Proceedings of
the 25th Workshop on General Relativity and Gravitation in Japan

7—11 December 2015

Yukawa Institute for Theoretical Physics, Kyoto University
Kyoto, Japan

Volume 4
Oral Presentations: Fourth Day

Contents

Oral Presentations: Fourth Day

Thursday 10 December

Plenary Session 7 [Chair: Tetsuya Shiromizu]

10:00 Richard Schoen (U. of California, Irvine) [Invited]
"Localizing solutions of the Einstein equations"
[JGRG25(2015)I09]

11:00-11:15 Caffee break

Parallel Session 6a [Chair: Jiro Soda]

11:15 Tsuyoshi Houri (Kobe U.)
"Prolongations of Killing-Stackel tensor equations"
[*]

11:30 Kei Yamada (Kyoto U.)
"Non-chaotic Evolution of Lagrange's orbit due to GW Radiation Reaction"
[JGRG25(2015)6a2]

11:45 Naoya Kitajima (APCTP)
"Disappearing inflaton potential"
[JGRG25(2015)6a3]

12:00 Shingo Kukita (Nagoya U.)
"Entanglement dynamics for two Unruh detectors in de Sitter space-time"
[JGRG25(2015)6a4]

12:15 Kunihito Uzawa (Kwansei Gakuin U.)
"Cosmic censorship in dynamical brane backgrounds"
[JGRG25(2015)6a5]

Parallel Session 6b [Chair: Teruaki Suyama]

11:15 Ryo Saito (APC)
"Modified gravity inside astrophysical bodies"
[JGRG25(2015)6b1]

11:30 Masato Minamitsuji (CENTRA, IST, UL)
"Slowly-rotating black hole solutions in Horndeski gravity"
[JGRG25(2015)6b2]

11:45 Ivan Arraut (TUS)
"V M in time-domains, graviton Higgs mechanism"
[JGRG25(2015)6b3]

12:00 Cancelled

12:15 Kohei Kamada (Arizona State U.)
"Higgs G-inflation and field-dependent cutoff scale"
[JGRG25(2015)6b5]

12:30-14:30 Lunch & poster view

Parallel Session 7a [Chair: Takeshi Chiba]

14:00 Gyula Fodor (Paris Observatory)
"Localized oscillating configurations formed by real scalar fields"
[JGRG25(2015)7a1]

14:15 Cancelled

14:30 Sadra Jazayeri (IPM)
"Anisotropies from fluctuations of a domain wall during inflation"
[JGRG25(2015)7a3]

14:45 Takafumi Kokubu (Rikkyo U.)
"Does the Gauss-Bonnet term stabilize wormholes?"
[JGRG25(2015)7a4]

15:00 Kota Ogasawara (Rikkyo U.)
"High energy particle emission form particle collision near an extremal Kerr black hole"
[JGRG25(2015)7a5]

15:15 Takayuki Ohgami (Yamaguchi U.)
"Wormhole shadows"
[JGRG25(2015)7a6]

Parallel Session 7b [Chair: Tsutomu Kobayashi]

14:00 Antonio Enea Romano (UDEA & UOC)
"Directional dependence of the local estimation of H0 and the non perturbative effects of primordial curvature perturbations"
[JGRG25(2015)7b1]

14:15 Daisuke Yamauchi (RESCEU)
"Probing primordial non-Gaussianity consistency relation with galaxy surveys"
[JGRG25(2015)7b2]

14:30 Ichihiko Hashimoto (YITP)
"Modeling redshift-space bispectrum from perturbation theory"
[JGRG25(2015)7b3]

14:45 Yuki Sakakihara (Kyoto U.)
"Scalar perturbations in Bimetric Gravity"
[JGRG25(2015)7b4]

15:00 Yasuho Yamashita (YITP)
"bigravity from gradient expansion in DGP 2-brane model"
[JGRG25(2015)7b5]

15:15 Daisuke Yoshida (TiTech)
"Perturbations of Cosmological and Black Hole Solutions in Massive Gravity and Bi-Gravity"
[JGRG25(2015)7b6]

15:30-16:30 Coffee break & poster view

Plenary Session 8 [Chair: Shinji Mukohyama]

16:30 Mordehai Milgrom (WIS) [Invited]
"Scale Invariance at low accelerations and the mass discrepancies in the Universe"
[JGRG25(2015)I10]

16:30 Masahide Yamaguchi (TiTech) [Invited]
"Beyond Inflation and Beyond Horndeski Theory"
[JGRG25(2015)I11]

“Localizing solutions of the Einstein equations”

by Richard Schoen (invited)

[JGRG25(2015)I09]

Localizing solutions of the Einstein equations

Richard Schoen

UC, Irvine and Stanford University

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The 25th Workshop on General Relativity
and Gravitation in Japan

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December 10, 2015

Plan of Lecture

The lecture will have four parts:

Part 1: Introduction

Part 2: Main theorem on localization of initial data

Part 3: Connections to the geometry of initial data sets

Part 4: Some features of the proof

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Part 3: Connections to the geometry of initial data sets

Part 4: Some features of the proof

Main results are joint with A. Carlotto and appear in paper at arXiv:1407.4766.

Part 1: Introduction

On a spacetime $\mathcal{S}^{n+1}$, the Einstein equations couple the gravitational field g (a Lorentz metric on $\mathcal{S}$) with the matter fields via their stress-energy tensor T

$$Ric(g) - \frac{1}{2}R\, g = T$$

where Ric denotes the Ricci curvature and $R = Tr_g(Ric(g))$ is the scalar curvature.

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where Ric denotes the Ricci curvature and $R = Tr_g(Ric(g))$ is the scalar curvature.

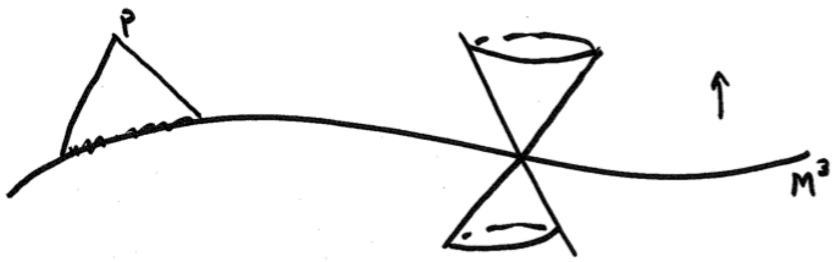
When there are no matter fields present the right hand side T is zero, and the equation reduces to

$$Ric(g) = 0.$$

These equations are called the vacuum Einstein equation.

Initial Data

The solution is determined by initial data given on a spacelike hypersurface M^n in $\mathcal{S}$.



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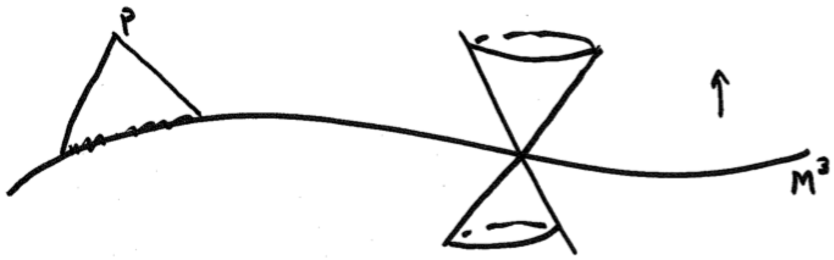

The initial data for g are the induced (Riemannian) metric, also denoted g, and the second fundamental form p. These play the role of the initial position and velocity for the gravitational field. An initial data set is a triple (M, g, p).

The constraint equations for vacuum solutions

It turns out that $n+1$ of the $(n+1)(n+2)/2$ Einstein equations can be expressed entirely in terms of the initial data and so are not dynamical. These come from the Gauss and Codazzi equations of differential geometry.

In case there is no matter present, the vacuum constraint equations become

$$R_M + Tr_g(p)^2 - \|p\|^2 = 0$$
$$\sum_{j=1}^{n} \nabla^j \pi_{ij} = 0$$

for $i = 1, 2, \ldots, n$ where R_M is the scalar curvature of M and $\pi_{ij} = p_{ij} - Tr_g(p) g_{ij}$.

The constraint equations with matter present

Using the Einstein equations with matter fields encoded in the stress-energy tensor T together with the Gauss and Codazzi equations, the constraint equations are

$$\mu = \frac{1}{2}(R_M + Tr_g(p)^2 - \|p\|^2)$$

$$J_i = \sum_{j=1}^{3} \nabla^j \pi_{ij}$$

for $i = 1, 2, \ldots, n$ where $\pi_{ij} = p_{ij} - Tr_g(p) g_{ij}$. Here the quantity μ is the observed energy density of the matter fields and J is the observed momentum density.

Energy Conditions

For spacetimes with matter, the stress-energy tensor is normally required to satisfy the **dominant energy condition**. For an initial data set this implies the inequality $\mu \geq \|J\|$.

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In the time symmetric case ($p = 0$) the dominant energy condition is equivalent to the inequality $R_M \geq 0$.

The initial value problem

Given an initial data set (M, g, p) satisfying the vacuum constraint equations, there is a unique local spacetime which evolves from that data. This result involves the local solvability of a system of nonlinear wave equations.

Asymptotic Flatness

We will consider asymptotically flat solutions. The requirement is that the initial manifold M outside a compact set be diffeomorphic to the exterior of a ball in R^n and that there be coordinates x in which g and p have appropriate falloff.

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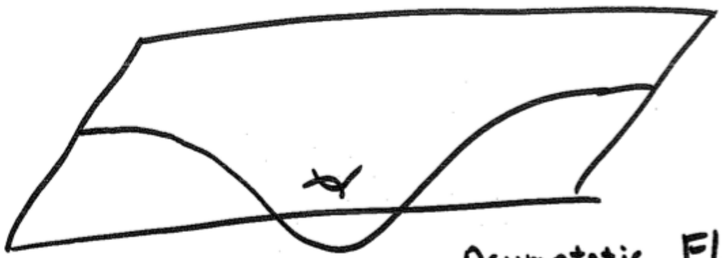

Asymptotic Flatness

Minkowski and Schwarzschild Solutions

The following are two basic examples of asymptotically flat spacetimes:

1) The Minkowski spacetime is R^{n+1} with the flat metric $g = -dx_0^2 + \sum_{i=1}^n dx_i^2$. It is the spacetime of special relativity.

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The following are two basic examples of asymptotically flat spacetimes:

1) The Minkowski spacetime is R^{n+1} with the flat metric $g = -dx_0^2 + \sum_{i=1}^n dx_i^2$. It is the spacetime of special relativity.

2) The Schwarzschild spacetime is determined by initial data with $p = 0$ and

$$g_{ij} = (1 + \frac{E}{2|x|^{n-2}})^{\frac{4}{n-2}} \delta_{ij}$$

for $|x| > 0$. It is a vacuum solution describing a static black hole with mass E. It is the analogue of the exterior field in Newtonian gravity induced by a point mass.

ADM Energy

For general asymptotically flat initial data sets there is a notion of total (ADM) energy which is computed in terms of the asymptotic behavior of g. For this definition we fix asymptotically flat coordinates x.

$$E = \frac{1}{2(n-1)\omega_{n-1}} \lim_{r\to\infty} \int_{|x|=r} \sum_{i,j=1}^{n} (g_{ij,i} - g_{ii,j})\nu_0^j \, d\sigma_0$$

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The limit exists under quite general asymptotic decay conditions. There is an analogous expression for the linear momentum in terms the asymptotic behavior of p.

The positive energy theorem

The positive energy theorem says that $E \geq 0$ whenever the dominant energy condition holds, and that $E = 0$ only if (M, g, p) can be isometrically embedded into the $(n+1)$-dimensional Minkowski space with p as its second fundamental form. In case $p = 0$, the assumption is $R_g \geq 0$, and equality implies that (M, g) is isometric to $\mathbb{R}^n$.

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The problem can be posed in any dimension, and it can be proven in various cases using mean curvature ideas (S & Yau) or using the Dirac operator approach developed by E. Witten. In three dimensions there is a third approach (for $p = 0$) which is the inverse mean curvature flow proposed by R. Geroch and made rigorous by G. Huisken and T. Ilmanen.

Part 2: Main theorem on localization of initial data

The Einstein equations lie somewhere between the wave equation and Newtonian gravity (or the stationary Einstein equations). For the wave equation one can localize initial data and reduce many questions to the study of compactly supported solutions.

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For Newtonian gravity the asymptotic behavior of the Newtonian potential is determined by the Poisson equation, and the asymptotic terms include the total mass and center of mass. The asymptotic form of the potential is rigidly determined and cannot be changed. It is similarly true for the Einstein equations that the asymptotic terms contain physical information such as energy, momentum, and center of mass. While this limits the asymptotic forms which are possible, it does not determine the form uniquely.

Asymptotic behavior

The energy and linear momentum can be shown to exist the under rather weak asymptotic decay

$$g_{ij} = \delta_{ij} + O_2(|x|^{-q}),\ \ p_{ij} = O_1(|x|^{-q-1})$$

for any $q > (n-2)/2$.

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for any $q > (n-2)/2$.

Clearly the positive energy theorem implies that there are no solutions of the constraint equations with compact support.

A further consequence of positive energy

If we let U denote the open subset of M consisting of those points at which the Ricci curvature of g is nonzero, then we have the following. It shows that under reasonable decay conditions the set U must include a positive 'angle' at infinity.

Proposition Assume that (M, g, p) satisfies the decay conditions

$$g_{ij} = \delta_{ij} + O_3(|x|^{2-n}),\ \ p_{ij} = O_2(|x|^{1-n}).$$

Unless the initial data is trivial, we have

$$\liminf_{\sigma \to \infty} \sigma^{1-n} Vol(U \cap \partial B_\sigma) > 0.$$

Proof of proposition

The energy can be written in terms of the Ricci curvature

$$E = -c_n \lim_{\sigma \to \infty} \sigma \int_{S_\sigma} Ric(\nu, \nu)\ da$$

for a positive constant c_n.

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$$E = -c_n \lim_{\sigma\to\infty} \sigma \int_{S_\sigma} Ric(\nu,\nu)\ da$$

for a positive constant c_n.

If our initial data is nontrivial, then we have $E > 0$, and so for any σ sufficiently large we have

$$E/2 < c_n\sigma \int_{S_\sigma} |Ric(\nu,\nu)|\ da \leq c\sigma^{1-n} Vol(U \cap \partial B_\sigma)$$

where the second inequality follows from the decay assumption.

□

Energy in terms of Ricci curvature

The energy formula used in the proposition is based on the identity

$$div(Ric(\cdot,X)^{\#}) = \frac{1}{2}\langle Ric, \mathcal{D}(X)\rangle + \frac{1}{n}Rdiv(X) + \frac{1}{2}\langle \nabla R, X\rangle$$

where $\mathcal{D}$ is the conformal Killing operator

$$\mathcal{D}(X) = L_X g - \frac{2}{n}div(X)g.$$

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$$\mathcal{D}(X) = L_X g - \frac{2}{n}div(X)g.$$

Note that under the decay assumption $g_{ij} = \delta_{ij} + O_3(|x|^{-q})$ for $q > (n-2)/2$ and $R = O_1(|x|^{-r})$ for $r > n$, the righthand side is integrable with $X = \sum x^i \partial_{x^i}$, so the limit exists

$$\lim_{\sigma\to\infty} \int_{S_\sigma} Ric(X, \nu)\ da = \lim_{\sigma\to\infty} \sigma \int_{S_\sigma} Ric(\nu, \nu)\ da.$$

To evaluate the limit we can do it in three steps.

Step 1: Compute it for the Schwarzschild metric

$$g_{ij} = (1 + \frac{E}{2|x|^{n-2}})^{\frac{4}{n-2}}\delta_{ij}$$

with $X = \sum_i x^i \frac{\partial}{\partial x^i}$. Since X satisfies $\mathcal{D}(X) = 0$ and $R = 0$ we see that the righthand side vanishes and the flux integral

$$\int_\Sigma Ric(\nu, X)\ da$$

is the same over any hypersurface Σ which is homologous to the horizon S which is the $|x| = (E/2)^{1/(n-2)}$ sphere. An easy calculation on the horizon shows that the value is $-c_n E$ where $c_n = (n-1)(n-2)2^{4/(n-2)}\sigma_{n-1}$ where $\sigma_{n-1} = Vol(S^{n-1})$. Note that $c_3 = 128\pi$.

Step 2: The same formula now follows for any initial data set for which g is Schwarzschild to leading order; that is,

$$g_{ij} = (1 + \frac{E}{2|x|^{n-2}})^{\frac{4}{n-2}} \delta_{ij} + O_3(|x|^{1-n}).$$

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Step 3: For the general asymptotic conditions

$$g_{ij} = \delta_{ij} + O_3(|x|^{-q}), \quad R = O_1(|x|^{-r})$$

with $q > (n-2)/2$ and $r > n$, we can now appeal to a density theorem which asserts that initial data with leading order Schwarzschild asymptotics is dense in those with general decay conditions in a norm in which the energy is continuous.

What are good asymptotic forms?

Since it is possible to achieve any chosen pair E, P by a suitably boosted slice in the Schwarzschild, people have assumed that this would be a natural asymptotic form for an asymptotically flat solution of the vacuum constraint equations.

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It was shown by J. Corvino ($p = 0$) and by Corvino and S. (also Chruściel and Delay) that the set of initial data which are identical to a boosted slice of the Kerr (generalization of Schwarzschild) spacetime are dense in a natural topology in the space of all data with reasonable decay.

Localizing in a cone

Let us consider an asymptotically flat manifold $(M, \breve{g})$ with $R_{\breve{g}} = 0$ and with decay

$$\breve{g}_{ij} = \delta_{ij} + O(|x|^{-\breve{q}})$$

where $(n-2)/2 < \breve{q} \leq n-2$.

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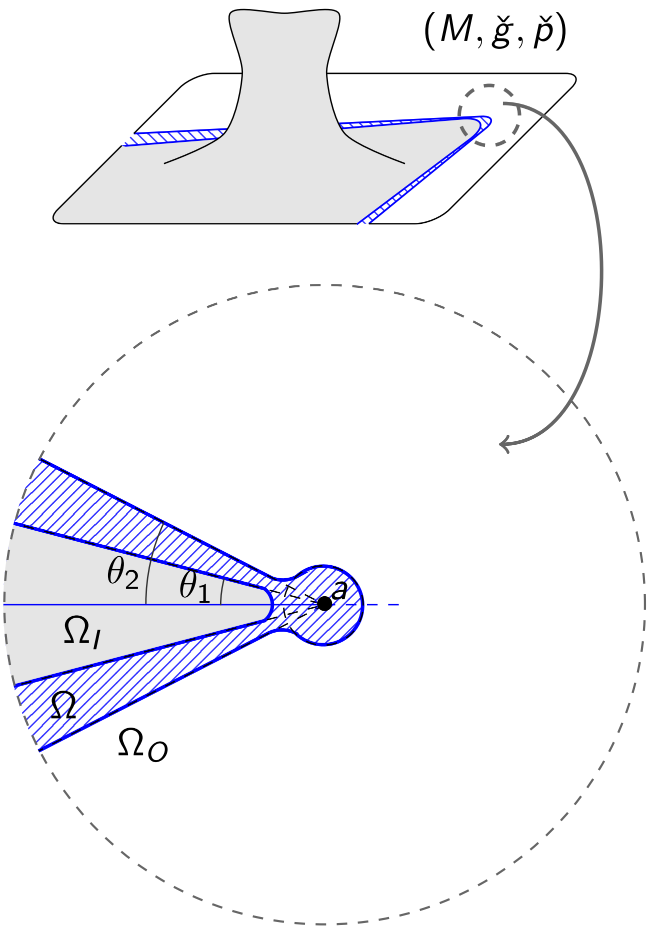
where $(n-2)/2 < \breve{q} \leq n-2$.

In joint work with A. Carlotto we have shown that there is a metric g which satisfies $R_g = 0$ with $g = \breve{g}$ inside a cone based at a point far out in the asymptotic region while $g = \delta$ outside a cone with slightly larger angle. Moreover g is close to $\breve{g}$ in a topology in which the energy is continuous, so E is arbitrarily close to $\breve{E}$. The metric g satisfies

$$g_{ij} = \delta_{ij} + O(|x|^{-q})$$

with $q < \breve{q}$.



Where is the energy?

Since there is very little contribution to the energy inside the region where $\bar{g} = g$ and none in the euclidean region, most of the energy resides on the transition region. This shows that one cannot impose too much decay on this region and makes the weakened decay plausible.

Construction of non-interacting solutions

Another interesting application of the construction is that it gives a method of 'adding together' initial data. If we have localized solutions we can super-impose them by putting them in disjoint cones. When we do this the energies and linear momenta add up. Since we can approximate a general solution on an arbitrarily large set and in a suitable topology, we can construct n-body initial data with bodies which are far separated.

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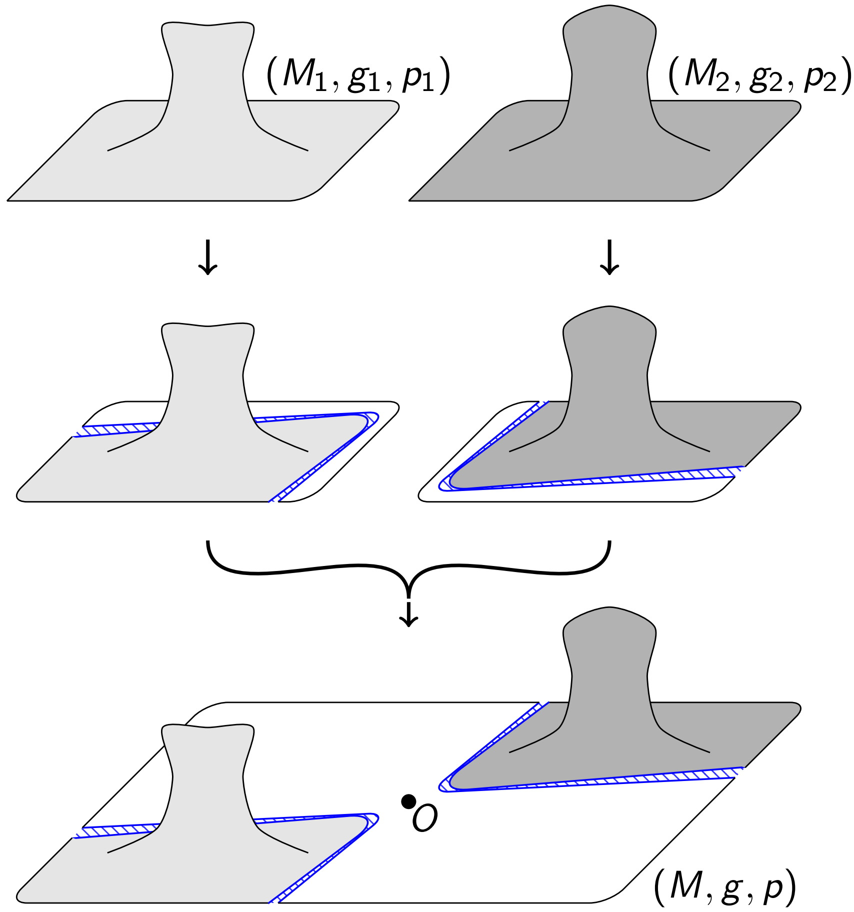
The constructions allows us to superimpose solutions in such a way that they do not interact at all for a fixed time period.



Part 3: Connections to the geometry of initial data sets

Certain geometric aspects of the initial data have important consequences for the spacetime. For example, the Penrose singularity theorem shows that if the initial data has an outer trapped surface then the spacetime cannot be null geodesically complete.

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The mean curvature proof of the positive energy theorem relies on the geometric theorem that an initial data set with strictly positive energy density cannot have an asymptotically planar stable minimal surface. The constructions we have made show that this is not true for nontrivial vacuum initial data sets (e.g. planes in the euclidean region are stable).

Minimal surfaces and MOTS

The notion of trapping naturally leads to the notion of a marginally outer trapped surface (MOTS). Such a surface would satisfy $H + Tr_\Sigma(p) = 0$, and if it is the boundary between surfaces that are outer trapped and untrapped, it satisfies a stability condition. For $p = 0$ this is the ordinary variational stability of the area functional (second variation nonnegative for all variations).

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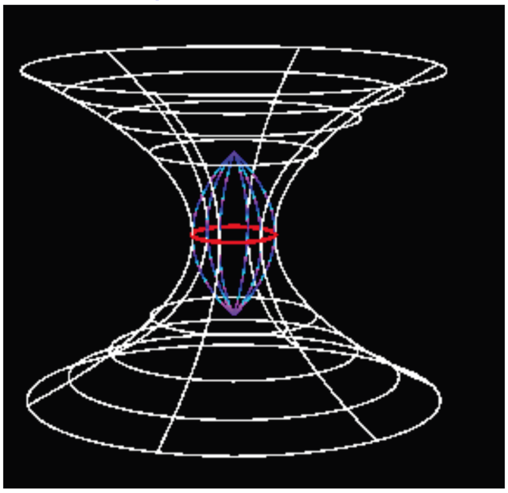
For example the Schwarzschild horizon is a stable minimal surface.



A question coming from the proof of PMT

A key ingredient of the mean curvature proof of the PET is the statement that for $n = 3$ there can be no complete asymptotically planar stable minimal surface ($p = 0$) or stable MOTS (general case) provided the dominant energy condition holds **strictly**. For $n \geq 4$ there is a corresponding statement for *strongly* stable MOTS.

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Question: Can there be a stable asymptotically planar minimal surface (or MOTS) in a nontrivial initial data set?

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Question: Can there be a stable asymptotically planar minimal surface (or MOTS) in a nontrivial initial data set?

Our localization construction shows that this same property is not true without the strictness of the energy conditions.

A positive result

The following theorem was proven by A. Carlotto (arXiv:1310.5118).

Theorem. If (M^3, g, p) is nontrivial, satisfies the dominant energy condition, and is asymptotic to leading order to a slice in the Schwarzschild spacetime, then there is no complete non-compact stable MOTS.

A positive result

The following theorem was proven by A. Carlotto (arXiv:1310.5118).

Theorem. If (M^3, g, p) is nontrivial, satisfies the dominant energy condition, and is asymptotic to leading order to a slice in the Schwarzschild spacetime, then there is no complete non-compact stable MOTS.

The construction we have made is limited in the decay which can be arranged, so the question is still open with $|x|^{2-n}$ decay. Some evidence for this was given by the result of A. Carlotto.

Isoperimetric properties of spheres in the euclidean region

Spheres in euclidean space are isoperimetric surfaces in that they have least area for their enclosed volume. If we consider a sphere in the euclidean region of a localized solution, it is natural to ask if it is an isoperimetric surface for the initial data set.

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We have observed that this is not the case for sufficiently large euclidean spheres. This is based on work of Fan, Miao, Shi, and Tam who gave a formula for the ADM energy in terms of a deficit in the isoperimetric profile for surfaces enclosing a large volume (an idea proposed by Huisken).

Area minimizing surfaces

The planes in the euclidean region are clearly stable, so it is natural to ask if they can be area minimizing in a nontrivial initial data set. The result for isoperimetric spheres suggests that they may not be. This was shown very recently by O. Chodosh and M. Eichmair who proved that a nontrivial time symmetric initial data set cannot contain a complete noncompact area minimizing surface.

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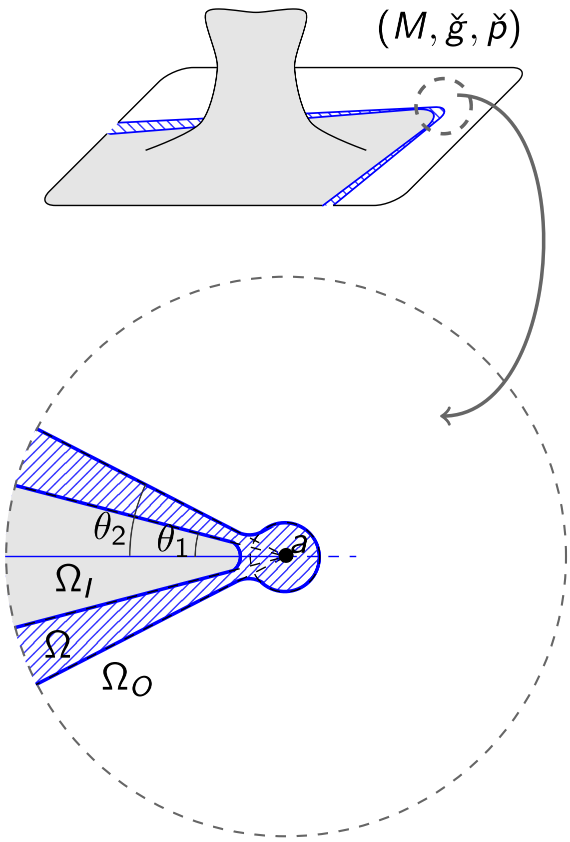
The mean curvature proof of the positive energy theorem shows that any asymptotically flat metric with negative mass does contain an area minimizing surface which is asymptotically planar. (The scalar curvature must be negative somewhere.)

Part 4: Some features of the proof



Outline of proof I

We first construct a metric $\tilde{g}$ of the form

$$\tilde{g} = \chi\check{g} + (1 - \chi)\delta$$

where $\chi(\phi)$ is a smooth cutoff function which is 1 in Ω_I of smaller angle and zero in Ω_O. Here ϕ is the angle function on the cone outside the unit ball extended so that it is constant on each component of $\partial\Omega$.

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where $\chi(\phi)$ is a smooth cutoff function which is 1 in Ω_I of smaller angle and zero in Ω_O. Here ϕ is the angle function on the cone outside the unit ball extended so that it is constant on each component of $\partial\Omega$.

We then seek a solution of the form $g = \tilde{g} + h$ with $R(g) = 0$ where h is supported in Ω. The equation can be written

$$R(g) = R(\tilde{g}) + \tilde{L}h + Q(h) = 0$$

where $\tilde{L}$ is the linearization of the scalar curvature map at $\tilde{g}$. Note that $R(\tilde{g}) = 0$ outside the transition region Ω.

Outline of proof II

We have the formula for the operator

$$\tilde{L}h = \delta\delta h - \Delta_{\tilde{g}}(Tr(h)) - \langle h, Ric(\tilde{g})\rangle$$

where computations are with respect to $\tilde{g}$. The adjoint operator is then

$$\tilde{L}^* u = Hess_{\tilde{g}}(u) - \Delta_{\tilde{g}}(u)\tilde{g} - uRic(\tilde{g}).$$

The composition is given by

$$\begin{aligned}\tilde{L}(\tilde{L}^* u) = (n-1)\Delta(\Delta u) + 1/2(\Delta\tilde{R})u + 3/2\langle\nabla\tilde{R}, \nabla u\rangle \\ + 2\tilde{R}(\Delta u) - \langle Hess(u), Ric(\tilde{g})\rangle\end{aligned}$$

Outline of proof III

We solve the equation

$$\tilde{L}h + Q(h) = f$$

using a Picard iteration scheme in spaces which impose decay of $|x|^{-q}$ at infinity and rapid decay near $\partial\Omega$. The proof involves first showing that $\tilde{L}$ is surjective in such spaces.

Outline of proof III

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$$\tilde{L}h + Q(h) = f$$

using a Picard iteration scheme in spaces which impose decay of $|x|^{-q}$ at infinity and rapid decay near $\partial\Omega$. The proof involves first showing that $\tilde{L}$ is surjective in such spaces.

The basic estimate which enables us to impose rapid decay near $\partial\Omega$ is

$$\|u\|_{2,-s,\Omega} \leq c\|\tilde{L}^* u\|_{0,-s-2,\Omega}$$

for any $s > 0$ where these are norms in L^2 Sobolev norms and no boundary condition is imposed on u.

Why do we need $q < n-2$?

We need to show surjectivity of $\tilde{L}$, and this follows from injectivity of $\tilde{L}^*$. The domain of $\tilde{L}^*$ is the dual space of the range of $\tilde{L}$, that is the dual of $H_{0,-2-q}$. This dual space is $H_{0,2+q-n}$ since we have

$$|\int_M f_1 f_2 \, d\mu| \leq (\int_M |f_1|^2 |x|^{-n+2(q+2)})^{1/2} (\int_M |f_2|^2 |x|^{n-2(q+2)})^{1/2},$$

and the right hand side is $\|f_1\|_{0,-q-2} \|f_2\|_{q+2-n}$.

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and the right hand side is $\|f_1\|_{0,-q-2} \|f_2\|_{q+2-n}$.

Since $q < n-2$ implies that $s = n-2-q > 0$, we can apply the basic estimate to get the injectivity estimate

$$\|u\|_{2,2+q-n} \leq c \|\tilde{L}^* u\|_{0,q-n}.$$

This bound is no longer true if $q \geq n-2$.

Surjectivity of L

The injectivity of L^* implies surjectivity of L between dual spaces. The L^2 dual to the decay of $|x|^{q-n}$ corresponds to the decay of order $|x|^{-q}$ at infinity. Since no decay is required near ∂U in the basic estimate we can impose rapid decay near ∂U as the dual condition. Thus we can construct solutions of $\tilde{L}h = f$ in spaces with such decay. Given sufficiently good estimates we can then solve the nonlinear equation $\tilde{L}h + Q(h) = f$ with the same decay.

Main technical issues

Some of the technical issues which need to be overcome to do this construction are the following:

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(1) The transition region is noncompact and this creates major difficulties. We are able to exploit the homogeneity to help overcome this difficulty. The noncompctness presents challenges both for getting the basic injectivity estimate and for higher order estimates. This is especially so for the general constraint equations since they are more complicated than the $p = 0$ case; for example, they are of mixed order.

Main technical issues

Some of the technical issues which need to be overcome to do this construction are the following:

(1) The transition region is noncompact and this creates major difficulties. We are able to exploit the homogeneity to help overcome this difficulty. The noncompctness presents challenges both for getting the basic injectivity estimate and for higher order estimates. This is especially so for the general constraint equations since they are more complicated than the $p = 0$ case; for example, they are of mixed order.

(2) There are two different decay rates which must be imposed on solutions. First the solutions must decay rapidly near the boundary of U in order to make the patched solution smooth enough. Secondly we must maintain the decay rate at infinity for the solutions. These are handled by working in spaces with double weights which impose the two decay conditions.

“Non-chaotic Evolution of Lagrange’s orbit due to GW Radiation Reaction”

by Kei Yamada

[JGRG25(2015)6a2]

Non-chaotic Evolution of Lagrange's orbit due to GW Radiation Reaction

Kei Yamada (Kyoto U.)

with H. Asada (Hirosaki U.)

arXiv:1512.01087 [gr-qc]

Contents

- Introduction
- GW radiation reaction force to Lagrange's orbit
- Evolution of Lagrange's Orbit
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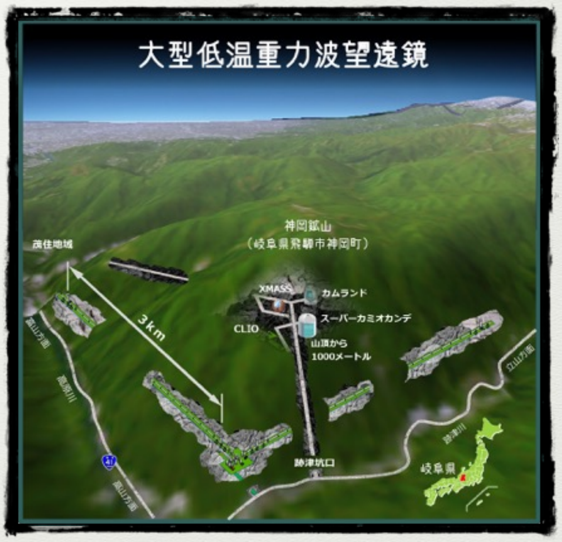
Gravitational Wave Detectors

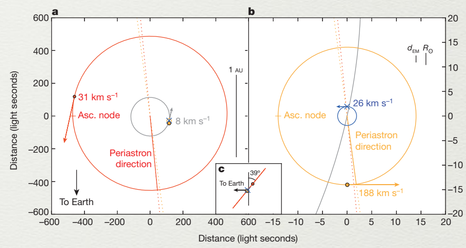
- Ground-based
  - aLIGO (USA)
  - aVIRGO (Italy, France)
  - KAGRA (Japan)
- Space-borne
  - eLISA (Europe)
  - DECIGO (Japan)



http://gwcenter.icrr.u-tokyo.ac.jp

Recent Works of Three-body Systems

- "A millisecond pulsar in stellar triple system" [Ranson et al., Nature (2014)]



[Ranson et al., Nature (2014)]

- GW & three-body interactions [Wen, ApJ (2003); Seto, PRL (2013)]
- PN triangular solution and its stability [KY & Asada, PRD (2012); KY, Tsuchiya, & Asada, PRD (2015)]

Lagrange's orbit in Newton

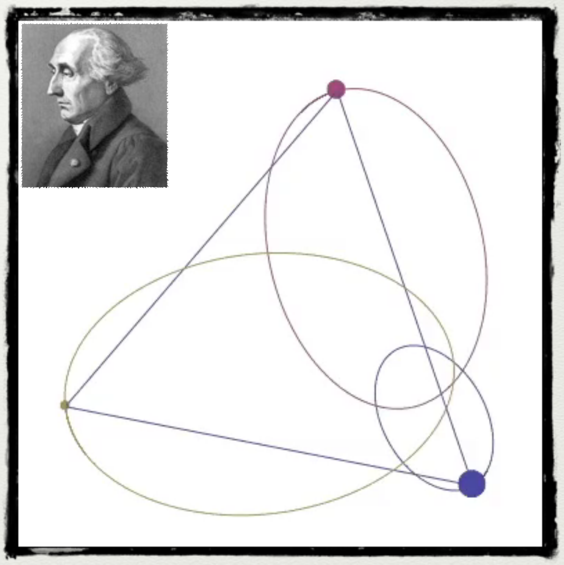
Newtonian EoM for $\boldsymbol{r}_I$ is

$$\ddot{\boldsymbol{r}}_I = -\frac{\mu_I}{r_I^3}\boldsymbol{r}_I .$$

$$\left(\mu_I \equiv \frac{(m_J^2 + m_J m_K + m_K^2)^{3/2}}{M^2}\right)$$

In circular motion,
the Kepler's third **like** law

$$\omega = \sqrt{\frac{M}{\ell^3}} . \quad (\ell : \text{separation})$$

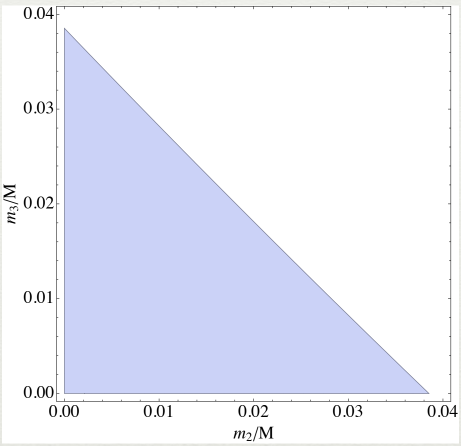


Lagrange's equilateral triangular orbit

The Linear Stability of Lagrange's orbit

This configuration is stable [Gascheau (1843)], if

$$(0 <) \quad \frac{m_1 m_2 + m_2 m_3 + m_3 m_1}{(m_1 + m_2 + m_3)^2} < \frac{1}{27}.$$



GWs from Lagrange's orbit

- Gravitational-wave forms from Lagrange's orbit [Torigoe et al., PRL (2009)]
- Parameter determinations [Asada, PRD (2009)] (based on energy balance argument)

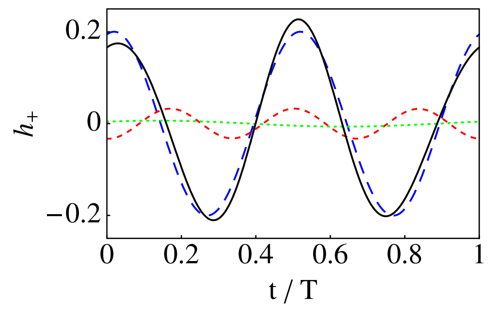


FIG. 2 (color online). Gravitational waves for a three-body system in Lagrange's orbit. Long-dashed (blue), short-dashed (red), and dotted (green) curves denote mass quadrupolar, octupolar, and current quadrupolar parts, respectively. The sold (black) one denotes the total wave forms. The vertical axis is in arbitrary units, and time in the horizontal axis is normalized by the orbital period. For simplicity, the mass ratio is assumed as m_1:m_2:m_3 = 1:2:3, though stability arguments prefer much larger ratios [24]. In order to exaggerate differences between

[Asada, PRD (2009)]

Energy Balance Argument (EBA)

- In linearized theory, $L_{\rm GW}$ and $S_{\rm GW}$ come from the source.
- Binaries: # of DOF = # of constants of motion.

$$\left(\frac{dE}{dt}, \frac{d\boldsymbol{L}}{dt}\right) \leftrightarrow \left(\frac{da}{dt}, \frac{de}{dt}\right)$$

However

- Triples: # of DOF > # of constants of motion.

Question

- Is Asada's assumption wrong?
  - Most 3-bodies orbits are chaotic.
- Study the evolution of Lagrange's orbit in circular motion

  ➡ by directly treating radiation reaction force.

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Gravitational radiation reaction

In the harmonic gauge, “reaction potential” is

$$h_{00}^{(\mathrm{react})} = -2\Phi^{(\mathrm{react})}, \quad \Phi^{(\mathrm{react})} = \frac{1}{5}\frac{d^5 \text{Ɨ}_{jk}}{dt^5} x^j x^k,$$

where, for point-like particles,

$$\text{Ɨ}_{jk} = \sum_J m_J \left(x_j^J x_k^J - \frac{1}{3}\delta_{jk} r_J^2 \right).$$

Gravitational radiation reaction force is

$$F_j^{(\mathrm{react})} = -\frac{\partial \Phi^{(\mathrm{react})}}{\partial x_j} = -\frac{2}{5}\frac{d^5 \text{Ɨ}_{jk}}{dt^5} x^k.$$

Radiation reaction on Lagrange’s orbit

In the orbital plane (*x*,*y*), non-zero components are

$$\frac{d^5 \text{Ɨ}_{xx}}{dt^5} = -16\omega^5 \sum_J m_J r_J^2 \sin(2\theta_J),$$

$$\frac{d^5 \text{Ɨ}_{xy}}{dt^5} = 16\omega^5 \sum_J m_J r_J^2 \cos(2\theta_J) = \frac{d^5 \text{Ɨ}_{yx}}{dt^5},$$

$$\frac{d^5 \text{Ɨ}_{yy}}{dt^5} = 16\omega^5 \sum_J m_J r_J^2 \sin(2\theta_J) = -\frac{d^5 \text{Ɨ}_{xx}}{dt^5}.$$

ω : orbital frequency, θ_J : direction of Jth body.

Radiation reaction on Lagrange's orbit

$$\frac{d^5 I\!\!\!-_{xx}}{dt^5} = -16\omega^5 \sum_J m_J r_J^2 \sin(2\theta_J),$$
$$\frac{d^5 I\!\!\!-_{xy}}{dt^5} = 16\omega^5 \sum_J m_J r_J^2 \cos(2\theta_J) = \frac{d^5 I\!\!\!-_{yx}}{dt^5},$$
$$\frac{d^5 I\!\!\!-_{yy}}{dt^5} = 16\omega^5 \sum_J m_J r_J^2 \sin(2\theta_J) = -\frac{d^5 I\!\!\!-_{xx}}{dt^5}.$$

$F_{Iz}^{(\mathrm{react})} \propto \dfrac{d^5 I\!\!\!-_{zk}}{dt^5} x_I^k = 0$: Orbital plane does not change.

Also, $\sum \boldsymbol{F}_I^{(\mathrm{react})} = 0$: CoM is not moved.

➡ Need 4 perturbations in the orbital plane.

Radiation Reaction Force

For convenience,
we consider the relative position $\boldsymbol{r}_{IJ} = \boldsymbol{r}_I - \boldsymbol{r}_J$.

GW radiation reaction force to $\boldsymbol{r}_{IJ}$ is

$$\boxed{\boldsymbol{F}_{IJ}^{(\mathrm{react})} = \frac{16}{5}\frac{M}{\ell^2}\varepsilon(A_{IJ}\boldsymbol{n}_{IJ} - B_{IJ}\boldsymbol{n}_{\perp IJ}),}$$

where $\varepsilon \equiv (M\omega)^{5/3}$.

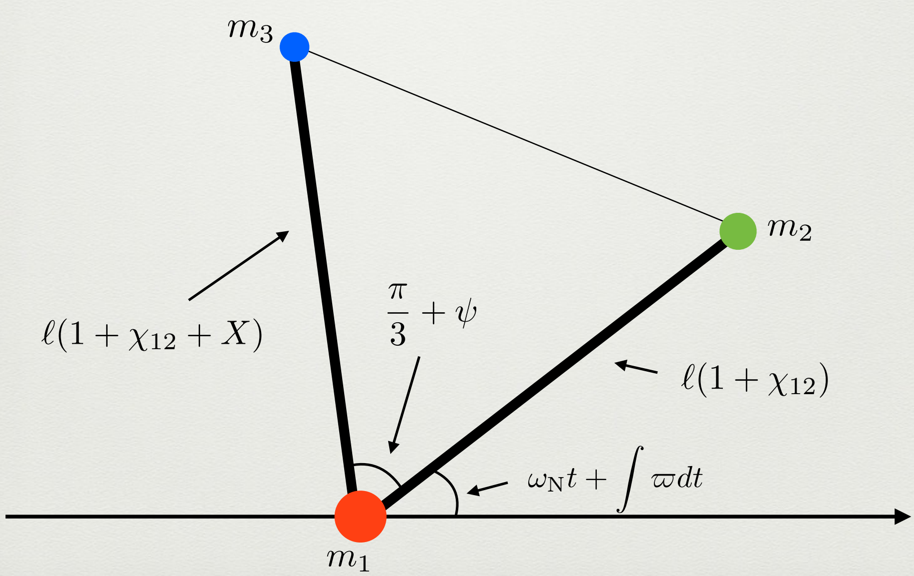
$$A_{IJ} = \sqrt{3}(\nu_I - \nu_J)\nu_K,\ B_{IJ} = \nu_I(\nu_J - \nu_K) + \nu_J(\nu_K - \nu_I).$$

$$\nu_I \equiv m_I/M,\ \boldsymbol{n}_{IJ} \equiv \boldsymbol{r}_{IJ}/\ell,\ \boldsymbol{n}_{\perp IJ} \equiv \boldsymbol{v}_{IJ}/\ell\omega$$

Contents

Perturbations in Lagrange's orbit

4 perturbations in the orbital plane: $(\chi_{12}, \varpi, X, \psi)$

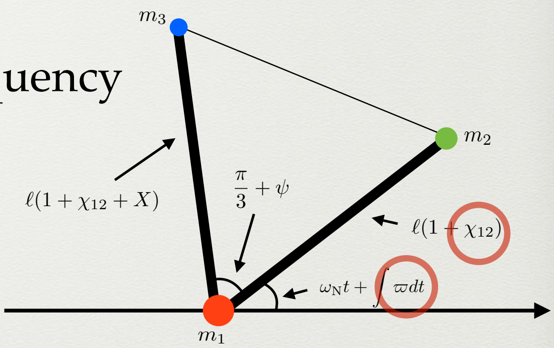


Perturbations in Lagrange's orbit

χ_{12} : a scale transformation

ϖ : a change of the orbital frequency

➡ $\left(\frac{d\ell}{dt}, \frac{dP_{\rm orbit}}{dt}\right)$



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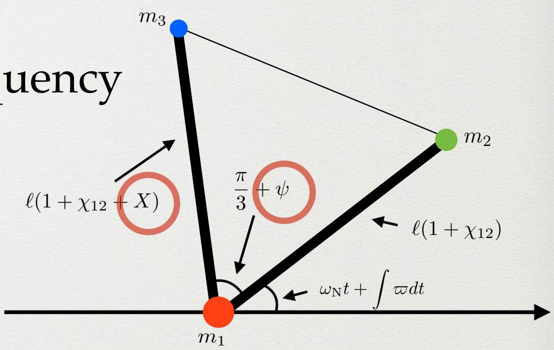

X, ψ : a shape change from equilateral triangle.

➡ Evolution of Lagrange's orbit is chaotic
if X, ψ increase with time.

