermodynamical behaviour of the BHs in AdS space.

Outline

Effective approach

- From the fact that each horizon of the BH can be treated as a thermal system separately.



Horizon-1 system

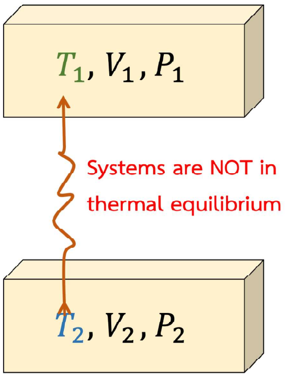
T_1, V_1, P_1

T_2, V_2, P_2

Horizon-2 system

Effective approach

- These systems are thermal <u>non-equilibrium</u> in which thermodynamics is not applicable.



Horizon-1 system

T_1, V_1, P_1

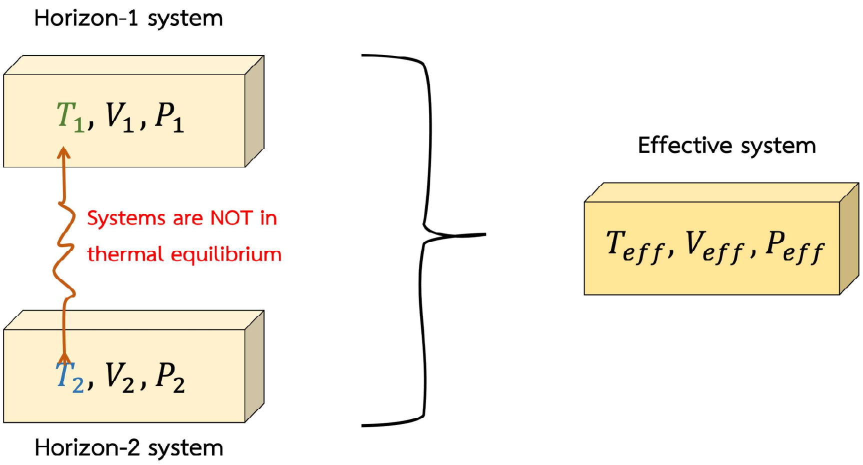
Systems are NOT in thermal equilibrium

T_2, V_2, P_2

Horizon-2 system

Effective approach

- Thought as an effective equilibrium system (called the effective approach) [Urano, Tomimatsu & Saida, 2009]



Horizon-1 system

T_1, V_1, P_1

Systems are NOT in thermal equilibrium

T_2, V_2, P_2

Horizon-2 system

Effective system

$T_{eff}, V_{eff}, P_{eff}$

Outline

This study

- We are going to study the thermodynamic behaviour of the multi-hotizon BH (Sch-dS) by using the non-extensive Rényi entropy and effective system approach.

The 1st law for the effective system

$$\delta M = T_{\text{eff}}\delta S + V_{\text{eff}}\delta P. \qquad (2)$$

→ Entropy: $S = S_{R(b)} + S_{R(c)} = \frac{1}{\lambda}\ln\left[\left(1+\lambda\pi r_b^2\right)\left(1+\lambda\pi r_c^2\right)\right].$

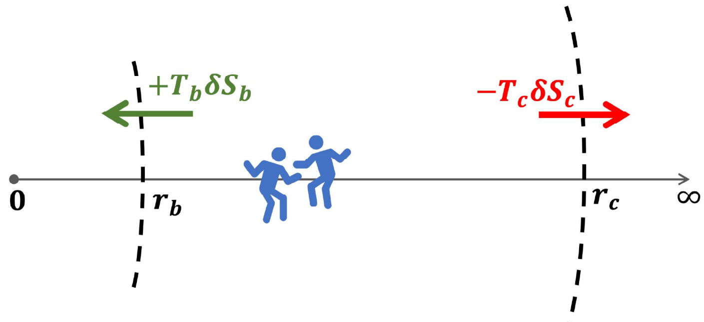
→ Pressure: $P = -\frac{\Lambda}{8\pi}.$

Introducing the negative sign in the effective approach

1st laws of the subsystems

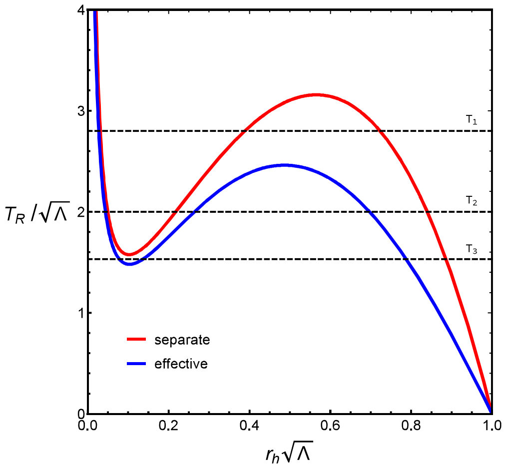
$$\delta M = T_b\delta S_b + V_b\delta P, \qquad (3)$$

$$\delta M = -T_c\delta S_c + V_c\delta P. \qquad (4)$$



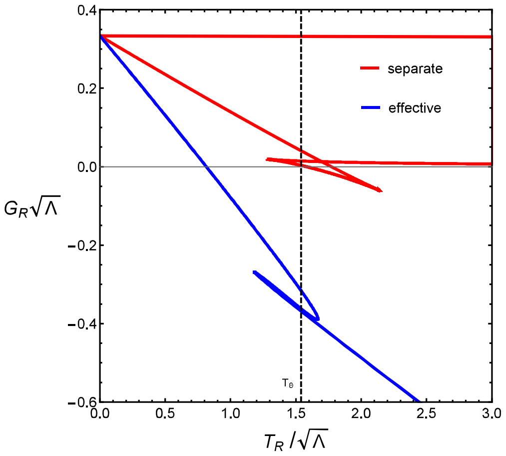
$$\delta S = \frac{\mathrm{d}S_b}{\mathrm{d}r_b}\delta r_b - \frac{\mathrm{d}S_c}{\mathrm{d}r_c}\delta r_c$$

Temperature



- At T_1, there exists only the stable BH described by separate approach.
- At T_3, there exists only the stable BH described by effective approach.
- At T_2, the stable BHs with both descriptions exist.
  → The effective BH is larger than separate BH.

Gibbs free energy



- The transition from the hot gas phase to the stable BH phase:
  → It is the 1st-order phase transition for BH described by separate approach.
  → It is the 0th-order phase transition for BH described by effective approach.

Outline

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Conclusion

- The Rényi entropy is used as the zeroth law compatible form of the BH entropy.
- In order to avoid the problem of non-equilibrium, the Schwarzschild-de Sitter BH is chosen to describe by the effective approach.
- The distinguishable features of the BH with both separate and effective descriptions are discussed.

Thank you
for
your attention

Session B3b 14:30–16:00

[Chair: Norihiro Tanahashi]

Emmanuel Frion

Helsinki Institute of Physics

“Testing the Equivalence Principle with Black Hole Shadows and Photon Rings”

(15 min.)

[JGRG30 (2021) 120733]

Black Hole Shadow and Photon Ring Frequency Drifts

Emmanuel Frion (Helsinki Institute of Physics)
In collaboration with L. Giani and T. Miranda
Based on Open J.Astrophys. 4 (2021) 1 (arXiv:2107.13536)

JGRG30, 7 December 2021

Assumptions

Strong Hints for an Expanding Universe

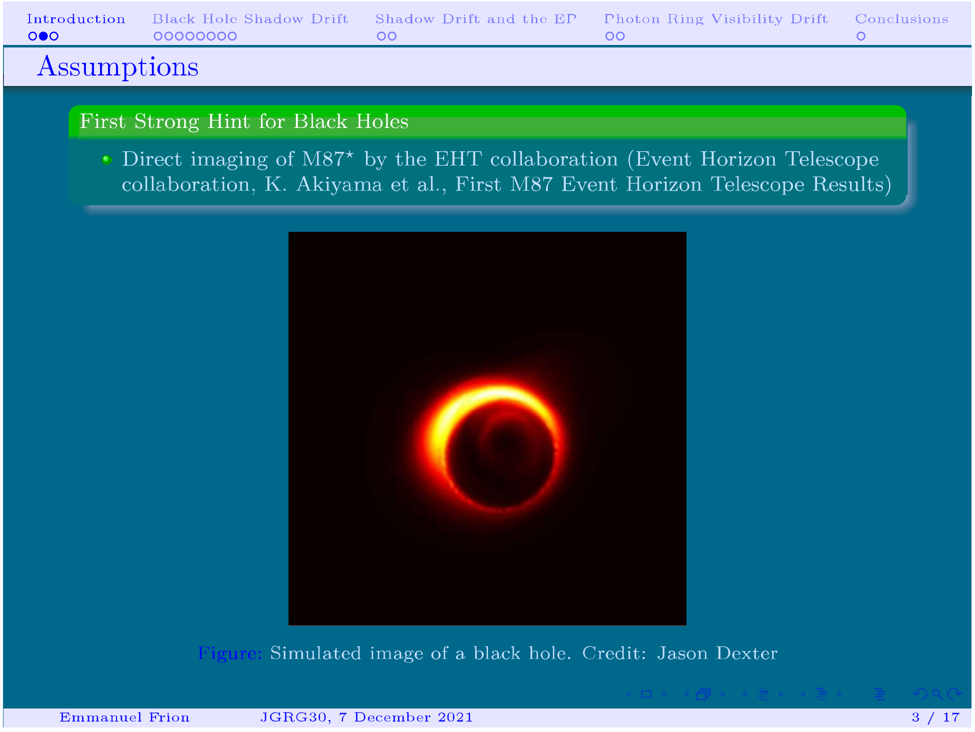
- Supernovae
- Cosmic Microwave Background
- Large-Scale Structures
- Cosmic Chronometers
- and more...

Direct Consequence on Redshift (Sandage, McVittie, The Astr. Journ. 136 (1962) 319)

- Redshift is dynamical: redshift drift.
- Cosmological drift driven by the expansion of the Universe: everything is time-dependent!

$$\frac{\mathrm{dz}}{\mathrm{dt}} = (1+\mathrm{z})\mathrm{H}_0 - \mathrm{H}(\mathrm{z}) \ . \tag{1}$$

- Very small at low redshift $\approx 10^{-10}$ per year. Target of big collaborations (ELT, SKA, Vera Rubin). Optimistic estimates show that these facilities could reach this precision.



Assumptions

First Strong Hint for Black Holes

- Direct imaging of M87* by the EHT collaboration (Event Horizon Telescope collaboration, K. Akiyama et al., First M87 Event Horizon Telescope Results)

Figure: Simulated image of a black hole. Credit: Jason Dexter

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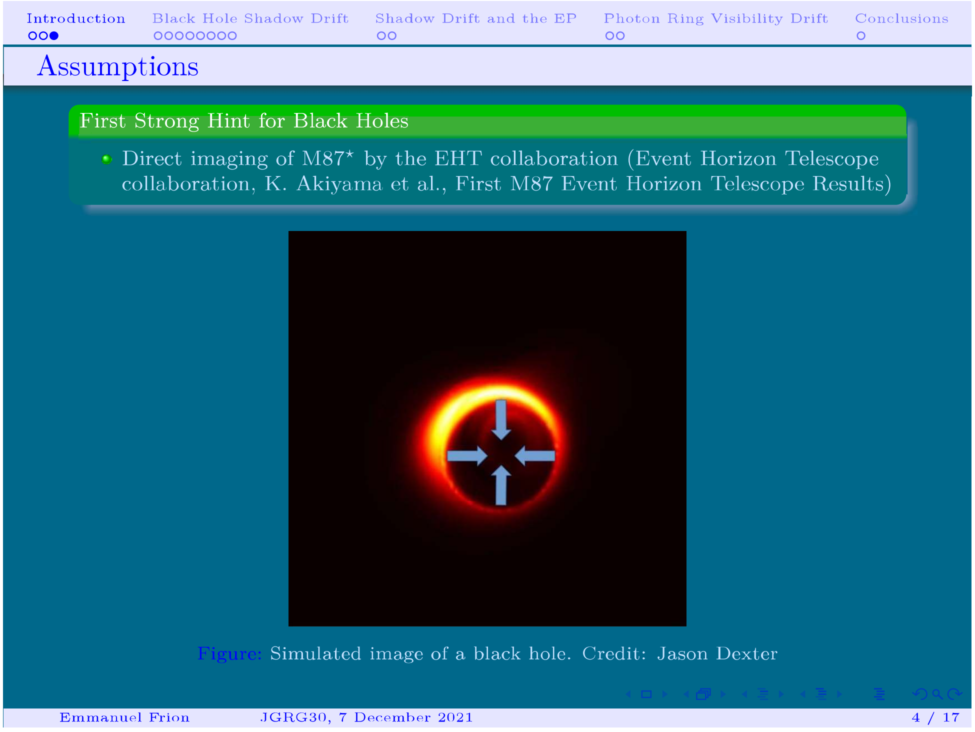

Assumptions

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- Direct imaging of M87* by the EHT collaboration (Event Horizon Telescope collaboration, K. Akiyama et al., First M87 Event Horizon Telescope Results)

Figure: Simulated image of a black hole. Credit: Jason Dexter

Schwarzschild black hole in an expanding universe

The McVittie metric (MNRAS 93 (1933) 325–339)

- Hybrid solution: Schwarzschild near BH, flat FLRW far away.

$$\mathrm{ds}^2 = -\left(\frac{1-\mu}{1+\mu}\right)^2 \mathrm{c}^2\mathrm{dt}^2 + (1+\mu)^4\, \mathrm{a}^2(\mathrm{t})\, \left(\mathrm{dl}^2 + \mathrm{l}^2\mathrm{d}\Omega^2\right) \,, \tag{2}$$

with

$$\mu := \frac{\mathrm{m}}{2\mathrm{a}(\mathrm{t})\mathrm{l}} \,, \quad \mathrm{m} = \frac{\mathrm{GM}}{\mathrm{c}^2} \,. \tag{3}$$

l: radial comoving distance, a(t): scale factor, M: BH's mass.

- Approximate BH shadows (Tsupko, Bisnovatyi-Kogan, arXiv:1912.07495):

$$\alpha_{\mathrm{appr}}(\mathrm{R_O}) = \underbrace{\alpha_{\mathrm{schw}}(\mathrm{R_O})}_{\text{near BH}} - \underbrace{\alpha_{\mathrm{overlap}}(\mathrm{R_O})}_{\text{intermediate region}} + \underbrace{\alpha_{\mathrm{cosm}}(\mathrm{R_O})}_{\text{cosmological scales}} \,, \tag{4}$$

$\mathrm{R_O}$: position of the observer.

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with

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$\mathrm{R_O}$: position of the observer.

IF OBSERVER FAR AWAY FROM BH: $\alpha_{\mathrm{overlap}} = \alpha_{\mathrm{schw}}$.

Apparent angular diameter in an expanding universe

BH shadow at cosmological distances

- Drift in the shadow angular radius

$$\frac{\dot{\alpha}_{\rm appr}}{\alpha_{\rm appr}} = \frac{\dot{\alpha}_{\rm cosm}}{\alpha_{\rm cosm}} \ . \tag{8}$$

- Cosmological solution is redshift-dependent (Bisnovatyi-Kogan, Tsupko, ☠:1805.03311)

$$\alpha_{\rm cosm}(R_O) = \frac{3\sqrt{3}m}{D_A(z)} \tag{9}$$

- Angular diameter distance in a flat universe

$$D_A(z) := \frac{c}{1+z}\int_0^z \frac{d\tilde{z}}{H(\tilde{z})} = \frac{1}{1+z}\chi \ , \tag{10}$$

χ: comoving distance to BH (time-independent).

Apparent angular diameter in an expanding universe

BH shadow at cosmological distances

- Redshift drift in FLRW (Sandage, McVittie)

$$\frac{dz}{dt} = H_0(1+z) - H(z) \ , \tag{11}$$

- Shadow drift

$$\frac{\dot{\alpha}}{\alpha} = H_0 - \frac{H(z)}{1+z} \ , \tag{12}$$

Black Hole Shadow Drift – Probe of Cosmology

Flat ΛCDM with matter Ω_{m0}, radiation Ω_{r0} and dark energy Ω_Λ densities

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_\Lambda}\ . \tag{13}$$

Black Hole Shadow Drift – Probe of Cosmology

Late-time flat ΛCDM with matter Ω_{m0}, ~~radiation Ω_{r0}~~ and dark energy Ω_Λ densities

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \cancel{\Omega_{r0}(1+z)^4} + \Omega_\Lambda}\ . \tag{14}$$

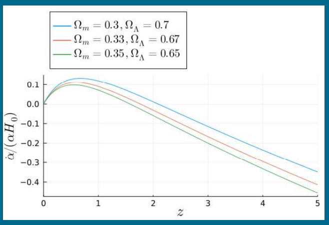


Figure: Different cosmological models expect different drifts

Shadow drift normalised with H_0

- Drift bigger with redshift...
- ... but surface brightness plummets $\propto (1+z)^{-4}$. At $z = 1$: reduced 16 times (Vagnozzi, Bambi, Visinelli, ✠:2001.02986).
- Better chance of observation around $z = 0.5$.

Black Hole Shadow Drift – Probe of local Hubble Rate

"Real" shadow drift (not relative), at $\mathrm{m} = \mathrm{cst}$

$$\dot{\alpha} = \alpha\left(\mathrm{H}_0 - \frac{\mathrm{H(z)}}{1+\mathrm{z}}\right) \ , \quad \alpha = \frac{3\sqrt{3}\mathrm{m}}{\mathrm{D_A(z)}} \qquad (15)$$

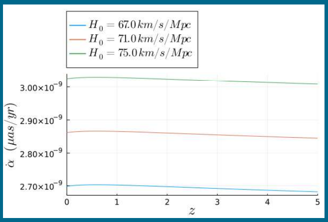


Figure: A measurement of H_0? $\Omega_{\mathrm{m}0} = 0.33$, $\Omega_{\mathrm{r}0} = 0$, $\Omega_\Lambda = 0.67$, and $\alpha = 40\mu\mathrm{as}$.

However, ...

- ... an independent measure of H_0 is possible only if given an independent measure of the black hole's mass!

Black Hole Shadow Drift – M87

Application to M87*

- M87*: BH at the centre of galaxy Messier 87.
- Estimated distance: $\mathrm{z} \sim 0.004$, very-low redshift, but still cosmological scales.
- $\Omega_{\mathrm{m}0} \simeq 0.31$, $\Omega_\Lambda \simeq 0.67$, $\mathrm{H}_0 = 67.8$ km/s/Mpc.
- Shadow drift

$$\frac{\dot{\alpha}}{\alpha} \approx 2\mathrm{H}_0 \times 10^{-3} \approx 3.8 \times 10^{-16}/\mathrm{day}\ . \qquad (16)$$

- A tiny variation, far from the EHT angular resolution on Earth, even for the future Plateau de Bure–South Pole baseline (R: 15 μas at 345 GHz). Earth–Moon baseline: 10 R. Earth-L2 Lagrange: 100 R. Still very low.

Shadow drift as a probe of the equivalence principle

Should we forget about shadow drifts? No!

- Can still be useful when $m = GM/c^2$ is time-dependent
- Shadow drift

$$\frac{\dot{\alpha}}{\alpha} = H_0 - \frac{H(z)}{1+z} + \frac{\dot{G}}{G}\,. \qquad (17)$$

- Non-observation of shadow drift = probe of the equivalence principle (but degenerate with mass...)

Shadow drift as a probe of the equivalence principle

Let's use data from M87*

- Three different pipelines (DIFMAP, eht-imaging and SMILI)
- Non-observation over a period of 6 days: $\dot{G}/G \lesssim 10^{-1} - 10^{-2}$ day^{-1} $\implies$ $10^{-3} - 10^{-4}$ year^{-1}.
- Far inferior to other probes (the strongest constraint gives $\dot{G}/G \leq 10^{-13} - 10^{-14}$ day^{-1}, see Table 1 from Alestas et al. : 2104.14481)...
- ...but almost model-independent probe of the EP (similar to Giani & Frion, :2005.07533 for strong lensing)

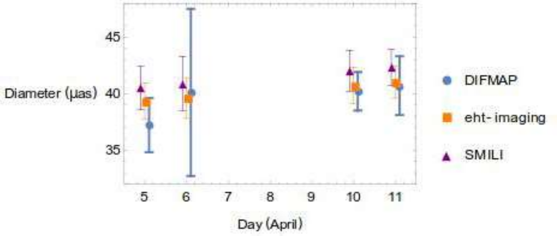


Figure: Shadow of M87* from April 5 to April 11. Data available at :1906.11241.

Photon Ring Visibility Drift

Maybe more luck with photon rings?

Visibility Amplitude dominated by

- gas/astrophysical processes for short baselines..
- ... but by photon rings otherwise!

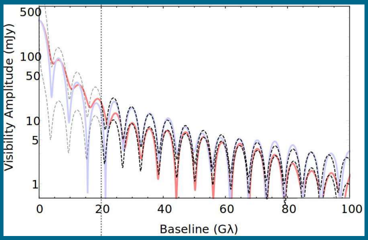


Figure: Visibility Amplitude dominated by photon rings for large baselines! Credit: Johnson et al. :1907.04329

Photon Ring Visibility Drift

Relative variation

- Rings infinitely thin and uniform: $\mathrm{V(u) = J_0(\pi du)}$ (d: ring's diameter, u: baseline length)
- Visibility drift:

$$\frac{\dot{\mathrm{V}}}{\mathrm{V}} = -\frac{1}{2}(\pi \mathrm{u} \mathrm{d})^2 \left(\mathrm{H_0} - \frac{\mathrm{H(z)}}{1+\mathrm{z}}\right) \tag{18}$$

- Best case: variation of 10^{-6} at $\mathrm{z} = 0.5$ for Earth-Moon array. Still small, but precision of spectroscopic measurements expected $\approx 10^{-9} - 10^{-10}$ over 10 years. Hope for interferometric measurements?

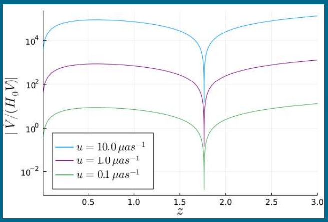


Figure: Visibility drift for a photon ring with diameter $\mathrm{d} = 40\mu$as, computed for three baselines (approx. EHT, Geostationary satellite, Moon).

Conclusions

Main purpose: quantify the impact of cosmological drift on BH shadow and photon rings.

- Shadow drift better seen at low redshift, $z \approx 0.5$
- Maximum drift is of order $10^{-1} H_0$ ($\approx 10^{-13}$ day^{-1})
- For M87* ($z \approx 4 \times 10^{-3}$): shadow drift $\approx 10^{-16}$ day^{-1}. Beyond resolution of EHT and forthcoming experiments.
- Still useful to probe the EP ($\dot{G}/G \approx 10^{-3} - 10^{-4}$ per year)
- Visibility drift might be a better target in the future, for both cosmological drift and the EP.

Session B3b 14:30–16:00

[Chair: Norihiro Tanahashi]

Alejandro García-Quismondo

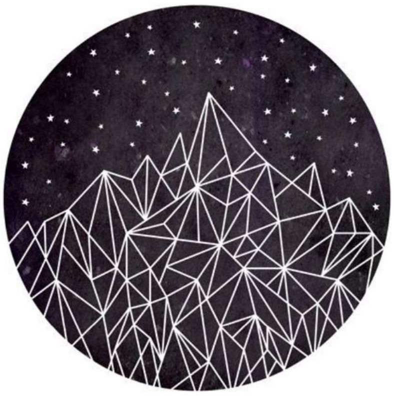
Institute for the Structure of Matter (IEM-CSIC)

“Investigating an alternative Hamiltonian derivation of the Ashtekar-Olmedo-Singh black hole solution ”

(15 min.)

[JGRG30 (2021) 120734]

Investigating an alternative Hamiltonian derivation of the Ashtekar-Olmedo-Singh black hole solution



Alejandro García-Quismondo
Instituto de Estructura de la Materia (IEM-CSIC)

JGRG30

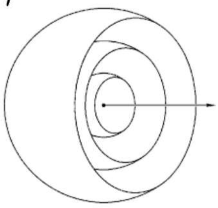
Introduction

- ✓ System: interior region of a Schwarzschild black hole.
- ✓ Large number of works within the framework of LQC.
- ✓ Focus on one particular model (AOS, Ashtekar-Olmedo-Singh).

$$\{\boxed{b, p_b}\} = G\gamma, \qquad \{\boxed{c, p_c}\} = 2G\gamma.$$

Radial sector   Angular sector

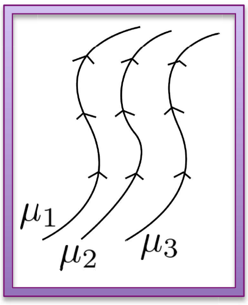
$$N = \frac{\gamma\delta_b\sqrt{|p_c|}}{\sin\delta_b b}, \qquad NH = \frac{L_o}{G}(\boxed{O_b} - \boxed{O_c}),$$



$$O_b = -\frac{1}{2\gamma}\left(\frac{\sin\delta_b b}{\delta_b} + \frac{\gamma^2\delta_b}{\sin\delta_b b}\right)\frac{p_b}{L_o}, \qquad O_c = \frac{1}{\gamma}\frac{\sin\delta_c c}{\delta_c}\frac{p_c}{L_o}.$$

- ✓ Key idea: select polymerisation parameters as constants of motion.
- ✓ Different approaches to this issue exist (AOS, Bodendorfer-Mele-Münch).

Parameters as constants of motion



$$O_b|_{\text{on-shell}} = O_c|_{\text{on-shell}} = m.$$

- ✓ AOS → $\delta_i = g_i(m)$ → constants set to constants of motion.
- ✓ BMM → $\delta_i = g_i(O_i)$ → nontrivial from the beginning.
- ✓ Can we reconcile these two approaches? → Explore related alternatives.

Parameters as constants of motion

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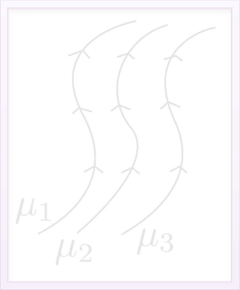

- ✓ Each partial Hamiltonian cannot be told apart on shell: $\delta_i = f_i(O_b, O_c)$.
- ✓ The radial and angular sectors no longer decouple!

$$\partial_t i = C_i \left[s_i \frac{L_o}{G} \{i, p_i\} \frac{\partial O_i}{\partial p_i} \right], \quad \partial_t p_i = C_i \left[-s_i \frac{L_o}{G} \{i, p_i\} \frac{\partial O_i}{\partial i} \right],$$

$$C_i = \frac{1 - \Delta_{jj} - \Delta_{ji}}{(1 - \Delta_{ii})(1 - \Delta_{jj}) - \Delta_{ij}\Delta_{ji}}, \quad j \neq i, \quad \Delta_{ij} = \frac{\partial O_i}{\partial \delta_i} \frac{\partial f_i}{\partial O_j}.$$

- ✓ A phase space dependent factor multiplies the AOS dynamical equations.

Time redefinitions

- ✓ These factors can be reabsorbed through time redefinitions:

$$dt_i = C_i dt.$$

- ✓ These redefinitions are sector dependent!
- ✓ Dynamical equations adopt the same form as AOS when written in terms of two a priori different time variables, implicitly related by

$$\int^{t_b} dt'_b[1 - \Delta_{bb}(t'_b) - \Delta_{bc}(t'_b)] = \int^{t_c} dt'_c[1 - \Delta_{cc}(t'_c) - \Delta_{cb}(t'_c)].$$

- ✓ Can the off-shell freedom be used to set $t_b|_{\text{on-shell}} = t_c|_{\text{on-shell}}$?

$$1 - F_c(p_c)\frac{\partial f_c(m,m)}{\partial m} = \alpha\left[1 - F_b(p_b)\frac{\partial f_b(m,m)}{\partial m}\right].$$

- ✓ The answer is in the negative, unless we consider constant parameters on the whole phase space.

Large black hole mass limit

- ✓ Solutions to the EOM are identical to those obtained in previous works when written in terms of the newly-defined time variables.
- ✓ We can use these solutions to perform an asymptotic expansion of $\dfrac{\partial O_i}{\partial \delta_i}(t_i)$.
- ✓ Such an expansion shows that $\lim\limits_{m\to\infty} F_i \dfrac{\partial f_i}{\partial m} = 0$.
- ✓ Therefore, the equality condition that could not be made work unless the parameters were taken as constants does hold in the limit of large masses.
- ✓ This is only true if $\alpha = 1$.
- ✓ In this limit, the relation between the two time variables is given by

$$t_c = t_b - \frac{1}{9}\gamma^2(-3t_b + 3\sinh t_b + \cosh t_b - 1)\delta_b^2 + o(\delta_b^2).$$

- ✓ The NLO term is of order m$^{-2/3}$.

$$\delta_i \sim m^{-1/3}$$

Conclusions

- ✓ Our objective is to explore alternatives that had not been explored before in order to see whether other proposals can be reconciled with the results of the original model.

- ✓ We propose that the parameters should capture the contributions of two phase space sectors that had been thought to be decoupled.

- ✓ Two distinct time variables arise as a result of this choice.

- ✓ These are found to coincide in the asymptotic limit of large black hole masses up to a term that goes as $m^{-2/3}$.

- ✓ Even if the original results of the AOS model can be recovered to a certain extent in this limit, the spacetime geometry is modified.

- ✓ This might open the door to an alleviation of some of the criticisms that the model has received.

References

- A. Ashtekar et al., Phys. Rev. Lett. **121**, 241301 (2018); Phys. Rev. D **98**, 126003 (2018).
- A. Ashtekar and J. Olmedo, Int. J. Mod. Phys. D **29**, 2050076 (2020).

- N. Bodendorfer et al., Class. Quantum Grav. **37**, 187001 (2019).

- A. G.-Q. and G. A. Mena Marugán, Front. Astron. Space Sci. **8**, 701723 (2021).

Acknowledgements:

"la Caixa" Banking Foundation has supported this work.

Session C1a 10:00–12:00

[Chair: Atsushi Nishizawa]

Anzhong Wang

Baylor University

"Testing Gravitational Theories with Broken Lorentz Symmetry by Gravitational Wave Observations"

(15 min.)

[JGRG30 (2021) 120802]

Decmber 8, 2021

Testing Gravitational Theories with broken Lorentz Symmetry by Gravitational Wave and Black Hole Observations

Anzhong Wang

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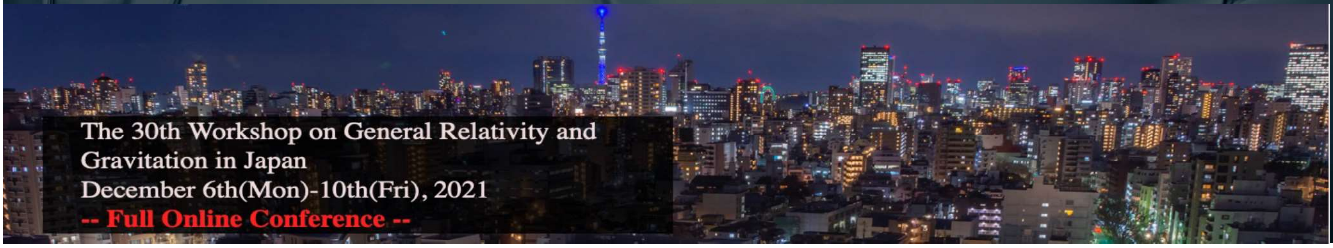
Baylor University



The 30th Workshop on General Relativity and
Gravitation in Japan
December 6th(Mon)-10th(Fri), 2021
-- Full Online Conference --

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1. Einstein-æther Theory

2. Observational Tests

3. Summary & Outlook

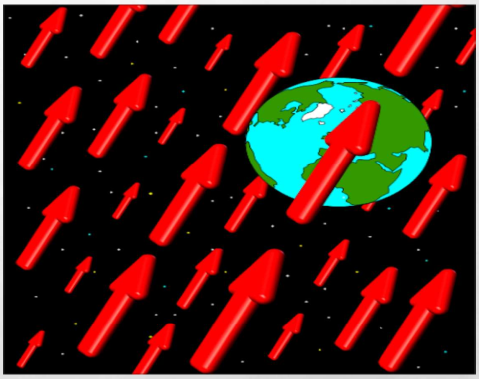
1. Einstein-æther Theory (T. Jacobson, arXiv:0801.1547)

æ-theory is a type of vector-tensor theories

$$(g_{\mu\nu}, u^{\mu})$$

It is different from GR in several aspects:

- There is a preferred reference frame, defined by the aether field u^{μ}
- Due to the presence of the unit timelike aether field (u^{μ}), it breaks the Lorentz symmetry locally, a conerstone of modern physics



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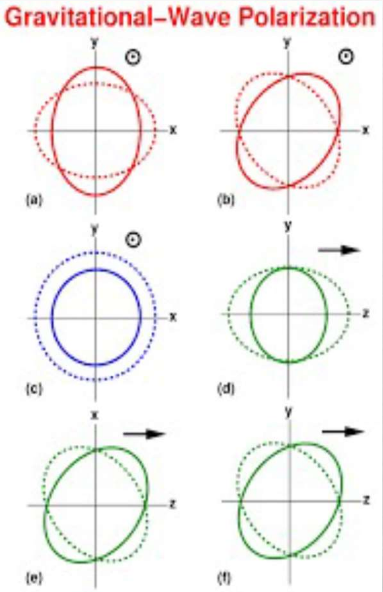
1. Einstein-æther theory (æ-theory)

Remarkable properties:

- Yet, it is self-consistent and consistent with all observations carried out so far (J. Oost, S. Mukohyama, AW, PRD97 (2018) 124023)
- Its Cauchy problem (Cauchy–Kovalevskaya theorem) is well-posed (O. Sarbach, E. Barausse, J.A. Preciado-López, Class. Quantum Grav. 36 (2019) 165007).
- Possesses all the gravitational radiation channels (scalar, vector, tensor) (T. Jacobson, D. Mattingly, PRD70 (2004) 024003)



1. Einstein-æther Theory

Action (T. Jacobson, arXiv:0801.1547):

$$S = S_{\text{æ}} + S_m,$$

$$\mathcal{L}_\lambda \equiv \lambda\left(g_{\alpha\beta}u^\alpha u^\beta + 1\right),$$

$$\mathcal{L}_{\text{æ}} \equiv R(g_{\mu\nu}) - M^{\alpha\beta}{}_{\mu\nu}\left(D_\alpha u^\mu\right)\left(D_\beta u^\nu\right),$$

$$\begin{aligned} S_{\text{æ}} &= \frac{1}{16\pi G_{\text{æ}}}\int\sqrt{-g}\,d^4x\Big[\mathcal{L}_{\text{æ}}\left(g_{\mu\nu},u^\alpha,c_i\right)+\mathcal{L}_\lambda\left(g_{\mu\nu},u^\alpha,\lambda\right)\Big],\\ S_m &= \int\sqrt{-g}\,d^4x\Big[\mathcal{L}_m\left(g_{\mu\nu},u^\alpha;\psi\right)\Big]. \end{aligned}$$

$$\begin{aligned} c_S^2 &= \frac{c_{123}(2-c_{14})}{c_{14}(1-c_{13})(2+c_{13}+3c_2)},\\ c_V^2 &= \frac{2c_1-c_{13}(2c_1-c_{13})}{2c_{14}(1-c_{13})}, \end{aligned}$$

$$M^{\alpha\beta}{}_{\mu\nu} \equiv c_1 g^{\alpha\beta}g_{\mu\nu} + c_2\delta^\alpha_\mu\delta^\beta_\nu + c_3\delta^\alpha_\nu\delta^\beta_\mu - c_4 u^\alpha u^\beta g_{\mu\nu},$$

$$c_T^2 = \frac{1}{1-c_{13}},$$

$$c_{ij} \equiv c_i + c_j$$

Four free parameters:

c_1, c_2, c_3, c_4

When $c_1= c_2= c_3= c_4=0$, œ-theory reduces to GR.

$$c_{ijk} \equiv c_i + c_j + c_k$$

2.1 Observational Tests of Einstein-æther theory

Imposing the follwoing conditions (J. Oost, S. Mukohyama, AW, PRD97 (2018) 124023):

- Free of ghosts (Coefficients of the kenetic terms of the scalar, vector and tensor modes are positive):

$$q_{S,V,T} > 0\,,$$

where

$$\begin{aligned} q_S &= \frac{(1-c_{13})\,(2+c_{13}+3c_2)}{c_{123}}\,,\\ q_V &= c_{14}\,,\\ q_T &= 1-c_{13}\,. \end{aligned}$$

- Free of gradient instability:

$$c^2_{S,V,T} \geq 0$$

2.1 Observational Tests

- Aviod the gravi-Cerenkov radiation [J. W. Elliott, G. D. Moore and H. Stoica, JHEP 0508, 066 (2005)]:

$$c^2_{S,V,T} \gtrsim 1$$

- Be consistent with Bib Bang Nucleosynthesis [S. M. Carroll and E. A. Lim, PRD70 (2004) 123525]:

$$\left|\frac{G_{\cos}}{G_N} - 1\right| \lesssim \frac{1}{8}. \qquad G_{\cos} = \frac{G_{æ}}{1+\frac{1}{2}(c_{13}+3c_2)}.$$

- Be consistent with GW170817 observations [LIGO/Virgo: PRL119 (2017) 161101]:

$$-3 \times 10^{-15} < c_T - 1 < 7 \times 10^{-16}$$

2.1 Observational Tests

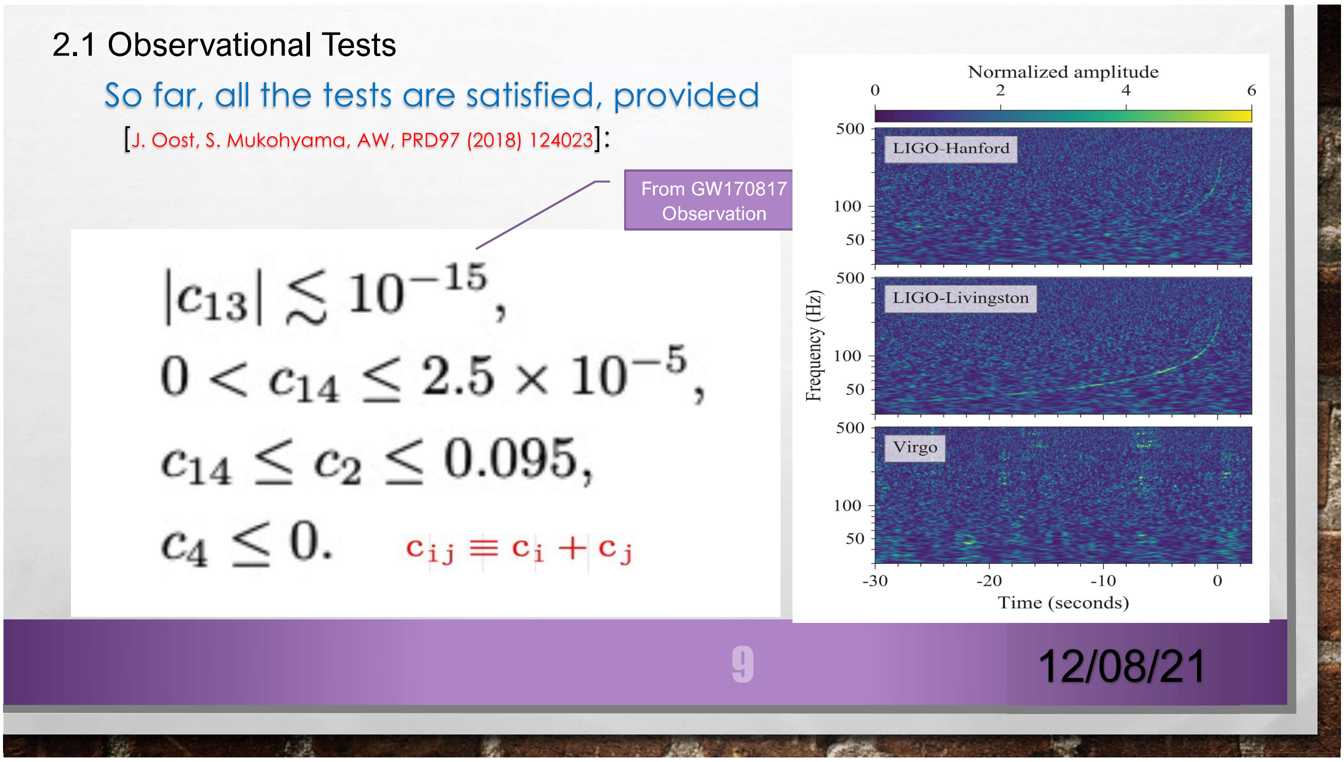
- Be Consistent with Solar Sysytem Observations from lunar laser ranging and solar alignment with the ecliptic [C.M. Will, Liv. Rev. Relativ. 9 (2006) 3]:

All the PPN parameters are the same as those given in GR for any given c_i, except the preferred frame parameters,

$$|\alpha_1| \le 10^{-4}, \quad |\alpha_2| \le 10^{-7}.$$

$$\alpha_1 = -\frac{8(c_3^2 + c_1 c_4)}{2c_1 - c_1^2 + c_3^2},$$

$$\alpha_2 = \frac{1}{2}\alpha_1 - \frac{(c_1 + 2c_3 - c_4)(2c_1 + 3c_2 + c_3 + c_4)}{c_{123}(2 - c_{14})}$$



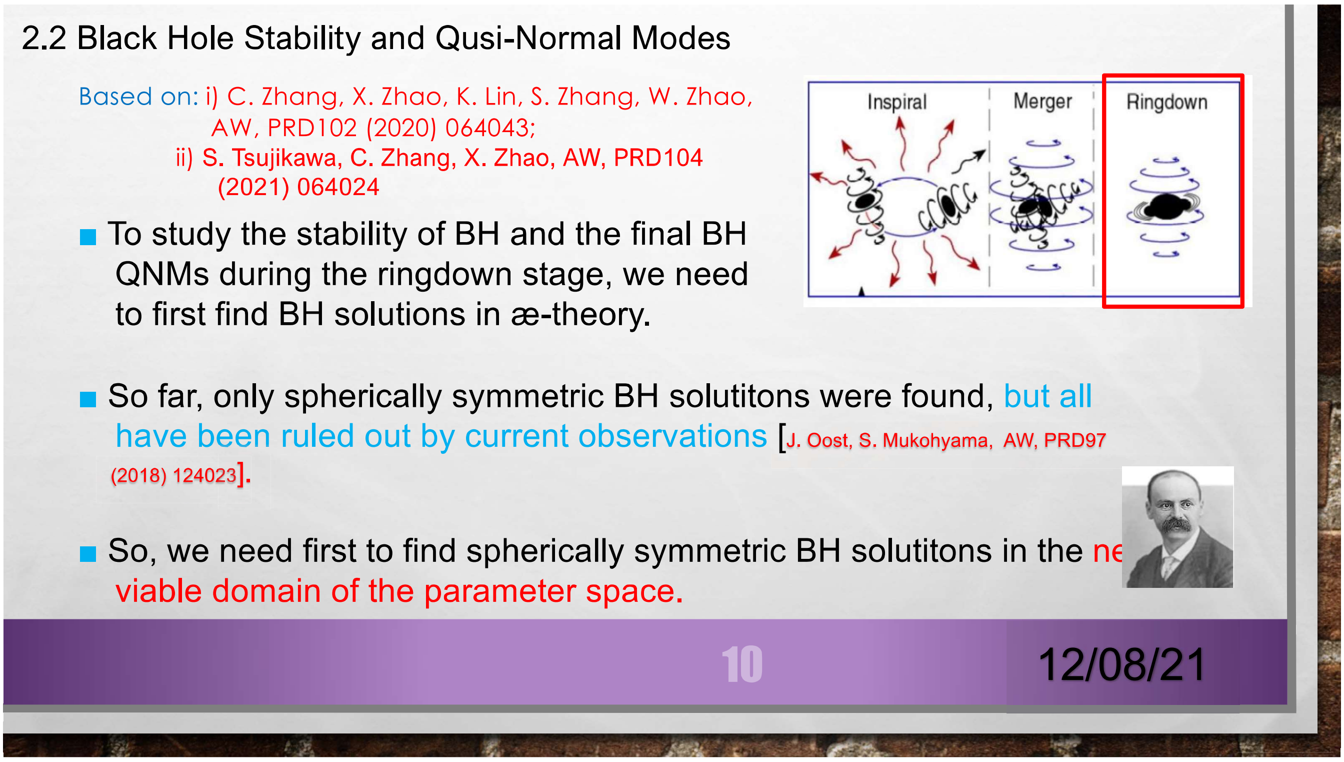
2.1 Observational Tests

So far, all the tests are satisfied, provided

[J. Oost, S. Mukohyama, AW, PRD97 (2018) 124023]:

From GW170817 Observation

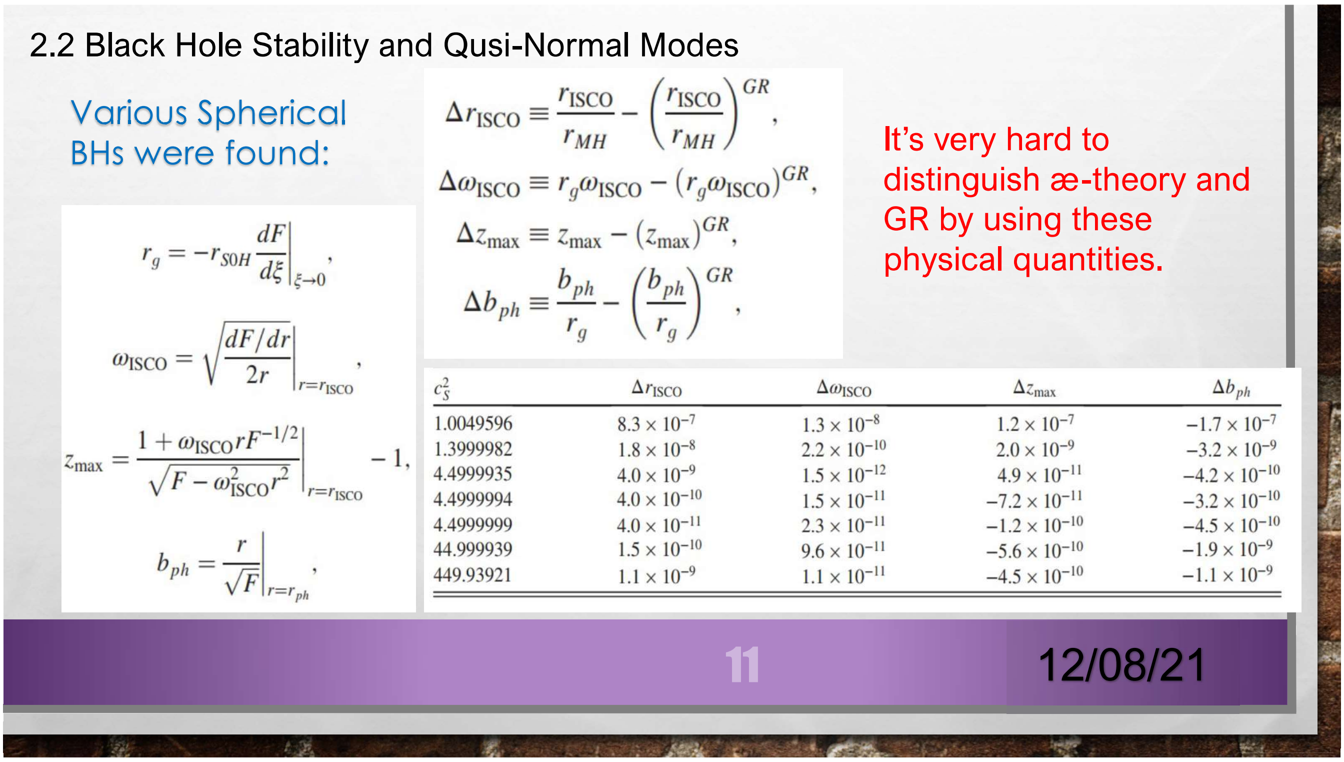
$$|c_{13}| \lesssim 10^{-15},$$
$$0 < c_{14} \lesssim 2.5 \times 10^{-5},$$
$$c_{14} \lesssim c_2 \lesssim 0.095,$$
$$c_4 \lesssim 0. \qquad c_{ij} \equiv c_i + c_j$$



2.2 Black Hole Stability and Qusi-Normal Modes

Based on: i) C. Zhang, X. Zhao, K. Lin, S. Zhang, W. Zhao, AW, PRD102 (2020) 064043;
ii) S. Tsujikawa, C. Zhang, X. Zhao, AW, PRD104 (2021) 064024

- To study the stability of BH and the final BH QNMs during the ringdown stage, we need to first find BH solutions in æ-theory.
- So far, only spherically symmetric BH solutitons were found, but all have been ruled out by current observations [J. Oost, S. Mukohyama, AW, PRD97 (2018) 124023].
- So, we need first to find spherically symmetric BH solutitons in the ne viable domain of the parameter space.



2.2 Black Hole Stability and Qusi-Normal Modes

Various Spherical BHs were found:

$$r_g = -r_{S0H} \left.\frac{dF}{d\xi}\right|_{\xi\to 0},$$

$$\omega_{\rm ISCO} = \sqrt{\left.\frac{dF/dr}{2r}\right|_{r=r_{\rm ISCO}}},$$

$$z_{\rm max} = \left.\frac{1+\omega_{\rm ISCO} r F^{-1/2}}{\sqrt{F-\omega_{\rm ISCO}^2 r^2}}\right|_{r=r_{\rm ISCO}} - 1,$$

$$b_{ph} = \left.\frac{r}{\sqrt{F}}\right|_{r=r_{ph}},$$

$$\Delta r_{\rm ISCO} \equiv \frac{r_{\rm ISCO}}{r_{MH}} - \left(\frac{r_{\rm ISCO}}{r_{MH}}\right)^{GR},$$

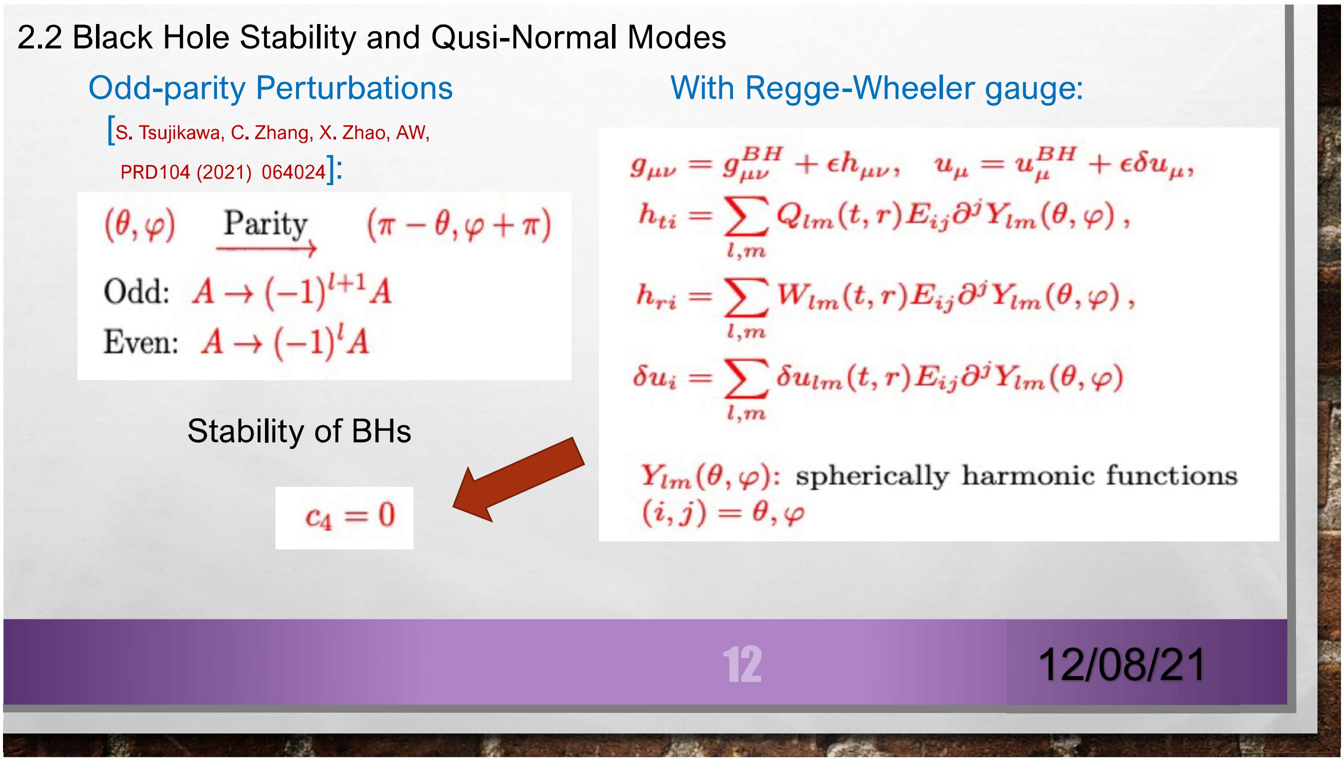
$$\Delta\omega_{\rm ISCO} \equiv r_g\omega_{\rm ISCO} - (r_g\omega_{\rm ISCO})^{GR},$$

$$\Delta z_{\rm max} \equiv z_{\rm max} - (z_{\rm max})^{GR},$$

$$\Delta b_{ph} \equiv \frac{b_{ph}}{r_g} - \left(\frac{b_{ph}}{r_g}\right)^{GR},$$

It's very hard to distinguish æ-theory and GR by using these physical quantities.

c_S^2	$\Delta r_{\rm ISCO}$	$\Delta\omega_{\rm ISCO}$	$\Delta z_{\rm max}$	Δb_{ph}
1.0049596	8.3×10^{-7}	1.3×10^{-8}	1.2×10^{-7}	-1.7×10^{-7}
1.3999982	1.8×10^{-8}	2.2×10^{-10}	2.0×10^{-9}	-3.2×10^{-9}
4.4999935	4.0×10^{-9}	1.5×10^{-12}	4.9×10^{-11}	-4.2×10^{-10}
4.4999994	4.0×10^{-10}	1.5×10^{-11}	-7.2×10^{-11}	-3.2×10^{-10}
4.4999999	4.0×10^{-11}	2.3×10^{-11}	-1.2×10^{-10}	-4.5×10^{-10}
44.999939	1.5×10^{-10}	9.6×10^{-11}	-5.6×10^{-10}	-1.9×10^{-9}
449.93921	1.1×10^{-9}	1.1×10^{-11}	-4.5×10^{-10}	-1.1×10^{-9}



2.2 Black Hole Stability and Qusi-Normal Modes

Odd-parity Perturbations

[S. Tsujikawa, C. Zhang, X. Zhao, AW, PRD104 (2021) 064024]:

$$(\theta,\varphi) \xrightarrow{\text{Parity}} (\pi-\theta, \varphi+\pi)$$

Odd: $A \to (-1)^{l+1}A$

Even: $A \to (-1)^{l}A$

Stability of BHs

$$c_4 = 0$$

With Regge-Wheeler gauge:

$$g_{\mu\nu} = g_{\mu\nu}^{BH} + \epsilon h_{\mu\nu}, \quad u_\mu = u_\mu^{BH} + \epsilon\delta u_\mu,$$

$$h_{ti} = \sum_{l,m} Q_{lm}(t,r)E_{ij}\partial^j Y_{lm}(\theta,\varphi),$$

$$h_{ri} = \sum_{l,m} W_{lm}(t,r)E_{ij}\partial^j Y_{lm}(\theta,\varphi),$$

$$\delta u_i = \sum_{l,m} \delta u_{lm}(t,r)E_{ij}\partial^j Y_{lm}(\theta,\varphi)$$

$Y_{lm}(\theta,\varphi)$: spherically harmonic functions

$(i,j) = \theta,\varphi$

2.3 GWs emitted by N-body Systems

- Based on: K. Lin, X. Zhao, C. Zhang, et al., PRD99, 023010 (2019).
- Previously:
  - B. Foster, PRD73 (2006) 104012;
  - K. Yagi, D. Blas, E. Barausse, and N. Yunes, PRD89 (2014) 084067.

- But, we rederived the results by correcting various errors.

Minkowski Background:

$$(\bar{g}_{\mu\nu}, \bar{u}^{\mu}) = (\eta_{\mu\nu}, \delta^{\mu}_{t})$$

$$\eta_{\mu\nu} = \text{diag}\,(-1, 1, 1, 1)$$

Linear Perturbations:

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}, \quad u^{\mu} = \delta^{\mu}_{t} + \epsilon w^{\mu}, \ (\epsilon \ll 1)$$

$h_{\mu\nu}$: 10 components; w^{μ} : 4 components

- Therefore, the perturbations have N= 14 (= 10 + 4) independent components.

2.3 GWs emitted by N-body Systems

Gauge conditions: $\phi_i = 0, \qquad \nu = \gamma = 0,$

Solutions of gauge invariants:

Linearized Field Equations ➡

$$\phi_{ij} = \frac{2G_{æ}}{R}(\ddot{Q}_{ij})^{TT},$$

$$\Psi^{\mathrm{I}}_{i} = (1 - c_{13})\nu_i, \qquad \Phi^{\mathrm{II}} = \frac{1}{2}F,$$

$$\nu_i = -\frac{2G_{æ}}{(2c_1 - c_{13}c_{-})R}\left(\frac{c_{13}\ddot{Q}_{ij}N_j}{(1 - c_{13})c_V} - 2\Sigma_i\right)^{T},$$

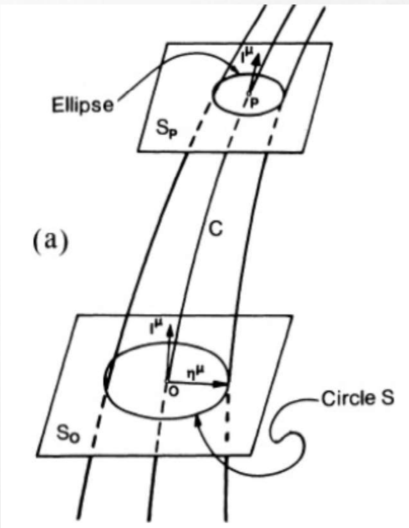
$$F = \frac{G_{æ}}{R}\frac{c_{14}}{2 - c_{14}}\left(6(Z - 1)\ddot{Q}_{ij}N_iN_j - \frac{8}{c_{14}c_S}\Sigma_iN_i + 2Z\ddot{I}\right),$$

2.3 GWs emitted by N-body Systems

Geodesic deviations:

$$\ddot{\zeta}_i = -R_{0i0j}\zeta^j \equiv \frac{1}{2}\ddot{\mathcal{P}}_{ij}\zeta^j,$$

$$\mathcal{P}_{ij} = \phi_{ij} - \frac{2c_{13}}{(1-c_{13})c_V}\Psi^{\mathrm{I}}_{(i}N_{j)} - \frac{c_{14}-2c_{13}}{c_{14}(c_{13}-1)c_S^2}\Phi^{\mathrm{II}}N_iN_j + \delta_{ij}\Phi^{\mathrm{II}}.$$



AW, PRD44 (1991) 1120

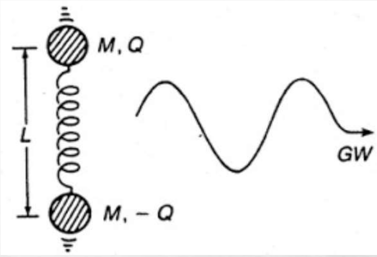
2.3 GWs emitted by N-body Systems

Construct polarized waveforms:

$$h_+ \equiv \frac{1}{2}(\mathcal{P}_{XX} - \mathcal{P}_{YY}), \qquad h_\times \equiv \frac{1}{2}(\mathcal{P}_{XY} + \mathcal{P}_{YX}),$$

$$h_b \equiv \frac{1}{2}(\mathcal{P}_{XX} + \mathcal{P}_{YY}), \qquad h_L \equiv \mathcal{P}_{ZZ},$$

$$h_X \equiv \frac{1}{2}(\mathcal{P}_{XZ} + \mathcal{P}_{ZX}), \qquad h_Y \equiv \frac{1}{2}(\mathcal{P}_{YZ} + \mathcal{P}_{ZY}),$$



h_b: Scalar polarization;
h_L: Longitudinal polarization; But, here **we always have** $h_b \propto h_L$.
h_x, h_y: Vector polarizations

2.3 GWs emitted by N-body Systems

- Due to the existence of the four extra modes, the radiation powers of a given system in Einstein-aether theory in general are quite from that of GR:

$$\dot{\mathcal{E}} = -G_N \left\langle \frac{\mathcal{A}}{5}\left(\dddot{Q}_{ij}\right)^2 + \mathcal{B}\left(\dddot{I}\right)^2 + \mathcal{C}\left(\dot{\Sigma}_i\right)^2 \right\rangle,$$

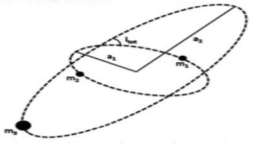
$\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$: functions of c_i's.

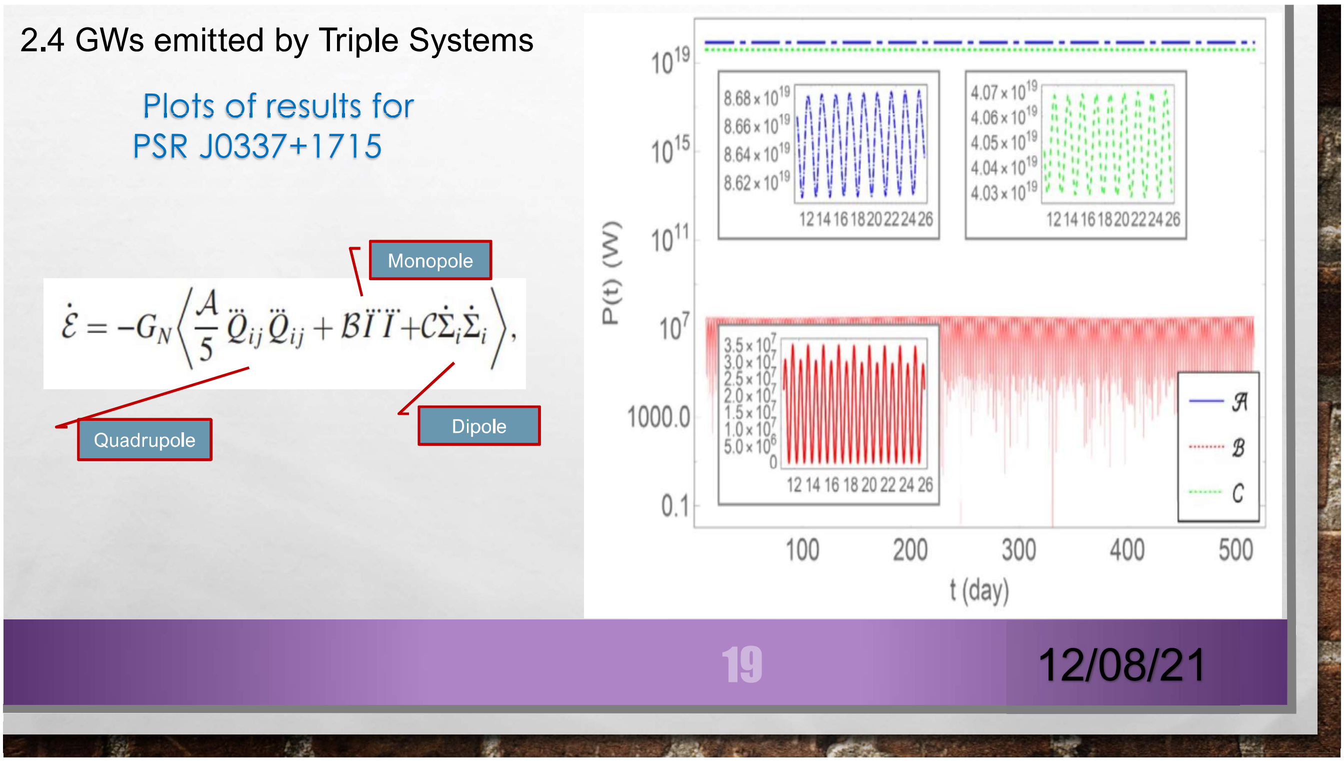
- $\mathcal{A}$ contains the contributions from all the six modes;
- $\mathcal{B}$, $\mathcal{C}$ contain the contributions only from the vector and scalar modes.
- In GR, we have $\mathcal{A}_{GR} = 1, \quad \mathcal{B}_{GR} = 0 = \mathcal{C}_{GR}$.

2.4 GWs emitted by Triple Systems

Based on: X. Zhao, C. Zhang, et al., PRD100, 083012

- Recently, we applied the above expression to the first observed relativistic triple system, PSR J0337 + 1715 [S.M. Ransom, et al, Nature 505 (2014) 520]:
  - The inner binary consists of a pulsar with mass $m_1 = 1.44M_\odot$ and a white dwarf with mass $m_2 = 0.20M_\odot$ in a 1.6 day orbit
  - The outer binary consists of the inner binary and a second dwarf with mass $m_3 = 0.41M_\odot$ in a 327 day orbit
  - Two orbits are very circular with $e_I = 6.9 \times 10^{-4}$ for the inner binary, and $e_O = 3.5 \times 10^{-2}$ for the outer orbit
  - Two orbital planes are remarkably coplanar with an inclination $i_{tot} \lesssim 0.01^\circ$





2.4 GWs emitted by Triple Systems

Plots of results for
PSR J0337+1715

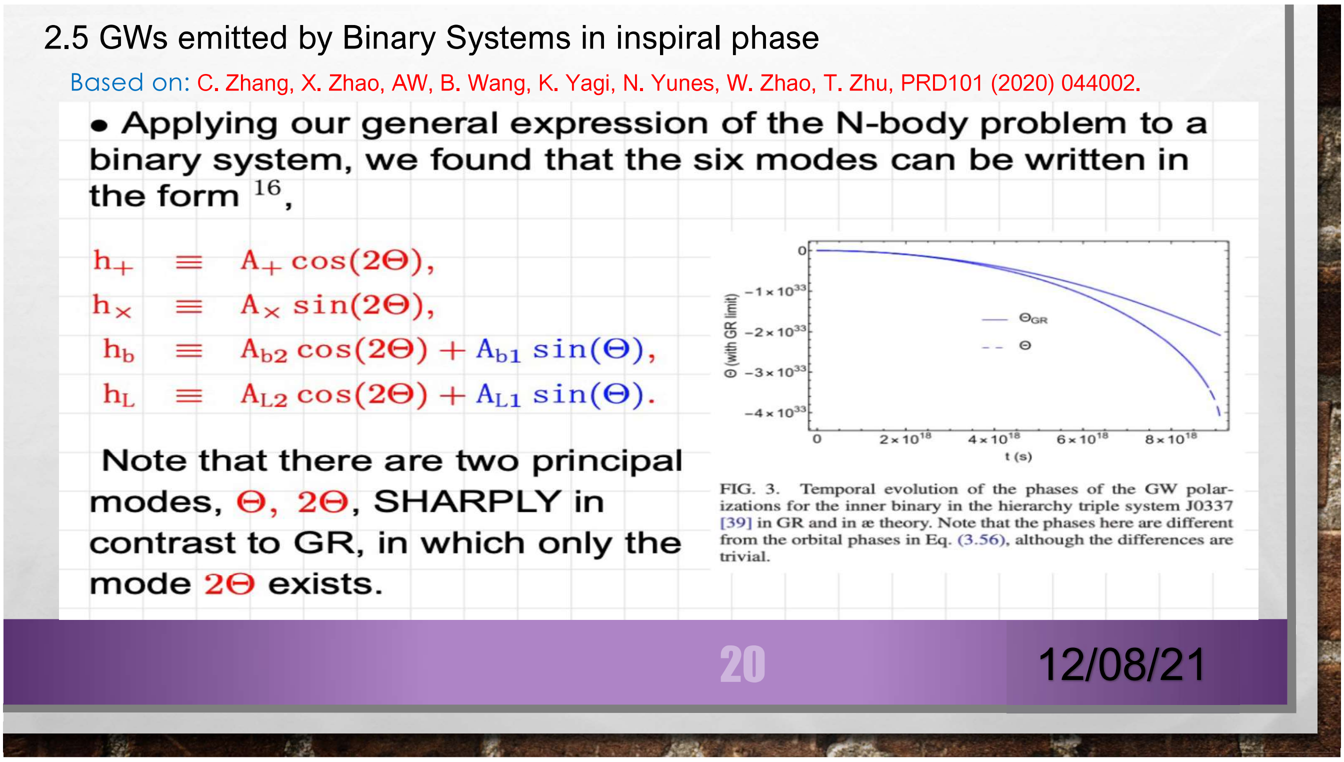
Monopole

$$\dot{\mathcal{E}} = -G_N\left\langle \frac{\mathcal{A}}{5}\dddot{Q}_{ij}\dddot{Q}_{ij} + \mathcal{B}\dddot{I}\dddot{I} + \mathcal{C}\dot{\Sigma}_i\dot{\Sigma}_i \right\rangle,$$

Quadrupole

Dipole

12/08/21



2.5 GWs emitted by Binary Systems in inspiral phase

Based on: C. Zhang, X. Zhao, AW, B. Wang, K. Yagi, N. Yunes, W. Zhao, T. Zhu, PRD101 (2020) 044002.

- Applying our general expression of the N-body problem to a binary system, we found that the six modes can be written in the form [16],

$$\begin{aligned}
h_{+} &\equiv A_{+}\cos(2\Theta),\\
h_{\times} &\equiv A_{\times}\sin(2\Theta),\\
h_{b} &\equiv A_{b2}\cos(2\Theta) + A_{b1}\sin(\Theta),\\
h_{L} &\equiv A_{L2}\cos(2\Theta) + A_{L1}\sin(\Theta).
\end{aligned}$$

Note that there are two principal modes, Θ, 2Θ, SHARPLY in contrast to GR, in which only the mode 2Θ exists.

FIG. 3. Temporal evolution of the phases of the GW polarizations for the inner binary in the hierarchy triple system J0337 [39] in GR and in æ theory. Note that the phases here are different from the orbital phases in Eq. (3.56), although the differences are trivial.

