Proceedings of
the 25th Workshop on General Relativity and Gravitation in Japan

7—11 December 2015

Yukawa Institute for Theoretical Physics, Kyoto University

Kyoto, Japan

Volume 5

Oral Presentations: Fifth Day

Poster Presentations

Contents

Oral Presentations: Fifrh Day

Friday 11 December

Plenary Session 9 [Chair: Antonio De Felice]

9:30 Wayne Hu (U. of Chicago) [Invited]
"Massive Gravity: Trouble with Metrics"
[JGRG25(2015)I12]

10:15-10:30 Caffee break

Session 8a [Chair: Hideki Asada]

10:30 Makoto Narita (NIT, Okinawa College)
"Global existence theorem for Gowdy symmetric spacetimes in supergravity theory"
[JGRG25(2015)8a1]

10:45 Hassan Firouzjahi (IPM)
"Anisotropic and Asymmetric Primordial Universe"
[JGRG25(2015)8a2]

11:00 Andrei Frolov (SFU)
"Results from Planck 2015"
[JGRG25(2015)8a3]

11:15 Teruaki Suyama (RESCEU)
"Spontaneous scalarization-induced dark matter and variation of the gravitational constant"
[JGRG25(2015)8a4]

11:30 Marcus Christian Werner (YITP)
"The Einstein-Struble Correspondence and Lorentz Invariance"
[JGRG25(2015)8a5]

11:45 Presentation awards

12:00 Closing

“Massive Gravity: Trouble with Metrics”

by Wayne Hu (invited)

[JGRG25(2015)I12]

Massive Gravity:
Trouble with Metrics

metric  imperial

Wayne Hu

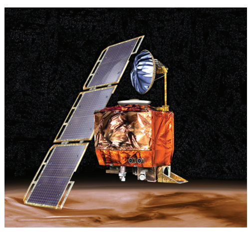
JGRG, December 2015

Massive Gravity:
Trouble with Metrics

NASA's metric confusion caused Mars orbiter loss

September 30, 1999
Web posted at: 1:46 p.m. EDT (1746 GMT)

(CNN) -- NASA lost a $125 million Mars orbiter because one engineering team used metric units while another used English units for a key spacecraft operation, according to a review finding.



Wayne Hu
JGRG, December 2015

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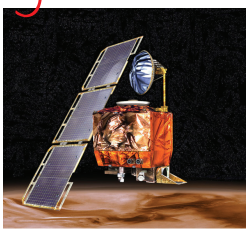

Wayne Hu JSPS Fellow
JGRG, December 2015

Pierre Gratia, Austin Joyce, Hayato Motohashi, Pavel Motloch, Mark Wyman

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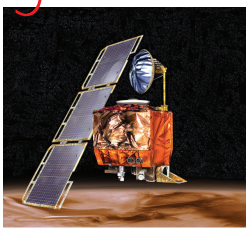
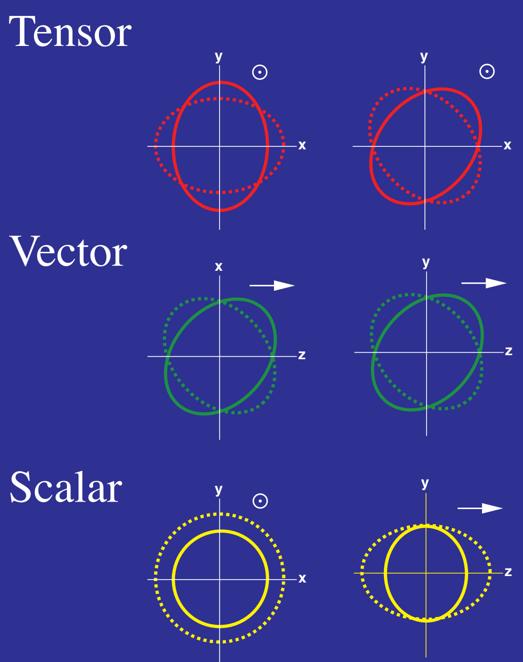


Wayne Hu PhD Student
JGRG, December 2015
Pierre Gratia, Austin Joyce, Hayato Motohashi, Pavel Motloch, Mark Wyman

Massive Gravity

- A generic theory of massive gravity propagates 6 polarization states: 5 for a massive spin-2 and 1ghost



General Relativity

New Degrees of Freedom

Massive Gravity

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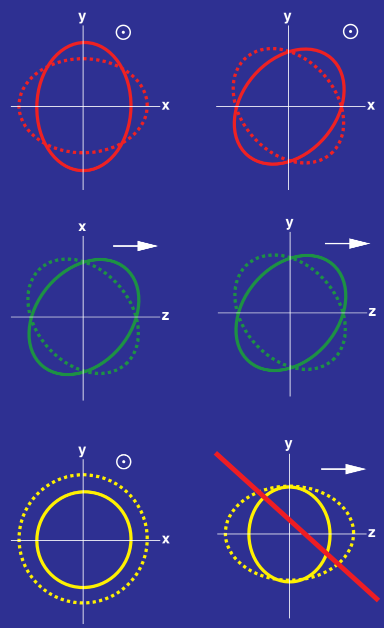
Tensor

Vector

Scalar

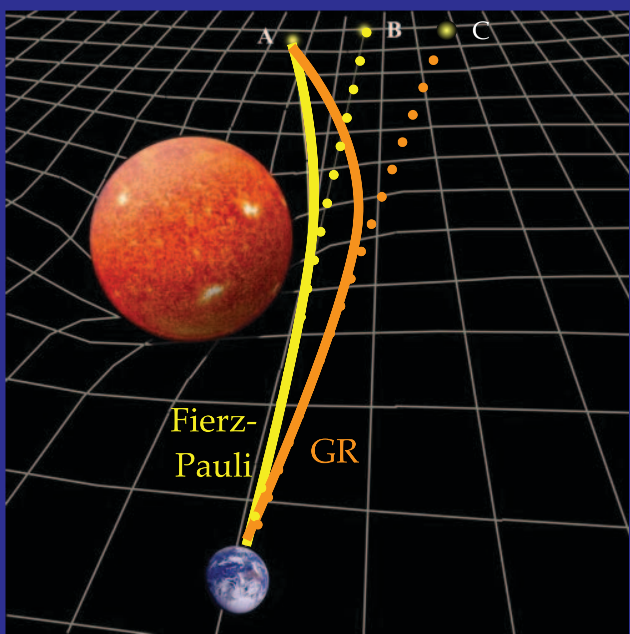


Fierz & Pauli (1939):
Ghost free linearized theory

$$m^2 \left(h_{\mu\nu}h^{\mu\nu} - h^2\right)$$

vDVZ Discontinuity

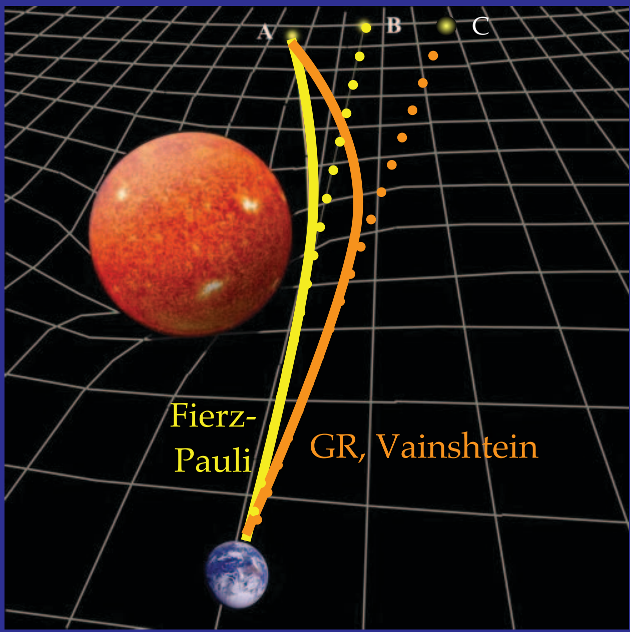
- Scalar mode coupled to matter changes space curvature per unit dynamical mass violating solar system lensing even as $m \to 0$



van Dam & Veltman (1970)
Zakharov (1970)

Vainshtein Mechanism

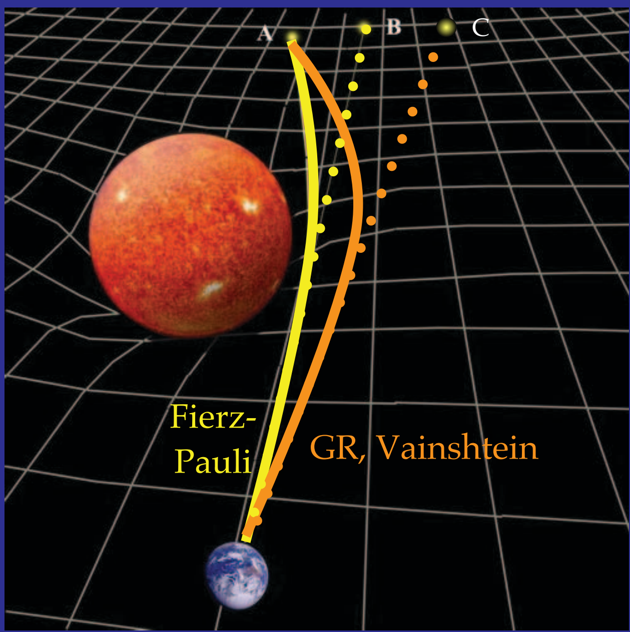
- Around massive sources, nonlinear interactions suppress scalar force

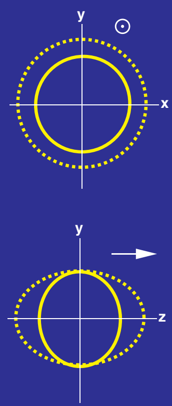


Vainshtein (1972)

Boulware-Deser Ghost

- But a generic nonlinear completion restores the 6th ghostly polarization





Bad trade!

Boulware & Deser (1972)

Massive Gravity

- de Rham, Gabadadze, Tolley (dRGT 2011) provided nonlinear completion to Fierz-Pauli that evades the Boulware-Deser ghost

$$S = \frac{M_p}{2} \int d^4X \sqrt{-g} \left[R - \frac{m^2}{2} \sum_{n=0}^{4} \frac{\beta_n}{n!} F_n(\sqrt{\mathbf{g}^{-1}\boldsymbol{\eta}}) \right]$$

where $\boldsymbol{\eta}$ is a fiducial metric, taken to be non-dynamical flat

$$ds_g^2 = g_{ab} dX^a dX^b, \quad ds_f^2 = \eta_{ab} dX^a dX^b = -dT^2 + dX_i^2$$

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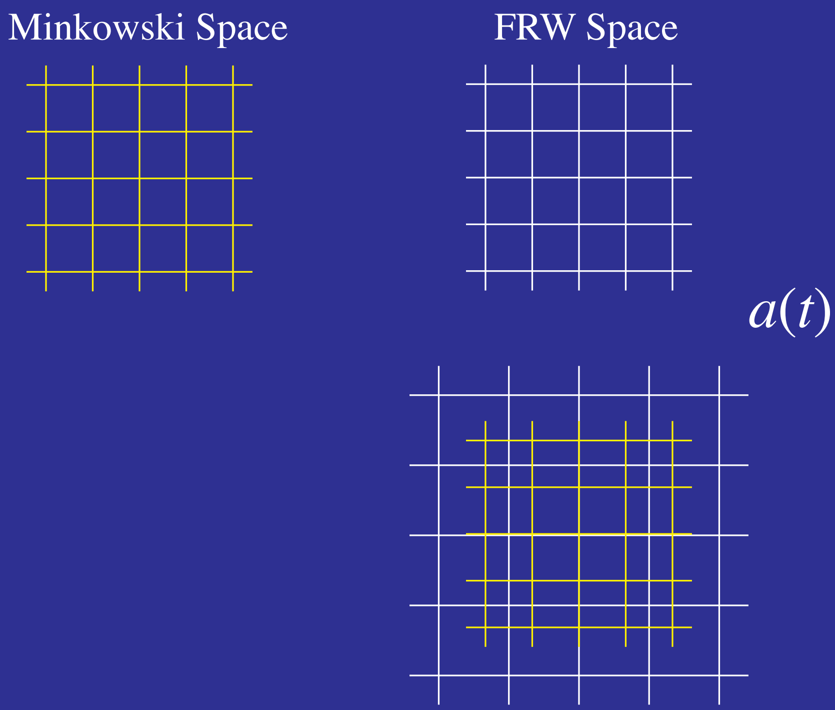
- Presence of fiducial metric breaks diffeomorphism invariance: a preferred unitary gauge where metric is standard Minkowski
- Diffeomorphism invariance can be restored by transforming from these preferred coordinates

$$\mathbf{g}^{-1}\boldsymbol{\eta} \to g^{\alpha\mu} \partial_\mu X^a \partial_\nu X^b \eta_{ab} = g^{\alpha\mu} f_{\mu\nu}$$

- Jacobian transformation represents fiducial metric covariantly $f_{\mu\nu}$
- Unitary gauge coordinates become 4 scalar Stückelberg fields

Spacetime Evolves from Minkowski

- Using Minkowski coordinates to chart the expanding spacetime



Spacetime Evolves from Minkowski

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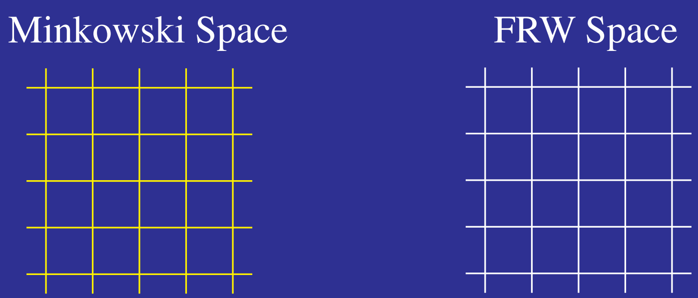
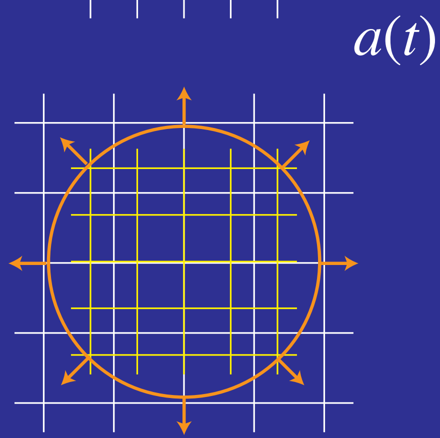




In spatially flat Minkowski coordinates the spacetime metric is superficially inhomogeneous but isotropic (H^2R^2 terms; static/physical vs comoving coordinates)

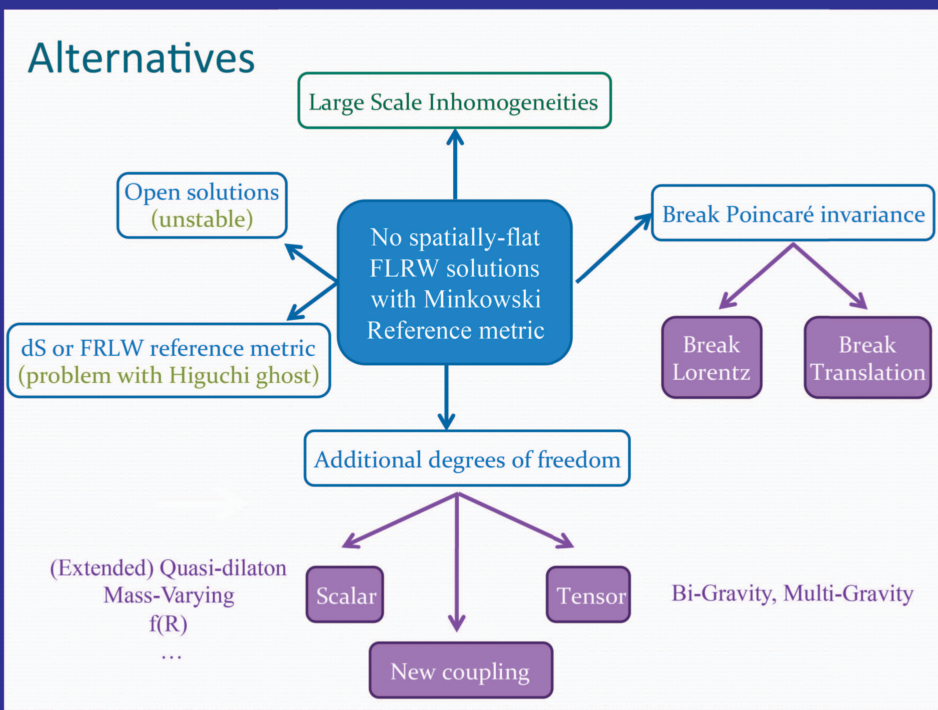
Homogeneity and Isotropy

- Coordinate problems take on geometric significance with two metrics
- Spatially flat slicing of Minkowski incompatible with homogeneous and isotropic FRW slicing of spacetime
  "no spatially flat FRW cosmologies" d'Amico et al (2011)

  = no single coordinates where both the spacetime and fiducial metric are simultaneously homogeneous and isotropic
- Open slicing of Minkowski (Milne) compatible with homogeneous and isotropic slicing of an open FRW spacetime
  Gumrukcuoglu, Lin, Mukohyama (2011)

  ...but these are generally are generally unstable
  Gumrukcuoglu, Lin, Mukohyama (2011); DeFelice, Gumrukcuoglu, Mukohyama (2012)
- Note: this does not preclude homogeneous and isotropic FRW spacetimes of any curvature or address their stability

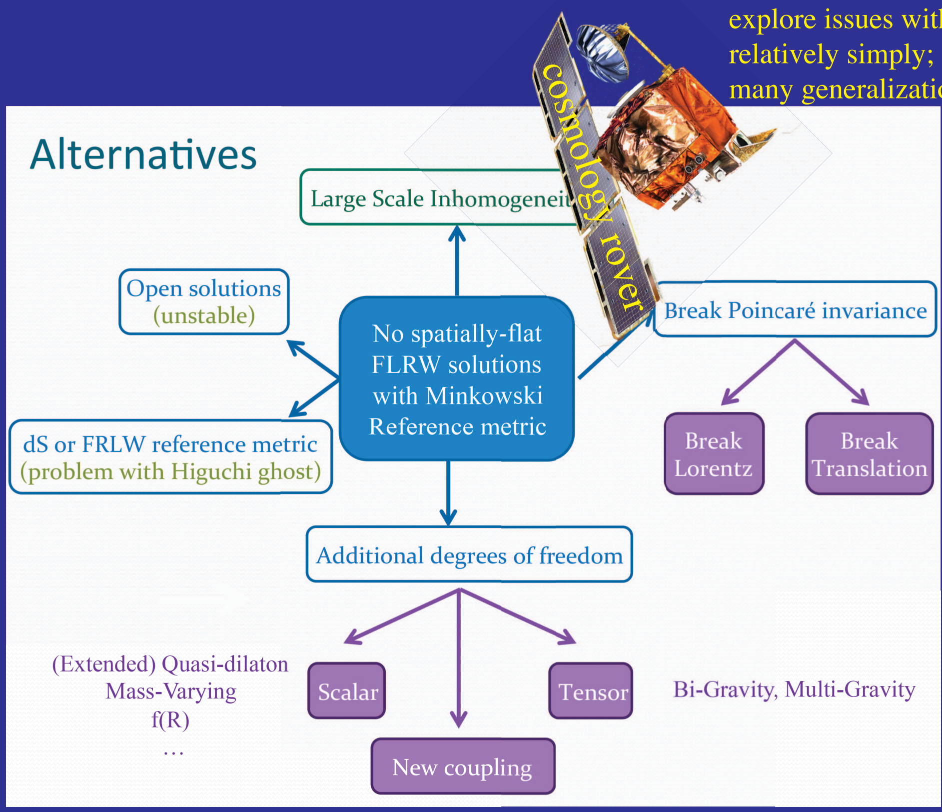
Massive Multiverse



de Rham (2015)

Massive Multiverse

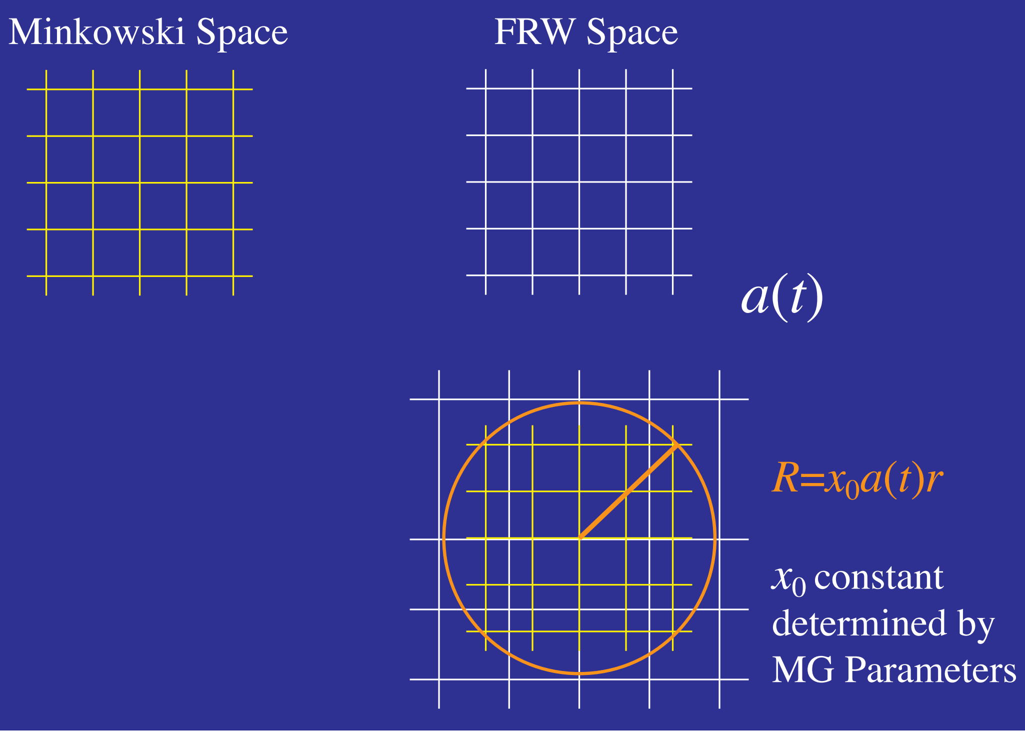
explore issues with 2 metrics relatively simply; common to many generalizations



de Rham (2015)

Self-Accelerating Solutions

- Allow the Minkowski coordinates T, R or Stuckelberg field to be inhomogeneous in isotropic FRW coordinates



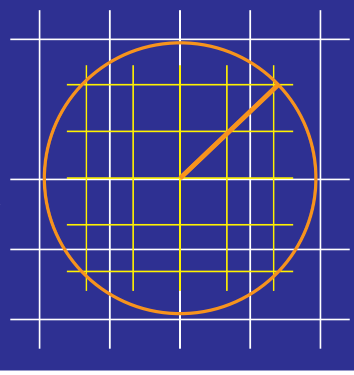
Self-Accelerating Solutions

- Allow the Minkowski coordinates T, R or Stuckelberg field to be inhomogeneous in isotropic FRW coordinates

Minkowski Space FRW Space

$a(t)$

All such constructions lead to an effective stress energy of a cosmological constant leaving remaining freedom in choosing Minkowski time $T(t,r)$



$R=x_0a(t)r$

x_0 constant determined by MG Parameters

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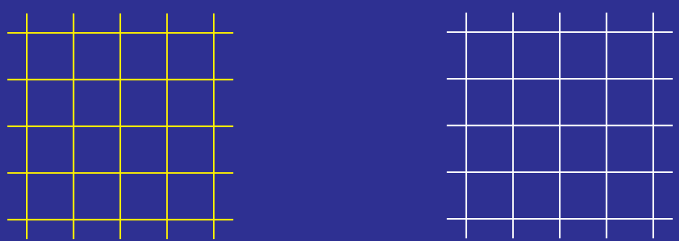
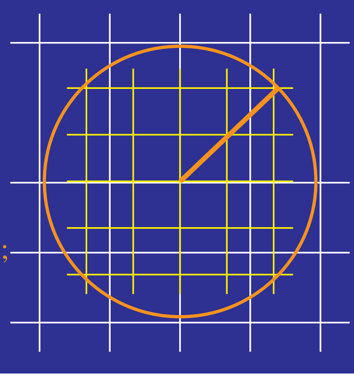
Minkowski Space FRW Space



$a(t)$

applies to any isotropic distribution of matter and unifies the description of all self-accelerating solutions
Gratia, Hu, Wyman (2012)

generalizes Koyama, Niz, Tasinato (2011); d'Amico et al (2012); Gumrukcouglu et al (2012); Berezhiani et al (2011);...

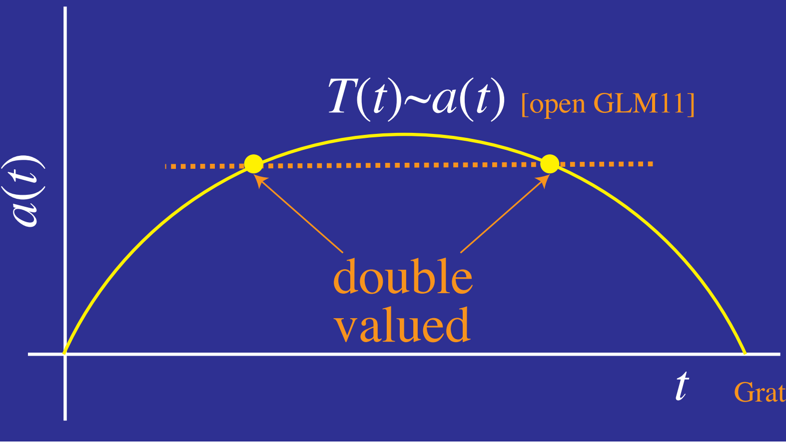


$R=x_0a(t)r$

x_0 constant determined by MG Parameters

Determinant Singularities

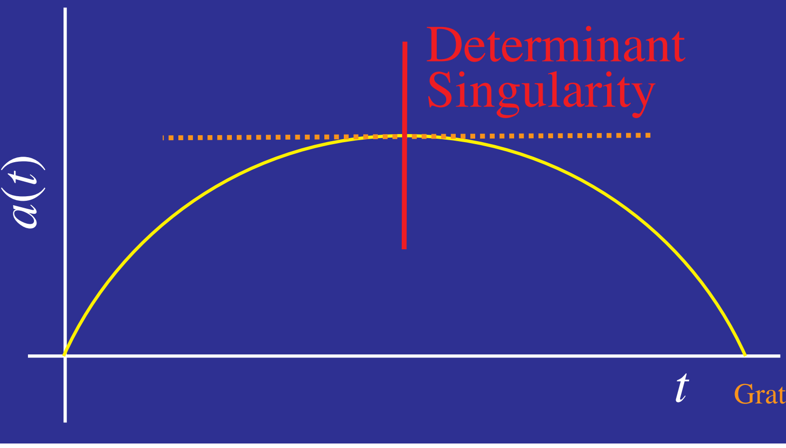
- Minkowski coordinates may not uniquely chart the whole spacetime - Jacobian between Minkowski and spacetime coordinates singular
- Fiducial metric has a determinant singularity where the spacetime metric does not or vice versa - ratio of determinants is a diffeomorphism invariant spacetime scalar
- Example: evolution to a det singularity



Gratia, Hu, Wyman (2013)

Determinant Singularities

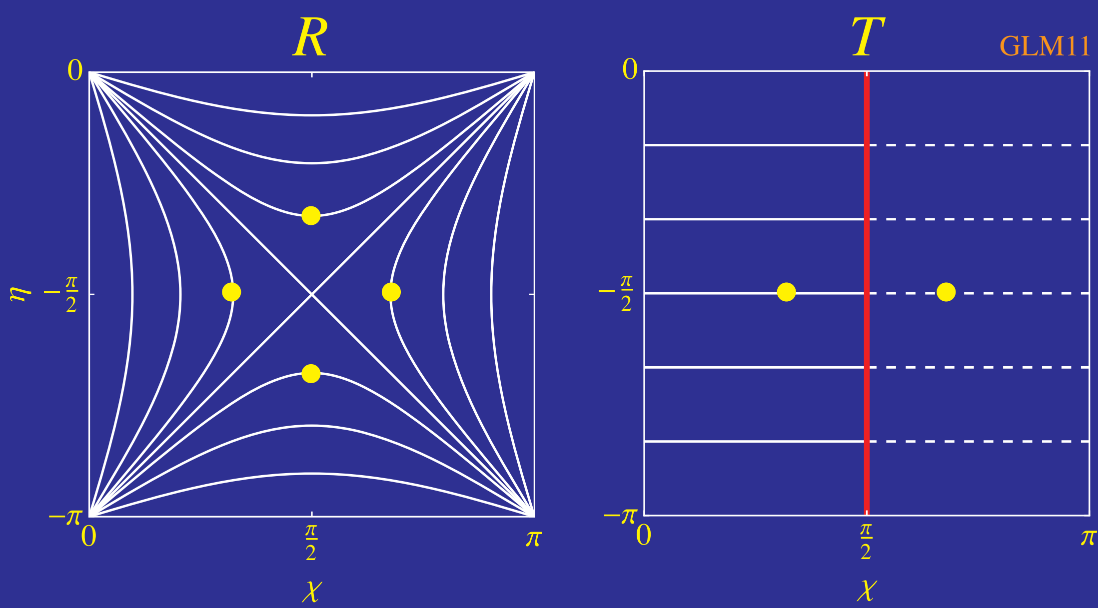
- No curvature singularity in the spacetime, normal matter sees only spacetime metric
- But requires ad hoc rules for smoothly joining charts for the massive gravity degrees of freedom; evolves into a singularity
- Occurs in more general bi-gravity models Gratia, Hu, Wyman (2014); Lagos & Ferreira (2014); Johnson & Terrana (2015) and extended quasi dilaton model (where smooth continuation fails) Motohashi & Hu (2014)



Gratia, Hu, Wyman (2013)

DeSitter Solutions

- Conformal diagram of de Sitter self-accelerating solutions
- Det=0 singularity when coordinates double valued



4 fold symmetric

Motloch, Hu, Joyce, Motohashi (2015)

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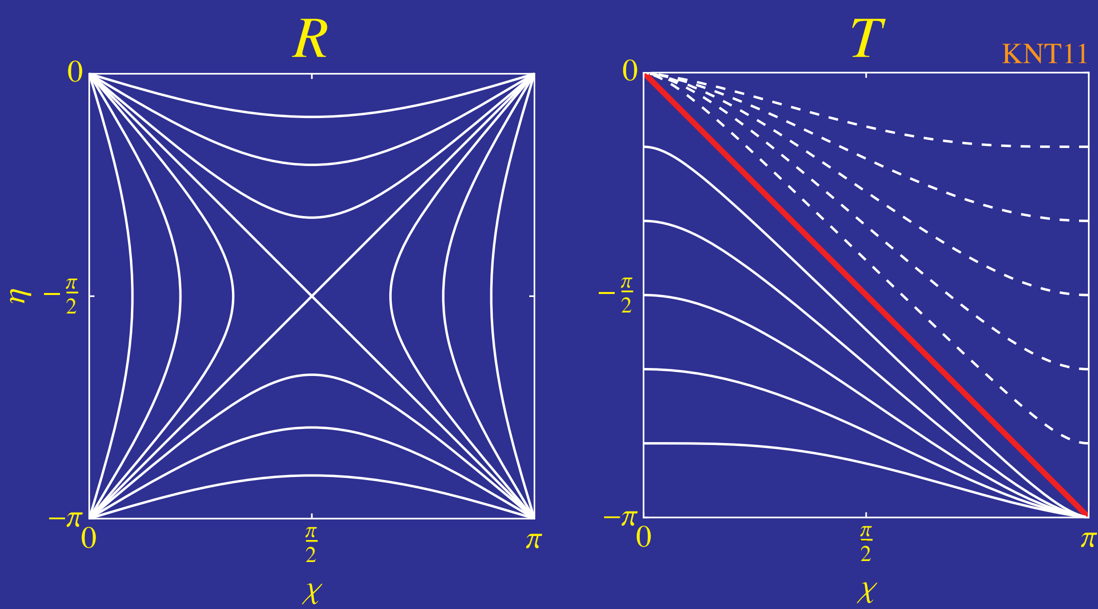

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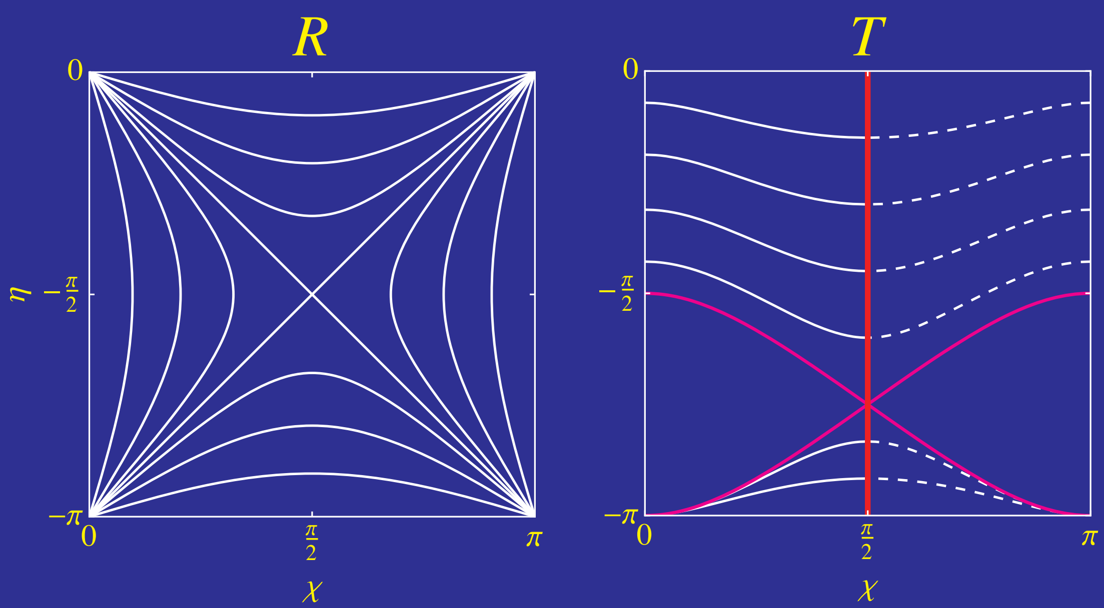
Motloch, Hu, Joyce, Motohashi (2015)

DeSitter Solutions

- Conformal diagram of de Sitter self-accelerating solutions
- Det=±∞ singularity where continuation flips signature



4 fold symmetric

Motloch, Hu, Joyce, Motohashi (2015)

Perturbations

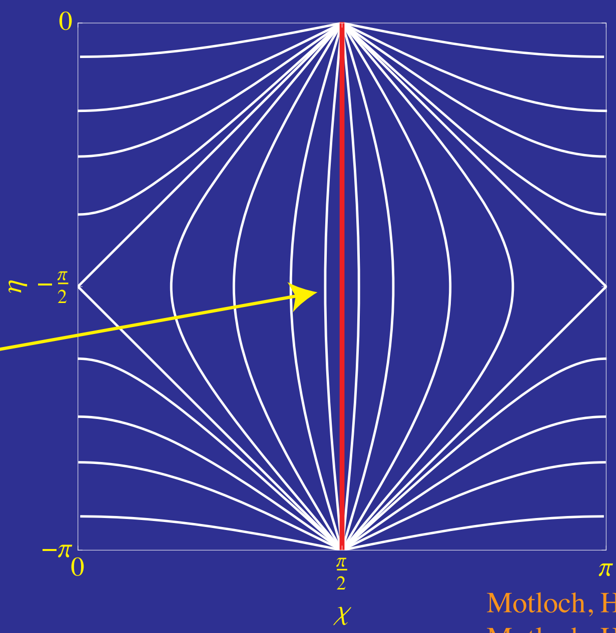
- Inhomogeneous Stuckelberg background complicates analysis
- Isotropic mode (scalar) not sourced by matter, carries stress energy, obeys first order equation of motion Wyman, Hu, Gratia (2011)

  simple system, analytic solutions
- Decoupling limit expectations for the helicity 0 and ±1 modes not obeyed, kinetic terms only at order curvature d'Amico (2011); Motloch & Hu (2014)

  In general 5 degrees of freedom (including open GLM solution, but 3 parabolic not hyperbolic)
- Fully covariant Stuckelberg-metric quadratic Lagrangian Motloch & Hu (2014)
- Specialize to vacuum unitary perturbation gauge: metric perts only Regge-Wheeler analysis of gw polarizations Motloch, Hu, Motohashi (2015)

Characteristics

- Characteristic curves of new degrees of freedom
- Example: "open FRW" solution of GLM11



characteristics run tangent to determinant singularities

Motloch, Hu, Joyce, Motohashi (2015)
Motloch, Hu, Motohashi (2015)

see also: Deser, Waldron, etal (2012-15); Izumi & Ong (2013)

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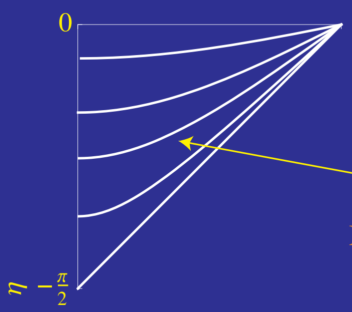

Constant Open Time

Motloch, Hu, Joyce, Motohashi (2015)

- Characteristics coincide with constant open time slices [no dynamics in open frame]
- Superluminal characteristics
- For monopole & dipole mode first order system: characterstics give all smooth and discontinuous front solutions
- Superluminal front and group velocity

Characteristics

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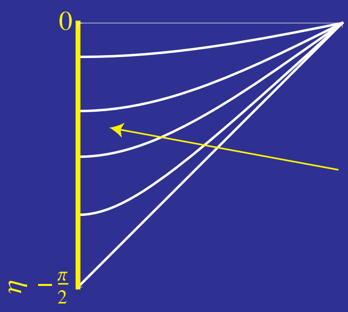

No Spacelike Cauchy Surface:
Spatial Boundary Conditions
Motloch, Hu, Joyce, Motohashi (2015)

- No spacelike surface intersect all characteristics
- For isotropic & dipole modes, second order system decouples into two first order systems, where a conditions on a single spatial boundary defines unique solution

Characteristics

- Characteristic curves of new degrees of freedom
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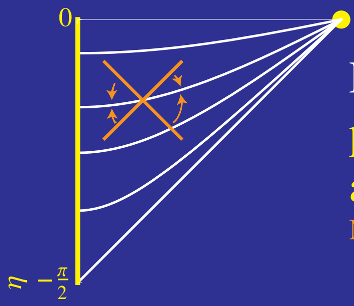
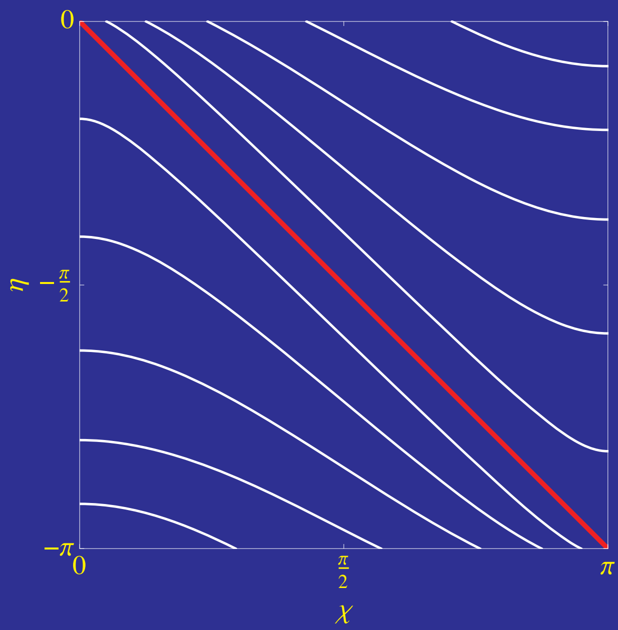


Lightcone degenerates:
parabolic equation for
anisotropic modes
Motloch, Hu, Motohashi (2015)

- Anisotropic $l \geq 2$ odd modes are second order and parabolic, not hyperbolic
- No wavelike solutions, similar to heat equation
- Requires two spatial boundary conditions to define unique solution

Characteristics

- Example: "SdS" solution of KNT11: characteristic curves run tangent to det singularities - information doesn't cross
- Spacelike surface do intersect characteristics defining initial value problem for isotropic & dipole modes
- Special case with luminal characteristics
- But $l \geq 2$ odd parity modes are still parabolic, requiring two boundary conditions: true of all self accelerating solutions



Motloch, Hu, Joyce, Motohashi (2015)
Motloch, Hu, Motohashi (2015)

Summary: Trouble with Metrics

- Self-accelerating dRGT massive gravity provides a relatively simple arena where Cauchy breakdown occurs at linear order in cosmological perturbations
(det singularities, parabolic/elliptic equations, no joint spacelike surface)
- In other cases where modes propagate on a separate metric similar problems occur on nonlinear backgrounds

Cosmological voids with cubic galileon [illegible] [illegible] [hyperbolic turns to elliptic]

Spherical collapse far from quasistatic approximation with DGP [illegible] [no joint spacelike Cauchy surface]

- Can be viewed as a strong coupling problem which may be solved by a UV completion of effective theory but occurs at relatively low densities and large scales from non pathological initial conditions

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  Cosmological voids with cubic galileon Barreira et al (2013); Winther & Ferreira (2015) [hyperbolic turns to elliptic]

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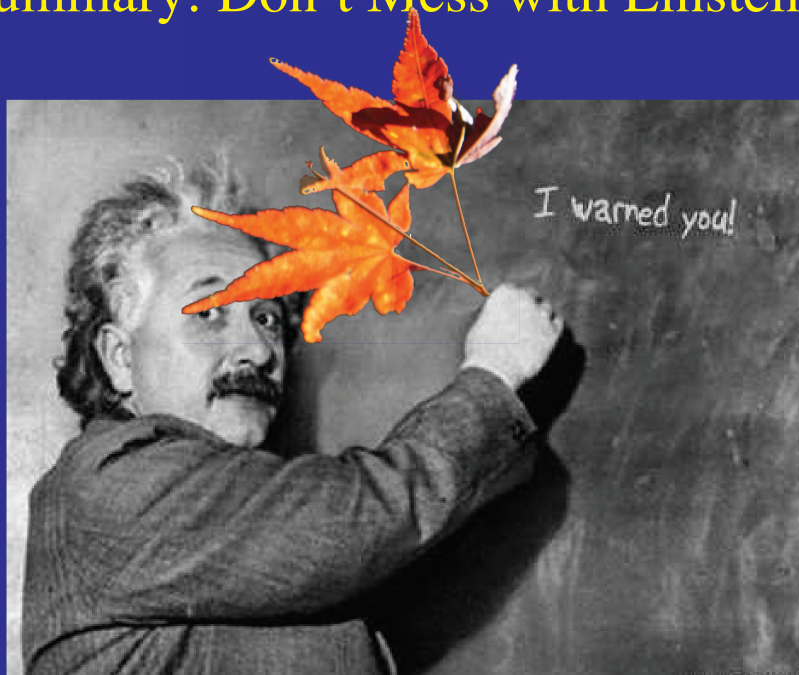
Summary: Don’t Mess with Einstein!



Happy 100th Birthday
GR

“Global existence theorem for Gowdy symmetric spacetimes in supergravity theory”

by Makoto Narita

[JGRG25(2015)8a1]

2015/Dec/11 @JGRG25, Kyoto

Global existence theorem for Gowdy symmetric spacetimes in supergravity theory

Makoto Narita

National Institute of Technology, Okinawa College

1 Introduction: Inflationary universe

It is believed that our universe has started from *initial singularity*, which is the Hot Big Bang model. However, there are problems (horizon problem, flatness problem, etc) in this model. These basic questions about the early universe can find a suitable and natural answer if the universe has *an inflationary era, a period of accelerated expansion*. Recently this constitutes the standard model of cosmology.

According the current observations, the degree of anisotropy at cosmological scales is very small (but NOT ZERO). Therefore, the accelerated expansion during inflation should be isotropic. Then the following conjecture was proposed:

Conjecture 1 (Gibbons-Hawking) The late-time behaviour of any accelerating universe is an isotropic universe.

This is the **cosmic no-hair conjecture**, which is an unsolved and important problem in General Relativity.

1 Introduction: Cosmic No-Hair Theorems 1

The following theorem supports the validity of cosmic no-hair conjecture:

Theorem 1 (Wald) The Bianchi models (except type IX) with the total energy-momentum tensor of the form

$$T_{\mu\nu} = -\Lambda_0 g_{\mu\nu} + \mathcal{T}_{\mu\nu},$$

and with a constant $\Lambda_0 > 0$ (cosmological constant) and $\mathcal{T}_{\mu\nu}$ satisfying the dominant and strong energy conditions, approach de Sitter space exponentially fast, within a few Hubble times $H^{-1} = \sqrt{\frac{3}{\Lambda_0}}$.

In the case of spatially inhomogeneous setting, Ringström has shown the following theorem:

Theorem 2 Consider a Gowdy symmetric solution the the Einstein-Vlasov system with a positive cosmological constant. Then, the solution is future asymptotically de Sitter.

To prove the above theorem, one needs to show

1. global existence,
2. future completeness,
3. cosmic no-hair (asymptotic behaviour).

1 Introduction: Cosmic No Hair Theorems 2

However, our universe is not de Sitter space. In addition, the recent observations suggest small anisotropy. Motivated by these, the next theorem has been shown:

Theorem 3 (Maleknejad-Sheikh-Jabbari) For general inflationary systems of all Bianchi type with the total energy-momentum tensor of the form

$$T_{\mu\nu} = -\Lambda(t) g_{\mu\nu} + \mathcal{T}_{\mu\nu},$$

where $\Lambda(t)$ is a cosmological term which decreases by time t and $\mathcal{T}_{\mu\nu}$ satisfies the dominant and strong energy conditions, *anisotropy may grow* nonetheless there is an upper bound on the growth of anisotropy.

To get anisotropic accelerated expansion of the universe, it is known that non-trivial coupling between scalar and *gauge fields* is important.

Such non-trivial coupling is suggested by some fundamental field theories.

Thus, we will consider the Einstein-Maxwell-Scalar system arising in supergravity theory, which is one candidate for the Unified Theory. .

1 Introduction: Supergravity Action and Einstein-Maxwell-Scalar equations

Four dimensional reduced action of the supergravity theory is of the form

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2}\nabla^{\mu}\phi\nabla_{\mu}\phi + V(\phi) + \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right],$$

where $g_{\mu\nu}$ is a Lorentzian metric on $(3+1)$-dimensional spacetime manifold M, $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar of g, ϕ is a scalar field, $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the field strength for a gauge field A_{μ}, $V(\phi) > 0$ is a potential of the scalar field and $f(\phi)$ is a *gauge kinetic function*.

Varying the action with respect to $g_{\mu\nu}$, A_{μ} and ϕ, we have the Einstein-Maxwell-Scalar equations as follows (*SUGRA EMS system*):

$$R_{\mu\nu} - 2\left[\partial_{\mu}\phi\partial_{\nu}\phi + V(\phi)g_{\mu\nu}\right] - 2f^2(\phi)\left[F_{\mu}^{\gamma}F_{\nu\gamma} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}g_{\mu\nu}\right] = 0,$$

$$\nabla_{\mu}F^{\mu\nu} + 2F^{\mu\nu}\partial_{\mu}\phi f^{-1}(\phi)\frac{\partial f(\phi)}{\partial\phi} = 0,$$

$$g^{\alpha\beta}\partial_{\alpha}\partial_{\beta}\phi - \Gamma^{\alpha}\partial_{\alpha}\phi - \frac{\partial V(\phi)}{\partial\phi} - \frac{1}{2}f(\phi)\frac{\partial f(\phi)}{\partial\phi}F_{\alpha\beta}F^{\alpha\beta} = 0.$$

1 Introduction: Hyperbolic reduction of SUGRA EMS system

To consider initial value problem for the system, we reduce this to a hyperbolic system.

- $$-\frac{1}{2}g^{\alpha\beta}\partial_{\alpha}\partial_{\beta}g_{\mu\nu} + \nabla_{(\mu}\mathcal{F}_{\nu)} + g^{\alpha\beta}g^{\gamma\delta}(\Gamma_{\alpha\gamma\mu}\Gamma_{\beta\delta\nu} + \Gamma_{\alpha\gamma\mu}\Gamma_{\beta\nu\delta} + \Gamma_{\alpha\gamma\nu}\Gamma_{\beta\mu\delta})$$
  $$-2\left[\partial_{\mu}\phi\partial_{\nu}\phi + V(\phi)g_{\mu\nu}\right] - 2f^2(\phi)\left[F_{\mu}^{\gamma}F_{\nu\gamma} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}g_{\mu\nu}\right] = 0,$$
- $$g^{\alpha\beta}\partial_{\alpha}\partial_{\beta}A_{\lambda} - \partial_{\lambda}(\mathcal{F}^{\mu}A_{\mu}) + (\partial_{\lambda}g^{\mu\nu})\partial_{\mu}A_{\nu}$$
  $$-g^{\alpha\beta}\Gamma^{\delta}_{\alpha\beta}(\partial_{\delta}A_{\lambda} - \partial_{\lambda}A_{\delta}) - g^{\alpha\beta}\Gamma^{\delta}_{\beta\lambda}(\partial_{\alpha}A_{\delta} - \partial_{\delta}A_{\alpha}) + 2F_{\lambda\nu}\partial^{\nu}\phi f^{-1}(\phi)\frac{\partial f(\phi)}{\partial\phi} = 0,$$
- $$g^{\alpha\beta}\partial_{\alpha}\partial_{\beta}\phi - \Gamma^{\alpha}\partial_{\alpha}\phi\frac{\partial V(\phi)}{\partial\phi} - \frac{1}{2}f(\phi)\frac{\partial f(\phi)}{\partial\phi}F_{\alpha\beta}F^{\alpha\beta} = 0,$$

where $\mathcal{F}_{\mu}$ is a *gauge source function*.

We can show existence of maximal globally hyperbolic developments:

Theorem 4 (Chquet-Bruhat-Geroch) Given initial data for the vacuum Einstein equations, there is a maximal globally hyperbolic development (MGHD) of the data which is unique up to isometry.

Remark 1 The above theorem can be generalized to the SUGRA EMS system.

1 Introduction: Anisotropic Inflation

Theorem 5 (Watanabe-Kanno-Soda (WKS)) Assume that all functions depend only on time t and $V = V_0 e^{\lambda\phi}$ and $f = f_0 e^{\rho\phi}$ where λ, ρ are constants. Suppose $\lambda^2 + 2\rho\lambda - 4 > 0$. Then, the following is a solution to the SUGRA EMS system:

$$\begin{aligned} ds^2 &= -dt^2 + e^{2a}\left[e^{-4b}dx^2 + e^{2b}(dy^2+dz^2)\right], \\ \frac{dA_x}{dt} &= Ct^{\gamma}, \\ \phi &= -\frac{2}{\lambda}\log t, \end{aligned}$$

where $a = \kappa \log t$, $b = \zeta \log t$, $\kappa = \dfrac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda+2\rho)}$, $\zeta = \dfrac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda+2\rho)}$ and $\gamma = \dfrac{4\rho}{\lambda} - \omega - 4\zeta$.

This spacetime is an axially symmetric Bianchi type I (homogeneous and anisotropic) one and has accelerating expansion.

Note that anisotropy grows during inflation in contrast to cosmic no-hair conjecture in this solution.

Next setting: spatially inhomogeneous case

2 Gowdy symmetric spacetimes: Metric and Gauge Potential

Assume that spatial topology is T^3 and there are two spacelike Killing vectors generated by ∂_x and ∂_y. Note that θ, x, y are coordinates on T^3 and translation in the x and y directions defines a smooth action of T^2. Also note that the area of the symmetry orbits is propotional to time t. The areal metric is given by

$$ds^2 = -e^{2(\eta-U)}(\alpha dt^2 + d\theta^2) + e^{2U}(dx^2 + W dy)^2 + e^{-2U}t^2 dy^2.$$

This spacetime is an inhomogeneous generalization of the axially symmetric Bianchi type I spacetimes.

Due to symmetry, only $A_x = \omega$ and $A_y = \chi$ are non-vanishing functions for A_μ.

Note that all functions $\alpha > 0, \eta, U, W, \omega, \chi, \phi$ depend only on time t and θ.

To get wave map form, the following transformation is used:

$$\begin{aligned} \sqrt{\alpha}W_\theta &= -te^{-4U}(\psi_t + 2\omega\xi_t), & W_t &= -\sqrt{\alpha}te^{-4U}(\psi_\theta + 2\omega\xi_\theta) \\ 2(\chi_t - W\omega_t) &= t\sqrt{\alpha}e^{-2U}\xi_\theta, & 2\sqrt{\alpha}(\chi_\theta - W\omega_\theta) &= te^{-2U}\xi_t, \end{aligned}$$

and we put $\sigma = 2\omega$ for convenience.

2 Gowdy symmetric spacetimes: Einstein constraint equations

$$\frac{\eta_t}{t} = U_t^2 + \alpha U_\theta^2 + \frac{e^{-4U}}{4}\left[(\psi_t + \sigma\xi_t)^2 + \alpha(\psi_\theta + \sigma\xi_\theta)^2\right] + \phi_t^2 + \alpha\phi_\theta^2 + \alpha e^{2(\eta-U)}V,$$

$$\frac{\eta_\theta}{t} = 2U_tU_\theta + \phi_t\phi_\theta + \frac{e^{-4U}}{2}(\psi_t + \sigma\xi_t)(\psi_\theta + \sigma\xi_\theta) + \frac{f^2}{2}e^{-2U}(\sigma_t\sigma_\theta + \xi_t\xi_\theta) - \frac{\alpha_\theta}{2t\alpha},$$

$$\alpha_t = -4t\alpha^2 e^{2(\eta-U)}V,$$

where $\eta_t = \dfrac{\partial \eta}{\partial t}$ and $\eta_\theta = \dfrac{\partial \eta}{\partial \theta}$.

2 Gowdy symmetric spacetimes: Einstein evolution equations

$$\begin{aligned} U_{tt} + \frac{1}{t}U_t - U_{\theta\theta} - \frac{\alpha_t U_t}{2\alpha} + \frac{\alpha_\theta U_\theta}{2} + \frac{e^{-4U}}{2}\left[(\psi_t + \sigma\xi_t)^2 - \alpha(\psi_\theta + \sigma\xi_\theta)^2\right] \\ + \frac{f^2}{16}e^{-2U}(\sigma_t^2 - \alpha\sigma_\theta^2 + \xi_t^2 - \alpha\xi_\theta^2) - \frac{1}{2}\alpha e^{2(\eta-U)}V = 0, \end{aligned}$$

$$\begin{aligned} \psi_{tt} + \frac{1}{t}\psi_t - \psi_{\theta\theta} - \frac{\alpha_t\psi_t}{2\alpha} + \frac{\alpha_\theta\psi_\theta}{2} - 4(U_t\psi_t - \alpha U_\theta\psi_\theta) - 2\sigma(U_t\xi_t - \alpha U_\theta\xi_\theta) \\ + (1 - 4f^{-2}\sigma^2 e^{-2U})(\sigma_t\xi_t - \alpha\sigma_\theta\xi_\theta) - 4f^{-2}\sigma e^{-2U}(\sigma_t\psi_t - \alpha\sigma_\theta\psi_\theta) \\ - \frac{2\sigma}{f}\frac{\partial f}{\partial \phi}(\phi_t\xi_t - \alpha\phi_\theta\xi_\theta) = 0. \end{aligned}$$

These equations describe time-evolution of gravitational waves.

2 Gowdy symmetric spacetimes: Maxwell equations

$$\xi_{tt} + \frac{1}{t}\xi_t - \xi_{\theta\theta} - \frac{\alpha_t \xi_t}{2\alpha} + \frac{\alpha_\theta \xi_\theta}{2} - 2(U_t \xi_t - \alpha U_\theta \xi_\theta) + \frac{2}{f}\frac{\partial f}{\partial \phi}(\phi_t \xi_t - \alpha \phi_\theta \xi_\theta)$$
$$+ 4 f^{-2} e^{-2U}(\sigma_t \psi_t - \alpha \sigma_\theta \psi_\theta) + 4 f^{-2} \sigma e^{-2U}(\sigma_t \xi_t - \alpha \sigma_\theta \xi_\theta) = 0,$$

$$\sigma_{tt} + \frac{1}{t}\sigma_t - \sigma_{\theta\theta} - \frac{\alpha_t \sigma_t}{2\alpha} + \frac{\alpha_\theta \sigma_\theta}{2} - 2(U_t \sigma_t - \alpha U_\theta \sigma_\theta) + \frac{2}{f}\frac{\partial f}{\partial \phi}(\phi_t \sigma_t - \alpha \phi_\theta \sigma_\theta)$$
$$- 4 f^{-2} e^{-2U}\left[\xi_t(\psi_t + \sigma \xi_t) - \alpha(\psi_\theta + \sigma \xi_\theta)\right] = 0.$$

These equations describe time-evolution of electromagnetic waves.

2 Gowdy symmetric spacetimes: Scalar field equation

$$\phi_{tt} + \frac{1}{t}\phi_t - \phi_{\theta\theta} - \frac{\alpha_t \phi_t}{2\alpha} + \frac{\alpha_\theta \phi_\theta}{2} - \frac{1}{4}e^{-2U} f \frac{\partial f}{\partial \phi}\left[\xi_t^2 + \sigma_t^2 - \alpha(\xi_\theta^2 + \sigma_\theta^2)\right] + \alpha e^{2(\eta - U)}\frac{\partial V}{\partial \phi} = 0.$$

In summary, the *SUGRA EMS Gowdy system* consist of five semi-linear wave equations with three constraint equations.

2 Gowdy symmetric spacetimes: Wave Map

We can find that five evolution equations for the SUGRA Gowdy system are described by the following wave map $u : (\mathcal{M}^{2+1}, \beta) \mapsto (\mathcal{N}^5, \tau)$, where $\mathcal{M}^{2+1}$ is a base manifold with Lorentzian metric β,

$$\beta = -dt^2 + \frac{1}{\alpha} d\theta^2 + t^2 d\delta^2,$$

and $\mathcal{N}^5$ is a target manifold with Riemannian metric τ,

$$\tau = 4dU^2 + e^{-4U}(d\psi + \sigma d\xi)^2 + d\phi^2 + \frac{1}{4} f^2 e^{-2U}(d\xi^2 + d\sigma^2).$$

The action for this wave map is given by

$$S_W = \int_{S^1} dt d\theta \sqrt{-\beta} \left(\beta^{\mu\nu} \tau_{AB} \partial_\mu u^A \partial_\nu u^B + 2\alpha e^{2(\eta - U)} V \right).$$

Note that only evolution equations are obtained from the action.

The energy-momentum tensor can be defined from the action and then we can define the *energy* of this system.

2 Gowdy symmetric spacetimes: Global Eexistence

Theorem 6 Let (M, g) be the MGHD of C^∞ initial data for the SUGRA EMS Gowdy system. Then, for $t_0 > 0$, M can be covered by compact Cauchy surfaces T^3 of constant areal time t with each value in the range (t_0, ∞).

The method of the proof is the standard energy estimate (*light cone estimate*).

Remark 2 We can show a existence theorem in the past direction $t \in (0, t_0)$.

Remark 3 Asymptotically velocity terms dominated solutions can be constructed near initial singularity $t = 0$.

3 Summary

1. As inhomogeneous generalization, Gowdy symmetric spacetimes in supergravity theory were analyzed.
2. A global existence theorem of solutions to SUGRA Gowdy system was shown.
3. Next questions:
   (a) Are solutions to SUGRA Gowdy system future complete?
   (b) Is generalized cosmic no-hair conjecture true in the case of the SUGRA Gowdy system?
   (c) Are solutions to SUGRA Gowdy system stable?

Thank you for your attention.

“Anisotropic and Asymmetric Primordial Universe”

by Hassan Firouzjahi

[JGRG25(2015)8a2]

Anisotropic and Asymmetric Primordial Universe

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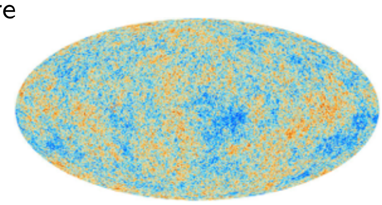
JGRG25, YITP, Dec. 2015

Motivation

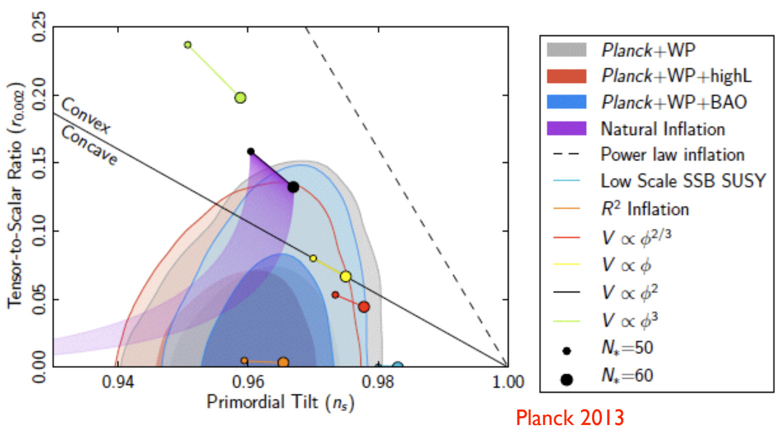
Inflation is the leading paradigm for early Universe and structure formations.

Basics predictions of inflation: The CMB perturbations are



- Nearly scale-invariant
- Nearly Gaussian
- Nearly adiabatic

These predictions are in good agreement with the Planck data.

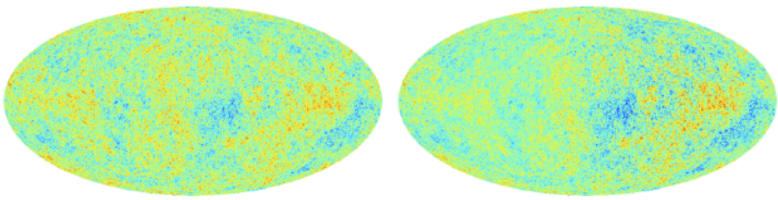


Planck 2013

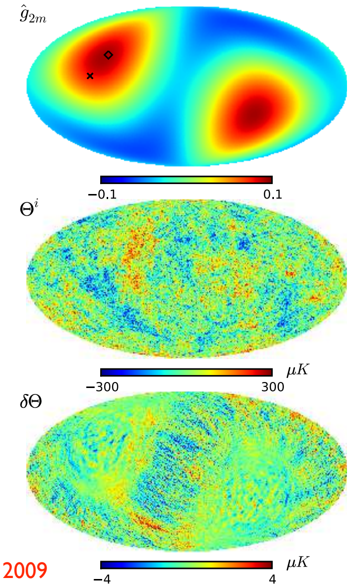
A Fundamental Question: Is the Universe isotropic?

- Planck has reported anisotropies on CMB map

  Hemispherical Power Asymmetry $\mathcal{P}_{\mathcal{R}} = \mathcal{P}^{(0)}_{\mathcal{R}}(1 + 2A\,\hat{\mathbf{n}}.\hat{\mathbf{p}})$
- Planck : $A = 0.07 \pm 0.02$ for $2 \ll \ell \lesssim 64$ with $(l, b) = (227°, -21°)$



A=0 Seljebotn, 2010 A=0.3



Hanson & Lewis, 2009

Statistical Anisotropy

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}) = \mathcal{P}^{(0)}_{\mathcal{R}}\left(1 + \sum_{LM} g_{LM} Y_{LM}(\hat{\mathbf{k}})\right)$$

quadrupole anisotropy: $L = 2$, $m = 0$

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}) = \mathcal{P}^{(0)}_{\mathcal{R}}(1 + g_* \,(\hat{\mathbf{p}}.\hat{\mathbf{k}})^2)$$

Observational bounds from Planck $|g_*| < 10^{-2}$

Anisotropic Inflation from Gauge Field Dynamics

The model contains a $U(1)$ gauge field minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{f^2(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\right]$$

Here $1/f(\phi)$ is the time-dependent gauge kinetic coupling.

We turn on the background gauge field $A_\mu = (0, A_x(t), 0, 0)$
The background metric is

$$\begin{aligned} ds^2 &= -dt^2 + e^{2\alpha(t)}\left(e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}(dy^2 + dz^2)\right) \\ &= -dt^2 + a(t)^2dx^2 + b(t)^2(dy^2 + dz^2) \end{aligned}$$

In this view $H \equiv \dot{\alpha}$ is the average Hubble expansion rate and

$$H_a \equiv \frac{\dot{a}}{a} \quad , \quad H_b \equiv \frac{\dot{b}}{b}$$

The anisotropy in the system is measured by

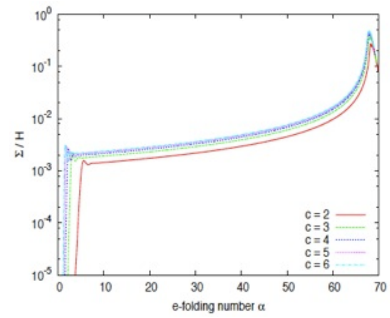
$$\frac{\dot{\sigma}}{H} \equiv \frac{H_b - H_a}{H}$$

The background equations are too complicated to be solved !

It is instructive to look at the ratio of gauge field energy density to total energy density

$$R = \frac{\rho_A}{V} = \frac{\dot{A}^2 f(\phi)^2 e^{-2N}}{2V} = \frac{p_A^2}{2V} f^{-2} e^{-4N}$$

Watanabe, Kanno, Soda, 09



In the absence of conformal coupling $f(\phi)$ R decays like a^{-4}.

Consider the chaotic potential $V = \frac{m^2}{2}\phi^2$. If one chooses

$$f(\phi) = \exp\left(\frac{c\phi^2}{2M_P^2}\right) = \left(\frac{a}{a_f}\right)^{-2c} \qquad (c > 1)$$

the system reaches the attractor regime:

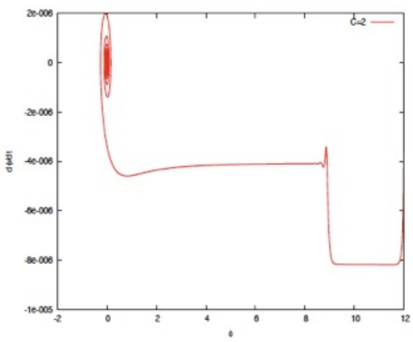
$$R = \frac{I}{2}\epsilon \quad , \quad I \equiv \frac{c-1}{c}$$

and

$$M_P^{-2}\frac{d\phi}{dN} \simeq -\frac{V_\phi}{V} + \frac{c-1}{c}\frac{V_\phi}{V}$$

and therefore

$$\phi^2 - \phi_e^2 = 4M_P^2 N(1-I)$$



δN *in anisotropic background*

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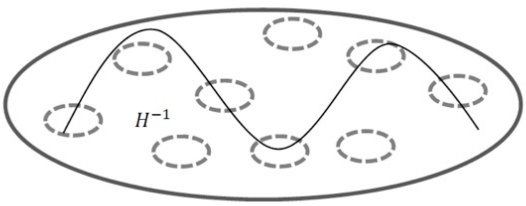
δN is a powerful method applicable to all order in perturbations.

We only need to solve the number of e-folds N as a function of background fields ϕ, A.



The revenant equations are

$$\phi^2 - \phi_e^2 = 4M_P^2 N(1-I)$$

and

$$R = \frac{\dot{A}^2 f(\phi)^2 e^{-2N}}{2V} = \frac{p_A^2}{2V} f^{-2} e^{-4N} = \frac{I}{2}\epsilon$$

We obtain

$$\delta N = -\frac{\phi}{2M_P^2}\delta\phi + 2I\,N\frac{\delta\dot{A}}{\dot{A}}$$

This is our result to first order in perturbations in $\delta\phi$ and $\delta\dot{A}$.

All we need to know is that at the time of horizon crossing

$$\frac{\delta \dot{A}}{\dot{A}} = \sum_\lambda \vec{\epsilon}_\lambda \frac{\sqrt{3}H}{\sqrt{2I\epsilon k^3}} \,.$$

Now we can calculate the power spectrum A.A. Abolhasani, R. Emami, J. Taghizadeh, H. F., 2013

$$\mathcal{R} = \delta N = -\frac{\phi}{2M_P^2}\delta\phi + 2IN\frac{\delta\dot{A}_x}{\dot{A}}.$$

The isotropic and the anisotropic parts are $\mathcal{P}_{\mathcal{R}} \equiv \mathcal{P}_0 + \Delta\mathcal{P}$ in which

$$\mathcal{P}_0 = \frac{H^2}{8\pi^2 M_P^2 \epsilon_H}.$$

To calculate the anisotropic power spectrum we note that $\delta\phi$ and $\delta\dot{A}$ are mutually uncorrelated so $\langle\delta\phi\delta\dot{A}\rangle|_* = 0$. As a results

$$\begin{aligned}\Delta\mathcal{P} &= \frac{k_1^3}{2\pi^2}4I^2N^2\left\langle\frac{\delta\dot{A}_x(k_1)}{A_x}\frac{\delta\dot{A}_x(k_2)}{A_x}\right\rangle \\ &= \frac{k_1^3}{2\pi^2}\frac{6IH^2}{\epsilon_H k_1^3}N^2\sin^2\theta = 24\, IN^2\mathcal{P}_0\sin^2\theta\end{aligned}$$

in which the angle θ is defined via $\cos\theta = \hat{n}.\hat{k}$. Now comparing this with the anisotropy factor g_* defined via

$$\mathcal{P}_{\mathcal{R}}(\vec{k}) = \mathcal{P}_0\left(1 + g_*(\hat{k}.\hat{n})^2\right).$$

we obtained

$$g_* = -24IN^2$$

Bispectrum

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The second oder δN is

$$\delta N = N_\phi\delta\phi + N_{\dot{A}}\delta\dot{A} + \frac{N_{\phi\phi}}{2}\delta\phi^2 + \frac{N_{\dot{A}\dot{A}}}{2}\delta\dot{A}^2 + N_{\phi\dot{A}}\delta\phi\delta\dot{A}_x$$

in which to leading order in I

$$N_\phi \simeq -\frac{\phi}{2M_P^2} \quad , \quad N_{\phi\phi} \simeq \frac{2f_{,\phi}^2}{f^2} + \frac{2f_{,\phi\phi}}{f} + \frac{\phi^2}{M_P^4} + \frac{4\phi}{M_P^2}\frac{f_{,\phi}}{f}$$

and

$$N_{,\dot{A}} \simeq \frac{2IN}{\dot{A}} \quad , \quad N_{,\dot{A}\dot{A}} \simeq \frac{2IN}{\dot{A}^2} \quad , \quad N_{,\phi\dot{A}} \simeq \frac{4IN}{\dot{A}}\frac{f_\phi}{f}$$

The leading contribution in bispectrum comes from $N_{\dot{A}\dot{A}}$ term and

$$\begin{aligned}&\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \\ &\simeq 4I^3N(k_1)N(k_2)N(k_3)\int\frac{d^3p}{(2\pi)^3}\langle\delta\dot{A}_x(\vec{k}_1)\delta\dot{A}_x(\vec{k}_2)\delta\dot{A}_i(\vec{p})\delta\dot{A}_i(\vec{k}_3-\vec{p})\rangle + 2\text{perm.}\end{aligned}$$

The bispectrum is

$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \simeq 288IN_{k_1}N_{k_2}N_{k_3}\left(C(\vec{k}_1,\vec{k}_2)P_0(k_1)P_0(k_2) + 2\text{perm.}\right)(2\pi)^3\delta^3(\sum\vec{k}_i)$$

in which

$$C(\vec{k}_1,\vec{k}_2) \equiv \left(1 - (\widehat{k}_1.\widehat{n})^2 - (\widehat{k}_2.\widehat{n})^2 + (\widehat{k}_1.\widehat{n})(\widehat{k}_2.\widehat{n})(\widehat{k}_1.\widehat{k}_2)\right)$$

Our result is in exact agreement with the results obtained from in-in formalism!

In the squeezed limit we get

$$\begin{aligned} f_{NL} &= 240 I N(k_1) N(k_2)^2 C(\vec{k}_1, \vec{k}_2) \qquad (k_1 \ll k_2 \simeq k_3) \\ &\simeq 10 N\, |g_*|\, C(\vec{k}_1, \vec{k}_2) \end{aligned}$$

Similarly, calculating the trispectrum $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle$ we obtain

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle \simeq 3456 I N_{k_1} N_{k_2} N_{k_3} N_{k_4} \Big(D(\vec{k}_3, \vec{k}_4, \vec{k}_1 + \vec{k}_3) P(k_3) P(k_4) P(|\vec{k}_1 + \vec{k}_3|) + 11\text{perm.} \Big) (2\pi)^3 \delta^3 \left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4 \right) ,$$

in which

$$\begin{aligned} D(\vec{k}_3, \vec{k}_4, \vec{k}_1 + \vec{k}_3) = 1 &- (\widehat{k}_4.\widehat{n})^2 - (\widehat{k}_3.\widehat{n})^2 - (\widehat{k_1 + k_3}.\widehat{n})^2 + (\widehat{k}_3.\widehat{n})(\widehat{k}_4.\widehat{n})(\widehat{k}_3.\widehat{k}_4) \\ &+ (\widehat{k}_4.\widehat{n})(\widehat{k_1 + k_3}.\widehat{n})(\widehat{k_1 + k_3}.\widehat{k}_4) + (\widehat{k}_3.\widehat{n})(\widehat{k_1 + k_3}.\widehat{n})(\widehat{k_1 + k_3}.\widehat{k}_3) \\ &- (\widehat{k}_3.\widehat{n})(\widehat{k}_4.\widehat{n})(\widehat{k_1 + k_3}.\widehat{k}_3)(\widehat{k_1 + k_3}.\widehat{k}_4) . \end{aligned}$$

In the collapsed limit $\vec{k}_1 + \vec{k}_3 = \vec{k}_2 + \vec{k}_4 = 0$

$$\tau_{NL}(k_1, k_2, k_3, k_4) \simeq 3456 I N(k_3)^2 N(k_4)^2 D(\vec{k}_3, \vec{k}_4, \vec{k}_1 + \vec{k}_3) .$$

Our result is in exact agreement with the results obtained from in-in formalism!

Shiriashi, Komatsu, Peloso, Barnaby 20123

Statistical Anisotropy: The Effective Field Theory Approach

- Effective Field Theory (EFT) provides a model-independent view of inflation. It helps to classify various inflationary models based on their predictions for power spectrum and bispectrum.
- Our goal is to study anisotropic inflation model-independently and capture the general predictions.
- In isotropic models based on a single field model, the evolution of $\phi(t)$ breaks the time diffeomorphism spontatnously. However, one still has the spatial diffeomorphism invariance
$$\xi^i \to \xi^i + \xi^i(x^\nu)$$
- In the presence of the gauge field, we also have to take into account the internal U(1) gauge symmetry:
$$A^\mu \to A^\mu + \nabla^\mu \mathcal{F}$$
- The choice of Unitary Gauge
$$\delta\phi = 0 \quad , \quad A^\mu = (0, A^1(t), 0, 0) .$$

A. A. Abolhasani, M. Akhshik, R. Emami, H. F. , 1511.03218

Challenge: Under a U(1) gauge transformation we can always undo the unitary gauge and turn on δA^μ.

Under a combined spatial and U(1) transofrmation we have

$$\delta A^\mu \to \delta A^\mu + \overline{A}^1 \partial_1 \xi^\mu + g^{\mu\alpha}\partial_\alpha \mathcal{F}.$$

Suppose

$$\xi^\mu = \nabla^\mu \xi_L + \xi^\mu_T = g^{\mu\alpha}\partial_\alpha \xi_L + \xi^\mu_T\,, \qquad \nabla_\mu \xi^\mu_T = 0$$

Then

$$\begin{aligned} \delta A^\mu \quad &\to \quad \delta A^\mu + \overline{A}^1 \partial_1 \xi^\mu_T + g^{\mu\alpha}\partial_\alpha \mathcal{F} + \overline{A}^1 \partial_1 \left(g^{\mu\alpha}\partial_\alpha \xi_L\right), \\ &= \quad \delta A^\mu + \overline{A}^1 \partial_1 \xi^\mu_T + \overline{A}^1 \left(\partial_1 g^{\mu\alpha}\right)\partial_\alpha \xi_L + \dot{\overline{A}}^1 g^{\mu 0}\partial_1 \xi_L + g^{\mu\alpha}\partial_\alpha \left(\overline{A}^1 \partial_1 \xi_L + \mathcal{F}\right). \end{aligned}$$

The remnant symmetry:

$$\xi^\mu_T = \xi^\mu_T(t,y,z), \qquad \left(\partial_1 g^{\mu\alpha}\right)\partial_\alpha \xi_L = \frac{\dot{\overline{A}}^1}{\overline{A}^1} g^{\mu 0}\partial_1 \xi_L, \qquad \text{(remnant symmetry)}$$

Now our building blocks are metric perturbations $\delta g_{\alpha\beta}$ and their derivatives subject to the above remnant symmetries.

The Quadratic action in Unitary gauge

A.A.Abolhasani, M.Akhshik, R.Emami, H.F. , 1511.03218

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Big[&\Lambda + \alpha_0 g^{00} + \frac{c_0}{4}\left(\delta g^{00}\right)^2 - \frac{1}{4}M_1\delta\left(G^{\alpha\beta}G_{\alpha\beta}\right) - \frac{1}{4}M_2\delta\left(G^{\alpha\beta}G_{\alpha\beta}\right)^2 \\ &- \frac{1}{4}M_3\delta\left(G^{\alpha\beta}\tilde{G}_{\alpha\beta}\right) - \frac{1}{4}M_4\delta\left(G^{\alpha\beta}\tilde{G}_{\alpha\beta}\right)^2 + \frac{1}{2}\lambda_1\delta g^{00}\delta\left(G^{\alpha\beta}G_{\alpha\beta}\right) \\ &+ \frac{1}{2}\lambda_2\delta g^{00}\delta\left(G^{\alpha\beta}\tilde{G}_{\alpha\beta}\right) + \ldots\Big]. \end{aligned}$$

in which

$$G_{\alpha\beta} \equiv \partial_\alpha g_{\beta 1} - \partial_\beta g_{\alpha 1} + \frac{\dot{\overline{A}^1}}{\overline{A^1}}\left(\delta^0_\alpha g_{\beta 1} - \delta^0_\beta g_{\alpha 1}\right).$$

Example: Anisotropic inflation in Maxwell theory: $L_{\rm Maxwell} = -\frac{f(\phi)^2}{4}F_{\mu\nu}F^{\mu\nu}$. Then we find $F_{\mu\nu} = \overline{A^1}G_{\mu\nu}$ and

$$M_1 = f^2\left(\overline{A^1}\right)^2 \propto a^{-2}, \qquad M_2 = M_3 = M_4 = \lambda_1 = \lambda_2 = c_0 = 0.$$

c_0: non-trivial sound speed c_s for inflaton

M_3: Parity violating interactions

λ_1: non-trivial interaction between gauge field and inflaton: $L = f(X)F_{\mu\nu}F^{\mu\nu}$.

M_2: The photon four interaction (Euler-Heisenberg model).

The Goldstone Bosons:

The Goldstone bosons are $x^\mu \to x^{\mu\prime} = x^\mu + \pi^\mu$.

As usual π^0 reprtesents the inflaton fluctuations which captures the curavture perturbations. In addition we have 3 Goldstone bosons π^i.

Upon restoring the Goldstone bosons we have

$$\delta A^i \to \delta A^{i\prime} = \delta A^i + \partial_1 \pi^i A^1 = \delta A^i + \delta X_i A^1 \,.$$

To fix the U(1) gauge, we impose the Coulomb-radiation gauge $A^0 = \partial_i A^i = 0$. This yields $\partial_i \delta X_i = 0$.

Decompose δX_i into the transverse and the longitudinal parts:

$$\delta X_i = \partial_i \delta X_L + \delta X_{Ti} \quad , \quad \partial_i \delta X_{Ti} = 0$$

Then from the condition $\partial_i \delta X_i = 0$ we obtain $\nabla^2 \delta X_L = 0$.

In conclusion we have left with the two transverse degrees δX_T .

In total, we have 3 physical Goldstone bosons: one associayted with $\delta\phi$, the other 2 associated with δA_μ transverse fluctuations.

Anisotropic Power Spectrum

After canonically normalizing the fields, we obtain three types of interactions

$$\begin{aligned}
L_1 &= a^2 \left[2\overline{M}_1 (n+2)(n-1) H^3 \right] \pi^0 \delta X_{T1} \\
L_2 &= -a \left[4\overline{\lambda}_1 H^2 n(2+n) + 2H^2 (2+n) \overline{M}_1 \right] \pi^{0\prime} \delta X_{T1}, \\
L_3 &= -4\overline{\lambda}_1 H (2+n) \pi^{0\prime} \delta X'_{T1}.
\end{aligned}$$

The anisotropic power specrum is:

$$\delta P_{ji} = -\int_{-\infty}^{\tau_e} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \Big\langle \Big[L_i(\tau_2), \Big[L_j(\tau_1), \pi^{0*}(\tau_e) \pi^{0*}(\tau_e) \Big] \Big] \Big\rangle,$$

We define the anisotrpy as: $P_{\mathcal{R}}(\mathbf{k}) = P_{\mathcal{R}}^{(0)} \left(1 + g_* (\widehat{\mathbf{n}} \cdot \widehat{\mathbf{k}})^2 \right)$

Case $M_2 = M_3 = M_4 = 0$:

$$g_* = 72 \frac{\overline{M}_1 c_s^5}{\epsilon M_P^2} \left(1 + \frac{6\overline{\lambda}_1}{\overline{M}_1} \right)^2 N^2 \,.$$

Example: $c_s = 1, \ \lambda_1 = 0$

$$g_* = 24 I \, N^2 \,, \qquad I = \frac{c-1}{c} \qquad \text{(Maxwell theory)}\,.$$

Case $M_2, M_4 \neq 0$:

The coupling M_2 does not appear in interactions. However, it changes the wavefunction of δX_T. It affects one of the linear polarizations, $X_T^{(1)c}$:

$$S_2^{X_T^{(1)}} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\delta\dot{X}_T^{(1)c})^2 - \frac{1}{2a^2}\Big(1+\frac{8\overline{M}_2H^2(2+n)^2\sin^2\theta}{\overline{M}_1 - 8\overline{M}_2H^2(2+n)^2}\Big)(\delta X_{T,j}^{(1)c})^2\Big]$$

This corresponds to a non-trivial speed of light for photons:

$$c_v^2 \simeq 1 + \frac{8\overline{M}_2H^2(2+n)^2}{\overline{M}_1}\sin^2\theta.$$

This is like the birefringence effect in optics.

$$\frac{\delta P}{P_{\pi^0}} = \frac{72\overline{M}_1}{\epsilon M_P^2}\frac{(1+\frac{6\overline{\lambda}_1}{\overline{M}_1})^2c_s^5}{c_v^3(1-\frac{c_v^2-1}{\sin^2\theta})}N^2\sin^2\theta$$

In simple case if we further assume $c_s = 1, \lambda_1 = 0$

$$\frac{\delta P}{P_{\pi^0}} = 24IN^2\sin^2\theta\left[1 - \frac{36H^2\overline{M}_2}{\overline{M}_1}(1-3\cos^2\theta)\right].$$

We have both $\ell = 2$ and $\ell = 4$ anisotropies.

Conclusion

- Primordial asymmetries and anisotropies are interesting both theoretically and observationally. There are evidences for hemispherical asymmetry on CMB maps.
- δN approach can be extended to anisotropic backgrounds. One can calculate the power spectrum, bispectrum, trispectrum etc which are in exact agreements with the results obtained from alternative in-in formalism.
- EFT provides a good platform to study various aspects of primordial statistical anisotropies model-independently. Our EFT approaches reproduces the know results. We also found new types of interactions.
- It will be very interesting to perform the bispectrum (non-Gaussianity) analysis in our EFT approach.