Perturbed Equations of Motion

Perturbed EoMs become

Reaction force

$$
\begin{aligned}
\ddot{\chi}_{12} - 3\chi_{12} - 2\varpi - \frac{9}{4}\nu_3 X - \frac{3\sqrt{3}}{4}\nu_3\psi & \boxed{- \frac{16}{5}\varepsilon A_{12}} = 0, \\
2\dot{\chi}_{12} + \dot{\varpi} - \frac{3\sqrt{3}}{4}\nu_3 X + \frac{9}{4}\nu_3\psi & \boxed{+ \frac{16}{5}\varepsilon B_{12}} = 0, \\
\ddot{\chi}_{12} - 3\chi_{12} - 2\varpi + \ddot{X} - \left(3 - \frac{9}{4}\nu_2\right) X - 2\dot{\psi} - \frac{3\sqrt{3}}{4}\nu_2\psi & \boxed{- \frac{16}{5}\varepsilon A_{31}} = 0, \\
2\dot{\chi}_{12} + \dot{\varpi} + 2\dot{X} - \frac{3\sqrt{3}}{4}\nu_2 X + \ddot{\psi} - \frac{9}{4}\nu_2\psi & \boxed{+ \frac{16}{5}\varepsilon B_{31}} = 0,
\end{aligned}
$$

Solve in the Newtonian stable case:

$$\nu_1\nu_2 + \nu_2\nu_3 + \nu_3\nu_1 < \frac{1}{27}.$$

Behavior of perturbations

We can solve EoMs and obtain

$$
\begin{aligned}
\chi_{12} &= -\frac{32}{5V}\varepsilon\left[\nu_1^2(\nu_2-\nu_3)^2 + \nu_2^2(\nu_3-\nu_1)^2 + \nu_3(\nu_1-\nu_2)^2\right]\omega t + (\text{oscillating terms}), \\
\varpi &= \frac{48}{5V}\varepsilon\left[\nu_1^2(\nu_2-\nu_3)^2 + \nu_2^2(\nu_3-\nu_1)^2 + \nu_3(\nu_1-\nu_2)^2\right]\omega t + (\text{oscillating terms}), \\
X &= -\frac{32\sqrt{3}}{15V}\varepsilon\left[\nu_2^2(\nu_3-\nu_1) - \nu_3^2(\nu_1-\nu_2)\right] + (\text{oscillating terms}), \\
\psi &= -\frac{32}{15V}\varepsilon(\nu_2-\nu_3)(2\nu_1 - V) + (\text{oscillating terms}),
\end{aligned}
$$

where $V \equiv \nu_1\nu_2 + \nu_2\nu_3 + \nu_3\nu_1$.

χ_{12}, ϖ increase with time.

X, ψ oscillate.

Evolution of Lagrange's Orbit

$$\chi_{12} = -\frac{32}{5V}\varepsilon\left[\nu_1^2(\nu_2-\nu_3)^2+\nu_2^2(\nu_3-\nu_1)^2+\nu_3(\nu_1-\nu_2)^2\right]\omega t + (\text{oscillating terms}),$$
$$\varpi = \frac{48}{5V}\varepsilon\left[\nu_1^2(\nu_2-\nu_3)^2+\nu_2^2(\nu_3-\nu_1)^2+\nu_3(\nu_1-\nu_2)^2\right]\omega t + (\text{oscillating terms}),$$
$$X = -\frac{32\sqrt{3}}{15V}\varepsilon\left[\nu_2^2(\nu_3-\nu_1)-\nu_3^2(\nu_1-\nu_2)\right] + (\text{oscillating terms}),$$
$$\psi = -\frac{32}{15V}\varepsilon(\nu_2-\nu_3)(2\nu_1-V) + (\text{oscillating terms}),$$

χ_{12}, ϖ increase with time.

➡ Triangle shrinks with increasing orbital frequency.

X, ψ oscillate. ➡ Lagrange's orbit is adiabatically kept.

Strikingly, the evolution is non-chaotic!

Contents

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Summary

- Evolution of Lagrange’s orbit
  by directly treating the reaction force.
  - The orbit is adiabatically kept.
  - It supports [Asada, (2009)].
    - ➡ Possibility of parameter determinations.
- What about elliptic cases…?

THANK YOU FOR YOUR ATTENTION

“Disappearing inflaton potential”

by Naoya Kitajima

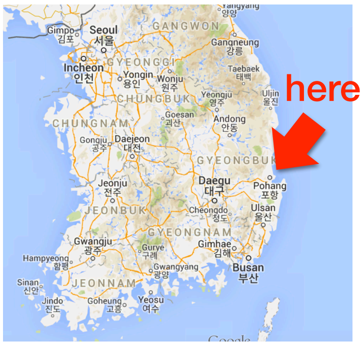
[JGRG25(2015)6a3]

Disappearing Inflaton Potential

by heavy field dynamics after inflation

Naoya Kitajima

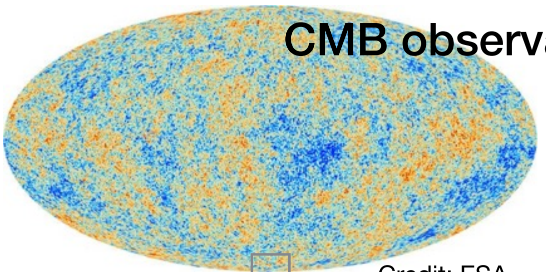
APCTP Asia Pacific Center for Theoretical Physics (Korea)



NK, F. Takahashi (Tohoku Univ.) arXiv:1509.01729

JGRG25, YITP Kyoto, Dec 7-11 2015

CMB observation



Credit: ESA

Primordial spectra

$$\mathcal{P}_\zeta = A_s \left(\frac{k}{k_0}\right)^{n_s-1}$$

$$\mathcal{P}_t = A_t \left(\frac{k}{k_0}\right)^{n_t}$$

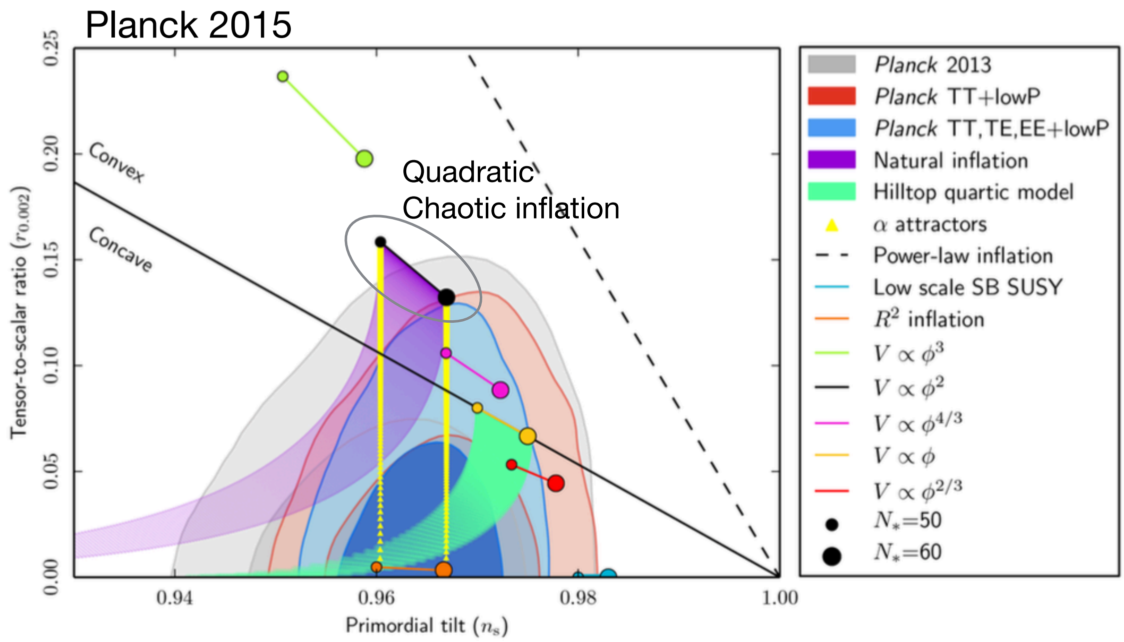
Inflaton potential

$$A_s = \frac{V^3}{2\sqrt{3}V'^2}$$

$$r = \frac{A_t}{A_s} = 8\left(\frac{V'}{V}\right)^2$$

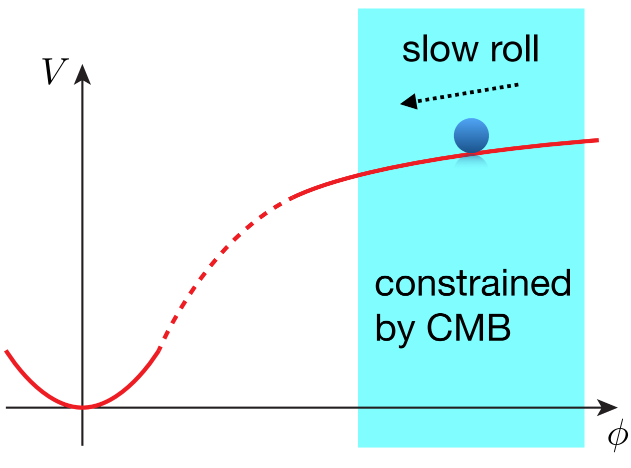
$$n_s = 1 + 2\frac{V''}{V} - 3\left(\frac{V'}{V}\right)^2$$

$$\boxed{A_s, n_s, r \longleftrightarrow V, V', V''}$$



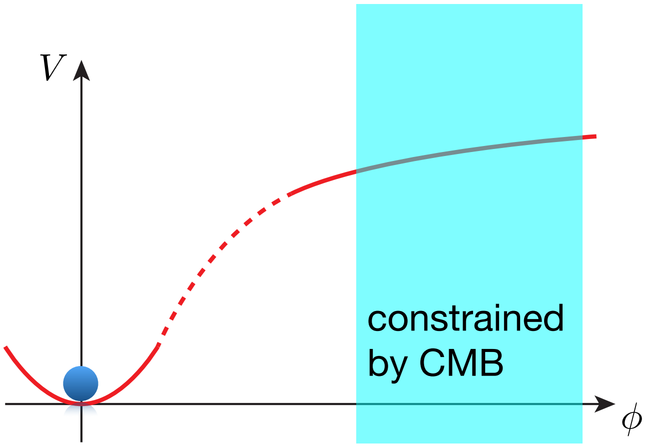
Quadratic chaotic inflation: $V = \frac{1}{2}m^2\phi^2 \quad \Rightarrow \quad m \sim 10^{13}\mathrm{GeV}$

During inflation



$$\boxed{A_s, n_s, r \longleftrightarrow V, V', V''}$$

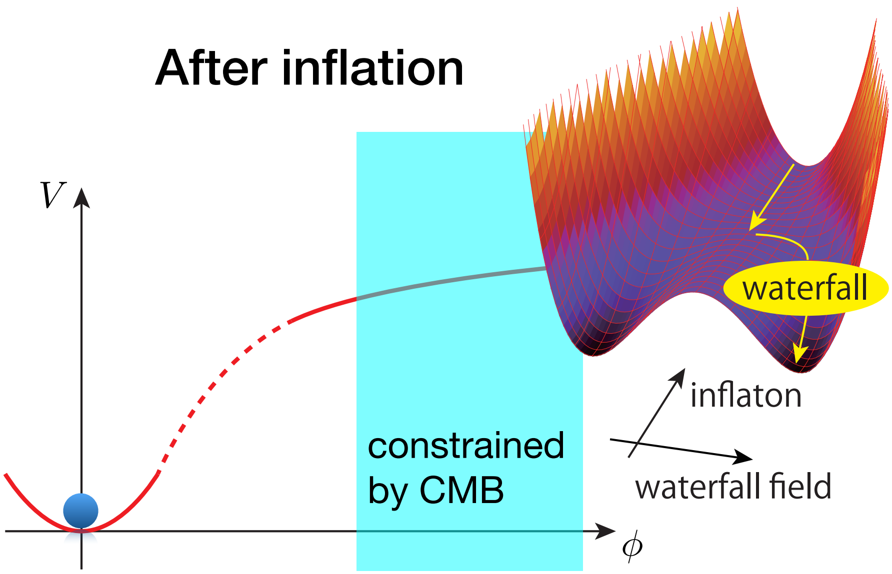
After inflation



oscillate -> reheating ⟶ Baryogenesis, Dark matter, …

$$\boxed{A_s, n_s, r \overset{??}{\longleftrightarrow} V, V', V''}$$

After inflation



oscillate -> reheating ⟶ Baryogenesis, Dark matter, …

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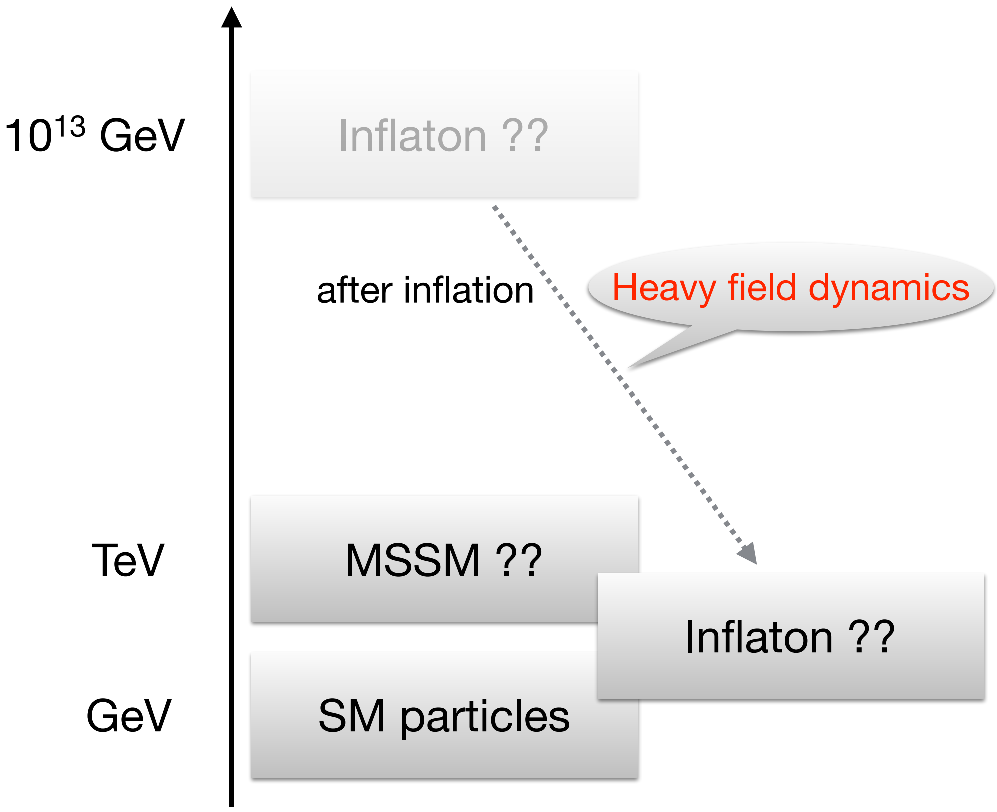
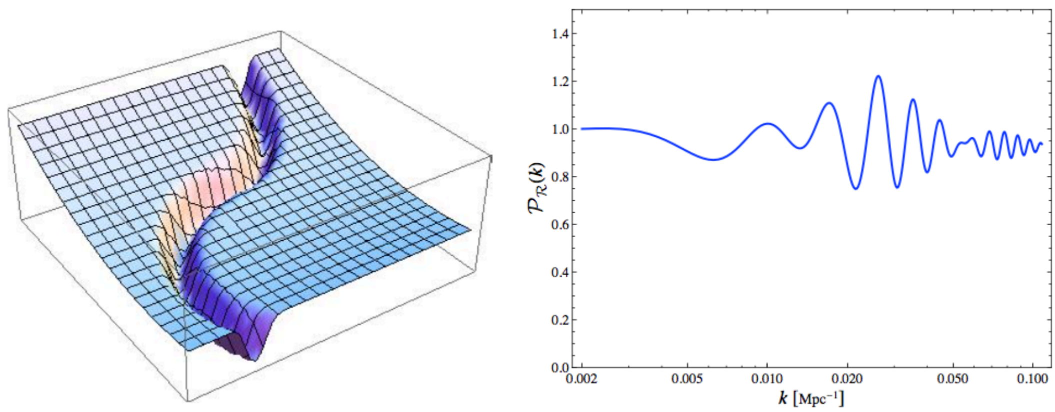


Inflaton + heavy field

Dong, Horn, Silverstein, Westphal, 1011.4521; Achucarro, Gong, Hardeman, Palma, Patil 1010.3693
Cespedes, Atal, Palma 1201.4848; Gao, Langlois, Mizuno 1205.5275; Buchmuller, Wieck, Winkler 1404.2275
Buchmuller, Dudas, Heurtier, Westphal, Wieck, Winker 1501.05812 Kumar, Sandora, Sloth 1501.06919
Dudas, Wieck 1506.01253; Harigaya, Ibe, Kawasaki, Yanagida 1506.05250; and …

Bended trajectory $\longrightarrow$ oscillating feature in primordial spectrum



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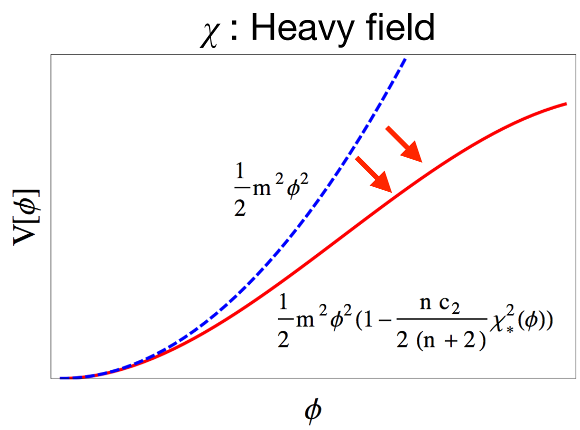
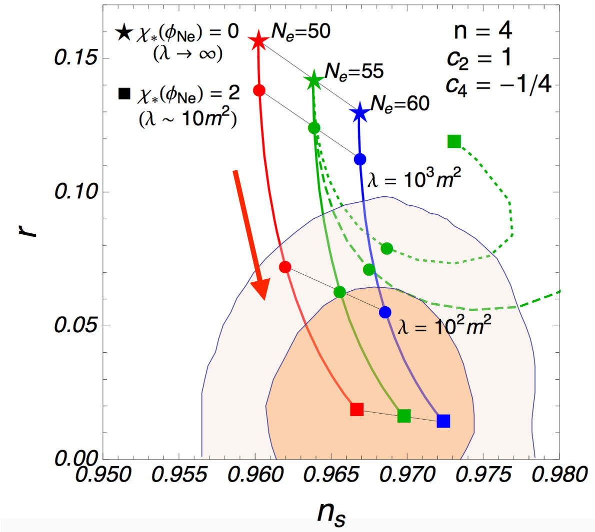




Harigaya, Ibe, Kawasaki, Yanagida 1506.05250

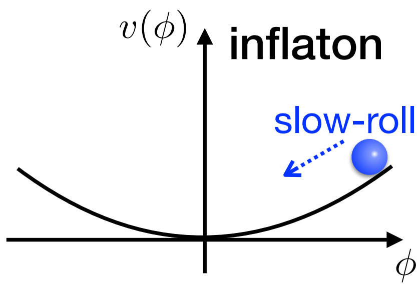
Disappearing Inflaton Potential by Heavy field dynamics

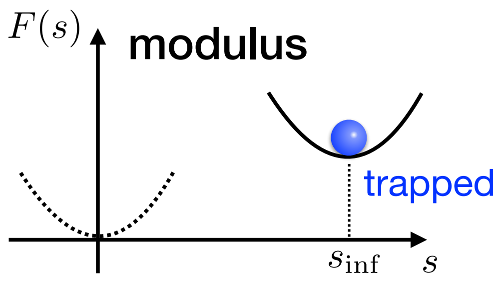
NK, F. Takahashi, arXiv:1509.01729

$$\mathcal{L} = \frac{1}{2}K(s)\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}G(s)\partial_\mu s\partial^\mu s - V(\phi, s)$$

$$\boxed{V(\phi, s) = F(s)v(\phi) + U(s)}$$

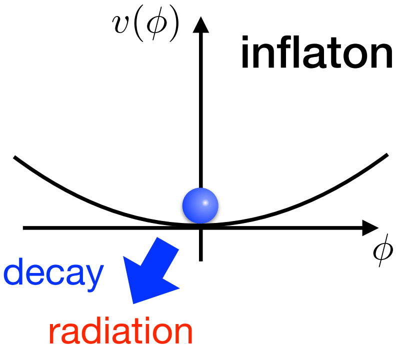
During inflation

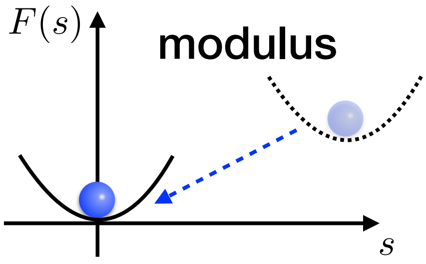




Large VEV of modulus —> large inflaton mass

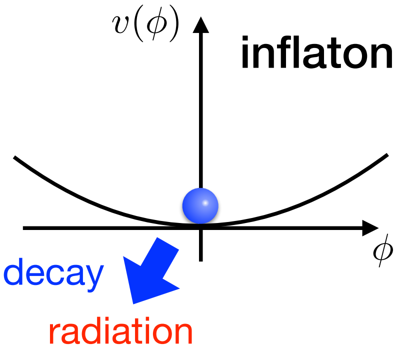
After inflation

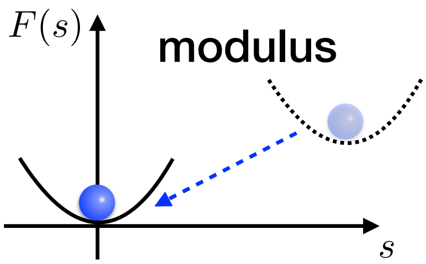




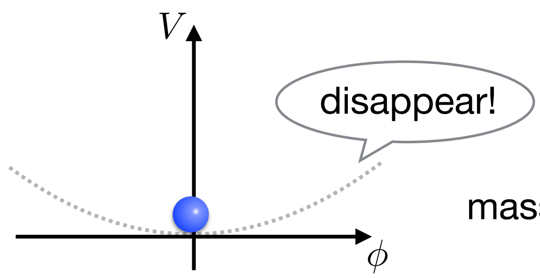
$$V(\phi, s) = F(s)v(\phi) + U(s)$$

After inflation





$$V(\phi, s) = F(s)v(\phi) + U(s)$$



massless / light inflaton

I. Chaotic Inflation in SUGRA

Kawasaki, Yamaguchi, Yanagida (2000)

Shift symmetry : $\Phi \to \Phi + iC$ ⇨ flat potential

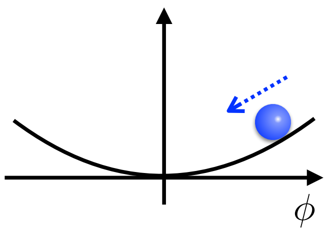
Kähler potential : $K(\Phi, \Phi^\dagger, X, X^\dagger) = \frac{1}{2}(\Phi + \Phi^\dagger)^2 + XX^\dagger + \ldots$

~~shift symmetry~~ ⇨ $\boxed{\text{Superpotential: } W = mX\Phi}$

Φ: inflaton, X: stabilizer

Scalar potential: $V = e^K \left[(D_i W) K^{i\bar{j}} (D_j W)^\dagger - 3|W|^2 \right]$

$$D_i W = \partial_i W + K_i W, \quad i = \Phi, X$$



$\boxed{V = \frac{1}{2} m^2 \phi^2}$ $\quad \Phi = \frac{1}{\sqrt{2}}(\eta + i\phi), \quad \eta, X \to 0$

I. Chaotic Inflation in SUGRA

NK, F. Takahashi 1509.01729

Φ: inflaton, X: stabilizer + S: modulus

$$m \to \lambda S$$

Superpotential : $W = \lambda S X \Phi$

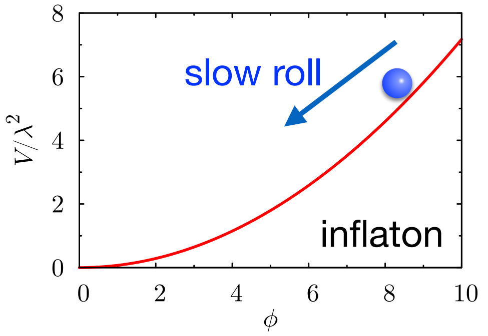
Kähler potential : $K = \frac{1}{2}(\Phi + \Phi^\dagger)^2 (1 + c_\Phi^{(2)} |S|^2 + c_\Phi^{(4)} |S|^4)$
$+ |X|^2 (1 + c_X^{(2)} |S|^2 + c_X^{(4)} |S|^4)$
$+ |S|^2 (1 + c_S^{(2)} |S|^2 + c_S^{(4)} |S|^4) + \cdots$

Scalar potential :

$$V(\phi, s) = e^K K^{X\bar{X}} |W_X|^2 = \boxed{\frac{1}{4}\lambda^2 \phi^2 s^2 \left(1 - \frac{1}{2} c_2 s^2 + \frac{1}{4} c_4 s^4 + \ldots \right)}$$

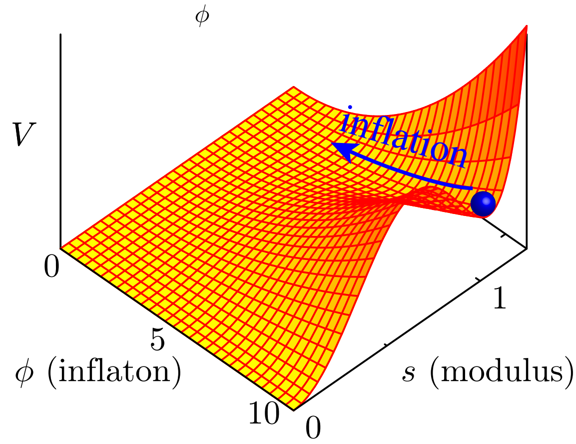
$$c_2 = c_X^{(2)} - 1, \quad c_4 = \frac{1}{2} - c_X^{(2)}(1 - c_X^{(2)}) - c_X^{(4)} + c_S^{(2)}$$

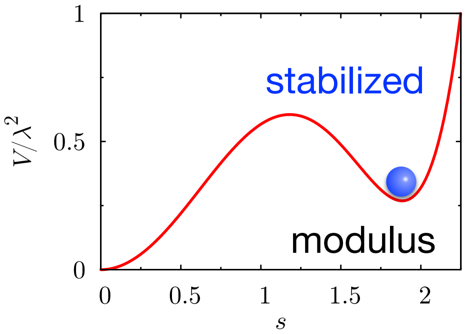
Inflaton potential



$$V(\phi, s) = \phi^2 F(s)$$

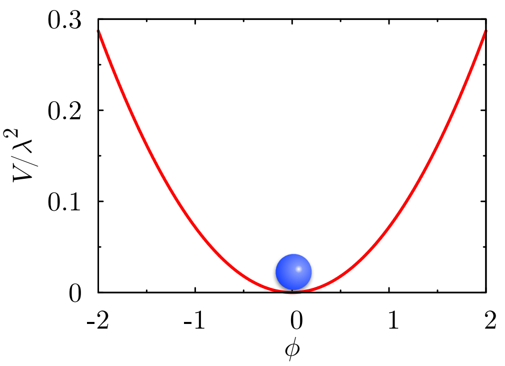
$$F(s) = \frac{1}{4}\lambda^2 s^2 \left(1 - \frac{1}{2}c_2 s^2 + \frac{1}{4}c_4 s^4 + \ldots\right)$$

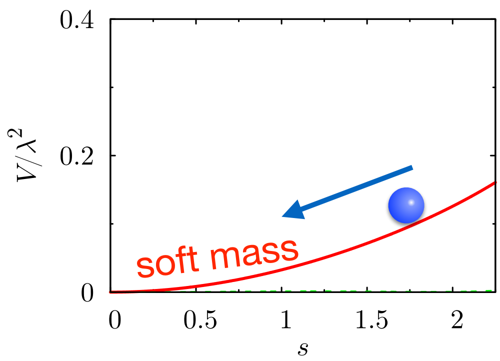


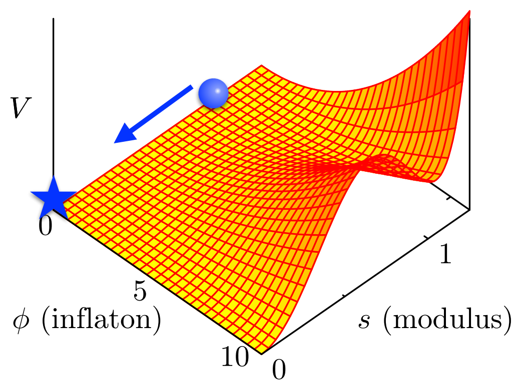


$$s_{\rm inf} \sim M_P \quad \rightarrow \quad \lambda \sim 10^{-5}$$

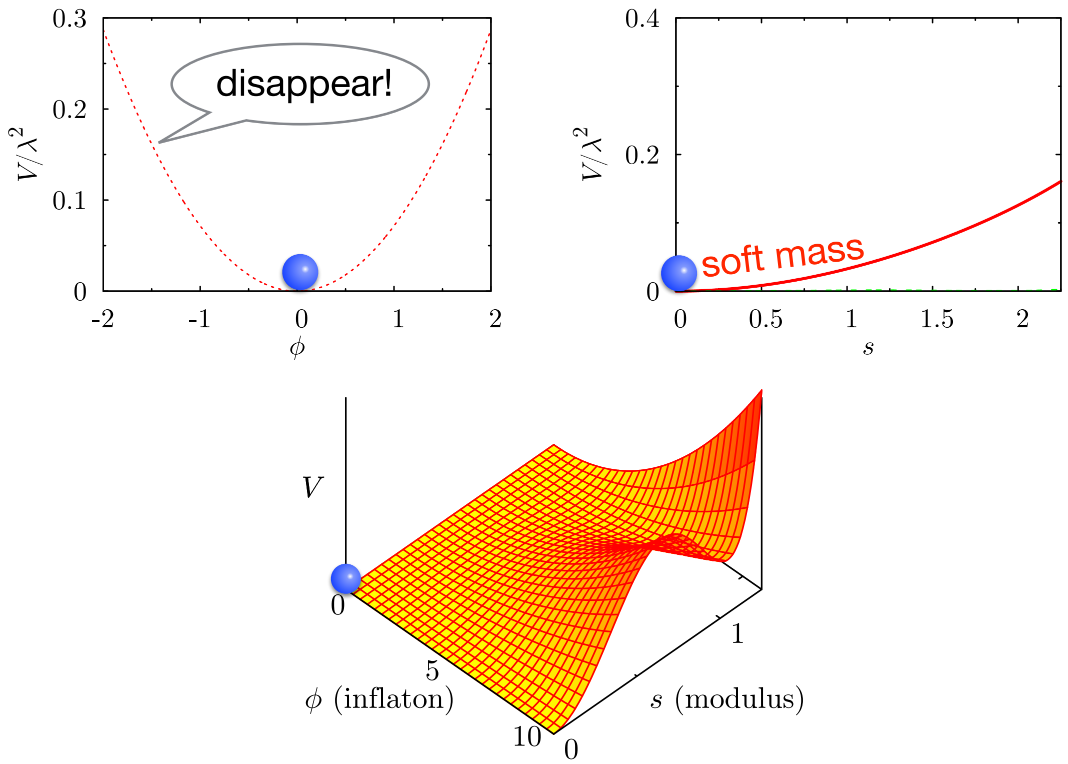
Post inflationary dynamics







Post inflationary dynamics

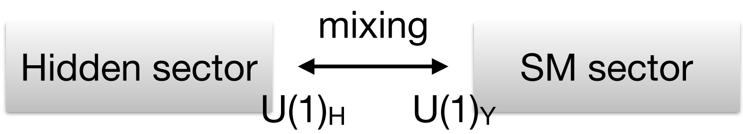


Reheating

Inflaton => right handed neutrinos : $W = y\Phi SNN + \gamma\Pi NN$

=> thermal / non thermal leptogenesis

Modulus (s) => Hidden light quarks : $W = hSQ\bar{Q}$



	Φ	S	X	N	Π	Q	$\bar{Q}$
$\mathrm{U}(1)_R$	0	0	2	1	0	1	1
Z_2	$-$	$-$	$+$	$+$	$+$	$-$	$+$
Z_{4B-L}	0	2	2	1	2	1	1

Reheating

Inflaton => right handed neutrinos : $W = y\Phi SNN + \gamma \Pi NN$

=> thermal / non thermal leptogenesis

Modulus (s) => Hidden light quarks : $W = hSQ\bar{Q}$

Hidden sector $\xleftrightarrow{\text{mixing}}$ SM sector

U(1)$_H$ U(1)$_Y$

	Φ	S	X	N	Π	Q	$\bar{Q}$
$\mathrm{U}(1)_R$	0	0	2	1	0	1	1
Z_2	$-$	$-$	$+$	$+$	$+$	$-$	$+$
Z_{4B-L}	0	2	2	1	2	1	1

inflaton is stable => dark matter

~~SUSY~~ $\rightarrow$ $m_{\phi,0}^2 \sim \frac{\lambda^2}{16\pi^2} m_{3/2}^2$

II. Hilltop /new inflation

Asaka+ (1999); Senoguz, Shafi (2004); Nakayama, F. Takahashi (2012)

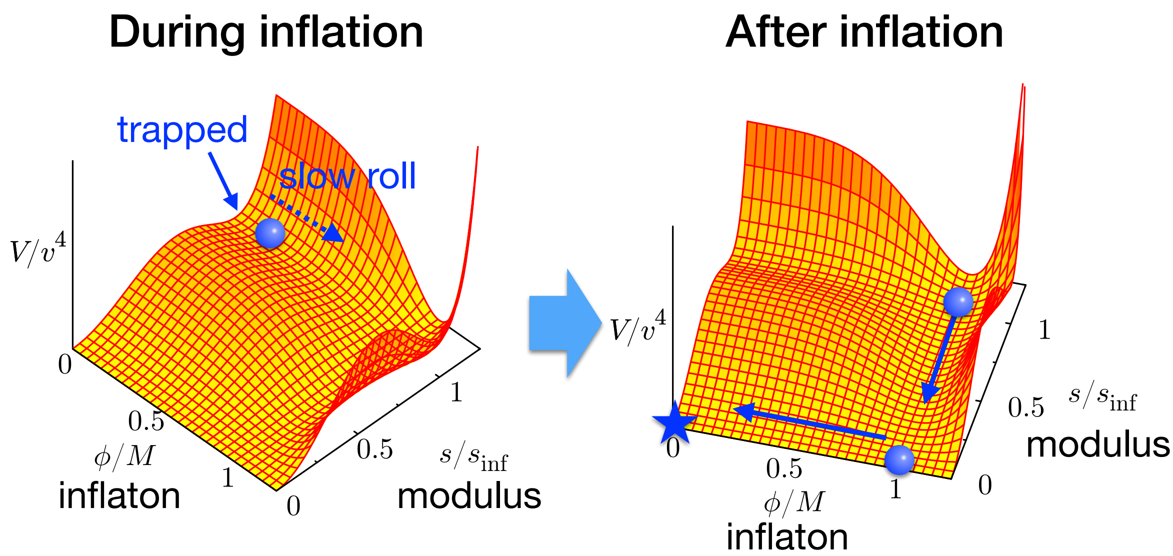
$$\boxed{W = XS\left(-\mu^2 + \frac{\Phi^{2m}}{M_*^{2m-2}}\right)}$$

additional

$$V = \frac{v^4 s^2}{2}\left(1 - \frac{2\phi^{2m}}{M^{2m}} + \frac{\phi^{4m}}{M^{4m}}\right)\left(1 - \frac{c_2 s^2}{2} + \frac{c_4 s^4}{4} - \frac{\beta\phi^2}{2} + \ldots\right)$$

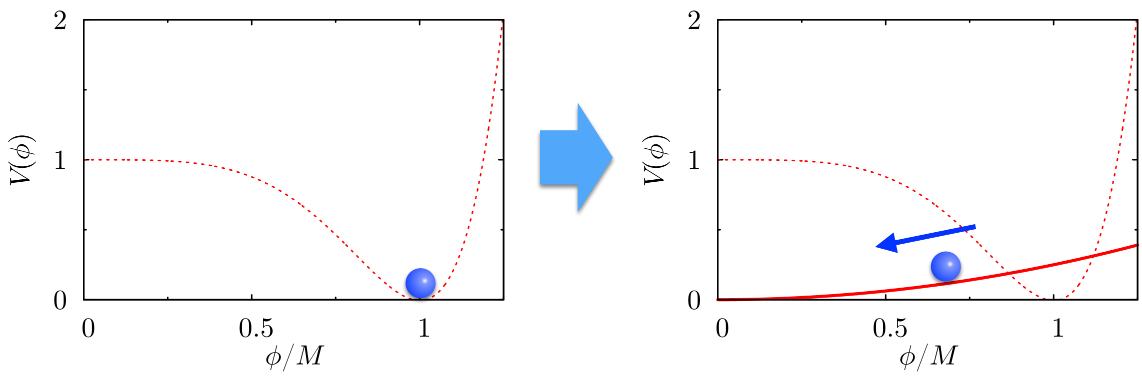
$$F(s) = \frac{1}{2}v^4 s^2\left(1 - \frac{1}{2}c_2 s^2 + \frac{1}{4}c_4 s^4 + \ldots\right),$$

$$v(\phi) = 1 - \frac{\beta\phi^2}{2} - \frac{2\phi^{2m}}{M^{2m}} + \frac{\phi^{4m}}{M^{4m}} + \ldots, \quad U(s) = 0.$$



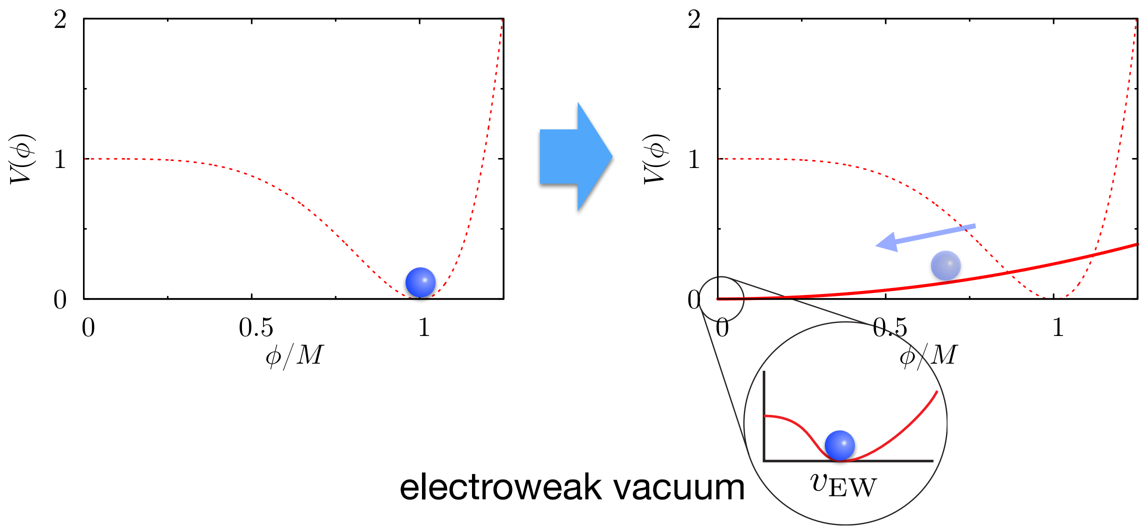
SUSY SM Higgs new inflation

$$W = XS\left(-v^2 + \frac{(H_u H_d)^2}{M_*^2}\right) + \mu H_u H_d$$



SUSY SM Higgs new inflation

$$W = XS\left(-v^2 + \frac{(H_u H_d)^2}{M_*^2} \right) + \mu H_u H_d$$



Summary

Inflaton potential may disappear by heavy field dynamics

I. Chaotic inflation in SUGRA

- Mass parameter is promoted to be a dynamical field : S
- <S> ≠ 0 during inflation & <S> = 0 after inflation, and then inflaton becomes (almost) massless.

=> Dark matter / dark radiation / searched by some experiments

II. New inflation in SUGRA

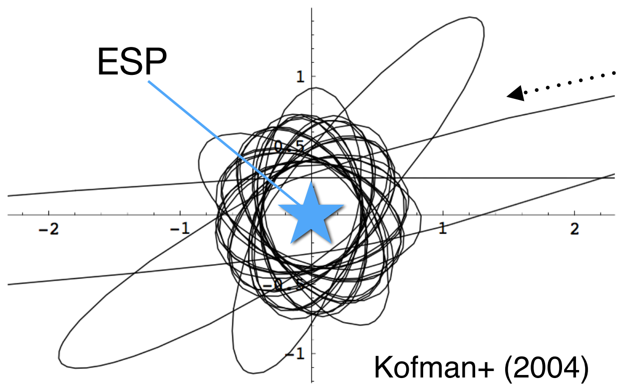
- Inflaton may return to the origin after reheating and rolls down to the electroweak vacuum

=> Inflaton can be MSSM Higgs field

ESP (enhanced symmetry point)

Kofman+ (2004)

Modulus is attracted to some symmetry restoration point at which some massless degrees of freedom appear.



Kinetic term

$$\mathcal{L}_K = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\left(1+\frac{1}{2}c_\Phi^{(2)}s^2+\frac{1}{4}c_\Phi^{(4)}s^4+\dots\right) + \frac{1}{2}\partial_\mu s\partial^\mu s\left(1+2c_S^{(2)}s^2+\frac{9}{4}c_S^{(4)}s^4+\dots\right)$$

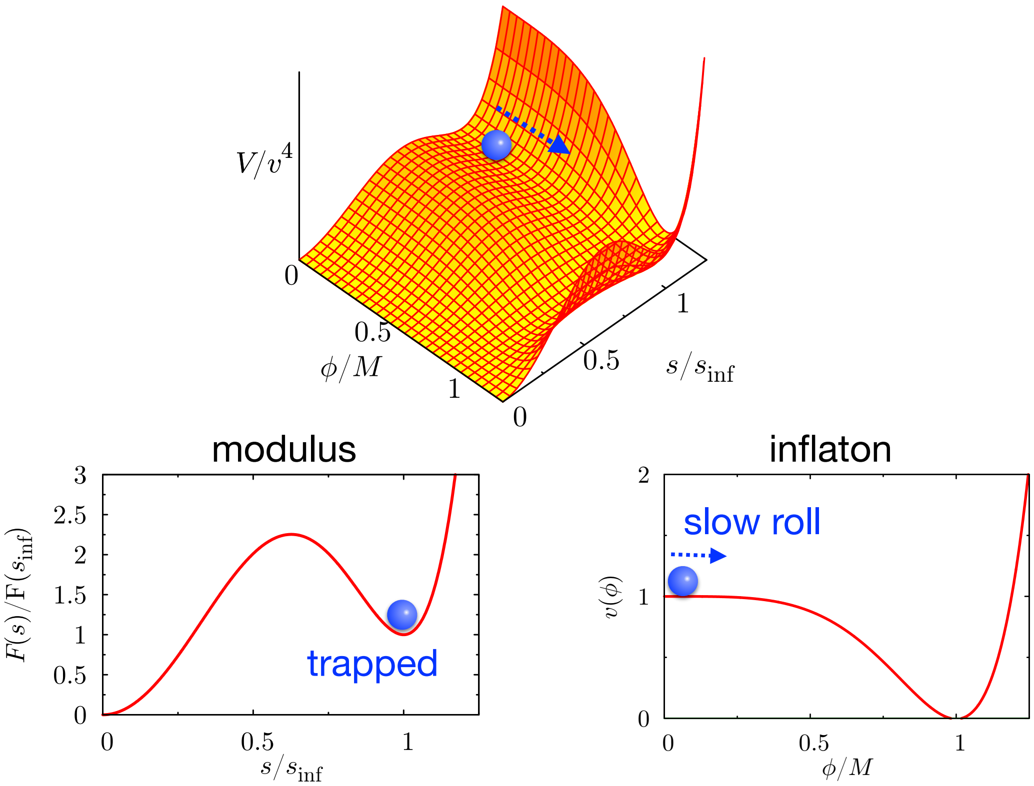
Scalar potential

$$V(\phi,s) = e^K K^{X\bar{X}}|W_X|^2, = \frac{1}{4}\lambda^2 s^2\phi^2\left(1-\frac{1}{2}c_2 s^2+\frac{1}{4}c_4 s^4+\dots\right)$$

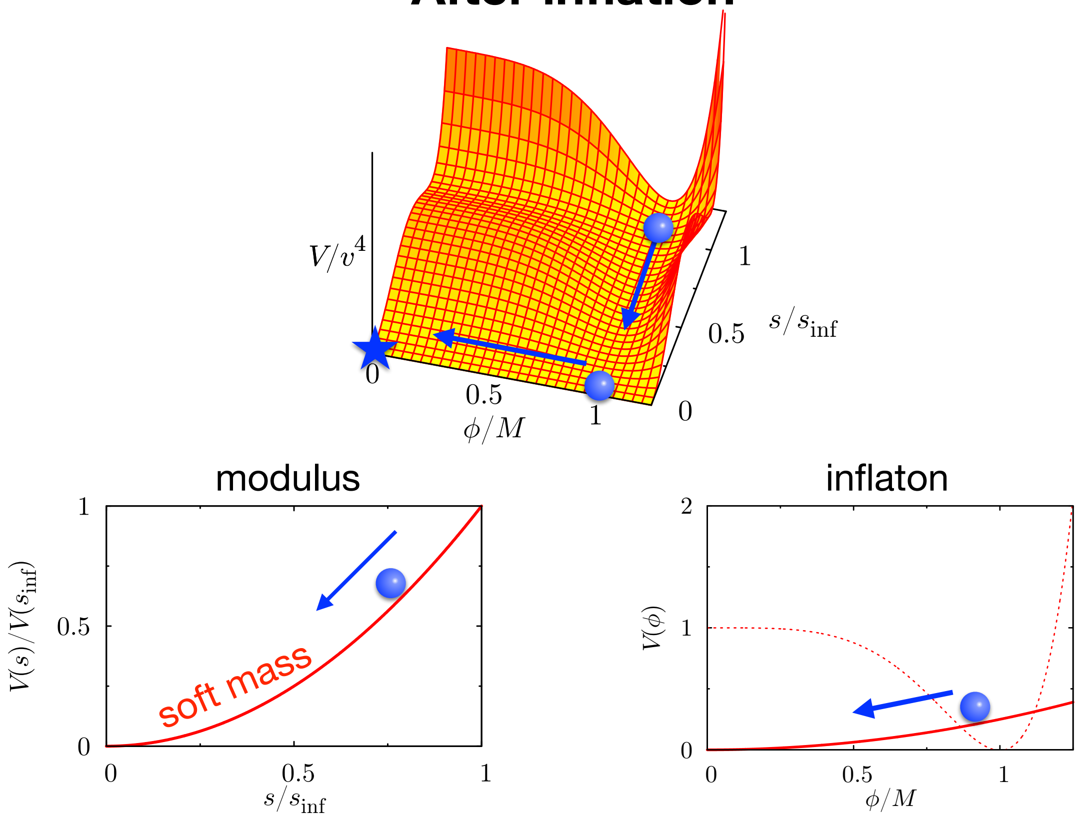
$$c_2 = c_X^{(2)}-1,\quad c_4=\frac{1}{2}-c_X^{(2)}(1-c_X^{(2)})-c_X^{(4)}+c_S^{(2)}$$

$$(\mathrm{U}(1)_\mathrm{R},\mathrm{U}(1)_\mathrm{S},\mathrm{Z}_2)\to\Phi(0,0,-),X(2,1,+),S(0,-1,-)$$

During inflation



After inflation



“Entanglement dynamics for two Unruh detectors in de Sitter space-time”

by Shingo Kukita

[JGRG25(2015)6a4]

Entanglement dynamics for two Unruh detectors on de Sitter space-time

SHINGO KUKITA(Nagoya Univ.)

YASUSADA NAMBU(Nagoya Univ.)

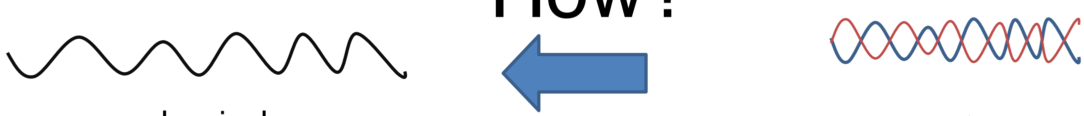
Motivation

fluctuations generated in the inflation can be represented by classical distribution function.

Our classical universe

How?

the origin of that structure



classical

quantum

A condition that quantum fluctuations become “classical”.

disappearance of entanglement(quantum correlation)

GOAL:
Understand how the entanglement of a quantum field in an expanding universe vanish.