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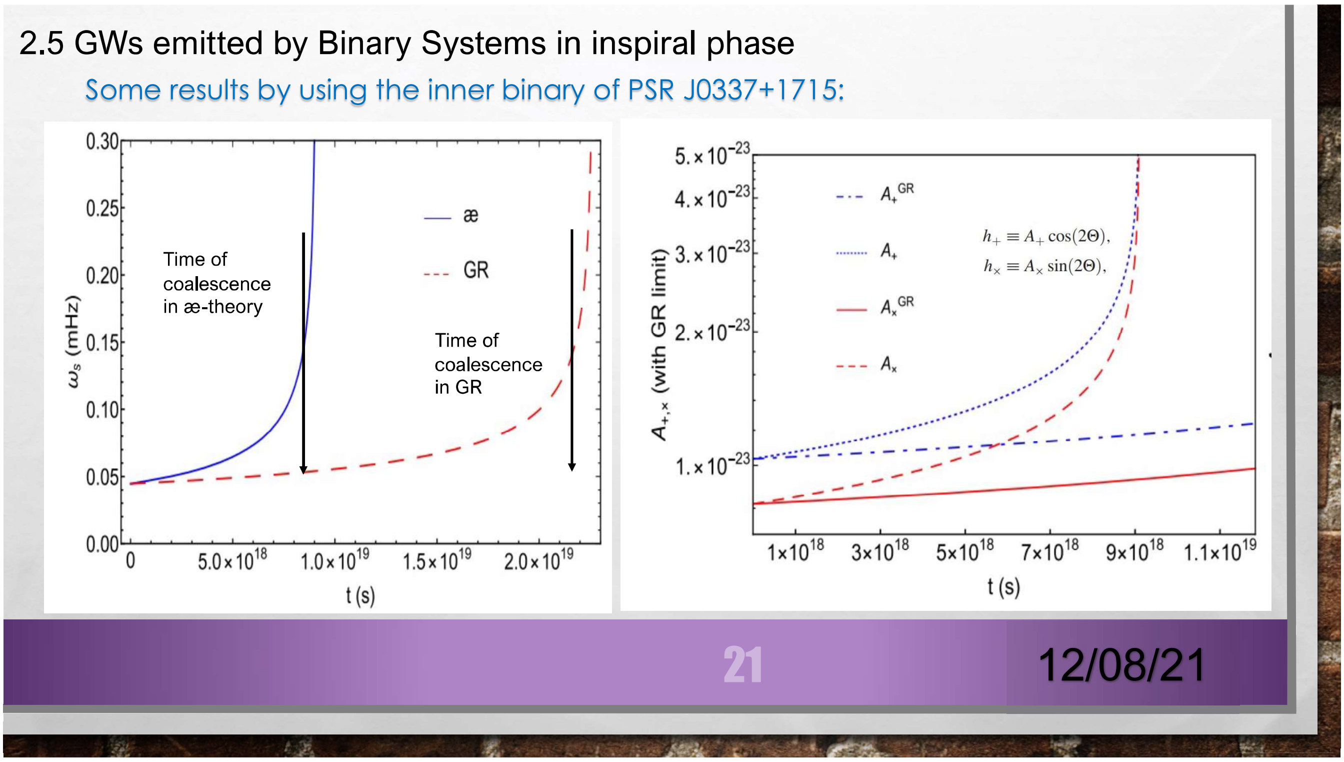
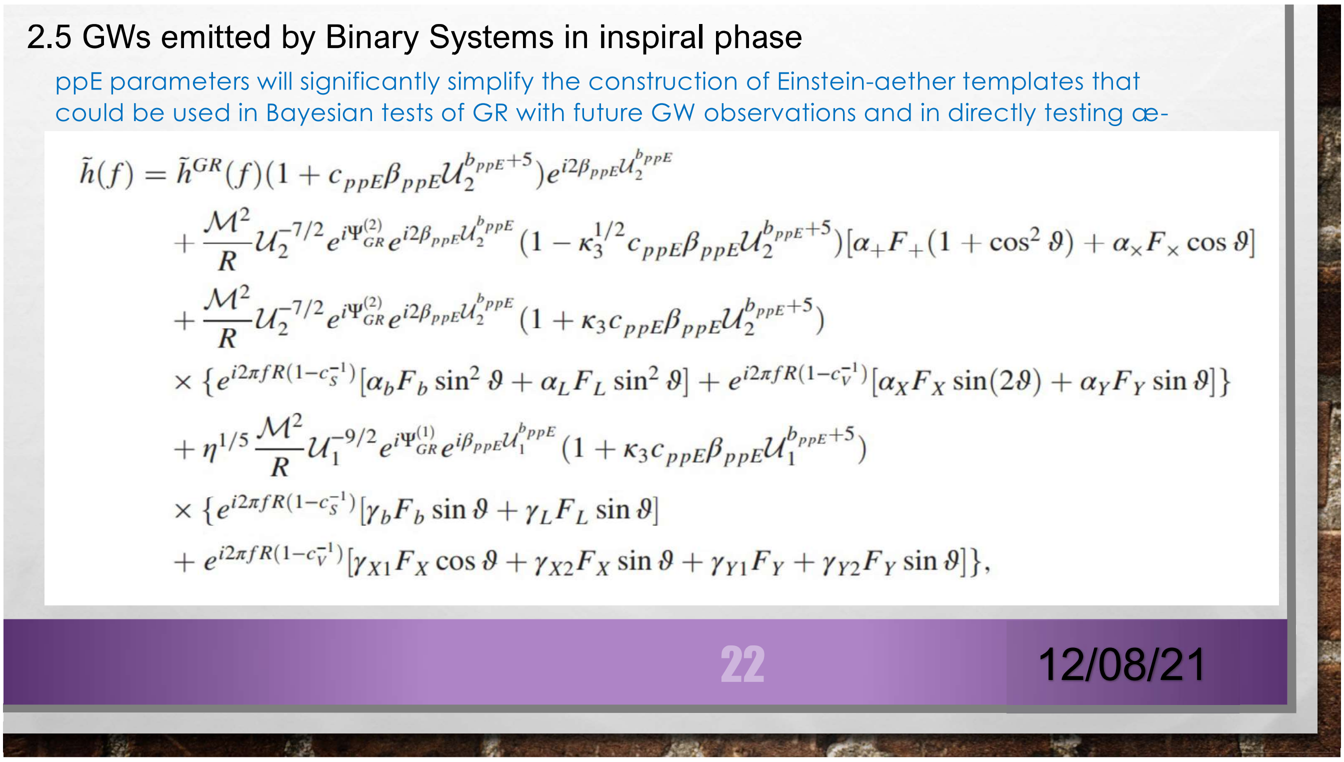


2.5 GWs emitted by Binary Systems in inspiral phase

Some results by using the inner binary of PSR J0337+1715:



2.5 GWs emitted by Binary Systems in inspiral phase

ppE parameters will significantly simplify the construction of Einstein-aether templates that could be used in Bayesian tests of GR with future GW observations and in directly testing æ-

$$
\begin{aligned}
\tilde{h}(f) = {} & \tilde{h}^{GR}(f)(1 + c_{ppE}\beta_{ppE}\mathcal{U}_2^{b_{ppE}+5})e^{i2\beta_{ppE}\mathcal{U}_2^{b_{ppE}}} \\
& + \frac{\mathcal{M}^2}{R}\mathcal{U}_2^{-7/2}e^{i\Psi^{(2)}_{GR}}e^{i2\beta_{ppE}\mathcal{U}_2^{b_{ppE}}}(1 - \kappa_3^{1/2}c_{ppE}\beta_{ppE}\mathcal{U}_2^{b_{ppE}+5})[\alpha_+F_+(1+\cos^2\vartheta) + \alpha_\times F_\times\cos\vartheta] \\
& + \frac{\mathcal{M}^2}{R}\mathcal{U}_2^{-7/2}e^{i\Psi^{(2)}_{GR}}e^{i2\beta_{ppE}\mathcal{U}_2^{b_{ppE}}}(1 + \kappa_3c_{ppE}\beta_{ppE}\mathcal{U}_2^{b_{ppE}+5}) \\
& \times\{e^{i2\pi fR(1-c_S^{-1})}[\alpha_bF_b\sin^2\vartheta + \alpha_LF_L\sin^2\vartheta] + e^{i2\pi fR(1-c_V^{-1})}[\alpha_XF_X\sin(2\vartheta) + \alpha_YF_Y\sin\vartheta]\} \\
& + \eta^{1/5}\frac{\mathcal{M}^2}{R}\mathcal{U}_1^{-9/2}e^{i\Psi^{(1)}_{GR}}e^{i\beta_{ppE}\mathcal{U}_1^{b_{ppE}}}(1 + \kappa_3c_{ppE}\beta_{ppE}\mathcal{U}_1^{b_{ppE}+5}) \\
& \times\{e^{i2\pi fR(1-c_S^{-1})}[\gamma_bF_b\sin\vartheta + \gamma_LF_L\sin\vartheta] \\
& + e^{i2\pi fR(1-c_V^{-1})}[\gamma_{X1}F_X\cos\vartheta + \gamma_{X2}F_X\sin\vartheta + \gamma_{Y1}F_Y + \gamma_{Y2}F_Y\sin\vartheta]\},
\end{aligned}
$$

2.5 GWs emitted by Binary Systems in inspiral phase

Calculating ppE parameters:

$$c_{ppE}=\frac{224}{3},$$

$$b_{ppE}=-7,$$

$$\beta_{ppE}=\frac{1}{2}\phi_1=-\frac{3}{448}\kappa_3^{-1}\eta^{2/5}\epsilon_x,$$

$$\alpha_+=\frac{\sqrt{5\pi}}{8\sqrt{6}}G_N^2e^{i2\varphi}(\kappa_3^{-1/2}-1)g_+,$$

$$\alpha_\times=-i\frac{\sqrt{5\pi}}{8\sqrt{6}}G_N^2e^{i2\varphi}(\kappa_3^{-1/2}-1)g_\times,$$

$$\alpha_b=\frac{\sqrt{5\pi}}{8\sqrt{6}}\kappa_3^{-1/2}G_N^2e^{i2\varphi}g_{b1},$$

$$\alpha_L=\frac{\sqrt{5\pi}}{8\sqrt{6}}\kappa_3^{-1/2}G_N^2e^{i2\varphi}g_{L1},$$

$$\gamma_b=-i\frac{\sqrt{5\pi}}{8\sqrt{3}}\kappa_3^{-1/2}\eta^{-1/5}G_N^2e^{i\varphi}(g_{b2}+g_{b4}),$$

$$\gamma_L=-i\frac{\sqrt{5\pi}}{8\sqrt{3}}\kappa_3^{-1/2}\eta^{-1/5}G_N^2e^{i\varphi}(g_{L2}+g_{L4}),$$

$$\alpha_X=\frac{\sqrt{5\pi}}{8\sqrt{6}}\kappa_3^{-1/2}G_N^2e^{i2\varphi}g_{X1},$$

$$\alpha_Y=-i\frac{\sqrt{5\pi}}{8\sqrt{6}}\kappa_3^{-1/2}G_N^2e^{i2\varphi}g_{Y1},$$

$$\gamma_b=-i\frac{\sqrt{5\pi}}{8\sqrt{3}}\kappa_3^{-1/2}\eta^{-1/5}G_N^2e^{i\varphi}(g_{b2}+g_{b4}),$$

$$\gamma_L=-i\frac{\sqrt{5\pi}}{8\sqrt{3}}\kappa_3^{-1/2}\eta^{-1/5}G_N^2e^{i\varphi}(g_{L2}+g_{L4}),$$

$$\gamma_{X1}=-i\frac{\sqrt{5\pi}}{8\sqrt{3}}\kappa_3^{-1/2}\eta^{-1/5}G_N^2e^{i\varphi}(g_{X2}+g_{X4}),$$

$$\gamma_{X2}=-i\frac{\sqrt{5\pi}}{8\sqrt{3}}\kappa_3^{-1/2}\eta^{-1/5}G_N^2e^{i\varphi}g_{X3},$$

$$\gamma_{Y1}=\frac{\sqrt{5\pi}}{8\sqrt{3}}\kappa_3^{-1/2}\eta^{-1/5}G_N^2e^{i\varphi}(g_{Y2}+g_{Y4}),$$

$$\gamma_{Y2}=-i\frac{\sqrt{5\pi}}{8\sqrt{3}}\kappa_3^{-1/2}\eta^{-1/5}G_N^2e^{i\varphi}g_{Y3},$$

3. Summary & Outlook

3.1 Summary

- We numerically found various BH solutions, which are the only ones that satisfy the current observational constraints.

- The strong field regime considerations results:

$c_{13}=0$ ← GW170817

$c_4=0$ ← Stability of BHs

} Strong Field Regime

3. Summary & Outlook

3.1 Summary

- In studying the relativistic triple system PSR J0337+1715, observed in 2014,
  we found that the dipole emission from the aether field can be as large as the quadrupole emission. This provides a very promising window to make severe constraints on æ-theory.
- We generalized the existing ppE framework to allow for different propagation speeds among the scalar, vector and tensor modes, which will significantly simplify the construction of Einstein-aether templates that could be used in tests of æ-theory.

---

3 Summary & Outlook

3.2 Outlook

- For GW signals during the inspiral stage, we would like to generalize the current calculations of waveforms, energy losses, ppE parameters, etc. to 2PN order, which is required by the 3rd-generation detectors, e.g., Cosmic Explorer, LISA, Tianqin and Taiji.
- For GW signals during the ringdown stage, we would like to calculate explicately QNMs of BHs.
- According to the GW observations, e.g., GW170729, BHs with non-zero spins are quite common. Therefore, we would also like to study rotating BHs, and then study their QNMs.

Thank you

Any question?

Session C1a 10:00–12:00

[Chair: Atsushi Nishizawa]

Nami Uchikata

ICRR Univ. of Tokyo

“Parameter estimation on superspinar binaries using gravitational waves”

(15 min.)

[JGRG30 (2021) 120803]

Parameter estimation on superspinar binaries using gravitational waves

Nami Uchikata and Tatsuya Narikawa
(ICRR, Univ. of Tokyo)

Based on Phys. Rev. D **104**, 024059 (2021)

Introduction: Black hole spin

- Black hole spin is bounded by $|\chi| \leq 1$.
  → The event horizon exists.

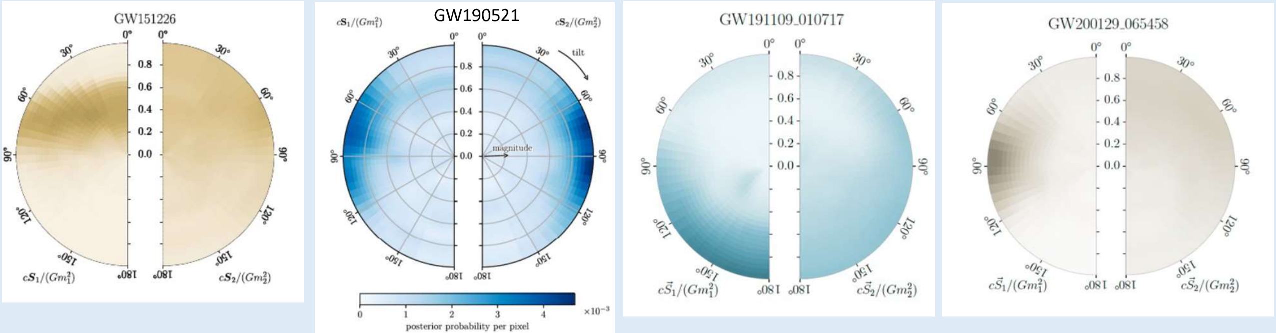
- String theory allows black hole spacetime with $|\chi| > 1$: ***superspinars***
  (Gimon & Horava 2009)

- Proposed as a possible source of high energy cosmic rays.

- Stability? → Can be stable against linear perturbations. (Nakao et al. 2017, Roy et al. 2019)
  (Complete physics is unknown.)

Black hole spins from gravitational wave observations

- 90 events observed by LIGO and Virgo. (GWTC-3)
- Some highly spinning objects were observed.



GWTC-1 Abbot et al. 2019

Abbot et al. 2020

GWTC-3 Abbot et al. 2021

Our study

- We have been imposing $|\chi| \leq 1$ in gravitational wave data analysis.
- How parameter estimation is biased by the prior $|\chi| \leq 1$, if we observe supserspinars?

→ Analyze simulated superspinar binary signals with two spin priors, $|\chi| \leq 1$ and $|\chi| \leq 1.5$, and compare the results.

Method of analysis

- Bayesian parameter estimation (`LALInference`) on
  - ✓simulated inspiral signals of superspinar binaries
  - ✓observed black hole binary event (GW190814)

$$\underset{\text{posterior}}{p(\theta|d)} = \frac{\overset{\text{Likelihood}}{\mathcal{L}(d|\theta)}\,\overset{\text{prior}}{\tilde{\pi}(\theta)}}{\underset{\text{evidence}}{Z}}$$

d : data
θ : parameter

- Template and injection waveform models :
  - TaylorF2 (frequency domain inspiral waveform using the post-Newtonian approximation)
  - phase : 3.5 PN
  - amplitude : 3 PN

$$\tilde{h}(f) = A(f)e^{i\Psi(f)}$$

$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\pi \underline{\mathcal{M}_c} f)^{-5/3}\sum_{i=0}^{7}\psi_i(\Xi)x^{i/3},$$

$$A(f) = \sqrt{\frac{5\pi}{24}}\frac{\underline{\mathcal{M}_c}}{d_L}(\pi \underline{\mathcal{M}_c} f)^{-7/6}\sum_{i=0}^{6}A_i(\Xi)x^{i/3}$$

$(x = \pi M f)$

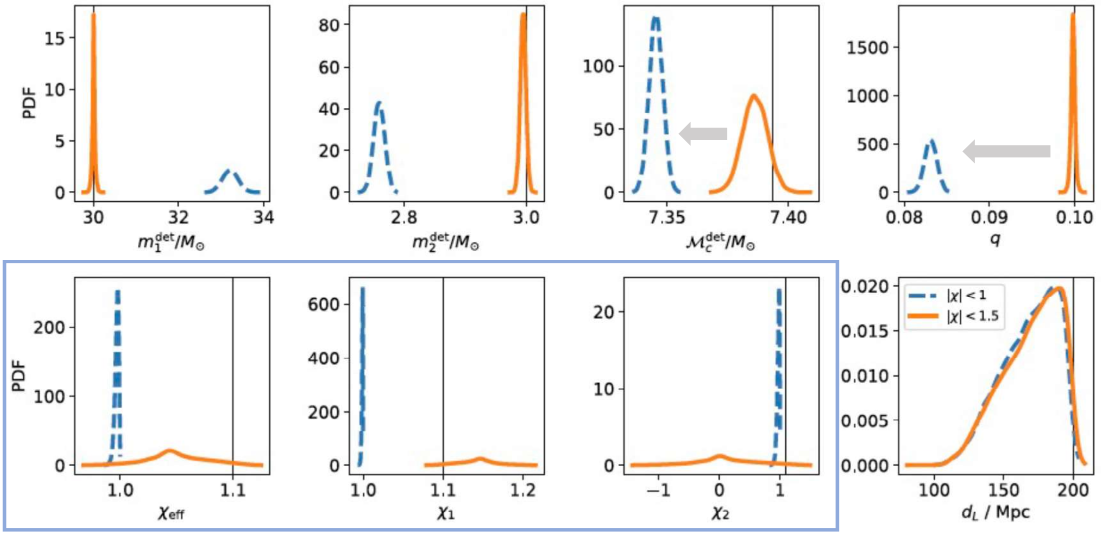
Chirp mass $M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

Signal injection

- At least one component of a binary is a superspinar.
- Hanford-Livingston-Virgo network (design sensitivities)
- Spin priors: $|\chi| \leq 1$ and $|\chi| \leq 1.5$
- Without Gaussian noise $\mathrm{d}(t) = h(t) + \cancel{n(t)}$
- Common injection parameters：
  $m_1 = 30M_\odot, m_2 = 3M_\odot, d_L = 200\text{Mpc}$, no precession

Result 1: Primary is a superspinar

- $\chi_1 = 1.1, \chi_2 = 1.1$ injection



SNR ~ 80

Log Bayes factor

|χ| < 1:   2960.43

|χ| < 1.5: 3122.01

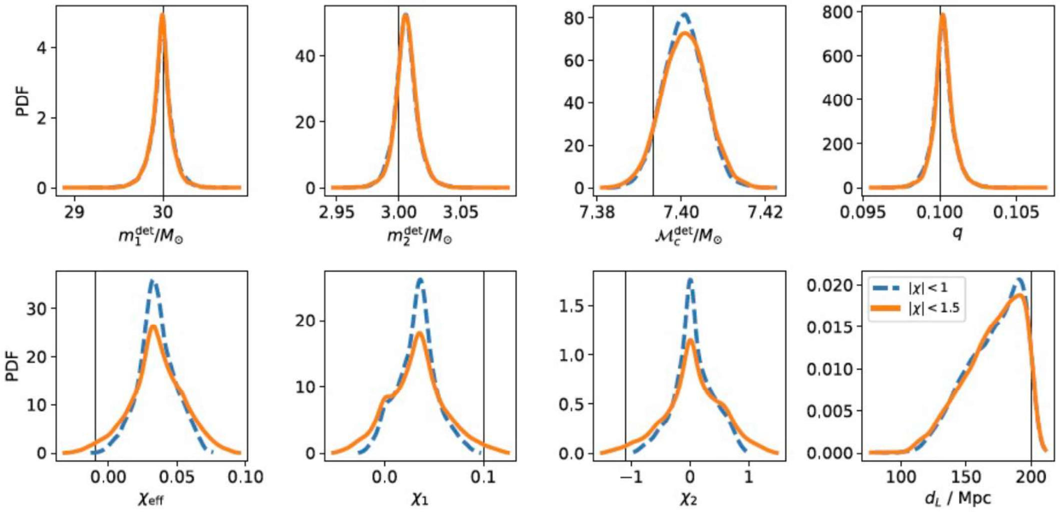
Extreme behavior might be a clue for of the detection?

$$\chi_{eff} = \frac{m_1\chi_1 + m_2\chi_2}{m_1 + m_2}$$

Similar tendency for other χ_2 values.

Result 2 : Primary is a black hole

- $\chi_1 = 0.1, \chi_2 = 1.1$ injection



SNR ~ 67

Log Bayes factor

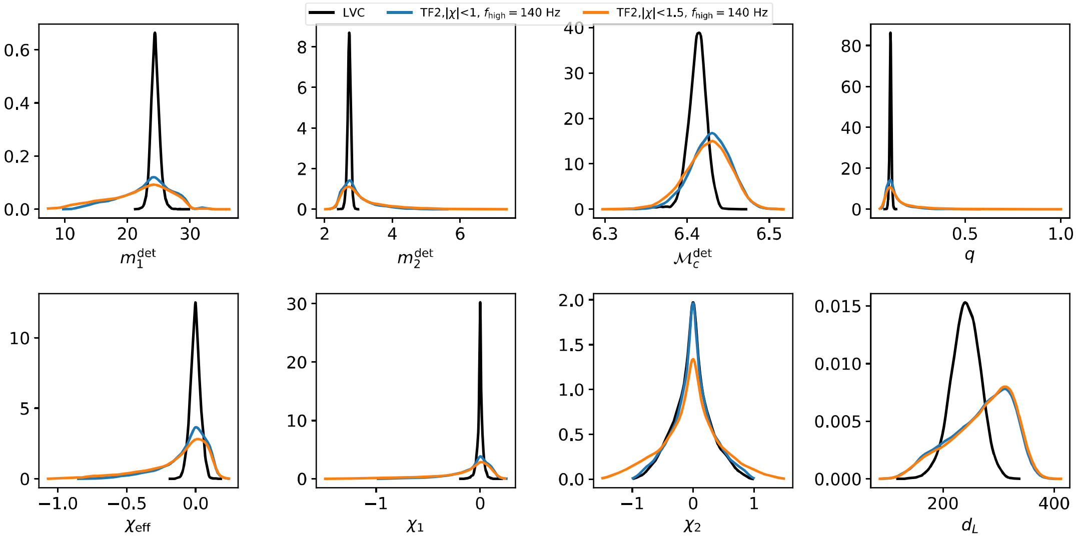
|χ| < 1:   2219.36

|χ| < 1.5: 2219.26

Similar tendency for other χ_1 values if $\chi_1 < 1$ or for $\chi_2 = -1.1$.

Result 3 : GW190814 (SNR~25)

TaylorF2 template is cutoff around at ISCO frequency (~ 140 Hz).



Log Bayes factor
$|\chi| < 1$: 207.114
$|\chi| < 1.5$: 206.999

(Large statistical errors may arise from the absence of higher modes and precession effects.)

Summary and future work

We have estimated binary parameters for superspinar binaries.

- Primary is a superspinar: **Mass** and **spin parameters are biased** by the prior $|\chi| \leq 1$.
- Primary is a black hole: No significant difference due to the spin prior.
  → Difficult to distinguish.
- GW190814: Primary spin may be small. Secondary spin is uninformative.

We are now working on for a search for superspinar binary signals in the real data.

Session C1a 10:00–12:00

[Chair: Atsushi Nishizawa]

Tatsuya Narikawa

ICRR, The University of Tokyo

“Gravitational-wave constraints on the GWTC-2 events by measuring the tidal deformability and the spin-induced quadrupole moment”

(15 min.)

[JGRG30 (2021) 120804]

C1a3 Wed. Dec 8, 2021

Gravitational-Wave Constraints on the GWTC-2 Events by Measuring the Tidal Deformability and the Spin-Induced Quadrupole Moment

arXiv:2106.09193
Phys. Rev. D 104, 084056 (2021)

Λ $\delta\kappa$

Tatsuya Narikawa
(ICRR, Univ. of Tokyo)

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JGRG30
Dec 6-10, 2021

KAGRA

Abstract

tidal deformability Λ **spin-induced quadrupole moment** $\delta\kappa$

We reanalyze GWs emitted from binary black hole (BBH) coalescences. Focusing on the influence of Λ and $\delta\kappa$, we provide model-independent constraints on deviations from the standard BBH case.

We report constraints on Λ and $\delta\kappa$ for six low-mass GWTC-2 events (long-inspiral regime): GW151226, GW170608, GW190707, GW190720, GW190728, GW190924

Previous works: focusing on only one of Λ and $\delta\kappa$

Λ - Tidal tests: Johnson-McDaniel+, 2020 (Constraints on Boson stars by future simulated observations)

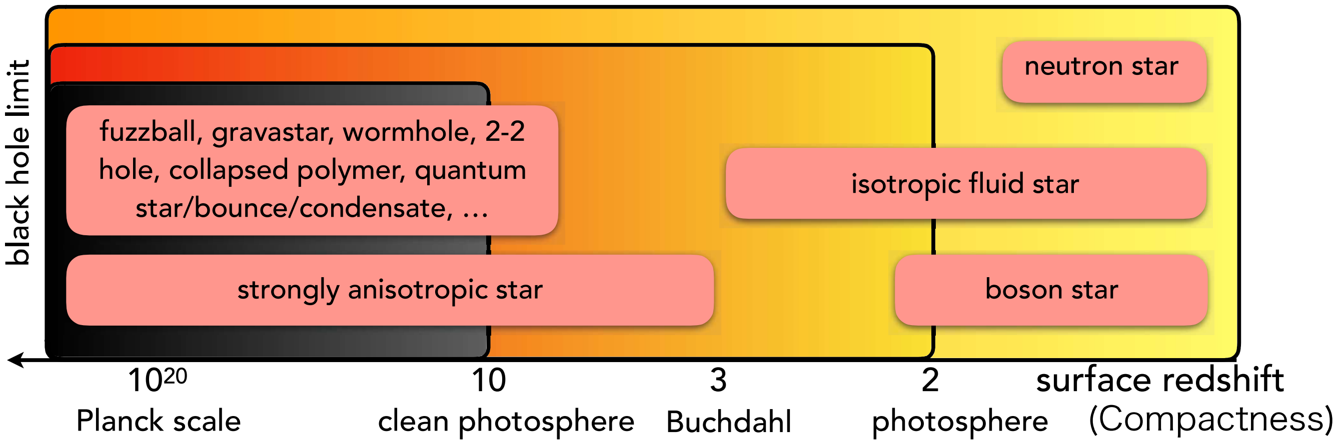
$\delta\kappa$ - SIQM tests: ① Krishnendu+, 2019 (GW151226 & GW170608); ② O3a Tests of GR paper, 2020 (GWTC-2 events)

Exotic compact objects (ECOs)

ECOs: alternatives to BH in GR

Motivation: e.g., to avoid spacetime singularity in BH, to solve information loss problem of BH, etc.

[GWIC-3G_science-case]



ECOs have largely different values of Λ and $\delta\kappa$ from those of BHs.

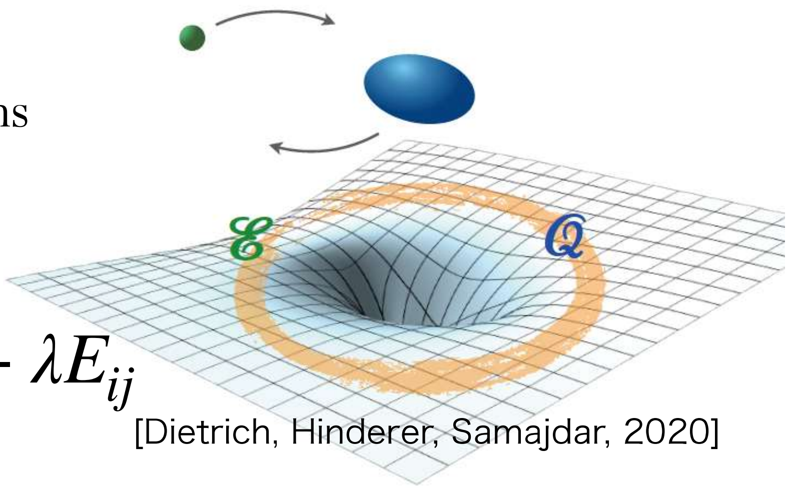
Aim: model-independent tests of strong-field gravity regimes from the measurements of Λ and $\delta\kappa$ via GWs.

We provide constraints on deviations from the BBH in GR.

—> those would provide evidence for existence of ECOs and/or hint for new physics.

Tidal deformability

When binary orbital separations are small, each star is tidally distorted by its companion.



$$Q_{ij} = -\lambda E_{ij}$$

[Dietrich, Hinderer, Samajdar, 2020]

tidal deformability: $\lambda = -\dfrac{Q_{ij}\text{: (tidal-induced) quadrupole moment}}{E_{ij}\text{: companion's tidal field}}$

the leading effect on GW phase:

binary tidal deformability, mass-weighted combination of $\Lambda_{1,2}$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(1+12q)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}$$

[Flanagan, Hinderer, 2007;
Hinderer 2008;
Vines, Flanagan, Hinderer 2011]

$\Lambda_{1,2} = \lambda_{1,2}/m_{1,2}^5$: individual ones

$q = m_2/m_2 \leq 1$: mass ratio

Λ

Tidal deformability

Λ

$$Q_{ij} = -\lambda E_{ij} \qquad \Lambda_{1,2} = \lambda_{1,2}/m^5_{1,2}$$

$\Lambda = 0$: **BH in GR**

(Schwarzschild BH [Binnington, Poisson, 2009; Damour, Nagar, 2009],
Kerr BH [Poisson, 2015; Pani+, 2015; Landry, Poisson, 2015]),

$\Lambda \sim 100 - 1000$: Neutron Stars (NSs) [Lattimer, Prakash2004].

($\Lambda < 900$ by GW170817 [LVC 2018])

$\Lambda \neq 0$: **exotic compact objects (ECOs),**

boson stars, gravastars, wormhole, quantum correction to BH

For gravastars, $\Lambda < 0$. [Uchikata, Yoshida, Pani, 2016]

Previous works: focusing on only Λ

Tidal tests: Johnson-McDaniel+, 2020 (Constraints on Boson stars by future simulated observations)

Spin-induced quadrupole moment (SIQM)

$\delta\kappa$

deformation due to compact object's spin

$$Q = -(1 + \delta\kappa)\chi^2 m^3$$

$\delta\kappa = 0$: BH [Poisson, 1998],

$\delta\kappa \sim 2 - 20$: spinning NS [Laarakkers, 1997; Pappas, 2012],

$\delta\kappa \sim 10 - 150$: **spinning boson stars** [Ryan 1997]**,**

For gravastar $\delta\kappa < 0$ **is possible** [Uchikata+2016]**.**

the leading effect: **symmetric combination of SIQM parameters** $\delta\kappa_{1,2}$**:**

$$\delta\kappa_s = (\delta\kappa_1 + \delta\kappa_2)/2$$

Previous works: focusing on only $\delta\kappa$

SIQM tests: ① Krishnendu+, 2019 (GW151226 and GW170608);
② LVC, "O3a Tests of GR" paper, 2020 (GWTC-2 events)

Our analysis setup - parameter estimation

- **Post-Newtonian (PN) inspiral waveform model:**
  ECO = BBH + Tidal + SIQM

Λ $\delta\kappa$

- Bayesian inference library: Nested sampling in LALSUITE (LALInferenceNest)

Bayes's theorem $$p(\theta|d) = \frac{\mathcal{L}(d|\theta)\pi(\theta)}{\mathcal{Z}},$$

Likelihood $$\mathcal{L}(d|\theta) \propto \exp\left[-\frac{\langle d-h(\theta)|d-h(\theta)\rangle}{2}\right]$$

Noise-weighted inner product $$\langle a|b\rangle := 4\mathrm{Re}\int_{f_{\rm low}}^{f_{\rm high}} df \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)},$$

Evidence $$\mathcal{Z} = \int d\theta \mathcal{L}(d|\theta)\pi(\theta).$$ **Bayes factor** $$\mathrm{BF}^{\rm ECO}_{\rm BBH} = \frac{\mathcal{Z}_{\rm ECO}}{\mathcal{Z}_{\rm BBH}}.$$

- Priors: uniform on $\tilde{\Lambda}$ and $\delta\tilde{\Lambda}$ for tidal, uniform on $\delta\kappa_{1,2}$ for SIQM.

Waveform models for inspiraling ECOs

- **Post-Newtonian (PN) inspiral waveform model:**
  ECO = BBH + Tidal + SIQM

Λ $\delta\kappa$

$$\tilde{h}_{\rm ECO}(f) = A(f)e^{i\Psi_{\rm BBH}(f)+\Psi_{\rm SIQM}(f)+\Psi_{\rm Tidal}(f)}$$

- Amplitude up to 3 PN for BBH (PP+spin), up to 5+1 PN (tidal)

- Phase

Adding higher-order PN terms prevent $\tilde{\Lambda}$ biasing

- **Point-particle: 0-5.5 PN (TF2g),**
- Spin (aligned-spin): SO:1.5-3.5 PN, SS:2-3 PN, **SIQM: 2-3 PN,**
- **Tidal: 5-7.5 PN.**

Spin terms at other PN orders help to break degeneracies, e.g., $q-\chi_{\rm eff}$

Refs.
- **TF2g** [Blanchet, 2014; Buonanno+, 2009; Messina+, 2019]
- **SIQM** [Krishnendu+, 2017]
- **Tidal** [Damour, Nagar, Villain, 2012; Henry, Faye, Blanchet, 2020]

The updated complete and corrected GW tidal phase

Narikawa, Uchikata, Tanaka https://arxiv.org/abs/2106.09193

We rewrite **the complete and corrected form** derived by Henry, Faye, and Blanchet (2020) only for the mass quadrupole interactions as a function of the dimensionless tidal deformability $\Lambda_{1,2}$, **in a convenient way.**

$$\Psi_{\mathrm{HFB}}(f) = \frac{3}{128\eta} x^{5/2} \sum_{A=1}^{2} \Lambda_A X_A^4 \Bigg[-24(12-11X_A) - \frac{5}{28}(3179 - 919X_A - 2286X_A^2 + 260X_A^3)x$$

$$+24\pi(12-11X_A)x^{3/2}$$

$$\underline{-5\left(\frac{193986935}{571536} - \frac{14415613}{381024}X_A - \frac{57859}{378}X_A^2 - \frac{209495}{1512}X_A^3 + \frac{965}{54}X_A^4 - 4X_A^5\right)} x^2 \quad \text{complete 7PN}$$

$$+\frac{\pi}{28}(27719 \underline{- 22415X_A + 7598X_A^2 - 10520X_A^3})x^{5/2}\Bigg], \quad \text{corrected 7.5PN}$$

$$X_A = m_A/(m_A + m_B)$$

—> We have implemented this and used it in our analyses.

Selected events

Low-mass events (long inspiral):
higher cutoff frequency $\gtrsim$ 120 Hz
and larger inspiral SNR $\gtrsim$ 9

f_{high} denotes the cutoff frequency divide the inspiral and post-inspiral regimes.

Event	f_{high} [Hz]	$\mathrm{SNR}_{\mathrm{inspiral}}$
GW151226	150	10.7
GW170608	180	14.7
GW190707	160	11.2
GW190720	125	9.3
GW190728	160	12.1
GW190814	140	22.0
GW190924	175	11.4

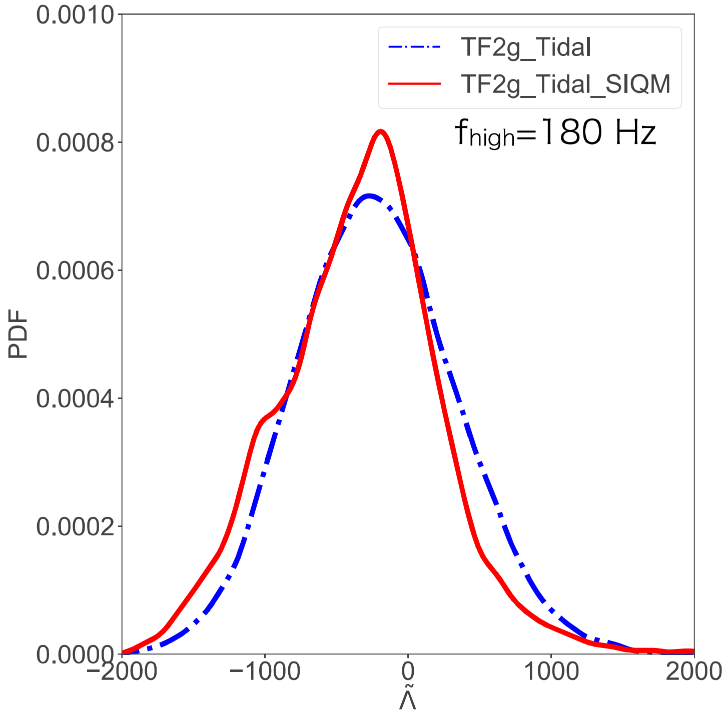
← the loudest inspiral SNR

First, we present the results for GW170608 in detail.

Tidal constraints on GW170608

Λ

The posterior PDF of $\tilde{\Lambda}$



Consistent with GR ($\tilde{\Lambda} = 0$) at the 90% CL

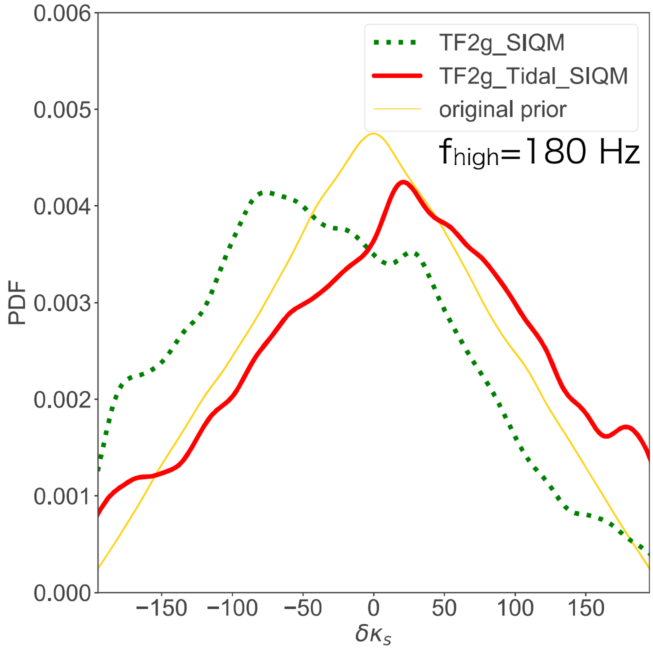
Adding the SIQM terms do not affect the constraint on the tidal deformability $\tilde{\Lambda}$.

The 90% symmetric credible range of $\tilde{\Lambda}$: [-1265, 565]

SIQM constraints on GW170608

$\delta\kappa$

The posterior PDF of $\delta\kappa_s$



Consistent with GR ($\delta\kappa_s = 0$) at the 90% CL

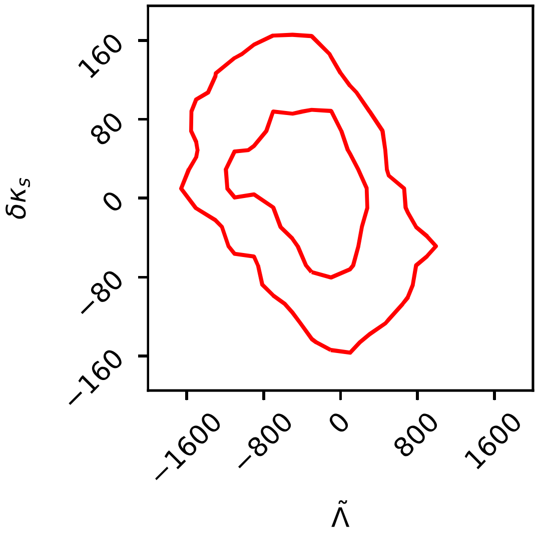
$\delta\kappa_s$ is poorly constrained for both waveform templates, which is consistent with the results shown in the previous studies by LIGO-Virgo.

They are weighted by dividing the original prior: uniform on $\delta\kappa_{1,2}$.

Tidal and SIQM constraints on GW170608

The corner plots of $\tilde{\Lambda}$-$\delta\kappa_s$ plane.

Λ $\delta\kappa$



Consistent with GR
($\tilde{\Lambda} = 0$ and $\delta\kappa_s = 0$)
at the 90% CL

We find weak negative correlation between $\tilde{\Lambda}$ and $\delta\kappa_s$.

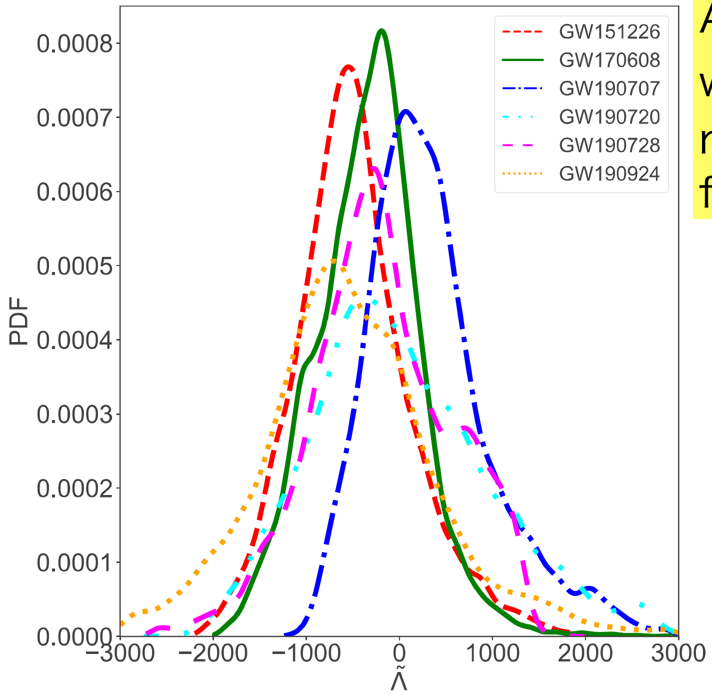
uniform priors on $\tilde{\Lambda}$, $\delta\tilde{\Lambda}$ and $\delta\kappa_{1,2}$.

— TF2g_Tidal_SIQM
f_{high}=180 Hz

Tidal constraints on six events

The posterior PDF of $\tilde{\Lambda}$ for six low-mass events.

Λ



TF2g_Tidal_SIQM waveform model

All events are consistent with BBH in GR ($\tilde{\Lambda} = 0$), no evidence of deviation from GR

Event	$\tilde{\Lambda}$
GW151226	[-1441, 649]
GW170608	[-1265, 565]
GW190707	[-590, 1661]
GW190720	[-1445, 1768]
GW190728	[-1432, 1078]
GW190924	[-2041, 1118]

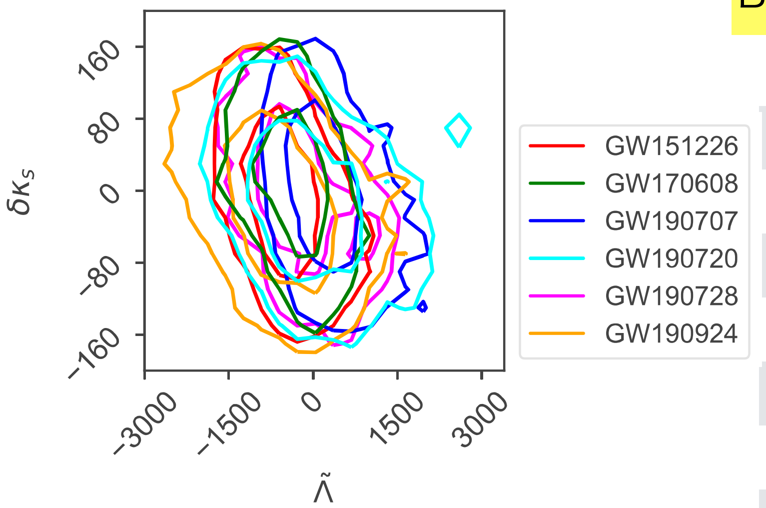
90% symmetric intervals

Tidal and SIQM constraints on six events

Λ $\delta\kappa$

The corner plots of $\tilde{\Lambda}$-$\delta\kappa_s$ plane for six low-mass events.

TF2g_Tidal_SIQM waveform model



We find weak negative correlation between $\tilde{\Lambda}$ and $\delta\kappa_s$.

All events are consistent with BBH in GR ($\tilde{\Lambda} = 0$ and $\delta\kappa_s = 0$)

Event	$\log_{10} \mathrm{BF}^{\mathrm{ECO}}_{\mathrm{BBH}}$
GW151226	-0.45
GW170608	-2.08
GW190707	-2.07
GW190720	-1.77
GW190728	-1.98
GW190924	-2.03
Combined	-10.38

The binary ECO model (with Tidal and SIQM) is disfavored compared to the BBH in GR.

Conclusion

Λ $\delta\kappa$

- We implemented the tidal and SIQM terms in TF2g.
- We analyzed six low-mass GWTC-2 events: GW151226, GW170608, GW190707, GW190720, GW190728, and GW190924 using TF2g_Tidal_SIQM waveform model.
- **The first constraints on $\tilde{\Lambda}$ of events classified as BBH**
- **We found that all events that we have analyzed are consistent with BBH mergers in GR ($\tilde{\Lambda} = 0$ and $\delta\kappa_s = 0$).**
- **The binary ECO model (with tidal and SIQM terms) is disfavored compared to the BBH in GR.**

Future work

- Improvement of waveform model by extension to post-inspiral regimes of binary ECOs

Session C1a 10:00–12:00

[Chair: Atsushi Nishizawa]

PRITI GUPTA

KYOTO UNIVERSITY

“Impact of tidal resonances in extreme-mass-ratio inspirals”

(15 min.)

[JGRG30 (2021) 120805]

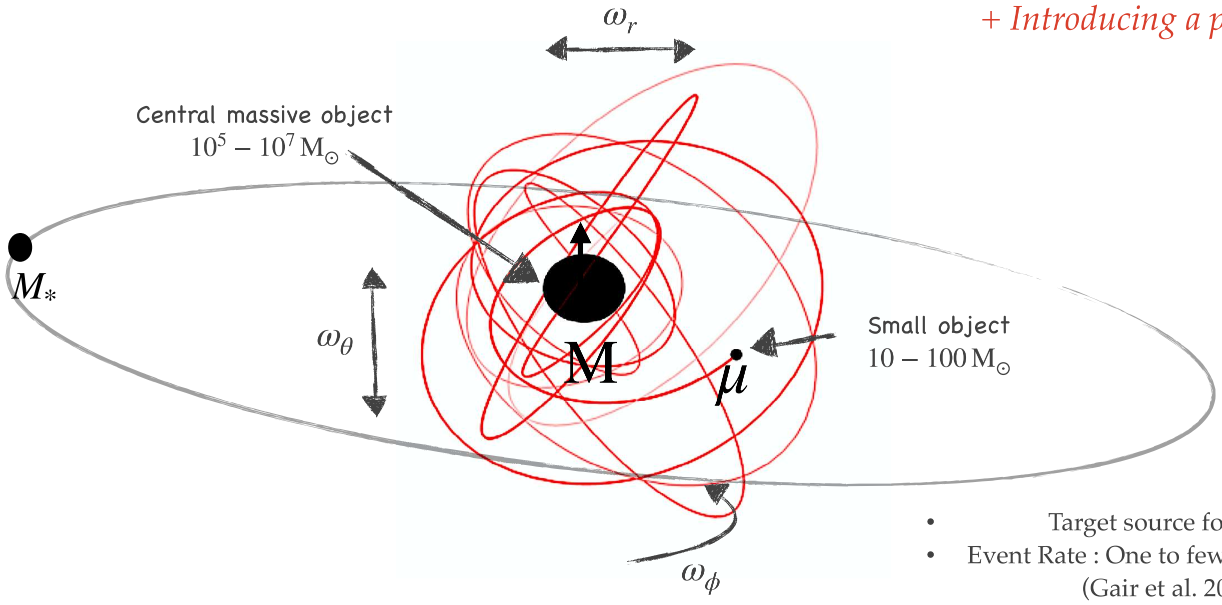
Impact of tidal resonances in EMRIs

Priti Gupta (Kyoto U.)
Béatrice Bonga (Radboud U.,Netherlands)
Alvin Chua (Caltech)
Takahiro Tanaka (Kyoto U.)

JGRG30 @ Waseda University (online), 08/12/2021

Extreme Mass Ratio Inspiral = EMRI

+ Introducing a perturber...



- Target source for LISA
- Event Rate : One to few tens per Year (Gair et al. 2017)

Motivation to introduce a perturber

Mass segregation + dynamical friction cause more massive black holes to sink to the centre [Emami & Loeb `19]:

$20-30\,\mathrm{M}_{\odot}$ black holes at distance ~ 2 - 5 AU from Sgr A* .

Rough estimate using EMRI merger rate:

$$\frac{1}{T_{EMRI}} \sim 0.3\left(\frac{M}{10^6 M_{\odot}}\right)^{0.19} Myr^{-1}$$

$$R \sim 4.3 \text{ AU} \left(\frac{M_{\star}}{10 M_{\odot}}\right)^{1/4} \left(\frac{M}{M_{\mathrm{SgrA}^*}}\right)^{0.45},$$

with $M_{\mathrm{SgrA}^*} = 4 \times 10^6 M_{\odot}$ the mass of Sagittarius A*.

---

q_i : angle variables

J_i : action variables

Action-angle formalism

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J})$$

$$\frac{dJ_i}{d\tau} = \mathbf{0}$$

Four constants of motion: $\{\mu, E, L_z, Q\}$

Tidal Resonance

❖ Adiabatic approximation ⟶ Fourier Domain + Averaging ⟶ Tidal Resonance condition

- $$\frac{dJ_i}{d\tau} \approx \epsilon \langle G^{(1)}_{i,\text{tide}}(q_\theta, q_r, q_\phi, \mathbf{J}) \rangle$$
- $$G_i^{(1)}(q_\phi, q_\theta, q_r, \mathbf{J}) = \sum_{m,k,n} G^{(1)}_{i,mkn}(\mathbf{J}) e^{i\underbrace{(mq_\phi + kq_\theta + nq_r)}_{\text{Rapidly oscillating for most index pairs m,}k,n}}$$
- $$\langle G^{(1)}_{i,\text{tide}}(q_\theta, q_r, q_\phi, \mathbf{J}) \rangle = G^{(1)}_{i,\text{tide},000}(\mathbf{J}) \quad + G^{(1)}_{\text{tide,nkm}}(\text{J})$$

$$\boxed{n\omega_r + k\omega_\theta + m\omega_\phi = 0}$$

Jump induced by resonances

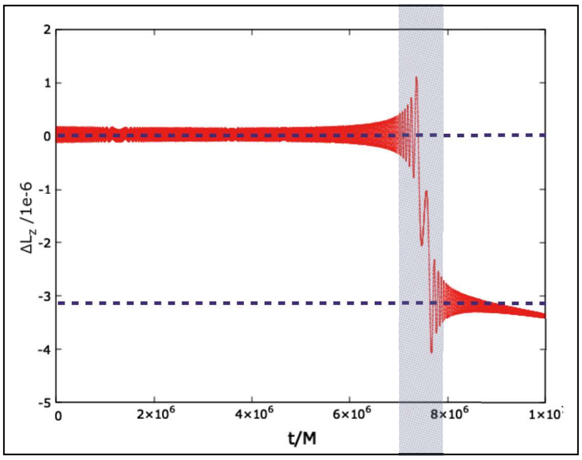
$$\langle G^{(1)}_{i,\text{tide}}(q_\theta, q_r, q_\phi, \mathbf{J}) \rangle = G^{(1)}_{i,\text{tide},000}(\mathbf{J}) \quad + G^{(1)}_{\text{tide,nkm}}(\text{J})$$

$M_* \sim 30M_\odot$

$R \sim 250\text{M}$

(n:k:m) = (3:0:-2)

$t_{res} \sim month$



$$\Delta L_z = L_z(\text{tidal+RR}) - L_z(\text{RR})$$

Treatment: Tidally perturbed Kerr

- We need perturbation to the central BH’s spacetime due to the tidal field.

\>>> Metric of tidally perturbed Kerr from [Gonzales + Yunes, 2005]

- For simplicity, we consider a stationary tidal perturber restricted to equatorial plane.

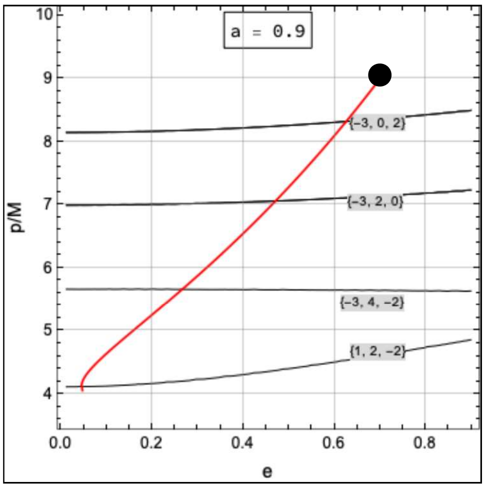
Given the metric, we can compute the induced acceleration and corresponding changes in L_z & Q.

$$a^\alpha = -\frac{1}{2}(g^{\alpha\beta}_{\text{Kerr}} + u^\alpha u^\beta)(2h_{\beta\lambda;\rho} - h_{\lambda\rho;\beta})u^\lambda u^\rho$$

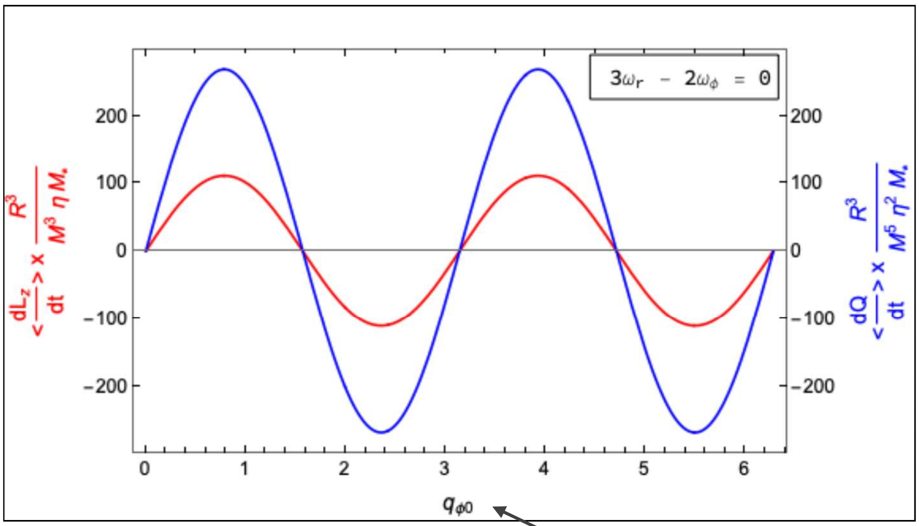
Resonances during inspiral

- Every inspiral encounters at least one of these resonances during final year of inspiral.

$$n\omega_r + k\omega_\theta + m\omega_\phi = 0$$



Sensitive dependence on phase



the orbital phase at which small object enters resonance

Impact on orbital phase of GWs

To estimate the effect, two orbits are evolved and compared

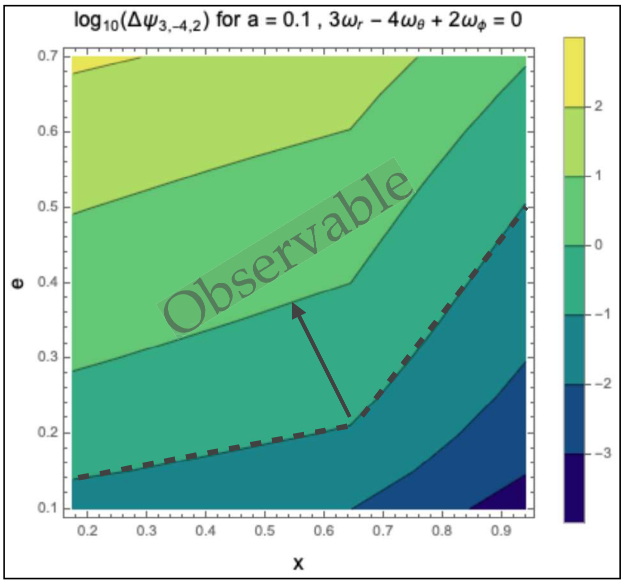
$$\{E, Q, L_z\} \to \omega_\phi^{(1)} \quad \text{versus} \quad \{E, Q + \Delta Q, L_z + \Delta L_z\} \to \omega_\phi^{(2)}$$

$$\Delta\Psi := \int_0^{T_{\text{plunge}}} 2\Delta\omega_\phi dt$$

Phase resolution of LISA. $\Delta\psi \sim \mathcal{O}(1)$

One example of parameter Survey

a = 0.1



$\mu = 30\, M_\odot, M = 4 * 10^6\, M_\odot$

$R \sim 250\text{M} \quad M_{*} \sim 30 M_\odot$

$$\Delta\Psi'_{nkm} = \Delta\Psi_{nkm} \left(\frac{M'}{M}\right)^{7/2} \left(\frac{\mu'}{\mu}\right)^{-3/2} \left(\frac{M'_\star}{M_\star}\right) \left(\frac{R'}{R}\right)^{-3} .$$

Summary and Ongoing Work

- Tidal resonances can change EMRI waveforms significantly depending on the distance and mass of the tidal perturbers ----- hamper detection rate.
- Important to understand such environmental effects when constraining deviations from GR.
- Opportunity to learn about distribution of stellar mass objects that are close to SMBHs.

- Making fast and effective resonance model to study mismatch and parameter estimation bias from tidal resonances.

Thank you for your attention & see you @ banquet!

Session C1a 10:00–12:00

[Chair: Atsushi Nishizawa]

Norichika Sago

Kyoto University

“Oscillations in the EMRI gravitational wave phase correction as a probe of reflective boundary of the central black hole”

(15 min.)

[JGRG30 (2021) 120806]

Oscillations in the EMRI gravitational wave phase correction as a probe of reflective boundary of the central black hole

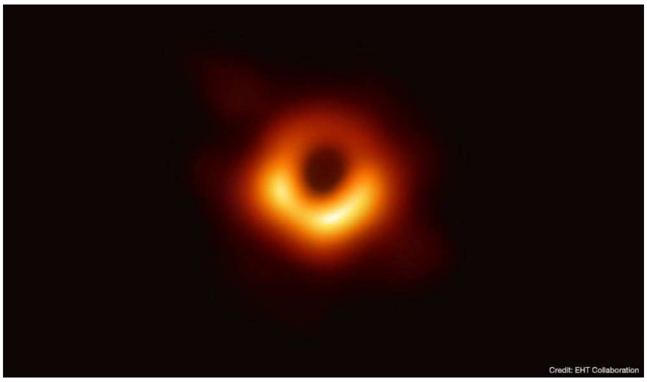
Norichika Sago (Kyoto U./Osaka City U.)
Collaborator: Takahiro Tanaka

Reference: PRD 100 064009 (arXiv:2106.07123)
2021.12.8 JGRG30

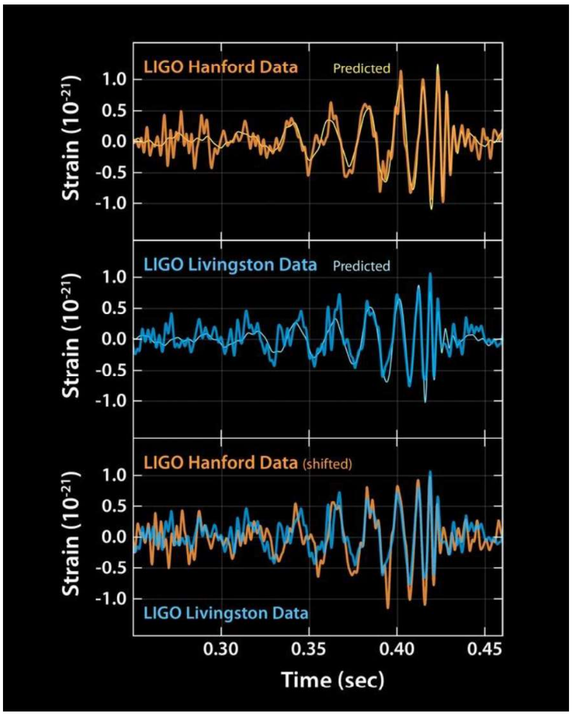
New era of BH observation

EHT and GW observatories are new windows to observe BHs.

- Event Horizon Telescope
  ⇒ Supermassive BH in M87
- Ground-based GW detectors
  ⇒ BBH mergers
- In future, massive BHs by LISA, …



Credit: EHT Collaboration



Credit: Caltech/MIT/LIGO Lab

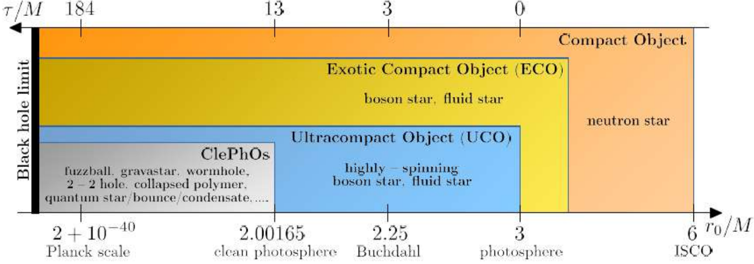
BH in GR or ECO?

More accurate observation of BHs raises a natural question as

"Are these objects really BHs predicted by GR?"

Other possibilities are discussed:

- BH mimicker?
  Ex) gravastars,
  boson stars,
  wormholes...



[Cardoso & Pani (2017)]

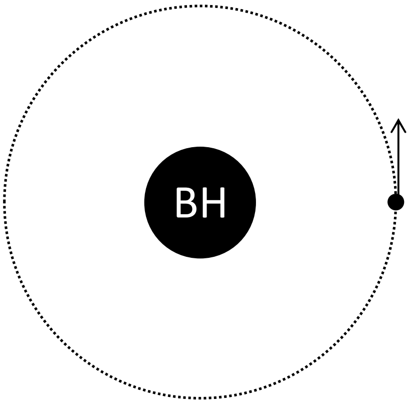
- BHs with Planck-scale structure near the horizon?
  Ex) fuzzball, firewall, ...

(Here I refer these objects as exotic compact objects (ECOs).)

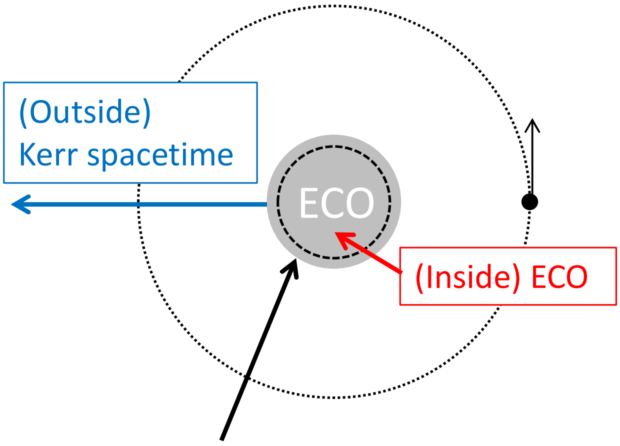
We consider GWs from extreme mass ratio inspiral (EMRI) to test the possibility of an alternative scenario.

Setup: circular equatorial EMRI

BH EMRI (BH)



ECO EMRI (modified)



(Outside)
Kerr spacetime

(Inside) ECO

Not pure ingoing wave (reflected wave included) on the boundary slightly outside the BH horizon.

Field equation (radial Teukolsky equation)

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR(r)}{dr}\right)+V(r)R(r)=T(r)$$

$$\Delta = r^2-2Mr+a^2$$

Asymptotic solution

$$R_{\mathrm{in}}=\begin{cases} r^{-1}e^{-i\omega r^*}+\mathcal{R}r^{-1-2s}e^{i\omega r^*} & (r^*\to+\infty)\\ \mathcal{T}\Delta^{-s}e^{-ikr^*} & (r^*\to-\infty)\end{cases}$$

$$R_{\mathrm{up}}=\begin{cases} \tilde{\mathcal{T}}r^{-1-2s}e^{i\omega r^*} & (r^*\to+\infty)\\ e^{ikr^*}+\tilde{\mathcal{R}}\Delta^{-s}e^{-ikr^*} & (r^*\to-\infty)\end{cases}$$

$$k=\omega-m\Omega_H,\qquad \Omega_H=\frac{am}{2Mr_+},\qquad \frac{dr^*}{dr}=\frac{r^2+a^2}{\Delta}$$

Green’s function (BH EMRI case)

$$G(r,r')=\frac{1}{W}\left[\underset{\uparrow}{\underline{R_{\mathrm{in}}(r)}}R_{\mathrm{up}}(r')\theta(r'-r)+\underset{\uparrow}{\underline{R_{\mathrm{up}}(r)}}R_{\mathrm{in}}(r')\theta(r-r')\right]$$

pure ingoing near the horizon $\sim\Delta^2e^{-ikr_*}$ pure outgoing at infinity $\sim e^{i\omega r_*}$

The solution is given as

$$R(r)=\int G(r,r')T(r')dr'$$

$$\longrightarrow\begin{cases} Z_H\mathcal{T}\Delta^{-s}e^{-ikr_*} & (r\to r_+)\\ Z_\infty\tilde{\mathcal{T}}r^{-1-2s}e^{i\omega r_*} & (r\to\infty)\end{cases}$$

$$Z_{H/\infty}=\frac{1}{W}\int_r^\infty R_{\mathrm{up/in}}(r')T(r')dr'$$

Modified Green's function (ECO EMRI case)

$$\tilde{G}(r,r') = \frac{1}{\widetilde{W}}\left[\underline{\tilde{R}_{\rm in}(r)}R_{\rm up}(r')\theta(r'-r) + \tilde{R}_{\rm in}(r)\underline{R_{\rm up}(r')}\theta(r-r')\right]$$

replace the boundary condition to

$\sim Be^{ikr_*} + \Delta^{-s}e^{-ikr_*}$

pure outgoing at infinity

$\sim e^{i\omega r_*}$

Change the boundary condition near the horizon:

$$R_{\rm in} \to \tilde{R}_{\rm in} = R_{\rm in} + \beta R_{\rm up} = \beta e^{ikr^*} + (\mathcal{T} + \beta\mathcal{R})\Delta^{-s}e^{-ikr^*}$$

The solution is given as

$$\beta = \mathcal{T}R_{\rm b}\frac{\epsilon_-}{\epsilon_+}\left(1 - \tilde{R}R_{\rm b}\frac{\epsilon_-}{\epsilon_+}\right)^{-1}$$

$$\tilde{R}(r) = \int \tilde{G}(r,r')T(r')dr'$$

$$\to \begin{cases} Z_H\left[\beta e^{ikr^*} + (\mathcal{T} + \beta\mathcal{R})\Delta^{-s}e^{-ikr^*}\right] & (r \to r_+) \\ (Z_\infty + \beta Z_H)\tilde{\mathcal{T}}r^{-1-2s}e^{i\omega r_*} & (r \to \infty) \end{cases}$$

Energy flux (ECO EMRI case)

$$F_{\rm mod}^{(\infty)} = \frac{\left|\tilde{\mathcal{T}}\right|^2 |Z_\infty + \beta Z_H|^2}{4\pi\omega^2} \qquad \text{(at infinity)}$$

$$F_{\rm mod}^{(H)} = \frac{|Z_H|^2}{4\pi\omega^2}\left(\epsilon_-^2|\mathcal{T} + \beta\mathcal{R}|^2 - \epsilon_+^2|\beta|^2\right) \qquad \text{(near the horizon)}$$

Correction of the flux

$$\delta F^{(\rm tot)}$$

$$= \left(F_{\rm mod}^{(\infty)} + F_{\rm mod}^{(H)}\right) - \left(F^{(\infty)} + F^{(H)}\right)$$

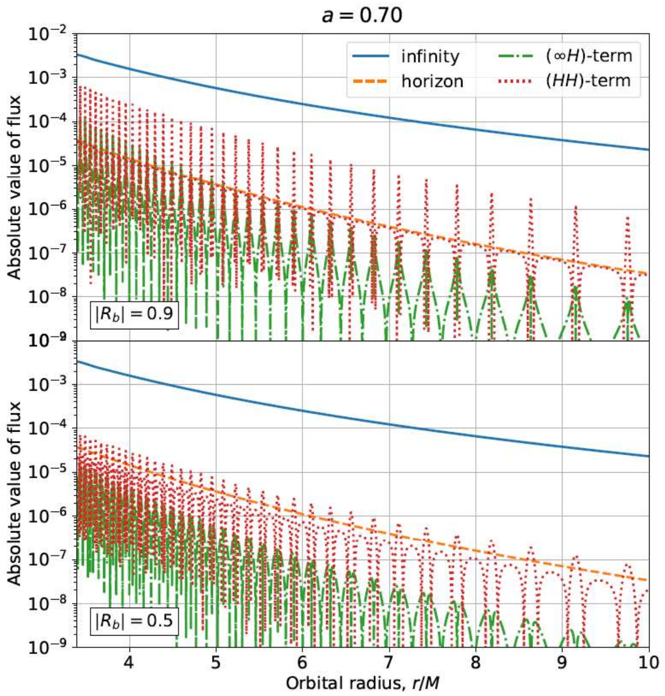
$$= \frac{|4Mr_+k - 2is(r_+ - M)|}{\omega}\sqrt{\left|F^{(\infty)}F^{(H)}\right|}\Re\left(\frac{Z_\infty^* Z_H\beta}{|Z_\infty Z_H \epsilon_-|}\right)$$

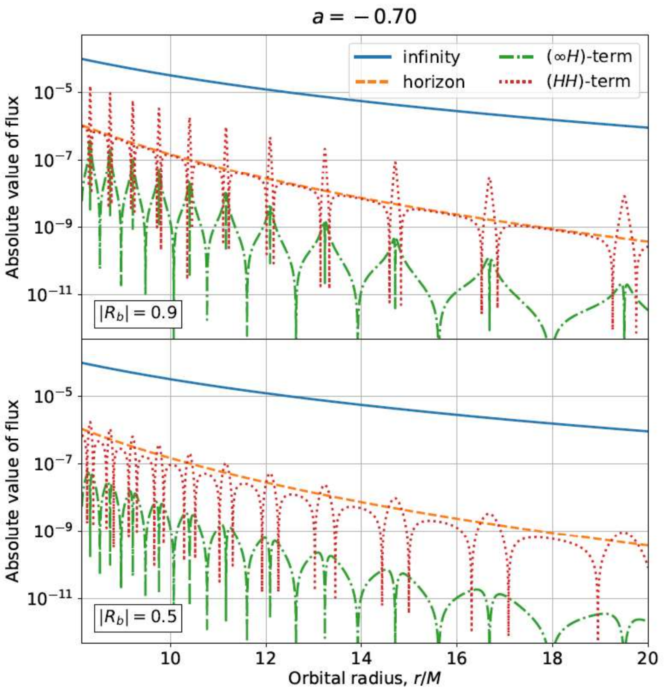
$$+2F^{(H)}\Re\left(\frac{\beta\tilde{\mathcal{R}}}{\mathcal{T}}\right) \longleftarrow \delta F^{HH}$$

$\delta F^{\infty H}$

Correction of the energy flux

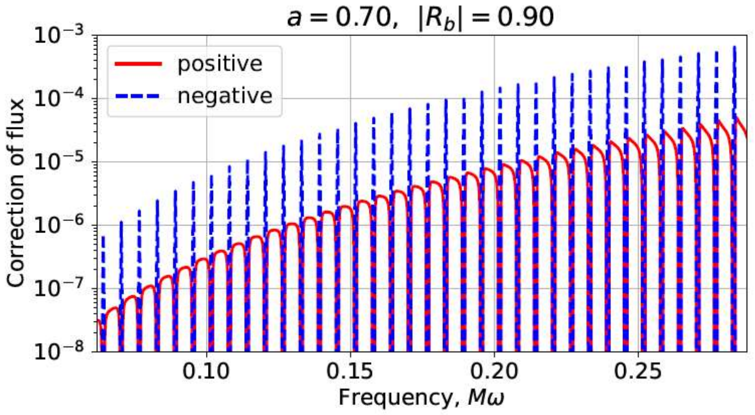
[NS and Tanaka (2021)]

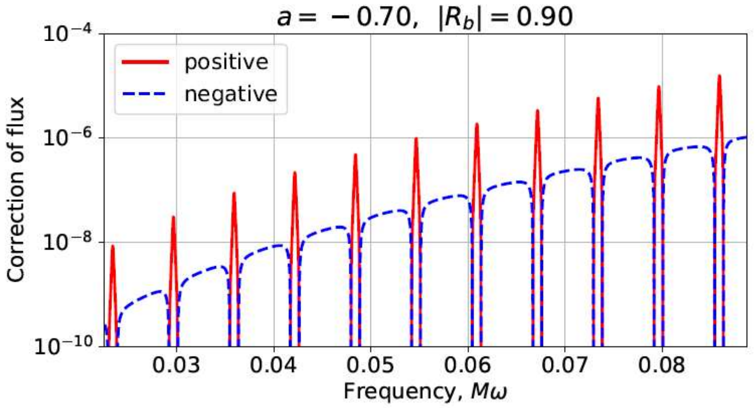




Correction of the energy flux

[NS and Tanaka (2021)]



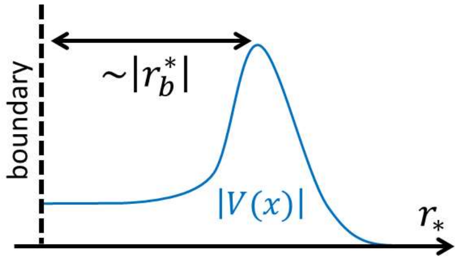


Period of the oscillation

$$e^{2i\Delta\omega|r_b^*|} \sim 1$$

$$\rightarrow\ \Delta\omega = \frac{\pi}{|r_b^*|}$$

r_b^* is the position of the boundary in the tortoise coordinate.