“Results from Planck 2015”

by Andrei Frolov

[JGRG25(2015)8a3]

Results from Planck 2015

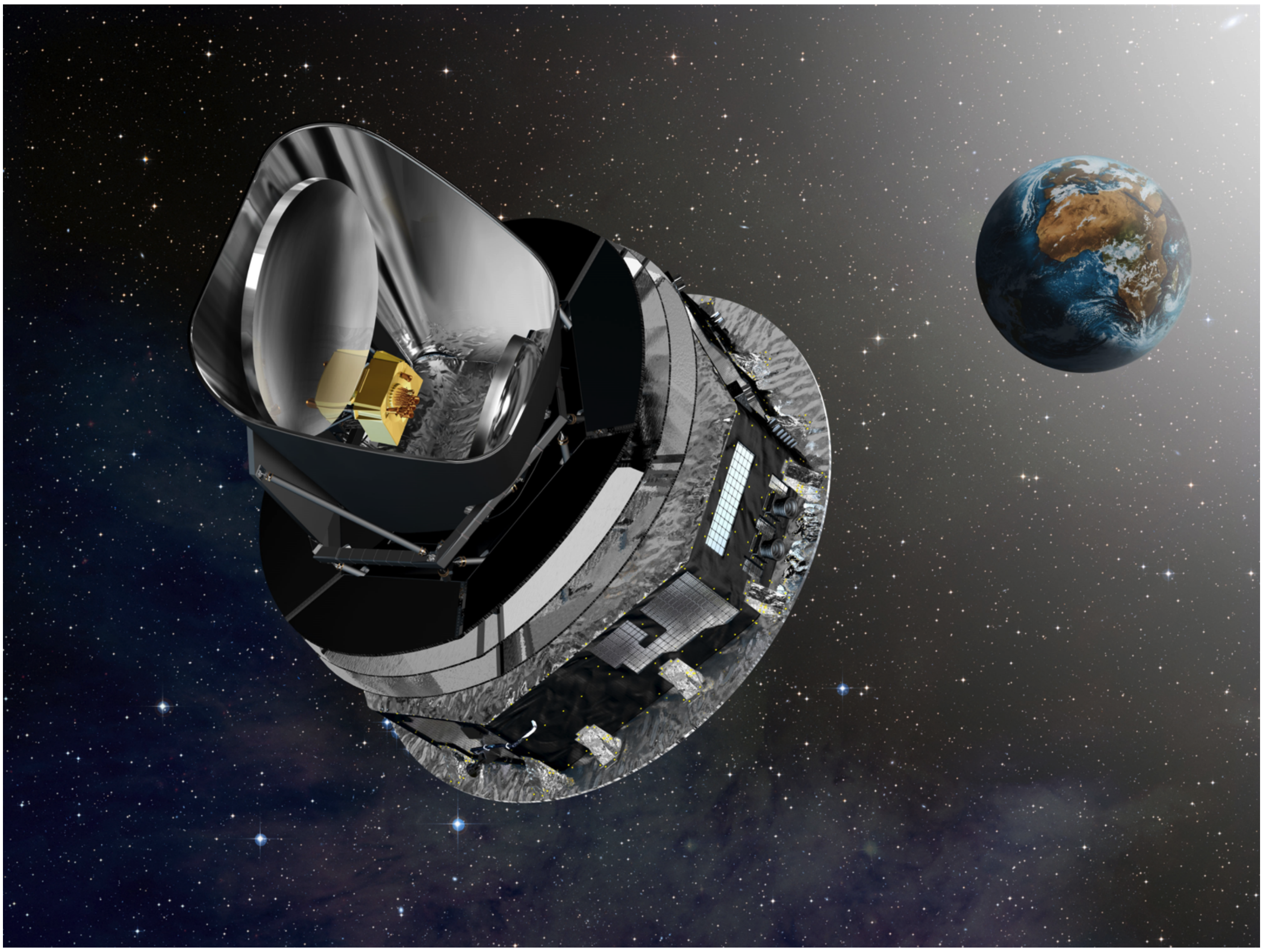
Andrei Frolov on behalf of Planck Collaboration

25th Workshop on General Relativity and Gravitation in Japan
Yukawa Institute for Theoretical Physics, Kyoto, Japan

11 December 2015

Outline

1. **Instrument and Mission Overview**
2. **Foregrounds and Component Separation**
3. **CMB Maps and Spectra**
4. **B-Modes and Dust**
5. **Implications for Inflation**
6. **Conclusions**



Planck before Launch

planck

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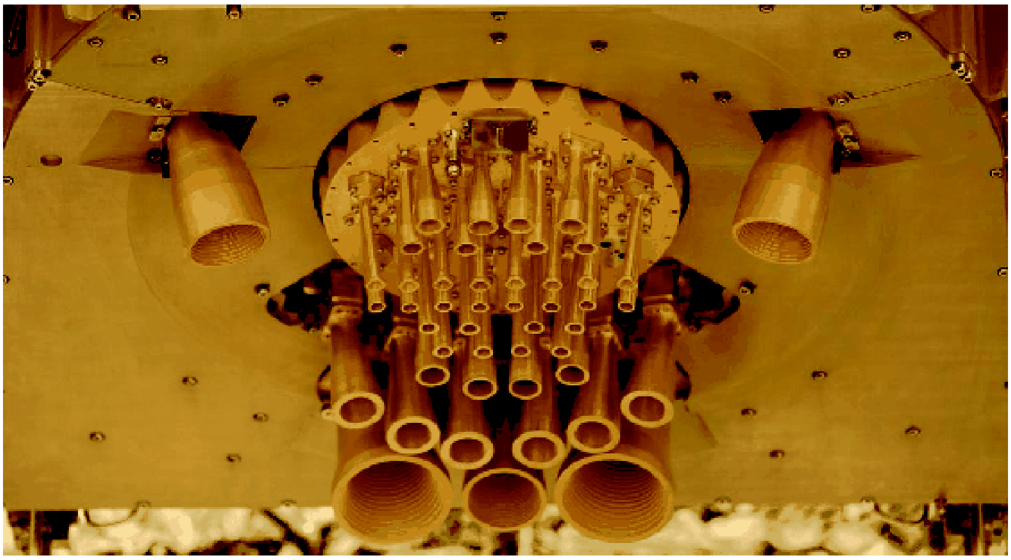
Planck Focal Plane

planck



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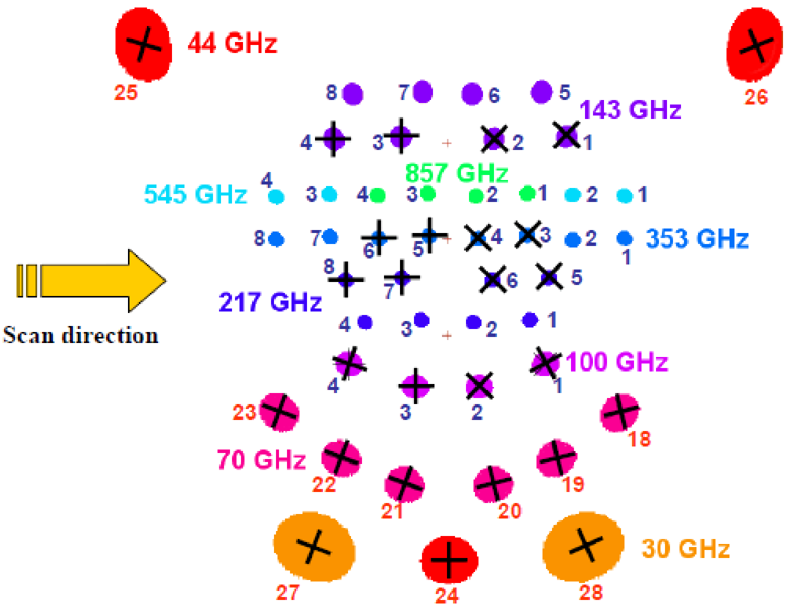
Planck Focal Plane Schematics

planck



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Planck 2015: What's New?

- **More data:** 48/29 months of LFI/HFI observations, enabling further checks
- **Improved data processing:** systematics removal, calibration, beam reconstruction
- **Improved foreground model:** larger sky-fraction used for analysis
- **More robust to systematics:** based on half-mission cross power spectra
- **The 2015 analysis includes polarization**

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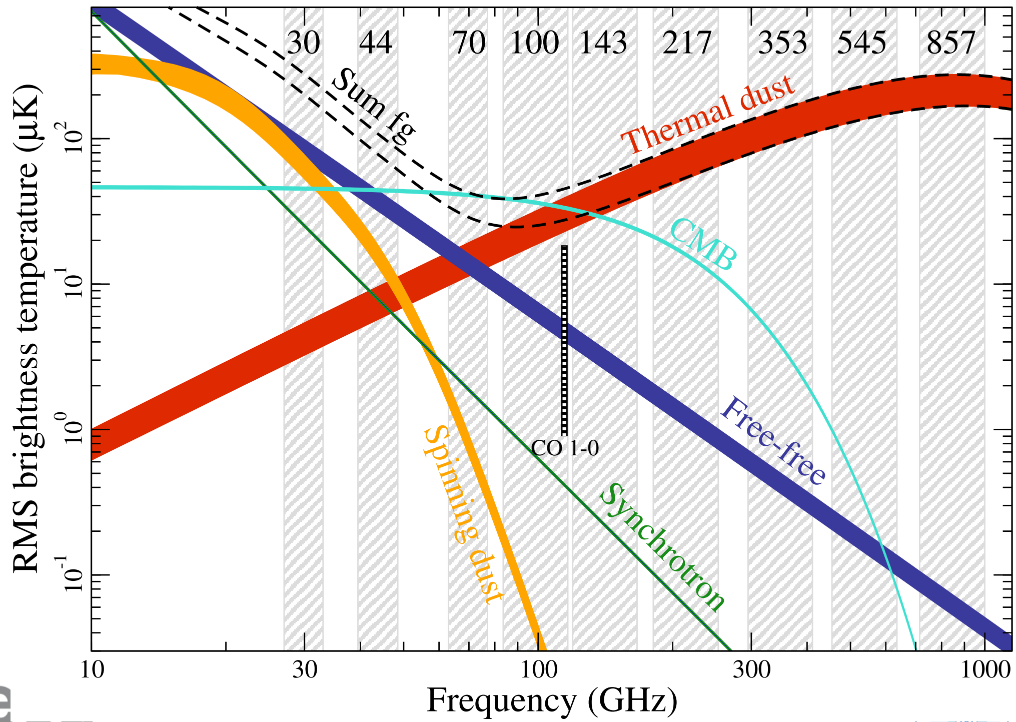
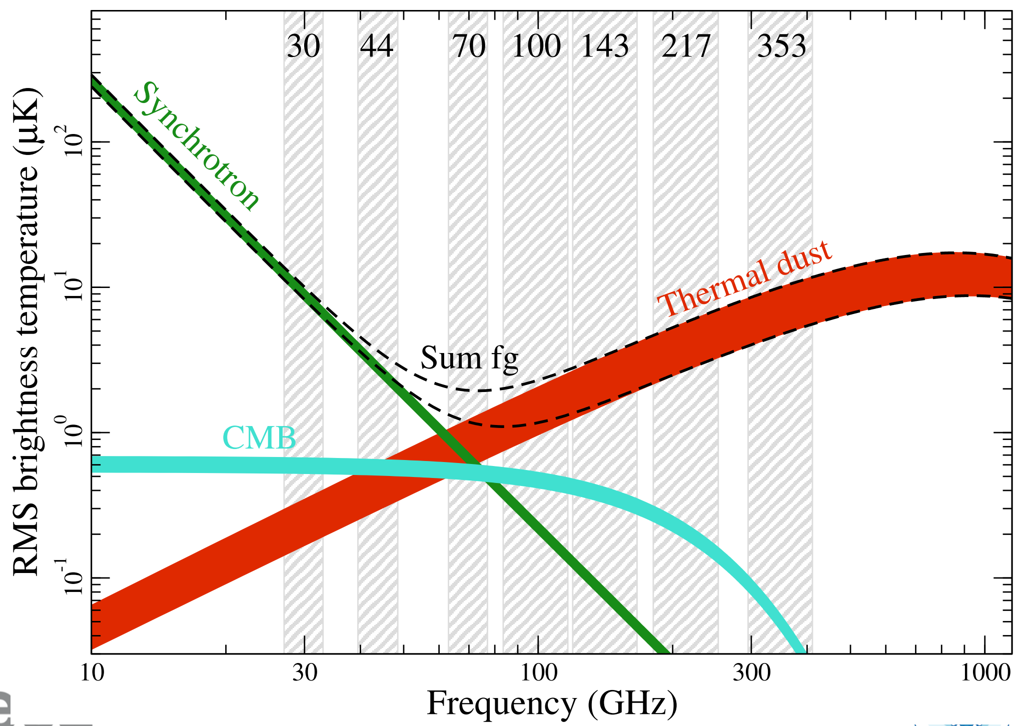
CMB vs. Astrophysical Foregrounds

○ Intensity ○ Polarization ○ Atmospheric Transmission ○



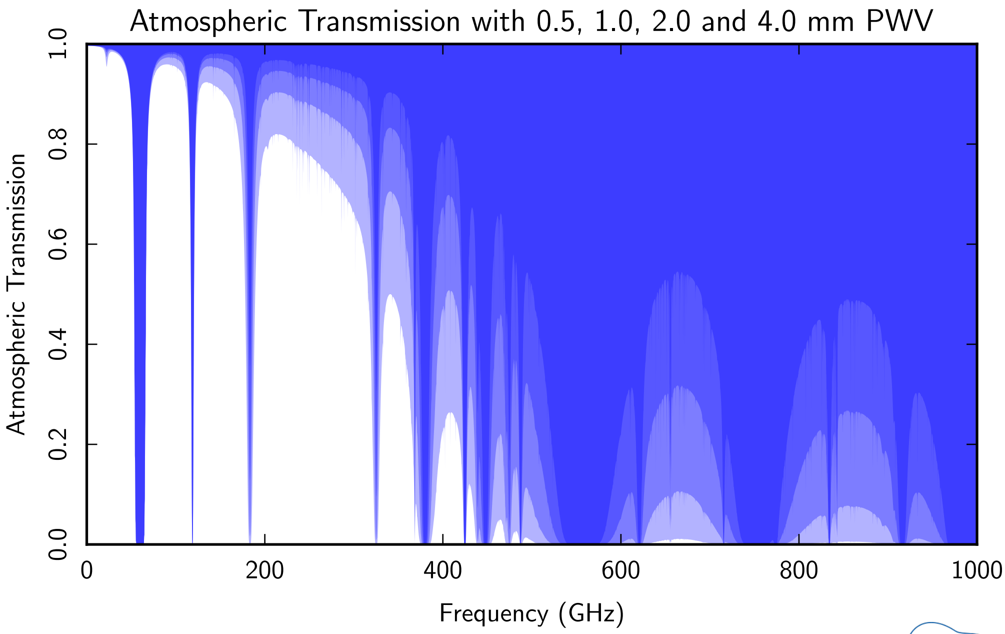
CMB vs. Astrophysical Foregrounds

○ Intensity ○ Polarization ○ Atmospheric Transmission ○



CMB vs. Astrophysical Foregrounds

◦ Intensity ◦ Polarization ◦ Atmospheric Transmission ◦

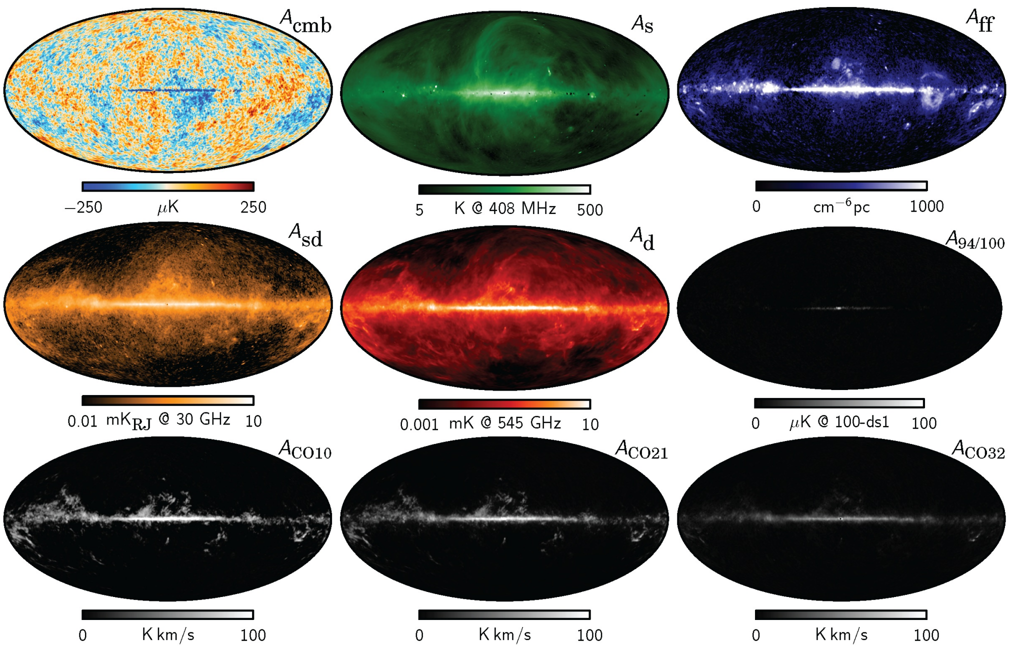


Component Separation Methods

Like in 2013, three CMB cleaning methods (SMICA, SEVEM, NILC) & 1 explicit Component Separation method (Commander).

Temperature Component Maps

planck



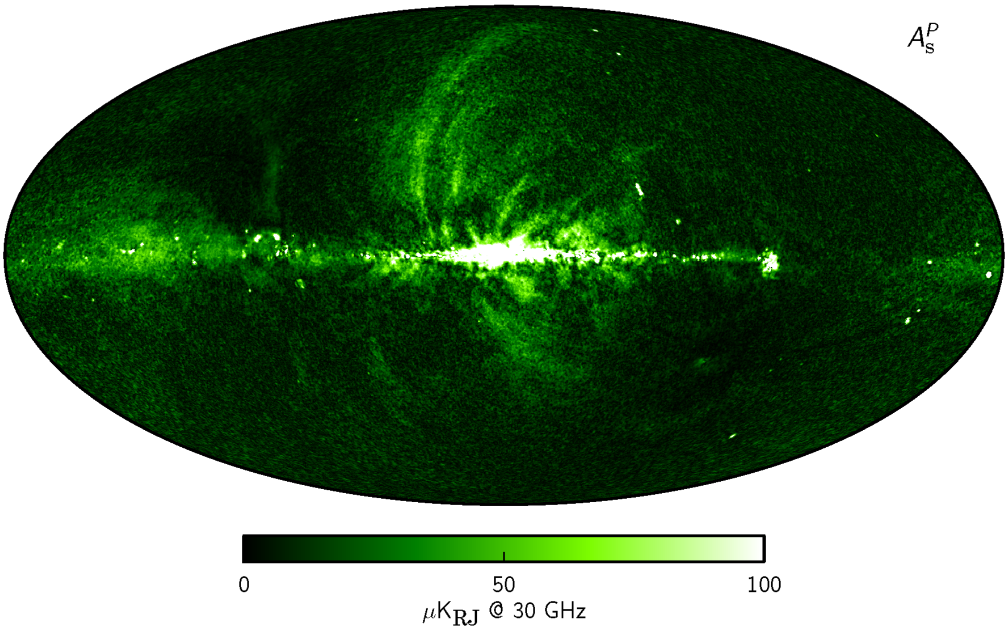
Polarization Component Maps

planck

- Two main foregrounds, synchrotron emission and thermal dust
- Amplitude of CMB polarization is less than foregrounds
- Dust emission is highly polarized (polarization fraction is up to 20%)

Synchrotron Polarization Amplitude

planck



$P = \sqrt{Q^2 + U^2}$, at 30 GHz, smoothed to $40'$

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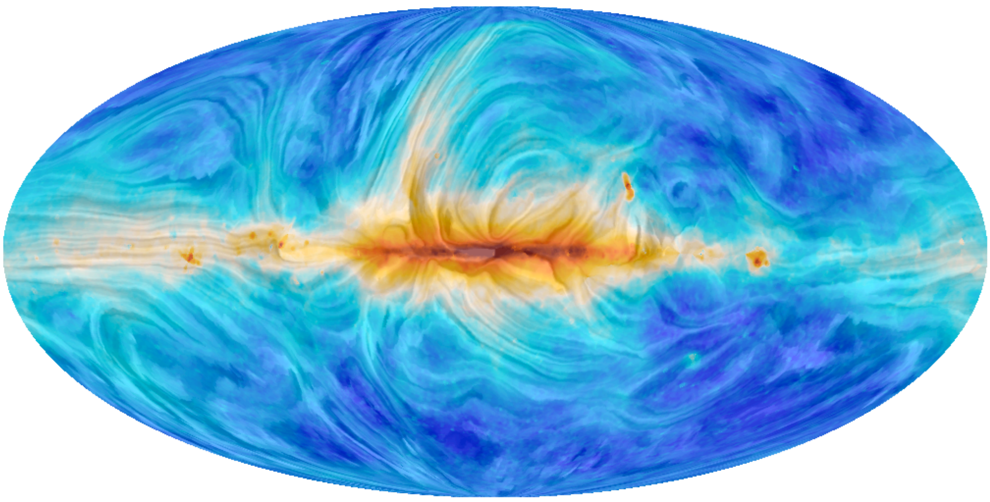
Magnetic Field and Total Intensity

planck



The colours represent intensity. The "drapery" pattern indicates the orientation of magnetic field projected on the plane of the sky, orthogonal to the observed polarization.

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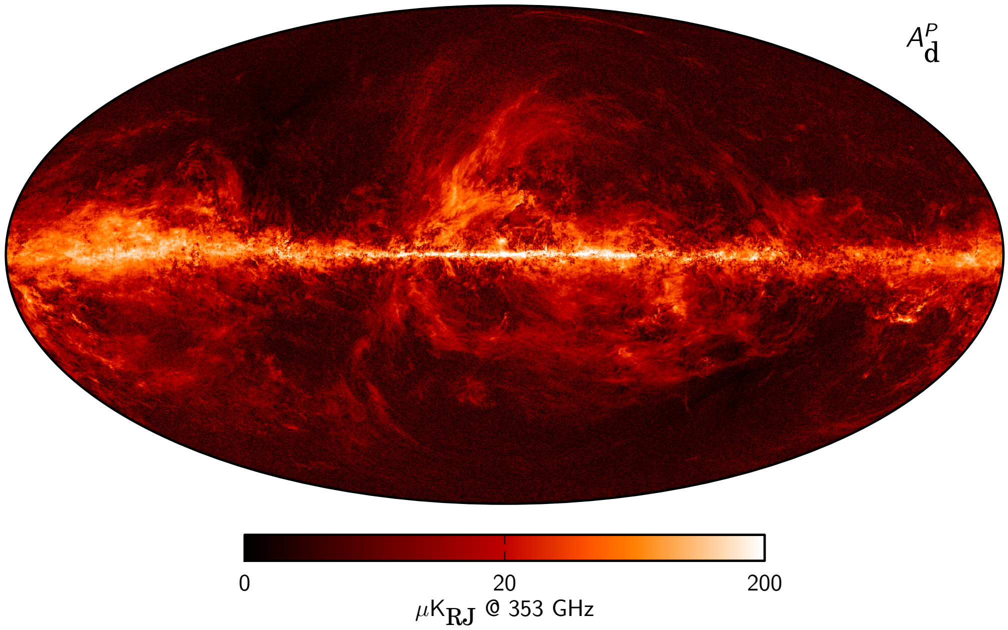
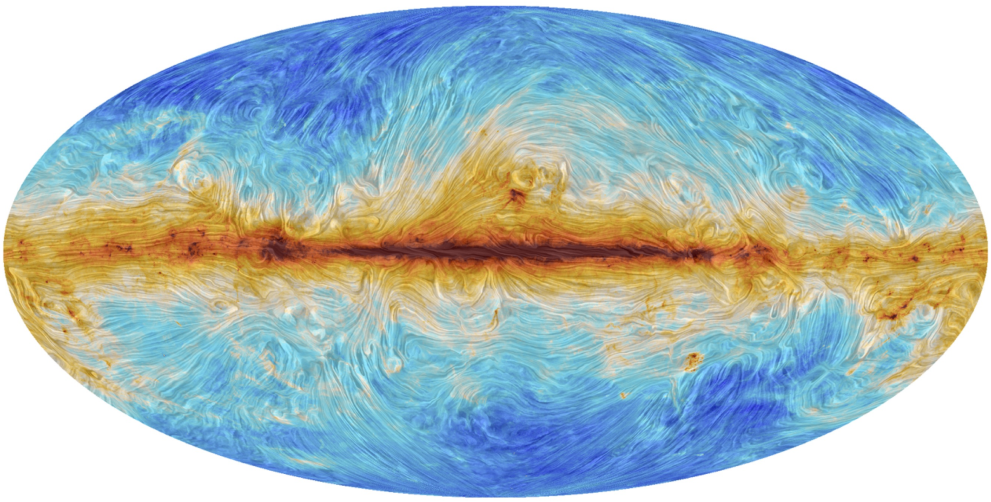
Dust Polarization Amplitude



$P = \sqrt{Q^2 + U^2}$, at 353 GHz, smoothed to $10'$

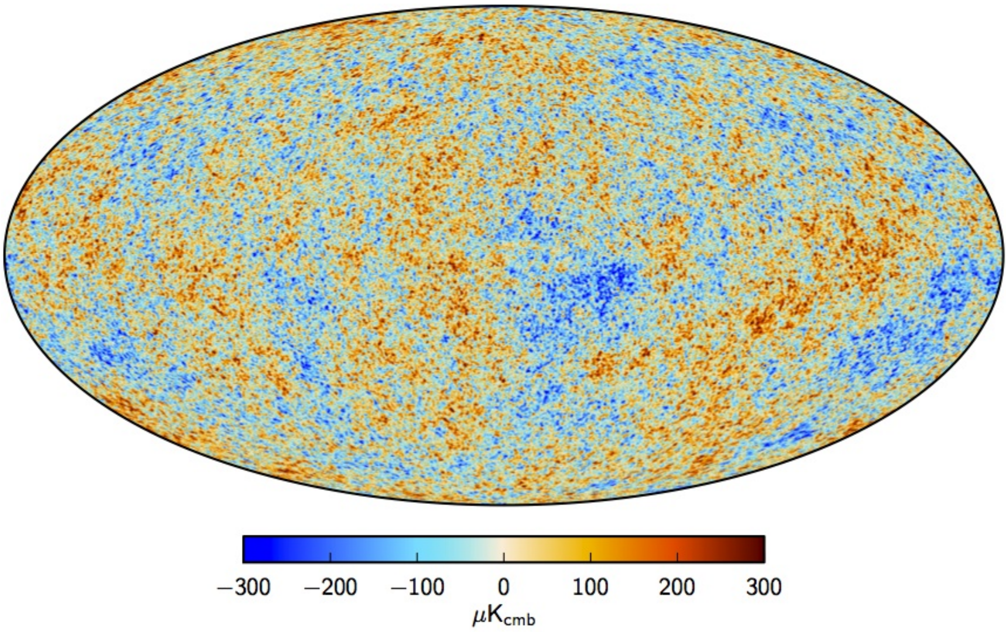
Magnetic Field and Total Intensity



The colours represent intensity. The “drapery” pattern indicates the orientation of magnetic field projected on the plane of the sky, orthogonal to the observed polarization.

CMB Intensity Map

planck



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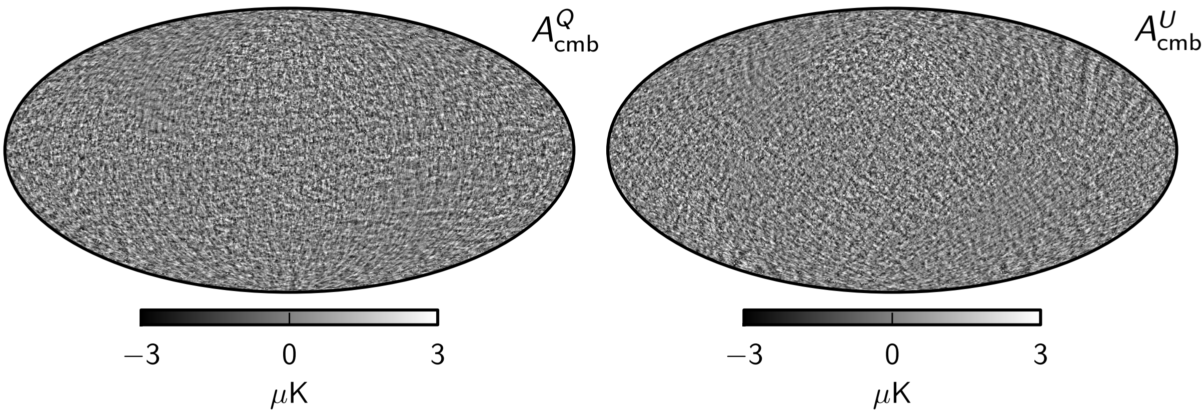
CMB Polarization Maps

planck

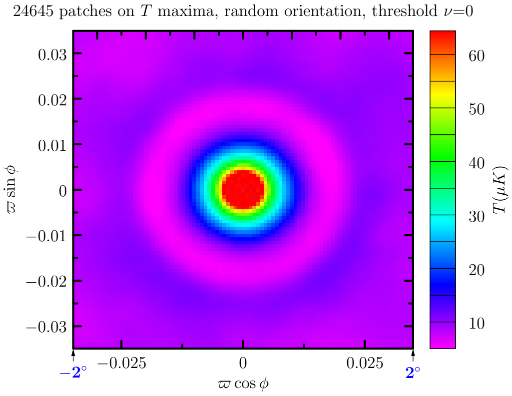


- Smoothed to 1 degree resolution
- High-pass filtered with l=20-40 cosine filter
- Galactic plane replaced with constrained Gaussian realization

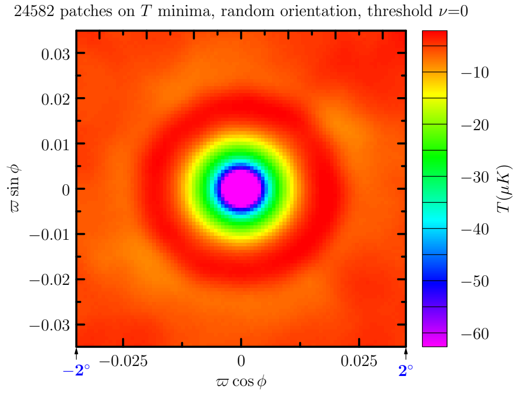
esa

Planck 2015: Stacking T & Q_r

T on hot spots



T on cold spots

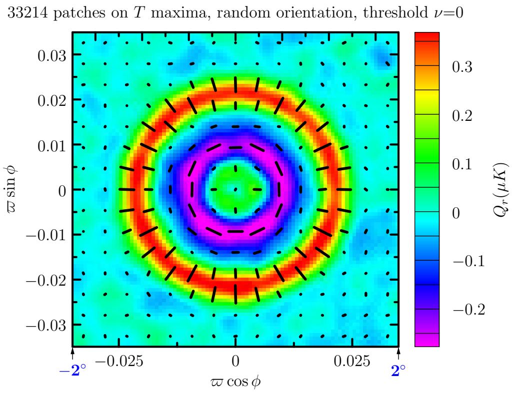


resolution: FWHM 15 arcmin

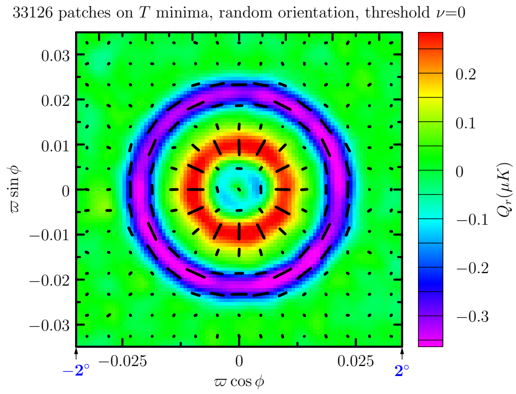
Peaks are selected above a threshold $|T_{\rm peak}| > \nu\sqrt{\langle T^2\rangle}$ ($\nu = 0$ here).

Planck 2015: Stacking T & Q_r

Q_r on hot spots



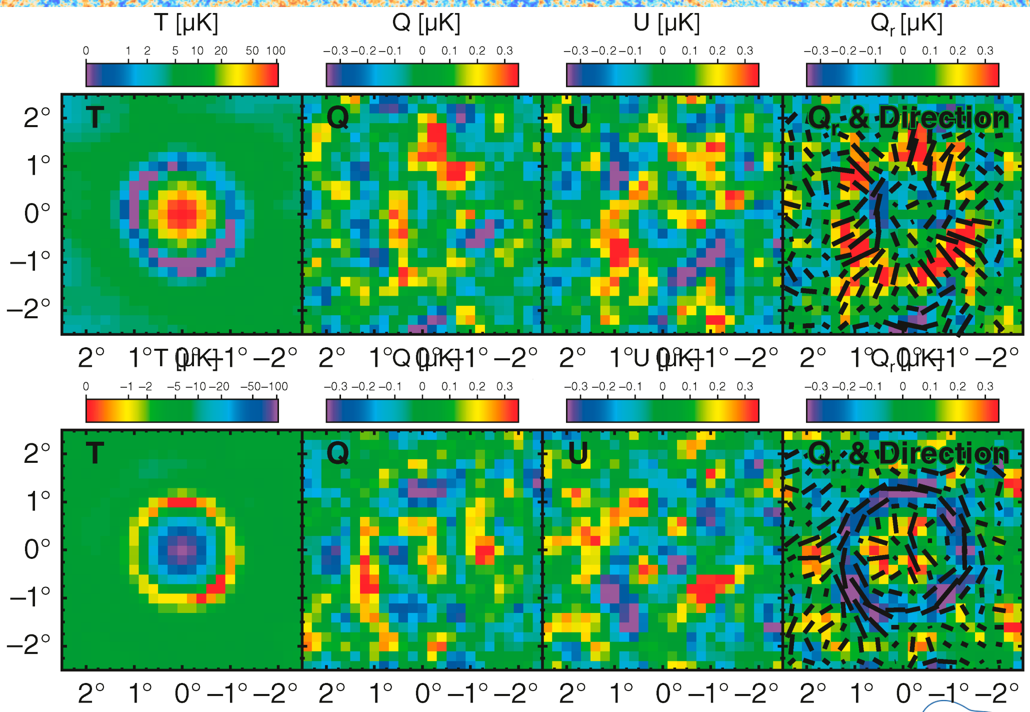
Q_r on cold spots



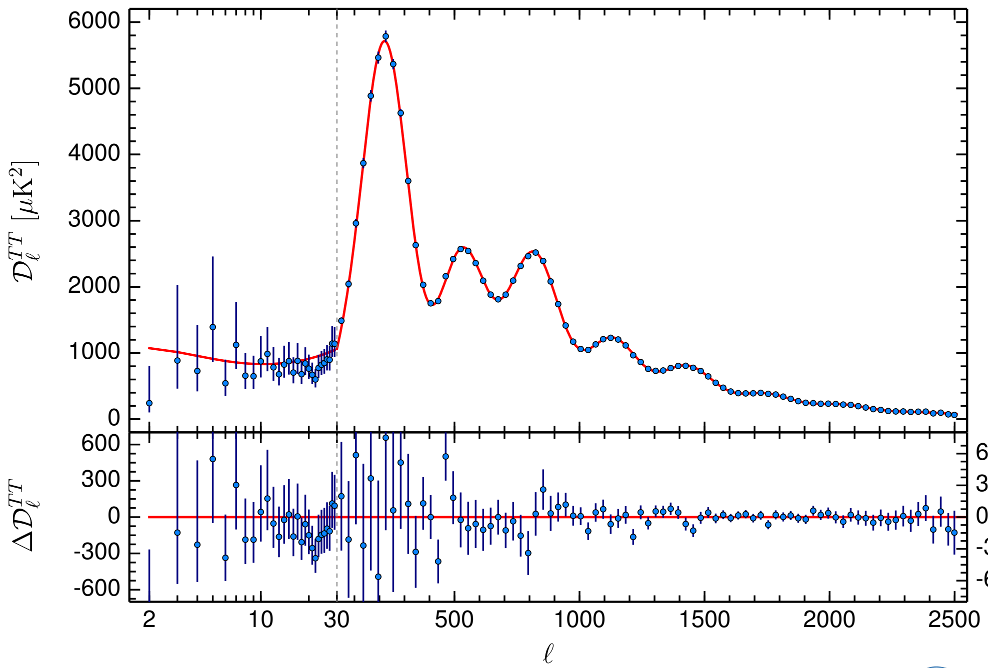
resolution: FWHM 15 arcmin

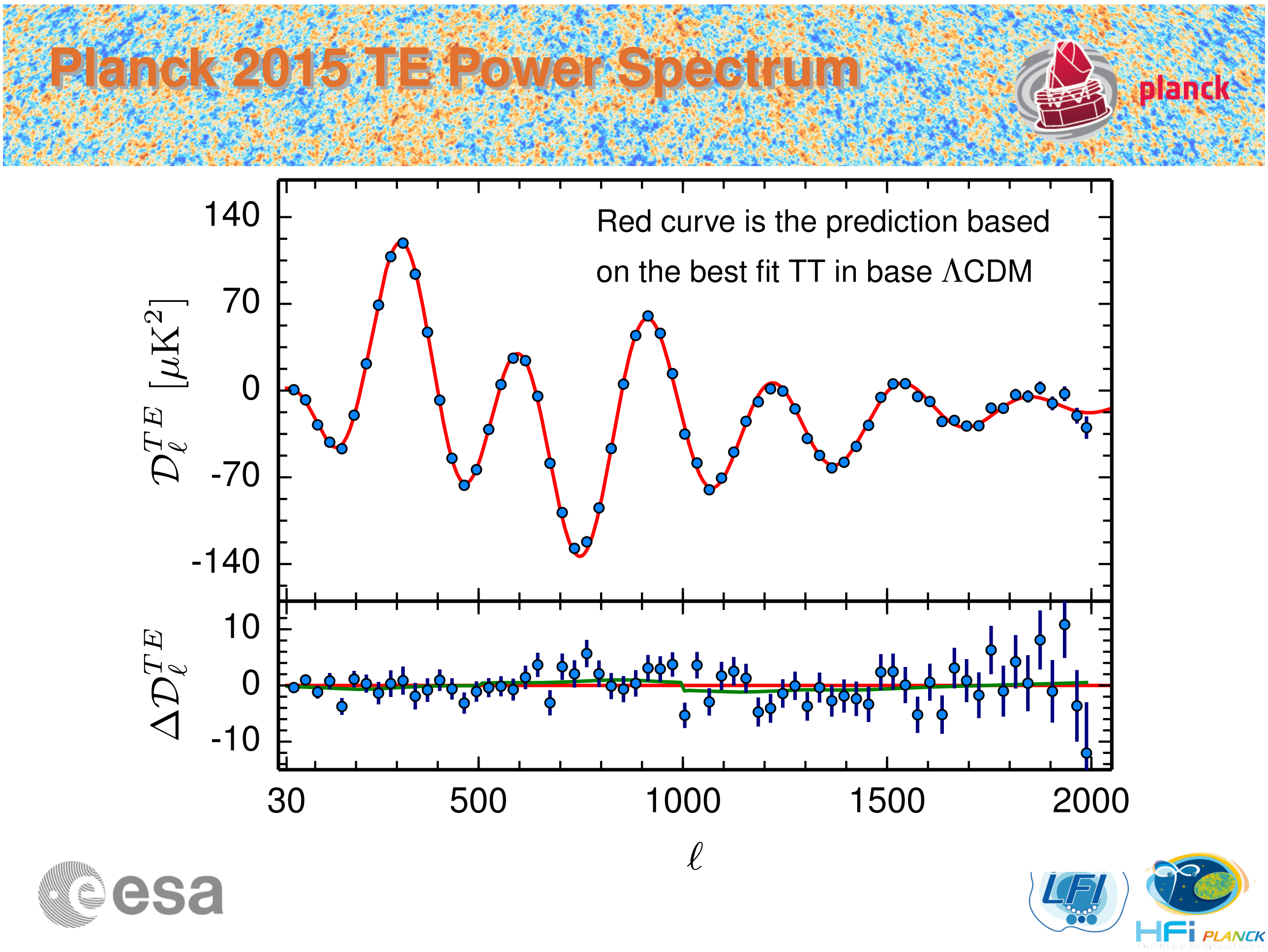
Peaks are selected above a threshold $|T_{\rm peak}| > \nu\sqrt{\langle T^2\rangle}$ ($\nu = 0$ here).

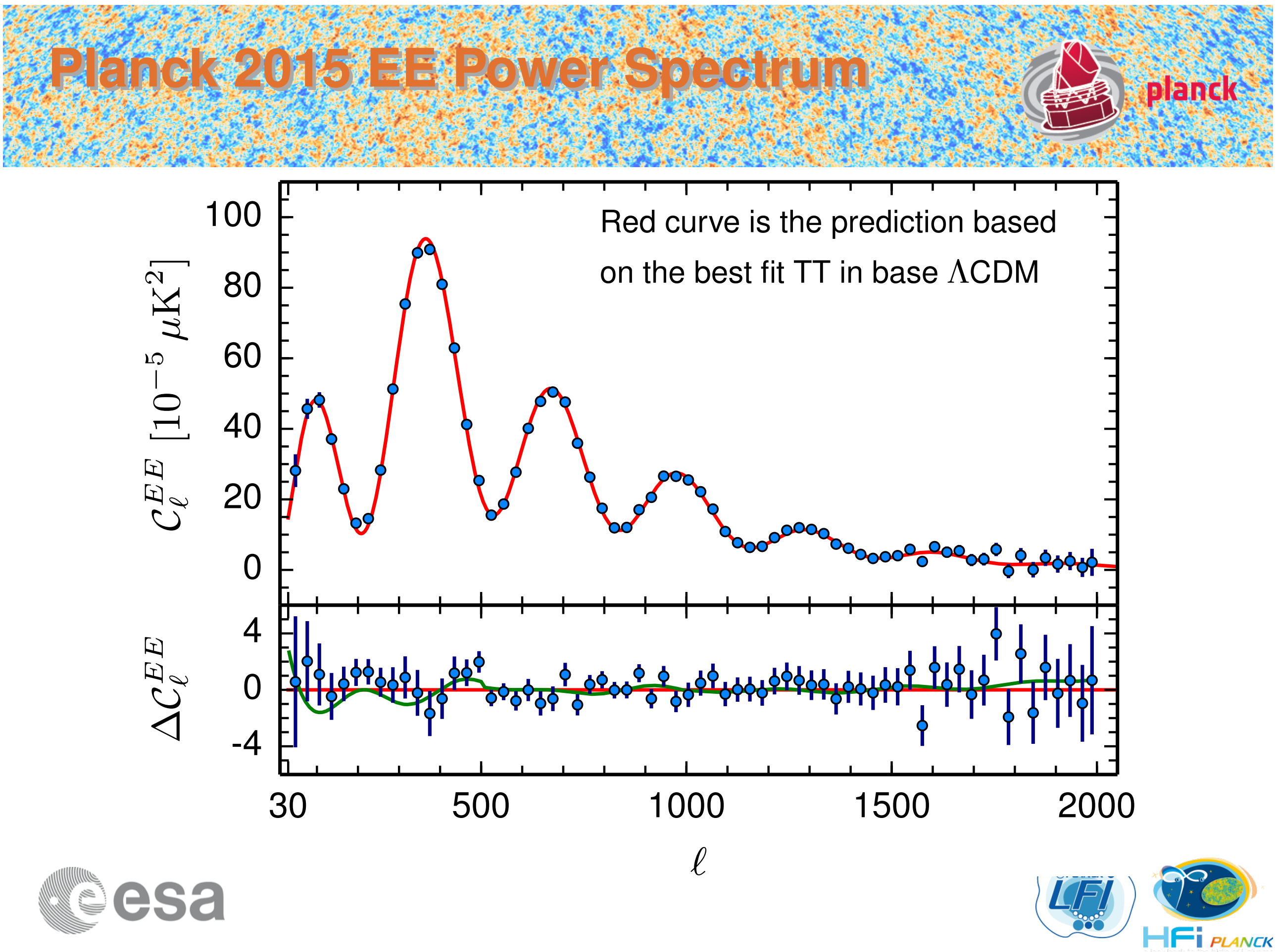
WMAP-7: Stacking T & Q_r



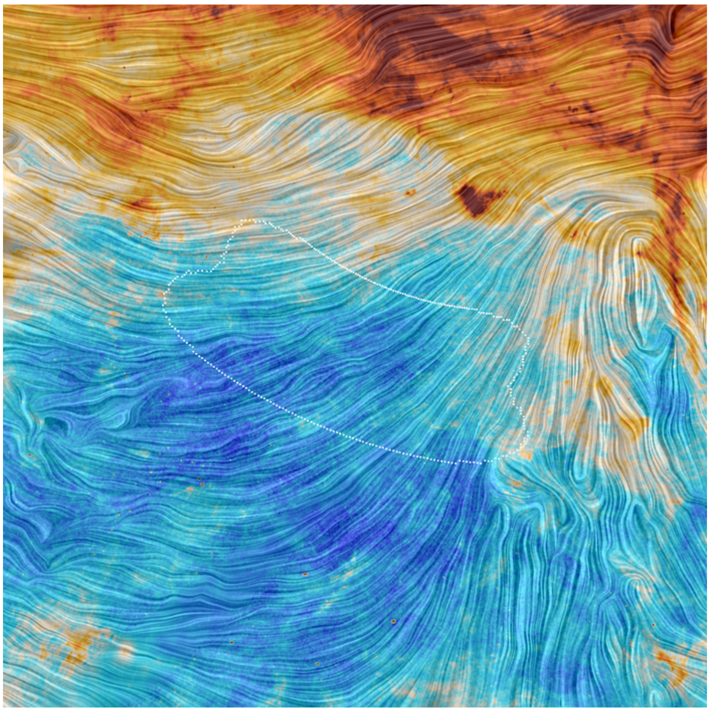
Planck 2015 TT Power Spectrum



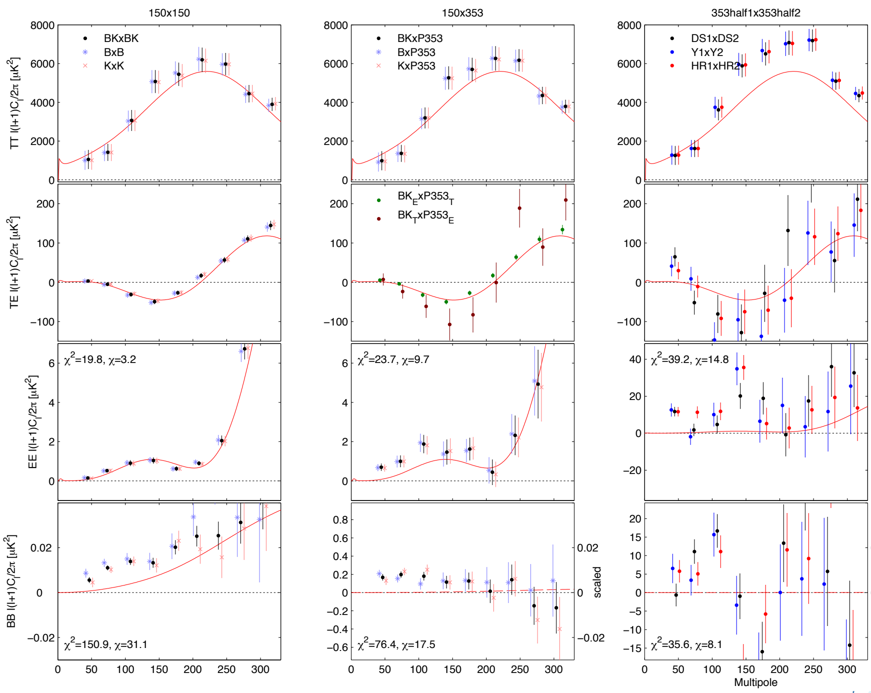

Planck 2015 TE Power Spectrum
planck
Red curve is the prediction based
on the best fit TT in base ΛCDM
$\mathcal{D}_\ell^{TE}\ [\mu K^2]$
140
70
0
-70
-140
$\Delta\mathcal{D}_\ell^{TE}$
10
0
-10
30
500
1000
1500
2000
ℓ
esa
LFI
HFI PLANCK


Planck 2015 EE Power Spectrum
planck
Red curve is the prediction based
on the best fit TT in base ΛCDM
$\mathcal{C}_\ell^{EE}\ [10^{-5}\ \mu K^2]$
100
80
60
40
20
0
$\Delta\mathcal{C}_\ell^{EE}$
4
0
-4
30
500
1000
1500
2000
ℓ
esa
LFI
HFI PLANCK

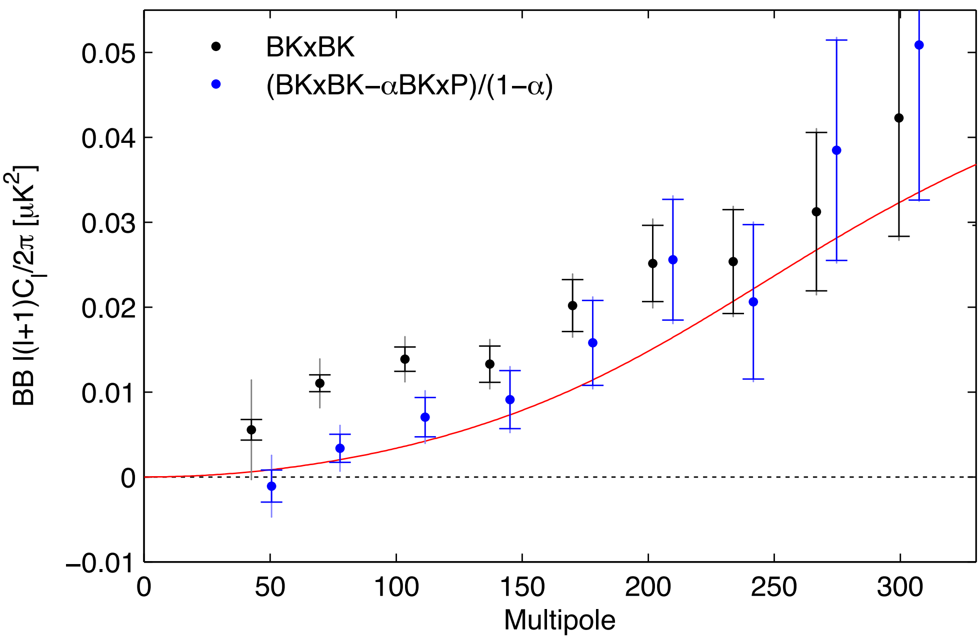
Planck View of BICEP2 Field



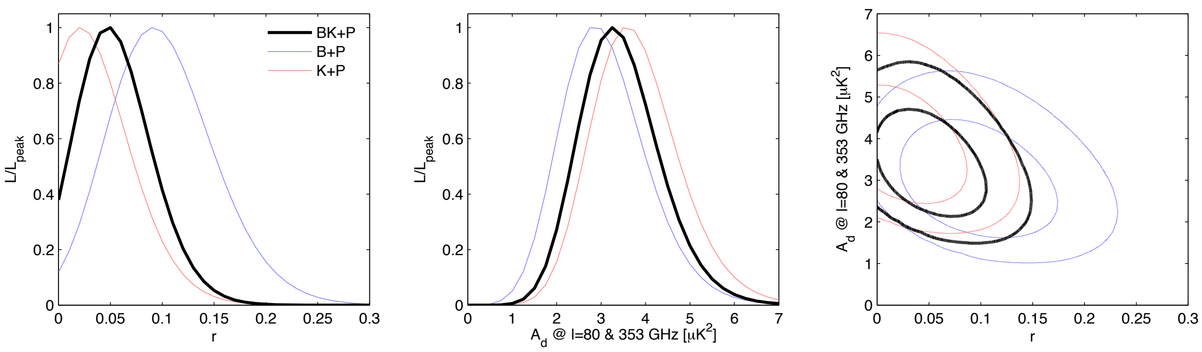
Planck x BICEP2/Keck Cross-Spectra



Cleaning Up Dust with Planck



Planck x BICEP2 Joint Likelihood



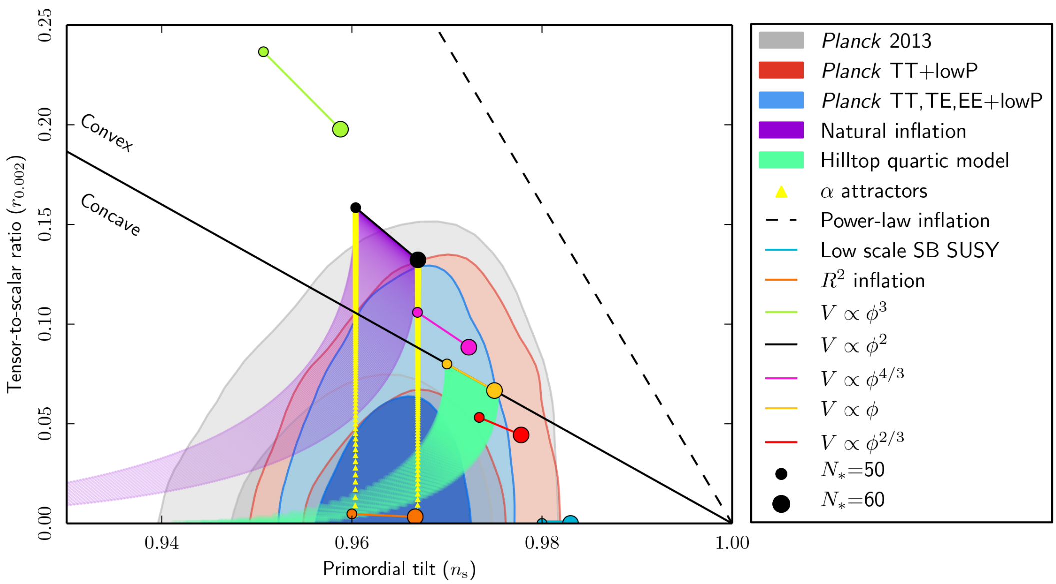
Likelihood results from a basic lensed-ΛCDM+r+dust model, fitting BB auto- and cross-spectra taken between maps at 150 (BICEP2/Keck) and 217 and 353 GHz (Planck).

A Gaussian prior is placed on the dust frequency spectrum parameter $\beta_d = 1.59 \pm 0.11$.

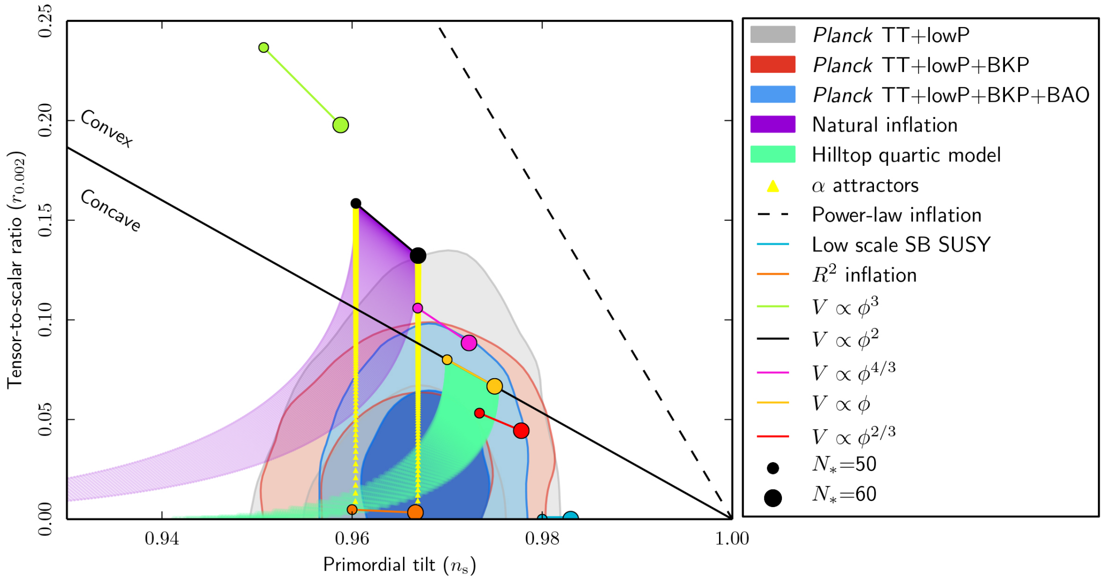
Datasets & Shorthands

- **Planck TT:** Planck TT for $2 < \ell < 2500$.
- **lowP:** low-ℓ Planck polarization, $2 < \ell < 30$. (For 2013 results, this will indicate low-ℓ WMAP polarization, WP)
- **Planck TE, EE:** Planck TE & EE at high-ℓ, $30 < \ell < 2000$.
- **Lensing:** Planck lensing potential at $40 < \ell < 400$ from 4-point correlation function (i.e. conservative)
- **External datasets:**
  - BAO (6dFGS, SDSS-MGS, BOSS-LOWZ, CMASS DR11)
  - JLA: Type Ia Supernovae (SNLS+SDSS+low z Sne)
  - H0: Hubble constant (Reanalysis by Efstathiou 2014 of Riess et al. 2011)
  - BKP: BICEP2/Keck x Planck joint likelihood
- Whenever not specified, we assume $N_{\text{eff}} = 3.046$, $\sum m_\nu = 0.06$eV (1 massive, two massless).

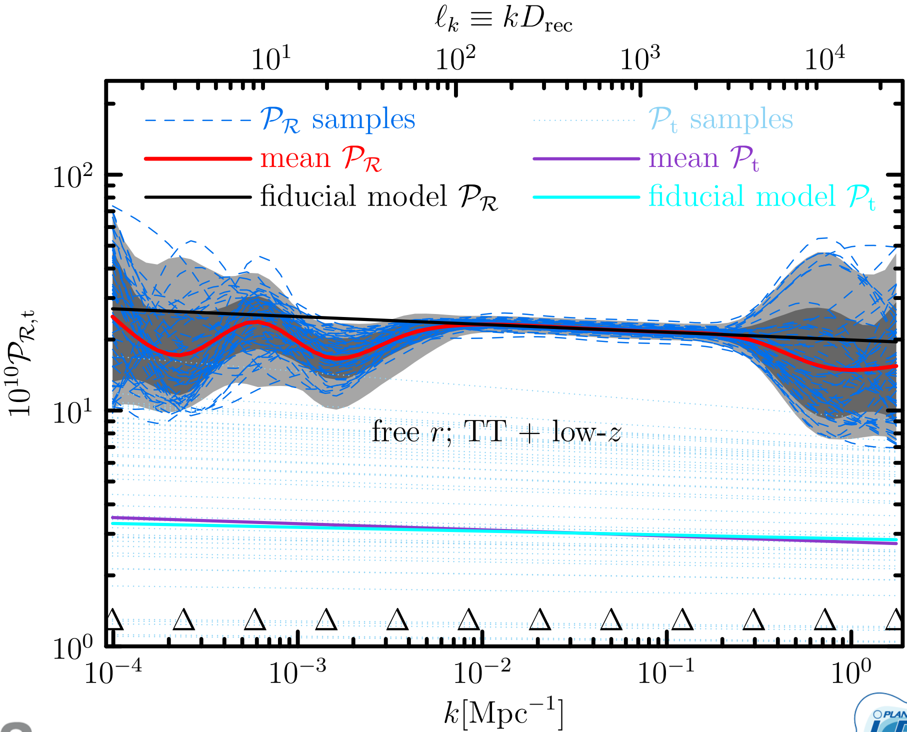
Planck 2015 Constraints on Inflation



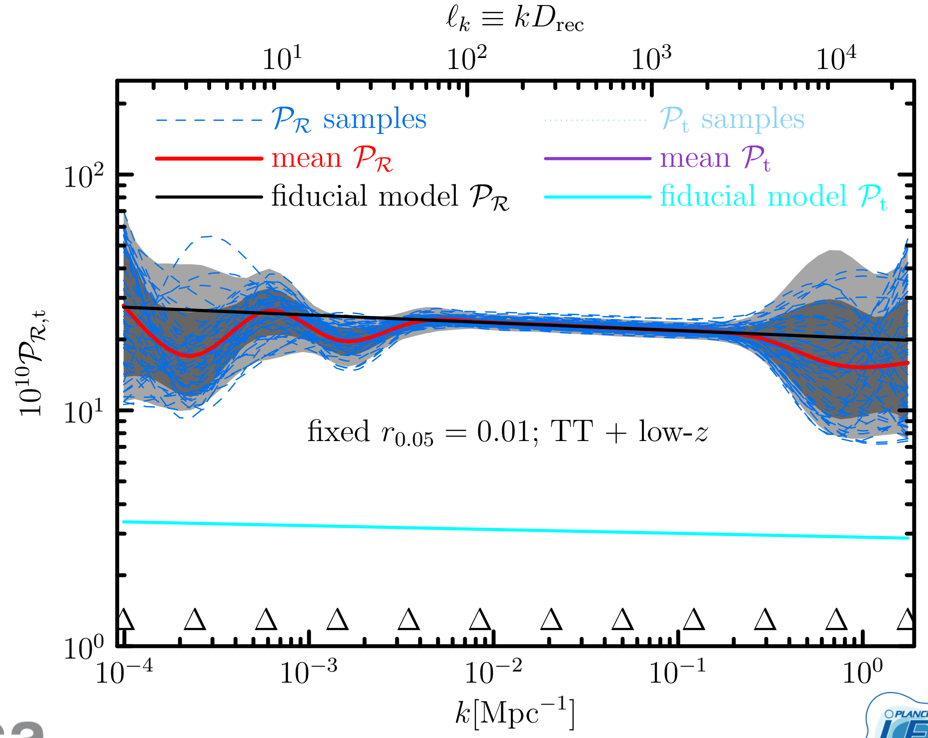
Planck + BICEP2 Constraints



Primordial Spectrum Reconstruction



Primordial Spectrum Reconstruction



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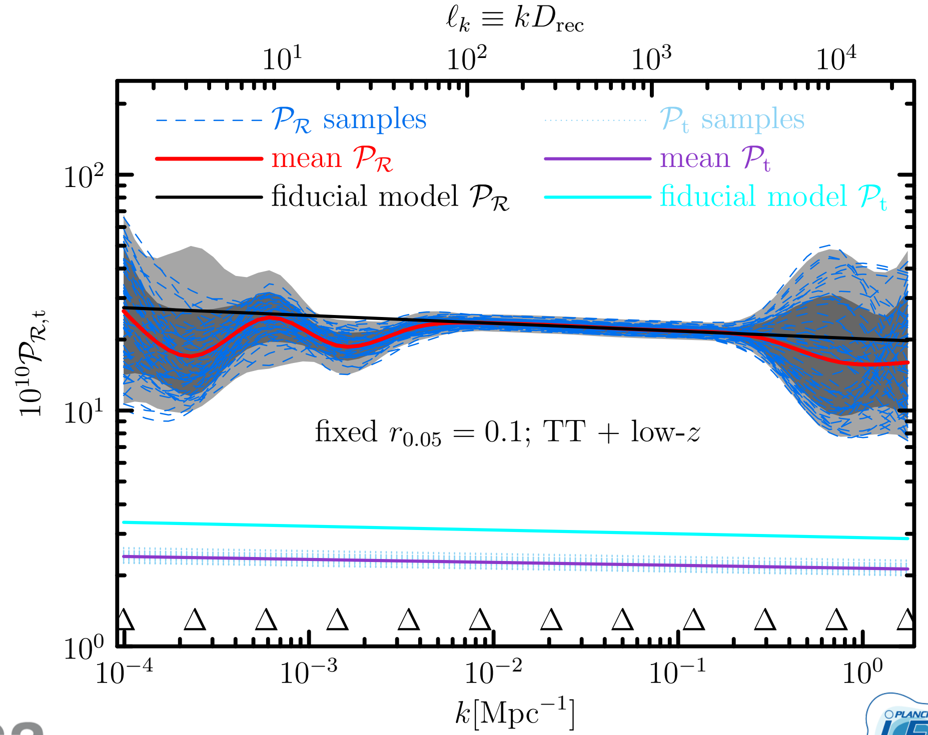
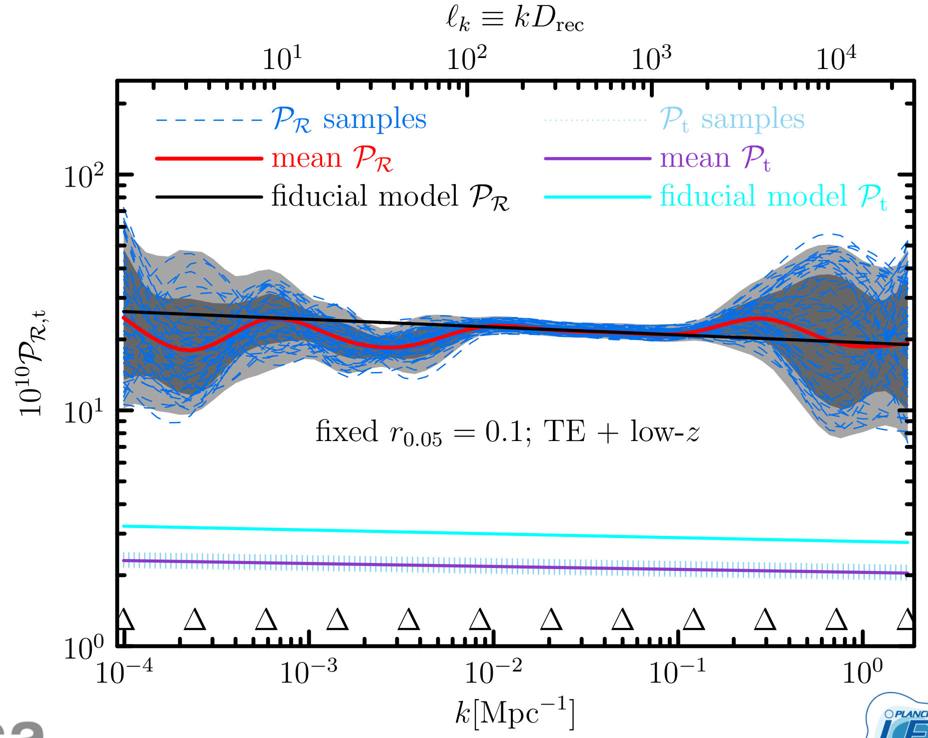


Primordial Spectrum Reconstruction



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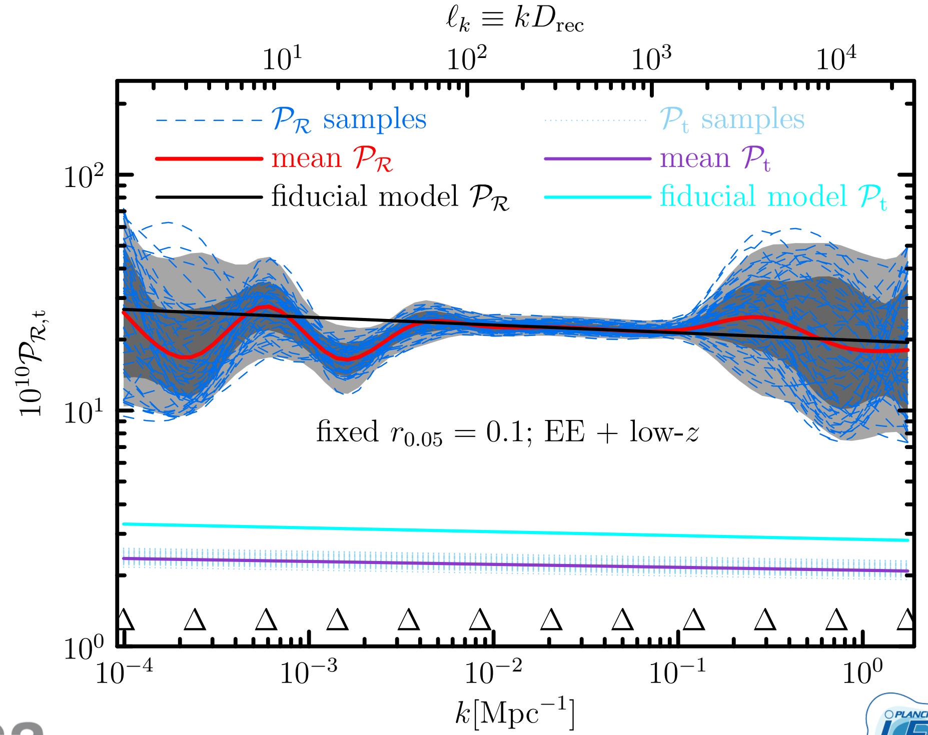
Primordial Spectrum Reconstruction



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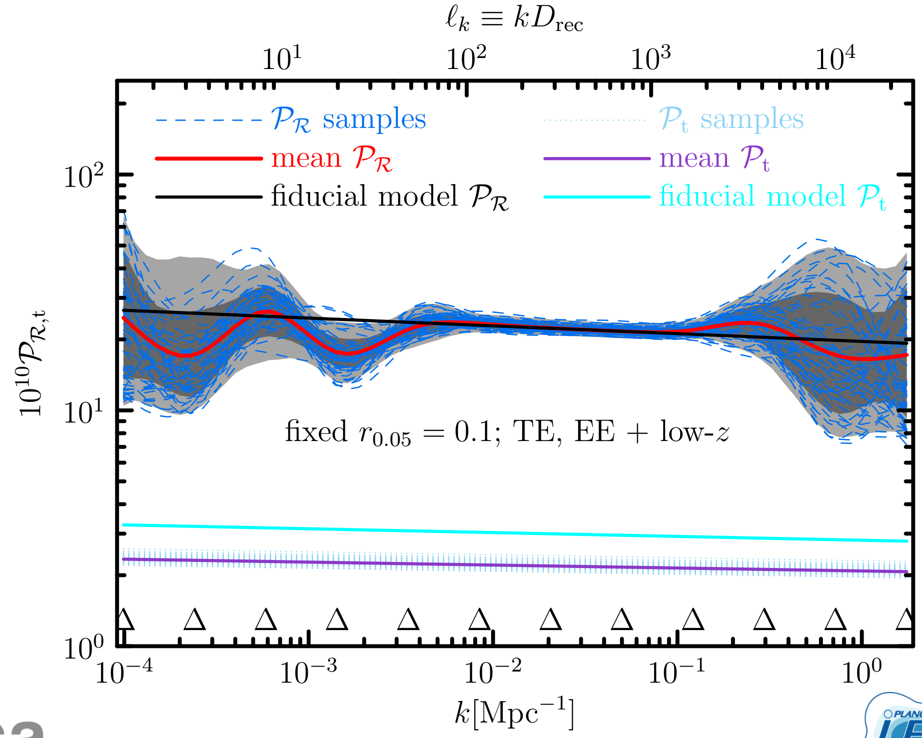
Primordial Spectrum Reconstruction

planck



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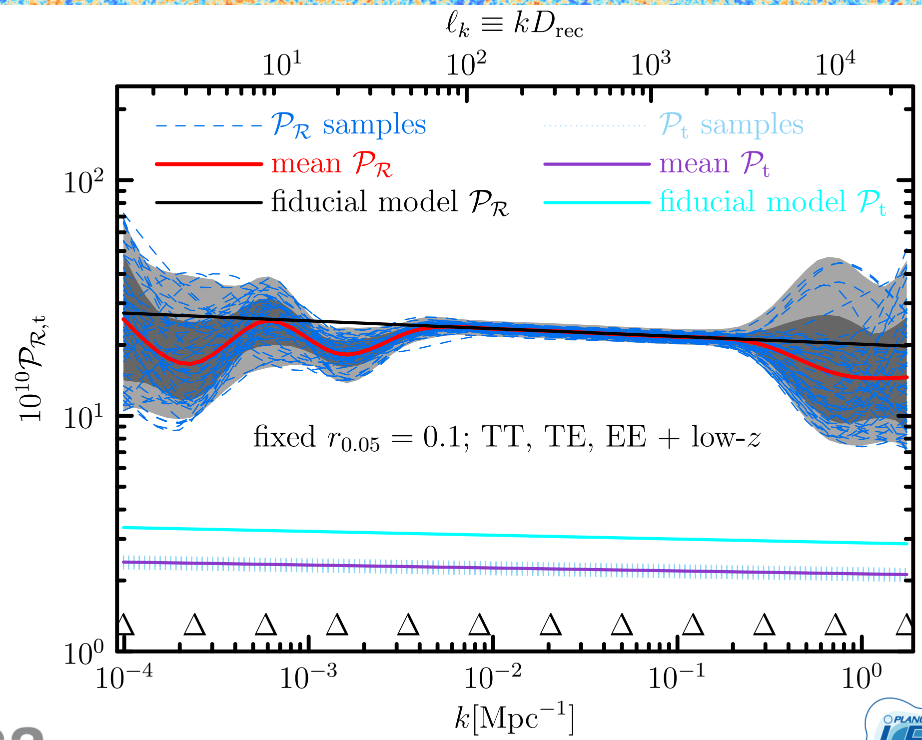
LFI PLANCK

HFI PLANCK

Primordial Spectrum Reconstruction

planck



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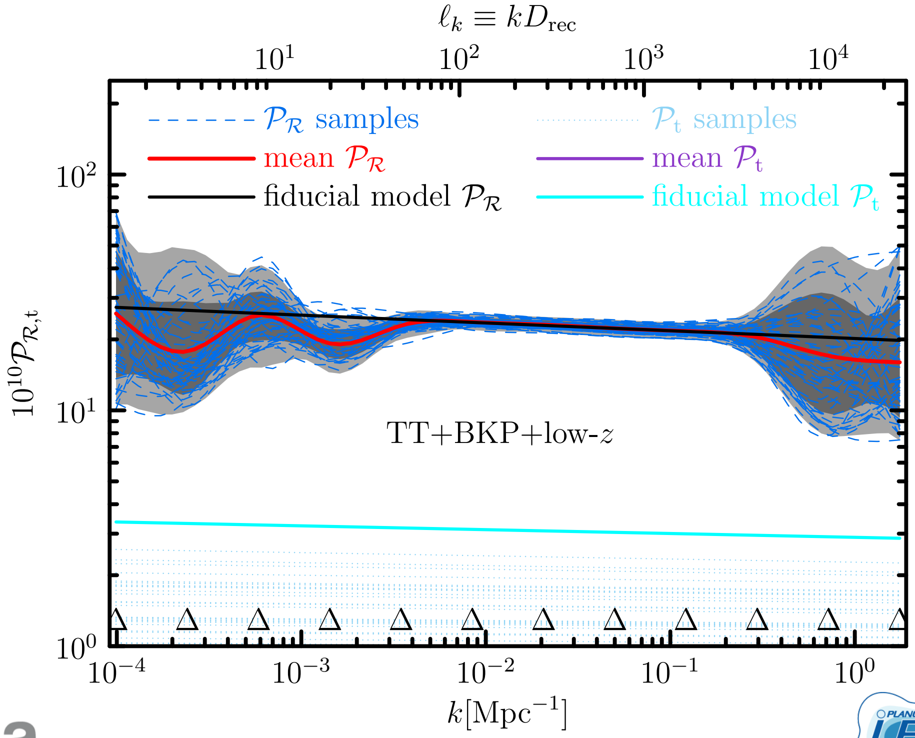
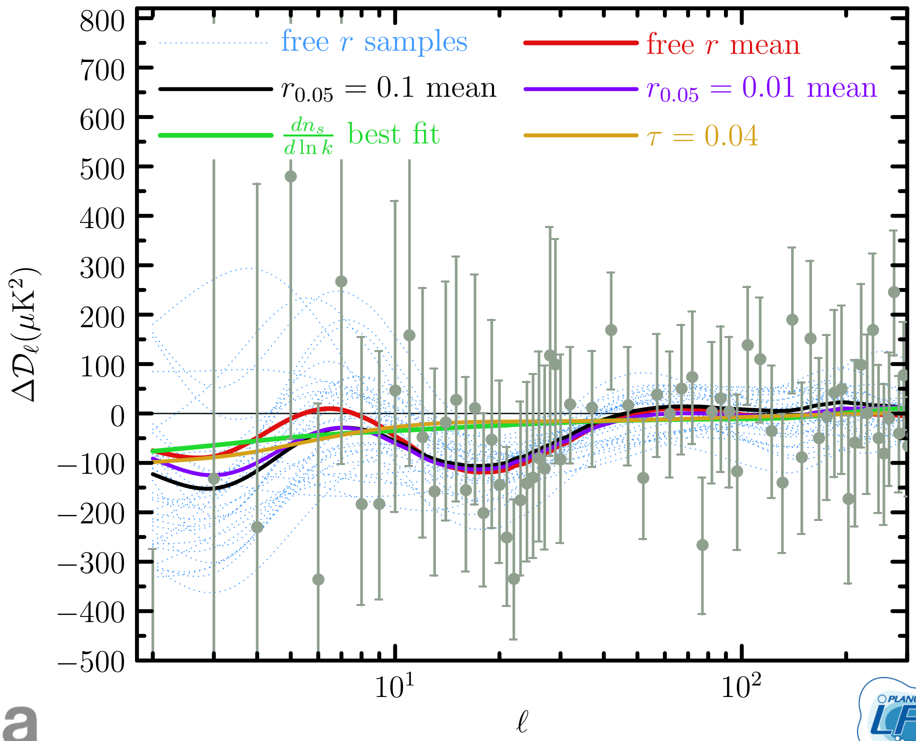
LFI PLANCK

HFI PLANCK

Primordial Spectrum Reconstruction

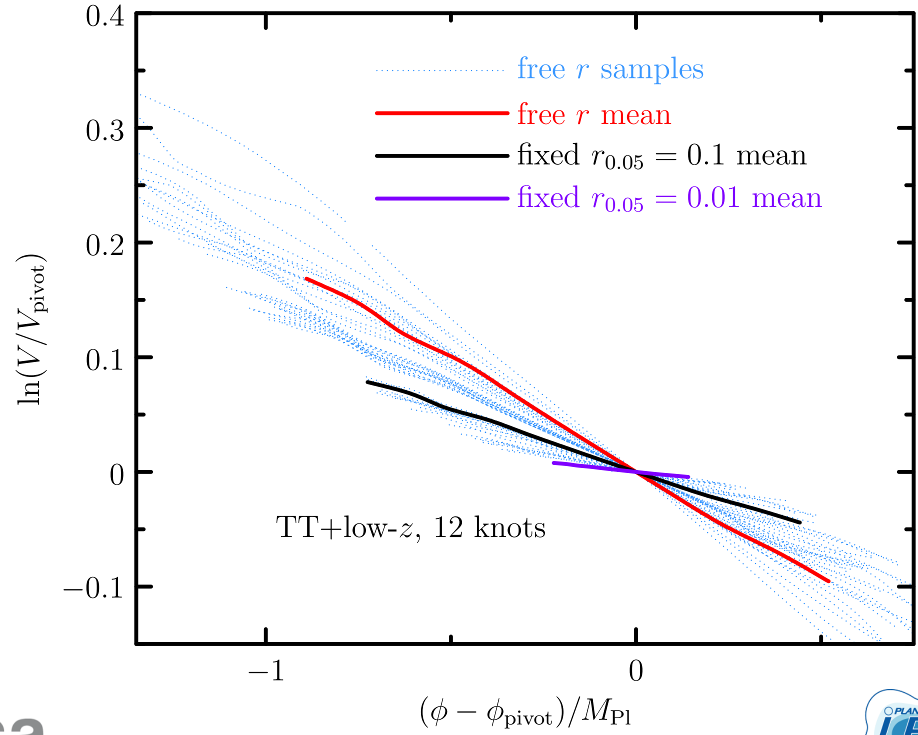


Running Spectral Index is Not a Good Fit!



Primordial Potential Reconstruction

planck



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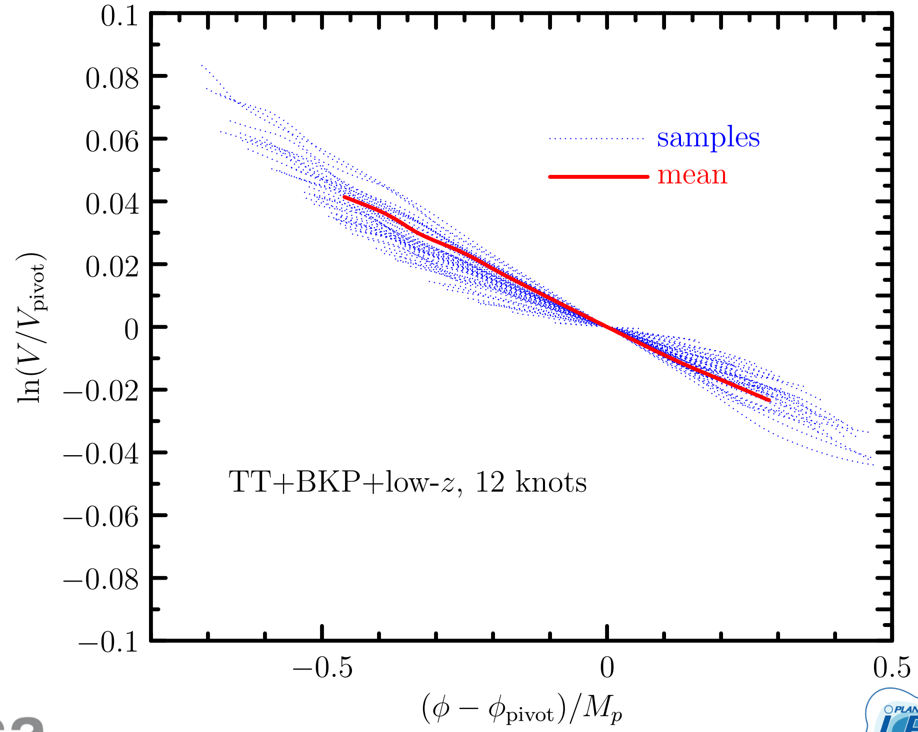
LFI PLANCK

HFI PLANCK

Primordial Potential Reconstruction

planck



esa

LFI PLANCK

HFI PLANCK

Conclusions

- A lot more and better processed and analyzed data.
- As in 2013, base ΛCDM continues to be a good fit to the Planck data, including polarization.
- Polarization has a degeneracy lifting capability often comparable to BAO.
- No convincing evidence for any simple extensions.
- Some tensions with astrophysical data that measure the amplitude of matter fluctuations.
- Planck constraints on r remain as in 2013 ($r < 0.11$ @ 95% CL), reminding that this constraint is model dependent.
- Scalar fluctuations consistent with pure adiabatic modes with a featureless tilted spectrum.

Conclusions

2015 papers and data are released!

+ more to come...

The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada.

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.

16th Canadian Conference on General Relativity and Relativistic Astrophysics

6-8 July 2016, SFU Segal Building, Vancouver

http://www.sfu.ca/physics/cosmology/CCGRRA-16/

"Spontaneous scalarization-induced dark matter and variation of the gravitational constant"

by Teruaki Suyama

[JGRG25(2015)8a4]

Spontaneous scalarization-induced dark matter and variation of the gravitational constant

Teruaki Suyama

RESCEU, the University of Tokyo

in collaboration with

Pisin Chen (LeCosPA), Jun'ichi Yokoyama (RESCEU)

References:
P.Chen, TS&J.Yokoyama, arXiv:1508.01384

Dawn of a new era of gravitational physics

Experimental tests of GR in strong gravity regime become possible soon.

Stringent solar-system constraints do not mean GR is correct.

Is there any model where large deviation from GR occurs only in strong gravity regime?

Damour–Esposito–Farese(DEF) model (Damour&Esposito–Farese, 1993)

Simplest class of the ST theory

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\mu^2}{2}\phi^2\right) + \int d^4x\sqrt{-\tilde{g}}\mathcal{L}_m(\tilde{g},\psi_m),$$

$\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$ is the Jordan–frame metric.