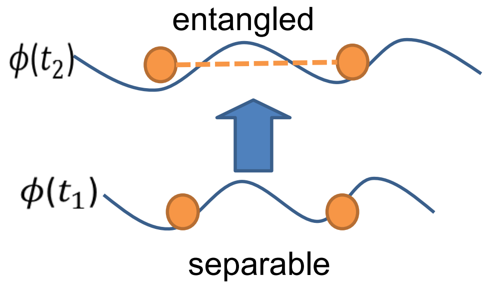
Detector model

calculating entanglement of a field is generally difficult.

quantum entanglement of the field

↓interpret

entanglement detection by “detectors”



Two comoving detector coupling with conformal massless scalar

$$H_S = \frac{\omega}{2}\sigma_3^{(1)} + \frac{\omega}{2}\sigma_3^{(2)} \qquad H_F = \int dx^3 \, (\partial\phi)^2 + \frac{1}{6}R\phi^2 \qquad H_{int} = \lambda(\sigma_1^{(1)}\phi(x_1) + \sigma_1^{(2)}\phi(x_2))$$

two level system $|\uparrow\rangle\ |\downarrow\rangle$ conformal massless scalar Assuming $\lambda \ll 1$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Method

We can use naive perturbation method to analyze this system.

However,This method can fail in long time.

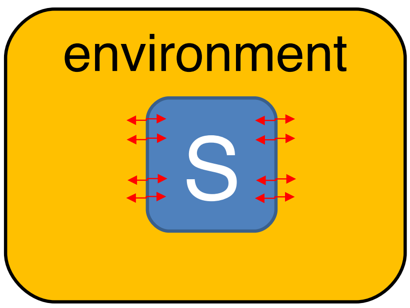
$$\left| \int dt\, H_{int}(t) \right|^2 << 1$$ If not, naive perturbation fails.

quantum master equation

Method to treat open quantum systems.

$$\dot{\rho}_S = i[H_{eff}, \rho_S] + \mathcal{L}[\rho_S]$$

$$\rho_S \equiv Tr_\phi \rho_{tot}$$



Method: quantum master equation

treat the background field as an environment(thermal bath)

- Thermalization and entanglement in Unruh effect (2004, Benatti et al)
- Thermalization in de Sitter space (2013, Fukuma et al)
- Entanglement in de Sitter space (2013, J. Hu H. Yu)

→Previous studies: impose rotating wave approximation (RWA)

The RWA master equation is easy to solve.
but
It can not detect quantum fluctuation of the field.

→need more general quantum master equation

Method: coarse graining approximation (schaller 2008)

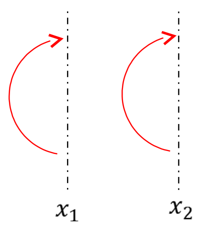
quantum master equation with coarse graining approximation

$$\dot{\rho}_S = -i[H_{eff}, \rho_S] - \int_t^{t+\Delta t} dt_1 dt_2 \, Tr_\phi[\tilde{H}_{int}(t_1), [\tilde{H}_{int}(t_2), \rho_S \otimes \rho_\phi]]$$

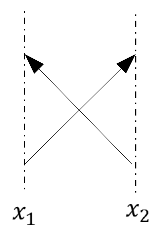
free parameter Δt $\Longleftrightarrow$ $\Delta E \Delta t > 1$ Heisenberg uncertainty

We use this quantum master equation.

this equation consists of two parts.



$$H_{int} = \lambda(\sigma_1^{(1)}\phi(x_1) + \sigma_1^{(2)}\phi(x_2))$$

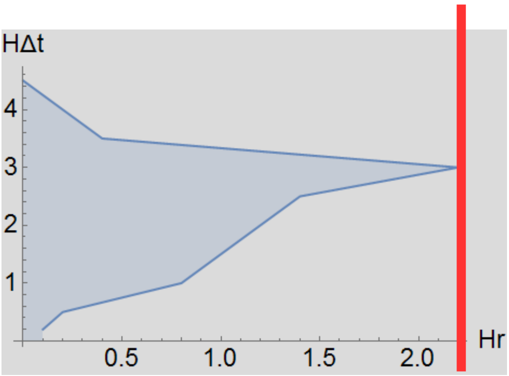


Self-correlation term
(~thermal fluctuation)

Two point
correlation

Result1: entanglement generation from the ground state

the region can be entangled from the ground state $|\downarrow\downarrow\rangle\langle\downarrow\downarrow|$



distance of the detectors

H/ω=1

λ=0.2

The region is compact.

There exists the critical distance r_c for all Δt.

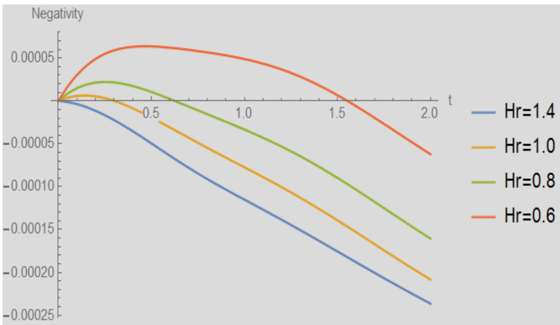
The ground state can be entangled for some parameters

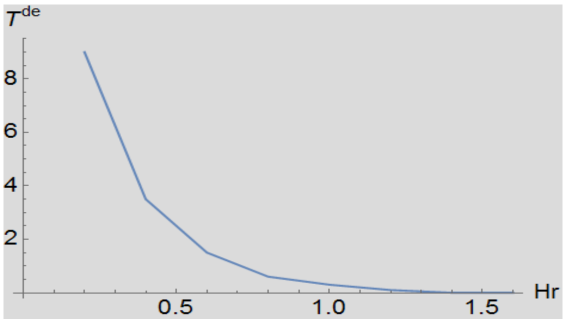
⟺ detect quantum fluctuations of the field

dynamics of entanglement ?

Result2: Disentanglement in long time dynamics

$H\Delta t = 2$





※We take negativity as a measure of quantum entanglement.
If the value of negativity is positive, the detectors is entangled.

The time of disentanglement T^{de} also depends on Hr.

entanglement vanishes for all parameters and initial states in the long-time evolution.

Result2: Disentanglement in long time dynamics

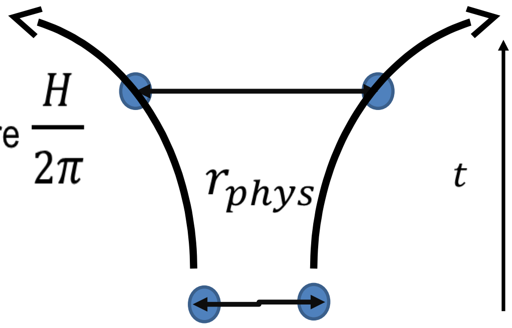
analytically explanation

The physical distance of the detectors →∞ as t →∞

・The correlations between detectors asymptote to 0.

・The self correlations remain finite and this destroy the entanglement.

Final state is the Gibbs state at the temperature $\dfrac{H}{2\pi}$



No entanglement

Summary and discussion

・We discussed entanglement and disentanglement process in an expanding universe by using the detector model.

In the analysis, we use CGA quantum master equation

・there exists the parameter region where the ground state never become entangled.

・The entanglement of the detectors finally disappears.

・Relation with the entanglement of field itself?

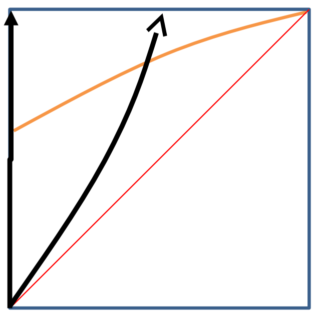
・Other condition for classicalization?

・How is the free parameter Δt decided in the realistic case?

APPENDIX

Negativity: one of the entanglement monotones.

$ds^2 = dt^2 - e^{2Ht} d\vec{r}^2$



two comoving detectors

“Cosmic censorship in dynamical brane backgrounds”

by Kunihito Uzawa

[JGRG25(2015)6a5]

Cosmic censorship in dynamical brane backgrounds

Kengo Maeda and Kunihito Uzawa

[arXiv:1510.01496 [hep-th]]

[1] Introduction

❖ String theory :

♠ This is the only viable unified fundamental theories at present.

♣ String theory contains p-branes ($p>1$) as well as strings.

❁ An innumerable number of static brane solutions have been discovered so far.

But …

☺ Cosmological brane solutions may also exist !

☞ Dynamical brane background

▶ "Dynamical" means time-dependent.

♦ Dynamical brane may be related to

- brane collision
  (Gibbons & Lu & Pope, Phys.Rev.Lett. 94 (2005) 131602)
- cosmic Big-Bang of our universe
  (Chen, et al., Nucl.Phys. B732 (2006) 118-135)
- black hole in expanding universe
  (Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)
  (Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)

☀ It is of great significance to understand the cosmological backgrounds profoundly.

☠ There is a naked singularity in the dynamical brane background due to ⋯

(i) the divergence of non-trivial dilaton (This appears in the static brane).

(ii) the time-dependence in the theory.

✍ The naked singularity in the 4-dim Einstein-Maxwell-dilaton theory with cosmological constant gives the violation of cosmic censorship.
(Horne & Horowitz, Phys.Rev. D48 (1993) 5457-5462)

☆ Question

Does the smooth initial data in the dynamical brane background evolve into the naked singularity?

☀ Cosmic censorship conjecture

(Penrose, Riv. Nuovo Cim. 1 (1969) 252-276)
(Penrose, "Singularities and time-asymmetry", (1979) 617-629)

- **Weak :**
  "Singularities have to be hidden by the event horizon of a black hole."

- **Strong :**
  "For smooth initial data with suitable matter systems, the maximal Cauchy development is not extendible."

★Outline my talk

∗ Geometry of dynamical brane background

∗ Cosmic censorship in dynamical M5-brane

∗ Summary and comments

[2] Geometry of dynamical brane background

(Gibbons & Lu & Pope, Phys.Rev.Lett. 94 (2005) 131602)
(Chen, et al., Nucl.Phys. B732 (2006) 118-135)

◆ Background

(1) The background has gravity, field strength, dilaton.
⇒ Einstein-Maxwell-dilaton theory

(2) This is a part of SUGRA.
ex) M-brane, D-brane

● The characteristics of M-brane :

- Classical solution of 11-dim SUGRA
- Static limit of M-brane : Black brane
- M-brane on time-dependent background
⇒ Black hole in expanding universe

(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)
(Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)

❑ Our results:

❀ The cosmic censorship is violated in dynamical M-brane background.

☞ This is similar to the result which has been obtained in Einstein-Maxwell-dilaton theory (with cosmological constant).

(Horne & Horowitz, Phys.Rev. D48 (1993) 5457-5462)

[3] Cosmic censorship in dynamical M5-brane

☆ Logic :

- We can set a regular and smooth initial data for the M5-brane.

- These initial data in the far past evolve into the curvature singularity.

- The cosmic censorship is violated.

◆ Dynamical M5-brane background

(Binetruy & Sasaki & Uzawa, Phys.Rev. D80 (2009) 026001)
(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)

(1+5)-dim spacetime

$$ds^2 = \boxed{\left(a\,t + b + \frac{M}{r^3}\right)^{-1/3} \eta_{\mu\nu} dx^\mu dx^\nu}$$

$$+ \boxed{\left(a\,t + b + \frac{M}{r^3}\right)^{2/3} \left(dr^2 + r^2 d\Omega_{(4)}\right)}$$

5-dim space

$$\left(a\,t + b + \frac{M}{r^3}\right) = 0 \ : \quad \text{curvature singularity}$$

✿ M5-brane

(Duff & Stelle, Phys.Lett. B253 (1991) 113-118)
(Güven, Phys.Lett. B276 (1992) 49-55)

✏ **matter (bosonic) :**
gravity, 4-form field strength

	0	1	2	3	4	5	6	7	8	9	10
M5	○	○	○	○	○	○					
x^N	t	x^1	x^2	x^3	x^4	x^5	r	y^1	y^2	y^3	y^4

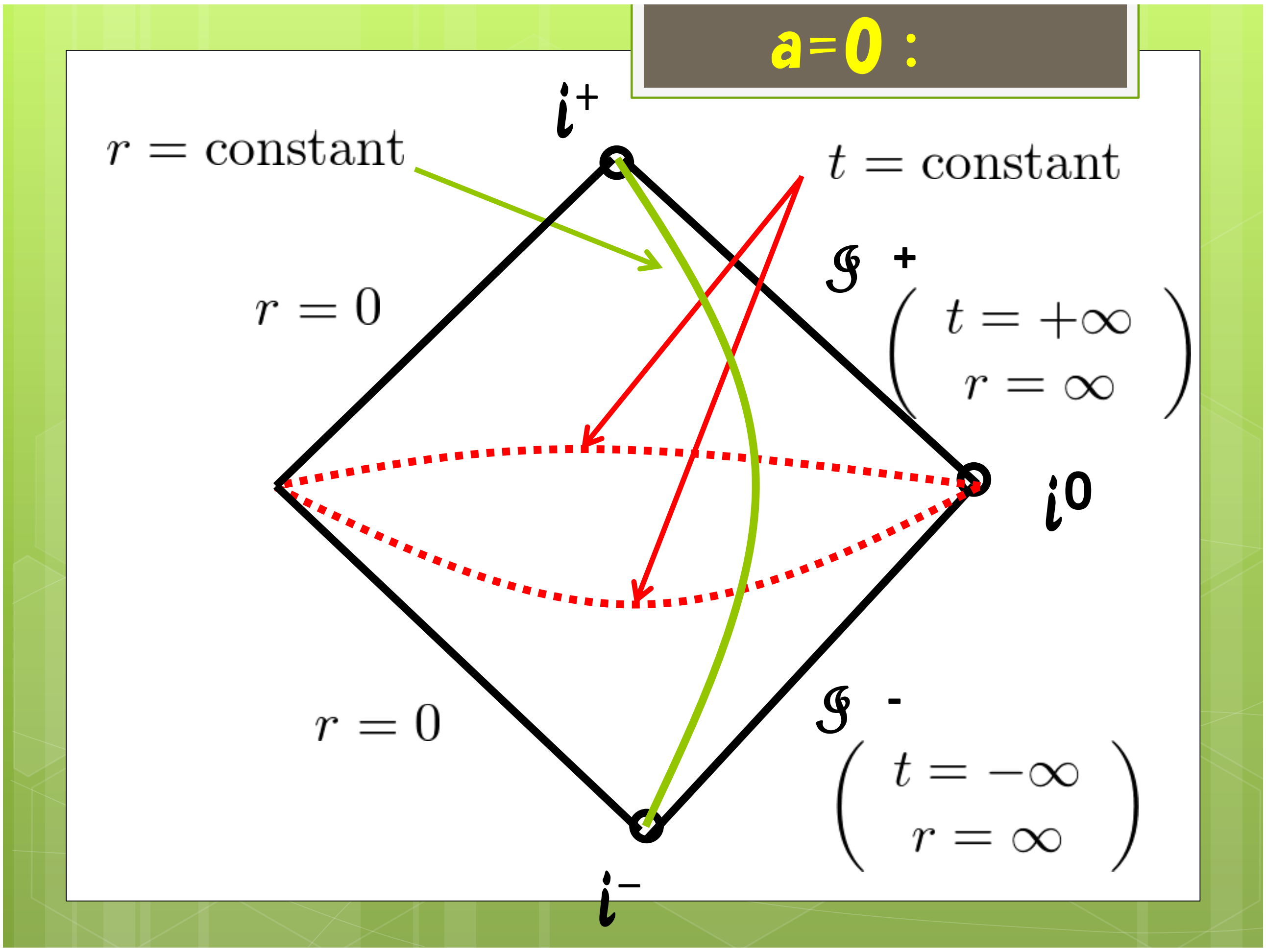
❈ The behavior of background

(i) Asymptotic region ($r \to \infty$): Kasner

$\Rightarrow$ Time dependent vacuum spacetime

(ii) Near horizon limit :
$t \to t/\varepsilon$, $r \to \varepsilon r$, $\varepsilon \to 0$

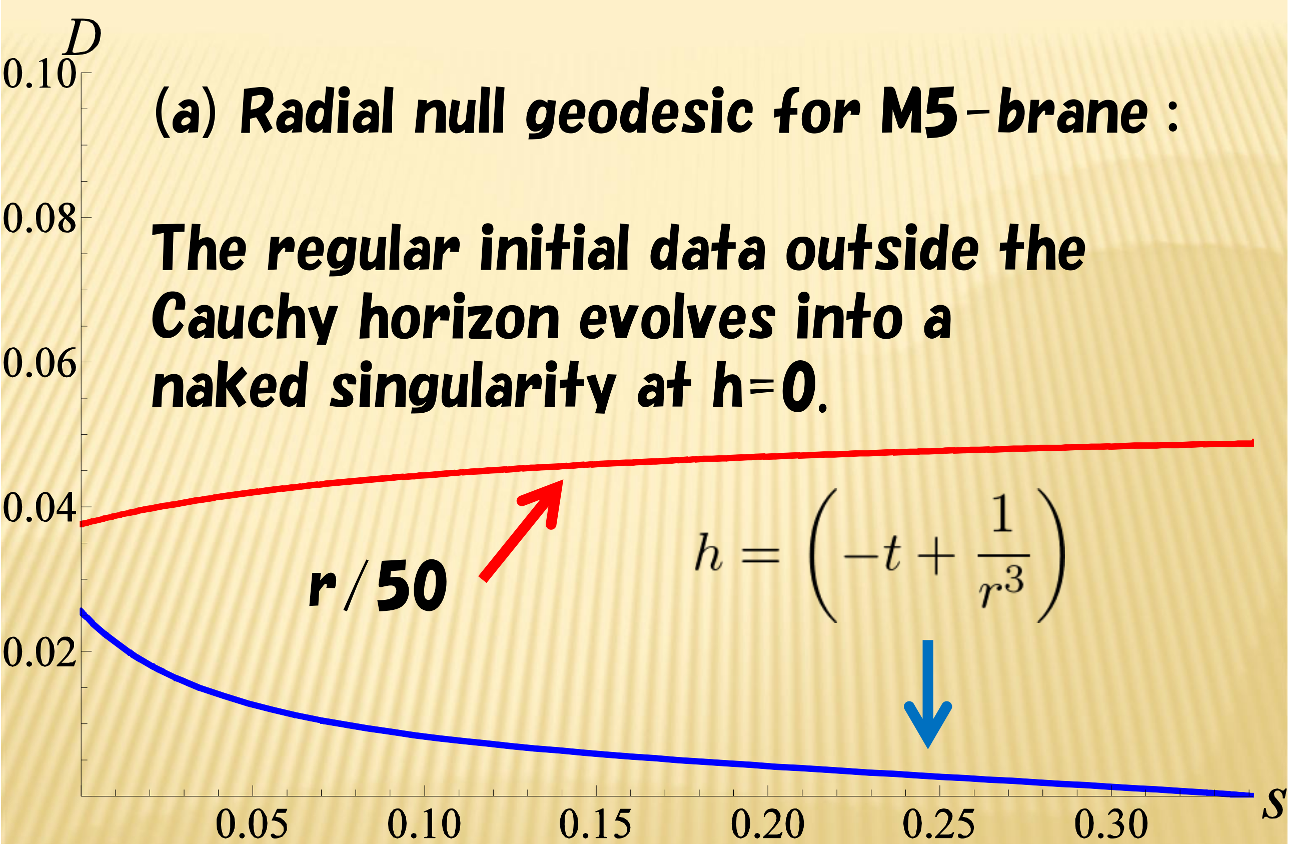
$\Rightarrow AdS_7 \times S^4$

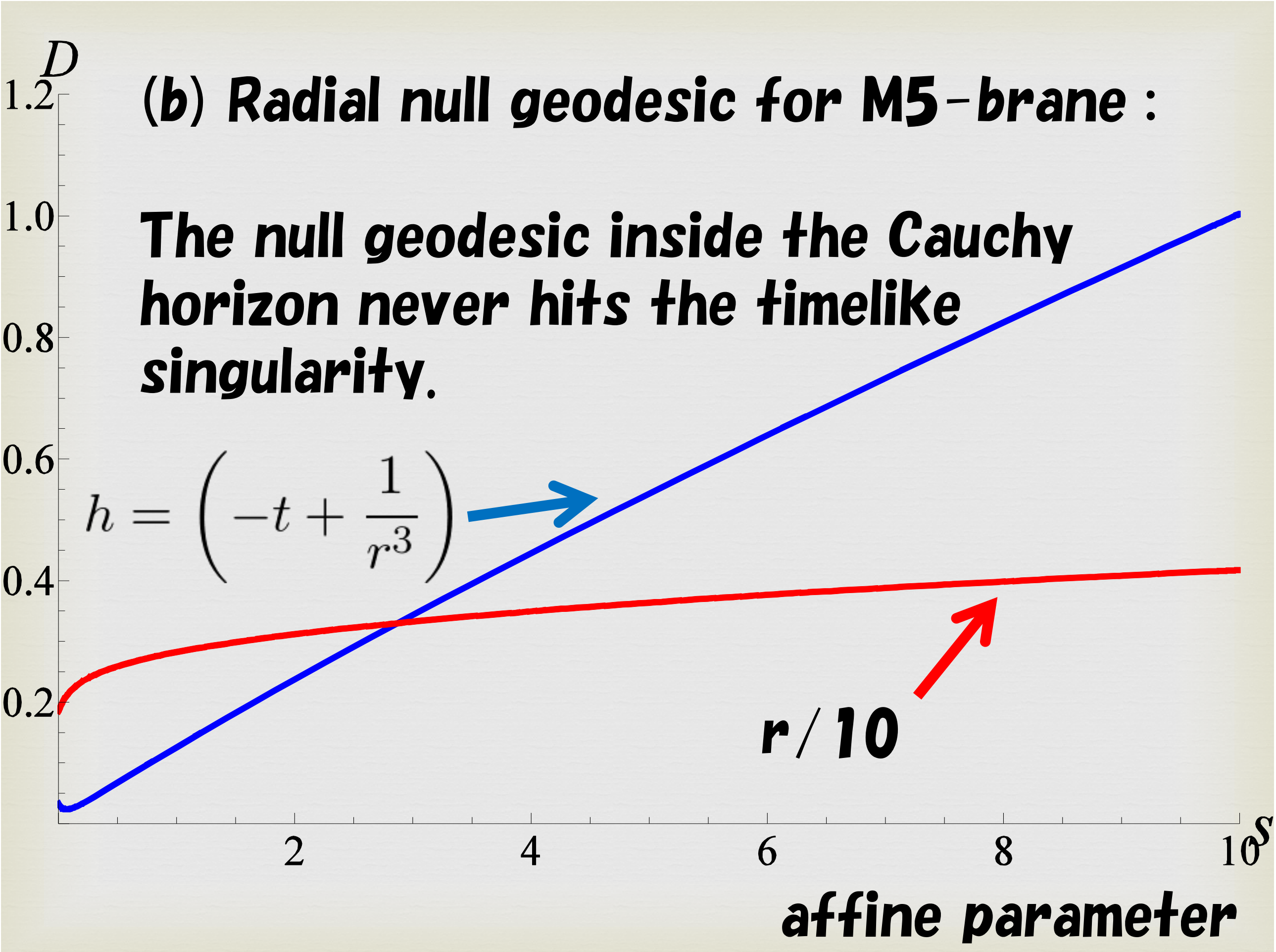


Geodesic equation :

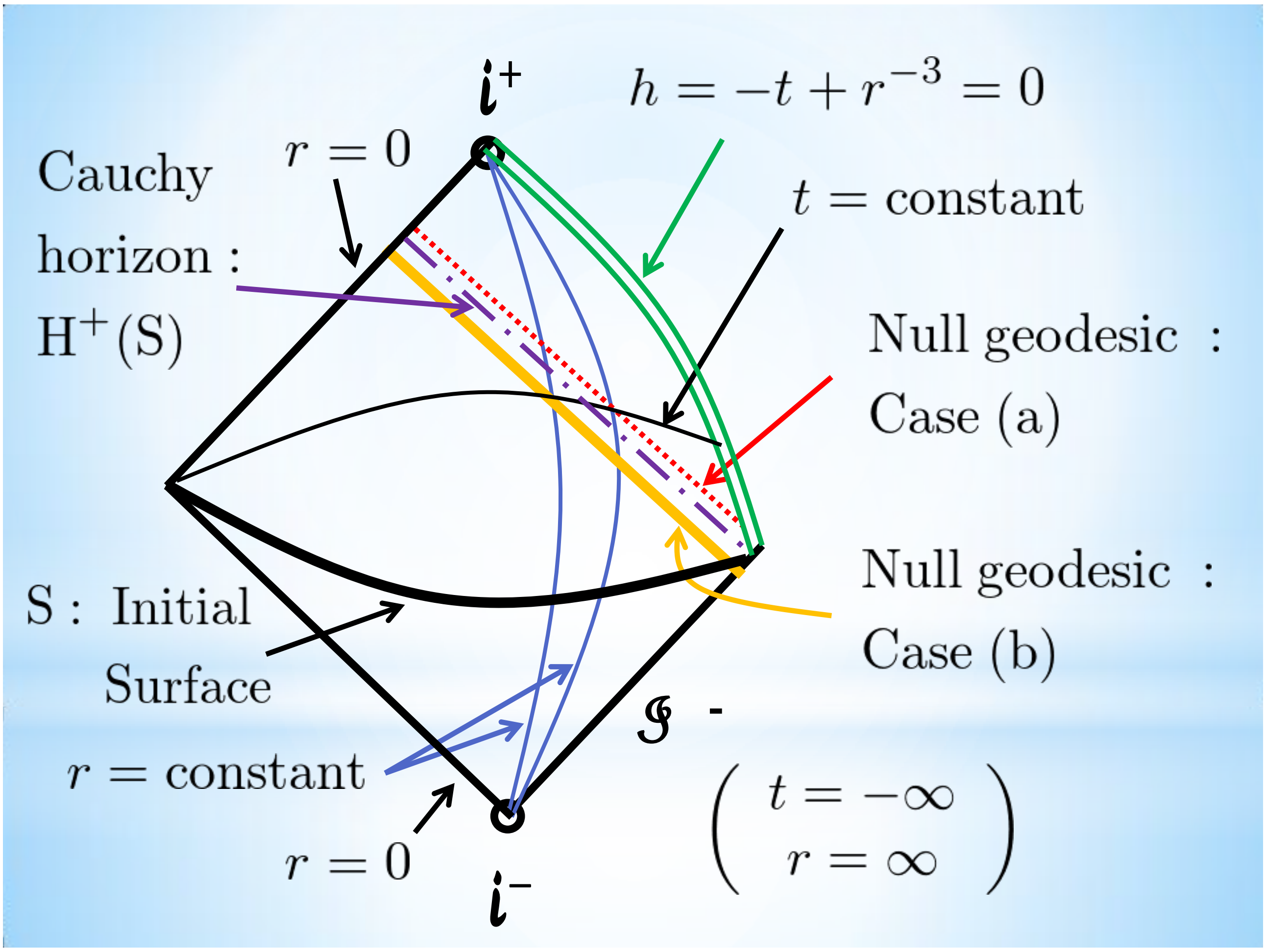
$$\frac{d^2 r}{ds^2} + \Gamma^r_{MN} \frac{dx^M}{ds} \frac{dx^N}{ds} = 0$$

- We can set a regular and smooth initial data for the M5-brane.
- The asymptotic behavior of the null curves depends crucially on whether r is inside or outside the Cauchy horizon.





D
1.2
1.0
0.8
0.6
0.4
0.2
(b) Radial null geodesic for M5-brane :
The null geodesic inside the Cauchy horizon never hits the timelike singularity.
$h = \left(-t + \frac{1}{r^3}\right)$
r / 10
2
4
6
8
10
S
affine parameter



i^+
$h = -t + r^{-3} = 0$
$r = 0$
Cauchy horizon : $H^+(S)$
$t =$ constant
Null geodesic : Case (a)
Null geodesic : Case (b)
S : Initial Surface
$r =$ constant
$\mathfrak{J}^-$
$\left(\begin{matrix} t = -\infty \\ r = \infty \end{matrix} \right)$
$r = 0$
i^-

[4] Summary and comments

(1) There is a singularity due to the time dependence in Einstein-Maxwell-dilaton theory.

(2) For dynamical M5-brane, we can set smooth initial data evolving into a timelike curvature singularity.

(3) For dynamical p-brane, the cosmic censorship is not violated by the non-trivial dilaton.

“Modified gravity inside astrophysical bodies”

by Ryo Saito

[JGRG25(2015)6b1]

JGRG25, Kyoto, 10th December 2015

Modified Gravity *inside* Astrophysical Bodies

Ryo SAITO (APC, Paris)

With Daisuke YAMUCHI, Shuntaro MIZUNO, Jerome GLEYZS, David LANGLOIS

JCAP **1506** (2015) 008

Infrared modification of gravity

- The expansion of the universe is accelerated today, **BUT** our best theory (GR [EH term] + SM) fails to explain it.

- Gravity should be a major player, **BUT** its nature is little known on cosmological scales;

  Gravity might not be described by GR.

Questions:

Is it possible **to modify GR on cosmological (IR) scales without spoiling its success in the solar-system observations?**

If possible**,**

is it possible **to test the modified gravity theories?**

Vainshtein mechanism

The first question is non-trivial.

Modified gravity models typically have

a light scalar dof universally coupled to matter

It mediates **a new long-range force (fifth force)** at short scales.

Screening mechanisms

Strong coupling for (nonlinear) derivative interactions.

➔ A fifth force sourced by an object is suppressed.

Vainshtein mechanism

[Galileon-type scalar-tensor theories, massive gravity,...]

Partial breaking of Vainshtein mechanism

In a very general class of scalar-tensor theories (GLPV theory [Gleyzes+ 2014])

the Vainshtein mechanism can be *partially* broken:

$$\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r} = G_N \left(\frac{M}{r^2} - \underline{\epsilon M''} \right)$$

ε : model-dependent (dimensionless) parameter

M : enclosed mass

T. Kobayashi, Y. Watanabe and D. Yamauchi (2015)

Gravity is modified ***inside*** a source but not outside.

Impact on the stellar structure

RS, D. Yamauchi, S. Mizuno, J. Gleyzes and D. Langlois, JCAP **1506** (2015) 008

Model for stellar interiors

Static, spherically symmetric, polytropic model

（Realistic stars [Koyama & Sakstein 2015]）

Basic equations

Force balance

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho \frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r}$$

Only the gravitational law is modified!

Poisson equation

$$\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r} = G_N \left(\frac{M}{r^2} - \epsilon M'' \right)$$

Equation of state

$$P = K\rho^{1+\frac{1}{n}}$$ (n=1: neutron star, n=3: sun)

Modified Lane-Emden equation

Closed equation for the density

$$\boxed{\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\xi^2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\chi-\epsilon\xi^2\chi^n\right)\right]=-\chi^n}$$

The variables were made dimensionless through

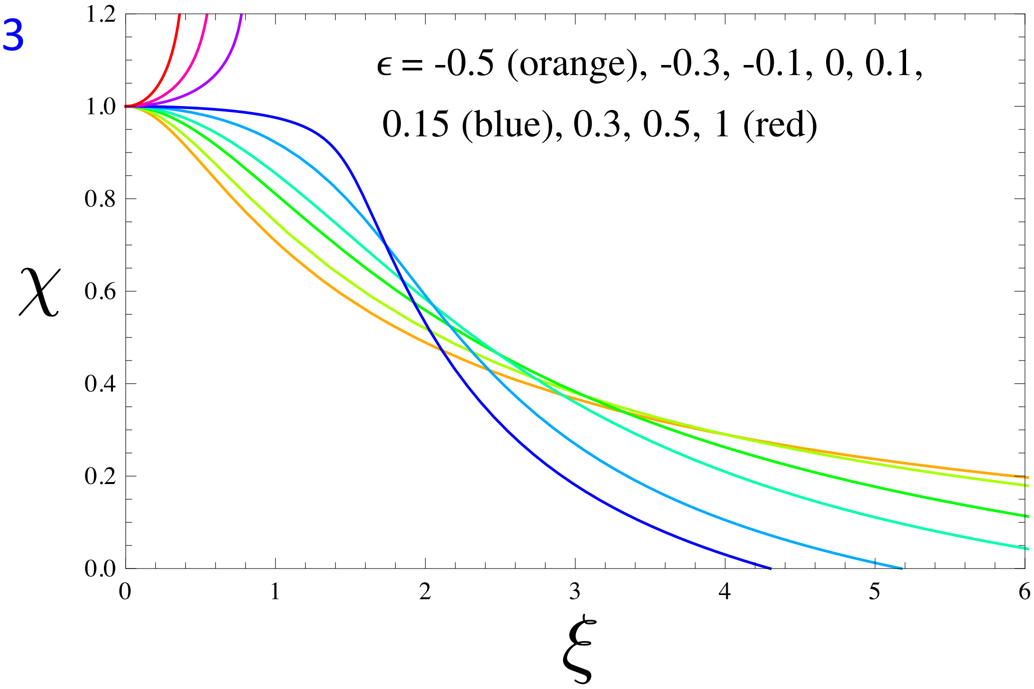
$$\xi\equiv r\sqrt{\frac{4\pi G_{\mathrm{N}}}{(n+1)K\rho_c^{-1+\frac{1}{n}}}},\qquad \rho=\rho_c\chi^n$$

It reduces to the standard Lane-Emden equation when $\epsilon\longrightarrow 0$

Solutions

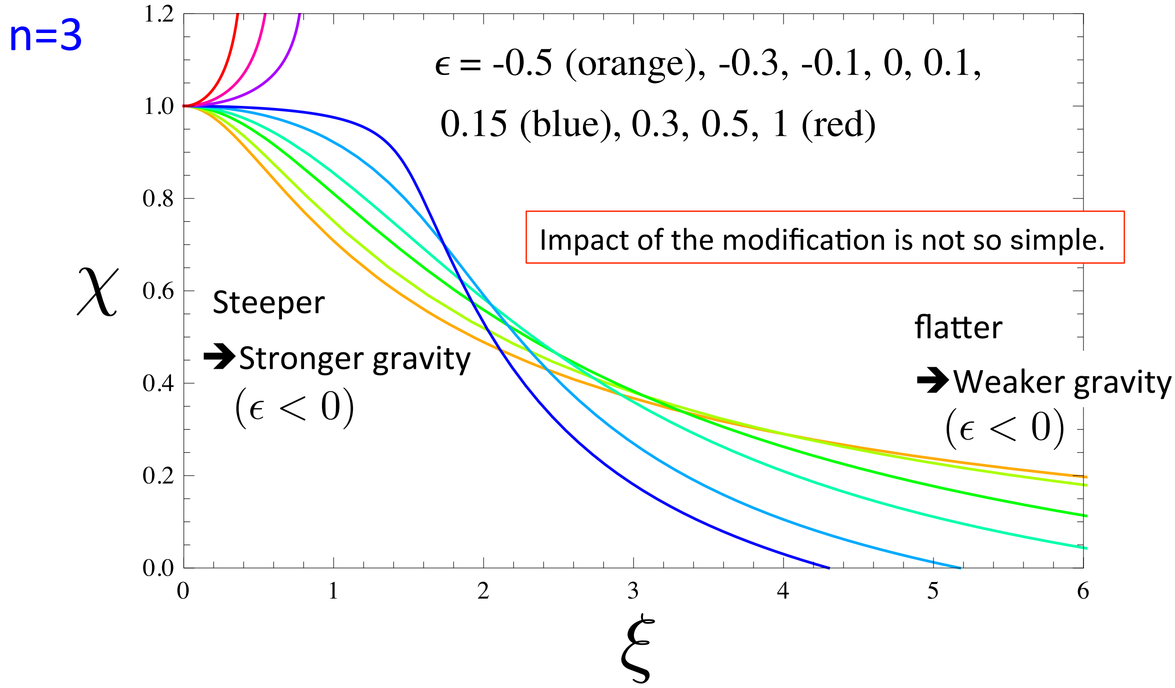
Numerical solutions of the modified LE equation:

n=3



Solutions

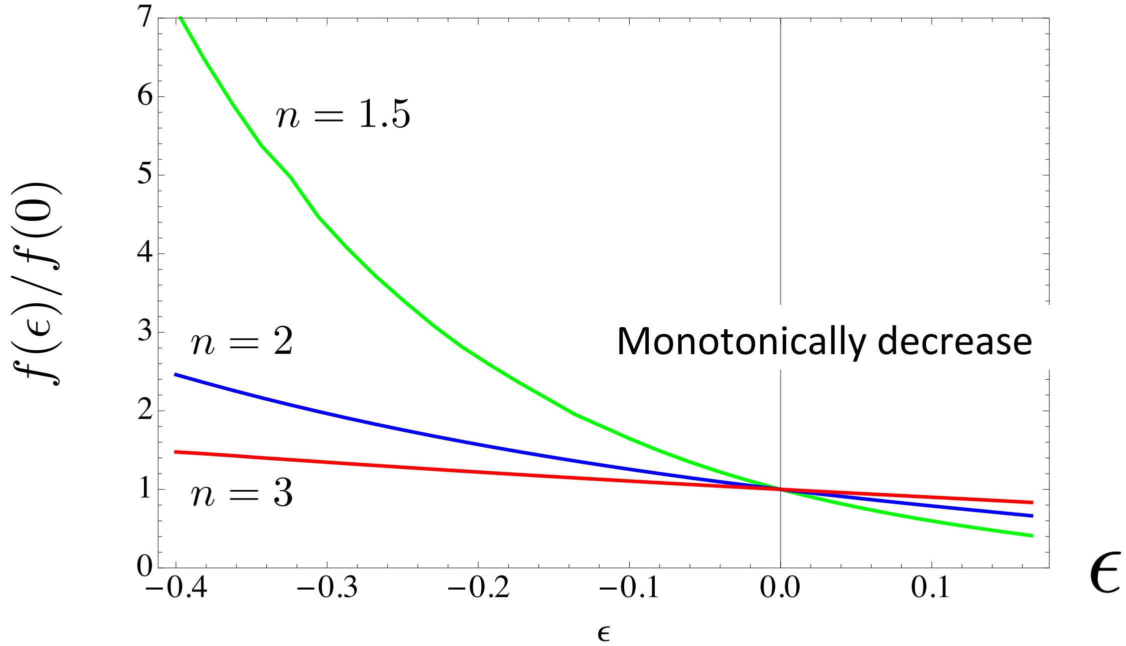
Numerical solutions of the modified LE equation:



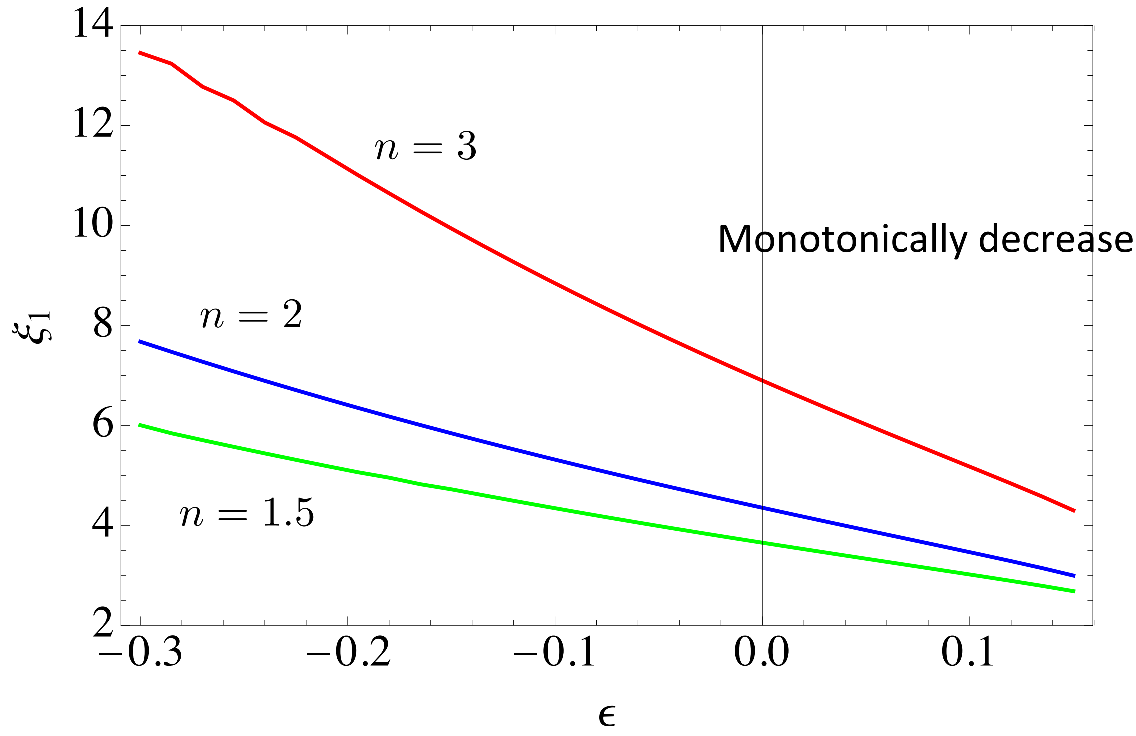
Mass−Radius relation

Mass-Radius relation is modified as:

$$M = f(\epsilon) R^{\frac{n-3}{n-1}}$$

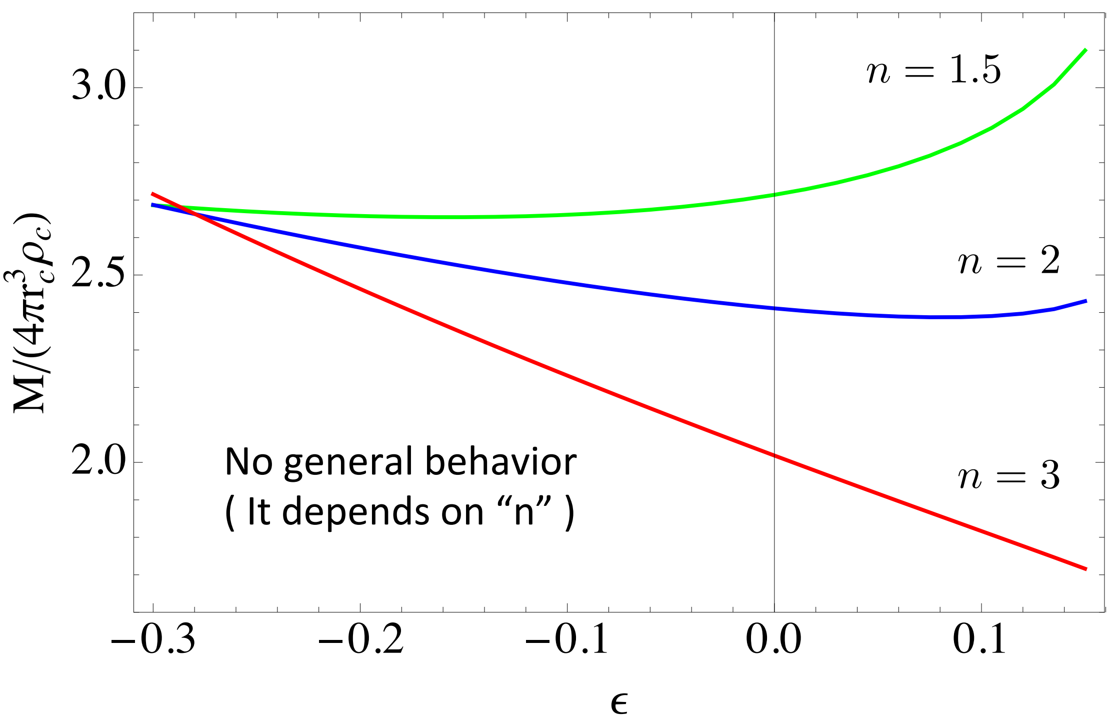


Radius



More sensitive to ε for softer equation of state.

Mass

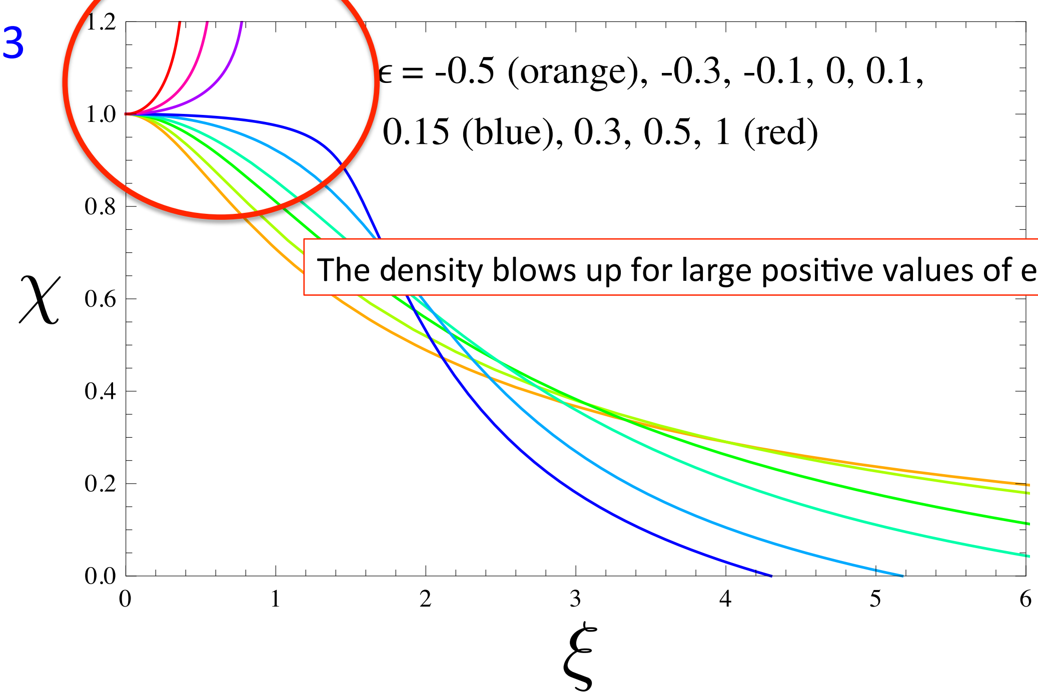


More sensitive to ε for softer equation of state.

Solutions

Numerical solutions of the modified LE equation:

n=3



Universal bound on the modification

Near the center,

$$\rho = \rho_c + \frac{1}{2}\rho_2 \left(\frac{r}{R}\right)^2 + \cdots \quad \Rightarrow \quad M = \frac{4\pi\rho_c r^3}{3} + \mathcal{O}(r^5)$$

Gravity becomes repulsive for $\epsilon > \frac{1}{6}$.

$$\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r} = G_N \left(\frac{M}{r^2} - \epsilon M''\right)$$
$$\simeq (1-6\epsilon) G_N \frac{M}{r^2}$$

→ The pressure has to increase → The density has to increase

(This is also true for larger radii.)

Very general bound $\epsilon < \frac{1}{6}$ (for any physically reasonable EoS)

Constraints from Red Dwarf stars

J. Sakstein [arXiv:1510.05964]

Minimum mass for hydrogen burning < Observed minimum RD mass

Lower limit on ε

$$\epsilon > -0.0068$$

Combining with our limit, ε should be in the range:

$$-0.0068 < \epsilon < 1/6$$

Implication of the bound

What is "ε"?

GLPV theories = Horndeski theories + disformal coupling

$$\mathcal{L} \supset \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi\, T^{\mu\nu};\;\; X \equiv (\partial\phi)^2$$

"ε" represents the amplitude of the derivative (X-dependent) coupling:

$$\epsilon \propto \left(\frac{\partial\Gamma}{\partial X}\right)^2$$

More generally, any derivative coupling can break the Vainshtein mechanism.
[RS, D. Langlois, D. Yamauchi, in preparation]

Constraints on "ε" → Constraints on derivative couplings

Summary

- Partial breaking of the Vainshtein screening mechanism;

  A deviation from GR *inside* a source but not outside.

Cosmology can be probed by seeing astrophysical objects.

- The stellar structure significantly changes by the modification of gravity without any conflict with solar-system constraints.
- A universal bound on the amplitude of the modification can be obtained, independently of the details of the equation of state.
- Derivative couplings to matter are tightly constrained.

“Slowly-rotating black hole solutions in Horndeski gravity”

by Masato Minamitsuji

[JGRG25(2015)6b2]

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

grit
gravitation in técnico

Slowly-rotating black hole solutions in Horndeski gravity

Masato Minamitsuji
(CENTRA, IST, U-Lisboa)

A. Maselli, H. O. Silva, M.M. and E. Berti
Physical Review D 92, 104049 (2015) [arXiv: 1508.03044]

Introduction

- GR has passed all the experimental tests in the weak-field/slow-motion regimes *with flying colors*.
- Observational/ theoretical issues with GR, such as the origins of Inflaton(s)/ DM/ DE, have motivated us to investigate modified theories of gravitation (MGs) in UV and IR regimes.

$\Longrightarrow$ Search of unambiguous signatures of MGs in the strong gravity regimes, in/around BHs and NSs, is a major goal of recent studies, in perspective of forthcoming electromagnetic and GW probes. Berti. et. al, 1501.07274

- Horndeski gravity, known as the most general single scalar-tensor gravity with 2nd order EOMs, includes most of familiar MG models, and so far has been analyzed mainly in the context of cosmology.

$\Longrightarrow$ *Slowly-rotating BH solutions* in Horndeski gravity.