Effect on GW phase

Cycle of GW: (l,m)=(2,2)

$$N = 2\pi \int f dt = \int_{r_{ISCO}}^{r(t)} \frac{\Omega_\phi}{\pi} \frac{dE/dr}{F} dr$$

$$N_{\mathrm{mod}} = \int_{r_{ISCO}}^{r(t)} \frac{\Omega_\phi}{\pi} \frac{dE/dr}{F_{\mathrm{mod}}} dr$$

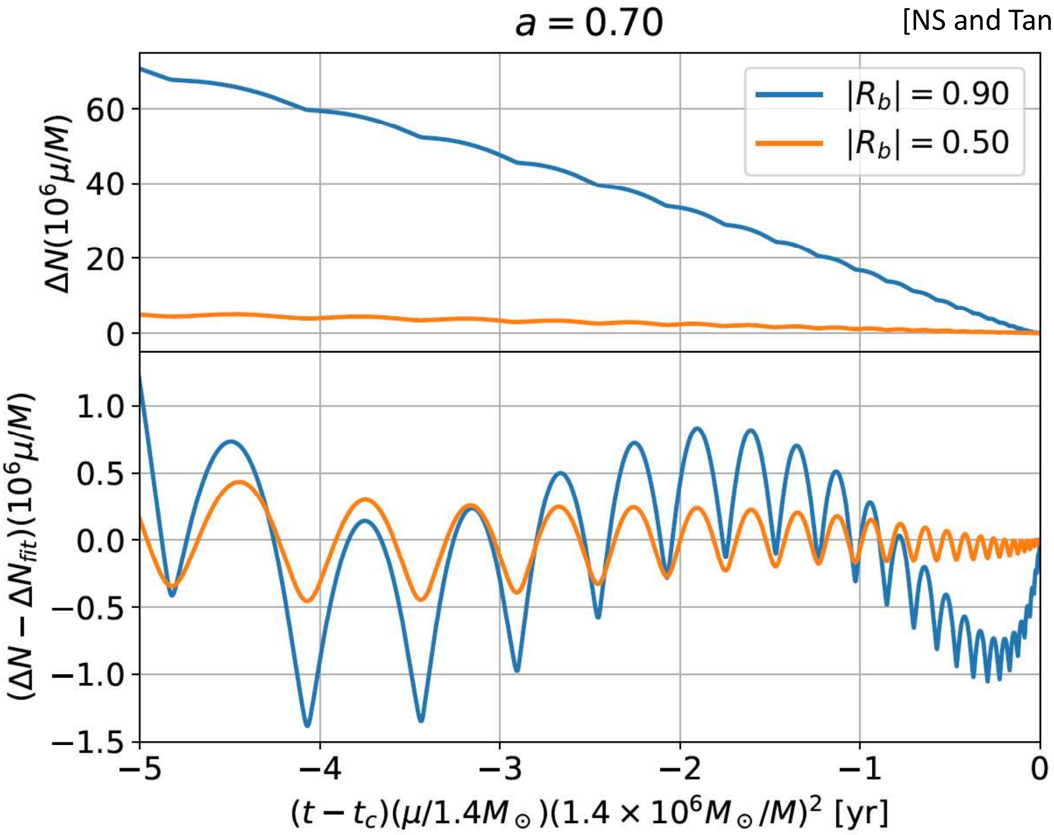
Here we take into account $(l,m) = (2, \pm 2)$ modes (leading order) of the flux.

Correction of GW cycle

$$\Delta N \equiv N_{\mathrm{mod}} - N$$

Correction of GW phase

[NS and Tanaka (2021)]



Search for the best fit parameters

Match between waveforms for BH and ECO EMRI cases

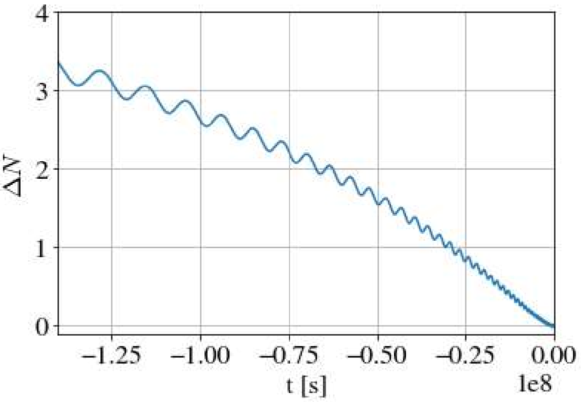
$$\mathcal{M}(t_0,\mu,M)=\frac{|(h_{\text{mod}}|h)|}{\sqrt{(h_{\text{mod}}|h_{\text{mod}})(h|h)}}\qquad (g|h)=\int\frac{\tilde{g}^*(f)\tilde{h}(f)}{S_n(f)}df$$

$S_n(f)$: LISA noise curve
[Robson et al. (2019)]

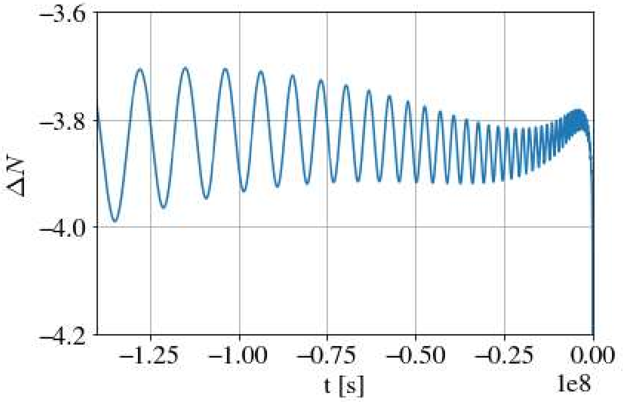
Search strategy

- Fix the parameters of the modified waveform for ECO EMRI.
- Change the parameters of the BH EMRI waveform.
  t_0: fiducial time, μ: companion's mass, M: BH's mass
- Search the best fit parameters so that the match is maximized.

Modulation in GW phase



$N_{\text{mod}}(t;\theta^i)-N(t;\theta^i)$



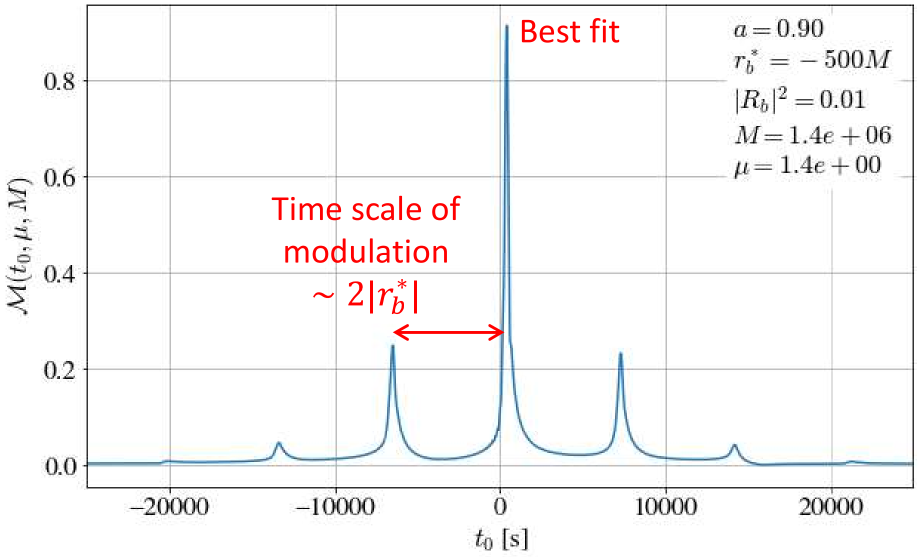
$N_{\text{mod}}(t;\theta^i)-N(t;\theta^i+\Delta\theta^i)$

The non-oscillatory part of the GW phase correction is suppressed for the best fit parameters.

The oscillatory part remains.
⇒ To test the ECO EMRI model by searching the oscillatory part.

Side peaks due to the oscillatory modulation

[NS and Tanaka in preparation]



Best fit

Time scale of modulation $\sim 2|r_b^*|$

Best-fitted match is reduced by ～10%.

The secondary peaks appear on the both sides of the largest one.

⇒ Signature of the reflective boundary near the horizon!

Summary

- ✓ We investigated the orbital evolution and the GW of a circular equatorial ECO EMRI, and evaluate the effect of the reflection near the horizon.
- ✓ We found that the correction in the GW phase is divided into two parts:
  oscillatory part and non-oscillatory part
- ✓ Non-oscillatory part can be suppressed by adjusting the parameters (masses, fiducial time and phase).
- ✓ The oscillatory part in the GW phase may be a smoking gun of ECOs. Further investigation required.

Session C1a 10:00–12:00

[Chair: Atsushi Nishizawa]

Alejandro Torres-Orjuela

TianQin Center for Gravitational Physics

“Detecting the motion of gravitational wave sources”

(15 min.)

[JGRG30 (2021) 120807]

Detecting the motion of gravitational wave sources

Alejandro Torres-Orjuela • TianQin Center @ Waseda University，Dec. 8 2021

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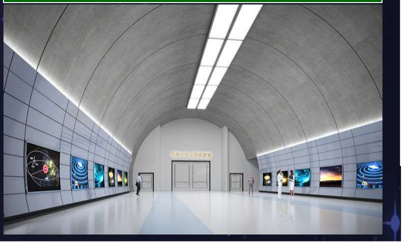


Content

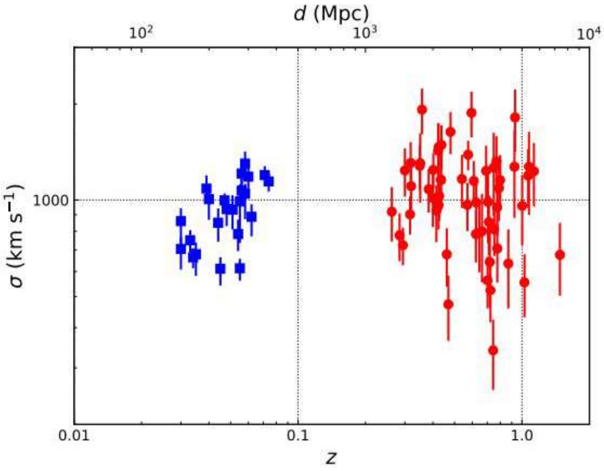
1、 Source motion

2、 Motion and GWs

3、 Effect for EMRIs

4、 Summary

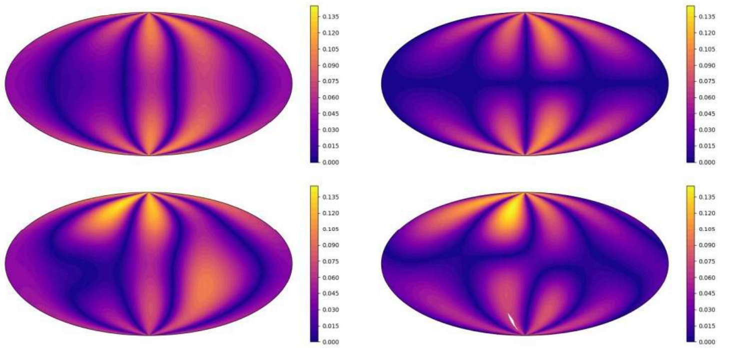
Topic 1: Velocity dispersion



Velocity dispersion of galaxy clusters
[Girardi+1996 & Ruel+2014]

- GW sources located inside galaxies!
- Galaxies in clusters with deep potentials
- Average velocity despersion around 1000 km/s

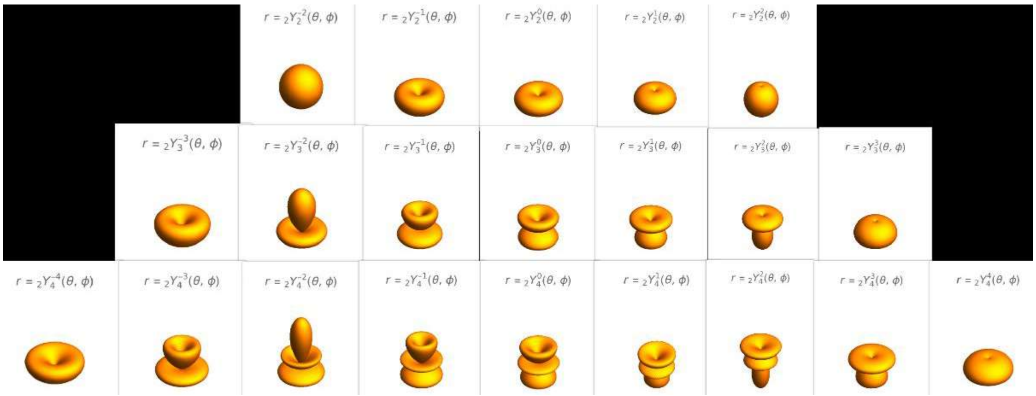
Topic 2: Radiation patterns



Radiation patterns for a source at rest and a moving one [Torres-Orjuela+2021a]

- Plane waves & Doppler shifted frequency → mass-redshift degeneracy
- GWs have structure → deformed by a CoM motion

Topic 2: GW modes

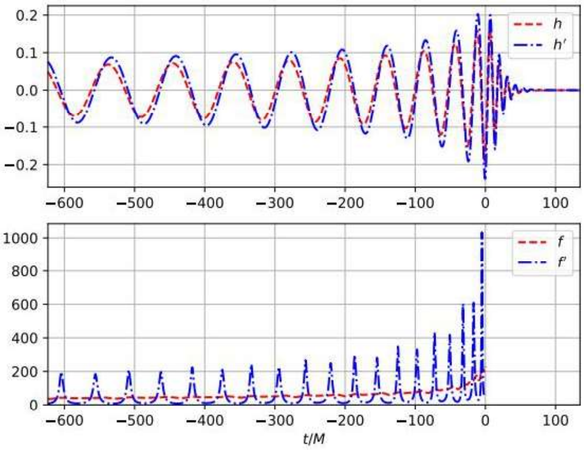


Spin-2 spherical harmonics [Wolfram Demonstration]

- Decompose GWs in modes using spin-2 spherical harmonics:

$$H^{l,m} := \int \left[h_p(\theta,\phi) - i\, h_c(\theta,\phi) \right]_{-2}\bar{Y}^{l,m}(\theta,\phi)\, d\Omega$$

Topic 2: GWs & moving source



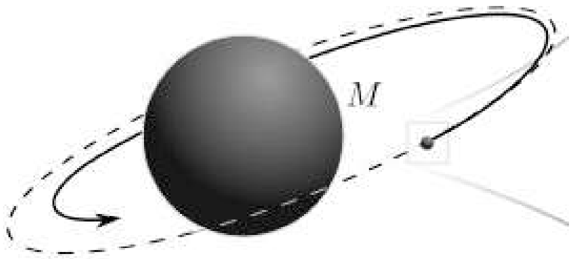
Signal and frequency for a source at rest and a moving one [Torres-Orjuela+2021a]

- Motion deforms radiation pattern → modes change:

$$H'^{l,m} = H^{l,m} + v \sum C^{l',m'} H^{l',m'}$$

- Amplitude and frequency change in a time dependent manner!

Topic 3: EMRIs

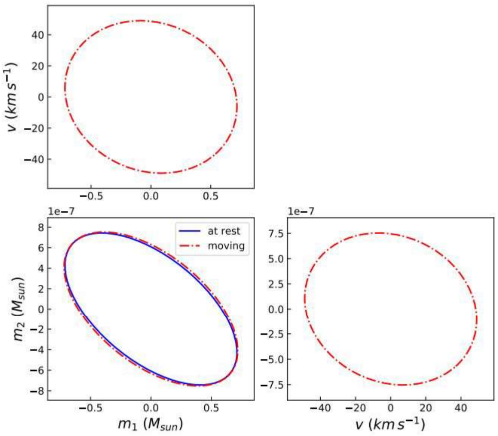


Extreme mass-ratio inspiral (EMRI)
[Barack+2019]

- Stellar mass BH (10 M_o) circling a SMBH (10^6 M_o)
- Outstanding target sources for TianQin & LISA
- Compare moving EMRI (1000 km/s) to one at rest with observation of 2 yr

---

Topic 3: Detection accuracy



Detection accuracy for the masses and the velocity [Torres-Orjuela+2021b]

- Peculiar velocity is detectable at percent level
- Ignoring velocity: decreasing accuracy of mass detection

Summary

- Peculiar velocity of host galaxies → GW sources move with a velocity of the order 1000 km/s
- Constant CoM velocity alters the modes of GWs → mass-redshift degeneracy is broken!
- Velocity of GW sources and their hosts can be detected

References

- Girardi+1996: M. Girardi et al., ApJ 457 (1996)
- Ruel+2014: J. Ruel et al.. ApJ 792 (2014)
- Scrimgeour+2016: M. I. Scrimgeour et al., MNRAS 455 (2016)
- Barack+2019: L. Barack & A. Pound, Rep. Prog. Phys. 82 (2019)
- Torres-Orjuela+2021a: A. Torres-Orjuela et al., arXiv:2010.15856 (2021) [accepted PRD]
- Torres-Orjuela+2021b: A. Torres-Orjuela et al., Phys. Rev. Lett. 127 (2021)

Thanks for your attention！

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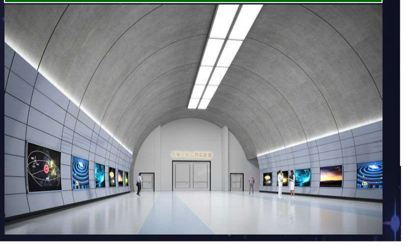

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□ Theoretical analysis and simulation

□ Space-based laser interferometry and laser ranging technology

□ Spacecraft and system technology

□ Contact: TianQin@sysu.edu.cn

Session C1a 10:00–12:00

[Chair: Atsushi Nishizawa]

Lu Yin

Sogang University

“Gravitational waves from the vacuum decay with LISA”

(15 min.)

[JGRG30 (2021) 120808]

Gravitational waves from the vacuum decay with LISA

Lu Yin
CQUeST
Sogang University

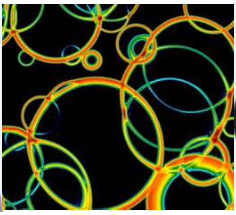
30th Workshop on General Relativity and Gravitation in Japan(JGRG30)
06-10 Dec, 2021

Based on: arXiv:2106.07430
Bum-Hoon Lee, Wonwoo Lee, Dong-han Yeom

Gravitational waves from the vacuum decay with LISA

Lu Yin

Scalar field

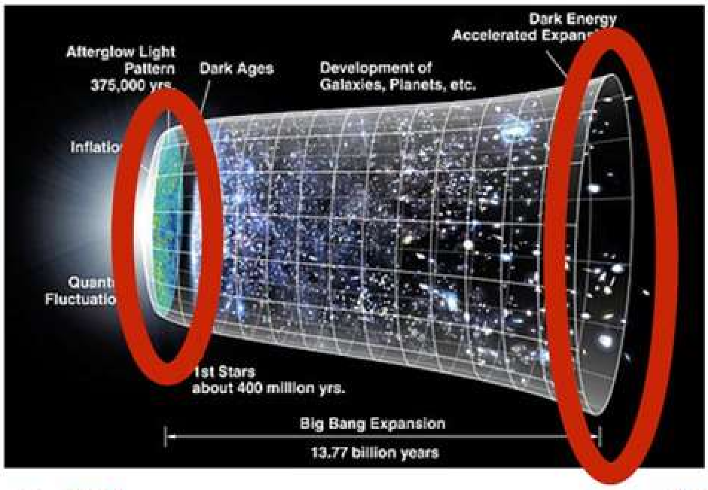


scalar with gravity field

30th Workshop on General Relativity and Gravitation in Japan(JGRG30)
06-10 Dec, 2021

Based on: arXiv:2106.07430
Bum-Hoon Lee, Wonwoo Lee, Dong-han Yeom

How can GW help to probe cosmology?



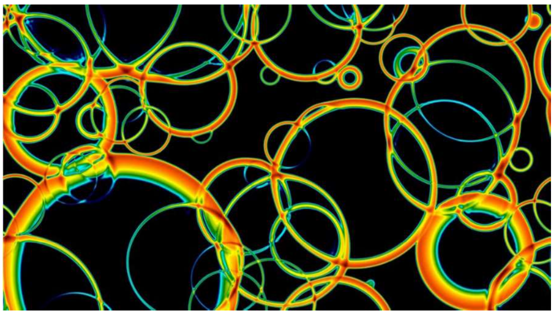
the stochastic GW background from primordial sources: test of early universe and high energy phenomena

use of GW emission from binaries to probe late-time dynamics and content of the universe

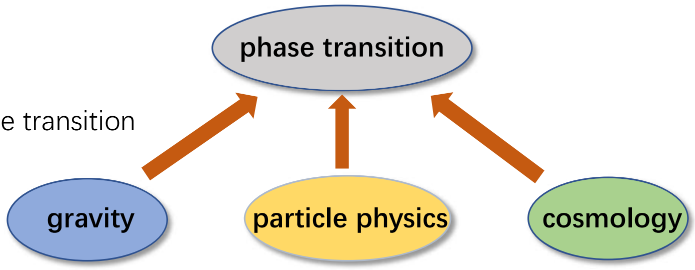
Outline

- **The bubble collision in first-order phase transition**
- Comparing the Gravitational Waves from bubble collision with LISA sensitivity
- Summary

Phase transition in the early Universe



Phase transition



phase transition

gravity

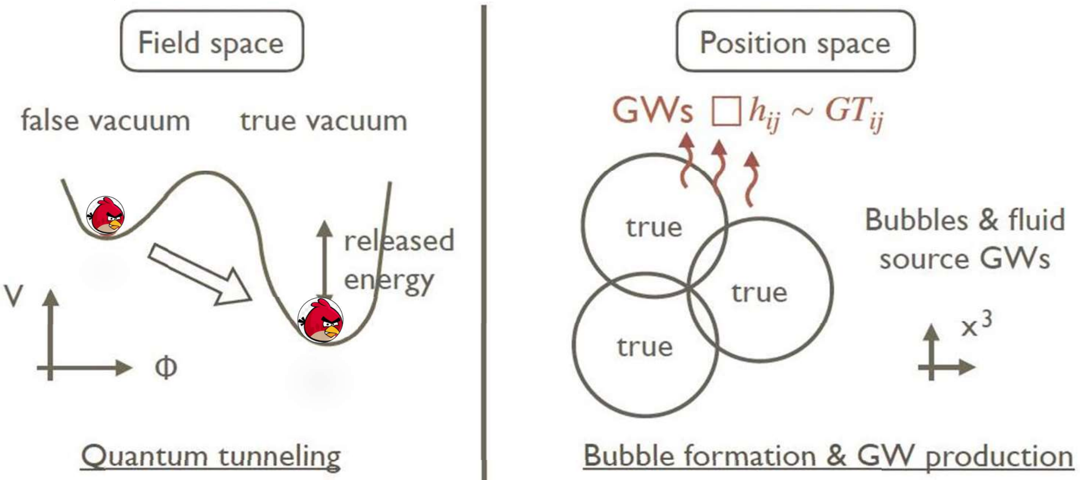
particle physics

cosmology

Electroweak and QCD phase transition

- **The cosmic phase transitions are of central interest in modern cosmology.**

Gravitational wave form bubble collision



Field space

false vacuum

true vacuum

released energy

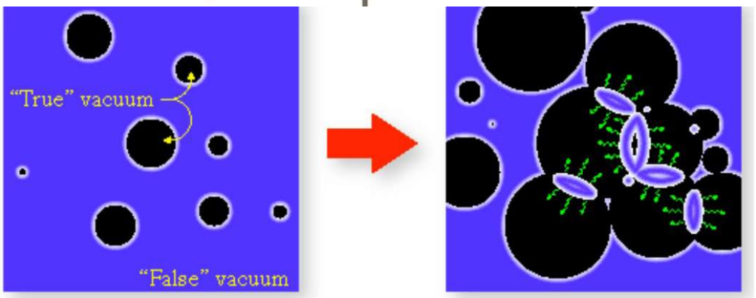
Quantum tunneling

Position space

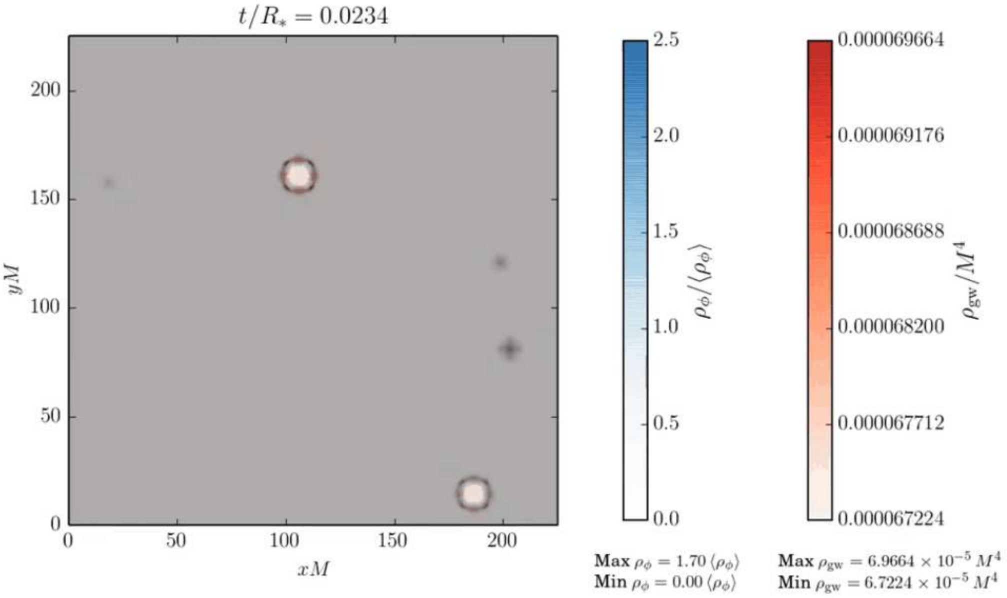
GWs □ $h_{ij} \sim GT_{ij}$

Bubbles & fluid source GWs

Bubble formation & GW production



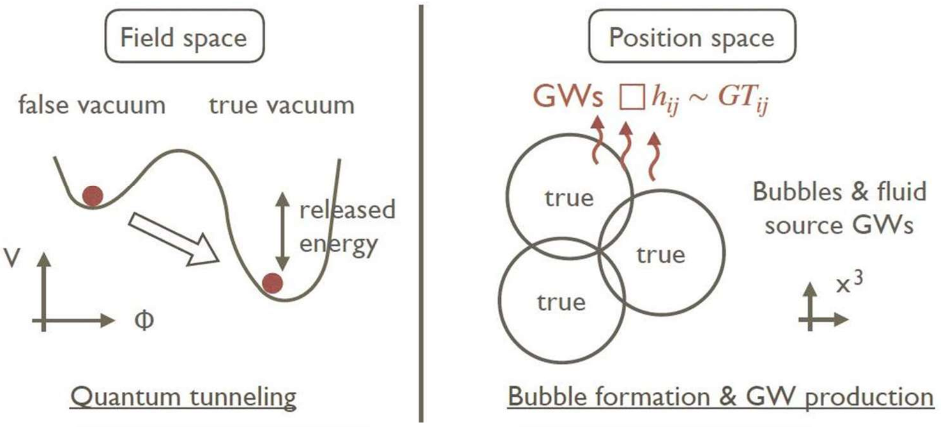
Numerical simulation for bubble collision

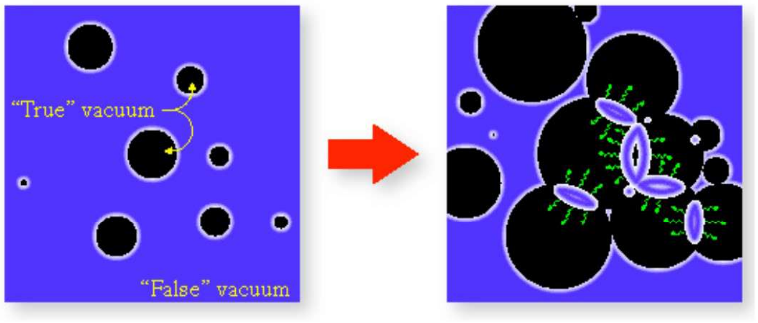


Daniel Cutting et al.
https://www.youtube.com/watch?v=uTz9lsvSr5A

the scalar field that is changing phase shown in blue,
and the energy density of gravitational waves is shown in red.

Gravitational wave form bubble collision





The potential in tunneling

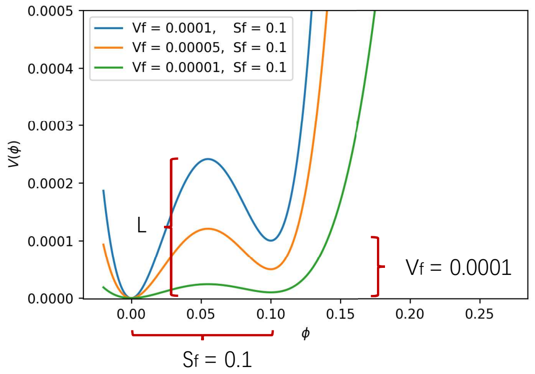
$$V(\phi) = \frac{6V_{\mathrm{f}}}{5\mathrm{S}_{\mathrm{f}}} \int_0^{\phi} \frac{\bar{\phi}}{\mathrm{S}_{\mathrm{f}}} \left(\frac{\bar{\phi}}{\mathrm{S}_{\mathrm{f}}} - 0.55 \right) \left(\frac{\bar{\phi}}{\mathrm{S}_{\mathrm{f}}} - 1 \right) d\bar{\phi}$$

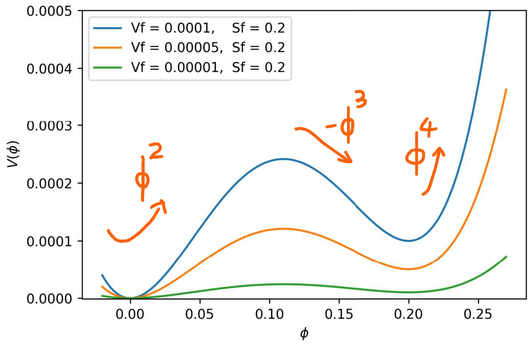
Notice:

Vf correspond to the released energy

If L ≫ Vf, the potential can use **thin wall limit**.

The small Vf correspond to the production of **large bubble**





Comparing the GW produced by different Euclidean action

Scalar field

$$\mathcal{S}_E = \int \mathrm{d}^4x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + V(\phi) \right]$$

$$\phi'' + \frac{3}{\rho}\phi' = \frac{dV(\phi)}{d\phi}$$

Scalar field with gravity

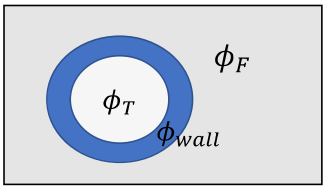
$$\mathcal{S}_E = \int \mathrm{d}^4x\sqrt{g} \left[-\frac{1}{16\pi}\boxed{R} + \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + V(\phi) \right]$$

$$\phi'' + \frac{3\boxed{\rho'}}{\rho}\phi' = \frac{dV(\phi)}{d\phi}$$

Vacuum bubble nucleation rate: $\Gamma(t) = A(t)e^{-B(t)}$

Coefficient in the vacuum decay amplitude: $B = S_E(\phi) - S_E(\phi_+)$

$$= B_{out} + B_{in} + B_{wall}$$



$$B_s = \frac{2\pi^2 S_1^4}{2\epsilon^3}$$

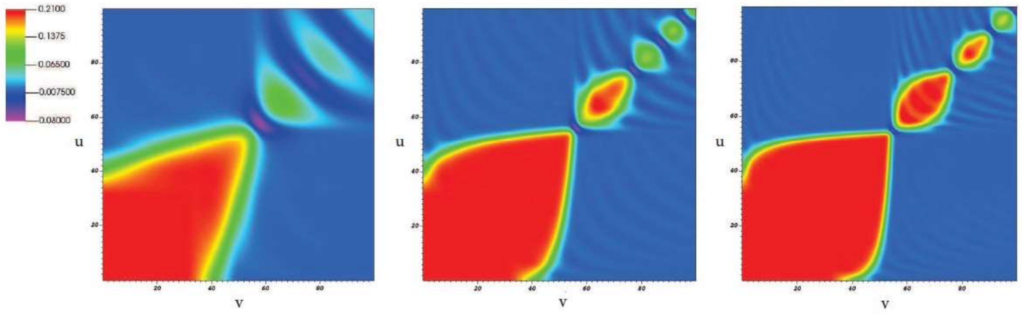
$$B_s \approx \boxed{4} B_{s+g}$$

$$B_{s+g} = \frac{B_0}{\left[1 + (\frac{\rho_0}{2\Lambda})^2\right]^2}$$

$B_0 = \frac{2\pi^2 S_1^2}{2\epsilon^3}$, $\rho_0 = 3S_1/\epsilon$ and $\Lambda^2 = \frac{3}{8\pi G\epsilon}$

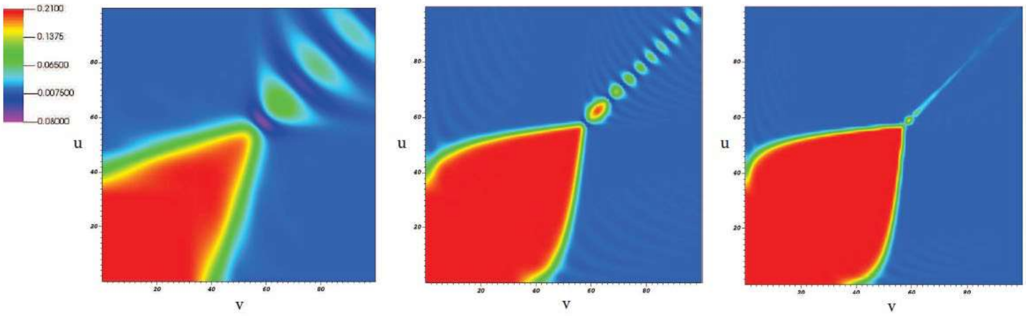
Numerical calculation for the potential in bubble collision

Scalar field with $S_f = 0.2$, $r_0 = 1$



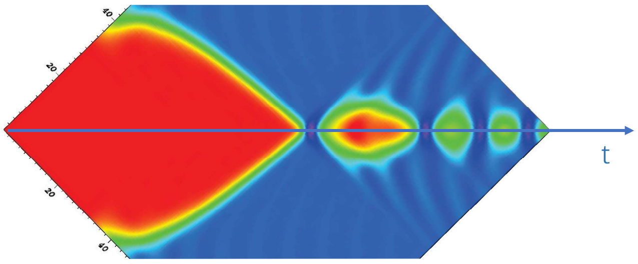
$V_f = 0.00001$ $V_f = 0.00005$ $V_f = 0.0001$

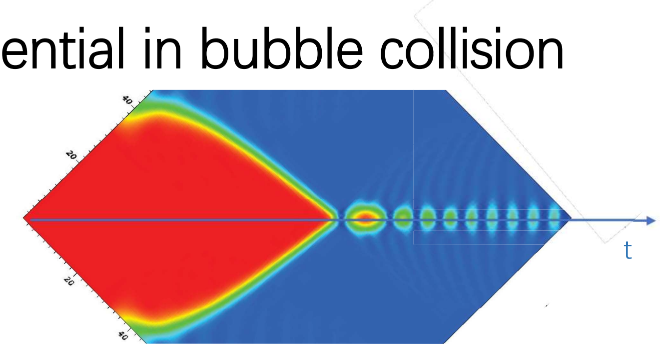
Scalar with gravity field with $S_f = 0.2$, $r_0 = 1$



$V_f = 0.00001$ $V_f = 0.00005$ $V_f = 0.0001$

Numerical calculation for the potential in bubble collision

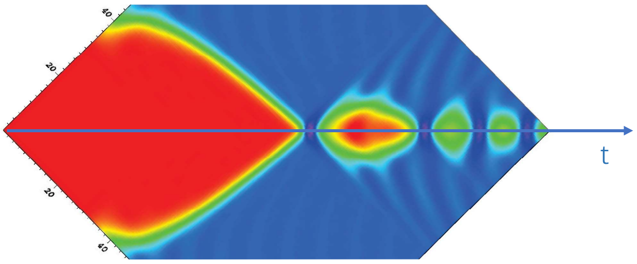




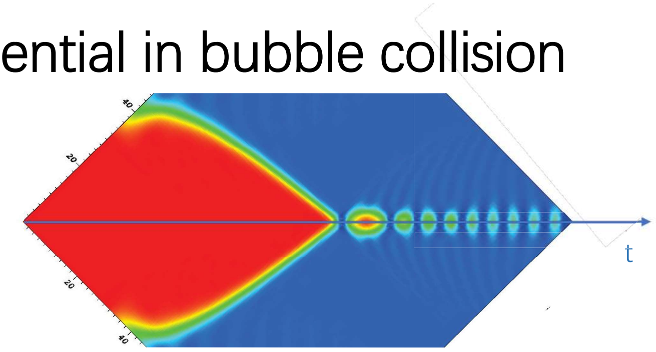
$$\mathcal{S}_E = \int \mathrm{d}^4x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + V(\phi)\right]$$

$$\mathcal{S}_E = \int \mathrm{d}^4x\sqrt{g}\left[-\frac{1}{16\pi}R + \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + V(\phi)\right]$$

Numerical calculation for the potential in bubble collision

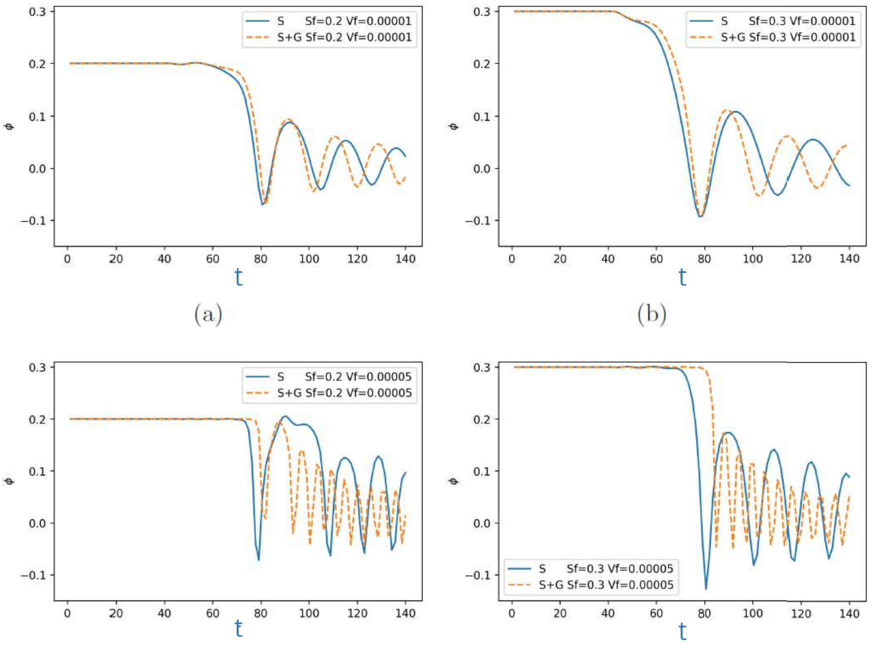


$$\mathcal{S}_E = \int \mathrm{d}^4x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + V(\phi)\right]$$



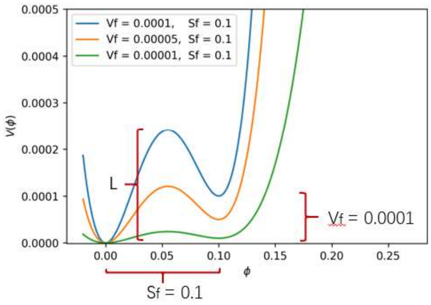
$$\mathcal{S}_E = \int \mathrm{d}^4x\sqrt{g}\left[-\frac{1}{16\pi}R + \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + V(\phi)\right]$$

The oscillation of the potential after bubble collision



(a) (b)

(c) (d)



$$V(\phi) = \frac{6V_f}{5S_f}\int_0^{\varphi}\frac{\phi}{S_f}\left(\frac{\phi}{S_f}-0.55\right)\left(\frac{\phi}{S_f}-1\right)d\bar{\phi}$$

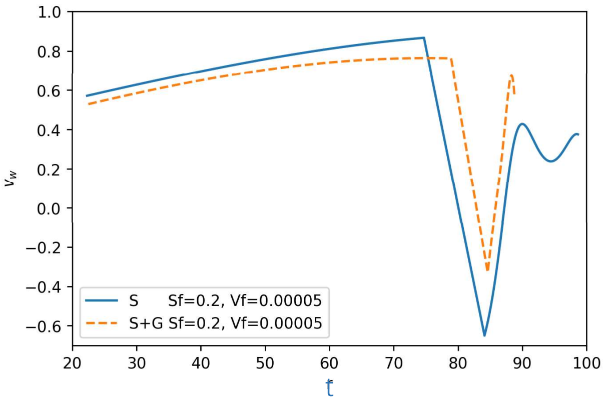
The collision moment with gravity will be late

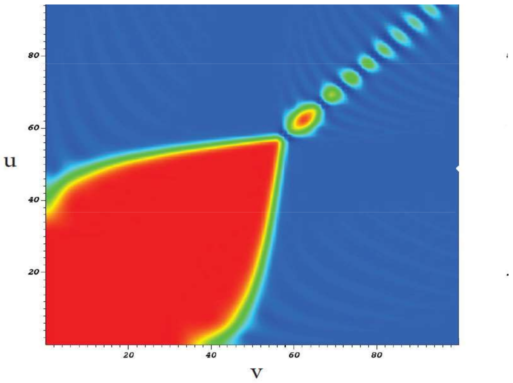
The S+G oscillation more quickly

The velocity of bubble wall

$$ds^2 = -\alpha^2(u,v)dudv + r^2(u,v)dH^2$$

$$v_w = \frac{\Delta \mathrm{v} - \Delta \mathrm{u}}{\Delta \mathrm{v} + \Delta \mathrm{u}}$$

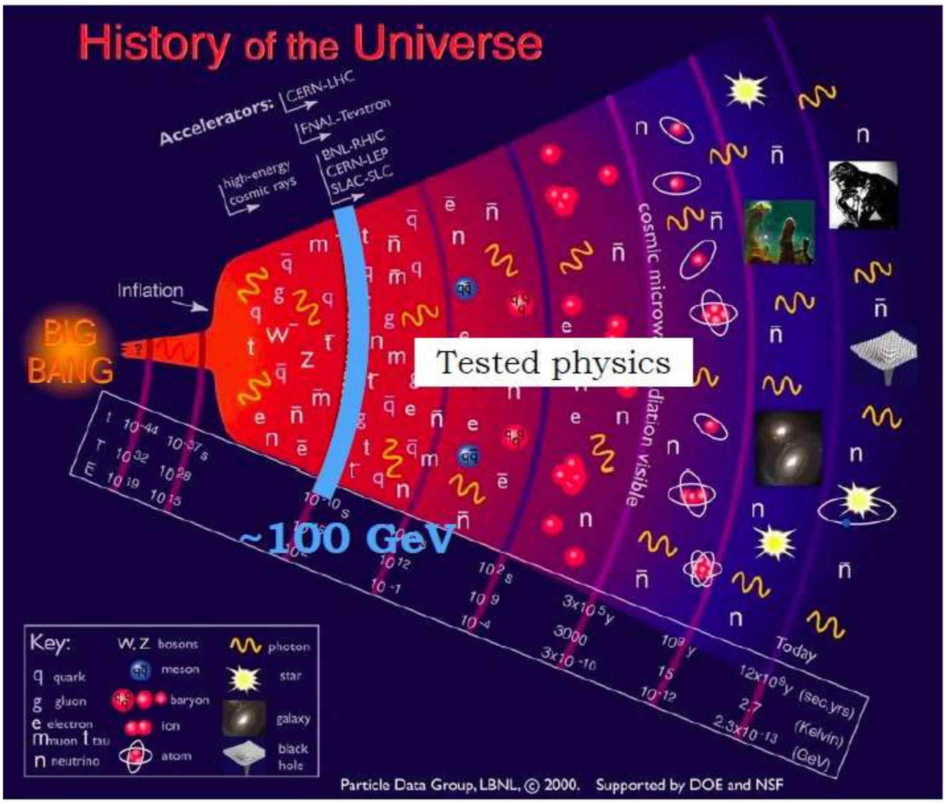




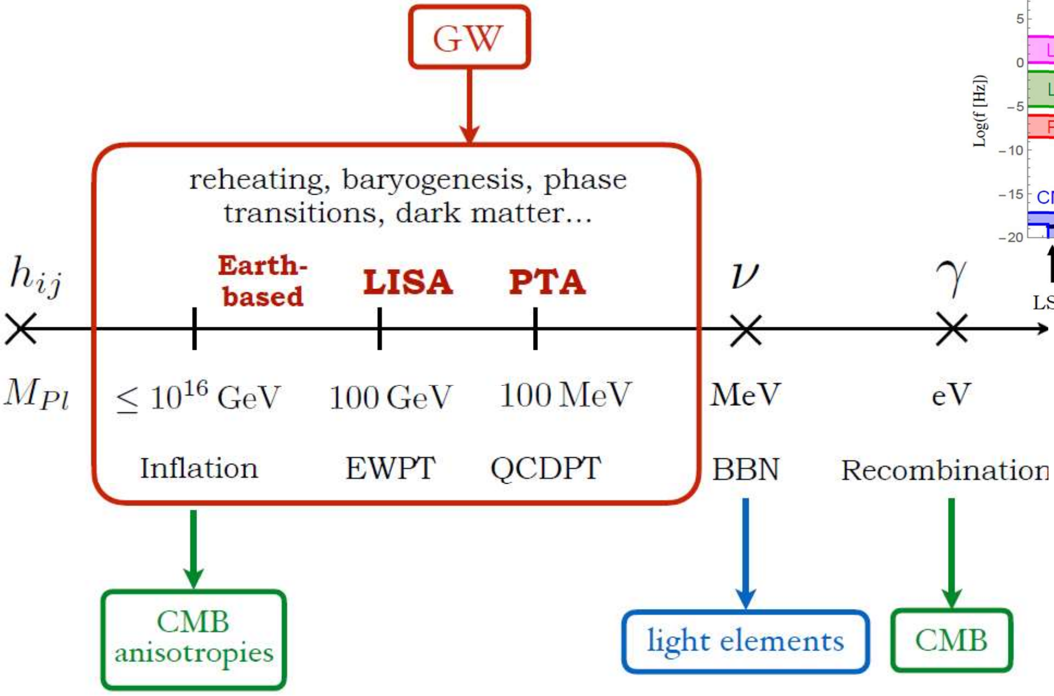
Value of v_w	$V_f = 0.00001$	$V_f = 0.00005$	$V_f = 0.0001$
Scalar field	0.895	0.867	0.620
Scalar with gravity field	0.875	0.761	0.578

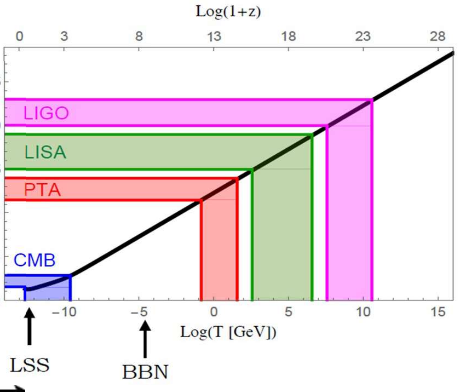
Outline

- The bubble collision in first-order phase transition
- **Comparing the Gravitational Waves from bubble collision with LISA sensitivity**
- Summary



The energy scale and detectable of GWs in the early Universe





The calculation of GW spectra

$$\frac{dE}{d\Omega} = \frac{r^2c^3}{16\pi G}\int_{-\infty}^{+\infty} dt\left(\dot{h}_+^2 + \dot{h}_\times^2\right)$$

$$= \frac{G}{2\pi^2c^7}\Lambda_{ij,kl}\left(\hat{\mathbf{k}}\right)\int_0^{+\infty} d\omega\omega^2\tilde{T}_{ij}\left(\omega, \frac{\omega\hat{\mathbf{k}}}{c}\right)\tilde{T}_{kl}^*\left(\omega, \frac{\omega\hat{\mathbf{k}}}{c}\right)$$

$$\Omega_{\mathrm{GW}*} = \omega\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}\omega}\frac{1}{E_{\mathrm{tot}}} = \kappa_\phi^2\left(\frac{H_*}{\beta}\right)^2\left(\frac{\alpha}{\alpha+1}\right)^2\Delta\left(\omega/\beta, v_{\mathrm{b}}\right)$$

Spectra in bubble collision time

$$\Omega_{\mathrm{GW}}h^2 = \Omega_{\mathrm{GW}*}\left(\frac{R_*}{R_0}\right)^4 H_*^2 = 1.67\times10^{-5}\left(\frac{100}{g_*}\right)^{1/3}\Omega_{\mathrm{GW}*}$$

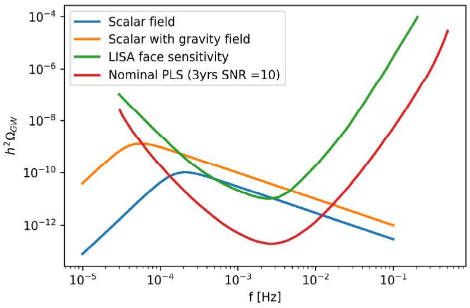
$$\Omega_{\mathrm{GW}}(f)h^2 = 1.67\times10^{-5}\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{\alpha+1}\right)^2\Delta S(f)$$

Spectra in present time

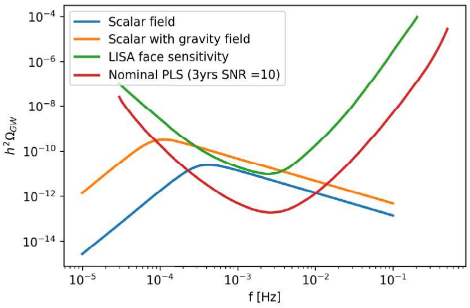
$$\Delta = \frac{0.11v_b^3}{0.42+v_b^2}$$

$$S(f) = \frac{3.8(f/f_0)^{2.8}}{1+2.8(f/f_0)^{3.8}}$$

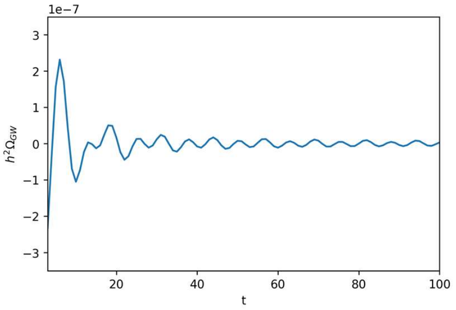
The GW spectrum from bubble collision

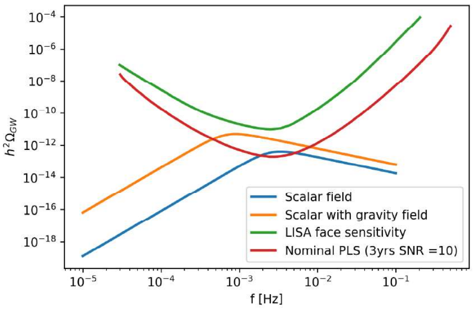


$\beta_s/H_* = 50$
$\beta_{s+g}/H_* = 12.5$ (a)

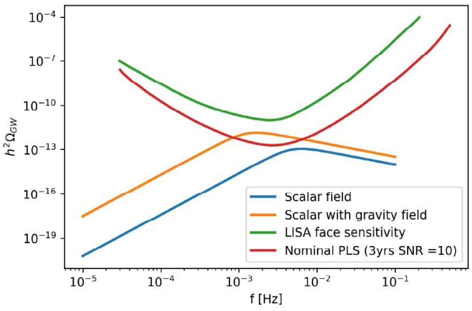


$\beta_s/H_* = 100$
$\beta_{s+g}/H_* = 25$ (b)





$\beta_s/H_* = 800$
$\beta_{s+g}/H_* = 200$ (c)



$\beta_s/H_* = 1500$
$\beta_{s+g}/H_* = 375$ (d)

$T_* \approx 100$ GeV $g_* \approx 106.75$
$\alpha = 1$ $\alpha_\infty = 0.3$

Summary

- For the bubble collision in first-order phase transition, the scalar field with **gravity** will have higher oscillation frequency than only scalar case.
- The velocity of bubble wall will be higher in the small difference of 2 local minima free energy (small Vf).
- The GW sensitivity from Scalar+Gravity field will be higher than only scalar case, and it is easier to be observed by LISA.

Session C1b 10:00–12:00

[Chair: Hayato Motohashi]

Asuka Ito

Tokyo institute of technology

“Effects of Earth’s gravity on electron (muon) g-2 measurements”

(15 min.)

[JGRG30 (2021) 120810]

Effects of Earth's gravity on electron (muon) g-2 measurements

Asuka Ito
from Tokyo institute of technology

(Ref: AI, Class.Quant.Grav. 38 (2021), [arXiv: 2011.11217])

12/8 2021 at JGRG

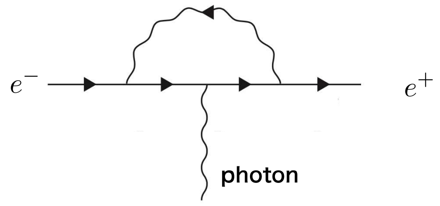
Introduction

~ The magnetic moment of an electron ~

An electron (muon) has a magnetic moment: $\boxed{H = -\mu_B \frac{g}{2} \boldsymbol{S} \cdot \boldsymbol{B}}$

$\Big($ Bour magneton: $\mu_B = \frac{e}{2m}$, g-factor: g , spin: $\boldsymbol{S}$, external magnetic field: $\boldsymbol{B}$ $\Big)$

At the tree level, g-factor is exactly equals to 2. However, It deviates from 2 due to loop corrections



$+ \mathcal{O}(\alpha^2)$ $\quad \Big(\alpha = e^2/4\pi \simeq 1/137 \Big)$

Now the g-factor has been calculated up to $\mathcal{O}(\alpha^4)$: $\boxed{\frac{g-2}{2} = a_e^{\text{SM}} = 1{,}159{,}652{,}181.61(23) \times 10^{-12}}$

(Hadronic and weak contributions have been included)

(G. Gabrielse, et al, PRL 97, 030802 (2006))

Introduction

~ The magnetic moment of an electron ~

The standard model prediction of the electron g-factor is

$$\boxed{\frac{g-2}{2} = a_e^{\mathrm{SM}} = 1{,}159{,}652{,}181.61(23) \times 10^{-12}}$$

(G. Gabrielse, et al, PRL 97, 030802 (2006))

However, it does not coincide with an experimental result:

$$\boxed{a_e^{\mathrm{exp}} = 1{,}159{,}652{,}180.73(28) \times 10^{-12}}$$

(D, Hanneke, et al, PRL 100, 120801 (2008))

$$\left(\Delta a_e = a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}} = -0.88(36) \times 10^{-12} \quad \text{at} \quad 2.5\sigma \right)$$

➡ Implication of a new physics? Something has been overlooked?

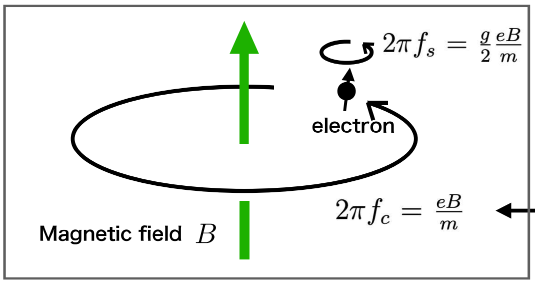
➡ Effects of Earth's gravity?

※ muon g-factor also has a discrepancy at 4.2σ

(T. Morishima, T. Futamase, H.M. Shimizu (2018), etc...)

How to measure the electron g-factor?

An electron experiences the cyclotron motion and the spin precession in the presence of an external magnetic field



← Spin level

← Landau level

Measuring the cyclotron frequency f_c and the spin precession frequency f_s , we can determine the g-factor

$$\boxed{\frac{g}{2} = \frac{f_s}{f_c}}$$

← How gravity affects f_s and f_c ?

Dirac equation in curved spacetime

In order to study gravitational effects on an electron, we consider the Dirac equation in curved spacetime

$$\boxed{i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu + \Gamma_\mu + ieA_\mu\right)\psi = m\psi}$$

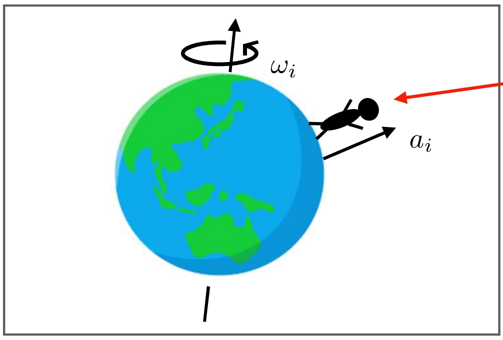
$\gamma^{\hat{\alpha}}$: gamma matrices , $e^{\mu}_{\hat{\alpha}}$: tetrad , A_μ : vector potential ,

$\Gamma_\mu = \frac{1}{2} e^{\hat{\alpha}}_{\nu} \sigma_{\hat{\alpha}\hat{\beta}} \left(\partial_\mu e^{\nu\hat{\beta}} + \Gamma^{\nu}_{\lambda\mu} e^{\lambda\hat{\beta}}\right)$: spin connection ,

$\sigma_{\hat{\alpha}\hat{\beta}} = \frac{1}{4}[\gamma_{\hat{\alpha}}, \gamma_{\hat{\beta}}]$: generator of the Lorentz group

What is the appropriate coordinate (metric) reflecting Earth's gravity...?

Dirac equation in curved spacetime



This observer is not freely falling due to the Earth

There are three kinds of gravitational effects from the Earth

- The linear acceleration a_i
- The rotation ω_i

) Inertial effects due to non-freely falling motion

- The tidal force ← Characterized by the Riemann tensor $R_{\mu\nu\lambda\sigma}$

Proper reference frame

An accelerating (a_i) and rotating (ω_i) observer in weak gravitational field ($R_{\mu\nu\lambda\sigma}$) can be characterized by the proper reference coordinate:
(up to leading order)

$$\begin{cases} g_{00} = -1 - 2a_i x^i - R_{0i0j}x^i x^j, \\ g_{0i} = -\omega_k \epsilon_{0ijk} x^j - \dfrac{2}{3} R_{0jik} x^j x^k, \\ g_{ij} = \delta_{ij} - \dfrac{1}{3} R_{ikjl} x^k x^l, \end{cases}$$

where Riemann tensors are evaluated at $\boldsymbol{x} = 0$

(W.-T. Ni, M. Zimmermann, PRD 17, 1473 (1978))

In the case of the Earth's gravity, we specifies

$$|\boldsymbol{a}| = 9.81\,\mathrm{m/s^2} \;, \qquad |\boldsymbol{\omega}| = 7.27 \times 10^{-5}\,\mathrm{rad/s} \;, \qquad R_{0i0j} = \left(G\frac{M}{r} \right)_{,ij} \;, \qquad \cdots$$

and substitutes it into the Dirac equation $\boxed{i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu + \Gamma_\mu + ieA_\mu \right) \psi = m\psi}$

Dirac equation in curved spacetime

We also take the non-relativistic limit of the Dirac equation $\boxed{i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu + \Gamma_\mu + ieA_\mu \right) \psi = m\psi}$

$$\xrightarrow{\text{rewriting}} \quad \begin{aligned} i\gamma^0 \partial_0 \psi &= \left[i\gamma^0 \left(\Gamma_0 + ieA_0 \right) - i\gamma^j \left(\partial_j - \Gamma_j - ieA_j \right) + m \right] \psi \\ &= \gamma^0 \underset{\uparrow}{\underline{H}} \psi \;, \end{aligned}$$

Identifying the non-relativistic Hamiltonian

An explicit calculation gives

$$\boxed{\begin{aligned} H = &-\frac{i}{2}\gamma^{\hat{0}}\gamma^{\hat{i}} \left(a_i + R_{0i0j}x^j \right) - \frac{i}{4}\gamma^{\hat{i}}\gamma^{\hat{j}} R_{0ikj}x^k - \frac{i}{8}\gamma^{\hat{0}}\gamma^{\hat{i}}\gamma^{\hat{j}}\gamma^{\hat{k}} R_{jkil}x^l - eA_0 \\ &+ \left[\gamma^{\hat{0}}\gamma^{\hat{i}} \left(\delta^j_i \left(1 + a_i x^i \right) + \theta^j_i \right) - \gamma^{\hat{i}}\gamma^{\hat{j}} \left(\omega_k \epsilon_{0ilk} x^l + \frac{1}{6} R_{ik0l} x^k x^l \right) + \frac{1}{2} R_{0kjl} x^k x^l \right] \left(-i\partial_j - eA_j \right) \\ &+ \left[\gamma^{\hat{0}} \left(1 + a_i x^i + \frac{1}{2} R_{0k0l} x^k x^l \right) - \gamma^{\hat{i}} \left(\omega_k \epsilon_{0ijk} x^j + \frac{1}{6} R_{ik0l} x^k x^l \right) \right] m \;, \end{aligned}}$$

Non-relativistic limit of Dirac equation

The obtained Hamiltonian is a 4×4 matrix for an electron and a positron

$$\boxed{\begin{aligned} H = &-\frac{i}{2}\gamma^{\hat{0}}\gamma^{\hat{i}}\left(a_i + R_{0i0j}x^j\right) - \frac{i}{4}\gamma^{\hat{i}}\gamma^{\hat{j}}R_{0ikj}x^k - \frac{i}{8}\gamma^{\hat{0}}\gamma^{\hat{i}}\gamma^{\hat{j}}\gamma^{\hat{k}}R_{jkil}x^l - eA_0 \\ &+\left[\gamma^{\hat{0}}\gamma^{\hat{i}}\left(\delta_i^j\left(1 + a_i x^i\right) + \theta_i^j\right) - \gamma^{\hat{i}}\gamma^{\hat{j}}\left(\omega_k\epsilon_{0ilk}x^l + \frac{1}{6}R_{ik0l}x^k x^l\right) + \frac{1}{2}R_{0kjl}x^k x^l\right]\left(-i\partial_j - eA_j\right) \\ &+\left[\gamma^{\hat{0}}\left(1 + a_i x^i + \frac{1}{2}R_{0k0l}x^k x^l\right) - \gamma^{\hat{i}}\left(\omega_k\epsilon_{0ijk}x^j + \frac{1}{6}R_{ik0l}x^k x^l\right)\right]m\ , \end{aligned}}$$

- separate an electron and a positron (block diagonalizing) $\psi = (\phi, \Phi)^T$
- take the non-relativistic limit (neglecting higher order of v/c & $1/mx$)

$$\boxed{\begin{aligned} H''' = &\left(1 + a_i x^i + \frac{1}{2}R_{0k0l}x^k x^l\right)m - eA_0 - \omega_k\epsilon_{0ijk}x^i\Pi_j - \omega_i S^i \\ &+\frac{1}{2m}\left[\delta_{ij}\left(1 + a_k x^k + \frac{1}{2}R_{0k0l}x^k x^l\right) + \frac{1}{3}R_{jkil}x^k x^l\right]\Pi_i\Pi_j \\ &-\frac{e}{m}S^i B^j\left[\delta_{ij}\left(1 + a_k x^k + \frac{1}{2}R_{0k0l}x^k x^l + \frac{1}{6}R_{mkml}x^k x^l\right) - \frac{1}{6}R_{ikjl}x^k x^l\right] \\ &+\frac{1}{2}\epsilon_{0ijl}S^l R_{ijk0}x^k + \frac{1}{2m}\epsilon_{0ijk}a^i\Pi^j S^k + \frac{1}{4m}\epsilon_{0ijk}S^k\left(R_{ijlm} + 2\delta_{jm}R_{0i0l}\right)x^l\Pi_m \end{aligned}}$$

Effects of Earth's gravity on g-factor

We have the Hamiltonian for a non-relativistic electron

$$\boxed{\begin{aligned} H''' = &\left(1 + a_i x^i + \frac{1}{2}R_{0k0l}x^k x^l\right)m - eA_0 - \omega_k\epsilon_{0ijk}x^i\Pi_j \underline{- \omega_i S^i} \\ &\underline{+\frac{1}{2m}\left[\delta_{ij}\left(1 + a_k x^k + \frac{1}{2}R_{0k0l}x^k x^l\right) + \frac{1}{3}R_{jkil}x^k x^l\right]\Pi_i\Pi_j} \\ &\underline{-\frac{e}{m}S^i B^j\left[\delta_{ij}\left(1 + a_k x^k + \frac{1}{2}R_{0k0l}x^k x^l + \frac{1}{6}R_{mkml}x^k x^l\right) - \frac{1}{6}R_{ikjl}x^k x^l\right]} \\ &+\frac{1}{2}\epsilon_{0ijl}S^l R_{ijk0}x^k + \underline{\frac{1}{2m}\epsilon_{0ijk}a^i\Pi^j S^k} + \underline{\frac{1}{4m}\epsilon_{0ijk}S^k\left(R_{ijlm} + 2\delta_{jm}R_{0i0l}\right)x^l\Pi_m} \end{aligned}}$$

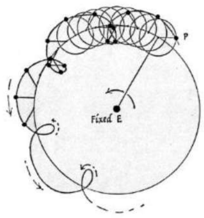
Kinetic terms

Spin interactions

$\left(\Pi_j = -i\partial_j - eA_j\right)$

Spin-orbit couplings

As a result, the Earth's gravity causes



- ● the cyclotron motion becomes an epicyclic orbit → $f_c + \delta f_c$
- ● modification to spin precession → $f_s + \delta f_s$
- ● appearance of fine structure due to the spin-obit couplings → $f_c + \delta f_c$ & $f_s + \delta f_s$

Effects of Earth's gravity on g-factor

Total leading order correction is

$$\left(\begin{aligned} f_c + \delta f_c &= f_c \left(1 + \cancel{a_i x^i_{cy}} \pm \frac{2\omega}{2\pi f_c} \cos\theta - \frac{1}{4\sqrt{2}} \frac{a}{\sqrt{(2\pi f_c) m}} + \frac{GM/x_0^3}{2(2\pi f_c)^2} \right) , \\ f_s + \delta f_s &= f_s \left(1 + \cancel{a_i x^i_{cy}} \pm \frac{\omega}{2\pi f_s} \cos\theta + \frac{1}{4\sqrt{2}} \frac{a}{\sqrt{(2\pi f_c) m}} \right) . \end{aligned} \right.$$

Therefore, gravitational correction to g-factor is

$$\boxed{\frac{\delta g}{2} = \frac{f_s + \delta f_s}{f_c + \delta f_c} - \frac{f_s}{f_c} \simeq \mp \frac{\omega}{2\pi f_c} \cos\theta + \frac{1}{2\sqrt{2}} \frac{a}{\sqrt{(2\pi f_c) m}} - \frac{GM/x_0^3}{(2\pi f_c)^2}} \qquad \left(2\pi f_c = \frac{eB}{m} \right)$$

※ Each term has different dependence on f_c & m

Effects of Earth's gravity on g-factor

The most accurate electron g-factor measurement was operated at Harvard Univ. in 2008.

(D, Hanneke, et al, PRL 100, 120801 (2008))

Using the experimental values $f_c = f_s = eB/m \simeq 150\,\mathrm{GHz}$, $\theta \simeq 0.674\,\mathrm{rad}$,

one can estimate the each gravitational correction:

Effects of Earth's rotation	$\frac{\omega}{2\pi f_c} \cos\theta$	5.2×10^{-17}
Spin-orbit coupling through a_i	$\frac{1}{2\sqrt{2}} \frac{a}{\sqrt{(2\pi f_c) m}}$	4.3×10^{-25}
Tidal effect	$-\frac{GM/x_0^3}{(2\pi f_c)^2}$	-1.7×10^{-30}
Sensitivity in the experiment (G. Gabrielse, et al, PRL 97, 030802 (2006))		2.8×10^{-13}

Summary

- The electron g-factor has been measured intensively to confirm the standard model prediction
- There is a discrepancy between an experimental result and the theoretical prediction at 2.5σ

- We explored the possibility that the discrepancy could be explained by effects of Earth's gravity

→ The leading correction comes from the rotation of the Earth, that is $\delta g/2 \simeq 5.2 \times 10^{-17}$

- Although the gravitational effect is smaller than the current sensitivity $\delta g/2 = 2.8 \times 10^{-13}$, it might be detectable in the future

→ Improving by 3 orders of magnitude is possible…?