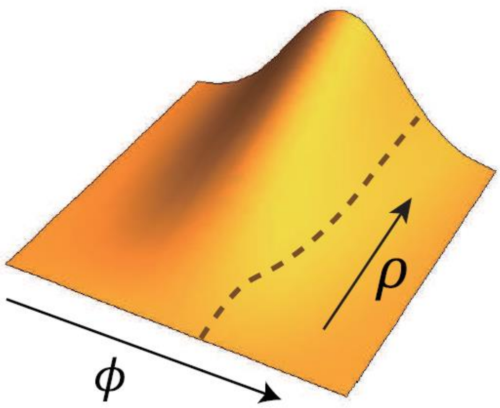
$$\Box\phi - V_{\mathrm{eff},\phi} = 0. \qquad V_{\mathrm{eff}}(\phi) = \frac{\mu^2}{2}\phi^2 - \frac{1}{4}A^4(\phi)\tilde{T},$$

$\frac{d}{d\phi}\ln A(\phi)$ measures the amount of deviation from GR (environment dependent).

(Brans–Dicke : $\mu = 0,\ A(\phi) = \exp(\sqrt{\frac{4\pi G_N}{2\omega_{BD}+3}}\,\phi)$) ($\omega_{BD} \gtrsim 5\times 10^4$)

DEF model : $\mu = 0,\ A(\phi) = \exp(2\pi G_N\beta\phi^2)$

Effective potential in the DEF model



$\phi = 0$ is GR.

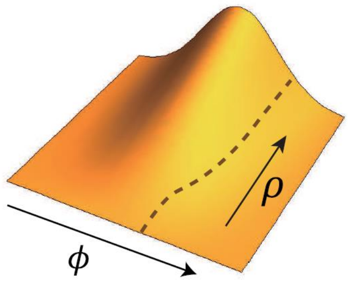
Even if the asymptotic value of ϕ satisfies the solar–system constraints, it is possible that significant deviation from GR appears at the vicinity and inside of the neutron star.

Many papers on this model.

Cosmology of the DEF model (e.g. Sampson et al, 2014)

GR limit corresponds to $\phi = 0$.

In the early Universe, the non-relativistic matter pushes ϕ away from the origin.



GR is not the cosmological attractor in the DEF model and fine-tuning of the initial value of ϕ is required to be consistent with the solar-system experiments.

Can we have a viable model?

Our new scalar-tensor model

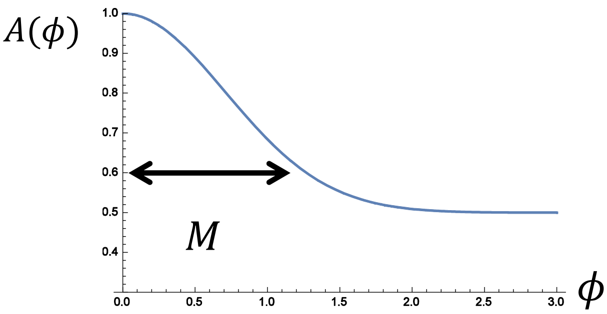
Two modifications

- Massive ϕ ($\mu \neq 0$)
- Decreasing function for $A(\phi)$

$$A^2(\phi) = 1 - \varepsilon + \varepsilon e^{-\frac{\phi^2}{2M^2}}$$

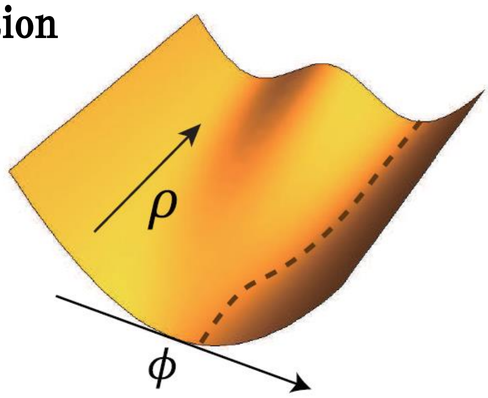
$$0 < \varepsilon < 1.$$

To derive quantitative results, we use this form.



Our new model does not suffer from the issue present in the DEF model. Furthermore, ϕ becomes a natural candidate of dark matter.

Spontaneous scalarization



$\rho_{PT} \equiv 2\mu^2 M^2/\varepsilon$ is the critical density.

If $\tilde{\rho}$ is above ρ_{PT}, spontaneous scalarization occurs and stable value $\bar{\phi}$ is given by

$$\frac{\bar{\phi}^2}{2M^2} = \ln f(\varepsilon, \rho_{\rm PT}/\tilde{\rho}), \qquad f(\varepsilon, \eta) \equiv \frac{2\varepsilon}{1-\varepsilon}\left(\sqrt{1+\frac{4\varepsilon\eta}{(1-\varepsilon)^2}}-1\right)^{-1}$$

Apart from the logarithmic factor, $\bar{\phi} \sim M$.

Symmetric phase ($\phi = 0$)

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\mu^2}{2}\phi^2\right) + \int d^4x\sqrt{-\tilde{g}}\mathcal{L}_m(\tilde{g}, \psi_m),$$

There is no difference between the Jordan-frame and the Einstein-frame metrics.

Interaction at the leading order is given by $\sim \frac{\phi^2}{M^2}T$. For our case of interest, M is much larger than TeV scale. We do not expect detectable signal of the existence of the ϕ field from the terrestrial experiments.

$$G_{\mu\nu} = 8\pi G_N\left[-\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{\mu^2}{2}\phi^2\right)g_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi + A^2(\phi)\tilde{T}_{\mu\nu}\right]$$

$\phi = 0$

$$G_{\mu\nu} = 8\pi G_N \tilde{T}_{\mu\nu}$$

Laws of gravity are just GR.

Spontaneous scalarization phase

In the spontaneous scalarization phase, field eqs for gravity are modified.

$$G_{\mu\nu} = 8\pi G_N \left[-\left(\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{\mu^2}{2}\phi^2\right) g_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi + A^2(\phi)\tilde{T}_{\mu\nu}\right]$$

Assuming no-excitation of the ϕ field,

$$\tilde{G}_{\mu\nu} + \boxed{\Lambda_{\rm eff}\tilde{g}_{\mu\nu}} = 8\pi \boxed{G_{\rm eff}} \tilde{T}_{\mu\nu},$$

Emergence of the effective cosmological constant (contrary to the Higgs mechanism)

Variation of the gravitational constant

$\tilde{g}_{\mu\nu}$ obeys the Einstein equations with the effective C.C. given by $\Lambda_{eff} > 0$ and with the gravitational constant given by $G_{eff} < G_N$.

$$\Lambda_{\rm eff} = 4\pi G_N \mu^2 \bar{\phi}^2 A^{-2}(\bar{\phi}) = 4\pi G_N \varepsilon \rho_{\rm PT} \ln f(\varepsilon, \rho_{\rm PT}/\tilde{\rho}) \left(1 - \varepsilon + \frac{\varepsilon}{f(\varepsilon, \rho_{\rm PT}/\tilde{\rho})}\right)^{-1}$$

$$G_{\rm eff} = A^2(\bar{\phi}) G_N = \left(1 - \varepsilon + \frac{\varepsilon}{f(\varepsilon, \rho_{\rm PT}/\tilde{\rho})}\right) G_N.$$

Spontaneous scalarization phase

For $\tilde{\rho} \gg \rho_{PT}$, we have

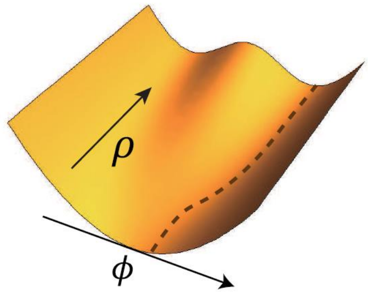
$$\Lambda_{\rm eff} \approx 4\pi G_N \frac{\varepsilon}{1-\varepsilon}\rho_{\rm PT} \ln\left((1-\varepsilon)\frac{\tilde{\rho}}{\rho_{\rm PT}}\right), \qquad G_{\rm eff} \approx (1-\varepsilon) G_N.$$

We find that Λ_{eff} is only logarithmically enhanced compared to ρ_{PT}.
Thus, the effective cosmological constant does not play a significant role in deep scalarization phase.

The gravitational constant is reduced from the one measured in the laboratory by the factor ε.

In the deep scalarization phase, the scalar force is suppressed and the dominant modification is the weakening of gravity while keeping the structure of GR.

Spontaneous scalarization

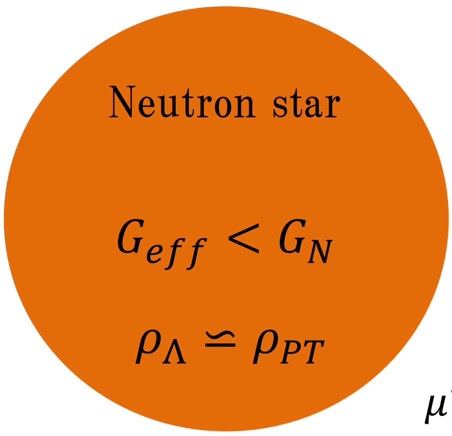


What is the value of ρ_{PT}?

It is a free parameter of the model.

Interesting case is $\rho_{PT} < \rho_{NS} \approx 3 \times 10^{-3} GeV^4$. Then, spontaneous scalarization occurs inside the compact objects such as the neutron stars.

It is possible that ϕ field constitutes the whole DM for such case.



$$\mu^{-1} = 2cm \left(\frac{\mu}{10^{-5} eV}\right)^{-1}$$

If SS occurs in compact stars, gravity becomes weaker inside the star.

Increase of NS mass?

We dubbed ϕ as *asymmetron*.

Spontaneous scalarization: asymmetron as dark matter

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[d] *Department of Physics, Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan*
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Abstract

We propose a new scalar-tensor model which induces significant deviation from general relativity inside dense objects like neutron stars, while passing solar-system and terrestrial experiments, extending a model proposed by Damour and Esposito-Farese. Unlike their model, we employ a massive scalar field dubbed asymmetron so that it not only realizes proper cosmic evolution but also can account for the cold dark matter. In our model, asymmetron undergoes spontaneous scalarization inside dense objects, which results in reduction of the gravitational constant by a factor of order unity. This suggests that observational tests of constancy of the gravitational constant in high density phase are the effective ways to look into the asymmetron model.

Disappearance of "asymmetron" in the title!

PHYSICAL REVIEW D **92**, 124016 (2015)

Spontaneous-scalarization-induced dark matter and variation of the gravitational constant

Pisin Chen,[1,2,3] Teruaki Suyama,[1] and Jun'ichi Yokoyama[1,4,5]
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We propose a new scalar-tensor model which induces significant deviation from general relativity inside dense objects like neutron stars, while passing the Solar System and terrestrial experiments, extending a model proposed by Damour and Esposito-Farese. Unlike their model, we employ a massive scalar field, dubbed the "asymmetron," that not only realizes proper cosmic evolution but can also account for the cold dark matter. In our model, the asymmetron undergoes spontaneous scalarization inside dense objects, which results in the reduction of the gravitational constant by a factor of order unity. This suggests that observational tests of the constancy of the gravitational constant in the high-density phase are effective ways to study the asymmetron model.

DOI: 10.1103/PhysRevD.92.124016 PACS numbers: 04.50.Kd, 95.35.+d, 98.80.-k

Symmetron Fields: Screening Long-Range Forces Through Local Symmetry Restoration

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We present a screening mechanism that allows a scalar field to mediate a long range (∼Mpc) force of gravitational strength in the cosmos while satisfying local tests of gravity. The mechanism hinges on local symmetry restoration in the presence of matter. In regions of sufficiently high matter density, the field is drawn towards $\phi = 0$ where its coupling to matter vanishes and the $\phi \to -\phi$ symmetry is restored. In regions of low density, however, the symmetry is spontaneously broken, and the field couples to matter with gravitational strength. We predict deviations from general relativity in the solar system that are within reach of next-generation experiments, as well as astrophysically observable violations of the equivalence principle. The model can be distinguished experimentally from Brans-Dicke gravity, chameleon theories and brane-world modifications of gravity.

arXiv:1001.4525v3 [hep-th] 21 Jun 2010

Scalar fields are the simplest of fields. Light, gravitationally coupled scalars are generically predicted to exist by many theories of high energy physics. These scalars may play a crucial role in dark energy as quintessence fields, and generically arise in infrared-modified gravity theories [1–7]. Despite their apparent theoretical ubiquity, no sign of such a fundamental scalar field has ever been seen, despite many experimental tests designed to detect solar system effects or fifth forces that would naively be expected if such scalars existed [8, 9].

Several broad classes of theoretical mechanisms have been developed to explain why such light scalars, if they exist, may not be visible to experiments performed near the Earth. One such class, the chameleon mechanism [5, 6], operates whenever the scalars are non-minimally coupled to matter in such a way that their effective mass depends on the local matter density. Deep in space, where the local mass density is low, the scalars would be light and would display their effects, but near the Earth, where experiments are performed, and where the local mass density is high, they would acquire a mass, making their effects short range and unobservable.

Another such mechanism, the Vainshtein mechanism [10], operates when the scalar has derivative self-couplings which become important near matter sources such as the Earth. The strong coupling near sources essentially cranks up the kinetic terms, which means, after

but is decoupled and screened in regions of high density.

This is achieved through the interplay of a symmetry-breaking potential, $V(\phi) = -\mu^2\phi^2/2+\lambda\phi^4/4$, and universal coupling to matter, $\phi^2\rho/2M^2$. In vacuum, the scalar acquires a VEV $\phi_0 = \mu/\sqrt{\lambda}$, which spontaneously breaks the $\mathbb{Z}_2$ symmetry $\phi \to -\phi$. In the presence of sufficiently high ambient density, however, the field is confined near $\phi = 0$, and the symmetry is restored. In turn, $\delta\phi$ fluctuations couple to matter as $(\phi_{\rm VEV}/M^2)\delta\phi\,\rho$, and so are weakly coupled in high density backgrounds and strongly coupled in low density backgrounds. Since the screening mechanism relies on the local restoration of a symmetry, we refer to the scalar as a *symmetron* field.

The model predicts a host of observational signatures. The solar light-deflection and time-delay deviations from general relativity (GR) are just below currents bound and within reach of next-generation experiments. Meanwhile, the expected signal from binary pulsars is much weaker, because neutron stars and their companions are screened. This is unlike standard Brans-Dicke (BD) theories, where solar system and binary pulsar signals are comparable. The symmetron observables are similarly distinguishable from standard chameleon and Vainshtein predictions. The symmetron also results in apparent violations of the equivalence principle between large (screened) galaxies and small (unscreened) galaxies [11].

There are key differences with [12, 13], with crucial

Disappearance of “symmetron” in the title!

PRL **104,** 231301 (2010) PHYSICAL REVIEW LETTERS week ending 11 JUNE 2010

Screening Long-Range Forces through Local Symmetry Restoration

Kurt Hinterbichler and Justin Khoury
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(Received 19 February 2010; published 10 June 2010)

We present a screening mechanism that allows a scalar field to mediate a long-range (∼ Mpc) force of gravitational strength in the cosmos while satisfying local tests of gravity. The mechanism hinges on local symmetry restoration in the presence of matter. In regions of sufficiently high matter density, the field is drawn towards $\phi = 0$ where its coupling to matter vanishes and the $\phi \to -\phi$ symmetry is restored. In regions of low density, however, the symmetry is spontaneously broken, and the field couples to matter with gravitational strength. We predict deviations from general relativity in the solar system that are within reach of next-generation experiments, as well as astrophysically observable violations of the equivalence principle. The model can be distinguished experimentally from Brans-Dicke gravity, chameleon theories and brane-world modifications of gravity.

DOI: 10.1103/PhysRevLett.104.231301 PACS numbers: 98.80.Cq

Scalar fields are the simplest of fields. Light, gravitationally coupled scalars are generically predicted to exist by many theories of high energy physics. These scalars may play a crucial role in dark energy as quintessence fields, and generically arise in infrared-modified gravity theories [1–7]. Despite their apparent theoretical ubiquity, no sign of such a fundamental scalar field has ever been seen, despite many experimental tests designed to detect solar system effects or fifth forces that would naively be expected if such scalars existed [8,9].

Several broad classes of theoretical mechanisms have been developed to explain why such light scalars, if they exist, may not be visible to experiments performed near the Earth. One such class, the chameleon mechanism [5,6], operates whenever the scalars are nonminimally coupled to matter in such a way that their effective mass depends on the local matter density. Deep in space, where the local mass density is low, the scalars would be light and would display their effects, but near Earth, where experiments are performed, and where the local mass density is high, they would acquire a mass, making their effects short range and

mass density, becoming large in regions of low mass density, and small in regions of high mass density. In addition, the coupling of the scalar to matter is proportional to the VEV, so that the scalar couples with gravitational strength in regions of low density, but is decoupled and screened in regions of high density.

This is achieved through the interplay of a symmetry-breaking potential, $V(\phi) = -\mu^2\phi^2/2 + \lambda\phi^4/4$, and universal coupling to matter, $\phi^2\rho/2M^2$. In vacuum, the scalar acquires a VEV $\phi_0 = \mu/\sqrt{\lambda}$, which spontaneously breaks the $\mathbb{Z}_2$ symmetry $\phi \to -\phi$. In the presence of sufficiently high ambient density, however, the field is confined near $\phi = 0$, and the symmetry is restored. In turn, $\delta\phi$ fluctuations couple to matter as $(\phi_{\rm VEV}/M^2)\delta\phi\rho$, and so are weakly coupled in high density backgrounds and strongly coupled in low density backgrounds. Since the screening mechanism relies on the local restoration of a symmetry, we refer to the scalar as a *symmetron* field.

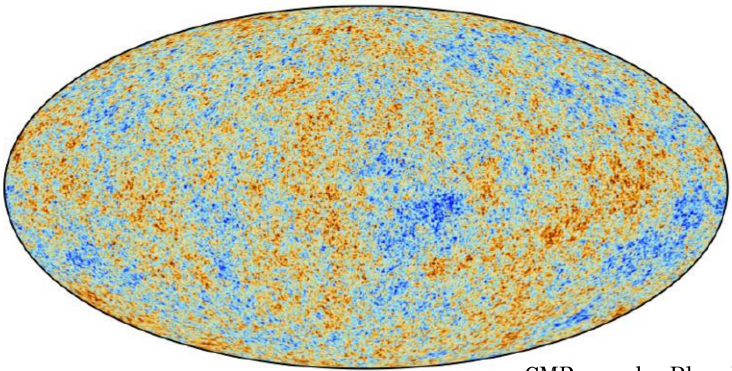
The model predicts a host of observational signatures. The solar light-deflection and time-delay deviations from general relativity (GR) are just below currents bound and

History repeats itself.

Asymmetron as dark matter

I show that asymmetron can behave as cold dark matter.

Asymmetron as dark matter



CMB map by Planck

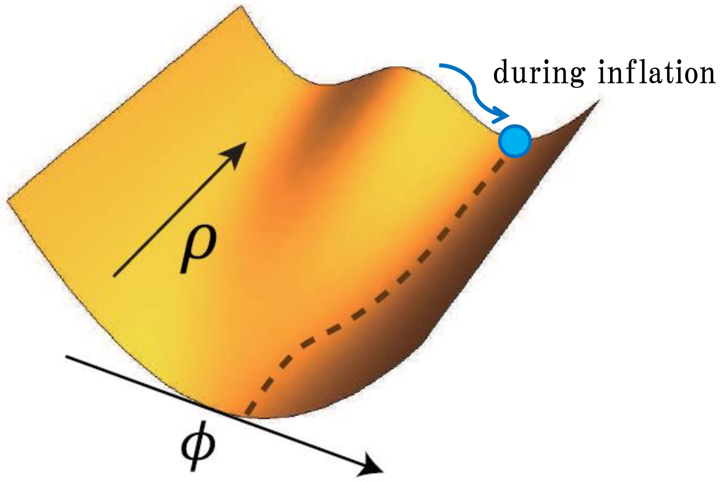
DM is known to exist in the Universe.

Inflation is known to have happened in the early Universe.

In the present model, DM is “seeded” during inflation.

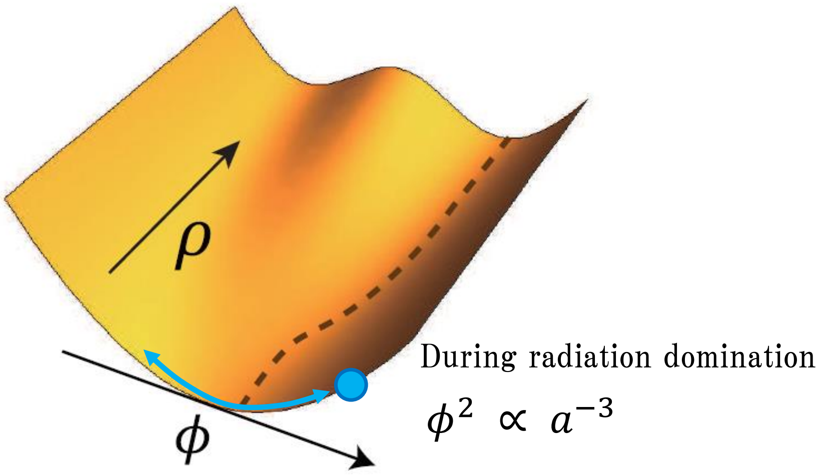
Asymmetron as dark matter

In our model, the ϕ field universally couples to all the rest of the fields including the inflaton.



During inflation, SS occurs by (large) inflaton energy density.

Asymmetron as dark matter



Since ϕ asymptotically approaches zero, GR is a cosmological attractor.

ϕ field behaves as non-relativistic matter.

ϕ field satisfies all the properties required for dark matter.

Asymmetron as dark matter

- Non-trivial two constraints

1. Non-observation of the CDM isocurvature perturbation in the CMB.

2. Non-observation of the fifth force.

Asymmetron as dark matter

1. Non-observation of the CDM isocurvature perturbation in the CMB.

Asymmetron DM contains uncorrelated CDM isocurvature perturbation.

Components: dark matter, baryons, radiations

Adiabatic perturbations: perturbations of all the components are the same

Isocurvature perturbations: perturbation of each components is independent

WMAP 9yr constraint

$$\frac{\mathcal{P}_{\rm CDM}}{\mathcal{P}_{\mathcal{R}}} < \frac{\alpha}{1-\alpha}, \qquad \alpha < 0.047 \quad (95\% \; CL)$$

Asymmetron as dark matter

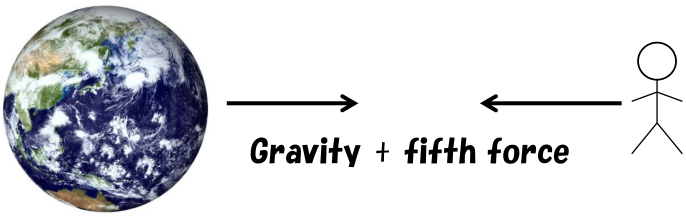
2. Non-observation of the fifth force.

Since ϕ field is still oscillating today, its has non-vanishing $<\phi^2>$.

$$\frac{1}{2}\mu^2 <\phi^2> = \rho_{DM,local}$$

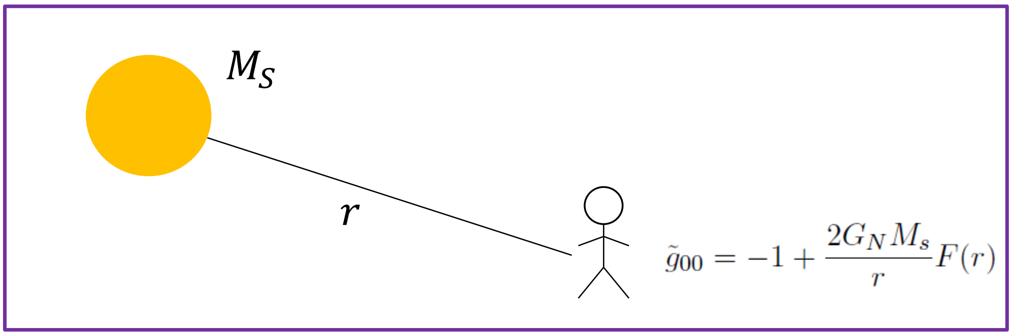
Since ϕ field has non-vanishing amplitude, it mediates fifth-force.
In particular, the strength of the fifth-force changes periodically in time.

$$\frac{2\pi}{\mu} \approx 4\times 10^{-4}\ s \left(\frac{\mu}{10^{-11}\ \mathrm{eV}}\right)^{-1}$$



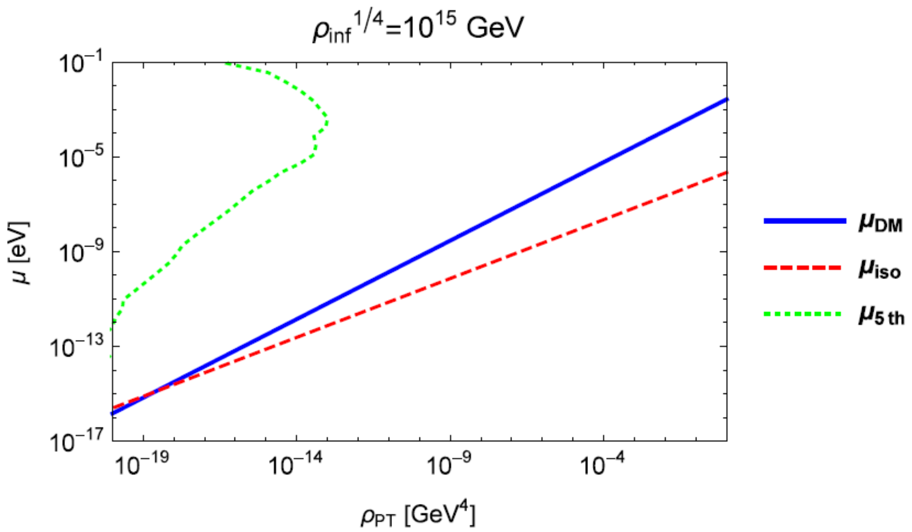
Asymmetron as dark matter

2. Non-observation of the fifth force.



$$F(r) = 1 + \boxed{\frac{\mu^2}{4\pi G_N \varepsilon}\frac{\rho_{\rm DM}}{\rho_{\rm PT}^2}e^{-\mu r}}$$

Correction due to the fifth-force



Fifth-force from the asymmetron dark matter is much smaller than the upper limit set by the experiments.

However, constraint from the non-observation of CDM isocurvature perturbation is strong. Asymmetron as dark matter is inconsistent with inflation models with energy scale as large as $10^{15}GeV$.

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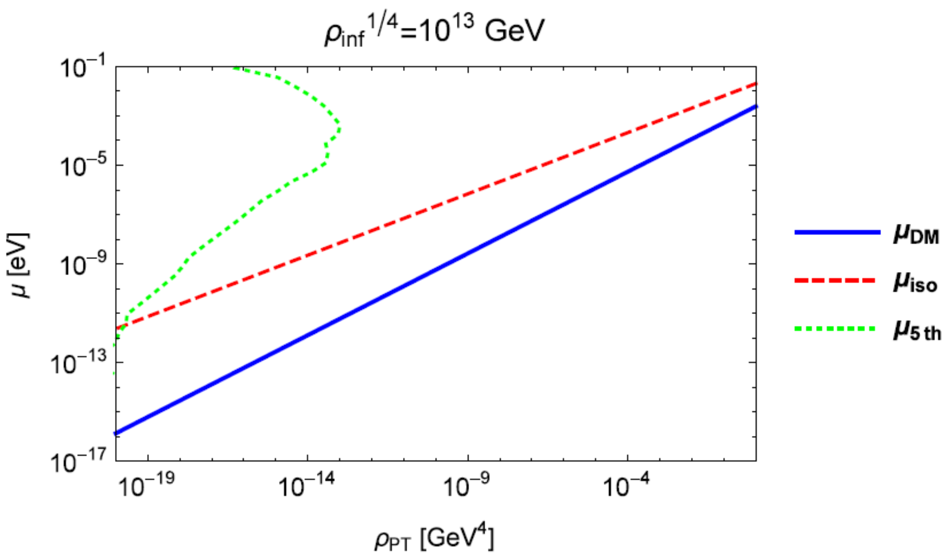


Asymmetron as dark matter is consistent with inflation models with energy scale $10^{13}GeV$.

Low energy inflation is consistent with asymmetron being DM. 26

Summary

We have proposed a new scalar-tensor model in which the scalar field undergoes the spontaneous scalarization above the critical density.

The scalar field can be the DM. Spontaneous scalarization also provides the mechanism to generate the initial abundance of DM. (additional production mechanism is not needed)

The scalar field is only very weakly interacting with our matter (in the symmetric phase) and detecting such a field on the Earth is quite hard.

In the spontaneous scalarization phase, the gravitational constant gets weakened compared to the one measured in the laboratory. This may happen inside the compact objects.

Opening of GW observations will enable us to test this scenario and first "detection" of DM may come from such observations.

Issues to be investigated

Astrophysics of the spontaneous scalarization

- How is the structure of neutron star changed?
- What happens when scalarized object collapses into a BH?
- Dynamics of spontaneous scalarization. Process to reach into the stable state.
- What kind of observations are the most effective to test this model?

“The Einstein-Struble Correspondence and Lorentz Invariance”

by Marcus Christian Werner

[JGRG25(2015)8a5]

The Einstein-Struble Correspondence and Lorentz Invariance

Marcus C. Werner, Kyoto University

11th December 2015
JGRG25 at Kyoto

Introduction

- 2015 marks the centennial of general relativity, and has been designated the *International Year of Light* by UNESCO.

United Nations Educational, Scientific and Cultural Organization
International Year of Light 2015

- I present hitherto unpublished correspondence of Einstein, Chandrasekhar and others with Struble, about an optical test of Lorentz invariance.
- This may be of scientific as well as historical relevance, given the recent renewed interest in Lorentz-violating theories, such as Hořava-Lifshitz gravity.

Outline

- Physical perspective: electromagnetism and Lorentz invariance, possible modifications.
- Historical perspective: special relativity versus emission theory, binary star test.
- The Struble effect, and correspondence with Chandrasekhar and Einstein.

Lorentz invariance from Maxwell

Consider (M, g) with arbitrary signature and test matter defined by vacuum Maxwell theory,

$$S = -\frac{1}{4}\int \omega_g \; g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu},$$

with volume form $\omega_g = \sqrt{|\det g|}\, d^4x$. Then the principal polynomial is quadratic,

$$P(x,p) = g^{-1}(x)^{\alpha\beta} p_\alpha p_\beta, \quad p \in T^*M,$$

and is hyperbolic iff g is Lorentzian.

Possible modifications

Now consider non-vacuum Maxwell theory with constitutive tensor G defining the medium,

$$S = -\frac{1}{8} \int \omega_G \; G^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta},$$

then the corresponding principal polynomial is quartic,

$$P(x, p) = \mathcal{G}(x)^{\alpha\beta\gamma\delta} p_\alpha p_\beta p_\gamma p_\delta, \quad p \in T^*M,$$

with the Fresnel tensor

$$\mathcal{G}^{\alpha\beta\gamma\delta} = -\frac{1}{24}(\omega_G)_{\kappa\lambda\mu\nu}(\omega_G)_{\rho\sigma\tau\upsilon} G^{\kappa\lambda\rho(\alpha} G^{\beta|\mu\sigma|\gamma} G^{\delta)\nu\tau\upsilon}.$$

In the standard theory, we assume $G = G(g)$ in vacuum.

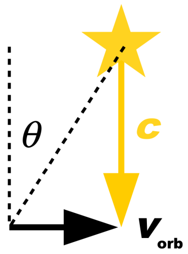
Possible modifications

- However, G instead of g may be taken as fundamental: premetric electromagnetism (e.g. Hehl, Obukhov, Rubilar 2002, Itin 2009) and area metric theory (e.g. Schuller, Witte, Wohlfarth 2010, Rätzel, Rivera, Schuller 2011).
- The hyperbolicity property is more complicated than in metric geometry, allowing, for instance, birefringence. Potentially interesting in cosmology as effective theory (e.g. Werner in prep.).
- Lorentz invariance is well established. Pulsar timing provides some of the best current evidence, i.e. absence of preferred frame effects. (e.g. Shao, Caballero, Kramer, Wex, Champion, Jessner 2013).

Prehistory of special relativity

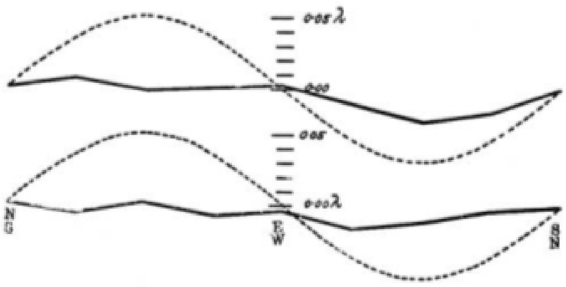
Discovery of the aberration of star light by James Bradley, 1727:

$$\Delta\theta \simeq \frac{2v_{\rm orb}}{c} \simeq 41''$$



→ Supposed aether stationary with respect to the Sun

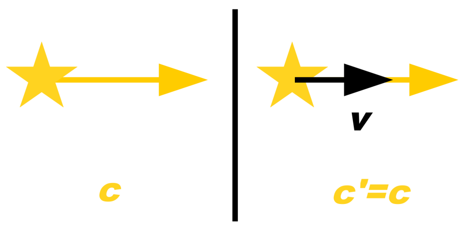
Interferometer experiment of Albert Michelson and Edward Morley at Cleveland, 1887



No diurnal variation observed → contradiction with aether theory

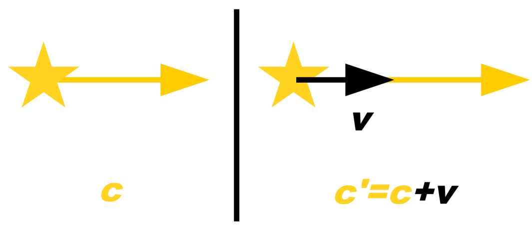
Two competitors

Albert Einstein (1879-1955): special relativity, 1905



Explains negative the result of the Michelson-Morley experiment

Walter Ritz (1878-1909): emission theory, 1908



Explains the negative result of the Michelson-Morley experiment as well!

Emission theory versus special relativity

- Interferometry experiment requires moving mirrors (Michelson 1913) or emitters (Tolman 1912), e.g. stars
- Negative result of the Michelson-Morley experiment using star light was achieved by Tomaschek, 1924
- Using binary star systems to test emission theory: Comstock (1910), de Sitter (1913), Freundlich (1913)

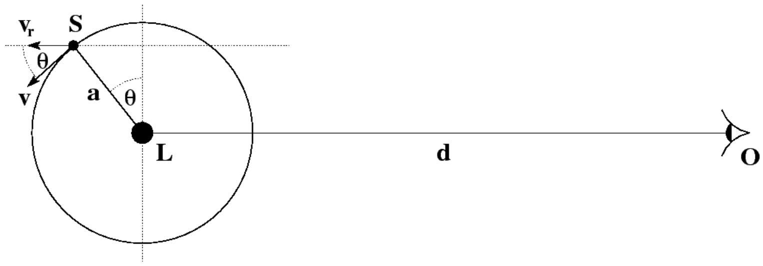
Emission theory versus special relativity

Letter by Einstein to Freundlich, 26 August 1913, mainly about gravitational lensing and its observability during solar eclipses:

> *[...] I am also very curious about the results of your investigations concerning the binary stars. If the speed of light depends on the speed of the light source even only in the slightest, then my entire theory of relativity, including the theory of gravity, is wrong.*

Binary star test

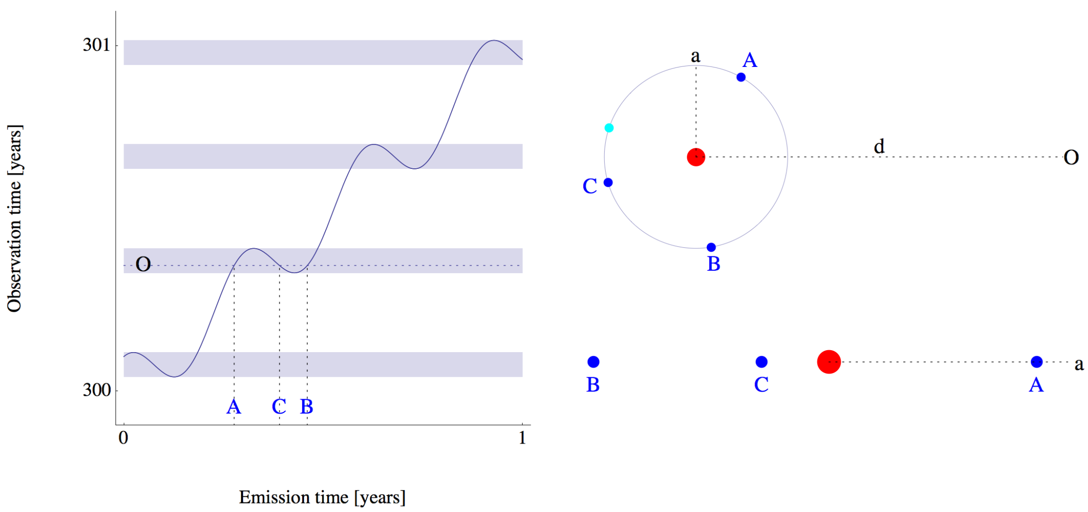
Binary star system L, S with circular orbit of radius a and orbital speed $v = \omega a$, observed by O from distance $d \gg a$:



Observation time t as a function of emission time t_0, at orbital position $\theta(t_0) = \omega t_0$, assuming emission theory:

$$t = t_0 + \frac{d + a \sin \omega t_0}{c - a\omega \cos \omega t_0}$$

Binary star test: no *ghost stars*

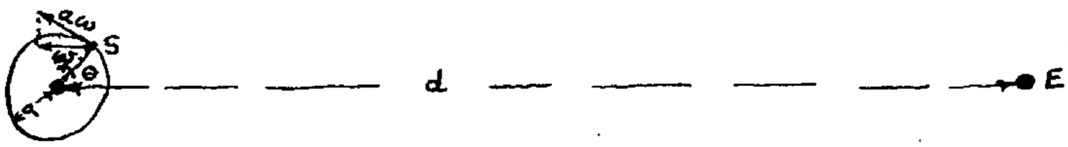


Revisiting emission theory

- Raimond Struble (1924-2013), mathematician at North Carolina State University, Raleigh
- Correspondence with Einstein and Chandrasekhar in 1947, as a student at Notre Dame University. Adviser: Karl Menger at Illinois Institute of Technology.
- Hitherto unpublished correspondence in private archive of the Struble Estate (PO Box 31346, Raleigh, NC 27622, USA)

Acceleration Doppler effect

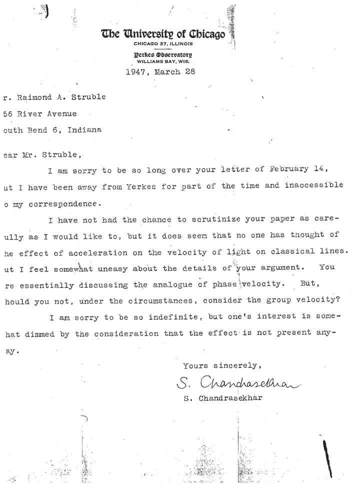
Struble found an acceleration Doppler effect hitherto overlooked in emission theory, providing a strong test of Lorentz invariance:



During $\Delta t_0 = \frac{\lambda_0}{c}$, we have $\Delta c = \Delta t_0 a \omega^2 \sin\theta$, and thus

$$\frac{\Delta\lambda}{\lambda} = -\Delta c \frac{d}{c} = -\lambda_0 \frac{a d \omega^2}{c^2} \sin\theta.$$

Struble and Chandrasekhar



The University of Chicago
CHICAGO 37, ILLINOIS
Yerkes Observatory
WILLIAMS BAY, WIS.

1947, March 28

r. Raimond A. Struble
56 River Avenue
outh Bend 6, Indiana

ear Mr. Struble,

I am sorry to be so long over your letter of February 14, ut I have been away from Yerkes for part of the time and inaccessible o my correspondence.

I have not had the chance to scrutinize your paper as care- ully as I would like to, but it does seem that no one has thought of he effect of acceleration on the velocity of light on classical lines. ut I feel somewhat uneasy about the details of your argument. You re essentially discussing the analogue of phase velocity. But, hould you not, under the circumstances, consider the group velocity?

I am sorry to be so indefinite, but one's interest is some- hat dimmed by the consideration that the effect is not present any- ay.

Yours sincerely,
S. Chandrasekhar
S. Chandrasekhar

The Struble Estate

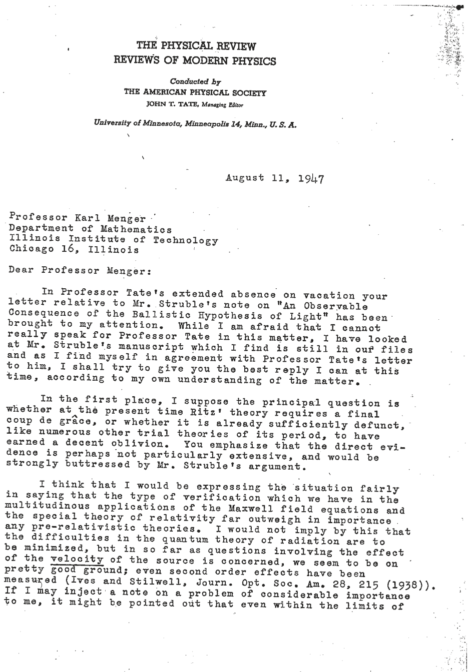
Struble wrote to Chandrasekhar at Yerkes Observatory, who noted that

> *no one has thought of the effect of acceleration on the velocity of light on classical lines,*

but pointed out the issue of phase versus group velocity, as well as de Sitter's work.

A first attempt to publish in *Phys. Rev.* failed. Then Menger tried to intercede on behalf of his student.

Menger and *Phys. Rev.*



THE PHYSICAL REVIEW
REVIEWS OF MODERN PHYSICS
Conducted by
THE AMERICAN PHYSICAL SOCIETY
JOHN T. TATE, *Managing Editor*
University of Minnesota, Minneapolis 14, Minn., U. S. A.

August 11, 1947

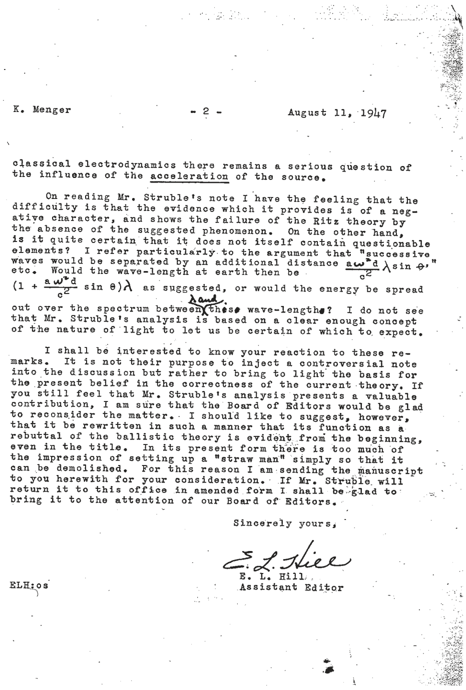
Professor Karl Menger
Department of Mathematics
Illinois Institute of Technology
Chicago 16, Illinois

Dear Professor Menger:

In Professor Tate's extended absence on vacation your letter relative to Mr. Struble's note on "An Observable Consequence of the Ballistic Hypothesis of Light" has been brought to my attention. While I am afraid that I cannot really speak for Professor Tate in this matter, I have looked at Mr. Struble's manuscript which I find is still in our files and as I find myself in agreement with Professor Tate's letter to him, I shall try to give you the best reply I can at this time, according to my own understanding of the matter.

In the first place, I suppose the principal question is whether at the present time Ritz' theory requires a final coup de grâce, or whether it is already sufficiently defunct, like numerous other trial theories of its period, to have earned a decent oblivion. You emphasize that the direct evi- dence is perhaps not particularly extensive, and would be strongly buttressed by Mr. Struble's argument.

I think that I would be expressing the situation fairly in saying that the type of verification which we have in the multitudinous applications of the Maxwell field equations and the special theory of relativity far outweigh in importance any pre-relativistic theories. I would not imply by this that the difficulties in the quantum theory of radiation are to be minimized, but in so far as questions involving the effect of the velocity of the source is concerned, we seem to be on pretty good ground; even second order effects have been measured (Ives and Stilwell, Journ. Opt. Soc. Am. 28, 215 (1938)). If I may inject a note on a problem of considerable importance to me, it might be pointed out that even within the limits of



K. Menger - 2 - August 11, 1947

classical electrodynamics there remains a serious question of the influence of the acceleration of the source.

On reading Mr. Struble's note I have the feeling that the difficulty is that the evidence which it provides is of a neg- ative character, and shows the failure of the Ritz theory by the absence of the suggested phenomenon. On the other hand, is it quite certain that it does not itself contain questionable elements? I refer particularly to the argument that "successive waves would be separated by an additional distance $\frac{a\omega^2 d}{c^2}\lambda \sin\theta$," etc. Would the wave-length at earth then be $(1 + \frac{a\omega^2 d}{c^2}\sin\theta)\lambda$ as suggested, or would the energy be spread out over the spectrum between λ and the~~se~~ wave-length~~s~~? I do not see that Mr. Struble's analysis is based on a clear enough concept of the nature of light to let us be certain of which to expect.

I shall be interested to know your reaction to these re- marks. It is not their purpose to inject a controversial note into the discussion but rather to bring to light the basis for the present belief in the correctness of the current theory. If you still feel that Mr. Struble's analysis presents a valuable contribution, I am sure that the Board of Editors would be glad to reconsider the matter. I should like to suggest, however, that it be rewritten in such a manner that its function as a rebuttal of the ballistic theory is evident from the beginning, even in the title. In its present form there is too much of the impression of setting up a "straw man" simply so that it can be demolished. For this reason I am sending the manuscript to you herewith for your consideration. If Mr. Struble will return it to this office in amended form I shall be glad to bring it to the attention of our Board of Editors.

Sincerely yours,
E. L. Hill
E. L. Hill
Assistant Editor

ELH:os

The Struble Estate

Finally, turning to Einstein

THE INSTITUTE FOR ADVANCED STUDY
SCHOOL OF MATHEMATICS
PRINCETON, NEW JERSEY

November 5,1947

Mr.Raimond A.Struble
556 River Ave.
South Bend 6,Indiana

Dear Sir:

Your result is essentially right. I was not able to grasp your calculation. I have done it as follows: ~~Is~~
Be t_0 the time on the emitting body then the time of arrival on the earth is (with sufficient precision):

$$t = t_0 + \frac{d}{c-v}$$

or again with sufficient precision

$$t = t_0 + \frac{d}{c}\left(1 + \frac{v}{c}\right) = t_0 + \frac{d}{c} + \frac{d}{c^2}v$$

The constant time interval $\frac{d}{c}$ is without influence.
One has therefore to put

$$t = t_0 + \frac{d}{c^2}v \qquad (v = a\Omega\cos(\Omega t_0))$$

$$t = t_0 + \frac{ad\Omega}{c^2}\cos(\Omega t_0) \quad \ldots (1)$$

The spectral oscillation is given by $\varphi = \cos\omega t_0$
At the earth this function expresses (approximately) the corresponding oscillation if one substitutes for t_0 its expression in t according to (1).
The frequency on earth is $\nu^2 = -\frac{\frac{d^2\varphi}{dt^2}}{\varphi}$. We have

$$\frac{d\varphi}{dt} = \frac{d\varphi}{dt_0}\frac{dt_0}{dt}; \quad \frac{d^2\varphi}{dt^2} = \frac{d^2\varphi}{dt_0^2}\frac{dt_0}{dt} + \frac{d\varphi}{dt_0}\frac{d^2t_0}{dt^2}$$

$$= \left(\frac{d^2\varphi}{dt_0^2}\cdot\frac{dt_0}{dt}\right)\cdot\frac{dt_0}{dt} + \frac{d\varphi}{dt_0}\cdot\frac{d^2t_0}{dt^2}$$

$$= \frac{d^2\varphi}{dt_0^2}\left(\frac{dt_0}{dt}\right)^2 + \frac{d\varphi}{dt_0}\cdot\frac{d^2t_0}{dt^2}$$

correction

THE INSTITUTE FOR ADVANCED STUDY
SCHOOL OF MATHEMATICS
PRINCETON, NEW JERSEY

-2-

The second term in the last formula is negligible relative to the first. It is therefore

$$\nu^2 = -\frac{\frac{d^2\varphi}{dt_0^2}}{\varphi}\left(\frac{dt_0}{dt}\right)^2 = \nu_0^2\left(\frac{dt_0}{dt}\right)^2$$

or

$$\nu = \nu_0\frac{dt_0}{dt}$$

According to (1) is

$$dt = dt_0 - \frac{ad\Omega^2}{c^2}\sin(\Omega t_0)\,dt_0$$

therefore

$$\nu = \nu_0\frac{1}{1 - \frac{ad\Omega^2}{c^2}\sin(\Omega t_0)}$$

Here t_0 has to be expressed as a function of t.
This is in accordance with your remarks. The effect is, of course, ~~mostly~~ in most cases of observation much greater than the ordinary Doppler-effect and without doubt incompatible with the experimental facts. (also if $\frac{ad\Omega^2}{c^2} < 1$)

I should be quite astonished if De Sitter would not have made this little calculation. In any case, you should carefully look into his paper before publishing something about it.

Sincerely yours,
A. Einstein
Albert Einstein.

The Struble Estate

Conclusions

- Einstein deemed Struble's effect *essentially right*, but also pointed out the possible priority of de Sitter, as mentioned above.
- Struble's effect was not discussed by de Sitter, but Struble did not publish and the effect is neglected in later critiques of emission theory (e.g. Fox 1965).
- Now given the renewed interest in Lorentz-violating theories, can we usefully apply Struble's effect in contemporary physics?

Poster Presentations

P01 Emel Altas Kiraci (METU)
"On Exact Solutions and the Consistency of 3D Minimal Massive Gravity"
[JGRG25(2015)P01]

P02 Shun Arai (Nagoya U.)
"Inflationary perturbations in the Lifshitz regime of Horava gravity"
[JGRG25(2015)P02]

P03 Cancelled

P04 Yu Furuya (Hirosaki U.)
"Perturbations of Kasner-de Sitter spacetime"
[JGRG25(2015)P04]

P05 Alexander Gallego (UdeA)
"Effects of local features of the inflaton potential on the spectrum and bispectrum of primordial perturbations"
[JGRG25(2015)P05]

P06 Cancelled

P07 Shinichi Hirano (Rikkyo U.)
"Large scale suppression with ultra slow-roll inflation scenario"
[JGRG25(2015)P07]

P08 Hideo Iguchi (Nihon U.)
"An Alternative Approach to Black Hole Thermodynamics: Renyi Entropy and Phase Transition"
[JGRG25(2015)P08]

P09 Taishi Ikeda (Nagoya U.)
"Spherical symmetric domain wall collapse by numerical simulation"
[JGRG25(2015)P09]

P10 Asahi Ishihara (Hirosaki U.)
"Bending angle of light in a non-asymptotically flat black hole"
[JGRG25(2015)P10]

P11 Mao Iwasa (Mao Iwasa)
"Orbital Evolution of Stars Around Shrinking Massive Black Hole Binaries"
[JGRG25(2015)P11]

P12 Ryo Kato (Kobe U.)
"Detection of Circular Polarization in Stochastic Gravitational Wave Background with Pulsar Timing Arrays"
[JGRG25(2015)P12]

P13 Daiki Kikuchi (Hirosaki U.)
"Possible orbiting gyroscope precession by a Chern-Simons modification to gravity"
[JGRG25(2015)P13]

P14 Shunichiro Kinoshita (Chuo U.)
"Conic D-branes"
[JGRG25(2015)P14]

P15 Cancelled

P16 Meguru Komada (Nagoya U.)
"Born-Infeld Gravity and Black Hole Formation"
[JGRG25(2015)P16]

P17 Ken Matsuno (Osaka City U.)
"Slowly rotating dilatonic black holes with exponential form of nonlinear electrodynamics"
[JGRG25(2015)P17]

P18 Takashi Mishima (Nihon U.)
"Behavior of the new cylindrically symmetric gravitational solitonic waves"
[JGRG25(2015)P18]

P19 Umpei Miyamoto (Akita Pref. Univ.)
"Vacuum excitation by sudden (dis-)appearance of a Dirichlet wall in a cavity"
[JGRG25(2015)P19]

P20 Shuntaro Mizuno (WIAS)
"Halo/Galaxy Bispectrum with Equilateral-type Primordial Trispectrum"
[JGRG25(2015)P20]

P21 Taisaku Mori (Nagoya U.)
"Two Dimensional Black Hole in Bigravity"
[JGRG25(2015)P21]

P22 Taro Mori (SOKENDAI/KEK)
"Multi-field effects on Non-Gaussianity in Starobinsky inflation"
[JGRG25(2015)P22]

P23 Hiroyuki Nakano (Kyoto U.)
"Possible golden events for ringdown gravitational waves -- total mass dependence --"
[JGRG25(2015)P23]

P24 Hiroyuki Negishi (Osaka City U.)
"Can we remove the systematic error due to isotropic inhomogeneities?"
[JGRG25(2015)P24]

P25 Tatsuya Ogawa (Osaka City U.)
"Cosmic string shielding of electric field of line charge"
[JGRG25(2015)P25]

P26 Seiju Ohashi (KEK)
"Causality and shock formation in general scalar-tensor theories"
[JGRG25(2015)P26]

P27 Cancelld

P28 Toshiaki Ono (Hirosaki U.)
"Sturm's theorem to marginal stable circular orbits"
[JGRG25(2015)P28]

P29 Hisaaki Shinkai (Osaka Inst. of Tech.)
"Singularity formation in n-dim Gauss-Bonnet gravity"
[JGRG25(2015)P29]

P30 Kiyoshi Shiraishi (Yamaguchi U.)
"Conditions on Scalar Potentials in Geometric Scalar Gravity"
[JGRG25(2015)P30]

P31 Tomohito Suzuki (Hirosaki U.)
"Marginal stable circular orbits for stationary and axially symmetric spacetimes"
[JGRG25(2015)P31]

P32 Masaaki Takahashi (Aichi U. of Education)
"MHD Wave Propagetion in a Black Hole Magnetosphere"
[JGRG25(2015)P32]

P33 Kenji Tomita (Kyoto U.)
"Cosmological entropy production, perturbations and CMB fluctuations in (1+3+6)-dimensional space-times"
[JGRG25(2015)P33]

P34 Jonathan White (KEK)
"Gravitational reheating after multi-field inflation"
[JGRG25(2015)P34]

P35 Kazuhiro Yamamoto (Hiroshima U.)
"Unruh radiation produced by a uniformly accelerating charged particle coupled to vacuum fluctuations"
[JGRG25(2015)P35]

P36 Masashi Yamazaki (Nagoya U.)
"Compact Objects in dRGT Massive Gravity"
[JGRG25(2015)P36]

P37 Yingli Zhang (NAOC)
"Theoretical Aspects of Nonlocal Gravity"
[JGRG25(2015)P37]

“On Exact Solutions and the Consistency of 3D Minimal Massive Gravity”

by Emel Altas Kiraci

[JGRG25(2015)P01]

On Exact Solutions and the Consistency of 3D Minimal Massive Gravity

Emel Altas Kiraci and Bayram Tekin

Middle East Technical University

Phys. Rev. D 92, 025033 – 20 July 2015

JGRG 25,December 7-11,2015

MIDDLE EAST TECHNICAL UNIVERSITY

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

ABSTRACT

We show that all algebraic Type-O, Type-N and Type-D and some Kundt-Type solutions of Topologically Massive Gravity are inherited by its holographically well-defined deformation, that is the recently found Minimal Massive Gravity. This construction provides a large class of constant scalar curvature solutions to the theory. We also study the consistency of the field equations both in the source-free and matter-coupled cases of the field equations.

INTRODUCTION

In the current work we shall consider two aspects of the MMG theory:The first being the consistency of both the vacuum and matter-coupled MMG equations and the second being the systematic construction of new solutions to the vacuum fiels equations that are inherited from the TMG.We shall upgrade all the algebraic Types O,N,D and some Kundt-Type solutions of TMG to be the solutions of MMG with simple modifications of the parameters.

1)CONSISTENCY OF FIELD EQUATIONS OF MMG

The matter-free field equation of the theory is defined as [1]

$$\varepsilon_{\mu\nu} = \bar{\sigma} G_{\mu\nu} + \bar{\lambda}_0 g_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \frac{\gamma}{\mu^2} J_{\mu\nu} = 0$$

with dimensionless parameters σ , γ and dimensionful ones μ and Λ_0. The Cotton tensor is given in terms of the Schouten tensor and the new ingredient is the J-tensor defined in terms of the products of two Schouten tensors as

$$C_{\mu\nu} = \eta_\mu{}^{\alpha\beta} \nabla_\alpha S_{\beta\nu} \qquad S_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R \qquad J_{\mu\nu} = \frac{1}{2} \eta_\mu{}^{\alpha\beta} \eta_\nu{}^{\rho\sigma} S_{\alpha\rho} S_{\beta\sigma}$$

The matter coupled field equations [2]

$$\eta G_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \frac{\gamma}{\mu^2} J_{\mu\nu} = \Theta_{\mu\nu}(T)$$

where the source term reads

$$\Theta_{\mu\nu}(T) = \frac{b}{\gamma} T_{\mu\nu} + \frac{b^2}{\mu\gamma} \eta_\mu{}^{\rho\sigma} \nabla_\rho \hat{T}_{\sigma\nu} - \frac{b^2}{\mu^2} \eta_\mu{}^{\rho\sigma} \eta_\nu{}^{\lambda\kappa} S_{\rho\lambda} \hat{T}_{\sigma\kappa} + \frac{b^4}{2\mu^2\gamma} \eta_\mu{}^{\rho\sigma} \eta_\nu{}^{\lambda\kappa} \hat{T}_{\rho\lambda} \hat{T}_{\sigma\kappa}$$

A)SOURCE FREE CASE

Consistency of the field equations requires that the first divergence vanishes but from direct substitution it doesn't .Which means the MMG field equations does not obey the Bianchi Identity and therefore cannot be obtained from an action with the metric being the only variable [1,3]. But the covariant divergence vanishes for metrics that are solutions to the full MMG equations. (Therefore, one has an "on-shell Bianchi Identity".) This is necessary condition for the consistency of the classical field equations but not a sufficient condition, since the rank-two tensor equations are susceptible to double-divergence. We show that for the source-free case the double-divergence of the field equations vanish for the solutions of the field equation.

$$\nabla_\mu J^{\mu\nu} = \eta^{\nu\rho\sigma} S_\rho^\lambda C_{\sigma\lambda} = 0 \quad \Longrightarrow \quad \nabla_\nu \nabla_\mu J^{\mu\nu} = \eta^{\nu\rho\sigma} S_\rho^\lambda \nabla_\nu C_{\sigma\lambda} + \eta^{\nu\rho\sigma} C_{\sigma\lambda} \nabla_\nu S_\rho^\lambda = 0$$

B)MATTER COUPLED CASE

For the consistency of the matter-coupled MMG, one should require the covariant divergence of the left-hand side and the right-hand side to be equal to each other when the field equations are used which was worked out to be the case in [2]. Once again, this is necessary but not sufficient and one should also check the double divergence.

$$\nabla_\mu \Theta^{\mu\nu}(T) = \frac{\gamma}{\mu} \nabla_\mu J^{\mu\nu} = \eta^{\nu\rho\sigma} S_\rho^\lambda \Theta_{\sigma\lambda}(T) \quad \Longrightarrow \quad \nabla_\nu \nabla_\mu \Theta^{\mu\nu}(T) = \frac{\gamma}{\mu} \eta^{\nu\rho\sigma} S_\rho^\lambda \nabla_\nu \Theta_{\sigma\lambda}(T) + \frac{\gamma}{\mu} C^{\sigma\lambda} \Theta_{\sigma\lambda}(T)$$

This shows that the double divergence of the left hand-side and the right hand-side of the field equations are equal to each other on shell hence the equations are consistent.

2) CONSTANT SCALAR CURVATURE SOLUTIONS

In three dimensions, classification of space-times can be done either using the *Cotton-tensor* (analogous to the four dimensional *Petrov* classification)
or using the
Traceless Ricci tensor (analogous to the four dimensional *Segre* classification).
To search for solutions, let us rewrite the source-free field equations as a trace part and a traceless part.

The trace part of TMG equations $\Longrightarrow$ R=6λ

traceless part $\Longrightarrow$ $\frac{1}{\mu} C_{\mu\nu} + \bar{\sigma} \tilde{R}_{\mu\nu} = 0$

The trace part of MMG equations $\Longrightarrow$ $R_{\mu\nu} R^{\mu\nu} - \frac{1}{24} R^2 + \frac{\mu^2}{\gamma} \bar{\sigma} R - 6 \frac{\mu^2}{\gamma} \lambda_0 = 0$

traceless part $\Longrightarrow$ $\frac{1}{\mu} C_{\mu\nu} + \bar{\sigma} \tilde{R}_{\mu\nu} + \frac{\gamma}{\mu^2} \tilde{J}_{\mu\nu} = 0$

A)TYPE-N

For Type-N space-times traceless Ricci tensor can be written as [4] $\tilde{R}_{\mu\nu} = \rho \xi_\mu \xi_\nu$
where ρ is a scalar function and $\xi\mu$ is a null vector(ξμξμ = 0) From the trace part of the MMG field equations, Ricci scalar is constant with two possible values and the traceless part of the field equation is:

$$\frac{1}{\mu} C_{\mu\nu} + (\bar{\sigma} - \frac{\gamma R}{12\mu^2}) \tilde{R}_{\mu\nu} = 0$$

which is nothing but the field equations of TMG with the simple replacement of the parameters as

$$\mu\bar{\sigma} \to \mu\bar{\sigma} - \frac{\gamma R}{12\mu}$$

B) TYPE –D

Type-D solutions split into two as Type-Dt and Type-Ds and both types have the traceless Ricci tensor as [5] $\tilde{R}_{\mu\nu} = p(g_{\mu\nu} - \frac{3}{a} \xi_\mu \xi_\nu)$, where $\xi\mu\xi\mu = \pm 1$, and p is a scalar function. Again we have constant curvature scalar with two possible solutions. Reducing the MMG equation to the TMG equation as,

$$\frac{1}{\mu} C_{\mu\nu} + (\bar{\sigma} - (\frac{\gamma}{\mu^2} p + \frac{R}{12})) \tilde{R}_{\mu\nu} = 0$$

which means all Type-D solutions of TMG solve MMG once the following replacement is made

$$\mu\bar{\sigma} \to \mu\bar{\sigma} - \frac{\gamma}{\mu}(p + \frac{R}{12})$$

Finally, let us note that the following restricted version of the general Kundt solution of TMG [6] also solves MMG.

$$ds^2 = 2dudv + (\frac{1}{2} R - \frac{1}{9} \mu^2 \bar{\sigma}^2) v^2 du^2 + (d\rho + \frac{2}{3} \mu\bar{\sigma} v du)^2 + du^2$$

C) TYPE-O

For Types O, N, III one has $\tilde{R}^\nu_\mu \tilde{R}^\mu_\nu = \tilde{R}^\nu_\mu \tilde{R}^\alpha_\nu \tilde{R}^\mu_\alpha = 0$. Let us first consider all the solutions of MMG that satisfy $\tilde{J}^{\mu\nu} = 0$,which boils down to all the solutions of TMG that has this property. Clearly Type-O solutions of TMG for which the canonical form of the traceless-Ricci and traceless-J tensor vanish, hence all such solutions of TMG, which are locally Einstein spaces, also solve MMG.

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“Inflationary perturbations in the Lifshitz regime of Horava gravity”

by Shun Arai

[JGRG25(2015)P02]

Inflationary perturbations in the Lifshitz regime of Horava gravity

Shun Arai (Nagoya University) arai.shun@a.mbox.nagoya-u.ac.jp
collaborators: Sergey Sibiryakov (CERN, INR RAS), Yuko Urakawa(Nagoya University)

Abstract

We study the evolution of scalar perturbations in inflationary epoch with a single Lifshitz scalar in the context of the BPSH theory, which generalizes the original non-projectable Horava Lifshitz gravity. In the previous studies, power spectrum of the curvature in the fixed de-Sitter background, the evolution in the IR regime and the evolution the isocurvature mode in the Einstein Aether theory have been obtained. In our study, we consistently solve the evolution of the coupled scalar graviton and the inflaton fluctuation. We will derive the curvature power spectrum and discuss the observational constraints on the Lorentz violation scale.

1.Introduction

Horava Lifshitz gravity (HL gravity) is the one of the modified theory of gravity which is **power-counting renormalizable** in the UV regime(*P.Horava 2009*).

GR

$$S_{EH}=\frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}R\longrightarrow\frac{M_{\rm pl}^2}{2}\int \underbrace{d^4x}_{\mathrm{dim}h=1}\{(\partial h)^2+\underbrace{h(\partial h)^2}_{\text{irrelevant term}}+\cdots\}$$

HL gravity

$$S_{HL}=\frac{M_{\rm pl}^2}{2}\int dtd^3x\sqrt{\gamma}N\{(K_{ij}K^{ij}-\lambda K^2)+R^{(3)}+\mathcal{V}\}$$

$\mathcal{V}$: $\Delta R, R^2, R_{ij}R^{ij}, (\nabla_i R)^2, (\nabla_i R_{jk})^2$

anisotropic scaling

$$\boxed{t\to b^z t,\ \mathbf{x}\to b\mathbf{x}}\Longrightarrow h\to b^{(3-z)/2}h \quad \mathrm{dim}h=\frac{3-z}{2}\le 0\ (z\ge 3)$$

But the anisotropic scaling law **violates the local Lorentz symmetry** in the UV regime. Then a new scalar degree of freedom appears called "***scalar graviton***" or "***khronon***".

Purposes of our study

- **Investigate consistent growth of the scalar graviton and the inflaton fluctuation in the Lifshitz regime.**
- **Derive the curvature perturbation and isocurvature perturbation.**
- **Compare the curvature & isocurvature power spectra with current CMB observation to constrain to LV scale and modulations of higher derivatives.**

2.Formulation

We introduce the general action of the BPSH theory including the transition energy scale **M_*** where **the local Lorentz symmetry is broken.**

Gravity sector

$$S_{HL}=\int dtd^3xN\sqrt{\gamma}\frac{M_*^2}{2}\left[\frac{1}{\alpha_1}K_{ij}K^{ij}-\frac{1}{\alpha_2}K^2+\frac{1}{\alpha_3}R+a_ia^i\right]$$
$$-\frac{1}{2}\left[\frac{R_{ij}R^{ij}}{\beta_1}+\frac{R^2}{\beta_2}-\frac{R\nabla_ia^i}{\beta_3}+\frac{a_i\Delta a^i}{\beta_4}\right]$$
$$-\frac{1}{2M_*^2}\left[\frac{(\nabla_iR_{jk})^2}{\gamma_1}+\frac{(\nabla_iR)^2}{\gamma_2}-\frac{\Delta R\nabla_ia^i}{\gamma_3}-\frac{a_i\Delta^2a^i}{\gamma_4}\right]$$

$ds^2=-N^2dt^2+\gamma_{ij}(dx^i+N^idt)(dx^j+N^jdt)$
$K_{ij}=\frac{1}{2N}(\dot\gamma_{ij}-\nabla_iN_j-\nabla_jN_i)$
$a_i=\partial_i\ln N$ non-projectable condition
R_{ij} 3-dimensional Ricci tensor
Δ 3-dimensional Laplacian

Inflaton sector

$$S_\Phi=\frac{1}{2\kappa^2}\int dtd^3\mathbf{x}N\sqrt{g}\left[\frac{(\dot\Phi-N^i\partial_i\Phi)^2}{2N^2}+\kappa_1\frac{\Phi\Delta\Phi}{2}-\kappa_2\frac{\Phi\Delta^2\Phi}{2{M_*}^2}+\kappa_3\frac{\Phi\Delta^3\Phi}{2{M_*}^4}-V(\Phi)\right].$$

Inflaton is a single Lifshitz scalar field and thus higher derivative terms appear in the UV regime.

Then we take the metric ansatz of perturbation

Metric ansatz

$$\boxed{N=e^\phi, N_i=\partial_iB, \gamma_{ij}=a^2e^{2\mathcal{R}}\delta_{ij}}$$

momentum & energy constraints → $\mathcal{R}$: scalar graviton φ : inflaton fluctuation

Parameter hierarchy(set up)

$M_*^2=\alpha_1M_{\rm pl}^2$
$\alpha_1-\alpha_2=2\alpha_1\bar\alpha=\mathcal{O}(\alpha_1^2)$
$\alpha_1/\alpha_3>0$
$\kappa_{1,2,3}=\mathcal{O}(1)$
$\beta_{1,2,3}\sim\gamma_{1,2,3}\sim\alpha_1$
$\beta_4,\gamma_4\le 1$

GR limit (z=1) → $\bar\alpha=0$, $\alpha_1/\alpha_3=1$, $\kappa_1=1$

dispersion relations of spin 2 mode & scalar graviton are both $k\sim\omega\sim M_*$ in the UV

3.Results

$$X\equiv\left(\frac{pL}{a}\right)^{2z}$$
$$\underbrace{L=(HM_*^{z-1})^{-1/z}}_{\text{sound horizon}}$$

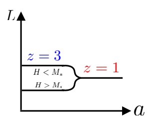

Fig1. Evolution of the sound horizon

3.1 WKB phase: $X\ge\frac{1}{\alpha_1\bar\alpha}$

$$\nu^2\equiv\left(\frac{3+2\varepsilon_1}{2\{z+(z-1)\varepsilon_1\}}\right)^2$$

$$\mathcal{R}=\sqrt{\frac{\alpha_1\bar\alpha}{2M_*^2}}e^{i\pi(\nu/2+/4)}\sqrt{\frac{\pi}{4\{z+(z-1)\varepsilon_1\}}}(1-\varepsilon_1)H(-\eta)^{3/2}H_\nu^{(1)}\left(\frac{1+\varepsilon_1}{z+(z-1)\varepsilon_1}\sqrt{\frac{2\bar\alpha X}{\alpha_1}}\right)$$

$$\varphi=e^{i\pi(\nu/2-3/4)}\sqrt{\frac{\pi}{4\{z+(z-1)\varepsilon_1\}}}(1-\varepsilon_1)H(-\eta)^{3/2}H_\nu^{(1)}\left(\frac{1+\varepsilon_1}{z+(z-1)\varepsilon_1}\sqrt{X}\right)$$

3.2 Intermediate phase: $\mathcal{O}(\varepsilon_1)<X<1$

$$\mathcal{R}=\sqrt{\frac{\alpha_1}{2M_*^2}}(\sqrt{\varepsilon_1}\psi_++\psi_-)\qquad\varphi=-\psi_++\sqrt{\varepsilon_1}\psi_-$$

$$\psi_+(\eta)\simeq e^{i\pi(\nu/2-1/4)}2^{\nu-2}\{z+(z-1)\varepsilon_1\}^{\nu-1/2}{\lambda_+}^{(\nu-1)/2}(1-\varepsilon_1)^{\nu+1}\frac{\Gamma(\nu)}{\Gamma(3/2)}H(-\eta)^{3/2}X^{-\frac{\nu}{2}}$$

$$\psi_-(\eta)=(2\alpha_1)^{-1/4}(1-\varepsilon_1)^{3/2}H(-\eta)^{3/2}\{-(1-\varepsilon_1)H\eta\}^{i\sqrt{2/\alpha_1}}X^{-1/2}$$

3.3 At large scale: $X\ll 1$

Large scale fluctuations decouple into a conserved mode and an isocurvature mode

$$\zeta=\mathrm{const.}$$

$$\varphi=-\frac{\dot\Phi}{H}\zeta+\underbrace{A_me^{i\int\frac{d\eta\mathcal{H}}{\sqrt{\alpha_1}}}\left(\frac{a}{a_*}\right)^{-(3-2z+2\varepsilon_1)/2}}_{\text{isocurvature mode}}$$

$z=3$ **→exponentially grow !**

4.Summary & Future works

We have obtained the solutions of the scalar graviton and inflaton fluctuation. But our analysis has yet to complete among the regime where the curvature fluctuation conserves. Therefore we will have to **calculate the perturbations in the whole regimes** and obtain the curvature power spectrum. Then we will obtain **constraints of the LV scale and modulations of higher derivatives** by using the PLANCK data.

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“Perturbations of Kasner-de Sitter spacetime”

by Yu Furuya

[JGRG25(2015)P04]

Perturbations of Kasner-de Sitter spacetime

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Abstract: We study perturbations of Kasner-de Sitter (KdS) spacetimes without axisymmetry to explore observational signatures of the isotropization process of inflation.We discuss directional dependences of the growth factor of gravitational waves relative to the axes of anisotropic expansion

1 Introduction

The early universe is thought to have begin in neither spatially homogeneous nor isotropic state. Kasner-de Sitter (KdS) spacetime, belonging to Bianchi type I, is a simple inflation model in which the early anisotropic expansion isotropizes due to a positive cosmological constant Λ. A gauge-invariant formulation for the perturbations of Bianchi I universe was constructed by Pereira et al. [1]. Time evolution of gravitational waves in axisymmetric Kasner-like expansion case was studied by Gümrükçüoğlu et al. [2]. We discuss directional dependences of the growth factor of gravitational waves relative to the axes of anisotropic expansion.

2 Gauge-invariant formalism for the perturbations of Bianchi I spacetimes by Pereira et al. [1]

General form of Bianchi I background metric

$$ds^2 = e^{2\alpha(\eta)}[-d\eta^2 + \gamma_{ij}dx^i dx^j], \quad \gamma_{ij} \equiv e^{2\beta_i(\eta)}\delta_{ij}, \quad \sum_{i=1}^{3}\beta_i = 0.$$

e^α: scale factor, γ_{ij}: spatial metric.

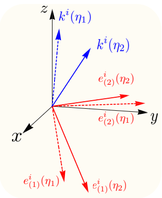
Shear tensor

$$\sigma_{ij} \equiv \frac{1}{2}(\gamma_{ij})' = \beta_i' \gamma_{ij}.$$

Polarization basis

$$\vec{k} : \text{wavevector}, \ k_i = \text{const}, \ k^i \equiv \gamma^{ij}k_j, \ k^2 \equiv k^i k_i, \ \hat{k}^i \equiv \frac{k^i}{k}$$

Because γ^{ij} depends on the time, k^i ($\equiv \gamma^{ij}k_j$) does so as well.



$\vec{k}$ is orthogonal to $\vec{e}_a$.

$$\epsilon^\lambda_{ij} = \frac{e^{(1)}_i e^{(1)}_j - e^{(2)}_i e^{(2)}_j}{\sqrt{2}}\delta^\lambda_+ + \frac{e^{(1)}_i e^{(2)}_j + e^{(2)}_i e^{(1)}_j}{\sqrt{2}}\delta^\lambda_\times,$$

$$\epsilon^\lambda_{ij}k^i = \epsilon^\lambda_{ij}\gamma^{ij} = \epsilon^\lambda_{[ij]} = 0, \ \epsilon^\lambda_{ij}\epsilon^{ij}_\mu = \delta^\lambda_\mu.$$

e^i_a $(a = (1),(2))$: vector basis, ϵ^{ij}_λ ($\lambda = +$: plus mode, $\times$: cross mode) : tensor basis

Based on the above construction, we can decompose the shear tensor into the scalar, vector and tensor components: $(\sigma_\parallel, \sigma_{\mathrm{V}a}, \sigma_{\mathrm{T}\lambda})$.

$$\sigma_{ij} = \frac{3}{2}\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\gamma_{ij}\right)\sigma_\parallel + 2\hat{k}_{(i}\sum_{a=(1),(2)} e^a_{j)}\sigma_{\mathrm{V}a} + \sum_{\lambda=+,\times}\epsilon^\lambda_{ij}\sigma_{\mathrm{T}\lambda}.$$

Perturbed metric in the "conformal Newtonian gauge"

$$(g_{\mu\nu} + \delta g_{\mu\nu})dx^\mu dx^\nu = e^{2\alpha}\left[-(1+2\Phi)d\eta^2 + 2\Phi_i dx^i d\eta + (\gamma_{ij} + h_{ij})dx^i dx^j\right],$$

$$h_{ij} = 2E_{ij} - 2(\gamma_{ij} + \Sigma_{ij})\Psi, \quad \Sigma_{ij} \equiv \frac{\sigma_{ij}}{\mathcal{H}}, \quad \mathcal{H} \equiv \alpha'.$$

$$(\partial_i\Phi^i = E_{[ij]} = \partial_i E^i{}_j = E^i{}_i = 0.)$$

$(\Phi, \Psi, \Phi_i, E_{ij})$ are the gauge-invariant variables.

Perturbed eoms for E_λ

$$E_{ij} = \sum_{\lambda=+,\times} E_\lambda \epsilon^\lambda_{ij},$$

$$\delta G^\mu{}_\nu = 0. \ (\because \text{GR + no matter})$$

Eventually, after eliminating Φ, Ψ and Φ_i, the eoms for gravitational waves are obtained as

$$E_\lambda'' + 2\mathcal{H}E_\lambda' + k^2E_\lambda - 2\eta_{\lambda(1-\lambda)}\sigma_{\mathrm{T}(1-\lambda)}\sum_\mu \eta_{\mu(1-\mu)}\sigma_{\mathrm{T}(1-\mu)}E_\mu$$
$$+ \sigma_{\mathrm{T}\lambda}\frac{12\mathcal{H}^2 - 2\sigma^2 - 2e^{-2\alpha}(e^{2\alpha}\sigma_\parallel)'}{(2\mathcal{H} - \sigma_\parallel)^2}\sum_\mu \sigma_{\mathrm{T}\mu}E_\mu$$
$$- \frac{2e^{-2\alpha}}{2\mathcal{H} - \sigma_\parallel}\sum_\mu [(e^{2\alpha}\sigma_{\mathrm{T}\lambda})' + (e^{2\alpha}\sigma_{\mathrm{T}\mu})'\sigma_{\mathrm{T}\lambda}]E_\mu - e^{-2\alpha}(e^{2\alpha}\sigma_\parallel)'E_\lambda = 0,$$

$$\eta_{\lambda\mu} \equiv \delta_{\lambda+}\delta_{\times\mu} - \delta_{\lambda\times}\delta_{+\mu}, \quad (1-\lambda) \equiv + \text{ if } \lambda = \times \text{ or } \times \text{ if } \lambda = +.$$

3 Gravitational waves in KdS without axisymmetry

KdS metric

$$ds^2 = -dt^2 + \sinh^{\frac{2}{3}}(3Ht)\sum_{i=1}^{3}\tanh^{2q_i}\left(\frac{3Ht}{2}\right)(dx^i)^2, \ \sum_i q_i = 0, \ \sum_i q_i^2 = \frac{2}{3},$$

$$q_1 = \frac{2}{3}\sin\left(\theta - \frac{2\pi}{3}\right), \ q_2 = \frac{2}{3}\sin\left(\theta - \frac{4\pi}{3}\right), \ q_3 = \frac{2}{3}\sin\theta,$$

$$e^\alpha = \sinh^{\frac{1}{3}}(3Ht), \quad \gamma_{ij} = \tanh^{2q_i}\left(\frac{3Ht}{2}\right)\delta_{ij}, \quad H \equiv \sqrt{\frac{\Lambda}{3}}.$$

Shear tensor

$$\sigma_{ij} = 3q_iH\frac{\tanh^{2q_i}\left(\frac{3Ht}{2}\right)}{\sinh^{\frac{2}{3}}(3Ht)}\delta_{ij}.$$

Other quantities

$$k^2 \equiv k^ik_i = \sum_i\left[\tanh^{-2q_i}\left(\frac{3Ht}{2}\right)(k_i)^2\right], \ \mathcal{H} = H\frac{\cosh(3Ht)}{\sinh^{\frac{2}{3}}(3Ht)}, \ \sigma_\parallel = \frac{3H}{e^{2\alpha}}\frac{\sum_i\left[q_i\tanh^{-2q_i}\left(\frac{3Ht}{2}\right)(k_i)^2\right]}{k^2},$$

$$\sigma_{\mathrm{V}(1)} = \frac{3H}{e^{2\alpha}}\frac{\sum_i\left[q_i\tanh^{-q_i}\left(\frac{3Ht}{2}\right)k_i\omega^i_{(1)}\right]}{k}, \quad \sigma_{\mathrm{V}(2)} = \frac{3H}{e^{2\alpha}}\frac{\sum_i\left[q_i\tanh^{-q_i}\left(\frac{3Ht}{2}\right)k_i\omega^i_{(2)}\right]}{k},$$

$$\sigma_{\mathrm{T}+} = \frac{3H}{\sqrt{2}e^{2\alpha}}\sum_i\left[q_i[(\omega^i_{(1)})^2 - (\omega^i_{(2)})^2]\right], \ \sigma_{\mathrm{T}\times} = \frac{6H}{\sqrt{2}e^{2\alpha}}\sum_i\left[q_i(\omega^i_{(1)}\omega^i_{(2)})\right],$$

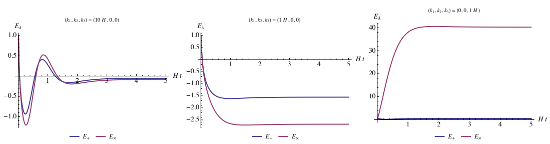
$$\omega^i_{(1)} = \begin{pmatrix} \cos\gamma\cos\beta\cos\alpha - \sin\alpha\sin\gamma \\ \cos\gamma\sin\alpha + \cos\alpha\cos\beta\sin\gamma \\ -\cos\alpha\sin\beta \end{pmatrix}, \ \omega^i_{(2)} = \begin{pmatrix} -\cos\gamma\cos\beta\sin\alpha - \cos\alpha\sin\gamma \\ \cos\gamma\cos\alpha - \sin\alpha\cos\beta\sin\gamma \\ \sin\alpha\sin\beta \end{pmatrix}.$$

(α, β, γ) are the Euler angles [3].

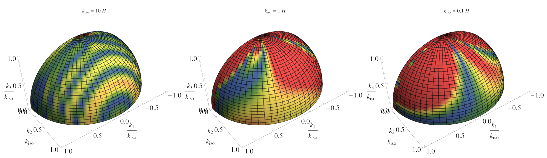
We integrate E_+ and $E_\times$ numerically substituting these quantities into the perturbed eoms for E_λ.

Results ($q_1 = -1/\sqrt{3}, \ q_2 = 1/\sqrt{3}, \ q_3 = 0$)

Time evolutions of E_+, $E_\times$ for some wavevectors are shown below.



Directional dependences of the growth of $E_\times$ on the wavevector (k_1, k_2, k_3) relative to the axes of anisotropic expansion. The color a exhibits relative growth factor $|E_\times(\infty)/E_\times(0)|$ (red: high, blue: low).



4 Conclusion/Discussion

- We could confirm that KdS spacetime without axisymmetry is stable, because amplitudes freeze in any direction.
- We need to consider initial conditions of eom for E_λ [3, 4].
- We might be able to probe anisotropy of the early universe if we can observe directional variation of the amplitude of gravitational waves over the whole sky.
- Application to Weyl gravity.

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"Effects of local features of the inflaton potential on the spectrum and bispectrum of primordial perturbations"

by Alexander Gallego

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Effects of local features of the inflaton potential on the spectrum and bispectrum of primordial perturbations

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Abstract

We study the effects of a class of features of the potential of slow-roll inflationary models corresponding to a step symmetrically dumped by an even power negative exponential factor. As a consequence, this type of features only affects the spectrum and bispectrum in a narrow range of scales which leave the horizon during the time interval corresponding to the modification of the potential.

We also compute the effects of the features on the CMB temperature spectrum. Due to the local nature of their effects, the features of this type could be used to model local glitches of the power spectrum without affecting other scales.