Horndeski Gravity

Horndeki (74)
Deffayet, Deser & Esposito-Farese (09)
Kobayashi, Yokoyama and Yamaguchi (11)

Most general single field scalar-tensor gravity with 2nd order EOMs

$$S = \sum_{i=2}^{5} \int d^4x\sqrt{-g}\mathcal{L}_i \qquad G_i = G_i(\phi, X) \qquad X = -\frac{1}{2}g^{\mu\nu}\phi_\mu\phi_\nu$$

$$\mathcal{L}_2 = G_2$$

$$\mathcal{L}_3 = -G_3\,\Box\phi$$

$$\mathcal{L}_4 = G_4\,R + G_{4X}\left[(\Box\phi\,)^2 - \phi_{\mu\nu}^2\right]$$

$$\mathcal{L}_5 = -G_5\,G_{\mu\nu}\phi^{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi\,)^3 + 2\phi_{\mu\nu}^3 - 3\phi_{\mu\nu}^2\Box\phi\right]$$

Horndeski Gravity

contains most of the simple and familiar MG/DE models

$$S = \sum_{i=2}^{5} \int d^4x\sqrt{-g}\mathcal{L}_i \qquad G_i = G_i(\phi, X) \qquad X = -\frac{1}{2}g^{\mu\nu}\phi_\mu\phi_\nu$$

$\mathcal{L}_2 = G_2$ $\Longrightarrow$ Quintessence $X - V(\phi)$, K-essence

$\mathcal{L}_3 = -G_3\,\Box\phi$ $\Longrightarrow$ DGP $G_3 = \alpha\,X$

$\mathcal{L}_4 = G_4\,R + G_{4X}\left[(\Box\phi\,)^2 - \phi_{\mu\nu}^2\right]$ $\Longrightarrow$ Nonminimal coupling $f(\phi)R$ / $f(R)$

GR $G_4 = \frac{1}{2}$

$$\mathcal{L}_5 = -G_5\,G_{\mu\nu}\phi^{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi\,)^3 + 2\phi_{\mu\nu}^3 - 3\phi_{\mu\nu}^2\Box\phi\right]$$

Shift-symmetric Horndeski Gravity

subclass of Horndeski gravity invariant under $\phi \to \phi + c$

$$S = \sum_{i=2}^{5} \int d^4x\sqrt{-g}\mathcal{L}_i \qquad G_i = G_i(X) \qquad X = -\frac{1}{2}g^{\mu\nu}\phi_\mu\phi_\nu$$

$$\mathcal{L}_2 = G_2$$

$$\mathcal{L}_3 = -G_3 \,\Box\phi$$

$$\mathcal{L}_4 = G_4\, R + G_{4X}\left[(\Box\phi\,)^2 - \phi_{\mu\nu}^2\right]$$

$$\mathcal{L}_5 = -G_5\, G_{\mu\nu}\phi^{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi\,)^3 + 2\phi_{\mu\nu}^3 - 3\phi_{\mu\nu}^2\Box\phi\right]$$

Several models in shift-symmetric Horndeski gravity

- Nonminimal derivative coupling gravity

Saridakis and Sushkov (10)
Gubitosi and Linder (11)

$$\mathcal{L} = \zeta R + 2\eta X + \beta G^{\mu\nu}\phi_\mu\phi_\nu - 2\Lambda_0 \implies \begin{aligned} G_2 &= -2\Lambda_0 + 2\eta X \\ G_4 &= \zeta + \beta X \\ G_3 &= G_5 = 0 \end{aligned}$$

- Einstein-dilaton Gauss-Bonnet (EdGB) gravity

Kanti, et.al (96)
Pani and Cardoso (09)…

$$\mathcal{L} = \frac{1}{2}R + X + \xi(\phi)R_{GB}^2 \implies \begin{aligned} G_2 &= X + 8\xi^{(4)}X^2(3 - \ln X) \\ G_3 &= 4\xi^{(3)}X(7 - \ln X) \\ G_4 &= \frac{1}{2} + 4\xi^{(2)}X(2 - \ln X) \\ G_5 &= -4\xi^{(1)}\ln X \end{aligned}$$

$$R_{GB}^2 = R_{\alpha\beta\mu\nu}^2 - 4R_{\mu\nu}^2 + R^2$$

$$\xi^{(n)} = \frac{\partial^n\xi}{\partial\phi^n}$$

This theory is *shift-symmetric* only for the linear coupling $\xi(\phi) = \alpha\phi$

Sotiriou and Zhou (14)

Black Holes in Scalar-Tensor Gravity

- Kerr solution is the unique endpoint of gravitational collapse in GR.
- GR BH solution is also the unique endpoint in
  - BD gravity Hawking (72)
  - ST gravity with potential, including $f(R)$ Sotiriou and Faraoni (11)
  - k- essence models Graham and Jha (14)
- EdGB gravity of $\xi(\phi) = e^{\phi}$ admits BHs with a nontrivial scalar hair
  - Static spherically symmetric BHs Kanti, et. al (96)
  - Rotating BHs Pani and Cardoso (09) Kleihaus, Kunz and Radu (11) Ayzenberg and Yunes (15), Maselli, et.al (15)
- In the *shift-symmetric* Horndeski gravity, BHs *cannot* have nontrivial scalar hair, except for the EdGB gravity with the linear coupling.
  Hui and Nicolis (11), Sotiriou and Zhou (14)

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No-Hair Theorem in Shift-symmetric Horndeski Gravity

Hui and Nicolis (11)

1) Assumptions:

a) Static and spherically symmetric spacetime

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2(d\theta^2 + sin^2\theta d\phi^2)$$

b) Asymptotic flatness $\Longrightarrow A \to 1$ and $B \longrightarrow 1$ as $r \to \infty$

c) Shift symmetry $\Longrightarrow$ Noether current J^{μ}; Scalar EOM $\Rightarrow \nabla_{\mu} J^{\mu} = 0$

d) Scalar field shares the same symmetry with the metric $\phi = \psi(r)$

$$\Longrightarrow J^{\mu} = (J^r, 0,0,0)$$

2) $J^2 = J^{\mu} J_{\mu} = \frac{(J^r)^2}{B} < \infty$ at $r \longrightarrow r_h \Longrightarrow J^r \to 0$ at $r \to r_h$ as $B \to 0.$

3) $\partial_r(r^2 J^r) = 0 \Longrightarrow J^r r^2 = const \underset{2)}{\Longrightarrow} J^r = 0 \quad \forall r$

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4) J^r can be schematically written as

$$J^r = B\psi' F(g, g', g'', \psi')$$

unspecified function

- Asymptotic flatness implies that $B \to 1$ and $\psi' \to 0$ as $r \to \infty$

- In the weak-field regime, the scalar kinetic term should be dominated by the quadratic one, $J_\mu \longrightarrow \partial_\mu \phi \Longrightarrow F \to const \neq 0$ as $r \to \infty$

- Moving "inward" from infinity toward the horizon, B and F will vary continuously and still be nonzero.

$$J^r = 0 \quad \forall r \Longrightarrow \psi' = 0 \quad \forall r \Longrightarrow \psi = 0 \quad \forall r$$

c) Shift symmetry

Hairy BH Solutions

- Relaxing Assumptions (a)-(d)

e.g.) Relaxing (b) and (d) in the case of nonminimal derivative coupling

d) $\not\Rightarrow$ Scalar field *doesn't* share the same symmetry with the metric

$$\phi = \psi(r) + qt \Rightarrow J^r = B\psi' F(g, g', g'', \psi')$$

Babichev and Charmousis (14), Kobayashi and Tanahashi (14)

EOMs$\Rightarrow F = 0 \Longrightarrow \psi' \neq 0$

necessary to ensure the regularity of the scalar field at horizon.

b) $\not\Rightarrow$ Asymptotically-locally AdS BHs Rinaldi (12), Minamitsuji (14)

- Finding loopholes in Steps 1)-4)

e.g.) In the EdGB gravity, Step 4) is not the case Sotiriou and Zhou (14)

$$J^r = -B\psi' - \alpha \frac{A' B (B-1)}{A r^2} = 0 \Longrightarrow \psi' \neq 0$$

No-Hair Theorem with Linear-in-time and Slow-Rotation

1) Assumptions:

a) With the leading-order correction in the BH angular velocity

Hartle and Thorne (68)

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2(d\theta^2 + sin^2\theta d\phi^2) - 2\omega(r)dtd\varphi$$

b) Asymptotic flatness $\Rightarrow A \longrightarrow 1$ and $B \longrightarrow 1$ as $r \to \infty$

c) Linear-in time $\phi = \psi(r) + qt \Longrightarrow J^\mu = (J^r, 0,0, J^t)$

$J^t = -\frac{q}{A} G_{2X} + \cdots$ does not depend on time.

At the linear order in rotation,

- $A(r), B(r), \psi(r)$ remain the same as in the static case
- $\mathcal{E}_{t\varphi} = 0 \Rightarrow$ 2nd order differential equation for $\omega(r)$

2) $J^2 = (J^r)^2/B(r) - A(r)(J^t)^2 < \infty$

$\Rightarrow J^r \to 0$ as $r \to r_h$, as long as $J^t = \text{finite} \Longleftarrow (B/A)' = \text{finite}$.

3) $\partial_\mu\left(\sqrt{-g}J^\mu\right) = 0 \Longrightarrow \partial_r(r^2 J^r) = 0 \Rightarrow J^r = 0 \quad \forall r$

$J^t = J^t(r)$ 2)

4) $J^r = B\psi' F(g, g', g'', \psi') = 0$

- Asymptotic flatness $B \to 1$ and $\psi' \to 0$ as $r \to \infty$
- In the weak field limit, $F \to const \neq 0$ as $r \to \infty$
- Moving "inward", F and B will still be nonzero

$J^r = 0 \quad \forall r \Longrightarrow \psi' = 0 \quad \forall r \Longrightarrow \psi = 0 \quad \forall r$

$\Longrightarrow$ BHs cannot have nontrivial scalar hair at the leading order in rotation.

Nonminimal Derivative Coupling Gravity

$$\mathcal{L} = \zeta R + 2\eta X + \beta G^{\mu\nu}\phi_\mu\phi_\nu - 2\Lambda_0$$

Case 1) $X = X_0 = const$ $\Longrightarrow$ Self-tuned BHs with $\Lambda_{\rm eff} = -\eta/\beta \neq \Lambda_0$

$q^2 = 2X_0$ Babichev and Charmousis (14)

$$A(r) = B(r) = 1 - \frac{\mu}{r} - \frac{\Lambda_{\rm eff}}{3}r^2 \qquad \psi'(r) = \frac{1}{A(r)}\sqrt{\frac{\zeta\eta + \beta\Lambda_0}{\beta\eta}\big(1 - A(r)\big)}$$

Case 2) $q = 0 \Rightarrow$ Asymptotically locally AdS BHs

Rinaldi (12), Minamitsuji (14)

$$A(r) = \frac{1}{12\beta\zeta^2\eta^2 r}\{r(\zeta\eta - \beta\Lambda_0)[\zeta\eta(9\beta^2 + \eta r^2) + \beta\Lambda_0(3\beta^2 - \eta r^2)] - 24\beta\zeta^2\eta^2\mu\} + \frac{\sqrt{\beta}(\zeta\eta + \beta\Lambda_0)^2 \tan^{-1}\left(\frac{\sqrt{\eta}r}{\sqrt{\beta}}\right)}{4\zeta^2\eta^{5/2}r}$$

$$B(r) = \frac{4\zeta^2(\beta + \eta r^2)^2}{(2\beta\zeta - \beta\Lambda_0 r^2 + \zeta\eta r^2)^2}A(r) \qquad \psi'(r) = \sqrt{-\frac{(\zeta\eta + \beta\Lambda_0)[r^3(\zeta\eta - \beta\Lambda_0) + 2\beta\zeta r]^2}{4\beta\zeta^2(\beta + \eta r^2)^3 A(r)}}$$

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- Frame-dragging equation $\mathcal{E}_{t\varphi} = 0$

$$\Longrightarrow \mathcal{g}\omega'' + \left[\mathcal{g}_X X' + \frac{1}{2}\left(\frac{8}{r} - \frac{A'}{A} + \frac{A'}{A}\right)\mathcal{g}\right]\omega' = 0$$

$$\mathcal{g}(X) = 2(\zeta - \beta X)$$

For both Cases 1) and 2)

$$\Rightarrow \omega'' + \frac{4}{r}\omega' = 0 \Longrightarrow \omega = c_1 + \frac{c_2}{r^3}$$

Hartle and Thorne (68)

$\Longrightarrow$ The leading-order rotational corrections are identical to the case of GR.

See also Ogawa, Kobayashi and Suyama (15)

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Summary

- We have studied leading-order rotational corrections to BH solutions in shift-symmetric Horndeski gravity.
- The no-hair theorem for shift-symmetric Horndeski gravity can be extended to BHs at the leading-order in rotation.
- For nonminimal derivative coupling gravity, the frame-dragging function is exactly identical to the case of GR for all known BH solutions.

Issues

- No-hair argument should not be hold at the second order in rotation.
- Slowly-rotating NS solutions Cisterna, Delsate and Rinaldi (15)

Thank you.

"V M in time-domains, graviton Higgs mechanism"

by Ivan Arraut

[JGRG25(2015)6b3]

V M in time-domains, graviton Higgs mechanism.

Ivan Arraut.

Tokyo University of Science (TUS).

1). Europhys.Lett. 111 (2015) 61001. arXiv:1509.08338 [gr-qc]

2). arXiv:1505.06215 [gr-qc]

Based on the sequence of papers:

3).PTEP 2014 (2014) 023E02, with Hideo Kodama.

4).Europhys.Lett. 109 (2015) 0002

5). arXiv:1504.00467 [gr-qc]

6). Phys.Rev. D90 (2014) 124082

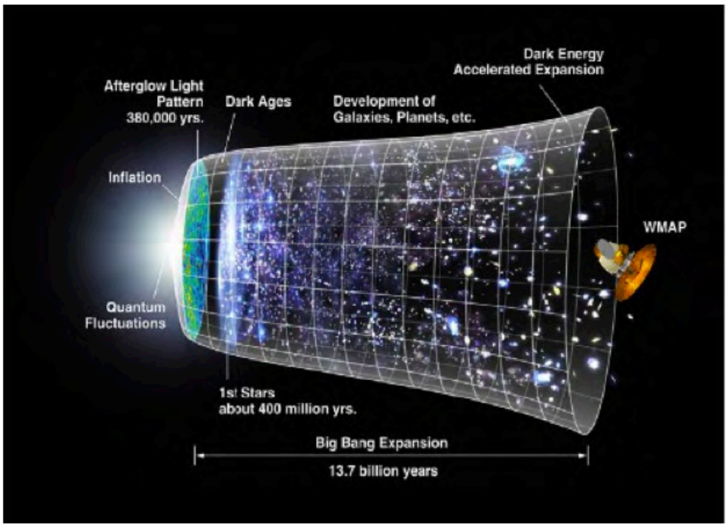
7). arXiv:1503.02150 [gr-qc]

8). Int.J.Mod.Phys. D24 (2015) 03, 1550022

Content.

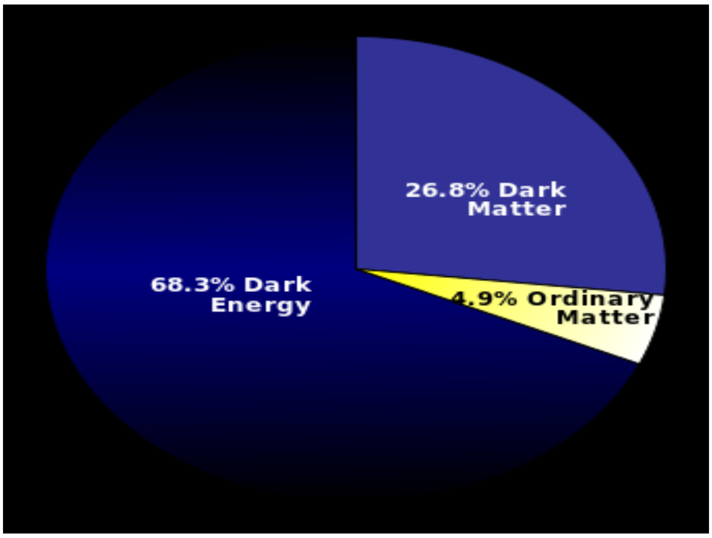
- 1). Motivation: Dynamical origin of the graviton mass.
- 2). What is the Vainshtein mechanism?
- 3). What is the connection between Vainshtein mechanism and the Nambu-Goldstone theorem?
- 4). What is the connection between Vainshtein mechanism and the Higgs mechanism?
- 5). Symmetries of the action.

Motivation. Inflation



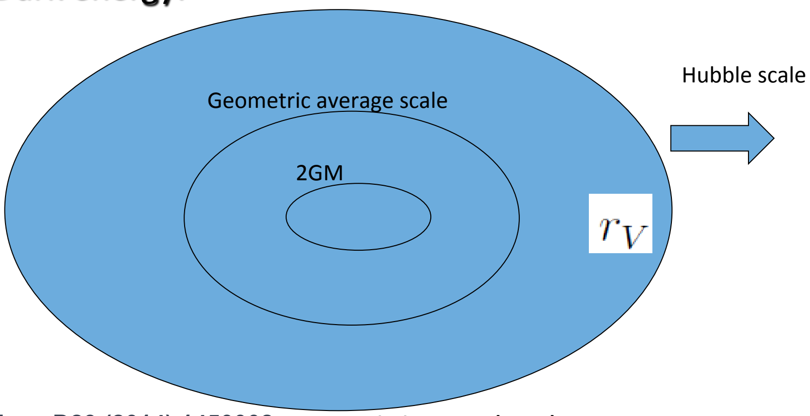
Motivation

- Actual composition of the universe in agreement with the standard model of Cosmology.



Motivation.

- **The scales involved in the problems of Dark Matter and Dark energy.**



Int.J.Mod.Phys. D23 (2014) 1450008, **Int.J.Mod.Phys. D24 (2015) 03, 1550022**.

Dark Energy

The easiest way for explaining the acceleraded expansion of the Universe, is introducing a cosmological constant

$$S_{EH} = \int d^4x\sqrt{-g}(R - 2\Lambda) \Longrightarrow R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Lambda g_{\mu\nu}$$

The theory is invariant under the transformation

$$\delta g_{\mu\nu} = -2\nabla_{(\mu}\zeta_{\nu)}$$

But then what's the problem with introducing the cosmological constant? We should be happy with that. What's going on?

The calculations from zero point quantum fluctuations provide a huge value in comparison with observations. Then we need a magical mechanism for explaining why CC is so small.

dRGT non-linear massive gravity.

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R + m^2 U(g,\phi)),$$

It is a ghost-free theory. The field equations are:

$$U(g,\phi) = U_2 + \alpha_3 U_3 + \alpha_4 U_4.$$

$$\alpha = 1 + 3\alpha_3,$$

$$\beta = 3(\alpha_3 + 4\alpha_4)$$

$$G_{\mu\nu} = -m^2 X_{\mu\nu},$$

$$X_{\mu\nu} = \frac{\delta U}{\delta g^{\mu\nu}} - \frac{1}{2} U g_{\mu\nu}.$$

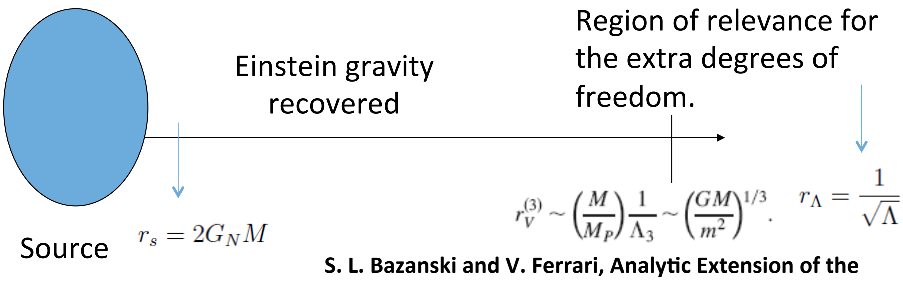
$$\nabla_\mu\left(\frac{\partial U}{\partial(\partial_\mu \phi^a)}\right) = 0.$$

Corresponding to the Bianchi identity in unitary gauge.

dRGT Massive gravity. De-Rham-Gabadadze-Tolley, 2011

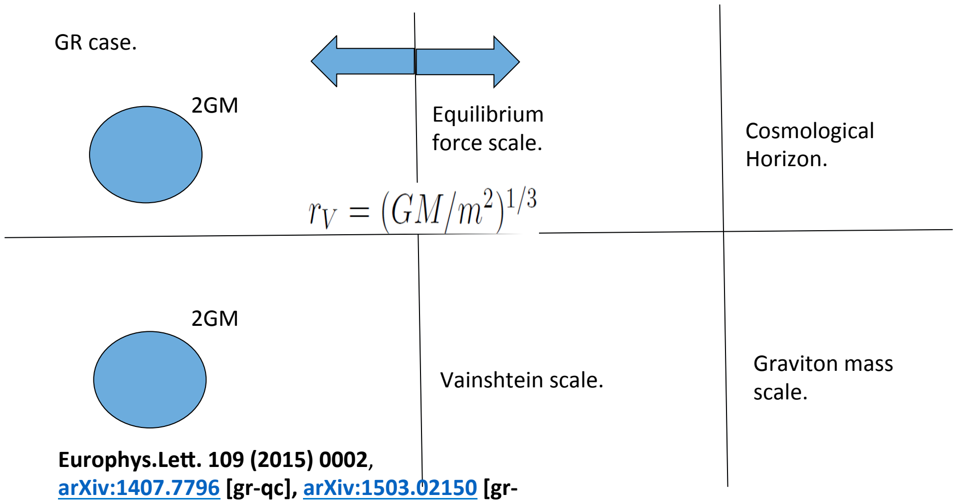
The Vainshtein mechanism in dRGT

- Every theory trying to reproduce the accelerated expansion of the universe reproduces at least three scales when the local physics is analyzed:



Check for example: **S. L. Bazanski and V. Ferrari, Analytic Extension of the Schwarzschild-de Sitter Metric, II Nuovo Cimento Vol. 91 B, N. 1, 11 Gennaio (1986).**

Analogy with GR



Europhys.Lett. 109 (2015) 0002, arXiv:1407.7796 [gr-qc], arXiv:1503.02150 [gr-qc].

The new mechanism operates through the non-linearities of the theory. The usual explanation goes to the decouping limit of the theory after finding the field equations.

Deffayet et al.; Kurt Hinterbichler: 2012.

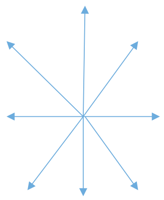
The mechanism in terms of Stuckelberg functions appears as a extremal condition of the dynamical metric.

$$dU(g,\phi) = \left(\frac{\partial U(g,\phi)}{\partial g}\right)_\phi dg = 0, \quad \Longrightarrow \quad dg_{\mu\nu} = \left(\frac{\partial g_{\mu\nu}}{\partial r}\right)_t dr + \left(\frac{\partial g_{\mu\nu}}{\partial t}\right)_r dt = 0,$$

These set of conditions, when applied to the dynamical metric, help us to evaluate the Vainshtein radius.

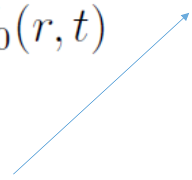
arXiv:1407.7796 [gr-qc], Int.J.Mod.Phys. D24 (2015) 03, 1550022

The extra-degrees of freedom reproduce the effect of a preferred time-direction.



Non-preferred notion of time.
Diffeomorphism invariance restored at this level.

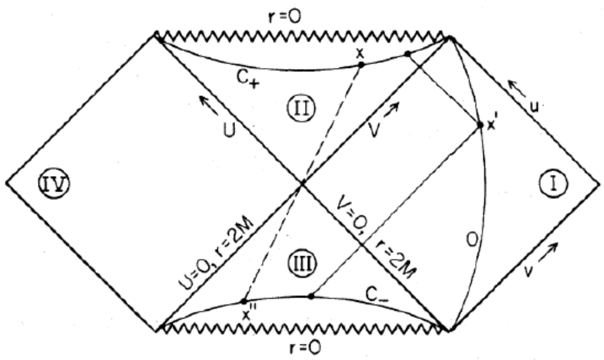
$T_0(r,t)$



Notion of preferred time-direction.
Diffeomorphism invariance explicitly broken.
Restored after introducing redundant variables.

Vainshtein scale

The extra-degrees of freedom reproduce the effect of a preferred time-direction.



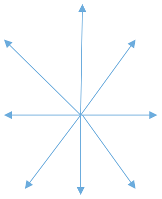
The notion of a preferred time-direction, affects the definition of particles in the same way as the curvature effects can reproduce the same effects, responsible of the Hawking radiation.

arXiv:1503.02150 [gr-qc] **Europhys.Lett. 109 (2015) 0002**

The path integral formulation

From the perspective of the path integrals, the mismatch between the periodicities of the propagators with respect to the ordinary time-coordinate and the same propagator defined and analytically extended with respect to the St\"uckelberg function, create the effect of extra-particle creation.

t → t-i4(\pi) M

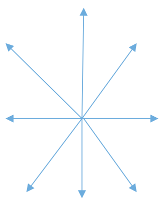


$$T_0(r,t) \to T_0(r,t) - i4\pi M$$

Vainshtein scale

From the perspective of the Bogolubov transformations.

We can use the same set of Bogolubov transformations, in order to demonstrate that the extra-degrees of freedom of the theory are able to reproduce an extra-particle creation process due to the ambiguity generated in the definition of vacuum.



$$T_0(r,t)$$

$$\hat{a}_i|0>=0,$$

$$\hat{a}_i|\bar{0}>=\Sigma_j\beta^*_{ji}|\bar{1}>\neq 0.$$

The notion of vacuum after the Vainshtein scale is different to the notion of vacuum before it.

The vacuum solution. Spherical symmetry

With the degrees of freedom inside the dynamical metric, the object created becomes diffeomorphism invariant.

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{rt}(drdt + dtdr) + r^2 d\Omega_2^2,$$

With:

$$g_{tt} = -f(Sr)(\partial_t T_0(r,t))^2, \qquad g_{rr} = -f(Sr)(\partial_r T_0(r,t))^2 + \frac{S^2}{f(Sr)},$$
$$g_{tr} = -f(Sr)\partial_t T_0(r,t)\partial_r T_0(r,t),$$

The Stückelberg fields enter here in the following

$$g_{\mu\nu} = \left(\frac{\partial Y^\alpha}{\partial x^\mu}\right)\left(\frac{\partial Y^\beta}{\partial x^\nu}\right) g'_{\alpha\beta}, \quad \Longrightarrow \quad Y^0(r,t) = T_0(r,t), \qquad Y^r(r,t) = r.$$

The vacuum degeneracy

The generic solution can be written as

$$ds^2 = -f(Sr)dT_0(t,r)^2 + \frac{S^2 dr^2}{f(Sr)} + S^2 r^2 d\Omega^2.$$

Two family of solutions in this theory.

1). Two free-parameters and the Stückelberg function given by

$$T_0(r,t) = St \pm \int^{Sr} \left(\frac{1}{f(u)} - 1\right).$$

Hideo Kodama and Ivan Arraut PTEP 2014 (2014) 023E02

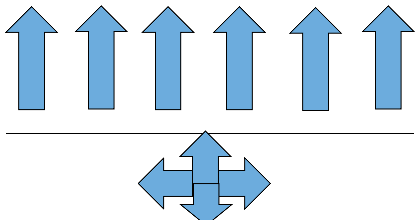
The equivalence principle is satisfied for this previous case. In other words, $\partial_r T_0(r,t)=0$ for free-falling observers. For the second family of solutions, we cannot assure the same.

2). One free-parameter and the Stückelberg function arbitrary. In this case, the Stückelberg function being arbitrary, operates as a free-paramater.

This second case is the interesting one.

The Stückelberg function defines the prefereed time direction.

$T_0(r,t)$



r_V

The procedure for analysis.

Perturbation theory. In a "free-falling" frame.

$$f(Sr)\to\ 1$$

And the dynamical metric of the theory becomes.

$$ds^2=S^2\left(-dt^2+dr^2\left[1-\left(\frac{T_0'(r,t)}{S}\right)^2\right]-2\frac{T_0'(r,t)}{S}dtdr+r^2d\Omega^2\right).$$

Here I have assumed the stationary condition for the metric: $\dot{T}_0(r,t)\ =\ S$. This assumption is unnecessary but simplifies the calculations.

Perturbation theory in a free-falling frame.

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R + m_g^2 U(g,\phi)).$$

Expanding the action up to second order in perturbations. The relevant quantity for our purposes is the potential term, defined as.

$$\sqrt{-g}U(g,\phi) \approx S^2\left(1 + \frac{1}{2}h - \frac{1}{4}h^{\alpha}{}_{\beta}h^{\beta}{}_{\alpha} + \frac{1}{8}h^2\right)(U(g,\phi)_{back} + \delta U(g,\phi))$$

After expansion up to second order.

$$\sqrt{-g}U(g,\phi) \approx S^2\left(1 + \frac{1}{2}h - \frac{1}{4}h^{\alpha}{}_{\beta}h^{\beta}{}_{\alpha} + \frac{1}{8}h^2\right)U(g,\phi)_{back} + S^2\left(1 + \frac{1}{2}h\right)\delta U(g,\phi).$$

$$\Downarrow$$

$$\sqrt{-g}U(g,\phi) = V(g,\phi)$$

Then we find the vacuum solutions, defined in agreement with.

$$\frac{dV(g,\phi)}{dh_{\mu\nu}} = 0,$$

The effective matrix mass for each mode is defined in agreement with the matrix

$$m^{\mu\nu}m^{\alpha\beta}=\frac{\partial^2 V(g,\phi)}{\partial h_{\mu\nu}\partial h_{\alpha\beta}},$$

In agreement with the second derivatives with respect to the field, evaluated at the vacuum level. The symmetries of the action are defined in agreement with:

$$\delta_g h_{\mu\nu} = \partial_\mu\zeta_\nu + \partial_\nu\zeta_\mu + \pounds_\zeta h_{\mu\nu},$$

$$\delta_g A_\mu = \partial_\mu\Lambda - \zeta_\mu - A^\alpha\partial_\alpha\zeta_\mu - \frac{1}{2}A^\alpha A^\beta\partial_\alpha\partial_\beta\zeta_\mu - \ldots,$$

$$\delta_g\phi = -\Lambda.$$ In terms of St\”uckelberg functions. Or equivalently:

$$g_{\mu\nu}\to\frac{\partial f^\alpha}{\partial x^\mu}\frac{\partial f^\beta}{\partial x^\nu}g_{\alpha\beta}(f(x)),\qquad Y^\mu(x)\to f^{-1}(Y(x))^\mu.$$

$$Y^\alpha(r,t)=x^\alpha+A^\alpha,\qquad A_\mu\to A_\mu+\partial_\mu\phi,$$

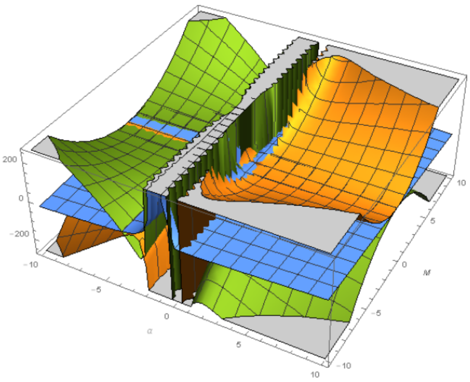
$$h_{oovac}=\frac{\alpha^2[2+\alpha(6+\alpha(6+\alpha))]}{2(1+\alpha)^5}+2T_0'(r,t)^2A(\alpha),$$

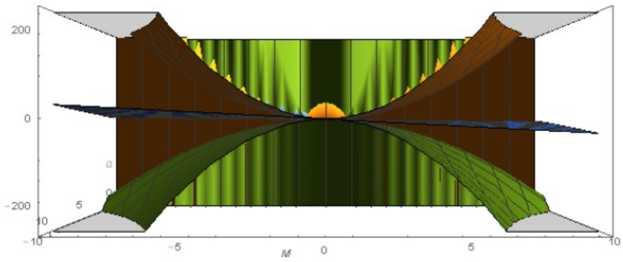
$$h_{rrvac}=-\frac{\alpha^2[2+\alpha(6+\alpha(6+\alpha))]}{2(1+\alpha)^5}+2T_0'(r,t)^2B(\alpha),$$

$$h_{0rvac}=\frac{T_0'(r,t)[-2(1+\alpha)^7+\alpha[1+3\alpha(1+\alpha)][2+\alpha(6+\alpha(6+\alpha))]]}{2(1+\alpha)^4(1+3\alpha(1+\alpha))},$$

$$|h_{\mu\nu}|_{vac}=C(\alpha)+T_0'(r,t)^2D(\alpha)=v.$$

This quantity is the parameter defining the real vacuum.
The solution is degenerate, the symmetry under time-translations is broken, but the spherical symmetry still remains.





$$SO(3,1) \quad (G) \rightarrow SO(3) \quad (H), \qquad G/H \longrightarrow U(1).$$

In a free-falling frame. We can compare the situation with the scalar case.

$$\pounds = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}\mu^2(\phi^i)^2 - \frac{\lambda}{4}[(\phi^i)^2]^2,$$

Where the action is invariant under

$$\phi^i \rightarrow R^{ij}\phi^j.$$

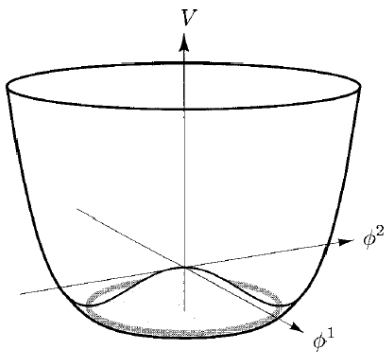
Has a minimum

And the potential

$$V(\phi^i) = -\frac{1}{2}\mu^2(\phi^i)^2 + \frac{\lambda}{4}[(\phi^i)^2]^2. \quad \Longrightarrow \quad (\phi_0^i)^2 = \frac{\mu^2}{\lambda}.$$

The Nambu-Goldstone theorem

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi_0)^a(\phi - \phi_0)^b \left(\frac{\partial^2}{\partial\phi^a\partial\phi^b}V\right)_{\phi_0} + ...,$$



$$G: \quad \phi_0^{a'} = U(g)\phi_0^a \neq \phi_0^a,$$

$$H: \quad \phi_0^{a'} = U(h)\phi_0^a = \phi_0^a,$$

$$G/H$$

The dimension of the coset defines the number of broken generators

Finally the physical perturbations in our case, have to be done with respect to

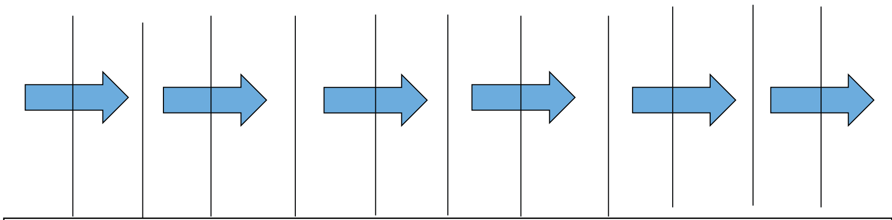
$$h_{\mu\nu} = \bar{h}_{\mu\nu} + h_{\mu\nu vac}, \qquad \bar{h}_{\mu\nu vac} = 0.$$

And the action becomes of the form.

$$\pounds = \pounds_{EH} + K(\mathrm{d}T_0(r,t)) + F(v,\alpha)(\Gamma\Gamma)_{gauge} + \bar{V}(g,\phi),$$

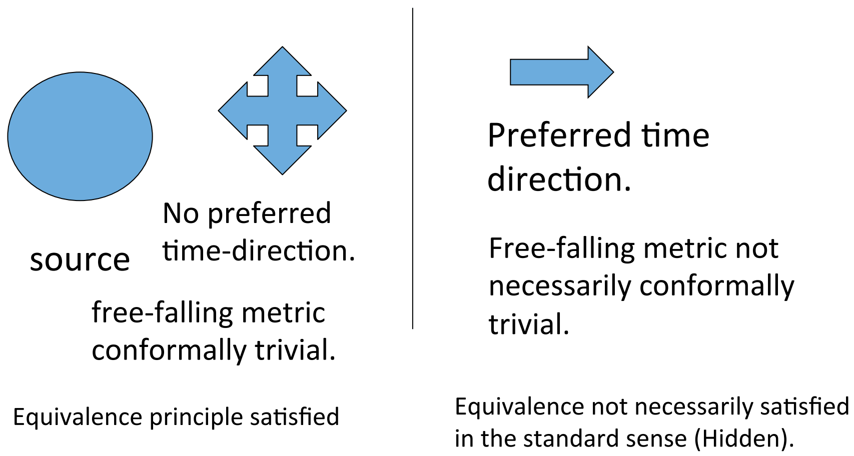
The connections become gauge fields due to their explicit dependence on the Stückelberg function. The Stückelberg function also appears in the explicit deefinition of the parameter generating the vacuum shift.

What happens if we include gravity now.



We then have a multiplicity of Vainshtein scales, all of them connected through the U(1) symmetry transformation. The analysis should be better done by defining the observer located at this scale. The same proceure as explained before applies.

Cartoon picture



Conclusions

- In the non-linear formulation of massive gravity, the Vainshtein mechanism in time-domains defines the dynamical origin of the graviton mass.
- The Stückelberg function defines the preferred time-direction of the theory. Breaking the symmetry for generators depending explicitly on time. The spherical symmetry remains.
- At the spatial domains, the Vainshtein scale defines a phase transition scale.
- The previous results explain why apparently the particle creation process of black-holes is affected by the presence of the extra-degrees of freedom. See for example:

Europhys.Lett. 109 (2015) 0002,

arXiv:1407.7796 [gr-qc], arXiv:1503.02150 [gr-qc].

“Higgs G-inflation and field-dependent cutoff scale”

by Kohei Kamada

[JGRG25(2015)6b5]

Higgs G-inflation and Field-dependent Cutoff Scale

based on: KK, T. Kobayashi, M.Yamaguchi & J. Yokoyama, PRD85 (2011) 043503
KK, PLB744 (2015) 347

See also: KK, T. Kobayashi, T. Takahashi, M. Yamaguchi & J. Yokoyama, PRD86 (2012) 023504
KK, T. Kobayashi, T. Kunimitsu, M. Yamaguchi & J. Yokoyama, PRD86 (2013) 123518

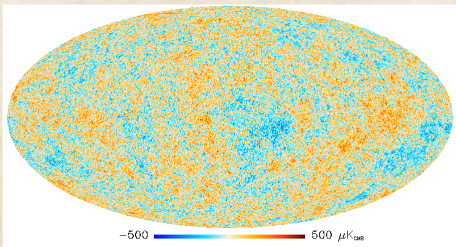
Kohei Kamada
(Arizona State University)

JGRG25, Kyoto, 12/10/2015

Courtesy H.Oide

Premise

Our Universe has experienced primordial inflation.



('13) Planck collaboration

since Planck and other CMB experiments strongly supports inflation and we do not have at present any other good mechanism to solve the cosmological problems such as the horizon, flatness and monopole problems.

Courtesy H.Oide

Question

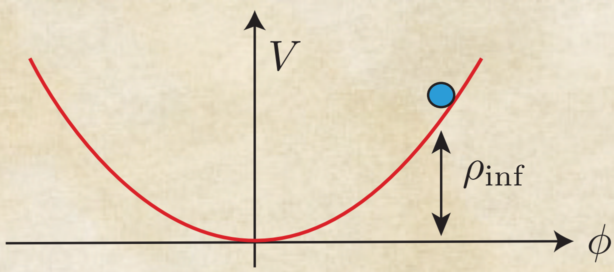
What drove inflation?

Courtesy H.Oide

Question

What drove inflation?

We usually assume that the potential energy of a scalar field drives inflation.



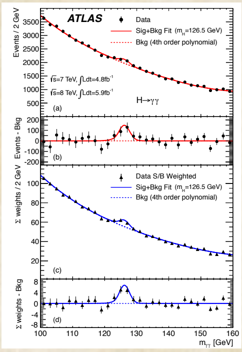
$$\cancel{\ddot{\phi}} + 3H\dot{\phi} + V' = 0,$$
$$3H^2 M_{\rm pl}^2 = \cancel{\frac{1}{2}\dot{\phi}^2} + \rho_{\rm inf}$$

Courtesy H.Oide

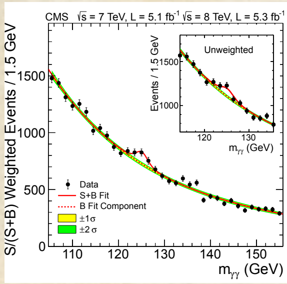
Question

What drove inflation?

Now we finally know that
the unique scalar field in the SM, Higgs, really exists.



ATLAS, 1207.7214



CMS, 1207.7235

Courtesy H.Oide

Question

What drove inflation?

Now we finally know that
the unique scalar field in the SM, Higgs, really exists.

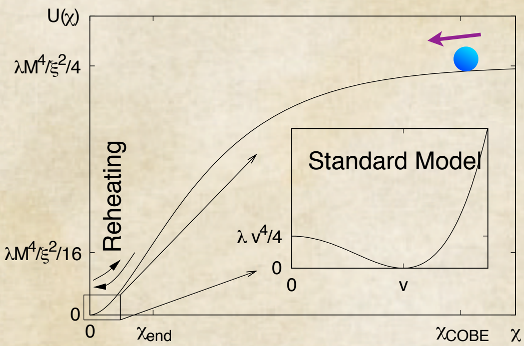
Why don't we use the Higgs as inflaton?

Courtesy H.Oide

The simplest, most elegant model of Higgs inflation would be $R|\mathcal{H}|^2$ Higgs inflation.

('08 Bezrukov & Shaposhnikov;
See also '84, Spokoiny; '89 Futamase & Maeda; '95 Cota; '98 Komatsu & Futamase)

$$\boxed{\Delta S = \int d^4x\sqrt{-g}\left[\frac{-\xi h^2}{2}R\right]}$$



Courtesy H.Oide

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$$\boxed{\Delta S = \int d^4x\sqrt{-g}\left[\frac{-\xi h^2}{2}R\right]}$$

$$R \sim (\partial\tilde{h})^2/M_{\rm pl}^2$$

- ✓ Apparently renormalizable.
- ✓ Self-consistent.
- ✓ Stable under quantum corrections.
- ✓ possible even in the case where Higgs potential once gets negative.

Nonminimal coupling is not really renormalizable...

($\tilde{h}$: canonically normalized graviton)

Courtesy H.Oide

If we allow non-renormalizable terms for Higgs inflation, there can be several variants, stimulated by Galileons.

('09, '10, '11, Deffayet+)

$$\mathcal{L}_2 = K(\phi, X),$$
$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi,$$
$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\Big[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\Big].$$

$$S = \int d^4x\sqrt{-g}\sum_{i=2}^{5}\mathcal{L}_i$$

$$X \equiv -\frac{1}{2}(\nabla\phi)^2, \quad G_{iX} \equiv \frac{\partial G_i}{\partial X}$$

Noncanonical derivative coupling changes the friction term in the equation of motion and the coupling to gravity changes the potential shape in the Einstein frame.

('10, Germani & Kehagias, '10, Nakayama & Takahashi, '11, Kamada+)

Courtesy H.Oide

Now we can construct a Higgs inflation model in the spirit of Generalized Galileons. => See KK+('12)

A simple, but non-trivial model is the one uses $\mathcal{L}_3$ term,

('11 KK+)

$$\Delta S = -\int d^4x\sqrt{-g}\left(\frac{\mathcal{H}^\dagger}{M^4}D_\mu D^\mu\mathcal{H} + \text{h.c.}\right)|D_\mu\mathcal{H}|^2$$
$$\to -\int d^4x\sqrt{-g}\frac{\phi\Box\phi}{2M^4}(\partial\phi)^2$$

$$\mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi \end{pmatrix}$$

which we name it Higgs G-inflation after Galileon.

Courtesy H.Oide

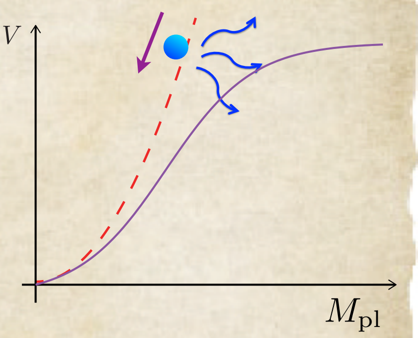
Features and consequences of Higgs G-inflation

Additional derivative coupling generates additional friction term in the EOM, and we have "potential driven additional friction term assisted inflation".

modified slow-roll equations

$$3H^2M_{\rm pl}^2 = \frac{\lambda}{4}\phi^4$$

$$\left(1 - \frac{3H\dot{\phi}\phi}{M^4}\right) \times 3H\dot{\phi} + \lambda\phi^3 = 0$$



As a result, sub-Planckian chaotic inflation is possible.

Courtesy H.Oide

For the self-consistency, one should check the interaction of fluctuations around the inflationary trajectory.

Note that the kinetic term of these fluctuations are not canonical and one must canonically normalize them.

$$S_{\rm kin}[\chi] = \int d^4x a^3(t) \left[\frac{1}{2}\left(1 + \frac{2\dot{\bar{\phi}}^2 - 6H\bar{\phi}\dot{\bar{\phi}}}{M^4}\right)\dot{\chi}^2 - \frac{1}{2}\left(1 - \frac{2\bar{\phi}(\ddot{\bar{\phi}} + 2H\dot{\bar{\phi}})}{M^4}\right)\frac{\delta^{ij}}{a^2(t)}\partial_i\chi\partial_j\chi\right]$$

=> For $H\bar{\phi}\dot{\bar{\phi}} \gg M^4$, the kinetic structure of $\chi(x)$ is much different from the original one.

Courtesy H.Oide

For the self-consistency, one should check the interaction of fluctuations around the inflationary trajectory.

Note that the kinetic term of these fluctuations are not canonical and one must canonically normalize them.

For the canonically normalized fluctuations χ , the interaction terms reads,

$$\int dt d^3\tilde{x} \frac{1}{G(t)^{3/4}F(t)^{3/4}} \frac{\dot{\bar{\phi}}}{M^4} \tilde{\chi}\dot{\tilde{\chi}}\Box\tilde{\chi} + \frac{1}{G(t)^{3/4}F(t)^{3/4}} \frac{\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}}}{2M^4} \tilde{\chi}(D_\mu\tilde{\chi})^2 - \frac{1}{G(t)^{3/4}F(t)^{3/4}} \frac{\dot{\bar{\phi}}}{2M^4} \Box\tilde{\chi}(D_\mu\tilde{\chi})^2 - \frac{1}{G(t)^{1/2}F(t)^{3/2}} \frac{1}{2M^4} \chi\Box\chi(D_\mu\chi)^2.$$

with $G(t) \equiv 1 + \dfrac{2\dot{\bar{\phi}}^2 - 6H\bar{\phi}\dot{\bar{\phi}}}{M^4}, \quad F(t) \equiv 1 - \dfrac{2\bar{\phi}(\ddot{\bar{\phi}} + 2H\dot{\bar{\phi}})}{M^4}$

Courtesy H.Oide

The system becomes strongly coupled at the scale,

$$E_{\rm sc} \equiv \min.\left\{ \frac{G(t)^{3/8}F(t)^{3/8}M^2}{\dot{\bar{\phi}}^{1/2}}, \frac{2G(t)^{3/4}F(t)^{3/4}M^4}{\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}}}, \frac{2^{1/3}G(t)^{1/4}F(t)^{1/4}M^{4/3}}{\bar{\phi}^{1/3}}, 2^{1/4}G(t)^{1/8}F(t)^{3/8}M \right\}.$$

For inflationary BG, we have

$$E_{\rm sc}(\bar{\phi}) = \frac{2^{1/3}G(t)^{1/4}F(t)^{1/4}M^{4/3}}{\bar{\phi}^{1/3}} \simeq \lambda^{1/4}\bar{\phi}^{2/3}M^{1/3}$$

This is found to be larger than the Hubble scale during inflation:

$H \simeq \dfrac{\lambda^{1/2}\bar{\phi}^2}{2\sqrt{3}M_{\rm pl}}$ when the COBE scale exited the horizon, $\bar{\phi} \sim \lambda^{-1/8}\sqrt{MM_{\rm pl}}$

Thus, this model is self-consistent for sufficiently small M.

Courtesy H.Oide

Features and consequences of Higgs G-inflation

We can examine the cosmological perturbation in this setup, and find that effectively the potential is flattened,

$$A_s = \frac{(2\mathcal{N}+1)^2}{8\pi^2}\left(\frac{3}{8}\right)^{1/2}\lambda^{1/2}\left(\frac{M}{M_{\rm pl}}\right)^2$$

=> For $M \sim (\lambda/0.01)^{-1/4}10^{13}$ GeV, $A_s \sim 10^{-9}$ is realized.