(G. Gabrielse, et al, arXiv:1904.06174 (2019), X. Fan, et al, arXiv:2011.08136 (2020))

(※ muon sensitivity: $\delta g/2 = 4.6 \times 10^{-7}$ modification from the rotation of the Earth: $\delta g/2 \sim 10^{-12}$

(B. Abi, et al, arXiv:2104.03281 (2021)))

Session C1b 10:00–12:00

[Chair: Hayato Motohashi]

Atsushi Naruko

CGP, YITP, Kyoto U

“Axion Cloud Decay due to the Axion-photon Conversion with Background Magnetic Fields”

(15 min.)

[JGRG30 (2021) 120812]

Axion Cloud Decay

due to the Axion-photon Conversion with Background Magnetic Fields

Atsushi Naruko

[**C**enter for **G**ravitational **P**hysics, YITP]

in collaboration with : **Chul-Moon Yoo,** Yusuke Sakurai,
Keitaro Takahashi, Yohsuke Takamori, Daisuke Yamauchi

accepted by PASJ (arXiv:2103.13227)

Axion

QCD axion, string axion, ... etc

- See Murata [B2a5], Omiya&Takahashi [D2b3&4]
 and Obata [D3a3] san's talks

$L^{QCD} \supset \boldsymbol{\theta}\, F^a_{\mu\nu}\, {}^*F^{a\,\mu\nu}$

- Solve the strong CP problem in QCD :
  θ must be extremely small though it is not zero..
  $\theta < 10^{-10} \leftrightarrow$ dynamics of θ = axion

- Axion-like particles (ALPs) from string theory
  with various masses and various couplings..

Axion

in cosmology

- a candidate of dark matter
- generation of (chiral) gravitational waves from inflation
  amplitude of GWs ≠ H_{inf} (energy scale of inflation)
  See Fujita+ [1705.01533] & Murata-san's talk [B2a5]
- indirect detection of ALPs !?
  from the observation of cosmic birefringence
  See Minami+ [2011.11254] & Obata-san's talk [D3a3]

Axion Cloud Decay

due to the Axion-photon Conversion with Background Magnetic Fields

Atsushi Naruko

[Center for Gravitational Physics, YITP]

in collaboration with : **Chul-Moon Yoo,** Yusuke Sakurai,
Keitaro Takahashi, Yohsuke Takamori, Daisuke Yamauchi

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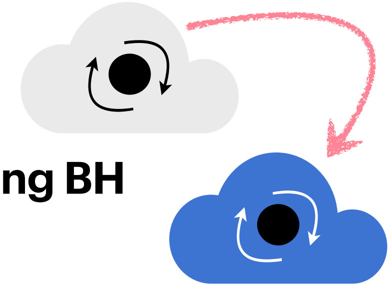
axion cloud



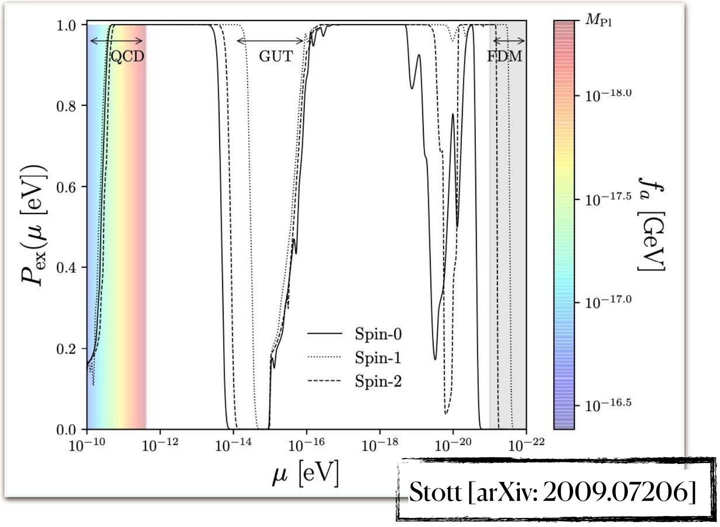
= **quasi bound state of axion around a rotating BH**

- axion cloud may grow by the superradiant instability.

- axion may efficiently extract the angular momentum of a BH

$\Rightarrow$ no rotating black holes in our universe ?

$\Rightarrow$ constraints on axion (its mass & coupling) from the existence of spinning BHs



Stott [arXiv: 2009.07206]

Axion Cloud Decay

due to the Axion-photon Conversion with Background Magnetic Fields

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accepted by PASJ (arXiv:2103.13227)

stability of the axion cloud

magnetic fields in the universe could affect the axion cloud ??

- Since the axion-BH system can be a GW source (cf. bosenova), the stability of the axion cloud draw much attention recently.

→ self-interaction of axion, gravitational back reaction, etc

See Omiya&Takahashi san's talks [D2b3&4]

- Although axion (massive particle) cannot escape from a BH, photon (massless particle) can escape from a BH !!

- Magnetic fields are ubiquitous in the universe... any effect ??

Axion Cloud Decay

due to the Axion-photon Conversion with Background Magnetic Fields

Atsushi Naruko

[Center for Gravitational Physics, YITP]

in collaboration with : **Chul-Moon Yoo,** Yusuke Sakurai,

Keitaro Takahashi, Yohsuke Takamori, Daisuke Yamauchi

accepted by PASJ (arXiv:2103.13227)

set-up

axion cloud around a BH with background magnetic fields

$$\mathcal{L} = -\frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}F_{\mu\nu}^2 - \underline{\frac{\kappa}{4}\phi\, F_{\mu\nu}{}^{*}F^{\mu\nu}}$$

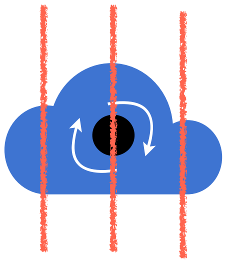
coupling b/w EM & axion ~ φ E・B

- We consider two types of configuration of BG M-fields

A, monopole magnetic field around a Sch. BH

B, uniform magnetic field along z axes

around a Sch. BH (Wald's solution)



laugh sketch of the analysis

EOM : $\boxed{(\Box - \mu^2)\phi = \kappa\, F_{\mu\nu}\tilde{F}^{\mu\nu}}$ & $\boxed{\nabla_\mu F^{\mu\nu} = -\kappa\, \tilde{F}^{\mu\nu}\nabla_\mu\phi}$

1, At BG, **axion cloud** form due to the effect of gravity of a BH

$$\boxed{(\Box^{\mathrm{Sch}} - \mu^2)\underline{\phi^{(0)}} = 0}$$

2, **Axion** could generate EM waves through the coupling

$$\boxed{\nabla_\mu \underline{F^{\mu\nu\,(1)}} = -\kappa\, \tilde{F}^{\mu\nu}\underline{\nabla_\mu\phi^{(0)}}}$$

3, **Generated EM waves** could backreact to **axion cloud**

through the same coupling -> axion cloud could decay

$$\boxed{(\Box - \mu^2)\underline{\phi^{(1)}} = -\kappa\, \tilde{F}^{\mu\nu}\underline{F_{\mu\nu}{}^{(1)}}}$$

results

axion cloud decay around a BH with BG magnetic fields

- Superradiant instability (growth of axion cloud)

$$\omega_{sr} \sim (GM\mu)^8 \mu \sim 10^{-17} s^{-1} \left(\mu/10^{-18}[eV]\right)^9 \left(M/10^6 M_\odot\right)^8$$

- axion decay with a monopole magnetic field

$$(\mathrm{Im}\,\omega/\omega_{\mathrm{s}})_{\mathrm{mono}} \sim (\kappa^2 q^2/a_0^2)\,(GM\mu)^{-5}$$

a_0 : Bohr radius
~ $1/(GM\mu^2)$

- axion decay with a uniform magnetic field

$$(\mathrm{Im}\,\omega/\omega_{\mathrm{s}})_{\mathrm{uni}} \sim (a_0^2 \kappa^2 B_0^2)\,(GM\mu)$$

$$\sim \left(\frac{\kappa}{10^{-12}\mathrm{GeV}^{-1}}\right)^2 \left(\frac{B_0}{10^3 G}\right)^2 \left(\frac{\mu}{10^{-18}\mathrm{eV}}\right)^{-3} \left(\frac{M}{10^6 M_\odot}\right)^{-1}$$

summary & discussion

Axion Cloud Decay with background magnetic fields

- We have considered cloud decay due to the axion-photon conversion with background magnetic fields

- Axion cloud may decay at the time scale same as that for the superradiant instability around a BH for the uniform M-field while the time scale is extremely large for the monopole M-field
  -> need to consider a realistic configuration of M-fields
  stay tuned for the further updates by Sakurai & Yoo !!

- In reality, due to the presence of plasma, photons cannot propagate from a BH -> other decay process ? Alfven wave ??

Thank you
for your attention

Session C1b 10:00–12:00

[Chair: Hayato Motohashi]

Paul Martens

YITP, Kyoto University

“Reheating after relaxation of large cosmological constant”

(15 min.)

[JGRG30 (2021) 120813]

Reheating after relaxation of some large cosmological constant

---

Paul Martens

Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University

General Relativity and Gravitation in Japan, JGRG30

December the 8th, 2021 — work in progress

in collaboration with Shinji Mukohyama & Ryo Namba

Motivations

The cosmological constant problem

- Why does the c.c. value obtained by using quantum principles **differs so much** from the one that drives the current expansion?

> "*the mother of all physics problems*"
>
> L. Süsskind

How to answer to it?

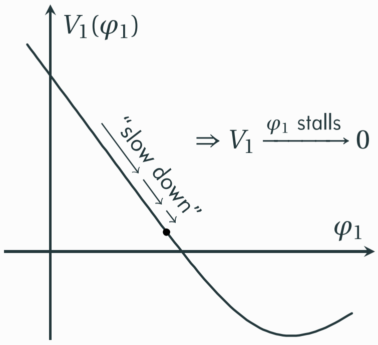
- Anthropic principle?
- Symmetry?
- Dynamic relaxation process?

Cosmological constant relaxation model

$$\overbrace{\underbrace{\frac{M_{\text{Pl}}^2}{2}}_{\mathscr{L}_{\text{E.-H.}}}R+\alpha R^2}^{\text{gravity}}+\underbrace{\frac{X_1}{f(R)}-V_1(\varphi_1)}_{\mathscr{L}_{\text{c.c. relax}}}$$

$$X_1 \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi_1\partial_\nu\varphi_1$$

$$f(R) \approx \left(\frac{R}{M_{\text{Pl}}^2}\right)^{2m} \quad \text{if } R \ll M_{\text{Pl}}^2 \quad \text{and} \quad m > \frac{3}{2}$$



As φ_1 stalls, $V_1(\varphi_1) \to 0$ and $\frac{X_1}{f(R)} \to 0$, but **more quickly**

The Universe is now empty! 😱

Mukohyama and Randall (2004)

Proposed model

Therefore, we include

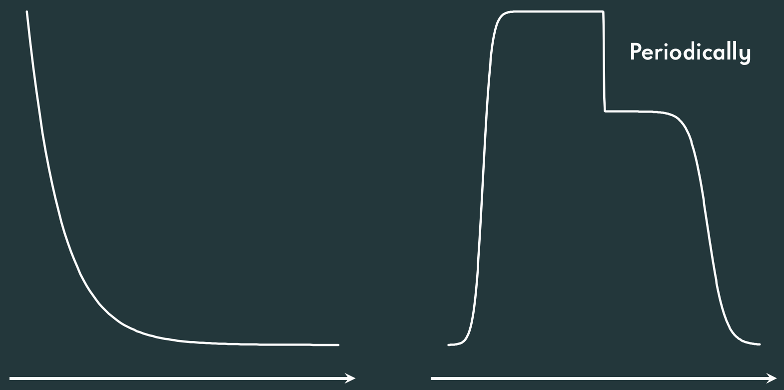
- Gravity
- A cosmological constant relaxation mechanism
- **A reheating mechanism**
  - Null energy condition violating field φ_2
  - Waterfall field φ_3

The new Lagrangian now reads

$$\mathscr{L}_{\text{E.-H.}} + \mathscr{L}_{\text{c.c. relax}}[\varphi_1, g] + \mathscr{L}_{\text{NECV+reh}}[\varphi_2, \varphi_3, g] \tag{1}$$

A staged history

① C.c. relaxation… …② NECV and reheating



Effective c.c. along time H along time

NEC violation and reheating

We choose

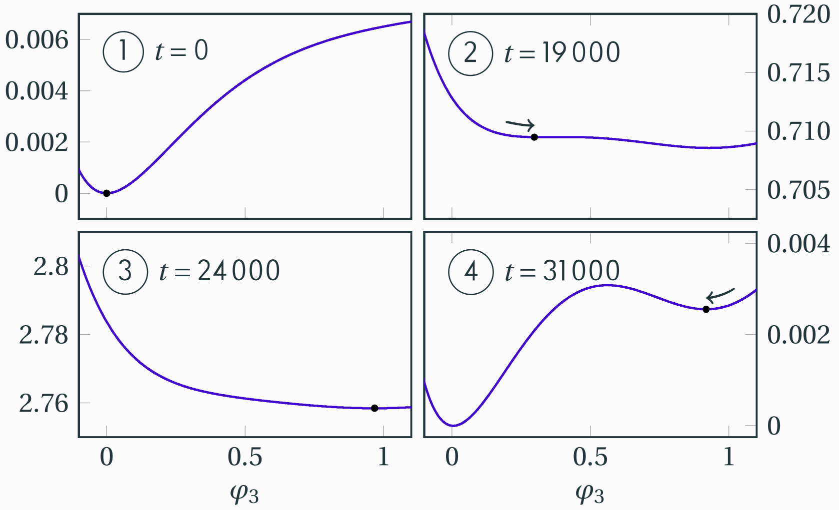
$$\mathscr{L}_{\text{NECV+reh}} = K(\varphi_2, X_2, \varphi_3) - G_3(\varphi_2, X_2, \varphi_3)\Box\varphi_2 + P(\varphi_3, X_3) \qquad (2)$$

- φ_2 invokes the NECV
- φ_3 is a waterfall field, that reheats the Universe
- K & G_3 are assumed periodic

The above is effectively decoupled from the other sectors, except through gravity.

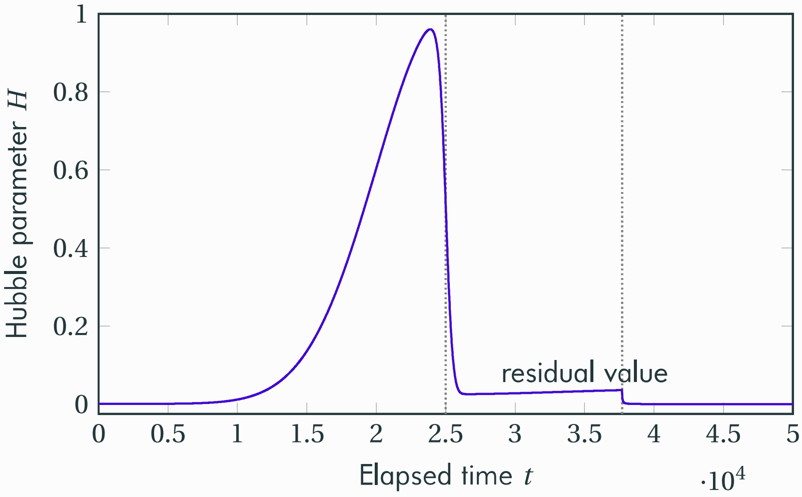
Kobayashi et al. (2011); Rubakov (2014)

The reheating potential

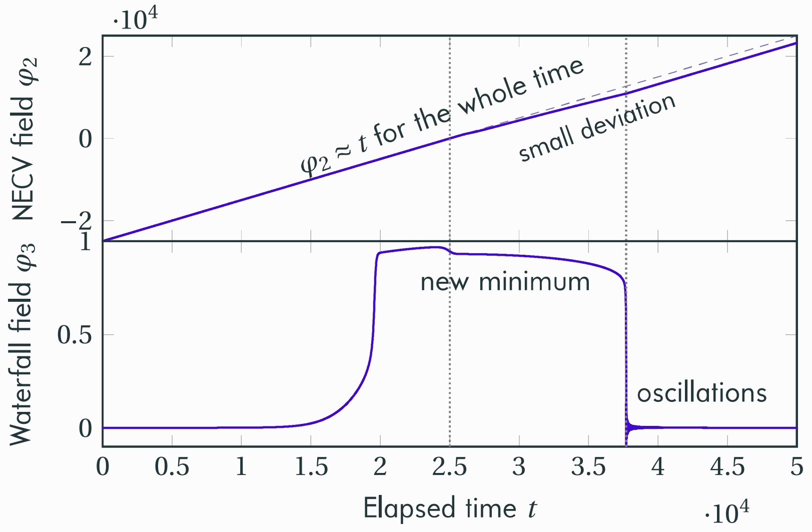


Alberte et al. (2016)

Hubble parameter



Null energy condition violating field φ_2 and reheating field φ_3



What's next

Limitations

- No inflationary phase is (yet?) incorporated
- The behavior is here only qualitatively observed

Outlook

- Should be linked to some more fundamental theory
- Variant models should also be possible

What has been achieved

Exhibited proof-of-concept model made out of 3 parts

- Gravity (general relativity)
- Cosmological constant relaxation via φ_1
- Reheating via
  - Null-energy condition violating field φ_2
  - Waterfall reheating field φ_3

The model is stable with no gradient or ghost instabilities

What has been achieved

Exhibited proof-of-concept model made out of 3 parts

- Gravity (general relativity)
- Cosmological constant relaxation via φ_1
- Reheating via
  - Null-energy condition violating field φ_2
  - Waterfall reheating field φ_3

The model is stable with no gradient or ghost instabilities

Thank you!

Check out the upcoming paper! 😃

More slides...

References

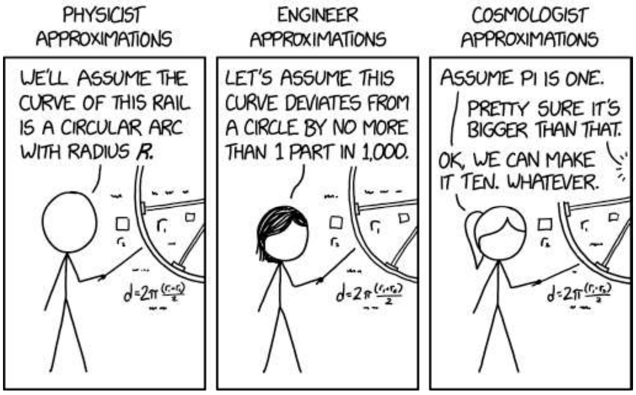
L. Alberte, P. Creminelli, A. Khmelnitsky, D. Pirtskhalava, and E. Trincherini. Relaxing the cosmological constant: A proof of concept. (arXiv:1608.05715), 2016.

T. Kobayashi, M. Yamaguchi, and J. Yokoyama. Generalized G-inflation: Inflation with the most general second-order field equations. (arXiv:1105.5723), 2011.

S. Mukohyama and L. Randall. A Dynamical approach to the cosmological constant. (arXiv:hep-th/0306108), 2004.

V. A. Rubakov. The null energy condition and its violation. (arXiv:1401.4024), 2014.

Types of approximation (xkcd 2205)



Session C1b 10:00–12:00

[Chair: Hayato Motohashi]

Benliang Li

southwest jiaotong university

“The Underlying Mechanisms of Time Dilation in Curved Space-Time”

(15 min.)

[JGRG30 (2021) 120814]

The underlying mechanisms of time dilation measured by different clocks

Benliang Li（李本良）

Southwest Jiaotong University（西南交通大学）

Traditional views on time and clock

- Time is real and unique in one reference frame
- If time in one reference frame slows, then every physical phenomena in that reference frame slows, and vice versa
- How time transforms depends on the theory of Relativity, irrespective to what types of clocks are used to measure it
- Clock measures the proper time in its reference frame

Outline

- What is clock? Discussions of the mechanisms of clocks
- Atomic clock
- Pendulum clock
- Two clocks version of twin paradox
- Study of the two clocks based on quantum field theory in curved spacetime
- Atomic clock vs pendulum clock
- What is time?

What is clock?

- Clocks are mechanical or electronic devices that keep the track of time by counting the number of the periodic phenomena occurred.
- The interval between two successive physical phenomena occurred is called the period of the clock, which is the basic unit of the time interval that the clock can display.
- The clock works as a special instrument to record the number of the periodic phenomena occurred, then the time displayed is equal to this number multiplied by the period.
- For the time dilation experiment conducted by two identical clocks, the period of the two clocks are set with the same value, then the time difference measured by two clocks is just the difference of the two numbers of the phenomena occurred counted by two clocks respectively.

Atomic clock

Example 1: Atomic clocks count the number of electronic oscillations which is in resonance with frequency of the transition of the atom.

We have two identical atomic clocks A and B, when N electronic oscillations are counted, the two clocks will display that 1 second has passed.

Therefore, the period for both of the two atomic clocks is set as $T = \frac{1}{N}$ (s), this period is fixed by the clock itself, and the value is artificially fixed when the clock is built.

A is on a planet with stronger gravity and B is on a planet with weaker gravity. After some time, we compare the time measured by two clocks. Assuming that the time displayed by clock A and clock B are 60 minutes and 61 minutes, respectively (since stronger gravity slows down the time measure by atomic clock).

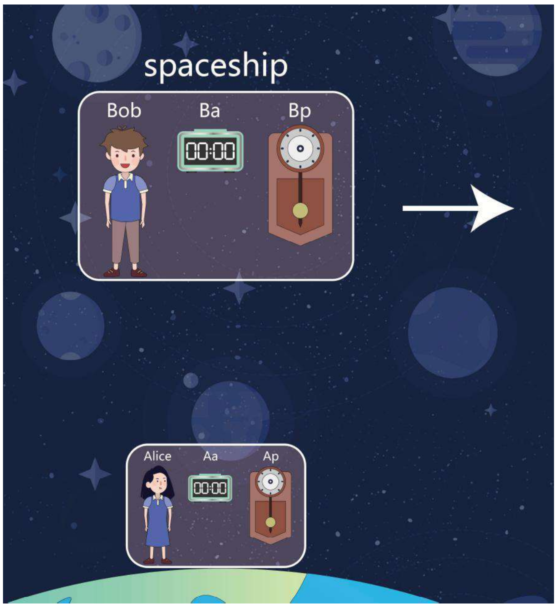
This just indicates that the number of electronic oscillations counted by clock A and clock B are 3600N and 3660N, respectively. And the ratio between the time measured by two clocks is just the ratio between the two numbers counted by two clocks, i.e., the time dilation is just

$$\frac{3600\text{N} * T}{3660\text{N} * T} = \frac{3600}{3660} = \frac{60}{61} = \frac{t_A}{t_B}$$

Pendulum clock

- Example 2: we have two pendulum clocks A and B, A is on earth and B is on the moon.
- The gravitation field strength on earth is stronger than it on the moon, as a result, the pendulum clock A swings faster than pendulum clock B, so the number of swings recorded by clock A is bigger than the number recorded by clock B.
- For time dilation experiment, the time difference measured by two clocks is just the difference of the two numbers of the phenomena occurred counted by two clocks respectively.
- This is in contradiction with the general relativity, which predicts time on earth is slower than time on the moon.

Two clocks version of twin paradox



Based on two atomic clocks, Alice is younger; however, based on two pendulum clocks, Bob is younger.

Moreover, the aging process may be a mixture of a vast range of microscopic and macroscopic physical phenomena involving of both the atoms' vertical movement influenced by gravity, and the atomic transitions or radiations.

The physical process of vertical movement become faster with stronger gravitational field while the physical process of atomic radiation become slower (i.e., the period of photon emitted is larger).

We do not know which physical process is dominating in the aging process, so we cannot make a judgement just based on the two clocks

---

Quantum field theory in curved space-time

Suppose there are two identical hydrogen atoms A and B located at two different locations XA and XB respectively in curved space-time, XA and XB are two locations far away with each other. In the vicinity of location XA , the Lagrangian density of quantum electrodynamics in curved space-time can be given as

$$L_A = \bar{\psi}_A(t,\vec{x})[\gamma_A^{\mu}\nabla_{\mu} + m_0]\psi_A(t,\vec{x}) + e_0 j_A^{\mu}(t,\vec{x})A_{\mu}^{A}(t,\vec{x}) - \frac{1}{4} F_{\mu\nu}^{A}(t,\vec{x})F_A^{\mu\nu}(t,\vec{x})$$

All the above quantities depend on the local metric value $\mathrm{g}_{\mathrm{A}}^{\mu\nu}$

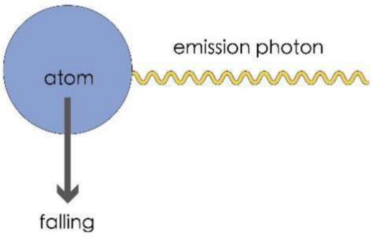
In the vicinity of location XB, the Lagrangian density of quantum electrodynamics in curved space-time can be written as

$$L_B = \bar{\psi}_B(t,\vec{x})[\gamma_B^{\mu}\nabla_{\mu} + m_0]\psi_B(t,\vec{x}) + e_0 j_B^{\mu}(t,\vec{x})A_B^{\mu}(t,\vec{x}) - \frac{1}{4} F_B^{\mu\nu}(t,\vec{x})F_{\mu\nu}^{B}(t,\vec{x})$$

All these quantities depend on the local metric value $\mathrm{g}_{\mathrm{B}}^{\mu\nu}$

[1] https://arxiv.org/abs/1607.03501

Atomic clock vs pendulum clock



An atom free falls in a gravitational field while emitting a photon. Free-falling and radiation are two different physical phenomena influenced by two different factors, one factor is the changing rate of the metric and the other one is the value of the metric [2].

$g_B^{\mu\nu}$ affect the pulse frequency counted by atomic clock

VEED.IO

B

Gravitational acceleration g_B (related with changing rate of metric) affect its swing rate

metric $g_A^{\mu\nu}$ affect the pulse frequency counted by atomic clock

VEED.IO

A

Gravitational acceleration g_A (related with changing rate of metric) affect its swing rate

[2] https://arxiv.org/abs/1802.10406

The relationship between clock and time

- In fact, some clocks (such as pendulum clocks) do not measure proper time as we discussed earlier.
- For any clocks such as atomic clocks, optical clocks, pendulum clocks, pulsar clocks or other timekeeping tools, they are just an instrument used to record the number of periodic phenomena. It has its own working principle and the slowing of time is not the cause of the slowing of the clock.
- For example, a pendulum clock becomes faster in a stronger gravitational field only because the stronger gravity makes the pendulum swing faster (the period of each swing becomes smaller), not because the time becomes faster.
- Likewise, the atomic clock becomes slower in stronger gravitational field is because the energy levels between ground state and the first excited state of the atoms become smaller [3], so the period of the clock becomes larger. Not because time becomes slower either.
- Time itself is not a force or an interaction that can affect the operation of devices or other physical phenomena, so it cannot make any physical process faster or slower by itself.

[3]: https://www.youtube.com/watch?v=xFaYiBlnLEk

What is time?

- Physicists never directly measured the space-time itself; instead, only the behavior of particles and the interactions among physical objects could be measured.
- Einstein field equations are not a complete theory without the statement that the physical objects' motion follow geodesics (and massless particles follow null geodesics).
- It is well-known that general relativity is a theory to describe the movement of objects in the presence of other objects, and Einstein field equations can be treated as an intermediate step that involves the structure of space-time which can be regarded as a mathematical tool.
- Even if for the gravitational wave detection experiments, physicists can only measure the changing of the trajectory of photons due to the influence of distant black holes, and gravitational waves just provides a mathematical model to explain this phenomenon.
- Time, itself, is a mathematical parameter to describe physical phenomena, and it only lives in mathematics and can never be detected. It does not make its own existence independently of the physical objects. In other words, it becomes meaningless when it is divorced from the physical phenomena.

Summary

- Some physical processes (such as atomic radiation) are influenced by the values of the 16 elements of the metric tensor while other physical processes (such as pendulum clock) are influenced by the changing rate of the 16 elements of the metric tensor
- Time is slower in stronger gravitational field predicted by general relativity does not indicate all physical process is slower
- Time is a parameter to describe the physical phenomena, whether it becomes faster or slower only depends on whether the physical phenomena becomes faster or slower correspondingly.

Thanks for your attention!

Email: libenliang732@swjtu.edu.cn

Session C1b 10:00–12:00

[Chair: Hayato Motohashi]

Jia-Hui Huang

South China Normal University

“Analytical study on superradiant stability of higher dimensional RN black holes”

(15 min.)

[JGRG30 (2021) 120815]

Analytical study on superradiant stability of higher dimensional RN black holes

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JGRG30, Waseda University, Tokyo (Dec. 8, 2021)

Based on arxiv: 2103.04227, 2109.04035

Introduction

- Superradiance for black hole and scalar perturbation systems [Brito-Cardoso-Pani]

$$0 < \omega < m_a \Omega_H + e\phi_H$$

- Black hole bomb mechanism [Press-Teukolsky]

- Necessary conditions for a black hole bomb [Hod,…]
  1. superradiant amplification of a bounded mode
  2. trapping potential well outside black hole horizon

- Superradiant stability of 4D RN black holes [Hod, Huang-Mai,…]

Introduction

- Superradiant stability of various higher dimensional black holes[Konoplya, Zhidenko,Ishibashi,Kodama,Kimura,Murata,...]

  1. RN in D=5,6,..,11 [Konoplya-Zhidenko]

  2. extremal RN in any D [Konoplya-Zhidenko]

- Analytical method based on Descartes' rule of signs

  Descartes' rule of signs tells us an upper bound on the number of positive real roots of a polynomial with real coefficients. The bound is equal to the number of sign changes in the sequence of coefficients of the polynomial.

KG equation in D-dimensional RN background

- D-dimensional RN black holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2}^2. \qquad f(r) = 1 - \frac{2m}{r^{D-3}} + \frac{q^2}{r^{2(D-3)}}$$

$$A = -\sqrt{\frac{D-2}{2(D-3)}}\frac{q}{r^{D-3}}dt$$

- KG equation for charged massive scalar perturbation

$$(D_\nu D^\nu - \mu^2)\phi = 0, \quad \phi(t,r,\theta_i) = e^{-i\omega t}R(r)\Theta(\theta_i)$$

radial EOM: $\Delta\frac{d}{dr}\left(\Delta\frac{dR}{dr}\right) + UR = 0 \qquad \Delta = r^{D-2}f(r)$

$$U = (\omega + eA_t)^2 r^{2(D-2)} - l(l+D-3)r^{D-4}\Delta - \mu^2 r^{D-2}\Delta$$

KG equation in D-dimensional RN background

- Define the tortoise coordinate y $dy = \frac{r^{D-2}}{\Delta} dr$

  new radial function $\tilde{R} = r^{\frac{D-2}{2}} R.$

  radial EOM $\frac{d^2 \tilde{R}}{dy^2} + \tilde{U}\tilde{R} = 0$

$$\tilde{U} = \frac{U}{r^{2(D-2)}} - \frac{(D-2)f(r)[(D-4)f(r) + 2rf'(r)]}{4r^2}$$

- Boundary conditions $r \to +\infty (y \to +\infty),\ \tilde{R} \sim e^{-\sqrt{\mu^2-\omega^2}y};$
  $r \to r_+ (y \to -\infty),\ \tilde{R} \sim e^{-i(\omega - e\Phi_h)y}.$

  bound state condition $\omega^2 < \mu^2.$

Effective potential and asymptotic analysis

- Schrodinger-like radial equation

$$\psi = \Delta^{1/2} R. \qquad \frac{d^2\psi}{dr^2} + (\omega^2 - V)\psi = 0$$

- Effective potential $V = \omega^2 + \frac{B}{A}$

$$A = 4r^2(r^{2D-6} - 2mr^{D-3} + m^2)^2 = 4r^2(r^{D-3} - m)^4$$

$$\begin{aligned} B = {} & 4(\mu^2 - \omega^2)r^{4D-10} + (2l + D - 2) \\ & \times (2l + D - 4)r^{4D-12} - 8(m\mu^2 - c_D e m \omega)r^{3D-7} \\ & -4m(2\lambda_l + (D-4)(D-2))r^{3D-9} \\ & +4m^2(\mu^2 - c_D^2 e^2)r^{2D-4} \\ & +2m^2(2\lambda_l + 3(D-4)(D-2))r^{2D-6} \\ & -4m^3(D-4)(D-2)r^{D-3} \\ & +m^4(D-4)(D-2), \qquad (\end{aligned}$$

Effective potential and asymptotic analysis

- Asymptotic analysis

effective potential V:

$$\begin{aligned} &V \to -\infty, \quad r \to r_h; \\ &V \to \mu^2, \quad r \to +\infty. \end{aligned}$$

derivative of V:

$$V'(r) \to \begin{cases} \frac{-(D-2)(D-4)-4\lambda_l-8m(\mu^2+c_D e\omega-2\omega^2)}{2r^3}, & D=5; \\ \frac{-(D-2)(D-4)-4\lambda_l}{2r^3}, & D \geqslant 6. \end{cases}$$

Effective potential and asymptotic analysis

- Based on the asymptotic analysis, there is one maximum outside the outer horizon for effective potential V.

- No potential well for V when $r > r_h$

⇔ V'(r) has only one real root when $r > r_h$.

D=5 extremal RN case

- Event horizon, superradiance condition, bound state condition

$$r_h = \sqrt{m}\,, \quad \omega < \frac{\sqrt{3}}{2}\mathrm{e} \approx 0.87\mathrm{e}\ \ , \ \omega < \mu$$

- Numerator of the derivative of the effective potential

$$\begin{aligned} n_5 = {} & 3m^5 - 15m^4r^2 + 2m^3r^4(15 - 2l(l+2)) \\ & + 2m^2r^6(-15 + 3e^2m - 4m\mu^2 + 2l(l+2)) \\ & + mr^8(15 + 6e^2m + 16m\mu^2 - 12\sqrt{3}em\omega + 4l(l+2)) \\ & - r^{10}(3 + 8m\mu^2 + 4m(\sqrt{3}e - 4\omega)\omega + 4l(l+2)). \end{aligned}$$

D=5 extremal RN case

- Let z=$r^2 - m$, numerator of the derivative of the effective potential

$$\begin{aligned} n_5 = {} & z^5(-3 - 4m(2\mu^2 + (\sqrt{3}e - 4\omega)\omega) - 4\lambda_l) \\ & + 2mz^4(3e^2m - 12m\mu^2 - 16\sqrt{3}em\omega + 40m\omega^2 - 8\lambda_l) \\ & + 2m^2z^3(15e^2m - 12m\mu^2 - 44\sqrt{3}em\omega + 80m\omega^2 - 10\lambda_l) \\ & + 2m^3z^2(27e^2m - 56\sqrt{3}em\omega - 4(m(\mu^2 - 20\omega^2) + \lambda_l)) \\ & + 2m^5z(21e^2 - 34\sqrt{3}e\omega + 40\omega^2) \\ & + 4m^6(3e^2 - 4\sqrt{3}e\omega + 4\omega^2) \\ = {} & \sum_{i=0}^{5} a_i z^i, \end{aligned}$$

$$\begin{aligned} a_5 &= -3 - 8m\mu^2 - 4m\omega(\sqrt{3}e - 4\omega) - 4\lambda_l, \\ a_4 &= 2m(3e^2m - 12m\mu^2 - 16\sqrt{3}em\omega + 40m\omega^2 - 8\lambda_l), \\ a_3 &= 2m^2(15e^2m - 12m\mu^2 - 44\sqrt{3}em\omega + 80m\omega^2 - 10\lambda_l), \\ a_2 &= 2m^3(27e^2m - 56\sqrt{3}em\omega - 4(m(\mu^2 - 20\omega^2) + \lambda_l)), \\ a_1 &= 2m^5(21e^2 - 34\sqrt{3}e\omega + 40\omega^2), \\ a_0 &= 4m^6(3e^2 - 4\sqrt{3}e\omega + 4\omega^2) = 4m^6(2\omega - \sqrt{3}e)^2. \end{aligned}$$

D=5 extremal RN case

- It is easy to see $a_0 > 0$

$$\begin{aligned} a_5 &= -3 - 8m\mu^2 - 4m\omega(\sqrt{3}e - 4\omega) - 4\lambda_l \\ &= -3 - 8m(\mu^2 - \omega^2) - 4m\omega(\sqrt{3}e - 2\omega) - 4\lambda_l < 0 \end{aligned}$$

- Other coefficients can be rewritten as

$$\begin{aligned} a_4 &= 2m(3e^2m - 12m\mu^2 - 16\sqrt{3}em\omega + 40m\omega^2 - 8\lambda_l) \\ &= 2m(-8\lambda_l + 12m\omega^2 - 12m\mu^2) \\ &\quad +2m^2e^2(3 - 16\sqrt{3}t + 28t^2), \qquad (\end{aligned}$$

$$\begin{aligned} a_3 &= 2m^2(15e^2m - 12m\mu^2 - 44\sqrt{3}em\omega + 80m\omega^2 - 10\lambda_l) \\ &= 2m^2(-10\lambda_l + 12m\omega^2 - 12m\mu^2) \\ &\quad +2m^3e^2(15 - 44\sqrt{3}t + 68t^2), \qquad (32) \end{aligned}$$

$$\begin{aligned} a_2 &= 2m^3(27e^2m - 56\sqrt{3}em\omega - 4(m(\mu^2 - 20\omega^2) + \lambda_l)) \\ &= 2m^3(-4\lambda_l - 4m\mu^2 + 4m\omega^2) \\ &\quad +2m^4e^2(27 - 56\sqrt{3}t + 76t^2), \qquad (3 \end{aligned}$$

$$\begin{aligned} a_1 &= 2m^5(21e^2 - 34\sqrt{3}e\omega + 40\omega^2) \\ &= 2m^5e^2(21 - 34\sqrt{3}t + 40t^2), \end{aligned}$$

where $0 < \mathrm{t} = \frac{\omega}{e} < 0.87$

D=5 extremal RN case

- Key results: sign changes in the sequence of coefficients are always 1

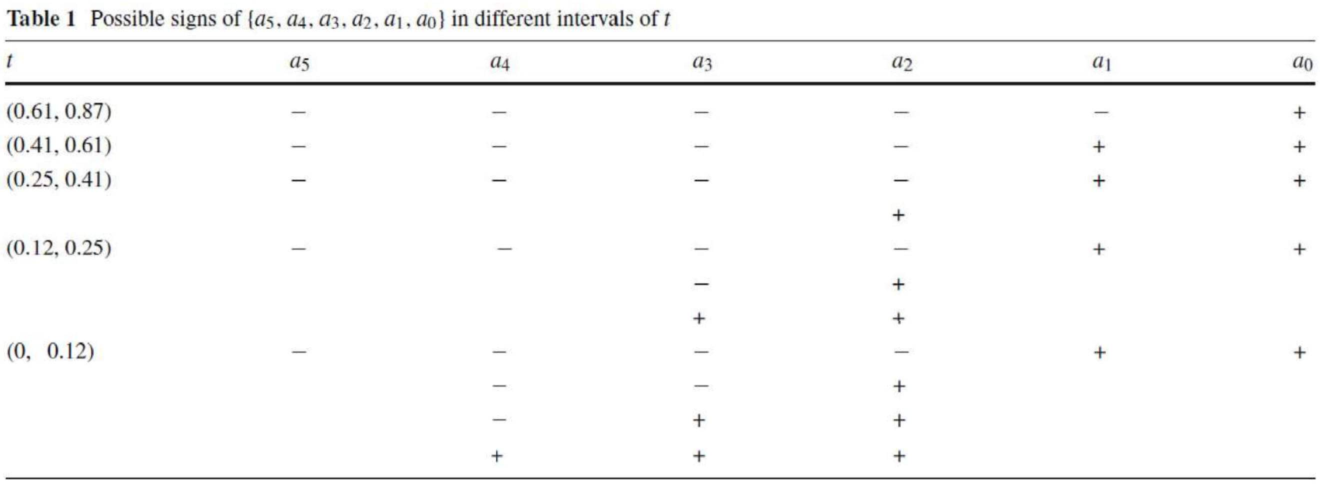

Table 1 Possible signs of $\{a_5, a_4, a_3, a_2, a_1, a_0\}$ in different intervals of t

t	a_5	a_4	a_3	a_2	a_1	a_0
(0.61, 0.87)	−	−	−	−	−	+
(0.41, 0.61)	−	−	−	−	+	+
(0.25, 0.41)	−	−	−	−	+	+
				+		
(0.12, 0.25)	−	−	−	−	+	+
			−	+		
			+	+		
(0, 0.12)	−	−	−	−	+	+
		−	−	+		
		−	+	+		
		+	+	+		

- No potential well for V $\Longrightarrow$ the system is superradiantly stable

D=5 extremal RN case

- Rescaling the coefficients and consider the differences

$$\frac{a_2}{8m^3} - \frac{a_3}{20m^2} = \frac{me^2}{20}(105 - 192\sqrt{3}t + 240t^2 + 4\mu^2/e^2) \qquad 0 < t < 0.25$$

$$\frac{a_3}{20m^2} - \frac{a_4}{16m} = \frac{3me^2}{40}(15 - 32\sqrt{3}t + 40t^2 + 4\mu^2/e^2). \qquad 0 < t < 0.12.$$

6D extremal RN case

- The numerator of the derivative of effective potential

$$\begin{aligned}
n^{(6)}(z) = {} & (-8 - 4\lambda_l)z^{15} + 4\Big(- 30m^{1/3} - 3m\mu^2 \\
& -\sqrt{6}em\omega + 6m\omega^2 - 15m^{1/3}\lambda_l\Big)z^{14} \\
& +4\Big(- 210m^{2/3} - 42m^{4/3}\mu^2 - 14\sqrt{6}em^{4/3}\omega \\
& +84m^{4/3}\omega^2 - 105m^{2/3}\lambda_l\Big)z^{13} \\
& +4\Big(- 900m - 273m^{5/3}\mu^2 - 91\sqrt{6}em^{5/3}\omega \\
& +546m^{5/3}\omega^2 - 455m\lambda_l\Big)z^{12} \\
& +4\Big(- 2610m^{4/3} + 2e^2m^2 - 1086m^2\mu^2 \\
& -367\sqrt{6}em^2\omega \\
& +2184m^2\omega^2 - 1365m^{4/3}\lambda_l\Big)z^{11} \\
& +4\Big(- 5346m^{5/3} + 22e^2m^{7/3} - 2937m^{7/3}\mu^2 \\
& -1034\sqrt{6}em^{7/3}\omega \\
& +6006m^{7/3}\omega^2 - 3003m^{5/3}\lambda_l\Big)z^{10}
\end{aligned}$$

6D extremal RN case

- Rescaling the coefficients $b'_8 = \frac{b_8}{39984m^3}, b'_9 = \frac{b_9}{25344m^{8/3}},$

$$b'_{10} = \frac{b_{10}}{12276m^{7/3}}, b'_{11} = \frac{b_{11}}{4392m^2}$$

$$b'_9 - b'_{10} = \frac{91e^2}{8928} + \frac{94}{93}\sqrt{\frac{2}{3}}e\omega - \frac{197e\omega}{96\sqrt{6}} + \frac{18565\lambda_l}{98208m^{2/3}} + \frac{5523}{10912m^{2/3}} + \frac{91}{1488}(\mu^2 - \omega^2)$$

- Key results: the sign changes of the sequence of coefficients are always 1
- 6D extremal RN and scalar perturbation system is superradiantly stable

Summary

- Introduce an analytical method for studying superradiant stability regime of higher dimensional RN black hole and scalar perturbation system

- 5D non-extremal RN case (√)

- D-dimensional extremal/non-extremal RN black hole cases (?)

- Other higher dimensional black hole cases(?)

Thank you!

Session C1b 10:00–12:00

[Chair: Hayato Motohashi]

Shin'ichi Hirano

Nagoya University

“Black holes in effective field theory extension of GR with parity violating terms and scalar field”

(15 min.)

[JGRG30 (2021) 120816]

Black holes in effective field theory extension of GR with parity violating terms and scalar field

Shin’ichi Hirano (Nagoya U.)

collaborators: M. Kimura (Rikkyo U.), M. Yamaguchi (Tokyo Tech.)

JGRG30, Des. 6-10th 2021

Intro

- **GWs from binary BHs merger (LIGO, VIRGO)**

  observed wave forms would correspond to that predicted by numerical relativity

  → general relativity (GR) is almost correct

- **GR might be corrected from UV physics**

  e.g.) GR is non-renormalizable. We need inflaton.

  → add higher curvature terms effectively (EFT approach)

  e.g.) $R^2, \frac{1}{\Lambda^2}R^3$

Intro

- **BH physics？**

corrections becomes efficient on very small scale due to $r_{\mathrm{H}} \gg \dfrac{1}{\Lambda_{\mathrm{cut}}}$?

- **BH soln. itself can be changed thanks to corrections**

$(\mathrm{Riemann})^4$: Cardoso+ (2018)

$(\mathrm{Riemann})^3$: de Rham+ (2020), Cano+ (2021)

Non-linear Maxwell: K. Nomura+ (in prep.?)

※ In standard works, one consider effects of EFT operators in a given background spacetime cf.) Franciolini+ (2018) 1810.07706

Image

combination of $M_{\mathrm{pl}},\ \Lambda_{cut},\ r_{\mathrm{g}}\ (:= 2M)$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \epsilon\,(\mathrm{corrections})$$

Let us consider solving above peturbatively as $g_{\mu\nu} = g^{\mathrm{GR}}_{\mu\nu} + \epsilon g^{\mathrm{EFT}}_{\mu\nu}$

Image

combination of $M_{\rm pl},\ \Lambda_{cut},\ r_{\rm g}\ (:= 2M)$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \epsilon\,(\text{corrections})$$

Let us consider solving above perturbatively as $g_{\mu\nu} = g_{\mu\nu}^{\rm GR} + \epsilon g_{\mu\nu}^{\rm EFT}$

$g_{\mu\nu} = g_{\mu\nu}^{\rm GR}$

$\mathcal{O}(\epsilon^0)$: $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

Image

combination of $M_{\rm pl},\ \Lambda_{cut},\ r_{\rm g}\ (:= 2M)$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \epsilon\,(\text{corrections})$$

Let us consider solving above perturbatively as $g_{\mu\nu} = g_{\mu\nu}^{\rm GR} + \epsilon g_{\mu\nu}^{\rm EFT}$

$g_{\mu\nu} = g_{\mu\nu}^{\rm GR}$

$\mathcal{O}(\epsilon^0)$: $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

$\mathcal{O}(\epsilon^1)$: $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \epsilon\,(\text{corrections})$$

$g_{\mu\nu} = \epsilon g_{\mu\nu}^{\rm EFT}$ $\qquad$ $g_{\mu\nu} = g_{\mu\nu}^{\rm GR}$

From 1st order EoM, we obtain $g_{\mu\nu}^{\rm EFT}$

Our work

- **Our work** *Hirano, Kimura, Yamaguchi, in prep.*

  - new cubic term with **parity violation**
  - **additional DoFs** as EFT of GR

    → as a first step we consider **scalar field**

    ※ bonus: we can consider other terms outside modified gravity

(1) construction of effective action

(2) static spherically symm. BG and perturbation

CONSTRUCTING EFFECTIVE ACTION

construction of action

Hirano+

- On $\mathcal{O}(\epsilon^0)$ terms, we can use Ricci flat (scalar vanishes in vacuum)

→ in EoM, we need not to consider R, $R_{\mu\nu}$, and its derivatives

e.g.) $R_{\mu\nu}^2$, R^3, $RR_{\mu\nu}^2$ in action

- RX, $R_{\mu\nu}Y^{\mu\nu}$ (X: scalar, $Y^{\mu\nu}$: tensor) can be pushed to higher order via $g_{\mu\nu} \to g_{\mu\nu} + \epsilon Z_{\mu\nu}$ ($Z_{\mu\nu}$: tensor)

[bonus] we can construct Z2-violating sol. via $g_{\mu\nu} \to g_{\mu\nu} + \epsilon Z_{\mu\nu}$

- We can choose one from terms proportional to same Weyl components

e.g.) $R_{abcd}R^{cd\mu\nu}R_{\mu\nu}{}^{ab}$, $R_{abcd}R^{cd\mu\nu}R_{\mu}{}^{a}{}_{\nu}{}^{b} \propto W_{abcd}W^{cd\mu\nu}W_{\mu\nu}{}^{ab}$

Effective action

Hirano+

$$\frac{\mathcal{L}_{\mathrm{EFT}}}{\sqrt{-g}} = \frac{M_{\mathrm{pl}}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$+\frac{b_1}{\Lambda}\phi R_{\mu\nu\alpha\beta}^2 + \frac{b_2}{\Lambda}\phi\tilde{R}_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

$$+\frac{c_1}{\Lambda^2}R_{\mu\nu\alpha\beta}^3 + \frac{c_2}{\Lambda^2}\tilde{R}_{\mu\nu ab}R^{abcd}R_{cd}{}^{\mu\nu}$$

$$+\frac{d_1}{\Lambda^3}\Box\phi R_{\mu\nu\rho\sigma}^2 + \frac{d_2}{\Lambda^3}\phi R_{\mu\nu\rho\sigma}^3$$

$$+\frac{e_1}{\Lambda^3}\Box\phi\tilde{R}^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu} + \frac{e_2}{\Lambda^3}\phi\tilde{R}^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{ab}R^{ab}{}_{\mu\nu} \quad + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

Effective action

Hirano+

converted to ϵ by combining with $M_{\rm pl}$ and r_g

$$\frac{\mathcal{L}_{\rm EFT}}{\sqrt{-g}} = \frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$+\frac{b_1}{\Lambda}\phi R^2_{\mu\nu\alpha\beta} + \frac{b_2}{\boxed{\Lambda}}\phi\tilde{R}_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

← scalar Gauss-Bonnet
/dynamical Chern-Simmons

$$+\frac{c_1}{\Lambda^2}R^3_{\mu\nu\alpha\beta} + \frac{c_2}{\Lambda^2}\underline{\tilde{R}_{\mu\nu ab}R^{abcd}R_{cd}}{}^{\mu\nu}$$

← **new term** at cubic order

$$\underline{+\frac{d_1}{\Lambda^3}\Box\phi R^2_{\mu\nu\rho\sigma} + \frac{d_2}{\Lambda^3}\phi R^3_{\mu\nu\rho\sigma}}$$

← **new** sub-leading terms

↓

$$\underline{+\frac{e_1}{\Lambda^3}\Box\phi\tilde{R}^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu} + \frac{e_2}{\Lambda^3}\phi\tilde{R}^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{ab}R^{ab}{}_{\mu\nu}} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

Effective action

Hirano+

converted to ϵ by combining with $M_{\rm pl}$ and r_g

$$\frac{\mathcal{L}_{\rm EFT}}{\sqrt{-g}} = \frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2$$

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← scalar Gauss-Bonnet
/dynamical Chern-Simmons

this talk

$$+\frac{c_1}{\Lambda^2}R^3_{\mu\nu\alpha\beta} + \frac{c_2}{\Lambda^2}\underline{\tilde{R}_{\mu\nu ab}R^{abcd}R_{cd}}{}^{\mu\nu}$$

← **new term** at cubic order

$$\underline{+\frac{d_1}{\Lambda^3}\Box\phi R^2_{\mu\nu\rho\sigma} + \frac{d_2}{\Lambda^3}\phi R^3_{\mu\nu\rho\sigma}}$$

← **new** sub-leading terms

↓

$$\underline{+\frac{e_1}{\Lambda^3}\Box\phi\tilde{R}^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu} + \frac{e_2}{\Lambda^3}\phi\tilde{R}^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{ab}R^{ab}{}_{\mu\nu}} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

HIGHER CURVATURE EFT

Static spherically symm. sol.

de Rham+ (2020)

$$\boxed{\frac{c_1}{\Lambda^2}R^3_{\mu\nu\alpha\beta}\ ,\ \frac{c_2}{\Lambda^2}\tilde{R}_{\mu\nu ab}R^{abcd}R_{cd}{}^{\mu\nu}} \longleftarrow$$

~~parity~~ term

- Ansatz $$ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$A(r) = 1 - \frac{r_g}{r} + \epsilon f(r),\ B(r) = 1 - \frac{r_g}{r} + \epsilon g(r)\ \ ,\ \epsilon := \frac{1}{\Lambda^2 M_{\rm pl}^2 r_g^4}$$

- Substituting ansatz into EoMs, we can determine unknown func. at $\mathcal{O}(\epsilon)$

$$f(r) = 10c_1\left(\frac{r_g}{r}\right)^7,\ g(r) = c_1\left[108\left(\frac{r_g}{r}\right)^6 - 98\left(\frac{r_g}{r}\right)^7\right]$$

$$\rightarrow \quad r_H = r_g(1 + 10c_1\epsilon)$$

Perturbations

- $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$\bar{g}^{\text{GR}}_{\mu\nu} + \epsilon \bar{g}^{\text{EFT}}_{\mu\nu}$

$h^{\text{GR}}_{\mu\nu} + \epsilon h^{\text{EFT}}_{\mu\nu}$
$= [1 + \epsilon F^{\text{EFT}}(r)] h^{\text{GR}}_{\mu\nu}$

higher derivatives vanish

→ substituting ansatz into EoMs and using GR equations at $\mathcal{O}(\epsilon)$
we obtain effective master equations including $\mathcal{O}(\epsilon)$ corrections

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Perturbations

- $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$\bar{g}^{\text{GR}}_{\mu\nu} + \epsilon \bar{g}^{\text{EFT}}_{\mu\nu}$

$h^{\text{GR}}_{\mu\nu} + \epsilon h^{\text{EFT}}_{\mu\nu}$
$= [1 + \epsilon F^{\text{EFT}}(r)] h^{\text{GR}}_{\mu\nu}$

higher derivatives vanish

→ substituting ansatz into EoMs and using GR equations at $\mathcal{O}(\epsilon)$
we obtain effective master equations including $\mathcal{O}(\epsilon)$ corrections

- $h^{(odd)}_{\mu\nu} dx^\mu dx^\nu = 2e^{-i\omega t} \sin\theta \partial_\theta Y_{l0} d\phi (h_0 dt + h_1 dr)$

  h_0 is dependent of h_1. Master eq. depends on h_1.

- $h^{(even)}_{\mu\nu} dx^\mu dx^\nu = e^{-i\omega t} Y_{l0} [-A(r) H_0 dt^2 + 2H_1 dt dr + B(r)^{-1} H_2 dr^2] + r^2 K (d\theta^2 + \sin^2\theta d\phi^2)$

  H_0, H_2 are non-dynamical. Master eq. depends on combination of H_1 and K.

$\frac{c_1}{\Lambda^2}R^3_{\mu\nu\alpha\beta}$ term

de Rham+ (2020)

$$\frac{d^2\Psi^{\rm o/e}}{dr_*^2}+\frac{w^2}{c_s^2}\Psi^{\rm o/e}-\sqrt{AB}(V^{\rm o/e}_{\rm GR}+\epsilon V^{\rm o/e}_{\rm EFT})\Psi_{\rm o/e}=0$$

$$c_s^2=1-288c_1\epsilon\frac{(r-r_{\rm g})r_{\rm g}^5}{r^6}\ ,\ \frac{dr}{dr_*}=\sqrt{AB}=1-\frac{r_g}{r}+\mathcal{O}(\epsilon)$$

※ $r_{\rm H}=r_{\rm g}(1+10c_1\epsilon)$

$$\Psi^{\rm o}=\frac{i\sqrt{AB}h_1}{r\omega}\left(1+\epsilon f_{h_1}\right),$$

$$\Psi^{\rm e}=\frac{1}{(j^2-2)r+3r_g}\left[-r^2K\left(1+\epsilon f_K\right)+\frac{i\sqrt{AB}rH_1}{\omega}\left(1+\epsilon f_{H_1}\right)\right].$$

$\frac{c_2}{\Lambda^2}\tilde{R}_{\mu\nu ab}R^{abcd}R_{cd}{}^{\mu\nu}$ term

Hirano+

Cano+ (2021)

Coupled Schrodinger type eqs.

$$\frac{d^2\mathbf{\Psi}}{dr_*^2}+\omega^2\mathbf{\Psi}-\left(1-\frac{r_g}{r}\right)\mathbf{V\Psi}=0,$$

$$\mathbf{\Psi}=\begin{pmatrix}\tilde{\Psi}^{\rm o}\\ \tilde{\Psi}^{\rm e}\end{pmatrix}\ ,\ \mathbf{V}=\begin{pmatrix}V^{\rm o}_{\rm GR} & \epsilon V_{\rm int}\\ \epsilon V_{\rm int} & V^{\rm e}_{\rm GR}\end{pmatrix}\ ,\ c_s^2=1\ ,\ \frac{dr}{dr_*}=1-\frac{r_g}{r}$$

$$\tilde{\Psi}^{\rm o}=\Psi^{\rm o}_{\rm GR}+\epsilon c_2\left[\cdots\Psi^{\rm e}_{\rm GR}+\cdots\frac{d\Psi^{\rm e}_{\rm GR}}{dr}\right],\ \tilde{\Psi}^{\rm e}=\Psi^{\rm e}_{\rm GR}+\epsilon c_2\left[\cdots\Psi^{\rm o}_{\rm GR}+\cdots\frac{d\Psi^{\rm o}_{\rm GR}}{dr}\right]$$

$\frac{c_2}{\Lambda^2}\tilde{R}_{\mu\nu ab}R^{abcd}R_{cd}{}^{\mu\nu}$ term

Hirano+

Cano+ (2021)

$$\frac{d^2\mathbf{\Psi}}{dr_*^2}+\omega^2\mathbf{\Psi}-\left(1-\frac{r_g}{r}\right)\mathbf{V}\mathbf{\Psi}=0,$$

$$\mathbf{\Psi}=\begin{pmatrix}\tilde{\Psi}^{\mathrm{o}}\\ \tilde{\Psi}^{\mathrm{e}}\end{pmatrix},\ \mathbf{V}=\begin{pmatrix}V^{\mathrm{o}}_{\mathrm{GR}} & \epsilon V_{\mathrm{int}}\\ \epsilon V_{\mathrm{int}} & V^{\mathrm{e}}_{\mathrm{GR}}\end{pmatrix},\ c_s^2=1,\ \frac{dr}{dr_*}=1-\frac{r_g}{r}$$

- Quasi-normal mode parametrized QMN method *Cardoso+ (2019)*

$l=2$, fundamental
$(r_g=1)$

$$V_{\mathrm{int}}=\frac{36c_2(r-1)\left(-3976r^3-572r^2+4182r+1920\right)}{r^{10}(4r+3)^2}$$

$$\rightarrow\quad \delta\omega=(-1.33565-6.29487i)c_2\epsilon$$

(preliminary)

SCALAR-TENSOR EFT

Static spherically sym. sol.

Hirano+

$$\frac{d_1}{\Lambda^3}\Box\phi R^2_{\mu\nu\rho\sigma},\ \frac{d_2}{\Lambda^3}\phi R^3_{\mu\nu\rho\sigma},\ \frac{e_1}{\Lambda^3}\Box\phi\tilde{R}^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu},\ \frac{e_2}{\Lambda^3}\phi\tilde{R}^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{ab}R^{ab}{}_{\mu\nu}$$

~~parity~~ terms

- Ansatz $ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

$$A(r) = 1 - \frac{r_g}{r} + \epsilon_s f(r),\ B(r) = 1 - \frac{r_g}{r} + \epsilon_s g(r)$$

$\phi = 0 + \epsilon_s\pi(r)$ (no hair at $\mathcal{O}(\epsilon_s^0)$) , $\epsilon_s := \frac{1}{\Lambda^3 M_{\rm pl}^2 r_{\rm g}^5}$

- Substituting ansatz into EoMs, we can determine unknown func. at $\mathcal{O}(\epsilon)$

$f(r) = g(r) = 0,\ \pi(r) = -12d_1\frac{r_g^7M_{\rm pl}^2}{r^6} + d_2\frac{r_g^2M_{\rm pl}^2}{r}$ (regular at $r \to r_{\rm g},\ \infty$)

Static spherically sym. sol.

Hirano+

(field eq.) $$(\Box - m^2)\overset{\overset{\epsilon_s\pi(r)}{\downarrow}}{\phi} = \epsilon_s(\nabla^n \overset{\overset{\bar{g}_{\mu\nu}}{\downarrow}}{R^{abcd}}\nabla_n R_{abcd} + \cdots)$$

(EoMs) $$R_{\mu\nu} - \underset{\underset{\epsilon_s f(r),\ \epsilon_s g(r)}{\uparrow}}{\frac{1}{2}}g_{\mu\nu}R = \underbrace{\epsilon_s(\overset{\overset{\epsilon_s\pi(r)}{\downarrow}}{\phi}R^3_{abcd}g_{\mu\nu} + \cdots)}_{\mathcal{O}(\epsilon_s^2)}$$

- Ans

- Substituting ansatz into EoMs, we can determine unknown func. at $\mathcal{O}(\epsilon)$

$f(r) = g(r) = 0,\ \pi(r) = -12d_1\frac{r_g^7M_{\rm pl}^2}{r^6} + d_2\frac{r_g^2M_{\rm pl}^2}{r}$ (regular at $r \to r_{\rm g},\ \infty$)

Perturbations

Hirano+

- $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \phi = \bar{\phi} + \delta\phi$

Perturbations

Hirano+

- $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \phi = \bar{\phi} + \delta\phi$

- ϕ-R coupling

(field eq.)

$$(\Box - m^2)\overset{\substack{\boxed{\text{even}}\\ \delta\phi \\ \downarrow}}{\phi} = \epsilon_s(\nabla^n \overset{\substack{\boxed{\text{even}}\\ h_{\mu\nu} \\ \downarrow}}{R^{abcd}} \nabla_n R_{abcd} + \cdots)$$

(EoMs)

$$R_{\mu\nu} - \frac{1}{2}\overset{\substack{h_{\mu\nu} \\ \downarrow}}{g_{\mu\nu}} R = \epsilon_s(\overset{\substack{\text{only } \delta\phi \\ \downarrow}}{\phi} R^3_{abcd} \underset{\substack{\uparrow \\ \text{only } \bar{g}_{\mu\nu}}}{g_{\mu\nu}} + \cdots)$$

Perturbations

Hirano+

- $g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}, \phi=\bar{\phi}+\delta\phi$
- ϕ-R coupling

same type as sGB

e.g.) Blázquez-Salcedo (2016)

$$\frac{d^2\Psi^{\mathrm{o}}_{\mathrm{GR}}}{dr_*^2}+\omega^2\Psi^{\mathrm{o}}_{\mathrm{GR}}-\left(1-\frac{r_g}{r}\right)V^{\mathrm{o}}_{\mathrm{GR}}\Psi^{\mathrm{o}}_{\mathrm{GR}}=0,$$

$$\frac{d^2\mathbf{\Psi}}{dr_*^2}+\omega^2\mathbf{\Psi}-\left(1-\frac{r_g}{r}\right)\mathbf{V}\mathbf{\Psi}=0$$

$$\mathbf{\Psi}=\begin{pmatrix}\tilde{\Psi}^{\mathrm{scalar}}\\ \tilde{\Psi}^{\mathrm{e}}\end{pmatrix}, \mathbf{V}=\begin{pmatrix}V_{\mathrm{scalar}} & \epsilon V_{\mathrm{int}}\\ \epsilon V_{\mathrm{int}} & V^{\mathrm{e}}_{\mathrm{GR}}\end{pmatrix}, c_s^2=1, \frac{dr}{dr_*}=1-\frac{r_g}{r}$$

Perturbations

Hirano+

- $g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}, \phi=\bar{\phi}+\delta\phi$
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$$\frac{d^2\Psi^{\mathrm{o}}_{\mathrm{GR}}}{dr_*^2}+\omega^2\Psi^{\mathrm{o}}_{\mathrm{GR}}-\left(1-\frac{r_g}{r}\right)V^{\mathrm{o}}_{\mathrm{GR}}\Psi^{\mathrm{o}}_{\mathrm{GR}}=0,$$

$$\frac{d^2\mathbf{\Psi}}{dr_*^2}+\omega^2\mathbf{\Psi}-\left(1-\frac{r_g}{r}\right)\mathbf{V}\mathbf{\Psi}=0$$

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- ϕ-$\tilde{R}$ coupling

even ⇄ odd same type as dCS *e.g.) Kimura (2018)*

Summary

- **GR might be corrected from UV physics**

→ BH solns. can be changed thanks to **corrections**

- **Our work**

new cubic term with $\tilde{R}_{abcd}$, **scalar-tensor couplings**

✔ effective action

✔ $\tilde{R}RR$: coupled Schrodinger eqs., QNM

✔ ST: stealth Schwarzschild, same-type master eqs. as sGB/dCS

- **Future prospects**

rotating case , overtone, matter coupling

Order estimate of ε

Hirano+

- Corrections ~ $\epsilon \left(\frac{r_g}{r}\right)^{\bullet}$, $r_g = 2M$

$$\epsilon := \frac{1}{\Lambda^2 M_{pl}^2 r_g^4}, \ \epsilon_s := \frac{1}{\Lambda^3 M_{pl}^2 r_g^5}$$

- Normalizing Λ by M_{pl} and M by $30M_\odot$

$$\epsilon \sim 10^{-162} \left(\frac{\Lambda}{M_{pl}}\right)^{-2} \left(\frac{M}{30M_\odot}\right)^{-4}, \quad \epsilon_s \sim 10^{-203} \left(\frac{\Lambda}{M_{pl}}\right)^{-3} \left(\frac{M}{30M_\odot}\right)^{-5}$$

→ large value if $\Lambda \sim 10^{-60} M_{pl}$ (dark energy) or PBH

Session C1b 10:00–12:00

[Chair: Hayato Motohashi]

Kazufumi Takahashi

YITP, Kyoto University

“Perturbations of stealth black holes in modified gravity”

(15 min.)

[JGRG30 (2021) 120817]

Perturbations of stealth black holes in modified gravity

Kazufumi Takahashi
(YITP, Kyoto University)

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

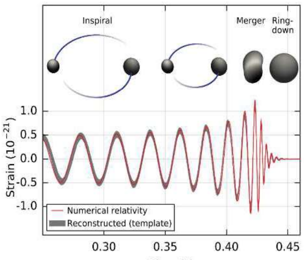
Based on

KT and Hayato Motohashi (Kogakuin Univ.)

"Black hole perturbations in DHOST theories: Master variables, gradient instability, and strong coupling," *JCAP* **08** (2021) 013 [arXiv: 2106.07128]

➢ Introduction (1)

- ■Observations of GWs from binary BHs/NSs & BH shadow
  ⋯ Used to test gravity at strong-field regimes
- ➢Important to study modified gravity for comparison with GR



(LIGO)

(EHT)

- ■Observations so far are consistent with Kerr BHs.
  ⋯ Essentially excludes all MG theories?

— No!

∵ MG theories can admit the metrics in GR as exact sol. (= stealth sol.)

➢ Introduction (2)

- What distinguishes stealth sol. from GR sol.?
  - — Features of MG would be encoded in perturbations.
    e.g. QNMs, Love number
  - → Underlying theory can be clarified with future obs.

- Which MG theories admit stealth BHs?
  - — One can derive existence conditions for a general class of scalar-tensor theories [KT, Motohashi (2020)]

➢ My talk: Stability of stealth BHs [KT, Motohashi (2021)]

cf. Keisuke Nakashi's talk on the 1st day:
Propagation of perturbations on stealth BH background
[Nakashi, Kimura, Motohashi, KT (in prep.)]

➢ Quadratic DHOST theories

- "Degenerate Higher-Order Scalar-Tensor" theories

[Langlois + (2015)]
[Crisostomi+ (2016)]

➢ A general class of ST theories up to $(\nabla\nabla\phi)^2$:

$$S=\int \mathrm{d}^4x\sqrt{-g}\Big[F_0+F_1\Box\phi+F_2R+A_1\phi_\mu^\nu\phi_\nu^\mu+A_2(\Box\phi)^2 \\ +A_3(\Box\phi)\phi^\mu\phi_\mu^\nu\phi_\nu+A_4\phi^\mu\phi_\mu^\nu\phi_\nu^\lambda\phi_\lambda+A_5\left(\phi^\mu\phi_\mu^\nu\phi_\nu\right)^2\Big]$$

with $F_\# = F_\#(\phi,X), A_\# = A_\#(\phi,X)$

"Degeneracy conditions"

$$A_2=-A_1\left(\neq-\frac{F_2}{X}\right),$$

$$A_4=\frac{1}{8(F_2-XA_1)^2}\{4F_2[3(A_1-2F_{2X})^2-2A_3F_2]-A_3X^2(16A_1F_{2X}+A_3F_2) \\ +4X(3A_1A_3F_2+16A_1^2F_{2X}-16A_1F_{2X}^2-4A_1^3+2A_3F_2F_{2X})\},$$

$$A_5=\frac{1}{8(F_2-XA_1)^2}(2A_1-XA_3-4F_{2X})[A_1(2A_1+3XA_3-4F_{2X})-4A_3F_2]$$

$\phi_\mu := \nabla_\mu\phi$
$\phi_{\mu\nu} := \nabla_\nu\nabla_\mu\phi$
$X := \phi^\mu\phi_\mu$

✓ The higher-derivative terms are degenerate.
→ No Ostrogradsky ghost

➢ Stealth BHs in DHOST

- The shift- & reflection-sym. subclass of DHOST theories,

$$F_\# = F_\#(X)$$
$$A_\# = A_\#(X)$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big[F_0 + F_2 R + A_1\,\phi^\nu_\mu\phi^\mu_\nu + A_2(\Box\phi)^2$$
$$+ A_3\,(\Box\phi)\phi^\mu\phi^\nu_\mu\phi_\nu + A_4\phi^\mu\phi^\nu_\mu\phi^\lambda_\nu\phi_\lambda + A_5\big(\phi^\mu\phi^\nu_\mu\phi_\nu\big)^2\Big]$$

admits stealth Sch-dS solutions with linearly time-dep. scalar hair,

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{A(r)} + r^2\mathrm{d}\Omega^2, \qquad A(r) \coloneqq 1 - \frac{\mu}{r} - \frac{\Lambda_{\mathrm{eff}}}{3}r^2$$
$$\phi = q\,t + \psi(r) \quad \text{and} \quad X \coloneqq g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -q^2,$$

where Λ_{eff} and q^2 are determined from the theory parameters as

$$F_0 + 2\Lambda_{\mathrm{eff}}(F_2 + q^2A_1) = 0,$$
$$2F_{0X} + \Lambda_{\mathrm{eff}}(8F_{2X} - 2A_1 - 4q^2A_{1X} - 3q^2A_3) = 0.$$

(see [**KT**, Motohashi, Minamitsuji (2019)] and [**KT**, Motohashi (2020)])

➢ Background coordinate system

- Instead of the Schwarzschild coordinates (t, r, θ, φ)

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{A(r)} + r^2\mathrm{d}\Omega^2, \qquad A(r) \coloneqq 1 - \frac{\mu}{r} - \frac{\Lambda_{\mathrm{eff}}}{3}r^2$$

we use the Lemaître coordinates $(\tau, \rho, \theta, \varphi)$

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + [1 - A(r)]\mathrm{d}\rho^2 + r^2\mathrm{d}\Omega^2,$$

where

$$\mathrm{d}\tau = \mathrm{d}t + \frac{\sqrt{1 - A(r)}}{A(r)}\mathrm{d}r, \qquad \mathrm{d}\rho = \mathrm{d}t + \frac{\mathrm{d}r}{A(r)\sqrt{1 - A(r)}}.$$

- $g_{\mu\nu}$ is not static:

$$\mathrm{d}(\rho - \tau) = \frac{\mathrm{d}r}{\sqrt{1 - A(r)}} \Rightarrow r = r(\rho - \tau)$$

- $\phi = q\tau$ ⋯ simplifies the perturbation analysis

✓Ready to study perturbations!