Introduction

An approximately scale invariant spectrum of curvature perturbation provides a good fit of CMB data. But recent analyses of the WMAP and Planck data have shown evidence of a feature around the scale $k = 0.002$ Mpc^{-1} in the power spectrum of primordial scalar fluctuations [1], that correspond to a dip in the CMB temperature spectrum at $l \simeq 20$. These features of the curvature perturbations spectrum provide an important observational motivation to find theoretical models able to explain it.

In this paper [2] we will consider the effects of local features (LF) which only modify the potential locally in field space, while leaving it unaffected sufficiently far from the feature. The important consequence is that also the effects of LF on the spectrum and bispectrum are local.

Inflation and the features

We consider inflationary models with a single scalar field governed by the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \quad (1)$$

where $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$ and the potential is given by [3]

$$V(\phi) = V_0(\phi) + V_F(\phi), \quad (2)$$
$$V_F(\phi) = \lambda e^{-\left(\frac{\phi-\phi_0}{\sigma}\right)^{2n}}, \; n > 0, \quad (3)$$

where V_0 is the featureless potential, and we call this type of modification of potential *local features* (LF). In this paper we will consider the case of power law inflation (PLI) to model the featureless behaviour $V_0(\phi) \propto \exp(-\phi/M_{Pl})$. While PLI is not in good agreement with CMB data due to high value of the predicted tensor-to-scalar ratio r, it can be used as good toy model to show qualitatively the general type of effects produced by LF. Future works may be devoted to test different potentials $V_0(\phi)$, for direct comparison with data. The definitions we use for the slow-roll parameters are

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H}. \quad (4)$$

Curvature perturbations

The linear equation of motion for the curvature perturbation ζ in Fourier space is given by [4]

$$\zeta_k'' + 2\frac{z'}{z}\zeta_k' + k^2\zeta_k = 0, \quad (5)$$

where $z \equiv a\sqrt{2\epsilon}$, k is the comoving wave number, and primes denote derivatives with respect to the conformal time $d\tau \equiv dt/a$. For the power spectrum of scalar perturbations we adopt the definition

$$P_\zeta(k) \equiv \frac{2k^3}{(2\pi)^2}|\zeta_k|^2. \quad (6)$$

Bispectrum of curvature perturbations

To study the non gaussianity we define a convenient quantity [5]

$$F_{NL}(k_1,k_2,k_3) \equiv \frac{10}{3(2\pi)^4}\frac{(k_1k_2k_3)^3}{k_1^3+k_2^3+k_3^3}\frac{B_\zeta}{P_\zeta^2}, \quad (7)$$

where k_* is the pivot scale at which the power spectrum is normalized, i.e. $P_\zeta(k_*) \approx 2.2 \times 10^{-9}$ and B_ζ is given by [5]

$$B_\zeta(k_1,k_2,k_3) = 2(2\pi)^3\Im\Big[\zeta_{k_1}(\tau_e)\zeta_{k_2}(\tau_e)\zeta_{k_3}(\tau_e) \int_{\tau_0}^{\tau_e} d\tau\eta\epsilon a^2\zeta_{k_1}^*(2\zeta_{k_2}'^*\zeta_{k_3}'^* - k_1^2\zeta_{k_2}^*\zeta_{k_3}^*) + \mathbf{2p}\Big], \quad (8)$$

where 2p means the two other permutations of k_1, k_2, and k_3. Our definition of F_{NL} reduces to the non linear parameter f_{NL} in the equilateral limit if the spectrum is approximately scale invariant. In this paper we study the equilateral limit of the bispectrum.

Effects of the parameter n

The parameter n is related to the dumping of the feature, and larger values are associated to a steeper change of the potential. As shown in fig.(1) ϵ, η, P_ζ, and F_{NL} show oscillations around the scale $k_0 = -1/\tau_0$ with an amplitude which increases for larger n.

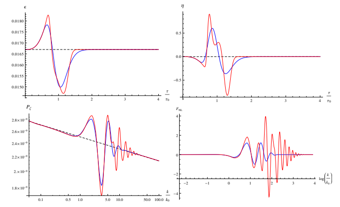

Figure 1: From left to right and top to bottom the numerically computed ϵ, η, P_ζ, and F_{NL} are plotted for $\lambda = 10^{-11}$, $\sigma = 0.05$ and $n = 1$ (blue) and $n = 2$ (red). The dashed black lines correspond to the featureless behavior.

Effects of the parameter σ

The parameter σ determines the size of the range of field values where the potential is affected by the feature. As shown in fig.(2) ϵ, η, P_ζ, and F_{NL} have oscillations around k_0, whose amplitude is larger for smaller σ, because in this case the potential changes faster.

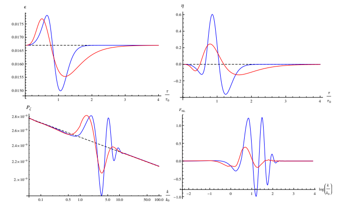

Figure 2: From left to right and top to bottom the numerically computed ϵ, η, P_ζ, and F_{NL} are plotted for $\lambda = 10^{-11}$, $\sigma = 0.05$ (blue) $\sigma = 0.1$ (red) and $n = 1$. The dashed black lines correspond to the featureless behavior.

Effects of the parameter λ

The parameter λ controls the magnitude of the potential modification. The sign of λ produce opposite and symmetric effects, since it implies an opposite sign for the derivative of the potential with respect to the field. As shown in figs.(3) and (4), larger absolute values of λ produce oscillations with larger amplitudes of ϵ, η, P_ζ, and F_{NL} around k_0.

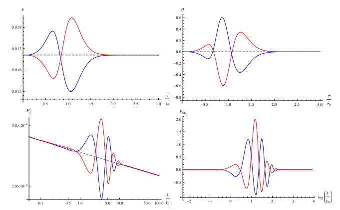

Figure 3: From left to right and top to bottom the numerically computed ϵ, η, P_ζ, and F_{NL} are plotted for $\lambda = 10^{-11}$ (blue) and $-\lambda = 10^{-11}$ (red), $\sigma = 0.05$ and $n = 1$. The dashed black lines correspond to the featureless behavior.

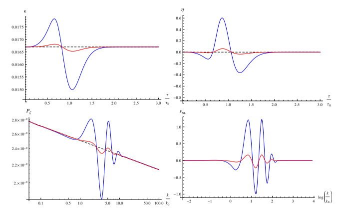

Figure 4: From left to right and top to bottom the numerically computed ϵ, η, P_ζ, and F_{NL} are plotted for $\lambda = 10^{-11}$ (blue) and $\lambda = 10^{-12}$ (red), $\sigma = 0.05$ and $n = 1$. The dashed black lines correspond to the featureless behavior.

Effects on the CMB temperature and polarization spectrum

To study how the feature impacts on the CMB spectra we modified CAMB to use the modified primordial power spectrum instead of the usual power-law expression $P_s(k) = A_s\,(k/k_*)^{n_s-1}$. In fig.(5) we show the CMB spectra obtained with different combinations of the parameters λ, σ and n: Feature A= 10^{-11}, $\sigma = 0.05$ and $n = 1$; Feature B= 10^{-11}, $\sigma = 0.05$ and $n = 2$; Feature C= -10^{-11}, $\sigma = 0.05$ and $n = 1$; Feature D= 10^{-11}, $\sigma = 0.1$ and $n = 1$; and Feature E= 10^{-12}, $\sigma = 0.05$ and $n = 1$. We show the spectra in terms of the quantity $D_\ell = \ell(\ell+1)C_\ell/(2\pi)$.

From the plot it is possible to see how the feature can change the predicted CMB spectra. In the TT spectrum the relative differences of the order of $10 - 15\%$ are visible. Choosing the values listed above for the parameters describing the feature, and $k_0 = 5 \cdot 10^{-4}$ Mpc^{-1}, $A_s = 2.2 \cdot 10^{-9}$ and $n_s = 0.967$ we can see from fig.(5) that the effects of some feature can partially reproduce the dip at $\ell \simeq 20$ in the TT spectrum.

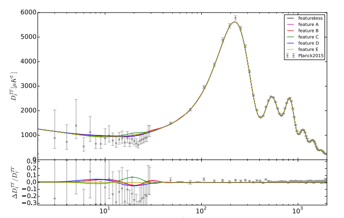

Figure 5: We plot the $D_\ell^{TT} = \ell(\ell+1)C_\ell^{TT}/(2\pi)$ spectra in units of μK^2 with respect to the multipole l, and the relative difference with respect to the featureless behavior. The solid black lines correspond to the featureless behavior.

Conclusions

- Local features only modify V in a limited range of the scalar field values, and consequently only affect P_ζ and B_ζ in a narrow range of scales.
- The P_ζ and B_ζ are affected by the feature, showing modulated oscillations which are dumped for scales larger or smaller than k_0.
- The effects of the features are larger when the potential modification is steeper, since in this case there is a stronger violation of the slow-roll conditions.
- We have shown that an appropriate choice of parameters can produce effects in qualitative agreement with the observational CMB data.
- Due to this local type effect these features could be used to model phenomenologically local glitches of the spectrum, without affecting other scales.

Forthcoming Research

Further studies involving data fitting can determine more accurately the values of the parameters which provide the best explanation of the observed deviation of the power spectrum from the power law form.

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[2] A. Gallego, A. Romano and S. Gariazzo. Effects of local features of the inflaton potential on the spectrum and bispectrum of primordial perturbations. 2015. arXiv:1508.05687 Submitted to EPJC.

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[5] A. Gallego and A. Romano. Effects of discontinuities of the derivatives of the inflaton potential. 2014. arXiv:1404.2985 Accepted for publication in EPJC.

“Large scale suppression with ultra slow-roll inflation scenario”

by Shinichi Hirano

[JGRG25(2015)P07]

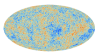
P07 Large scale suppression with Ultra Slow-Roll inflation scenario

Rikkyo Univ. Shinichi Hirano (M1) collaborators Tsutomu Kobayashi (Rikkyo)
Shuichiro Yokoyama (Rikkyo)

In preparation

§ Introduction

・ C_l of temperature fluctuation and Large scale suppression



P_ζ + Λ-CDM → **good prediction for almost observation date**

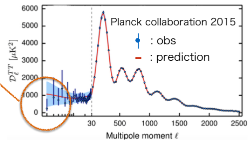
curvature perturbation (inflation) — Bing-Bang theory

But in C_l of CMB temperature fluctuation, it seems to be a suppression at large scale. One don't explain this suppression with standard inflation + Λ-CDM

It is called "Large scale suppression"

Models which explain this suppression

chaotic inflation, PBH remnants
topological defeats, solid inflation
hybrid inflation, …



・How to explain the suppression

curvature perturbation $\mathcal{R} \sim H\delta\phi/|\dot\phi|$ → $|\dot\phi|$ large → $P_\mathcal{R}$ is suppressed (C_l)

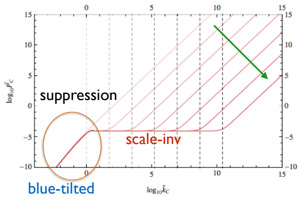
Adding some era before inflationary era → pre-inflation (ex: kinetic term dominat era)

§ Multi-stage and non-causal fluctuation

P. Chen, et al. (2015) ← they consider multi-stage inflation with canonical Lagrangian

Multi-stage inflation which can explain the suppression・・・

→ kinetic era (pre-inflation) + slow-roll era



kinetic era
||
decelerating expansion universe in canonical Lagrangian case
↓
non-causal fluctuation …
we don't solve three problems naturally

What is model for having an suppression and causal fluctuation ?

→ **Ultra Slow-Roll inflation** Inoue, Yokoyama (2002), Kinny (2005)

・inflaton potential is flat → Lagrangian has an shift symmetry w.r.t \phi

・slow-roll parameter $\eta = o(\epsilon) + \mathrm{const}$

→ constant term makes kinetic era and slow-roll era !

§ USR k- and G- inflation, large scale suppression

・k- and G-inflation

scalar-tensor theory

GR + scalar field (canonical)

↓ second-order EoM, non-canonical terms

inflation theory

k-essence, Galieon term → (inflaton) k- and G- inflation
C.A-Picon, et al. (1999)
T. Kobayashi, et al. (2010)

・USR k-inflation and G-inflation formalism

Lagrangian $\mathcal{L} = \frac{M_{\rm pl}^2}{2}R - V_0 + K(X) - G(X)\Box\phi$

Field equations
$$3M_{\rm pl}^2H^2 = V_0 - K + 2XF$$
$$M_{\rm pl}^2\dot H = -XF + XG_X\ddot\phi$$
$$F := K_X + 3G_XH\dot\phi\,,\quad \frac{d}{dt}\left(a^3\dot\phi F\right) = 0$$

now consider potential driven case $V_0 \gg |K|,\ |XF|$

・k-inflation case $(G=0)$

Our Lagrangian
$$\mathcal{L} = \frac{M_{pl}^2}{2}R + X + \frac{1}{M^{4n-4}}X^n - V_0$$
$$F = 1 + \frac{nX^{n-1}}{M^{4n-4}}\,,\quad \frac{d}{dt}\left[a^3\dot\phi\left(1+\frac{nX^{n-1}}{M^{4n-4}}\right)\right] = 0$$
$$3M_{\rm pl}H^2 = X + \frac{2n-1}{M^{4n-4}}X^n + V_0$$
$$M_{\rm pl}^2\dot H = -X - \frac{n}{M^{4n-4}}X^n$$

Slow-roll parameters
$$\epsilon_H := -\frac{\dot H}{H^2} = \frac{3\left(X + \frac{n}{M^{4n-4}}X^n\right)}{\left(X + \frac{2n-1}{M^{4n-4}}X^n + V_0\right)} \ll 1$$
$$\eta_H := \frac{\dot\epsilon_H}{H\epsilon_H} = 2\epsilon_H - 6\left[\frac{1+\frac{n^2}{M^{4n-4}}X^{n-1}}{1+\frac{n(2n-1)}{M^{4n-4}}X^{n-1}}\right]$$

Two era ① $X \ll \frac{1}{M^{4n-4}}X^n \Rightarrow \eta_H = 2\epsilon_H - \frac{6n}{2n-1}\,,\ c_s^2 \simeq \frac{1}{2n-1}$
② $X \gg \frac{1}{M^{4n-4}}X^n \Rightarrow \eta_H = 2\epsilon_H - 6\,,\ c_s^2 \simeq 1$

①, ② eras are quasi-de Sitter. → causal fluctuation !!

Quadratic action $S_\zeta^{(2)} = \frac{1}{2}\int d\tau d^3x\, z^2\left[\zeta'^2 - c_s^2(\partial\zeta)^2\right]$

Muhanov-Sasaki eq $u_k'' + \left(c_s^2k^2 - \frac{z''}{z}\right)u_k = 0,$
$$\frac{z''}{z} := \frac{1}{(-\tau)^2}\left[\left(2-\epsilon_H+\frac{\eta_H}{2}\right)\left(1+\frac{\eta_H}{2}\right)+\frac{\dot\eta_H}{2H}\right] = \frac{1}{(-\tau)^2}(2+q)$$

Powor spectrum in super-horizon scale
$$P_\zeta \simeq \frac{1}{2M_{\rm pl}^2\epsilon_H}2^{2\nu-3}\left(\frac{H}{2\pi}\right)^2\left[\frac{\Gamma(\nu)}{\Gamma(3/2)}\right]^2\left(\frac{c_sk}{aH}\right)^{3-2\nu},\quad \nu^2 = \frac{9}{4}+q$$

Era ①: $\nu \simeq \frac{3}{2(2n-1)} - \frac{5n+2}{3}\epsilon_H$
$$P_\zeta \simeq \frac{1}{2M_{\rm pl}^2\epsilon_H}\left(\frac{H}{2\pi}\right)^2 2^{-3(1-\frac{1}{2n-1})}\left[\frac{\Gamma(\frac{3}{2(2n-1)})}{\Gamma(3/2)}\right]^2\left(\frac{c_sk}{aH}\right)^{3(1-\frac{1}{2n-1})-\frac{2(5n+2)}{3}\epsilon_H}$$
blue-tilted

Era②: $\nu \simeq \frac{3}{2} - \frac{7}{3}\epsilon_H$
$$P_\zeta \simeq \frac{1}{2M_{\rm pl}^2\epsilon_H}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-\frac{14}{3}\epsilon_H}$$
scale-inv

similar behavior as kinetic era + slow-roll era

we can induce the suppression !!

・USR G-inflation driven by potential

Our Lagrangian $\mathcal{L} = \frac{M_{\rm pl}^2}{2}R + X - V_0 - \frac{1}{M^{4n-1}}X^n\ddot\phi$

Field equations
$$F = 1 + \frac{3n}{M^{4n-1}}H\dot\phi X^{n-1},\quad \frac{d}{dt}\left[a^3\left(\dot\phi + \frac{6n}{M^{4n-1}}HX^n\right)\right] = 0$$
$$3M_{\rm pl}^2H^2 = V_0 + X + \frac{6n}{M^{4n-1}}HX^n\dot\phi$$
$$M_{\rm pl}^2\dot H = -X - \frac{3n}{M^{4n-1}}H\dot\phi X^n + \frac{n}{M^{4n-1}}X^n\ddot\phi$$

Quadratic action
$$S_\zeta^{(2)} = \frac{1}{2}\int d\tau d^3x\, z_G^2\left[\zeta'^2 - c_s^2(\partial\zeta)^2\right]$$
$$c_s^2 := \frac{\epsilon_s}{q_s}\,,\ z_G := a\sqrt{2M_{\rm pl}^2q_s}$$
$$q_s := \delta_{PX} + 2\delta_{PXX} + 6\delta_{GX} + 6\delta_{GXX}$$
$$\epsilon_s := \delta_{PX} + 4\delta_{GX}$$

Slow-roll variables $\delta_{KX} := 3\frac{XK_X}{3M_{\rm pl}^2H^2} = \frac{3X}{3M_{\rm pl}^2H^2} > 0\,,\ \delta_{KXX} := 3\frac{X^2K_{XX}}{3M_{\rm pl^2}H^2} = 0$
$$\delta_{GX} := 3\frac{G_X\dot\phi XH}{3M_{\rm pl}^2H^2} = \frac{3n}{M^{4n-1}}\frac{\dot\phi HX^n}{3M_{\rm pl}^2H^2}\,,\ \delta_{GXX} := 3\frac{G_{XX}\dot\phi XH}{3M_{\rm pl}^2H^2} = (n-1)\delta_{GX}.$$

Usual slow-roll para $\epsilon_H := -\frac{\dot H}{H^2} = \delta_{PX} + 3\delta_{GX}\,,\ \eta_H := \frac{\dot\epsilon_H}{H\epsilon_H}$

Condition for avoiding ghost and Laplacian instability $\dot\phi > 0$

Two eras ① $\dot\phi \ll \frac{1}{M^{4n-1}}HX^n \quad c_s^2 \simeq \frac{2}{3n}\,,\ \delta_{KX} \ll \delta_{GX},\ \epsilon_H \simeq \delta_{KX}$
② $\dot\phi \gg \frac{1}{M^{4n-1}}HX^n \quad c_s^2 \simeq 1\,,\ \delta_{KX} \gg \delta_{GX},\ \epsilon_H \simeq 3\delta_{GX}$

Muhanov-Sasaki eq $u'' + \left(c_s^2k^2 - \frac{z_G''}{z_G}\right)u = 0\,,$
$$\frac{z_G''}{z_G} = (aH)^2\left[2 + \frac{3\eta_H}{2} + \frac{\eta_H^2}{4} - 2\epsilon_H - \frac{1}{2}\epsilon_H\eta_H\right] = \frac{1}{(-\tau)^2}(2+q)$$

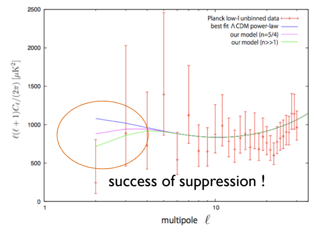
This equation is same form as k-inflation case.
In both cases , we can take a solution by using k-inflation solution.

Era①: $\nu \simeq \frac{3}{4n} + \frac{n(9-4n)}{9}\epsilon_H$
$$P_\zeta \simeq \frac{1}{2M_{\rm pl}^2\epsilon_H}\left(\frac{H}{2\pi}\right)^2 2^{-3(1-\frac{1}{2n})}\left[\frac{\Gamma(\frac{3}{4n})}{\Gamma(3/2)}\right]^2\left(\frac{c_sk}{aH}\right)^{3(1-\frac{1}{2n})-\frac{2n(9-4n)}{9}\epsilon_H}$$
blue-tilted

Era②: $\nu \simeq \frac{3}{2} + \frac{4}{3}\epsilon_H$
$$P_\zeta \simeq \frac{1}{2M_{\rm pl}^2\epsilon_H}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-\frac{8}{3}\epsilon_H}$$
scale-inv

So we can also induce the suppression !!

By using "Class" of Boltzmann code, we sketch C_l .



§ Summary & future direction

〈 Summary 〉

We consider USR k- and G-inflation driven by potential. In certain Lagrangian cases, we can construct the models which explain large scale suppression of C_l .

〈 Future direction〉

・Blue tensor

In USR k- and G-inflation driven by kinetic term, we consider the phase slightly away from attractor. Then it may be possible for us to take $\epsilon < 0$ without having ghost and Laplacian instability.

・Large scale oscillation in C_l

c_s changes at transition. So it may be possible to explain this oscillation in power spectrum by numerical calculations.

“An Alternative Approach to Black Hole Thermodynamics: Renyi Entropy and Phase Transition”

by Hideo Iguchi

[JGRG25(2015)P08]

JGRG25 P08

An Alternative Approach to Black Hole Thermodynamics: Rényi Entropy and Phase Transition

(based on arXiv:1511.06963/Phys. Lett. B752, 306-310 (2016))

Viktor G. Czinner (Nihon University)

Hideo Iguchi (Nihon University)

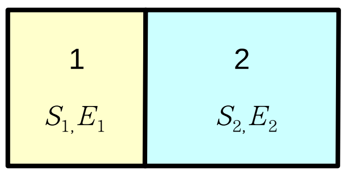
We study the thermodynamic stability problem of Schwarzschild black holes described by the Rényi formula which is an equilibrium compatible entropy function. It is shown that Schwarzschild black holes can be in stable equilibrium with thermal radiation at fixed temperature within this approach. This implies that the canonical ensemble exists just like in AdS space, and nonextensive effects can stabilize the black holes in a very similar way as it is done by the gravitational potential of an AdS space. It is also shown that a Hawking--Page-like black hole phase transition occurs at a critical temperature.

T. S. Biró and P. Ván, PRE83, 061147(2011)

Zeroth law (extensive)

Energy (extensive)

$$E_{12}(E_1, E_2) = E_1 + E_2$$



Entropy (extensive)

$$S_{12}(E_1, E_2) = S_1(E_1) + S_2(E_2)$$

Equilibrium → Maximum entropy principle (+ energy conservation)

$$dS_{12}(E_1, E_2) = \frac{\partial S_1}{\partial E_1} dE_1 + \frac{\partial S_2}{\partial E_2} dE_2 = (S'_1 - S'_2)\, dE_1 = 0$$

$\longrightarrow$ $S'_1(E_1) = S'_2(E_2)$ (factorize)

$\longrightarrow$ temperature $\frac{1}{T} = S'(E)$

T. S. Biró and P. Ván, PRE83, 061147(2011)

Zeroth law (non-extensive)

Composition rule

$$E_{12}=E_{12}(E_1,E_2) \qquad S_{12}=S_{12}(S_1,S_2)$$

EOS

$$S_1(E_1),S_2(E_2)$$

Maximum entropy principle

$$dS_{12}=\frac{\partial S_{12}}{\partial S_1}S_1'dE_1+\frac{\partial S_{12}}{\partial S_2}S_2'dE_2=0$$

$$dE_{12}=\frac{\partial E_{12}}{\partial E_1}dE_1+\frac{\partial E_{12}}{\partial E_2}dE_2=0$$

$$\Rightarrow \quad \frac{\partial E_{12}}{\partial E_2}\frac{\partial S_{12}}{\partial S_1}S_1'=\frac{\partial E_{12}}{\partial E_1}\frac{\partial S_{12}}{\partial S_2}S_2'$$

Formal logarithm

$$\boxed{\begin{aligned} L_{12}(E_{12})&=L_1(E_1)+L_2(E_2)\\ \hat{L}_{12}(S_{12})&=\hat{L}_1(S_1)+\hat{L}_2(S_2)\end{aligned}} \quad \Leftrightarrow \quad \begin{aligned} E_{12}&=\Phi(L_1+L_2)\\ S_{12}&=\Psi(\hat{L}_1+\hat{L}_2)\end{aligned}$$

$$\frac{\partial \hat{L}_1(S_1)}{\partial L_1(E_1)}=\frac{\partial \hat{L}_2(S_2)}{\partial L_2(E_2)} \quad \Rightarrow \quad \text{temperature} \quad \frac{1}{T}=\frac{\partial \hat{L}(S)}{\partial L(E)}$$

(factorize)

Abe formula and formal logarithm

Abe formula (the most general non-additive entropy composition rule)

S. Abe, PRE63, 061105 (2001)

$$H_\lambda(S_{12})=H_\lambda(S_1)+H_\lambda(S_2)+\lambda H_\lambda(S_1)H_\lambda(S_2)$$

Formal logarithm – zeroth law compatible entropy function

T. S. Biró and P. Ván, PRE83, 061147(2011)

$$L(S)=\frac{1}{\lambda}\ln(1+\lambda H_\lambda(S))$$

Origin of parameter λ

- Finite size reservoir corrections in the canonical approach

  T. S. Biró, Physica A 392, 3132 – 3139 (2013)
  T. S. Biró et. al., Eur. Phys. J. A 49, 110 (2013)

- Quantum corrections to micro black holes

  T. Biro and V. G. Czinner, Phys. Lett. B 726, 861-865 (2013)

Tsallis and Rényi entropy

T. S. Biró and P. Ván, PRE83, 061147(2011)

Non-additive composition rule (Tsallis) $H_\lambda(S)=S$ (leading order of λ)

$$S_{12}=S_1+S_2+\lambda S_1 S_2$$

Formal logarithm

$$1+\lambda S_{12}=(1+\lambda S_1)(1+\lambda S_2) \longrightarrow \hat{L}(S)=\frac{1}{\lambda}\ln(1+\lambda S)$$

Tsallis entropy

$$\boxed{S_T=\frac{1}{1-q}\sum_i(p_i^q-p_i)}=\frac{1}{\lambda}\left(\sum_i p_i^{1-\lambda}-p_i\right) \qquad \text{where} \quad \lambda=1-q$$

Formal logarithm

Rényi entropy

$$\hat{L}(S_T)=\frac{1}{\lambda}\ln(1+\lambda S_T)=\frac{1}{\lambda}\ln\left(\sum_i p_i^{1-\lambda}\right)=\boxed{\frac{1}{1-q}\ln\left(\sum_i p_i^q\right)=S_R}$$

Schwarzschild Black Hole in Renyi model

T. Biro and V. G. Czinner, Phys. Lett. B726 861-865 (2013)
V. G. Czinner and HI, Phys. Lett. B752, 306-310(2016)

Entropy $S=\frac{A}{4}=4\pi M^2$ $\longrightarrow$ Non-additive – Tsallis composition law $S=S_T$

Energy $E=M$ $\longrightarrow$ additive

Rényi entropy
(formal logarithm of S_T)

$$S_R=\frac{1}{\lambda}\ln(1+\lambda S_T)=\frac{1}{\lambda}\ln(1+4\pi\lambda M^2)$$

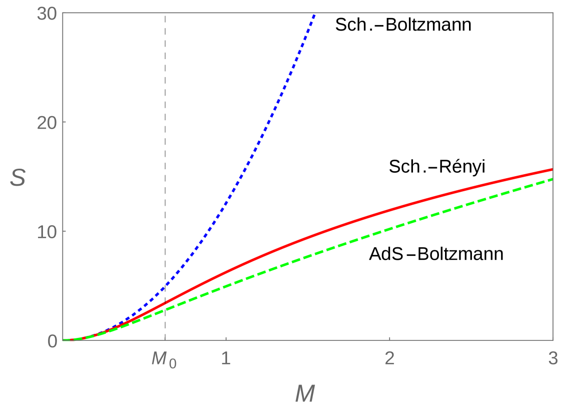
Temperature $\frac{1}{T_R}=S'_R(M)=\frac{8\pi M}{1+4\pi\lambda M^2} \xrightarrow{\lambda\to 0} \frac{1}{T}=8\pi M$

Heat capacity $C_R=\frac{-S'^2_R(M)}{S''_R(M)}=\frac{8\pi M^2}{4\pi\lambda M^2-1} \xrightarrow{\lambda\to 0} C=-8\pi M^2$

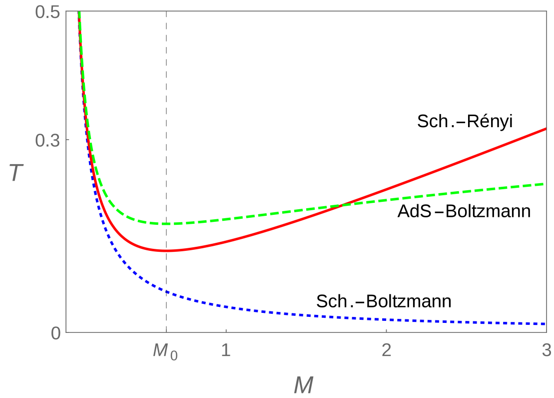
(Standard Boltzmann)

Thermodynamic property

Entropy v.s. Energy



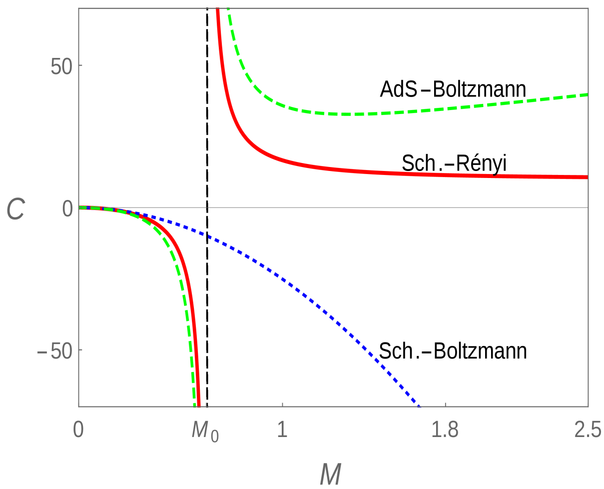
Temperature v.s. Energy



- Minimum temperature
- Similarity (correspondence?) with AdS BH

$$\lambda \Leftrightarrow \frac{3}{\pi l^2}$$

Heat capacity



$M < M_0$ Lower mass — $C_R < 0$ Negative heat capacity

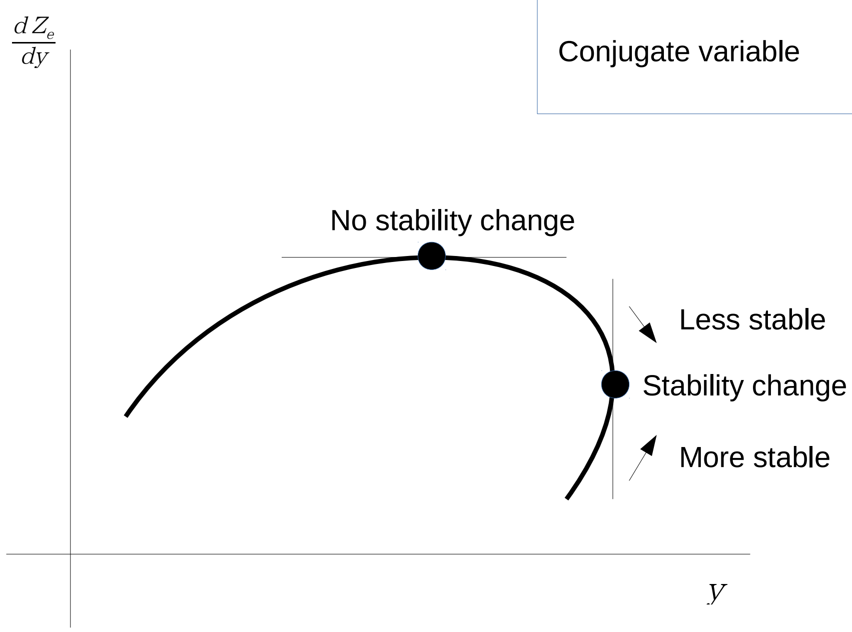
$M > M_0$ Higher mass — $C_R > 0$ Positive heat capacity

O. Kaburaki, I. Okamoto and J. Katz, PRD47, 2234(1993)

Turnig point (Poincaré) method

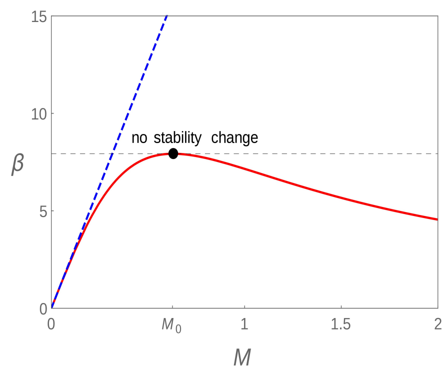
Massieu function	$Z_e(y)$
Control parameter	y
Conjugate variable	$\frac{dZ_e}{dy}$

Stability curve

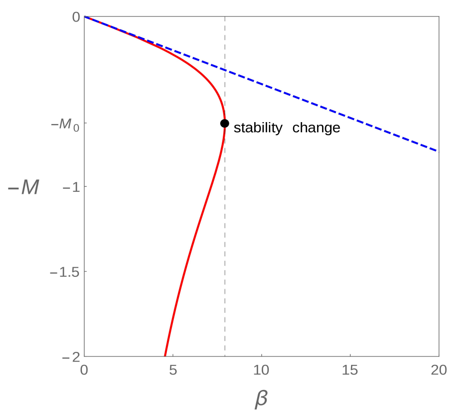


Stability curves

	microcanonical	canonical
Massieu function	S	$S-\beta M=-\beta F$
Control parameter	M	β
Conjugate variable	$\beta(M)=\frac{\partial S}{\partial M}$	$-M(\beta)$



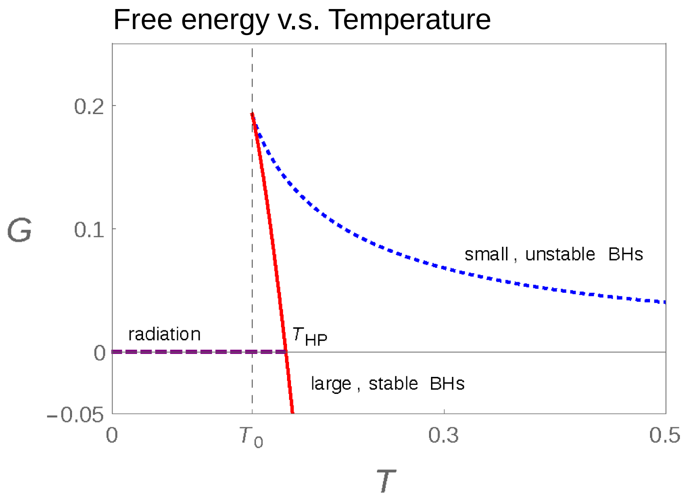
Isolated BH is stable.



Stability change occurs.

Small mass – unstable
Large mass – stable

Phase transition



$T<T_{HP}$	Pure radiation
$T>T_{HP}$	Large and stable black hole configuration

Summary

We investigated thermodynamics of black hole in the parametric extended Rényi entropy formula

- T_R has a minimum
- Hawking-Page-like Phase transition
- Stability change – stable equilibrium with thermal radiation
- Similarity with AdS BH

Future work

- Kerr black hole
- Another composition rule
- Non-extensive energy
- Higher dimensional BH

“Spherical symmetric domain wall collapse by numerical simulation”

by Taishi Ikeda

[JGRG25(2015)P09]

P09 Spherical symmetric domain wall collapse by numerical simulation

○ Taishi Ikeda
Chulmoon Yoo (Nagoya Univ.)

Introduction

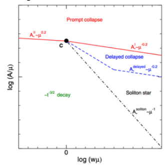
- The domain wall is one of the topological defect.
- It might exit in early universe, and become the Primordial BH by gravitational collapse.
- In this study, we research the property of domain wall collapse.
- Furthermore, by detailed numerical simulation, we will discuss whether the critical behavior is found.

- As in the case of massive scalar field, there may be rich behavior.
- Critical behavior of gravitational collapse
  - type 1 :

$$M \sim M_* \neq 0$$

  - type 2 :

$$M \sim |p - p_*|^\beta$$



phase diagram In the case of massive scalar[1]

Set up

- As suggested in ref[1], we consider the following metric and extrinsic curvature:

$$ds^2 = -\alpha^2 dt^2 + \psi^4 \eta_{ij} dx^i dx^j$$
$$K_{ij} = \frac{1}{3}\psi^4 \eta_{ij} K$$

where η_{ij} is the Minkowski 3-metric in spherical coordinate.

- Then, from the Einstein equation, we can get the following equations :

$$\dot{\psi} = -\frac{1}{6}\alpha\psi K$$
$$\dot{K} = -\frac{\Delta\alpha}{\psi^4} - \frac{2}{\psi^5}\psi'\alpha' + \frac{1}{3}\alpha K^2 + 4\pi\alpha(2\Pi^2 - 2V(\Phi))$$
$$\dot{\Phi} = -\alpha\Pi$$
$$\dot{\Pi} = \alpha\Pi K - \frac{\alpha}{\psi^4}\Delta\Phi - \frac{\alpha'\Phi'}{\psi^4} - \frac{2\alpha}{\psi^5}\psi'\Phi' + \alpha V'(\Phi)$$
$$\frac{\Delta\psi}{\psi^5} - \frac{K^2}{12} + \pi\{\Pi^2 + \psi^{-4}\Phi'^2 + 2V(\Phi)\} = 0$$
$$\frac{2}{3}K' + 8\pi\Pi\Phi' = 0$$

where $\quad V(\Phi) = -\frac{\mu^2}{2}\Phi^2 + \frac{\lambda}{24}\Phi^4 + V_0$

- We solve the above equations by numerical calculation.
  - spatial derivative : 4th order accurate
  - integration in time : 4th order Runge-Kutta scheme
- Initial data :

$$K(r,t=0) = 0,\ \Pi(r,t=0) = 0,\ \alpha(r,t=0) = 1 \quad \text{(momentary static)}$$

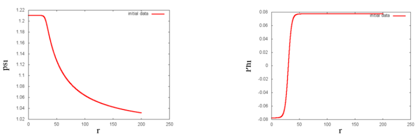
$$\phi(r,t=0) = \sigma\tanh\left(\frac{r-r_0}{l}\right)$$

r_0 : position of domain wall

l : width of domain wall

$$\sigma = \sqrt{\frac{6\mu^2}{\lambda}} \qquad l = \left(\frac{\lambda}{2}\right)^{-1/2}\sigma^{-1}$$

- Boundary condition : asymptotically Schwarzschild space time

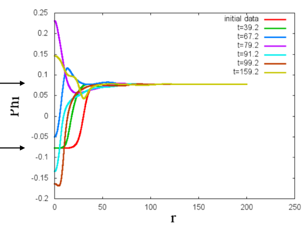


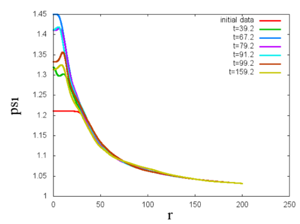
initial profile

Numerical simulation

σ

$-\sigma$





$\lambda = 10.0, \mu^2 = 0.01 \quad M_{ADM} = 12.7$

$r_0 = 30 \quad$ no apparent horizon

Conclusion and discussion

- We calculate the time evolution of domain wall collapse.
- We showed the example of disperse of scalar filed.

Future work

- The above metirc is approximate ansatz.
- In general, we must use the following ansatz :

$$ds^2 = -\alpha^2 dt^2 + \psi^4\{\gamma^{-2}(dr + r\beta dt)^2 + \gamma r^2 d\Omega^2)\}$$

- Furthermore, traceless part of extrinsic curvature does not vanish.

$$A^i_j = K^i_j - \frac{1}{3}K\delta^i_j \neq 0$$

- We will calculate the time evolution under these ansatz.

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“Bending angle of light in a non-asymptotically flat black hole”

by Asahi Ishihara

[JGRG25(2015)P10]

Bending angle of light in a non-asymptotically flat black hole

Asahi Ishihara

Hirosaki University, Japan
with Y. Suzuki, T. Ono, T. Kitamura and H. Asada (Hirosaki)

JGRG25 in Kyoto Dec. 7 - 11, 2015

HIROSAKI UNIVERSITY

Abstract: We propose a new method for calculations of the bending angle of light in a non-asymptotically flat black hole. Moreover, we carry out the calculation of the bending angle for some black hole spacetimes.

1 Introduction

[Motivation]

Bending angle of light in an asymptotically flat spacetime is well-known.But, there are no established method for calculating bending angle in a non-asymptotically flat spacetime. The main reason is that source and receiver must not be at the null infinity.

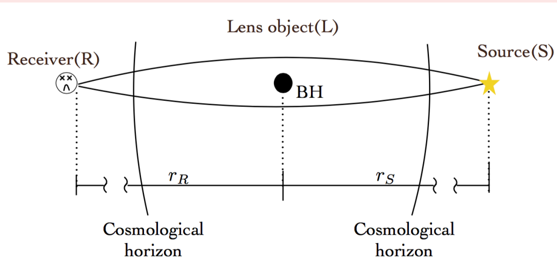

Figure 1: Previous works

[Setting in this poster]

- $G = c = 1$
- $r = r(\phi)$(Circumference radius), $r_g \equiv 2M$
- ϕ:azimuth at the lens, θ:angle of tangent, b:impact parameter, Λ:cosmological constant and α:bending angle of light

[Proposal for an alternative method]

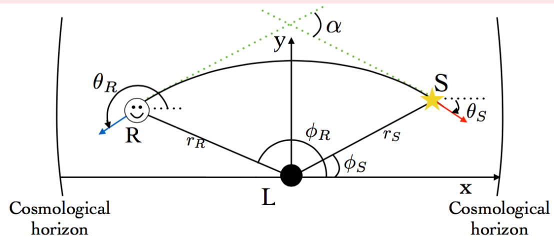

Figure 2: This study

$$\begin{cases} x = r\sin\phi \\ y = r\cos\phi \end{cases}, u \equiv \frac{1}{r}, \ \frac{du}{d\phi} \equiv u'.$$

$$\frac{y}{x} = \tan\phi \longrightarrow \frac{dy}{dx} = \tan\theta.$$

For simplicity, $\phi_0 = \frac{\pi}{2}$ and $\theta_0 = 0$.

$$\tan\theta_S = \frac{\left(\frac{du}{d\phi}\right)_S \sin\phi_S - u_S\cos\phi_S}{\left(\frac{du}{d\phi}\right)_S \cos\phi_S + u_S\sin\phi_S}, \quad (1)$$

$$\alpha = 2(\theta_0 - \theta_S) = -2\theta_S. \quad (2)$$

※ Eq.(1) is symmetric under ϕ-rotation and hence it may be invariant.

$$\tan(\theta_R - \theta_S) = \frac{B + A\tan(\phi_R - \phi_S)}{A - B\tan(\phi_R - \phi_S)}, \quad (3)$$

$$A \equiv u'_R u'_S + u_R u_S, \quad B \equiv u'_R u'_S - u_R u_S$$

$$\alpha = \theta_R - \theta_S - \pi. \quad (4)$$

2 Example 1

[The Schwarzschild metric]

$$ds^2 = -\left(1 - \frac{r_g}{r}\right)dt^2 + \left(1 - \frac{r_g}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \quad (5)$$

[Orbit equation and Weak field approx.]

$$\left(\frac{du}{d\phi}\right)^2 = \frac{1}{b^2} - u^2 + r_g u^3, \quad r_g \ll r_0 \le r \le r_S. \quad (6)$$

Iterative calculations give

$$u_S = \frac{1}{b}\sin\phi_S + \frac{r_g}{2b^2}(1 + \cos^2\phi_S) + O\left(\frac{r_g^2}{b^3}\right), \quad (7)$$

$$\sin\phi_S = bu_S - \frac{r_g}{b}\left[1 - \frac{(bu_S)^2}{2}\right] + O\left(\frac{r_g^2}{b^2}\right), \quad (8)$$

$$\cos\phi_S = \sqrt{1 - (bu_S)^2} + r_g\frac{u_S\left[1 - \frac{(bu_S)^2}{2}\right]}{\sqrt{1-(bu_S)^2}} + O\left(\frac{r_g^2}{b^2}\right), \quad (9)$$

$$\left(\frac{du}{d\phi}\right)_S = \frac{1}{b}\sqrt{1 - (bu_S)^2} + r_g\frac{1}{2}\frac{bu_S^3}{\sqrt{1-(bu_S)^2}} + O\left(\frac{r_g^2}{b^3}\right). \quad (10)$$

Therefore,

$$\tan\theta_S = -\frac{r_g}{b\sqrt{1-(bu_S)^2}}\left[1 - \frac{1}{2}(bu_S)^2 - \frac{1}{2}(bu_S)^4\right] + O\left(\frac{r_g^2}{b^2}\right), \quad (11)$$

$$\alpha = \frac{2r_g}{b\sqrt{1-(bu_S)^2}}\left[1 - \frac{1}{2}(bu_S)^2 - \frac{1}{2}(bu_S)^4\right] + O\left(\frac{r_g^2}{b^2}\right). \quad (12)$$

$$\downarrow bu_S \ll 1$$

$$\alpha = \frac{2r_g}{b} + O\left(\frac{r_g^2}{b^2}, (bu_S)^4\right). \quad (13)$$

This recovers previous study.

3 Example 2

[The Schwarzschild-de Sitter metric]

$$ds^2 = -\left(1 - \frac{r_g}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \left(1 - \frac{r_g}{r} - \frac{\Lambda}{3}r^2\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \quad (14)$$

[Orbit equation and Weak field approx.]

$$\left(\frac{du}{d\phi}\right)^2 = \frac{1}{b^2} - u^2 + r_g u^3 + \frac{\Lambda}{3}, \quad r_g \ll r_0 \le r \le r_S \ll \sqrt{\frac{3}{\Lambda}}. \quad (15)$$

Therefore, bending angle of light is

$$\begin{aligned} \alpha = & \frac{2r_g}{b\sqrt{1-(bu_S)^2}}\left[1 - \frac{1}{2}(bu_S)^2 - \frac{1}{2}(bu_S)^4\right] \\ & + \frac{r_g\Lambda b}{3\{1-(bu_S)^2\}^{\frac{3}{2}}}\left[1 - \frac{3}{2}(bu_S)^2 + \frac{3}{2}(bu_S)^4 - (bu_S)^6\right] \\ & + O\left(\frac{rg^2}{b^2}, rg\Lambda^2 b^3\right). \end{aligned} \quad (16)$$

$$\downarrow \frac{1}{B^2} \equiv \frac{1}{b^2} + \frac{\Lambda}{3}$$

$$\alpha = \frac{2r_g}{B\sqrt{1 - B^2u_s^2}}\left[1 - \frac{1}{2}(Bu_s)^2 - \frac{1}{2}(Bu_s)^4\right] + O\left(\frac{rg^2}{B^2}\right). \quad (17)$$

This bending angle of light recovered eq.(12).

4 Conclusion

- We take account of the finiteness of the source and observer distance to obtain the bending angle of light. Therefore, our result can treat even a non-asymptotically flat spacetime.
- Future work:Application to astronomical observations.
  Application to other spacetimes.
  More rigorous definition of our " θ ".

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“Orbital Evolution of Stars Around Shrinking Massive Black Hole Binaries”

by Mao Iwasa

[JGRG25(2015)P11]

Orbital Evolution of Stars Around Shrinking Massive Black Hole Binaries

M.Iwasa(Kyoto) , N.Seto(Kyoto)

abstract

Based on a simple geometrical approach, we analyze the evolution of the Kozai-Lidov mechanism for star around shrinking massive black hole binary. We find that, due to a peculiar bifurcation pattern induced by the general relativistic effects and the Newtonian potential of stellar clusters, the orbit of star could become highly eccentric. Our approach would be also useful for studying the Kozai-Lidov mechanism in various astrophysical contexts.

1.Introduction

Massive Black Hole (MBH) binary : strong source of gravitational waves to be detected by eLISA .

・The infall rate of stars to MBH could be enhanced significantly associated with shrinkage of MBH binary. ⟶ emitting gravitational waves and electromagnetic waves

・Stars infalling to MBHs must have an eccentricity unity

↑

Kozai-Lidov (KL) mechanism could play an important role.

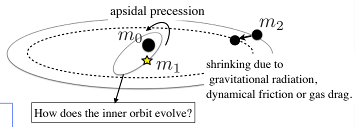
KL mechanism : for hierarchical triple systems, oscillation of inner eccentricity and inclination due to the exchange of the inner and outer angular momenta

suppress ↑

general relativistic effects, stellar potential, etc

In this poster

We study the evolution of the KL mechanism that is initially suppressed by the inner apsidal precession (due to general relativistic effects, stellar potential, etc) but later works efficiently along with the adiabatic contraction of the outer body.



m_0, m_2: MBH
m_1 : stellar mass component

2.KL mechanism for Post-Newtonian Systems

・Numerical Experiments

We first examine the evolution of relativistic hierarchical triple systems, only including gravitational interaction with the Post-Newtonian approximation at 2.5PN order.

Initial Condition

$e_{1(2)}$: inner (outer) eccentricity $a_{1(2)}$: inner (outer) semi-major axis $g_{1(2)}$: inner (outer) argument of perienter

s-I : $(e_1, a_2, e_2) = (0.001, 6300, 0.0001)$ s-III : $(e_1, a_2, e_2) = (0.3, 5700, 0.0001)$

s-II : $(e_1, a_2, e_2) = (0.001, 6300, 0.3)$

$m_0 : m_1 : m_2 = 0.3 : 0.3 \times 10^{-6} : 0.7$ $a_1 = 400$ $J_1 = \sqrt{1-e_1^2}\cos i = 0.150$ $g_1 = \pi/2$ $g_2 = 0$

$(c = G = m_0 + m_1 + m_2 = 1)$

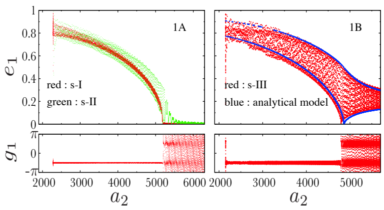


FIG. 1
Evolution of the inner orbital elements as a function of the outer semi-major axis.
red and green points
:result of direct three body calculation
blue points
:result of analytical model

◯ Inner eccentricity suddenly starts to increase.
◯ Inner argument of pericenter turns from circulation to libration.

We explain this interesting behaviors to use the double averaged Hamiltonian with respect to the inner and outer orbital periods.

・Averaged Hamiltonian

Averaged Hamiltonian $\mathcal{H}_{\mathrm{T,1PN}} = \mathcal{H}_{\mathrm{qp}} + \mathcal{H}_{\mathrm{1PN}}$

$$\mathcal{H}_{\mathrm{qp}} \equiv -3G_1^2 - 15\frac{J_1^2}{G_1^2} - 15(1-G_1^2)\left(1-\frac{J_1^2}{G_1^2}\right)\cos 2g_1 \qquad \mathcal{H}_{\mathrm{1PN}} \equiv -\frac{3\gamma}{G_1}$$

$(g_1, G_1 = \sqrt{1-e_1^2})$: canonical variable

$\gamma = 16\frac{m_0^2 a_2^3}{m_2 a_1^4}$: representing the superiority between outer orbital effect and 1PN effect.

・Geometrical Approach

Structure of phase space

The points $G_1 = J_1, 1$ are fixed points

(P1)For $\gamma > 12 - 20J_1^2 \equiv \gamma_c$ (Fig. 2A)
There are only circulating trajectories.

(P2) At $\gamma = \gamma_c$ (between Fig. 2A and 2B)
The fixed points $G_1 = 1$ transits from stable point to an unstable point (pitchfork bifurcation).

(P3) For $\gamma < \gamma_c$ (Fig. 2B and 2C)
There are circulating and librating trajectories, segregated by separatrixes

Adiabatic invariant

The Hamiltonian $\mathcal{H}_{\mathrm{T,1PN}}$ is not conserved because γ decreases.
How do we trace the evolution of the phase space?

Let us consider a periodic motion described by a Hamiltonian $\mathcal{H}(q, p; \lambda)$ including a parameter
If λ changes more slowly than the period of motion, the area surrounded by the periodic trajectory

$$S = \oint p dq$$

is conserved.

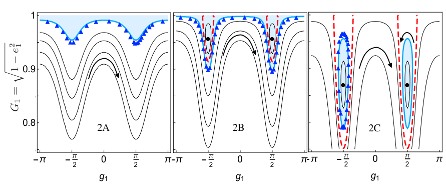


FIG. 2
Evolution of the phase space.
circles
: stable fixed points
blue triangles
: results of direct calculation
colored regions
: adiabatic invariants
red dashed curve
: separatrix
arrows
: direction of flow

3.KL mechanism for Newtonian Potential of Stellar Cluster

We apply geometrical approach to the evolution of individual stars in a stellar cluster around MBH with an outer infalling tertiary MBH (we assume that the infalling rate is sufficiently small).

The local density profile of the stellar cluster : $\rho = \rho_1 \left(\frac{r}{a_1}\right)^{-\beta}$

・Averaged Hamiltonian

Averaged Hamiltonian $\mathcal{H}_{\mathrm{T,SP}} = \mathcal{H}_{\mathrm{qp}} + \mathcal{H}_{\mathrm{SP}}$

$$\mathcal{H}_{\mathrm{SP}} = \eta e_1^2 \left[1 + \beta(-1+\beta)\left(\frac{e_1^2}{16} + \mathcal{O}(e_1^4)\right)\right]$$

$\eta \equiv 16\pi\frac{\rho_1 a_2^3}{m_2}$: including all the information of the cluster and decreasing

・Numerical Experiments

We calculate the Hamiltonian equations with $\dot{\eta} = -10^{-3}$, $\beta = 3/2$

Initial Condition at $\eta = 200$

s-IV : $(g_1, G_1, e_2) = (0, 0.95, 0)$
s-V : $(g_1, G_1, e_2) = (0, 0.6, 0)$
$J_1 = 0.5$

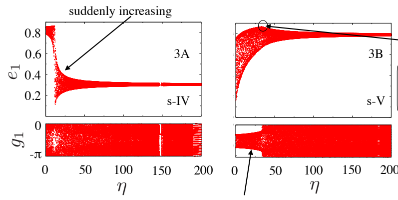


FIG. 3
Evolution of the inner orbital elements as a function of η

・Geometrical Approach

Structure of phase space

(P1') For $\eta > -18 + 30/J_1^2 \equiv \eta_{c1}$ (Fig. 4A)
There are only circulating trajectories.

(P2') For $\eta = \eta_{c1}$ (between Fig 4A and 4B)
Pitchfork bifurcation similar to (P2) occurs.
The fixed point $G_1 = J_1$ turns from a stable point to an unstable point.
The new stable fixed point is generated at $g_1 = \pi/2$.

(P3') For $\eta_{c2} \equiv 12 < \eta < \eta_{c1}$ (Fig. 4B)
There are circulating and librating trajectories.
The area surrounded by the separatrix increases with time.

(P4') For $\eta = \eta_{c2}$ (Fig. 4C)
There are only librating trajectories.
The fixed points exist on the vertical lines $g_1 = 0, \pi$.

(P5') For $\eta_{c3} \equiv -18 + 30J_1^2 < \eta < \eta_{c2}$ (Fig. 4D)
The stability of the two points $G_1 = J_1, 1$ interchanges.

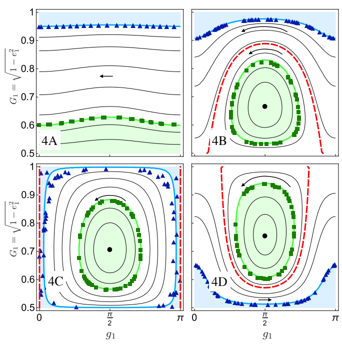


FIG. 4
Evolution of the phase space
blue triangle : result of the numerical experiments of s-IV
green square : results of the numerical experiments of s-V
(We plot the absolute value of g_1.)
colored regions : adiabatic invariants

4.Summary

・We consider the evolution of a star around a shrinking massive black hole binary

・We find that the orbit of a star becomes highly eccentric, due to the Non-Keplerian potential.

・For the initial distribution of the cluster biased to small eccentricity, reaching the maximum eccentricity during (P3') could help to refill the loss cone.

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“Detection of Circular Polarization in Stochastic Gravitational Wave Background with Pulsar Timing Arrays”

by Ryo Kato

[JGRG25(2015)P12]

Detection of circular polarization in stochastic gravitational wave background with Pulsar Timing Arrays

Ryo Kato and Jiro Soda, Particle Theory and Cosmology Group, Kobe University.

Introduction

We generalize the overlap reduction function (ORF) so that we can detect circular polarized stochastic gravitational wave background (SGWB) with Pulsar Timing Arrays (PTAs). In this poster presentation, I explain the ORF for circular polarization, which describe the angular sensitivity of PTAs.

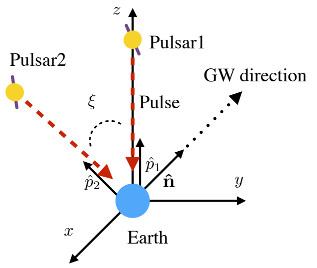
The signal

The pulsar red shift integrated over $\hat{\mathbf{n}}$ (the signal) is given by

$$z(t,\hat{\mathbf{n}}) = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\mathbf{n}}\cdot\hat{p}}[h_{ij}(t_p,\hat{\mathbf{n}}) - h_{ij}(t_e,\hat{\mathbf{n}})].$$

$h_{ij}(t_e,\hat{\mathbf{n}})$: a single GW at the Earth, $(t_e,\mathbf{x}_e) = (t,0)$.

$h_{ij}(t_p,\hat{\mathbf{n}})$: a single GW at a pulsar, $(t_p,\mathbf{x}_p) = (t-L, L\hat{p})$.



Formulation

In the TT gauge, metric perturbation $h_{ij}(t,\mathbf{x})$ with a given propagation direction $\hat{\mathbf{n}}$ can be expanded as

$$h_{ij}(t,\mathbf{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty} df \int_{S^2} d^2\hat{\mathbf{n}}\ \tilde{h}_A(f,\hat{\mathbf{n}})\ e^A_{ij}(\hat{\mathbf{n}})\ \mathrm{e}^{-2\pi i f(t-\hat{\mathbf{n}}\cdot\mathbf{x})}.$$

$\tilde{h}_A(f,\hat{\mathbf{n}})$: the Fourier amplitudes

$e^A_{ij}(\hat{\mathbf{n}})$: the polarization tensors

Assumptions for SGWB: stationary, Gaussian, ~~isotropic~~, ~~unpolarized~~

The ensemble average of the two Fourier amplitudes can be written as

$$\langle \tilde{h}^*_A(f,\hat{\mathbf{n}})\tilde{h}_{A'}(f',\hat{\mathbf{n}}')\rangle = \delta(f-f')\delta^2(\hat{\mathbf{n}},\hat{\mathbf{n}}')S_h^{AA'}(f,\hat{\mathbf{n}}),$$

where

$$S_h^{AA'}(f,\hat{\mathbf{n}}) = \begin{pmatrix} I(f,\hat{\mathbf{n}})+Q(f,\hat{\mathbf{n}}) & U(f,\hat{\mathbf{n}})-iV(f,\hat{\mathbf{n}}) \\ U(f,\hat{\mathbf{n}})+iV(f,\hat{\mathbf{n}}) & I(f,\hat{\mathbf{n}})-Q(f,\hat{\mathbf{n}}) \end{pmatrix}.$$

Using the spherical harmonics Y_{lm} and the spin-weighted harmonics $_{\pm 4}Y_{lm}$, we can expand the Stokes parameters:

$$I(f,\hat{\mathbf{n}}) = I(f)\sum_{l=0}^{\infty}\sum_{m=-l}^{l} c^I_{lm}Y_{lm}(\hat{\mathbf{n}}),$$

$$V(f,\hat{\mathbf{n}}) = V(f)\sum_{l=0}^{\infty}\sum_{m=-l}^{l} c^V_{lm}Y_{lm}(\hat{\mathbf{n}}),$$

$$(Q+iU)(f,\hat{\mathbf{n}}) = P^+(f)\sum_{l=4}^{\infty}\sum_{m=-l}^{l} c^{P+}_{lm}\,{}_4Y_{lm}(\hat{\mathbf{n}}),$$

$$(Q-iU)(f,\hat{\mathbf{n}}) = P^-(f)\sum_{l=4}^{\infty}\sum_{m=-l}^{l} c^{P-}_{lm}\,{}_{-4}Y_{lm}(\hat{\mathbf{n}}).$$

For simplicity, we consider the case of $l = 0,1,2,3$.

The correlation of the signals is calculated as follows:

$$\langle z_1(t)z_2(t')\rangle = \int_{-\infty}^{\infty} df\int_{-\infty}^{\infty} df' \langle \tilde{z}_1^*(f)\tilde{z}_2(f')\rangle \mathrm{e}^{i2\pi(ft-f't')}$$

$$= \int_{-\infty}^{\infty} df \sum_{l=0}^{3}\sum_{m=-l}^{l}\left[c^I_{lm}I(f)\Gamma^I_{lm} + c^V_{lm}V(f)\Gamma^V_{lm}\right]\mathrm{e}^{i2\pi f(t-t')},$$

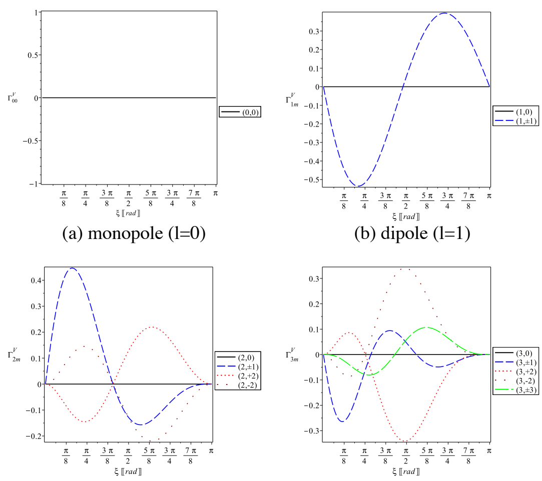
where

$$\Gamma^I(f) \equiv \int_{S^2} d\hat{\mathbf{n}}\ \kappa(f,\hat{\mathbf{n}})\sum_{lm}^{l=3} c^I_{lm}Y_{lm}(\hat{\mathbf{n}})\left\{F_1^{*+}(\hat{\mathbf{n}})F_2^{+}(\hat{\mathbf{n}}) + F_1^{*\times}(\hat{\mathbf{n}})F_2^{\times}(\hat{\mathbf{n}})\right\},$$

$$\Gamma^V(f) \equiv -i\int_{S^2} d\hat{\mathbf{n}}\ \kappa(f,\hat{\mathbf{n}})\sum_{lm}^{l=3} c^V_{lm}Y_{lm}(\hat{\mathbf{n}})\left\{F_1^{*+}(\hat{\mathbf{n}})F_2^{\times}(\hat{\mathbf{n}}) - F_1^{*\times}(\hat{\mathbf{n}})F_2^{+}(\hat{\mathbf{n}})\right\},$$

$$\kappa(f,\hat{\mathbf{n}}) \equiv (\mathrm{e}^{-2\pi i f L(1+\hat{\mathbf{n}}\cdot\hat{p}_1)}-1)(\mathrm{e}^{2\pi i f L(1+\hat{\mathbf{n}}\cdot\hat{p}_2)}-1) \overset{fL\gg 1}{\simeq} 1,$$

$$F^A(\hat{\mathbf{n}}) \equiv \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\mathbf{n}}\cdot\hat{p}}e^A_{ij}(\hat{\mathbf{n}}).$$



(a) monopole (l=0) (b) dipole (l=1)

(c) quadrupole (l=2) (d) octupole (l=3)

Fig.1 Plots of the circular polarization of the generalized ORFs as a function of the angular separation between the two pulsars.

The functions Γ^I and Γ^V called the generalized ORF. Γ^V describes the angular sensitivity of the circular polarization in the SGWB. Integrating Γ^V, we obtain the following form:

$$\Gamma^V_{00} = 0,$$

$$\Gamma^V_{10} = 0,$$

$$\Gamma^V_{11} = -\frac{\sqrt{6\pi}}{3}\sin\xi\left[1+3\left(\frac{1-\cos\xi}{1+\cos\xi}\right)\log\left(\sin\frac{\xi}{2}\right)\right],$$

$$\Gamma^V_{1-1} = \Gamma^V_{11},$$

$$\Gamma^V_{20} = 0,$$

$$\Gamma^V_{21} = \frac{\sqrt{30\pi}}{6}\frac{\sin\xi}{1+\cos\xi}\left[2+(1-\cos\xi)\left\{\cos\xi + 6\log\left(\sin\frac{\xi}{2}\right)\right\}\right],$$

$$\Gamma^V_{2-1} = \Gamma^V_{21},$$

$$\Gamma^V_{22} = -\frac{\sqrt{30\pi}}{6}(1-\cos\xi)\left[2-\cos\xi+6\left(\frac{1-\cos\xi}{1+\cos\xi}\right)\log\left(\sin\frac{\xi}{2}\right)\right],$$

$$\Gamma^V_{2-2} = -\Gamma^V_{22}.$$

Fig.1 shows these generalized ORFs.

Conclusion

We show the angular sensitivity of the PTAs for circular polarization. As you can see by looking at the Fig.1, in isotropic (l=0) case, we cannot detect the circular polarized SGWB with PTAs. On the other hand, when we consider anisotropic ($l \neq 0$) ORF, it is worth that we take into account polarization, otherwise it might be behave like a noise.

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“Possible orbiting gyroscope precession by a Chern-Simons modification to gravity”

by Daiki Kikuchi

[JGRG25(2015)P13]

Possible orbiting gyroscope precession by a Chern-Simons modification to gravity

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JGRG25 in Kyoto Dec. 7 - 11, 2015

Abstract: Alexander and Yunes [1] discussed a possible constraint on a Chern-Simons modified gravity theory by using Gravity Probe B experiment [2]. We will reexamine the constraint in more details.

1 Motivation

The Chern-Simons (CS) correction is one of the most interesting modified gravity models. Then it leads to some effects distinct from general relativity (GR) .

- The CS modification motivated by both string theory and quantum gravity and introduces the parity violation into the gravity theory.
- K. Konno *et al.*[3] investigated the CS correction of the spacetime of a slowly rotating black hole and explained the flatness of rotation curves without the existence of the dark matter.
- A possible constraint by interferometers [4, 5], or Gravity Probe B (GPB) experiment [1, 6] has recently been studied.

We investigate the gyroscopic spin precession in CS modified gravity using the solution by Alexander and Yunes (AY) [1]. Then, We reexamine the constraint on a CS gravity toward data fitting of the GPB experiment in more detail.

2 Spin precession analogies in GR

○The spin drift rate $d\vec{S}/dt$ and the spin precession $\delta\vec{S}/S$ per a certain time ($t=p$)

$$\frac{d\vec{S}}{dt} = \vec{\Omega}\times\vec{S} \quad\Rightarrow\quad \frac{\delta_p\vec{S}}{S} = \frac{1}{S}\int_0^p \vec{\Omega}\,dt\times\vec{S}. \qquad \vec{S}\ :\ \text{spin vector} \tag{1}$$

○The angular velocity $\vec{\Omega}$

$$\vec{\Omega}_{GR} = \frac{1}{2}\vec{\nabla}\times\vec{g}_{0i}^{\;GR},\quad \vec{g}_{0i}^{\;GR} = (g_{01}, g_{02}, g_{03})\,. \tag{2}$$

$$\Rightarrow\ \vec{\Omega}_{LT} = -\frac{G}{c^2 {r_g}^3}\left[\vec{J}_E - 3\vec{n}_g\left(\vec{n}_g\cdot\vec{J}_E\right)\right].\ \langle\ \textbf{Lense-Thirring (LT) effect}\ \rangle \tag{3}$$

$$\vec{\Omega}_{GE} = \frac{3}{2}\vec{v}_g\times\vec{\nabla}\left(\frac{Gm_E}{c^2 r_g}\right) = \frac{3Gm_E}{2c^2{r_g}^2}\vec{n}_g\times\vec{v}_g.\ \langle\ \textbf{Geodetic (GE) effect}\ \rangle \tag{4}$$

$\vec{J}_E$: spin angular momentum of Earth, $\vec{v}_g$: velocity of the gyroscope
r_g : distance to the gyroscope, $\vec{n}_g(=\vec{r}_g/r_g)$: unit vector pointing to the gyroscope
m_E : mass of Earth

3 Chern-Simons gravity

○The action of CS gravity theory

$$S = \frac{c^4}{16\pi G}\sqrt{-g}\int d^4x\left[\boldsymbol{R} + \frac{f}{4}{}^*\boldsymbol{RR}\right],\quad \boldsymbol{R} = R_{\alpha\beta\gamma\delta},\quad {}^*\boldsymbol{R} = \frac{1}{2}\boldsymbol{\epsilon}^{\alpha\beta\mu\nu}R^{\gamma\delta}{}_{\mu\nu}. \tag{5}$$

f : scalar field ($[f] = \mathrm{L}^2$), $\boldsymbol{\epsilon}^{\alpha\beta\mu\nu}(=\epsilon^{\alpha\beta\mu\nu}/\sqrt{-g})$: Levi-Civita tensor density

○The CS correction to the metric as the weak-field solution
Considering Sun-Earth system in the standard PPN approximation, the CS correction to g_{0i} is given by [1]

$$\vec{g}_{0i}^{\;CS} = \vec{g}_{0i} - \vec{g}_{0i}^{\;GR} = 2\dot{f}\frac{G}{c^3 r_g^3}\left[m_E r_g\left(\vec{v}_E\times\vec{n}_g\right) - \frac{1}{2}\vec{J}_E + \frac{3}{2}\vec{n}_g\left(\vec{n}_g\cdot\vec{J}_E\right)\right], \tag{6}$$

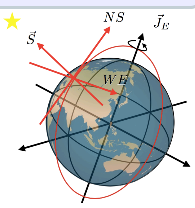
$\vec{v}_E$: velocity of the Earth, $\dot{f}$: CS coupling parameter ($\dot{}$ stands for time derivative) where we neglect both the velocity and the spin angular momentum of the sun.

○The CS angular velocity $\vec{\Omega}_{CS}$

$$\vec{\Omega}_{CS} = -\frac{1}{2}\vec{\nabla}\times\vec{g}_{0i}^{\;CS} = \dot{f}\frac{Gm_E}{c^3 r_g^3}\left[\vec{v}_E - 3\vec{n}_g\left(\vec{v}_E\cdot\vec{n}_g\right)\right].\ \langle\ \textbf{Chern-Simons (CS) effect}\ \rangle \tag{7}$$

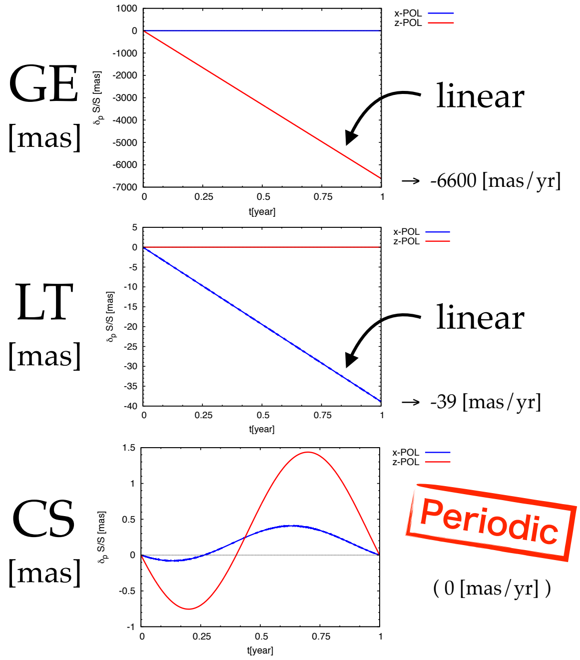
4 GPB experiment

○GPB mission final result [2]
・Lense-Thirring effect : -37.2 ± 7.2[mas/yr]
(GR prediction -39.2[mas/yr])
・Geodetic effect : -6601.8 ± 18.3[mas/yr]
(GR prediction -6606.1[mas/yr])



⋆ GPB coordinate system
$\vec{S}$: the direction of the guide star (IM Pegasi) at first
WE : East-West direction
NS : North-South direction

5 Gyro precession



6 Possible constraint on $\dot{f}$ by GPB

The gyro drift rate per a year given by taking the average by the orbital period of gyro
○ NS direction ⇐ by GE effect

$$\left\langle\frac{(d\vec{S})_{CS}}{Sdt}\right\rangle_{P_g} \sim 6.7\times10^5\left(\frac{\dot{f}}{c}\right) \lesssim 18.3[\text{mas/year}] \ \Rightarrow\ \left(\frac{\dot{f}}{c}\right) \lesssim 2.7\times10^{-5}[\text{s}]. \tag{8}$$

○ WE direction ⇐ by LT effect

$$\left\langle\frac{(d\vec{S})_{CS}}{Sdt}\right\rangle_{P_g} \sim 1.5\times10^5\left(\frac{\dot{f}}{c}\right) \lesssim 7.2[\text{mas/year}] \ \Rightarrow\ \left(\frac{\dot{f}}{c}\right) \lesssim 4.8\times10^{-5}[\text{s}]. \tag{9}$$

These bounds are better than the AY's estimation ($\frac{\dot{f}}{c}\lesssim10^{-3}$[s]) [1] or Smith *et al.*'s estimation (${m_{CS}}^{-1}\lesssim10^3$[km]) [6].

7 Conclusion

We investigated the gyroscopic precession in CS gravity.

- The spin precessions caused by CS gravity periodically oscillates at one year period, although that of LT effects or GE effect increases linearly.
- The CS parameter bound may be improved by considering in more details.

⇒ We wish to make data fitting for testing CS separately taking statistics , for example, every four months out of all data table.

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“Conic D-branes”

by Shunichiro Kinoshita

[JGRG25(2015)P14]

JGRG25 The 25th Workshop on General Relativity and Gravitation in Japan

Conic D-branes

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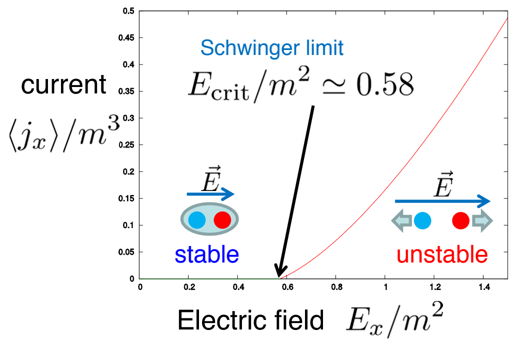
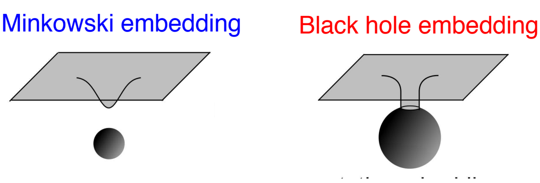
Based on PTEP 2015 (2015) 8, 083B04
(arXiv:1505.04506)

D3/D7 system

Karch, Katz (2002), Grana, Polchinski (2002), Bertolini *et al.* (2002)

- Holographic dual to $\mathcal{N}=2$ SQCD
  - In large N_c limit, a probe D7-brane is embedded in $\mathrm{AdS}_5 \times S^5$ geometry
  - Fluctuations of the D7-brane = "meson" excitations
- Phase transition by applying electric fields
  - Dielectric breakdown due to Schwinger effect

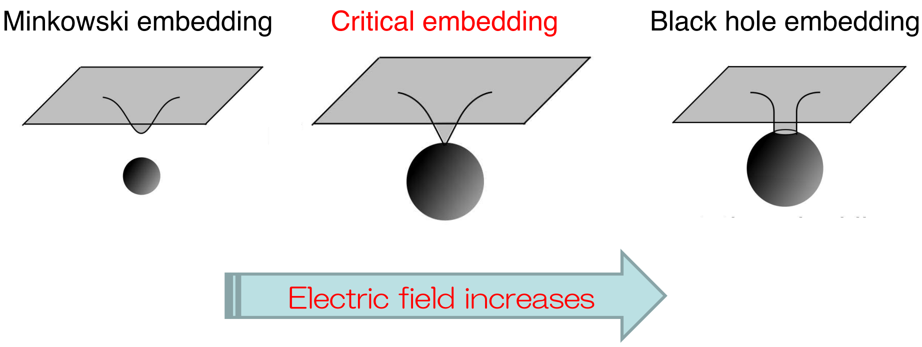




Karch, O'Bannon (2007)
Erdmenger, Meyer, Shock (2007)
Albash, Filev, Johnson, Kundu (2007)

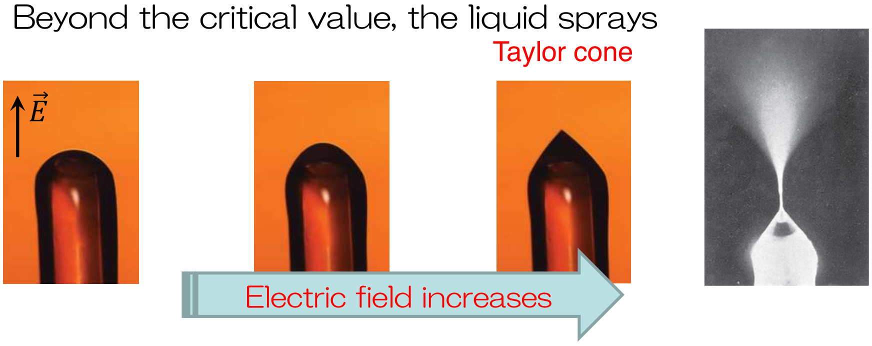
Critical embedding in the D3/D7 system

- A phase boundary between the Minkowski embeddings and the BH embeddings
  - Two series of the solutions merge
  - The shape of the D7-brane is conical



Taylor cone

- A hydrodynamic phenomena, which are used in electrospray in material/industrial science
- As an electric field increases the surface of a conductive liquid is sharpening, and at a critical electric field a cone is formed
  - Beyond the critical value, the liquid sprays



Ref. R.Krpoun "Micromachined Electrospray Thrusters for Spacecraft Propulsion" (2009)

- The first theoretical model of this phenomena is given by Taylor (1964)

G.Taylor Proc. R. Soc. Lond. A 280, 383 (1964)

  - He assumed the liquid was a perfect conductor and the cone was formed when the surface tension and the electrostatic stress equilibrated on the liquid surface
  - Repulsive forces between the induced charges cancel surface tension forces
- A half-cone angle $49.29°$ predicted by Taylor is very close to experimental results
  - This angle is determined by a zero of the Legendre polynomial

Can we find something like universal properties for conic D-branes?

RR flux background

- D2-brane in a constant Ramond-Ramond (RR) flux background in flat spacetime
  - The d-dim. bulk spacetime

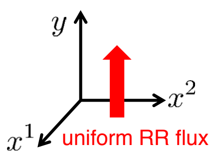
$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + (dx^1)^2 + (dx^2)^2 + d\vec{y}^2_{d-3}$$

  - Embedding function

$$y = \phi(\rho) \qquad \rho = \sqrt{(x^1)^2 + (x^2)^2}$$

  - RR field

$$C_\mu dx^\mu = C_0 dt = cy\,dt \quad (c = \text{const.})$$

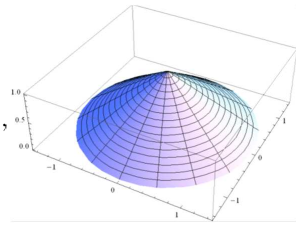


- The action is a DBI action with a coupling to the RR field

$$S = -\mathcal{T}_2 \int dx^0 dx^1 dx^2 \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + \partial_a\phi\partial_b\phi)}$$
$$- \frac{\mathcal{T}_2}{2} \int dx^0 dx^1 dx^2 \; 2\pi\alpha' F_{ab} C_c \epsilon^{abc}$$

Conic solution

- The equations of motion are



$$\partial_i \left[\frac{\partial_i \phi}{\sqrt{1+(2\pi\alpha' F_{12})^2+(\partial_1\phi)^2+(\partial_2\phi)^2}} \right] + 2\pi\alpha' \frac{dC_0[\phi]}{d\phi} F_{12} = 0\,,$$

$$\partial_i \left[\frac{2\pi\alpha' F_{12}}{\sqrt{1+(2\pi\alpha' F_{12})^2+(\partial_1\phi)^2+(\partial_2\phi)^2}} \right] - \partial_i C_0[\phi] = 0$$

The second equation can be integrated $2\pi\alpha' F_{12} = C_0[\phi]\sqrt{\frac{1+(\partial_i\phi)^2}{1-(C_0[\phi])^2}}\,.$

$$\partial_i \left[\frac{\partial_i \phi}{\sqrt{1+(\partial_j\phi)^2}} \sqrt{1-(C_0[\phi])^2} \right] + \frac{dC_0[\phi]}{d\phi} C_0[\phi] \sqrt{\frac{1+(\partial_j\phi)^2}{1-(C_0[\phi])^2}} = 0\,.$$

This equation is singular when $C_0[\phi] = c\phi(\rho) = \pm 1$

- If we expand $\phi(\rho)$ around this point, we have a critical (conical) solution

$$\phi(\rho) = \frac{1}{c} - \frac{1}{\sqrt{2}}\rho + \cdots$$

The apex of the cone is located at $\rho = 0$

Half-cone angle

$$\boxed{\theta_{\rm cone} = \arctan\sqrt{2}}$$

Other examples

- NSNS flux background
  - Dp-brane in a constant NSNS flux

topology of the cone: $\mathbf{R}_+ \times S^{p-2}$

$$\boxed{\theta_{\rm cone} = \arctan\sqrt{2(p-2)}}$$

- D3/D7
  - Probe D7-brane with worldvolume gauge fields in $\mathrm{AdS}_5\mathrm{-Schwarzschild} \times S^5$

topology of the cone: $\mathbf{R}_+ \times S^3$

$$\boxed{\theta_{\rm cone} = \arctan\sqrt{6}}$$

The cone angle is unique independent of three parameters $(E_i, B_i, r_{\rm h})$

Universal formula?

- We have three conical D-brane solutions for different external forces and couplings
  - RR flux, NSNS flux, gravitational field (AdS curvature)
- It is expected that the half-cone angle is determined as

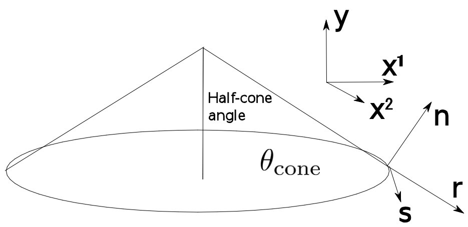
$$\boxed{\theta_{\text{cone}} = \arctan\sqrt{2(d_{\text{cone}}-1)}}$$

topology of the cone: $\mathbf{R}_+ \times S^{d_{\text{cone}}-1}$

- What mechanism determines the angle of conic D-branes?
- Where is the factor of 2 in the square root coming from?