$\phi_{\rm inf} \gtrsim \lambda^{-1/8}\sqrt{MM_{\rm pl}}$

It also predicts

$$n_s \simeq 0.967$$
$$r \simeq -\frac{32\sqrt{6}}{9}n_T \sim 0.14$$

and slow-roll suppressed NG.

Non trivial consistency relation!

Courtesy H.Oide

Features and consequences of Higgs G-inflation

BICEP2 / *Keck Array* **VI: Improved Constraints On Cosmology and Foregrounds When Adding 95 GHz Data From** *Keck Array*

Keck Array and BICEP2 Collaborations: P. A. R. Ade,[1] Z. Ahmed,[2,3] R. W. Aikin,[4] K. D. Alexander,[5] D. Barkats,[5] S. J. Benton,[6] C. A. Bischoff,[5] J. J. Bock,[4,7] R. Bowens-Rubin,[5] J. A. Brevik,[4] I. Buder,[5] E. Bullock,[8]

of the sky. This analysis yields an upper limit $r_{0.05} < 0.09$ at 95% confidence, which is robust to variations explored in analysis and priors. Combining these B-mode results with the (more model-dependent) constraints from *Planck* analysis of CMB temperature and other evidence yields a combined limit $r_{0.05} < 0.07$ at 95% confidence. These are the strongest constraints to date on inflationary gravitational waves.

arXiv:1510.09217

p, d.

$\phi_{\rm inf} \gtrsim \lambda^{-1/8}\sqrt{MM_{\rm pl}}$

It also predicts

$$n_s \simeq 0.967$$
$$r \simeq -\frac{32\sqrt{6}}{}n_T \sim 0.14$$

...sed NG.

...ation!

Radiative corrections or more complicated setup may relax the situation. But it is more sincere to take the model as a toy study to read off the general feature of this type of models. (See '15 Kunimitsu et al.)

Courtesy H.Oide

Summary

✓Higgs G-inflation is (was?) one of the possible candidates of Higgs inflation and is found to be self-consistent.

✓Its strong coupling scale depends not only field values but also its derivative and environmental parameters such as the Hubble parameter.

✓The strong coupling scale is smaller than the field value, the knowledge of UV completion would be necessary to connect the results of low energy experiments to the inflationary predictions.

✓Since the model allows superluminal propagation of fluctuations in a specific BG, Lorentz-invariant UV completion may not exist (e.g. '06 Adams+).

UV *extension* may be possible? See '14 Ivanov+, '15 Barbon+. See also '15 Keltner+.

Courtesy H.Oide

“Localized oscillating configurations formed by real scalar fields”

by Gyula Fodor

[JGRG25(2015)7a1]

Localized oscillating configurations formed by real scalar fields

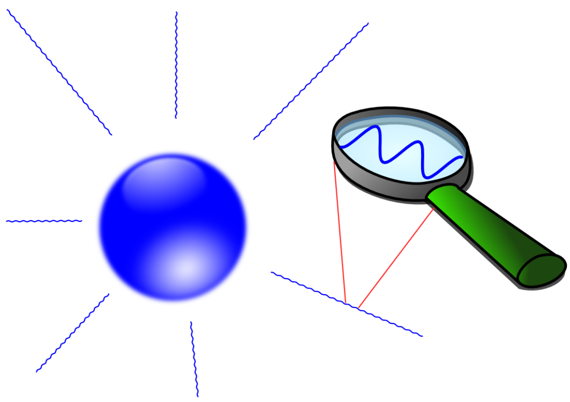
Gyula Fodor

Observatoire de Paris, Meudon
Wigner Research Centre for Physics, Budapest

Péter Forgács (Wigner Research Centre, Tours University)
Philippe Grandclément (Observatoire de Paris, Meudon)

The 25th Workshop on General Relativity and Gravitation in Japan
Kyoto, 10 December 2015

Gravitational attraction forms a spherically symmetric star-like object from a scalar field



For a complex scalar field it is called boson star

For a real scalar field it is called oscillaton

Extremely long living and stable, but for oscillatons the mass decreases very slowly because of a tiny scalar field radiation

Small amplitude oscillatons

G. Fodor, P. Forgács and M. Mezei, *Phys. Rev. D,* **81,** 064029 (2010)

- amplitude $\sim \varepsilon^2$
- size $\sim \frac{1}{m\varepsilon}$
- mass $M = \frac{1}{m}\left[1.753\,\varepsilon - 2.117\,\varepsilon^3\right]$
- mass loss rate

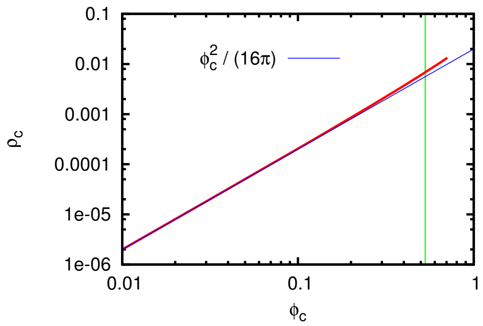
$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{30.0}{\varepsilon^2}\exp\left(-\frac{22.4993}{\varepsilon}\right)$$

extension of the mode equations to the complex plane, study the behavior near the pole, Borel summation

For large amplitude oscillatons the radiative tail can be calculated numerically by spectral methods

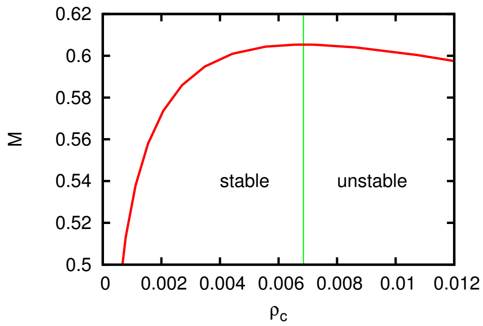
P. Grandclément, G. Fodor and P. Forgács, *Phys. Rev. D,* **84,** 065037 (2011)

There is a one-parameter family of oscillatons



Time averaged central density is a monotonically increasing function of the scalar field central amplitude

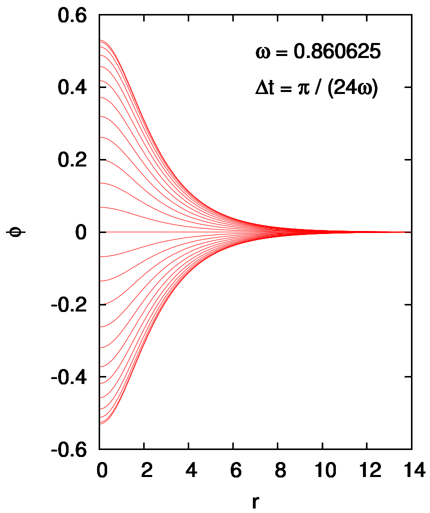
– for small amplitudes it is proportional to ϕ_c^2



In general, a star-like astrophysical object is stable if the total mass increases with increasing central density

There is a maximal amplitude for stable oscillatons

For $\Lambda = 0$ oscillatons oscillate almost periodically, with an extremely slowly increasing frequency



Behavior of the scalar field during one oscillation period for the largest amplitude stable oscillaton

Scalar field decreases exponentially up to $r = 32$, where a radiative tail begins, with amplitude 10^{-8}

For smaller amplitude oscillatons the tail is even much smaller – the tail decreases exponentially with decreasing central amplitude

Mass lost since the early universe

Start with a maximal mass oscillaton 13.7 billion years ago, choosing various scalar field masses m we list how big part of the mass is lost by now

$m\frac{c^2}{eV}$	$\frac{M_{\max}-M}{M_{\max}}$	$\frac{M}{M_\odot}$
10^{-25}	$3.16 \cdot 10^{-5}$	$8.09 \cdot 10^{14}$
10^{-15}	0.0896	$7.36 \cdot 10^{4}$
10^{-5}	0.263	$5.96 \cdot 10^{-6}$
10^{5}	0.401	$4.85 \cdot 10^{-16}$
10^{15}	0.500	$4.04 \cdot 10^{-26}$

the last column gives the resulting mass M in solar mass units

P. Grandclément, G. Fodor and P. Forgács, *Phys. Rev. D*, **84,** 065037 (2011)

$\Lambda < 0$: asymptotically anti-de Sitter case

Negative cosmological constant acts as an effective attractive force

Exactly periodic solutions exist for real scalar fields (oscillatons)

– we call them AdS breathers
there is no radiative tail, similarly to the sine-Gordon breather

There are breather solutions even for massless free scalar fields

– their size is determined by the cosmological constant $\sim 1/\sqrt{-\Lambda}$

– for $\Lambda = 0$ massive fields the size is $\sim \frac{1}{m\varepsilon}$

Rest of the talk: massless Klein-Gordon field minimally coupled to Einstein's gravity, with $\Lambda < 0$

AdS breathers – massless minimally coupled real scalar

G.Fodor, P. Forgács and P. Grandclément, *Phys. Rev. D* **92,** 025036 (2015)

We apply three methods:

- Spectral code for constructing time-periodic solutions
- Time-evolution code to study stability
- High-order small-amplitude expansion to get analytical results

Extension of the results of M. Maliborski and A. Rostworowski, *Phys. Rev. Lett.* **111,** 051102 (2013)
– methods that work well only for $2n + 1$ spacetime dimensions
– results presented only for $4 + 1$ dimensions

We give $3 + 1$ and $4 + 1$ results, can reach higher amplitudes, find maximal mass state and higher amplitude unstable states

$d+1$ dimensional Einstein's equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad , \quad T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}$$

the contracted Bianchi identity gives the wave equation

$$\nabla^{\mu}\nabla_{\mu}\phi = 0$$

usually ϕ is rescaled to make $8\pi G = d-1$

We look for spherically symmetric solutions with metric

$$\mathrm{d}s^2 = \frac{L^2}{\cos^2 x}\left(-Ae^{-2\delta}\mathrm{d}t^{\,2} + \frac{1}{A}\mathrm{d}x^2 + \sin^2 x\, \mathrm{d}\Omega^2_{d-1}\right)$$

where $L^2 = -\frac{d(d-1)}{2\Lambda}$, A and δ are functions of t and x

– anti-de Sitter corresponds to $A = 1$ and $\delta = 0$

Small-amplitude expansion

ε is chosen as the central amplitude of ϕ at $t = 0$

$$\phi = \sum_{\substack{n=1 \\ odd}}^{\infty} \phi^{(n)}\varepsilon^n \ , \quad A = 1 + \sum_{\substack{n=2 \\ even}}^{\infty} A^{(n)}\varepsilon^n \ , \quad \delta = \sum_{\substack{n=2 \\ even}}^{\infty} \delta^{(n)}\varepsilon^n$$

To linear order metric remains AdS, and there are periodic localized solutions for the scalar field $\phi^{(1)} = p_n \cos(\omega_n t)$

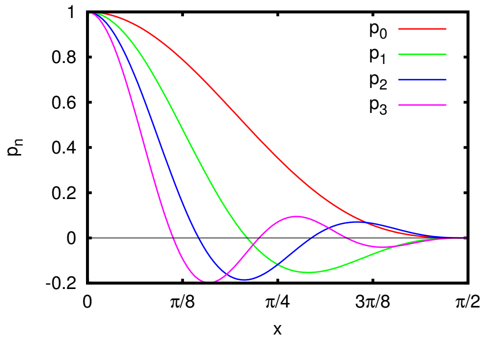
$$p_n = \frac{n!}{(d/2)_n} \cos^d x \; P_n^{(d/2-1,d/2)}(\cos(2x))$$

$P_n^{(\alpha,\beta)}$ is the Jacobi polynomial,
$(\alpha)_n = \alpha(\alpha+1)\ldots(\alpha+n-1)$ is the Pochhammer symbol

frequency: $\omega_n = d + 2n$

fully resonant spectrum – turbulent instability

$3+1$ dimensional spacetime ($d=3$)



$$p_0 = \cos^3 x$$

$$p_1 = \frac{\cos^3 x}{3}\left[4\cos(2x)-1\right]$$

$$p_2 = \frac{\cos^3 x}{3}\left[3\cos(4x)-2\cos(2x)+2\right]$$

$$p_3 = \frac{\cos^3 x}{15}\left[12\cos(6x)-9\cos(4x)+18\cos(2x)-5\right]$$

Combination with arbitrary amplitudes and phases is a valid solution of the linearized problem

$$\phi^{(1)} = \sum_{n=0}^{\infty} a_n \cos(\omega_n t + b_n) p_n \quad , \quad \omega_n = 3 + 2n$$

but to ε^3 order, there are $t\sin(\omega t)$ secular terms in $\phi^{(3)}$ if more then one a_n is nonzero

There is a one-parameter family of solutions emerging from each p_n linearized mode

We investigate the family emerging from the nodeless solution p_0

Initial guess for numerical code: linearized solution $p_0 \cos(3t)$

KADATH library developed by Philippe Grandclément at Observatoire de Paris - Meudon

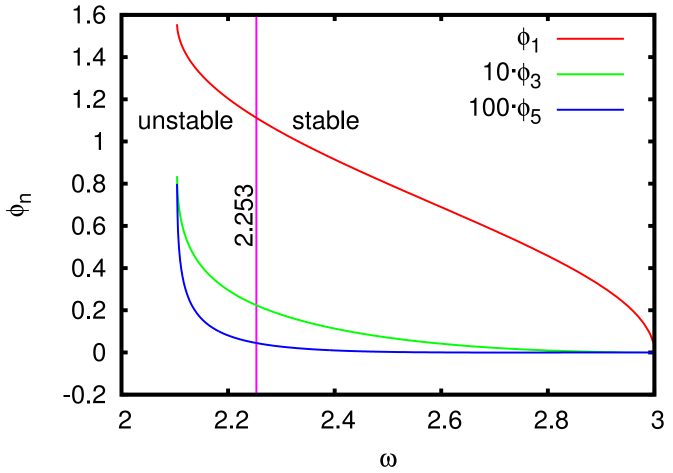
– multidomain spectral method

– radial direction: Chebyshev polynomials

– time direction: Fourier decomposition

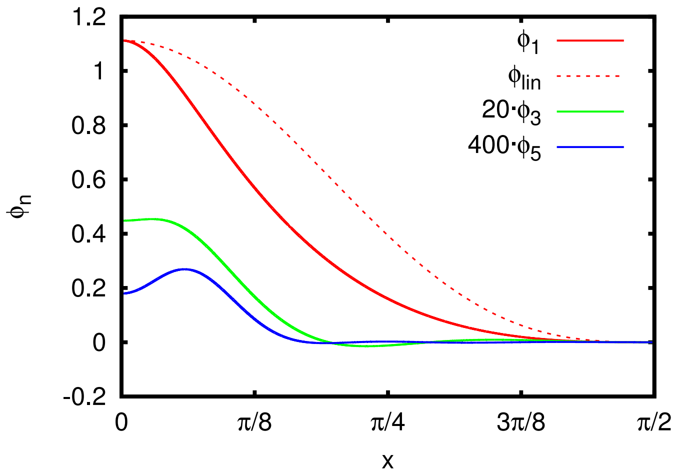
$$\phi = \sum_{\substack{k=1\\ odd}}^{\infty} \phi_k \cos(k\omega t) \ , \quad A = \sum_{\substack{k=0\\ even}}^{\infty} A_k \cos(k\omega t) \ , \quad \delta = \sum_{\substack{k=0\\ even}}^{\infty} \delta_k \cos(k\omega t)$$

Central values of the $\cos(\omega t)$, $\cos(3\omega t)$, $\cos(5\omega t)$ Fourier modes as function of oscillation frequency



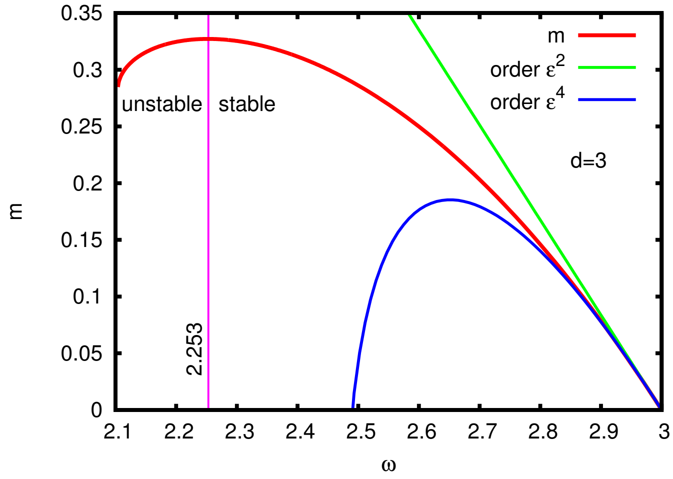
Using the solution as initial data for a time-evolution code:
AdS breathers with frequency $\omega < 2.253$ are unstable
– collapse into black holes

Radial profile for the first three modes of the scalar field for the largest amplitude stable AdS breather (maximal mass)



– more compact than the linear solution, but similar shape

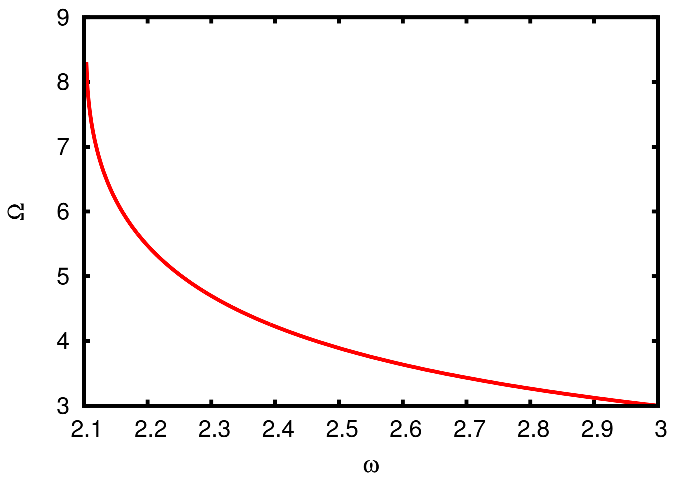
Mass as function of the oscillation frequency



AdS breather becomes unstable when the total mass starts to decrease with increasing central density

First two orders of the small-amplitude expansion is also plotted
– in order to get ε^4 order results one has to calculate to ε^6 order
(to fix coefficients of homogeneous solutions)

Central frequency Ω as function of asymptotic frequency ω



As the amplitude grows

the frequency observed outside the breather decreases

the frequency measured by a central observer grows

Concluding remarks for $\Lambda < 0$

Periodic solutions, up to a certain amplitude, are on
"stability islands"
– general configurations collapse into black holes

AdS/CFT correspondence
– periodic solutions correspond to states that never thermalize

There are other asymptotically AdS localized regular configurations

- static axially symmetric electromagnetic states
  Herdeiro and Radu, *Phys. Lett. B* **749**, 393 (2015)
- vacuum gravitational wave geons
  Dias, Horowitz and Santos, *CQG* **29**, 194002 (2012)
  – helical symmetry

“Anisotropies from fluctuations of a domain wall during inflation”

by Sadra Jazayeri

[JGRG25(2015)7a3]

Anisotropies *from fluctuations of a* **Domain Wall** *during* **inflation**

Sadra Jazayeri
Institute for Research in Fundamental Sciences
(**IPM**)- Tehran

Works in Progress: In Collaboration with H. Firouzjahi and M. Akhshik & Y. Akrami
Some parts of the talk from: **1408.3057** with Y. Wang et al

Contents

- Inflationary Cosmology
- Motivation for the work: CMB Anomalies
- Inflation in presence of a 2-Brane
- Gravitational reaction of the brane on the inflaton
- Branion-Inflaton interaction

Contents

- Inflationary Cosmology
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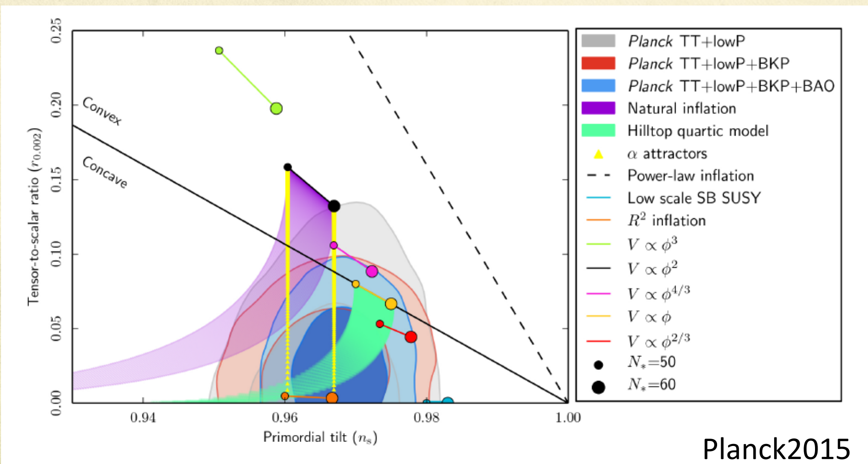
- *Branion-Inflaton interaction*

Inflationary Cosmology

- Currently Inflation is the main paradigm of early universe Cosmology.
- It acts both as a solution to Big bang Puzzles (Flatness & Horizon problems) and a source for producing primordial seeds for structures.
- Data still can not distinguish between enormous bunch of existing models



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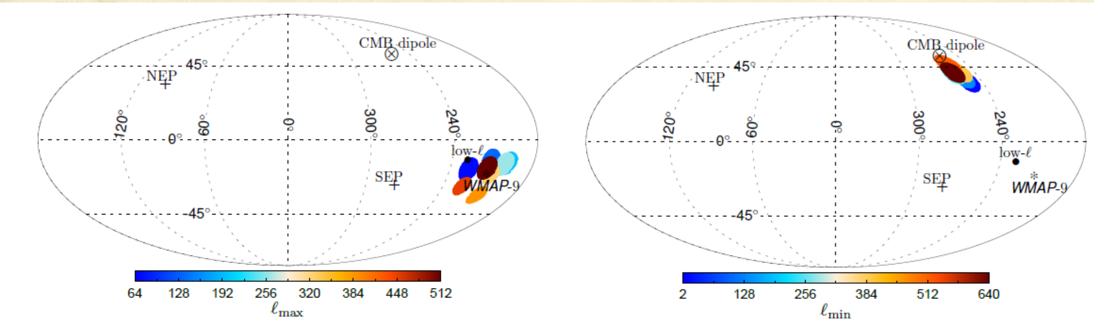
CMB Anomalies

- Planck 2013 & 2015: Quadropole-Octopole alignment, Dipole Asymmetry, Parity Violation etc.

- Dipole Power Asymmetry

$$\Delta T/T(n) = (1 + A\, n.p)(\Delta T/T)\downarrow iso \qquad l < 600$$



Planck2015

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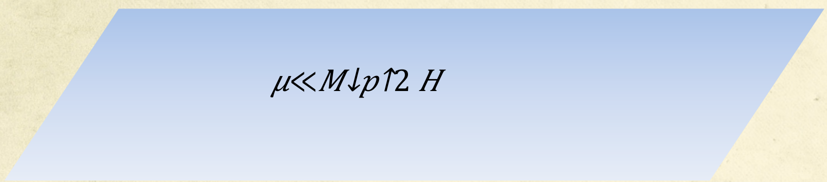
Inflation in presence of a domain wall

Inflaton



- The space time of a cosmological constant and a flat wall could be described by the following metric

$$ds^2 = -dt^2 + e^{2Ht}\, dx^i\, dx_i - 2\beta e^{Ht}\, sgn(z)\, dt dz + O(\beta, \epsilon)$$

- Where $\beta = \mu / M_p^2 H$ is a dimensionless perturbative quantity.

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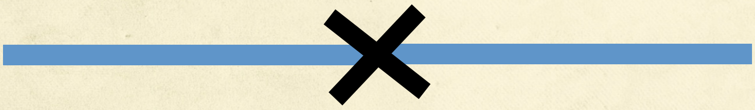
Gravitational reaction of the domain on the inflaton

- There are two contributions from our brane into inflaton **2 point** fluctuations: First one is the background geometry induced by the domain and the second is the probable Wall perturbations.

$$\sqrt{-g}\, g^{\mu\nu}\, \partial_\mu \phi \partial_\nu \phi \supset \beta\sqrt{-g}\, sgn\,(z)\partial_z\, \delta\phi\, \delta\phi'$$



- The varied 2pt due to this contribution has the following form:

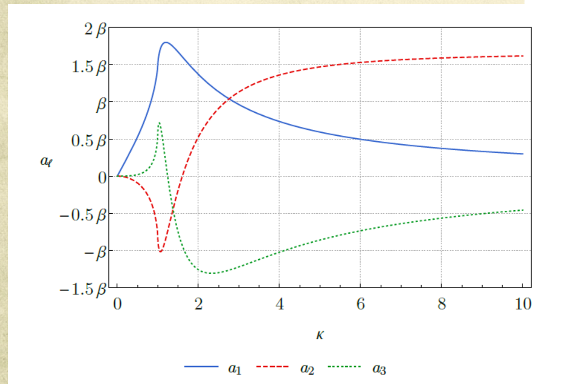
$$\delta\left\langle \delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{q}} \right\rangle = -\frac{\beta H^2}{4k^3q^3}\frac{k^2q_z+q^2k_z}{k_z+q_z}(2\pi)^3\delta^2(\mathbf{k}_{||}+\mathbf{q}_{||})$$

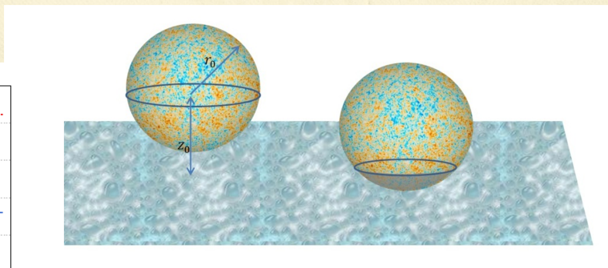
The 2Dim delta function signals the violation of a translational invariance due to the Brane.

- The background interaction produces a $O(\beta)$ variance asymmetry.

$$\langle \Delta T/T^2\,(n)\rangle = \langle \Delta T/T^2\rangle_{iso} \sum a_l P_l\,(cos\theta)$$

* This is a different signal for modeling the power asymmetry on the CMB. (**1402.0870** and **Planck 2015**)



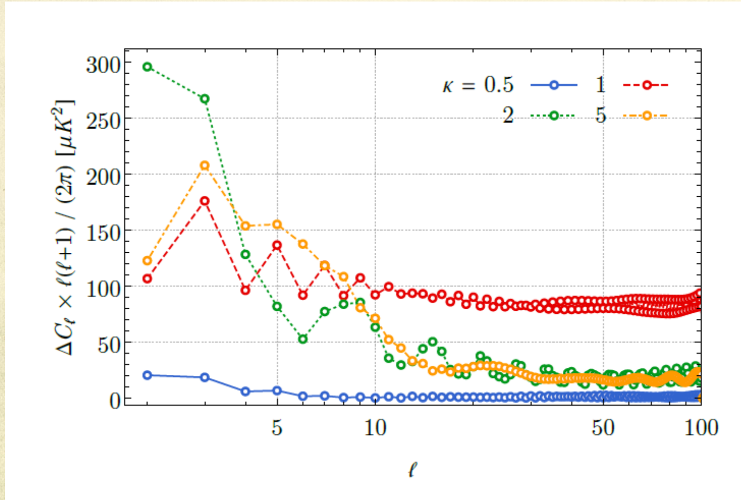


- The background interaction also affects CMB spectrum. Now there are many non zero **off-diagonal** terms thanks to the breaking of translational invariance.

$$\langle a_{l_1 m_1} a_{l_2 m_2} \rangle = C_{l_1 l_2}^{m}$$

- As expected the variation to C_l decays like $\sim 1/momentum$.

l_2



$m=0,\ l_1 = l_2$

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Branion –Inflaton interactions

- In principle our Wall could be either a defect or a fundamental brane.
- We consider some direct coupling between 2-brane and inflaton . We neglect inflaton dependent tension, consequently the leading operators are of the form:

$$S_{Brane}=\underbrace{\mu\int\sqrt{-h}\,d^3\zeta}_{\text{Nambu-Goto action}}+\Lambda\int\sqrt{-h}\;n^\mu\,\partial_\mu\phi\,d^3\zeta+1/\Lambda'\int\sqrt{-h}\,(n^\mu\,\partial_\mu\phi)^2\,d^3\zeta$$

- By choosing an approprate meshing on the brane we can set $\zeta^0=t$, $\zeta^1=x$, $\zeta^2=y$. The Height of the brane could be treated as a scalar field living on the brane. This brane perturbations (**branion**) acts as a **mediator** between inflaton legs.

$$S_\pi=4\beta M_p^2 H\int a(t)^4 d^2x_{||}dt\frac{1}{2}\left[\dot{\pi}^2-\frac{1}{a^2}(\partial_I\pi)^2\right]$$

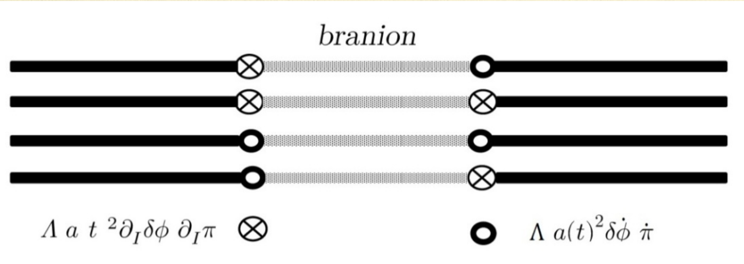
$\pi_c(k,\tau)\propto\sqrt{-k\tau}\,H_2(k\tau)$

- Power spectrum of density perturbations would be modified due to this new interactions.

$$\Lambda \int \sqrt{-h} d^3\zeta n^\mu \partial_\mu \phi \supset \Lambda \int a^3 d^4 x \delta(z) \Big[-\dot{\pi}\delta\dot{\phi} + \frac{1}{a^2}\partial_I \pi \partial_I \delta\phi \Big]$$

$$n_\mu = \frac{(\dot{\pi}, \partial_I \pi, -1)}{\Big[-\dot{\pi}^2 + \frac{1}{a^2}((\partial_I \pi)^2 + 1) \Big]^{1/2}}$$

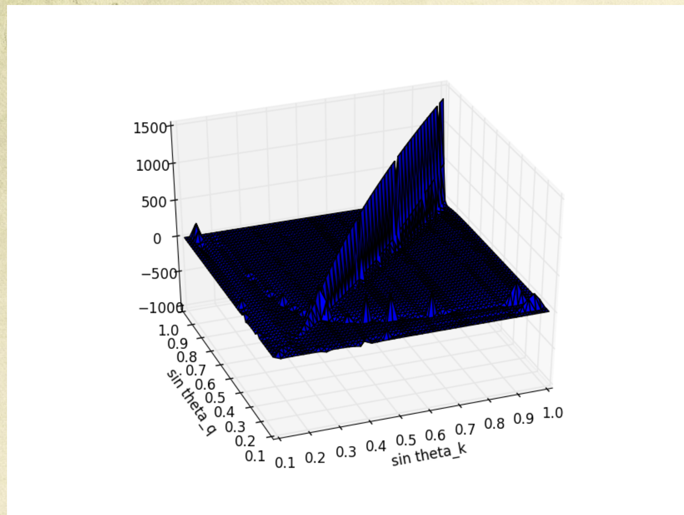
$$\delta^{(ij)} \langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{q}} \rangle = -\int_0^{t_e} dt_2 \int_0^{t_2} dt_1 \Big\langle \Big[H_I^i(t_1), [H_I^j(t_2), \delta\phi_k(t_e)\delta\phi_q(t_e)] \Big] \Big\rangle$$

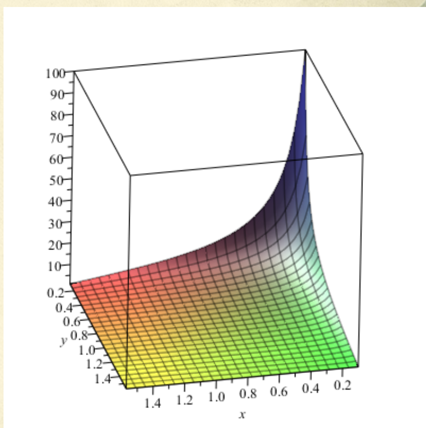


- The In-In integrals should be carried numerically. The result for the power looks like:

$$\boxed{\langle \delta\phi_{\mathbf{k}}(\tau_e) \delta\phi_{\mathbf{q}}(\tau_e) \rangle = (2\pi)^2 \delta^2(\mathbf{K} + \mathbf{Q}) \Big(\frac{\sqrt{\pi}\Lambda}{4\sqrt{\beta} M_p} \Big)^2 \frac{H^2 K^2}{k^3 q^3} \mathcal{S}(\sin\theta_k, \sin\theta_q)}$$

$$\begin{aligned}
\mathcal{S}(\sin\theta_k, \sin\theta_q) &:= \int_0^\infty \frac{dx_2}{x_2^2} \int_{x_2}^\infty \frac{dx_1}{x_1^2} \times \Bigg(\\
& Im\Big\{ x_1^2 x_2^2 H_2(x_1) H_2^*(x_2) (1 - \frac{ix_1}{\sin\theta_k}) \exp(\frac{ix_1}{\sin\theta_k}) \Big\} Im\Big\{ (1 - \frac{ix_2}{\sin\theta_q}) \exp(\frac{ix_2}{\sin\theta_q}) \Big\} \\
& + Im\Big\{ [x_1^2 H_2(x_1)]' [x_2^2 H_2^*(x_2)]' [(1 - \frac{ix_1}{\sin\theta_k}) \exp(\frac{ix_1}{\sin\theta_k})]' \Big\} Im\Big\{ [(1 - \frac{ix_2}{\sin\theta_q}) \exp(\frac{ix_2}{\sin\theta_q})]' \Big\} \\
& - Im\Big\{ x_1^2 H_2(x_1) [x_2^2 H_2^*(x_2)]' (1 - \frac{ix_1}{\sin\theta_k}) \exp(\frac{ix_1}{\sin\theta_k}) \Big\} Im\Big\{ [(1 - \frac{ix_2}{\sin\theta_q}) \exp(\frac{ix_2}{\sin\theta_q})]' \Big\} \\
& - Im\Big\{ [x_1^2 H_2(x_1)]' x_2^2 H_2^*(x_2) [(1 - \frac{ix_1}{\sin\theta_k}) \exp(\frac{ix_1}{\sin\theta_k})]' \Big\} Im\Big\{ (1 - \frac{ix_2}{\sin\theta_q}) \exp(\frac{ix_2}{\sin\theta_q}) \Big\} \Bigg) + k \leftrightarrow q
\end{aligned}$$





$$\langle \delta\phi_{\mathbf{k}}(\tau_e)\delta\phi_{\mathbf{q}}(\tau_e)\rangle = (2\pi)^2\delta^2(\mathbf{K}+\mathbf{Q})\left(\frac{\sqrt{\pi}\Lambda}{4\sqrt{\beta}M_p}\right)^2\frac{H^2K^2}{k^3q^3}\mathcal{S}(\sin\theta_k,\sin\theta_q)$$

- *After taking into account the natural values for couplings together with the resonant shape of the above function, we conclude that this contribution is as important as the background induced 2pt. The shape of the power asymmetry or any other observable is completely distinguishable from the former.*

- There are some bounds induced from Strong coupling limit of $\delta\mathcal{L}_2$. For having a perturbative region in our model we need:

$$\Lambda/\sqrt{4\beta}\, M_p \ll 1 \qquad\qquad 1/\Lambda' \ll \sqrt{2\beta}/\epsilon \;\; 1/H$$

This constraint are important for estimating the amount of Power asymmetry and PNG.

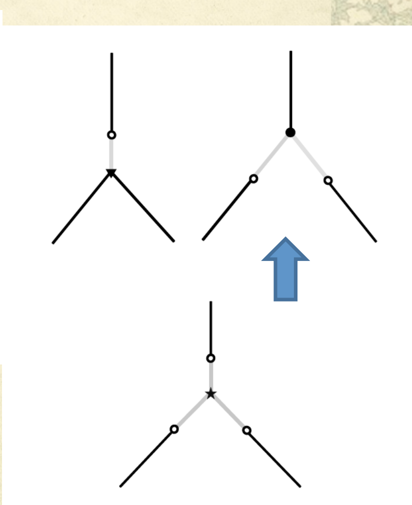
- There are many things interesting about Primordial Non-Gaussianities in our set up. The Shape of PNG is very non trivial and completely anisotropic.

$$(i):\Lambda/2\int dtd^2x\left[a(\partial_I\pi)^2\partial_z\delta\phi - a^3\dot{\pi}^2\partial_z\delta\phi\right]\subset\Lambda\int n^\mu\partial_\mu\phi\sqrt{-h}d^3\zeta$$

$$(ii):\Lambda\dot{\phi}\int\frac{1}{2}a^4dtd^2x\left[\dot{\pi}^3-\dot{\pi}(\partial_I\pi)^2\right]\subset\Lambda\int n^\mu\partial_\mu\phi\sqrt{-h}d^3\zeta$$

$$(iii):\frac{2}{\Lambda'}\int dtd^2x\left[\dot{\pi}\delta\dot{\phi}\partial_z\delta\phi-\frac{1}{a^2}\partial_I\pi\partial_I\delta\phi\partial_z\delta\phi\right]\subset\frac{1}{\Lambda'}\int(n^\mu\partial_\mu\phi)^2\sqrt{-h}d^3\zeta$$

$$(iv):\frac{\dot{\phi}}{\Lambda'}\int a^2dtd^2x\delta(z)\left(\dot{\pi}^2\delta\dot{\phi}-\frac{1}{a^2}\dot{\pi}\partial_I\pi\partial_I\delta\phi\right)\subset\frac{1}{\Lambda'}\int(n^\mu\partial_\mu\phi)^2\sqrt{-h}d^3\zeta$$



For instance : $f_{NL}^i \sim \langle\zeta^3\rangle/\langle\zeta^2\rangle\langle\zeta^2\rangle \sim (\Lambda/\sqrt{\beta}\, M_p)^3\sqrt{\epsilon}/\beta$

Conclusion and prospective

- We have proposed a model for explaining some of the CMB anomalies. The set up consists of an inflationary epoch happening in vicinity of a massive domain wall.
- There are two kinds of interactions between inflaton and the brane: the background gravitational induced interaction in the bulk and the **localized** interactions on the boundary due to **branion.**
- The former induces a sufficient amount of variance asymmetry on the CMB sphere, as well as variation of angular power spectrum on large scales.
- For a fundamental brane these two contributions to 2pt are comparable and have completely different shapes.
- The PNG in this set up is appealing. Especially in $\beta \to 0$ case, could it be related to the recent idea of **"Cosmological Collider physics? 1503.08043"** which concerns inflation as a laboratory for studying the effects of different fields and branes on primordial perturbations?
- What happens for Primordial Gravitational waves in presence of the brane?
- So far we have neglected the gravitational reaction of branions. In what regimes this assumption is justified?

“Does the Gauss-Bonnet term stabilize wormholes?”

by Takafumi Kokubu

[JGRG25(2015)7a4]

JGRG25 @ YITP

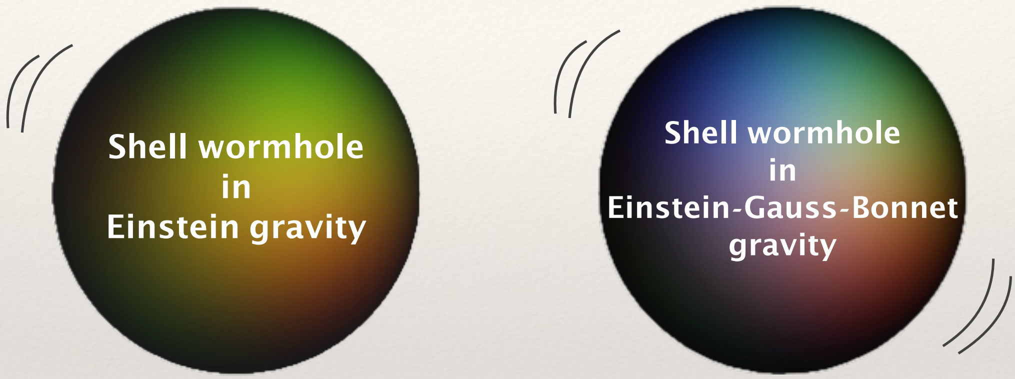
Does the Gauss-Bonnet term stabilize wormholes?

Takafumi Kokubu
Hideki Maeda
Tomohiro Harada

Classical and Quantum Gravity, 32, (2015) 235021

Rikkyo University
Hokkai-Gakuen University

Today's Talk



- **STABILITY against RADIAL PERTURBATIONS.**
- **Compare the two wormhole (ANALYTICALLY).**

  → Reveal the EFFECT of the GB term on STABILITY.

Motivation

Wormholes are fascinating compact objects.

$\rightarrow$ space-time short cut, time travel.

Problem: instability, use of exotic matter, great tidal force…

Stability is the first priority for wormhole study.

Einstein-Gauss-Bonnet gravity

$$S = \frac{1}{16\pi G}\int \mathrm{d}^d x\sqrt{-g}(R - 2\lambda + \alpha L_{GB}) \qquad L_{\mathrm{GB}} := R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

The Gauss-Bonnet term appears in the action as the ghost-free quadratic curvature correction term in the low-energy limit of heterotic superstring theory in ten dimensions (together with a dilaton).

Einstein gravity

$$\boxed{S = \frac{1}{16\pi}\int \mathrm{d}^d x\sqrt{-g}(R - 2\Lambda)}$$

Vacuum solution

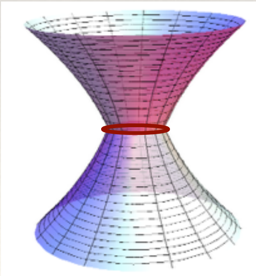
$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\Omega^k_{d-2})^2, \qquad f(r) = k - \frac{m}{r^{d-3}} - \tilde{\Lambda}r^2$$

Setup

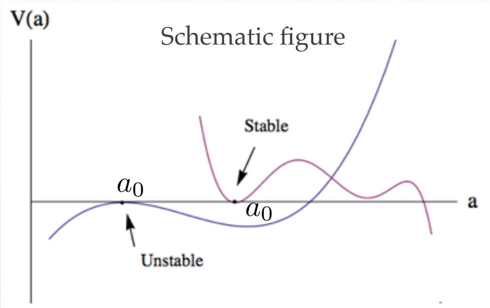
Junction conditions : $S^i{}_j = -\frac{1}{8\pi G}([K^i{}_j] - \delta^i{}_j[K])$

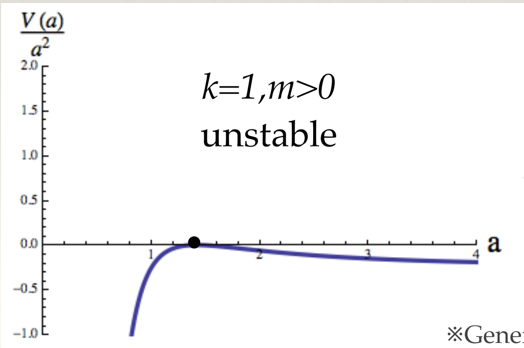
Imposition for matter :
negative tension $(\sigma < 0)$
$S^i{}_j = \mathrm{diag}(-\sigma, -\sigma, -\sigma, \cdots, -\sigma)$

Imposition for symmetry :
Z_2 **symmetry**



Master equation & Stability





Birkhoff's theorem → no GW for radial motion

Master eq for radial motion : $\dot{a}^2 + V(a) = 0,$

$$V(a) = f(a) - \left(\frac{4\pi\sigma}{d-2}\right)^2 a^2 \qquad \sigma : \text{brane-tension}$$

Stable: $V''(a_0) > 0$

Unstable: $V''(a_0) < 0$

$$\boxed{V''(a_0) = -\frac{2(d-3)k}{a_0^2}} \qquad k = \pm 1, 0$$

One static unstable solution

※Generalization for Einstein-Maxwell system→ T.K, Harada, 2015

Einstein-Gauss-Bonnet gravity

$$\boxed{S = \frac{1}{16\pi G}\int d^d x\sqrt{-g}(R - 2\lambda + \alpha L_{GB})} \qquad (d \geq 5)$$

$$L_{\rm GB} := R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

α : coupling constant, inverse string tension

Alpha>0

Vacuum solution

$$ds_d^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\gamma_{AB}dz^A dz^B$$

$$f(r) := k + \frac{r^2}{2\tilde{\alpha}}\left(1 \mp \sqrt{1 + \frac{4\tilde{\alpha}m}{r^{d-1}} + 4\tilde{\alpha}\tilde{\Lambda}}\right)$$

$$\tilde{\Lambda} := \frac{2\Lambda}{(d-1)(d-2)}$$

$$\tilde{\alpha} := (d-3)(d-4)\alpha$$

$-$: the GR branch
$+$: non-GR branch

in GR branch, $\alpha \to 0$

$$f(a) = k - \frac{m}{r^{d-3}} - \tilde{\Lambda}r^2$$

GR branch

Setup

Junction conditions: $[K^i{}_j]_\pm - \delta^i{}_j[K]_\pm + 2\alpha\Big(3[J^i{}_j]_\pm - \delta^i{}_j[J]_\pm - 2P^i{}_{kjl}[K^{kl}]_\pm\Big) = -\kappa_d^2 S^i{}_j$

$J_{ij} := \frac{1}{3}\left(2KK_{ik}K^k{}_j + K_{kl}K^{kl}K_{ij} - 2K_{ik}K^{kl}K_{lj} - K^2K_{ij}\right), \quad P_{ikjl} := \mathcal{R}_{ikjl} + 2h_{i[l}\mathcal{R}_{j]k} + 2h_{k[j}\mathcal{R}_{l]i} + \mathcal{R}h_{i[j}h_{l}$

Imposition for matter: **negative tension**

Imposition for symmetry: Z_2 **symmetry**

EGB Master equation& Stability criterion

Birkhoff's theorem → no GW for radial motion

Master eq. for radial motion: $\dot{a}^2 + V(a) = 0 \qquad V(a) := f(a) - J(a)a^2,$

$$J(a) := \frac{\left(B(a) - A(a)^{1/2}\right)^2}{4\tilde{\alpha}B(a)}, \quad B(a) := \left\{18\tilde{\alpha}\Omega^2 + A(a)^{3/2} + 6\sqrt{\tilde{\alpha}\Omega^2(9\tilde{\alpha}\Omega^2 + A(a)^{3/2})}\right\}^{1/3}. \quad \Omega := \frac{16\pi^2\sigma^2}{(d-2)^2}$$

Stability criterion

$$V''(a_0) = -\frac{2kP(a_0)}{a_0^2(a_0^2 + 2k\tilde{\alpha} + 2\tilde{\alpha}f_0)(a_0^2 + 2k\tilde{\alpha} - 2\tilde{\alpha}f_0)},$$

$$P(a_0) := 4\tilde{\alpha}^2 f_0\left\{6k - (d-3)f_0\right\} + (a_0^2 + 2k\tilde{\alpha})\left\{(d-3)a_0^2 + 2(d-5)k\tilde{\alpha}\right\}$$

$\longrightarrow \boxed{V''(a_0) \propto -kP(a_0)}$

$k = \pm 1, 0$

Instability for $k = 1$ with $m > 0$

$$V''(a_0) \propto -kP(a_0) \longrightarrow V''(a_0) \propto -P(a_0)$$

$$P(a_0) := 4\tilde{\alpha}^2 f_0\left\{6k - (d-3)f_0\right\} + (a_0^2 + 2k\tilde{\alpha})\left\{(d-3)a_0^2 + 2(d-5)k\tilde{\alpha}\right\}$$

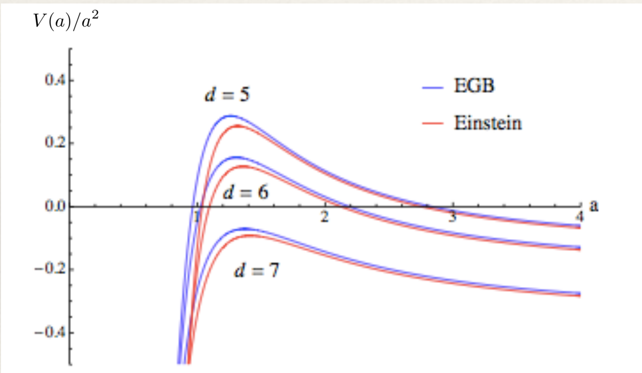


Figure: The potential $\bar{V}(a)$ for $d = 5, 6, 7$ in Einstein and Einstein-Gauss-Bonnet (EGB) gravity with $k = 1$, $\alpha = 0.02$, $m = 1$, $\Lambda = 1$ and $\sigma = -0.1$.