➢ Some history (1)

- The first goal: Find master variables among 10+1 pert. variables
  → Further analysis (QNMs, Love number)

- Perturbations of spherically sym. BHs ⋯ Decomposed into odd- & even-parity perturbations. [Regge, Wheeler (1957)]

➢ BH perturbation in GR

- ✓ Both odd and even modes consist of 1 DOF. (2DOFs in total)
  - Odd modes [Regge, Wheeler (1957)] Even modes [Zerilli (1970)]
- ✓ Linearly stable

➢ BH perturbation in ST theories

- The even modes contain an additional DOF (i.e., scalar waves), which makes the analysis for even modes complicated…
- Stability is nontrivial. (Stealth sol. could be unstable)

➢ Some history (2)

- Many works on perturbations of spherically sym. BHs in ST theories
  - BHs w/ $\phi = \phi(r)$ in Horndeski theories
    - Odd modes [Kobayashi, Motohashi, Suyama (2012)]
    - Even modes [Kobayashi, Motohashi, Suyama (2014)]
  - BHs w/ $\phi = qt + \psi(r)$ in shift-sym. Horndeski theories
    - Odd modes [Ogawa+ (2015)] [**KT**, Suyama (2016)]
    - Monopole & dipole modes for stealth BHs [Khoury+ (2020)]
  - BHs w/ $\phi = qt + \psi(r)$ in shift-sym. DHOST theories
    - Odd modes [**KT**, Motohashi, Minamitsuji (2019)] [Tomikawa+ (2021)]
    - Even modes (master equation for $\delta\phi$) [de Rham+ (2019)]

➢ Our work: stealth BHs w/ $\phi = qt + \psi(r)$ in shift-sym. DHOST theories

- ✓ Completed the analysis for both odd & even modes for the first time!

➢ Our work: Overview

- Expand the perturbations about the stealth BHs in terms of $Y_{\ell m}(\theta,\varphi)$ and separate them into odd & even modes
  - Odd modes
    - Dipole $(\ell = 1)$ $\cdots$ nondynamical (related to slow rot. of BH)
    - Multipole $(\ell \geq 2)$ $\cdots$ gravitational waves
  - Even modes
    - Monopole $(\ell = 0)$ $\cdots$ scalar waves [see also Keisuke's talk]
    - Dipole $(\ell = 1)$ $\cdots$ scalar waves
    - Multipole $(\ell \geq 2)$ $\cdots$ gravitational waves + scalar waves

➢Today, we focus on $\ell \geq 2$ where GWs are present.

✓Master variables

✓Stability

➢ Quadratic Lagrangian: Odd modes

- Quadratic Lagrangian in terms of the master variable χ

$$\boxed{\mathcal{L}_{\mathrm{odd}} = s_1 \dot{\chi}^2 - s_2 \chi'^2 - [\ell(\ell+1)s_3 + V]\chi^2}$$

where

$$s_1 \propto \frac{(F_2 + q^2 A_1)^2}{F_2}, \qquad s_2, s_3 \propto F_2 + q^2 A_1.$$

NB F_2, A_1 are evaluated at $X = -q^2$.

✓Squared sound speed

$$c_\rho^2 = c_\theta^2 = \frac{F_2}{F_2 + q^2 A_1} =: c_{\mathrm{GW}}^2$$

cf. c_{GW}^2 on a cosmological BG:

$$c_{\mathrm{GW}}^2 = \frac{F_2}{F_2 - X A_1}$$

✓No ghost/gradient instabilities if $s_1 > 0$ and $c_{\mathrm{GW}}^2 > 0$, i.e.,

$$F_2 > 0 \ \text{ and } \ F_2 + q^2 A_1 > 0$$

(For details, see [KT, Motohashi (2021)])

➢ Quadratic Lagrangian: Even modes

- Quadratic Lagrangian in terms of master variables v^I $(I = 1,2)$

$$\boxed{\mathcal{L}_{\rm even} = \sum_{I,J=1}^{2} \left(\frac{1}{2}\mathcal{K}_{IJ}\dot{v}^I\dot{v}^J + \mathcal{M}_{IJ}\dot{v}^I v^{J\prime} - \frac{1}{2}\mathcal{G}_{IJ} v^{I\prime} v^{J\prime} - \frac{1}{2}\mathcal{W}_{IJ} v^I v^J \right)}$$

✓Squared radial sound speed (gravitational waves + scalar waves)

$$c_{\rm GW}^2 = \frac{F_2}{F_2 + q^2 A_1}, \qquad c_{\rm SW}^2 = \frac{q^4 \Phi\Pi}{\Psi(F_2 + q^2 A_1)^2} \frac{1-A}{r^2}$$

✓No ghost/gradient instabilities only if $\det \mathcal{K}_{IJ} > 0$ and $c_{\rm SW}^2 > 0$, i.e.,

$$\Phi\Pi > 0 \ \text{ and } \ \Psi > 0.$$

$$\Phi(X) \coloneqq F_2 - XF_{2X} - \frac{3}{2}XA_1 - 2X^2 A_{1X} - \frac{3}{4}X^2 A_3, \qquad \Pi(X) \coloneqq F_2 A_3 + 2(F_2 A_1)_X$$

$$\Psi(X) \coloneqq 2F_0 - 2XF_{0X} + X^2 F_{0XX} - X^2 F_0 \frac{(F_2 - XA_1)_{XX}}{F_2 - XA_1} + \frac{X^3}{F_2 - XA_1}\left(\frac{F_0 \Phi}{X^2}\right)_X$$

(For details, see [KT, Motohashi (2021)])

➢ Gradient instability

- What we have found so far:
  - Odd modes: no ghost & $c_\rho^2, c_\theta^2 > 0$ if
    $F_2 > 0$ and $c_{\rm GW}^2 > 0$
  - Even modes: no ghost & $c_\rho^2 > 0$ if
    $\det \mathcal{K}_{IJ} > 0$ and $c_{\rm SW}^2 > 0$

	Odd	Even
No ghost	✓	✓
$c_\rho^2 > 0$	✓	✓
$c_\theta^2 > 0$	✓	✗

- c_θ^2 for even modes:

$$c_{\theta,+}^2 \simeq c_{\rm GW}^2, \qquad c_{\theta,-}^2 \simeq -\frac{1}{2}c_{\rm SW}^2$$

  - One of the c_θ^2's becomes negative so long as $c_{\rm SW}^2 > 0$.
    → Gradient instability in the θ direction

- This problem can be alleviated if $c_{\rm SW}^2$ is vanishing. However, …

➢ Strong coupling & scordatura

- A tiny $c_{\rm SW}^2$ signals the strong coupling:

$$E_{\rm sc} \propto c_{\rm SW}^p, \qquad p > 0.$$

→ Gradient instability/strong coupling would be unavoidable!

- Some ways out:

➢ Incorporate the "scordatura" effect [Motohashi, Mukohyama (2019)]

⋯ A weak and controlled violation of the degeneracy

$$S = S_{\rm DHOST} + \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{\alpha}{2M^2}(\Box\phi)^2\right]$$

✓ Renders $E_{\rm sc}$ sufficiently high

✓ Maintains the ghost-free nature at low energy

"scordatura"
= mistuning

➢ Consider theories with nondynamical ϕ

- "cuscuton" [Afshordi+ (2006)] [Iyonaga, **KT**, Kobayashi (2018)]
- "minimally modified gravity" [Lin, Mukohyama (2017)]

➢ Summary

- DHOST: General framework of ST theories without Ostrogradsky ghost

- A large subclass of DHOST admits stealth BHs

- Perturbations about the stealth Sch-dS BHs
  - Master variables for odd & even modes
  - Gradient instability/strong coupling problem
    - ✓ Cured by scordatura effect
    - ✓ Intrinsically absent in models w/ nondynamical ϕ

Session C2a 15:30–16:45

[Chair: Tsutomu Kobayashi]

Masroor Chandhanapparambil Pookkillath

Yukawa Institute for Theoretical Physics, Kyoto University

“Minimal theory of massive gravity and constraints on the graviton mass”

(15 min.)

[JGRG30 (2021) 120818]

Minimal theory of massive gravity and constraints on the graviton mass

Masroor C. Pookkillath
In collaboration with: Antonio De Felice, Shinji Mukohyama

08 December, 2021

Center for Gravitational Physics,
Yukawa Institute for Theoretical Physics, Kyoto University, Japan

arXiv:2110.01237, JCAP 12(2021)011

Motivation

- Modified theories of gravity has a wide motivation.
  - Cosmological Constant problem
  - Origin of late time acceleration
- GR can be considered as a unique theory of spin-2 massless particle.
- One possible alternative is to consider massive spin-2 theory: MTMG theory as a viable one.
- MTMG breaks Lorentz invariance.
- The constraint for mass of graviton from LIGO-VIRGO collaboration is $\mathrm{m} \sim 10^{-22}\,\mathrm{eV}$.
- Let us look if mass of the graviton gets strongly bounded form cosmological data.

MTMG: Cosmology

- The first Friedmann equation

$$3M_{\rm P}^2H^2 = \rho_X + \sum_I \rho_I\,.$$

- The second Friedmann equation

$$2M_{\rm P}^2\frac{\dot H}{N} = \sum_I(\rho_I + P_I) - (\rho_X + P_X)$$

- We also have a constraint equation

$$\mathcal{E}_\lambda = \underbrace{(c_1X^2 + 2c_2X + c_3)}_{=0\,\text{self-accelerating branch}}\underbrace{\left(\tfrac{\dot X}{N} + HX - H\tfrac{M}{N}\right)}_{=0\,\text{normal branch}} = 0$$

- This fixes

$$M = \frac{\dot X}{H} + NX \qquad\qquad X = \frac{\tilde a}{a}$$

[A. De Felice, S. Mukohyama PLB 752 2016]

MTMG: Cosmology

- Energy density

$$\rho_X = \frac{1}{2}m^2M_{\rm P}^2(c_1X^3 + 3c_2X^2 + 3c_1X + c_4).$$

- Pressure

$$p_X = \frac{P_X}{3M_{\rm p}^2} = \frac{1}{3}H_0^2\theta\epsilon_X - \varrho_X$$

- where

$$\theta = \frac{1}{2}X(\bar c_1X^2 + 2\bar c_2X + \bar c_3)$$

$$\epsilon_X = \frac{\dot X}{NHX}$$

MTMG: Background

$$\frac{H^2}{H_0^2} = \underbrace{\Omega_{\mathrm{m}0}(1+z)^3 + \Omega_{\mathrm{r}0}(1+z)^4 + (1-\Omega_{\mathrm{m}0}-\Omega_{\mathrm{r}0})}_{\frac{H^2_{\Lambda\mathrm{CDM}}}{H_0^2}} \underbrace{+[f(z)-1]\Delta + \mathcal{O}(\Delta^2)}_{\text{Exact } \Lambda\text{CDM } \Delta\to 0}$$

with

$$f(z) = \frac{1+\tanh\frac{A_2-z}{A_2A_3}}{1+\tanh A_3^{-1}}$$

To realize this profile of $H(z)$ we can introduce

$$X(z) = 1 + (A_1-1)f(z)$$

with

$$\Delta = \frac{1}{6}[\bar{c}_1(A_1^3-1) + 3\bar{c}_2(A_1^2-1) + 3\bar{c}_3(A_1-1)] \qquad \bar{c}_i = c_i\frac{m^2}{H_0^2}$$

Finally the mass of the graviton is given by

$$\mu^2 = \frac{1}{2}H_0^2X\left[c_2X + c_3 + \frac{M}{N}(c_1X+c_2)\right]$$

[A. De Felice, S. Mukohyama,MCP PLB 816,2021]

MTMG: Perturbation

- The matter equations of motion

$$\dot{\delta}_I = -3aH(c_{sI}^2 - w_I)\,\delta_{\mathrm{I}} - (1+w_I)\,\theta_I + 3(1+w_I)\,\dot{\phi}\,,$$

$$\dot{\theta}_I = aH(3c_{sI}^2-1)\,\theta_I + k^2\psi + \frac{c_{sI}^2k^2}{1+w_I}\,\delta_I - k^2\sigma_I\,.$$

- Dynamical equation in Boltzmann solver

$$\dot{\phi} + \frac{3a\theta Y\left(\Gamma - \frac{\epsilon_X(Y\theta-2)H^2}{3}\right)}{2H\,(Y\theta-2)}\,\phi + Ha\,\psi + \frac{3a^2}{k^2(Y\theta-2)}\sum_I \Gamma_I\theta_I = 0\,,$$

- Similarly, the second dynamical equation in Boltzmann solver looks,

$$\psi \quad + \quad \frac{9\,a^2}{2k^2}\sum_I \Gamma_I\sigma_I - \mathcal{A}\,\phi + \frac{9a^3Y\left(\eta_X H^2\epsilon_X - \frac{3\theta\Gamma}{2}\right)}{2H\,k^4\,(Y\theta-2)}\sum_I\Gamma_I\theta_I$$
$$-\frac{9a^2Y\theta}{2k^2\,(Y\theta-2)}\sum_I c_{s,I}^2\,\varrho_I\,\delta_I = 0\,.$$

MTMG: Perturbation

- Equation of motion for the dust fluid in the high-k limit is

$$\ddot{\delta}_c + aH\dot{\delta}_c - \frac{3}{2}\frac{G_{\rm eff}}{G_N}\varrho_c a^2 \delta_c = 0.$$

- The mass term is non-standard and is given as

$$\frac{G_{\rm eff}}{G_N} = \frac{2}{2 - Y\theta} - \frac{3Y\theta\Omega_m}{(Y\theta - 2)^2} + \frac{2\eta_X \epsilon_X Y}{(Y\theta - 2)^2}\,.$$

- The ISW effect can set constraint. The relation between $\psi_{\rm ISW} = \phi + \psi$ and the matter density profile δ_c is

$$\psi_{\rm ISW} = -\frac{3H_0^2\Omega_{m0}}{k^2}\frac{\Sigma\delta_c}{a}\,,$$

where in the high-k limit we can find

$$\Sigma = \frac{8 + [2\eta_X\epsilon_X - (4 + 3\Omega_m)\theta]Y}{2(Y\theta - 2)^2}\,.$$

MTMG: Perturbation

- In the high-k regime, for scalar field we have following no-ghost and no-Laplacian-instability conditions

$$\begin{aligned} Q_I &= \frac{\rho_I^2}{(\rho_I + P_I)}\frac{a^2}{k^2} > 0\,, \\ c_{s,I}^2 &= \frac{\dot{P}_I}{\dot{\rho}_I} \geq 0\,. \end{aligned}$$

- For tensor mode we have the following equations motion

$$\ddot{h}_f = -2\,\frac{\dot{a}}{a}\,\dot{h}_f - (k^2 + \mu^2\,a^2)\,h_f\,,$$

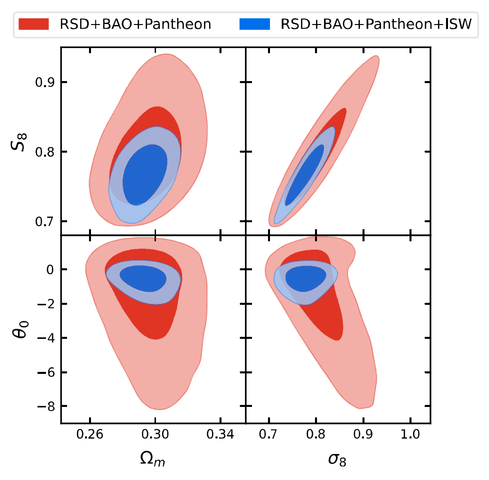
where $f \in \{+, \times\}$, and

$$\mu^2 = \frac{H_0^2\,[(\theta^2 Y - 2\eta_X)\,\epsilon_X + 4\theta]}{4}\,.$$

MTMG: Result

$$\theta_0 \equiv \frac{\mu_0^2}{H_0^2} = \frac{1}{2} A_1 [\bar{c}_1 A_1^2 + 2\bar{c}_2 A_1 + \bar{c}_3] \, .$$

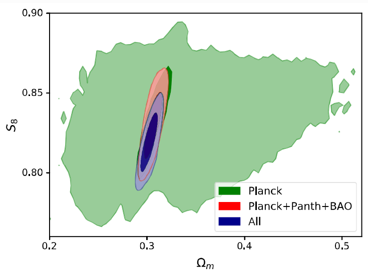
ΛCDM limit: $A_1 \to 1$ or $\bar{c}_i \to 0$

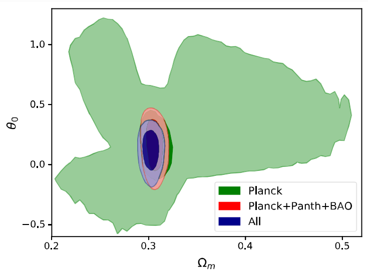


MTMG: Result

	Planck	Planck+BAO+Pantheon	All joint analysis
Ω_m	$0.318^{+0.17}_{-0.068}$	$0.306^{+0.012}_{-0.012}$	$0.302^{+0.011}_{-0.011}$
S_8	$0.832^{+0.040}_{-0.040}$	$0.830^{+0.028}_{-0.027}$	$0.819^{+0.023}_{-0.024}$
Δ	$-0.4^{+2.7}_{-4.2}$	$-0.4^{+2.5}_{-4.1}$	$-0.1^{+1.3}_{-1.5}$
θ_0	$0.18^{+0.64}_{-0.40}$	$0.16^{+0.27}_{-0.28}$	$0.12^{+0.21}_{-0.22}$

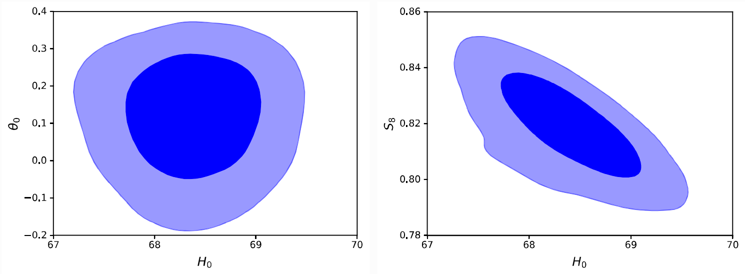
Does not exclude ΛCDM





MTMG: Result

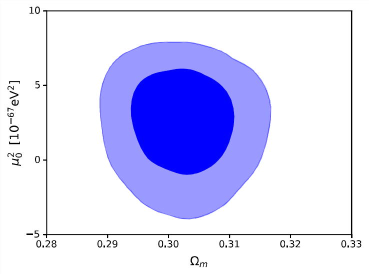
	Planck	Planck+BAO+Pantheon	All joint analysis
H_0	67^{+8}_{-10}	$68.11^{+0.92}_{-0.92}$	$68.37^{+0.87}_{-0.93}$
S_8	$0.832^{+0.040}_{-0.040}$	$0.830^{+0.028}_{-0.027}$	$0.819^{+0.023}_{-0.024}$



Predictions given by the theory of MTMG regarding the observables θ_0, S_8 and H_0.

MTMG: Result

We arrive at the strongest bound for the graviton mass



LIGO: $\mu < 1.2 \times 10^{-22}$ eV

$\mu < 8.4 \times 10^{-34}$ eV

Summary

- We have studied the normal branch of MTMG with a specific choice of background.
- We find that ΛCDM is still inside the allowed parameter.
- We find strongest bound to the mass of the graviton $\mu < 8.4 \times 10^{-34}\,\mathrm{eV}$.
- Confronting the theory with both late-time and early universe data MTMG does not feel any internal tension.
- Even though we have 5 additional parameter from that of ΛCDM Planck 2018 still find strong bound to the mass of the graviton.

Thank You

Session C2a 15:30–16:45

[Chair: Tsutomu Kobayashi]

Reginald Christian Bernardo

Institute of Physics, Academia Sinica

“Towards well-tempered dark energy and teleparallel gravity”

(15 min.)

[JGRG30 (2021) 120819]

Towards well-tempered dark energy and teleparallel gravity

[2107.08762 & 2108.02500]

Reggie Bernardo *with* Jackson Levi Said, Maria Caruana, Stephen Appleby
Institute of Physics, Academia Sinica

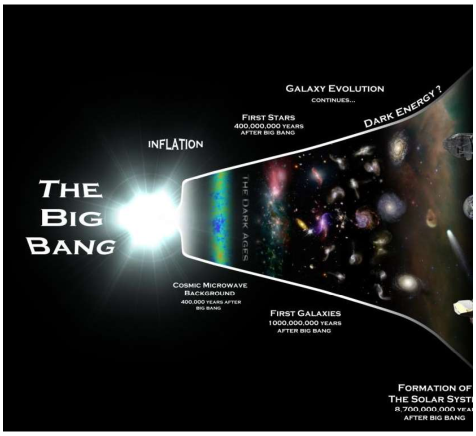
08December2021@JGRG30

Outline

1. Motivation
2. *Well-tempered* cosmology
   - ○ Self-tuning fields, degenerate states, Fab Four
3. Recent teleparallel gravity extensions
   - ○ Teledeski gravity
   - ○ The well-tempered recipe
   - ○ Dynamics in a well-tempered de Sitter model
4. (*In progress*) Observational status

> *The effort to understand the Universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy."*
>
> \- **S.W.**

The Cosmological Constant Problem

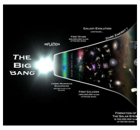


Our Universe

VS

Λ

The Cosmological Constant Problem



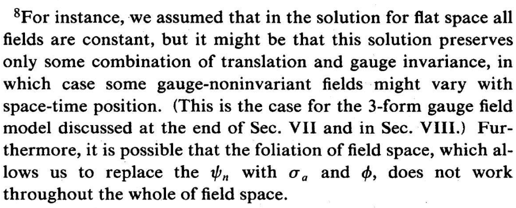
$$\ll\ll\ \ldots \ll\ll 120 \sim \Lambda$$

Our Universe

Steven Weinberg, *The cosmological constant problem*, Rev. Mod. Phys. 61 (1989) 1.
Antonio Padilla, *Lectures on the Cosmological Constant Problem*, arXiv:1502.05296.

Self-tuning fields

$$\phi(t) \quad \text{vs} \quad \Lambda$$



[8] For instance, we assumed that in the solution for flat space all fields are constant, but it might be that this solution preserves only some combination of translation and gauge invariance, in which case some gauge-noninvariant fields might vary with space-time position. (This is the case for the 3-form gauge field model discussed at the end of Sec. VII and in Sec. VIII.) Furthermore, it is possible that the foliation of field space, which allows us to replace the ψ_n with σ_a and ϕ, does not work throughout the whole of field space.

footnote 8, page 11

Well-tempered cosmology

- Use $\phi(t)$ to design a low energy vacuum state $\left(H(t) = h, \phi(t)\right)$ s.t.

$$\boxed{\begin{gathered} 3H^2 = \rho_\Lambda + \rho_\phi \\ 2\dot{H} + 3H^2 = -P_\Lambda - P_\phi \\ \ddot{\phi} f + \dot{\phi} g + \phi k = j \end{gathered}}$$

Vacuum State : $\phi(t)$ vs $\boldsymbol{\Lambda}$

- Result: Screen $\boldsymbol{\Lambda}$ with *theory constants* of order *unity*.
- Price: E.g., in KGB, $K(X), G(X) \to q[K(X), G(X)]$

S. Appleby and E. V. Linder, *The Well-Tempered Cosmological Constant*, JCAP 07 (2018) 034 [1805.00470].

Recent teleparallel gravity extensions

- ***TEGR***: Teleparallel Equivalent of GR
- ***Teledeski***: ***Tele***parallel Analogue of Horn***deski*** gravity

$$\mathcal{L}_{\rm Tele} := G_{\rm Tele}\left(\underbrace{\phi, X}_{\phi(t)}, \underbrace{T, T_{\rm ax}, T_{\rm vec}}_{T(t)}, \underbrace{I_2, J_1, J_3, J_5, J_6, J_8, J_{10}}_{\phi - T \text{ couplings}}\right)$$

- ***Well-tempered Teledeski models***: 2107.08762 & 2108.02500

Well-tempered cosmology

- $H = h =$ constant overconstrain the dynamical system

$$\dot{H} = \ddot{\phi} Z(\phi, \dot{\phi}, H) + Y(\phi, \dot{\phi}, H)$$
$$0 = \ddot{\phi} D(\phi, \dot{\phi}, H) + C(\phi, \dot{\phi}, H, \dot{H})$$

- Utilize *degeneracy*:
  - ○ $D(\phi, \dot{\phi}, H) = C(\phi, \dot{\phi}, H) = 0$, **Fab Four**
  - ○ $Z(\phi, \dot{\phi}, H) \propto D(\phi, \dot{\phi}, H)$ *and* $Y(\phi, \dot{\phi}, H) \propto C(\phi, \dot{\phi}, H, \dot{H})$, **Well-tempering**

Dynamics in a Well-tempered de Sitter model

$\boldsymbol{\rho_\Lambda / h^2 \sim 10^{10}}$, Λ is ten(!) orders of magnitude > de Sitter vacuum

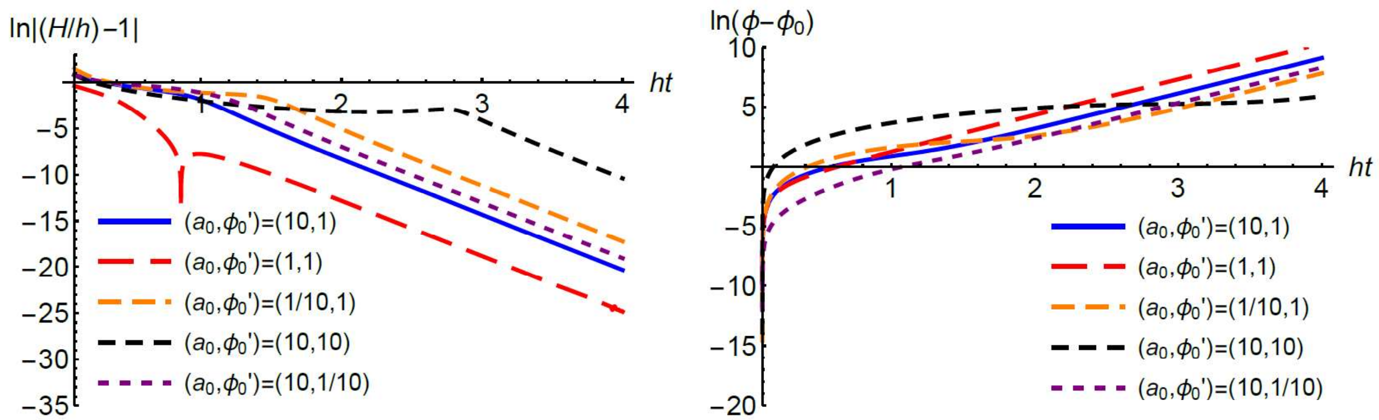


Figure 1. Results of numerical integration in well-tempered model with $\rho_\Lambda / h^2 \sim 10^{10}$ with theory constants of order unity.

Dynamics in a Well-tempered de Sitter model

$\boldsymbol{\rho_\Lambda/h^2} \sim \mathbf{10^{10}}$, Λ is ten(!) orders of magnitude > de Sitter vacuum

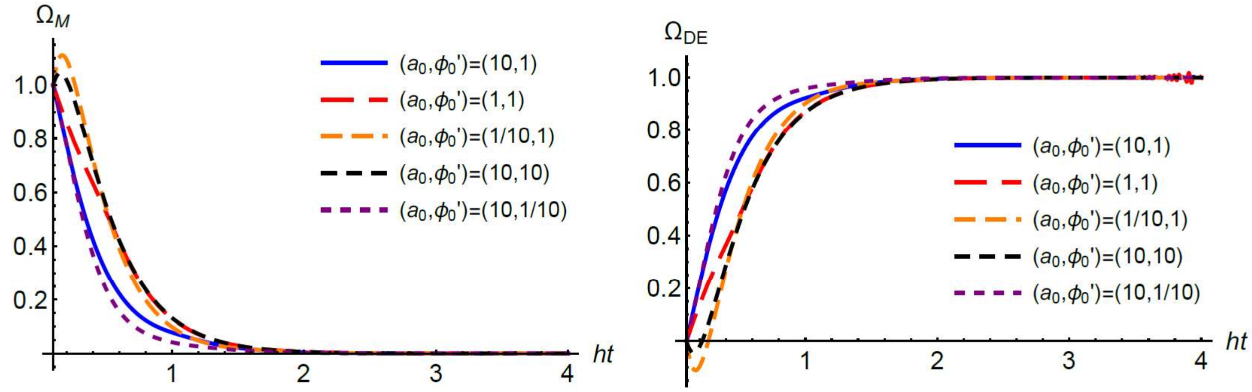
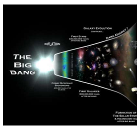


Figure 2. Results of numerical integration in well-tempered model with $\rho_\Lambda/h^2 \sim 10^{10}$ with theory constants of order unity.

(*In progress*) Observational status

A well-tempered dark energy?



$\ll \cdots \ll \sim \boldsymbol{\Lambda}$

Consider an example:

$$S_g[e,\phi] = \int d^4x\, e \left[l\phi - \frac{l}{18h^2}\left(\frac{2\gamma}{\sqrt{X}} + \ln X\right)\partial^2\phi + \frac{\boldsymbol{M_{\rm Pl}^2}}{\mathbf{2}}(-\boldsymbol{T} + \boldsymbol{B})\right]$$

where e = tetrad, ϕ = scalar field, $M_{\rm Pl}^2 = 1/8\pi G$

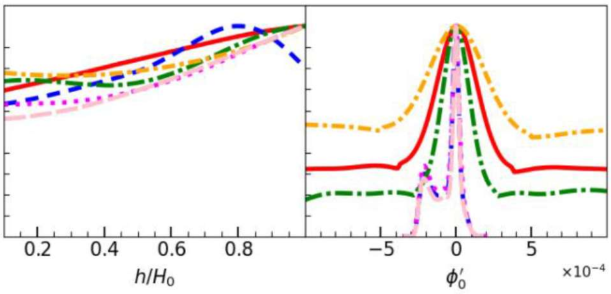
(*In progress*) Observational status

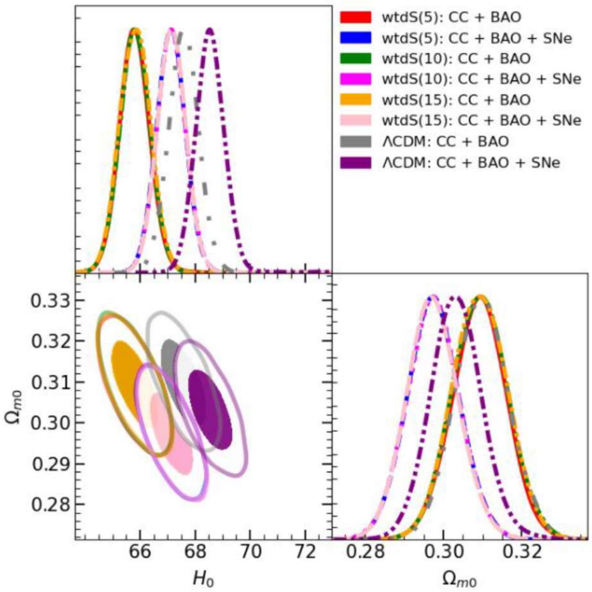
A well-tempered dark energy?

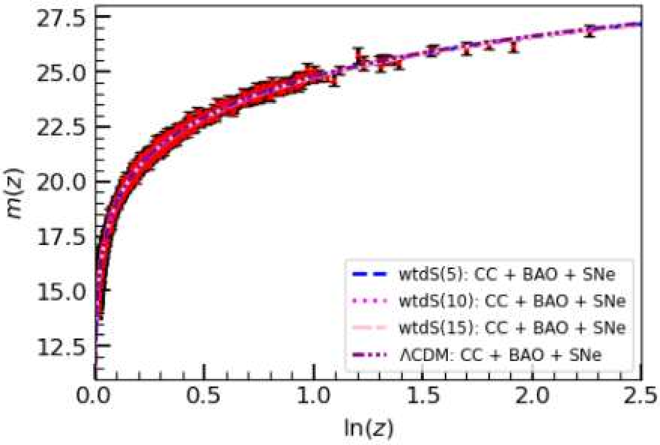
Preliminary: w/ CC + BAO + SNe

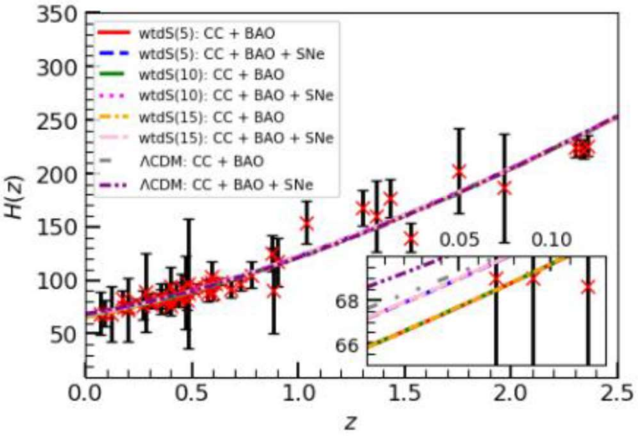
wtdS(*X*): $\rho_\Lambda/h^2 \sim 10^X$

theory constants $\sim O(1)$



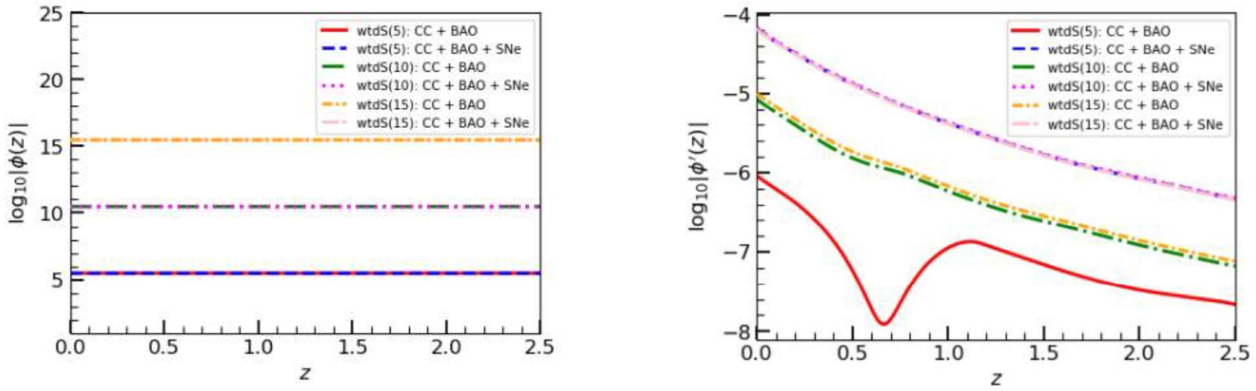






(*In progress*) Observational status

Best fit Hubble function and SNe apparent magnitudes for well-tempered de Sitter models wtdS(*X*) and ΛCDM.



(*In progress*)
Observational status

Best fit scalar field and its first derivative for the well-tempered de Sitter models and ΛCDM.

Outlook

- Well-tempered cosmology
  - Screening an *arbitrary large* Λ with $\phi(t)$ to obtain a late-time, low energy state
  - Can be achieved in models with scalar field potentials: **Horndeski/Teledeski**
- (*In progress*) Is dark energy well-tempered?

References

[1] Steven Weinberg, *The cosmological constant problem*, Rev. Mod. Phys. 61 (1989) 1.

[2] Antonio Padilla, *Lectures on the Cosmological Constant Problem*, arXiv:1502.05296.

[3] S. Appleby and E. V. Linder, *The Well-Tempered Cosmological Constant*, JCAP 07 (2018) 034 [1805.00470].

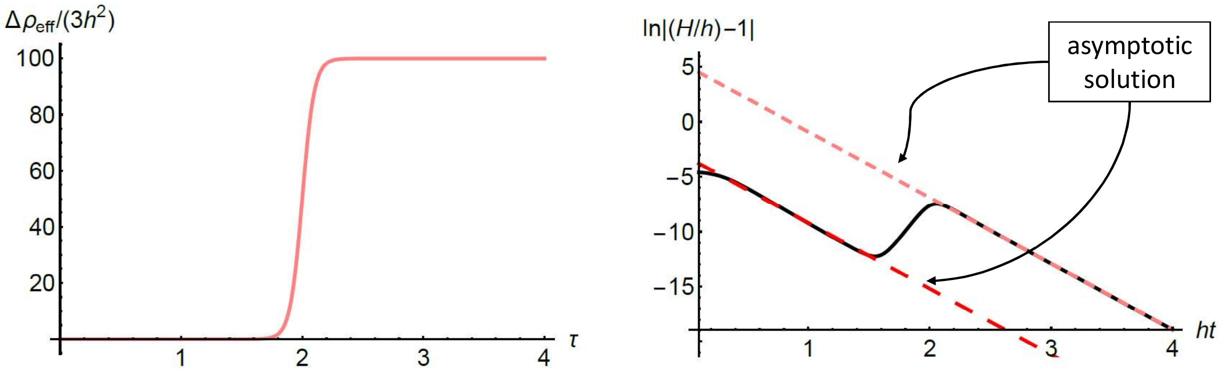
[4] **RCB**, J. Levi Said, M. Caruana, S. Appleby, *Well-Tempered Teleparallel Horndeski Cosmology: A Teleparallel Variation to the Cosmological Constant Problem*, arXiv:2107.08762.

[5] **RCB**, J. Levi Said, M. Caruana, S. Appleby, *Well-Tempered Minkowski Solutions in Teleparallel Horndeski Theory*, arXiv:2108.02500.

Extra slides

Phase transition in a well-tempered model

$\boldsymbol{\rho_\Lambda/h^2} \sim \mathbf{10^{10}}$, Λ is ten(!) orders of magnitude > de Sitter vacuum



asymptotic solution

Figure 3. Phase transition in well-tempered model with $\rho_\Lambda/h^2 \sim 10^{10}$ with theory constants of order unity.

Session C2a 15:30–16:45

[Chair: Tsutomu Kobayashi]

Pheiroijam Suranjoy Singh

Bodoland University

“Is the cosmic doomsday inevitable when the dark energy EoS parameter is less than -1?”

(15 min.)

[JGRG30 (2021) 120820]

Is the cosmic doomsday inevitable when the dark energy EoS parameter is less than -1?

Based on: Chin. J. Phys., DOI - 10.1016/j.cjph.2021.05.022 (2021)
Article in Press

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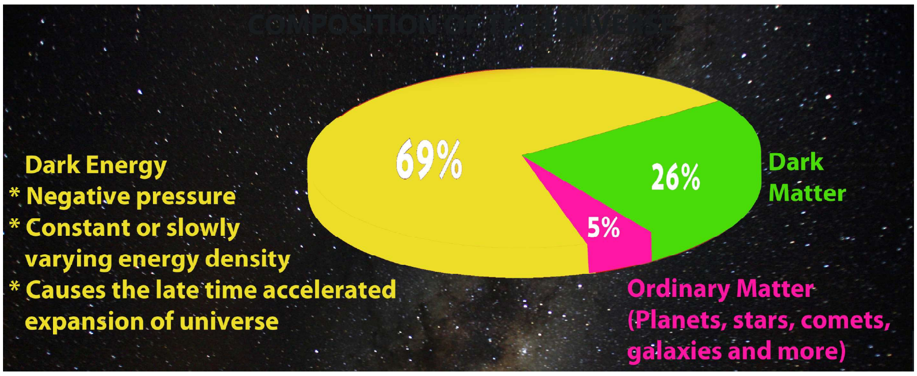
The 30th Workshop on General Relativity and Gravitation in Japan(JGRG30)

December 8, 2021

Overview

Introduction



Introduction

- Dark energy (DE) was discovered in 1998 [1,2].
- DE nature and properties still remain a mystery.
- Equation of state (EoS) parameter $\lambda = \frac{p_{de}}{\rho_{de}}$ classifies DE into specific categories.
  - $\lambda < -1 \implies$ Phantom energy [3].
  - $\lambda = -1 \implies$ Cosmological constant [3].
  - $-1 < \lambda \implies$ Quintessence [4].
- According to the latest Planck 2018 results [5], $\lambda = -1.03 \pm 0.03$ $\implies$ DE component dominating the universe is of phantom type.

---

1. Riess A. G. et al., Astron. J., 116 (1998) 1009.
2. Perlmutter S. et al., Astrophys. J., 517 (1999) 565.
3. Caldwell R. R., Phys. Lett. B, 545 (2002) 23.
4. Steinhardt P. J. et al., Phys. Rev. D, 59 (1999) 123504.
5. Collaboration P. et al., A&A 641 (2020) A6.

Introduction

- Phantom energy $\implies$ cosmic doomsday [6].
- The phantom energy will rip apart the matter of the universe, from stars and galaxies to atoms and subatomic particles, and even space-time at a certain time in the future, until distances between particles become infinite.

---

6. Caldwell R. R. et al., Phys. Rev. Lett. 91 (2003) 071301.

Introduction

Higher dimensional cosmological model

- Nordstorm proposed the first theory involving extra dimensions [7].
- Kaluza and Klein introduced higher dimension in GR [8,9].

Why higher dimension?

- The extra dimension in 5D might correspond to DE and DM [10].
- Makes HDE models more complete and consistent [11].
- Describes the late time expanding phenomenon [12].
- Higher dimensional phase during the early evolution [13].

---

7. Nordstrom G., Phys. Z., 15 (1914) 504–506.

8. Kaluza T., Sitzungsberichte Der Königlich Preussen Akademie Der Wissenschaften (Berlin),1921 (1921) 966–972.

9. Klein O., Z. Phys., 37 (1926) 895–906.

10. Chakraborty S. & Debnath U., Int. J. Theor. Phys. 49 (2010) 1693.

11. Zhang X. , Phys. Lett. B, 683 (2010) 81.

12. Banik S. K. & Bhuyan K., Pramana J. Phys. 88 (2017) 26.

13. Singh G. P. et al., Pramana J. Phys. 63 (2004) 937.

Introduction

Saez-Ballester theory [14]: The action for the SBT is given by

$$S = \int d^5x\sqrt{-g}\left[\varphi R - \omega\varphi^n g^{ij}\varphi_{,i}\varphi_{,j}\right] + 8\pi L_m \tag{1}$$

where φ is the scalar field, R is the curvature scalar corresponding to the 5D metric g_{ij}, and L_m is the 5D Lagrangian of matter fields.

Field equations:

$$R_{i\,j} - \frac{1}{2}g_{i\,j}R - \omega\varphi^n\left(\varphi_{,\,i}\varphi_{,\,j} - \frac{1}{2}g_{i\,j}\varphi_{,\,k}\varphi^{,\,k}\right) = -\left(T_{i\,j} + S_{i\,j}\right), \tag{2}$$

where $T_{i\,j}$ and $S_{i\,j}$ are the energy momentum tensors for matter and HDE respectively, ω is the SB coupling parameter, and R_{ij} is the tensor. φ satisfies

$$2\varphi^n\varphi^{;i}_{;i} + n\varphi^{n-1}\varphi_{,k}\varphi^{,k} = 0, \tag{3}$$

where n is an arbitrary constant.

---

14. Saez D. &, Ballester V. J., Phys. Lett. A 113 (1986) 467.

Introduction

Why Saez-Ballester theory?

- One of scalar-tensor theories (STT's) of gravitation.
- STT's can be considered as perfect candidates for DE [15].
- φ in SBT can lead to the emergence of an anti-gravity phase [16].
- φ can also illustrate prodigies like DE and DM [17].

---

15. Mandal, R. et al., JHEP, 05 (2018) 078.
16. Rao V. U. M. et al., Astrophys. Space Sci., 337 (2012) 499–501.
17. Aditya Y. et al., Indian J. Phys., 95 (2021) 383–389.

Introduction

Spherically symmetric metric [18]

$$ds^2 = dt^2 - e^{\mu}\left(dr^2 + r^2 d\Theta^2 + r^2 \sin^2 \Theta d\phi^2\right) - e^{\delta} dy^2 \tag{4}$$

where $\mu = \mu(t)$ and $\delta = \delta(t)$ are cosmic scale factors.

Why is spherically symmetric (SS) space-time important?

- Comparative simplicity.
- The space-time used in relativistic cosmology, including the space-time of the de-Sitter and the Einstein universes, is SS [19].
- The Robertson-Walker model depicting the expanding cosmos is also SS [20].

---

18. Samanta G. C., & Dhal S. N., Int. J. Theor. Phys., 52 (2013) 1334–1344. .

19. Takeno H., Prog. Theor. Phys., 8 (1952a) 317–326.

20. Karade T. M., Indian J. Pure Appl. Math., 11 (1980) 1202–1209.

Formulation of problem

Considerations in the study

- Interacting HDE and DM.
- 5D SS space-time
- Saez-Ballester theory

If the cosmic doomsday is inevitable when the DE EoS parameter λ is less than -1?

- HDE $\implies$ all physical quantities inside the universe including the energy density of DE can be described by some quantities on the boundary of the universe [20].
- The expansion of the universe at an accelerated rate is a pure manifestation of the interaction of DE and DM [21].

---

20. Wang S. et al., Phys. Rep. 696 (2017) 1.

21. Zimdahl W., AIP Conf. Proc., 1471 (2012) 51.

Formulation of problem

Energy momentum tensor for DM:

$$T_{i\,j} = \rho_m u_i u_j \tag{5}$$

Energy momentum tensor for HDE:

$$S_{i\,j} = (\rho_{de} + p_{de})\, u_i u_j - g_{ij} p_{de} \tag{6}$$

ρ_m = energy density of matter.
ρ_{de} = energy density of HDE.
p_{de} = pressure of the HDE.

Formulation of problem

Surviving field equations:

$$\frac{3}{4}\left(\dot{\mu}^2 + \dot{\mu}\dot{\delta}\right) + \frac{\omega}{2}\varphi^n \dot{\varphi}^2 = \rho \tag{7}$$

$$\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 + \frac{\ddot{\delta}}{2} + \frac{\dot{\delta}^2}{4} + \frac{\dot{\mu}\dot{\delta}}{2} - \frac{\omega}{2}\varphi^n \dot{\varphi}^2 = -p_{de} \tag{8}$$

$$\frac{3}{2}\left(\ddot{\mu} + \dot{\mu}^2\right) - \frac{\omega}{2}\varphi^n \dot{\varphi}^2 = -p_{de} \tag{9}$$

Formulation of problem

The conservation equation $T^{ij}_{;j} + S^{ij}_{;j} = 0$ takes the form

$$\rho_m \left(\frac{3\dot{\mu} + \dot{\delta}}{2} \right) + \dot{\rho}_m + \dot{\rho}_{de} + \rho_{de} \left(1 + \lambda\right) \left(\frac{3\dot{\mu} + \dot{\delta}}{2} \right) = 0. \tag{10}$$

Considering minimal interaction between HDE and DM, by [22,23],

$$\rho_m \left(\frac{3\dot{\mu} + \dot{\delta}}{2} \right) + \dot{\rho}_m = 0 \tag{11}$$

$$\rho_{de} \left(1 + \lambda\right) \left(\frac{3\dot{\mu} + \dot{\delta}}{2} \right) + \dot{\rho}_{de} = 0 \tag{12}$$

---

22. Sarkar S., Astrophys. Space Sci. 349 (2014) 985.

23. Sarkar S., Astrophys. Space Sci. 352 (2014) 245.

Formulation of problem

Cosmic scale factors:

$$\mu = l_1 - \log\left(k - t\right)^{\frac{2}{3}}, \tag{13}$$

$$\delta = m_1 - \log\left(k - t\right)^{\frac{2}{3}}, \tag{14}$$

where l_1, m_1 and k are arbitrary constants.

Discussion

Throughout the discussion,
$l_0 = l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

Discussion

$$\text{Spatial volume:}v = e^{\frac{3l_1+m_1}{2}}(k-t)^{-\frac{4}{3}} \tag{15}$$

$$\text{Scalar expansion:}\theta = \frac{4}{3}(k-t)^{-1} \tag{16}$$

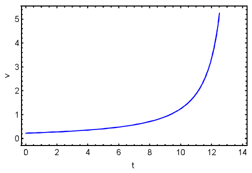


Figure: 1. v with t when when when $l_1 = m_1 = 1, k = 13.80497512437811$.

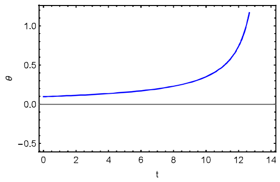


Figure: 2. θ with t when $k = 13.80497512437811$.

- v and θ increase $\Longrightarrow$ accelerated expansion.

Discussion

Energy density of DE:

$$\rho_{de} = m_0 e^{-\frac{1}{2}(1+\lambda)(3l_1+m_1)}\left(k-t\right)^{\frac{4}{3}(1+\lambda)} \tag{17}$$

Energy density of DM:

$$\rho_m = l_0 e^{-\frac{1}{2}(3l_1+m_1)}\left(k-t\right)^{\frac{4}{3}} \tag{18}$$

Discussion

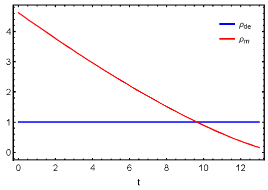


Figure: 3. ρ_{de} and ρ_m with t when $l_0 = l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

- ρ_m decreases, whereas ρ_{de} remains constant throughout.
- Due to the expansion, galaxies move apart from each other, leading the DM density to diminish gradually [24], whereas DE varies slowly or remains unchanged with time [24-27].

---

24. Carroll S. M., arXiv:astro-ph/0107571v2 (2001)
25. Carroll S. M., Living Rev. Rel. 4 (2001) 1
26. M. H. Chan M. H., J. Gravity 2015 (2015) 384673.
27. Peebles P. J. & Ratra B., Rev. Mod. Phys. 75 (2003) 559.

Discussion

Hubble parameter:

$$H = \frac{1}{3}(k-t)^{-1} \tag{19}$$

$t = 13.8 \implies H = 67$
$\approx H_0 = 67.36 \pm 0.54$ kms^{-1}Mpc^{-1} of the latest Planck 2018 result [5].

Anisotropic parameter:

$$A_h = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2 = 0 \tag{20}$$

---

5. Collaboration P. et al., A&A 641 (2020) A6.

Discussion

DE EoS parameter λ:

$$l_0 e^d (k-t)^{\frac{4}{3}} + m_0(1+\lambda)e^{(1+\lambda)d}(k-t)^{\frac{4}{3}(1+\lambda)} = 0, \tag{21}$$

where $d = -\frac{1}{2}(3l_1 + m_1)$.

$t = 13.8 \implies \lambda = -1.00011$
$\approx \lambda = -1.03 \pm 0.03$ of the latest Planck 2018 result [5].

---

5. Collaboration P. et al., A&A 641 (2020) A6.

Discussion

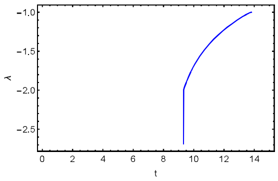


Figure: 4. λ with t when $l_0 = l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

- HDE is of phantom type $\implies$ cosmic doomsday [6].
- $\lambda \to -1 \implies$ DE departs from phantom-like nature to cosmological constant in the future.
- The model will bypass the cosmic doomsday, leading to the de-Sitter phase (a phase of late inflation).
- It will continue to expand, driven by cosmological constant, and all the matter and radiation will be diluted [28,29].

---

6. Caldwell R. R. et al., Phys. Rev. Lett. 91 (2003) 071301.

28. Zhang X. & Huang Y., Journal of Modern Physics, 8 (2017) 1234-1256.

29. Ridpath I., *A Dictionary of Astronomy (2nd ed.)*, Oxford University Press (2012).

Discussion

$$\text{DE pressure:} p_{de} = \lambda\, m_0 e^{-\frac{1}{2}(3l_1+m_1)(1+\lambda)} \left(k-t\right)^{\frac{4}{3}(1+\lambda)} . \qquad (22)$$

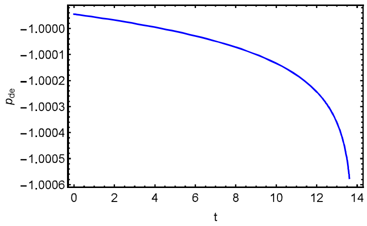


Figure: 5. p_{de} with t when when $l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

- DE pressure p_{de} lies in the negative plane.

Discussion

Shear scalar:

$$\sigma^2 = \frac{2}{9}\left(\frac{1}{k-t} - 1\right)^2 \qquad (23)$$

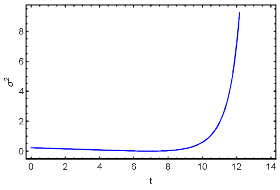


Figure: 6. σ^2 with t when $k = 13.80497512437811$.

- σ^2 shows the rate of deformation of the matter flow within the massive cosmos [16]
- Initially, σ^2 appears to decreases, and then it tends to diverge.
- The model universe expands with a slow and uniform change of size in the early evolution, whereas the change tends to become faster at late times. This is in agreement with the present observation.

---

16. Ellis G. F. R. & Elst H. V., NATO Adv. Study Inst. Ser. C. Math. Phys. Sci., 541 (1999) 1.

Discussion

$$\text{Sacalr field:}\varphi = \left((6+3n)\,(k-t)^{\frac{7}{3}} - 14c_1\right)^{\frac{2}{2+n}} c_2, \qquad (24)$$

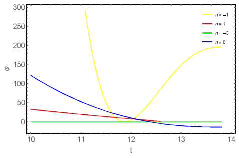


Figure: 7. φ with t when $c_1 = c_2 = 1$ (both positive).

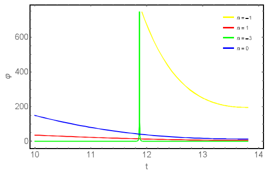


Figure: 8. φ with t when $c_1 = -1, c_2 = 1$ (opposite sign).

- When $n = -1$, φ tends to attain almost the same large positive constant, which might be the reason for the phantom-like nature of the DE at present. This observation is similar to that of [16].

---

15. Naidu R. L. et al., Heliyon 5 (2019) e01645.

Conclusions

- HDE of the accelerating isotropic model is of phantom type.
- In the far future, the DE departs from phantom-like nature to a cosmological constant, thereby bypassing the cosmic doomsday, and ultimately leading to the de-Sitter phase.
- The present values of the Hubble parameter and the DE EoS parameter are found to be $H = 67$ and $\lambda = -1.00011$, which agree with the respective values of the latest Planck 2018 result.
- The model expands with a slow and uniform change of size in the early evolution, whereas the change tends to become faster at late times.

The cosmic doomsday is evitable when the DE EoS parameter is less than -1.

Thank You

Session C2a 15:30–16:45

[Chair: Tsutomu Kobayashi]

Tiago Gonçalves

Institute of Astrophysics and Space Sciences, Faculty of Sciences of
the University of Lisbon

“Accelerated cosmological expansion in f(R,T) gravity”

(15 min.)

[JGRG30 (2021) 120821]

JGRG30
8th December 2021

ACCELERATED COSMOLOGICAL EXPANSION IN *f(R,T)* GRAVITY

Tiago B. Gonçalves
(IA-U.Lisboa)

Based on work with

João Luís Rosa
(Institute of Physics, U. Tartu)

Francisco S. N. Lobo
(IA-U.Lisboa)

[arXiv: **2112.02541**]

Acknowlegments: UIDB/04434/2020 & UIDP/04434/2020, PTDC/FIS-OUT/29048/2017, CEECIND/04057/201, CERN/FIS-PAR/0037/2019, MOBJD647.

$$\mathcal{L}_g \propto R$$

curvature scalar

$$\mathcal{L}_g \propto R \longrightarrow \mathcal{L}_g \propto f(R)$$

curvature scalar

$$\mathcal{L}_g \propto R \longrightarrow \mathcal{L}_g \propto f(R) \longrightarrow \mathcal{L}_g \propto f(R,T)$$

curvature scalar

trace stress-energy

[Harko+ 1104.2669]

$$S = \frac{1}{2\kappa^2}\int f(R,T)\sqrt{-g}d^4x + \int \mathcal{L}_m\sqrt{-g}d^4x$$

Modified field equations

GR: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu}$

$$f_R R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R,T) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R = \kappa^2 T_{\mu\nu} - f_T(T_{\mu\nu} + \Theta_{\mu\nu})$$

$$f_R = \frac{\partial f(R,T)}{\partial R}$$

$$f_T = \frac{\partial f(R,T)}{\partial T}$$

$$T_{\mu\nu} + \Theta_{\mu\nu} = \frac{\delta\left(g^{\rho\sigma}T_{\rho\sigma}\right)}{\delta g^{\mu\nu}}$$

Scalar-tensor representation

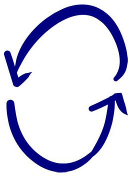
[Rosa 2103.11698]

- Define two scalar fields & potential

$$\varphi \equiv \frac{\partial f}{\partial R} \qquad \psi \equiv \frac{\partial f}{\partial T} \qquad V(\varphi, \psi) \equiv -f(R,T) + \varphi R + \psi T$$

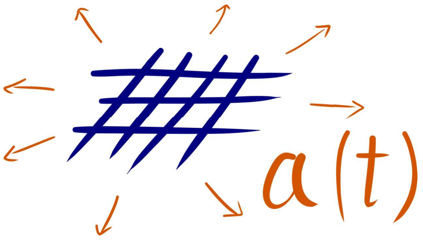
$$S = \frac{1}{2\kappa^2}\int \left[\varphi R + \psi T - V(\varphi,\psi)\right]\sqrt{-g}d^4x + \int \mathcal{L}_m\sqrt{-g}d^4x$$

Reconstruction methods



- Function *f(R,T)* not specified *a priori*
- Start from observed evolution of the universe
- Check if there are consistent solutions.

Assumptions



- FLRW metric
- Perfect fluid $p = w\rho$
- Conservation of stress-energy $\nabla^{\mu} T_{\mu\nu} = 0$

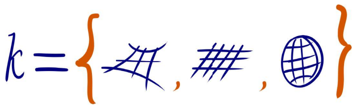
[Gonçalves+ 2112.02541]

Constraints

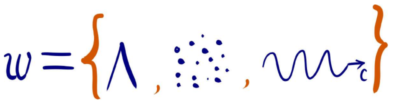
- Scale factor (exponential and power-laws)

$$a \propto \left\{ e^{t},\ t^{2/3},\ t^{1/2} \right\}$$

- Curvature parameter k={-1,0,1}



- Equation of state w={-1,0,1/3}



[Gonçalves+ 2112.02541]

Results

In general:

$$\rho = \rho_0 \, a(t)^{-3(1+w)} \qquad \psi = \psi_0 \, a(t)^{3(1-w)/2}$$

[Gonçalves+ 2112.02541]

Results

In general:

$$\rho = \rho_0 \, a(t)^{-3(1+w)} \qquad \psi = \psi_0 \, a(t)^{3(1-w)/2}$$

Choosing $a(t) = a_0 \, e^{\sqrt{\Lambda}(t-t_0)}$:

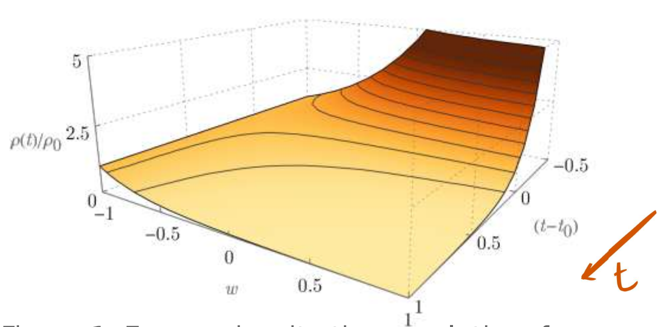


Figure 1. Energy density time evolution, for a range of values of the equation of state.

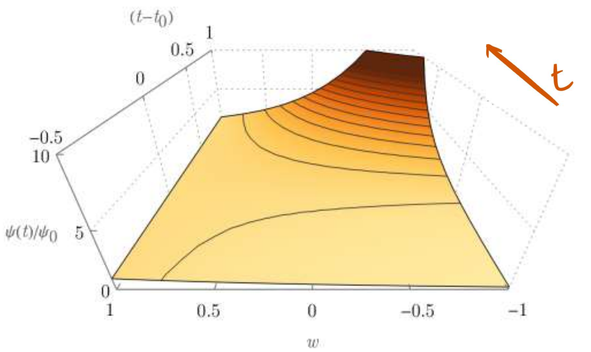


Figure 2. Scalar field ψ time evolution, for a range of values of the equation of state.

Results

Exponential expansion $a(t) = a_0 \, e^{\sqrt{\Lambda}(t-t_0)}$

\+ spatially-flat $k=0$

\+ matter-dominated $w=0$

$$V(\varphi,\psi) = V_0 + 12\Lambda\varphi(t) + \frac{\rho_0\psi_0^2}{\psi(t)}$$

$$\varphi(t) = \varphi_0 \, a(t) - \frac{2\rho_0}{3\Lambda}\left[\pi \, a(t)^{-3} + \frac{2\psi_0}{5} a(t)^{-3/2}\right] - \frac{V_0}{6\Lambda}$$

$$f(R,T) = g(R) + 2\psi_0\sqrt{-\rho_0 T}$$

$g(R)$ is an arbitrary function of R.
Symbols with a subscript 0 are arbitrary integration constants.

Results

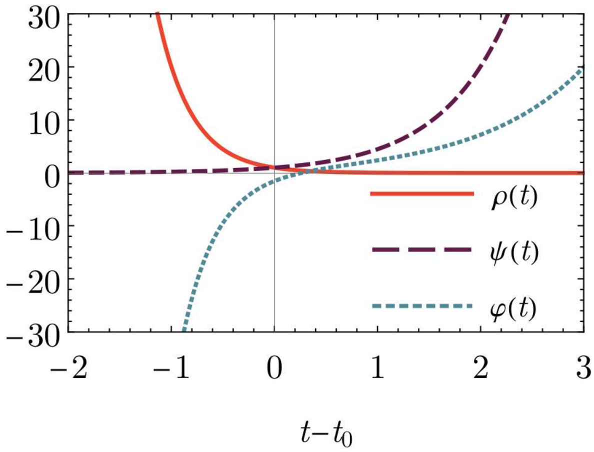


Figure 3. Solutions of the energy density and the two scalar fields. With exponential scale factor, k=0 and w=0 and all constants set to 1.

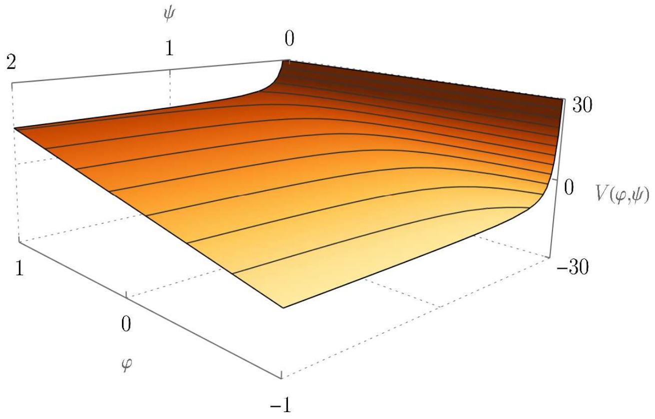


Figure 4. Potential dependence on the two scalar fields. With exponential scale factor, k=0, w=0 and all constants set to 1.

Conclusion

- Scalar-tensor *f(R,T)* gravity

$$S = \frac{1}{2\kappa^2}\int\left[\varphi R + \psi T - V(\varphi,\psi)\right]\sqrt{-g}d^4x + \int \mathcal{L}_m\sqrt{-g}d^4x$$

- We have given here one example of a solution where dust is the total contribution to energy density and the universe expansion is exponential.
- This is possible due to the extra gravitational components which act as effective dark energy.
- We are currently studying whether there can be future singularities.

Thank you!

[Gonçalves+ **2112.02541**]

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Acknowlegments: UIDB/04434/2020 & UIDP/04434/2020, PTDC/FIS-OUT/29048/2017, CEECIND/04057/201, CERN/FIS-PAR/0037/2019, MOBJD647.

Session C2a 15:30–16:45

[Chair: Tsutomu Kobayashi]

Ricardo Landim

Technical University of Munich

“Fractional Dark Energy”

(15 min.)

[JGRG30 (2021) 120822]

30th Workshop on General Relativity and Gravitation - Dec. 8 2021

Fractional Dark Energy

Ricardo Landim
Technical University of Munich

Based on:
- 2101.05072 (PRD 103, 2021)
- 2106.15415 (PRD 104, 2021)

Unterstützt von / Supported by
Alexander von Humboldt
Stiftung / Foundation

Thermodynamics of a dark fluid

Using the second law of thermodynamics: [Lima, Alcaniz, PLB 600, 191 (2004)]

$$\rho \propto T^{\frac{1+w}{w}} \propto V^{-(1+w)}$$

$$\rho = C_0 \int_0^\infty \frac{\varepsilon^{\frac{1}{w}}}{e^{\beta\varepsilon}+1} d\varepsilon$$

Constant *w* and valid only for fermions (bosons give a negative ρ)

Fractional dark energy

Density of states: $D(\varepsilon) \propto \varepsilon^{\frac{1}{w}-1}$

Non-canonical kinetic term:

$$\varepsilon \approx m + \frac{p^2}{2m} + \underbrace{\frac{C}{p^{-3w}}}_{Cp^{3w} \gg m}$$

$$\Big\downarrow$$

$$\varepsilon \approx \frac{C}{p^{-3w}}$$

$$N_\varepsilon = -\frac{C^{-\frac{1}{w}} gV}{6\pi^2 w} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \frac{\varepsilon^{\frac{1}{w}-1}}{e^{\beta\varepsilon}+1} d\varepsilon$$

$\varepsilon_{\min} \sim m$

$\varepsilon_{\max}$ to avoid divergence when $p \to 0$

Fractional dark energy

$$n = -\frac{C^{-\frac{1}{w}} g}{6\pi^2 w} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \frac{\varepsilon^{\frac{1}{w}-1}}{e^{\beta\varepsilon}+1} d\varepsilon$$

$$= -\frac{C^{-\frac{1}{w}} g}{6\pi^2 w} \beta^{-\frac{1}{w}} \mathscr{F}^{u_{\max}}_{u_{\min}, \frac{1}{w}-1}$$

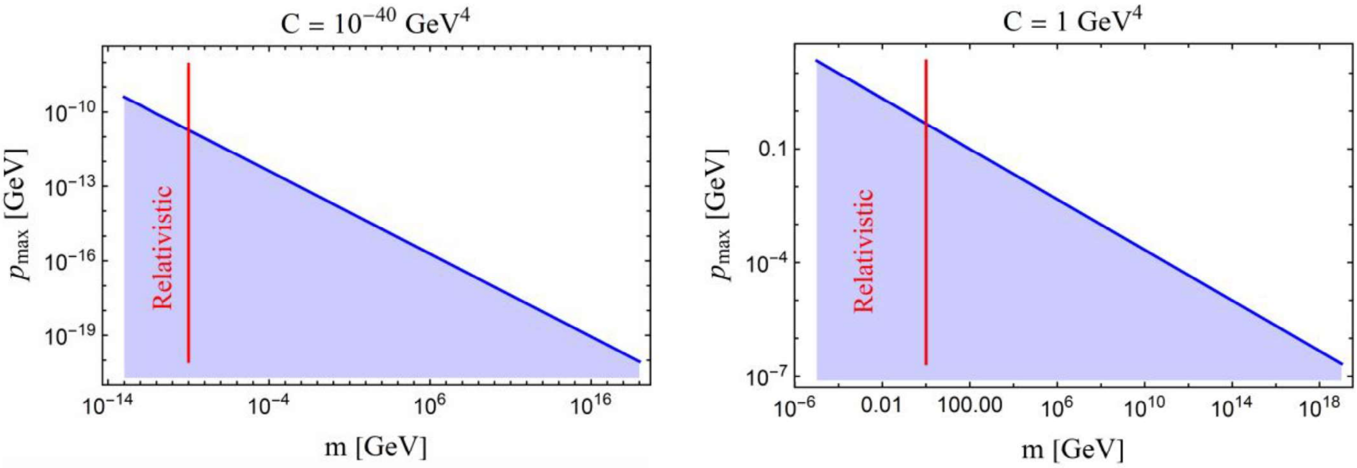
$$\mathscr{F}^{u_{\max}}_{u_{\min}, a} \equiv \int_{u_{\min}}^{u_{\max}} \frac{u^{\frac{1}{a}}}{e^u+1} du$$

$$\rho = -\frac{C^{-\frac{1}{w}} g}{6\pi^2 w} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \frac{\varepsilon^{\frac{1}{w}}}{e^{\beta\varepsilon}+1} d\varepsilon$$

$$= -\frac{C^{-\frac{1}{w}} g}{6\pi^2 w} \beta^{-\frac{1+w}{w}} \mathscr{F}^{u_{\max}}_{u_{\min}, \frac{1}{w}} = 10^{-47} \text{ GeV}^4$$

$$\rho = \beta^{-1} \frac{\mathscr{F}^{u_{\max}}_{u_{\min}, \frac{1}{w}}}{\mathscr{F}^{u_{\max}}_{u_{\min}}, \frac{1}{w}} n$$

Fractional dark energy



$u_{\min} = 10$ (left) and $u_{\min} = 100$ (right)

Fractional Quantum Mechanics

- Developed by N. Laskin in 2000
- Generalization of QM using fractional calculus

- Applied to several QM problems

Riemann-Liouville derivative:

$$_aD_x^\alpha f(x) = \frac{1}{\Gamma(n+1-\alpha)}\frac{d^{n+1}}{dx^{n+1}}\int_a^x (x-y)^{n-\alpha} f(y)dy \qquad n \leq \alpha < n+1$$

$$_aD_x^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-y)^{\alpha-1} f(y)dy\,, \quad \alpha > 0$$

$$_aD_b^\alpha({}_aD_b^{-\alpha} f(x)) \;=\; f(x)$$

$$_aD_b^{\pm\alpha}({}_aD_b^{\pm\beta} f(x)) = {}_aD_b^{\alpha\pm\beta} f(x)$$

Fractional Quantum Mechanics

- Riesz fractional derivative

$$\left(-i\hbar\frac{\partial}{\partial x}\right)^{-\alpha} \equiv \frac{1}{2}\left({}_{-\infty}D_x^{-\alpha} +_x D_\infty^{-\alpha}\right)$$

- fractional Laplacian operator

$$(-\hbar^2\Delta)^{\alpha/2}\psi(\mathbf{r},t) = \frac{1}{(2\pi\hbar)^3}\int d^3p e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}|\mathbf{p}|^\alpha\varphi(\mathbf{p},t)$$

- fractional Schrödinger equation for FDE

$$C = \lambda^6/M_{Pl}^2,$$

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = C(-\hbar^2\Delta)^{3w/2}\psi(\mathbf{r},t) \longrightarrow \varepsilon \approx \frac{C}{p^{-3w}}$$

λ~ 0.5- 10^{19} GeV

Negative Absolute Temperature

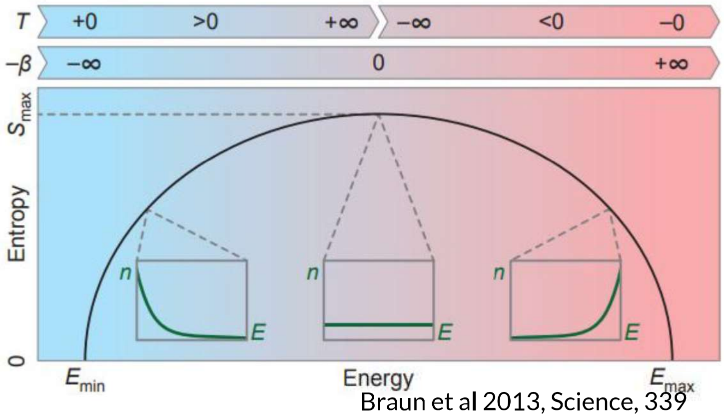
Late 1940’s and 1950’s Pound, Purcell, Onsager and Ramsey studied experimentally and theoretically NAT
[Il Nuovo Cimento 6, 1949, Physical Review, 81, 1951, 103, 1956.]