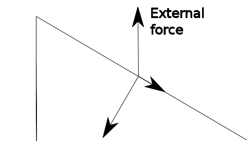
Force balance in Newtonian mechanics

- We have two force balance conditions:
  - Normal direction (extrinsic dynamics)
  - Radial direction (intrinsic dynamics)



$$\boxed{|F_n| = -T_{ss}\frac{1}{r\tan\theta_{\text{cone}}}}$$

Young-Laplace eq.



$$|F_n| = |F_r|\tan\theta_{\text{cone}}$$

$$\boxed{F_r} = -\partial_r T_{rr}$$

Hydrodynamic (elastic) equilibrium

$$T_{ss} = (\tan\theta_{\text{cone}})^2 r\frac{d}{dr}T_{rr}$$

Assuming that the tension is isotropic and its distribution behaves as

$$T_{ss} = T_{rr} \simeq Ar^{\alpha} \quad (r\sim 0)$$

$$\boxed{\theta_{\text{cone}} = \arctan\sqrt{\frac{1}{\alpha}}}$$

Equations of motion for generic membranes

- Extrinsic and intrinsic dynamics

$$T^{ab}{K^{\mu}}_{ab} = -\mathcal{F}^{\mu}_{\mathrm{n}},$$
$$D_a T^{ab} = \mathcal{F}^{b}_{\mathrm{t}},$$

External force
$$\mathcal{F}^{\mu} = \mathcal{F}^{\mu}_{\mathrm{n}} + \mathcal{F}^{a}_{\mathrm{t}}{h_a}^{\mu}$$

Induced metric: $h_{ab} \equiv g_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu}$

Extrinsic curvature: $-{K^{\mu}}_{ab} \equiv ({g^{\mu}}_{\lambda} - {h^{\mu}}_{\lambda}){h_a}^{\nu}\nabla_{\nu}{h_b}^{\lambda}$

$$= D_a D_b X^{\mu} + {\Gamma^{\mu}}_{\alpha\beta} D_a X^{\alpha} D_b X^{\beta},$$

Embedding functions: $x^{\mu} = X^{\mu}(y^a)$

Nambu-Goto brane
$T^{ab} = -\sigma h^{ab} \quad (\sigma = \mathrm{const.})$ $\qquad \mathrm{Tr}K^{\mu} = 0$ Extremal surface

In general, the energy density is not equal to the tension (negative pressure).

Force balance in curved spacetimes

- A membrane in an "axisymmetric" spacetime

Bulk metric: $g_{\mu\nu}dx^{\mu}dx^{\nu} = \mathsf{A}_{ij}(\rho,\zeta)dy^i dy^j + \mathsf{B}(\rho,\zeta)(d\rho^2 + d\zeta^2) + \mathsf{C}(\rho,\zeta)d\Omega^2_{d-1}$
$$+ \mathsf{D}_{kl}(\rho,\zeta)dw^k dw^l$$

Induced metric on the memebrane:

$$h_{ab}dy^a dy^b = \mathsf{A}_{ij}(\rho,\phi(\rho))dy^i dy^j + [1+\phi'(\rho)^2]\mathsf{B}(\rho,\phi(\rho))d\rho^2 + \mathsf{C}(\rho,\phi(\rho))d\Omega^2_{d-1}$$

Embedding functions
$\zeta = \phi(\rho),\ w^k = \mathrm{const.}$

Topology of the cone $\mathbf{R}_+ \times S^{d-1}$

We assume the membrane has an isotropic tension on the cone

stress-energy tensor $T_{ab} = \tau_{ab} - \sigma(r_a r_b + s_{ab})$

tension

induced metric on the cone

$$\boxed{\begin{aligned} T^{ab}{K^{\mu}}_{ab} &= -\mathcal{F}^{\mu}_{\mathrm{n}}, \\ D_a T^{ab} &= \mathcal{F}^{b}_{\mathrm{t}}, \end{aligned}} \quad \frac{\sin\theta}{\sqrt{-\mathsf{AB}}}\frac{d}{d\rho}(\sqrt{-\mathsf{A}}\sigma\sin\theta) - \sigma\cos\theta n^{\mu}\partial_{\mu}\log(\sqrt{\mathsf{B}}\mathsf{C}^{(d-1)/2}) + \frac{1}{2\sqrt{\mathsf{B}}}\tau^{ij}\partial_{\rho}\mathsf{A}_{ij} = 0$$

If the external force is along the axis of the cone, we can combine two equations.

- If we assume that the bulk spacetime is regular at the membrane (the membrane does not touch event horizons or some singularities) and the tension σ plays a dominant role, then we have

$$(\sin\theta_{\rm cone})^2\frac{d\sigma}{d\rho}\simeq(d-1)(\cos\theta_{\rm cone})^2\frac{\sigma}{\rho}$$

- If the tension behaves as $\sigma\sim\rho^\alpha$ near the apex of the cone $\rho\sim 0$, the angle of the cone becomes

$$\boxed{\theta_{\rm cone}=\arctan\sqrt{\frac{d-1}{\alpha}}}$$

- The dimension of the spherical part of the cone
- The power of the stress distribution

Stress-energy tensor of the various D-branes

- D2-brane in the RR flux

isotropic tension

$$T^0{}_0=-\frac{1}{\sqrt{1-(C_0[\phi])^2}},\quad \boxed{T^\theta{}_\theta=T^r{}_r=-\sqrt{1-(C_0[\phi])^2}}\qquad \mathbf{R}_+\times S^1$$

- Dp-brane in the NSNS flux

isotropic tension

$$T^0{}_0=T^1{}_1=-\frac{1}{\sqrt{1-c^2\phi^2}},\quad \boxed{T^r{}_r=-\sqrt{1-c^2\phi^2},\quad T^m{}_n=-\sqrt{1-c^2\phi^2}\delta^m{}_n}$$

$$\mathbf{R}_+\times S^{p-2}$$

- D7-brane in $\mathrm{AdS}_5\times S^5$

$$T^0{}_0=-\frac{1}{\sigma}(1+\frac{\mathbf{B}^2}{g^2}),\quad T^0{}_i=\frac{1}{\sigma}\frac{1}{g^2h}\epsilon_{ijk}E^jB^k,\quad T^i{}_0=-\frac{1}{\sigma}\frac{1}{g^2}\epsilon^{ijk}E_jB_k,$$

$$T^i{}_j=-\frac{1}{\sigma}\left[(1-\frac{\mathbf{E}^2}{g^2h})\delta^i{}_j+\frac{1}{g^2}(h^{-1}E^iE_j+B^iB_j)\right],$$

$$\boxed{T^r{}_r=-\sigma,\quad T^m{}_n=-\sigma\delta^m{}_n}$$ isotropic tension $\mathbf{R}_+\times S^3$

$$\left(\sigma^2\equiv 1-\frac{1}{g^2}(h^{-1}\mathbf{E}^2-\mathbf{B}^2)-\frac{1}{g^4h}(\mathbf{E}\cdot\mathbf{B})^2,\quad h(u)\equiv\left(\frac{u^4-r_{\rm h}^4/4}{u^4+r_{\rm h}^4/4}\right)^2,\quad g(u)\equiv\frac{1}{2\pi\alpha' R^2}\frac{u^4+r_{\rm h}^4/4}{u^2}\right)$$

Mechanism for the conic D-branes

- When the isotropic tension vanishes, a cone is formed.
- The angle of the cone is universally determined by the dimension of the cone and the power of the distribution of the tension.
  - For the conic D-branes, the power is $\frac{1}{2}$ independent of the background fields, which comes from the square root of the DBI action

$$\boxed{\theta_{\rm cone} = \arctan\sqrt{2(d_{\rm cone}-1)}}$$

topology of the cone: $\mathbf{R}_+ \times S^{d_{\rm cone}-1}$

Summary

- We found various conic D-brane solutions, whose cone angles obey an universal formula
  - The cone is formed at a critical point where the brane tension is canceled
- In general, the cone angles are determined by simply the local force balance
  - It is expected that many conic D-branes other than our limited examples exist and our formula is valid
- Beyond the critical value, what happens?
  - Spray solution? Funnel solution?

“Born-Infeld Gravity and Black Hole Formation”

by Meguru Komada

[JGRG25(2015)P16]

Born-Infeld Gravity and Black Hole Formation

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arXiv: 1409.1663
Nagoya University QG lab

Palatini Formalism

Usually, a gravitation theory has only metric degrees of freedom.

But, there is an another formalism which adopts connection as independent degrees of freedom.

Take the variation of the Einstein-Hilbert action in this formalism. Then

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 0 \qquad (1)$$

$$\nabla_\alpha(-g)^{1/2}g_{\mu\nu} = 0 \qquad (2)$$

The connection which satisfies the second equation is Levi-Civita connection Itself (that is a rewriting of the definition of the Levi-Civita connection).
So, metric and Palatini formalism are equivalent to each other.

but

This property does not hold except the Einstein gravity. In general gravity theories, metric and Palatini formalism describe different theories.

Black hole formation by collapse of dust

Now we assume that the collapsing dust is uniform and spherical symmetric and Outside of the dust ball spacetime is Schwarzschild spacetime.

Let R be the initial radius(large enough) of the dust ball its e-folding is $N = N_0$.
Then the total mass is given by

$$M = \frac{4\pi}{3}R^3\rho_0 e^{-3N_0} \qquad (15)$$

The condition is obtained by comparing the bouncing radius and Schwarzschild radius. If the bouncing radius is smaller than Schwarzschild radius, then black hole horizon can appear. By using(13) and (14), the condition is

(i) $b\kappa^2\rho_0 \gg 1$ $\boxed{M^2 > \frac{18\pi b}{\kappa^4}}$ (16)

(ii) $b\kappa^2\rho_0 \ll 1$ $\boxed{M^2 > \frac{27\pi b}{4\kappa^4}}$ (17)

Palatini Born-Infeld Gravity

The Born-Infeld gravity in the Palatini formalism is also a theory different from the Born-Infeld gravity in the metric formalism [1] [2] [3].

The Palatini Born Infeld gravity can avoid the ghost with simplest action.

$$S = \frac{1}{\kappa^2 b}\int d^4x\left\{\sqrt{|\det(g_{\mu\nu}+bR_{\mu\nu})|} - \sqrt{|\det(g_{\mu\nu})|}\right\} + S_{\text{matter}} \qquad (3)$$

$\kappa \equiv \sqrt{8\pi G}$ b : constant

Take the variation and derive equation of motion. Then

$$\boxed{\sqrt{-P}\left(P^{-1}\right)^{\mu\nu} - \sqrt{-g}g^{\mu\nu} = \kappa^2 T^{\mu\nu}} \qquad (4)$$

$$\boxed{\nabla_\lambda([-P]^{1/2}(P^{-1})^{\mu\nu}) = 0} \qquad (5)$$

where

$$P_{\mu\nu} = g_{\mu\nu} + bR_{\mu\nu}$$

After BH formation

The black hole which has a regular core and coincide with usual black hole asymptotically is considered(Fig.2) [4].

It appears from the collapse of the star with the quantum effect (This star bounces too.) [5][6].
Futhermore the remnant object "Planck star" appears after the collapse.

There might be potentiality that the same mechanism work in black hole formation of the Palatini Born-Infeld gravity (classical remnant?).

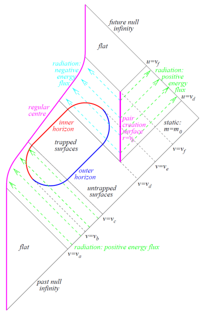

Fig.2 Quotation from Hayward[4].
Formation and evaporation of Regular BH

FLRW universe with dust

Consider the shrinking FLRW dust universe by using this theory.

$$ds^2 = -dt^2 + a(t)^2\sum_{i=1,2,3}(dx^i)^2 \qquad (6)$$

$$\Gamma^t_{tt} = A(t) \qquad \Gamma^t_{ij} = a(t)^2B(t)\delta ij \qquad (7)$$

$$\Gamma^i_{jt} = \Gamma^i_{tj} = C(t)\delta^i{}_j \qquad (8)$$

Substituting these above assumptions to EoM (4) and (5).

$$b\kappa^2\rho = \left\{1+b\left(\dot{B}+3HB\right)\right\}^{\frac{1}{2}}\left\{1+3b\left(\dot{C}+2C^2-CH\right)\right\}^{-\frac{1}{2}} - 1 \qquad (9)$$

$$0 = \left\{1+b\left(\dot{B}+3HB\right)\right\}^{\frac{1}{2}}\left\{1+3b\left(\dot{C}+2C^2-CH\right)\right\}^{\frac{1}{2}} - 1 \qquad (10)$$

$$-C+H = \frac{1}{2}\frac{d}{dt}\left\{\ln\left\{1+3b\left(\dot{C}+2C^2-CH\right)\right\}\right\} \qquad (11)$$

$$B = \left\{1+3b\left(\dot{C}+2C^2-CH\right)\right\}^{-1} \times\left\{2H+b\left\{5H\dot{B}+6H^2B+3\dot{H}B+\ddot{B}\right\}\right\} \qquad (12)$$

ρ: density of dust

By solving above equations, we obtain the behavior of shrinking dust universe. The behavior is very different from the case of the general relativity.
The universe does not collapse to a point, but bounces back and switches to expand at finite size.

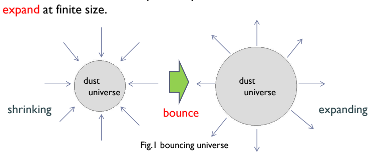

Fig.1 bouncing universe

We calculated the size of scale factor when the universe bounce at.

(i) $b\kappa^2\rho_0 \gg 1$ $\boxed{e^{3N} \sim \frac{3}{8}b\kappa^2\rho_0}$ (13)

(ii) $b\kappa^2\rho_0 \ll 1$ $\boxed{e^{3N} \sim \frac{9}{64}b\kappa^2\rho_0}$ (14)

b>0
$\rho = \rho_0 a^{-3}$
$a = e^N$

Summary & Future outlook

1. The Palatini Born-Infeld gravity has a characteristic action and is different from the metric Born-Infeld gravity(Palatini Born-Infeld action is simpler).

2. By the combination of bounce FLRW dust universe and Schwarzschild spacetime, we obtained the condition black hole can be formed.

3. The new kind remnant object(like the Planck star?) might appear after the formation and the evaporation of black hole in the Palatini Born-Infeld gravity.

4. To obtain a more correct estimation of condition that black hole can be formed, we should treat the junction condition directly in the Palatini Born-Infeld gravity and should derive the formula correspond to (15).

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“Slowly rotating dilatonic black holes with exponential form of nonlinear electrodynamics”

by Ken Matsuno

[JGRG25(2015)P17]

Slowly rotating dilatonic black holes with exponential form of nonlinear electrodynamics

Ken Matsuno

S.H. Hendi, A. Sheykhi, M. Sepehri Rad

Gen. Relativ. Gravit. ***47***, *117 (2015)*

Introduction

- Born-Infeld nonlinear electrodynamics:
  Removing divergence of electric field at the origin in classical Maxwell theory

✓ Other models of nonlinear electrodynamics:

$$\left.\begin{array}{l}\text{Power-law Maxwell}\\ \text{Logarithmic form}\\ \text{Exponential form}\end{array}\right\}\ \text{can modify electric field}$$

We add small angular momentum to 4D exact charged static dilaton black hole solutions with exponential form of nonlinear electrodynamics

Models of nonlinear electrodynamics

- Born-Infeld nonlinear electrodynamics $(\mathcal{F} = F_{\mu\nu}F^{\mu\nu})$

$$L_{\mathrm{BI}} = 4\beta^2 \left(1 - \sqrt{1 + \frac{\mathcal{F}}{2\beta^2}}\right)$$

- Logarithmic form of nonlinear electrodynamics

$$L_{\mathrm{LN}} = -4\beta^2 \ln\left(1 + \frac{\mathcal{F}}{4\beta^2}\right)$$

- Exponential form of nonlinear electrodynamics

$$L_{\mathrm{EN}} = 4\beta^2 \left[\exp\left(-\frac{\mathcal{F}}{4\beta^2}\right) - 1\right]$$

✓ $\beta \to \infty$ limit:

$$L_{\mathrm{BI}} = L_{\mathrm{LN}} = L_{\mathrm{EN}} = -\mathcal{F} + \frac{\mathcal{F}^2}{8\beta^2} + O\left(\beta^{-4}\right)$$

standard linear Maxwell field

Four-dimensional Einstein-dilaton system with exponential form of nonlinear electrodynamics

- Action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi) + \mathcal{L}(\mathcal{F},\Phi)\right)$$

$V(\Phi) = 2\Lambda_0 e^{2\xi_0\Phi} + 2\Lambda e^{2\xi\Phi}$: two Liouville-type dilaton potentials

➢ Exponential nonlinear electrodynamics with dilaton field

$$\mathcal{L}(\mathcal{F},\Phi) = 4\beta^2 e^{2\alpha\Phi}\left[\exp\left(-\frac{e^{-4\alpha\Phi}\mathcal{F}}{4\beta^2}\right) - 1\right]$$

$$\mathcal{L}(\mathcal{F},\Phi) \begin{cases} \xrightarrow{\beta\to\infty} -\mathcal{F}e^{-2\alpha\Phi} & \text{: Maxwell-dilaton system} \\ \xrightarrow{\alpha\to 0} 4\beta^2\left[\exp\left(\frac{-\mathcal{F}}{4\beta^2}\right) - 1\right] & \text{: exponential nonlinear electrodynamics} \end{cases}$$

Field equations

$$
\begin{cases}
G_{\mu\nu} = 2\left(\nabla_\mu \Phi\right)\left(\nabla_\nu \Phi\right) - \left((\nabla\Phi)^2 + \frac{V(\Phi)}{2} + \Pi\right) g_{\mu\nu} \\
\qquad - 2e^{-2\alpha\Phi}\left(F_{\mu\lambda}F_\nu{}^\lambda - \frac{\mathcal{F}}{2} g_{\mu\nu}\right)\frac{\partial \mathcal{L}(Y)}{\partial Y} \\
\partial_\mu \left(\sqrt{-g}e^{-2\alpha\Phi}\frac{\partial \mathcal{L}(Y)}{\partial Y} F^{\mu\nu}\right) = 0 \\
\nabla^2 \Phi = \frac{1}{4}\frac{dV(\Phi)}{d\Phi} + \alpha\Pi
\end{cases}
$$

$$
\Pi = 2\beta^2 e^{2\alpha\Phi}\left[2Y\frac{\partial \mathcal{L}(Y)}{\partial Y} - \mathcal{L}(Y)\right]
$$

$$
\mathcal{L}(Y) = e^{-Y} - 1, \qquad Y = \frac{e^{-4\alpha\Phi}\mathcal{F}}{4\beta^2}
$$

Ansatz of slowly rotating charged dilatonic BHs

- Metric

$$
ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 R^2(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right) + 2aG(r)\sin^2\theta dt d\phi
$$

- Gauge potential

$$
A_\mu dx^\mu = h(r)dt + aqC(r)\sin^2\theta d\phi
$$

- Dilaton field

$$
\Phi = \Phi(r)
$$

✓ Slowly rotating solution:
Solving field equations up to linear order of angular momentum parameter a

Electric field

$$\begin{cases} E(r) = \dfrac{dh(r)}{dr} = \dfrac{qe^{2\alpha\Phi}\exp\left[-\frac{1}{2}L_W\right]}{r^2R^2(r)} \\ L_W = \mathrm{Lambert}W\left(\dfrac{q^2}{\beta^2 r^4 R^4(r)}\right) \end{cases}$$: Lambert W function

$$\begin{cases} \mathrm{Lambert}W(x)\exp\left[\mathrm{Lambert}W(x)\right] = x \\ \mathrm{Lambert}W(x) = x - x^2 + 3x^3/2 - 8x^4/3 + \ldots \quad \text{for } |x| < 1 \end{cases}$$

✓ $\beta \to \infty$ limit:

$$E(r) \simeq e^{2\alpha\Phi}\left[\frac{q}{r^2R^2(r)} - \frac{q^3}{2\beta^2 r^6 R^6(r)} + O\left(\frac{q^5}{\beta^4 r^{10} R^{10}(r)}\right)\right]$$

same as dilatonic Reissner–Nordström black holes

4D slowly rotating dilatonic black holes with exponential form of nonlinear electrodynamics

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2R^2(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right) + 2aG(r)\sin^2\theta dt d\phi$$

$$A_\mu dx^\mu = h(r)dt + aqC(r)\sin^2\theta d\phi, \qquad \Phi(r) = \frac{\alpha}{1+\alpha^2}\ln\left(\frac{b}{r}\right)$$

$$F(r) = \left[(1+\alpha^2)\int r^{\frac{-2\alpha^2}{\alpha^2+1}} H(r)dr - m\right] r^{\frac{\alpha^2-1}{\alpha^2+1}}, \qquad R(r) = e^{\alpha\Phi(r)}$$

$$H(r) = 4q\beta\left(\frac{1}{\sqrt{L_W}} - \sqrt{L_W}\right) - \left[\Lambda_0\left(\frac{b}{r}\right)^{\frac{2\alpha\xi_0}{\alpha^2+1}} + \Lambda\left(\frac{b}{r}\right)^{\frac{2\alpha\xi}{\alpha^2+1}} + 4\beta^2\left(\frac{b}{r}\right)^{\frac{2\alpha^2}{\alpha^2+1}}\right] r^2 + \left(\frac{b}{r}\right)^{\frac{-2\alpha^2}{\alpha^2+1}}$$

$$G(r) = F(r) - \frac{1+\alpha^2}{1-\alpha^2}\left(\frac{b}{r}\right)^{\frac{-2\alpha^2}{\alpha^2+1}}$$

$$h = \int \frac{qe^{2\alpha\Phi}\exp\left(-L_W/2\right)}{r^2R^2(r)}dr, \quad C = \frac{r^2(1+\alpha^2)(dG/dr) - 2rG}{8q^2(1+\alpha^2)}\left(\frac{b}{r}\right)^{\frac{2\alpha^2}{\alpha^2+1}}$$

$$\left(\Lambda_0 = \frac{\alpha^2}{\left(\alpha^2-1\right)b^2}, \quad \xi_0 = \frac{1}{\alpha}, \quad \xi = \alpha\right)$$

Limits

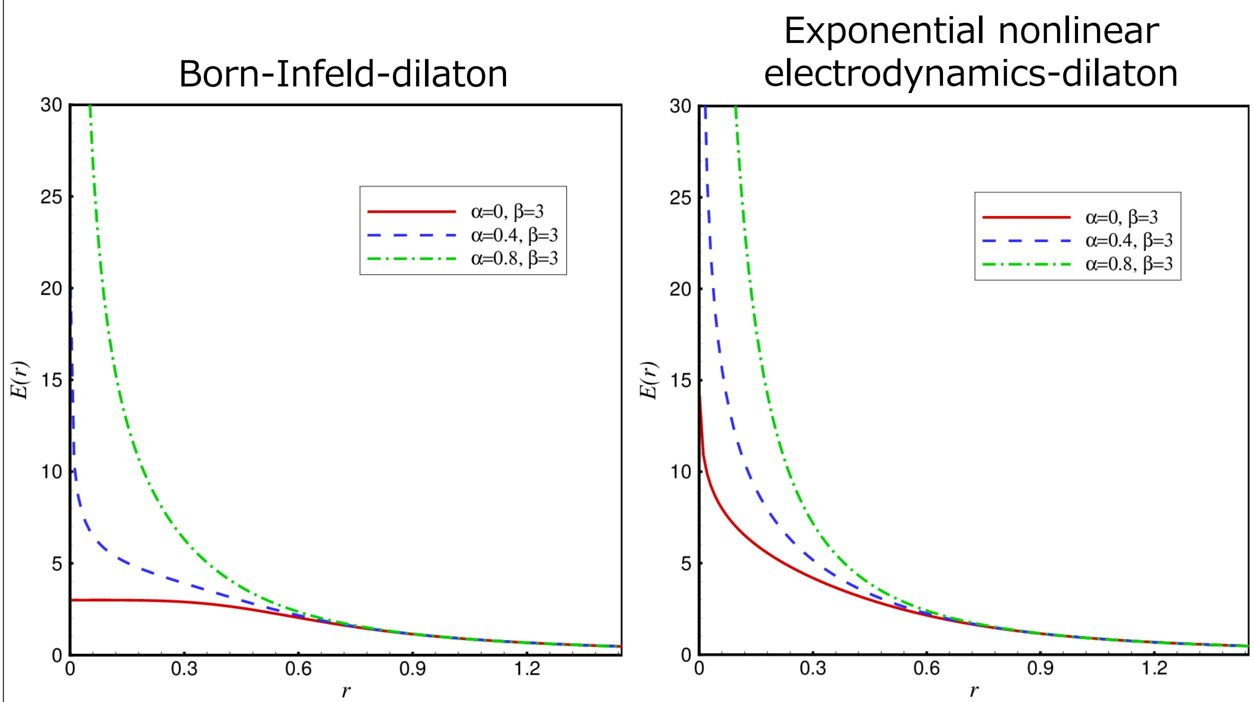
- $a \to 0$: Charged static dilaton black holes with exponential form of nonlinear electrodynamics
- $\alpha \to 0$: Slowly rotating nonlinearly charged black holes without dilaton field
- $\beta \to \infty$: Slowly rotating Einstein-Maxwell-dilaton black holes

$$F(r) \simeq \left(\frac{1+\alpha^2}{1-\alpha^2} + \frac{2(1+\alpha^2)q^2}{r^2}\right)\left(\frac{b}{r}\right)^{\frac{-2\alpha^2}{1+\alpha^2}} - mr^{\frac{\alpha^2-1}{1+\alpha^2}} + \frac{\left(1+\alpha^2\right)^2 \Lambda r^2}{\alpha^2-3}\left(\frac{b}{r}\right)^{\frac{2\alpha^2}{1+\alpha^2}} - \frac{\left(1+\alpha^2\right)^2 q^4}{2\beta^2 r^6(\alpha^2+5)}\left(\frac{b}{r}\right)^{\frac{-6\alpha^2}{1+\alpha^2}} + O\left(\beta^{-4}\right)$$

- $\alpha \to 0,\ \beta \to \infty$: Slowly rotating Kerr-Newman-AdS black holes

$$F(r) \simeq 1 - \frac{m}{r} + \frac{2q^2}{r^2} - \frac{\Lambda r^2}{3} - \frac{q^4}{10\beta^2 r^6} + O\left(\beta^{-4}\right)$$

Behaviors of electric fields



In both cases, electric fields are finite near the origin but diverge exactly at r=0, due to presence of dilaton field

Physical properties

- $r=0$: Curvature singularity ($R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\to\infty$)
- Neither asymptotically flat nor (A)dS, because of dilaton field
- $r=r_{\pm}(>0)$: Two horizons ($g^{rr}=F(r_{\pm})=0$)
- Entropy : $S = \dfrac{\omega b^{\frac{2\alpha^2}{1+\alpha^2}} r_+^{\frac{2}{1+\alpha^2}}}{4}$ (ω: area of unit S^2)
- Hawking temperature :

$$T_+ = -\frac{\alpha^2+1}{4\pi} r_+^{\frac{\alpha^2-1}{\alpha^2+1}} \left[\frac{b^{-\frac{2\alpha^2}{\alpha^2+1}}}{\alpha^2-1} + b^{\frac{2\alpha^2}{\alpha^2+1}}(4\beta^2+\Lambda) r_+^{\frac{2(1-\alpha^2)}{\alpha^2+1}} + 4q\beta r_+^{-\frac{2\alpha^2}{\alpha^2+1}} \left(\sqrt{L_{W+}} - \frac{1}{\sqrt{L_{W+}}} \right) \right]$$

$$L_{W+} = \mathrm{LambertW}\left(\frac{q^2}{\beta^2 r_+^4 R^4(r_+)} \right)$$

($T_+=0 \Leftrightarrow r_+=r_-$: extremal black holes)

Physical quantities

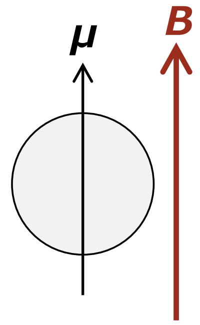
$$\begin{cases} \mathcal{M} = \dfrac{\omega b^{\frac{2\alpha^2}{1+\alpha^2}} m}{8\pi(1+\alpha^2)} & \text{: Mass} \\ J = \dfrac{\omega(3-\alpha^2) b^{\frac{2\alpha^2}{1+\alpha^2}} ma}{24\pi(1+\alpha^2)} & \text{: Angular momentum} \\ Q = \dfrac{\omega q}{4\pi} & \text{: Electric charge} \\ \mu = \dfrac{\omega q a}{4\pi} = Qa & \text{: Magnetic dipole moment} \end{cases}$$

Up to linear order of angular momentum parameter

effect of nonlinear electrodynamics specified by β
does not appear in physical quantities

Gyromagnetic ratio (g-factor) *g*

- Interaction between magnetic dipole moment of electron $\boldsymbol{\mu}$ and external magnetic field $\boldsymbol{B}$



$$H_{\mathrm{int}} = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

✓ Compared with Dirac equation ⇒ $\boldsymbol{\mu} = \dfrac{e}{m}\boldsymbol{S}$

- g-factor: Ratio of gyromagnetic ratio $\boldsymbol{\mu}/\boldsymbol{S}$ to Bohr magneton μ_{B}

$$\boldsymbol{\mu} = g\mu_{\mathrm{B}}\frac{\boldsymbol{S}}{\hbar} = g\frac{e\boldsymbol{S}}{2m}$$

$$g = 2$$

➢ Analogy: Electron ⇔ Charged rotating black holes (Carter, 1968)

$$\mu = g\frac{QJ}{2\mathcal{M}}$$ (μ=Qa : “Magnetic dipole moment”)

Gyromagnetic ratios of rotating black holes

- 4D Kerr-Newman black hole (Carter, 1968)

$$g = 2$$

- *n*-dim. slowly rotating Einstein-Maxwell black hole (Aliev, 2006)

$$g = n - 2$$

- 4D slowly rotating dilatonic black hole with exponential form of nonlinear electrodynamics

$$g = \frac{2\mu\mathcal{M}}{QJ} = \frac{6}{3-\alpha^2}$$

Only dilaton field modifies
gyromagnetic ratio of rotating black holes

Summary

We construct 4D slowly rotating black hole solutions in Einstein-dilaton system with

$\begin{cases} \text{exponential form of nonlinear electrodynamics} \\ \text{two Liouville-type dilaton potentials} \end{cases}$

- Asymptotic structure: neither flat nor (A)dS
  ($\because$ presence of dilaton field)
- Mass, Angular momentum, Gyromagnetic ratio:
  No modification by nonlinear electrodynamics

“Behavior of the new cylindrically symmetric gravitational solitonic waves”

byTakashi Mishima

[JGRG25(2015)P18]

Dec. 2015 / JGRG25 at Kyoto Univ.

Behavior of the New Cylindrically Symmetric Gravitational Solitonic Waves

T. Mishima (Nihon Univ.)
& S. Tomizawa (Tokyo Univ. of Tech.)

This contribution is based on the works: **[1]** Phys. Rev. **D90,** 044036 (2014),
[2] Phys. Rev. **D91,** 124058 (2015).

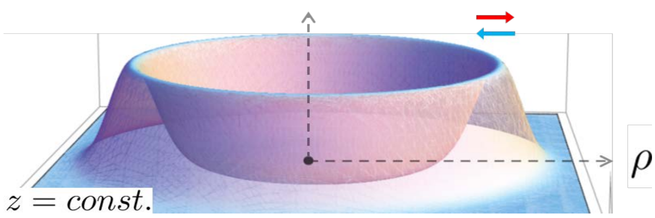
I. Introduction

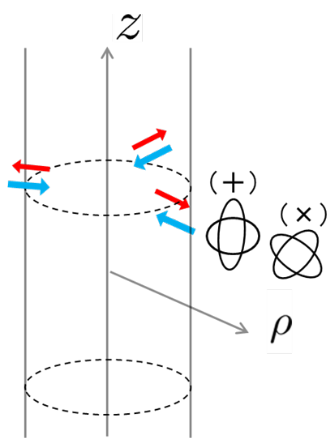
「In the previous works[1,2], we showed ***new gravitational wave solitons*** **can be constructed by the inverse scattering method equipped with the procedure introduced by Pomeranski ['05].」**

「We demonstrate ***some characteristic nonlinear behavior of the new solutions*****, especially two-soliton solutions presented in the work[2]」**

The solitonic waves treated here **:**

- **Cylindrically symmetric and regular packet-like waves**
- **Having nonlinearly interacting two modes: (+) and (×)**
- **Coming into the symmetric axis and reflecting off**





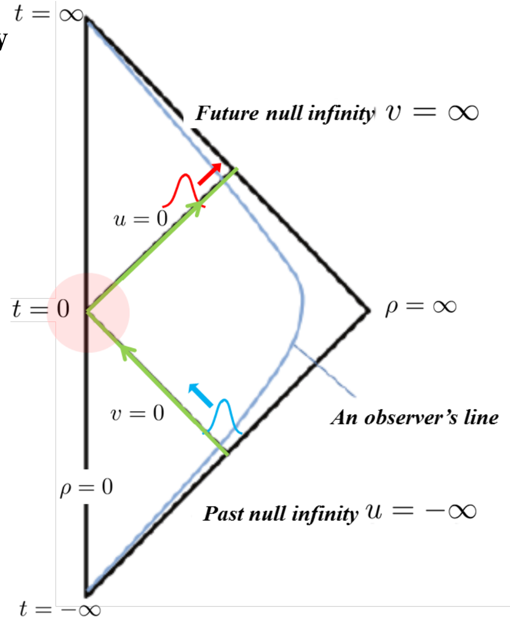
< Purpose >

「 **We clarify the peculiar nonlinearity of the new solution, compared with the soliton previously constructed by Economou and Tsoubelis ['88] , with the original Belinski-Zakharov IST (we call this ET-soliton).** 」

「 **We focus our concern on the following two points** 」

1. Comparison of in-going and out-going waves near null infinites : ***1.1 shape changing*** ***1.2 mode conversion***

2 Difference of non-linearity between new sol. And ET-sol.



「 **Schematic spacetime diagram (z = const. , φ = const.)** 」

(in-going) **(out-going)**

< Preparation−1 : metric, amplitudes and basic equations >

<u>**Following Piran, Safier and Stark['85]**</u>

(Kompaneets – Jordan_Ehlers metric for cylindrically symmetric spacetimes)

$$ds^2 = e^{2\psi}(dz + \omega d\phi)^2 + \rho^2 e^{-2\psi} d\phi^2 + e^{2(\gamma-\psi)}(d\rho^2 - dt^2)$$

(The metric depends only on ρ and t)

(Amplitudes used in the rest)

$$A_+ = 2\psi_{,v}, \quad B_+ = 2\psi_{,u} \qquad A_\times = \frac{e^{2\psi}\omega_{,v}}{\rho}, \quad B_\times = \frac{e^{2\psi}\omega_{,u}}{\rho}$$

$$A = \sqrt{A_+^2 + A_\times^2} \qquad B = \sqrt{B_+^2 + B_\times^2} \qquad \left(u = \frac{1}{2}(t-\rho), \quad v = \frac{1}{2}(t+\rho) \right)$$

(Basic equations for the amplitudes: deduced from vacuum Einstein equation)

$$A_{+,u} = \frac{A_+ - B_+}{2\rho} + A_\times B_\times, \quad B_{+,v} = \frac{A_+ - B_+}{2\rho} + A_\times B_\times$$

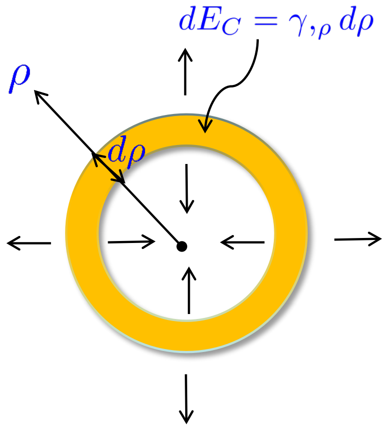
$$A_{\times,u} = \frac{A_\times + B_\times}{2\rho} - A_+ B_\times, \quad B_{\times,v} = -\frac{A_\times + B_\times}{2\rho} - A_\times B_+$$

(nonlinear term)

< Preparation－2 : C-energy based energy density and fluxes >

- C−energy: Thorne['65]

$$E_C(\rho') = \int_0^{\rho'} \gamma_{,\rho}\, d\rho$$



(Energy density and fluxes)

$$\gamma_{,\rho} = \frac{\rho}{8}\left(A_+^2 + B_+^2 + A_\times^2 + B_\times^2\right)$$

$$\gamma_{,t} = \frac{\rho}{8}\left(A_+^2 - B_+^2 + A_\times^2 - B_\times^2\right) = \frac{1}{2}(\gamma_{,u} + \gamma_{,v})$$

$$\gamma_{,u} = -\frac{\rho}{4}(B_+^2 + B_\times^2) \quad : \textit{outward null flux}$$

$$\gamma_{,v} = \frac{\rho}{4}(A_+^2 + A_\times^2) \quad : \textit{inward null flux}$$

II. Construction of new solutions and the metric form

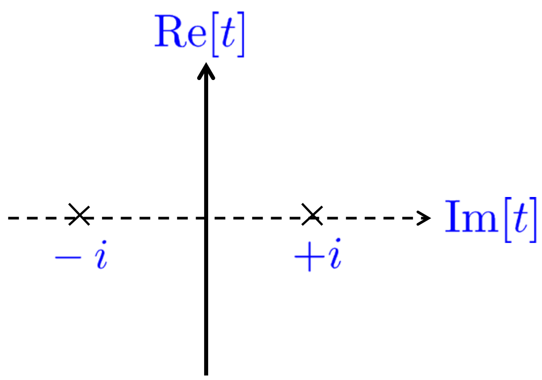
< Procedure to generate new type of gravitational solitons >

Pomeranski ['05]

(1) Choose Minkowski spacetime as a seed solution.

(2) Remove two solitons with trivial Belinski-Zakharov parameter $(1, 0)$ at $t = \pm i$

(3) Adding back the same solitons with non-trivial BZ-parameter set $(1,\ a)$ and $(1,\ \bar{a})$ $[\ a = a_r + a_i i = ke^{i\theta}\ ]$



< New Solution >

Introducing pseud-spheroidal coordinates : $t = xy, \quad \rho = \sqrt{(x^2+1)(y^2-1)}$

$(\infty > x > -\infty, \quad y > 1)$

(metric)

$$ds^2 = e^{2\psi}(dz+\omega d\phi)^2 + \rho^2 e^{-2\psi} d\phi^2 + e^{2(\gamma-\psi)}(d\rho^2 - dt^2)$$

$$e^{2\psi} = \frac{\mathcal{Y}}{\mathcal{X}}, \quad \omega = \frac{\mathcal{X}}{\mathcal{Y}}$$

$$\left\{\begin{array}{rcl}
\mathcal{X} &=& k^4(y^2-1)^2(y+1)^4 + 128k^2(y+1)^2(x^2+y^2)^3 + 4096(x^2+y^2)^4 \\
&& +128k^2(y+1)^2\left[\,(x^6 - x^4(6y^2-8y+1) + x^2y^2(y^2-8y+6) - y^4\,)\cos 2\theta \right. \\
&& \left. -2x(x^2-y^2+2y)(x^2(2y-1)+y^2)\sin 2\theta\,\right] \\
\mathcal{Y} &=& k^4(y^2-1)^4 + 128k^2(y^2-1)(x^2+y^2)^3 + 4096(x^2+y^2)^4 \\
&& +128k^2(x^2+1)(y^2-1)\left[\,(x^4-6x^2y^2+y^4)\cos 2\theta - 4xy(x^2-y^2)\sin 2\theta\,\right] \\
\mathcal{Z} &=& 32k(y+1)\left\{\,(64(x^2+y)(x^2+y^2)^3 \right. \\
&& -k^2(y^2-1)(y+1)^2[\,x^4-3x^2y(y-1)-y^3\,]\cos\theta \\
&& \left. -x(y-1)[\,64(x^2+y^2)^3 - k^2(y+1)^3(x^2(3y-1)-y^2(y-3)\,)\,]\sin\theta\,\right\}
\end{array}\right.$$

- **Regular and packet-like solitonic waves**

< ET solution : Economou –Tsoubelis['88] >

- **Regular and packet-like wave soliton**
- **Generated by using original Belinski-Zakharov ISM**

  (Adding two complex conjugate solitons to Minkowski seed simply) $\sim$ **(pure 2-soliton)**

(metric)

$$\left\{\begin{array}{rcl}
\mathcal{X} &=& 16(y-1)^2 + k^4(y+1)^2 + 8k^2(x^2+y^2) + 8k^2[(x^2-1)\cos 2\theta + 2x\sin\theta] \\
\mathcal{Y} &=& -k^4 - 16 + 8k^2x^2 + (k^2+4)^2y^2 + 8k^2(x^2+1)\cos 2\theta \\
\mathcal{Z} &=& 8k[(k^2+4)x^2 + (k^2-4)(x^2+1)y + (k^2+4)y^2]\cos\theta - 8k(k^2+4)x(y^2-1)\sin\theta
\end{array}\right.$$

- **New solution may be interpreted two ways …?**

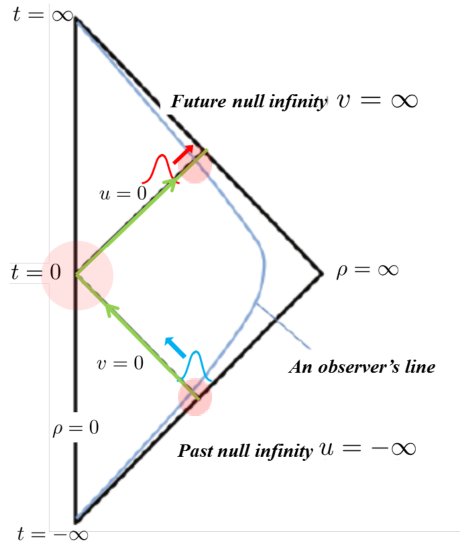
  - **2-soliton + anti 2-soliton = 4-soliton**
  - **2-soliton - 2-soliton = 0-soliton**

III. Comparison of the behavior of two solitons

1.1 Comparison of the shapes of in-going and out-going waves near null infinites

Incident wave : (: **blue**)
(*in-going*)

Reflecting wave : (: **red**)
(*out-going*)



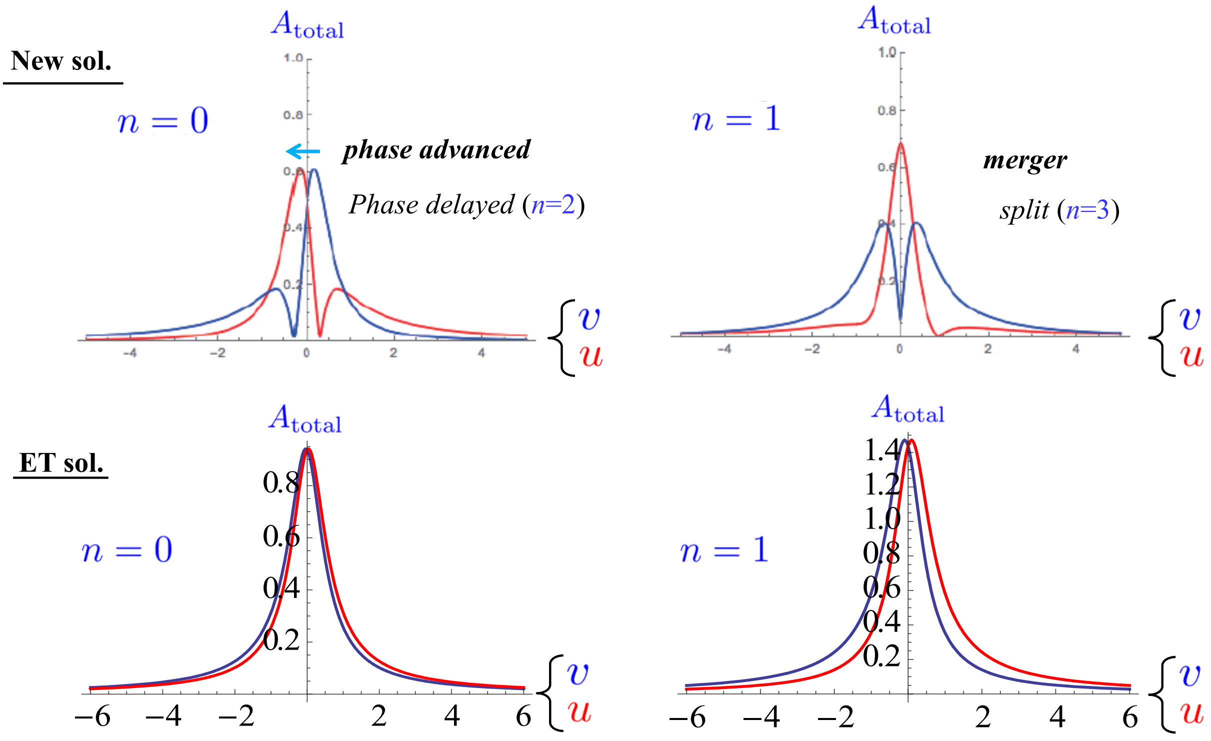
Graphs in the next slide :

Vertical axes : total amplitude

$$\left(A_{\text{toal}} = \sqrt{A_+^2 + A_\times^2 + B_+^2 + B_\times^2} \right)$$

Horizontal axes : retarded and advanced null coordinates
(v **and** u)

<e.g. $k=2\,,\quad \theta=\dfrac{n}{4}\pi$ **>**

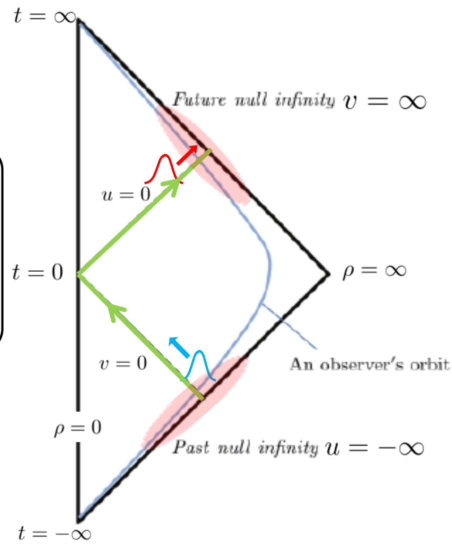


■ **New solutions have noticeable behaviors: phase shift, merger, split, …**

1.2 *Comparison of the ratios of mode conversion from the in-going wave soliton to the out-going wave soliton*

- **Calculate the following total fluxes for out-going $+$ and $\times$ modes and ingoing $+$ and $\times$ modes respectively.**

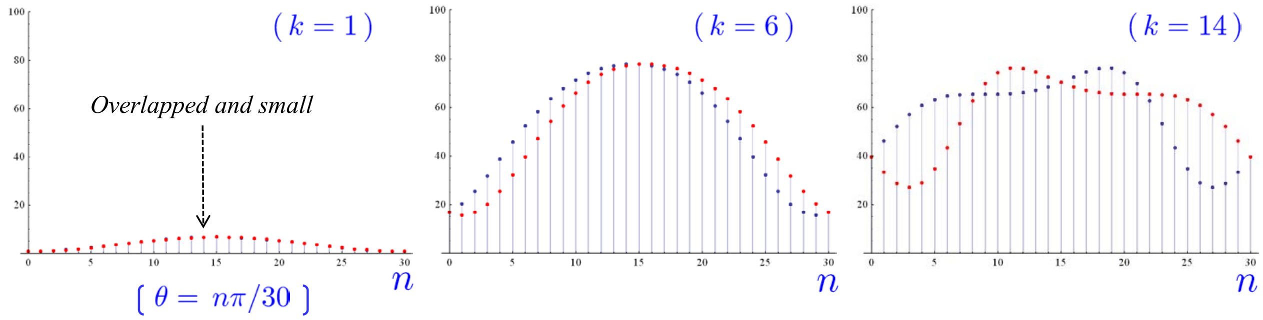
$$
\left(
\begin{array}{l}
I_+ = \displaystyle\int_{-\infty}^{\infty} \left[\frac{\rho}{4} A_+^2\right]_{u=-\infty} dv \;,\; I_\times = \displaystyle\int_{-\infty}^{\infty} \left[\frac{\rho}{4} A_\times^2\right]_{u=-\infty} dv \\
O_+ = \displaystyle\int_{-\infty}^{\infty} \left[\frac{\rho}{4} B_+^2\right]_{v=\infty} du \;,\; O_\times = \displaystyle\int_{-\infty}^{\infty} \left[\frac{\rho}{4} B_\times^2\right]_{v=\infty} du
\end{array}
\right)
$$



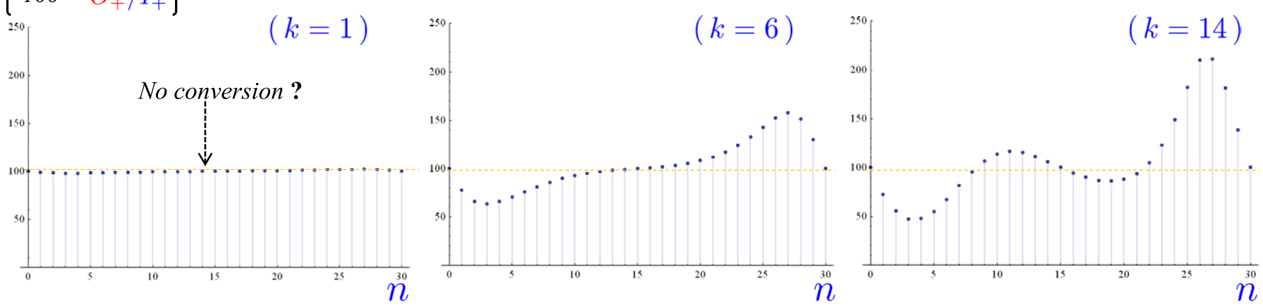
Next slides **show the ratios of I_+ (O_+) to total Flux and also O_+/I_+ for some values of the BZ-parameter's modulus: k = 1, 6, 14.**

(New sol.)

$\left\{ \begin{array}{l} 100 \times I_+/\text{Total Flux} : \textbf{Blue} \\ 100 \times O_+/\text{Total Flux} : \textbf{red} \end{array} \right\}$



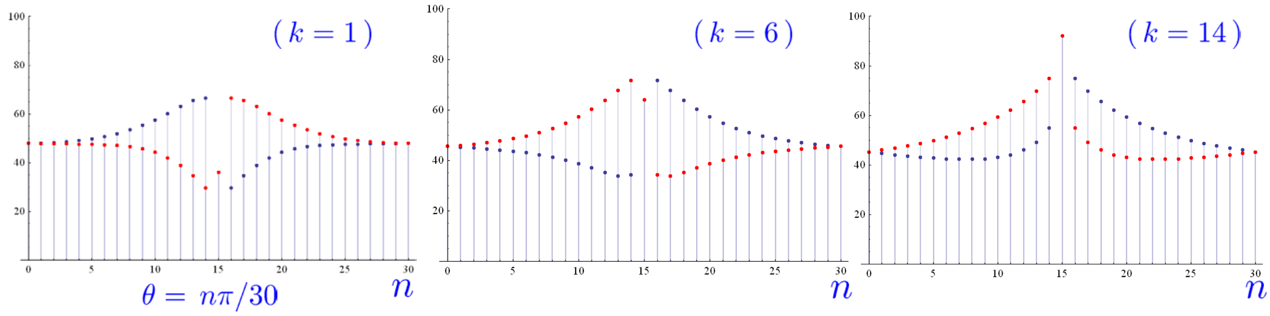
$\{ 100 \times O_+/I_+ \}$



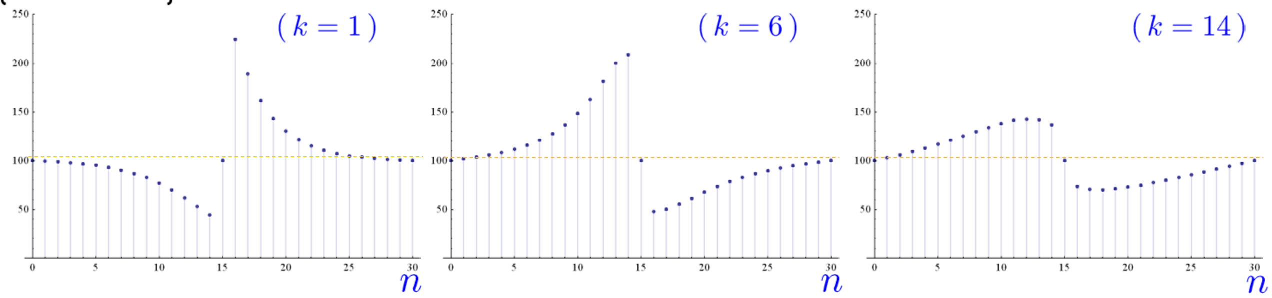
- **For small k, the mode conversion between $+$ and $\times$ seems not to occur.**

(ETT sol.)

$\left\{ \begin{array}{l} \boxed{100 \times I_+/\text{Total Flux}} : \textbf{Blue} \\ \boxed{100 \times O_+/\text{Total Flux}} : \textbf{red} \end{array} \right\}$



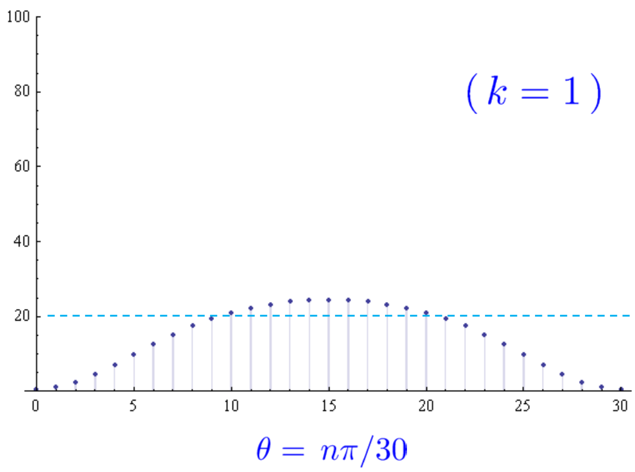
$\left\{ 100 \times \boxed{O_+/I_+} \right\}$

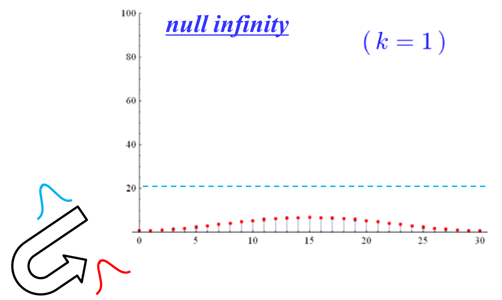


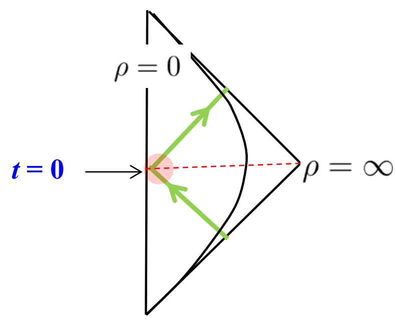
- **Simpler behavior of mode conversion are seen thorough all *k*.**

< Does the mode mixing occur for k = 1 in the new solution? >

<u>Ratio of + mode at t = 0 , when the soliton exists near axis.</u>







- **The + mode is enhanced when the soliton comes close to the axis !**

$(\rho = 0)$

2. Different non-linear behavior of new sol. and ET-sol.

「 To specify the region where the nonlinearity of the waves cannot be neglected, we plot the following *'ratios' of (the nonlinear term^2) to (linear term^2 + nonlinear term^2) of the basic equations* in the cases of new solution and ET solution, respectively. 」

$$R_{+} := \frac{(A_{\times}B_{\times})^2}{\left(\frac{A_{+}-B_{+}}{2\rho}\right)^2 + (A_{\times}B_{\times})^2} \qquad R_{\times} := \frac{(A_{\times}B_{+})^2}{\left(\frac{A_{\times}+B_{\times}}{2\rho}\right)^2 + (A_{\times}B_{+})^2}$$

$$\left(\begin{array}{ll} A_{+,u} = \dfrac{A_{+}-B_{+}}{2\rho} + \boxed{A_{\times}B_{\times}}, & B_{+,v} = \dfrac{A_{+}-B_{+}}{2\rho} + \boxed{A_{\times}B_{\times}} \\ A_{\times,u} = \dfrac{A_{\times}+B_{\times}}{2\rho} - \boxed{A_{+}B_{\times}}, & B_{\times,v} = -\dfrac{A_{\times}+B_{\times}}{2\rho} - \boxed{A_{\times}B_{+}} \end{array}\right)$$

(nonlinear term)

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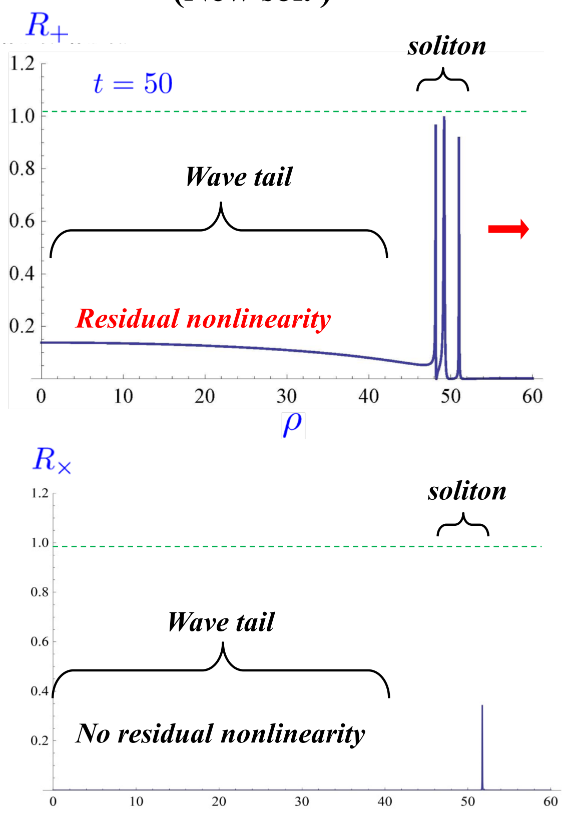
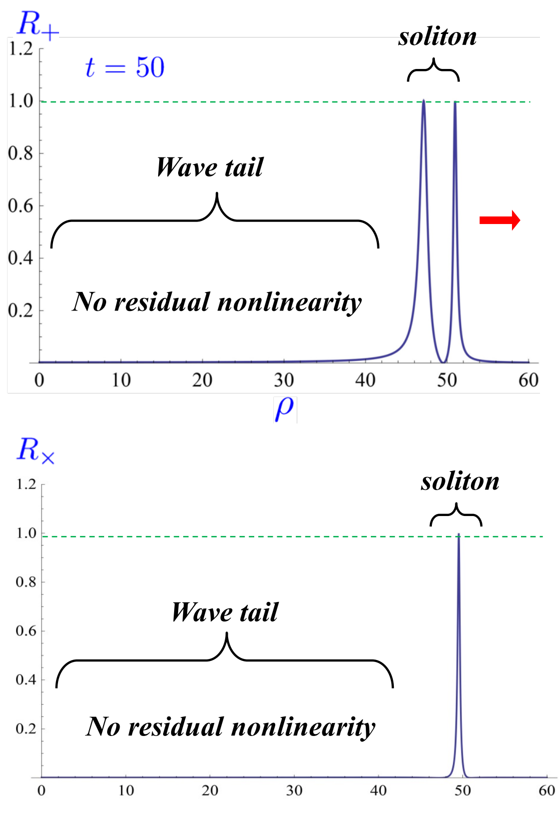
(e.g. : θ= π/4, *k*=1)

(New sol.)



(ETT sol.)



16

- For the new solution, nonlinearity of + mode seems to be nonlocal while nonlinearity of the ET solution is localized. These tendencies cause the difference of the asymptotic behavior of the solitons near time-like infinities.

new Solution	***ET Solution***
$A_+ \simeq B_+ \simeq \dfrac{k^2 \cos^2\theta}{2\,t^3} + \mathcal{O}(t^{-4})$	$A_+ \simeq B_+ \simeq \dfrac{2\tan\theta}{t^2} + \mathcal{O}(t^{-3})$
$A_\times \simeq -B_\times \simeq \dfrac{k\cos\theta}{2t^2} + \mathcal{O}(t^{-3})$	$A_\times \simeq -B_\times \simeq -\dfrac{(k^2-4)\sec\theta}{2k\,t^2} + \mathcal{O}(t^{-3})$

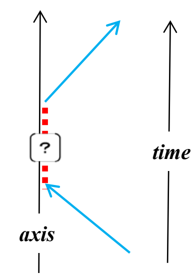
cf. *Einstein-Rosen waves* have the same asymptotic behavior as ET solutions.

→ (***+ mode only,*** ***basic equations are linear :*** $A_{+,u} = \dfrac{A_+ - B_+}{2\rho}, \quad B_{+,v} = \dfrac{A_+ - B_+}{2\rho}$)

IV. Summary

Characteristics of the new gravitational solitonic waves

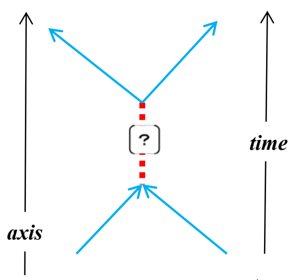
- ✓ **Clear Sign of phase shifts (both advanced and delayed)**
- ✓ **Large and complicated mode-conversion between + and × modes as time passes**
- ✓ **Non-local nonlinearity in wave tail**



(*Self-interaction*)

As further investigations,

「 Systematic analysis of scattering and collision of cylindrically symmetric higher multi-solitonic waves 」



(*2-body interaction*)

Deep understanding of the nonlinearity of gravity

“Vacuum excitation by sudden (dis-)appearance of a Dirichlet wall in a cavity”

by Umpei Miyamoto

[JGRG25(2015)P19]

Vacuum excitation by sudden (dis-)appearance of a Dirichlet wall in a cavity

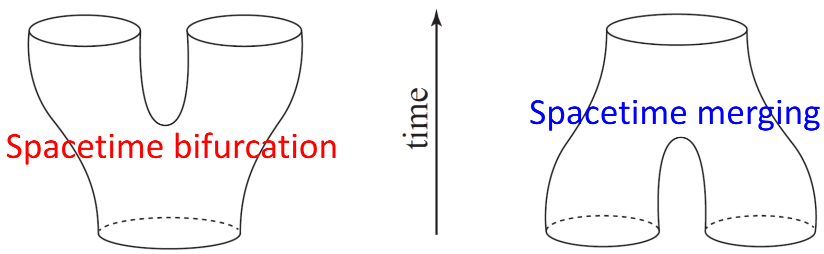
Umpei Miyamoto (Akita Prefectural U.)
with
Tomohiro Harada (Rikkyo U.)
Shunichiro Kinoshita (Chuo U.)

1

Abstract

- We investigate the vacuum excitation of a test scalar field by the sudden INSERTION (appearance) and REMOVAL (disappearance) of a both-sided Dirichlet wall in a 1D cavity.

- These systems can serve as toy models of a bifurcating spacetime and merging spacetime (see below)



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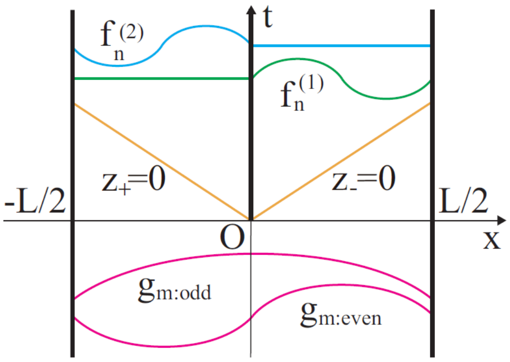
System (1): Sudden INSERTION of a Dirichlet wall



Massless scalar field in a 1D cavity

$$(-\partial_t^2 + \partial_x^2)\phi(t,x) = 0,$$

Dirichlet BCs at both ends

$$\phi(t, \pm\frac{L}{2}) = 0, \quad -\infty < t < \infty.$$

A Dirichlet wall is INSERTED at (t,x)=(0,0)

$$\phi(t,0) = 0, \quad t > 0,$$

Quantization of scalar field

The quantum field is expanded by two sets of mode functions (related by a Bogoliubov transformation each other)

$$\phi = \sum_{m=1}^{\infty} (\boldsymbol{b}_m g_m + \boldsymbol{b}_m^\dagger g_m^*) = \sum_{\gamma=1}^{2} \sum_{n=1}^{\infty} (\boldsymbol{a}_n^{(\gamma)} f_n^{(\gamma)} + \boldsymbol{a}_n^{(\gamma)\dagger} f_n^{(\gamma)*}),$$

The quantum field is in the vacuum defined by

$$\boldsymbol{b}_m |0_g\rangle = 0, \quad \forall m \in \mathbf{N}.$$

Calculate the Vacuum expectation value of EM tensor

$$\boldsymbol{T}_{\pm\pm} = (\partial_\pm \boldsymbol{\phi})^2, \quad z_\pm := t \pm x.$$

Double null coordinates

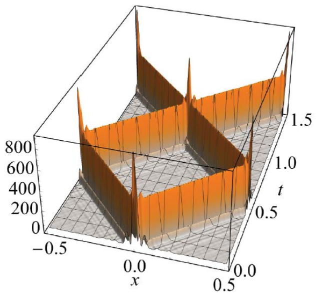
Result: Thunderbolt (explosive flux)

shifted Casimir energy

$$\langle 0_g|\boldsymbol{T}_{\pm\pm}|0_g\rangle_{t>0} = \frac{1}{\pi}\sum_{\substack{m=1\\ m:\mathrm{odd}}}^{\infty}\frac{1}{m}\sum_{\ell=-\infty}^{\infty}\delta(z_\pm-\ell L)^2 + \begin{cases} \dfrac{\pi}{4L^2}\displaystyle\sum_{\substack{m=2\\ m:\mathrm{even}}}^{\infty} m & (z_\pm=\ell L,\ \ell\in\mathbf{Z}) \\ \dfrac{\pi}{4L^2}\displaystyle\sum_{m=1}^{\infty} m & (\mathrm{otherwise}) \end{cases},$$

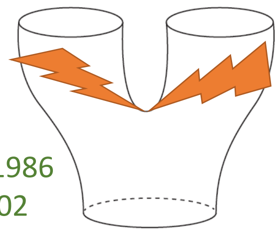
“thunderbolt”

ambient Casimir energy



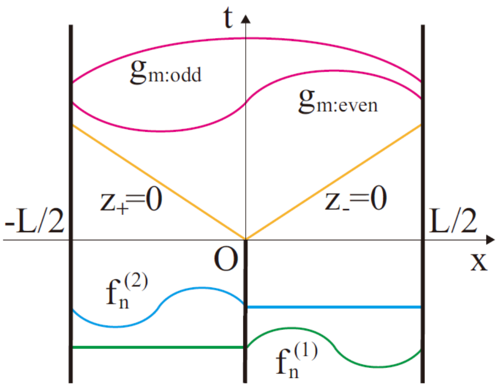
$\langle 0_g|(\boldsymbol{T}_{--}+\boldsymbol{T}_{++})|0_g\rangle_{t>0}$

Cf
Anderson-DeWitt, Found.Phys.1986
Ishibashi-Hosoya, Phys.Rev.D2002



The backreaction of thunderbolts will prevent the spacetime from bifurcating

System (2): Sudden REMOVAL of a Dirichlet wall



Dirichlet wall is REMOVED at (t,x)=(0,0)

$$\phi(t,0)=0,\quad t<0.$$

The quantum field is in the vacuum defined by

$$\boldsymbol{a}_n^{(\gamma)}|0_f\rangle = 0,$$

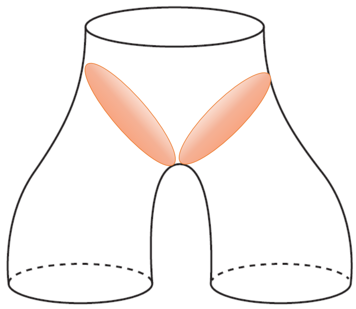
$$\forall\gamma\in\{1,2\},\quad \forall n\in\mathbf{N}.$$

Result: finite flux only to shift the Casimir energy

shifted Casimir energy

$$\langle 0_f | \boldsymbol{T}_{\pm\pm} | 0_f \rangle_{t>0} = \begin{cases} \dfrac{\pi}{2L^2}\displaystyle\sum_{n=1}^{\infty} n & (z_\pm = \ell L,\ \ell \in \mathbf{Z}) \\ \dfrac{\pi}{L^2}\displaystyle\sum_{n=1}^{\infty} n & (\text{otherwise}) \end{cases} .$$

ambient Casimir energy



The backreaction of weak flux will NOT prevent the spacetime from merging

Summary and future directions

- Particle creation by the sudden INSERTION of a wall is EXPLOSIVE.
  ➔ Spacetime bifurcation will be prevented by the backreaction.

- Particle creation by the sudden REMOVAL of a wall is WEAK.
  ➔ Spacetime merging will be not prevented by the backreaction.

- Future directions:
  - Smooth INSERTION/REMOVAL
  - Various BCs (e.g. Robin-type: ϕ'=aϕ)
  - Proposal of Lab. Experiments (cf: Wilson et al. Nature 2011)
  - … and many

“Halo/Galaxy Bispectrum with Equilateral-type Primordial Trispectrum”

by Shuntaro Mizuno

[JGRG25(2015)P20]

25th JGRG @ Yukawa Institute

7 ~ 11 December 2015

Halo/Galaxy bispectrum with Equilateral-type Primordial Trispectrum

Shuntaro Mizuno (Waseda)

With Shuichiro Yokoyama (Rikkyo)

Phys. Rev. D 91, 123521 (arXiv: 1504.05505 [astro-ph.CO])

Primordial non-Gaussianity

$$\mathcal{P}_\zeta^2\, f_{\rm NL}\, \mathcal{S}(k_1,k_2,k_3)$$

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = (2\pi)^3\delta^3(k_1+k_2+k_3)\boxed{B_\zeta(k_1,k_2,k_3)}$$

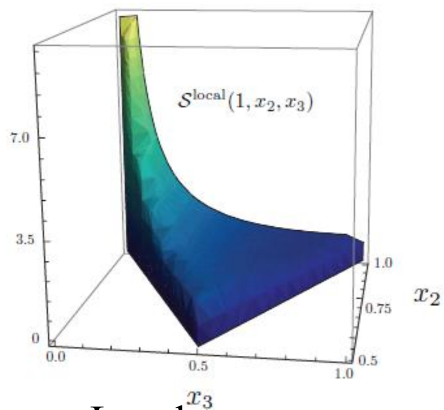
Local and equilateral shapes

$x_2 \equiv k_2/k_1$

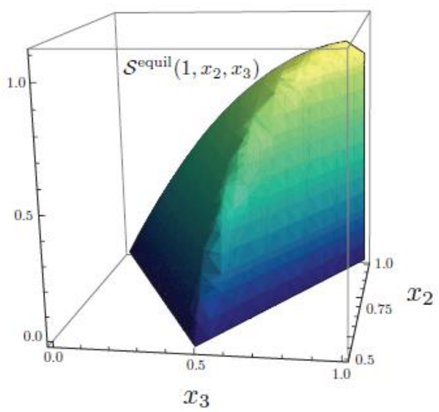
$x_3 \equiv k_3/k_1$

$k_3 < k_2 < k_1$

$k_2 + k_3 > k_1$



Local-type



Equilateral-type

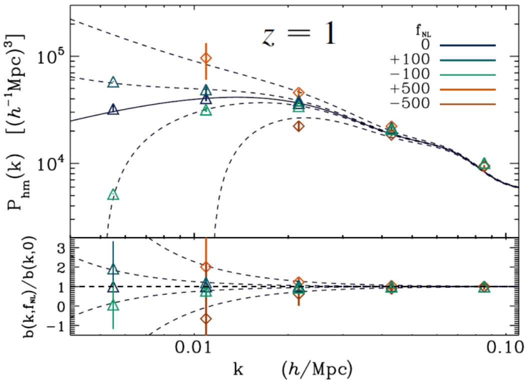
Planck constraint (68% CL)

$$\boxed{f_{\rm NL}^{\rm local} = 0.8 \pm 5.0}$$

$$\boxed{f_{\rm NL}^{\rm equil} = -4 \pm 43}$$

Constraints on local-type NG from LSS

Dalal et al `08



$1.6\times10^{13}M_\odot < M < 3.2\times10^{13}M_\odot$

Constraints on bispectrum

Giannatonio et al `13

$$\boxed{-36 < f_{\rm NL}^{\rm local} < 45 \quad (95\%\ \mathrm{CL})}$$

Future constraints:

Yamauchi et al `14

SKA (Square Km Array)

$|f_{NL}| < 0.1$?

Constraints on trispectrum

Desjacques and Seljak `10

$$\zeta(x) = \zeta_G(x) + \frac{9}{25}\underline{g_{\rm NL}^{\rm local}\zeta_G^3(x)}$$

$$\boxed{-3.5\times10^5 < g_{\rm NL}^{\rm local} < 8.2\times10^5 \quad (95\%\ \mathrm{CL})}$$

How about equilateral-type NG ?

Integrated Perturbation Theory (iPT)

Matsubara `12, `13, Bernardeau et al `08

・Multi-point propagator of biased objects

$$\left\langle \frac{\delta^n \delta_X(\mathbf{k})}{\delta\delta_{\rm L}(\mathbf{k}_1)\delta\delta_{\rm L}(\mathbf{k}_2)\cdots\delta\delta_{\rm L}(\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta(\mathbf{k}_1+\mathbf{k}_2+\cdots+\mathbf{k}_n)\underline{\Gamma_X^{(n)}(\mathbf{k}_1,\mathbf{k}_2,\cdots,\mathbf{k}_n)}$$

Gravitational evolution

Lagrangian bias, ….

δ_X : number density field of the biased objects

$\delta_{\rm L}$: linear density field which is related with primordial curvature perturbation ζ through

$$\boxed{\delta_{\rm L}(k) = \mathcal{M}(k)\zeta(k);\ \mathcal{M}(k) = \frac{2}{3}\frac{D(z)}{D(z_*)(1+z_*)}\frac{k^2T(k)}{H_0^2\Omega_{\rm m0}}}$$

$D(a)$: growth factor $\quad T(k)$: transfer function

⇒ spectra of biased objects (Halo/Galaxy) systematically !!

Multi-point propagators on large scales

Matsubara `12

$$\Gamma_X^{(1)}(\mathbf{k}) \approx 1 + \underline{c_1^{\mathrm{L}}(k)}$$

$$\Gamma_X^{(2)}(\mathbf{k}_1,\mathbf{k}_2) \approx F_2(\mathbf{k}_1,\mathbf{k}_2) + \left(1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_2^2}\right)\underline{c_1^{\mathrm{L}}(\mathbf{k}_1)} + \left(1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1^2}\right)\underline{c_1^{\mathrm{L}}(\mathbf{k}_2)} + \underline{c_2^{\mathrm{L}}(\mathbf{k}_1,\mathbf{k}_2)}$$

$$F_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{10}{7} + \left(\frac{k_2}{k_1}+\frac{k_1}{k_2}\right)\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2} + \frac{4}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\right)^2$$

c_n^{L} : renormalized bias function defined in Lagrangian space

$$\left\langle \frac{\delta^n\delta_X^{\mathrm{L}}(\mathbf{k})}{\delta\delta_{\mathrm{L}}(\mathbf{k}_1)\delta\delta_{\mathrm{L}}(\mathbf{k}_2)\cdots\delta\delta_{\mathrm{L}}((\mathbf{k}_n)}\right\rangle = (2\pi)^{3-3n}\delta(\mathbf{k}_1+\mathbf{k}_2+\cdots+\mathbf{k}_n)c_n^{\mathrm{L}}(\mathbf{k}_1,\mathbf{k}_2,\cdots,\mathbf{k}_n)$$

The other parts include the information of displacement field in Lagrangian perturbation theory

Renormalized bias function

For the mass function, we adopted Sheth-Tormen model given by

$$f_{\mathrm{ST}}(\nu) = A(p)\sqrt{\frac{2}{\pi}}\left[1+(q\nu^2)^{-p}\right]\sqrt{q}\nu e^{-q\nu^2/2}$$

$$p = 0.3,\ q = 0.707$$

$$A(p) = \left[1+\Gamma(1/2-p)/(\sqrt{\pi}2^p)\right]^{-1}$$

$$\Rightarrow \quad c_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) \approx b_n^{\mathrm{L}}(M) \qquad (|\boldsymbol{k}_i| \to 0)$$

$$b_1^{\mathrm{L}}(M) = \frac{1}{\delta_{\mathrm{c}}}\left[q\nu^2 - 1 + \frac{2p}{1+(q\nu^2)^p}\right]$$

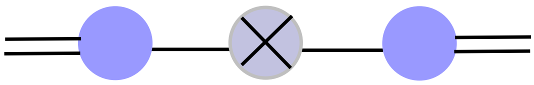
$$b_2^{\mathrm{L}}(M) = \frac{1}{\delta_{\mathrm{c}}{}^2}\left[q^2\nu^4 - 3q\nu^2 + \frac{2p(2q\nu^2+2p-1)}{1+(q\nu^2)^p}\right]$$

no scale-dependence on large scales

Effects on Halo/galaxy power spectrum

・Diagrams for the power spectrum of the biased objects

P_0

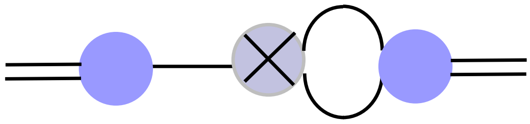


$\Gamma_X^{(1)}(\mathbf{k})\,P_{\rm L}(k)\,\Gamma_X^{(1)}(-\mathbf{k})$

large scale limit $\Rightarrow \propto \mathcal{M}(k)^2 P_\zeta(k) \underline{\propto k}$

$k \ll \underline{p}$ typical scale of the biased objects

$P_{\rm bis}$



$\Gamma_X^{(1)}(\mathbf{k})\,B_{\rm L}(k,p,|\mathbf{p}+\mathbf{k}|)\,\Gamma_X^{(2)}(\mathbf{p},-\mathbf{p}-\mathbf{k})$

large scale limit

$$\begin{cases} \propto \mathcal{M}(k)k^{-3} \underline{\propto k^{-1}} & \text{for } B_\zeta^{\rm local} \\ \propto \mathcal{M}(k)k^{-1} \underline{\propto k} & \text{for } B_\zeta^{\rm equil} \end{cases}$$

enhancement (for $B_\zeta^{\rm local}$)

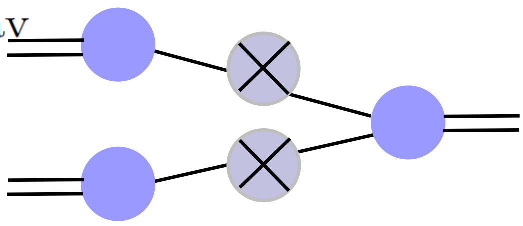
no enhancement (for $B_\zeta^{\rm equil}$)

Effects on Halo/galaxy bispectrum

Yokoyama, Matsubara, Taruya `13

・Diagrams for the bispectrum of the biased objects

$B_{\rm grav}$

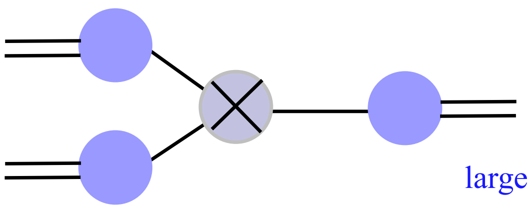


large scale limit $\Rightarrow \propto \mathcal{M}(k)^4 P_\zeta(k)^2 \underline{\propto k^2}$

$k \equiv k_1 = k_2 = k_3 \ll p$

$\Gamma_X^{(1)}(\mathbf{k}_1)P_L(k_1)\Gamma_X^{(1)}(\mathbf{k}_2)P_L(k_2)\Gamma_X^{(2)}(-\mathbf{k}_1,-\mathbf{k}_2)$

$B_{\rm bis}$

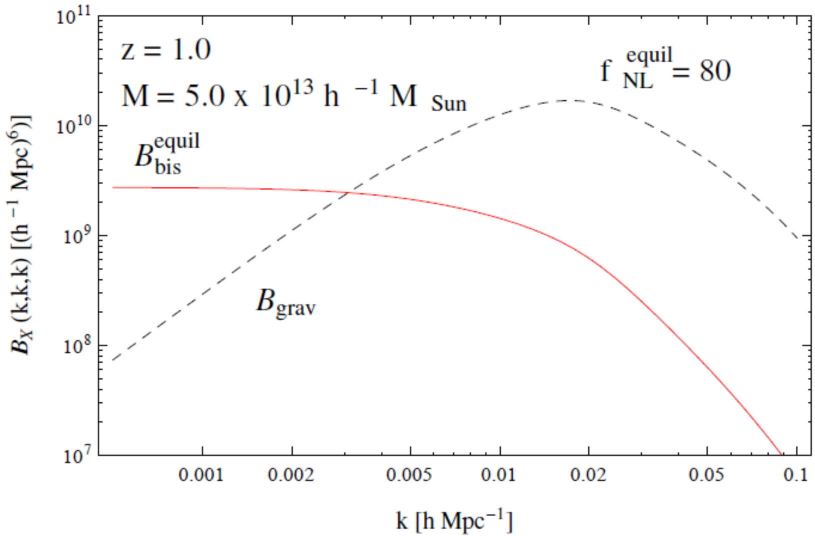


large scale limit $\Rightarrow \propto \mathcal{M}(k)^3 P_\zeta(k)^2 \underline{\propto k^0}$

for $B_\zeta^{\rm equil}$

$\Gamma_X^{(1)}(\mathbf{k}_1)\Gamma_X^{(1)}(\mathbf{k}_2)B_{\rm L}(k_1,k_2,k_3)\Gamma_X^{(1)}(\mathbf{k}_3)$

Halo/galaxy bispectrum with $f_{\rm NL}^{\rm equil}$



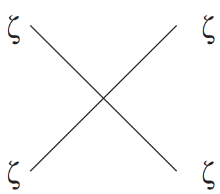
$\Delta f_{\rm NL}^{\rm equil} < 20$ by future surveys !!

Sefusatti and Komatsu `07

Primordial trispectrum in general k-inflation

Arroja, SM, Koyama, Tanaka `09, Chen et al `09, (Smith, Senatore, Zaldarriaga `15)

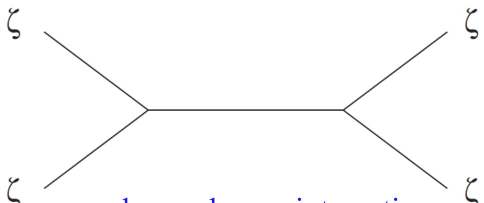
$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{\rm Pl}^2 R + 2P(X,\phi)] \quad X \equiv -(1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$



contact interaction

$$\begin{aligned}&\langle\Omega|\delta\phi(0,\mathbf{k}_1)\delta\phi(0,\mathbf{k}_2)\delta\phi(0,\mathbf{k}_3)\delta\phi(0,\mathbf{k}_4)|\Omega\rangle^{\rm CI}\\ &= -i\int_{-\infty}^{0} d\eta\langle 0|[\delta\phi_I(0,\mathbf{k}_1)\delta\phi_I(0,\mathbf{k}_2)\delta\phi_I(0,\mathbf{k}_3)\\ &\quad\times\delta\phi_I(0,\mathbf{k}_4), \underline{H_I^{(4)}(\eta)}]|0\rangle,\end{aligned}$$

$$H_I^{(4)}(\eta) = \int d^3x[\underline{\beta_1\delta\phi_I'^4} + \underline{\beta_2\delta\phi_I'^2(\partial\delta\phi_I)^2} + \underline{\beta_3(\partial\delta\phi_I)^4}],$$



scalar-exchange interaction

$$\begin{aligned}&\langle\Omega|\delta\phi(0,\mathbf{k}_1)\delta\phi(0,\mathbf{k}_2)\delta\phi(0,\mathbf{k}_3)\delta\phi(0,\mathbf{k}_4)|\Omega\rangle^{\rm SE}\\ &= -\int_{-\infty}^{0} d\eta\int_{-\infty}^{\eta} d\tilde\eta\langle 0|[[\delta\phi_I(0,\mathbf{k}_1)\delta\phi_I(0,\mathbf{k}_2)\\ &\quad\times\delta\phi_I(0,\mathbf{k}_3)\delta\phi_I(0,\mathbf{k}_4), \underline{H_I^{(3)}(\eta)}], \underline{H_I^{(3)}(\tilde\eta)}]|0\rangle,\end{aligned}$$

$$H_I^{(3)}(\eta) = \int d^3x[\underline{Aa\delta\phi_I'^3} + \underline{Ba\delta\phi_I'(\partial\delta\phi_I)^2}],$$

$$T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = T_\zeta^{\dot\sigma^4} + T_\zeta^{\dot\sigma^2(\partial\sigma)^2} + T_\zeta^{(\partial\sigma)^4} + T_\zeta^{\dot\sigma^6} + T_\zeta^{\dot\sigma^4(\partial\sigma)^2} + T_\zeta^{\dot\sigma^2(\partial\sigma)^4}$$

Trispectra from contact interactions

・Concrete expressions

$$\frac{T_\zeta^{\dot\sigma^4}}{(2\pi^2\mathcal{P}_\zeta)^3} = \frac{221184}{25}\frac{g_{\rm NL}^{\dot\sigma^4}}{(\sum k_i)^5 k_1k_2k_3k_4}$$

$$\frac{T_\zeta^{\dot\sigma^2(\partial\sigma)^2}}{(2\pi^2\mathcal{P}_\zeta)^3} = -\frac{27648}{325}g_{\rm NL}^{\dot\sigma^2(\partial\sigma)^2}\left[\frac{k_1^2k_2^2(\mathbf{k}_3\cdot\mathbf{k}_4)}{(\sum k_i)^3\Pi k_i^3}\left(1+3\frac{(k_3+k_4)}{\sum k_i}+12\frac{k_3k_4}{(\sum k_i)^2}\right)+\text{perms.}\right]$$

$$\frac{T_\zeta^{(\partial\sigma)^4}}{(2\pi^2\mathcal{P}_\zeta)^3} = \frac{165888}{2575}g_{\rm NL}^{(\partial\sigma)^4}\frac{[(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_3\cdot\mathbf{k}_4)+\text{perms.}]}{\sum k_i\Pi k_i}$$
$$\times\left(1+\frac{\sum_{i<j}k_ik_j}{(\sum k_i)^2}+3\frac{\Pi k_i}{(\sum k_i)^3}\sum\frac{1}{k_i}+12\frac{\Pi k_i}{(\sum k_i)^4}\right)$$

These trispectra also appear in effective field theory of inflation !!