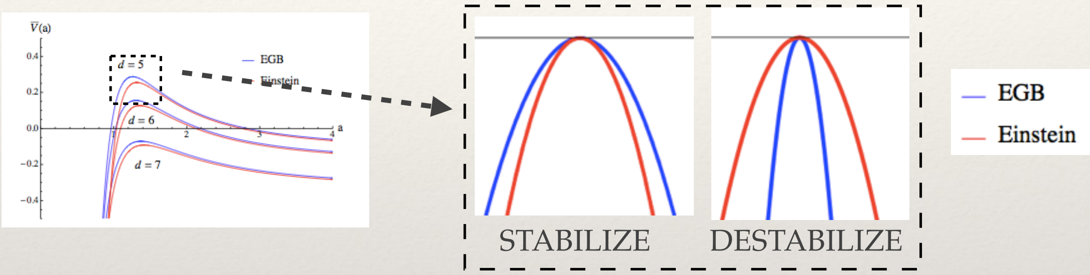
$$\begin{aligned}
\mathbf{P(a_0)} &= 4\tilde{\alpha}^2 f_0\left\{6 - (d-3)f_0\right\} \\
&\quad + (a_0^2 + 2\tilde{\alpha})\left\{(d-3)a_0^2 + 2(d-5)\tilde{\alpha}\right\} \\
&> 4\tilde{\alpha}^2 f_0\left\{6 - (d-3)f_0\right\} + 2\tilde{\alpha}f_0\left\{(d-3)a_0^2 + 2(d-5)\tilde{\alpha}\right\} \\
&= 2\tilde{\alpha}f_0\left\{2(d-3)\tilde{\alpha}\left(\frac{a_0^2}{2\tilde{\alpha}} - f_0\right) + 2(d+1)\tilde{\alpha}\right\} \\
&> 2\tilde{\alpha}f_0\left\{-2(d-3)\tilde{\alpha} + 2(d+1)\tilde{\alpha}\right\} = 16\tilde{\alpha}^2 f_0 > \mathbf{0}.
\end{aligned}$$

$\longrightarrow P(a_0) > 0$

the wormhole is unstable

Does the small Gauss-Bonnet term stabilize wormholes?

GB shell wormhole turned out to be unstable.
But, how it unstable is when compared with Einstein shell wormhole?



α is sufficiently small : $\epsilon := \tilde{\alpha}/a_{\rm E}^2 \ll 1$

Perturbative analysis : $a_0 = a_{\rm E} + a_{(1)}\epsilon + a_{(2)}\epsilon^2 + \ldots.$

ϵ expansion up to 1st order : $V''_{\rm GB}(a_0) \simeq V''_{\rm Einstein}(a_{\rm E}) - k\dfrac{8 f_{\rm E}(a_{\rm E})}{a_{\rm E}^2}\epsilon$

k=*1*:Destabilize

Summary

- We construct thin-shell wormholes made of its tension in Einstein and EGB gravity.
- This is the best setup to analyze stability as a pure gravitational effect because such a thin shell does not suffer from the matter instability.
- Shell wormhole both in Einstein and EGB gravity is unstable.
- When α is small, the GB term destabilizes spherically symmetric shell wormhole.

Future Work :
Since Kanti *et al.* (2012) numerically reported stable wormholes in EGB + dilaton system,
The Effect of Dilaton is the Next Target.

“High energy particle emission form particle collision near an extremal Kerr black hole”

by Kota Ogasawara

[JGRG25(2015)7a5]

High energy particle emission from particle collision near an extremal Kerr black hole

No. 7a5

Kota Ogasawara, Rikkyo U.

arXiv 1511.00110 [gr.qc]

K.O., T. Harada, and U. Miyamoto

High energy particle emission from particle collision near an extremal Kerr black hole

energy extraction process from the black hole

Question

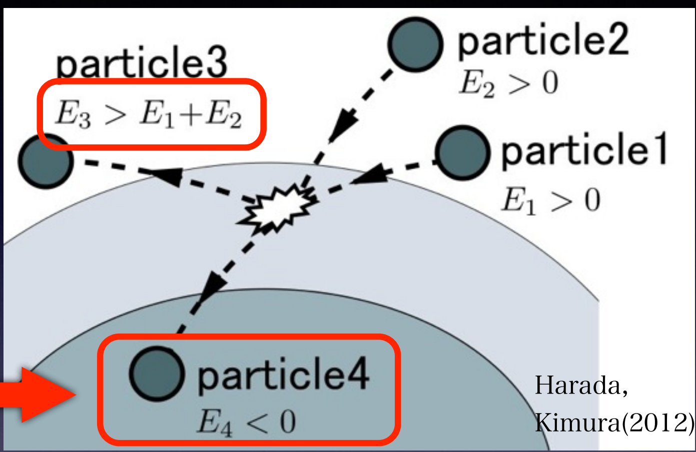
「Can we extract energy
from the black hole !?」

If the answer is yes.

⇒ 「How long can we extract
energy from the black hole !?」

Collisional Penrose process

- particle collision in the ergoregion
- energy conservation
  $E_1 + E_2 = E_3 + E_4$
  negative energy



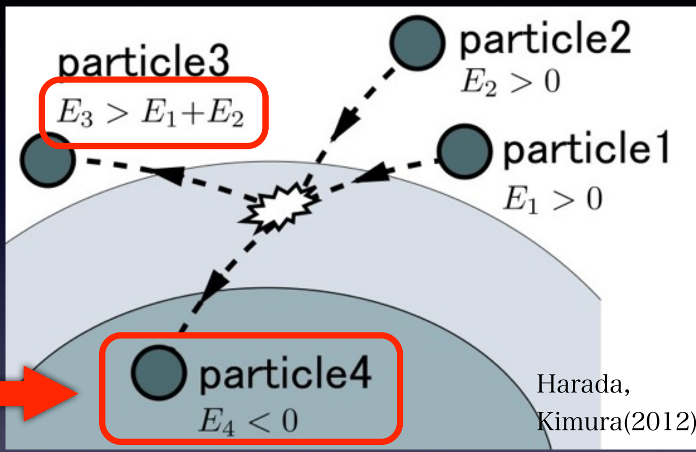
- energy extraction efficiency

$$\eta := \frac{E_3}{E_1 + E_2}$$

1

Collisional Penrose process

- particle collision
  in the ergoregion



- energy conservation
  $E_1 + E_2 = E_3 + E_4$

  negative energy

- energy extraction efficiency

  $\eta > 1$

  energy extraction !!

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Why Collisional Penrose process?

- more realistic process than original Penrose process (divide → collision)
- ultra-high-energy cosmic ray
- near horizon Physics
- B.S.W. effect (2009)
  ・・arbitrarily high CM energy ⇐ talk later
- super-Penrose process (2014)
  ・・arbitrarily high energy extraction (on going)

Outline

1. Introduction ⇐ finish
2. (Usual) collisional Penrose process
3. B.S.W. effect and heavy particle production
4. Summary

Outline

1. Introduction ⇐ finish
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Kerr spacetime

- conserved quantity

$E := -g_{\mu\nu}\xi^\mu p^\nu$: energy

$L := g_{\mu\nu}\psi^\mu p^\nu$: angular momentum

- geodesic eq.s. $(\theta = \pi/2,\ a = M)$

$$\frac{1}{2}(p^r)^2 + V(r) = 0 \quad \Leftarrow \text{1D potential problem}$$

$$V(r) = -\frac{m^2}{r/M} + \frac{\tilde{L} - E^2 + m^2}{2(r/M)^2} - \frac{(\tilde{L} - E)^2}{(r/M)^3} - \frac{E^2 - m^2}{2}$$

where $\tilde{L} := L/M$

Particle collision and reaction

- particle collision : $p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$

- conservation equations

$$E_1 + E_2 = E_3 + E_4, \quad \tilde{L}_1 + \tilde{L}_2 = \tilde{L}_3 + \tilde{L}_4$$

$$\sigma_1|p_1^r| + \sigma_2|p_2^r| = \sigma_3|p_3^r| + \sigma_4|p_4^r|, \ (p^r = \sigma|p^r|)$$

where $\sigma = \pm 1$

$$|p^r| = \left[\frac{2m^2}{r/M} - \frac{\tilde{L}^2 - E^2 + m^2}{2(r/M)^2} + \frac{2(\tilde{L} - E)^2}{(r/M)^3} + E^2 - m^2\right]^{\frac{1}{2}}$$

Particle collision and reaction

parameters

E : energy

$\tilde{L}$: angular momentum

m : mass

σ : sign of p^r

→ 16 unknown

$E_1,\ E_2,\ E_3,\ E_4$

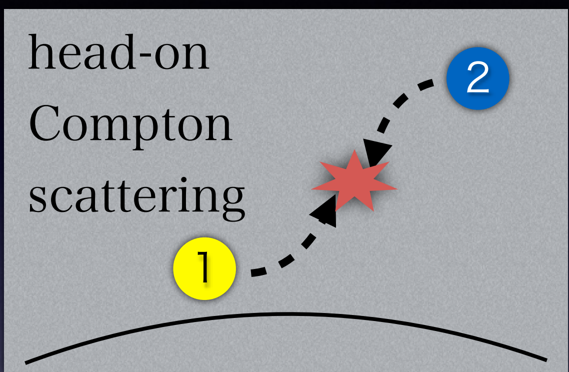
$\tilde{L}_1,\ \tilde{L}_2,\ \tilde{L}_3,\ \tilde{L}_4$

$m_1,\ m_2,\ m_3,\ m_4$

$\sigma_1,\ \sigma_2,\ \sigma_3,\ \sigma_4$

$$|p^r| = \left[\frac{2m^2}{r/M} - \frac{\tilde{L}^2 - E^2 + m^2}{2(r/M)^2} + \frac{2(\tilde{L}-E)^2}{(r/M)^3} + E^2 - m^2\right]^{\frac{1}{2}}$$

Particle collision and reaction



←

$\boxed{E_1,\ E_2,}\ E_3,\ E_4$

$\boxed{\tilde{L}_1,\ \tilde{L}_2,}\ \tilde{L}_3,\ \tilde{L}_4$

$\boxed{m_1,\ m_2,}\ m_3,\ m_4$

$\boxed{\sigma_1,\ \sigma_2,}\ \sigma_3,\ \sigma_4$

if we choose

$\sigma_1 = 1,\ \sigma_2 = -1$

$m_1 = 0,\ m_2 \neq 0$

the parameter of particle 1 and 2 gives it by hand

Near-horizon behavior

- using radial momentum conservation
  to estimate the upper limit of efficiency : η

$$\sigma_1|p_1^r| + \sigma_2|p_2^r| = \sigma_3|p_3^r| + \sigma_4|p_4^r|$$

$$|p^r| = \left[\frac{2m^2}{r/M} - \frac{\tilde{L}^2 - E^2 + m^2}{2(r/M)^2} + \frac{2(\tilde{L}-E)^2}{(r/M)^3} + E^2 - m^2\right]^{\frac{1}{2}}$$

horizon : $r = M$

critical : $\tilde{L} = 2E$

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Near-horizon behavior

- using radial momentum conservation
  to estimate the upper limit of efficiency : η

$$\sigma_1|p_1^r| + \sigma_2|p_2^r| = \sigma_3|p_3^r| + \sigma_4|p_4^r|$$

$$|p^r| = \left[\frac{2m^2}{r/M} - \frac{\tilde{L}^2 - E^2 + m^2}{2(r/M)^2} + \frac{2(\tilde{L}-E)^2}{(r/M)^3} + E^2 - m^2\right]^{\frac{1}{2}}$$

near horizon : $r = \dfrac{M}{1-\epsilon},\ 0 < \epsilon \ll 1$

near critical : $\tilde{L} = 2E(1+\delta),\ \delta = \delta_{(1)}\epsilon + \delta_{(2)}\epsilon^2 + O(\epsilon^3)$

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Near-horizon behavior

- using radial momentum conservation
  to estimate the upper limit of efficiency : η

$$\sigma_1|p_1^r| + \sigma_2|p_2^r| = \sigma_3|p_3^r| + \sigma_4|p_4^r|$$

$$|p^r| = p_{(0)}^r + p_{(1)}^r\epsilon + p_{(2)}^r\epsilon^2 + O(\epsilon^3)$$

expanded
radial momentum

near horizon : $r = \dfrac{M}{1-\epsilon},\ 0 < \epsilon \ll 1$

near critical : $\tilde{L} = 2E(1+\delta),\ \delta = \delta_{(1)}\epsilon + \delta_{(2)}\epsilon^2 + O(\epsilon^3)$

5

Near-horizon behavior

- e.g.) 1=critical, 2=subcritical,
  3=near critical, 4=negative energy

$$\sigma_1|p_1^r| + \sigma_2|p_2^r| = \sigma_3|p_3^r| + \sigma_4|p_4^r|$$

$$|p^r| = p_{(0)}^r + p_{(1)}^r\epsilon + p_{(2)}^r\epsilon^2 + O(\epsilon^3)$$

expanded
radial momentum

$$|p_1^r| = \sqrt{3E_1^2 - m_1^2}\epsilon + O(\epsilon^2)$$

$$|p_2^r| = (2E_2 - \tilde{L}_2) - 2(E_2 - \tilde{L}_2)\epsilon + O(\epsilon^2)$$

$$|p_3^r| = \sqrt{E_3^2\left[4(1-\delta_{(1)})^2 - 1\right] - m_3^2}\epsilon + O(\epsilon^2)$$

$$|p_4^r| = (2E_2 - \tilde{L}_2) - \left[2(E_2 - \tilde{L}_2) + 2E_3(1-\delta_{(1)}) - 2E_1\right]\epsilon + O(\epsilon^2)$$

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Expanded conservation eq.s.

- e.g.) 1=critical, 2=subcritical,
  3=near critical, 4=negative energy

$$\sigma_1|p_1^r| + \sigma_2|p_2^r| = \sigma_3|p_3^r| + \sigma_4|p_4^r|$$

$O(\epsilon^0)$ → estimate σ_i

$$|p_1^r| = \sqrt{3E_1^2 - m_1^2\epsilon + O(\epsilon^2)}$$
$$|p_2^r| = (2E_2 - \tilde{L}_2) - 2(E_2 - \tilde{L}_2)\epsilon + O(\epsilon^2)$$
$$|p_3^r| = \sqrt{E_3^2\left[4(1-\delta_{(1)})^2 - 1\right] - m_3^2\epsilon + O(\epsilon^2)}$$
$$|p_4^r| = (2E_2 - \tilde{L}_2) - \left[2(E_2 - \tilde{L}_2) + 2E_3(1-\delta_{(1)}) - 2E_1\right]\epsilon + O(\epsilon^2)$$

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6

Expanded conservation eq.s.

- e.g.) 1=critical, 2=subcritical,
  3=near critical, 4=negative energy

$$\sigma_1|p_1^r| + \sigma_2|p_2^r| = \sigma_3|p_3^r| + \sigma_4|p_4^r|$$

$O(\epsilon)$ → estimate the upper limit of E_3

$$|p_1^r| = \sqrt{3E_1^2 - m_1^2\epsilon + O(\epsilon^2)}$$
$$|p_2^r| = (2E_2 - \tilde{L}_2) - 2(E_2 - \tilde{L}_2)\epsilon + O(\epsilon^2)$$
$$|p_3^r| = \sqrt{E_3^2\left[4(1-\delta_{(1)})^2 - 1\right] - m_3^2\epsilon + O(\epsilon^2)}$$
$$|p_4^r| = (2E_2 - \tilde{L}_2) - \left[2(E_2 - \tilde{L}_2) + 2E_3(1-\delta_{(1)}) - 2E_1\right]\epsilon + O(\epsilon^2)$$

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The upper limit of E_3

- $O(\epsilon)$ terms of p^r conservation

$$\sigma_1\sqrt{3E_1^2 - m_1^2} + 2E_1 - 2E_3(1-\delta_{(1)}) = \sigma_3\sqrt{E_3^2\left[4(1-\delta_{(1)})^2 - 1\right] - m_3^2}$$

$$\Rightarrow \quad E_{3,\mathrm{max}} = (2+\sqrt{3})^2 E_1$$

where

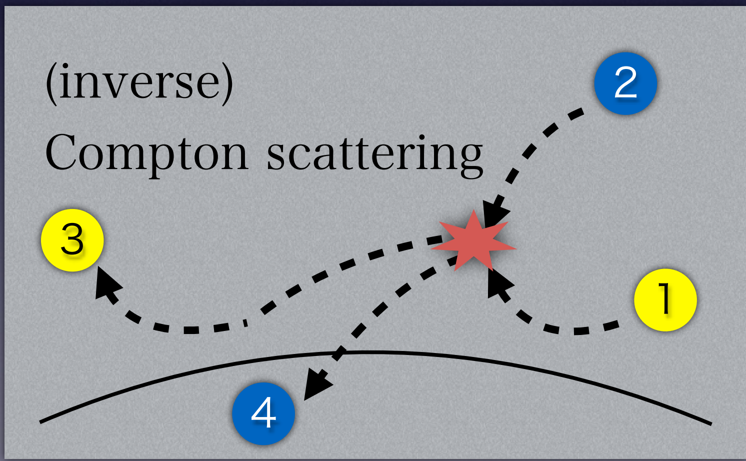
we have assumed

$m_1 = m_3 = 0$

$\sigma_1 = 1$

$\sigma_2 = \sigma_3 = \sigma_4 = -1$

$\delta_{(1)} = 0$



Energy extraction efficiency

- $O(\epsilon)$ terms of p^r conservation

  $\Rightarrow \quad E_{3,\mathrm{max}} = (2+\sqrt{3})^2 E_1$

- $O(\epsilon^2)$ terms of p^r conservation

  $\Rightarrow$ it gives the lower limit of E_2

$$\eta := \frac{E_3}{E_1 + E_2}$$

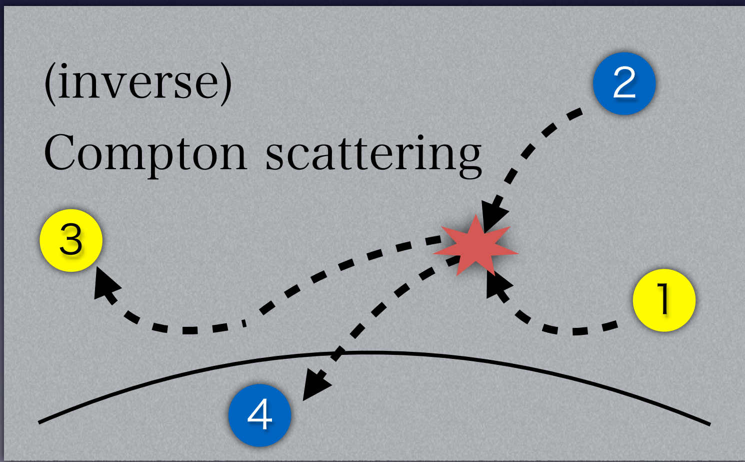
Energy extraction efficiency

- $O(\epsilon)$ terms of p^r conservation

  $\Rightarrow \quad E_{3,\max} = (2+\sqrt{3})^2 E_1$

- $O(\epsilon^2)$ terms of p^r conservation

$\eta_{\max} \simeq 2.19$



$m_1 = m_3 = 0$

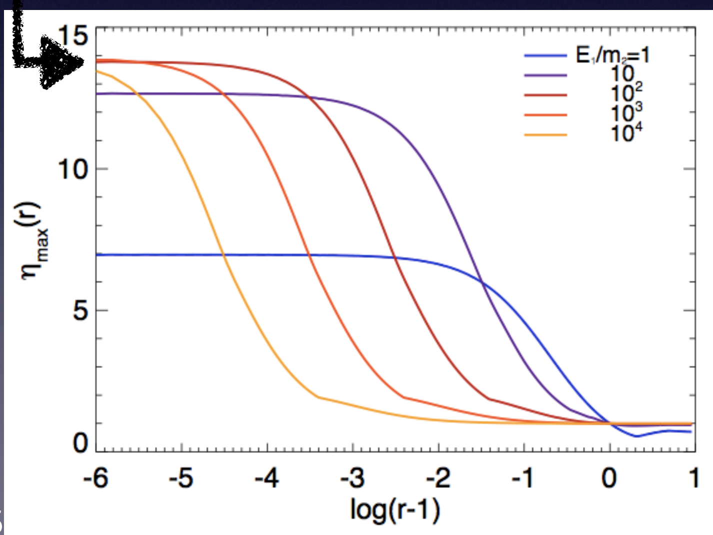
$\sigma_1 = 1$

$\sigma_2 = \sigma_3 = \sigma_4 = -1$

Energy extraction efficiency

inconsistent !?

$\eta_{\max} \simeq 14 \neq \eta_{\max} \simeq 2.19$



$m_1 = m_3 = 0$

$\sigma_1 = 1$

$\sigma_2 = \sigma_3 = \sigma_4 = -1$

Schnittman (2014)

Outline

1. Introduction ⇐ finish

2. Usually collisional Penrose process ⇐ finish

3. B.S.W. effect and heavy particle production

4. Summary

B.S.W. effect

- CM energy of particle 1 and 2

$$E_{\rm cm}^2 := -g_{\mu\nu}(p_1^\mu + p_2^\mu)(p_1^\nu + p_2^\nu)$$
$$\simeq \frac{2(2E_1 - \sqrt{3E_1^2 - m_1^2})(2E_2 - \tilde{L}_2)}{\epsilon} + O(\epsilon^0)$$

where $2E_1 - \tilde{L}_1 = 0$: 1=critical

$2E_2 - \tilde{L}_2 > 0$: 2=subcritical

$r = \dfrac{M}{1-\epsilon}$: near horizon

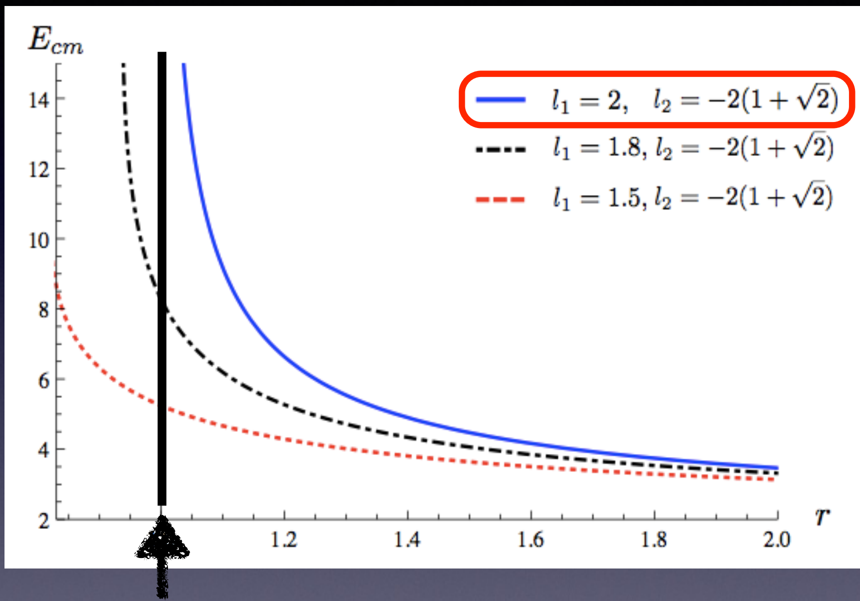
$\epsilon \to 0$: horizon limit

$\Rightarrow E_{\rm CM} \to \infty$: arbitrarily high CM energy!!

B.S.W. effect

Bañados, Silk, West (2009)

$$E_{\rm cm} \propto \frac{1}{\sqrt{\epsilon}}$$



$\epsilon \to 0$: horizon limit

$\Rightarrow E_{\rm CM} \to \infty$: arbitrarily high CM energy!!

B.S.W. effect

$$E_{\rm cm} \propto \frac{1}{\sqrt{\epsilon}}$$

- The arbitrarily high CM energy can produce very energetic and/or heavy particle

$E \propto 1/\sqrt{\epsilon}$ and/or $m \propto 1/\sqrt{\epsilon}$

high energy collision !!

Heavy particle production

- using the momentum conservation

$$m_4^2 = -(p_1^\mu + p_2^\mu - p_3^\mu)(p_{1\mu} + p_{2\mu} - p_{3\mu})$$
$$= E_{\rm cm}^2 + m_3^2 + 2(p_1^\mu + p_2^\mu)p_{3\mu}$$
$$O(1/\sqrt{\epsilon}) \qquad O(\epsilon^0)$$

- we can assume particle 4 is very massive as

$m_4^2 = \dfrac{\mu_4}{\epsilon} + \nu_4$ where μ_4 (> 0) and ν_4 are constants

only in this case, we can obtain $\eta_{\rm max} \simeq 14$
if the energy, mass and μ_4 are fine tuned

Energy extraction efficiency

- $O(\epsilon)$ terms of p^r conservation

$\Rightarrow$ $E_{3,\rm max} = (2+\sqrt{3})^2 E_1$

$\cancel{\eta_{\rm max} \simeq 2.19}$

- $O(\epsilon^2)$ terms of p^r conservation

$\Rightarrow$ it gives the ~~lower limit~~ of E_2 ($m_4 = O(\epsilon^0)$ case)

$\Rightarrow$ it using estimate ν_4 ($m_4 = O(1\sqrt{\epsilon})$ case)

Energy extraction efficiency

- $O(\epsilon)$ terms of p^r conservation

  $\Rightarrow \quad E_{3,\mathrm{max}} = (2+\sqrt{3})^2 E_1$

- $O(\epsilon^2)$ terms of p^r conservation

  $\Rightarrow$ it gives the ~~lower limit of~~ E_2 ($m_4 = O(\epsilon^0)$ case)

  $\Rightarrow$ it using estimate ν_4 ($m_4 = O(1\sqrt{\epsilon})$ case)

- if $E_1 E_2 \gg \mu_4$

$$\eta_{\mathrm{max}} = \frac{(2+\sqrt{3})^2 E_1}{E_1 + E_2}$$

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Energy extraction efficiency

- $O(\epsilon)$ terms of p^r conservation

  $\Rightarrow \quad E_{3,\mathrm{max}} = (2+\sqrt{3})^2 E_1$

- $O(\epsilon^2)$ terms of p^r conservation

  $\Rightarrow$ it gives the ~~lower limit of~~ E_2 ($m_4 = O(\epsilon^0)$ case)

  $\Rightarrow$ it using estimate ν_4 ($m_4 = O(1\sqrt{\epsilon})$ case)

- if $E_1 E_2 \gg \mu_4$

- moreover, for $E_1 / E_2 \gg 1$

$$\eta_{\mathrm{max}} \simeq 14$$

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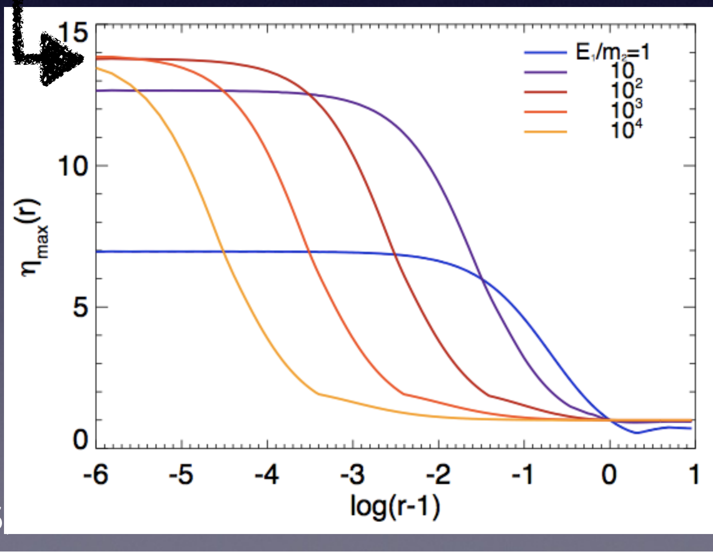
Energy [illegible] iency

consistent !!

$\eta_{\max} \simeq 14$ $=$ $\eta_{\max} \simeq 14$



$(\, E_1/E_2 \gg 1\,)$

Schnittman (2014)

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In other cases

- e.g.) energetic escaping particle case

$E_3^2 = \dfrac{\rho_3}{\epsilon} + \phi_3 \;\Rightarrow\; |p_3^r|$ and $|p_4^r|$ have $O(\sqrt{\epsilon})$ terms

LHS no $O(\sqrt{\epsilon})$

$$\sigma_1|p_1^r| - |p_2^r| = \sigma_3|p_3^r| - |p_4^r|$$

$$\Rightarrow \quad 0 = \sigma_3\sqrt{\rho_3[4(1-\delta_{(1)})^2 - 1]} - 2(1-\delta_{(1)})\sqrt{\rho_3}$$

there is no solution for ρ_3 under the assumption of $\rho_3 > 0$

$\Rightarrow$ there is no energy extraction

Outline

1. Introduction ⇐ finish

2. Usually collisional Penrose process ⇐ finish

3. B.S.W. effect and heavy particle production
⇐ finish

4. Summary

Question

「Can we extract energy
from the black hole !?」

The answer is 「Yes !!」

⇒ 「How long can we extract
energy from the black hole !?」

So far, upper limit of the efficiency is 「14 !!」

Summary

- Upper limit of the energy extraction efficiency of particle collision in the ergoregion
  - critical+subcritical+ $m_4 = O(\epsilon^0)$: 220%
  - critical+subcritical+ $m_4 = O(1/\sqrt{\epsilon})$: 1400%
- If η as large as 10 is observed for this process this strongly suggest the production $m_4 = O(1/\sqrt{\epsilon})$ as a result of the collision of high CM energy
- Two subcritical particles collision might realize more high efficiency or arbitrarily high efficiency

(on going)

“Wormhole shadows”

by Takayuki Ohgami

[JGRG25(2015)7a6]

Wormhole Shadows

Graduate School of Science and Engineering
Yamaguchi University
Takayuki Ohgami and Nobuyuki Sakai

Contents

1. Background : Black hole shadows
2. Ellis wormhole
3. Numerical calculation
4. Results
5. Conclusion

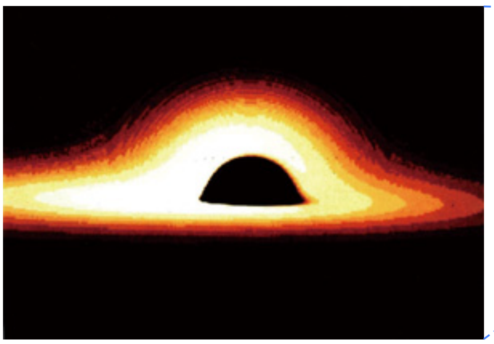
+ Background

3

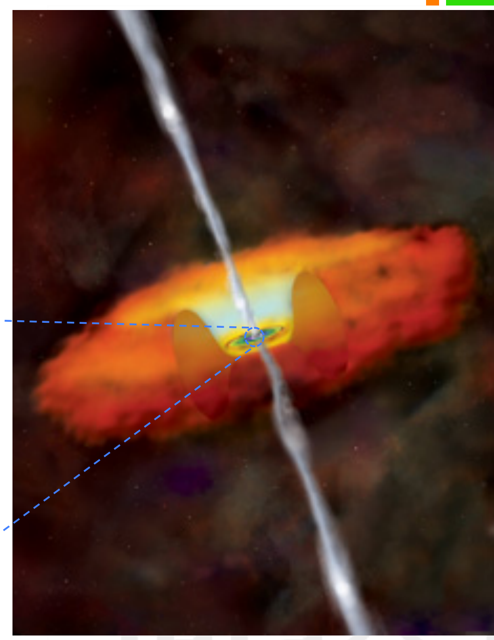
4

+ Black hole shadows

- Observation of black hole horizon is attempted.
- Black hole shadows



Fukue+ (1988)



Aurore Simonnet,
Sonoma State Univ.

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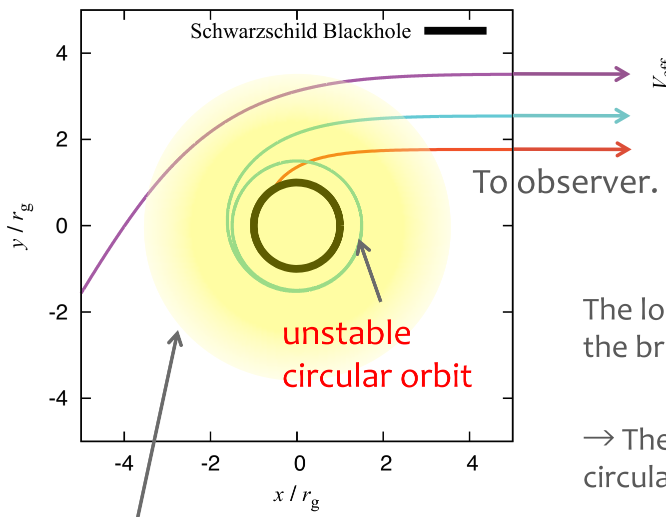
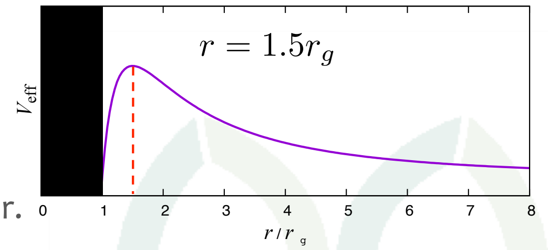
- cause of black hole shadow
- photon orbits from source to observer in Schwarzschild spacetime



Touch up Ohgami+ 2015.

Interstellar medium (light source)



Effective potential of photon.

The longer orbit pass through the source, the brighter light becomes.

→ The light passing through unstable circular orbit is observed the most brightest.

bright ring

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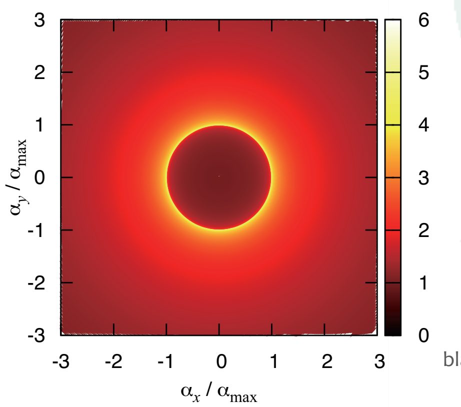
- cause of shadow
  - Light ray passing through unstable circular orbit is the brightest.
  - Inner rays are darker because black hole horizon arrest light rays.
  - Shadow is given by contrast of brightness.



black hole shadow.

Ellis wormhole

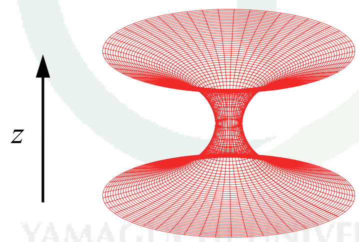
Ellis wormhole spacetime

$$ds^2 = -dt^2 + dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

- a : throat radius

- About wormhole.
  - tunnel-like structure which connects two distant or disconnected regions.
  - Warp drive (long-distance movement at short times), Time travel

- traversable wormhole



Ohgami+ 2015

Embedding diagram of Ellis wormhole.

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- Ellis wormholes are unstable.
  - Shinkai and Hayward, Phys. Rev. D **66**, (2002)
  - Gonzalez et al., Classical Quantum Gravity **26**, (2009)

- Exotic matter could contribute to supporting the Ellis geometry.
  - Das and Kar, Classical Quantum Gravity **22**, (2005)

- Even if other wormhole solutions are applied, we could discuss similarly by using method in this study.

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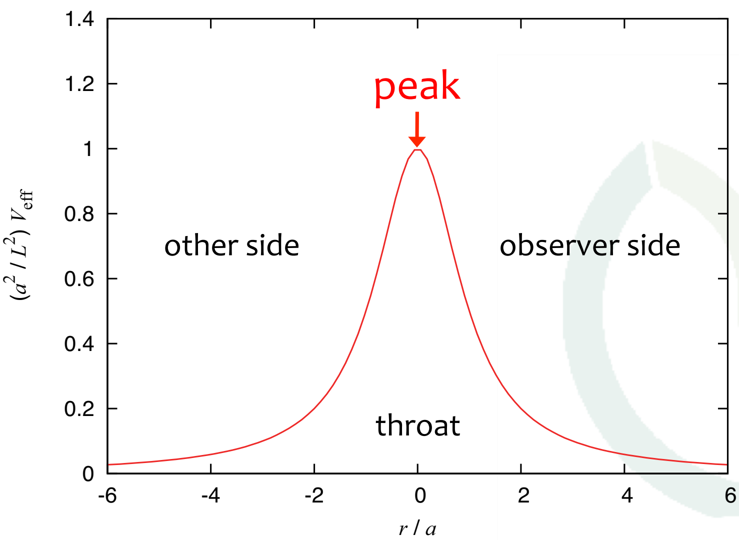
- Effective potential of photon.



Touch up Ohgami+ 2015.

- It has unstable circular orbits in throat.
- shadow appears.

Numerical calculation

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Intensity distribution

- intensity distribution of light source
- Intensity : radiative transfer equation

$$\frac{d\mathcal{J}}{d\lambda} = \frac{\eta(\nu)}{\nu^2} - \nu\chi(\nu)\mathcal{J} \qquad \mathcal{J} \equiv \frac{I(\nu)}{\nu^3}$$

  - interstellar medium emit only

$$\eta(\nu)d\lambda = -H(\nu)\rho u_\mu dx^\mu, \quad \chi(\nu) = 0$$

depend on the condition of interstellar medium
ρ : energy density, u^μ : four-velocity

- Relativistic primitive equations (dust approximation)

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- continuity equation

$$(nu^{\mu})_{;\mu} = 0$$

m : mass of constituent particle
n : number density

- Euler equation

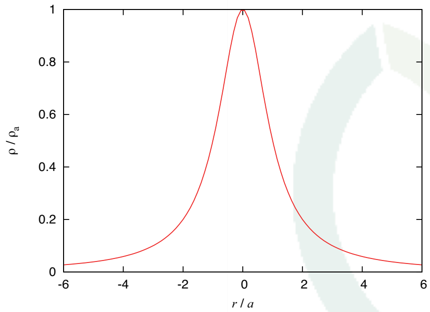
$$nmu_{\mu;\nu}u^{\nu} = 0$$

- Solutions

- Spherical symmetry

$$u^r = \text{const.}(\neq 0)$$

$$\rho \propto \frac{1}{r^2 + a^2}$$



Distribution of dust.

Ohgami+ 2015

- Axial symmetry

$$u^{\varphi} = u_0^{\varphi}\frac{a^2}{r^2 + a^2} \qquad u^r = \pm\sqrt{(au_0^{\varphi})^2\left(1 - \sin^2\theta\frac{a^2}{r^2 + a^2}\right) + (u_0^r)^2}$$

$$\dot{M} = 4\pi mnu^r(r^2 + a^2) = \text{constant}$$

$$\rho = \rho_0\frac{u_0^r}{u^r}\frac{a^2}{r^2 + a^2}$$

$u^r \neq 0$

subscript 0 means value in $r = 0$ and $\theta = \pi/2$.

Result

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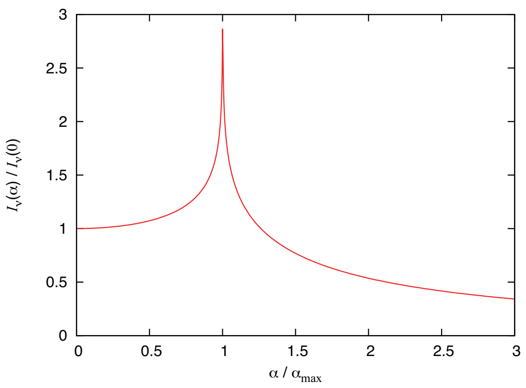
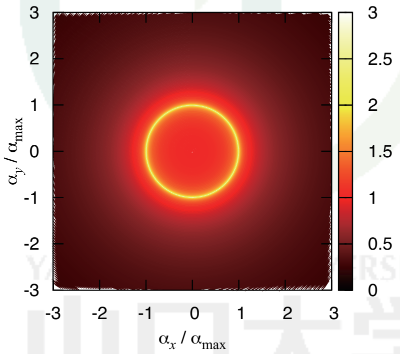
- Spherical symmetry
  - Wormhole shadow
  - Intensity distribution has a similar shape as black hole.

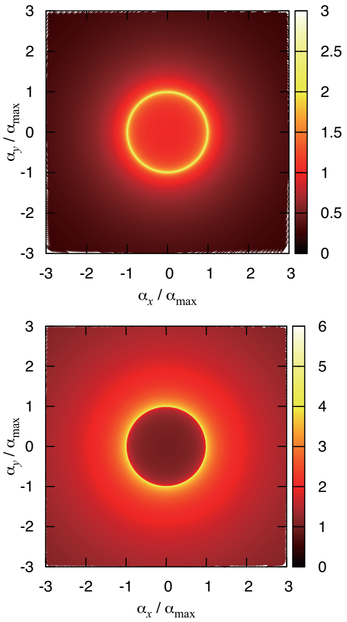


Ohgami+ 2015



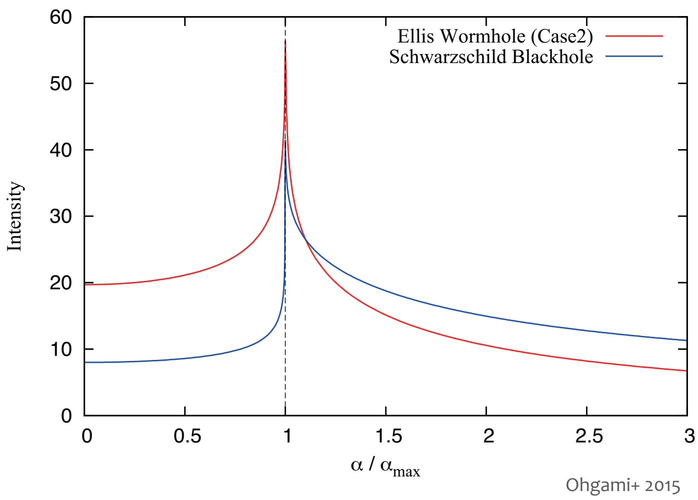
wormhole shadow.

- Comparison with Schwarzschild black hole.
  - Intensity distributions have a peak in common.
  - Black hole : Inner region < Outer region.
  - Wormhole : Inner region > Outer region.



wormhole shadow.

black hole shadow.



Ohgami+ 2015

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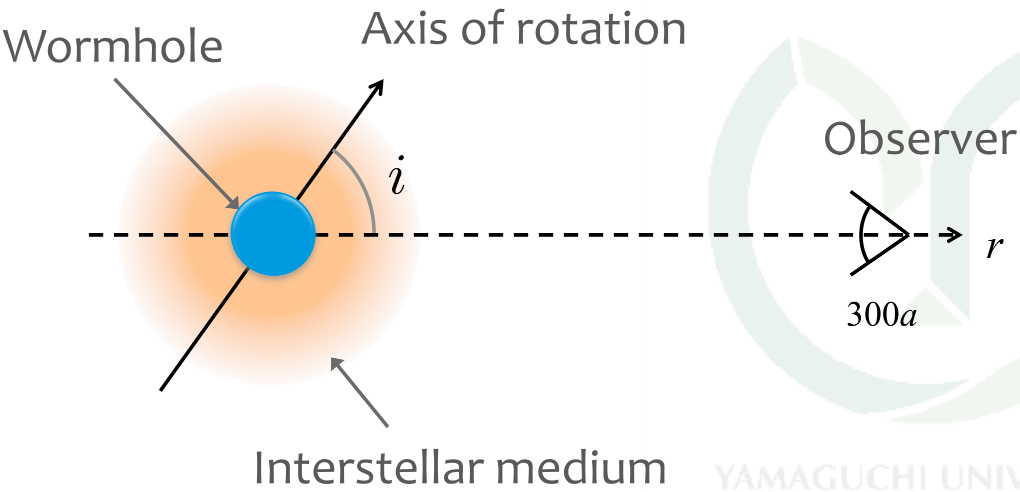
❖ Axial symmetry



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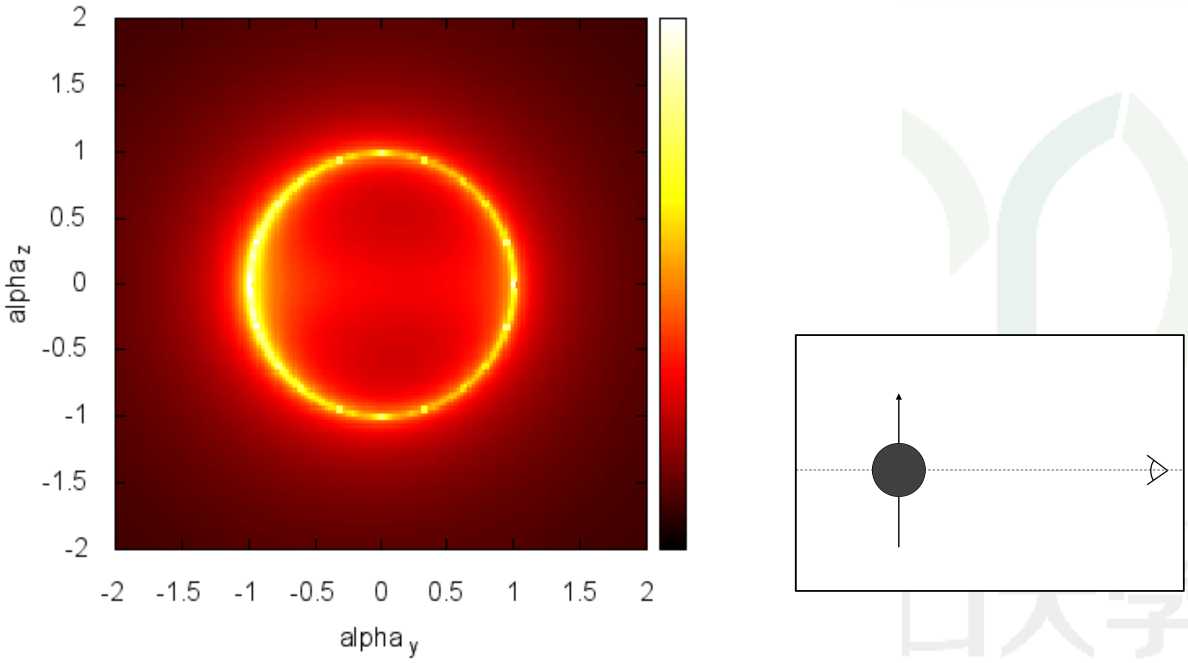
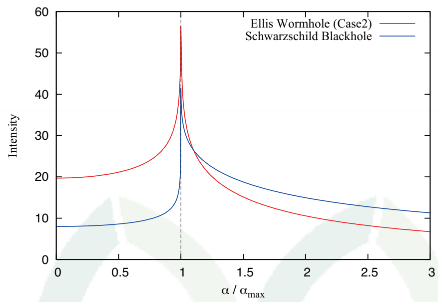
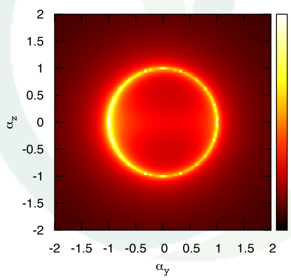
❖ Result : change direction of axis.



+ Conclusion

- We calculated intensity distribution of dust flow around Ellis wormhole.
- spherical symmetry
  - similar shape as black hole
  - intensity contrast are quite different.
- axial symmetry
  - bright ring : beaming effect
  - weakly luminous ring





- We could detect Ellis wormholes by these properties.
- It needs to use high-resolution observations (e.g. VLBI).

“Directional dependence of the local estimation of H0 and the non perturbative effects of primordial curvature perturbations”

by Antonio Enea Romano

[JGRG25(2015)7b1]

Directional dependence of the local estimation of H_0 and the non perturbative effects of primordial curvature perturbations

Antonio Enea Romano[1]

[1]UDEA, University of Crete
Based on work in collaboration with
Alexei Starobinsky, Misao Sasaki, Sergio Peña,Sergio Sanes
Partially based on
Eur.Phys.J. C72 (2012) 2242, Europhys.Lett. 106 (2014) 69002, Europhys.Lett. 109 (2015) 3, 39002 .

Outline

Outline

H_0 estimation tension
Luminosity distance in an inhomogeneous space: LTB case
Effects of non perturbative evolution of primordial curvature
Consequences of ignoring large scale inhomogeneities

H_0 tension

- A 3σ tension has been claimed between the local(low red-shift Supernovae at $z \approx 0.04$,(Riess, Astrophys.J. **730**, 119, 2011) and cosmological (Planck CMB data) estimation of the Hubble parameter
- $H^{app}_{0,SN} \approx 1.09 H^{app}_{0,CMB}$, where $H^{app}_{0,SN}$ and $H^{app}_{0,CMB}$ are the values estimated from fitting respectively low-redshift supernovae and CMB observations
- Taking into account the effects of metallicity on the P-L relation for the Cepheid can reduce substantially the discrepancy (Efstathiou , MNRAS 2014)
- Could this apparent tension be the result of the effects local structure on the luminosity distance of low red-shift supernovae ?