- Crystals, lasers, motional degrees of freedom, etc. → Negative pressures!

- Lord Kelvin introduced the concept of absolute temperature where absolute zero is the point where particles don't move

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N} \qquad P_i \propto e^{-E_i/k_B T}$$

+0 K, … , +300 K, … , +∞ K, −∞ K, … , −300 K, … , −0 K.



Braun et al 2013, Science, 339

NAT in Cosmology

J. Vieira, C. Byrnes, and A. Lewis. JCAP 2016

$$
\begin{aligned}
n(T,\mu) &= \int_m^\Lambda D(\epsilon)\mathcal{N}(T,\epsilon,\mu)d\epsilon \\
\rho(T,\mu) &= \int_m^\Lambda \epsilon D(\epsilon)\mathcal{N}(T,\epsilon,\mu)d\epsilon \\
P(T,\mu) &= \beta^{-1}\int_m^\Lambda \epsilon D(\epsilon)\ln(1+e^{-\beta(\epsilon-\mu)})d\epsilon
\end{aligned}
$$

Using properties of FD distribution:

$$
\begin{aligned}
\mathcal{N}(T,\epsilon,\mu) &= \frac{1}{e^{\beta(\epsilon-\mu)}+1} = 1-\frac{1}{e^{-\beta(\epsilon-\mu)}+1} \\
&= 1-\mathcal{N}(-T,\epsilon,\mu) \\
\ln\left[1+e^{-\beta(\epsilon-\mu)}\right] &= -\beta(\epsilon-\mu)+\ln\left[1+e^{\beta(\epsilon-\mu)}\right]
\end{aligned}
$$

NAT and FDE

$$
\begin{aligned}
n(T,\mu) &= n_{\max} - n(-T,\mu) \\
\rho(T,\mu) &= \rho_{\max} - \rho(-T,\mu) \\
P(T,\mu) &= -\rho_{\max} + \mu n_{\max} - P(-T,\mu)
\end{aligned}
$$

T = -0 K

$$
\begin{aligned}
n_{\max} &= \int_m^\Lambda D(\epsilon)d\epsilon = \frac{C^{-\frac{1}{w}}g}{6\pi^2}\left(m_0^{\frac{1}{w}} - \Lambda^{\frac{1}{w}}\right) \\
\rho_{\max} &= \int_m^\Lambda \epsilon D(\epsilon)d\epsilon \overset{w=-1}{=} \frac{Cg}{6\pi^2}\left[\ln(\Lambda) - \ln(m_0)\right]
\end{aligned}
$$

$$
\left.
\begin{aligned}
n_h &= -\frac{C^{-\frac{1}{w}}g}{6\pi^2 w}|\beta|^{-\frac{1}{w}}\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},\,\frac{1}{w}-1} \\
\rho_h &= -\frac{C^{-\frac{1}{w}}g}{6\pi^2 w}|\beta|^{-\frac{1+w}{w}}\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},\,\frac{1}{w}}
\end{aligned}
\right\rbrace \text{Similar to } +T
$$

NAT and FDE

$$\dot{n}_p + 3Hn_p = 0$$
$$\dot{n}_h + 3H(n_h - n_{\max}) = 0$$
$$\dot{\rho}_p + 3H(1+w)\rho_p = 0$$
$$\dot{\rho}_h + 3H(1+w)\rho_h = 0$$

$$\rho_h = |\beta|^{-1} \frac{\mathscr{F}^{u_{\max},\, \beta\mu}_{u_{\min},\, \frac{1}{w}}}{\mathscr{F}^{u_{\max},\, \beta\mu}_{u_{\min},\, \frac{1}{w}-1}} n_h$$

Taking the time derivative:

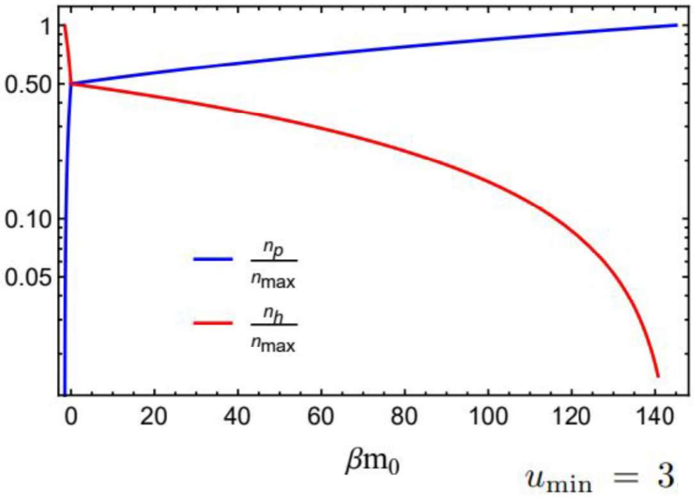
$$\dot{\rho}_p = -3H(1+w)\rho_p$$
$$\dot{\rho}_h = -3H(1+w)\rho_h + 3H(1+w)\rho_h \frac{n_{\max}}{n_h}$$

$$\downarrow$$

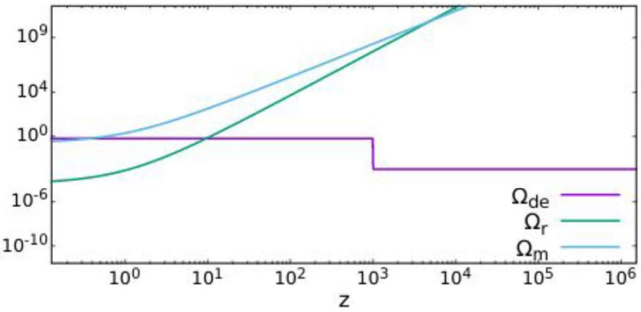
$$w \approx -1$$

FDE and NAT

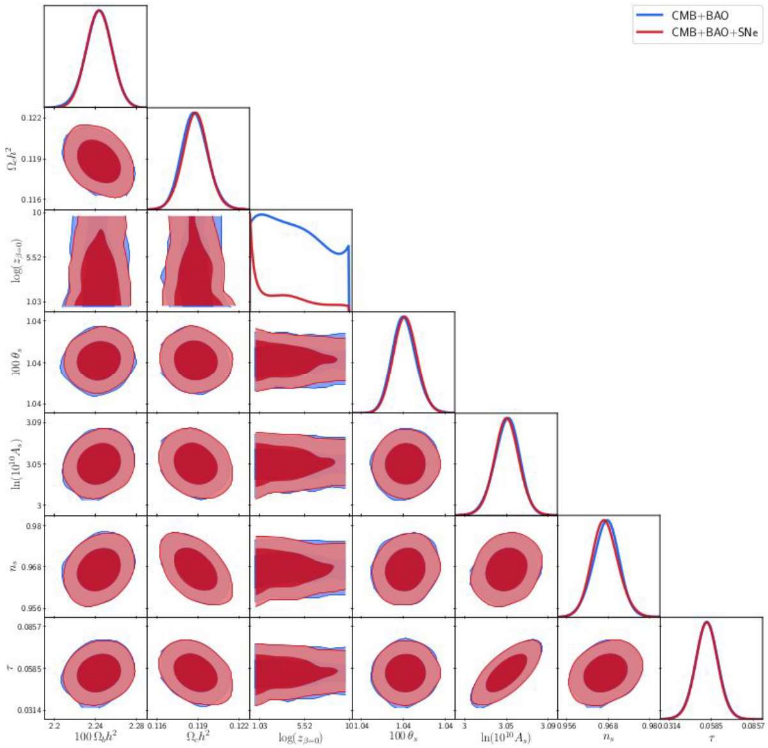
$$n_p = \frac{Cg}{6\pi^2} \beta \mathscr{F}^{u_{\max},\, +\beta}_{u_{\min},\, -2} + \frac{n_{\max}}{2}$$
$$n_h = \frac{Cg}{6\pi^2} |\beta| \mathscr{F}^{u_{\max},\, -\beta}_{u_{\min},\, -2} + \frac{n_{\max}}{2}$$



$u_{\min} = 3$



FDE and NAT



Conclusions

- FDE is a fluid with non-canonical kinetic term and it is described by FQM
- Gives a cosmological constant
- Connection between FDE and NAT

Thank you!!!

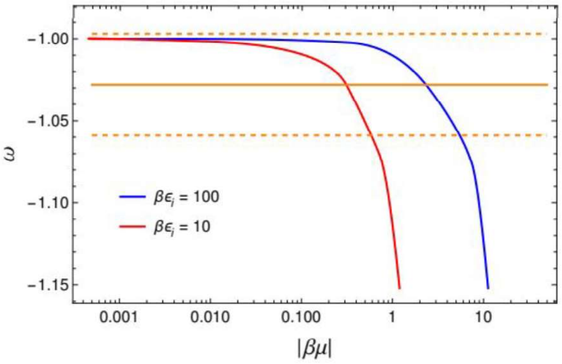
Chemical Potential

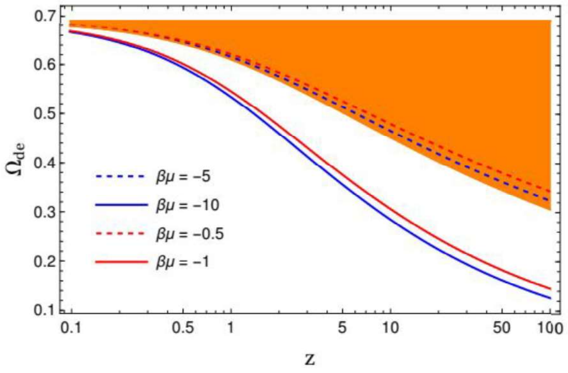
$$Ts = \rho + P - \mu n = (1 + w)\rho - \mu n$$

$$\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},a} \equiv \int_{u_{\min}}^{u_{\max}} \frac{(u + \beta\mu)^a}{e^u \pm 1} du$$

$$w \geq -1 + \frac{\mu n}{\rho}$$

$$w = -1 - \frac{|\mu_0| n_0}{\rho_0} = -1 - |\beta\mu| \frac{\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},\,\frac{1}{w}-1}}{\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},\,\frac{1}{w}}}$$





Session C2b 15:30–16:45

[Chair: Teruaki Suyama]

Kota Hayashi

YITP, Kyoto U.

“General-relativistic neutrino-radiation magnetohydrodynamics simulations of black hole-neutron star merger”

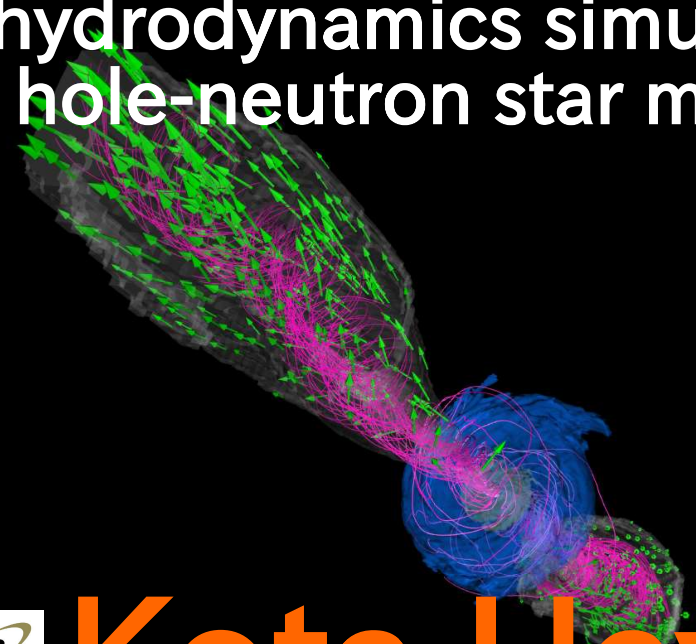
(15 min.)

[JGRG30 (2021) 120823]

The 30th Workshop on General Relativity and Gravitation in Japan, Dec. 2021

General-relativistic neutrino-radiation magnetohydrodynamics simulations of black hole-neutron star mergers

arXiv:2111.04621



YITP, D2

Kota Hayashi

Collaborators :
Sho Fujibayashi (AEI) , Kenta Kiuchi (AEI) , Koutarou Kyutoku (Kyoto U.) ,
Yuichiro Sekiguchi (Toho U.) , Masaru Shibata (AEI・YITP)

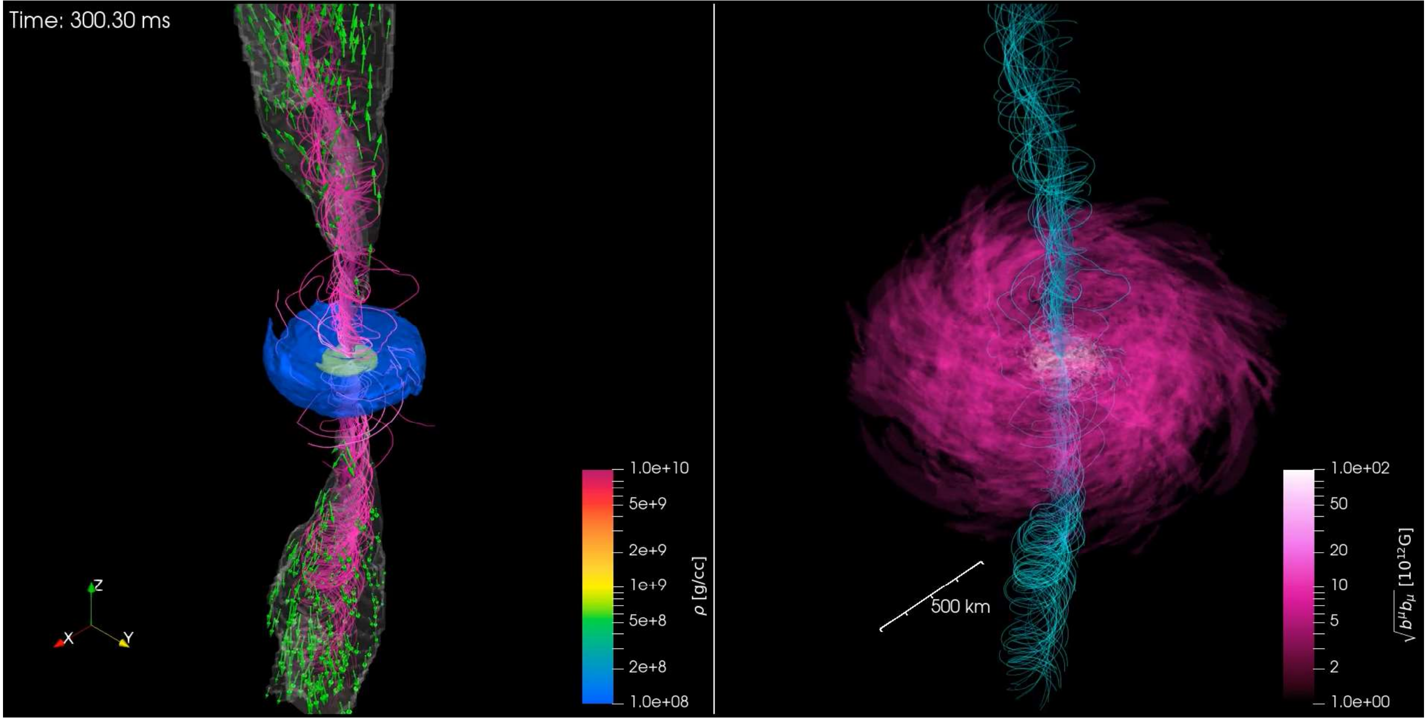
3D GRRMHD simulations of
Black hole-Neutron star merger and
BH-accretion disk system formed after the merger

We calculated the evolution of the system from inspiral stage (10ms) to post merger stage (2s)

Results associated to **sGRB**

- **Magnetosphere** near the rotational axis of the BH is developed, and mass outflow is driven
- **Poynting flux** consistent with sGRB is generated

3D GRRMHD simulations of Black hole-Neutron star merger and BH-accretion disk system formed after the merger



https://www2.yukawa.kyoto-u.ac.jp/˜kota.hayashi/Q4B5L-3D.mp4

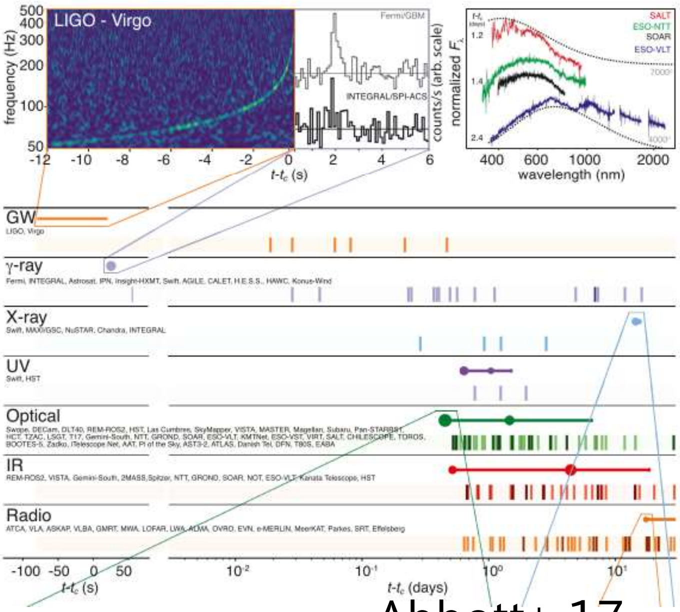
Introduction

Observing Compact Binary Merger using Gravitational Wave

GW170817:

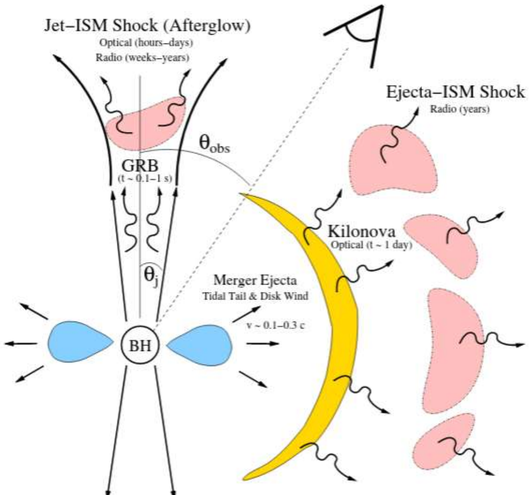
GW event from the **NSNS merger**

Electromagnetic counterparts were detected



Abbott+ 17

GRB170817A



AT2017gfo

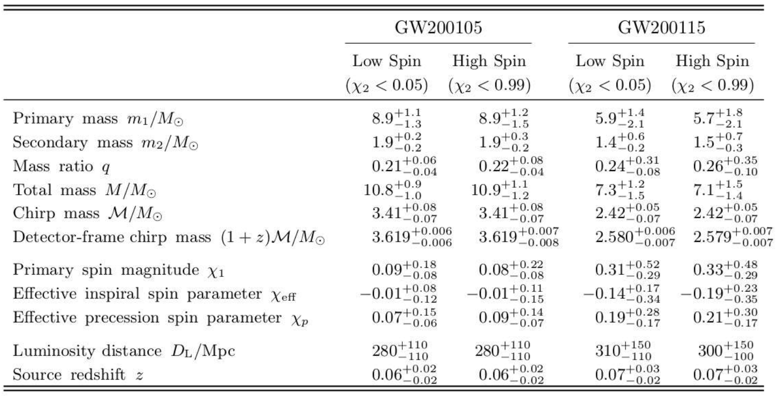
Metzger & Berger 12

Observing Compact Binary Merger using Gravitational Wave

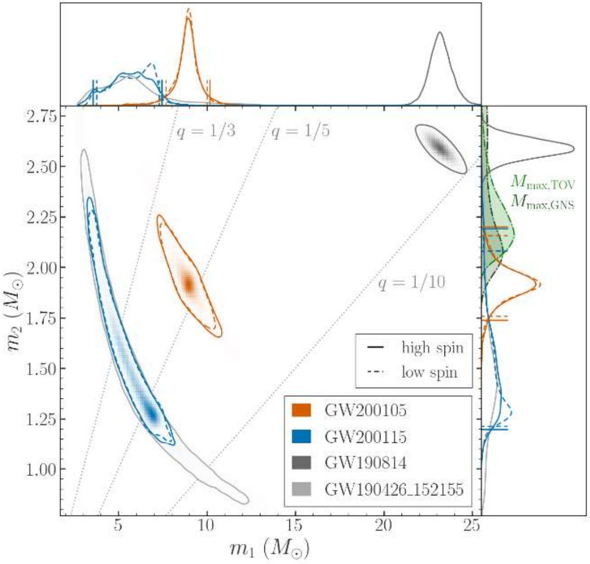
GW200115, (GW200105) :

GW events from the **BHNS merger**

NO Electromagnetic counterparts were detected



	GW200105		GW200115	
	Low Spin ($\chi_2 < 0.05$)	High Spin ($\chi_2 < 0.99$)	Low Spin ($\chi_2 < 0.05$)	High Spin ($\chi_2 < 0.99$)
Primary mass $m_1/M_\odot$	$8.9^{+1.1}_{-1.3}$	$8.9^{+1.2}_{-1.5}$	$5.9^{+1.4}_{-2.1}$	$5.7^{+1.8}_{-2.1}$
Secondary mass $m_2/M_\odot$	$1.9^{+0.2}_{-0.2}$	$1.9^{+0.3}_{-0.2}$	$1.4^{+0.6}_{-0.2}$	$1.5^{+0.7}_{-0.3}$
Mass ratio q	$0.21^{+0.06}_{-0.04}$	$0.22^{+0.08}_{-0.04}$	$0.24^{+0.31}_{-0.08}$	$0.26^{+0.35}_{-0.10}$
Total mass $M/M_\odot$	$10.8^{+0.9}_{-1.0}$	$10.9^{+1.1}_{-1.2}$	$7.3^{+1.2}_{-1.5}$	$7.1^{+1.5}_{-1.4}$
Chirp mass $\mathcal{M}/M_\odot$	$3.41^{+0.08}_{-0.07}$	$3.41^{+0.08}_{-0.07}$	$2.42^{+0.05}_{-0.07}$	$2.42^{+0.05}_{-0.07}$
Detector-frame chirp mass $(1+z)\mathcal{M}/M_\odot$	$3.619^{+0.006}_{-0.006}$	$3.619^{+0.007}_{-0.008}$	$2.580^{+0.006}_{-0.007}$	$2.579^{+0.007}_{-0.007}$
Primary spin magnitude χ_1	$0.09^{+0.18}_{-0.08}$	$0.08^{+0.22}_{-0.08}$	$0.31^{+0.52}_{-0.29}$	$0.33^{+0.48}_{-0.29}$
Effective inspiral spin parameter $\chi_{\rm eff}$	$-0.01^{+0.08}_{-0.12}$	$-0.01^{+0.11}_{-0.15}$	$-0.14^{+0.17}_{-0.34}$	$-0.19^{+0.23}_{-0.35}$
Effective precession spin parameter χ_p	$0.07^{+0.15}_{-0.06}$	$0.09^{+0.14}_{-0.07}$	$0.19^{+0.28}_{-0.17}$	$0.21^{+0.30}_{-0.17}$
Luminosity distance $D_{\rm L}/{\rm Mpc}$	280^{+110}_{-110}	280^{+110}_{-110}	310^{+150}_{-110}	300^{+150}_{-100}
Source redshift z	$0.06^{+0.02}_{-0.02}$	$0.06^{+0.02}_{-0.02}$	$0.07^{+0.03}_{-0.02}$	$0.07^{+0.03}_{-0.02}$



Abbott+ 21,GWTC-3

Black hole Neutron star merger

Orbital evolution associated with GW radiation

Deformation of NS by BH tidal force

· · · determined qualitatively by
tidal radius vs. ISCO

Tidal Disruption
Possibility of disk formation
and mass ejection

Plunge
Matter falls in to the
BH without disruption

small ——— Mass ratio（$M_{\mathrm{BH}}/M_{\mathrm{NS}}$）——→ **Large**

Large ←—— NS radius ——— small

Large ←—— BH spin ——— small

Method

Method

3D GRRMHD simulations of
Black hole-Neutron star merger and
BH-accretion disk system formed after the merger

- Full GR
  (BSSN formalism)
  [Shibata & Nakamura 95, Baumgarte & Shapiro 99]

- Neutrino radiation transport
  （Truncated moment formalism + Leakage-based source term）
  [Shibata+ 11, Sekiguchi+ 12]

- Ideal MHD
  (constraint transport scheme)
  [Shibata & Sekiguchi 05]

Method

3D GRRMHD simulations of
Black hole-Neutron star merger and
BH accretion disk system formed after the merger

- Time scale of the Dynamical mass ejection is <10ms

- Time scale of the Post-merger mass ejection is ～ 1s

- Time scale of sGRB is ～ 1-2s

↓

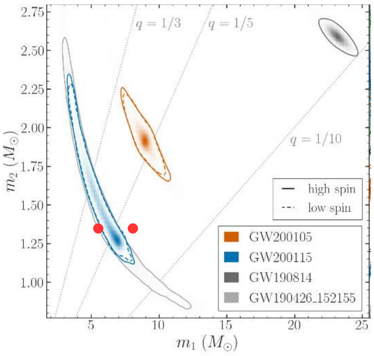
- **We calculated the evolution of the system from inspiral stage (10ms) to post merger stage (2s)**
- **Several factor longer than existing simulations**

Model parameters

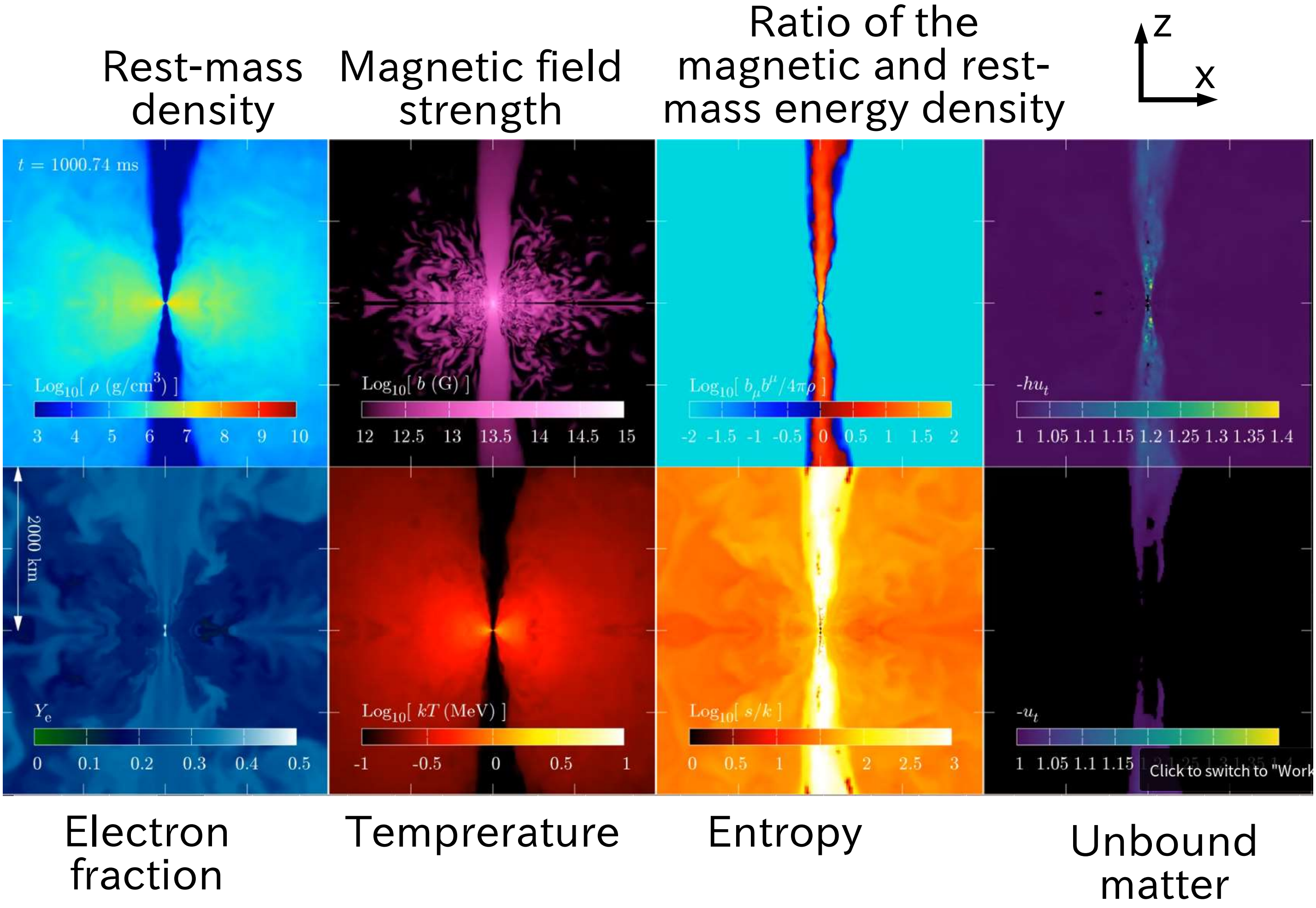
- Black hole mass: 5.4M_{sun}, 8.1M_{sun}
- Black hole spin: 0.75
- Neutron star mass: 1.35M_{sun}
- Neutron star radius: 13.2km (EoS:DD2)
- Max Magnetic field strength:
  3x10^{16}G, 5x10^{16}G
- Max resulution: 400m, 270m

- Confined poloidal magnetic field is superimposed on neutron star initially

- Equatorial symmetry



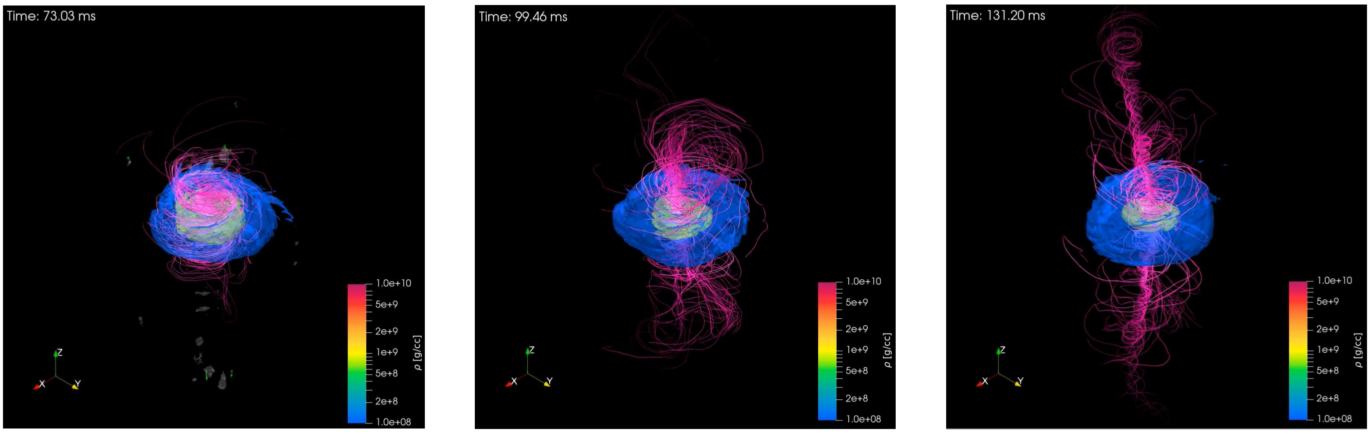
Results



https://www2.yukawa.kyoto-u.ac.jp/˜kota.hayashi/Q4B5L-2000a.mp4

Magnetosphere near the BH rotation axis

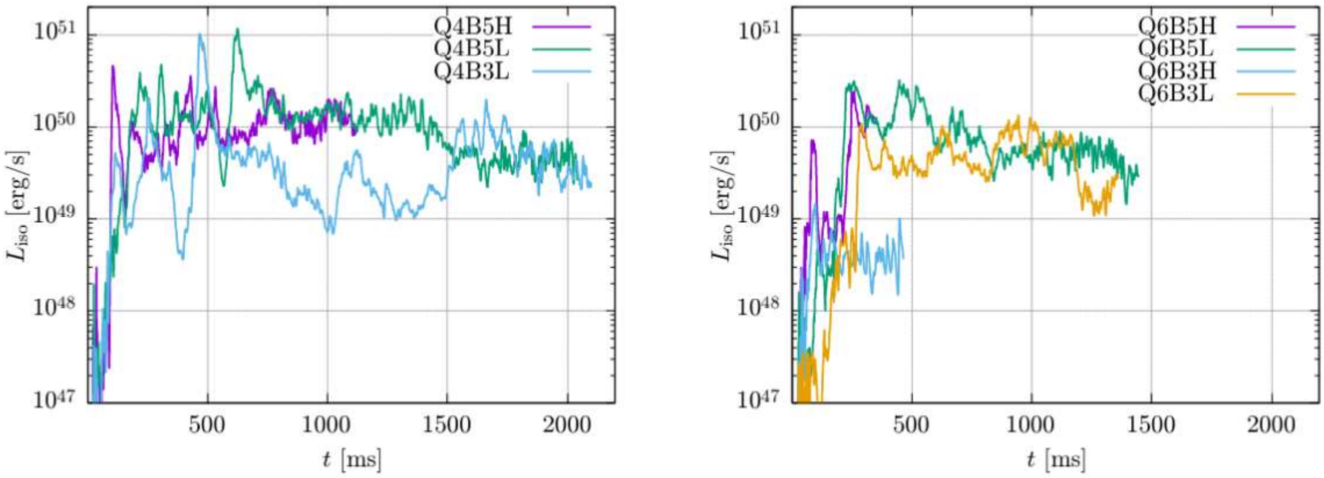
- **Magnetic tower effect** play a role in forming **global magnetic field** in the polar region:
  Magnetic pressure is enhanced by the twisting due to the BH spin, and the matter is pushed away resulting in the magnetic field dominance



https://www2.yukawa.kyoto-u.ac.jp/˜kota.hayashi/Q4B5L-3D.mp4

Poynting Flux generated near the BH rotation axis

- Poynting flux consistent with sGRB:
  - ・collimated with the opening angle ～10°
  - ・L_{iso} ～ 10^{50}erg/s



Poynting Flux generated near the BH rotation axis

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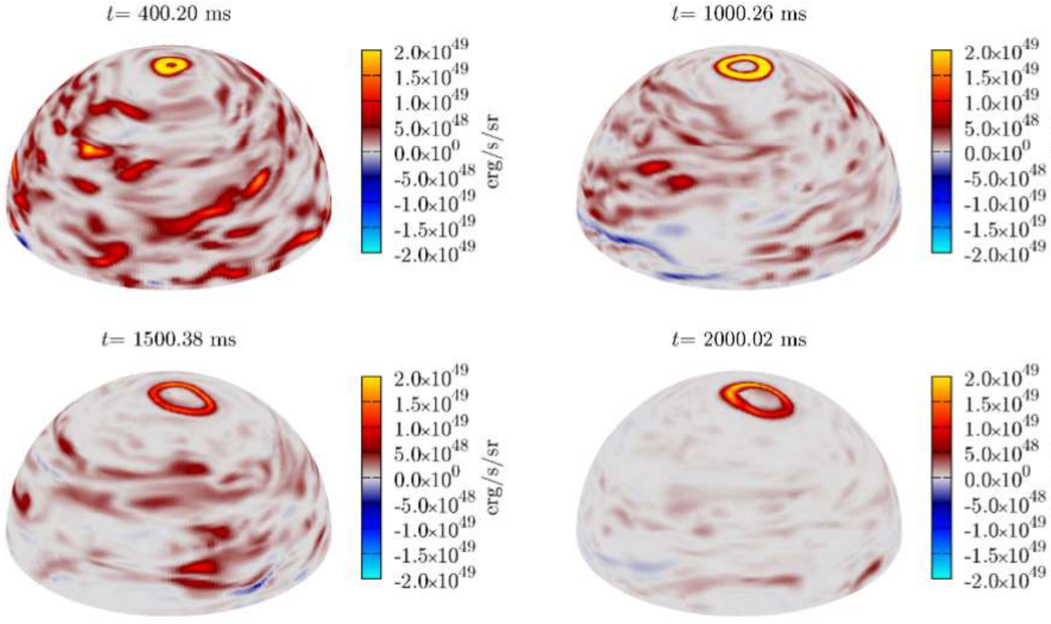

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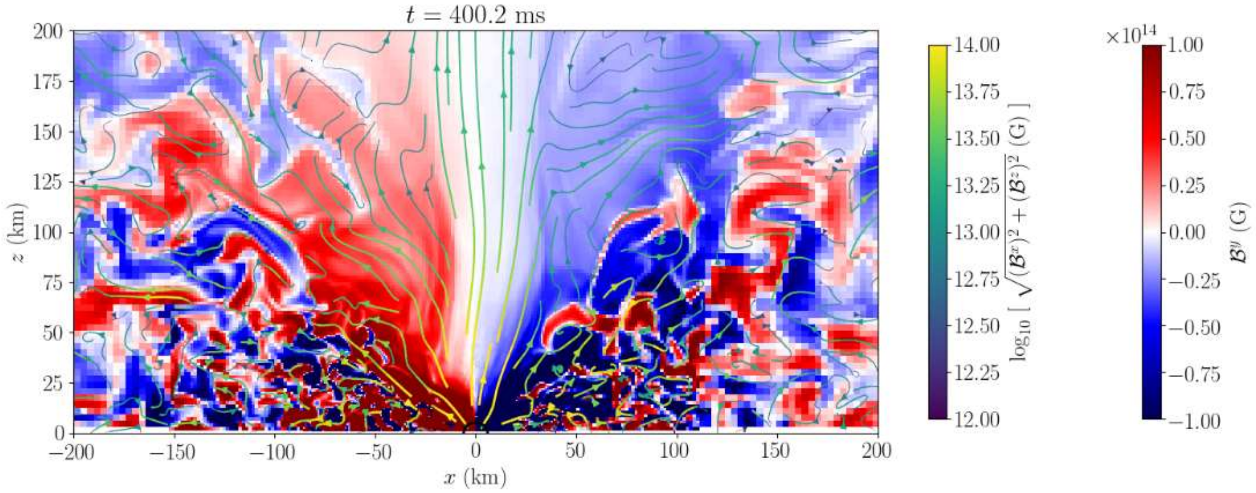
Energy extraction by the Blandford-Znajck mechanism

- Twisted structure of the magnetic field near the BH implies the energy extraction by the **Blandford-Znajck mechanism**
- Extracted energy is transferred in the form of torsional Alfven waves in the magnetosphere



https://www2.yukawa.kyoto-u.ac.jp/~kota.hayashi/Q4B5L-mf.mp4

Summary

3D GRRMHD simulations of Black hole-Neutron star merger and BH-accretion disk system formed after the merger

We calculated the evolution of the system from inspiral stage (10ms) to post merger stage (2s)

Results associated to **sGRB**
- **Magnetosphere** near the rotational axis of the BH is developed, and mass outflow is driven
- **Poynting flux** consistent with sGRB is generated

Session C2b 15:30–16:45

[Chair: Teruaki Suyama]

Tomohiro Harada

Rikkyo University

“Spins of primordial black holes formed in the radiation-dominated phase of the universe: first-order effect”

(15 min.)

[JGRG30 (2021) 120824]

Spins of primordial black holes formed in the radiation-dominated phase of the universe: first-order effect

T. Harada (Rikkyo U)
with C.-M. Yoo, K. Kohri, Y. Koga, T. Monobe

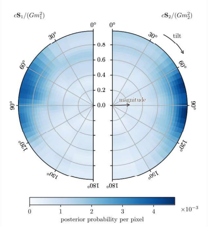
JGRG30, 6-9/12/2021 @ Online, organized by Waseda U.
Reference: ApJ 908 (2021) 2, 140 [arXiv://2011.00710]

PBH spin

- Nondimensional Kerr parameter

$$a_* := c\sqrt{S^2}/(GM^2)$$

- Information on BBH spins observed by LIGO/Virgo (Abbott et al. (2020))



- Important information for the origin of BBHs: e.g. Strong evidence for primordial population (Franciolini et al., `arXiv://2105.03349`)

Initial spin of PBH

- PBHs formed from primordial perturbation in radiation domination
- Studies on initial spins of PBHs
  Chiba & Yokoyama (2017), He & Suyama (2019), De Luca et al. (2019), Mirbabayi et al. (2020), etc.
- De Luca et al. (2019) gives a clear expression.

  $$\sqrt{\langle a_*^2 \rangle} \simeq (\Omega_{\rm dm}/\pi)\sigma_H \sqrt{1-\gamma^2} \simeq O(0.01)\sqrt{1-\gamma^2},$$

  where $\Omega_{\rm dm} := (\rho_{\rm dm}/\rho_c)_0$, σ_H is the density fluctuation at the horizon entry and $\gamma \simeq 1$.
- We follow essentially the same approach but improve the result.

Expression of angular momentum

- Background: flat FLRW

  $$ds^2 = a^2(-d\eta^2 + dx^2 + dy^2 + dz^2)$$

- Truncated Taylor series at the peak

  $$\begin{aligned} \delta &\simeq \delta_{pk} - \frac{1}{2}\sigma_2 \sum_{i=1}^{3} \lambda_i((x - x_{pk})^i)^2 \\ v^i &\simeq v^i_{pk} + v^i_j (x - x_{pk})^j, \end{aligned}$$

  where $\lambda_1 \geq \lambda_2 \geq \lambda_3$,

  $$\sigma_j^2 := \int \frac{d^3\mathrm{k}}{(2\pi)^3} k^{2j} |\delta_{\mathrm{k}}|^2,\ v^k_{\ l} := \left.\frac{\partial v^k}{\partial x^l}\right|_{x=x_{pk}}.$$

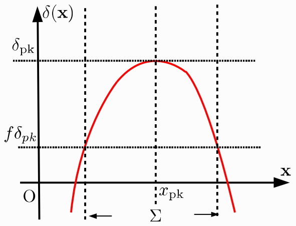
Angular momentum for a triaxial ellipsoid

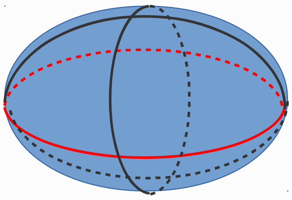
- Assumption: $\Sigma = \{\mathrm{x} | \delta(\mathrm{x}) > f\delta_{pk}\}$ $(0 \leq f < 1)$
- Σ is a triaxial ellipsoid with the three axes

$$a_i = \sqrt{2\frac{\sigma_0}{\sigma_2}\frac{1-f}{\lambda_i}\nu}, \quad \nu := \frac{\delta_{pk}}{\sigma_0}.$$

- Angular momentum within Σ in $\beta^i = 0$ gauge

$$S_i(\Sigma) \quad \simeq \quad (\rho_b + p_b)a^4\epsilon_{ijk}\int_\Sigma (x - x_{pk})^j(v - v_{pk})^k d^3x$$





Angular momentum around the peak

- Heavens & Peacock (1988)

$$\begin{aligned}
\sqrt{\langle S^2\rangle} &= S_{\mathrm{ref}}(\eta)\sqrt{\langle s_e^2\rangle}, \\
S_{\mathrm{ref}} &= (1+w)(a^4\rho_b)(\eta)g(\eta)(1-f)^{5/2}R_*^5, \\
s_e &= \frac{16\sqrt{2}\pi}{135\sqrt{3}}\left(\frac{\nu}{\gamma}\right)^{5/2}\frac{1}{\sqrt{\Lambda}}(-\alpha_1\tilde{v}_{23}, \alpha_2\tilde{v}_{13}, -\alpha_3\tilde{v}_{12}), \\
\alpha_1 &= |\lambda_3^{-1} - \lambda_2^{-1}|, ..., \Lambda = \lambda_1\lambda_2\lambda_3, \; R_* = \sqrt{3}\sigma_1/\sigma_2,
\end{aligned}$$

where $\tilde{v}^k_{\ l}$ is time-independent and $\gamma := \sigma_1^2/(\sigma_0\sigma_2)$.

- The function $g(\eta)$ is the time-dependence of $v^k_{\ l}(\eta)$.
- Since $(a^4\rho_b)(\eta)$ is time-independent for $w = 1/3$, only $g(\eta)$ is time-dependent.

Narrow power spectrum

- Assuming a narrow power spectrum, we find $\gamma \simeq 1$ and

$$g(\eta) \simeq \frac{1}{6} T_v(k_0, \eta) k_0 \sigma_H,$$

where $T_v(k, \eta)$ is a transfer function of the velocity field.

- Peak theory (Bardeen et al. (1986)) implies for $\nu = \delta_{pk}/\sigma_0 \gg 1$, δ is a sinc function

$$\delta(\eta, \mathrm{r}) = \delta_{pk}(\eta)\psi(r), \quad \psi(r) = \frac{\sin(k_0 r)}{k_0 r}$$

and Σ is quasi-spherical

$$a_i \simeq r_f(1 + O(\nu^{-1})), \; r_f = \sqrt{6(1-f)}\sigma_0/\sigma_1.$$

PBH threshold and turn around

- To estimate the PBH's a_*, we should estimate it at the turnaround (or maximum expansion) of the threshold perturbation, after which it will be roughly constant.
- The PBH formation threshold is at $\bar{\delta}_H \simeq 0.768$ in the CMC slice by numerical relavitity (Shibata & Sasaki (1999), Musco & Miller (2013), Harada, Yoo, Namaka, Koga (2015), ...).
- The turnaround can be identified by $\delta = 1$ in the CMC slice. Extrapolating a linear perturbation solution at the threshold, we can numerically obtain $x_{\mathrm{ta}} := k_0 \eta_{\mathrm{ta}}$ and $T_v(k_0, \eta_{\mathrm{ta}})$ in the conformal Newtonian gauge.

Estimate of a_*

- Reference spin value ($\sqrt{\langle a_*^2\rangle} = A_{\rm ref}\sqrt{\langle s_e^2\rangle}$)

$$\begin{aligned} A_{\rm ref}(\eta_{\rm ta}) &:= S_{\rm ref}(\eta_{\rm ta})/(GM_{\rm ta}^2) \\ &\simeq \frac{x_{\rm ta}^2|T_v(k_0,\eta_{\rm ta})|\sigma_H}{24\sqrt{3}\pi\sqrt{1-f}} \end{aligned}$$

- Peak theory (Heavens & Peacock (1988)) gives

$$\sqrt{\langle s_e^2\rangle} \simeq 5.96\nu^{-1}\sqrt{1-\gamma^2}.$$

- Putting the obtained values for $x_{\rm ta}$, $T_{v{\rm CN}}$ and $\bar{\delta}_H$ with $\nu = (5/2)(\bar{\delta}_H/\sigma_H)$ and $\gamma \simeq 1$, we find

$$\sqrt{\langle a_*^2\rangle} \simeq 4.01\times 10^{-3}\left(\frac{M}{M_H}\right)^{-1/3}\sqrt{1-\gamma^2}\left(\frac{\nu}{8}\right)^{-2},$$

where ν is identified with $\nu_{\rm th}$.

Physical implication

- $\nu_{\rm th}$ can be written in terms of $\beta_0(M_H)$ and therefore $f_{\rm PBH} = (\Omega_{\rm PBH}/\Omega_{\rm dm})_0$ through Carr's formula.

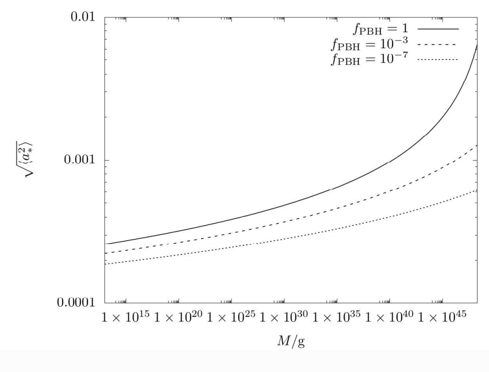


Figure 1: $\sqrt{\langle a_*^2\rangle}$ vs M for $\gamma = 0.85$ and $M = M_H$.

Summary

- The initial PBH spin is obtained

$$\begin{aligned}\sqrt{\langle a_*^2\rangle} &\simeq 4.0\times 10^{-3}\left(\frac{M}{M_H}\right)^{-1/3}\sqrt{1-\gamma^2}\\ &\times \left[1-0.072\log_{10}\left(\frac{\beta_0(M_H)}{1.3\times 10^{-15}}\right)\right]^{-1}.\end{aligned}$$

- We have basically confirmed De Luca et al. (2019) but removed an unnecessary factor $\Omega_{\rm dm}$ and revealed a new mass-dependence.
- Spins are suppressed by $\nu_{\rm th}^{-2} \sim \sigma_H^2$ and apparently second order (cf. Mirbabayi et al. (2020)).
- $\sqrt{\langle a_*^2\rangle} \lesssim O(10^{-3})$ if $M \simeq M_H$, while can be much larger if $M \ll M_H$, which may be realised in the critical collapse.

Session C2b 15:30–16:45

[Chair: Teruaki Suyama]

Michael Zantedeschi

Max-Planck Institute for Physics, Munich

“Primordial Black Holes from Confinement”

(15 min.)

[JGRG30 (2021) 120826]

Primordial black holes from confinement *

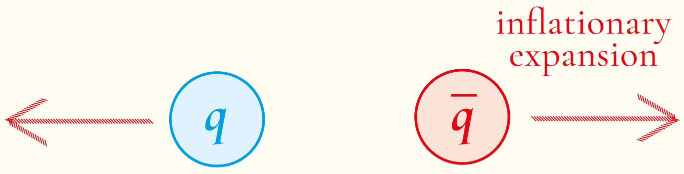
Michael Zantedeschi, MPI

work with Gia Dvali and Florian Kühnel

Key: The same force responsible for the confinement of hadrons can explain dark matter

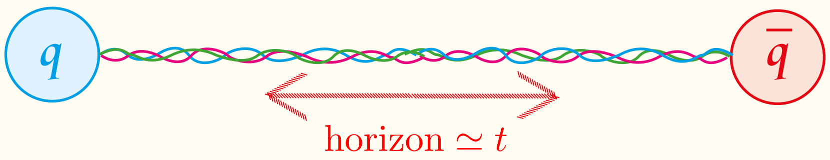
Mechanism

Inflationary fluctuations produce quarks which are diluted by inflationary expansion



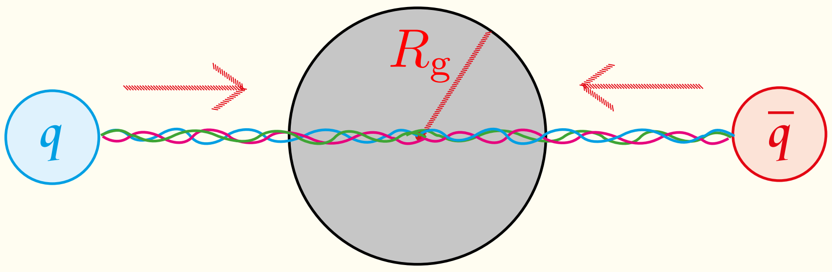
- By the end of inflation $d \propto e^{N_e}$ N_e being the number of e-folds

Quarks are confined at energy scale Λ_c



- Coloured flux tubes (strings) connecting them form
- Collapse cannot be immediate due to causality $d \gg$ horizon
- String stability $\Longrightarrow$ $\boxed{\Lambda_c \gtrsim m_{\text{quark}}}$

Eventually quarks enter in casual contact and accelerate relativistically toward each other

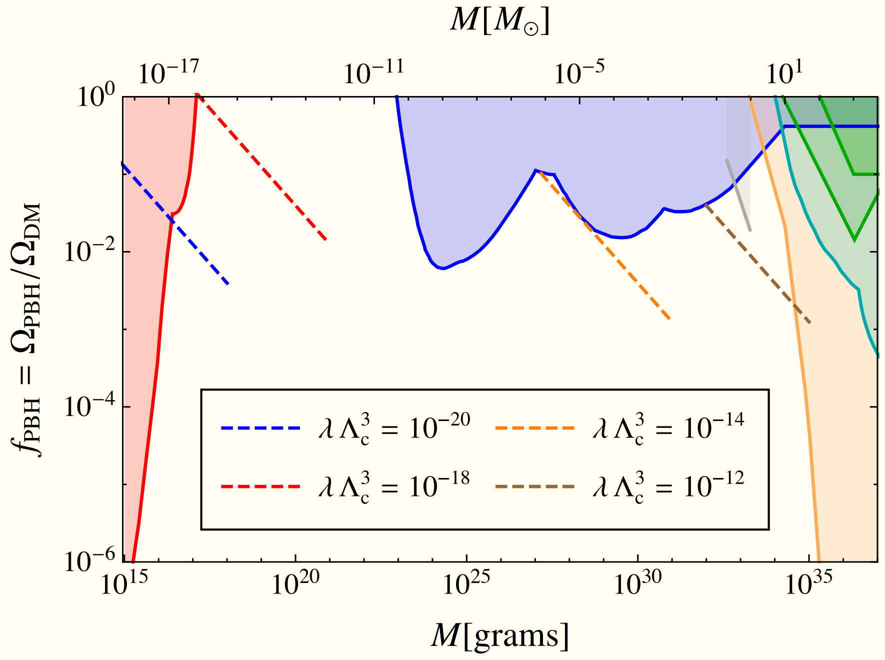


- Configuration Schwarzschild radius is

$$\boxed{R_g \simeq l_{\text{Pl}}^2\, \Lambda_c^2\, t \gg \Lambda_c^{-1}}$$

$\Longrightarrow$ Primordial black hole forms

Dark matter



$$\boxed{f_{\text{PBH}} = \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}} \propto \Lambda_c^3\, M_{\text{PBH}}^{-1/2}}$$

- Filled areas correspond to pheno. costraints
- 100 % of dark matter @ $M_{\text{PBH}} \sim 10^{17}\text{g}$
- Naturally explains supermassive black holes in galactic centres $f_{\text{PBH}} \propto M_{\text{PBH}}^{-1/2}$
- Lighter black holes can be maximally spinning due to impact parameter induced by the string fluctuations
- Compatible with QCD if during fomation

  $$\Lambda_c \gtrsim m_{\text{quark}}$$

  generic in string theory (moduli)
- Formation generates unique gravitational - waves signal

  Ω_{GW} is flat

  interesting for NANOGrav and LISA

* arXiv.2018.09471

Session C2b 15:30–16:45

[Chair: Teruaki Suyama]

Albert Escrivà

ULB (Brussels University)

“Numerical simulations of Primordial Black Holes with non-Gaussianities”

(15 min.)

[JGRG30 (2021) 120827]

Numerical simulation of Primordial Black Holes with non-Gaussianities

Albert Escrivà

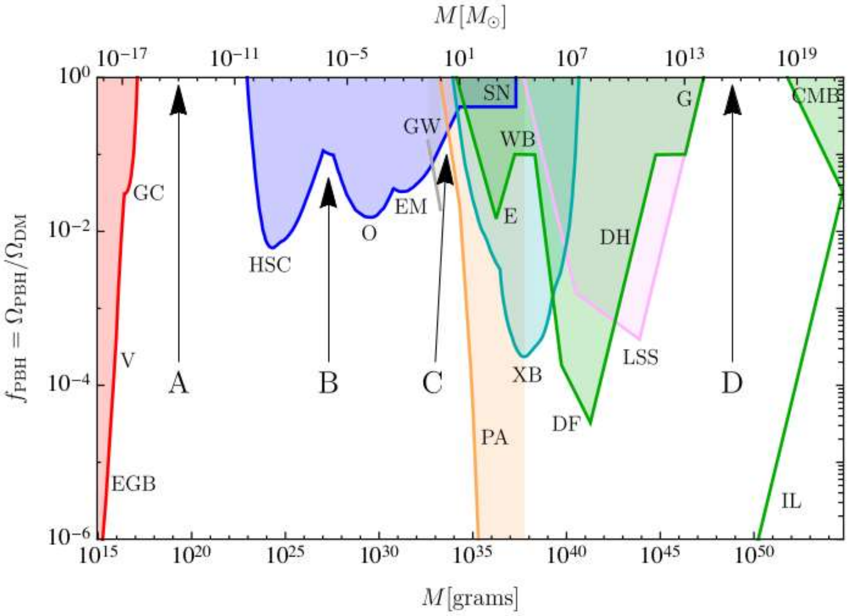
Based on:

A. Escrivà, N. Kitajima, Y. Tada, S. Yokoyama and C. Yoo. <u>In preparation</u>

JGRG30
8th December, 2021

Motivation

- There is some missing matter in the Universe with an unknown composition
- The existence of dark matter can be explained through compact objects like Black Holes with a primordial origin



Golden Age
for PBH scenario

B. Carr and F. Kühnel 2020

Motivation

- Some fluctuations generated during inflation could be sufficiently large (rare events) and collapse during radiation epoch in the very early Universe

- These rare fluctuations will have roughly spherical symmetry (spherical peaks)

J. M. Bardeen, J. R. Bond, N. Kaiser and A. S. Szalay 1986

- **But ,** the effetc of non-gaussianities could be important!

N. Kitajima, Y. Tada, S. Yokoyama and C. Yoo. *JCAP* 10 (2021) 053

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Motivation

- It was found an analytical criterion using the critical averaged compaction function, useful to determine the threshold of PBH formation

A. Escrivà, C. Germani and R. K. Sheth.
Phys. Rev. D 101, 044022 (2020)

applied in the context of non-gaussianities

N. Kitajima, Y. Tada, S. Yokoyama and C. Yoo. *JCAP* 10 (2021) 053

Our aim is to numerically check this estimation for the model considered previously and compare it with the simulations

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Basics on primordial black hole formation

- PBHs could be formed by sufficiently large cosmological perturbations collapsing after re-entering the cosmological horizon. Assuming spherical symmetry, such regions can be described by the following approximate form of the metric at super-horizon scales (gradient expansion approach)

Shibata,Sasaki. ArXiv:grqc/9905064

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(r)}(dr^2 + r^2 d\Omega^2)$$

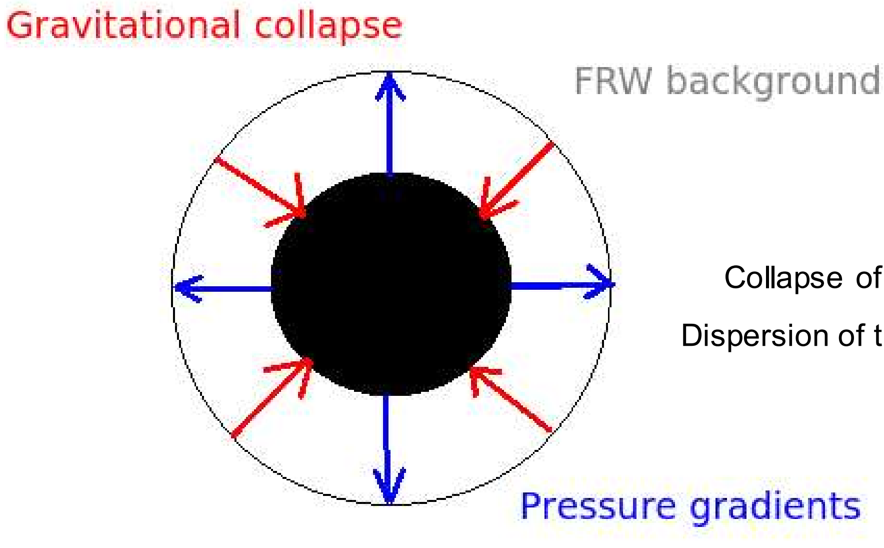
- The curvature profile characterize the cosmological perturbation

$$\zeta(r) \Rightarrow \frac{\rho - \rho_b}{\rho_b}$$

We consider perturbations at super-horizon scales

Mechanism: hydrodynamic collapse

Spherical Collapse of cosmological perturbations leading to PBH formation



Threshold δ_c
:

Collapse of the perturbation: $\delta_m > \delta_c$

Dispersion of the perturbation: $\delta_m < \delta_c$

$p = \omega\rho$ Perfect fluid

Condition for PBH formation

- The compaction function is an essential magnitude to characterize the cosmological perturbation, in particular its peak.

$$\mathcal{C} = \frac{M - M_b}{R} \quad \xrightarrow{\text{At Super-horizon scales}} \quad \mathcal{C} = \frac{2}{3}\left[1 + (1 - r\zeta')^2\right]$$

- The peak of the compaction function is considered as the threshold for PBH formation

$$\mathcal{C}_c(r_m) = \delta_c$$

the lengthscale of the perturbation is precisely given by: r_m

Shibata,Sasaki. ArXiv:grqc/9905064

- The peak of the compaction function must be greater than a given threshold to form a PBH

$$\mathcal{C}(r_m) \equiv \delta_m > \mathcal{C}_c(r_m)$$

We focus on PBH type I $\Rightarrow$ $R' > 0$

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Non-gaussian template

- We consider the inclusion of the non-gaussian term with a monochromatic power spectrum

$$\mathcal{P}_\zeta(k) = \sigma_0^2 k_* \delta(k - k_*) \qquad \sigma_0^2 = \int \frac{dk}{k}\mathcal{P}_\zeta(k)$$

$$\psi(r) = \frac{1}{\sigma_0^2} < \zeta_g(r)\zeta_g(0) >= \frac{1}{\sigma_0^2}\int \mathcal{P}_\zeta(k) sinc(kr)\frac{dk}{k}$$

$$\zeta_g = \mu\,\psi(r) = \mu\, sinc\,(k_* r)$$

Local-non gaussianity $\quad \zeta = \zeta_g + \frac{3}{5} f_{\rm NL}\, \zeta_g^2$

Assumption on the correspondence of the peaks $\xrightarrow{\text{Doubted if}}$ $\frac{3}{5}\mu f_{\rm NL} < -\frac{1}{2}$

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Analytical estimate for the threshold

Analytical estimate for the threshold of PBH formation (radiation fluid)

$$\bar{\mathcal{C}}_c = 2/5$$

A. Escrivà, C. Germani and R. K. Sheth.
Phys. Rev. D 101, 044022 (2020)

$$q = -\frac{r_m^2 \mathcal{C}''(r_m)}{4\mathcal{C}(r_m)}$$

$$\delta_c = \frac{4}{15} e^{-\frac{1}{q}} \frac{q^{1-5/2q}}{\Gamma(5/2q) - \Gamma(5/2q, 1/q)}$$

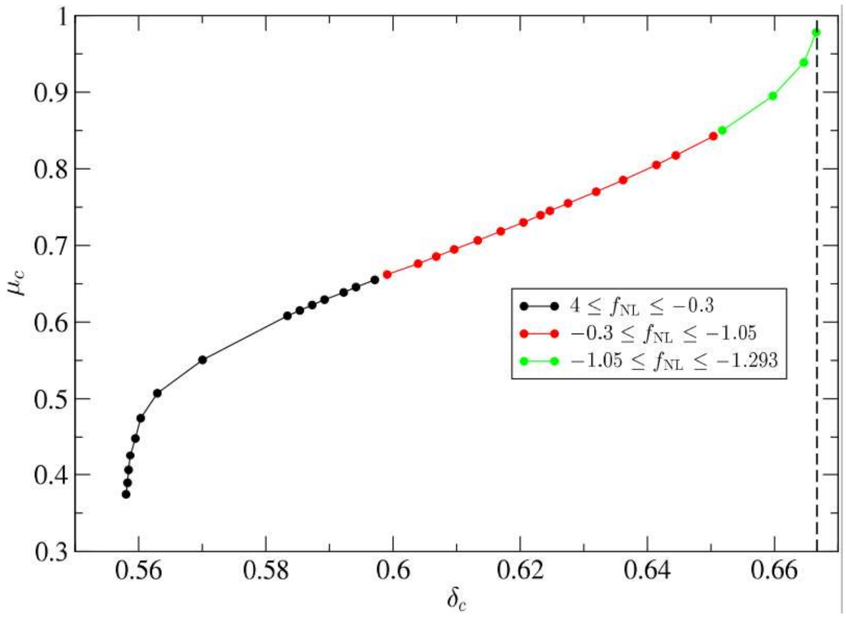
$$q = -\frac{1}{4} r_m^2 \frac{\mathcal{C}''(r_m)}{\mathcal{C}(r_m)\left(1-\frac{3}{2}\mathcal{C}(r_m)\right)}$$

I.Muso, G. Franciolini, V. de Luca and A. Riotto
Phys. Rev. D 103, 063538 (2021)

Numerical results

We perform numerical simulations using pseudospectral methods
A. Escrivà. Phys.Dark Univ. 27 (2020) 100466

With a bisection method we obtain the thresholds $\delta_c(\mu_c, f_{\rm NL})$



We have found formation of PBH type I for:

$$f_{\rm NL} \gtrsim -1.2$$

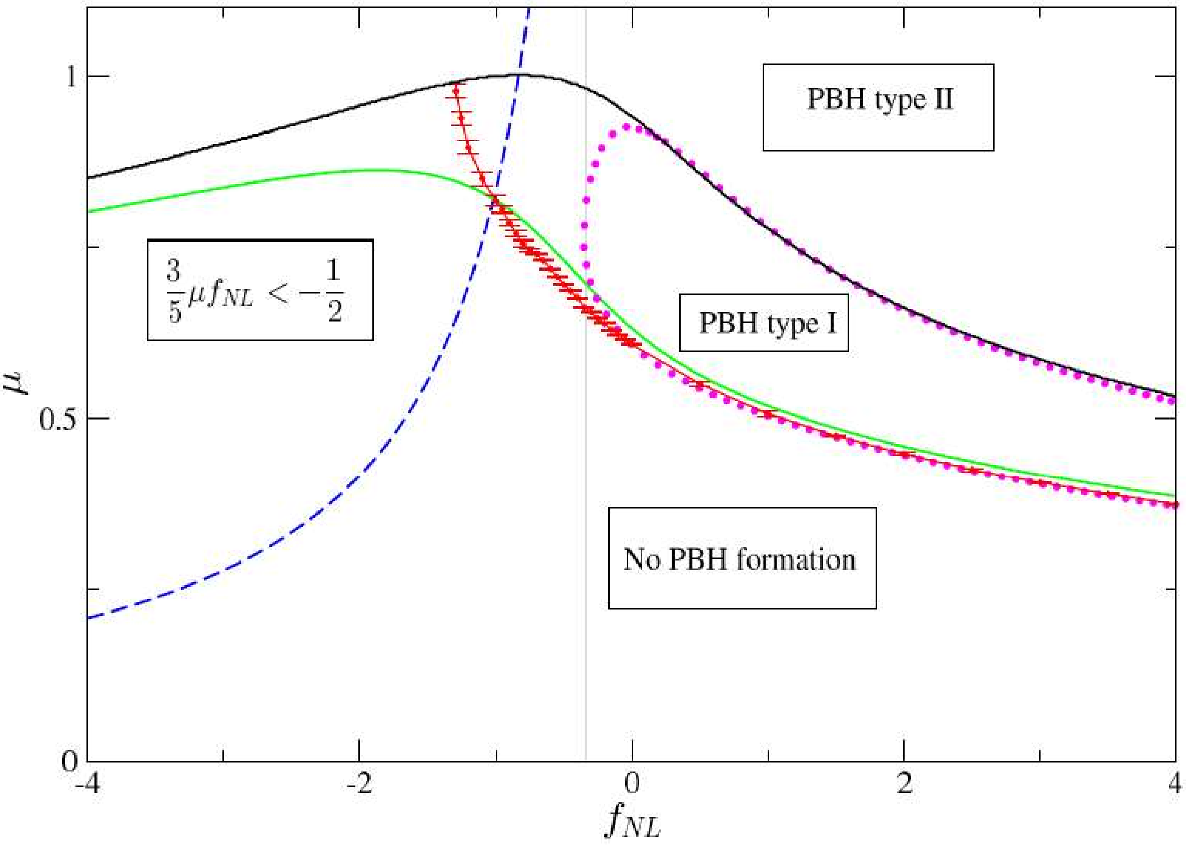
Numerical evolution:Supercritical

MOVIE(not available in PDF format)

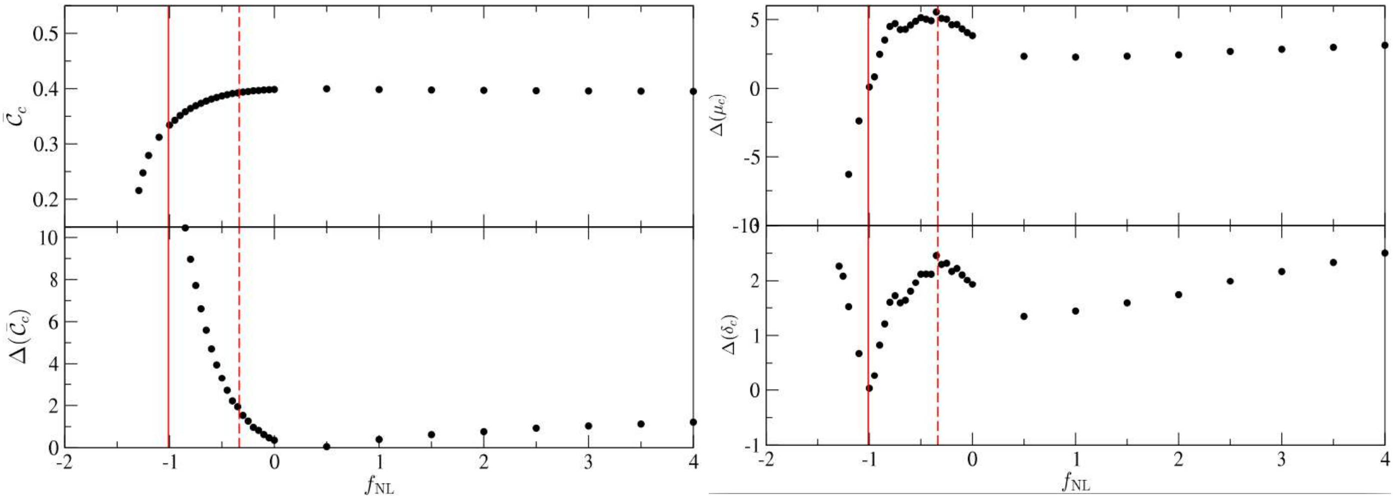
Numerical evolution:Subcritical

MOVIE (not available in PDF format)

Diagram result



Compare with the analytical estimations



$$\Delta(\bar{\mathcal{C}}_c) = 100 \times \frac{\bar{\mathcal{C}}_c - 0.4}{0.4}$$

$$\Delta(\mu) = 100 \times \frac{\mu_c^N - \mu_c^A}{\mu_c^N}$$

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Conclusions

- We have found the existence of PBH formation of type I in a new region in terms of the non-gaussian parameter, in contrast with the previous analytical results.