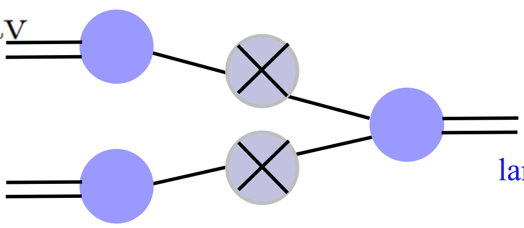
・Constraints from CMB (95 % CL) Smith, Senatore, Zaldarriaga `15

$$\boxed{(-9.38\times10^6) < g_{NL}^{\dot\sigma^4} < (2.98\times10^6)}\quad\boxed{(-2.34\times10^6) < g_{NL}^{(\partial\sigma)^4} < (0.19\times10^6)}$$

Effects on Halo/galaxy bispectrum

・Diagrams for the bispectrum of the biased objects

$B_{\rm grav}$



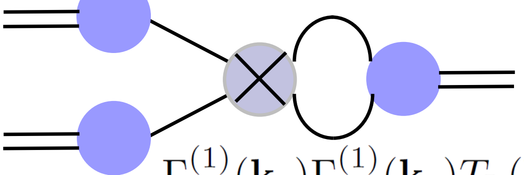
$\propto \mathcal{M}(k)^4P_\zeta(k)^2 \propto k^2$

large scale limit & equilateral config.

$k\equiv k_1=k_2=k_3\ll p$

typical scale of the biased objects

$B_{\rm tri}$



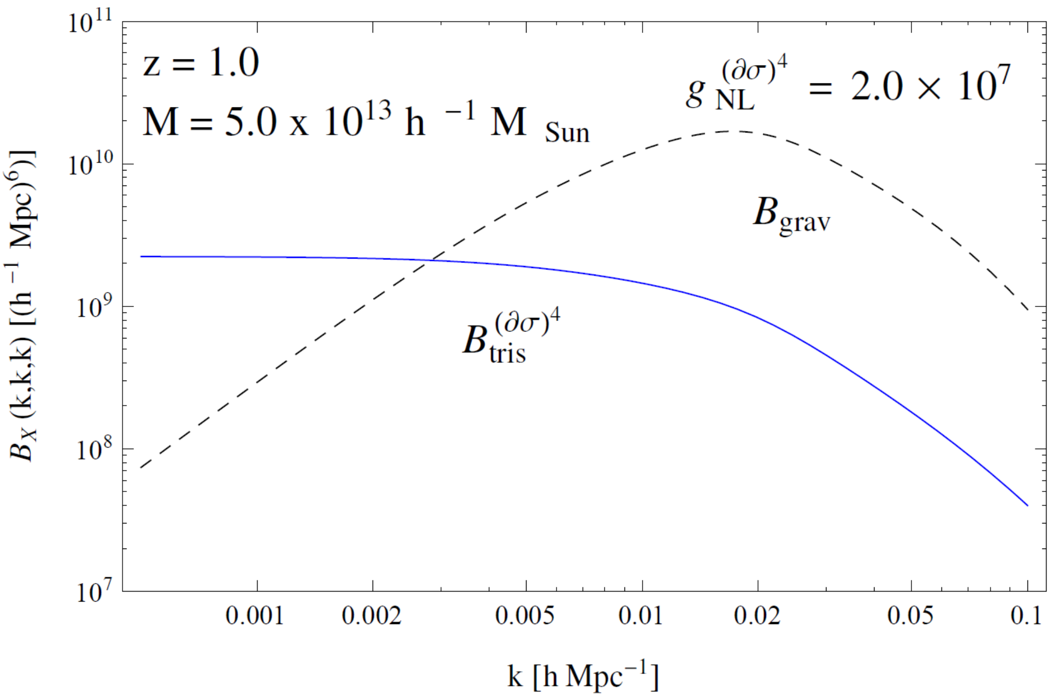
$$\Gamma_X^{(1)}(\mathbf{k}_1)\Gamma_X^{(1)}(\mathbf{k}_2)T_{\rm L}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p},\mathbf{k}_3-\mathbf{p})\Gamma_X^{(2)}(\mathbf{p},\mathbf{k}_3-\mathbf{p})$$

large scale limit & equilateral config.

$$\begin{cases}\propto\mathcal{M}(k)^2k^{-2}\propto k^2 & \text{for } g_{\rm NL}^{\dot\sigma^4}\\ \propto\mathcal{M}(k)^2k^{-4}\propto k^0 & \text{for } g_{\rm NL}^{\dot\sigma^2(\partial\sigma)^2},\ g_{\rm NL}^{(\partial\sigma)^4},\ \text{(same as } f_{\rm NL}^{\rm equil}\text{)}\end{cases}$$

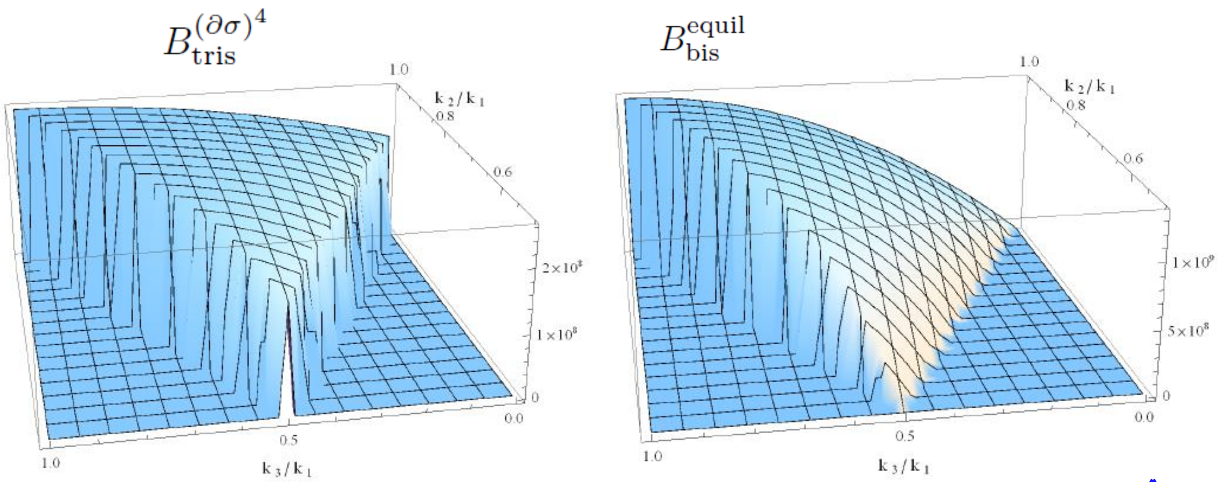
Halo/galaxy bispectrum with $g_{\rm NL}^{(\partial\sigma)^4}$

Adopting maximum allowed values by CMB observations

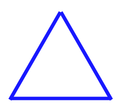


Contributions from $g_{\rm NL}^{(\partial\sigma)^4}$ and $f_{\rm NL}^{\rm equil}$

- Shape-dependence of Halo/galaxy bispectrum



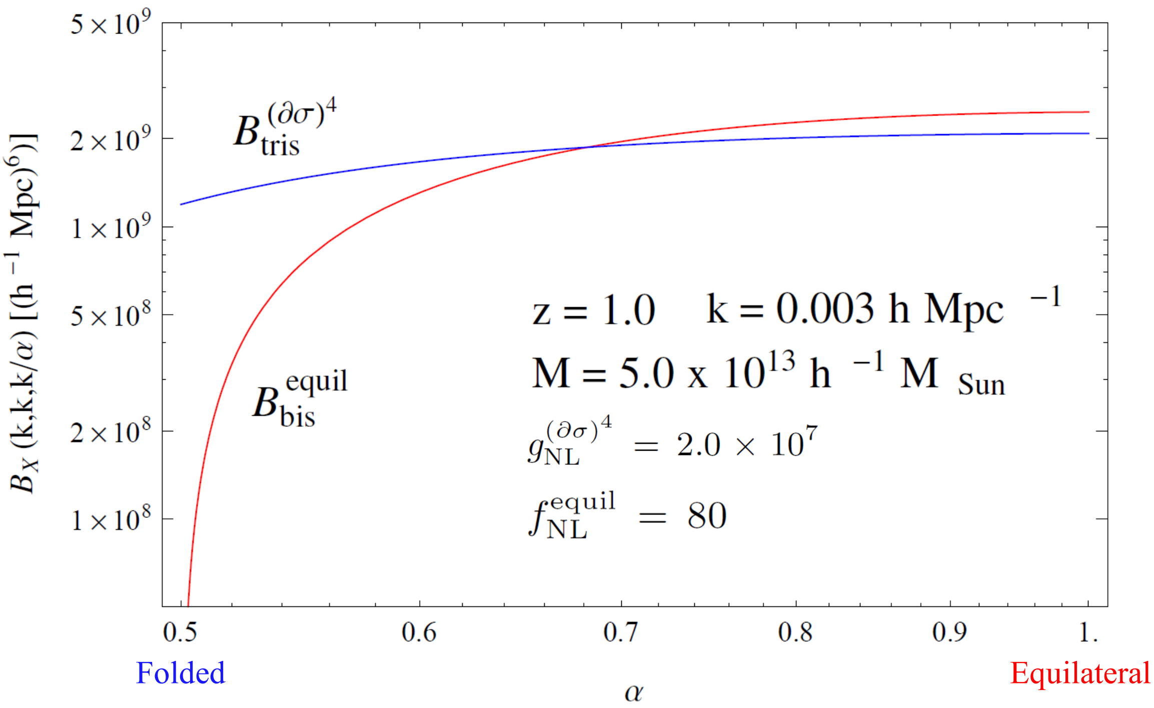
So far, we have limited the equilateral configuration ($k_1 = k_2 = k_3 = k$)



But the folded configuration ($k_1/2 = k_2 = k_3 = k$)
is also helpful to distinguish $B_{\rm tris}^{(\partial\sigma)^4}$ with $B_{\rm bis}^{\rm equil}$

Halo/galaxy bispectrum with $g_{\rm NL}^{(\partial\sigma)^4}$ and $f_{\rm NL}^{\rm equil}$

isosceles configuration given by $k \equiv k_1 = k_2 = \alpha k_3$



Conclusions and Discussions

- Halo/galaxy bispectrum was shown to be useful tool to distinguish equilateral-type NG from gravitational nonlinearity.

$$B_{\rm grav} \propto k^2, \quad B_{\rm bis}^{\rm equil} \propto k^0 \quad \Rightarrow \quad \Delta f_{\rm NL}^{\rm equil} < 20$$

- We can also constrain primordial trispectrum generated by general k-inflation based on halo/galaxy bispectrum.

$$B_{\rm tris}^{\dot\sigma^2(\partial\sigma)^2,\,(\partial\sigma)^4} \propto k^0 \quad \Rightarrow \quad \Delta g_{\rm NL}^{\dot\sigma^2(\partial\sigma)^2,\,(\partial\sigma)^4} = \mathcal{O}(10^6)\ ?$$

- Constraints on more general class of inflation models which give equilateral-type bispectrum

  k-inflation (scalar-exchange interaction)

  Ghost inflation, Lifshitz scalar, Galileon inflation,….

“Two Dimensional Black Hole in Bigravity”

by Taisaku Mori

[JGRG25(2015)P21]

Two Dimensional Black Hole in Bigravity

Taisaku Mori(QG-Lab. Nagoya University)

In collaboration with S.Nojiri, T.Katsuragawa and H. Suenobu

Abstract

Two dimensionally reduced gravity theory is investigated as an effective theory of four dimensional gravity, which may describe the behavior of four dimensional black hole and its evaporation. Especially we consider the model in bigravity and study the classical solutions and their stability.

The bigravity model may give predictions different from those in general relativity because the bigravity includes massive spin-2 mode besides usual massless graviton.

Bigravity

Bigravity theory (Hassan-Rosen) incudes two independent metric tensor fields, *g* and *f* . The action of the bigravity is given by

$$S_{\mathrm{bigravity}} = M_g^2 \int d^4x \sqrt{-\det(g)} R(g) + M_f^2 \int d^4x \sqrt{-\det(f)} R(f) - 2m_0^2 M_{\mathrm{eff}}^2 \int d^4x \sqrt{-\det(g)} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1}f}\right)$$

- Two dynamical metrics: $g_{\mu\nu}$ and $f_{\mu\nu}$ $\qquad \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\rho} \left(\sqrt{g^{-1}f}\right)^{\rho}{}_{\nu} = g^{\mu\rho} f_{\rho\nu}$
- Planck mass scales: $\dfrac{1}{M_{\mathrm{eff}}} = \dfrac{1}{M_g} + \dfrac{1}{M_f}$
- Free parameters: β_n, Mass of spin-2 field (massive graviton) : m^0

$$[\mathrm{X}] = X^{\mu}_{\mu}, \quad e_0(\mathrm{X}) = 1\,, \quad e_1(\mathrm{X}) = [\mathrm{X}]\,, \quad e_2(\mathrm{X}) = \frac{1}{2}\left([\mathrm{X}]^2 - [\mathrm{X}^2]\right)\,, \quad e_3(\mathrm{X}) = \frac{1}{6}\left([\mathrm{X}]^3 - 3[\mathrm{X}][\mathrm{X}^2] + 2[\mathrm{X}^3]\right)\,,$$

$$e_4(\mathrm{X}) = \frac{1}{24}\left([\mathrm{X}]^4 - 6[\mathrm{X}]^2[\mathrm{X}^2] + 3[\mathrm{X}^2]^2 + 8[\mathrm{X}][\mathrm{X}^3] - 6[\mathrm{X}^4]\right) = \det(\mathrm{X})\,, \quad e_k(\mathrm{X}) = 0 \quad \text{for} \quad k > 4$$

When we regard the bigravity as an alternative theory of gravity, it is interesting to apply this theory to the two dimensional model coupled with dilaton.

CGHS Model

In general relativity, imposing spherical symmetry on the four dimensional space-time corresponds to the two dimensional gravity theory coupled with dilaton. This model is called CGHS (Callan–Giddings–Harvey–Strominger) model. The degrees of freedom in this model is simplified. In our work, we focus on the black hole solution.

$$g^{(4)}_{\mu\nu} dx^\mu dx^\nu = g_{ab} dx^a dx^b + S_{AB} dx^A dx^B$$

$$S_{AB} dx^A dx^B = \lambda^{-2} e^{-2\phi} \Omega_{AB} dx^A dx^B\,, \quad \Omega_{AB} dx^A dx^B = d\theta^2 + \sin^2\theta d\varphi^2$$

$$\text{dilaton}: \ \phi = \phi(x^0, x^1) \qquad \lambda = \text{const}, \ \dim\lambda = [L^{-1}]$$

The action of CGHS model is given by,

$$S_{CGHS} = \frac{1}{2\pi} \int \sqrt{-g} e^{-2\phi} (R^{(2)} + 4\nabla_a \phi \nabla^a \phi + 4\lambda^2) - \frac{1}{4\pi} \int d^2x \sqrt{-g} \nabla_a f \nabla^a f$$

- matter field : f · 2D Ricci scalar : $R^{(2)}$

In vacuum ($f = 0$) ,

- E.o.M for δg_{ab} : $\nabla_a \nabla_b \phi + g_{ab}\left[(\nabla\phi)^2 + \nabla^2\phi - \lambda\right] = 0$
- E.o.M for $\delta\phi$: $R^{(2)} + 4\lambda^2 + 4\nabla^2\phi - 4(\nabla\phi)^2 = 0$

There are three components in the two dimensional metric. Two of those are gauge freedom followed by covariance. For simplicity, we consider the conformal gauge and light cone coordinate.

- Conformal gauge : $g_{01} = g_{10} = 0, \quad g_{00} = g_{11} = e^{2\rho(x^0, x^1)}$
- Light cone coordinate : $x^+ = x^0 + x^1, \quad x^- = x^0 - x^1$
- Line element : $ds^2 = -\frac{1}{2} e^{2\rho} dx^+ dx^-$, · Ricci scalar : $R^{(2)} = 8e^{-2\rho} \partial_+ \partial_- \rho$

In this gauge and coordinate, E.o.M becomes,

$$\rho - \phi = f_+(x^-) + f_-(x^+) \qquad\qquad \partial_+\partial_-(\phi - \rho) = \partial_+\partial_- e^{-2\phi} + \lambda^2$$

$$ds^2 = -\frac{1}{2} e^{2\rho} dx^+ dx^-$$

coordinate transformation

$$ds^2 = -\frac{1}{2} e^{2\phi} d\tilde{x}^+ d\tilde{x}^-$$

$$f_\pm = 0, \quad \rho = \phi$$

$$\partial_+\partial_- e^{-2\rho} = -\lambda^2$$

$$\Longleftrightarrow e^{-2\rho} = e^{-2\phi} = \frac{M_{ADM}}{\lambda} - \lambda^2 x^+ x^-$$

$$\therefore \ R^{(2)} = \frac{4\lambda M_{ADM}}{M_{ADM}/\lambda - \lambda^2 x^+ x^-}$$

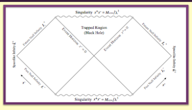
It has singularity at $\boxed{x^+ x^- = \dfrac{M_{ADM}}{\lambda^3}}$

Dilaton field corresponds to the surface area of sphere in 4D. So the event horizons can be expressed as

$$\partial_\pm \left(e^{-2\phi}\right) = 0$$

$$\Longleftrightarrow x^\pm = 0$$

Event horizon



Black hole solution in CGHS model

CGHS Model in Bigravity

We consider the CGHS model in bigravity. This motivation is that if the massive spin-2 graviton which is predicted by string theory run up to the planck scale, we can see the evaporation of the black hole as an effective theory of massive spin-2 field.

In order to study the dynamics of two-dimensional reduced bigravity, we introduce the following two metrics in conformal gauge

$$g^{(4)}_{\mu\nu} dx^\mu dx^\nu = g_{ab} dx^a dx^b + S^g_{AB} dx^A dx^B\,,$$
$$f^{(4)}_{\mu\nu} dx^\mu dx^\nu = f_{ab} dx^a dx^b + S^f_{AB} dx^A dx^B\,.$$

$$\text{Topology}: \ S^1 \times S^1, \ a, b = 0, 1, \ A, B = \theta, \varphi, \ g_{ab} = g_{ab}(x^0, x^1), \ f_{ab} = f_{ab}(x^0, x^1)$$

$$S^{g(f)}_{AB} dx^A dx^B = \lambda^{-2}_{g(f)} e^{-2\phi_{g(f)}} \Omega_{AB} dx^A dx^B\,, \quad \Omega_{AB} dx^A dx^B = d\theta^2 + \sin^2\theta d\varphi^2$$

$$\text{Dilaton}: \ \phi_{g(f)} = \phi_{g(f)}(x^0, x^1), \ \lambda_{g(f)} = \text{const}, \ \dim\lambda_{g(f)} = [L^{-1}]$$

Simplified action inspired by CGHS model. (vacuum)

$$\begin{aligned} S = &\frac{4\pi M_g^2}{\lambda_g^2} \int d^2x \sqrt{-\det(g)} e^{-2\phi_g} \left[R_g + 2\left(\nabla_{(g)a}\phi_g\right)^2 + 2\lambda_g^2 e^{2\phi_g}\right] \\ &+ \frac{4\pi M_f^2}{\lambda_f^2} \int d^2x \sqrt{-\det(f)} e^{-2\phi_f} \left[R_f + 2\left(\nabla_{(f)a}\phi_f\right)^2 + 2\lambda_f^2 e^{2\phi_f}\right] \\ &- m_0^2 \frac{4\pi M_g^2}{\lambda_g^2} \int d^2x \sqrt{-\det(g)} e^{-2\phi_g} \left[3 - \lambda \mathrm{tr}\left(\sqrt{g^{-1}f}\right) - 2\lambda e^\xi + \lambda^2 e^{2\xi} \det\left(\sqrt{g^{-1}f}\right)\right] \end{aligned}$$

- $M_g = M_f$, · $\lambda = \dfrac{\lambda_g}{\lambda_f}$, · $\xi = \phi_g - \phi_f$
- Minimal model : $\beta_0 = 3, \ \beta_1 = -1, \ \beta_2 = 0, \ \beta_3 = 0, \ \beta_4 = 1$

In this Similarly, in conformal gauge and light cone coordinate,

- Line element : $g_{ab} dx^a dx^b = -\frac{1}{2} e^{2\rho_g} dx^+ dx^-, \quad f_{ab} dx^a dx^b = -\frac{1}{2} e^{2\rho_f} dx^+ dx^-$
- Ricci scalar : $R^{(2)}_g = 8e^{-2\rho_g} \partial_+ \partial_- \rho_g, \quad R^{(2)}_f = 8e^{-2\rho_f} \partial_+ \partial_- \rho_f$

E.o.M after coordinate transformation

$$g_{ab} dx^a dx^b = -\frac{1}{2} e^{2\rho_g} dx^+ dx^- \quad \Longrightarrow \quad -\frac{1}{2} e^{2\phi_g} d\tilde{x}^+ d\tilde{x}^-$$

$$\Longrightarrow \begin{cases} \cdot \ \rho_f = 2\phi_g - \phi_f \\ \cdot \ \partial_+\partial_- e^{-2\phi_g} = -\lambda_g^2 + \dfrac{3m_0^2}{4}\left(1 - \lambda e^\xi\right) \\ \cdot \ \partial_+\partial_- (\phi_g - \phi_f) = -\dfrac{m_0^2}{8} \lambda^2 e^{2(2\phi_g - \phi_f)} \\ \cdot \ \partial_+\partial_- (\phi_g - \phi_f) = -\dfrac{1}{2} \lambda_f^2 e^{2\phi_g - \phi_f} + \dfrac{m_0^2}{8} \lambda e^{(\phi_g + \phi_f)} \end{cases}$$

How to solve it ?

Since this equations are very complicated, we cannot find general solution…

For simply, consider the special case

$$f_{\mu\nu} = C^2 g_{\mu\nu}, \qquad C : \ \text{const}$$

But this proportional relation leads to usual CGHS model…

If we assume m_0 to be very small, it also corresponds to the CGHS model that we performed.

Summary and Discussion

We search the black hole solution of this model, but it is very difficult to solve these equation. There are two way both simple and not to be a trivial solution.

・First, since we use the same coordinate transformation as we perform the usual CGHS model, this difficulty might be able to removed by taking the proper coordinate transformation.

・Second, we consider the case where the two metric tensors are proportional to each other and we find it becomes trivial solution, there may be a possibility of another simple relation between these metrics.

We consider the simplified action motivated by CGHS model. Bigravity theory is, however, constructed not to exist ghost mode, it might be suffer from ghost in CGHS formalism. We confirm that this action do not exist ghost mode.

This model cannot be solved analytically. But we find that the difference of this model and usual CGHS model are the overall factor of mass of massive spin-2. So we conclude that the dynamics of the simple two- dimensional system is modified by the non trivial mode.

“Multi-field effects on Non-Gaussianity in Starobinsky inflation”

by Taro Mori

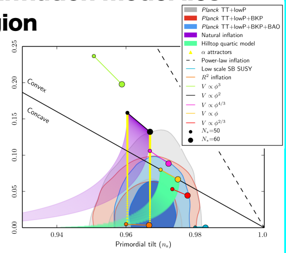
[JGRG25(2015)P22]

Multi-field effects on Non-Gaussianity in Starobinsky inflation

SOKENDAI/KEK Taro Mori

Introduction

- Recent CMB observation strongly suports inflationary cosmology
- In particular, Starobinsky inflation model lies deep inside of allowed region



n_s and r are related to 2-point fct. of curvature perturbation

But f_{NL} **is also important to evaluate inflation models. This is related to 3-point fct.**

$$\langle \zeta_{(\mathbf{k_1})}\zeta_{(\mathbf{k_2})}\zeta_{(\mathbf{k_3})}\rangle$$

Theory ··· $f_{NL}^{loc} \sim \mathcal{O}(10^{-2})$

Observation ··· $f_{NL}^{loc} = 0.8 \pm 5.0 \sim \mathcal{O}(1)$

Planck collaboration, 2015

Almost all single field inflation predict tiny f_{NL}

The formula of f_{NL}

Using δN- formalism

f_{NL} **is given by···**

$$\boxed{f_{NL} = -\frac{5}{6}\frac{N_{,}{}^{A}N_{,}{}^{B}N_{AB}}{N_{,}{}^{I}N_{,I}}}$$

N : **e-folding**

But if kinetic terms in Lagrangian are NOT canonically normalized, such as···

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{pl}^2}{2}R - \frac{1}{2}\mathcal{G}(\phi)_{IJ}\partial_\mu\phi^I\partial_\mu\phi^J - V(\phi)\right]$$

Then the field-space has non-trivial geometry

Line element of field sp. :

$$d\sigma^2 = \mathcal{G}(\phi)_{IJ}d\phi^I d\phi^J$$

So we have to modify the formula with replacing derivatives to covariant derivatives

$$\boxed{f_{NL} = -\frac{5}{6}\frac{N_{,}{}^{A}N_{,}{}^{B}\mathcal{D}_A\mathcal{D}_B N}{(N_{,I}N_{,}{}^{I})^2}}$$

where···

$$\mathcal{D}_J A^I \equiv \partial_J A^I + \Gamma^I_{JK}A^K$$

constructed by field space metric $\mathcal{G}_{IJ}$

Multi-field model

***Supergravity or Superstring theory may provide many scalar fields so let us concern Multi-field inflation model**

• Kaiser's model (Example) D.Kaiser *et al*, arXiv:1210.7487

Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left[f(\phi^I)R - \frac{1}{2}\mathcal{G}_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$

non-minimal coupling fct: $f(\phi,\chi) = \frac{1}{2}[m_{pl}^2 + \xi_\phi\phi^2 + \xi_\chi\chi^2]$

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g\phi^2\chi^2 + \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_\chi}{4}\chi^4$$

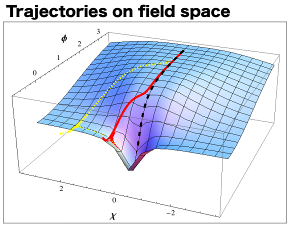
Einstein frame

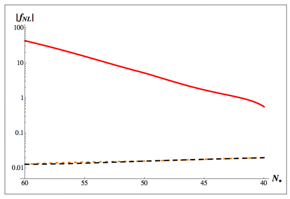
$$S_{\mathrm{E}} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{m_{\mathrm{pl}}^2}{2}\tilde{R} - \frac{1}{2}\tilde{\mathcal{G}}_{IJ}\tilde{g}^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \tilde{V}(\phi^I)\right]$$

$$\tilde{V}(\phi,\chi) = \frac{m_{\mathrm{pl}}^4}{4}\frac{\left(2m_\phi^2\phi^2 + 2m_\chi^2\chi^2 + 2g\phi^2\chi^2 + \lambda_\phi\phi^4 + \lambda_\chi\chi^4\right)}{\left[m_{\mathrm{pl}}^2 + \xi_\phi\phi^2 + \xi_\chi\chi^2\right]^2}$$

In Multi-field case the trajectory of inflation is NOT unique there can be various trajectories corresponding with various initial conditions.

Trajectories on field space





initial condition

1. $(\phi,\chi) = (3.1, 1.1\times10^{-4})$
2. $(\phi,\chi) = (3.1, 1.1\times10^{-3})$
3. $(\phi,\chi) = (3.1, 1.1\times10^{-2})$

f_{NL} could be enhanced by Multi-field effects??

• $R^2+\chi^2$ model C.de Bruck and L.E. Paduraru arXiv:1505.01727

Jordan frame

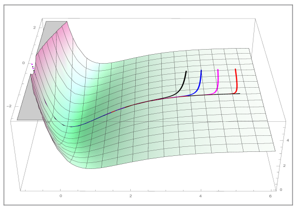
$$S_J = \int d^4x\sqrt{-g}\left[\frac{m_{pl}^2 R}{2} + \frac{\mu}{2}R^2\right] + \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}m_\chi^2\chi^2\right]$$

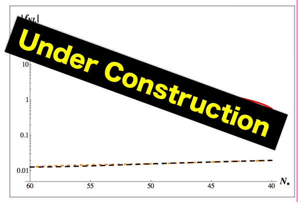
Just a combination of Starobinsky-type action and quadratic potential

Einstein frame

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{m_{pl}^2\tilde{R}}{2} - \frac{\tilde{g}^{\mu\nu}}{2}(\tilde{\partial}_\mu\psi)(\tilde{\partial}_\nu\psi) - \frac{1}{2}\tilde{g}^{\mu\nu}e^{-2\alpha\psi}(\tilde{\partial}_\mu\chi)(\tilde{\partial}_\nu\chi) - \tilde{V}\right] \quad \tilde{V} = \frac{m_{pl}^4}{8\mu}(1-e^{2\alpha\psi})^2 + \frac{1}{2}m_\chi^2 e^{-4\alpha\psi}\chi^2$$

$$e^{2\alpha\psi} = 1 + \frac{2\mu R}{m_{pl}^2} \quad \alpha = \frac{1}{\sqrt{6}m_{pl}}$$





Summary:

- The formula to compute f_{NL} should be modified if Lagrangian contains non-canonical kinetic terms.
- In Multi-field inflation inflaton can goes along various trajectories in field space.
- f_{NL} could be enhanced by such Multi-field effects.

“Possible golden events for ringdown gravitational waves -- total mass dependence --”

by Hiroyuki Nakano

[JGRG25(2015)P23]

POSSIBLE GOLDEN EVENTS FOR RINGDOWN GRAVITATIONAL WAVES -- TOTAL MASS DEPENDENCE --

Hiroyuki Nakano (Kyoto University)

Ref.) H. Nakano, T. Tanaka and T. Nakamura,
Phys. Rev. D92, 064003 (2015), [arXiv:1506.00560]

JGRG25, YITP, Kyoto University

MOTIVATION

- Black hole (BH) singularities appear unavoidably in GR.
- Unphysical!?
- The allowance of the presence of singularities will not be acceptable even though they are hidden behind the event horizon.
- Various possibilities of the singularity avoidance have been discussed.

GR or not?

- Approach: calculate parameter estimation errors by using Fisher information matrix.

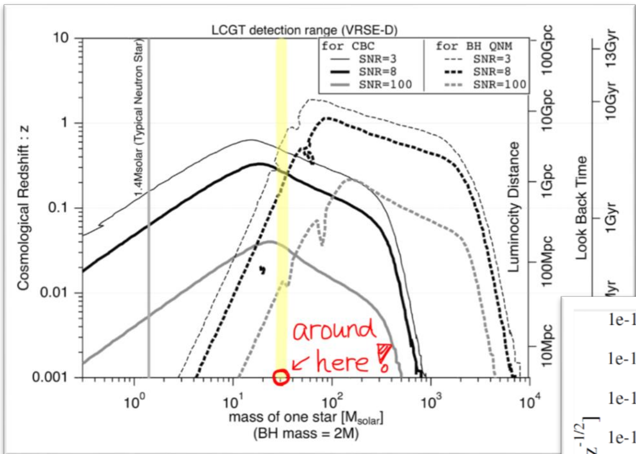
POP III BBHS

- **Kinugawa**, Inayoshi, Hotokezaka, Nakauchi and Nakamura, MNRAS 442, 2963 (2014) [arXiv:1402.6672].
- **Kinugawa**, Miyamoto, Kanda and Nakamura, arXiv:1505.06962.

Pop III binary black holes (BBHs):
Typical chirp mass ～30M_sun
Typical total mass ～60M_sun

- Total mass: 60M_sun, equal mass is characteristic one.
- (restrict to nonspinning BBHs in this poster)

GWS FROM POP III BBHS



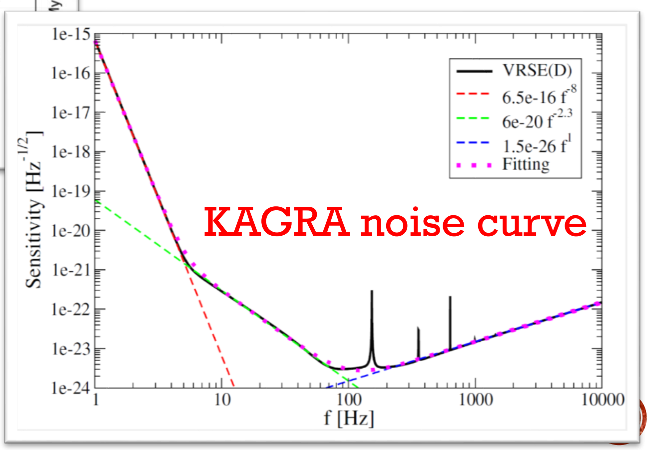
Kanda,

the LCGT collaboration,

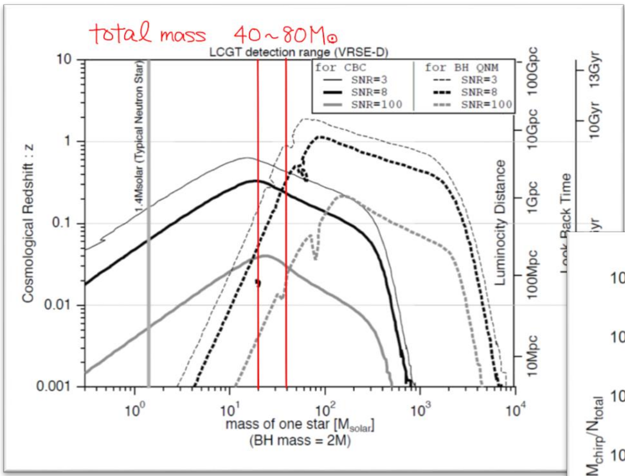
arXiv:1112.3092

30M_sun-30M_sun

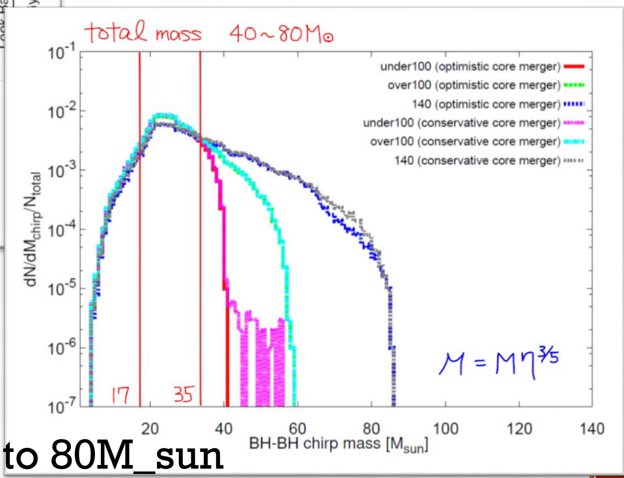
Similar SNR for the inspiral and ringdown phases



RANGE FOR TOTAL MASS OF BBHS



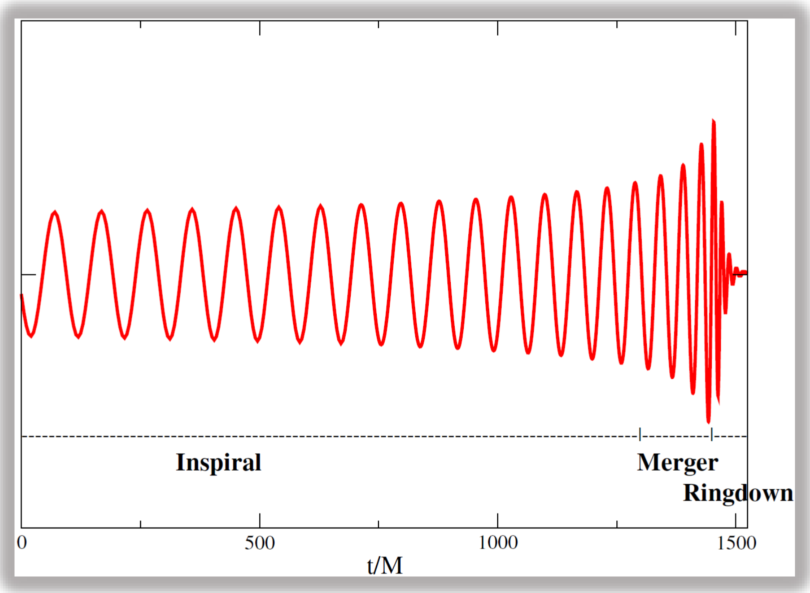
From 40M_sun to 80M_sun



Previous study:
30M_sun-30M_sun
This poster:
Total mass from 40M_sun to 80M_sun

BBH INSPIRAL, MERGER AND RINGDOWN

- Use the inspiral and ringdown data analysis.
- Treat the merger phase as a black box.



INSPIRAL (POST NEWTONIAN)

- TaylorF2 waveform (L=2, m=2) in frequency domain

$$\tilde{A}_{\ell m} e^{i\psi_{\ell m}}$$

$$\psi_{22}(v) = 2\,\frac{t_c}{M} v^3 - 2\Phi_c - \frac{\pi}{4} + \frac{3}{128\,\eta\,v^5}\left[1 + \mathcal{O}(v^2)\right] ,$$

$$\tilde{A}_{22} = A_{22}\sqrt{\frac{\pi M}{3v^2\dot{v}}} ,$$

- Parameters in the inspiral phase: $\{M,\ \eta,\ t_c,\ \Phi_c\}$
- Summary of the formulation:

  Ajith et al., arXiv:0709.0093

- Terminate at the ISCO frequency

MERGER (BLACK BOX, NUMERICAL RELATIVITY)

- Much progress in NR.
- The whole GW waveforms are also well modeled in the effective-one-body approach.

  Taracchini, Buonanno et al., Phys. Rev. D 89, 061502 (2014) [arXiv:1311.2544].

- Simply use the phenomenological fitting formulas for the remnant mass and spin:

$$\frac{M_{\rm rem}}{M} = (4\eta)^2\left(M_0 + K_{2d}\,\delta m^2 + K_{4f}\,\delta m^4\right) + \left[1 + \eta(\tilde{E}_{\rm ISCO} + 11)\right]\delta m^6 ,$$

$$\alpha_{\rm rem} = \frac{S_{\rm rem}}{M_{\rm rem}^2} = (4\eta)^2\left(L_0 + L_{2d}\,\delta m^2 + L_{4f}\,\delta m^4\right) + \eta\tilde{J}_{\rm ISCO}\delta m^6 ,$$

Healy, Lousto and Zlochower, Phys. Rev. D 90, 104004 (2014) [arXiv:1406.7295].

RINGDOWN (BLACK HOLE PERTURBATION)

- Ringdown waveform:

$$h(f_c,\, Q,\, t_0,\, \phi_0;\, t) = \begin{cases} e^{-\frac{\pi\, f_c\,(t-t_0)}{Q}} \cos(2\,\pi\, f_c\,(t-t_0) - \phi_0) & \text{for} \quad t \geq t_0\,, \\ 0 & \text{for} \quad t < t_0\,, \end{cases}$$

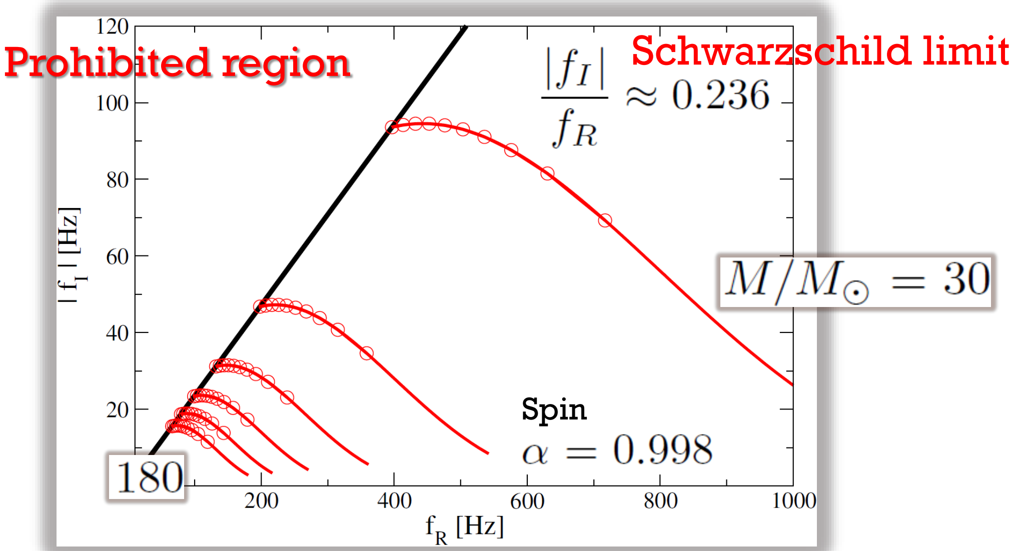
- The dominant (L=2, m=2), least-damped (n=0) mode:

$$\begin{aligned} f_c &= \frac{1}{2\pi M_{\rm rem}} \left[1.5251 - 1.1568(1-\alpha_{\rm rem})^{0.1292}\right] \\ &= 538.4 \left(\frac{M}{60 M_\odot}\right)^{-1} \left[1.5251 - 1.1568(1-\alpha_{\rm rem})^{0.1292}\right] \; [\mathrm{Hz}]\,, \\ Q &= 0.7000 + 1.4187(1-\alpha_{\rm rem})^{-0.4990}\,. \end{aligned}$$

- Parameters for the ringdown phase: $\{f_c,\, Q,\, t_0,\, \phi_0\}$

  Berti, Cardoso and Will, Phys. Rev. D 73, 064030 (2006) [gr-qc/0512160].

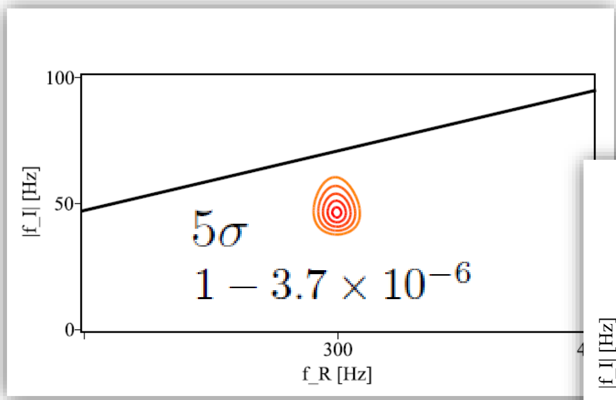
RINGDOWN (REAL AND IMAGINARY FREQ.)



Real and imaginary parts of ringdown frequency

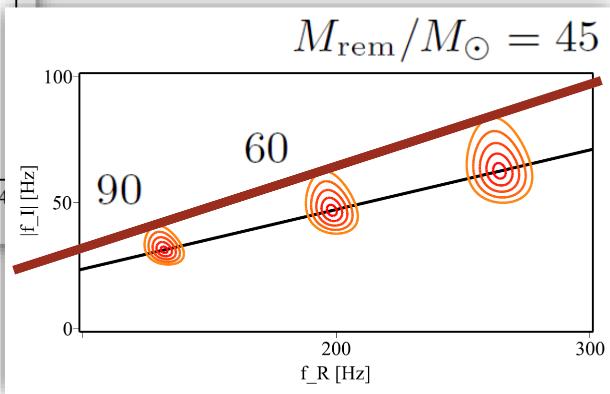
$$f_R = f_c\,, \qquad f_I = -\frac{f_c}{2Q}\,,$$

ONLY RINGDOWN



Prohibited region

NEW PHYSICS

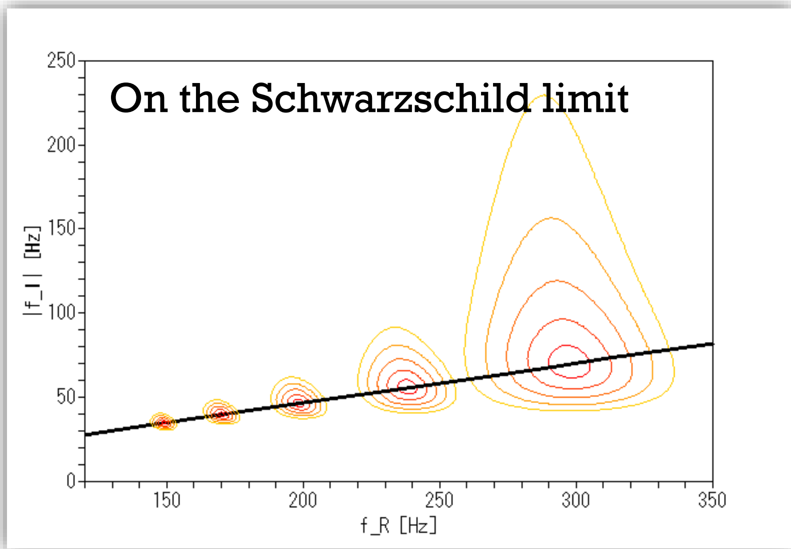


- SNR = 50 (200Mpc) for the typical Pop III BBH
- SNR = 50 (200Mpc), Schwarzschild case

$$(M = 60M_\odot\,,\ \eta = 1/4) \to (M_{\rm rem} = 57.09M_\odot\,,\ \alpha_{\rm rem} = 0.6867)$$
$$f_R = 299.5{\rm Hz}\,,\ f_I = -46.34{\rm Hz}\,, (f_c = 299.5{\rm Hz}\,,\ Q = 3.232)$$

ONLY RINGDOWN

Mass (M_sun)	40	50	60	70	80
SNR	18.74	32.86	50 (fix)	68.53	86.68



CONSISTENCY ANALYSIS WITH I/R

- By combining the data analysis for the (I)nspiral and (R)ingdown GWs.
- We use the PN waveform for the inspiral phase to extract the binary parameters.

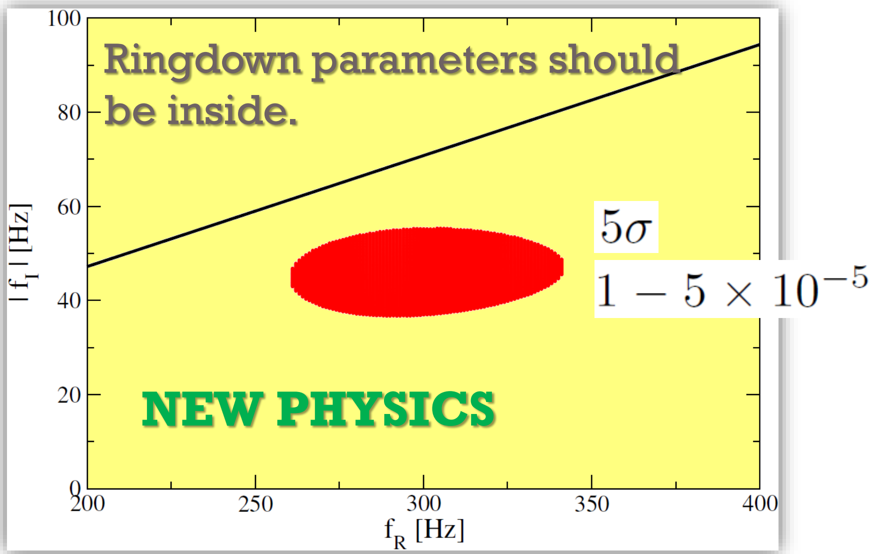
$\{M,\, \eta,\, t_c,\, \Phi_c\}$

- The remnant formulas are applied to obtain the GR prediction for the parameters of the remnant BH.

$\{f_c,\, Q,\, t_0,\, \phi_0\}$

- We find the expected parameter region of the QNM for given inspiral parameters.

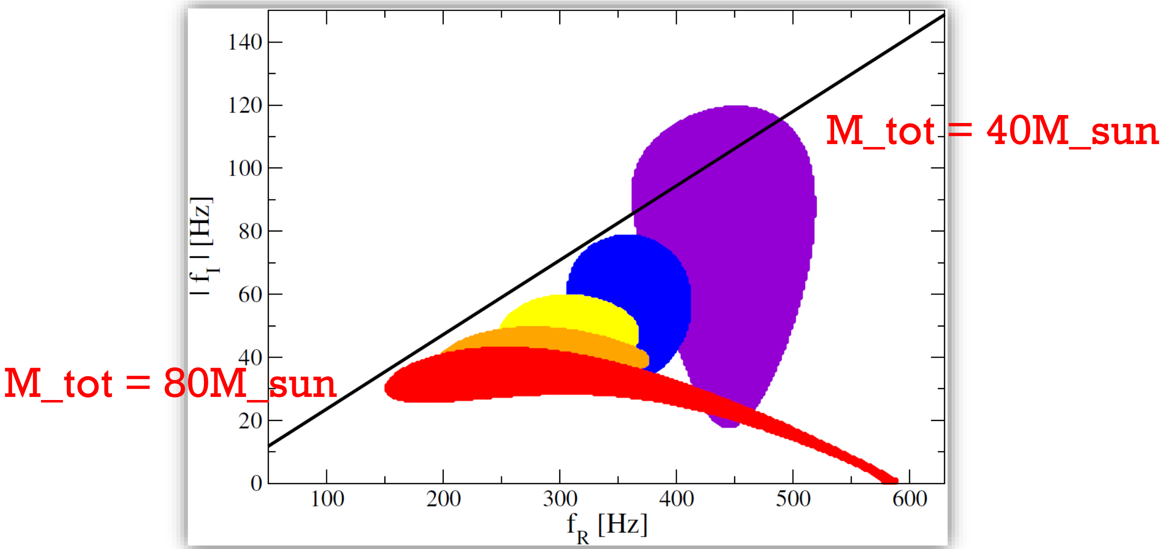
CONSISTENCY ANALYSIS WITH I/R



- SNR = 50 for both the inspiral and ringdown phases

$$(M = 60 M_\odot\,,\ \eta = 1/4) \rightarrow (M_{\rm rem} = 57.09 M_\odot\,,\ \alpha_{\rm rem} = 0.6867)$$
$$f_R = 299.5\text{Hz}\,,\ f_I = -46.34\text{Hz}\,, (f_c = 299.5\text{Hz}\,,\ Q = 3.232)$$

CONSISTENCY ANALYSIS WITH I/R



Total mass $[M_\odot]$	40	50	60	70	80
$\mathrm{SNR_{Insp}}$	47.60308136	43.45077593	35 (fix)	24.76985161	18.94621580
$\mathrm{SNR_{Ring}}$	13.56269312	23.70717338	35 (fix)	42.85387322	45.72790073

Fixed $SNR = \sqrt{35^2 + 35^2}$

DISCUSSION

- **Kinugawa**, Miyamoto, Kanda and Nakamura, arXiv:1505.06962.

  The event rate (inspiral+ringdown) with **SNR > 35**

$$3.2 \text{ events yr}^{-1}(\mathrm{SFR_p}/(10^{-2.5}\ \mathrm{M_\odot\ yr^{-1}\ Mpc^{-3}})) \cdot ([\mathrm{f_b}/(1+\mathrm{f_b})]/0.33) \cdot \mathrm{Err_{sys}}$$

- Great chance to confirm GR in the strong gravity by observing the expected QNMs!

- Although lower Q ringdown waves are difficult in DA, there may be a new physics!

- Next, mass ratio and spin dependence!

“Can we remove the systematic error due to isotropic inhomogeneities?”

by Hiroyuki Negishi

[JGRG25(2015)P24]

1

Can we remove the systematic error due to isotropic inhomogeneities?

Hiroyuki Negishi

Osaka City University, Japan

JGRG25@Kyoto(YITP)

Topics

2

In cosmology

Usually

- We assume that our universe is homogeneous and isotropic.
- Our universe is filled with non-relativistic matter and dark energy.

In this poster

- We assume that our universe is inhomogeneous and isotropic.
- Our universe is filled with non-relativistic matter and positive cosmological constant.
- There are large-scale isotropic inhomogeneities.

We compare these two cosmological model.

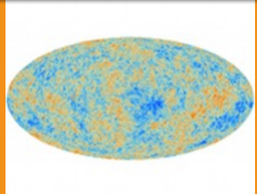
Introduction

Usually modern cosmology adopt

Copernican principle

we do not live in the privileged domain in the universe

Cosmic microwave background (CMB)



The observed high isotropy of the CMB

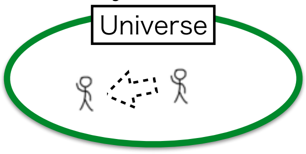
・The high homogeneity and isotropy of our universe in the globally averaged sense.

But

・The Copernican principle can not be directory confirmed by observations.

reason

・We can't move to the other clusters of galaxies.
・Our observations are confined on a past light cone.



There are possibility of non-negligible large-scale isotropic inhomogeneities in our universe.

Introduction

What is happen, if there are isotropic inhomogeneities in our universe?

The existence of isotropic inhomogeneities around us affects the determination of the cosmological parameters.

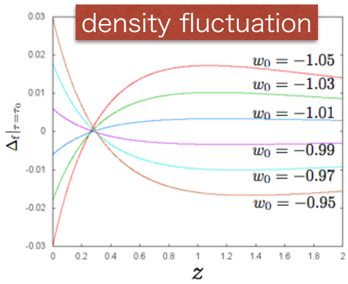
Example

We consider two universe model.

・FLRW universe model filled with dust and dark energy $(w \neq -1)$.
・Inhomogeneous isotropic universe model filled with dust and cosmological constant.

From only the distance-redshift relation, we can't distinguish the inhomogeneous isotropic universe from the homogeneous isotropic one.

Isotropic inhomogeneities affect the determination of the cosmological parameters.



There is systematic error due to isotropic inhomogeneities

Introduction 5

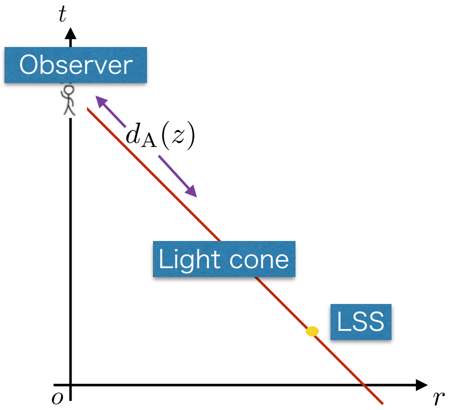
We want to know exact the equation of state of dark energy.

We want to remove a systematic error due to isotropic inhomogeneities.

Why is there a systematic error?

Our observations are confined on a past light cone, and we use only one observable.

In this poster, we use multiple observables.



Our purpose

We study whether we can distinguish the inhomogeneous isotropic universe model from the FLRW universe model and remove the systematic error due to isotropic inhomogeneities, if we use multiple observables.

※In particular, we focus on the equation of state of dark energy.

FLRW universe model 6

Metric

$$ds^2 = -dt^2 + a^2_{(\mathrm{FLRW})}(t)\delta_{ij}dx^i dx^j$$

Matter

Cold dark matter $\Omega^{(\mathrm{FLRW})}_{\mathrm{CDM}}$

Baryon $\Omega^{(\mathrm{FLRW})}_{\mathrm{b}}$

Dark energy $\Omega^{(\mathrm{FLRW})}_{\mathrm{d}}$

equation of state of dark energy

$$w = \sum_{m=0} w_m (1-a)^m$$

$$w_0 \neq -1 \text{ or } w_1 \neq 0$$

$$w_m = 0 \quad (m > 2)$$

Power spectrum

$$P_{(\mathrm{FLRW})} = R_{0(\mathrm{FLRW})}\left(\frac{k}{k_0}\right)^{n_{\mathrm{s(FLRW)}}}$$

We fix
$$\begin{cases} H_0^{(\mathrm{FLRW})} = 70\ \mathrm{km/s/Mpc} \\ \Omega^{(\mathrm{FLRW})}_{\mathrm{CDM}} = 0.25 \\ \Omega^{(\mathrm{FLRW})}_{\mathrm{b}} = 0.05 \\ n_{\mathrm{s(FLRW)}} = 0.96 \\ R_0 = 2.2 \times 10^{-9} \end{cases}$$

Inhomogeneous isotropic universe model

Metric

FLRW universe + inhomogeneous isotropic perturbation

$$ds^2 = -dt^2 + a^2(t)\left[(1 + A(t,r))\delta_{ij} + \partial_i\partial_j B(t,r)\right] dx^i dx^j$$
$$= -dt^2 + a^2\left[(1 + A + \partial_r^2 B)dr^2 + \left(1 + A + \frac{1}{r}\partial_r B\right) r^2 d\Omega^2\right],$$

We fix the gauge

$$A = -h(r) \qquad B = -\frac{h(r)}{H_0^2} D_+ \qquad D_+(t) := H_0^2 \left(H(a)\int^a \frac{1}{b^3 H^3(b)} db\right)$$

arbitrary function

Matter

Cold dark matter ρ_{CDM0}

Baryon ρ_{b0}

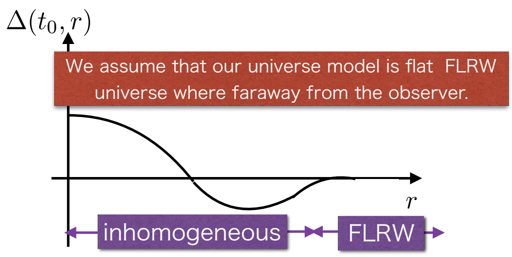
Cosmological constant Λ

Stress energy tensor

$$T_{\mu\nu} = \bar{\rho}(t)(1 + \Delta(t,r))u_\mu u_\nu$$
$$\Delta = \frac{1}{2H_0^2} D_+ \partial_i \partial^i h$$

Power spectrum

$$P(k) = R_0 \left(\frac{k}{k_0}\right)^{n_s}$$



There are three parameters ρ_{CDM0}, ρ_{b0} and Λ and one arbitrary function $h(r)$ in the inhomogeneous isotropic universe model.

Geodesic equation

We derive the null geodesic equations.

Assumption

・The observer stays at the symmetry center r = 0

Both k^θ and k^φ should vanish.

Trajectory

$$t = \bar{t}(z) + \delta t(z) \qquad r = \bar{r}(z) + \delta r(z)$$

Background

$$\frac{d\bar{t}}{dz} = -\frac{1}{(1+z)\bar{H}} \qquad \frac{d\bar{r}}{dz} = \frac{1}{\bar{H}}$$
$$\bar{H} = H_0\sqrt{\Omega_{\mathrm{m}}(1+z)^3 + \Omega_\Lambda}$$

Perturbation

$$\frac{d\delta t}{dz} = \frac{1}{(1+z)\bar{H}^2}\left(\frac{\ddot{\bar{a}}}{\bar{a}} - \bar{H}^2\right)\delta t + \frac{1}{2H_0^2}\frac{dD_+}{dz}\left(\bar{H}\frac{d^2h}{dz^2} + \frac{d\bar{H}}{dz}\frac{dh}{dz}\right)$$
$$\frac{d\delta r}{dz} = \frac{h}{2\bar{H}} + \frac{D_+}{2H_0^2}\left(\bar{H}\frac{d^2h}{dz^2} + \frac{d\bar{H}}{dz}\frac{dh}{dz}\right) - \delta t - (1+z)\frac{d\delta t}{dz}$$

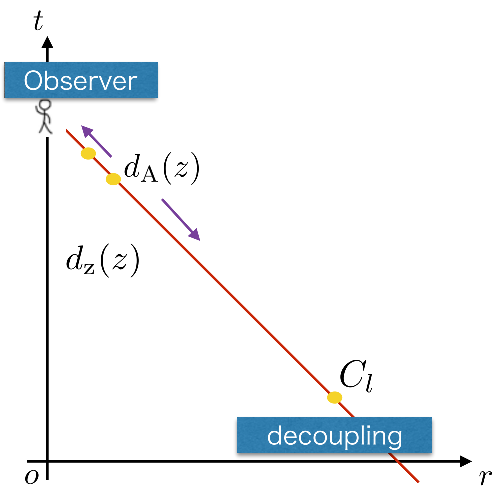
Boundary condition

$$\bar{t}(0) = t_0 \qquad \bar{r}(0) = \delta t(0) = \delta r(0) = 0$$

Observables

We use three observables

- Angular diameter distance-redshift relation $d_{\rm A}(z)$ $0 < z < 2$
- Baryon acoustic oscillation(BAO) scale $d_{\rm z}(z)$ $z = 0.2, 0.35$
- CMB angular power spectrum C_l $l \gg 1$



Observables in inhomogeneous isotropic universe

Angular diameter distance-redshift relation

Areal radius

$$d_{\rm A}(z) = a\left[1 + \frac{1}{2}A + \frac{1}{2r}\partial_r B\right] r\bigg|_{t=t(z),\ r=r(z)}$$

$$\approx a(\bar{t})\bar{r} + a(\bar{t})\left[\delta r - \left(\frac{1}{2}h(\bar{r}) + \frac{D_+(\bar{t})}{2\bar{r}\bar{H}_0^2}\partial_r h(\bar{r}) - \bar{H}\delta t\right)\bar{r}\right]$$

We use angular diameter distance-redshift relation in the range $0 < z < 2$

Solving the geodesic equations determine $\bar{t}, \bar{r}, \delta t$ and δr.

➡ Angular diameter distance-redshift relation depends on two parameters $\bar{\rho}_{\rm m0}, \Lambda$ and one arbitrary function $h(r)$ $(0 < z < 2)$.

Observables in inhomogeneous isotropic universe 11

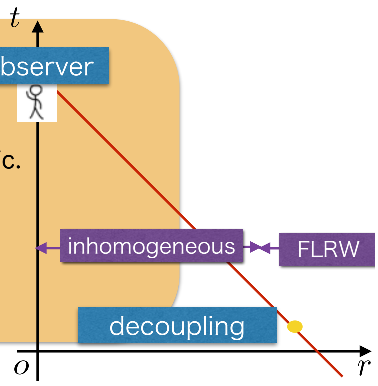
Assumption

- For simplicity, we ignore the Integrated Sachs-Wolfe effect.

  ➡ We discard low multipoles in our analysis.

- We regard spacetime faraway from observer as homogeneous and isotropic.

  ➡ We assume that the linearly anisotropy perturbed FLRW universe with Standard Model particle physics and cold dark matter is fulfilled in decoupling epoch..



CMB angular power spectrum

$$C_l = S^{-2} C^{(\mathrm{EdS})}_{S^{-1}l}$$

$$S = \frac{d_{\mathrm{A}}(z_{\mathrm{dec}})}{d_{\mathrm{A}}^{(\mathrm{EdS})}(z_{\mathrm{dec}})}$$

C_l^{EdS} : CMB angular power spectrum of Einstein de Sitter (EdS) Universe which has the same values of $\bar{\rho}_{\mathrm{CDM}}(t_{\mathrm{dec}})$ $\bar{\rho}_{\mathrm{b}}(t_{\mathrm{dec}})$ and P(k) in inhomogeneous isotropic universe model

$d_{\mathrm{A}}^{(\mathrm{EdS})}(z)$: Angular diameter distance of EdS universe

※We choose the photon temperature today T_0 = 2.725K and n_s = 0.96.

CMB angular power spectrum depends on $\bar{\rho}_{\mathrm{CDM}}(t_{\mathrm{dec}})$, $\bar{\rho}_{\mathrm{b}}(t_{\mathrm{dec}})$, $d_{\mathrm{A}}(z_{\mathrm{dec}})$ and R_0 .

Observables in inhomogeneous isotropic universe 12

Baryon acoustic oscillation scale

$$d_z(z) = \left(\Delta\theta^2_{\mathrm{BAO}} \frac{\Delta z_{\mathrm{BAO}}}{z}\right)^{1/3}$$

Δz_{BAO} is the redshift interval the BAO scale engraved in line-of-sight direction.

$$\Delta z_{\mathrm{BAO}}(t(z), r(z)) = \int \frac{dz}{dr} dr$$

$\Delta\theta_{\mathrm{BAO}}$ is the angle that observer seen the BAO scale in the transverse direction.

$$\Delta\theta_{\mathrm{BAO}}(t(z), r(z)) = \frac{L^{\mathrm{T}}_{\mathrm{BAO}}(t(z), r(z))}{d_{\mathrm{A}}(z)}$$

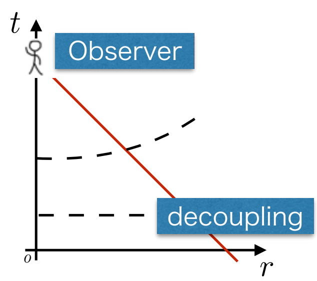
In our universe model

$$d_z(z) = \frac{L_{\mathrm{BAO(dec)}}}{a(t_{\mathrm{dec}})}\left(\frac{1}{d_{\mathrm{A}}^2}\frac{\bar{H}}{z(1+z)^2}\right)^{1/3}\left[1 + \frac{2}{3}\bar{H}\delta t + \frac{1}{3\bar{H}}\frac{\ddot{a}}{a}\delta t - \frac{1}{2}h - \frac{\bar{H}D_+}{3\bar{r}H_0^2}\frac{dh}{dz} - \frac{1}{6H_0^2}\left((1+z)\frac{dD_+}{dz} + D_+\right)\left(\bar{H}^2\frac{d^2h}{dz^2} + \bar{H}\frac{d\bar{H}}{dz}\frac{dh}{dz}\right)\right]$$

Assumption

We approximate that isotropic density fluctuation is very small at decoupling epoch.

➡ $L_{\mathrm{BAO(dec)}} = const$



The BAO scale depends on ρ_{CDM0}, ρ_{b0}, Λ, $h(r)$ and $a(t_{\mathrm{dec}})$.

Fitting Observations

Our purpose is to study whether the observational data in FLRW universe model can also be explained by the inhomogeneous isotropic universe model.

We try to construct the inhomogeneous isotropic universe model whose distance-redshift relation, temperature fluctuation of CMBR and BAO scale are identical with those of the FLRW universe model with dark energy of $w \neq -1$.

Conditions

$$d_\mathrm{A}(z) = d_{\mathrm{A(FLRW)}}(z) \qquad (0 < z < 2)$$

$$d_\mathrm{z}(z) = d_{\mathrm{z(FLRW)}}(z) \qquad (z = 0.2, 0.35)$$

$$C_l = C_{l\mathrm{(FLRW)}} \qquad (l \gg 1)$$

With these conditions, null geodesic should be regarded as the system of differential equations to determine the inhomogeneous isotropic universe model.

If there is a solution in null geodesic equations we can't distinguish inhomogeneous isotropic universe model from FLRW universe model, otherwise we can.

Result

Result

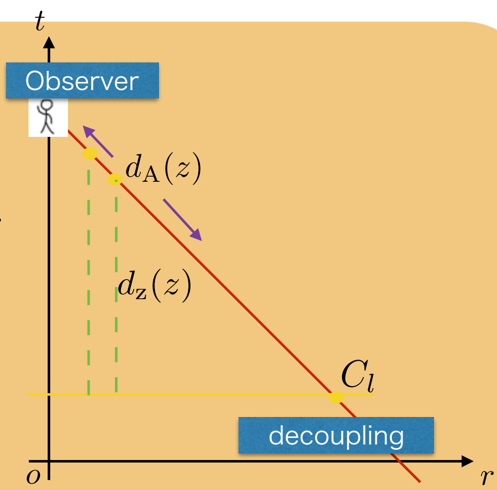
We find that there are no solutions.

It is mean that, we can remove the systematic error due to isotropic inhomogeneities, if adopt appropriate observables.

Why are there no solutions?

- $d_\mathrm{A}(z)$ restrict isotropic inhomogeneity at the range $0 < z < 2$.
- C_l restrict the BAO scale at decoupling epoch.
- There is no degree of freedom to fit the BAO scale on the light cone.

Inhomogeneous isotropic universe model has one arbitrary function, but it is not enough.



Summary

・We compare two universe model which is inhomogeneous isotropic universe model and FLRW model.

・We use three observables which is distance-redshift relation, BAO scale and CMB angular power spectrum.

・Observables in FLRW universe model and inhomogeneous isotropic universe model are not coincided.

・We can't construct the inhomogeneous isotropic universe model whose distance-redshift relation, temperature fluctuation of CMBR and BAO are identical with those of the FLRW universe model with dark energy of $w \neq -1$.

"Cosmic string shielding of electric field of line charge"

by Tatsuya Ogawa

[JGRG25(2015)P25]

Cosmic string shielding of electric field of line charge

Tatsuya Ogawa and Hideki Ishihara

Department of Mathematics and Physics, Graduate School of Science, Osaka City University

Abstract

We consider a system consists of a complex scalar field with a potential which causes spontaneous symmetry breaking and U(1) gauge field. It is well known that there exist the Nielsen-Olsen string solutions in this system. We put a line charge in this field. In the symmetric phase , electric field appears whose strength is in proportion to 1/r. In a broken phase, on the other hand, we show that a cosmic string is possibly formed along the line charge, where the electric field is shielded by rotating phase of the complex scalar field. The energy density of scalar field and electromagnetic field is localized in a finite thickness of the string, then the surface integral of energy density on a surface $t=\text{const.}, z=\text{const.}$ plane is finite. We discuss physical properties of the phase rotating string in comparison to the Nielsen-Olsen string.

1.Introduction

If there is a line charge in a complex scalar field coupled to U(1) gauge field in the symmetric phase , a long-range potential of U(1) field appears. On the other hand , if there is a line charge in the broken phase , what kind of the field configuration is allowed ? In this poster , we introduce a new cosmic string solution which we call phase rotating string , and explain the physical differences from the Nielsen-Olsen string solution.

2.Nielsen-Olsen string ; brief review

We consider the Lagrangian

$$\mathscr{L}=-g^{\mu\nu}(\bar{D}_\mu\bar{\phi})(D_\nu\phi)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-V(\phi,\bar{\phi}),$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ ,\ V(\phi,\bar{\phi})=\frac{\lambda}{4}(\bar{\phi}\phi-\eta^2)^2,$$

where D_μ is the covariant derivative operator and j^ν is the conserved current density:

$$D_\mu\phi=(\partial_\mu-ieA_\mu)\phi\quad,\quad j^\nu=-2eIm[\bar{\phi}(\partial^\nu-ieA^\nu)\phi]\,.$$

The potential $V(\phi,\bar{\phi})$ causes spontaneous symmetry breaking.
We assume static and axisymmetric fields

$$\phi(r,\theta)=e^{in\theta}f(r)\quad,\quad A_\mu(r)=(0,0,n\tilde{A}_\theta(r),0)\ ,$$

where we put $e=1$ for simplicity, and n is winding number of the complex scalar field. If we require boundary conditions

$$f(r)\to 0: r\to 0\quad,\quad f(r)\to\eta: r\to\infty,$$
$$\tilde{A}_\theta(r)\to 0: r\to 0\quad,\quad \tilde{A}_\theta(r)\to 1: r\to\infty\ .\qquad\cdots(1)$$

We find a solution called Nielsen-Olsen string , (see fig.1).

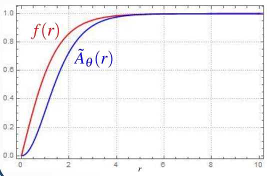


Fig.1. The Nielsen-Olsen string solution in the case $n=\lambda=\eta=1$.

3.Basic equations

(A)Symmetric phase

In the case $\eta^2=0$, we have the symmetric vacuum $\langle 0|\bar{\phi}\phi|0\rangle=0$.

If we put a line charge as an external field,

we have a solution $\phi=0, A_t(r)\propto\log(r)$.

(B) Broken phase

In the case with non-vanishing V.E.V. , $\langle 0|\bar{\phi}\phi|0\rangle=\eta^2\neq 0$.

We assume stationary and axisymmetric fields

$$\phi(t,r,\theta)=e^{i(\omega t+n\theta)f(r)}\ ,\quad A_\mu(r)=(A_t(r),0,n\tilde{A}_\theta(r),0)\ .$$

The complex scalar field configuration is expected to be the one which is illustrated in fig.2.

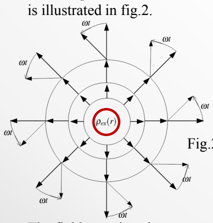


Fig.2. The complex scalar field configuration and the external line charge density $\rho_{ex}(r)$ which is placed at origin in two-dimensional physical space. Each arrow represents amplitude and phase of the complex scalar field.

The field equations become

$$\frac{d^2f}{dr^2}+\frac{1}{r}\frac{df}{dr}-\frac{n^2}{r^2}f(\tilde{A}_\theta-1)^2+fq^2-\frac{\lambda}{2}f(f^2-\eta^2)^2=0,$$

$$\frac{d^2\tilde{A}_\theta}{dr^2}-\frac{1}{r}\frac{d\tilde{A}_\theta}{dr}-2f^2(\tilde{A}_\theta-1)=0,$$

$$\frac{d^2q}{dr^2}+\frac{1}{r}\frac{dq}{dr}-2f^2q+\rho_{ex}(r)=0,$$

where $q(r)\equiv A_t(r)-\omega$, and $\rho_{ex}(r)$ is the external charge density.

4.Solution of phase rotating string

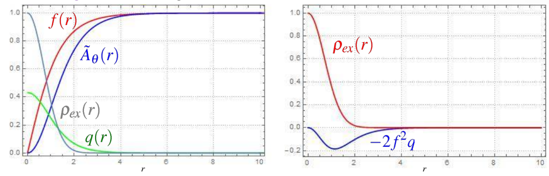
The charge density $\rho_{ex}(r)$ is assumed as the Gaussian function ,

$$\rho_{ex}(r)=\rho_0 e^{-(r/r_s)^2}\,.$$

If we require boundary conditions (1) and

$$\frac{dq(r)}{dr}\to 0: r\to 0\ ,\ q(r)\to 0: r\to\infty,$$

We find cosmic string solutions which we call phase rotating strings. The string solution screen out the electric field of the line charge as shown in fig.3.



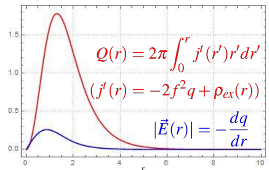


Fig.3. Upper left panel: The phase rotating string solution for the external charge density $\rho_{ex}(r)$.
Upper right panel: The phase rotation of the scalar field induces the charge density $-2f^2q$.
Lower panel: The charge $Q(r)$within radius r , and the amplitude of the electric field $E(r)$.

The energy-momentum tensor is given by

$$T_{\mu\nu}=(\bar{D}_\mu\bar{\phi})(D_\nu\phi)-\frac{1}{2}g_{\mu\nu}\left\{g^{\alpha\beta}(\bar{D}_\alpha\bar{\phi})(D_\beta\phi)+\frac{\lambda}{4}(\bar{\phi}\phi-\eta^2)^2\right\}+\frac{1}{2}\left\{g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}-\frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right\},$$

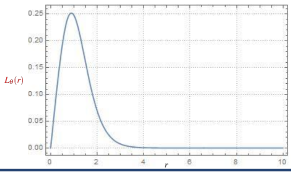
and averaged line energy density , i.e. the integral of energy density on a surface $t=\text{const.}, z=\text{const.}$ plane , becomes

$$\varepsilon=2\pi\int_0^\infty\left[\frac{1}{2}\left\{f^2q^2+\left(\frac{df}{dr}\right)^2+\frac{n^2}{r^2}(\tilde{A}_\theta-1)^2f^2+\frac{\lambda}{4}(f^2-\eta^2)^2\right\}+\frac{1}{4}\left\{\left(\frac{dq}{dr}\right)^2+\frac{n^2}{r^2}\left(\frac{d\tilde{A}_\theta}{dr}\right)^2\right\}\right]rdr.$$

The first term is the kinetic energy part and the fifth term is the electric field part.
Tension of the phase rotating string is given by

$$\tau=2\pi\int_0^\infty\left[\frac{1}{2}\left\{-f^2q^2+\left(\frac{df}{dr}\right)^2+\frac{n^2}{r^2}(\tilde{A}_\theta-1)^2f^2+\frac{\lambda}{4}(f^2-\eta^2)^2\right\}+\frac{1}{4}\left\{-\left(\frac{dq}{dr}\right)^2+\frac{n^2}{r^2}\left(\frac{d\tilde{A}_\theta}{dr}\right)^2\right\}\right]rdr.$$

Since the terms related to $q(r)$ exist unlike the Nielsen-Olsen string solution ,the phase rotating string solution have tension which is different from the averaged line energy density.
The phase rotating string solution have angular momentum around the symmetric axis which is given by

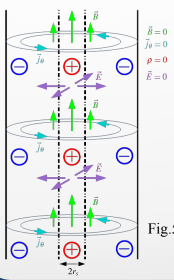


$$L_\theta(r)=-2nq(\tilde{A}_\theta-1)f^2-\frac{n^2}{r^2}\left(\frac{dq}{dr}\right)\left(\frac{d\tilde{A}_\theta}{dr}\right).$$

Fig.4. The angular momentum around the symmetric axis of the phase rotating string.

5.Conclusion

We found the phase rotating string solution which is screening of electric field of line charge, and analyzed the differences from the Nielsen-Olsen string solution. The analysis results are summarized in table, and the inner structure of the phase rotating string is illustrated in fig.5.



	The Nielsen-Olsen string	The phase rotating string
Electric field	Not exist	Exist(localized)
Charge density	Not exist	Screened
Angular momentum	Not exist	Exist(localized)
Relation between tension and energy density	Equal	Different

Table. The differences of the phase rotating string solution from the Nielsen-Olsen string solution.

Fig.5. The inner structure of the phase rotating string.

“Causality and shock formation in general scalar-tensor theories”

by Seiju Ohashi

[JGRG25(2015)P26]

Causality and Shock Formation in General Scalar-Tensor Theories

Seiju Ohashi(KEK)
Collaboration with
Norihiro Tanahashi(DAMTP)

1.1 Motivation

- **General scalar(s)-tensor theory**
  - -Modified gravity theories.
  - -Frequently discussed to study inflation, dark energy, dark matter
    - Horndeski(1974)
    - The generalized multi-Galileon theory(2013) …
    - The most general bi-scalar-tensor theories(2015)
- **They have non-canonical kinetic terms**
  - -Non-canonical kinetic terms may imply superluminal motion and shock formation

1.2 Motivation

- Reveal the peculiarities of the theories

-Superluminal motion

- Gauss-Bonnet gravity, K.Izumi(2014)
- Lovelock gravity, H.Real, N.Tanahashi & B.Way(2014)
- Special class of general scalar-tensor theory, Minamitsuji(2015)

-Shock formation

- Gauss-Bonnet & Lovelock gravity, H.Real, N.Tanahashi & B.Way (2014)

What about general scalar-tensor theories ???

1.3 Motivation

- Focusing on peculiarities in the most general bi-scalar-tensor theory
  S.O, N.Tanahashi, T.Kobayashi & M.Yamaguchi(2015)

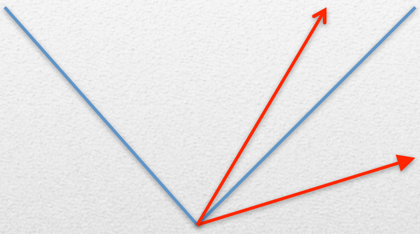
$$
\begin{aligned}
\mathcal{G}^a_b = {}& A\delta^a_b + \left[-2\mathcal{F}_{,I} - 4\mathcal{W}_{,I} + 2\left(D_{JKI} + 8J_{J[K,I]}\right)X^{JK} - 8E_{JKLMI}X^{JK}X^{LM}\right]\delta^{ac}_{bd}\phi^{I|d}_{|c} \\
& + \left(-2\mathcal{F}_{,I,J} - 4\mathcal{W}_{,I,J} + A_{,IJ} + 2D_{IKJ,L}X^{KL} - 16E_{KIMNJ,L}X^{KL}X^{MN} - 16J_{K[I,L],J}X^{KL}\right)\phi^{(I|a}\phi^{J)}_{|b} \\
& + D_{IJK}\delta^{ace}_{bdf}\phi^{I}_{|c}\phi^{J|d}\phi^{K|f}_{|e} + E_{IJKLM}\delta^{aceg}_{bdfh}\phi^{I}_{|c}\phi^{J|d}\phi^{K}_{|e}\phi^{L|f}\phi^{M|h}_{|g} + \left(\frac{1}{2}\mathcal{F} + \mathcal{W}\right)\delta^{ace}_{bdf}R_{ce}{}^{df} + \mathcal{F}_{,IJ}\delta^{ace}_{bdf}\phi^{I|d}_{|c}\phi^{J|f}_{|e} \\
& + J_{IJ}\delta^{aceg}_{bdfh}\phi^{I}_{|c}\phi^{J|d}R_{eg}{}^{fh} + 2J_{IJ,KL}\delta^{aceg}_{bdfh}\phi^{I}_{|c}\phi^{J|d}\phi^{K|f}_{|e}\phi^{L|h}_{|g} + K_I\delta^{aceg}_{bdfh}\phi^{I|d}_{|c}R_{eg}{}^{fh} + \frac{2}{3}K_{I,JK}\delta^{aceg}_{bdfh}\phi^{I|d}_{|c}\phi^{J|f}_{|e}\phi^{K|h}_{|g}.
\end{aligned}
$$

-A,D,E,F,J,K are arbitrary functions of ϕ^I and X^{IJ} where $\phi^I_{|a} \equiv \nabla_a\phi^I$, $X^{IJ} \equiv -\frac{1}{2}\phi^I_{|a}\phi^{J|a}$

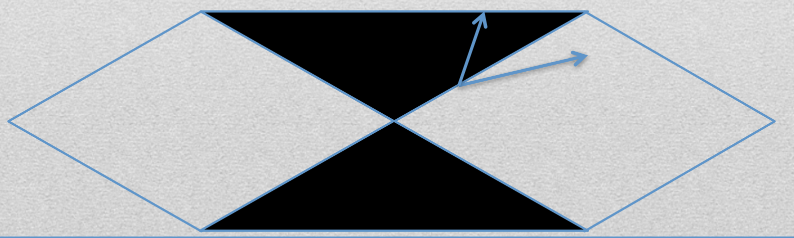
- Superluminality ?
- Killing Horizon (BH horizon) ?
- Shock formation ?

2.1 Speed of Propagation

- ☐ Is propagation speed faster (or slower) than light ?



- ☐ Is Killing horizon the causal edge (characteristic surface) ?