- The metric plays the role of the gravitational potential in GR and the effects of a spatially homogeneous potential **with time evolution determined by dark energy** can be partially **mimicked in red-shift space by the effects of a spatially inhomogeneous potential without dark energy**
- In the case of the luminosity distance the same red-shift can be associated to the **expansion** of the Universe (assuming spatial homogeneity) or to the propagation through a **spatially inhomogeneous potential** (metric)
- The effects can be important even for relatively small inhomogeneities compatible with inflation theory
- The assumption of homogeneity can **mistakenly lead to the conclusion of an evolving dark energy** with $w(z)$ while in fact there is only a cosmological constant with $w = -1$.

LTB metric

- The LTB metric can be written as:

$$ds^2 = -dt^2 + \frac{(R_{,r})^2 \, dr^2}{1 + 2\,E(r)} + R^2 d\Omega^2 \,, \tag{1}$$

- Introducing the following variables

$$a(t,r) = \frac{R(t,r)}{r}, \quad k(r) = -\frac{2E(r)}{r^2}, \quad \rho_0(r) = \frac{6M(r)}{r^3}$$

$$ds^2 = -dt^2 + a^2\left[\left(1 + \frac{a_{,r} r}{a}\right)^2 \frac{dr^2}{1 - k(r)r^2} + r^2 d\Omega_2^2\right]$$

$$\left(\frac{\dot a}{a}\right)^2 = -\frac{k(r)}{a^2} + \frac{\rho_0(r)}{3a^3} + \frac{\Lambda}{3}$$

Outline

H_0 estimation tension
Luminosity distance in an inhomogeneous space: LTB case
Effects of non perturbative evolution of primordial curvature

- The **luminosity distance** in a LTB space-time is
  $D_L(z) = (1+z)^2 R\left(t(z), r(z)\right) = (1+z)^2 r(z) a\left(\eta(z), r(z)\right)$,
  where $\Big(t(z), r(z)\Big)$ or $\Big((\eta(z), r(z)\Big)$ is the solution of the radial geodesic equation as a function of z.
- The past-directed radial null geodesics is given by

$$\frac{dt}{dr} = -\frac{R_{,r}(t,r)}{\sqrt{1+2E(r)}} \, .$$

  from which we can get:

$$\frac{dr}{dz} = \frac{\sqrt{1+2E(r(z))}}{(1+z)\dot{R}_{,r}[r(z),t(z)]} \, ,$$

$$\frac{dt}{dz} = -\frac{R_{,r}[r(z),t(z)]}{(1+z)\dot{R}_{,r}[r(z),t(z)]} \, .$$

Outline

- In order to make a connection with the **early universe** we introduce the metric which describes a **spherically symmetric space-time after inflation** at scales much exceeding the Hubble one:

$$ds^2 = -dt^2 + a_F^2(t)e^{2\zeta(r)}(dr^2 + r^2 d\Omega^2)\,.$$

-

$$ds^2 = -dt^2 + \frac{(R_{,r})^2\, dr^2}{1+2\,E(r)} + R^2 d\Omega^2\,, \qquad (2)$$

$R = a_F(t)e^{\zeta} r$, we find the exact relation:

$$1 + 2E(r) = [1 + r\zeta'(r)]^2.$$

- In the linear approximation, this reduces to

$$k(r) = -2\frac{\zeta'(r)}{r}\,.$$

- We define apparent the value of H_0^{app} obtained assuming homogeneity, and H_0^{true} the one obtained taking the inhomogeneity into account
- In order to resolve the tension we will assume that H_0^{CMB} is the correct estimation. This can be achieved if two conditions are satisfied
- Fit the low red-shift luminosity distance with $H_{0,CMB}^{app}$

$$D_L^{inh}(z_{SN}, H_{0,CMB}^{true}) = D_L^{obs}(z_{SN}) = D_L^{hom}(z_{SN}, H_{0,SN}^{app}) \,. \quad (3)$$

- The distance to the last scattering surface should be unaffected by the inhomogeneity

$$D^{hom}(H_{0,CMB}^{true}, z_{LS}) \approx D^{inh}(H_{0,CMB}^{true}, z_{LS}) \Rightarrow H_{0,CMB}^{app} = H_{0,CMB}^{true} \quad (4)$$

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- We consider an ansatz for the curvature function of this type

$$k(r) = Ae^{-\left(\frac{r-r_0}{\sigma}\right)^2} \,, \quad (5)$$

- By an appropriate choice of parameters it is possible to **modify the luminosity distance** relation only in the vicinity of z_{SN} while leaving **unchanged the distance to the last scattering**

- Luminosity density data show that the local universe is inhomogeneous and anisotropic for $z < 0.15$
- Both the Cepheid used for calibration and the low red-shift supernovae are in this range
- This implies both SN fitting and calibration could have a direction dependence, and will definitely be affected by the local inhomogeneity

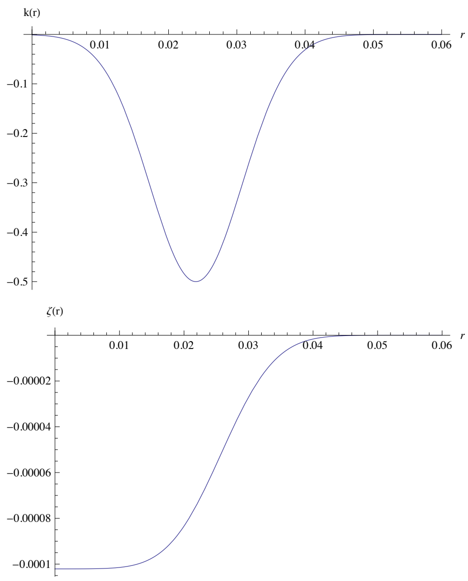

Figure: The primordial curvature perturbation $\zeta(r)$ and the function $k(r)$ are plotted for $A = -0.5$, $r_0 = r_{SN} * 0.8$, $\sigma = r_0/2.5$, and $r_{SN} = r^{\Lambda CDM}$. The quantities A and $k(r)$ are in units of $(H^{true}_{0,CMB})^2$, while r_0 is in units of $(H^{true}_{0,CMB})^{-1}$.

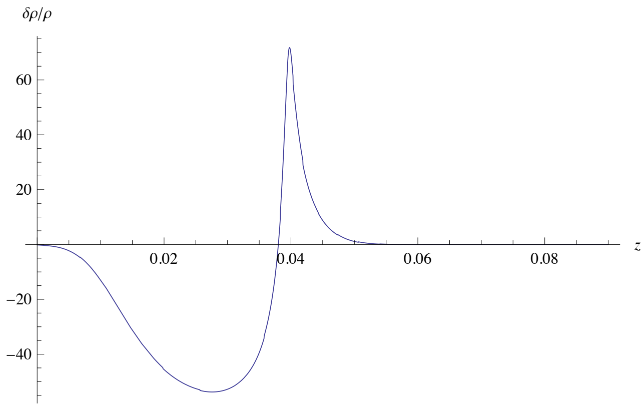

Figure: On the top the percentage density contrast $\frac{\delta\rho}{\rho} = 100(1 - \frac{\rho^{LTB}(z)}{\rho^{\Lambda CDM}(z)})$ is plotted as a function of the redshift, showing how contrary to the linear theory approximation, when non perturbative effects are taken into account, not only underdense regions but also overdense regions can be associated to the decrease of the luminosity distance necessary to explain observations.

- For the luminosity distance in a LTB space

$$D_L^{inh}(z, H_0^{true}) = D^{\Lambda LTB}(z) = (1+z)^2 R(t(z), r(z)) \tag{6}$$

where we set the parameters of the LTB solution so that

$$H_0^{LTB} = \frac{2}{3}\frac{\dot{R}(t_0,0)}{R(t_0,0)} + \frac{1}{3}\frac{\dot{R}'(t_0,0)}{R'(t_0,0)} = H_0^{true}. \tag{7}$$

- For homogeneous cosmological models we assume a flat ΛCDM solution according to

$$\begin{aligned} H(z) &= H_0^{app}\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}\,, \\ D_L^{hom}(z, H_0^{app}) &= (1+z)\int_0^z \frac{dx}{H(x)}\,. \end{aligned} \tag{8}$$

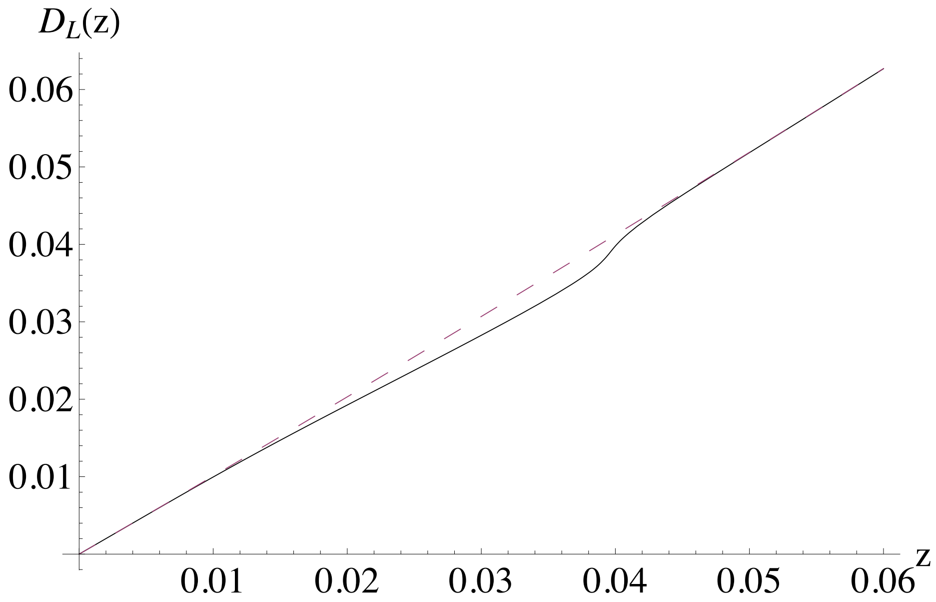

Figure: The dashed line is the plot of $D^{hom}(H^{true}_{0,CMB}, z)$ while the solid line is for $D^{inh}(H^{true}_{0,CMB}, z)$, both in units of $(H^{app}_{0,CMB})^{-1}$.

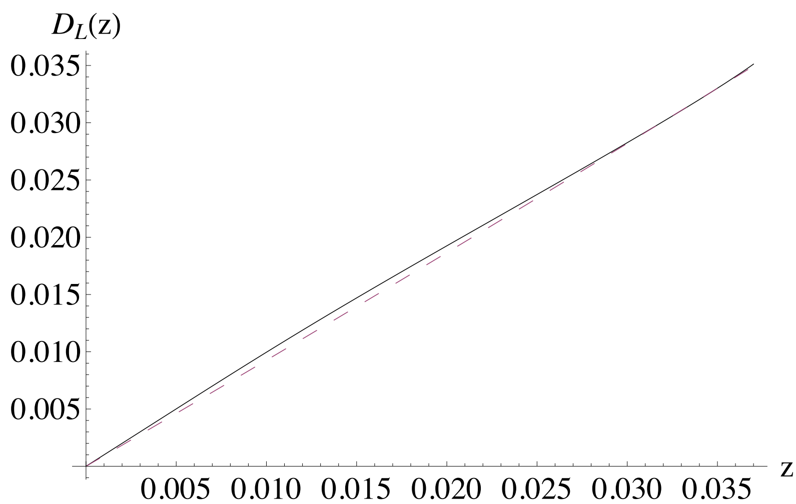

Figure: $D^{hom}(H^{app}_{0,SN}, z)$ for a homogeneous model and $D^{inh}(H^{true}_{0,CMB}, z)$ for an inhomogeneous model are plotted in units of $(H^{app}_{0,CMB})^{-1}$ respectively with a dashed and solid line.

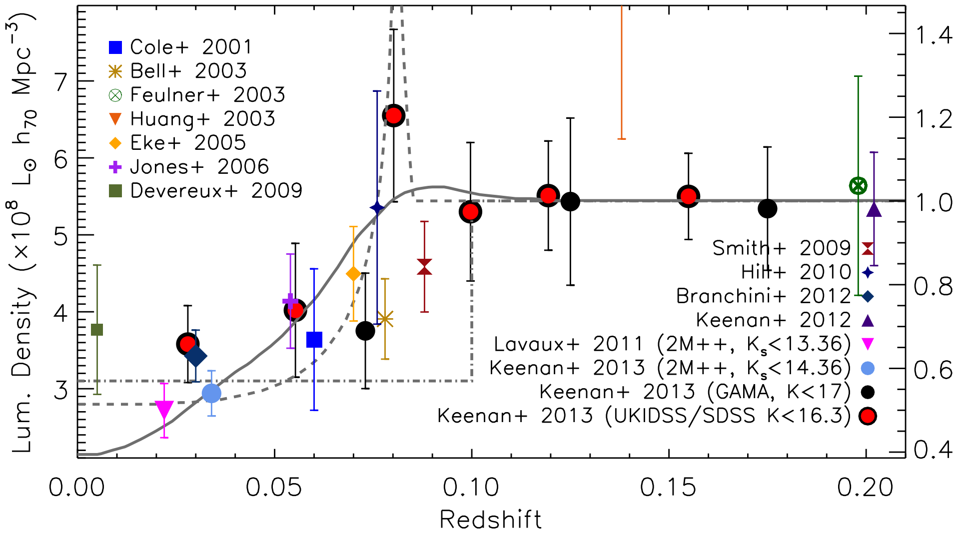

Figure: Density contrast in different directions as a function of red-shift(fig 11,Astrophys. J. **775**, 62 (2013)). The subregion 2 is approximately in the same direction of the supernovae set used by Riess(same RA≅azimuth range, wider DEC≅zenith).The density contrast profile is similar to the one determined theoretically.

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LTB metric and primordial curvature fluctuations
H_0 tension
Effects on the cosmological constant

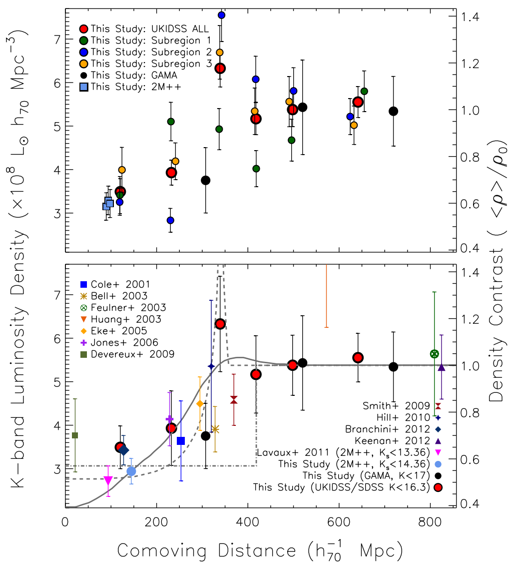

Figure: Density contrast in different directions as a function of the comoving distance(fig 11,Astrophys. J. **775**, 62 (2013)). The subregion 2 is approximately in the same direction of the supernovae set used by Riess(same RA≅azimuth range, wider DEC≅zenith).The density contrast profile is similar to the one determined theoretically.

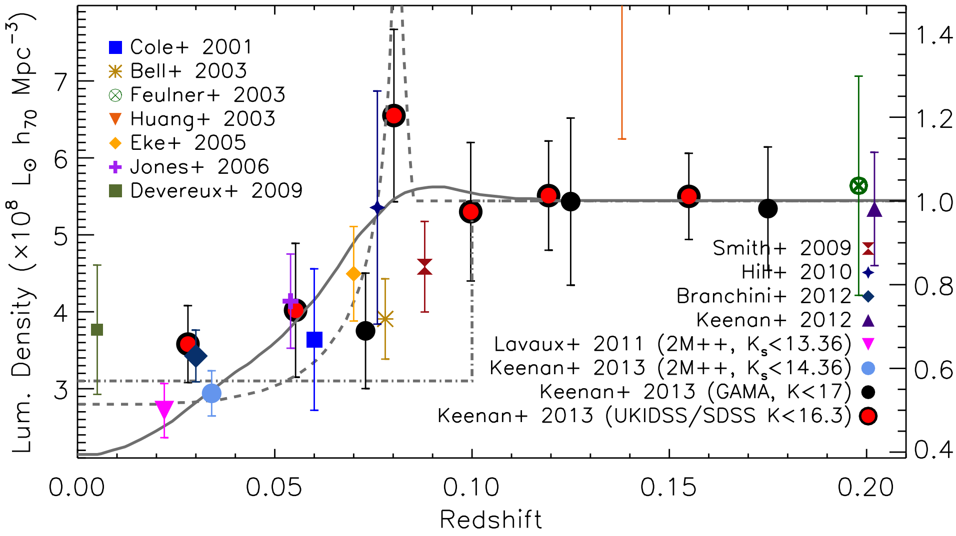

Figure: Density contrast in different directions as a function of red-shift(fig 11,Astrophys. J. **775**, 62 (2013)). The subregion 2 is approximately in the same direction of the supernovae set used by Riess(same RA≃azimuth range, wider DEC≃zenith).The density contrast profile is similar to the one determined theoretically.

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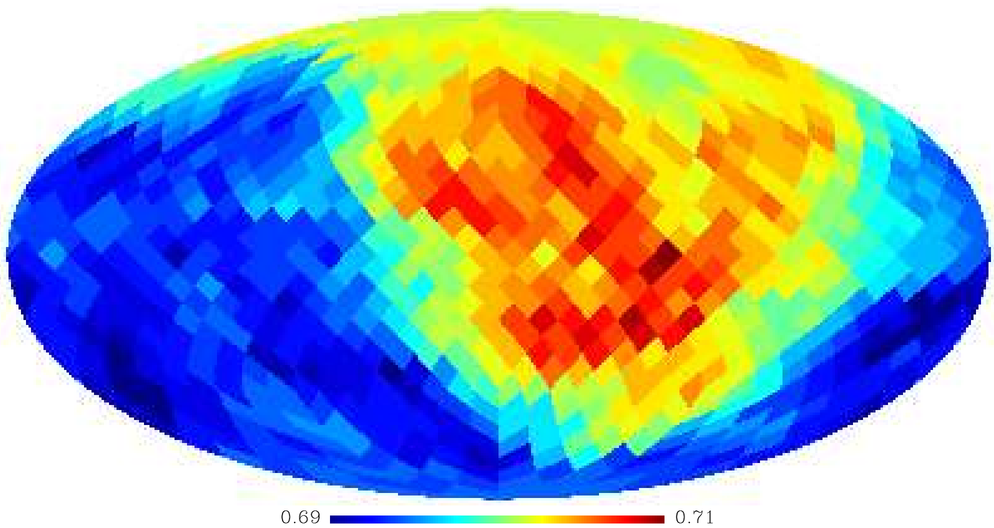


Figure: The H_0 parameter mapped through the celestial sphere, namely hubble-map, given in terms of $H_0/100$. The lowest and highest values obtained are $H_0 = 68.9 \pm 0.5\ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$ and $71.2 \pm 0.7\ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, respectively, yielding $\delta H_0 = 2.3\ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$.(arXiv:1510.05545)

Effects on the cosmological constant estimation

- We use the relation
$$k(r) = -2\frac{\zeta'(r)}{r}$$
to determine the LTB metric given $\zeta(r)$ according to the ansatz
$$\zeta(r) = Ae^{-\left(\frac{r}{\sigma}\right)^2}$$
- CMB observations give $A \approx 510^{-5}$
- The parameter σ controls both the **physical size** of the inhomogeneity and **the density contrast**
- The gaussian ansatz for the primordial curvature perturbations is not related to the gaussianity of the field, it is only a convenient profile, and since the **primordial curvature perturbation are approximately scale invariant** there is **no preferred** valued of σ a priory.

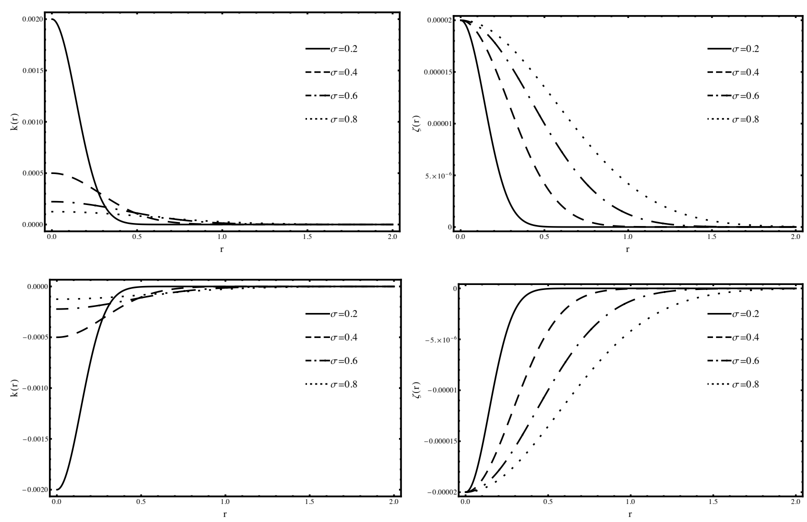
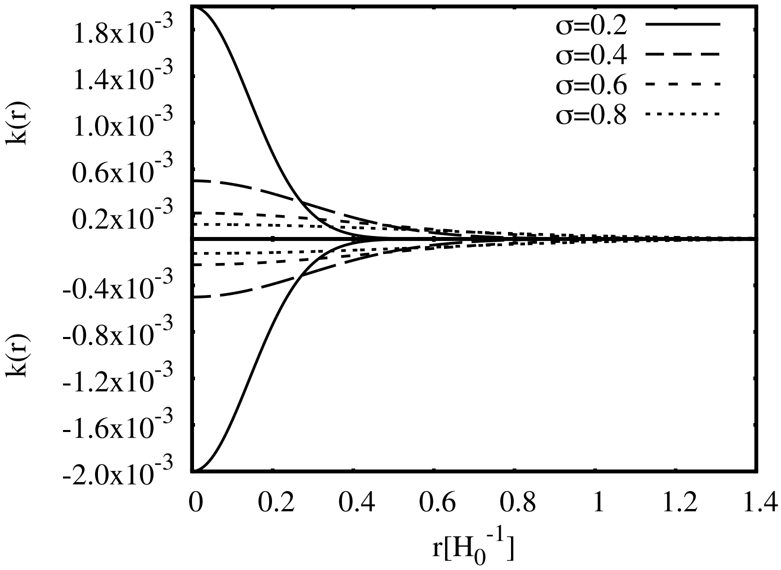


Figure: In the top figures $\zeta(r)$ and $k(r)$ are plotted for $A = 2 \times (5 \times 10^{-5})$ for different values of σ . In the bottom figures $\zeta(r)$ and $k(r)$ are plotted for $A = 2 \times (5 \times 10^{-5})$ for different values of σ .



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Effects on the cosmological constant

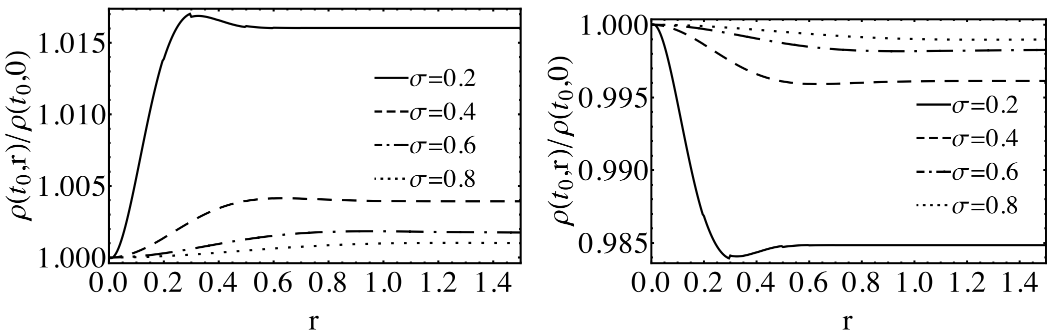


Figure: The energy density ratio $\rho(t_0, r)/\rho(t_0, 0)$ at the time observation t_0 is plotted as function of the radial coordinate for $A = -2 \times (5 \times 10^{-5})$ on the left and $A = 2 \times (5 \times 10^{-5})$ on the right. As it can be seen positive primordial curvature perturbations, correspond to a central overdensity, and negative primordial curvature perturbations correspond to a central underdensity. Another important feature is that larger values of σ correspond to smaller levels of inhomogeneity. The radial coordinate r and σ are expressed in units of H_0^{-1}.

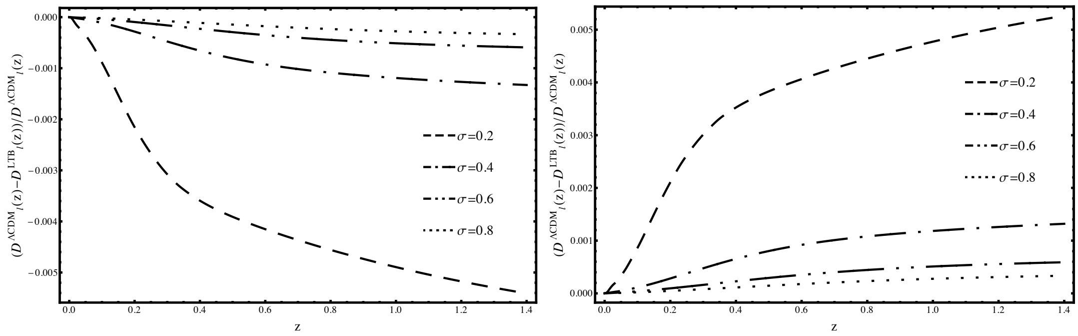


Figure: The relative difference $\Delta(z) = (D_L^{\Lambda CDM}(z) - D_L^{\Lambda LTB}(z))/D_L^{\Lambda CDM}(z)$ of the luminosity distance between the ΛLTB case and ΛCDM is plotted for different values of σ, where the latter is in units of H_0^{-1}. The left figure corresponds to $A = -2 \times (5 \times 10^{-5})$ and the right to $A = 2 \times (5 \times 10^{-5})$. A local underdensity, corresponding to $A < 0$, is associated to a larger luminosity distance respect to the homogeneous case, while local overdensities give a smaller distance. σ is expressed in units of H_0^{-1}.

H_0 estimation tension
Luminosity distance in an inhomogeneous space: LTB case
Effects of non perturbative evolution of primordial curvature
LTB metric and primordial curvature fluctuations
H_0 tension
Effects on the cosmological constant

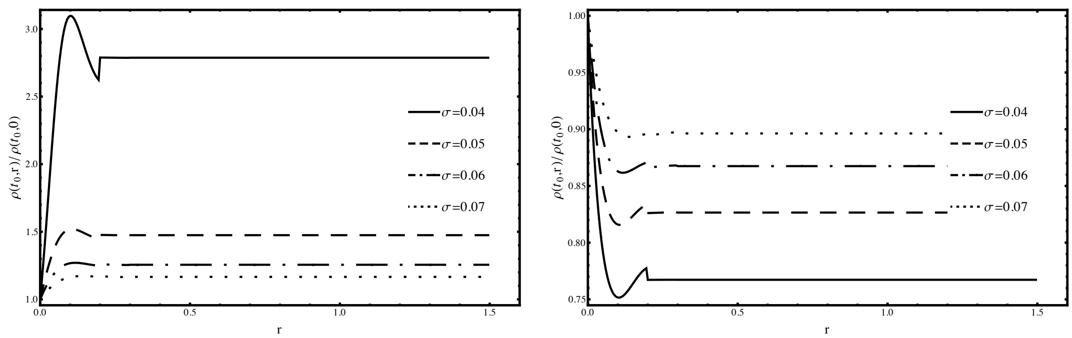


Figure: The energy density ratio $\rho(t_0, r)/\rho(t_0, 0)$ at the time observation t_0 is plotted as function of the radial coordinate for $A = -2 \times (5 \times 10^{-5})$ on the left and $A = 2 \times (5 \times 10^{-5})$ on the right. As it can be seen small values of σ correspond to very large levels of inhomogeneity, making them incompatible with observations. The radial coordinate r and σ are expressed in units of H_0^{-1}.

Antonio Enea Romano

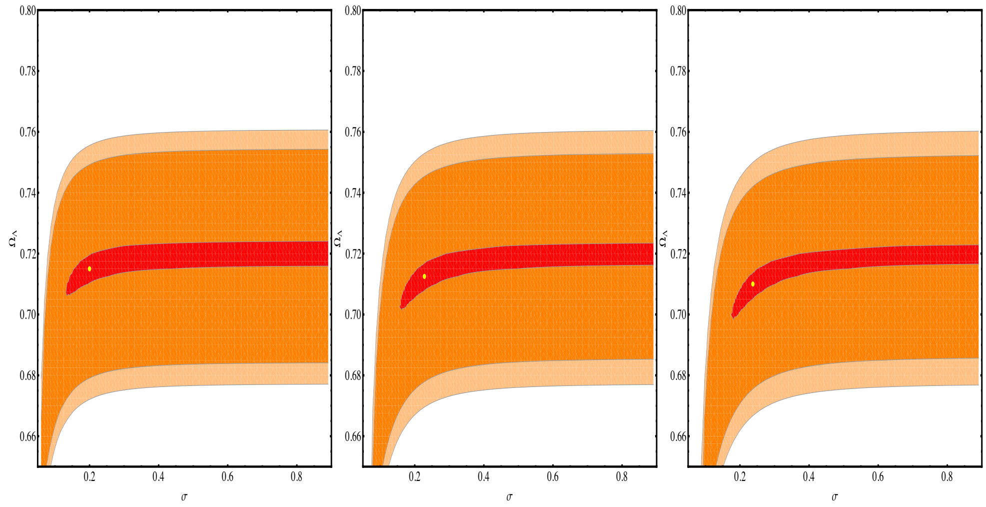


Figure: The contour plots for the luminosity distance χ^2 are shown for the parameters Ω_Λ and σ, expressed in units of H_0^{-1}. For the top figures $A = 1 \times 5 \times 10^{-5}$, $A = 2 \times 5 \times 10^{-5}$ and $A = 3 \times 5 \times 10^{-5}$, from left to right respectively.

$A/(5 \times 10^{-5})$	σ	Ω_Λ	χ^2_{min}	t_0
3	1.64	0.7204	562.242	0.983023
2	1.212	0.7204	562.242	0.982992
1	0.864	0.7204	562.242	0.982995
0		0.72	562.242	0.982778
-1	0.209	0.7155	562.217	0.981357
-2	0.228	0.7124	562.202	0.980357
-3	0.232	0.709	562.190	0.979288

Table: The table shows the values of σ, expressed in units of H_0^{-1}, and Ω_Λ minimizing the χ^2 for different values of the amplitude A, where the latter is expressed in integer multiples of the 5×10^{-5}, the value of the standard deviation of the primordial curvature perturbations implied by CMB observations. Positive values of A do not improve appreciably the value of χ^2, neither affect greatly the best fit values for Ω_Λ, while negative values improve the χ^2 and affect Ω_Λ.

Conclusions

- Ignoring local inhomogeneities can cause misestimation of cosmological parameters
- Perturbation theory is not always able to account for these effects
- The H_0 discrepancy between the local and Planck estimation could be explained (1403.2034) as the result of local structure
- Low red-shift supernovae data should be analyzed in order to check the compatibility with the local inhomogeneity profile coming from luminosity distance and number counts
- Preliminary result show a degradation of χ^2. How to explain this? **Evolution, k-correction, selections effects** could substantially bias the inhomogeneity profile detection
- A more realistic model requires to go beyond the spherical symmetry and take into account directional dependence

“Probing primordial non-Gaussianity consistency relation with galaxy surveys”

by Daisuke Yamauchi

[JGRG25(2015)7b2]

2015/12/10 JGRG25 @ YITP

Probing primordial non-Gaussianity consistency relation with galaxy surveys

YAMAUCHI, Daisuke

(RESCEU, U. Tokyo)

DY and K. Takahashi(Kumamoto), 1509.07585

A critical test of primordial Universe

➢ One of the most powerful tests of inflation

→ *Primordial non-Gaussianity*

= Possible departures from a purely Gaussian distribution of primordial density fluctuations

- Hint about a mechanism for generating primordial fluctuations
- More generally key to understanding the extreme high-energy physics

$f_{NL}, \tau_{NL}, g_{NL}, \cdots$

➢Primordial bispectrum (3-pt. fn.)

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3 B_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\, \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

(amplitude) × (shape dependent fn)
f_{NL}

➢Primordial trispectrum (4-pt. fn.)

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4)\rangle = (2\pi)^3 T_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\, \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$$

(amplitude) × (shape dependent fn)
τ_{NL}, g_{NL}

PNG consistency relation

All inflationary models predict that (if $f_{NL} \neq 0$)
the trispectrum must necessarily exist with

$$\boxed{\tau_{NL} \geqq ((6/5) f_{NL})^2}$$

[Suyama+Yamaguchi (2010)]

The confirmation of the inequality would
indicate the presence of complicated dynamics
in the primordial Universe.

It should be the target in future experiments!

Current constraints from CMB

- (local-form) bispectrum

f_{NL} = 0.8+-5.0 (68%CL) [Planck 2015]

- (local-form) trispectrum

g_{NL} = (-9.0+-7.7) × 10^4 (68%CL)

[Planck 2015]

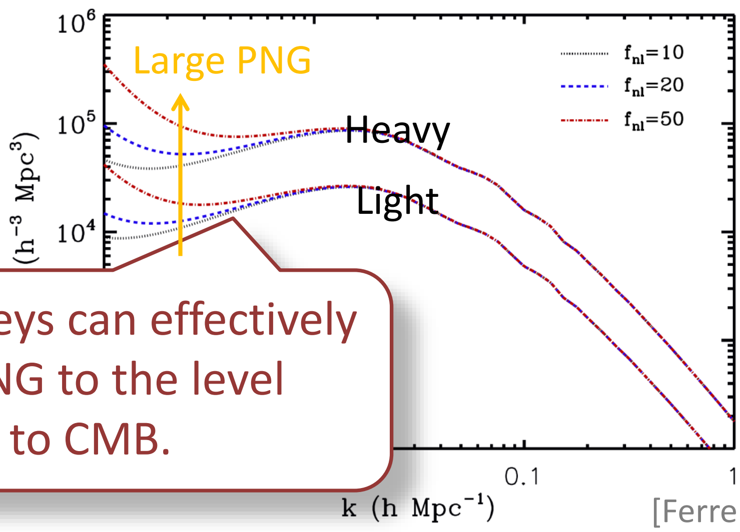
τ_{NL} < 2800 (95%CL) [Planck 2013]

Almost all models are still consistent, though model parameters are severely constrained.

PNG in large-scale structure

- PNG induces the scale dependent-bias such that the effect dominates at very large scales:

$$P_{gal} = [b_L(M,z) + f_{NL}\beta_f(M,z)/k^2 D_+(z)]^2\, P_\delta$$



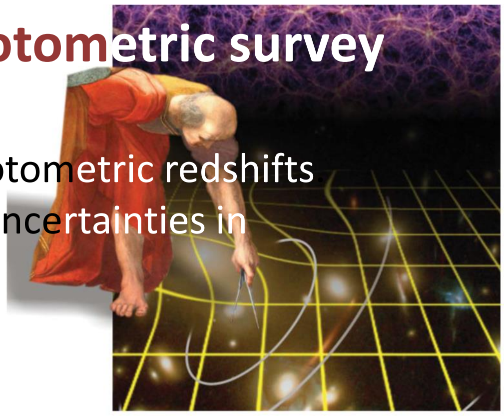
Galaxy surveys can effectively constrain PNG to the level comparable to CMB.

[Ferremacho+(2014)]

Survey design

➢ SKA : radio continuum survey

- Covers 30,000 [deg^2] out to z~5.
- The redshift information is not available.
- Halo mass can be estimated from the galaxy type.
- 8 nuisance parameters for mass inference.

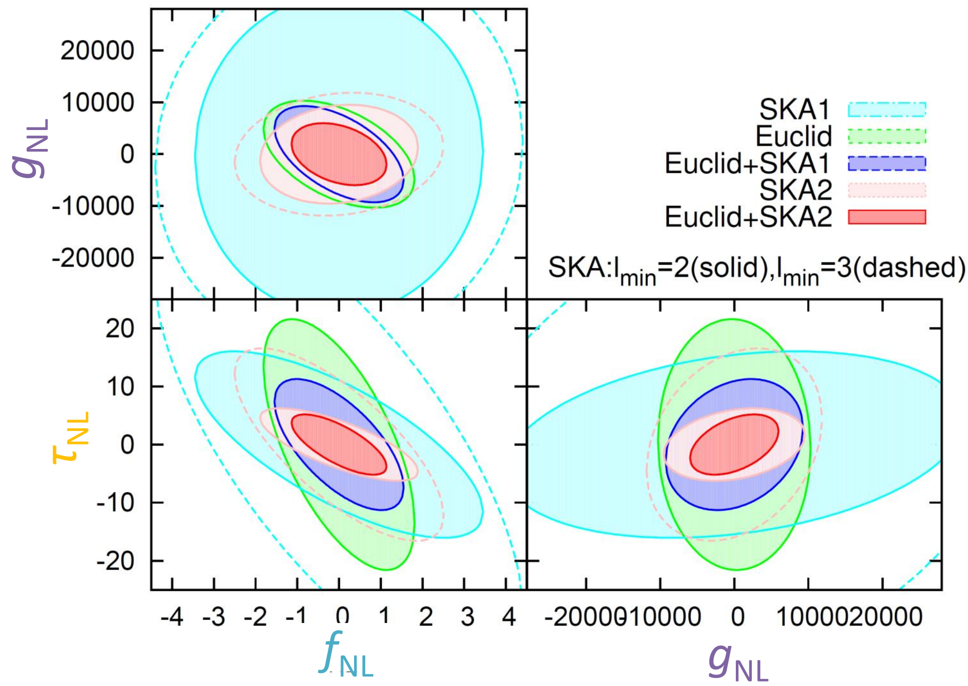


➢ Euclid : optical/infrared photometric survey

- Covers 15,000 [deg^2] out to z~2.7.
- Provides redshift information via photometric redshifts
- 14 nuisance parameters to include uncertainties in mass inference from data

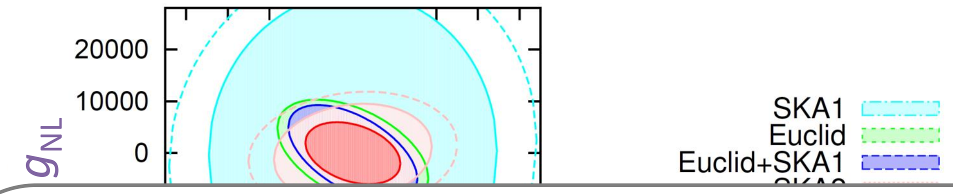
➢ SKA+Euclid : 9,000 [deg^2]

[**DY**+Takahashi 1509.07585,SKA-JP Science Book]



Complementary information from SKA and Euclid helps to break the parameter degeneracy and the joint analysis are quite effective to constrain PNG.

[**DY**+Takahashi 1509.07585,SKA-JP Science Book]

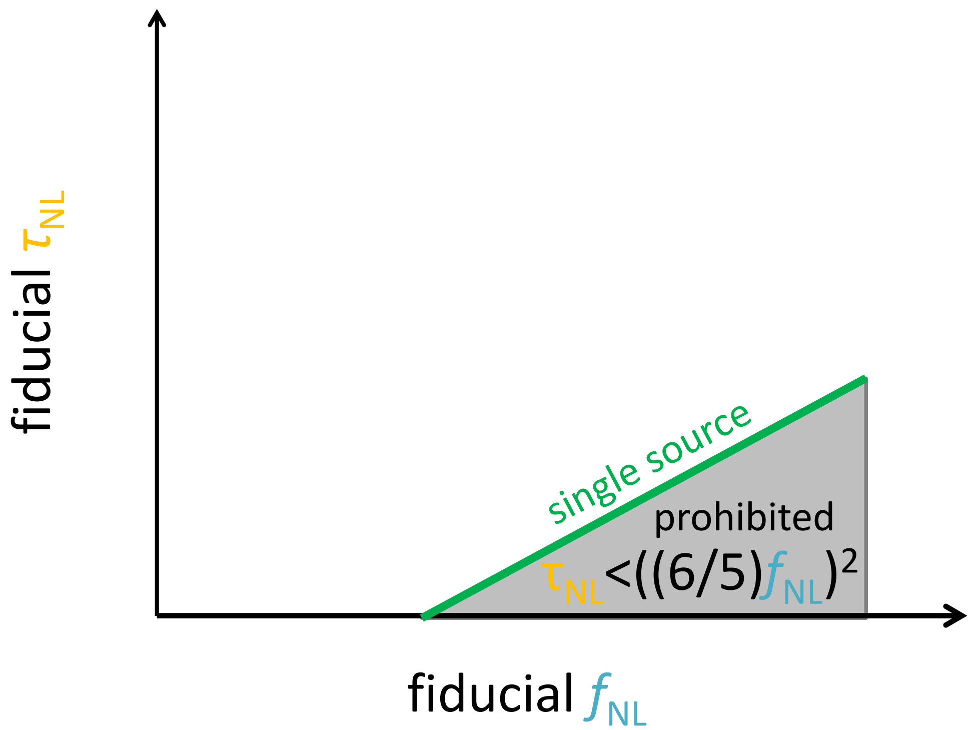


	Planck	this work	$\sigma_{Planck}/\sigma_{ours}$
$\sigma(f_{NL})$	5	1	5
$\sigma(\tau_{NL})$	10^3	10	10^2
$\sigma(g_{NL})$	10^5	5×10^3	20

We expect >100 times improvement of FoM in future!

break the parameter degeneracy and the joint analysis are quite effective to constrain PNG.

Can we confirm $\tau_{NL} \geqq ((6/5)f_{NL})^2$?



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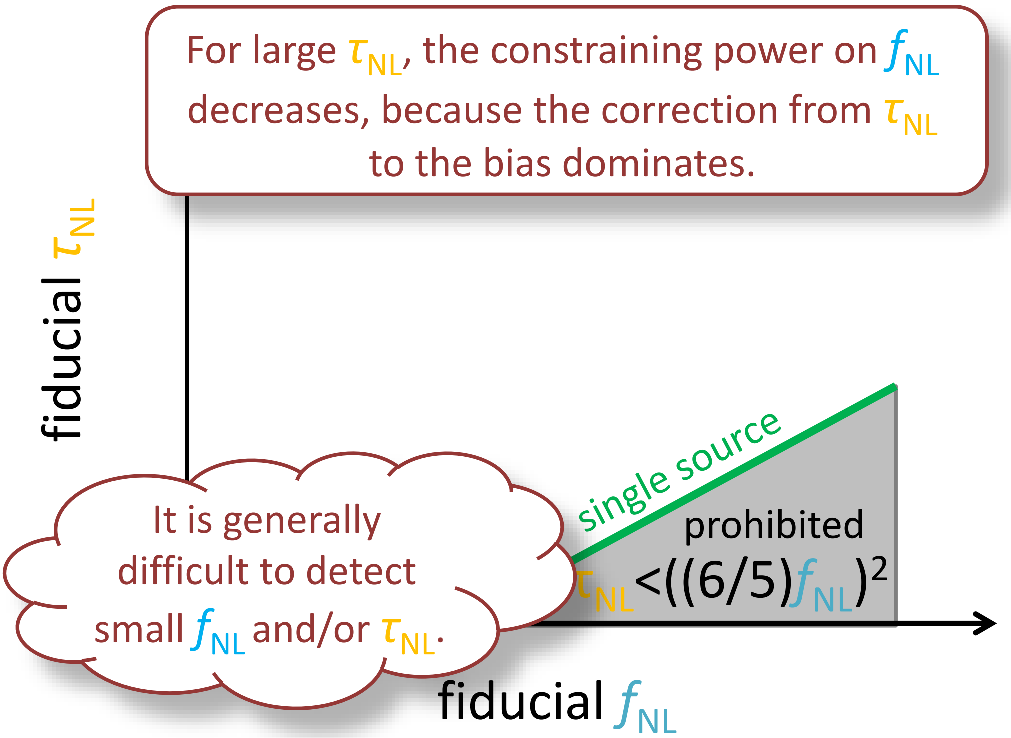
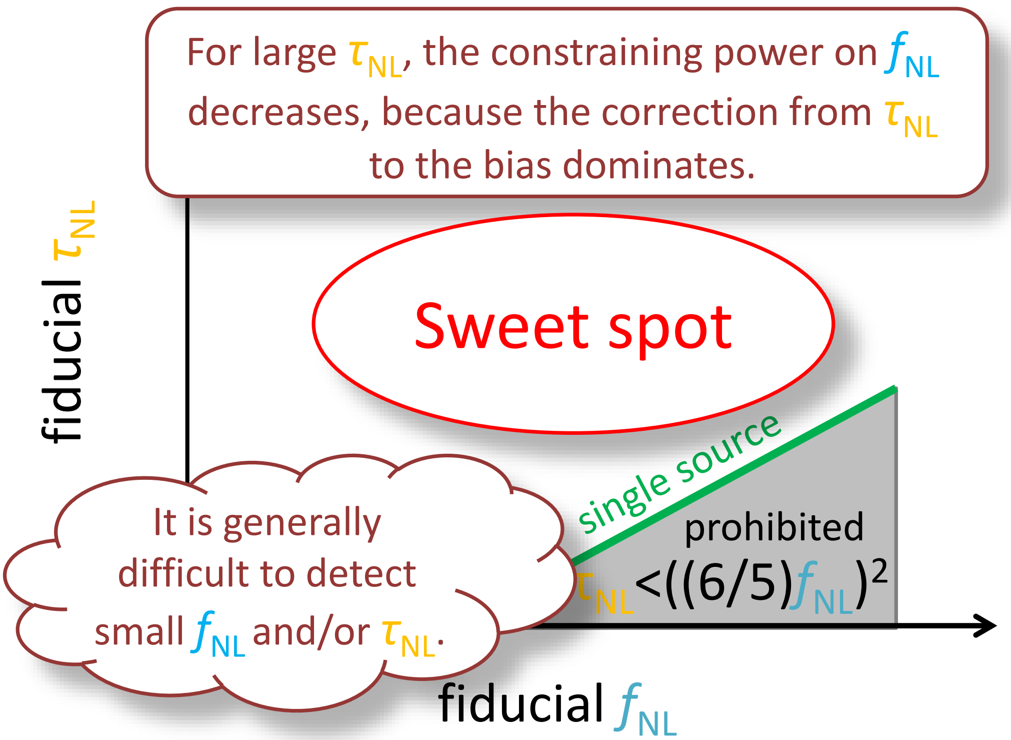
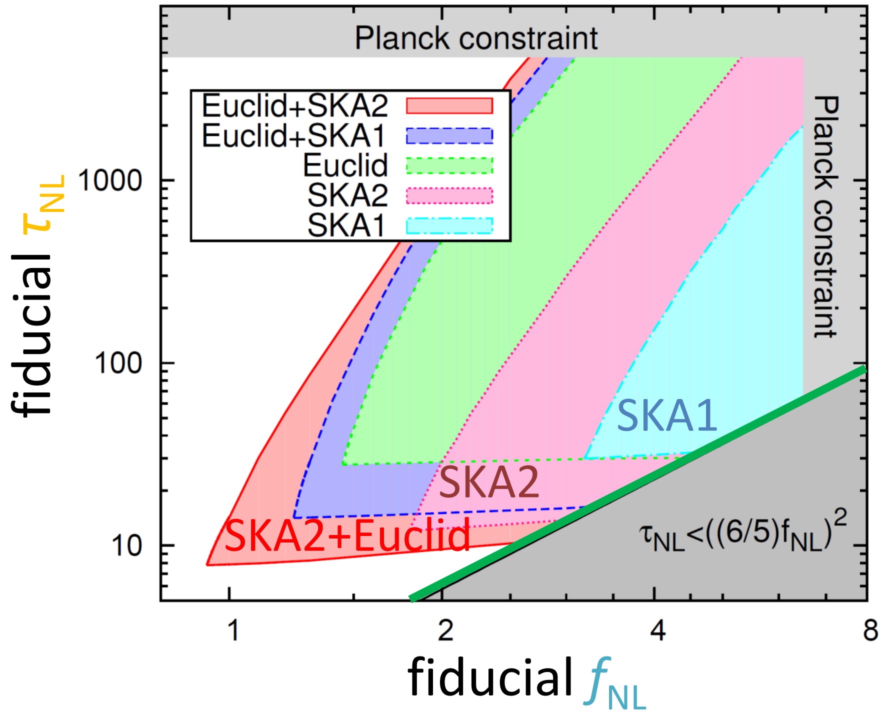


Can we confirm $\tau_{NL} \geqq ((6/5)f_{NL})^2$?