- The analytical estimation of the average of the critical compaction function seems to fail for the profiles considered. One should take this into account when considering "rare" profiles.

- Although that, the estimation using the q-procedure directly seems more robust (at least for the profiles considered)

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ご清聴ありがとうございました

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Session D1a 9:00–10:30

[Chair: Katsuki Aoki]

Aya Iyonaga

Rikkyo University

“Distinguishing modified gravity with just two tensorial degrees of freedom from general relativity: cosmology”

(15 min.)

[JGRG30 (2021) 120902]

JGRG30@Waseda(online) Dec. 09, 2021

Distinguishing modified gravity with just two tensorial degrees of freedom from general relativity: cosmology

Phys. Rev. D 104, 124020 (2021)

Aya Iyonaga (Rikkyo Univ.)

Collaborator: Tsutomu Kobayashi (Rikkyo Univ.)

Two-DOF Scalar-Tensor Theory

Scalar-tensor theory

Modifies gravity by using additional scalar fields

- Require 4-dim. covariance (single-scalar)

  …3 dynamical DOFs $\begin{cases} \text{2 tensor modes} \\ \text{1 scalar mode} \end{cases}$

- Require only spatial covariance

  …Can modify GR without a dynamical scalar mode
  (= same #DOFs as in GR)

"Two-DOF scalar-tensor theory"

- Timelike $\partial_\mu \phi$ breaks time diffeomorphism
- The scalar field exists as an auxiliary field, and does not propagate

General Action

General two-DOF scalar-tensor theory with quadratic K_{ij} in the ADM formalism:

Gao, Yao (2020)

$$S=\frac{1}{2}\int \mathrm{d}t\ \mathrm{d}^3x N\sqrt{\gamma}\left[\frac{\beta_0 N}{\beta_2+N}K_{ij}K^{ij}-\frac{\beta_0}{3}\left(\frac{2N}{\beta_1+N}+\frac{N}{\beta_2+N}\right)K^2\right.$$

$$\left.+\alpha_1+\alpha_2 R+\frac{1}{N}(\alpha_3+\alpha_4 R)\right]$$

$$\alpha_A=\alpha_A(t)$$
$$\beta_A=\beta_A(t)$$

$\downarrow$ require $\gamma_{\mathrm{PPN}}=1,\ c_{\mathrm{GW}}=1$
+ field redefinitions

- ✓ solar-system tests
- ✓ bound on the speed of GW

$$S=\frac{1}{2}\int \mathrm{d}t\ \mathrm{d}^3x N\sqrt{\gamma}\left[K_{ij}K^{ij}-\frac{1}{3}\left(\frac{2N}{\beta(t)+N}+1\right)K^2+R+\alpha_1(t)+\frac{\alpha_3(t)}{N}\right]$$

Can we distinguish this theory from GR by some observations?

Distinguishing from GR

$$S=\frac{1}{2}\int \mathrm{d}t\ \mathrm{d}^3x N\sqrt{\gamma}\left[K_{ij}K^{ij}-\frac{1}{3}\left(\frac{2N}{\beta(t)+N}+1\right)K^2+R+\alpha_1(t)+\frac{\alpha_3(t)}{N}\right]$$

In such a model, we investigated…

1. Black hole solutions → *Tsutomu Kobayashi's poster [P18]*

✓ This model has the Schwarzschild & Kerr solutions!

2. Cosmological dynamics with matter (this talk)

✓ Background evolution

✓ Density perturbations of matter

in a homogeneous & isotropic universe

Scalarless Theory with Matter

Action

$$S = \frac{1}{2}\int \mathrm{d}t\ \mathrm{d}^3x N\sqrt{\gamma}\left[K_{ij}K^{ij} - \frac{1}{3}\left(\frac{2N}{\beta(t)+N}+1\right)K^2 + R + \alpha_1(t) + \frac{\alpha_3(t)}{N} + 2P(Y)\right]$$

Two-DOF scalar-tensor sector

Matter sector

Mimics a barotropic perfect fluid

$Y := -\frac{1}{2}(\partial\varphi)^2$ φ : matter field

$P_{,Y} := \frac{dP}{dY}$

$\longrightarrow$ $p = P,\ \ \rho = 2YP_{,Y} - P,\ c_s^2 = \frac{P_{,Y}}{P_{,Y} + 2YP_{,YY}}$

Background sp.

Homogeneous & isotropic spacetime:

$$\underline{N = \bar{N}(t),}\quad N_i = 0,\quad \gamma_{ij} = a^2(t)\delta_{ij},\quad \varphi = \varphi(t)$$

No time reparametrization sym. in spatial covariant theories

$\longrightarrow$ $\bar{N}(t) \neq 1$ in general

Background Equations

Hamiltonian constraint: $\frac{3H^2}{(\beta/\bar{N}+1)^2} + \frac{\alpha_1}{2} = \rho$

Evolution eq.: $-\frac{3H^2}{\beta/\bar{N}+1} - \frac{2}{\bar{N}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{H}{\beta/\bar{N}+1}\right) - \frac{1}{2}\left(\alpha_1 + \frac{\alpha_3}{\bar{N}}\right) = P$

Conservation law: $\frac{1}{\bar{N}}\frac{\mathrm{d}\rho}{\mathrm{d}t} + 3H(\rho + P) = 0$

$H := \frac{1}{\bar{N}}\frac{\mathrm{d}\ln a}{\mathrm{d}t}$

Choosing the parameter functions $\beta(t), \alpha_1(t)$ and $\alpha_3(t)$

$\longrightarrow$ We can realize some interesting evolutions of the background spacetime

Background Dynamics—Example

Model and EOS

$$S = \frac{1}{2}\int \mathrm{d}t\ \mathrm{d}^3x N\sqrt{\gamma}\left[K_{ij}K^{ij} - \frac{1}{3}\left(\frac{2N}{\beta(t)+N}+1\right)K^2 + R + \alpha_1(t) + \frac{\alpha_3(t)}{N}\right]$$

with $\beta = const.,\quad \alpha_1 = 6h_0^2\left[\frac{1}{\xi} - \frac{1}{(1+\beta)^2}\right]\coth^2\left[\frac{3}{2}(1+w)h_0 t\right] - \frac{6h_0^2}{\xi}$

$$\alpha_3 = 6h_0^2\left[\frac{1+w(1+\beta)}{(1+\beta)^2} - \frac{1+w}{\xi}\right]\mathrm{csch}^2\left[\frac{3}{2}(1+w)h_0 t\right] - \frac{6\beta h_0^2}{(1+\beta)^2} \qquad \xi, h_0 = const.$$

EOS: $w = P/\rho = const.$

Evolution eqs.

$3H^2 = \xi\rho + 3h_0^2,\quad -2\dot{H} = \xi(\rho + P)$ $\cdots$Same as in the ΛCDM model besides ξ

↘ $8\pi G_{\rm cos} = \xi$ (= a free parameter in this model)

The present model has $8\pi G_N = 1 \quad \Rightarrow \quad \frac{G_{\rm cos}}{G_N} = \xi$

observational constraint:
$|\xi - 1| \lesssim 0.1$

This model can $\left\{\begin{array}{c}\text{evade the observational constraint} \\ \text{or even} \\ \text{be identical to that in the } \Lambda\text{CDM model}\end{array}\right\}$ by choosing ξ

Cosmological Perturbations

Scalar perturbations: $N = \bar{N}(1+\delta n),\quad N_i = \bar{N}\partial_i\chi,\quad \gamma_{ij} = a^2 e^{-2\psi}\delta_{ij}$

$\varphi = \varphi(t) + \delta\varphi$

(Gauge-invariant) density fluctuation: $\delta = \frac{\rho+P}{\rho c_s^2}\left(\frac{\delta\dot{\varphi}}{\dot{\varphi}} - \delta n\right) - \frac{3(\rho+P)}{\rho}\psi$

Quadratic Lagrangian

After some procedure, we get

$$\mathcal{L}^{(2)}(\delta) = a^3\left[\mathcal{A}(t,\Delta)\dot{\delta}^2 + \mathcal{B}(t,\Delta)\delta^2\right]$$

EOM for δ: $\ddot{\delta} + \left(3H + \frac{\dot{\mathcal{A}}}{\mathcal{A}}\right)\dot{\delta} - \frac{\mathcal{B}}{\mathcal{A}}\delta = 0$

Gravitational potentials: $\begin{cases}\Phi := \delta n + \dot{\chi} \\ \Psi := \psi - H\chi\end{cases}$

How do these behave in the dust limit?
$(P \to 0,\ c_s^2 \to 0)$

Short/Long Wavelength Limit

Short wavelength limit: $\Delta/a^2 \gg \rho$

$\longrightarrow$ EOM for δ:

$$\ddot{\delta} + 2H\dot{\delta} = \frac{1}{2}\rho\delta$$

$\left(\text{Present model: } 4\pi G_N = \frac{1}{2}\right)$

$\xrightarrow{\rho \propto a^{-3}}$ Gravitational potentials:

$$\Delta\Phi = \Delta\Psi = \frac{a^2}{2}\rho\delta$$

same as in GR

……

Long wavelength limit: $\Delta/a^2 \ll \rho$

$\longrightarrow$ EOM for δ:

$$\ddot{\delta} + 3H\dot{\delta} = -\frac{\Delta}{3a^2}\delta$$

$\longrightarrow$ Gravitational potentials:

$$\Phi = \Psi = -\frac{\delta_0(\vec{x})}{3}\left[1 - \frac{H}{a}\int^t a\left(t'\right)\mathrm{d}t'\right]$$

$\delta_0(\vec{x})$: time-independent function

same as in GR (with pressureless fluid & Λ)

The perturbation dynamics can be modified
only through the modification of the b.g. evolution

Summary

We considered the cosmology in "**two-DOF scalar-tensor theory**"

spatially-covariant theory with only 2 tensorial DOFs

✓ Background

We can realize some interesting evolutions
by choosing the parameter functions

e.g.) b.g. evolution very close to, or identical to the ΛCDM model

✓ Density perturbation

In both the short/long wavelength limits, the perturbation dynamics can be modified only through the modification of the b.g. evolution

……

Futurework

The perturbation dynamics on intermediate scales might be modified

Session D1a 9:00–10:30

[Chair: Katsuki Aoki]

Zhi-Bang Yao

Sun Yat-sen University

“Minimal theory of single and bi- metric gravity with multiple auxiliary constraints”

(15 min.)

[JGRG30 (2021) 120905]

JGRG30 @ Waseda University

Minimal theory of single and bi-metric gravity with multiple auxiliary constraints

Speaker: Zhi-Bang Yao

Collaborators: Michele Oliosi, Xian Gao
and Shinji Mukohyama

Department of Physics and Astronomy
Sun Yat-sen University

Date: Dec. 09th, 2021

Based on: 2112.XXXXX [gr-qc]

Outline

- Introduction
- MMG with Auxiliary Constraints
- Minimal theory of bigravity
- Summary & Outlook

Introduction

The uniqueness of GR

The Lovelock theorem: GR is the unique theory

$$S^{(\mathrm{GR})} = \frac{M_{\mathrm{Pl}}^2}{2}\int d^4x\sqrt{-g}\left({}^{(4)}R - 2\Lambda\right) \qquad \#_{\mathrm{dof}} = \frac{1}{2}\Big(10\times 2 - \underbrace{8\times 2}_{4-\mathrm{diff.}}\Big) = 2_{\mathrm{t}}$$

- 4-dim spacetime
- General covariance
- Metric theory & Locality
- 2nd-order EoM

[D. Lovelock, JMP, 1971]

Is GR still unique when we assume:

- 4-dim spacetime
- Spatial covariance
- Metric theory & Locality
- 2 physical tensorial d.o.f.

No, we have minimally modified gravity (MMG) theories.

Motivations of looking for MMG theories:

- Candidates for the tensorial polarizations signals from gravitational waves events;
- Addressing problems in cosmology (dark energy, Hubble tension, etc.)

Introduction

The MMG theories

2007 Cuscuton
[N. Afshordi, D. J. H. Chung and G. Geshnizjani, PRD, 2007]

$$\mathcal{L}^{(\mathrm{cus.})} = K_{ij}K^{ij} - K^2 + R_{ij} + \frac{\mu^2}{N} + V(t)$$

2017 Minimally modified gravity
[C. Lin and S. Mukohyama, JCAP, 2017]

$$\mathcal{L}^{(\mathrm{m.m.g.})}\left(h_{ij}, K_{ij}, R_{ij}, \nabla_i, t\right)$$

2018 Extended Cuscuton
[A. Iyonaga, K. Takahashi and T. Kobayashi, JCAP, 2018]

$$\mathcal{L}^{(\mathrm{e.c.})}\left(N, h_{ij}, K_{ij}, R_{ij}, t\right)$$

2020 SCG with TTDOF
[X. Gao and Z.-B. Yao, PRD, 2020]

$$\mathcal{L}^{(\mathrm{ttdof})}\left(N, h_{ij}, K_{ij}, R_{ij}, \nabla_i, t\right)$$

2021 SCG with dynamical lapse function
[J. Lin, Y. Gong, Y. Lu and F. Zhang, PRD, 2021]

$$\mathcal{L}^{(\mathrm{d.l.})}\Big(N, h_{ij}, K_{ij}, \dot{N}, R_{ij}, \nabla_i, t\Big)$$

Must be satisfied the two tensorial d.o.f. (TTDOF) conditions which are difficult to be solved.

Introduction

Two approaches for MMG

Lagrangian approach:

Lagrangian
- Legendre trans. (straightforwardly)

Hamiltonian
- Primary constraints

Constraints analysis
- Conditions for Lagrangian (complicated)

Theory desired
- In configuration space

Hamiltonian approach:

Hamiltonian
- Auxiliary constraints

Constraints analysis
- Conditions for Hamiltonian (simplified)

Theory desired
- In phase space

Lagrangian
- Inverse Legendre trans. (perturbatively)

MMG with AC

Hamiltonian construction

Total Hamiltonian takes the following form:

$$H_{\mathrm{T}} = \int \mathrm{d}^3x \left[\mathrm{Functions}(\underbrace{N, N^i, h_{ij},}_{\text{ADM-var.}} \underbrace{\pi, \pi_i, \pi^{ij}}_{\text{conj.-mmta.}}; \nabla_i) + \text{some constriants}\right]$$

Determine the constraints:

$$\underbrace{(N, \pi, N^i, \pi_i, h_{ij}, \pi^{ij})}_{\text{20-dim phase space}} = \underbrace{\begin{cases} 8_{\mathrm{s}} \to 4_{\mathrm{s}} \\ 8_{\mathrm{v}} \to 0_{\mathrm{v}} \\ 4_{\mathrm{t}} \to 4_{\mathrm{t}} \end{cases}}_{\underbrace{\mathcal{H}_i \approx 0_i, \pi_i \approx 0_i}_{\text{3-diff., 1st-calss}}} + \underbrace{\begin{cases} 4_{\mathrm{s}} \to 0_{\mathrm{s}} \\ 0_{\mathrm{v}} \to 0_{\mathrm{v}} \\ 4_{\mathrm{t}} \to 4_{\mathrm{t}} \quad \text{MMG} \end{cases}}_{\underbrace{\mathcal{S}^{1\sim4} \approx 0}_{\text{2nd-calss}}}$$

A consistent framework:

$$H_{\mathrm{T}} = \int \mathrm{d}^3x \Big[\underbrace{\mathscr{H}(N, \pi, h_{ij}, \pi^{ij}; \nabla_i)}_{\text{free function}} + \underbrace{\mu_{\mathrm{n}} \mathcal{S}^{\mathrm{n}}}_{\text{aux.}} + \underbrace{N^i \mathcal{H}_i + \lambda^i \pi_i}_{\text{3-diff}}\Big] \text{ with } 1 \leq \mathrm{n} \leq 4$$

Evolutions of the auxiliary constraints: $\underbrace{\overbrace{\mathcal{S}^{\mathrm{n}} \approx 0^{\mathrm{n}}}^{\text{primary}} \Rightarrow \overbrace{\dot{\mathcal{S}}^{\mathrm{n}} \approx 0^{\mathrm{n}}}^{\text{secondary}} \Rightarrow \cdots}_{\text{total number } = \#^{\mathrm{s}}_{\mathrm{1st}} + \#^{\mathrm{s}}_{\mathrm{2nd}} \geq \mathrm{n}}$

$$\#_{\mathrm{dof}} = 2_{\mathrm{t}} + \frac{1}{2} \underbrace{(4_{\mathrm{s}} - \#^{\mathrm{s}}_{\mathrm{1st}} \times 2 - \#^{\mathrm{s}}_{\mathrm{2nd}})}_{0 \Rightarrow \text{MMG}}$$

Minimalizing conditions are needed

MMG with AC

Case of n=1

MMG theories with one auxiliary constraint:

$$H_{\mathrm{T}}^{(\mathrm{n=1})} = \int \mathrm{d}^3x \left[\mathscr{H}\left(N,\pi,h_{ij},\pi^{ij};\nabla_i\right) + \mu_1\mathcal{S}^1 + N^i\mathcal{H}_i + \lambda^i\pi_i\right]$$

The minimalizing conditions:

$$\left[\mathcal{S}^1(\vec{x}),\mathcal{S}^1(\vec{y})\right]\approx 0,\ \left[\mathcal{S}^1(\vec{x}),\dot{\mathcal{S}}^1(\vec{y})\right]\approx 0,\ \left[\dot{\mathcal{S}}^1(\vec{x}),\dot{\mathcal{S}}^1(\vec{y})\right]\approx 0$$

Example: $\mathscr{H} = \mathcal{V}\left(h_{ij},\pi^{ij};\nabla_i\right) + N\mathcal{H}_0\left(h_{ij},\pi^{ij};\nabla_i\right),\ \mathcal{S}^1 = \pi \approx 0$

The minimalizing conditions: $\left[\mathcal{H}_0(\vec{x}),\mathcal{H}_0(\vec{y})\right]\approx 0$

[S. Mukohyama and K. Noui, JCAP, 2019]

MMG with AC

Case of n=2

MMG theories with two auxiliary constraints:

$$H_{\mathrm{T}}^{(\mathrm{n=2})} = \int \mathrm{d}^3x \left[\mathscr{H}\left(N,\pi,h_{ij},\pi^{ij};\nabla_i\right) + \mu_{\mathrm{n}}\mathcal{S}^{\mathrm{n}} + N^i\mathcal{H}_i + \lambda^i\pi_i\right]$$

The minimalizing conditions:

$$\left[\mathcal{S}^{\mathrm{n}}(\vec{x}),\mathcal{S}^{\mathrm{n}'}(\vec{y})\right]\approx 0 \quad \text{or} \quad \left[\mathcal{S}^1(\vec{x}),\mathcal{S}^{\mathrm{n}}(\vec{y})\right]\approx 0\ \&\ \left[\mathcal{S}^1(\vec{x}),\dot{\mathcal{S}}^1(\vec{y})\right]\approx 0$$

Example: $\mathcal{S}^1 = \pi \approx 0$

The minimalizing conditions:

$$\frac{\delta\mathcal{S}^2(\vec{y})}{\delta N(\vec{x})}\approx 0,\ \int \mathrm{d}^3z\left(\frac{\delta\mathcal{S}^2(\vec{x})}{\delta h_{mn}(\vec{z})}\frac{\delta\mathcal{S}^2(\vec{y})}{\delta\pi^{mn}(\vec{z})} - (\vec{x}\leftrightarrow\vec{y})\right)\approx 0$$

$$\text{or}\ \frac{\delta\mathcal{S}^2(\vec{y})}{\delta N(\vec{x})}\approx 0,\ \int \mathrm{d}^3z\,\frac{\delta^2\mathscr{H}(\vec{z})}{\delta N(\vec{x})\,\delta N(\vec{y})}\approx 0$$

[Z.-B. Yao, M. Oliosi, X. Gao and S. Mukohyama, PRD, 2021]

MMG with AC

Case of n=3 & n=4

MMG theories with three auxiliary constraints:

$$H_{\mathrm{T}}^{(\mathrm{n=3})} = \int \mathrm{d}^3x \Big[\mathscr{H}\left(N,\pi,h_{ij},\pi^{ij};\nabla_i\right) + \mu_{\mathrm{n}}\mathcal{S}^{\mathrm{n}} + N^i\mathcal{H}_i + \lambda^i\pi_i \Big]$$

The minimalizing conditions: $\left[\mathcal{S}^1\left(\vec{x}\right),\mathcal{S}^{\mathrm{n}}\left(\vec{y}\right)\right] \approx 0$

MMG theories with four auxiliary constraints:

$$H_{\mathrm{T}}^{(\mathrm{n=4})} = \int \mathrm{d}^3x \Big[\mathscr{H}\left(N,\pi,h_{ij},\pi^{ij};\nabla_i\right) + \mu_{\mathrm{n}}\mathcal{S}^{\mathrm{n}} + N^i\mathcal{H}_i + \lambda^i\pi_i \Big]$$

None minimalizing conditions are needed!

A concrete example of n=4:

$$\begin{aligned} H_{\mathrm{T}}^{(\mathrm{n=4})} &= \int \mathrm{d}^3x \Big[\mathscr{H}^{(\mathrm{C.H.})}\left(N,\mathscr{R}^{\mathrm{A}},\mathit{\Pi}^{\mathrm{A}}\right) + N^i\mathcal{H}_i \\ &\quad + \lambda^i\pi_i + \lambda\pi + \mu_{\mathrm{A}}\left(\mathscr{Q}^{\mathrm{A}} - \mathscr{P}^{\mathrm{A}}\left(N\right)\right) \Big] \end{aligned}$$

$$\begin{aligned} \mathscr{R}^{\mathrm{A}} &\equiv \left\{R^i_i, R^i_jR^j_i, R^i_jR^j_kR^k_i\right\} \\ \mathit{\Pi}^{\mathrm{A}} &\equiv \left\{\pi^i_i, \pi^i_j\pi^j_i, \pi^i_j\pi^j_k\pi^k_i\right\} \\ \mathscr{Q}^{\mathrm{A}} &\equiv \left\{R^i_j\pi^j_i, R^i_j\pi^j_k\pi^k_i, R^i_jR^j_k\pi^k_i\right\} \end{aligned}$$

Minimal theory of bigravity

A consistent framework

The total Hamiltonian the minimal theories of bigravity with spatial covariance and multiple auxiliary constraints:

$$H_{\mathrm{T}} = \int \mathrm{d}^3x \Big[\mathscr{H} + \underbrace{M^i\mathcal{H}_i^{\mathrm{tot}} + \lambda^ip_i}_{\text{3-diff.}} + \underbrace{\mu_{\mathrm{n}}\mathcal{S}^{\mathrm{n}} + \nu^i_{\mathrm{m}}\mathcal{V}^{\mathrm{m}}_i}_{\text{auxiliary}} \Big] \text{ with } \mathrm{n} \le 8,\ \mathrm{m} \le 4$$

The generalized MTBG: $\#_{\mathrm{dof}} = 2_{\mathrm{t}} + 2_{\mathrm{t}}$

$$\begin{aligned} H_{\mathrm{T}} &= \int \mathrm{d}^3x \Big\{ \mathcal{V}_0 + M\mathcal{H} + M^i\mathcal{H}_i^{\mathrm{tot}} + \lambda^ip_i + \lambda p \\ &\quad + \tilde{\mathcal{V}}_0 + \tilde{M}\tilde{\mathcal{H}} + \tilde{M}^i\tilde{\mathcal{H}}_i^{\mathrm{anti}} + \tilde{\lambda}^i\tilde{p}_i + \tilde{\lambda}\tilde{p} \\ &\quad + \mu\left(\mathcal{C}_0 - \tilde{\mathcal{C}}_0\right) + \nu^i\left(\mathcal{C}_i - \tilde{\mathcal{C}}_i\right) \\ &\quad + \tilde{\mu}\left[\sqrt{h}h^{ij}\nabla_i\nabla_j\left(\frac{\mathcal{C}_0}{\sqrt{h}}\right) + \sqrt{\tilde{h}}\tilde{h}^{ij}\tilde{\nabla}_i\tilde{\nabla}_j\left(\frac{\tilde{\mathcal{C}}_0}{\sqrt{\tilde{h}}}\right)\right] \Big\} \end{aligned}$$

Free functions: $\left\{\mathcal{V}_0, \tilde{\mathcal{V}}_0, \mathcal{H}, \tilde{\mathcal{H}}, \mathcal{C}_0, \tilde{\mathcal{C}}_0, \mathcal{C}_i, \tilde{\mathcal{C}}_i\right\}$

[A. De Felice, F. Larrouturou,
S. Mukohyama and M. Oliosi,
JCAP, 2021]

Summary and outlook

- The frameworks for the minimal theory of single and bi-metric gravity with multiple auxiliary constraints in the Hamiltonian approach.
- The minimalizing conditions for the MMG theories.
- A concrete example with four auxiliary constraints.
- Generalizing the minimal theory of bigravity.
- Coupling with matter and applying to cosmology.

Thank you very much!

Session D1a 9:00–10:30

[Chair: Katsuki Aoki]

Yu-min Hu

Sun Yat-sen University

“Building ghost-free scalar-tensor theories from spatially covariant gravity”

(15 min.)

[JGRG30 (2021) 120906]

Spatially Covariant Gravity：

Building Ghost-free higher derivative scalar-tensor theory

Email: huym25@mail2.sysu.edu.cn

胡钰敏 Yu-Min Hu

School of Physics and Astronomy,
Sun Yat-sen University

Spatially Covariant Gravity：

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Email: huym25@mail2.sysu.edu.cn

Yu-min Hu
胡钰敏

School of Physics and Astronomy,
Sun Yat-sen University

（arxiv:2111.08652）

background： ghost-free HOST

- **3-dof ghost-free higher order derivative scalar-tensor theory:**

$$L = L(\phi;\ \varepsilon_{abc}, g_{ab}, R_{abcd}, \nabla_a)$$

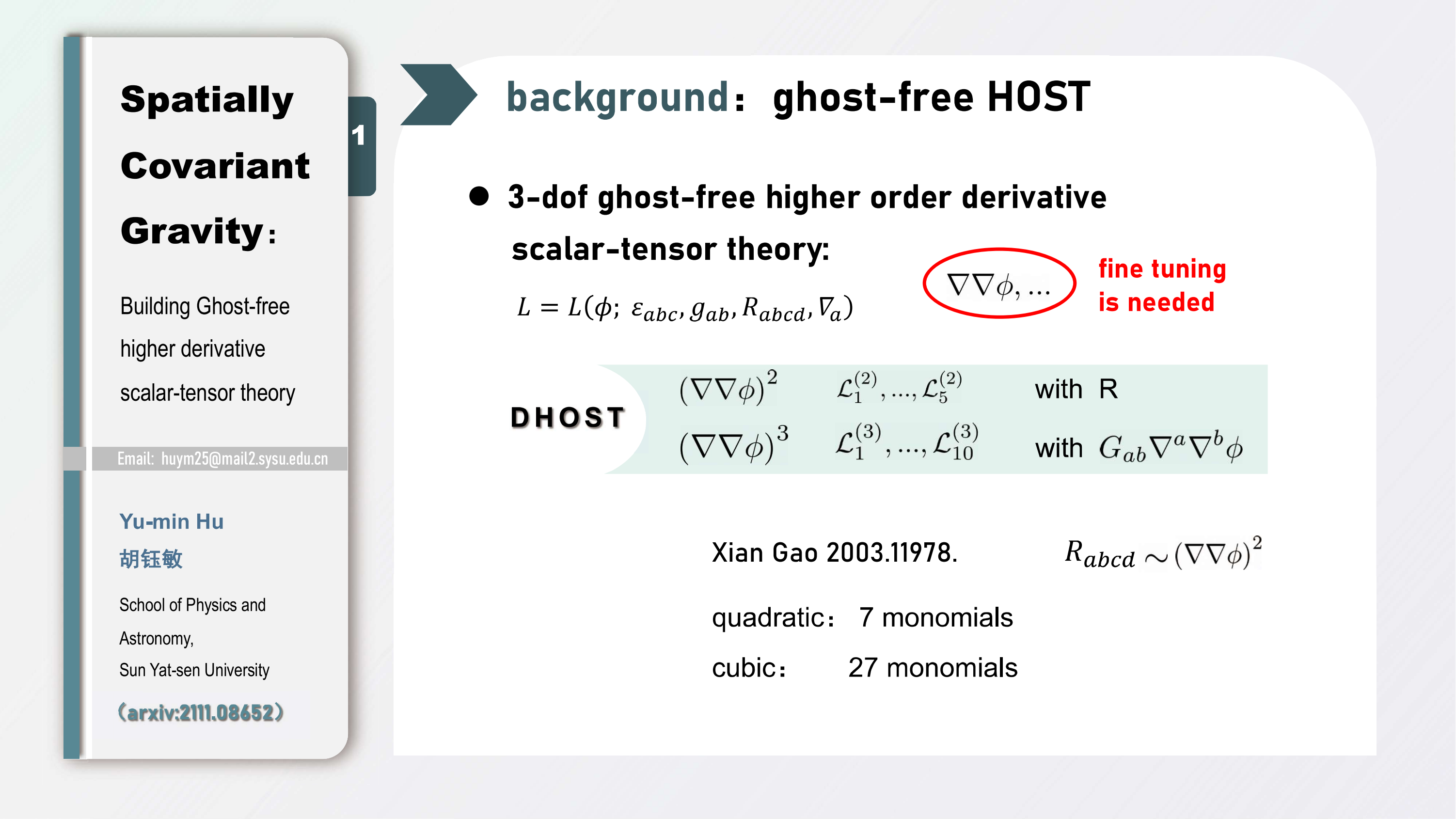
$\nabla\nabla\phi, \dots$ **fine tuning is needed**

☐ Horndeski theory/ generalized Galileons:

2nd order equations of motion

☐ Degenerate HOST (DHOST):

with additional **degenerate conditions**



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$\nabla\nabla\phi, \dots$

fine tuning is needed

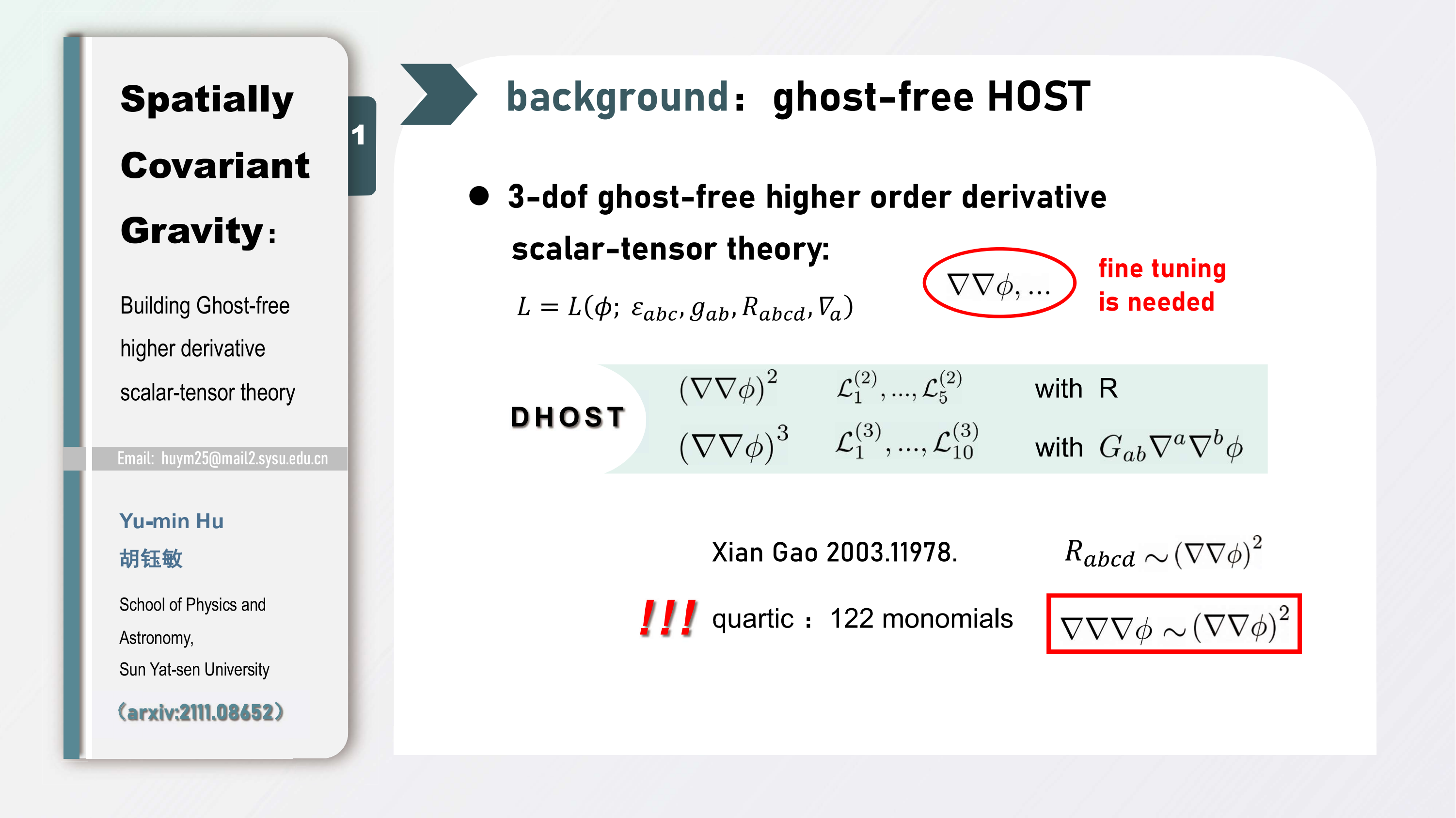
DHOST	$(\nabla\nabla\phi)^2$	$\mathcal{L}_1^{(2)}, \dots, \mathcal{L}_5^{(2)}$	with R
	$(\nabla\nabla\phi)^3$	$\mathcal{L}_1^{(3)}, \dots, \mathcal{L}_{10}^{(3)}$	with $G_{ab}\nabla^a\nabla^b\phi$

Xian Gao 2003.11978.

$R_{abcd} \sim (\nabla\nabla\phi)^2$

quadratic： 7 monomials

cubic： 27 monomials



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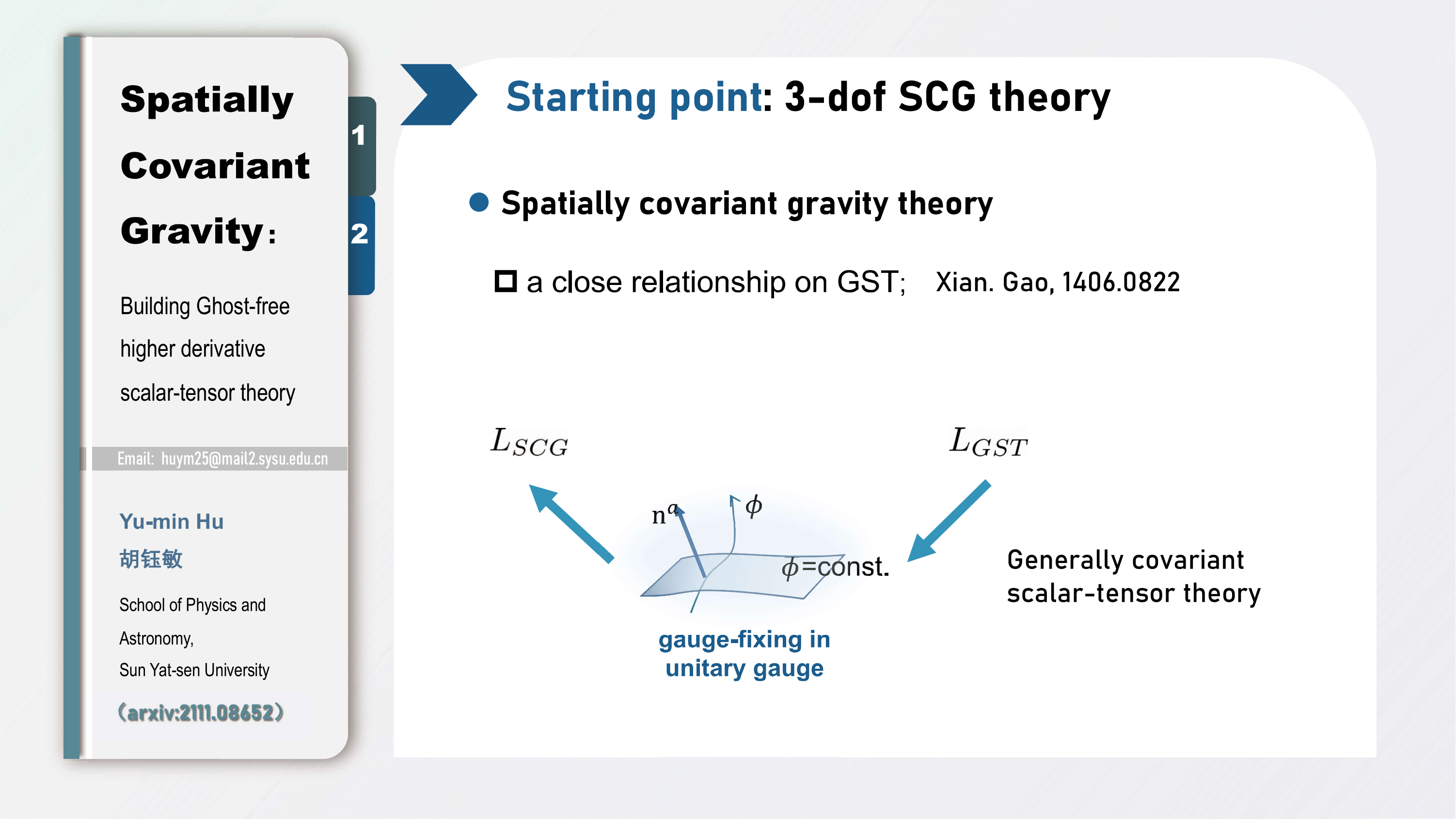
	$(\nabla\nabla\phi)^3$	$\mathcal{L}_1^{(3)}, \dots, \mathcal{L}_{10}^{(3)}$	with $G_{ab}\nabla^a\nabla^b\phi$

Xian Gao 2003.11978.

$R_{abcd} \sim (\nabla\nabla\phi)^2$

!!! quartic ： 122 monomials

$\boxed{\nabla\nabla\nabla\phi \sim (\nabla\nabla\phi)^2}$



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Starting point: 3-dof SCG theory

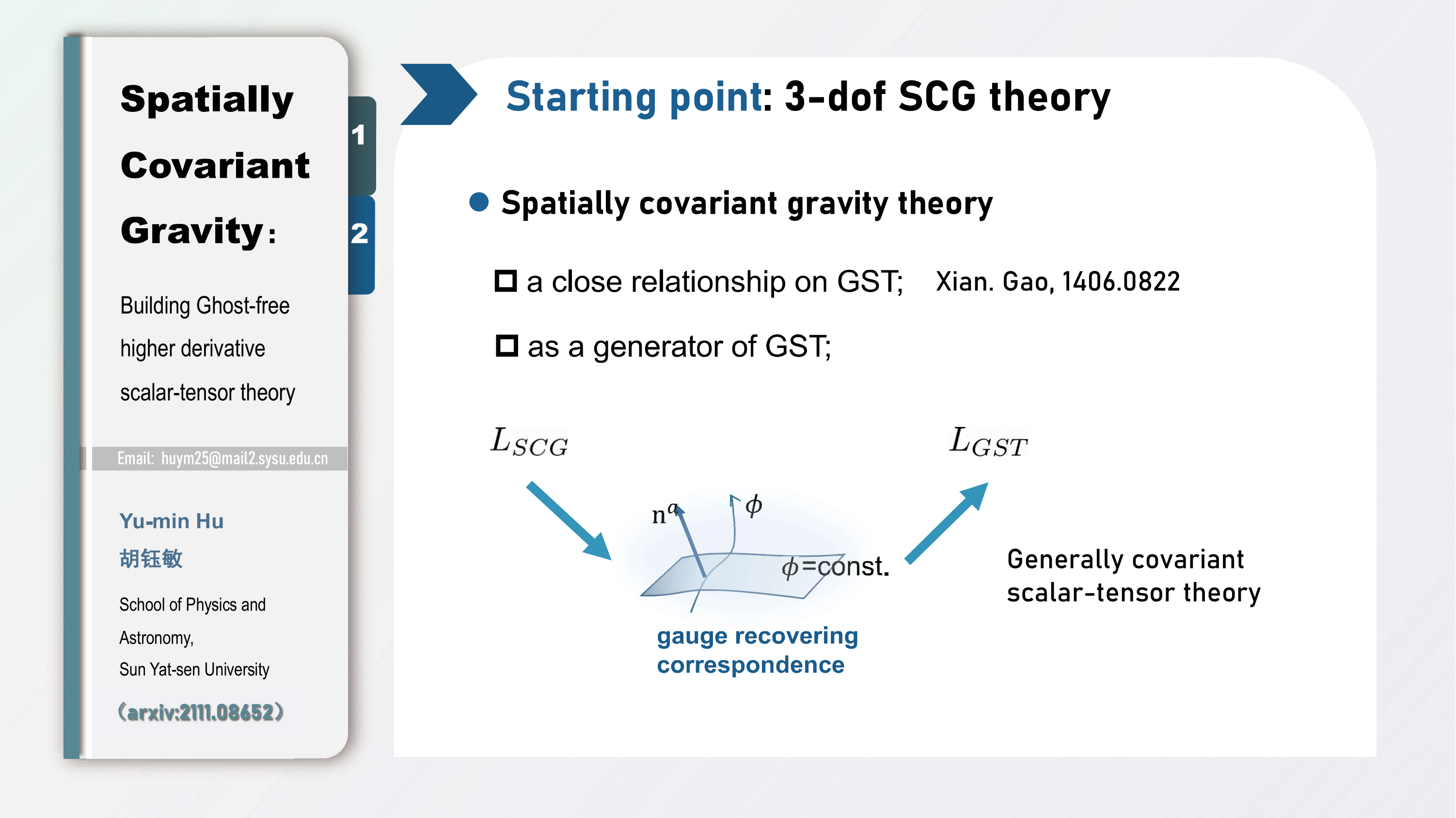
- **Spatially covariant gravity theory**
  - ☐ a close relationship on GST; Xian. Gao, 1406.0822

L_{SCG} L_{GST}

n^a ϕ ϕ=const.

gauge-fixing in unitary gauge

Generally covariant scalar-tensor theory



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- **Spatially covariant gravity theory**
  - ☐ a close relationship on GST; Xian. Gao, 1406.0822
  - ☐ as a generator of GST;

L_{SCG} L_{GST}

n^a ϕ ϕ=const.

gauge recovering correspondence

Generally covariant scalar-tensor theory

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1
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Starting point: 3-dof SCG theory

- **Spatially covariant gravity theory**
  - ☐ a close relationship on GST;   Xian. Gao, 1406.0822
  - ☐ as a generator of GST;

Especially, for our purpose

3-dof SCG → **Corresponding unitary degenerate GST**

Xian. Gao, Yu-min Hu, 2004. 07752

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1
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Starting point: 3-dof SCG theory

- **Obvious Advantages of 3-dof SCG**
  - ☐ the **decomposition form** of SCG:
    - ➢ Make degenerate/ constraints analysis directly without covariant 3+1 decomposition

Generally covariant decomposition (COD) form

For degenerate/ constraints analysis

Spatially Covariant Gravity:

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1

2

Starting point: 3-dof SCG theory

- **Obvious Advantages of 3-dof SCG**

☐ the **decomposition form** of SCG:

- Make degenerate/ constraints analysis directly without covariant 3+1 decomposition
- Easier to build 3-dof SCG

Spatially Covariant Gravity:

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Starting point: 3-dof SCG theory

- The general action: Xian. Gao, 1406.0822

$$S_{SCG}= \int dtd^3x\, N\sqrt{h}\, \mathcal{L}(t, N, h_{ij}, K_{ij}, R_{ij}, a_i, \varepsilon_{ijk}, \nabla_i),$$

- The specific model: Xian. Gao, Yu-Min Hu. 2004.07752

$K_{ij}\sim\nabla\nabla\phi$
$R_{ij}\sim(\nabla\nabla\phi)^2$
$a_i\sim\nabla\nabla\phi$

$$\mathcal{L}^{(2)}_{SCG} = c_1 K_{ij}K^{ij} + c_2 K^2 + c_3\, {}^3R + c_4 a_i a^i$$

$$\mathcal{L}^{(3)}_{SCG} = c_1^{(0;3,0)} K_{ij}K^{jk}K^i_k + c_2^{(0;3,0)} K_{ij}a^j a^i + c_3^{(0;3,0)} K K_{ij}K^{ij} + c_4^{(0;3,0)} K a_i a^i + c_5^{(0;3,0)} K^3 \\ + c_1^{(0;1,1)} K_{ij}\nabla^i a^j + c_2^{(0;1,1)} K\nabla_i a^i + c_1^{(1;1,0)} R^{ij}K_{ij} + c_2^{(1;1,0)} RK$$

$$\mathcal{L}^{(4)}_{SCG} \sim RR, \nabla_i K \nabla_j K, \ldots$$

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☐ as a generator of GST;

Especially, for our purpose

3-dof SCG → **Corresponding unitary degenerate GST**

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- The general action: Xian. Gao, 1406.0822

$$S_{SCG}=\int dtd^3x\, N\sqrt{h}\,\mathcal{L}(t,N,h_{ij},K_{ij},R_{ij},a_i,\varepsilon_{ijk},\nabla_i),$$

- The specific model: Xian. Gao, Yu-Min Hu. 2004.07752

$K_{ij}\sim\nabla\nabla\phi$
$\mathrm{R}_{ij}\sim(\nabla\nabla\phi)^2$
$a_i\sim\nabla\nabla\phi$

$$\mathcal{L}^{(2)}_{SCG}=c_1^{(0;2,0)}K_{ij}K^{ij}+c_2^{(0;2,0)}a_ia^i+c_3^{(0;2,0)}K^2+c_4^{(1;0,0)}R$$

$$\begin{aligned}\mathcal{L}^{(3)}_{SCG}\;&=c_1^{(0;3,0)}K_{ij}K^{jk}K^i_k+c_2^{(0;3,0)}K_{ij}a^ja^i+c_3^{(0;3,0)}KK_{ij}K^{ij}+c_4^{(0;3,0)}Ka_ia^i+c_5^{(0;3,0)}K^3\\&+c_1^{(0;1,1)}K_{ij}\nabla^ia^j+c_2^{(0;1,1)}K\nabla_ia^i+c_1^{(1;1,0)}R^{ij}K_{ij}+c_2^{(1;1,0)}RK\end{aligned}$$

$$\mathcal{L}^{(4)}_{SCG}\sim RR,\nabla_iK\nabla_jK,\ldots$$

$\boxed{\nabla\nabla\nabla\phi}$

already have a healthy higher derivative U-degenerate theory

A. D. Felice, et.al. . 1803.06241

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（arxiv:2111.08652）

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2

Starting point: 3-dof SCG theory

- **Obvious Advantages of 3-dof SCG**

☐ the **decomposition form** of SCG:

- Make degenerate/ constraints analysis directly without covariant 3+1 decomposition
- Easier to build 3-dof in SCG formalism

☐ the **simplified formulation** from SCG;

- the U-degenerate property

necessary condition !!!

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the covariant 3+1 correspondence

3-dof SCG → correspondence relations → **U-degenerate GST**

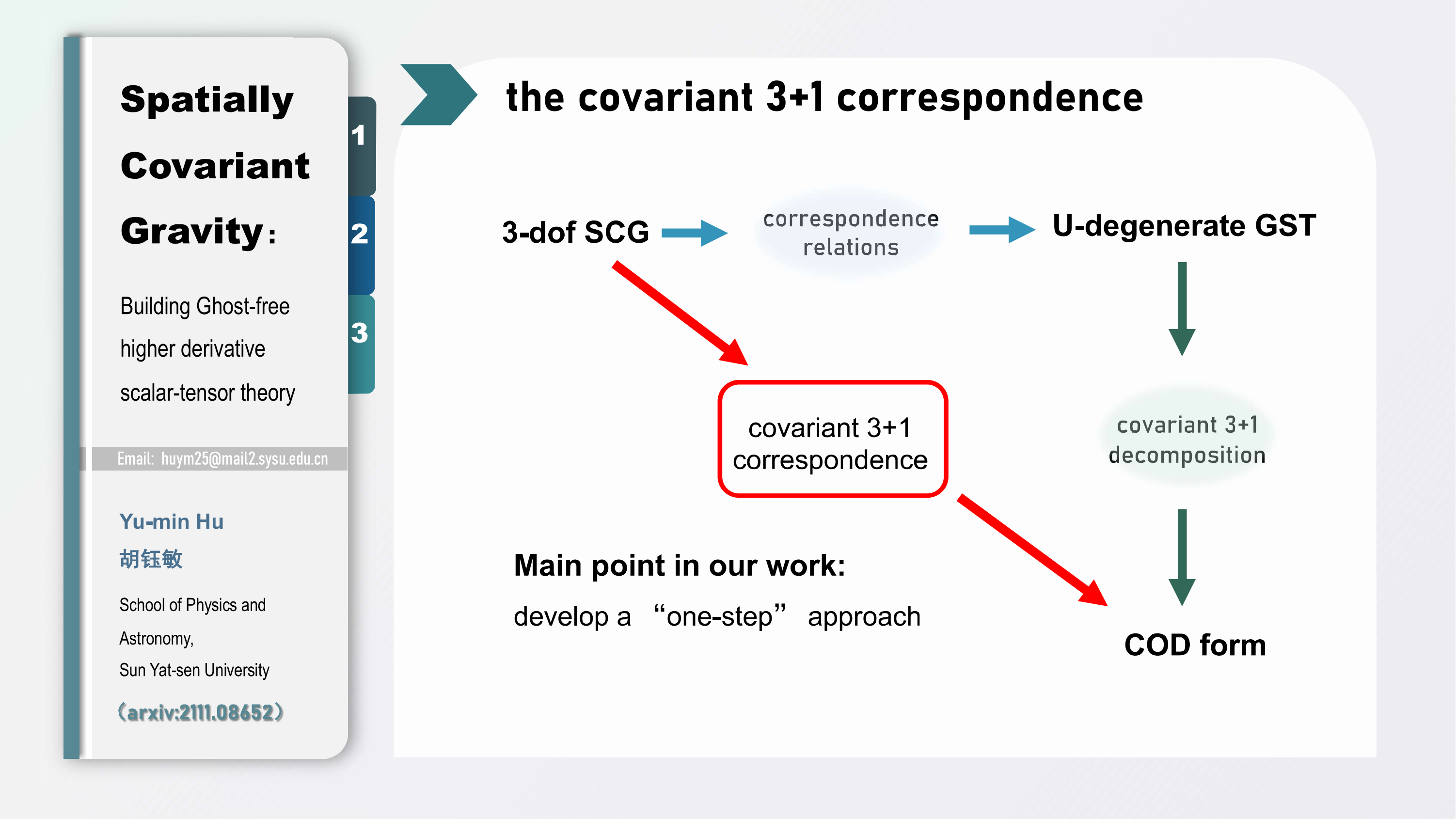
↓

covariant 3+1 decomposition

↓

COD form

!!! Further conditions are also need to get a degenerate GST even when unitary gauge is not accessible.



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the covariant 3+1 correspondence

3-dof SCG → correspondence relations → **U-degenerate GST**

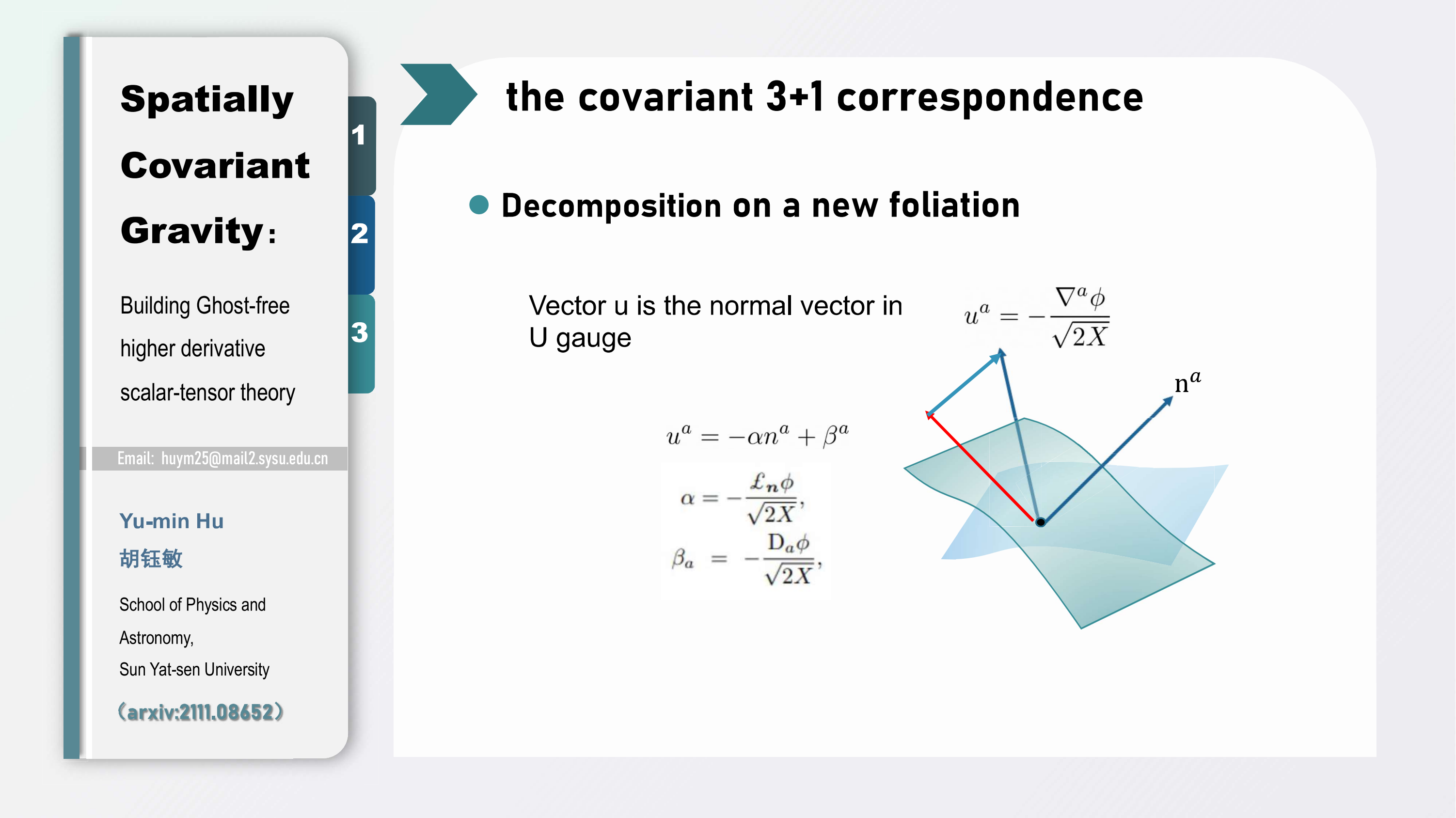
covariant 3+1 correspondence

covariant 3+1 decomposition

COD form

Main point in our work:

develop a “one-step” approach



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(arxiv:2111.08652)

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the covariant 3+1 correspondence

- **Decomposition on a new foliation**

Vector u is the normal vector in U gauge

$$u^a = -\frac{\nabla^a \phi}{\sqrt{2X}}$$

$$u^a = -\alpha n^a + \beta^a$$

$$\alpha = -\frac{\pounds_n \phi}{\sqrt{2X}},$$

$$\beta_a = -\frac{\mathrm{D}_a \phi}{\sqrt{2X}},$$

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4

Apply in 3-dof SCG model

- the specific SCG model

☐ d=2: $\mathcal{L}^{(2)}_{SCG} = c_1 K_{ij}K^{ij} + c_2 K^2 + c_3\,{}^3R + c_4 a_i a^i$

$$\boxed{\cancel{\dot{X}^2},\ \cancel{\dot{X}K}}$$

the unique solutions for the coefficients:

Consistent with Horndeski theory

$$\mathcal{L}^{(2)}_{SCG} = \left(c_3 - 2X\frac{\partial c_3}{\partial X}\right)\left(K_{ij}K^{ij} - K^2\right) + c_3\,{}^3R$$

$$c_1 = -c_2 = c_3 - 2X\frac{\partial c_3}{\partial X},\ c_4 = 0$$

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1
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Apply in 3-dof SCG model

- the specific SCG model

☐ d=3:

$$\begin{aligned}\mathcal{L}^{(3)} = {} & c_1^{(0;3,0)} K_{ij}K^{jk}K^i_k + c_3^{(0;3,0)} K K_{ij}K^{ij} + c_5^{(0;3,0)} K^3 \\ & + c_2^{(0;3,0)} K_{ij}a^j a^i + c_4^{(0;3,0)} K a_i a^i \\ & + c_1^{(0;1,1)} K_{ij}\nabla^i a^j + c_2^{(0;1,1)} K\nabla_i a^i \\ & + c_1^{(1;1,0)} R^{ij}K_{ij} + c_2^{(1;1,0)} RK\end{aligned}$$

$$\boxed{\cancel{\dot{X}^3},\ \cancel{\dot{X}^2K},\ \cancel{X\dot{K}},\ \cancel{K\dot{K}},\ \cancel{\dot{X}K^2}}$$

Consistent with Horndeski theory

$$c_2^{(1;1,0)} = -\frac{1}{2}c_1^{(1;1,0)} \qquad c_3^{(0;3,0)} = -\frac{3}{2}c_1^{(0;3,0)} \qquad c_5^{(0;3,0)} = \frac{1}{2}c_1^{(0;3,0)}$$

$$3c_1^{(0;3,0)} + 2X\frac{\partial c_1^{(1;1,0)}}{\partial X} = 0,$$

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(arxiv:2111.08652)

Summary:

- We develop a **"one-step" approach called the "covariant 3+1 correspondence"**; A COD form is obtained directly from the SCG;
- We demonstrate how we **regain Horndeski** theory from the 3-dof SCG model and **prove validity** of our method.
- Future plan: start with more general 3-dof SCG to search for more general ghost-free scalar-tensor theory.

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In the end:

Thanks for your attention

Session D1b 9:00–10:30

[Chair: Umpei Miyamoto]

Kazushige Ueda

Kyushu University

"Numerical Investigation of Quasi-normal Mode in Kerr-AdS_5 Black Hole"

(15 min.)

[JGRG30 (2021) 120907]

Oral D1 b1

Numerical Investigation of Quasi-Normal Mode in Kerr-AdS$_5$

Research ongoing

@JGRG30(online), 2021/12/09, 9:00~9:15

Outline

- Introduction
- Set up (Kerr AdS$_5$)
- Numerical Analysis
- Summary

Collaborators

Issei Koga
@Kyushu Univ.

Naritaka Oshita
@ Riken.

Contents are base the paper in preparation
Issei Koga, Naritaka Oshita, Kazushige Ueda
(To be announced)

Kyushu University
Theoretical Astrophysics Group D2

Kazushige Ueda

Introduction

Higher dimension "A Large Mass Hierarchy from a Small Extra Dimension " L. Randall and R. Sundrum, PRL. 83, 3370 (1999).

After L. Randall and R. Sundrum used the brane model in the context of hierarchy problem in particle physics, many types of higher dimensional model is considered in the area of string theory, brane cosmology, the holographic principle and so on.

Higher dimensional gravity (especially AdS$_5$) attracted many attentions.

・Holographic principle　　・Brane-world scenario

"Anti De Sitter Space And Holography", E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998)
"Brane-world cosmology", D. Ida, JHEP 09, 014 (2000).

BH in AdS$_5$

・Thermality of CFT　　・Source of dark radiation

"Conformal Field Theory Interpretation of Black Hole Quasi-normal Modes ," Danny Birmingham+, PRL 88, 151301 (2002)

S. Hod, Phys. Rev. Lett. 81, 4293 (1998), arXiv:gr-qc/9812002 .

Stability is not guaranteed. (cf. super radiant instability)

"The instability of anti-de Sitter space-time" Grégoire Martinon, arXiv:1708.05600v3
"Superradiant instability of five-dimensional rotating charged AdS black holes ", Alikram N+, PRD 79, 024013 (2009)

Quasi normal mode analysis

"QNMs of scalar fields on small Reissner-Nordstrom-AdS5 black holes", J. B. Amado+, PRD104, 084051 (2021)

"Scalar quasinormal modes of Kerr-AdS5", J. B. Amado+, PRD 99, 105006 (2019)

Procedure QNMs analysis in Kerr AdS$_5$

Prescription to find QN frequency “Scalar quasinormal modes of Kerr-AdS5”, J. B. Amado+, PRD 99, 105006 (2019)

Step1. describing field equation with the Kerr AdS$_5$ metric

Step2. separating radial equation and angular equation

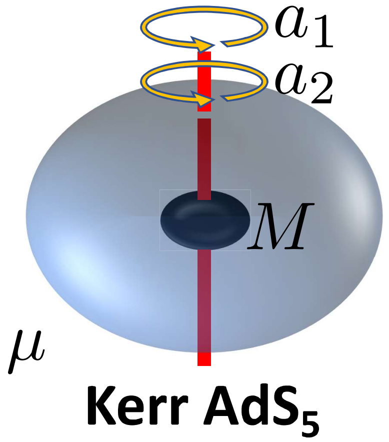
Step3. performing valuable transformation to obtain Heun equations

Step4. setting boundary condition at AdS boundary and event horizon

Step5. finding complex frequency which gives 0 Wronskian for solution at AdS boundary and solution event horizon with Wolfram Mathematica 12.1

“Quasinormal modes of Kerr-de Sitter black holes via the Heun function”, Yasuyuki Hatsuda,arXiv:2006.08957

Repeat Step1 〜 Step5 for various parameter region
(BH mass, field mass, spin, spin ratio…)



Procedure QNMs analysis in Kerr AdS$_5$

Prescription to find QN frequency “Scalar quasinormal modes of Kerr-AdS5”, J. B. Amado+, PRD 99, 105006 (2019)

Step1. describing field equation with the Kerr AdS$_5$ metric

$$\left[\nabla_\mu \nabla^\mu - \mu^2\right] \Phi = 0$$

The background metric is Kerr-AdS_5 metric with no back reaction.

$$\begin{aligned} ds^2 = &-\frac{\Delta_r}{\rho^2}\left(dt - \frac{a_1 \sin^2\theta}{1-a_1^2}d\phi - \frac{a_2\cos^2\theta}{1-a_2^2}d\psi\right)^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(a_1 dt - \frac{r^2+a_1^2}{1-a_1^2}d\phi\right)^2 \\ &+ \frac{1+r^2}{r^2\rho^2}\left(a_1 a_2 dt - \frac{a_2(r^2+a_1^2)\sin^2\theta}{1-a_1^2}d\phi - \frac{a_1(r^2+a_2^2)\cos^2\theta}{1-a_2^2}d\psi\right)^2 \\ &+ \frac{\Delta_\theta\cos^2\theta}{\rho^2}\left(a_2 dt - \frac{r^2+a_2^2}{1-a_2^2}d\psi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 \end{aligned}$$

where

$$\Delta_r \equiv \frac{1}{r^2}(r^2+a_1^2)(r^2+a_2^2)(1+r^2) - 2M,$$

$$\Delta_\theta \equiv 1 - a_1^2\cos^2\theta - a_2^2\sin^2\theta,$$

$$\rho^2 \equiv r^2 + a_1^2\cos^2\theta + a_2^2\sin^2\theta$$

ADM Mass, angular momentum

$$\mathcal{M} \equiv \frac{\pi M(2\Xi_1 + 2\Xi_2 - \Xi_1\Xi_2)}{4\Xi_1^2\Xi_2^2}$$

$$\mathcal{J}_\phi \equiv \frac{\pi M a_1}{2\Xi_1^2\Xi_2} \qquad \mathcal{J}_\psi \equiv \frac{\pi M a_2}{2\Xi_1\Xi_2^2}$$

$$\Xi_i \equiv 1 - a_i^2 \qquad a_i < 1$$

Procedure QNMs analysis in Kerr AdS$_5$

Prescription to find QN frequency “Scalar quasinormal modes of Kerr-AdS5”, J. B. Amado+, PRD 99, 105006 (2019)

Step1. describing field equation with the Kerr AdS$_5$ metric

Step2. separating radial equation and angular equation

$$\Psi = e^{-i\omega t + i m_1 \phi + i m_2 \psi}\Theta(\theta)\Pi(r)$$

Radial field equation

$$\frac{1}{r}\frac{d}{dr}\left(r\Delta_r\frac{d\Pi(r)}{dr}\right) - \left[\lambda + \mu^2 r^2 + \frac{1}{r^2}\left(a_1 a_2\omega - a_2(1-a_1^2)m_1 - a_1(1-a_2^2)m_2\right)^2\right]\Pi(r)$$
$$+\frac{(r^2+a_1^2)^2(r^2+a_2^2)^2}{r^4\Delta_r}\left(\omega - \frac{m_1 a_1(1-a_1^2)}{r^2+a_1^2} - \frac{m_2 a_2(1-a_2^2)}{r^2+a_2^2}\right)^2\Pi(r) = 0$$

Angular field equation

$$\frac{1}{\sin\theta\cos\theta}\frac{d}{d\theta}\left(\sin\theta\cos\theta\Delta_\theta\frac{d\Theta(\theta)}{d\theta}\right) - \left[-\lambda\omega^2 + \frac{(1-a_1^2)m_1^2}{\sin^2\theta} + \frac{(1-a_2^2)m_2^2}{\cos^2\theta}\right.$$
$$\left. - \frac{(1-a_1^2)(1-a_2^2)}{\Delta_\theta}(\omega + m_1 a_1 + m_2 a_2)^2 + \mu^2(a_1^2\cos^2\theta + a_2^2\sin^2\theta)\right]\Theta(\theta) = 0$$

λ Separation constant that makes $\Theta(\theta)$ is non-singular.

Procedure QNMs analysis in Kerr AdS$_5$

Prescription to find QN frequency “Scalar quasinormal modes of Kerr-AdS5”, J. B. Amado+, PRD 99, 105006 (2019)

Step1. describing field equation with the Kerr AdS$_5$ metric

Step2. separating radial equation and angular equation

Step3. performing valuable transformation to obtain Heun equations

Valuable transformation for radial equation

$$r \to z \equiv \frac{r^2 - r_-^2}{r^2 - r_0^2},$$
$$\Pi(r) \to R(z) \equiv z^{\theta_-/2}(z-z_0)^{\theta_+/2}(z-1)^{-\Delta/2}\Pi(z)$$

Valuable transformation for angular equation

$$\sin^2\theta \to u \equiv \frac{\sin^2\theta}{\sin^2\theta - \chi_0}, \text{ with } \chi_0 \equiv \frac{1-a_1^2}{a_2^2 - a_1^2},$$
$$\Theta(\theta) \to S(u) \equiv u^{-m_1/2}(u-1)^{-\Delta/2}(u-u_0)^{-m_2/2}\Theta(u)$$

Procedure QNMs analysis in Kerr AdS$_5$

Prescription to find QN frequency “Scalar quasinormal modes of Kerr-AdS5”, J. B. Amado+, PRD 99, 105006 (2019)

Step1. describing field equation with the Kerr AdS$_5$ metric

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Equation of radial part

$$\frac{d^2R}{dz^2}+\left[\frac{1-\theta_-}{z}+\frac{-1+\Delta}{z-1}+\frac{1-\theta_+}{z-z_0}\right]\frac{dR}{dz}+\left(\frac{\kappa_1\kappa_2}{z(z-1)}-\frac{K}{z(z-1)(z-z_0)}\right)R=0$$

Heun equations

Equation of angular part

$$\frac{d^2S}{du^2}+\left[\frac{1+m_1}{u}+\frac{-1+\Delta}{u-1}+\frac{1+m_2}{u-u_0}\right]\frac{dS}{du}+\left(\frac{q_1q_2}{u(u-1)}-\frac{Q}{u(u-1)(u-u_0)}\right)S=0$$

Procedure QNMs analysis in Kerr AdS$_5$

Prescription to find QN frequency “Scalar quasinormal modes of Kerr-AdS5”, J. B. Amado+, PRD 99, 105006 (2019)

Step1. describing field equation with the Kerr AdS$_5$ metric

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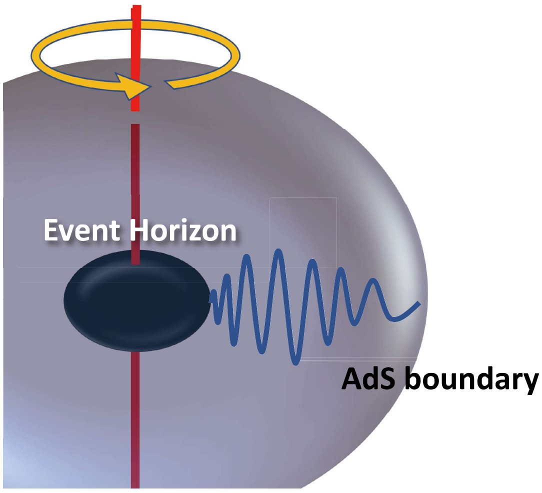
Step4. setting boundary condition at AdS boundary and event horizon

Kerr AdS



Boundary condition for radial component

$$\Pi(z)\sim\begin{cases}(z-z_0)^{-\theta_+/2} & \text{for } z\to z_0\ (r\to r_+),\\ (z-1)^{\Delta/2} & \text{for}z\to 1\ (r\to\infty),\end{cases}$$

Asymptotic behavior

Boundary condition for angular component

$$\Theta(u)\sim\begin{cases}u^{|m_1|/2} & \text{for } u\to 0\ (\theta\to 0),\\ (u-u_0)^{|m_2|/2} & \text{for } u\to u_0\ (\theta\to\pi/2).\end{cases}$$

Asymptotic behavior

Procedure QNMs analysis in Kerr AdS$_5$

Prescription to find QN frequency “Scalar quasinormal modes of Kerr-AdS5”, J. B. Amado+, PRD 99, 105006 (2019)

Step1. describing field equation with the Kerr AdS$_5$ metric

Step2. separating radial equation and angular equation

Step3. performing valuable transformation to obtain Heun equations

Step4. setting boundary condition at AdS boundary and event horizon

Step5. finding complex frequency which gives 0 Wronskian for solution at AdS boundary and solution event horizon with Wolfram Mathematica 12.1

Quasinormal modes of Kerr-de Sitter black holes via the Heun function, Yasuyuki Hatsuda,arXiv:2006.08957

QNM with Small BH

For small holes, QN frequencies are localized near the real axis of the complex ω-plane. **We call it as type-I QNM.**

The super radiant instability is caused by the resonance between the outer horizon and AdS boundary. The stability of the black hole can be confirmed by the positivity of the imaginary part ω.