2.2 Speed of Propagation

- ☐ Canonical scalar field case

$$g^{AB}\nabla_A\nabla_B\phi + V(\phi) = 0 \quad \xrightarrow{\text{fourier transform}} \quad g^{AB}k_A k_B \phi_k = 0$$

- Speed of the fastest mode is speed of light

- ☐ Non-canonical scalar field case

$$\left(g^{AB} + \alpha\nabla^A\psi\nabla^B\psi\right)\nabla_A\nabla_B\tilde{\phi} + V(\tilde{\phi}) = 0 \quad \xrightarrow{\text{fourier transform}} \quad \left(g^{AB} + \alpha\nabla^A\psi\nabla^B\psi\right)\tilde{k}_A\tilde{k}_B\tilde{\phi}_{\tilde{k}} = 0$$

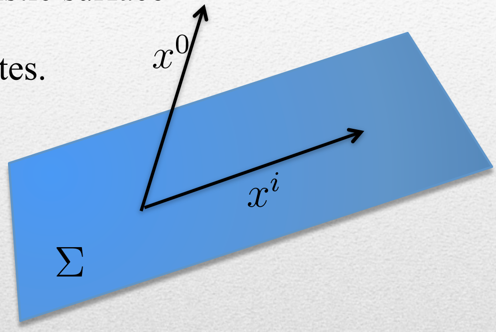
- Speed of the fastest mode is **not** speed of light

- ☐ Coefficients of second-order term determine the speed of fastest mode

2.3 Characteristic surfaces

-Causality is determined by the characteristic surface

-3D surfaces where fastest mode propagates.



- Assuming quasi-linear EoM

$$E_I\left(g, \partial g, \partial^2 g\right) = 0$$

Initial data= g_I and $\partial_\mu g_I$

$$\frac{\partial^2 E_I}{\partial(\partial_0^2 g_J)}\partial_0^2 g_J + \cdots = 0$$

- $\partial_0^2 g_J$ is uniquely determined if P is invertible

$$P(x,\xi)_I{}^J = \frac{\partial E_I}{\partial(\partial_\mu \partial_\nu g_J)}\xi_\mu \xi_\nu \qquad \text{where} \qquad \xi = dx^0$$

- Surface Σ is characteristic if and only if P is not invertible

$$\det P(x,\xi) = 0$$

2.4. General Scalar(s)-Tensor Theories

- ☐ Characteristic equation, assuming Σ is null

$$\mathcal{A}^{ij,kl} g_{kl,00} + \mathcal{B}_I^{ij}\phi_{I,00} = 0$$

$$\mathcal{B}_I^{ij} g_{ij,00} + \mathcal{C}_{IJ}\phi_{J,00} = 0$$

where

$$\begin{aligned}\mathcal{A}^{ij,kl} =& -2\left(\frac{1}{2}\mathcal{F}+\mathcal{W}\right) g^{p(i}\delta^{j)0(k}_{pmf} g^{l)f} g^{0m} - 2J_{IJ} g^{p(i}\delta^{j)c0(k}_{pdmf} g^{l)f} g^{0m}\phi^I_{|c}\phi^{J|d} \\ & - 2K_I g^{p(i}\delta^{j)c0(k}_{pdmf} g^{l)f} g^{0m}\phi^{I|d}_{|c},\end{aligned}$$

$$\begin{aligned}\mathcal{B}_I^{ij} =& \tilde{B}_I g^{l(i}\delta^{j)0}_{lm} g^{0m} + D_{JKI} g^{l(i}\delta^{j)c0}_{ldm} g^{0m}\phi^J_{|c}\phi^{K|d} + E_{JKLMI} g^{l(i}\delta^{j)ce0}_{ldfm} g^{0m}\phi^J_{|c}\phi^{K|d}\phi^L_{|e}\phi^{M|f} \\ & + 2\mathcal{F}_{,IJ} g^{l(i}\delta^{j)c0}_{ldm} g^{0m}\phi^{J|d}_{|c} + 4J_{JK,LI} g^{l(i}\delta^{j)ce0}_{ldfm} g^{0m}\phi^J_{|c}\phi^{K|d}\phi^{L|f}_{|e} \\ & + K_I g^{l(i}\delta^{j)0ce}_{lmdf} g^{0m} R_{ce}{}^{df} + 2K_{I,JK} g^{l(i}\delta^{j)ce0}_{ldfm} g^{0m}\phi^{J|d}_{|c}\phi^{K|f}_{|e}.\end{aligned}$$

C_{IJ} are extremely complicated

- ☐ **Not degenerate in general. Characteristic surfaces in general scalar-tensor theory are not null in general.**
- ☐ Speed of graviton & scalar field(s) are **faster (or slower) than light**

2.5 Killing horizon in Scalar(s)-Tensor Theories

- Can Killing horizon be the causal edge ?
  - If not, some information can escape from black holes

-Killing condition

$$\partial_1 g_{ij} = 0, \qquad \partial_1^2 g_{ij} = 0 \qquad \text{and} \qquad \partial_1 \partial_k g_{ij} = 0$$

-Characteristic equations are still invertible in general

-Killing horizon can not be the characteristic surface

-We need additional condition on **scalar fields** in order for KH to be characteristic

$$\partial_1 \phi_I = 0, \quad \partial_1^2 \phi_I = 0, \quad \partial_1 \partial_a \phi_I = 0$$

3.1 General description -Propagation of discontinuity

- Consider solution smooth everywhere except across Σ
  - continuous $g_I, \partial_\mu g_I, \partial_i \partial_\mu g_I$
  - discontinuous $\partial_0^2 g_{\mu\nu}$

-EoM $E_I\left(g, \partial g, \partial^2 g\right) = 0$

$$\Rightarrow \quad A_{IJ}(\mathbf{g}_{ij}, \mathbf{g}_0, \mathbf{g}_i, \mathbf{g}, x)(g_J)_{00} + b_I(\mathbf{g}_{0i}, \mathbf{g}_{ij}, \mathbf{g}_0, \mathbf{g}_i, \mathbf{g}, x) = 0$$

-Discontinuity across Σ . [f] is discontinuity of f across Σ

$$A_{IJ}[\partial_0^2 g_J] = 0$$

- $[\partial_0^2 g_J]$ is kernel of A_{IJ}
- We define Π, r_I and l_I as $[(g_I)_{00}] = \Pi r_I$

$$l_I A_{IJ} = A_{IJ} r_J = 0$$

3.2 General description
-Evolution of discontinuity

-To derive the evolution, take the time derivative of characteristic equation

$$\dot{\Pi} + N\Pi^2 + M\Pi = 0$$

-General solutions of transport equation

$$\Pi(s) = \Pi(0)e^{-\Phi(s)}\left(1 + \Pi(0)\int_0^s N(s')e^{-\Phi(s')}ds'\right)^{-1} \quad \text{where} \quad \Phi(s) = \int_0^s M(s')ds'$$

-Π diverges if $1 + \Pi(0)\int_0^s N(s')e^{-\Phi(s')}ds' \to 0$

- Non-linear effect, this happens if N≠0

-We can find whether or not shock forms **by checking the value of N**.

3.3 N in General Scalar(s)-Tensor theories

-EoM

$$\mathcal{A}^{ij,kl}g_{kl,00} + \mathcal{B}_I^{ij}\phi_{I,00} = 0$$

$$\mathcal{B}_I^{ij}g_{ij,00} + \mathcal{C}_{IJ}\phi_{J,00} = 0$$

-N

$$N = r_{ij}\frac{\partial\mathcal{A}^{ij,kl}}{\partial g_{mn,0}}r_{kl}r_{mn} + r_{ij}\frac{\partial\mathcal{B}_I^{ij}}{\partial\phi_{,0}^J}r_I r_J + r_I\frac{\partial\mathcal{B}_I^{ij}}{\partial g_{kl,0}}r_{ij}r_{kl} + r_I\frac{\partial\mathcal{C}_{IJ}}{\partial\phi_{,0}^K}r_J r_K$$

-N does not vanish in general

-Shock generically forms in general scalar(s)-tensor theories

4. Summary

- ☐ We study the causal structures in general scalar(s)-tensor theories
  - ✓ Propagation speed is not speed of light
  - ✓ Killing horizon can not be causal edge in general
  - ✓ If scalar fields have Killing symmetry, KH is causal edge
- ☐ We show that the shock forms generally in general scalar(s)-tensor theories
  - ✓ Shock forms in general
  - ✓ We should clarify in which background shock forms

“Sturm's theorem to marginal stable circular orbits”

by Toshiaki Ono

[JGRG25(2015)P28]

Sturm's theorem to marginal stable circular orbits

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JGRG25 in Kyoto Dec. 7 - 11, 2015

Abstract: Based on our recent work (Ono et al. EPL, 111, 30008, 2015), in terms of Sturm's theorem, we reexamine a marginal stable circular orbit (MSCO) such as the innermost stable circular orbit (ISCO) of a timelike geodesic in any spherically symmetric and static spacetime. Strum's theorem is explicitly applied to the Kottler spacetime. Moreover, we analyze MSCOs for an exact solution in Weyl conformal gravity.

1 Introduction

In general relativity, the orbital radius has a lower bound that is called innermost stable circular orbit (ISCO).

ISCOs may play key roles in astrophysics as well as in gravity theory.

- In gravitational-waves astronomy, ISCOs are thought to be the location at the transition from the inspiralling phase to the merging one, especially when a compact object is orbiting around a massive black hole probably located at a galactic center [2].
- In high-energy astrophysics, ISCOs are related to the existence for the inner edge of an accretion disk around a black hole [3].

Outermost stable circular orbit (OSCO) of a test body is possible in the Kottler (often called the Schwarzschild-de Sitter) spacetime [5].

The ISCO and OSCO are a boundary between a stable region and an unstable one. Hereafter, we call it a marginal stable circular orbit (MSCO).

Throughout this poster, we use the unit of $G = c = 1$.

2 Equation for a location of a MSCO

A general form of the metric for spherically symmetric and static spacetimes :

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where we consider $g_{tt} \equiv -A(r) < 0$, $g_{rr} \equiv B(r) > 0$, $g_{\theta\theta} \equiv C(r) > 0$.

A *necessary condition* for the existence of a MSCO is expressed as [4]

$$\frac{d}{dr}\left(\frac{1}{A(r)}\right)\frac{d^2}{dr^2}\left(\frac{1}{C(r)}\right) - \frac{d}{dr}\left(\frac{1}{C(r)}\right)\frac{d^2}{dr^2}\left(\frac{1}{A(r)}\right) = 0. \tag{2}$$

Hereafter, we call eq. (2) **MSCO equation**.

Given a root for eq. (2), E^2(E: specific energy) and L^2(L: specific angular momentum) :

$$E^2 = -\frac{1}{\Delta}\frac{d}{dr}\left(\frac{1}{C(r)}\right), \quad L^2 = -\frac{1}{\Delta}\frac{d}{dr}\left(\frac{1}{A(r)}\right), \tag{3}$$

$$\Delta \equiv \begin{vmatrix} \frac{1}{A(r)} & -\frac{1}{C(r)} \\ \frac{d}{dr}\left(\frac{1}{A(r)}\right) & -\frac{d}{dr}\left(\frac{1}{C(r)}\right) \end{vmatrix}. \tag{4}$$

A *sufficient condition* as $0 \le E^2 \le \infty$ and $0 \le L^2 \le \infty$.

3 Sturm's theorem

$p(r)$ denote a polynomial. Applying Euclid's algorithm to $p(r)$ and its derivative, Sturm's sequence

$$\begin{aligned} p_0(r) &\equiv p(r), \\ p_1(r) &\equiv p'(r), \\ p_2(r) &\equiv p_1(r)q_0(r) - p_0(r), \\ &\vdots \\ 0 &= p_n(r)q_{n-1}(r) - p_{n-1}(r), \end{aligned} \tag{5}$$

where $q_i(r)$ is the quotient of $p_i(r)$ by $p_{i+1}(r)$.

$\mathcal{V}(a)$ denote the number of the sign changing (ignoring zeros) in Sturm's sequence at $r = a$.

$\mathcal{V}(a) - \mathcal{V}(b)$ gives the number of distinct roots of $p(r)$ between a and b, where $a < b$.

4 Kottler (Schwarzschild-de Sitter) spacetime

The Kottler spacetime[5]:

$$ds^2 = -\left(1 - \frac{r_g}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{dr^2}{1 - \frac{r_g}{r} - \frac{\Lambda}{3}r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{6}$$

where r_g = Schwarzschild radius and Λ = the cosmological constant. For this spacetime, Eqs.(2) and (3) become

$$8\lambda x^4 - 15\lambda x^3 - x + 3 = 0, \quad E^2 = \frac{2(\lambda r^3 - r_g^2 r + r_g^3)^2}{r_g^2 r(2r - 3r_g)}, \quad L^2 = \frac{r^2(r_g^3 - 2\lambda r^3)}{r_g^2(2r - 3r_g)}, \tag{7}$$

where $x \equiv r/r_g$ and $\lambda \equiv \Lambda r_g^2/3$.

Sturm's theorem (as necessary condition) and the positive E^2 and L^2 (as sufficient condition) tell us that there are four cases:

Case1: $\lambda = 0$ (Schwarzschild case). Single MSCO,it's corresponding to the ISCO.

Case2: $0 < \lambda < 16/16875$. Two MSCOs, where one is corresponding to the ISCO and the other is the OSCO.

Case3: $16/16875 \le \lambda$. No MSCO (after the ISCO and the OSCO merge at $\lambda = 16/16875$).

Case4: $\lambda < 0$ (anti-de Sitter case). Single MSCO.

5 Spherically symmetric, static and vacuum solution in Weyl conformal gravity

the metric [6]:

$$\begin{aligned} ds^2 &= -B(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \\ B(r) &= \sqrt{1 - 6m\gamma} - \frac{2m}{r} + \gamma r - kr^2, \end{aligned} \tag{8}$$

where m = black hole mass, γ and k are the integration constants to the vacuum equation in Weyl conformal gravity. We focus on $k = 0, \gamma \neq 0$ in this poster.

For this spacetime, Eqs.(2) and (3) become

$$-\bar{\gamma}^2\bar{r}^4 - 3\gamma\sqrt{1 - 3\bar{\gamma}}\bar{r}^3 + 6\bar{\gamma}\bar{r}^2 - \sqrt{1 - 3\bar{\gamma}}\bar{r} + 3 = 0, \tag{9}$$

$$E^2 = \frac{2(\bar{\gamma}\bar{r}^2 + \sqrt{1 - 3\bar{\gamma}}\bar{r} - 1)^2}{\bar{r}(\bar{\gamma}\bar{r}^2 + 2\sqrt{1 - 3\bar{\gamma}}\bar{r} - 3)}, \quad L^2 = \frac{4m^2\bar{r}^2(\bar{\gamma}\bar{r}^2 + 1)}{\bar{\gamma}\bar{r}^2 + 2\sqrt{1 - 3\bar{\gamma}}\bar{r} - 3}, \tag{10}$$

Sturm's theorem and the positive E^2 and L^2 tell us that there are four cases:

Case1: $0 \le \bar{\gamma} < 1/3$. Single MSCO,it's corresponding to the ISCO.

Case2: $(45 - 32\sqrt{2})23 < \bar{\gamma} < 0$. Two MSCOs, where one is corresponding to the ISCO and the other is the OSCO.

Case3: $(45 - 32\sqrt{2})23 = \bar{\gamma}$. The ISCO and the OSCO merge.

Case4: $-1 < \bar{\gamma} < (45 - 32\sqrt{2})23$. No MSCO.

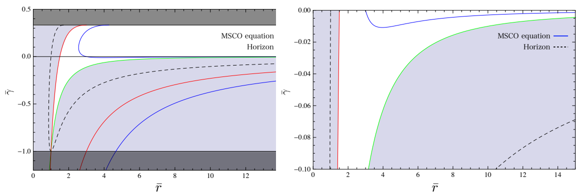

Figure 1: $\bar{\gamma}$-dependence of the MSCO radius.The horizontal axis denotes the MSCO radius. The vertical axis denotes the $\bar{\gamma}$ parameter.The shaded parts denote the prohibited regions where $E^2 < 0$, $L^2 < 0$, $\bar{\gamma} < -1$ or $\bar{\gamma} > 1/3$. Left: $\bar{\gamma} \in [-1.2, 0.5]$. Right:$\bar{\gamma} \in [-0.1, 0]$

6 Conclusion

We reexamined, in terms of Sturm's theorem, MSCOs of a time-like geodesic in Kottler and spherically symmetric, static and vacuum black-hole solution in Weyl conformal gravity.

Sturm's theorem is applicable for classifying MSCOs for some spacetime, if the MSCO eq. is a polynomial.

Extension to axisymmetric spacetime is in progress (Suzuki's poster).

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"Singularity formation in n-dim Gauss-Bonnet gravity"

by Hisaaki Shinkai

[JGRG25(2015)P29]

P29

Singularity Formation in n-dim Gauss-Bonnet gravity

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真貝寿明，鳥居隆（大阪工業大）

1. Motivation

Dynamics in Gauss-Bonnet gravity?

- Action

$$S=\int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R}+\alpha_2\mathcal{L}_{\rm GB}\}+\mathcal{L}_{\rm matter}\Big]$$

where $\mathcal{L}_{GB}=\mathcal{R}^2-4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}+\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu}+\alpha_2 H_{\mu\nu}+g_{\mu\nu}\Lambda=\kappa^2 T_{\mu\nu}$$

where $H_{\mu\nu}=2[\mathcal{R}\mathcal{R}_{\mu\nu}-2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}_{\ \nu}-2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta}+\mathcal{R}_{\mu}^{\ \alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}]-\frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

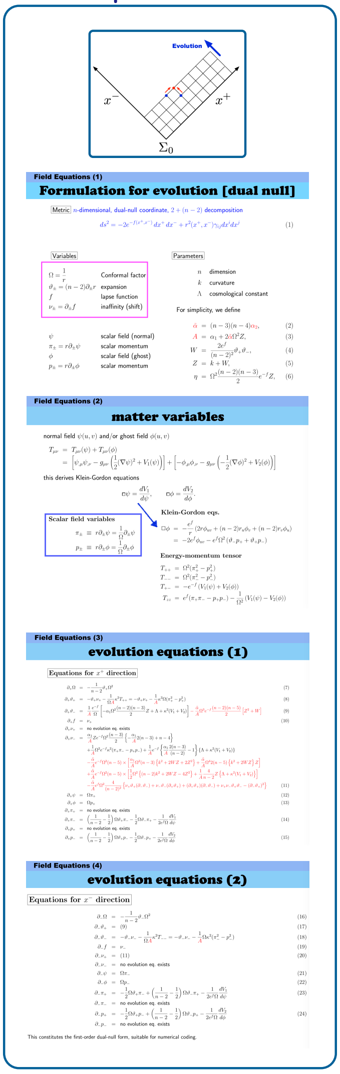
- **has GR correction terms from String Theory**
- **has two solution branches (GR/non-GR).**
- **is expected to have singularity avoidance feature. (but has never been demonstrated.)**

- **new topic in numerical relativity.**
  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496
- **much attentions in WH community**
  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

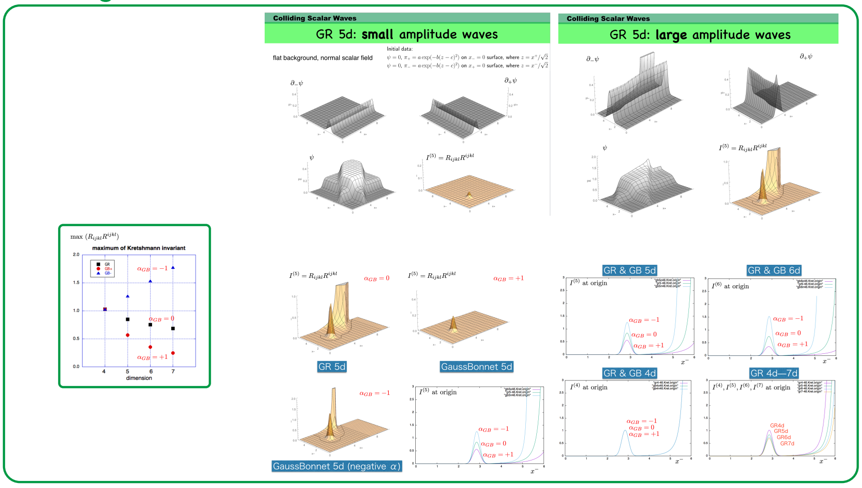
Outline & Summary

We numerically investigated how the dynamics depend on the dimensionality and how the higher-order curvature terms affect to singularity formation in two models:
(i) colliding scalar pulses in planar space-time, and (ii) perturbed wormhole in spherical symmetric space-time.
Our numerical code uses dual-null formulation, and we compare the dynamics in 5, 6 and 7-dimensional General Relativity and Gauss-Bonnet (GB) gravity.
(1) For scalar wave collisions, we observe that curvarure evolutions (Kretschmann invariant) are milder in the presence of GB term and/or in higher-dimensional space-time.
(2) For wormhole dynamics, we observe that the perturbed throat will be easily enhance in the presence of GB term.
Both suggest that the thresholds for the singularity formation become higher in higher dimension and/or in presence of GB terms, although it is not evitable.

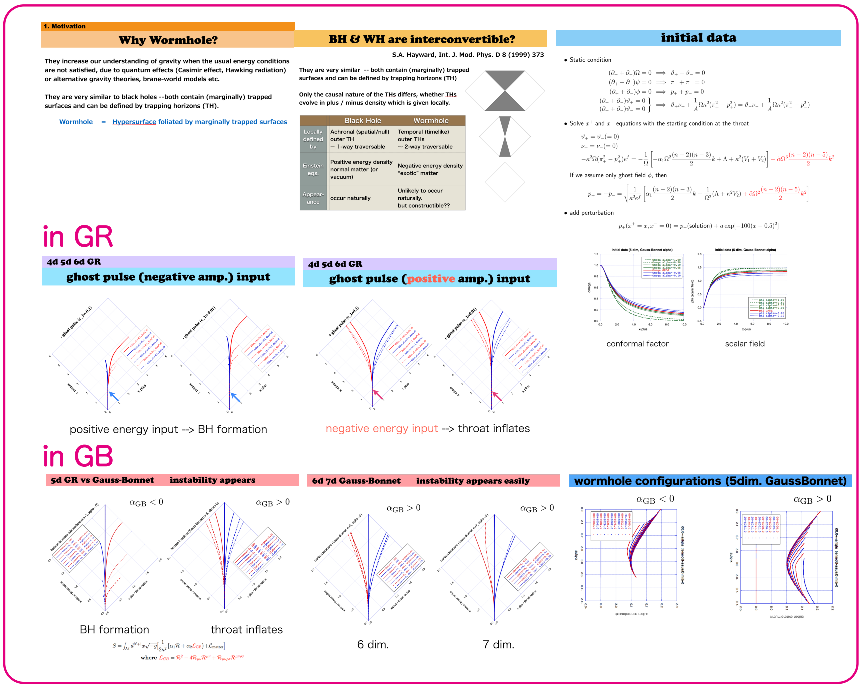
Field Eqs.



Colliding Scalar Waves



Wormhole Evolutions



Black Hole, Expanding Universe, and Gravitational Wave
--- 100 years of General Relativity

H.Shinkai (2015 Kobunsha)
[Korean ver. (2016 Kachi Pub.)]

"Conditions on Scalar Potentials in Geometric Scalar Gravity"

by Kiyoshi Shiraishi

[JGRG25(2015)P30]

Conditions on Scalar Potentials in Geometric Scalar Gravity

Kiyoshi Shiraishi (Yamaguchi U.)
arXiv:1508.02827 [gr-qc] (with Nahomi Kan (NIT, Gifu))

We discuss a generic form of the scalar potential in
Geometric Scalar Gravity
(M. Novello et al., JCAP1306:014)

FAQ

Q. Is it difficult to describe Gravity by the single scalar field?

A. It could be considered as an exercise for Modified Gravities (TeVeS, Mimetic, ・・).

§ 1. Old scalar gravity

Theory of Nordström (1912) $g_{\mu\nu} = e^{2\Phi} \eta_{\mu\nu}$

dynamical equation: $\Box e^{\Phi} = 0$ (in vacuum)

a solution: $ds^2 = \left(1 - \frac{GM}{r}\right)^2 (dt^2 - dr^2 - r^2 d\Omega^2)$

coupling to matter: $\Box e^{\Phi} \propto T$, $T \equiv T^{\lambda}_{\lambda}$

- Newton limit: $\nabla^2 \Phi = 4\pi G\rho$, (ρ : energy density) OK
- Deflection of light; conflicts with observation NG

related work

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§ 2. Review of Geometric Scalar Gravity

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arXiv:1412.4227 [gr-qc].

E. Bittencourt, M. Novello, U. Moschella, E. Goulart, J. M. Salim and J. D. Toniato,
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J. D. Toniato, "A teoria geometrica-escalar da gravitacao e sua aplicacao a cosmologia",
Tese de Doutorado (Rio de Janeiro, 2014).

I. C. Jardim and R. R. Landim,
"About the cosmological constant in geometric scalar theory of gravity",
ArXiv:1508.02665 [gr-qc]

6

Dynamic metric:

$$q_{\mu\nu} = e^{2\Phi}\left[\eta_{\mu\nu} - \frac{e^{-4\Phi}V(\Phi)-1}{e^{-4\Phi}V(\Phi)}\frac{\partial_\mu\Phi\partial_\nu\Phi}{w}\right]$$

$$w \equiv \eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi$$

$\eta_{\mu\nu}$: flat metric

$V(\Phi)$: scalar potential

inverse: $$q^{\mu\nu} = e^{-2\Phi}\left[\eta^{\mu\nu} + \frac{e^{-4\Phi}V(\Phi)-1}{w}\eta^{\mu\rho}\eta^{\nu\sigma}\partial_\rho\Phi\partial_\sigma\Phi\right]$$

$\sqrt{\text{determinant}}$: $$\sqrt{-q} = \sqrt{-\det q_{\mu\nu}} = \frac{e^{6\Phi}}{\sqrt{V(\Phi)}}\sqrt{-\eta}$$

action for Scalar: $S_\Phi = \frac{1}{\kappa}\int \sqrt{-q}\sqrt{V(\Phi)}q^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi d^4x$

variation: $\delta S_\Phi = -\frac{2}{\kappa}\int \sqrt{-q}\sqrt{V(\Phi)}(\Box\Phi)\delta\Phi d^4x$

Note:

$$\Box\Phi \equiv \frac{1}{\sqrt{-q}}\partial_\mu(\sqrt{-q}q^{\mu\nu}\partial_\nu\Phi) = e^{-6\Phi}V(\Phi)\left[\frac{1}{\sqrt{-\eta}}\partial_\mu(\sqrt{-\eta}\eta^{\mu\nu}\partial_\nu\Phi) + \frac{w}{2}\frac{d}{d\Phi}\ln V(\Phi)\right]$$

action for matter: $S_m = \int \sqrt{-q}\mathcal{L}_m d^4x$

$$\delta S_m = -\frac{1}{2}\int \sqrt{-q}\,T^{\mu\nu}\delta q_{\mu\nu}d^4x \text{, ここで } T_{\mu\nu} \equiv \frac{2}{\sqrt{-q}}\frac{\partial\sqrt{-q}\mathcal{L}_m}{\partial q^{\mu\nu}}$$

expressed as the variation of the scalar field,

$$T^{\mu\nu}\delta q_{\mu\nu} = \left[2T + \left(4 - \frac{1}{V}\frac{dV}{d\Phi}\right)E\right]\delta\Phi - 2C^\lambda\partial_\lambda\delta\Phi$$

ここで $T = T^{\mu\nu}q_{\mu\nu}\,,\quad E = \frac{T^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi}{\Omega}\,,\quad C^\lambda = \frac{e^{-4\Phi}V - 1}{\Omega}(T^{\lambda\nu} - Eq^{\lambda\nu})\partial_\nu\Phi$

$\Omega \equiv q^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi = e^{-6\Phi}Vw$, thus

$$\delta S_m = -\int\left[T + \left(2 - \frac{1}{2V}\frac{dV}{d\Phi}\right)E + \nabla_\lambda C^\lambda\right]\delta\Phi\sqrt{-q}d^4x$$

action for gravity+matter system:

equation of motionderived from $S = S_\phi + S_m$:

$$\sqrt{V(\Phi)}\Box\Phi = \kappa\chi\,,$$

where

$$\chi = -\frac{1}{2}\left[T + \left(2 - \frac{1}{2V}\frac{dV}{d\Phi}\right)E + \nabla_\lambda C^\lambda\right]$$

Newtonian Approximation :

$$T_{00} \approx \rho\,, \qquad g_{00} = e^{2\Phi} \approx 1 + 2\Phi_N$$

$$\sqrt{V(\Phi_\infty)}\nabla^2\Phi = \frac{\kappa}{2}\rho, \text{ where } \frac{\kappa}{\sqrt{V(\Phi_\infty)}} = 8\pi G$$

Φ_∞ : the value of the scalar at an infinite distance from the origin

Novello et al. adopted the following as V (Φ):

$$V(\Phi) = V_{\rm N}(\Phi) = \frac{1}{4}e^{2\Phi}(1 - 3e^{2\Phi})^2$$

(This produces an exact Schwarzschild spacetime)

What geometry from generic forms of the scalar potential？

§ 3. Weak Gravity Limit (post-Newtonian)

spherical, static spacetime

non-dynamical flat metric: $\eta_{\mu\nu}dx^\mu dx^\nu = dT^2 - dR^2 - R^2 d\Omega^2$

ansatz: $\Phi = \Phi(R)$

$$ds^2 = q_{\mu\nu}dx^\mu dx^\nu = e^{2\Phi}dT^2 - \frac{e^{6\Phi}}{V(\Phi)}dR^2 - e^{2\Phi}R^2 d\Omega^2$$

new variables : $t \equiv e^{\Phi_\infty}T \quad r \equiv e^{\Phi}R$

Sandard Form : $ds^2 = B(r)dt^2 - A(r)dr^2 - r^2 d\Omega^2$

with

$$B(r) = e^{2(\Phi - \Phi_\infty)}\,,$$

$$A(r) = \frac{e^{4\Phi}}{V(\Phi)}\left(1 - r\frac{d\Phi}{dr}\right)^2$$

Asymptotic Flatness⇄

$$\lim_{r\to\infty}\Phi(r) = \Phi_\infty\,, \quad \lim_{r\to\infty} r\Phi'(r) = 0\,, \quad e^{-4\Phi_\infty}V(\Phi_\infty) = 1$$

expand gravitational potential from the asymptotically flat spacetime: (g is a constant)

$$\Phi_N \equiv \Phi - \Phi_\infty = -\frac{GM}{r} - g\frac{G^2M^2}{r^2} + O((GM/r)^3)$$

expand the scalar potential: (k is a constant)

$$e^{-4\Phi}V(\Phi) = 1 + 4k\Phi_N + O(\Phi_N^2)$$

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Using the parameters, the line element can be written as

$$B(r) = 1 - \frac{2GM}{r} + \frac{2(1-g)G^2M^2}{r^2} + O((GM/r)^3)\,,$$
$$A(r) = 1 + \frac{2(2k-1)GM}{r} + O((GM/r)^2)\,.$$

PPN (leading) (see texts by Weinberg, Hartle,...)

$$B(r) = 1 - \frac{2GM}{r} + \frac{2(\beta-\gamma)G^2M^2}{r^2} + O((GM/r)^3)\,,$$
$$A(r) = 1 + \frac{2\gamma GM}{r} + O((GM/r)^2)\,,$$

by comparison, we get $\beta = 2k - g$, $\gamma = 2k - 1$

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$$\sqrt{\frac{B(r)}{A(r)}}r^2\frac{d\Phi}{dr} = GM + \frac{2(g-k)G^2M^2}{r} + O(GM(GM/r)^2)$$

From this e.o.m. of scalar in vacuum -> g=k

+also, From observation, (up to 6 digits) $\beta = \gamma = 1 \Rightarrow k=1$

in conclusion,

$$e^{-4\Phi}V(\Phi) = 1 + 4\Phi_N + O(\Phi_N^2)$$

or

$$\frac{1}{V(\Phi)}\frac{dV(\Phi)}{d\Phi} = 8 \quad \text{at } \Phi = \Phi_\infty$$

is the condition for weak gravity in post-Newtonian App. OK

§ 4. Strong Gravity (horizon)

From e.o.m. in vacuum $\sqrt{\frac{b(r)}{A(r)}} r^2 \frac{d\Phi}{dr} = \frac{1}{2} \frac{1}{\sqrt{A(r)b(r)}} r^2 \frac{db(r)}{dr} = GMe^{\Phi_\infty} > 0$

where $b(r) \equiv e^{2\Phi(r)}$, $g_{00} = B(r) = b(r)e^{-2\Phi_\infty}$.

$$\frac{\sqrt{b(r)e^{-4\Phi}V(\Phi(r))}}{\left|2 - \frac{rb'(r)}{b(r)}\right|} r^2 \frac{b'(r)}{b(r)} = \frac{\sqrt{V(\Phi(r))/b(r)}}{|2b(r) - rb'(r)|} r^2 b'(r) = GMe^{\Phi_\infty}$$

If $b > 0$ and $\sqrt{V(\Phi(r))/b(r)}$ is finite everywhere, then

$$\frac{b'(r)}{b(r)} \to \infty \quad \text{as} \quad r \to 0$$

singularity at r=0, no horizon

A horizon at $r = r_g$ can exists if

$$0 < \lim_{r \to r_g} \sqrt{A(r)b(r)} < \infty \quad \text{or} \quad 0 < \lim_{\Phi \to -\infty} \sqrt{e^{-2\Phi}V(\Phi)} < \infty .$$

also, at $r = r_0 > r_g$, $2b - rb' = 0$ holds, then

$V(\Phi(r_0)) = 0$ is necessary.

From these, if a horizon exists, b(r) should satisfy

$$\frac{r}{2} \frac{b'(r)}{b(r)} = \left(1 - r\frac{\sqrt{e^{-2\Phi(r)}V(\Phi(r))}}{GMe^{\Phi_\infty}}\right)^{-1} \quad \text{for } r_g < r < r_0 ,$$

$$\frac{r}{2} \frac{b'(r)}{b(r)} = \left(1 + r\frac{\sqrt{e^{-2\Phi(r)}V(\Phi(r))}}{GMe^{\Phi_\infty}}\right)^{-1} \quad \text{for } r > r_0 ,$$

Moreover, if we require b(r) at r = r $_0$ be a smoothfunction, the scalar potential should be written as

$$\boxed{e^{-2\Phi}V(\Phi) = [F(b)]^2}$$

then, from the condition for Asymptotic Infinity,

$$\frac{GMe^{\Phi_\infty}}{r_g} = |F(0)| \equiv F_0$$

⬆ in general, $r_g \neq 2GM$

Example. the Linear function

$$F(b) = F_0\,(1 - f_1 b)$$

$$F_0^2 = \frac{3}{4f_1}$$

$$V(\Phi) = \frac{3}{4f_1}e^{2\Phi}\left(1 - f_1 e^{2\Phi}\right)^2 = \frac{e^{4\Phi_\infty}}{4}e^{2(\Phi-\Phi_\infty)}\left[1 - 3e^{2(\Phi-\Phi_\infty)}\right]^2$$

If $\Phi_\infty = 0$: $V(\Phi) = V_{\mathrm{N}}(\Phi) = \frac{1}{4}e^{2\Phi}(1 - 3e^{2\Phi})^2$

This is one that Novello et al. adopted.

The exact Schwarzschild spacetime is obtained.

Example2. the Case easily solved

$$F(b) = F_0\,(1 - f_p b^p)$$

this leads to the spacetime solution

$$ds^2 = \left(1 - \frac{r_g}{r}\right)^{1/p} dt^2 - \left(1 - \frac{r_g}{r}\right)^{-2+1/p} + r^2 d\Omega^2$$

but singularity at $r=r_g$!

To get horizon at $r=r_g$, $F(b)=F_0(1-f(b))$

where $f(b)$ must be (prop. to) $b + O(b^2)$

＊Inverse Problem＊

Q. What scalar potential leads to the metric:

$$b = e^{2\Phi} = e^{2\Phi_\infty}\left[1 - \frac{2GM}{r} + \frac{4}{3}\delta\frac{(2GM)^3}{r^3}\right]$$

?

A. (using the new parameters below,)

$$e^{2\Phi_\infty} = \frac{3}{f}\left(1 + \frac{2}{3}h\right), \quad \delta = \frac{h}{4}\frac{\left(1+\frac{2}{3}h\right)^2}{(1+h)^3},$$

$$\frac{r_g}{2GM} = \frac{e^{\Phi_\infty}}{2F_0} = \frac{1+\frac{2}{3}h}{1+h}, \quad F_0^2 = \frac{3}{4f}\frac{(1+h)^2}{1+\frac{2}{3}h}$$

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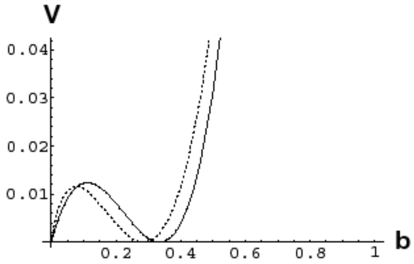
$$F(b) = F_0(1 - f\varphi(b))\,,$$

where $$f\varphi(b) = Y(b) + \frac{2fb}{3\left[1 + h(2Y(b) - Y(b)^2)\right]}\,,$$

$$Y(b) = 1 - \frac{2\sqrt{1+h}}{\sqrt{h}} \sin\left[\frac{1}{3}\arcsin\left\{\frac{3\sqrt{h}\,(1+\frac{2}{3}h)}{2(1+h)^{3/2}}\left(1 - \frac{f}{3+2h}b\right)\right\}\right] \quad \text{for } h > 0$$



A Schematic view of the S. P.

⬅ Unfortunately, very small difference between two cases with
h=0(solid) and h=∞(broken).

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§ 5. Summary

In Geometric Scalar Gravity,
we found physical conditions on the scalar potential V
by considering Weak Gravity and Strong Gravity

future subjects

- rotating BHs
- Gravitational waves
- Cosmology

§ 6. Charged solution

$$\mathcal{L}_{EM} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}$$

$$T_{\mu\nu} = -\frac{1}{4\pi}\left(F_{\mu\nu}^2 - \frac{1}{4}F^2 q_{\mu\nu}\right)$$

$$\nabla_\mu F^{\mu\nu} = 0 \qquad F_{0r} = \frac{Q}{r^2}\sqrt{B(r)A(r)}$$

$$T = 0\,, \quad E = \frac{1}{8\pi}\frac{Q^2}{r^4}\,, \quad C^\lambda = 0$$

$$\sqrt{\frac{V(\Phi(r))}{A(r)B(r)}}\frac{1}{r^2}\frac{d}{dr}\left[\sqrt{\frac{B(r)}{A(r)}}r^2\frac{d\Phi(r)}{dr}\right] = \frac{\kappa}{16\pi}\left(2 - \frac{1}{2V}\frac{dV}{d\Phi}\right)\frac{Q^2}{r^4}$$

Using $\frac{1}{V(\Phi)}\frac{dV(\Phi)}{d\Phi} = 8$ at $\Phi = \Phi_\infty$, asymptotically

$$\sqrt{\frac{V(\Phi_\infty)}{A(r)B(r)}}\frac{1}{r^2}\frac{d}{dr}\left[\sqrt{\frac{B(r)}{A(r)}}r^2\frac{d\Phi(r)}{dr}\right] \approx -\frac{\kappa}{8\pi}\frac{Q^2}{r^4}$$

solution is, $B(r) = 1 - \frac{2GM}{r} - \frac{GQ^2}{r^2} + O((GM/r)^3)$

RN black hole with wrong sign in front of $O(r^{-2})$!

§ 7. Spherical Star

e.m.tensor for a perfect fluid : $T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p q_{\mu\nu}$

$$q_{\mu\nu} u^\mu u^\nu = 1$$

$$T = \rho - 3p \,, \quad E = -p \,, \quad C^\lambda = 0$$

$$\sqrt{\frac{V(\Phi(r))}{A(r)B(r)}} \frac{1}{r^2} \frac{d}{dr} \left[\sqrt{\frac{B(r)}{A(r)}} r^2 \frac{d\Phi(r)}{dr} \right]$$

$$= \frac{V(\Phi)}{r^2 e^{3\Phi} |1 - r\Phi'|} \left[\frac{\sqrt{V(\Phi)} r^2 \Phi'}{e^\Phi |1 - r\Phi'|} \right]' = \frac{\kappa}{2} \left[\rho(r) - \left(5 - \frac{1}{2V} \frac{dV}{d\Phi} \right) p(r) \right]$$

eq. of conservation : $\nabla_\mu T^{\mu\nu} = \frac{1}{\sqrt{-q}} \partial_\mu (\sqrt{-q}\, T^{\mu\nu}) + \Gamma^\nu_{\rho\sigma} T^{\rho\sigma} = 0$

where $\Gamma^\nu_{\rho\sigma} = \frac{1}{2} q^{\nu\lambda} (\partial_\rho q_{\lambda\sigma} + \partial_\sigma q_{\lambda\rho} - \partial_\lambda q_{\rho\sigma})$

$$p'(r) + [\rho(r) + p(r)] \Phi'(r) = 0$$

∗ Inverse problem ∗ de Sitter like interior metric

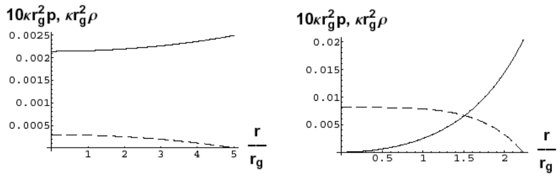
$$b(r) = b_*(r) \equiv b_e(r_*) - \frac{b_e'(r_*)}{2} \left(1 - \frac{r^2}{r_*^2} \right) \equiv e^{2\Phi_*(r)} \,, \quad \text{for } r < r_*$$

$$b(r) = e^{2\Phi(r)} = b_e(r) = 1 - \frac{2GM}{r} + \frac{4\delta}{3} \frac{(2GM)^3}{r^3} \,, \quad \text{for } r > r_*$$

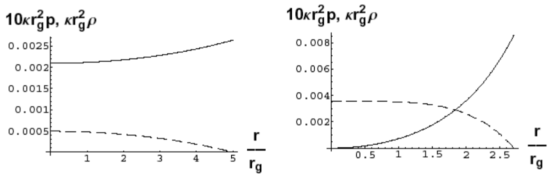
energy density and pressure for this metric

$$\left[e^{6\Phi_*(r)}\frac{1}{\sqrt{V(\Phi_*(r))}}\kappa p(r)\right]' = -\frac{2e^{3\Phi_*(r)}\sqrt{V(\Phi_*(r))}\Phi_*'(r)}{r^2(1-r\Phi_*'(r))}\left[\frac{\sqrt{V(\Phi_*(r))}r^2\Phi_*'(r)}{e^{\Phi_*(r)}(1-r\Phi_*'(r))}\right]'$$

h=0



h=∞



§ 8. Appendix

Inverse of Metric: $q^{\mu\nu} = \alpha\eta^{\mu\nu} + \frac{\beta}{w}\eta^{\mu\rho}\eta^{\nu\sigma}\partial_\rho\Phi\partial_\sigma\Phi$

where $w = \eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi$

then, $q_{\mu\nu} = \frac{1}{\alpha}\eta^{\mu\nu} - \frac{\beta}{\alpha(\alpha+\beta)w}\partial_\mu\Phi\partial_\nu\Phi$

Note that $q^{\mu\nu}\partial_\nu\Phi = (\alpha+\beta)\eta^{\mu\nu}\partial_\nu\Phi$

$$\frac{1}{\sqrt{-q}}\partial_\mu(\sqrt{-q}q^{\mu\nu}\partial_\nu\Phi) = \frac{1}{\sqrt{-q}}\partial_\mu(\sqrt{-q}(\alpha+\beta)\eta^{\mu\nu}\partial_\nu\Phi)$$

$$= (\alpha+\beta)\left[\partial_\mu(\eta^{\mu\nu}\partial_\nu\Phi) + \frac{w}{\sqrt{-q}(\alpha+\beta)}\frac{d\sqrt{-q}(\alpha+\beta)}{d\Phi}\right]$$

$$= (\alpha+\beta)\left[\partial_\mu(\eta^{\mu\nu}\partial_\nu\Phi) + \frac{w}{2}\frac{d\ln[\alpha^{-(D-1)}(\alpha+\beta)]}{d\Phi}\right]$$

Here, we have used

$$\frac{1}{\sqrt{-q}}\frac{d\sqrt{-q}}{d\Phi} = \frac{1}{2}q^{\mu\nu}\frac{dq_{\mu\nu}}{d\Phi} = -\frac{1}{2}q_{\mu\nu}\frac{dq^{\mu\nu}}{d\Phi} = -\frac{1}{2}\left[(D-1)\frac{1}{\alpha}\frac{d\alpha}{d\Phi} + \frac{1}{\alpha+\beta}\frac{d(\alpha+\beta)}{d\Phi}\right]$$

and $\sqrt{-q} \propto \frac{1}{\sqrt{\alpha^{D-1}(\alpha+\beta)}}$

Assume: $\mathcal{L} = V(\Phi)\eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi = V(\Phi)w$

Equation of Motion: $\partial_\mu(\eta^{\mu\nu}\partial_\nu\Phi) + \frac{w}{2}\frac{d}{d\Phi}\ln V = 0$

This is equivalent to $\frac{1}{\sqrt{-q}}\partial_\mu(\sqrt{-q}q^{\mu\nu}\partial_\nu\Phi) = 0$

provided that $\alpha+\beta = \alpha^{D-1}V$

($\sqrt{-q} \propto \frac{1}{\alpha^{D-1}\sqrt{V}}$)

Eliminating β , $q^{\mu\nu} = \alpha\eta^{\mu\nu} + \alpha\frac{\alpha^{D-2}V-1}{w}\eta^{\mu\rho}\eta^{\nu\sigma}\partial_\rho\Phi\partial_\sigma\Phi$

and $q_{\mu\nu} = \frac{1}{\alpha}\eta^{\mu\nu} - \frac{\alpha^{D-2}V-1}{\alpha^{D-1}Vw}\partial_\mu\Phi\partial_\nu\Phi$

The 25th JGRG (Kyoto), Dec. 2015

“Marginal stable circular orbits for stationary and axially symmetric spacetimes”

by Tomohito Suzuki

[JGRG25(2015)P31]

Marginal stable circular orbits for stationary and axially symmetric spacetimes

Tomohito Suzuki
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JGRG25 in YITP, Kyoto Dec. 7 - 11, 2015

Abstract: Continuing the earlier work[1], in this poster, we extend our formulations for a marginal stable circular orbit(MSCO) of a test particle to stationary and axially symmetric spacetimes.

HIROSAKI UNIVERSITY

1 Motivation

The earlier work[1](spherically symmetric and static spacetimes)
→ this poster(stationary and axially symmetric spacetimes: ex. Kerr metric)
In Kerr-like metric, the locations of the innermost stable circular orbit change significantly from the Kerr metric[2].

2 Timelike geodesic in stationary and axially symmetric spacetimes

A general form ($G = c = 1$)

$$\begin{aligned} ds^2 = -A(y^2,y^3)dt^2 - 2H(y^2,y^3)dtd\varphi + B(y^2,y^3)(dy^2)^2 \\ +C(y^2,y^3)(dy^3)^2 + D(y^2,y^3)d\varphi^2 \,, \end{aligned} \tag{1}$$

where $0 < \varphi < 2\pi$.
This spacetimes contain quasi-cylindrical Weyl-Papapetrou coordinates[3] in $y^2 = \rho\,,\ y^3 = z\,,\ B(\rho,z) = C(\rho,z)$, Boyer-Lindquist coordinates in $y^2 = r\,,\ y^3 = \theta$.
We assume a symmetry at $y^3 = y^3_C$

$$\left.\frac{\partial g_{\mu\nu}}{\partial y^3}\right|_{y^3=y^3_c} = 0\,. \tag{2}$$

The Lagrangian

$$\mathscr{L} \equiv -A\dot{t}^2 - 2H\dot{t}\dot{\varphi} + B(\dot{y}^2)^2 + D\dot{\varphi}^2\,,\quad \dot{} \equiv \frac{d}{d\tau}\,. \tag{3}$$

Two constants of motion

$$\varepsilon \equiv -A\dot{t} - H\dot{\varphi}\,,\quad \ell \equiv -H\dot{t} + D\dot{\varphi}\,. \tag{4}$$

3 MSCO conditions

Radial MSCO condition is well known as [3]

$$\left[-\tilde{A}\ell^2 + \tilde{D}\varepsilon^2 + 2\tilde{H}\varepsilon\ell\right]_{y^3=y^3_C} = 1\,, \tag{5}$$

$$\left[-\tilde{A}_{,y^2}\ell^2 + \tilde{D}_{,y^2}\varepsilon^2 + 2\tilde{H}_{,y^2}\varepsilon\ell\right]_{y^3=y^3_C} = 0\,, \tag{6}$$

$$\left[-\tilde{A}_{,y^2y^2}\ell^2 + \tilde{D}_{,y^2y^2}\varepsilon^2 + 2\tilde{H}_{,y^2y^2}\varepsilon\ell\right]_{y^3=y^3_C} = 0\,. \tag{7}$$

where

$$\tilde{A} = \frac{A}{AD+H^2}\,,\quad \tilde{D} = \frac{D}{AD+H^2}\,,\quad \tilde{H} = \frac{H}{AD+H^2}\,,$$

$\tilde{A}_{,y^2} = d\tilde{A}/dy^2$.
By Eqs.(6,7), we obtain MSCO equation

$$\begin{aligned} &\left(\tilde{D}_{,y^2}\tilde{A}_{,y^2y^2} - \tilde{A}_{,y^2}\tilde{D}_{,y^2y^2}\right)^2_{y^3=y^3_C} \\ =&4\left[\left(\tilde{A}_{,y^2}\tilde{H}_{,y^2y^2} - \tilde{H}_{,y^2}\tilde{A}_{,y^2y^2}\right)\left(\tilde{D}_{,y^2}\tilde{H}_{,y^2y^2} - \tilde{H}_{,y^2}\tilde{D}_{,y^2y^2}\right)\right]_{y^3=y^3_C}\,. \end{aligned} \tag{8}$$

Eq.(8) corresponds to previous work[3] of Eq.(40).
By Eqs.(5,6),

$$\varepsilon^2 = \frac{\tilde{A}^2_{,y^2}}{\left[-\tilde{A}_{,y^2}\left(\tilde{A}\tilde{D}_{,y^2} - \tilde{A}_{,y^2}\tilde{D}\right) + 2\left(\tilde{A}_{,y^2}\tilde{H} - \tilde{A}\tilde{H}_{,y^2}\right)\left(\tilde{H}_{,y^2} \mp \sqrt{\tilde{H}^2_{,y^2} + \tilde{A}_{,y^2}\tilde{D}_{,y^2}}\right)\right]} \tag{9}$$

$$\ell^2 = \frac{\left[\tilde{A}_{,y^2}\tilde{D}_{,y^2} + 2\tilde{H}_{,y^2}\left(\tilde{H}_{,y^2} \mp \sqrt{\tilde{H}^2_{,y^2} + \tilde{A}_{,y^2}\tilde{D}_{,y^2}}\right)\right]}{\left[-\tilde{A}_{,y^2}\left(\tilde{A}\tilde{D}_{,y^2} - \tilde{A}_{,y^2}\tilde{D}\right) + 2\left(\tilde{A}_{,y^2}H - \tilde{A}\tilde{H}_{,y^2}\right)\left(\tilde{H}_{,y^2} \mp \sqrt{\tilde{H}^2_{,y^2} + \tilde{A}_{,y^2}\tilde{D}_{,y^2}}\right)\right]}\,. \tag{10}$$

Axial stable condition

$$\left(\tilde{D}_{,y^3y^3}\varepsilon^2 - \tilde{A}_{,y^3y^3}\ell^2 + 2\tilde{H}_{,y^3y^3}\varepsilon\ell\right)_{y^3=y^3_C} > 0\,. \tag{11}$$

The radius of the MSCO must satisfy not only the root of MSCO equation Eq.(8) but also $0 \le \varepsilon^2 < \infty$ and $0 \le \ell^2 < \infty$ and Eq.(11).

4 Examples

4.1 Kerr spacetime

Boyer-Lindquist coordinates

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{4aMr\sin^2\theta}{\Sigma}dtd\varphi \\ &+ \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2a^2Mr\sin^2\theta}{\Sigma}\right)d\varphi^2\,, \end{aligned} \tag{12}$$

$$\Sigma = r^2 + a^2\cos^2\theta\,,\quad \Delta = r^2 - 2Mr + a^2\,. \tag{13}$$

MSCO equation

$$\bar{r}^4 - 12\bar{r}^3 - 6\bar{a}^2\bar{r}^2 + 36\bar{r}^2 - 28\bar{a}^2\bar{r} + 9\bar{a}^4 = 0\,, \tag{14}$$

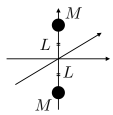
where $\bar{r} = r/M\,,\ \bar{a} = a/M$.

4.2 Majumdar-Papapetrou(MP) solution

Two charged point particles($Q = M$) in z axis[4]

$$ds^2 = -\Omega^{-2}dt^2 + \Omega^2(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2)\,, \tag{15}$$

$$\Omega = 1 + \frac{M}{\sqrt{r^2 + L^2 - 2Lr\cos\theta}} + \frac{M}{\sqrt{r^2 + L^2 + 2Lr\cos\theta}}\,.$$



MSCO equation

$$\begin{aligned} r^6 - 6(M\sqrt{r^2+L^2} - L^2)r^4 + L^2(10M\sqrt{r^2+L^2} + 9L^2)r^2 \\ +16L^4M\sqrt{r^2+L^2} + 4L^4(L^2+4M^2) = 0\,. \end{aligned} \tag{16}$$

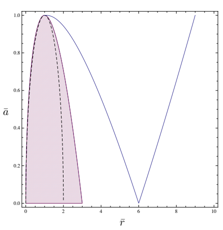


Figure 1: Kerr spacetime
Blue curve:Roots of the MSCO equation
Dashed curve:the horizon
Shaded region:ℓ^2 and ε^2 are the complex number

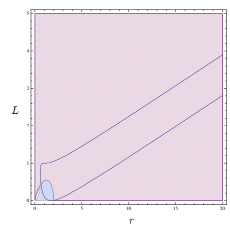


Figure 2: MP solution
Blue curve:Roots of the MSCO equation
Magenta Shaded region:$\ell^2 < 0,\ \varepsilon^2 < 0$
Blue Shaded region:not satisfied axial condition

5 Conclusion

We studied a MSCO of a timelike geodesic in stationary and axially symmetric spacetimes.
We consider not only radial stability, but also axial stability.
Future work: Axial condition should be studied.

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[3] M. Stute and M. Camenzind, Mon. Not. R. Astron. Soc. **336**, 831-840 (2002).

[4] S. D. Majumdar, Phys. Rev. **72**, 390 (1947), A. Papapetrou, Proc. Roy. Irish Acad. **A52**, 11 (1948).

“MHD Wave Propagetion in a Black Hole Magnetosphere”

by Masaaki Takahashi

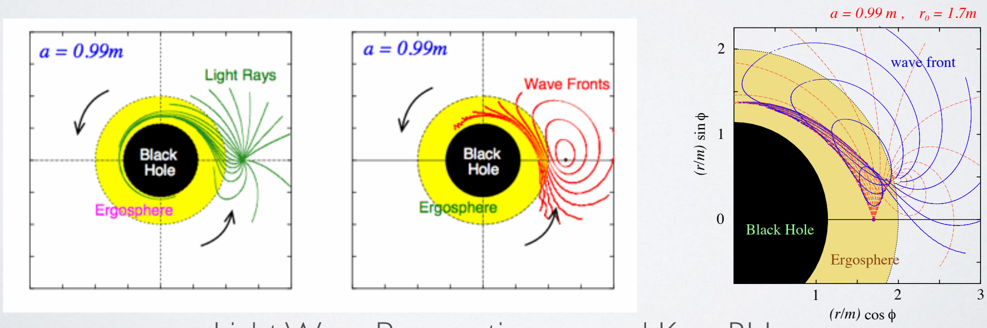
[JGRG25(2015)P32]

P32

The 25th workshop on General Relativity and Gravitation

MHD Wave Propagation in a Black Hole Magnetosphere

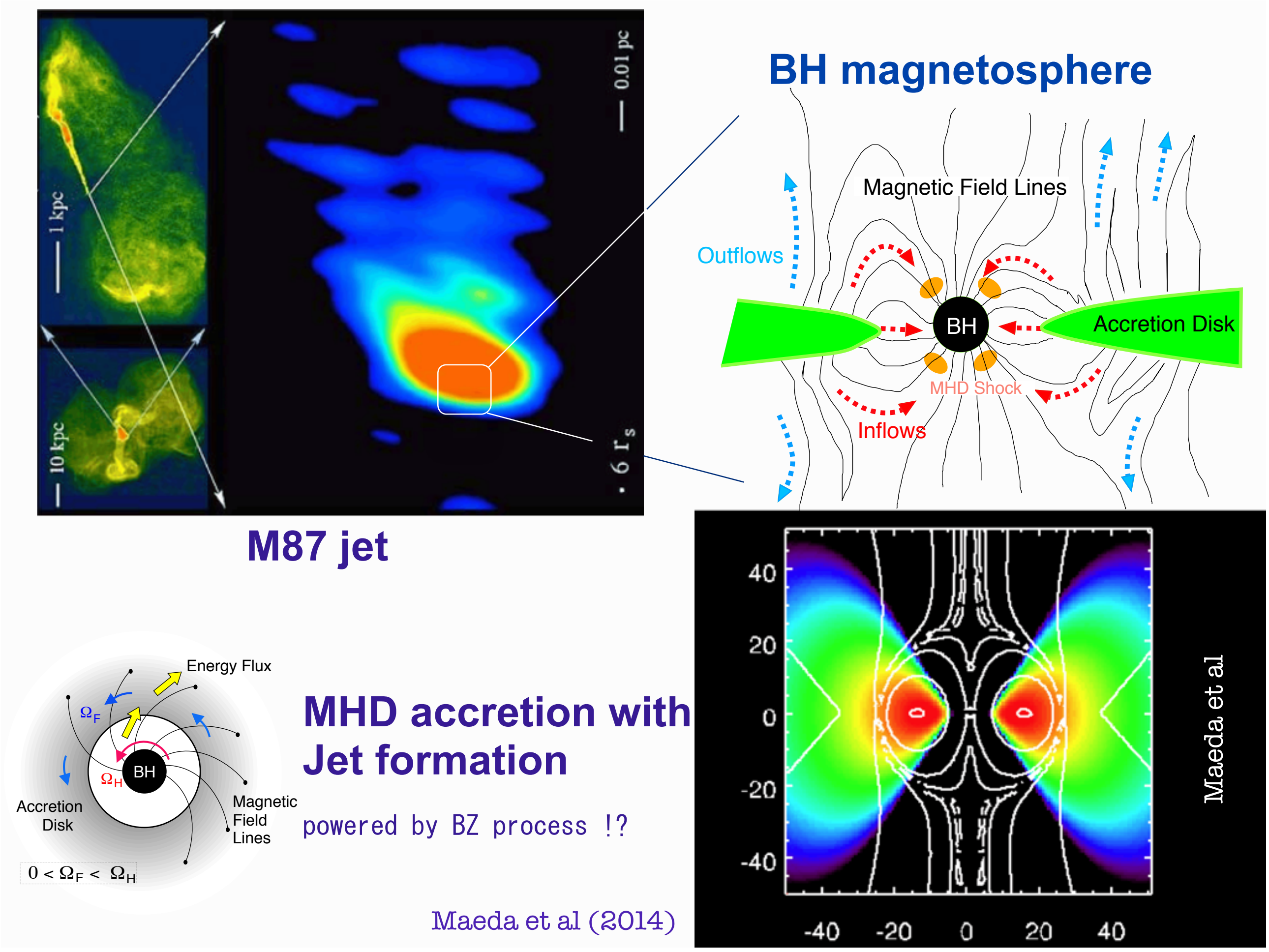
Masaaki Takahashi, Sho Izumaru
Aichi University of Education



Light Wave Propagation around Kerr BH

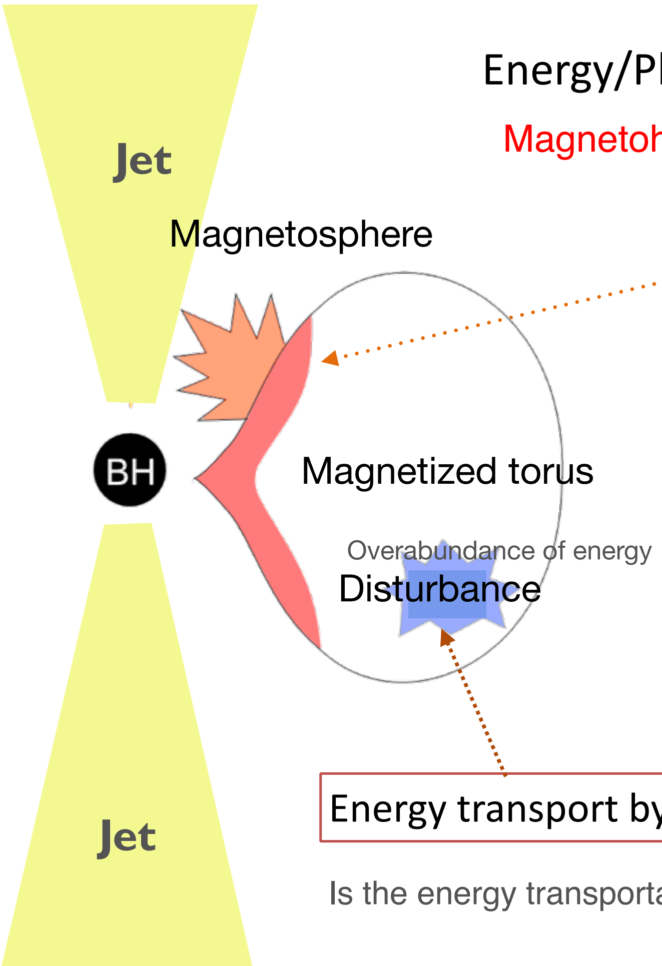
Abstract :

The focusing effects of the energy and momentum by **magnetohydrodynamical(MHD) wave** is studied in a black hole magnetosphere. By using a canonical type formulation for the propagation of MHD disturbances in magnetized accretion disk, the basic properties of MHD wave propagation and the numerical calculations of motion of the locus of simultaneous fronts of wave packets are presented. We define the ``**magnetosonic metric**'' for the propagation of MHD wave, and then we can discuss the ``magnetosonic horizon'', which corresponds to the magnetosonic critical point, and ``**magnetosonic ergoregion**''. The collimation mechanism of the relativistic jet by MHD wave which is emitted from the magnetosphere's plasma is also discussed.



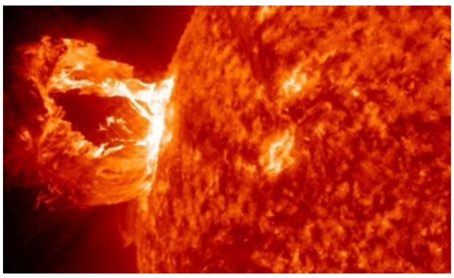
Energy/Plasma supply to the Jet

Magnetohydrodynamical phenomena



Mass ejection / Flare ?

Analogy of the sun surface



SUN: coronal mass ejection (2012.4.)

Energy transport by MHD waves is discussed.

Is the energy transportation to the inside domain possible?

How to calculate the wave fronts

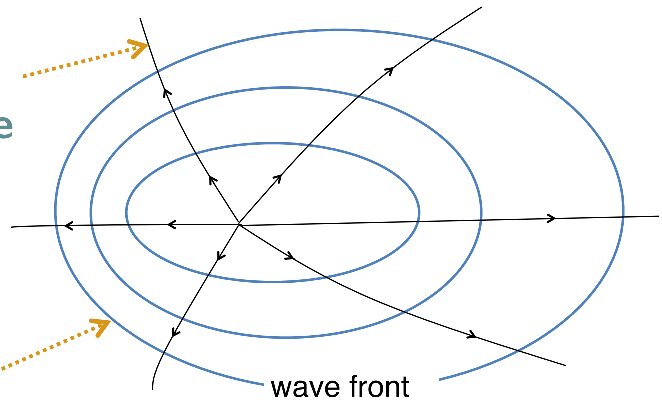
geometric optic approximation
The approximation that a wavelength considers smaller enough than characteristic length of the physical quantity of the system

Wave equation → **Eikonal equation** → **Canonical form**

Lichnerowicz (1967)

Eikonal equation gives the characteristic curve of the wave

Time=constant surface
Wave fronts



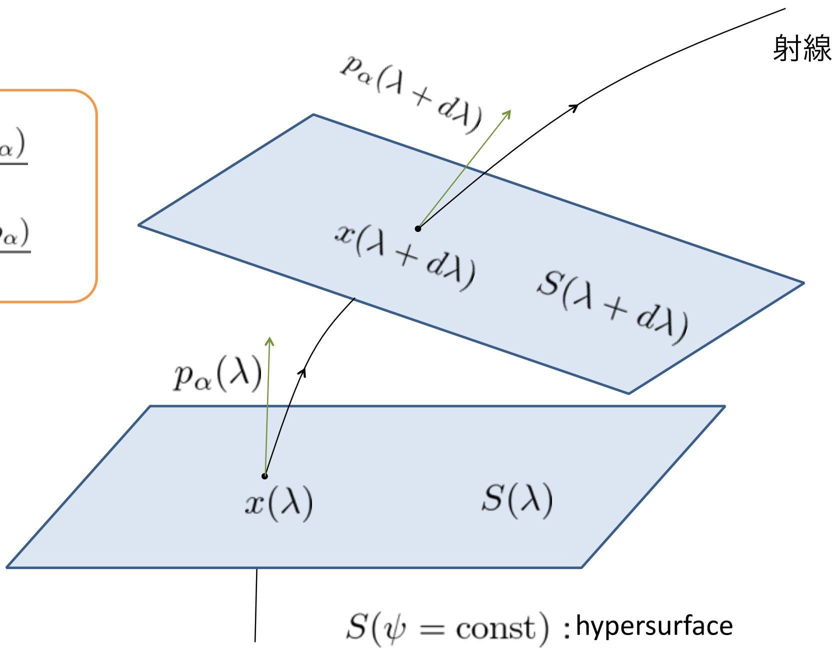
$$x^0(\lambda) = \psi(x^\alpha(\lambda)) = \mathrm{constant}$$

射線の求め方

正準形式

$$\dot{x}^\alpha(\lambda) = \frac{\partial H(x^\alpha, p_\alpha)}{\partial p_\alpha}$$

$$\dot{p}_\alpha(\lambda) = -\frac{\partial H(x^\alpha, p_\alpha)}{\partial x^\alpha}$$



$S(\psi = \mathrm{const})$:hypersurface

射線：初期値より λ に沿って x^α を求める

波面：何本も射線を走らせて 位相 $\psi(x^\alpha)$ の分布を調べる

$*$ Wave propagation is discussed under the Eikonal approximation.

notations & definitions

- n : the proper particle number density
- μ $[\,=(P+\varepsilon+n)/n\,]$: the specific enthalpy
- k : $(=1/4\pi$ for cgs units)
- $|\boldsymbol{h}|^2 \equiv -h_\alpha h^\alpha > 0$
- $h_\alpha \equiv \frac{1}{2}\eta_{\alpha\beta\gamma\delta}u^\beta F^{\gamma\delta}$: the magnetic field 4-vector in fluid's comoving flame [9]
- $e_\alpha \equiv F_{\alpha\beta}u^\beta$: electric field / We require infinite conductivity $e_\alpha \equiv F_{\alpha\beta}u^\beta = 0$.
- u^α : the fluid's 4-velocity
- h_n : the magnetic field in spatial direction of propagation of the waves

$$h_n{}^2 = \frac{-(h^\alpha \psi_{,\alpha})^2}{(g^{\alpha\beta} - u^\alpha u^\beta)\psi_{,\alpha}\psi_{,\beta}}$$

- V : the wave velocity

$$\frac{V^2}{c^2} = \frac{-(u^\alpha \psi_{,\alpha})^2}{(g^{\alpha\beta} - u^\alpha u^\beta)\psi_{,\alpha}\psi_{,\beta}}$$

First, we describe the hypersurface Σ by the equation x_0, where the index '0' denotes time coordinate, (We can chose it without any loss of generality). Next, we will change the coordinate system such that the equation for Σ is x_0 = constant to another coordinate system x^α, where the hypersurface is given by $x^0 = \psi(x^\alpha)$ = constant.

(1) the continuity equation :
The particle number conservation is

$$\boxed{\nabla_\alpha(nu^\alpha) = 0\ ,}$$

where n is the number density of the plasma and u^μ is the fluid 4-velocity. Using the thermodynamical relation $TdS = d\mu - dP/n$, we find the relation $n = n(\mu, S)$. For the adiabatic flow $dS = 0$, we have $\partial_0 n = (dn/d\mu)\partial_0\mu$. Then, the continuity equation can be expressed as

velocity enthalpy

$$\boxed{\Rightarrow \quad n\partial_0 u^0 + u^0 \frac{dn}{d\mu}\partial_0\mu = F_1(u^\alpha, \mu, g^{\alpha\beta})}$$

where the value of F_1 is known from the Cauchy data on the hyper surface Σ [see, Takahashi+(1990), p.877].

initial value problem

(2) the equation of stream line :
The ideal MHD condition is $u^\nu F_{\mu\nu} = 0$, where $F_{\mu\nu}$ is the electromagnetic tensor.

coordinate transformation : $\quad x^0(\lambda) = \psi(x^\alpha(\lambda)) = \text{constant}$

(3) the equation of stream line :

$$\boxed{\nabla_\alpha T^{\alpha\beta} = 0 \ ,}$$

where

$$T_{\alpha\beta} = (n\mu + k|\boldsymbol{h}|^2)u_\alpha u_\beta - \left(\frac{p}{c^2} + \frac{1}{2}k|\boldsymbol{h}|^2\right) g_{\alpha\beta} - k h_\alpha h_\beta \ .$$

By using the relations,

$$\begin{aligned} u^\beta \nabla_\alpha T^\alpha{}_\beta &= 0 \ , \\ \nabla_\alpha (u^\alpha h^\beta - u^\beta h^\alpha) &= 0 \qquad \text{the Maxwell eqs.} \ , \\ h^\beta u_\beta &= 0 \qquad \text{the ideal MHD } (\sigma = \infty) \ , \end{aligned}$$

velocity

we obtain the following relations from equation (80),

$$\Rightarrow \quad n\mu u^0 \partial_0 u^0 - [g^{00} - (u^0)^2] n \partial_0 \mu - \frac{1}{2} k g^{00} \partial_0 |\boldsymbol{h}|^2 = F_2(u^\alpha, \mu, g^{\alpha\beta})$$

$$\Rightarrow \quad \mu \partial_0 h^0 + h^0 \partial_0 \mu = F_3(u^\alpha, \mu, g^{\alpha\beta})$$

Futhermore, from the Maxwell eqs., we obtain

magnetic field enthalpy

$$\frac{1}{2}(u^0)^2 \partial_0 |\boldsymbol{h}|^2 + |\boldsymbol{h}|^2 u^0 \partial_0 u^0 - h^0 \partial_0 h^0 = F_4(u^\alpha, \mu, g^{\alpha\beta}) \ .$$

Then, equation (85) can be reduce to

$$\Rightarrow \quad [n\mu(u^0)^2 + k|\boldsymbol{h}|^2 g^{00}] u^0 \partial_0 u^0 - (u^0)^2 [g^{00} - (u^0)^2] n \partial_0 \mu - k g^{00} h^0 \partial_0 h^0 = F_5(u^\alpha, \mu, g^{\alpha\beta}) \ .$$

From equations (79), (86), (88) , we have

$$\begin{pmatrix} [n\mu(u^0)^2 + k|\boldsymbol{h}|^2 g^{00}]u^0 & -(u^0)^2[g^{00} - (u^0)^2] & -(1/\mu)kg^{00}h^0 \\ n & (u^0/n)(dn/d\mu) & 0 \\ 0 & (h_0/n) & 1 \end{pmatrix} \begin{pmatrix} \partial_0 u^0 \\ n\,\partial_0 \mu \\ \mu\,\partial_0 h^0 \end{pmatrix} = \begin{pmatrix} F_5 \\ F_1 \\ F_3 \end{pmatrix}$$

If the determinant of above matrix is zero; that is,

$$\begin{vmatrix} [n\mu(u^0)^2 + k|\boldsymbol{h}|^2 g^{00}]u^0 & -(u^0)^2[g^{00} - (u^0)^2] & -(1/\mu)kg^{00}h^0 \\ n & (u^0/n)(dn/d\mu) & 0 \\ 0 & (h_0/n) & 1 \end{vmatrix}$$

$$\boxed{= \left(\mu \frac{dn}{d\mu} - n\right)(u^0)^4 + \left(n + k|\boldsymbol{h}|^2 \frac{1}{n}\frac{dn}{d\mu}\right) g^{00}(u^0)^2 - \frac{1}{\mu} k g^{00} (h^0)^2 \ = \ 0 \ ,}$$

we cannot determine the value of u^0 and μ on the hypersurface. That is, this condition describes a singular hypersurface, which is tangent to the elementary cone.

Discontinuous condition

The wave fronts satisfy the condition.

Changing the coordinate system to another coordinate system x^α, equation is expressed as

$$H_{\rm MW}(x^\mu, p_\mu) \equiv P^{\lambda\mu\nu\sigma}\partial_\lambda\psi\partial_\mu\psi\partial_\mu\psi\partial_\sigma\psi = 0\,,$$

where

$$P^{\lambda\mu\nu\sigma} = (\mu n' - n)u^\lambda u^\mu u^\nu u^\sigma + \left(n + k|\boldsymbol{h}|^2\frac{n'}{n}\right)g^{(\lambda\mu}u^\nu u^{\sigma)} - k\left(\frac{1}{\mu}\right)g^{(\lambda\mu}h^\nu h^{\sigma)}$$

Such a characteristic surface can be built up from a family of elements (characteristic strips), which depends only on one parameter λ and is characterized by its loaction $x^\alpha(\lambda)$ and its normal vector $p_\alpha \equiv \partial_\alpha\psi(x^\mu(\lambda))$. The strip shows the null-geodesic of the metric (i.e.., rays of waves), and is obtained from ordinary differential equations

$$\begin{aligned}\dot{x}^\alpha(\lambda) &= \frac{\partial H(x^\alpha, p_\alpha)}{\partial p_\alpha}\,,\\ \dot{p}_\alpha(\lambda) &= -\frac{\partial H(x^\alpha, p_\alpha)}{\partial x^\alpha}\,.\end{aligned}$$

factorization

11

wave velocity

speed of magnetosonic wave

$$\left[\frac{-(u^\alpha p_\alpha)^2}{(g^{\alpha\beta} - u^\alpha u^\beta)p_\alpha p_\beta} - \left(\frac{V_{\rm MW}^2}{c^2}\right)_+\right]\left[\frac{-(u^\alpha p_\alpha)^2}{(g^{\alpha\beta} - u^\alpha u^\beta)p_\alpha p_\beta} - \left(\frac{V_{\rm MW}^2}{c^2}\right)_-\right] = 0\,,$$

fast magnetosonic wave

slow magnetosonic wave

Then, Hamiltonians for the fast and slow magnetosonic waves are given by

$$H_{\rm FM} \equiv M_{\rm FM}^{\mu\nu}\,p_\mu p_\nu \equiv \left[g^{\mu\nu} - \left(1 - \frac{c^2}{V_{\rm FM}^2}\right)u^\mu u^\nu\right]p_\mu p_\nu = 0$$

and

$$H_{\rm SM} \equiv M_{\rm SM}^{\mu\nu}\,p_\mu p_\nu \equiv \left[g^{\mu\nu} - \left(1 - \frac{c^2}{V_{\rm SM}^2}\right)u^\mu u^\nu\right]p_\mu p_\nu = 0\,,$$

respectively, where $V_{\rm FM} \equiv (V_{\rm MW})_+$ is the fast magnetosonic wave speed and $V_{\rm SM} \equiv (V_{\rm MW})_-$ is the slow magnetosonic wave speed. We will call the functions $M_{\rm FM}^{\mu\nu}(x^\alpha)$ and $M_{\rm SM}^{\mu\nu}(x^\alpha)$ to the "fast-magnetosonic metric" and "slow-magnetosonic metric" , respectively. For a weak magnetic field limit ($h^\alpha = 0$), we see $Y^{\alpha\beta} = (a_s^2/c^2)\,(g^{\alpha\beta} - u^\alpha u^\beta)$ and $X^{\alpha\beta\gamma\delta} = 0$, and then we have $M_{\rm FM}^{\mu\nu} = S^{\mu\nu}$ for the fast magnetosonic wave (i.e., the fast magnetosonic wave speed $V_{\rm FM}$ becomes the sound wave speed a_s) .

Eikonal eq. & Effetive Potential

General expression

$$H(x^\alpha, p_\alpha) = A^{\alpha\beta} p_\alpha p_\beta = 0$$

$A^{\alpha\beta}$: metric p_α : wave's momentum

Canonical form

$$\dot{x}^\alpha(\lambda) = \frac{\partial H(x^\alpha, p_\alpha)}{\partial p_\alpha}$$

$$\dot{p}_\alpha(\lambda) = -\frac{\partial H(x^\alpha, p_\alpha)}{\partial x^\alpha}$$

Light wave (Kerr BH metric)

$$A^{\alpha\beta} = g^{\alpha\beta}$$

Sound wave (sound metric)

$$A^{\alpha\beta} = s^{\alpha\beta} = g^{\alpha\beta} - \left(1 - \frac{1}{a_s^2}\right) u^\alpha u^\beta$$

a_s : sound velocity

$$a_s^2 = (\gamma - 1)\frac{h-1}{h}$$

u_α : fluid's 4-velocity

MHD wave (magnetosonic metric) New

$$A^{\alpha\beta} = M^{\alpha\beta} = g^{\alpha\beta} - \left(1 - \frac{1}{V_{\mathrm{MW}}^2}\right) u^\alpha u^\beta$$

V_{MW} : MHD wave velocity

We define the effective potential for waves.

$$V_\pm^{1/2} \equiv \frac{E}{|L|} = \frac{\pm A^{t\phi} + \sqrt{(A^{t\phi})^2 - A^{tt}A^{\phi\phi}}}{A^{tt}}$$

We can discuss the rough spread direction of the wave.

the speeds of the magnetosonic waves & the Alfven wave

$$V_{\mathrm{MW}}^2 = \frac{1}{2}\left(Z \pm \sqrt{Z^2 - 4a_s^2 V_{\mathrm{AW}}^2}\right), \quad Z \equiv (1 - a_s^2)\left[\frac{k|\boldsymbol{h}|^2}{nh + k|\boldsymbol{h}|^2} + C_s^2(1 + V_{\mathrm{AW}}^2)\right]$$

$$V_{\mathrm{AW}}^2 = \frac{kh_n^2}{nh + k|\boldsymbol{h}|^2}$$

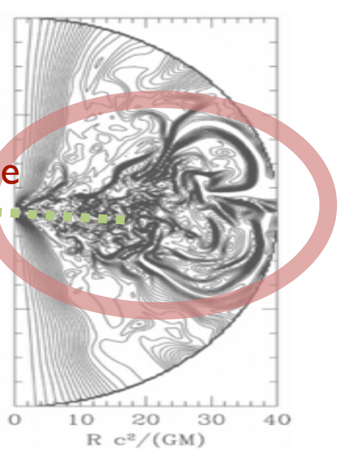
h_n : the magnetic field in spatial direction of propagation of the waves

$$h_n^2 = -\frac{(h^\alpha p_\alpha)^2}{(g^{\alpha\beta} - u^\alpha u^\beta)p_\alpha p_\beta}$$ direction dependence !

↓ **Average**

$$h_n = \frac{1}{\sqrt{2}}|\boldsymbol{h}|$$

turbulence



C. McKinney and F. Gammie *2004*

I check the spread direction of the wave depending on magnetic field strength.