Preliminary

The angular [redacted]

Spin dependence of the QN instability

The angular modes are fixed as l= 1,m1= 1,m2= 0 that is equivalent to l= 1,m1= 0,m2= 1.

Preliminary

When spin get large, the system becomes more unstable.

Spin-hierarchy dependence of the unstable QNMs

Small mass limit

Next, we break the symmetry of spin parameter.

$$U(2) \to U(1) \times U(1)$$

Super radiant condition

$$\mathrm{Re}(\omega_{lm_1m_2n}) < m_1\Omega_1 + m_2\Omega_2 \equiv \Omega$$

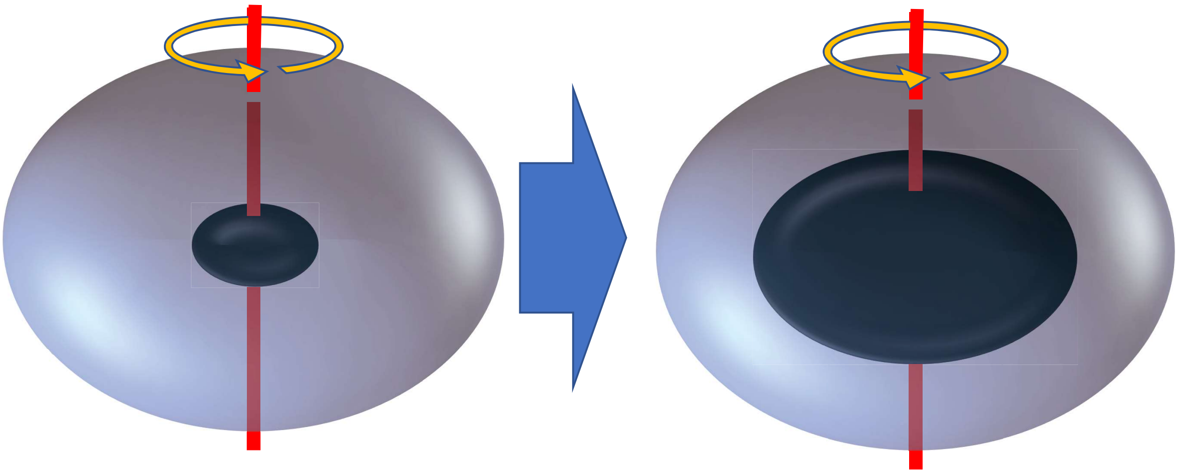
Preliminary

Asymmetry of the spin parameter destabilize the QN mode.

Research Question.

What happens when horizon becomes large?

Intuitively, the QNM gets will be stable with large mass BH.



Mass dependence of Hawking temperature in AdS$_5$

From now on, we check the BH mass dependence of QNMs since the mass dependence of Hawking temperature in AdS$_5$ is different from usual case.

Preliminary

Mass dependence of type-I QNMs

Type I becomes stable when BH mass becomes large.

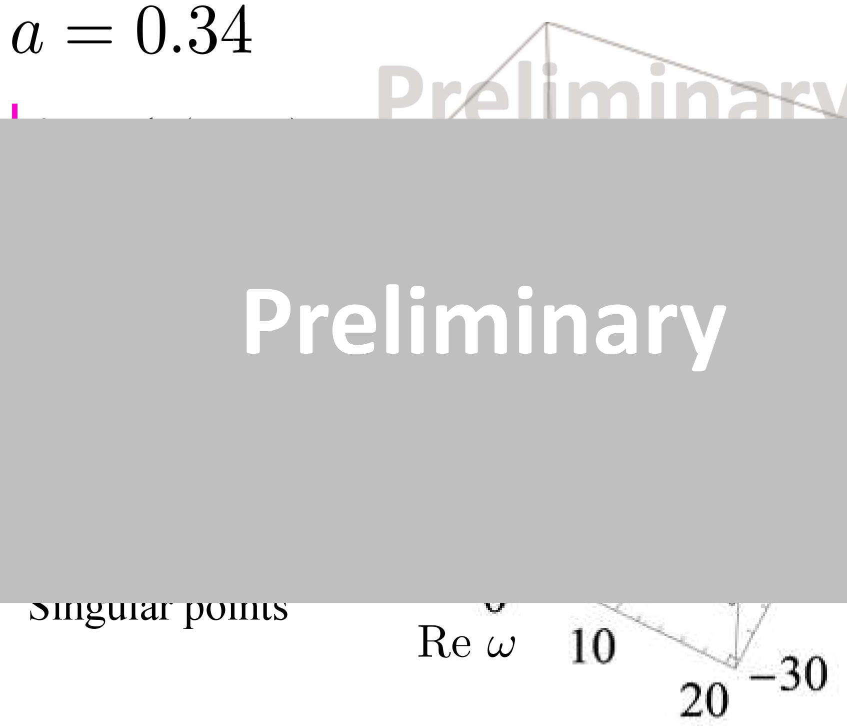
Preliminary

The trajectory of the QNM with the change of ADM Mass of BH.

Stable QNMs with equal spin parameters

Large mass limit

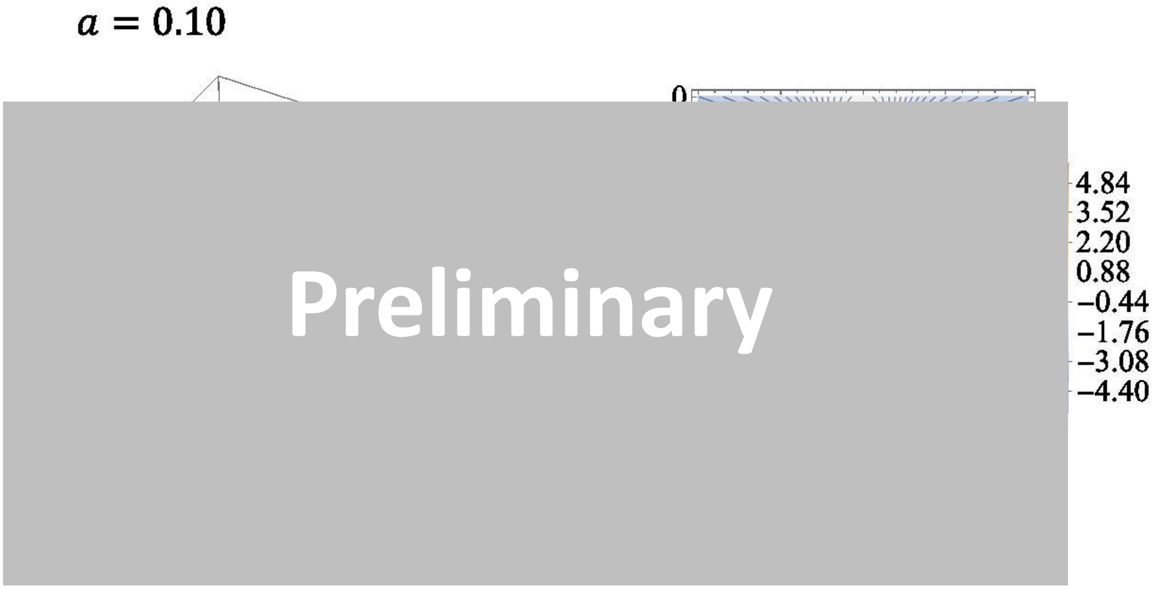
$a = 0.34$



Preliminary

There are 2 types of QNMs.

Spin dependence of QNMs for with large mass BH



Behavior of two types of QNMs

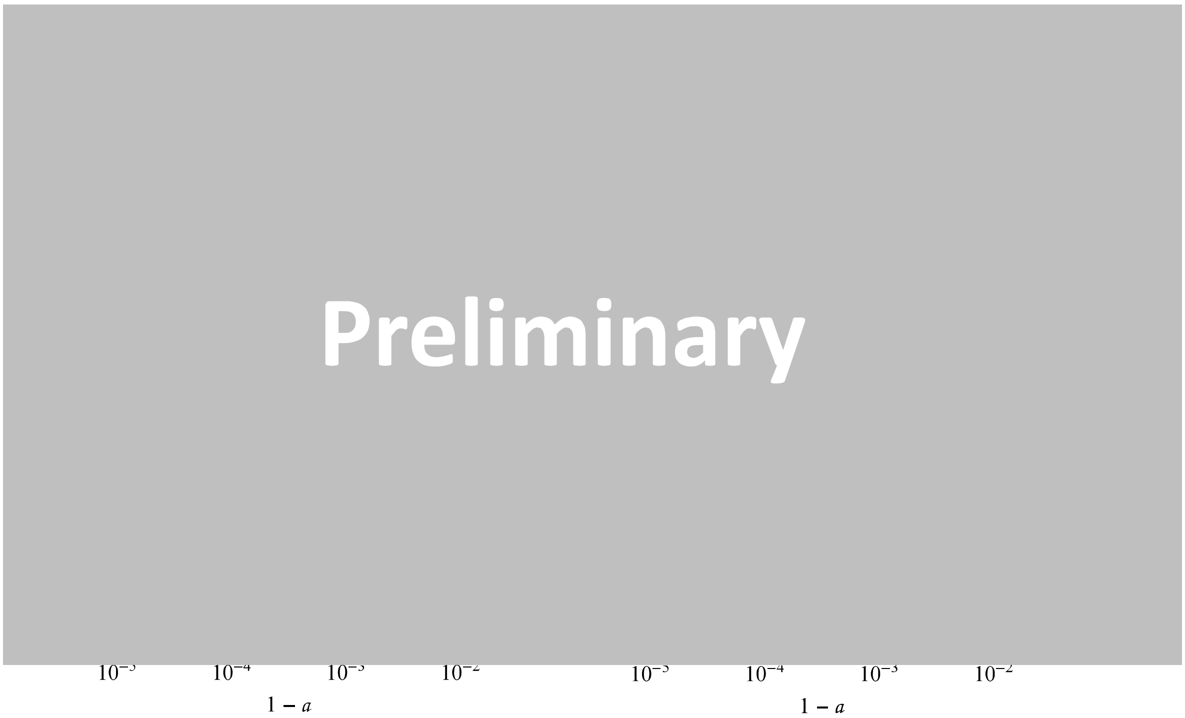
Type I and type II have different response to the change of spin parameter

●: Type-I ●: Type-II

Intervals of the (n+1)-th and n-th QNMs

When the scalar field mass becomes large, the dispersion of gaps of imaginary part also becomes large.

Intervals of higher overtones reduce to the 2π times Hawking temperature.



Imaginary part of QNM near super entropic limit

Real part of the QN frequency in super entropic limit.

Preliminary

Would be relevant to the area quantization (cf. Hod's conjecture).

S. Hod, Phys. Rev. Lett. 81, 4293 (1998), arXiv:gr-qc/9812002 .

TYPE-II in small BH mass case for small spin

E. Berti+, PRD 68, 124018 (2003) E. Berti+, PRD 69, 124018 (2004)

Preliminary

Super radiance condition

$$\Omega_{k,1} = \frac{a_1(1-a_1^2)}{r_k^2+a_1^2}, \qquad \Omega_{k,2} = \frac{a_2(1-a_2^2)}{r_k^2+a_2^2}, \qquad \mathrm{Re}(\omega_{lm_1m_2n}) < m_1\Omega_1 + m_2\Omega_2 \equiv \Omega$$

Summary & Conclusion

We investigated QNM in Kerr AdS$_5$

Investigation for small BH mass

1. **The instability of QNMs are dominated by l=1 angular mode.**
2. **The asymmetry of the spin parameter destabilize QNMs.**

Investigation for large BH mass

3. **There are 2 types of QNM (stable QN modes).**
   We call the class of QNMs which is localized around real axis in small mass limit as type I, and another class which is localized around imaginary axis in large mass limit as type II.
4. **Type II QNM has intervals of 2π times Hawking temperature.**
   →Which would be dual to the pole structure of green function in CFT

“Conformal Field Theory Interpretation of Black Hole Quasi-normal Modes”,
D. Birmingham+, PRL. 88, 151301 (2002)

Thank you for your listening.

Session D1b 9:00–10:30

[Chair: Umpei Miyamoto]

Ryotaku Suzuki

Toyota Technological Institute

“Squashed black holes at large D”

(15 min.)

[JGRG30 (2021) 120908]

Squashed black holes at large D

Ryotaku Suzuki

with

Shinya Tomizawa

Toyota Technological Institute

Based on arXiv: 2111.04962

JGRG30, Online, 6-10 December, 2021

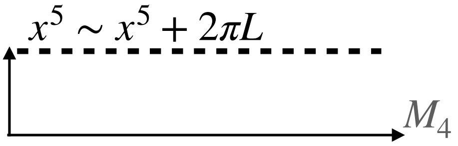
Introduction

String Theory predicts D>4 higher dimensions

To obtain our 4D universe, extra dimensions have to be “compactified”

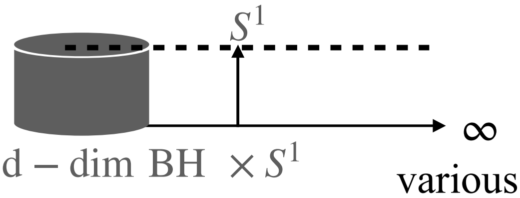
A simple idea is **Kaluza-Klein compactification**

$$M_5 \to M_4 \times S^1$$



Black holes are important to understand compactified spacetime

ex) Kaluza-Klein black holes



various types of compactification exist

We focus on BHs with compactification called “squashed Kaluza-Klein”

Squashed Kaluza-Klein BH

Hopf fibration

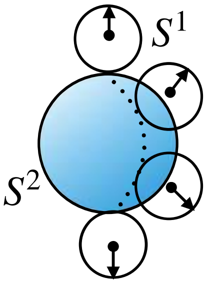
S^1 is twisted

$$d\Omega_3^2 = \frac{1}{4}\left((d\psi + \cos\theta d\phi)^2 + d\Omega_2^2\right)$$

base space

$S^3 \neq S^1 \times S^2$ **but** S^3 **is** S^1 fiber *on* S^2

$S^3 \sim$



Squashed transformation

D=5 Schwarzschild $ds_5^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2, \quad f(r) = 1 - r^{-2}$

Squashed Kaluza-Klein (SqKK) black hole

$$ds_5^2 = -f(r)dt^2 + \frac{k(r)^2 dr^2}{f(r)} + \frac{r^2}{4}\left((d\psi + \cos\theta d\phi)^2 + k(r) d\Omega_2^2\right)$$

k(r) : squashing function

Near horizon : D=5 BH / Far region ($k(r \to r_\infty) \to \infty$) : Kaluza-Klein

---

Squashed Kaluza-Klein BH in D=5

So far many SqKK BHs were found

- Myers-Perry → Rotating SqKK BH ($J_1 = J_2$) Dobiasch-Maison (1982)
- Myers-Perry → Rotating SqKK BH ($J_1 \neq J_2$) Rasheed (1995)
- Reissner-Nordstrom → Charged SqKK BH Ishihara-Matsuno (2005)
- (Charged) Rotating-Godel SqKK BH Tomizawa-Ishihara-Matsuno-Nakagawa (2008)
  Tomizawa-Ishibashi (2008)
- SqKK SUSY BH Gaiotto-Strominger-Yin (2006)

etc…

But these are with squashed S^1 over S^2 in D=5

Can we have higher dimensional generalization ?

SqKK BH in higher dimensions

Actually, in D=2n+3 Einstein-Maxwell theory
charged extremal SqKK BH with S^1 fiber over CP^n has been solved

Tatsuoka-Ishihara-Kimura-Matsuno (2011)

Hopf fibration of S^{2n+1}

$S^3 \sim S^1$ over S^2

$\longrightarrow \quad S^{2n+1} \sim S^1$ over CP^n $\qquad d\Omega^2_{2n+1} = (d\phi + \mathscr{A}_n)^2 + d\Sigma^2_{CP^n}$

$\mathscr{A}_n$: Kahler potential of CP^n

But we also have charged SqKK BH in D=5 Einstein-Maxwell

Ishihara-Matsuno (2005)

Can we find the non-extremal generalization ?

(Or higher dimensional generalization)

Strategy

We assume D=2n+3 metric with squashed S^1 over CP^n

$$ds^2 = G_{tt}dt^2 + G_{rr}dr^2 + G_{\phi\phi}(d\phi + \mathscr{A}_n)^2 + r^2 d\Sigma_n^2$$

CP^n

Extremal case has enhanced symmetry

Tatsuoka-Ishihara-Kimura-Matsuno (2011)

Einstein-Maxwell eq. → Laplace eq.

We cannot expect such simplification in non-extremal case

Instead, we solve the metric in the Large D limit (large n limit)

Large D limit

Emparan, RS, Tanabe (2013)

$$S_{EH} = \int dx^D R$$

- Large spacetime dimension "D" ($D \rightarrow \infty$)
  (mostly large D owes to S^{D-p})
- BH dynamics → Effective Theory (D=∞) + O(1/D) correction
  (analogy) Large N limit of SU(N) Super Yang-Mills

Large D limit of Schwarzschild

Emparan,RS,Tanabe 2013

D-dim Schwarzschild BH

$$ds^2 = -\left(1 - \frac{r_0^{D-3}}{r^{D-3}}\right) dt^2 + \left(1 - \frac{r_0^{D-3}}{r^{D-3}}\right)^{-1} dr^2 + r^2 d\Omega_{D-2}^2$$

2 ways of the large D limit

A. Far region: Fix r, r_0

➡ $$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{D-2}^2 + \mathcal{O}((r_0/r)^{D-3})$$

Minkowski + perturbation

B. Near horizon: Fix $\mathsf{R} := (r/r_0)^{D-3}$

➡ $$ds^2 = -(1 - \mathsf{R}^{-1}) dt^2 + \frac{r_0^2}{D^2} \frac{d\mathsf{R}}{\mathsf{R}(\mathsf{R}-1)} + r_0^2 d\Omega_{D-2}^2 + \mathcal{O}(\ln \mathsf{R}/D)$$

Horizon exists

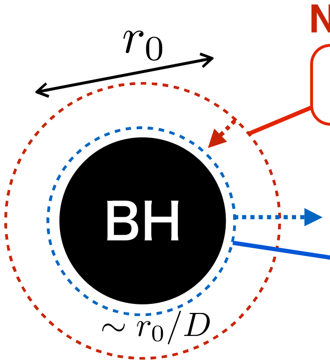
$\approx$ (2D BH) $\times S^{D-2}$

$r \simeq r_0\left(1 + \frac{1}{D}\log \mathsf{R}\right)$ → **B describes near horizon region $r - r_0 \sim r_0/D$**

1/D expansion

new small scale $r_0/D \ll r_0$

$$\mathsf{R} = (r/r_0)^{D-3}$$



Near horizon

$r - r_0 \ll r_0$

$$\phi(\mathsf{R}) = \phi_0(\mathsf{R}) + \frac{1}{D}\phi_1(\mathsf{R}) + \ldots$$

→1/D expansion

Far region

$r - r_0 \gg r_0/D$

$$\phi(r) = \phi_0^{(far)}(r) + \frac{\phi_1^{(far)}(r)}{r^{D-3}} + \ldots$$

→~Minkowski

Overlap region $r_0/D \ll r - r_0 \ll r_0$ $(1 \ll \mathsf{R},\ \log \mathsf{R} \ll D)$

→ new Matching region

matched in double expansion $1/D$ and $1/\mathsf{R}$

Squashed background at large D

Large D limit has been used to solve near horizon dynamics of BHs

Large D limit is also useful in the study of squashed background

Here we assume Ricci-flat background space with S^1 over CP^n

$$ds^2 = Fdr^2 + G_{ab}dx^a dx^b + 2G_{a\phi}dx^a(d\phi + \mathcal{A}_n) + G_{\phi\phi}(d\phi + \mathcal{A}_n)^2 + r^2 d\Sigma_n^2,$$

At the limit $n \to \infty$, the Einstein eq. leads

$$\begin{cases} F = 1 + \mathcal{O}\left(n^{-1}\right), \\ \partial_r G_{AB} = \dfrac{2G_{A\phi}G_{B\phi}}{r^3} + \mathcal{O}\left(n^{-1}\right) \end{cases}$$

$A, B = a, \phi$

$$G_{\phi\phi} = \frac{L^2 r^2}{r^2 + L^2},\ G_{ab} = \text{cosnt}.,\ G_{a\phi} = \text{cosnt}.$$

L<∞ gives squasching behavior in S^1

a geometric flow due to the hopf fibration

$$G_{\phi\phi} \to L^2 \quad (r \to \infty)$$

Squashed background at large D

Let us consider more specific ansatz

$$ds^2 = -dt^2 + h_{rr}(r)dr^2 + h_{\phi\phi}(r)(d\phi + \mathcal{A}_n)^2 + r^2 d\Sigma_n^2.$$

Solving in 1/n expansion

$$h_{\mu\nu} = h_{\mu\nu}^{[0]} + \frac{1}{n}h_{\mu\nu}^{[1]} + \cdots$$

$$h_{rr}(r) = 1 - \frac{3r^2}{2n(L^2+r^2)} + \frac{3r^2\left(L^2+2r^2\right)}{4n^2\left(L^2+r^2\right)^2} + \mathcal{O}\left(n^{-3}\right),$$

$$h_{\phi\phi}(r) = \frac{L^2r^2}{L^2+r^2} + 0 \times n^{-1} - \frac{3r^4L^4}{2n^2(L^2+r^2)^3} + \mathcal{O}\left(n^{-3}\right).$$

Solving in 1/r expansion

$$h_{rr}(r) = \frac{2n-1}{2n+2} + \frac{3(2n-1)^2}{4(n+1)^2(2n-3)}\frac{L^2}{r^2} + \mathcal{O}\left(\frac{L^4}{r^4}\right),$$

$$h_{\phi\phi}(r) = L^2\left(1 - \frac{n(2n-1)}{(n+1)(2n-3)}\frac{L^2}{r^2} + \mathcal{O}\left(\frac{L^4}{r^4}\right)\right).$$

consistent in
double expansion
with $1/n$ and $1/r$

squashed background is solved in 1/n-expansion for r<∞

Near horizon analysis

Assume near horizon region is put in the squashed background

$$ds^2 = -A(r)dt^2 + \underline{h_{rr}(r)}\frac{B(r)}{A(r)}dr^2 + \underline{h_{\phi\phi}(r)}H(r)(d\phi + \mathcal{A}_n)^2 + r^2 d\Sigma_n^2$$

BG already solved 1/n expansion

Introduce near horizon variable at large D $\quad r := \mathsf{R}^{\frac{1}{2n}} \simeq 1 + \frac{1}{2n}\log \mathsf{R}$

$$A = \sum_{i=0}^{\infty}\frac{A_i(\mathsf{R})}{n^i}, \quad B = 1 + \frac{1}{n}\sum_{i=0}^{\infty}\frac{B_i(\mathsf{R})}{n^i}, \quad H = 1 + \frac{1}{n}\sum_{i=0}^{\infty}\frac{H_i(\mathsf{R})}{n^i}.$$

boundary condition $\quad A = 1 + \mathcal{O}\left(\mathsf{R}^{-1}\right), \quad B = 1 + \mathcal{O}\left(\mathsf{R}^{-1}\right), \quad H = 1 + \mathcal{O}\left(\mathsf{R}^{-1}\right)$

Leading order solution

horizon at $\mathsf{R} = m$

$$ds^2 \simeq -\left(1 - \frac{m}{\mathsf{R}}\right)dt^2 + h_{rr}\left(1 - \frac{m}{\mathsf{R}}\right)^{-1}dr^2 + h_{\phi\phi}(d\phi + \mathscr{A}_n)^2 + r^2 d\Sigma_n^2$$

metric solutions are obtained up to $\mathcal{O}(1/n^3)$

Physical properties

Thermodynamics (M,T,A,κ) are calculated in 1/D expansion

ex) ADM mass

$$\mathcal{M} = \frac{Lm}{\sqrt{L^2+1}}\left[1+\frac{2L^2-2\log m+3}{4\left(L^2+1\right)n}+\frac{\left(12-24L^2\right)\log^2 m+8\left(\pi^2-9\right)L^2-36\log m+9}{96\left(L^2+1\right)^2 n^2}\right]$$

1st law and Smarr's formula are satisfied up to relevant order

$$\left.\begin{aligned} d\mathcal{M} &= \kappa\, d\mathcal{A}_H + \mathcal{T} dL, \\ 2n\mathcal{M} &= (2n+1)\kappa\mathcal{A}_H + \mathcal{T}L, \\ \mathcal{M}_K &= \frac{2n+1}{2n}\kappa\mathcal{A}_H, \end{aligned}\right\} \longrightarrow \mathcal{M}-\mathcal{M}_K = \frac{\mathcal{T}L}{2n},$$

Komar mass

Squashing function

Squashing effect on the horizon

$$k_{\text{sq}} := r^2/g_{\phi\phi}\Big|_H = 1 + \frac{r_H^2}{L^2} + \frac{0}{n} + \mathcal{O}(n^{-2})$$

size of CP^n on the horizon

$$r_H := m^{\frac{1}{2n}}$$

- Squashing is dissolved for $r_H/L \to 0$
- squashed horizon has “oblate” shape as in D=5

$CP^n > S^1$

Charged solution

Charged case is also solved similarly but with gauge field

$$A_\mu dx^\mu = \Phi dt, \qquad \Phi = \sum_{i=0}^{\infty} n^{-i}\Phi_i .$$

Leading order solution

horizon at $\mathsf{R} = \rho_+$

$$\Phi_0 = \frac{\sqrt{\rho_+\rho_-}}{\sqrt{2}\mathsf{R}}, \quad A_0 = 1 - \frac{\rho_+ + \rho_-}{\mathsf{R}} + \frac{\rho_+\rho_-}{\mathsf{R}^2},$$

$$H_0 = \frac{1}{1+L^2}\log\left(1 - \frac{\rho_-}{\mathsf{R}}\right), \quad B_0 = -\frac{\rho_-}{(1+L^2)(\mathsf{R}-\rho_-)}.$$

Thermodynamic variables are calculated in 1/D expansion

Extremal limit at large D

Extremal limit is given by

$$\kappa = \frac{\rho_+ - \rho_-}{\rho_+} + \frac{2L^2\rho_- - 2\left(L^2+1\right)\left(\rho_+ - \rho_-\right)\log\rho_+ + \rho_- + \rho_+}{4\left(L^2+1\right)n\rho_+} \quad \to 0$$

➡ $$\rho_+ = \widetilde{\rho}_- := \left(1 - \frac{1}{2n} + \frac{1}{4n^2} + \mathcal{O}\left(n^{-3}\right)\right)\rho_- \simeq \frac{2n}{2n+1}\rho_-$$

This limit seems to break 1/n expansion

$$\mathcal{M} = \frac{L}{\sqrt{1+L^2}}\left[\rho_+ + \rho_- + \frac{\rho_+\left(2L^2+3\right) + \rho_- - 2(\rho_+ + \rho-)\log(\rho_+ - \rho_-)}{4\left(L^2+1\right)n}\right.$$
$$+\frac{1}{n^2}\left(-\frac{\rho_-^2}{2(\rho_+ - \rho_-)(1+L^2)} + \frac{\left(4\left(2\pi^2-27\right)L^2 - 75\right)\rho_- + \left(8\left(\pi^2-9\right)L^2+9\right)\rho_+}{96(1+L^2)^2}\right.$$
$$\left.\left.-\frac{1}{8(1+L^2)^2}\left(\left(2L^2-1\right)\left(\rho_- + \rho_+\right)\log^2\left(\rho_+ - \rho_-\right) + \left(-2L^2\rho_- + \rho_- + 3\rho_+\right)\log\left(\rho_+ - \rho_-\right)\right)\right)\right]$$

But divergent terms can be absorbed in extremal parameter $\chi := (1 - \widetilde{\rho}_-/\rho_+)^{\frac{1}{2n}}$

to give regular function at $\chi \to 0$ $\quad \mathcal{M} = r_H^{2n}\left[\widetilde{\mathcal{M}}\left(\frac{r_H\chi}{L}\right) + (1-\chi^{2n})\widetilde{\mathcal{M}}_-\left(\frac{r_H\chi}{L}\right)\right]$

Summary

Summary

- D=2n+3 SqKK BH was studied in the large D limit with/without Maxwell charge
- Near horizon and Background structure was solved in 1/D expansion
- Thermodynamics are obtained in 1/D expansion

Future work

- (In)stability of SqKK : are there GL instabilities ?
- More general setup: ex) $S^{4n+3} \to S^3$ over HP^n
- Rotating SqKK ← Equally rotating Myers-Perry

Session D1b 9:00–10:30

[Chair: Umpei Miyamoto]

Daiki Saito

Nagoya University

“False Vacuum Decay in Rotating Spacetimes”

(15 min.)

[JGRG30 (2021) 120909]

JGRG 30
Dec. 9th

False Vacuum Decay in Rotating Black Hole Spacetimes

arXiv:2109.04051
To be appeared in PRD

Daiki Saito (Nagoya Univ.) collaboration with **Chul-Moon Yoo** (Nagoya Univ.)

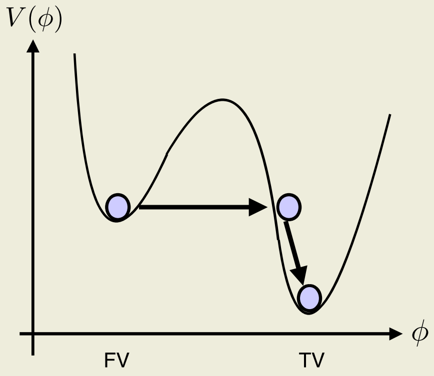
Introduction

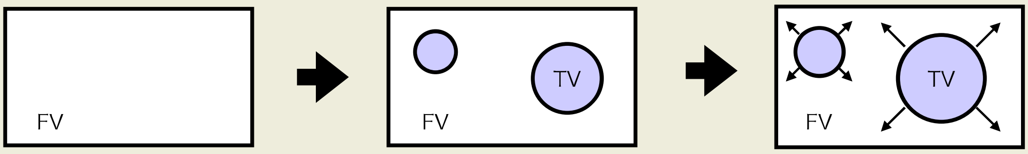
Introduction : What is false vacuum decay ?

Transition of fields : metastable → stable
False Vacuum(FV) True Vacuum(TV)

Induced by e.g) quantum tunneling
thermal jump

True vacuum regions nucleate in spacetime
(vacuum phase transition)



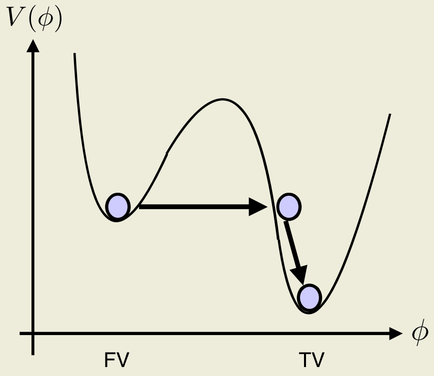


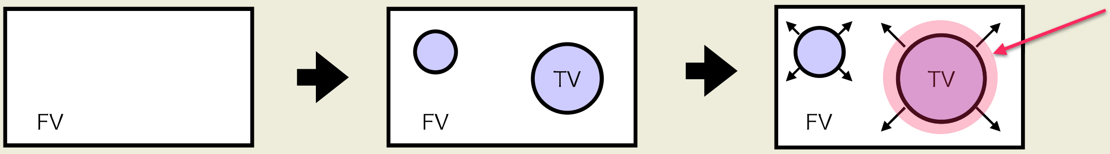
Introduction : What is false vacuum decay ?

Transition of fields : metastable → stable
False Vacuum(FV) True Vacuum(TV)

Induced by e.g) quantum tunneling
thermal jump

True vacuum regions nucleate in spacetime
(vacuum phase transition)





Vacuum bubble

Introduction : Motivation

Gregory et.al (2014) Vacuum decay in Schwarzschild spacetimes

“Existence of BH promotes the decay”

⟶ Various applications, extensions

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However, there is room for discussion

- Physical meaning of decay promotion is unclear
- Most researches focus on the static cases BH has spin in general, insufficient

Our motivation : get insight / wider application range by **adding spin**

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→ Various applications, extensions

However, there is room for discussion

- Physical meaning of decay promotion is unclear
- Most researches focus on the static cases BH has spin in general, insufficient

Our motivation : get insight / wider application range by **adding spin**

Kerr spacetimes (non-spherical) : less symmetric → **BTZ** as a toy model!

Set up & Calculation

Set up

FV & TV : different BTZ spacetimes

BTZ spacetime

(2+1)D asymptotic AdS BH solution

$$ds^2 = -f_\pm(r)dt^2 + \frac{1}{f_\pm(r)}dr^2 + r^2\left(d\varphi - \frac{4J_\pm}{r^2}dt\right)^2 \quad f_\pm(r) = -8M_\pm - \Lambda_\pm r^2 + \frac{16J_\pm^2}{r^2}$$

+:FV, -:TV

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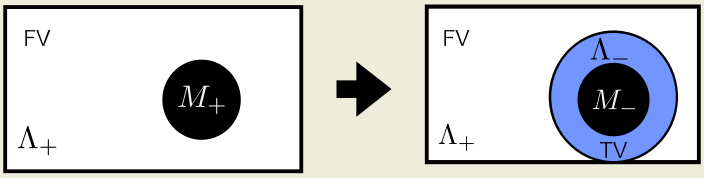
+:FV, -:TV

Consider coexistence system of FV/TV as a consequence of the nucleation

Assume concentric nucleation

Use **thin shell approximation**



$(\Lambda_- < \Lambda_+)$

Set up

BTZ spacetime

$$ds^2 = -f_\pm(r)dt^2 + \frac{1}{f_\pm(r)}dr^2 + r^2\left(d\varphi - \frac{4J_\pm}{r^2}dt\right)^2 \quad f_\pm(r) = -8M_\pm - \Lambda_\pm r^2 + \frac{16J_\pm^2}{r^2}$$

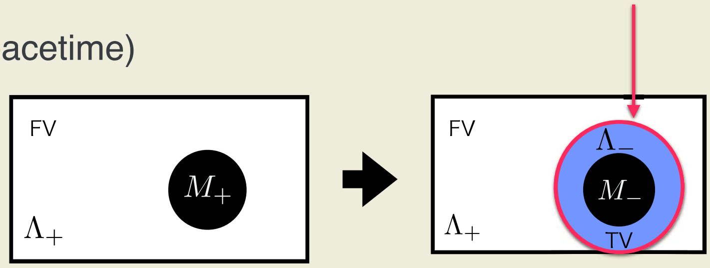
+:FV, -:TV

thin shell approximation

- bubble wall ~ spherical shell
- stepwise jump of the field (spacetime)

Λ corresponds to a potential

Thin wall of condensed matter



Set up

BTZ spacetime

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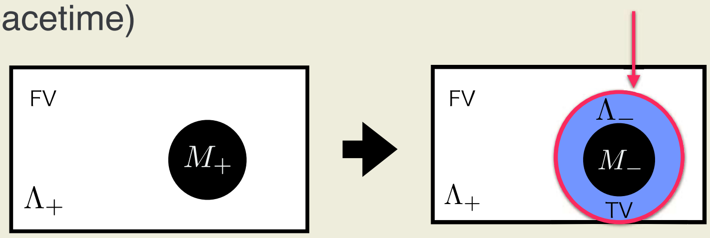


Instead of solving the field eqs,
focus on its dynamics

Set up

How to evaluate the decay [Coleman (1976)]

Decay rate per unit time volume $\Gamma \propto e^{-\mathcal{B}}$

$$\mathcal{B} = S_E - S_E$$

S_E : Euclidean action with sol. of Euclidean EoM **Bounce solution**
S_{E0}: Euclidean action with trivial sol. (FV)

To analyze the nucleation, we need to

1. Solve Euclidean EoMs
2. Calculate the (on-shell) Euclidean action

Calculation : Equations of motion

A motion of the Euclidean shell : given by **Israel's junction conditions**

1st cond. $[h_{Eab}]_{\pm} = 0$

2nd cond. $[K_{Eab}]_{\pm} = -8\pi\left(S_{Eab} - h_{Eab}S_E\right)$ $\quad [A]_{\pm} := A_+ - A_-$

h_{Eab} : induced metric $\quad K_{Eab}$: extrinsic curvature

S_{Eab} : energy momentum on the shell

Assume **pure tension** $\quad S_{Eab} = -\sigma h_{Eab} \quad \sigma$: tension

$$[K_{E\tau\phi}]_{\pm} = 0 \quad \xrightarrow{K_{E\tau\phi}|_{\mathcal{W}} = -\frac{4J_E}{R}} \quad J_+ = J_-$$

(angular momentum conservation)

$$[K_{E\phi\phi}]_{\pm} = -8\pi\sigma R^2 \xrightarrow{K_{E\phi\phi} = \sqrt{f_E - \dot{R}^2}} \sqrt{f_{E+} - \dot{R}^2} - \sqrt{f_{E-} - \dot{R}^2} = -8\pi\sigma R$$

(shell EoM)

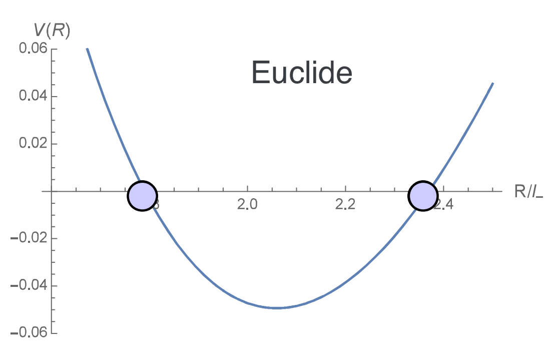
Calculation : Equations of motion

$$\sqrt{f_{E+} - \dot{R}^2} - \sqrt{f_{E-} - \dot{R}^2} = -8\pi\sigma R. \longrightarrow -\frac{1}{2}\dot{R}^2 = V(R)$$

1-D potential problem

$\dot{R} = \frac{dR}{d\tau}$ $\quad \tau$: proper time on the shell

Euclidean shell : oscillate in $V(R) < 0$ region



Calculation : Equations of motion

$$\sqrt{f_{E+} - \dot{R}^2} - \sqrt{f_{E-} - \dot{R}^2} = -8\pi\sigma R. \longrightarrow -\frac{1}{2}\dot{R}^2 = V(R)$$ 1-D potential problem

$\dot{R} = \frac{dR}{d\tau}$ τ : proper time on the shell

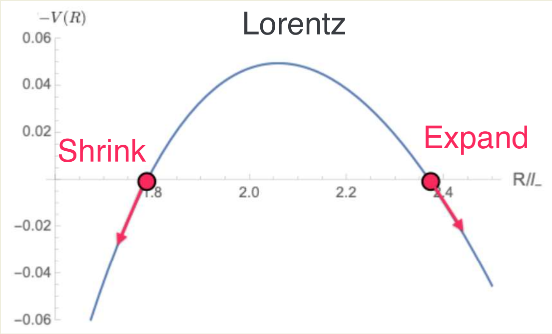
Euclidean shell : oscillate in $V(R) < 0$ region

Lorentzian shell :

emerge as a stationary configuration $\left(\dot{R} = 0\right)$

$\longrightarrow$ emerge with the root of $V(R) = 0$

Double root case : “static” shell



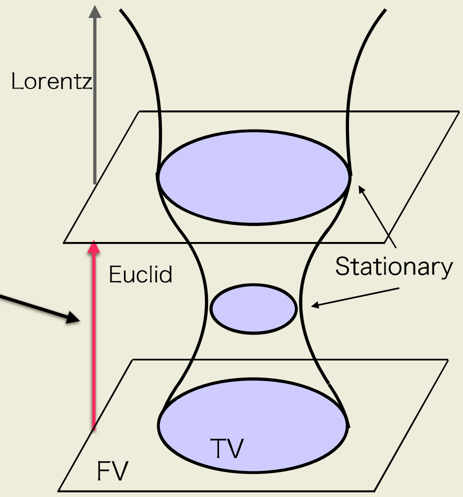
Calculation of the Euclidean action

$\Gamma \propto e^{-\mathcal{B}}$ $\mathcal{B} = S_E - S_E$: difference of the action

Calculate the action over <u>1 period</u>

$$S_E = S_{\mathcal{H}} + S_{\mathcal{M}_+} + S_{\mathcal{M}_-} + S_{\mathcal{W}}$$

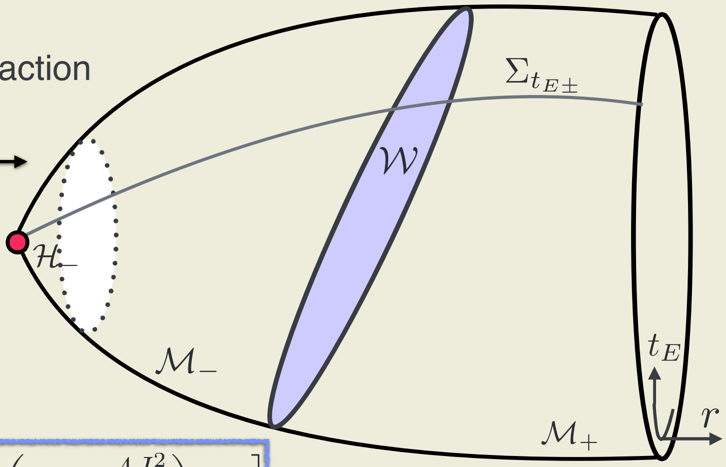
horizon bulk shell



Calculation of the Euclidean action

$\Gamma \propto e^{-\mathcal{B}}$ $\mathcal{B} = S_E - S_E$: difference of the action

Calculate the action over 1 period →



$$S_E = \underbrace{S_{\mathcal{H}}}_{\text{horizon}} + \underbrace{S_{\mathcal{M}_+} + S_{\mathcal{M}_-}}_{\text{bulk}} + \underbrace{S_{\mathcal{W}}}_{\text{shell}}$$

$$\therefore \mathcal{B} = \underbrace{\boxed{\frac{A_{\mathcal{H}_+}}{4} - \frac{A_{\mathcal{H}_-}}{4}}}_{\text{Area difference}} - 2\underbrace{\boxed{\oint d\tau \left[\left(M_+ - \frac{4J^2}{R^2}\right) t_{E+} - \left(M_- - \frac{4J^2}{R^2}\right) t_{E-}\right]}}_{\text{Vanishes in the static case}}$$

Calculation of the Euclidean action

$\Gamma \propto e^{-\mathcal{B}}$

Aim: Evaluate $\mathcal{B}$, especially effects of $\tilde{a}_+$

Parameters

M_+	M_-	$\tilde{a}_+$	L_+	s
mass		spin	AdS length	tension

(Taking l_- unit)

Strategy

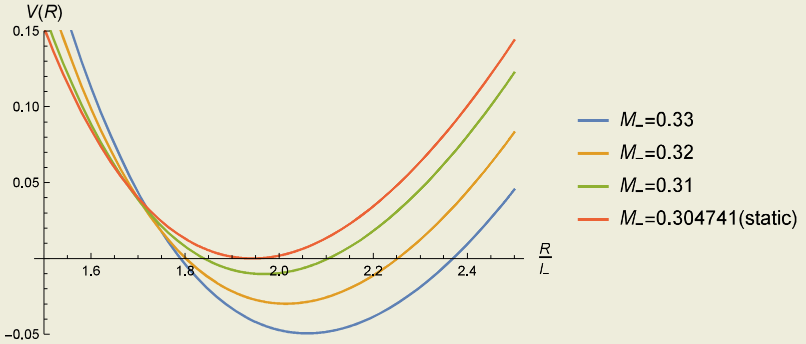
1. Set $\tilde{a}_+ = 0$ and alter M_- (fix the others), seek the dominant case
2. See whether the dominant case in 1. dominates even when spinning case
3. Alter $\tilde{a}_+$ in the dominant case

Evaluate of $\mathcal{B}$ by comparing $\mathcal{B}_{CDL}$ (pure AdS case)

Results

Results : $\tilde{a}_+ = 0$

M_- dependence of the potential (M_+, L_+, s fixed)



The distance of the roots increases with M_-

The lowest allowed M_- gives the static shell

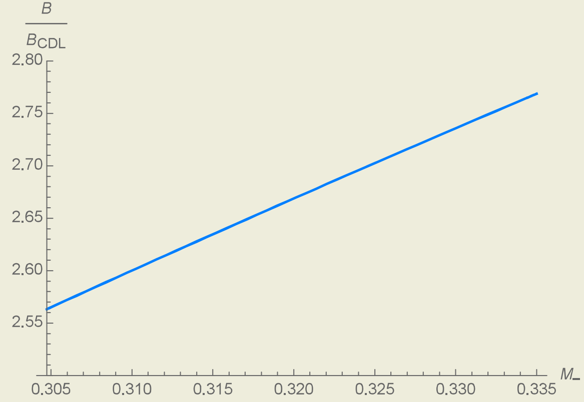
Results : $\tilde{a}_+ = 0$

$\mathcal{B}$ increases with M_-

The smallest M_- case (**static case**) dominates

$\mathcal{B}/\mathcal{B}_{CDL}$ with fixed L_+, s, M_+

L_+=2.0, s=0.2, $\tilde{a}_+$=0.0, M_+=0.1



Results : $\tilde{a}_+ = 0$

$\mathcal{B}$ increases with M_-

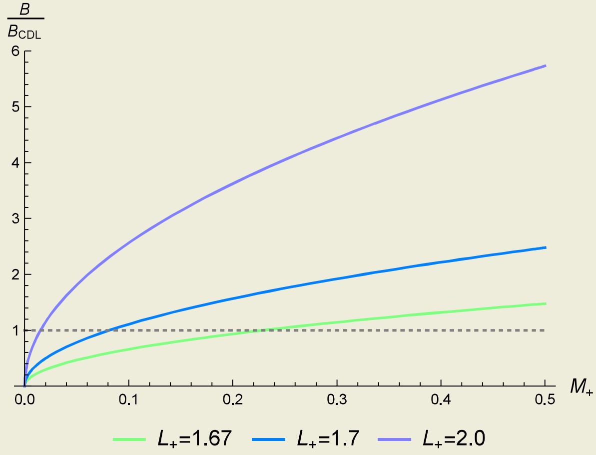
The smallest M_- case (**static case**) dominates

In the static case,

$\mathcal{B}$ monotonic increase with M_+

M_+ dependence of the rate

s=0.2, $\tilde{a}_+$=0.0, static



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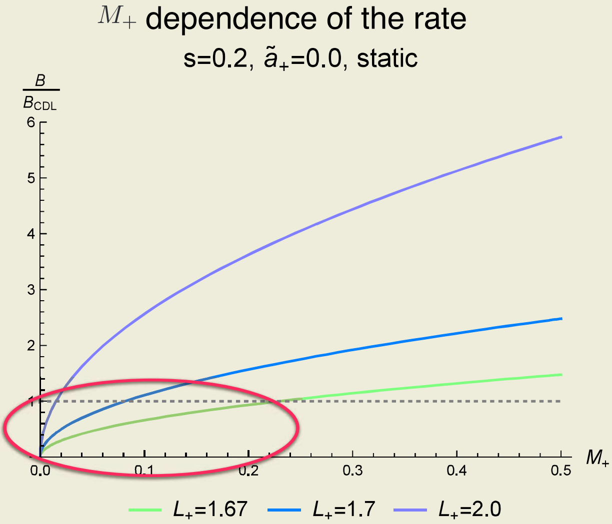
BHs with small M_+ promote the nucleation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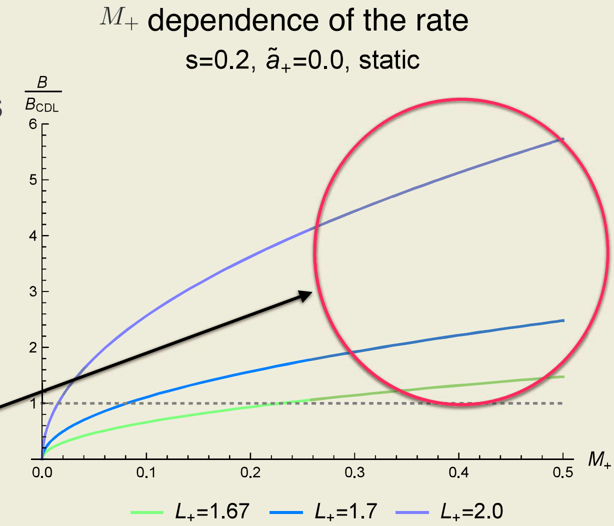
BHs with small M_+ promote the nucleation

BHs with large M_+ prevent the nucleation

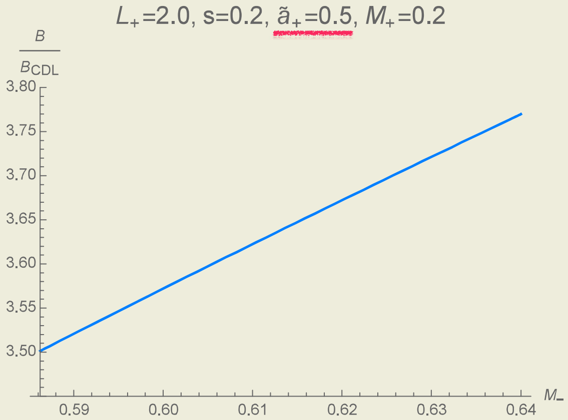

M_+ dependence of the rate
s=0.2, $\tilde{a}_+$=0.0, static

Results : $\tilde{a}_+ \neq 0$

$\mathcal{B}$ increases with M_-

The static case dominates even in spinning case

$\mathcal{B}/\mathcal{B}_{CDL}$ with fixed L_+, s, M_+



Results : $\tilde{a}_+ \neq 0$

$\mathcal{B}$ increases with M_-

The static case dominates even in spinning case

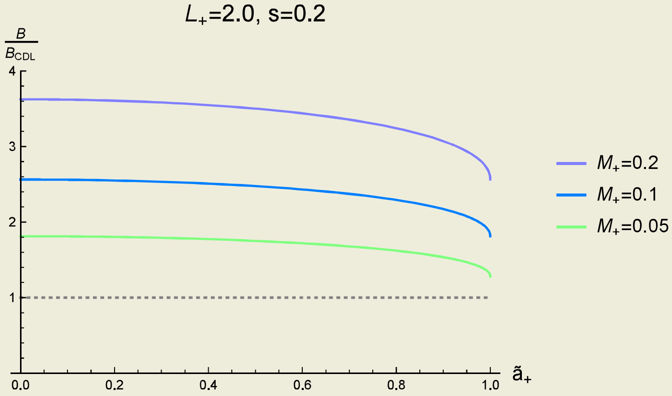
In the static case,

$\mathcal{B}$ monotonic decrease with $\tilde{a}_+$

$(\tilde{a}_+ := J/(M_+ l_+) \leq 1)$

Spins promote the nucleation

$\tilde{a}_+$ dependence of the rate



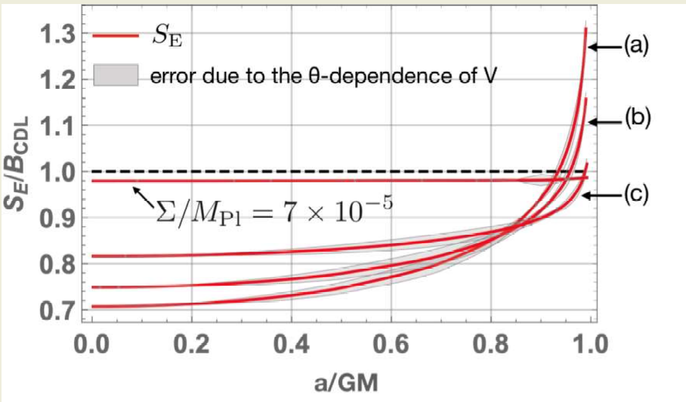
Results : the previous research in Kerr

Oshita, Ueda, Yamaguchi (2020)
: Kerr ⟶ Kerr-AdS

Make some assumptions

- $a_\pm^2 << l^2, a_+ = a_-, M_+ = M_-$
- the shell has anisotropic pressure

Spins of BHs suppress the decay



What made the difference ?

Dimensionality? Energy momentum of the shell?

Summary

Summary

- Analyzed the vacuum decay in BTZ spacetime
- The decay dominates in the static case
- The rate decreases with the seed mass
- The rate increases with the spin
- Future/ongoing work : Kerr

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Thank you for the attention !

Session D1b 9:00–10:30

[Chair: Umpei Miyamoto]

Sebastian Bahamonde

Tokyo Institute of Technology

“Black holes solutions in metric-affine gravity with dynamical torsion and nonmetricity”

(15 min.)

[JGRG30 (2021) 120910]

Black holes solutions in metric-affine gravity with dynamical torsion and nonmetricity

Sebastián Bahamonde

Postdoctoral Researcher at Tokyo Institute of Technology, Japan

JGRG30

Based on JCAP **09** (2020), 057, Eur.Phys.J.C **81** (2021) 6, 495;
arXiv:2108.12414 (to appear in JCAP); Jointly with Jorge Gigante Valcarcel.

Outline

1. Brief introduction to Metric-affine gravity
   - Basic quantities
   - Dynamics
2. MAG models with dynamical torsion and nonmetricity
   - Spherical symmetry
   - Observational constraints
   - Axial symmetry

Fundamental variables and characteristic tensors

- In the most general metric-affine setting, the fundamental variables are a **metric** $g_{\mu\nu}$ (10 comp.) as well as the coefficients $\tilde{\Gamma}^{\rho}{}_{\mu\nu}$ (64 comp.) of an **affine connection**.

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Connection decomposition

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$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \overbrace{\Gamma^{\lambda}{}_{\mu\nu}}^{\text{Levi-Civita}} + \overbrace{\frac{1}{2}T^{\lambda}{}_{\mu\nu} - T_{(\mu}{}^{\lambda}{}_{\nu)}}^{\text{Torsion part}}$$

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Fundamental variables and characteristic tensors

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Curvature	$\tilde{R}^{\mu}{}_{\nu\rho\sigma} = \partial_{\rho}\tilde{\Gamma}^{\mu}{}_{\nu\sigma} - \partial_{\sigma}\tilde{\Gamma}^{\mu}{}_{\nu\rho} + \tilde{\Gamma}^{\mu}{}_{\tau\rho}\tilde{\Gamma}^{\tau}{}_{\nu\sigma} - \tilde{\Gamma}^{\mu}{}_{\tau\sigma}\tilde{\Gamma}^{\tau}{}_{\nu\rho}$
Torsion	$\tilde{T}^{\mu}{}_{\nu\rho} = \tilde{\Gamma}^{\mu}{}_{\rho\nu} - \tilde{\Gamma}^{\mu}{}_{\nu\rho}$
Nonmetricity	$\tilde{Q}_{\mu\nu\rho} = \tilde{\nabla}_{\mu}g_{\nu\rho} = \partial_{\mu}g_{\nu\rho} - \tilde{\Gamma}^{\sigma}{}_{\nu\mu}g_{\sigma\rho} - \tilde{\Gamma}^{\sigma}{}_{\rho\mu}g_{\nu\sigma}$

Dynamics of metric-affine geometry

- Gravitational action with dynamical torsion and nonmetricity:

$$S = \int d^4x\sqrt{-g}\left[\mathcal{L}_m - \frac{1}{16\pi}\mathcal{L}_g(\tilde{\mathcal{R}},\mathcal{T},\mathcal{Q})\right]. \qquad (2)$$

Dynamics of metric-affine geometry

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- Correspondence between geometry and matter:

$$\frac{\delta S_g}{\delta e^a{}_\nu} = 16\pi\theta_a{}^\nu\,, \tag{3}$$

$$\frac{\delta S_g}{\delta\omega^a{}_{b\nu}} = 16\pi\Delta_a{}^{b\nu}\,. \tag{4}$$

Here $\theta_a{}^\nu$ is the energy-momentum tensor (canonical) and $\Delta_a{}^{b\nu}$ is the hypermomentum density tensor.

MAG models with dynamical torsion and nonmetricity

- Quadratic gravitational action with dynamical torsion and nonmetricity in Weyl-Cartan geometry ($Q_{\lambda\mu\nu}=g_{\mu\nu}W_\lambda$)

$$\begin{aligned}S=\int d^4x\sqrt{-g}\Big\{\mathcal{L}_{\rm m}+\frac{1}{64\pi}\Big[&-4R-6d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\lambda[\rho\mu\nu]}\\ &-9d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\mu[\lambda\nu\rho]}+8\,d_1\tilde{R}_{[\mu\nu]}\tilde{R}^{[\mu\nu]}+\frac{1}{8}\left(32e_1+8e_2+17d_1\right)\tilde{R}^\lambda{}_{\lambda\mu\nu}\tilde{R}^\rho{}_\rho{}^{\mu\nu}\\ &-7d_1\tilde{R}_{[\mu\nu]}\tilde{R}^\lambda{}_\lambda{}^{\mu\nu}+3\left(1-2a_2\right)T_{[\lambda\mu\nu]}T^{[\lambda\mu\nu]}\Big]\Big\}.\end{aligned}$$

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[1] S. Bahamonde and J. G. Valcarcel, JCAP **09**, 057 (2020).

[2] S. Bahamonde and J. G. Valcarcel, arXiv: 2108.12414, to be published in JCAP.

MAG models with dynamical torsion and nonmetricity

- Quadratic gravitational action with dynamical torsion and nonmetricity in Weyl-Cartan geometry ($Q_{\lambda\mu\nu} = g_{\mu\nu}W_{\lambda}$)

$$S = \int d^4x\sqrt{-g}\Big\{\mathcal{L}_{\rm m} + \frac{1}{64\pi}\Big[-4R - 6d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\lambda[\rho\mu\nu]} - 9d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\mu[\lambda\nu\rho]} + 8\,d_1\tilde{R}_{[\mu\nu]}\tilde{R}^{[\mu\nu]} + \frac{1}{8}\,(32e_1 + 8e_2 + 17d_1)\,\tilde{R}^{\lambda}{}_{\lambda\mu\nu}\tilde{R}^{\rho}{}_{\rho}{}^{\mu\nu} - 7d_1\tilde{R}_{[\mu\nu]}\tilde{R}^{\lambda}{}_{\lambda}{}^{\mu\nu} + 3\,(1-2a_2)\,T_{[\lambda\mu\nu]}T^{[\lambda\mu\nu]}\Big]\Big\}\,.$$

- Absence of a general Birkhoff's theorem in MAG: new spherically and axially symmetric vacuum solutions with independent dynamical torsion and nonmetricity fields[1,2].

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[1] S. Bahamonde and J. G. Valcarcel, JCAP **09**, 057 (2020).

[2] S. Bahamonde and J. G. Valcarcel, arXiv: 2108.12414, to be published in JCAP.

Spherical symmetry

- Metric, torsion and nonmetricity in spherically symmetric space-times ($\#2 + \#8 + \#2 = \#12$):

$$\mathcal{L}_{\xi}g_{\mu\nu} = \mathcal{L}_{\xi}T^{\lambda}{}_{\mu\nu} = \mathcal{L}_{\xi}W_{\mu} = 0 \implies \mathcal{L}_{\xi}\tilde{R}_{\lambda\rho\mu\nu} = 0$$

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- Metric, torsion and nonmetricity in spherically symmetric space-times ($\#2 + \#8 + \#2 = \#12$):

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi W_\mu = 0 \implies \mathcal{L}_\xi \tilde{R}_{\lambda\rho\mu\nu} = 0$$

- By solving these equations we find that torsion and nonmetricity behave as

$$\begin{aligned} &T^t{}_{tr} = a(r)\,, \quad T^r{}_{tr} = b(r)\,, \quad T^{\theta_k}{}_{t\theta_k} = f(r)\,, \quad T^{\theta_k}{}_{r\theta_k} = g(r)\\ &T^{\theta_k}{}_{t\theta_l} = e^{a\theta_k}\, e^b{}_{\theta_l}\, \epsilon_{ab}\, d(r)\,, \quad T^{\theta_k}{}_{r\theta_l} = e^{a\theta_k}\, e^b{}_{\theta_l}\, \epsilon_{ab}\, h(r)\,,\\ &T^t{}_{\theta_k\theta_l} = \epsilon_{kl}\, k(r)\, \sin\theta_1\,, \quad T^r{}_{\theta_k\theta_l} = \epsilon_{kl}\, l(r)\, \sin\theta_1\,,\\ &W_\lambda = (w_1(r), w_2(r), 0, 0)\ , \end{aligned}$$

whereas the metric is in the standard spherically symmetric form:

$$ds^2 = \Psi_1(r)\, dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 \left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2\right)\ .$$

Here, ϵ_{kl} is the Levi-Civita symbol in two dimensions.

Spherical symmetry - Solving the field equations

The field eqs are very involved. To solve them we use the following strategy:

1. **Imposing regularity:** In general, the solutions can have a singular behaviour. To ensure regularity, one can analyse the torsion/nonmetricity tensors referred to the rotated basis $\vartheta^a = \Lambda^a{}_b e^b$.

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   One can write the gauge curvature $\mathcal{F}^a{}_{bc} = \vartheta^a{}_\lambda \vartheta_b{}^\mu \vartheta_c{}^\nu T^\lambda{}_{\nu\mu}$ related to the torsion/nonmetricity tensor in this orthogonal coframe.

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   Regularity restricts the initial arbitrariness of the torsion components and the Weyl vector by imposing the relations

$$\begin{aligned} b(r) &= a(r)\sqrt{\Psi_1(r)\Psi_2(r)}\,, \quad & f(r) &= -g(r)\sqrt{\Psi_1(r)\Psi_2(r)}\,,\\ d(r) &= -h(r)\sqrt{\Psi_1(r)\Psi_2(r)}\,, \quad & l(r) &= k(r)\sqrt{\Psi_1(r)\Psi_2(r)}\,,\\ w_1(r) &= -w_2(r)\sqrt{\Psi_1(r)\Psi_2(r)}\,. \end{aligned}$$

Spherical symmetry - Solving the field equations

2. **Solve the weak field limit:** The weak field limit of the field equations become

$$\begin{aligned} \nabla_\rho \nabla_\lambda T^{\lambda\rho}{}_\mu + \nabla_\rho \nabla^\rho T^\lambda{}_{\mu\lambda} - \nabla_\rho \nabla_\mu T^{\lambda\rho}{}_\lambda &= 0\,,\\ \nabla_\mu \tilde{R}^\lambda{}_\lambda{}^{\mu\nu} &= 0\,. \end{aligned}$$

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These equations can be solved, yielding

$$w_1(r) = -\,\kappa_d \int \sqrt{\frac{\Psi_1(r)}{\Psi_2(r)}}\,\frac{dr}{r^2}\,,$$

$$b(r) = r f'(r) + f(r) + \frac{\kappa_d}{2r}\sqrt{\frac{\Psi_1(r)}{\Psi_2(r)}}\,,$$

where κ_d is an integration constant which represents the dilaton charge.

Spherical symmetry - Solving the field equations

1. The final solution for the metric behaves as Reissner-Nordström

$$g_{tt} = -1/g_{rr} \equiv \Psi(r) = 1 - \frac{2m}{r} + \frac{d_1 \kappa_{\rm s}^2 - 4e_1 \kappa_{\rm d,e}^2}{r^2} \, . \qquad (5)$$

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$$W_\mu = \frac{\kappa_{\rm d,e}}{r} \left(1, -1/\Psi(r), 0, 0\right) \, . \qquad (6)$$

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$$\bar{\mathcal{T}}_2^{abc} = \frac{\kappa_{\rm s}}{3r}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}\,. \tag{8}$$

Dilation and spin charges

What do κ_s (dilation charge) and $\kappa_{d,e}$ (spin charge) physically represent?

Point 1 - Hypermomentum density

In the geometric scheme of MAG, not only an energy-momentum tensor of matter arises as source of curvature, but also a hypermomentum density tensor which operates as source of torsion and nonmetricity.

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Point 2 - Dilation and spin charges

In Weyl-Cartan geometry, hypermomentum density tensor splits into spin and dilation currents, which carry their own charges and provide a RN solution.

Dilation and spin charges

When these charges might be important?

Significant effects are contemplated only around **extreme gravitational systems**, such as **neutron stars** with intense magnetic fields and sufficiently oriented elementary spins or **black holes** endowed with spin and dilation charges.

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Quantum nature

The **intrinsic hypermomentum of matter** is purely quantum since in its vanishes in the rest of ordinary matter sources (e.g. Dirac fermions).

Observational constrains

- Let us now consider the case where the effect of torsion dominates over the contribution of nonmetricity.

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[3] S. Bahamonde and J. Gigante Valcarcel, Eur. Phys. J. C **81** (2021) no.6, 495.

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- Indeed, due to the presence of a magnetic field in white dwarfs, it is expected that Sirius B can have sufficiently oriented elementary spins in comparison with an effective dilation charge, therefore, $\kappa_{s,\mathrm{SiriusB}} \gg \kappa_{d,\mathrm{SiriusB}}$.

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- **Perihelion shift+ Gravitational redshift:** Assuming the same approximation in Sgr A* and considering the universality of the coupling constant d_1, we find[3]

$$1.396 \cdot 10^{10} \leq \frac{\kappa_{s,\mathrm{SgrA*}}}{\kappa_{s,\mathrm{SiriusB}}} \leq 1.688 \cdot 10^{10} \,.$$

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- To the best of our knowledge, this bound provides the first observational comparison between the spin charges of a supermassive black hole and a degenerate star.

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Extension to axisymmetric space-times

- Metric, torsion and nonmetricity tensors in symmetric space-times:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^{\lambda}{}_{\mu\nu} = \mathcal{L}_\xi Q^{\lambda}{}_{\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^{\lambda}{}_{\rho\mu\nu} = 0\,. \qquad (9)$$

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- Stationary and axisymmetric space-times[4]:

$$\#10 \to \#4 \begin{cases} ds^2 = \Psi_1(r,\vartheta)\,dt^2 - \frac{dr^2}{\Psi_2(r,\vartheta)} \\ -\,r^2\Psi_3(r,\vartheta)\Big[d\vartheta^2 + \sin^2\vartheta(d\varphi - \Psi_4(r,\vartheta)dt)^2\Big] \end{cases} ;$$

$$\#24 \left\{ T^\lambda{}_{\mu\nu} = T^\lambda{}_{\mu\nu}(r,\vartheta) \right. \tag{10}$$

$$\#4 \left\{ W_\mu = (W_t(r,\vartheta), W_r(r,\vartheta), W_\vartheta(r,\vartheta), W_\varphi(r,\vartheta))\,. \right. \tag{11}$$

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Extension to axisymmetric space-times

- Rotating Kerr-Newman metric structure:

$$\begin{aligned} ds^2 = {} & \Psi(r,\vartheta)\,dt^2 - \frac{r^2 + a^2\cos^2\vartheta}{(r^2 + a^2\cos^2\vartheta)\,\Psi(r,\vartheta) + a^2\sin^2\vartheta}\,dr^2 \\ & - \left(r^2 + a^2\cos^2\vartheta\right)d\vartheta^2 + 2a\,(1 - \Psi(r,\vartheta))\sin^2\vartheta\,dt\,d\varphi \\ & - \sin^2\vartheta\left[r^2 + a^2 + a^2\,(1 - \Psi(r,\vartheta))\sin^2\vartheta\right]d\varphi^2\,, \end{aligned} \tag{12}$$

$$\Psi(r,\vartheta) = 1 - \frac{\left[2mr + 4e_1(\kappa^2_{\rm d,e} + \kappa^2_{\rm d,m}) - d_1\kappa^2_{\rm s}\right]}{r^2 + a^2\cos^2\vartheta}\,. \tag{13}$$

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- Field strength tensors:

$$\begin{aligned} \bar{R}_{[\mu\nu]} &= \frac{1}{12}\varepsilon^{\lambda}{}_{\sigma\mu\nu}\nabla_{\lambda}\bar{S}^{\sigma} + \frac{1}{2}\nabla_{\lambda}\bar{t}^{\lambda}{}_{\mu\nu}\,; \quad \tilde{R}^{\lambda}{}_{\lambda\mu\nu} = 4\nabla_{[\nu}W_{\mu]}\,; \\ \bar{R}^{\lambda}{}_{[\mu\nu\rho]} &= \frac{1}{6}\varepsilon^{\lambda}{}_{\sigma[\rho\nu}\nabla_{\mu]}\bar{S}^{\sigma} + \nabla_{[\mu}\bar{t}^{\lambda}{}_{\rho\nu]} + \frac{1}{4}\varepsilon^{\lambda}{}_{\omega\sigma[\rho}\mathring{t}_1^{\sigma}{}_{\mu\nu]}\bar{S}^{\omega} \\ &- \frac{1}{18}\varepsilon_{\sigma\mu\nu\rho}\mathring{T}_1^{\lambda}\bar{S}^{\sigma}\,. \end{aligned} \tag{14}$$

Extension to axisymmetric space-times

- Nonmetricity sector:

$$\begin{aligned} w_1(r,\vartheta) &= \frac{\kappa_{\mathrm{d,e}}r - a\,\kappa_{\mathrm{d,m}}\cos\vartheta}{r^2 + a^2\cos^2\vartheta}\,, \quad w_3(r,\vartheta) = 0\,, \\ w_2(r,\vartheta) &= -\,\frac{\kappa_{\mathrm{d,e}}r}{(r^2 + a^2\cos^2\vartheta)\Psi(r,\vartheta) + a^2\sin^2\vartheta}\,, \\ w_4(r,\vartheta) &= \kappa_{\mathrm{d,m}}\left(\frac{r^2 + a^2}{r^2 + a^2\cos^2\vartheta}\cos\vartheta - \gamma\right) - a\,\frac{\kappa_{\mathrm{d,e}}r\sin^2\vartheta}{r^2 + a^2\cos^2\vartheta}\,. \end{aligned} \tag{15}$$

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$$\bar{\mathcal{T}}_2^{\,abc} = \frac{\kappa_{\rm s}}{3r}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(a\kappa_{\rm s})\,. \tag{17}$$

Gravitational spin-orbit interaction

- We found a solution in the decoupling limit $a\kappa_s \ll 1$, which ensures that the Maxwell equation and closure conditions are fulfilled by the field strength tensors of torsion

$$\nabla_\lambda \tilde{R}^\lambda{}_{[\rho\mu\nu]} = \nabla_\mu \tilde{R}^{[\mu\nu]} = 0\,, \quad \nabla_{[\sigma}\tilde{R}_{\lambda[\rho\mu\nu]]} = \nabla_{[\lambda}\tilde{R}_{[\mu\nu]]} = 0\,.$$

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The dynamics of torsion and nonmetricity alters the geometry of the space-time $\Longrightarrow$

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The dynamics of torsion and nonmetricity alters the geometry of the space-time $\Longrightarrow$ Additional modifications provided by a strong coupling between the orbital and the spin angular.

- **Gravitational spin-orbit interaction:**

$$\mathcal{H}_{\rm LS} = \frac{1}{m_{\rm e}^2 r}\frac{\partial V}{\partial r}\mathbf{L}\cdot\mathbf{S} \approx \frac{d_1}{2r}\frac{\partial g_{tt}}{\partial r}a\kappa_{\rm s}\cos\vartheta \tag{18}$$

Conclusions

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- In progress: we are constructing Plebanski-Demianski uniformly accelerated rotating black hole solutions with NUT parameter, electromagnetic charges and a cosmological constant.
- Future: search of a gravitational spin-orbit interaction in MAG beyond the Kerr-Newman space-time (MAG is the main candidate to describe a spin-orbit interaction beyond GR).