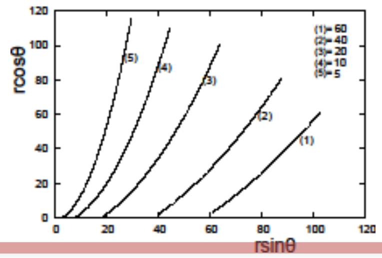
一般相対論的磁場強度分布 （Blandford & Znajek 1977）

Paraboloidal磁場

ベクトルポテンシャル

$$A_\phi = \frac{C}{2}[r(1-\cos\theta)+2M(1+\cos\theta)(1-\ln(1+\cos\theta))]$$

C=2.0M

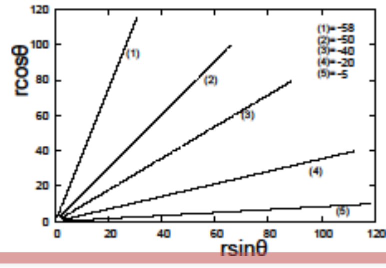


Split-monopole磁場

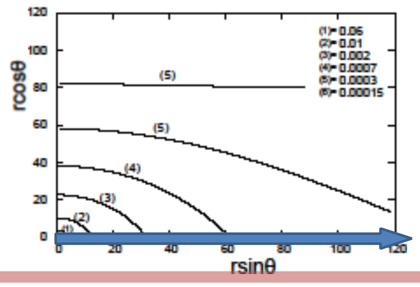
ベクトルポテンシャル

$$A_\phi = -C\cos\theta$$

C=70M

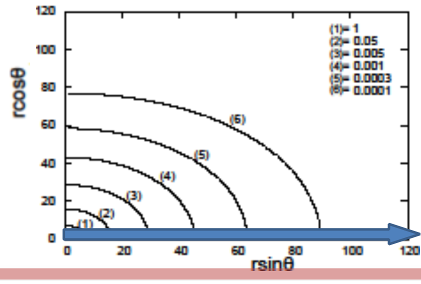


磁場強度分布 $|\boldsymbol{B}|^2 = B_p^2 + B_\phi^2$



$\sim \frac{1}{r}$

磁場強度分布 $|\boldsymbol{B}|^2 = B_p^2 + B_\phi^2$



$\sim \frac{1}{r^3}$

一般相対論的流体分布（Fishbone 1977）

✿ stationary and axisymmetric perfect fluid torus

✿ angular momentum per baryon = constant
energy = constant

Fluid distribution (enthalpy distribution)

$$\frac{h}{mc^2} = \frac{[(\omega-\omega_0)^2 e^{2\nu} - e^{-2\phi}]^{1/2}}{[(\omega-\omega_0)^2 e^{2\nu} - e^{-2\phi}]^{1/2}_{\mathrm{rin},\pi/2}}$$

$$e^{2\nu} = \Sigma\Delta/A \qquad e^{2\psi} = \sin^2\theta A/\Sigma$$

$$A \equiv (r^2+a^2)^2 - a^2\Delta\sin^2\theta$$

$$\Sigma \equiv r^2 + a^2\cos^2\theta$$

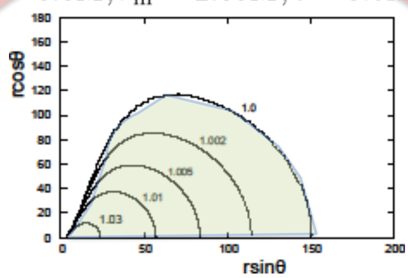
$$\Delta \equiv r^2 - 2Mr + a^2$$

$$\omega = 2Mar/A$$

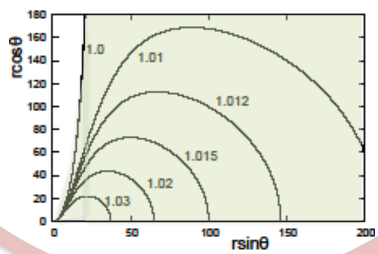
$$\omega_0 = \omega_0(\ell, r_{\mathrm{in}}, \pi/2) = [\pm\{e^{2\nu}(\ell^{-2} + e^{-2\phi})^{1/2}\} + \omega]_{r_{\mathrm{in}},\pi/2}$$

r_{in} ：inner-edge radius

$a = 0.8M, r_{\mathrm{in}} = 2.85M, \ell = 3.0M$



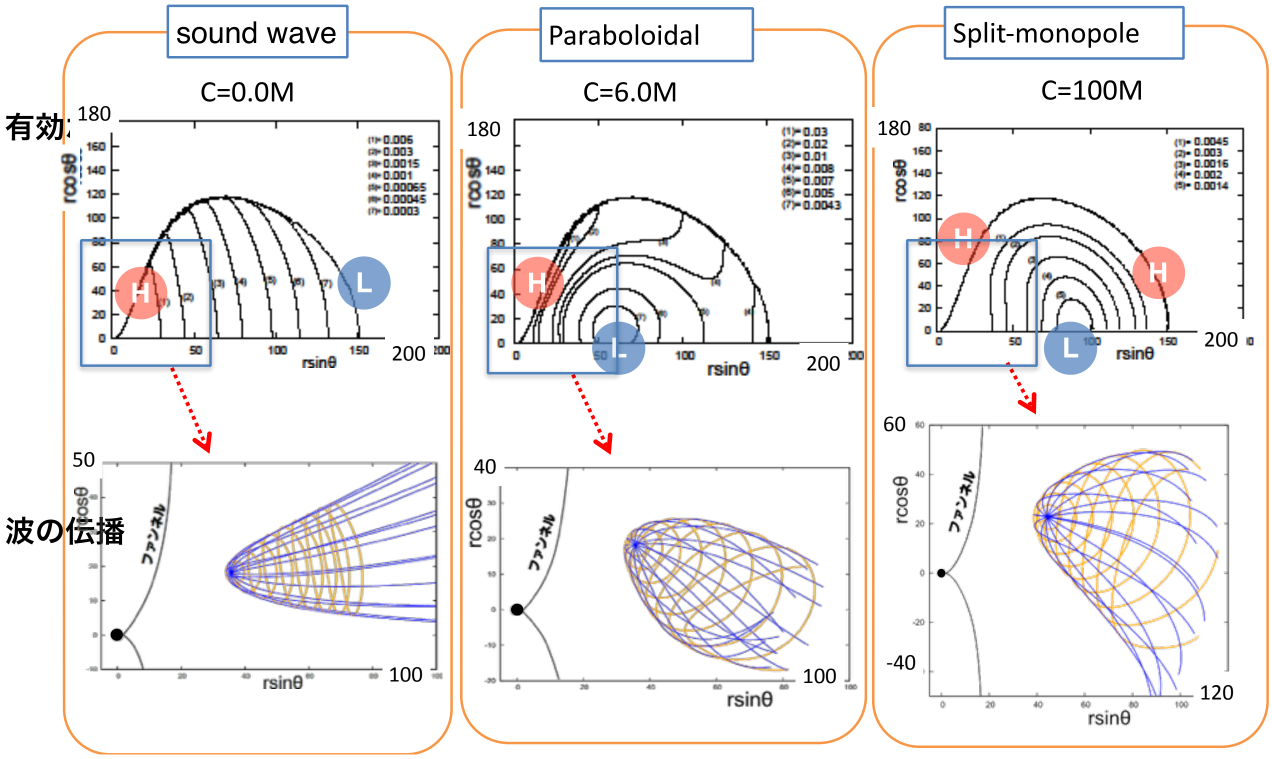
$a = 0.8M, r_{\mathrm{in}} = 2.85M, \ell = 3.06M$



Effective Potential and Wave Propagation （L > 0）

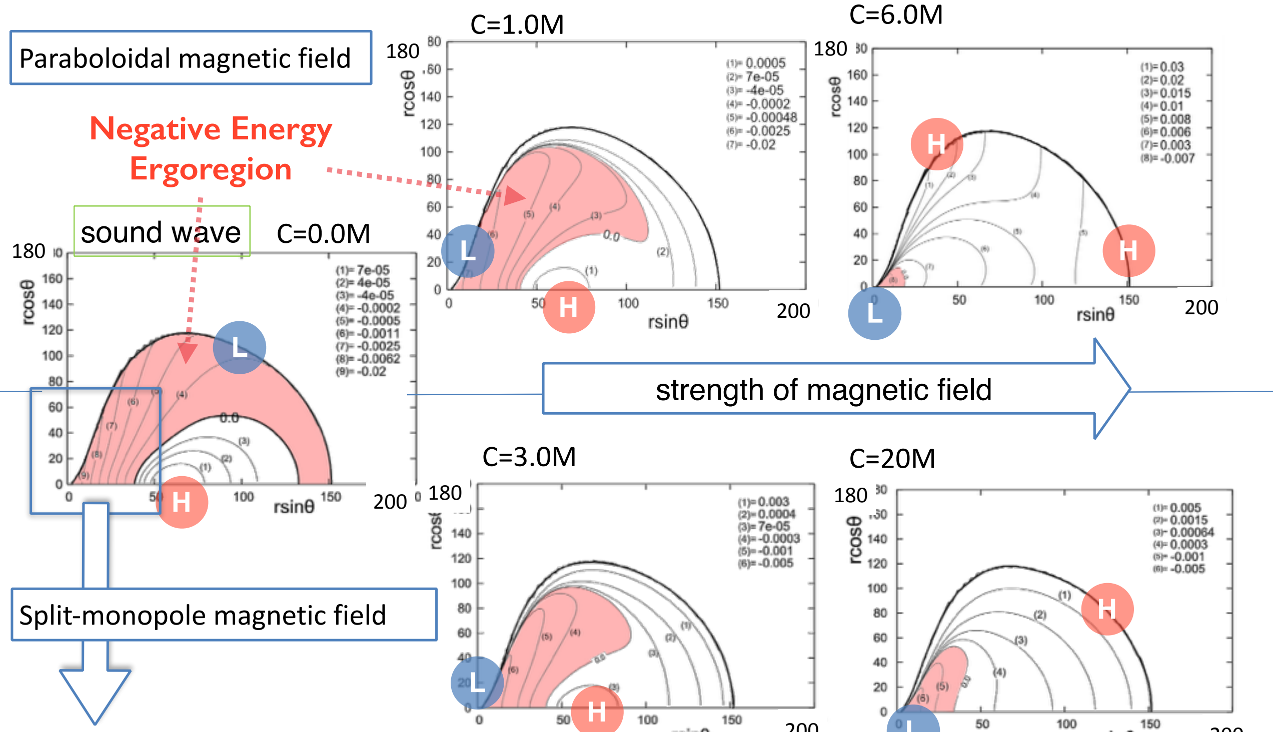
Blue : paths
Yellow : wave fronts

fluid's parameters $a = 0.8M, \ell = 3.0M, r_{\rm in} = 2.85M, \gamma = 4/3$



Effective Potential for MHD wave （L < 0）

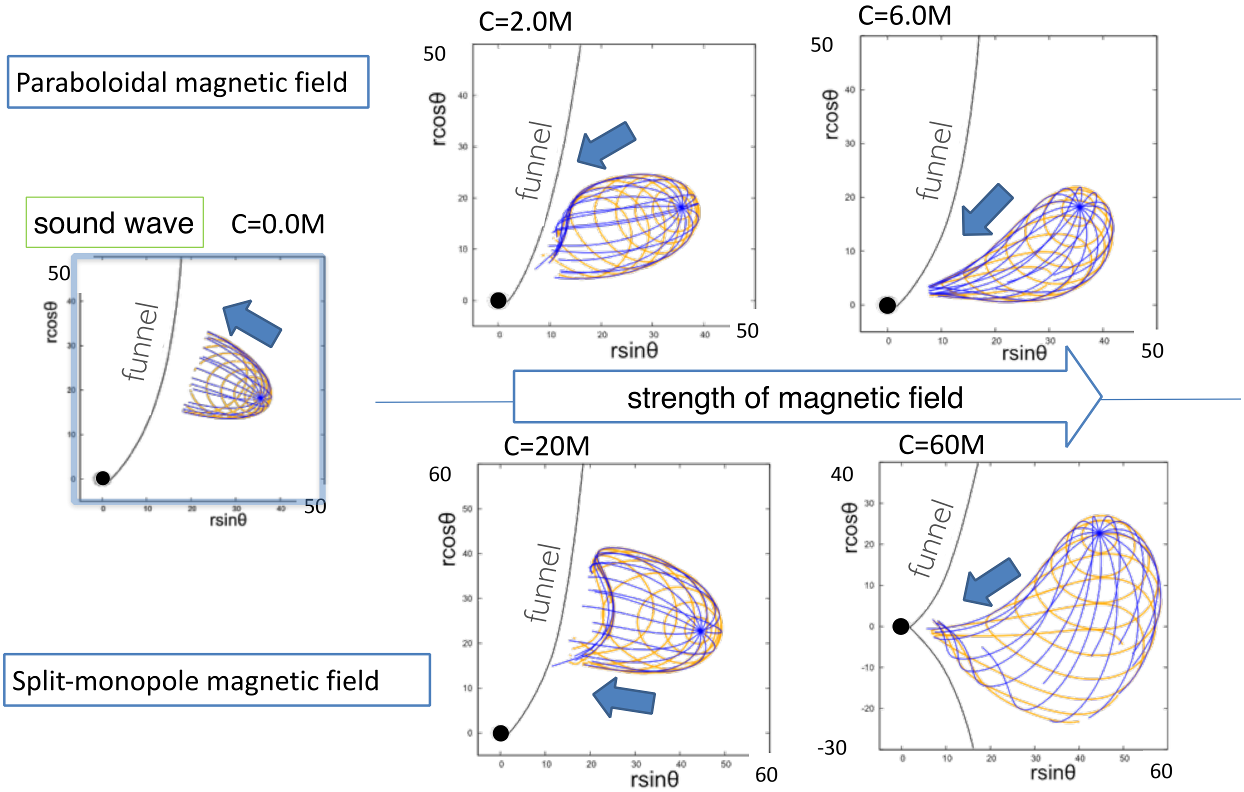
fluid's parameters $a = 0.8M, \ell = 3.0M, r_{\rm in} = 2.85M, \gamma = 4/3$



MHD wave propagation （L < 0）

Blue : paths　Yellow : wave fronts

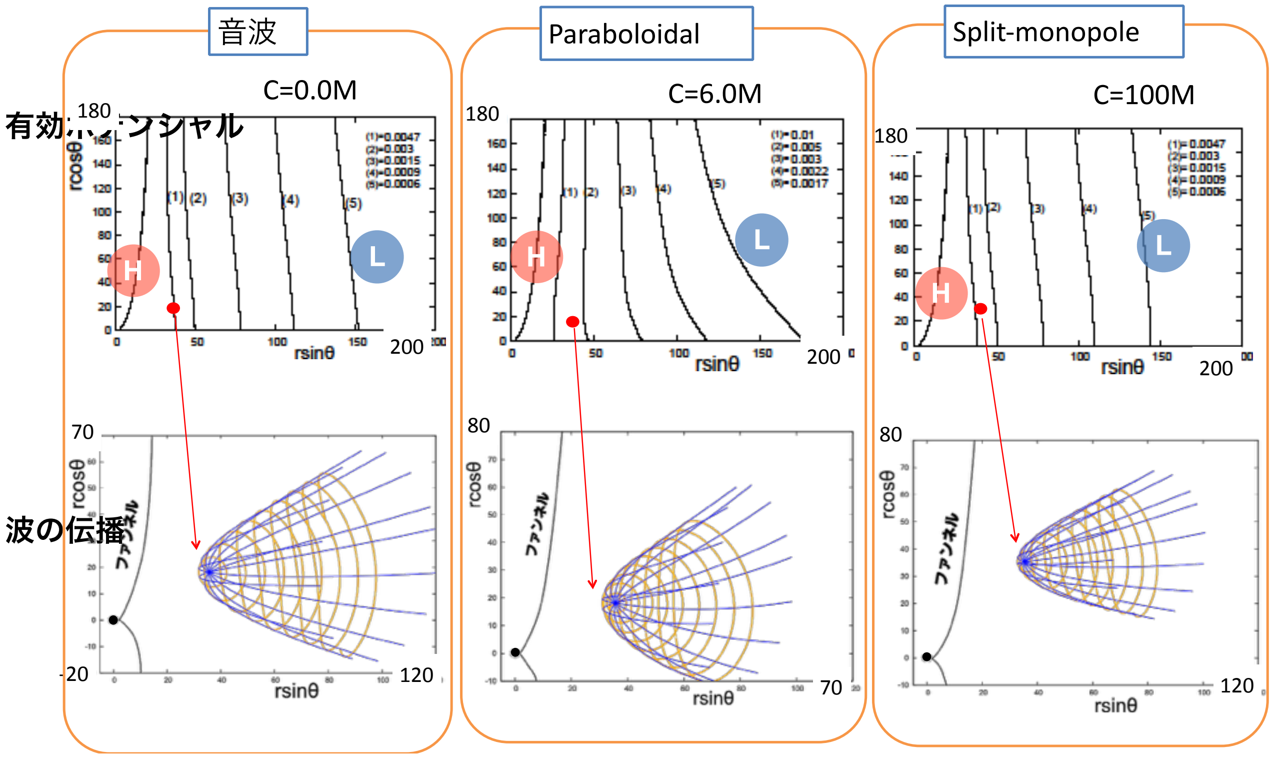
fluid's parameters　$a = 0.8M, \underline{\ell = 3.0M,} r_{\rm in} = 2.85M, \gamma = 4/3$



有効ポテンシャルと波の伝播（順方向：L > 0）

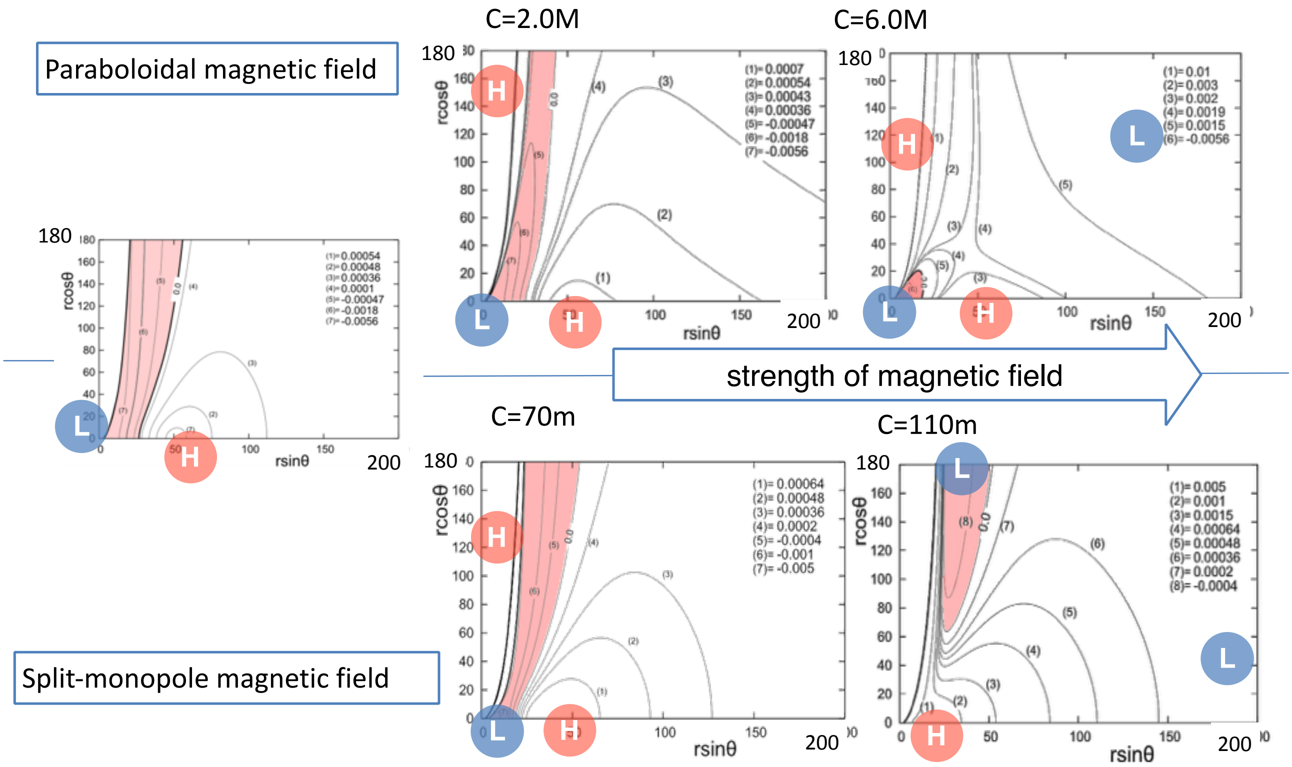
青：射線　橙：波面

流体のパラメータ：$a = 0.8M, \underline{\ell = 3.06M}, r_{\rm in} = 2.85M, \gamma = 4/3$



Effective Potential for MHD wave （L < 0）

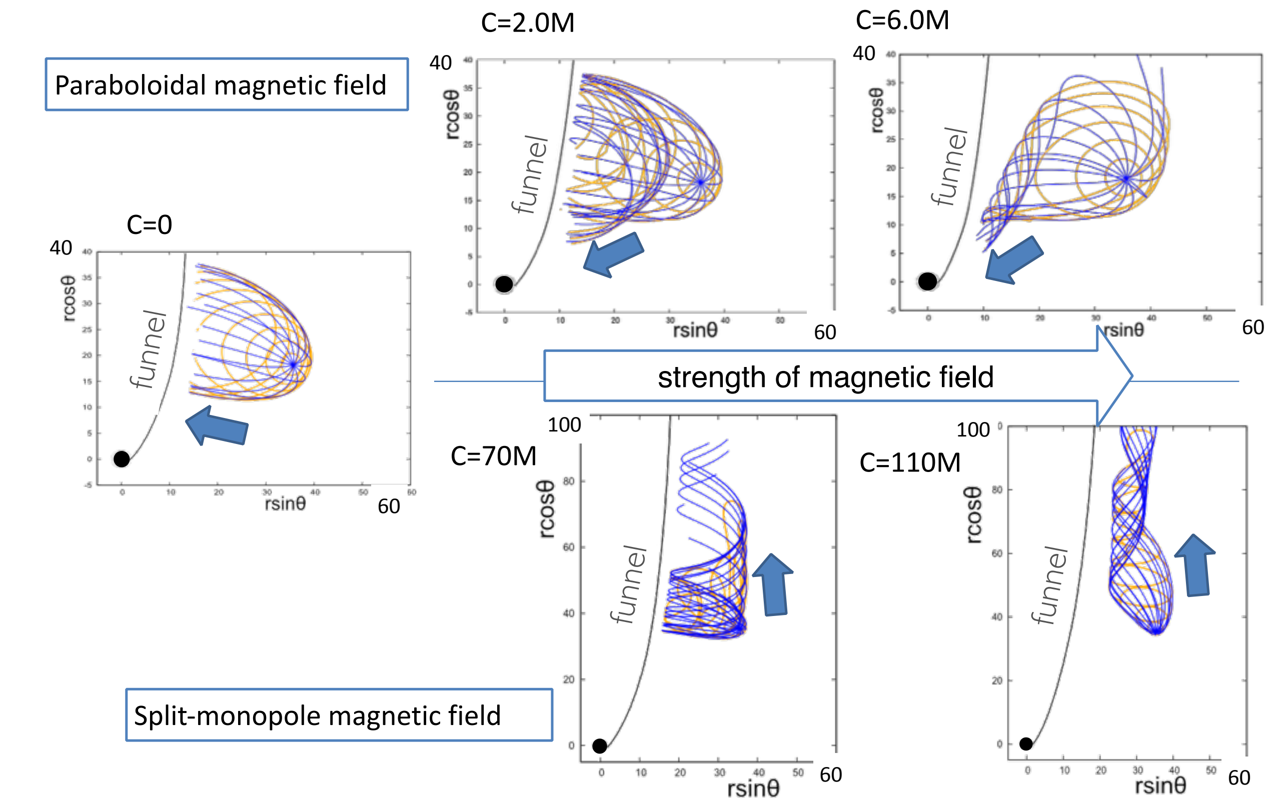
fluid's parameters $a = 0.8M, \underline{\ell = 3.06M}, r_{\rm in} = 2.85M, \gamma = 4/3$



MHD wave propagation （L < 0）

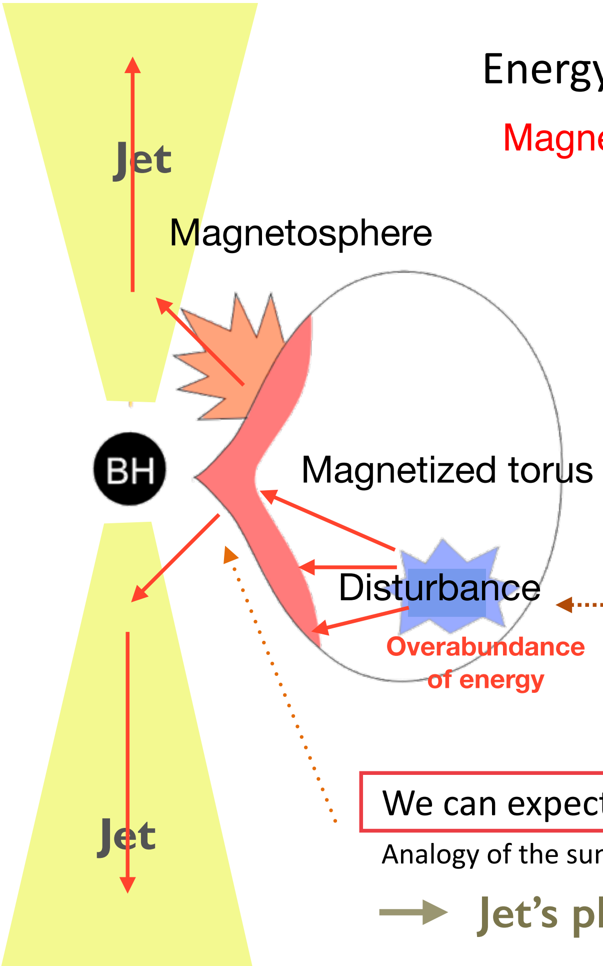
Blue : paths　Yellow : wave fronts

fluid's parameters $a = 0.8M, \underline{\ell = 3.06M}, r_{\rm in} = 2.85M, \gamma = 4/3$



Energy/Plasma supply to the Jet

Magnetohydrodynamical phenomena



Eikonal approximation :
* MHD metric
* Effective Potential for MHD wave
* MHD Ergoretion (E<0)
* MHD Horizon

Counter-rotating wave can transport its energy toward the BH or disk's funnel region.

We can expect Mass ejection / Flare !

Analogy of the sun surface

→ Jet's plasma source (injection surface)

“Cosmological entropy production, perturbations and CMB fluctuations in (1+3+6)-dimensional space-times”

by Kenji Tomita

[JGRG25(2015)P33]

Cosmological entropy production, perturbations and CMB fluctuations in (1+3+6)-dimensional space-times

Kenji Tomita *

Cosmologsical evolution of the 10-dimensional space-times is considered in which the 3-dim. Inflating space section evolves to the Friedmann universe, after the 6-dim. Space section collapses to within the Planck length and decouple. It is shown that significant entropy production and CMB fluctuation may be possible to the observational levels.

* Prof. Emer. YITP, Kyoto Univ., ketomita@ybb.ne.jp

$1. 10-dim. background model

It is assumed that our universe was born as the (1+3+6)-dim space-time and evolved to the Friedmann universe after the decoupling.

$$ds^2 = -dt^2 + r^2(t)\overset{d}{g}_{ij}(x^k)\,dx^i dx^j + R^2(t)\overset{d}{g}_{ab}(X^c)\,dX^a dX^b$$

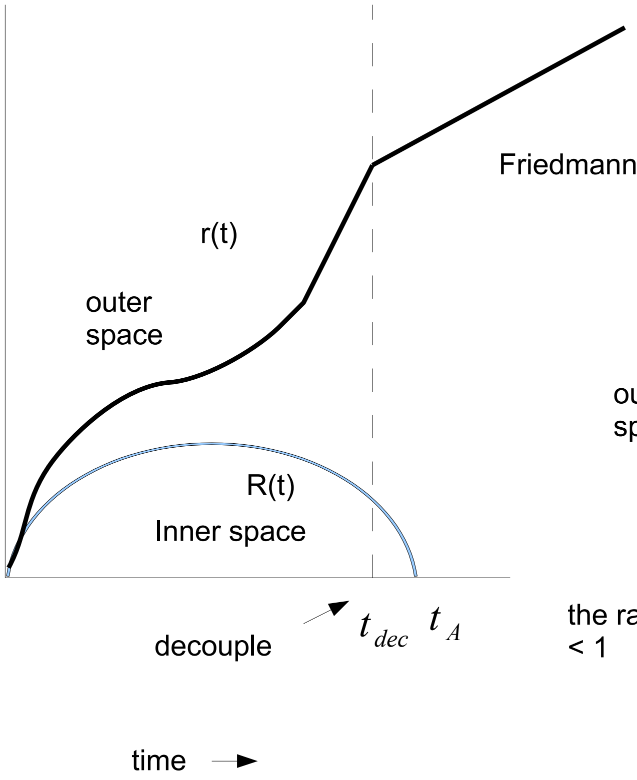
r(t) : scale-factor in 3-dim outer space
R(t) : scale-factor in 6-dim inner space

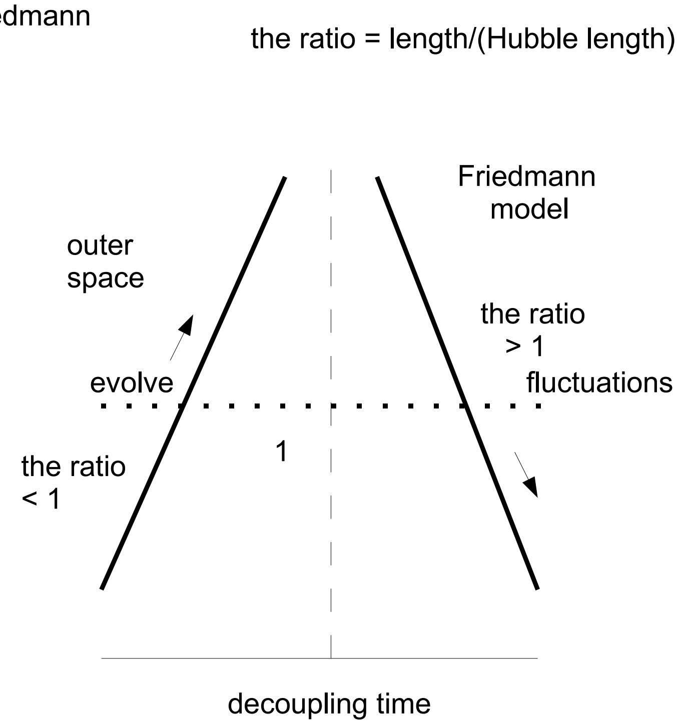
at the initial epoch, $r(t) = R(t)$: isotropic
at the final epoch, $r(t) = r_0(t_A - t)^{-1/3}, R(t) = R_0(t_A - t)^{1/3}$
: highly anisotropic (t_A is the epoch of the singular point)

at the decoupling epoch (near the final epoch),
the radius of the inner space (≈ R) is smaller than the Planck length and lose its Interaction with the outer space (**decoupling**)

after the decoupling epoch, the outer space behaves as the space section of the Friedmann universe.

t**he ratio** = (physical size of perturbations with constant wave- number k) / (the Hubble length 1/H) in the outer space and the Friedmann model





$2. Entropy production (see ref(1))

1. non-viscous case
10-dim. total entropy is conserved

6-dim entropy → **3-dim entropy**
(collapsing section) (inflating section)
decrease **increase**

2. viscous case → **total entropy** increases
Viscosity due to 10-dim gravitational waves (see ref(5))
More **3-dim entropy**

3-dim entropy within the horizon l_h of the 3-dim outer space

$$S_3 = s_3 (l_h)^3$$

where s_3 is 3-dim entropy density

Planck length (in the outer space) r_{pl} is defined by

$$G = [R(t)]^6 [r_{pl}]^2$$

where G is 10-dim gravitational constant.

Near the singularity (R = 0), we can find that there exists an epoch t_{dec} when the following two conditions A and B are satisfied at the same time

Condition A: $\left(S_3\right)_{dec}=10^{88}$ (entropy in Guth level)

Condition B: $R\left(t_{dec}\right)=r_{pl}$ (decoupling in Planck length)

Units c = h = k = 1

This 10-dim model is supported by the present super-string theory in a matrix model (4).

$3. Perturbations

(in the non-viscous case, see ref (2))

3 modes of perturbations

scalar mode SS

vector mode SV, VS, VV

tensor mode ST, TS

ST means (Scalar and Tensor in the outer and inner spaces)

under-bars mean main perturbations

1. Scalar mode SS

two independent gauge-invariant curvature perturbations Φ_h and Φ_H which are caused by perturbations of curvatures in the outer and inner spaces, respectively

$x=k_r\tau/r(t)$ $y=k_R\tau/R(t)$ $(\tau=t_A-t)$

outside the horizon [x << 1 and y <<1] $\Phi_h \propto x^{-2}$ $\Phi_H \propto x^{-1}$

inside the horizons [x >>1 and y >>1] : wavy behavior

$$\mu/x \equiv [(k_R/R(t))/(k_r/r(t))]^2$$

$\mu/x \ll 1$ -> waves in the outer space

$\mu/x \gg 1$ -> waves in the inner space : disappear after decoupling

So we pay attention to the case $\mu/x \ll 1$

2. Tensor mode TS

single gauge-invariant perturbation h_T

for x << 1, $h_T=a+b\ln\tau$ (a, b : const)

for x >> 1, h_T is wavy (Bessel function)

\$4. Fluctuations of CMB appearing in the outer space (in the non-viscous case, see ref(3))

Quantum fluctuations (before the decoupling) which are caused inside the horizon in the outer space under the condition

x >> 1, y >> 1 and $\mu / x \ll 1$

(other fluctuations are disturbed or erased at the decoupling epoch)

Quantization procedure in the Weinberg formalism (Ref. (5))

two curvature perturbations are expressed as

$$\Phi_H = -\frac{1}{3}\tau^{14/9} k_r^{1/2} \exp(ix) + \alpha \tau^{-10/9} k_r^{-3/2} (4r_0/3)^2 \exp(ix/3)$$

$$\Phi_h = \tau^{14/9} k_r^{1/2} \exp(ix) + \alpha \tau^{-10/9} k_r^{-3/2} (4r_0/3)^2 \exp(ix/3)$$

$(\alpha : arbitrary)$

two conserved quantities (which are constant outside horizons in the outer space)

$$R_h = (\tau / \tau_{dec})^{8/3} \Phi_h \qquad R_H = (\tau / \tau_{dec})^{4/3} \Phi_H$$

combination of R_h and R_H ->

single quantity R_{10} (with 1 freedom)

$$R_{10} = R_H [\lambda_0 + \lambda_1 R_h / R_H + \lambda_2 (R_h / R_H)^2]$$

comparison with the CMB observation -> adjustment of the free parameter

\$5. Comparison with other inflationary theories

1. Inflationary models due 4-dim Einstein theory of gravitation with a curvature perturbation as the conserved quantity : slow-roll parameters, e-fold number and coupling parameter are adjusted so as to be consistent with CMB observation

2. 4-dim inflationary models due to 4-dim modeified theory of gravitation with R + R^2 as the action with a curvature perturbation as the conserved quantity : e-fold number is adjusted

   H. Nariai and K. Tomita (1971), A.A. Starobinsky,, (1980)

3. (Present) inflationary model due to 10-dim Einstein theory with two independent curvature perturbations as the conserved quantities (Ref (1), Ref(2), Ref (3)) : one of curvature perturbations is adjusted.

References

Ref(1) K. Tomita, PTEP **2014**, 053E01 (2014)
Ref(2) K. Tomita, PTEP **2014**, 123E01 (2014)
Ref(3) K. Tomita, PTEP in press
Ref(4) S.-W. Kim et al. Phys. Rev. Lett. **108** 011601 (2012) for 10-dim models in the super-string theory (in the matrix model).
Ref(5) K. Tomita and H. Ishihara, Phys. Rev. **D32**, 1935 (1985) for 10-dim gravitational wave viscosity
Ref(6) S. Weinberg, Cosmology (2008), p470.

PTEP = Prog. Theor. Exp. Phys.

Appendix

De Sitter type solution in the modified gravity theory with R + R^2 was first derived by

(1) H. Nariai and K. Tomita
Prog. Theor. Phys. **46**, 776 (1971)

and derived 9 years later similarly again by
(2) A.A. Starobinsky
Phys. Lett. **91B**, 99 (1980).

We hope (1) will be cited together with (2).

“Gravitational reheating after multi-field inflation”

by Jonathan White

[JGRG25(2015)P34]

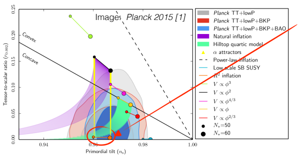
Gravitational reheating after multi-field inflation

Jonathan White KEK, Tsukuba, Japan

Based on Phys. Rev. D 92, 023504 (2015) with Yuki Watanabe NIT, Gunma, Japan

Introduction

- Current CMB data is very precise, allowing us to constrain inflation models:



- **A large class of models with non-minimal gravity sectors are in good agreement with observations**, e.g. R², Higgs, α-attractors

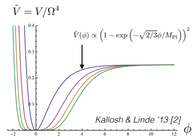
$$S_{f(R)} = \int d^4x\sqrt{-g}f(R) \Rightarrow \int d^4x\sqrt{-g}(\Phi R + ...) \qquad (\Phi = df/dR)$$

$$S_{Higgs} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(M_{pl}{}^2 + \xi h^2\right)R + ...\right]$$

These models are easiest to analyse in the so-called Einstein frame, which we reach by making a conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \Rightarrow \quad S = \int d^4x\sqrt{-\tilde{g}}\left(\frac{M_{pl}^2}{2}\tilde{R} - \frac{1}{2}s(\varphi)(\partial\varphi)^2 - \tilde{V}\right)$$

$$\tilde{V} = V/\Omega^4$$



The potential for the canonically normalised field in the Einstein frame takes a common form for large field values and is ideal for inflation (see figure to the right), and **this large class of models make the same observational predictions in terms of N_*:**

$$n_s = 1 - \frac{2}{N_*} \qquad r = \frac{12}{N_*^2}$$

where N_* is the number of e-folds before the end of inflation that the observable scales left the horizon.

- Despite having the same predictions for n_s and r in terms of N_*, the predictions of a given model depend on the value of N_*. This means that **in order to constrain individual inflation models we need to know how the universe evolves after inflation, including during the reheating epoch [3]:**

$$N_* \approx 67 - \ln\left(\frac{k}{a_0 H_0}\right) + \frac{1}{4}\ln\left(\frac{V_*^2}{M_{Pl}^4\rho_{end}}\right) + \frac{\Delta N_{rh}}{4}(3w_{rh} - 1) - \frac{1}{12}\ln g_{rh}$$

We need:

1. Effective equation of state
2. Duration $\Rightarrow$ we need to determine the inflaton decay rate Γ

- In addition to affecting N_*, **the temperature of the Universe after reheating may also vary from model to model, depending on Γ:**

$$\rho_{rh} = \frac{1}{3}\Gamma^2 M_{Pl}^2 = \frac{\pi^2}{30}g_*(T_{rh})T_{rh}^4$$

- Moreover, in the context of unifying theories it is natural to expect the presence of multiple fields during inflation. **In multi-field inflation** the curvature perturbation ζ is not conserved on super-horizon scales, so **we must follow the evolution of ζ through reheating and until an adiabatic limit is reached [4].**

- In light of the above, we consider reheating in the context of multi-field inflation models with non-minimal coupling, with the general form of action given as:

$$S = \int d^4x\sqrt{-g}\left\{f(\vec{\phi})R - \frac{1}{2}h_{ab}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V\right\} + S_m\left(g_{\mu\nu}, X_m\right)$$

Background dynamics

- To determine the background dynamics of the inflaton fields during reheating we work in the Einstein frame, where the fields are minimally coupled:

$$g_{\mu\nu} = \frac{M_{pl}^2}{f}\tilde{g}_{\mu\nu} \quad \Rightarrow \quad S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_{Pl}^2}{2}\tilde{R} - \frac{1}{2}S_{ab}\tilde{g}^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - \tilde{V}(\vec{\phi})\right] + S_m\left(\frac{M_{Pl}^2}{f(\vec{\phi})}\tilde{g}_{\mu\nu}, X_m\right)$$

$$S_{ab} = \frac{M_{pl}^2}{f}\left(h_{ab} + \frac{3f_a f_b}{2f}\right) \qquad \tilde{V} = \frac{M_{pl}^4 V}{f^2}$$

We assume the fields oscillate about a quadratic minimum of $\tilde{V}(\vec{\phi})$ at the end of inflation and decompose $\phi^a = \phi^a_{vev} + \sigma^a$

Expanding the Einstein frame action to second order in σ^a, introducing the mass eigen-basis $\sigma^a = \alpha^A e^a_A$, considering an FLRW background and **assuming $m_A^2 \gg \tilde{H}^2,\ d\tilde{H}/d\tilde{t}$**

$$\frac{d^2}{d\tilde{t}^2}\left(\tilde{a}^{3/2}\alpha^A\right) + \left[m_A^2 - \left(\frac{9}{4}\tilde{H}^2 + \frac{3}{2}\frac{d\tilde{H}}{d\tilde{t}}\right)\right]\left(\tilde{a}^{3/2}\alpha^A\right) = 0 \quad \Rightarrow \quad \alpha^A \simeq \frac{\alpha_0^A}{\tilde{a}^{3/2}}\cos[m_A\tilde{t} + d_A]$$

$$\tilde{H}^2 = \frac{1}{6M_{Pl}^2}\sum_A\left[\left(\frac{d\alpha^A}{d\tilde{t}}\right)^2 + m_A^2(\alpha^A)^2\right] = \sum_A\frac{(\alpha_0^A)^2 m_A^2}{6M_{Pl}^2\tilde{a}^3}\left(1 + \frac{3\tilde{H}}{2m_A}\sin(2(m_A\tilde{t} + d_A)) + \mathcal{O}(\tilde{H}^2/m_A^2)\right)$$

We thus **find that the Hubble rate in the Einstein frame evolves like that of a matter-dominated Universe i.e. $\omega_{rh} = 0$, and the oscillatory component of $\tilde{H}$ is sub-leading order in $\tilde{H}/m_A$**

- Under the conformal transformation we have: $(\Omega^2 = f(\phi)/M_{pl}^2)$

$$d\tilde{s}^2 = -d\tilde{t}^2 + \tilde{a}^2(\tilde{t})\delta_{ij}d\tilde{x}^i d\tilde{x}^j = \Omega^2 ds^2 = \Omega^2\left(-dt^2 + a^2(t)\delta_{ij}dx^i dx^j\right)$$

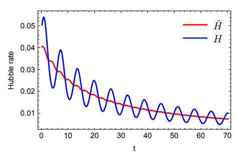
$$\Rightarrow \quad \tilde{a} = \Omega a, \quad d\tilde{t} = \Omega dt, \quad d\tilde{x}^i = dx^i \quad \text{and} \quad \tilde{H} = \frac{1}{\Omega}\left(H + \frac{\dot{\Omega}}{\Omega}\right)$$

Assuming $f|_{vev} = M_{Pl}^2$ and expanding:

$$f = M_{pl}^2\left(1 + \frac{f_A\alpha^A}{M_{Pl}^2} + \frac{1}{2}\frac{f_{AB}\alpha^A\alpha^B}{M_{Pl}^2} + ...\right)$$

$$\Downarrow$$

$$H \simeq \tilde{H}\left(1 + \frac{1}{\tilde{H}}\sum_A\frac{f_A}{2M_{Pl}^2}\frac{\alpha_0^A m_A}{\tilde{a}^{3/2}}\sin\left(m_A\tilde{t} + d_A\right)\right)$$



So **on average, evolution of H in the Jordan frame is like that of a matter-dominated universe, but there is an oscillatory component that is not suppressed**

Gravitational particle production

- Due to the non-minimal gravitational coupling, we **get reheating even in the absence of direct couplings** between $\vec{\phi}$ and matter, i.e. $S_m = S_m\left(g_{\mu\nu}, X_m\right)$

- Let us consider matter in the form of a scalar field, with the action in the Jordan frame given as:

$$S_\chi = \int d^4x\sqrt{-g}\left\{-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi)\right\}$$

In quantising the field we use conformal time $ad\eta = dt$ and define the canonically normalised field $u = a\chi$

$$S_u = \int d\eta d^3x\frac{1}{2}\left[u'^2 - (\nabla u)^2 - \left(a^2m_\chi^2 - \frac{a''}{a}\right)u^2\right] \quad \Rightarrow \quad \begin{array}{l} u_k'' + w_k^2 u_k = 0 \\ w_k^2 = k^2 + a^2 m_\chi^2 - \dfrac{a''}{a} \end{array} \longleftarrow \frac{a''}{a} = a^2(\dot{H} + 2H^2)$$

u has a rapidly time-varying mass due to the oscillating gravitational background, and this gives rise to gravitational particle production. If we take [5]:

$$u_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2w(\eta)}}\exp\left[-i\int_{-\infty}^{\eta}d\eta' w_k(\eta')\right] + \frac{\beta_k(\eta)}{\sqrt{2w(\eta)}}\exp\left[i\int_{-\infty}^{\eta}d\eta' w_k(\eta')\right]$$

then for the equations of motion and commutation relations to be satisfied we require

$$\alpha_k'(\eta) = \frac{w_k'}{2w_k}\exp\left[2i\int_{-\infty}^{\eta}d\eta' w_k(\eta')\right]\beta_k(\eta), \qquad \beta_k'(\eta) = \frac{w_k'}{2w_k}\exp\left[-2i\int_{-\infty}^{\eta}d\eta' w_k(\eta')\right]\alpha_k(\eta) \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

Furthermore, the Hamiltonian is diagonalised, with $E_k = w_k(1/2 + |\beta_k(\eta)|^2)$, so we **interpret $|\beta_k(\eta)|^2$ as the number density of particles**

- **Considering the perturbative regime** where $f_A\alpha^A/(2M_{Pl}^2) \ll 1$, $\beta_k(\eta) \ll 1$ and $\alpha_k(\eta) - 1 \ll 1$ we can evaluate $\beta_k(\eta)$ using the stationary phase approximation. Given that we have *n* oscillating fields, there are *n* stationary points corresponding to when

$$\frac{k^2}{\tilde{a}^2(\eta_k^A)} = \frac{m_A^2}{4}\left(1 - \frac{4m_\chi^2}{m_A^2}\right) \quad \Rightarrow$$

Since $k/\tilde{a}$ corresponds to the momentum of the produced particle, this agrees with what we expect from kinematics

Using our result for $\beta_k(\eta)$ in the continuity equation for ρ_χ we determine:

$$\tilde{\Gamma}_{\alpha^A\rightarrow\chi\chi} = \frac{\left[f_A(2m_\chi^2 + m_A^2)\right]^2}{128\pi M_{pl}^4 m_A}\left(1 - \frac{4m_\chi^2}{m_A^2}\right)^{1/2}$$

- It is almost trivial to see that **the calculation is the same if we start in the Einstein frame as the canonically normalised fields are the same**:

$$\tilde{u} = \tilde{a}\tilde{\chi} \ \ldots\text{but } \tilde{a} = \Omega a \text{ and } \tilde{\chi} = \chi/\Omega \quad \Rightarrow \quad \tilde{u} = u$$

As such, whichever frame we start in, ultimately we need to solve $u_k'' + w_k^2 u_k = 0$. However, **the interpretation in the Jordan and Einstein frames are different:**

Jordan Frame: $w_k^2 = k^2 + a^2m_\chi^2 - \frac{a''}{a}$ $\Rightarrow$ time-dependence of w_k due to oscillatory H

Einstein Frame: $w_k^2 = k^2 + \tilde{a}^2\left(m_\chi^2 - \frac{f_A\alpha^A}{2M_{Pl}^2}(m_A^2 + 2m_\chi^2)\right) - \frac{\tilde{a}''}{\tilde{a}}$ $\Rightarrow$ time-dependence of w_k due to explicit interaction terms: $S_m = S_m(M_{Pl}^2\tilde{g}_{\mu\nu}/f(\vec{\phi}), X_m)$

- We have performed a similar analysis **for fermions**, obtaining:

$$\tilde{\Gamma}_{\alpha^A\rightarrow\bar{\psi}\psi} = \frac{(f_A)^2 m_\psi^2 m_{\hat{A}}}{32\pi M_{Pl}^4}\left(1 - \frac{4m_\psi^2}{m_{\hat{A}}^2}\right)^{3/2}$$

R² and Higgs inflation

- As a simple single-field example, let us consider Starobinsky's R² inflation model, which can be expressed as a scalar-tensor theory:

$$S_S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\left(R + \frac{1}{6M^2}R^2\right) \quad \Leftrightarrow \quad S_S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}\left(1 + \frac{\xi\phi}{M_{Pl}^2}\right)R - \lambda^2\phi^2\right]$$

To see the equivalence: Eom gives $\phi = \frac{\xi R}{4\lambda^2}$. Substitute back into action and take $M^2 = \frac{4\lambda^2}{3\xi^2}M_{Pl}^2$

The relevant quantities are then:

$$f = M_{Pl}^2\left(1 + \frac{\xi\phi}{M_{Pl}^2}\right) \qquad \tilde{V} = \frac{\lambda^2\phi^2}{\left(1 + \frac{\xi\phi}{M_{Pl}^2}\right)^2} \qquad S_{\phi\phi} = \frac{3\xi^2}{2M_{Pl}^2\left(1 + \frac{\xi\phi}{M_{Pl}^2}\right)^2} \qquad \phi_{vev} = 0$$

$$S^{\phi\phi}\tilde{V}_{\phi\phi}|_{vev} = M^2 \qquad \alpha = \sqrt{\frac{3}{2}}\frac{\xi}{M_{Pl}}\phi \qquad f_\alpha = \sqrt{\frac{2}{3}}M_{Pl}$$

- Assuming daughter particles to be light, the dominant decay channel is into scalars:

$$\tilde{\Gamma} \simeq \frac{N_s f_\alpha^2 M^3}{128\pi M_{Pl}^4} \quad \Rightarrow \quad T_{rh} \sim 10^9\text{GeV}$$

Compare this with Higgs inflation, where $T_{rh} \sim 10^{13}$GeV due to non-gravitational coupling [6], i.e. **the reheating temperatures differ by four orders of magnitude,** which could have important consequences

- The number of e-folds before the end of inflation that observable scales left the Horizon is also affected [6]:

$$N_H = 57.66, \qquad N_{R^2} = 54.37. \quad \Rightarrow \quad \begin{array}{r} \text{Higgs-inflation: } n_s = 0.967, \quad r = 0.0032, \\ R^2\text{-inflation: } n_s = 0.965, \quad r = 0.0036. \end{array}$$

The differences are not currently observable.

Summary & Conclusions

- CMB data is now so precise that in order to constrain inflationary models we need to correctly determine how long before the end of inflation observable scales left the horizon. This in turn requires us to know about the post-inflationary evolution of the universe, including reheating.

- Inflation models with non-minimal gravitational coupling are well motivated and observationally favoured, so it is important to study reheating in this class of models. Interestingly, even in the absence of direct couplings, reheating can take place in this class of models due to so-called gravitational particle production.

- We have developed a formulation of multi-field gravitational particle production using the Bogoliubov approach. The analysis can be performed in either the Jordon or Einstein frame, and whilst ultimately equivalent, the interpretation in the Jordan- and Einstein-frames are different:

  Jordan Frame - There are no direct couplings between the inflaton and ordinary matter, but the gravitational background is highly oscillatory and gives rise to particle production.

  Einstein Frame - The gravitational background evolution is that of a matter-dominated universe and can essentially be neglected. However, explicit interaction terms provide the decay channels.

- The curvature perturbation is not conserved on super-horizon scales in the presence of multiple fields, so we need to use our results to investigate how ζ evolves through reheating and until an adiabatic limit is reached.

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“Unruh radiation produced by a uniformly accelerating charged particle

coupled to vacuum fluctuations”

by Kazuhiro Yamamoto

[JGRG25(2015)P35]

Unruh radiation produced by a uniformly accelerating charged particle coupled to vacuum fluctuations *

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Abstract

A particle in a uniformly accelerated motion exhibits Brownian random motions around the classical trajectory due to the coupling to the field vacuum fluctuations. Previous works show that the Brownian random motions satisfy the energy equipartition relation. Because this thermal property is understood as the consequence of the Unruh effect, this quantum radiation is termed Unruh radiation. We investigate the properties of Unruh radiation produced by a uniformly accelerating particle undergoing thermal random motions, which originate from the coupling to the vacuum fluctuations of a massless scalar field as well as an electromagnetic field. The energy flux of Unruh radiation is negative and smaller than that of Larmor radiation by one order in a/m, where a is the constant acceleration and m is the mass of the particle. Thus, the Unruh radiation appears to be a suppression of the classical Larmor radiation. The result is consistent with the previous studies on the quantum effect on the Larmor radiation.

*This presentation is based on the works reported in Phys. Rev. D 92 045027 (2015) and arXiv:1509.03038 by N. Oshita, K. Yamamoto, S. Zhang.

1 Introduction

Phenomena related to quantum fields associated with an event horizon are one of the central problems of theoretical physics. The Hawking effect predicts radiation with a thermal spectrum from a black hole, for which the existence of the event horizon is responsible. The Unruh effect is the theoretical prediction that an accelerating observer sees the Minkowski vacuum as a thermally excited state with the Unruh temperature $T_U = a/2\pi$ as the natural unit, where a is the acceleration. The accelerating observer will perceive a horizon, which is linked to the prediction of the Unruh effect. Therefore, both the Unruh effect and the Hawking effect are rooted in the same physical phenomenon associated with the horizon.

Although direct experimental verification of the Hawking effect seems to be difficult, that of the Unruh effect might be possible. One such argument is initiated the work by Chen and Tajima [1], who proposed a possible detectable signal in the radiation from a charged particle in an accelerated motion, which can be realized in an intense laser field. These studies suggested that the Unruh effect may give rise to Unruh radiation from an accelerating charged particle. However, the problem is not entirely straightforward; it has been argued that the naively expected Unruh radiation from the detector models cancels out due to the interference effect. It has been pointed out that such a cancellation partially occurs in the Unruh radiation produced by a uniformly accelerating particle coupled to vacuum fluctuations [2].

In the present work, we re-investigated the quantum radiation from a uniformly accelerating charged particle coupled to vacuum fluctuations. We investigate two models: One model is consisting of a particle and a massless scalar field and the other model is consisting of a charged particle and an electromagnetic field. It has been shown that random motions of a particle in the transverse direction, perpendicular to the direction of the acceleration, satisfies the energy equipartition relation [2, 3]. Then the quantum radiation from the random motions of particle can be termed Unruh radiation. We verified that although the naively expected Unruh radiation cancels out, but the remaining interference terms may give rise to a unique signature of the Unruh effect contained in the energy flux.

2 Basic Formulas

We first consider the theoretical model consisting of a particle and a massless scalar field [2, 3]. The action of which is given by

$$S = S_{\mathrm{P}}(z) + S_{\phi}(\phi) + S_{\mathrm{int}}(z, \phi),$$

where $S_{\mathrm{P}}(z)$ and $S_{\phi}(\phi)$ are the action for the free particle and field, and $S_{\mathrm{int}}(z, \phi)$ describes the interaction,

$$\begin{aligned}
S_{\mathrm{P}}(z) &= -m \int d\tau \sqrt{\eta_{\mu\nu} \dot{z}^{\mu} \dot{z}^{\nu}}, \\
S_{\phi}(\phi) &= \int d^4x \frac{1}{2} \partial^{\mu}\phi \partial_{\mu}\phi, \\
S_{\mathrm{int}}(z, \phi) &= e \int d\tau d^4x \sqrt{g_{\mu\nu}(x) \dot{z}^{\mu} \dot{z}^{\nu}} \phi(x) \delta^4 \left(x - z(\tau)\right),
\end{aligned}$$

where e is the charge of the particle. Note that $x^{\mu} = z^{\mu}(\tau)$ denotes the trajectory of a particle, which obeys

$$m\ddot{z}^{\mu} = e \left(\ddot{z}^{\mu}\phi + \dot{z}^{\mu}\dot{z}^{\alpha} \frac{\partial \phi}{\partial x^{\alpha}} - \eta^{\mu\alpha} \frac{\partial \phi}{\partial x^{\alpha}} \right) \bigg|_{x=z(\tau)} + F^{\mu},$$

where F^{μ} is a force for a uniformly accelerated motion, while the equation of motion for the scalar field is

$$\partial^{\mu}\partial_{\mu}\phi(x) = e \int d\tau \sqrt{\eta_{\mu\nu} \dot{z}^{\mu} \dot{z}^{\nu}} \delta^4(x - z(\tau)).$$

The field equation has the solution,

$$\phi(x) = \phi_{\mathrm{h}}(x) + \phi_{\mathrm{inh}}(x),$$

where $\phi_{\rm h}$ and $\phi_{\rm inh}$ are the homogeneous solution and the inhomogeneous solution, respectively. The homogeneous solution satisfies $\partial^\mu\partial_\mu\phi_{\rm h}=0$, which we regard as the quantized vacuum field, while the inhomogeneous solution is written as

$$\begin{aligned}\phi_{\rm inh}(x) &= \int d^4x' G_R(x,x') e \int d\tau' \sqrt{\eta_{\mu\nu}\dot{z}^\mu\dot{z}^\nu}\,\delta^4(x'-z(\tau')) \\ &= e\int^\tau d\tau' G_R(x,z(\tau')),\end{aligned}$$

where $G_R(x,y)$ denotes the retarded Green function satisfying $\partial^\mu\partial_\mu G_R(x)=\delta^4(x)$. The term of the inhomogeneous solution $\phi_{\rm inh}$ gives rise to a radiation reaction force, and we have the stochastic equation of motion,

$$m\ddot{z}^\mu = \frac{e^2}{12\pi}\left(\dddot{z}^\mu + \dot{z}^\mu\left(\ddot{z}\right)^2\right) + e\left(\ddot{z}^\mu\phi_{\rm h} + \dot{z}^\mu\dot{z}^\alpha\frac{\partial\phi_{\rm h}}{\partial x^\alpha} - \eta^{\mu\alpha}\frac{\partial\phi_{\rm h}}{\partial x^\alpha}\right)\bigg|_{x=z(\tau)} + F^\mu.$$

We consider a particle in an accelerated motion with a uniform acceleration a in the absence of the coupling to the quantum field. The equation of motion for random motions around the classical motion is solved by using the following perturbative method. Assuming that the trajectory of a particle is written as

$$z^\mu = \bar{z}^\mu + \delta z^\mu,$$

where $\bar{z}^\mu = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$ describes the classical trajectory with a uniformly acceleration, and δz^μ does the random motion due to the coupling to the quantum field.

Since the *transverse* motions satisfy the energy equipartition relation, then we consider the perturbative equation of motion for the transverse fluctuations [2],

$$m\delta\ddot{z}^i = \frac{e^2}{12\pi}(\delta\dddot{z}^i - a^2\delta\dot{z}^i) + e\frac{\partial\phi_{\rm h}}{\partial x^i}\bigg|_{x=z(\tau)}.$$

The thermal property of the random motions, which are obtained as solutions of this equation, has been demonstrated in Ref. [2, 3]. In the present work, for simplicity, we drop the third-order time derivative term of the radiation reaction force. The contribution of this term to the solution of δz^i is small, which is suppressed by the order of $\mathcal{O}\big((a/m)^2\big)$. Now we have

$$m\delta\ddot{z}^i = -\frac{e^2a^2}{12\pi}\delta\dot{z}^i + e\frac{\partial\phi_{\rm h}}{\partial x^i}\bigg|_{x=z(\tau)}.$$

The solution of the above equation is written as

$$\delta\dot{z}^i(\tau) = \int d\omega \frac{\partial_i\varphi(\omega)}{a\sigma - i\omega}e^{-i\omega\tau},$$

where we defined

$$\sigma = \frac{e^2a}{12\pi m},$$
$$\partial_i\varphi(\omega) = \int \partial_i\phi_{\rm h}(z(\tau))e^{i\omega\tau}d\tau.$$

We can demonstrate that the solution satisfies the energy equipartition relation with the Unruh temperature $T_U = a/2\pi$ [2],

$$\frac{m}{2}\big\langle\delta\dot{z}^i(\tau)\delta\dot{z}^j(\tau)\big\rangle = \frac{\delta_{ij}}{2}\frac{a}{2\pi}\left(1+\mathcal{O}\left(\frac{a^2}{m^2}\right)\right)$$

by using the Weightman function

$$\big\langle\phi_{\rm h}(x)\phi_{\rm h}(y)\big\rangle = -\frac{1}{4\pi^2}\frac{1}{(x^0-y^0-i\epsilon)^2-(\mathbf{x}-\mathbf{y})^2}.$$

Thus the random motions of a particle exhibit the thermal property that the transverse motions satisfy the energy equipartition relation. Therefore, we expect that the quantum radiation from the random motions of a particle can be investigated if it existed.

Using the expression of the retarded Green function for the massless scalar field, $G_R(x-y) = \theta(x^0-y^0)\delta_D((x-y)^2)/2\pi$, where $\delta_D(z)$ denotes the Dirac delta function, we have

$$\phi_{\rm inh}(x) = e\int d\tau G_R(x-z(\tau)) = \frac{e}{4\pi\rho(x)}$$

with

$$\rho(x) = \dot{z}_\mu(\tau_-^x)(x^\mu - z^\mu(\tau_-^x)),$$

where τ_-^x is the solution of $(x - z(\tau_-^x))^2 = 0$. Up to the first order of perturbations, $z^\mu = \bar{z}^\mu + \delta z^\mu$, the inhomogeneous solution is given by

$$\phi_{\rm inh}(x) \simeq \frac{e}{4\pi\rho_0(x)}\left(1 - \frac{\delta\rho(x)}{\rho_0(x)}\right).$$

where we defined

$$\rho_0(x) = \dot{\bar{z}}(\tau_-^x)\cdot(x - \bar{z}(\tau_-^x)), \qquad \delta\rho(x) \simeq \delta\dot{z}(\tau_-^x)\cdot(x - \bar{z}(\tau_-^x)),$$

where τ_-^x is redefined to satisfy $(x - \bar{z}(\tau_-^x))^2 = 0$. Here, we also introduced τ_+^x, which satisfies $(x - \bar{z}(\tau_+^x))^2 = 0$. The meaning of τ_-^x and τ_+^x is explained in Figure 1.

3 Energy Momentum Tensor

It is straightforward to evaluate the two-point function;

$$\begin{aligned}&\langle\phi(x)\phi(y)\rangle - \langle\phi_{\rm h}(x)\phi_{\rm h}(y)\rangle \\ &\qquad = \langle\phi_{\rm inh}(x)\phi_{\rm h}(y)\rangle + \langle\phi_{\rm h}(x)\phi_{\rm inh}(y)\rangle + \langle\phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle,\end{aligned}$$

the explicit expression for the symmetrized two-point function with respect to x and y is

$$\begin{aligned}[\langle\phi(x)\phi(y)\rangle - \langle\phi_{\rm h}(x)\phi_{\rm h}(y)\rangle]_S &= \frac{e^2}{(4\pi)^2}\frac{1}{\rho_0(x)\rho_0(y)} + \frac{-iae^2}{2m(4\pi)^2}\frac{x^i}{\rho_0^2(x)}\frac{y^i}{\rho_0^2(x)} \\ &\times\left[\frac{aL_x^2}{2\rho_0(x)}(I_3(x,y) - I_1(x,y)) + \frac{i}{a}I_2(x,y)\right] + (x \leftrightarrow y),\end{aligned}$$

where we defined

$$\begin{aligned}I_1(x,y) &= -\frac{i}{2\pi\sigma} + \frac{i}{\pi}\log(1 + e^{-a|\tau_-^y - \tau_+^x|}) + \frac{i}{\pi}a(\tau_-^y - \tau_+^x)\theta(\tau_-^y - \tau_+^x) + \mathcal{O}(\sigma), \\ I_2(x,y) &= -\frac{a}{\pi}\frac{1}{e^{a(\tau_+^x - \tau_-^y)} + 1} + \mathcal{O}(\sigma), \\ I_3(x,y) &= -\frac{i}{2\pi\sigma} + \frac{i}{\pi}\log(1 - e^{-a|\tau_-^y - \tau_-^x|}) + \frac{i}{\pi}a(\tau_-^y - \tau_-^x)\theta(\tau_-^y - \tau_-^x) + \mathcal{O}(\sigma),\end{aligned}$$

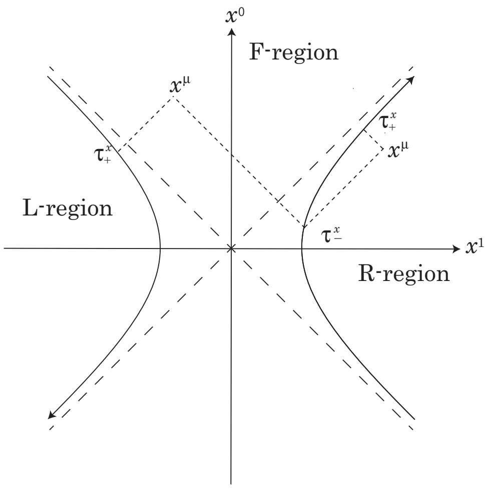

Figure 1: The hyperbolic curve in the R-region is the trajectory of a uniformly accelerating particle. The hyperbolic curve in the L-region is the hypothetical trajectory obtained by an analytic continuation of the true trajectory. For an observer at point x^μ in the R-region, τ_-^x is defined by the proper time of the particle's trajectory intersecting with the past light cone, while τ_+^x is similarly defined with the future light cone when x^μ is in the R-region. For an observer in the F-region, τ_+^x is the proper time of the hypothetical trajectory in the L-region intersecting with the past light cone.

up to the order of $\mathcal{O}(\sigma)$, for the F-region $x^0 > |x^1|$ (figure 1).

The energy flux can be computed as follows. Assuming that the energy flux is observed far from the particle, i.e., $r \gg z^1(\tau_-^x) > 1/a$, at the leading order of $1/r^2$ and σ, we have

$$\begin{aligned} T_{0i}(x) &= \lim_{y \to x} \frac{\partial}{\partial x^0} \frac{\partial}{\partial y^i} [\langle \phi(x)\phi(y) \rangle - \langle \phi_{\mathrm{h}}(x)\phi_{\mathrm{h}}(y) \rangle]_S \\ &= T_{0i}^{\mathrm{C}} + T_{0i}^{\mathrm{Q}}, \end{aligned}$$

where T_{0i}^{C} and T_{0i}^{Q} are the classical part and the quantum part, respectively. The energy flux in the laboratory frame is related to the energy momentum tensor by $f = -T_{0i}n^i$ with $n^i = x^i/r$. Here we consider the energy flux in the F-region. The energy flux for the classical part and the quantum part are given by

$$f^{\mathrm{C}} = \frac{1}{r^2}\frac{a^2e^2}{(4\pi)^2}\frac{G(q)}{\sin^4\theta}\theta(t-x^1),$$
$$f^{\mathrm{Q}} = \frac{1}{r^2}\frac{2a^3e^2}{(4\pi)^3 m}\frac{F(q)}{\sin^4\theta}\theta(t-x^1),$$

respectively, where $G(q)$ and $F(q)$ are defined as

$$\begin{aligned}
G(q) &= \frac{q^2}{(1+q^2)^3},\\
F(q) &= \frac{1}{(1+q^2)^3}\Bigg[-\frac{4q(2q^2-1)}{\sqrt{1+q^2}^3}\Bigg\{\log a\varepsilon - \log\Big(1+e^{-a|\tau_- - \tau_+|}\Big)\\
&\quad - a(\tau_- - \tau_+)\theta(\tau_- - \tau_+)\Bigg\} - \frac{2(8q^2-1)}{(1+q^2)(e^{a(\tau_+ - \tau_-)}+1)} - \frac{1}{1+q^2}\\
&\quad - \frac{q}{\sqrt{1+q^2}}\frac{2}{(a\varepsilon)^2} + \frac{q}{\sqrt{1+q^2}}\frac{5}{2}\frac{1}{\cosh^2(a(\tau_+ - \tau_-)/2)} - \frac{1}{2}\frac{\tanh(a(\tau_+ - \tau_-)/2)}{\cosh^2(a(\tau_+ - \tau_-)/2)}\Bigg]
\end{aligned}$$

with

$$q(t,r,\theta) = \frac{a}{\sin\theta}\left(t - r - \frac{1}{2a^2 r}\right) \sim \frac{a}{\sin\theta}\left(t-r\right),$$
$$a(\tau_+ - \tau_-) = \log\left[\frac{-q+\sqrt{1+q^2}}{+q+\sqrt{1+q^2}}\right],$$

for the F-region. Note that f^{Q} is smaller than the classical part f^{C} by the order of a/m and f^{Q} includes the divergent terms in the coincidence limit $\varepsilon \to 0$. We may understand that this divergence comes from the short-distance motion of a particle, originated from our formulation based on the point particle. The divergence coming from the short-distance motion of the particle could be removed by taking a finite size effect of the particle into account. Here, we simply omit the divergent terms.

The left panel of figure 2 shows the angular distribution of the classical energy flux and the quantum energy flux with fixing $\tau_-^x = 0$. The blue dotted curve is $\sin^{-4}\theta G(q(\tau_-^x, \theta))$, while the black solid curve (red dashed curve) is positive (negative) values of $\sin^{-4}\theta F(\tau_-^x, \theta)$. This polar plot shows the energy flux emitted in the direction of θ from the particle at the proper time $\tau = 0$.

The classical energy flux has the radiation power in the direction of acceleration, which is the consequence of the scalar field model. The quantum energy flux is negative in almost direction, which is described by the red curve. But it does not mean that one should observe a negative energy flux from an accelerated particle. Only the sum of the classical and the quantum flux is observed. The total energy flux is positive as long as $a/m \ll 1$.

4 Model of a particle and electromagnetic field

We have repeated the same investigation for the model consisting a particle and an electromagnetic field, the action of which is given by

$$S = S_{\mathrm{P}}(z) + S_{\mathrm{EM}}(A) + S_{\mathrm{int}}(z, A),$$

where $S_{\mathrm{EM}}(A)$ and $S_{\mathrm{int}}(z, A)$ are defined by

$$\begin{aligned} S_{\mathrm{EM}}(A) &= -\frac{1}{4}\int d^4x F^{\mu\nu}F_{\mu\nu}, \\ S_{\mathrm{int}}(z, \phi) &= -e\int d\tau \int d^4x \delta_D^4\left(x - z(\tau)\right) \dot{z}^\mu(\tau) A_\mu(x), \end{aligned}$$

and $F_{\mu\nu}(= \partial_\mu A_\nu - \partial_\nu A_\mu)$ is the field strength. We found the similar results to the case of the scalar field model. The expression of the energy flux is given by replacing the function $G(q)$ and $F(q)$ with

$$\begin{aligned} G(q) &= \frac{1}{(1+q^2)^3}, \\ F(q) &= \frac{1}{(1+q^2)^3}\left[\frac{6q(q^2-1)}{\sqrt{1+q^2}^3}\left\{\log a\varepsilon - \log(1 + e^{-a|\tau_- - \tau_+|})\right.\right. \\ &\quad \left. - a(\tau_- - \tau_+)\theta(\tau_- - \tau_+)\right\} + \frac{2}{(a\varepsilon)^2}\frac{q}{\sqrt{1+q^2}} \\ &\quad \left. + 2\frac{(3 - e^{a(\tau_+ - \tau_-)})(2 - e^{a(\tau_+ - \tau_-)}(9 - e^{a(\tau_+ - \tau_-)}))}{(1 + e^{a(\tau_+ - \tau_-)})^3}\right]. \end{aligned}$$

As in the case of the massless scalar field, the quantum part f^Q is smaller than the classical counterpart f^C by one order in a/m. The right panel of figure 2 shows the angular distribution of the classical energy flux and the quantum energy flux with fixing $\tau = 0$. The classical energy flux f^C of the Larmor radiation is dominantly emitted perpendicular to the direction of acceleration. The Unruh radiation flux is almost entirely negative. The emission directions in the dominant regions are similar to those of the classical radiation. This is understood as the suppression of the Larmor radiation due to the quantum effect, which is consistent with the predictions of the model based on a particle and a massless scalar field.

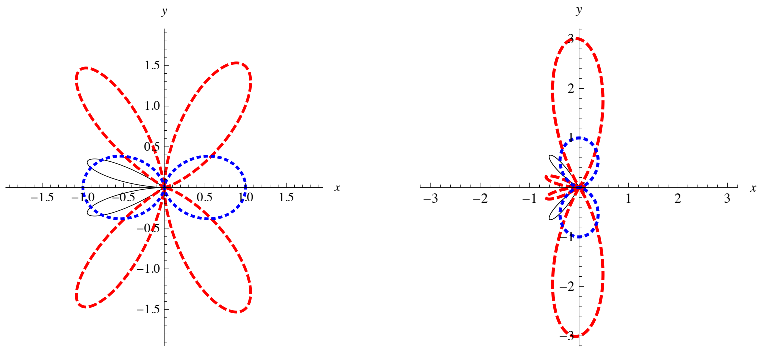

Figure 2: Angular distribution of the classical radiation $\sin^{-4}\theta G(\tau_-^x, \theta)$ (blue dotted curve) and the Unruh radiation $\sin^{-4}\theta F(\tau_-^x, \theta)$ (black solid: positive values; red dashed curve: negative values) at $a\tau_-^x = 0$. The coordinates x and y are x^1 and $\sqrt{(x^2)^2 + (x^3)^2}$, respectively. The **left** panel is the **massless scalar field** model, while the **right** panel is the **electromagnetic field** model.

5 Conclusions

We have scrutinized the theoretical features of the energy flux of the quantum fields coupled to the random thermal motions of an accelerated particle, where we focused on transverse motions in the direction perpendicular to the acceleration of the particle, which are demonstrated to exhibit the energy equipartition relation. Within our model, the energy flux of the radiation is obtained as the sum of the classical part and the quantum part. The quantum part can be considered as the quantum radiation coming from the random thermal motions around a uniformly accelerated motion. The energy flux of the quantum part is smaller than the classical part by the order of a/m, and the angular distribution can be a unique signature of the Unruh effect. However, the sign of the energy flux of the quantum part is almost negative. The results can be understood as a suppression of the total radiation flux by the quantum effect. These results are the common features for the scalar field model and the electromagnetic field model. This conclusion is consistent with the previous works [6, 7], which demonstrated that the quantum correction to the Larmor radiation suppresses the total radiation.

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“Compact Objects in dRGT Massive Gravity”

by Masashi Yamazaki

[JGRG25(2015)P36]

Compact Objects in dRGT Massive Gravity

Masashi Yamazaki (Nagoya University QG Lab.)

arXiv:1512.00660v1 [gr-qc]

In collaboration with T. Katsuragawa, S. Nojiri and S.D. Odintsov

INTRODUCTION

To explain the current expansion of the universe **without cosmological constant**, we can consider non-minimal extensions of the general relativity.
The de Rham-Gabadadze-Tolley (dRGT) massive gravity is one of such extensions and ghost-free theories with interacting massive spin-2 field.
If we regard the dRGT massive gravity as an alternative theory of gravity, it is interesting that we apply this theory to not only the accelerated expansion of the universe but also **astrophysical phenomena**.
Then it is significant to study the **both** of cosmological and astrophysical models and compare them with observational data.
In particular, it is very difficult to construct the general framework which quantifies the deviations from the prediction of the general relativity **in strong-gravity field** because the non-perturbative effects depends on the detail of each theories and parametric treatment is not suitable.
Therefore it is indispensable to study the compact objects in the dRGT massive gravity for a theoretical test of the modified gravity in strong-gravity regime.

de Rham-Gabadadze-Tolley (dRGT) MASSIVE GRAVITY

The action and equations of motion of the dRGT massive gravity are given as follows :

$$S_{\text{dRGT}} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\det(g)}\left[R - 2m_0^2\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)\right] + S_{\text{matter}}$$

$$\to G_{\mu\nu} + m_0^2 I_{\mu\nu} = \kappa^2 T_{\mu\nu},\ \nabla_\mu I^{\mu\nu} = 0$$

- $g_{\mu\nu},\ f_{\mu\nu}$: dynamical and reference metrics
- $\kappa^2 = 8\pi G$: gravitational coupling
- $\beta_n,\ m_0$(graviton mass) : free parameters
- $e_n(X)$: some polynomials of the eigenvalues of X

We set $m_0^2 = \Lambda$ so that the model could explain the expansion of the universe and has the compatibility of astrophysical and cosmological applications.
And for convenience, we choose a minimal model for other free parameters and reference metric is taken as the Minkowski spacetime.
Of cource we use static and spherical dynamical metric ansatz for considering compact stars.

MODIFIED TOV EQUATIONS

For numerical analysis, we use the dimensionless variables.
And we obtain modified TOV equations and constraints :

$$r_g \equiv GM_\odot,\ r \to rr_g,$$
$$M \to mM_\odot,\ m_0 \to \alpha M_\odot,$$
$$\rho \to \tilde{\rho}(M_\odot/r_g^3),\ P \to p(M_\odot/r_g^3)$$

$$\nabla_\mu I^{\mu\nu} = 0 \Leftrightarrow \left(1 - \frac{2m(r)}{r}\right)^{1/2} = \left(1 - \frac{1}{2}\frac{p'(r)}{p(r)+\tilde{\rho}(r)}r\right)^{-1}$$

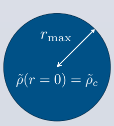
$$q(r) \equiv \frac{p'(r)}{p(r)+\tilde{\rho}(r)},$$

$$m'(r) = 4\pi\tilde{\rho}(r)r^2 + \frac{1}{2}\alpha^2\left(r_g M_\odot\right)^2 r^2\left[1 - \left(1 - \frac{1}{2}q(r)r\right)^{-1}\right],$$

$$\begin{aligned}
&8\pi p(r)q(r)r^3\left(1 - \frac{1}{2}q(r)r\right)^3 \\
&= q(r)r\left(1 - \frac{1}{2}q(r)r\right)(1 - 2q(r)r) - q(r)r\left(1 - \frac{1}{2}q(r)r\right)^3 \\
&+ \alpha^2\left(r_g M_\odot\right)^2 q(r)r^3\left(1 - \frac{1}{2}q(r)r\right)^3 + 8\pi p'(r)r^3\left(1 - \frac{1}{2}q(r)r\right)^3 \\
&+ 2\left(1 - \frac{1}{2}q(r)r\right)(1 - 2q(r)r) - r\left(q'(r)r + q(r)\right)(1 - 2q(r)r) \\
&+ 2r\left(1 - \frac{1}{2}q(r)r\right)\left(q'(r)r + q(r)\right) - 2\left(1 - \frac{1}{2}q(r)r\right)^3
\end{aligned}$$

Our numerical caluculation methods are as follows :

$p(\tilde{\rho}(r_{\max})) = 0$



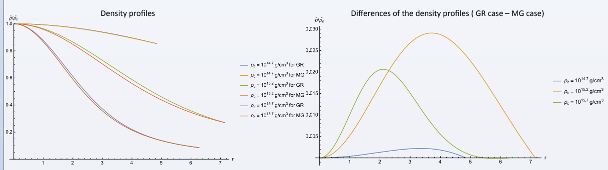
1. Impose an equation of state: $p(r) = p(\tilde{\rho}(r)) \Rightarrow q(r) = q(\tilde{\rho}(r))$.
2. Solve the last equation as $\tilde{\rho}(r)$'s 2nd order ODE from the center of stars $r = 0$ to the surface of stars $r = r_{\max}$ s.t. $p(\tilde{\rho}(r_{\max})) = 0$.
3. Choose the initial value $p''(r=0)$ so that the radius of star becomes identical with that in the GR.
4. Integrate the $m(r)$'s 1st order ODE using the solution of $\tilde{\rho}(r)$.

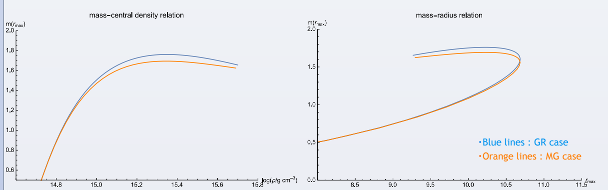
NUMERICAL ANALYSIS

◎ Quark star cases (MIT bag model)

$$P(\rho(r)) = c(\rho(r) - 4B)$$

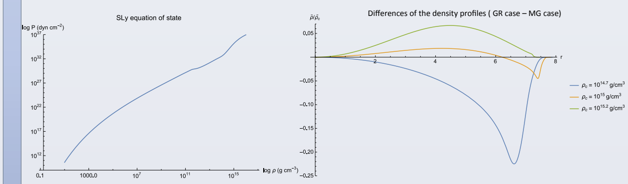
- c : It depends on chosen mass of strange quark. We used $c = 0.28$.
- B : bag constant. We use $B = 60\text{MeV}/\text{fm}^3$.

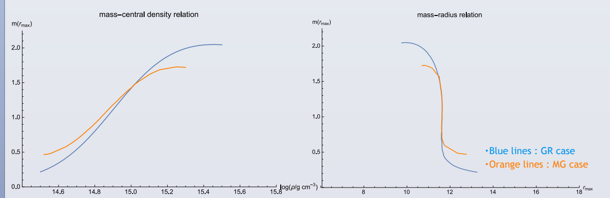




The results are similar to the case in the general relativity, but the density profiles and total mass are **smaller** than those in the general relativity.

◎ Neutron star cases (SLy EoS)





We can see that the density becomes **higher than that in the general relativity for small central density** and **lower for larger central density**.
The region of total mass becomes **narrow** compared with that in the general relativity.

SUMMARY AND DISCUSSION

We have concluded that the TOV equation is corrected by the term which is proportional to the graviton mass results from the potential term of massive graviton in the action, and **one constraint equation** appears if we assume the conservation of energy-momentum tensor. The correction is **very small** if we consider the light graviton mass against massive object.
From the numerical simulation, we found these.

- The basic properties are almost same as those in the general relativity but slightly different.
- Mass-Radius relation is **more constrained** rather than that in the general relativity.
- For quark star, the maximal mass gets **smaller** than that in the general relativity.
- For neutron star, **the maximal mass gets smaller** and **the minimal mass gets larger** than that in the general relativity.

Therefore, the massive neutron star can be **no more explained** in the dRGT massive gravity than it is in the general relativity.
However, our work does not result in that the dRGT massive gravity could be excluded by the observation because we

- assumed **the standard equations of state** and
- used **the minimal model** of the other parameters and referece metric.

Thus, the massive neutron star could be possible if we choose these suitably.

“Theoretical Aspects of Nonlocal Gravity”

by Yingli Zhang

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Theoretical Aspects of Nonlocal Gravity

arxiv:1512.XXXXX(?)

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Initial Motivation for Nonlocal Gravity

Nonlocal Gravity was initially proposed by N. Arkani-Hamed et. al to relieve the Cosmological Constant Problem by changing the Newtonian Constant

e.g. $G_{\rm eff} = \dfrac{G}{1+\mathcal{A}(\beta)}\,, \qquad \mathcal{A}(\beta)\begin{cases} \ll 1, & \text{for } \beta \gg 1 \iff \text{small scale;} \\ \gg 1, & \text{for } \beta \ll 1 \iff \text{large scale.}\end{cases}$

A simple choice is $\mathcal{A}(\beta) \propto \beta^{-n}, \quad \beta \equiv l^2\Box$, so the Ricci scalar can be found as

$$R = -\frac{4\rho_{\rm V}}{M_{\rm pl}^2(1+\mathcal{A}(\beta))} \approx -\frac{4\rho_{\rm V}}{M_{\rm pl}^2(1+\mathcal{A}(0))} \ll -\frac{4\rho_{\rm V}}{M_{\rm pl}^2}$$

i.e. provided that A(0) is large enough, we can obtain a large curvature radius while keeping the value of $\rho_{\rm V}$ predicted by quantum field theory.

Acausality Problem with Scalar Field

Up to now, most of the researches were done in the scalar-tensor presentation. Extra degrees of freedom appear in this method, which may cause either ghost instabilites or structure formation problem. We hope to do in its original form.

However, acausality problem arises…

A simple example: $S_\phi = \int d^4x\sqrt{-g(x)}\phi(x)\left(\Box^{-1}\phi\right)[x]$

Since $G_R(x,x') \leftrightarrow G_A(x',x)$, a naive variation principle will result in the advanced Green's function in EOM:

$$\begin{aligned}
&\int d^4x\sqrt{-g(x)}\phi(x)\left(\Box_R^{-1}\frac{\delta\phi}{\delta\phi(y)}\right)[x] \\
&= \int\int d^4x\; d^4x'\sqrt{-g(x)}\sqrt{-g(x')}\phi(x)G_R(x,x')\frac{\delta\phi(x')}{\delta\phi(y)} \\
&= \int\int d^4x'\; d^4x\sqrt{-g(x')}\sqrt{-g(x)}\phi(x')G_R(x',x)\frac{\delta\phi(x)}{\delta\phi(y)} \\
&= \int d^4x\sqrt{-g(x)}\delta(x-y)\int d^4x'\sqrt{-g(x')}\phi(x')G_R(x',x) \\
&= \int d^4x\sqrt{-g(x)}\delta(x-y)\int d^4x'\sqrt{-g(x')}\phi(x')G_A(x,x') \\
&= \int d^4x\sqrt{-g(x)}\delta(x-y)\left(\Box_A^{-1}\phi\right)[x]\,.
\end{aligned}$$

i.e. the variation principle symmetrizes the properties of Green's function, so that

$$\frac{\delta S_\phi}{\delta\phi(y)} = \sqrt{-g(y)}\left(\Box_R^{-1}\phi + \Box_A^{-1}\phi\right)[y]$$

Conclusion: Advanced Green's function always appears in EOM

Case of Gravitation

Consider a linear case of nonlocal gravity

$$S_{NL} = \int d^4x\sqrt{-g(x)}R(x)\left(\Box_R^{-1}R\right)[x]$$

In a homogeneous background, we can express the nonlocal operator explicitly:

$$\left(\Box_R^{-1}\phi\right)[t] = -\int_{-\infty}^{t}\frac{dt'}{\sqrt{-g(t')}}\int_{-\infty}^{t'}dt''\phi(t'')\sqrt{-g(t'')}$$

Then this part causes problem

$$\int d^4x\sqrt{-g(x)}R(x)\frac{\delta\left(\Box_R^{-1}R\right)[x]}{\delta g^{\mu\nu}(\tilde{x})}$$

so that the EOM via variation principle is expressed as

$$\begin{aligned}
\frac{\delta S_{NL}}{\delta g^{\mu\nu}} &= \sqrt{-g}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \nabla_\mu\nabla_\nu\right)\left[\left(\Box_R^{-1}+\Box_A^{-1}\right)R\right] + 2\sqrt{-g}\,g_{\mu\nu}R \\
&\quad - \frac{g_{\mu\nu}}{2\sqrt{-g}}\int_t^{+\infty}dt'\sqrt{-g(t')}R(t')\int_{-\infty}^{t}dt''\sqrt{-g(t'')}R(t'')\,.
\end{aligned}$$

As expected, advanced Green's function appears.
Moreover, another new "troublesome" term appears…

Implications (or possible solutions?)

1. Let us assume FLRW background and power-law scale factor $H(t) = s/t$, then

$$\Box_A^{-1}R = \frac{6s(2s-1)}{3s-1}\left\{\frac{1}{1-3s}\left[\left(\frac{t}{t_A}\right)^{1-3s} - 1\right] - \ln\left(\frac{t}{t_A}\right)\right\}$$

A possible implication can be that s=1/2 or s=0 after some cutoff time $t_A \sim T_{\rm cut}$, so the future of universe will go back to radiation dominated or Minkowskian?

2. The effective Newtonian Constant

$$G_{\rm eff} = \frac{G}{1+\Box_R^{-1}R+\Box_A^{-1}R}$$

Since $(\Box_R^{-1}R)[t] < 0$, in order to have a weakened gravitational force on large scales, $\Box_A^{-1}R$ should not only cancel the retarded Green's function, but also contribute a positive value. This can be possible if $s > 1/2$.

In this sense, the advanced Green's function can play a positive role? We do not know…Problems still remain…