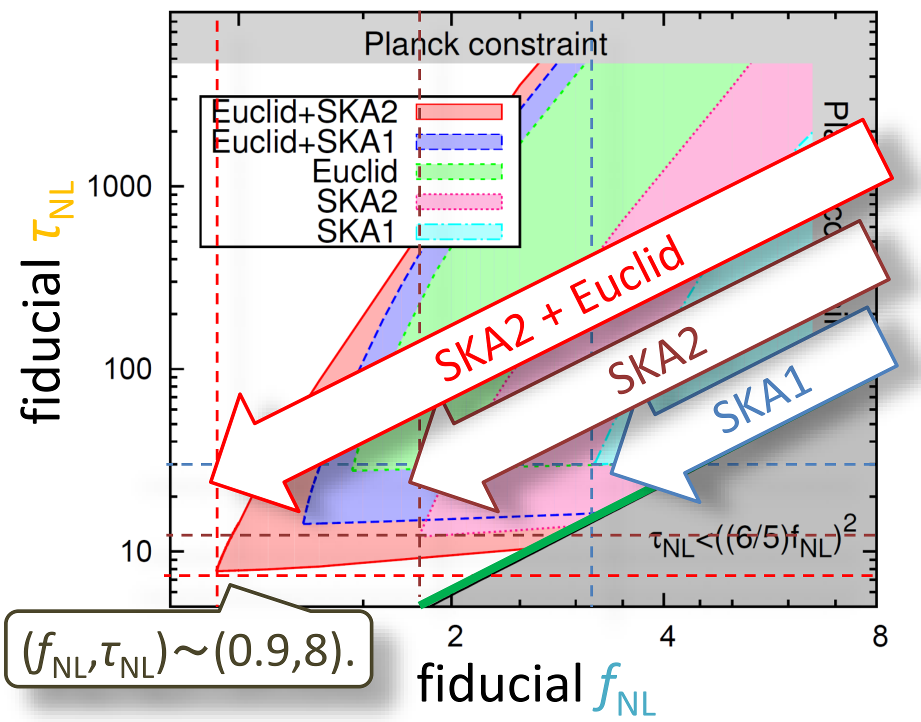
[DY+Takahashi 1509.07585,SKA-JP Science Book]

Accessible region: $f_{NL}/\sigma(f_{NL})>1\&\tau_{NL}/\sigma(\tau_{NL})>1$



[DY+Takahashi 1509.07585,SKA-JP Science Book]

Accessible region: $f_{NL}/\sigma(f_{NL})>1\&\tau_{NL}/\sigma(\tau_{NL})>1$



Summary

- The information from both Euclid and SKA is quite essential to break the degeneracy between the PNG.
- The combination of SKA2 and Euclid can detect the consistency inequality in the wide parameter region at more than 1σ level, though for a single survey it is still hard to confirm when f_{NL}<1.5.

Thank you!

“Modeling redshift-space bispectrum from perturbation theory”

by Ichihiko Hashimoto

[JGRG25(2015)7b3]

Modeling redshift-space bispectrum from perturbation theory

Yukawa institute of theoretical physics
Ichihiko Hashimoto

Collaborators :
Atsushi Taruya (Kyoto U), Yann Rasera (Paris Observatory)

Origin of accelerated expansion

Dark energy or Modified gravity ? ?

How to distinguish ?

Structure formation history offers cosmological test of gravity

Growth rate

$$f(z) \equiv \frac{d \ln D_+}{d \ln a}$$

a : scale factor

D_+ : linear growth factor of density fluctuation

GR case : $f(z) \simeq \{\Omega_\mathrm{m}(z)\}^{0.55}$ e.g. Linder ('05)

Redshift-space distortions (RSD) can be unique probe to measure the growth rate

Redshift-space distortion

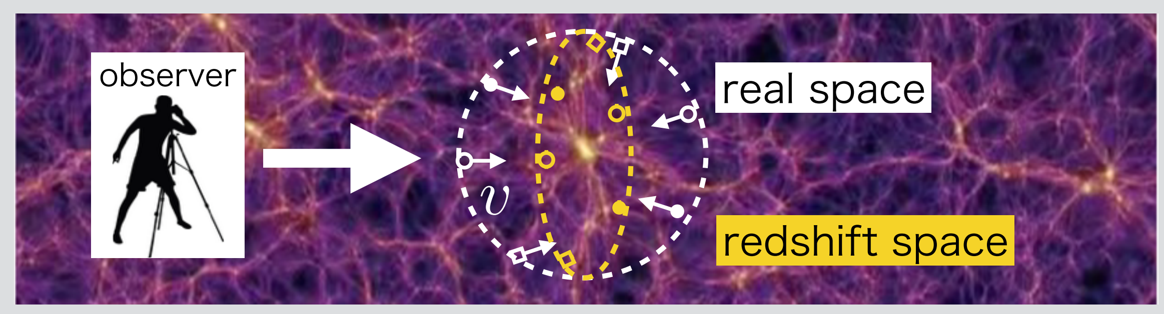
The apparent anisotropies of galaxy clustering due to peculiar velocity : $\boldsymbol{v}$

redshift space　　real space

$$\boldsymbol{s} = \boldsymbol{r} + \underline{\frac{v_z(\boldsymbol{r})}{aH(z)}}\hat{\boldsymbol{z}}$$

red-/blue- shift　　$\hat{\boldsymbol{z}}$: line of site

<u>On large scales,</u>

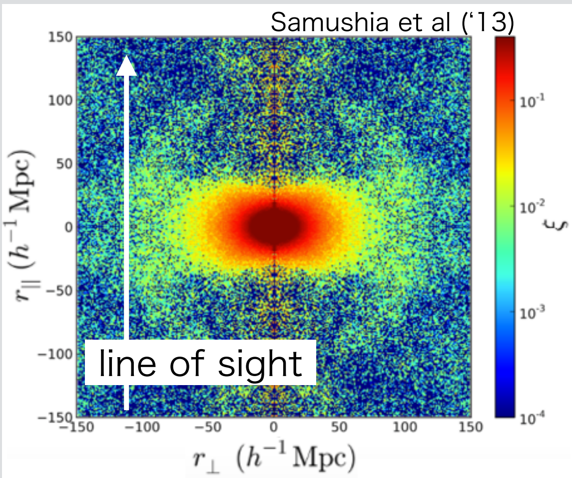


The strength of RSD is proportional to growth rate

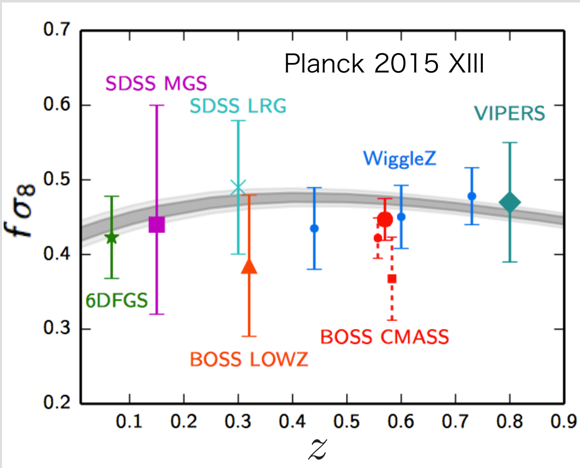
$$v \propto f(z)$$

Current status

Correlation function



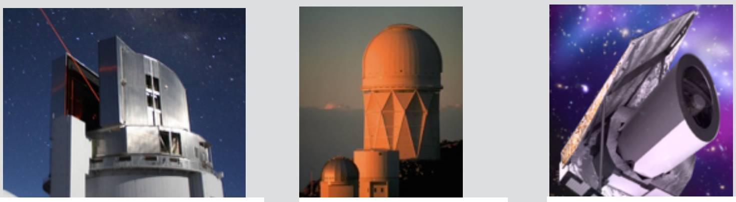
Constraints



- Consistent with ΛCDM at ~10% level
- Most of the constraints is based on the measurement of two point correlation or power spectrum

Era of precision cosmology

Future redshift surveys will release huge data set



Theoretical issues

- Precision theoretical-models reducing nonlinear systematics
- Combining higher-order statistics with power spectrum

In this talk

Precision modeling of bispectrum in redshift space

$$\langle \delta^{(\mathrm{s})}(\boldsymbol{k}_1)\delta^{(\mathrm{s})}(\boldsymbol{k}_2)\delta^{(\mathrm{s})}(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta_{\mathrm{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B^{(\mathrm{s})}(k_1,k_2,\theta_{12},\mu,\phi)$$

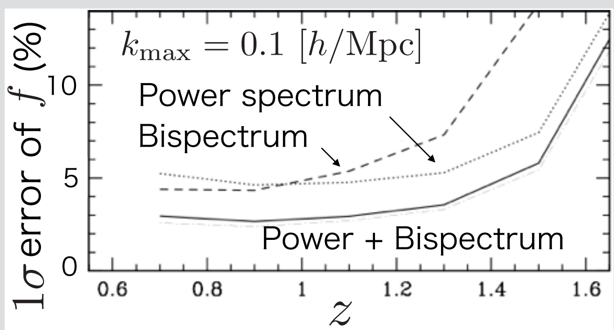
Impact of combining bispectrum

Sefusatti et al. ('06), Kayo & Takada ('13) ..

- Combination of power spectrum and bispectrum improves the constraints on cosmological parameters

Song et al. ('15)

- Forecast constraint on growth rate based on Scoccimarro et al. ('98)
- Combining bispectrum improves the constraint by a factor of two for DESI



Bispectrum helps to reduce statistical errors
→ How about systematics errors ?

Aim of this work

Modeling redshift-space bispectrum, taking account of nonlinear effects on RSD & gravitational evolution

Perturbation theory in redshift space

1. Solving Poisson and fluid eqs order by order

density fluctuation : $\delta \simeq \delta_1 + \delta_2 + \cdots$

velocity field : $v \simeq \delta v_1(\delta_1) + \delta v_2(\delta_1) + \cdots$

2. Computing $\delta^{(\mathrm{s})}$ based on the mapping formula ;

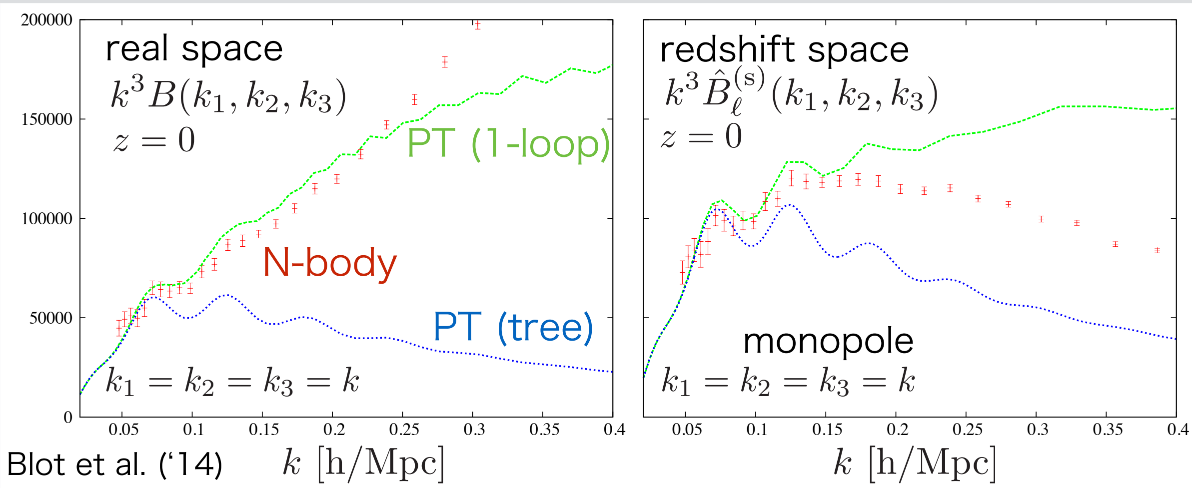
$$\underbrace{(1+\delta^{(\mathrm{s})})d^3s}_{\boxed{\text{redshift space}}} = \underbrace{(1+\delta)d^3x}_{\boxed{\text{real space}}} \qquad \boldsymbol{s} = \boldsymbol{r} + \frac{v_z(\boldsymbol{r})}{aH(z)}\hat{\boldsymbol{z}}$$

3. Substituting $\delta^{(\mathrm{s})}$ into bispectrum $\langle \delta^{(\mathrm{s})}\delta^{(\mathrm{s})}\delta^{(\mathrm{s})} \rangle = (2\pi)^3 \delta_{\mathrm{D}} B^{(\mathrm{s})}$

$$B^{(\mathrm{s})} \simeq \underbrace{B_{\mathrm{tree}}}_{\substack{\text{Previous works} \\ O((\delta_1)^4)}} + \underbrace{B_{1-\mathrm{loop}}}_{\substack{\text{Our work} \\ O((\delta_1)^6)}} + \cdots$$

Result : PT vs simulations

Equilateral triangles



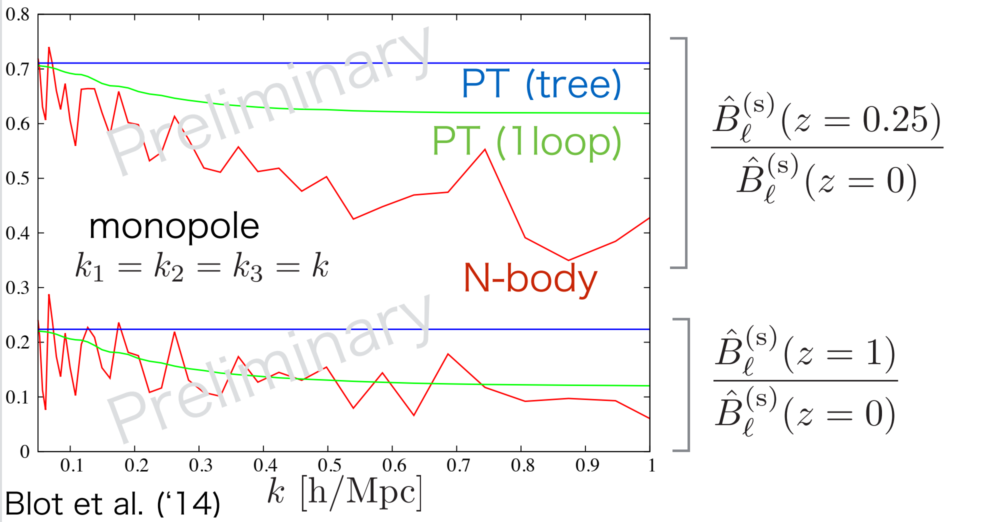
Blot et al. ('14)

Box size : 656.25 Mpc/h, # of particles : 1024^3, # of realizations : 64

In redshift space, 1-loop correction can give a moderate enhancement in bispectrum amplitude

Result : redshift dependence

Equilateral triangles



Blot et al. ('14)

Box size : 648 Mpc/h, # of particles : 1024^3, # of realizations : 1

Perturbation theory seems to reproduce simulation well at high redshift and large scales

Summary

Purpose

Modeling redshift-space bispectrum taking account of nonlinear effects on gravity and RSD

What we did

We calculated bispectrum up to 1-loop order in perturbation theory and compared it with N-body simulation

- In redshift space, 1-loop correction can give a moderate enhancement in bispectrum amplitude, while the simulation results show a rather mild enhancement
  → range of agreement becomes narrower at low-z
- Improved PT modeling would be essential (future task)

"Scalar perturbations in Bimetric Gravity"

by Yuki Sakakihara

[JGRG25(2015)7b4]

2015/12/10 JGRG25 YITP(Maskawa Hall) 14:45-15:00

Scalar perturbations in bimetric gravity

Yuki Sakakihara (Kyoto University)

This research is collaborated with Takahiro Tanaka (Kyoto University)
and it was motivated by Jiro Soda (Kobe University).
We are now preparing for the preprint version.

Massive Gravitons

General relativity well describes many of observations and experiments.

However, we have unknown things such as dark components of the universe. dark matter, dark energy...

Can we have some alternative theories to general relativity which help us to understand the origin of these unknown components?

One way to seek such theories is IR modification.

The effects arising from dark matter and dark energy are low-energy phenomena.

If gravitons have their mass, that works as an IR modification.

We know very few about graviton's features. Are gravitons massless? How many species do they have?

Massive Gravity

Then, how can we obtain theories including massive gravitons?

We need to introduce another spin-2 field (another spin-2 tensor).
in order to give gravitons their mass.

mass term $\sim$ $g_{\mu\nu} g^{\mu\nu} =$ const.

$$\downarrow$$

$$(g_{\mu\nu} f^{\mu\nu})^2 \neq \text{const.}$$

Bimetric Gravity (Bigravity)

(de Rham et. al., 2011, Hassan and Rosen, 2012)

$$\underbrace{\frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g_{\mu\nu}]}_{\text{massless}} + \underbrace{\frac{M_f^2}{2}\int d^4x\sqrt{-f}R[f_{\mu\nu}]}_{\text{massless}}$$

g and f are independently transformed.

$\downarrow$ by adding interaction terms (symmetry breaking) $\sim \; g^{\mu\lambda} f_{\lambda\nu}$

massive graviton $\sim f - g$ $+$ massless graviton (orthogonal to the massive mode) $\sim g + f$

- The interaction terms are restricted in order not to produce ghost instability.
- The dynamics of both of the metrics are determined by equations of motion.

Bimetric Gravity is realistic?

- Bimetric gravity is consistent with observations and it is a candidate which realizes the history of the universe?

  We focus on inflation among cosmological phenomena.

  Observable quantities related to inflation are measured (and are to be measured) with high precision. CMB temperature anisotropy, B-mode polarization, gravitational wave background …

  It is interesting to consider inflation which is an high-energy phenomenon in an theory with low-energy modifications.

- Is bimetric gravity both theoretically and observationally consistent with inflation?

Inflation in Bimetric Gravity

- Can we construct inflationary solutions with an inflaton as in the case of GR?

  Inflationary solutions are found.

- Are they stable solutions?

  We have stable one.

- What is the feature of gravitational waves and curvature perturbations generated during inflation?

  The amplitude are suppressed due to the decay of massive graviton's modes. The spectral index is also modified.

- How the points determined by an inflaton potential on the r - n_s plane are moved?

  Their behavior is rather restrictive than we expected.

Bimetric Action with an Inflaton

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g_{\mu\nu}] + \int d^4x\sqrt{-g}\Big(-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V[\varphi]\Big)$$
$$+\frac{\kappa M_g^2}{2}\int d^4x\sqrt{-f}R[f_{\mu\nu}] - m^2M_g^2\int d^4x\sqrt{-g}\sum_{n=0}^{4}c_nV_n[Y^\mu_\nu]\ ,$$

where

$$Y^\mu_\nu = \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$$

$$\begin{aligned} V_0 &= 1, \quad V_1 = [Y], \quad V_2 = [Y]^2 - [Y^2], \\ V_3 &= [Y]^3 - 3[Y][Y^2] + 2[Y^3], \\ V_4 &= [Y]^4 - 6[Y]^2[Y^2] + 8[Y][Y^3] + 3[Y^2]^2 - 6[Y^4], \end{aligned}$$

c_n : Coefficients of the interaction terms (Theoretical Parameters)

κ. : The ratio of the Planck Constant of the other metric to that of the physical metric

φ : Inflaton

Reduction of Bimetric Action

…

“bigravity from gradient expansion in DGP 2-brane model”

by Yasuho Yamashita

[JGRG25(2015)7b5]

bigravity from gradient expansion in DGP 2-brane model

YITP, Kyoto University
Yasuho Yamashita

in collaboration with T. Tanaka

ghost-free bigravity

bigravity : gravitational theory which contains two gravitons interacting each other

$$S = \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}\,[R + V(g,\tilde{g})] + \frac{\chi M_{pl}^2}{2}\int d^4x\sqrt{-\tilde{g}}\tilde{R}$$

For general interaction V, an extra DoF whose kinetic term has wrong sign appears.

… **Boulware-Deser ghost**

Boulware and Deser (1972)

To avoid BD ghost, V should be tuned as

$$V = m^2\sum_{n=0}^{4} c_n \epsilon^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}\mathcal{K}^{\nu_1}_{\mu_1}\ldots\mathcal{K}^{\nu_n}_{\mu_n}\,,\ \mathcal{K}^{\nu}_{\mu} = \sqrt{g^{\nu\rho}\tilde{g}_{\rho\mu}}$$

de Rham, Gabadadze, Tolley (2011)
Hassan and Rosen (2012)

- We can construct a realistic cosmological model at low energies.
- The gravitational wave has a characteristic feature.

… two gravitons cause "graviton oscillation" like neutrino oscillation

Questions in ghost-free bigravity

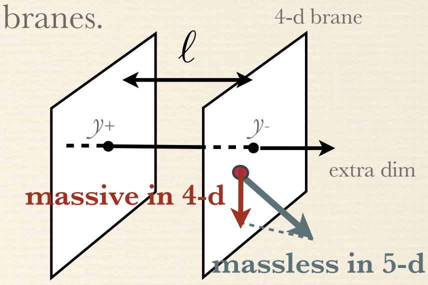
- What is the hidden metric?
- The form of the interaction is derived technically and artificially.
  How the fine-tuning of the interaction term can be realized?

embed ghost-free bigravity
to higher dimensional gravity.

Why higher dimensional theory?

Consider 5-dim braneworld model sandwiched by two branes.

$$S = \frac{M_5^3}{2} \int d^5x \sqrt{-g}\, R \; + \text{(boundary term)}$$



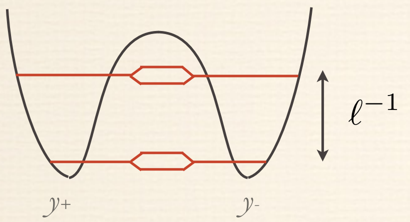
- There is no BD ghost.
- two metrics induced on two branes ⇔ two metrics in bigravity
- 5-dim massless graviton
  = 1 massless and infinite # of massive gravitons on the branes

The 4-d effective theory contains **one massless graviton**,
infinite # of massive gravitons and **one scalar** (radion=brane separation).

Model

In order to get bigravity, only one massive mode is to be kept at low energies.

effective potential by gravity



steep potential wells (ℓ << depth)

→ nearly degenerate two small mass

Dvali-Gabadadze-Poratti model

$$S = \frac{M_{pl}^2}{2}\left[\frac{1}{2r_c^{(+)}}\int dx^5\sqrt{-{}^5g}\,{}^5R + \underline{\int d^4x\sqrt{-g_+}\,(R_+ - 2\sigma_+) + \chi\int d^4x\sqrt{-g_-}\,(R_- - 2\sigma_-)}\right]$$

potential depth **induced gravity terms = potential wells**

Gradient expansion

To obtain bigravity, the parameter is to be tuned as $\frac{\ell}{r_c^{(\pm)}} \ll 1 \quad \to \quad m^2 \simeq \frac{1}{r_c^{(\pm)}\ell}$

The solution has two branches; one is ghost free and one has Higuchi/radion ghost.

$\left|K_\pm r_c^{(\pm)}\right| \lesssim 1$ should be satisfied for the ghost-free branch.

YY and Tanaka (2014)

gradient expansion

We calculate the effective action at the leading order the expansion in $K\ell \ll 1$

$$\Delta g_{\mu\nu} := g_{\mu\nu}^{(+)} - g_{\mu\nu}^{(-)} \sim \mathcal{O}(K\ell)$$

$$r_c^{(\pm)} \sim \mathcal{O}(1/K)\ , \quad m^2 \sim k^2 \sim \mathcal{O}(K/\ell)$$

Strategy to obtain the effective action

We solve the bulk equations for given boundary metrics $g_{\mu\nu}^{(\pm)}$

$$\frac{1}{N}\partial_y K_{\mu\nu} = -2K^{\rho}_{\mu}K_{\rho\nu} + KK_{\mu\nu} + \frac{4}{\ell_\Lambda^2}g_{\mu\nu} - R_{\mu\nu} + \frac{1}{N}\nabla_\mu\nabla_\nu N$$

$$K^2 - K^{\mu}_{\nu}K^{\nu}_{\mu} = -\frac{12}{\ell_\Lambda^2} + R \qquad K_{\mu\nu} = -\frac{1}{2N}\partial_y g_{\mu\nu}$$

gauge fix: $\partial_y N = 0\,,\ N^\mu = 0$

Then we can obtain the effective action written in $g_{\mu\nu}^{(\pm)}$ from

$$S = \frac{M_{pl}^2}{2r_c^{(+)}}\oint d^5x\sqrt{-g}(R + K^2 - K^{\mu}_{\nu}K^{\nu}_{\mu} - \frac{12}{\ell_\Lambda^2}) + (\text{induced gravity term})$$

by substituting back the bulk metric solution $g_{\mu\nu}(y)$ and integrating this along y.

➡ The bulk degrees of freedom is integrated out and we obtain bigravity.

Result

At the **leading order of gradient expansion**,

$$S = \frac{M_{pl}^2}{2}\frac{2}{r_c^{(+)}}\int d^4x\sqrt{-g}\left[\frac{\Delta g^2 - \Delta g_{\mu\nu}\Delta g^{\mu\nu}}{16\Phi} + \frac{\Phi}{3}\left(\nabla^\mu\nabla^\nu - g^{\mu\nu}\Box - R^{\mu\nu}\right)\left((\nabla_\mu\Phi)(\nabla_\nu\Phi) - \frac{\Phi^2}{\ell_\Lambda^2}g_{\mu\nu}\right)\right]$$
$$+ (\text{induced gravity terms})$$

ℓ_Λ : 5-d cosmological constant

∇ is the covariant differentiation with respect to $g_{\mu\nu}$,

which is **indistinguishable** from $g_{\mu\nu}^{(+)}$, $g_{\mu\nu}^{(-)}$, and $\frac{1}{2}\left(g_{\mu\nu}^{(+)} + g_{\mu\nu}^{(-)}\right)$.

$\Phi := \frac{1}{2}N\ell$ is determined by the Hamiltonian constraint C=0:

$$C := \bar{R} - \frac{12}{\ell_\Lambda^2} - \frac{\Delta g^2 - \Delta g_{\mu\nu}\Delta g^{\mu\nu}}{16\Phi^2} = 0 \qquad \text{where} \quad \bar{g}_{\mu\nu} = \frac{g_{\mu\nu}^{(+)} + g_{\mu\nu}^{(-)}}{2} + \Phi\nabla_\mu\nabla_\nu\Phi + \frac{\Phi^2}{\ell_\Lambda^2}g_{\mu\nu}$$

Result

treat Φ as an independent variable by adding λC
and eliminate Lagrange multiplier λ by use of EOM for Φ

absorbed by conformal trsf.

$$S = \frac{M_{pl}^2}{2}\left[\int d^4x\sqrt{-g_+}R_{(+)} + \chi\int d^4x\sqrt{-g_-}R_{(-)} + \frac{2}{r_c^{(+)}}\int d^4x\sqrt{-g}\left\{\frac{\Delta g^2 - \Delta g_{\mu\nu}\Delta g^{\mu\nu}}{32\Phi} - \frac{1}{2\ell_\Lambda^2}\Phi^2\left(\Box + \frac{4}{\ell_\Lambda^2}\right)\Phi + \frac{\Phi}{2}\left(R - \frac{12}{\ell_\Lambda^2}\right) - \frac{1}{6}\underbrace{(\nabla_\mu\Phi)(\nabla_\nu\Phi)(\nabla^\mu\nabla^\nu - g^{\mu\nu}\Box - R^{\mu\nu})\Phi}_{\text{cubic Galileon}}\right\}\right]$$

We obtain a well-known ghost-free system with two interacting gravitons and a scalar.

At the leading order of the gradient expansion, we cannot examine
the form of mass interactions at higher order of $\sqrt{g_{(+)}^{\mu\rho}g_{(-)\,\rho\nu}} - \delta^\mu_\nu$

Summary

- We want to derive the ghost-free bigravity from some more fundamental theory which is valid at high energies … **DGP 2-brane model**
- We calculate the effective action under **gradient expansion**, in which the brane separation is so small that the metric does not change significantly along *y*-direction, by solving bulk equations and integrating out the bulk degrees of freedom.
  We obtain a well-known ghost-free bigravity + one scalar system.
- The extension to the higher order of gradient expansion is difficult because it will produce complicated and higher-derivative interactions, which may correspond to the appearance of the other massive KK modes.

Stabilization mechanism (Goldberger & Wise)

There is an extra scalar d.o.f. corresponding to the brane separation.

… We should remove it to reproduce bigravity !

We introduce a stabilization scalar field to fix the brane separation.

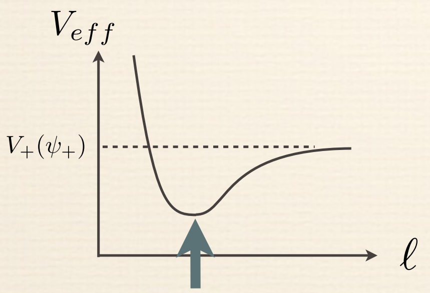
$$S_s = \int d^5x\sqrt{-g}\left(-\frac{1}{2}g^{ab}\psi_{,a}\psi_{,b} - V_B(\psi) - \sum_{\sigma=\pm} \underline{V_{(\sigma)}(\psi)\delta(y-y_\sigma)}\right)$$

$$\downarrow$$

$\psi(y_\pm)$: fixed

$$\downarrow$$

$$\partial_y\psi \to \infty \quad \text{as} \quad \ell \to 0$$



The distance between two branes are stabilized.

DGP 2-brane model with stabilization mechanism which reproduces bigravity

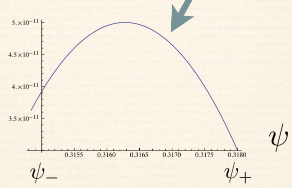
parameters $M_5 = 1.00$

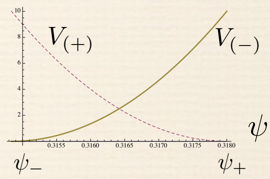
$r_c^{(\pm)} = 1.00 \times 10^5\,, \quad \ell = 1.00$

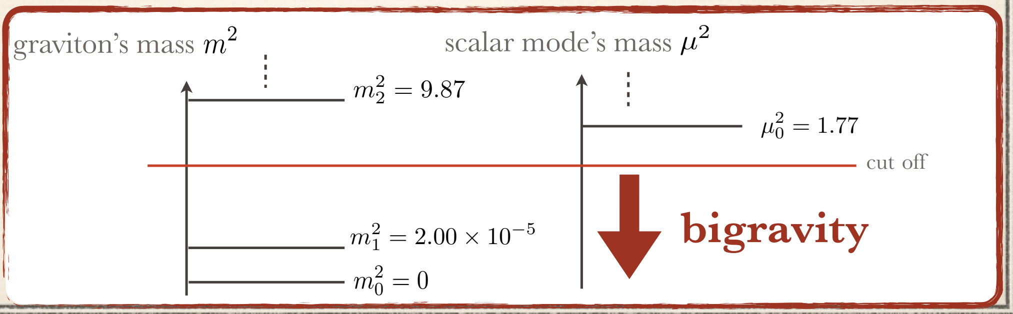
(**brane separation l**) << (**strength of induced gravity $r_c^{(\pm)}$**)

potential of scalar field

$$S_s = \int d^5x\sqrt{-g}\left(-\frac{1}{2}g^{ab}\psi_{,a}\psi_{,b} - V_B(\psi) - \sum_{\sigma=\pm} V_{(\sigma)}(\psi)\delta(y-y_\sigma)\right)$$







Correspondence between ghost-free bigravity and DGP 2-brane model with stabilization mechanism

When the two branes are almost flat,

DGP 2-brane model is identical to ghost-free bigravity.

ghost-free bigravity		**DGP 2-brane model**
two metrics	⟷	two metrics induced on the two branes
graviton's mass	⟷	the mass of the lowest massive mode

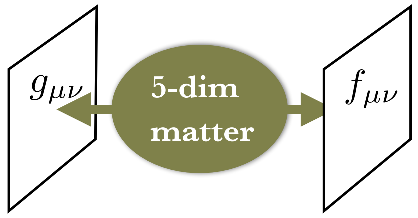
YY and Tanaka (2014)

However, can we really embed bigravity to braneworld setup?

Consider **doubly coupled matter** to test this idea.

doubly coupled matter

brane model

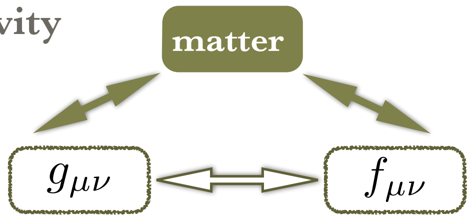


Introducing 5-d matter, we can naturally obtain a matter field which couples to both metrics.

$\rightarrow$ **BD ghost seems absent.**

contradiction

bigravity



coupling through the matter generally detunes the ghost-free structure of the interaction.

$\rightarrow$ **BD ghost appears?**

…There seems to be a difficulty in our attempt.

Seeking for models with doubly coupled matter which have no BD ghost

Introduce a k-essence scalar field

$$\mathcal{L}_m = \sqrt{-g}\,P(X,\phi) + \sqrt{-f}\,\tilde{P}(\tilde{X},\phi)$$

$$X = -\frac{1}{2} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\,, \quad \tilde{X} = -\frac{1}{2} f^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$$

Consider perturbation around FLRW and Bianchi type-1 spacetime

and evaluate the determinant and the eigenvalues of the kinetic matrix A.

When $\det A \neq 0$, an extra d.o.f. exists.

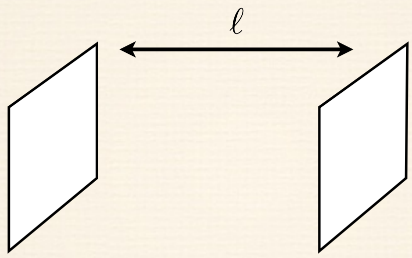
their signs clarify whether the d.o.f. is a ghost mode or not.

BD ghost appears unless $\quad \tilde{P} = \tilde{P}(\phi)$ or $P = P(\phi)$

YY, De Felice and Tanaka (2014)

Radion as a doubly coupled matter

Radion: a degree of freedom which corresponds to the brane separation



We will check how radion couples to the two metrics in 4-dim effective theory.

…We can obtain a ghost free model in bigravity with doubly coupled matter or find how the correspondence breaks between ghost-free bigravity and braneworld model.

bigravity and Boulware-Deser ghost

bigravity : gravity which contains two interacting gravitons

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}\left[R^{(g)} + \underline{2m^2 V(g,\, f)}\right] + \cancel{\frac{M_f^2}{2}\int d^4x\sqrt{-f}R^{(f)}}$$

fix ***f***

The interaction term breaks general covariance for *g*

→ $\underbrace{\text{GR (helicity-2) + 4 gauge breaking (helicity-1, helicity-0,}}_{\text{massive graviton}}$ helicity-0)

This mode's kinetic term has opposite sign!!

Boulware-Deser ghost

Boulware and Deser (1972)

In order to obtain healthy bigravity, we have to tune the interaction form so that the ghost mode is removed by constraints.

ghost-free bigravity

Choosing the form of the interaction as

$$\boxed{V = \sum_{n=0}^{4} c_n \epsilon^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} K^{\nu_1}_{\mu_1}\ldots K^{\nu_n}_{\mu_n} \qquad K^{\nu}_{\mu} = \sqrt{g^{\nu\rho}f_{\rho\mu}}}$$

de Rham, Gabadadze, Tolley (2011)

ADM decomposition
$$N^{-2} = -g^{00}\,, \quad N_i = g_{0i}\,, \quad \gamma_{ij} = g_{ij}\,,$$
$$L^{-2} = -f^{00}\,, \quad L_i = f_{0i}\,, \quad {}^3f_{ij} = f_{ij}\,.$$

define new shift-like vector n^i
and rewrite N^i with n^i

Then Hamiltonian becomes linear in $N,\, L,\, L^i$.

$$H = N\mathcal{C} + L\mathcal{C}^L + L^i\mathcal{C}^L_i\,.$$

$\mathcal{C},\, \mathcal{C}^L,\, \mathcal{C}^L_i$ are functions of $\{\gamma_{ij},\, \pi^{ij},\, {}^3f_{ij},\, p^{ij}\}$

conjugate momentum

→ One of the Hamiltonian constraints kills BD ghost.

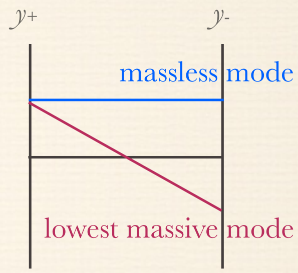
Hassan and Rosen (2012)

graviton's mass spectrum

massless mode always exists

the lowest massive mode

For $\ell \ll r_c$, eigenfunctions become



junction condition:

$$K_{\mu\nu}^{(\pm)} = r_c^{(\pm)} \left(G_{\mu\nu}^{\pm(4)} - \frac{1}{3} G^{\pm(4)} g_{\mu\nu} \right) \xrightarrow{\ell \ll r_c} g_{\mu\nu}/\ell \simeq r_c \Box^{(4)} g_{\mu\nu} = r_c m_1^2 g_{\mu\nu}$$

$$\Rightarrow \quad m_1^2 \simeq \frac{1}{r_c \ell} \ll \frac{1}{\ell^2} \simeq m_2^2$$ **: hierarchy**

mass spectrum (scalar mode)

stabilization mechanism $\rightarrow$ **no massless mode**

If stabilization is weak: $\left| \frac{\partial_y \mathcal{H}}{\mathcal{H}^2} \right| \sim \frac{(\partial_y \psi)^2}{M_5^3 \mathcal{H}^2} \ll 1$

the lowest mass becomes

$$\mu^2 \approx \frac{2 \int_{y_+}^{y_-} \frac{dy}{a^2} + \sum_\sigma \frac{2 r_c^{(\sigma)}}{a_\sigma^2} \frac{1}{1 - \sigma 2 r_c^{(\sigma)} \mathcal{H}_\sigma}}{\int_{y_+}^{y_-} \frac{dy}{a^4 (-\mathcal{H}')}}$$

$\mathcal{H}$: 5-d curvature scale

- stronger stabilization ($\boxed{\text{large } |\mathcal{H}'|}$) $\Rightarrow$ **large μ^2**
- $\boxed{1 \mp 2 r_c^{(\pm)} \mathcal{H}_\pm < 0}$ make μ^2 negative

$\Rightarrow$ corresponds to the **self accelerating branch**

: K.Izumi et al. (2007)

ghost in DGP model

the regularity on +brane imposes

H : 4-dim comoving curvature scale

$$2\left(\sum_i \frac{u_i^2(y_+)}{m_i^2 - 2H^2}\right) + \frac{1}{H_+^2(2r_c\mathcal{H}_+ - 1)}\left(\frac{2\kappa^2}{3H_+^2(2r_c\mathcal{H}_+ - 1)}\left(\sum_i \frac{v_i^2(y_+)}{\mu_i^2 + 4H^2}\right) + \mathcal{H}_+\right) = 0$$

diverges as $m^2 \to 2H^2$: Higuchi bound

diverges as $\mu^2 \to -4H^2$

: critical mass that scalar ghost appears

$2r_c\mathcal{H}_+ - 1 > 0$: self-accelerating branch

$\mu_i^2 + 4H^2 \to \mp\epsilon$ means $m_i^2 - 2H^2 \to \pm\epsilon$

➡ **ghost never disappears** K.Izumi et. al. (2007)

$2r_c\mathcal{H}_+ - 1 < 0$: normal branch

The same identity prohibits m_i^2 & μ_i^2 from crossing their critical masses

➡ **no ghost**

collapse of the structure in DGP model

junction condition

$$K_{\mu\nu}^{(\pm)} = r_c^{(\pm)}\left(G_{\mu\nu}^{\pm(4)} - \frac{1}{3}G^{\pm(4)}g_{\mu\nu}\right)$$

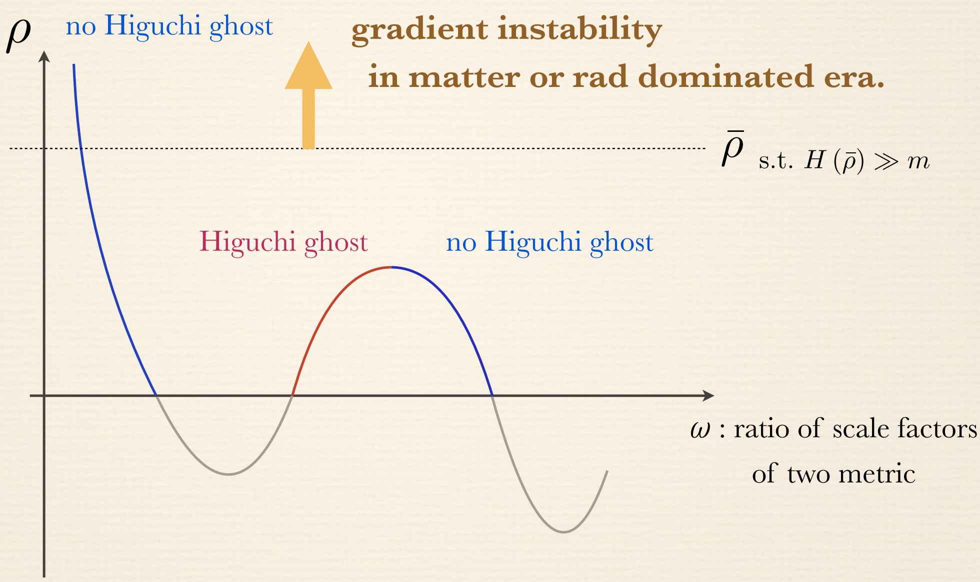
When we consider to increase the energy scale on the branes, the curvature scale also increase.

On the other hand,

$|\mathcal{H}| \lesssim \dfrac{1}{r_c^{\pm}}$ must be satisfied to avoid scalar-mode instability

➡ slightly curved branes cause instability and break the stabilization!

Cosmological solution in ghost-free bigravity



Higuchi ghost in dRGT bigravity

In dRGT model, equation for the de Sitter solution insists

$$\frac{\kappa_4^2}{m^2}\rho_m = \frac{c_1}{\chi\omega} + \left(\frac{6c_2}{\chi} - c_0\right) + \left(\frac{18c_3}{\chi} - 3c_1\right)\omega + \left(\frac{24c_4}{\chi} - 6c_2\right)\omega^2 - 6c_3\omega^3 \equiv f(\omega)$$

ω : ratio of scale factor of two metric

effective mass for massive graviton

$$m_{eff}^2 = m^2(1 + (\chi\omega^2)^{-1})\Gamma(\omega) = -\frac{m^2\omega}{3}\underline{f'(\omega)} + 2H^2$$

this sign determines the ghost appearance

$$\Gamma(\omega) \equiv c_1\omega + 4c_2\omega^2 + 6c_3\omega^3$$

For flat vacuum solution, $H \rightarrow 0$ as $\omega \rightarrow \omega_0$ where $\rho_m(\omega_0) \rightarrow 0$,

$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$ **negative** when $\Gamma > 0$ i.e. $m_{eff}^2 > 0$

→ **no Higuchi ghost**

Higuchi ghost in dRGT bigravity

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$$\frac{\kappa_4^2}{m^2}\rho_m = \frac{c_1}{\chi\omega} + \left(\frac{6c_2}{\chi} - c_0\right) + \left(\frac{18c_3}{\chi} - 3c_1\right)\omega + \left(\frac{24c_4}{\chi} - 6c_2\right)\omega^2 - 6c_3\omega^3 \equiv f(\omega)$$

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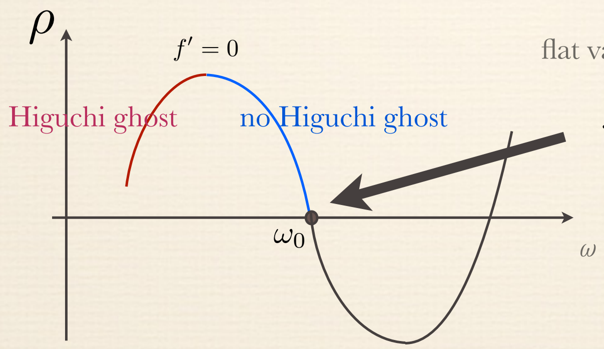


flat vacuum $H = 0, \rho = 0$

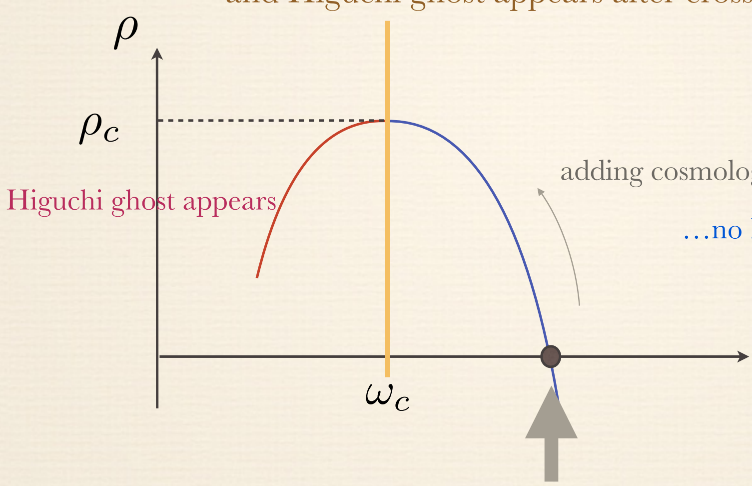
$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

...negative

when $\Gamma > 0 \Leftrightarrow m_{eff}^2 > 0$

Higuchi ghost in dRGT bigravity

de Sitter solution does not exist above this critical density, and Higuchi ghost appears after crossing the critical ω.



adding cosmological const. little by little

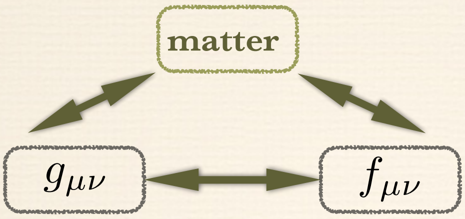
...no Higuchi ghost

ω : ratio of scale factor of two metric

choose the branch connected to the vacuum flat spacetime with positive graviton mass

doubly coupled matter

However,



coupling through the matter generally detunes the ghost-free structure of the interaction.

$\rightarrow$ **BD ghost?**

Consider a free scalar field which couples to both metric:

$$\mathcal{L}_m = \sqrt{-g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) + \sqrt{-f}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right)$$

conjugate momentum $\pi_\phi \sim \left(\frac{1}{N}+\frac{1}{L}\right)\partial_t\phi$

Hamiltonian $\mathcal{H} \ni \frac{NL}{N+L}\pi_\phi^2$ …nonlinear in the lapse fcns $\rightarrow$ **BD ghost!**

Seeking for models with doubly coupled matter which have no BD ghost

- another ghost-free model motivated by the quasi-dilaton massive gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_g^2 R^{(g)}}{2} + 2m^2 M_{\rm eff}^2 \sum_n c_n e_n\left(\sqrt{g^{\mu\nu}(f_{\mu\nu}+\alpha\partial_\mu\phi\,\partial_\nu\phi)}\right)\right] + \int d^4x\sqrt{-f}\left[\frac{M_f^2 R^{(f)}}{2} - \frac{1}{2}f^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right]$$

YY, De Felice and Tanaka (2014)

- matter which couples to an effective metric

$$g_{\mu\nu}^{\rm eff} = a^2 g_{\mu\nu} + 2ab\, g_{\mu\alpha}\sqrt{g^{\alpha\beta}f_{\beta\nu}} + b^2 f_{\mu\nu}$$

This model has BD ghost, but it appears beyond the strong coupling scale.

de Rham, Heisenberg and Rebeiro (2014)

The model of doubly coupled matter is considerably restricted.

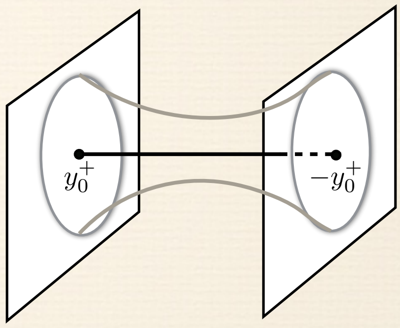
… inconsistent with the intuition in braneworld models.

Before going to the nonlinear theory

For simplicity, we consider the perturbation around de Sitter brane solution, whose curvature scale is given as H.

$$ds^2 = dy^2 + a^2(y)\gamma_{\mu\nu}dx^\mu dx^\nu$$

$$\mathcal{H}^2 := \left(\frac{\partial_y a}{a}\right)^2 = -\frac{1}{\ell_\Lambda^2} + H^2/a^2$$



$$g_{\mu\nu}^{(\pm)} = a^2(\pm y_0^+)\left(\gamma_{\mu\nu} + h_{\mu\nu}^{(\pm)}\right)$$

$$\tilde{y}^+ = y_0^+(1+x)$$

Result

$$S = \frac{M_{pl}^2}{2}\left[\int d^4x\sqrt{-\gamma}\, m_*^2\left\{\Delta h^2 - \Delta h^{\mu\nu}\Delta h_{\mu\nu} - \frac{3}{4}\Phi\left(1-\alpha^2H^2\left(\Box+4H^2\right)\right)\Phi\right\}\right.$$
$$\left.+\int d^4x\sqrt{-g_{(+)}}\left(R_{(+)} - \frac{6H^2}{a_+^2}\right) + \chi\int d^4x\sqrt{-g_{(-)}}\left(R_{(-)} - \frac{6H^2}{a_-^2}\right)\right]$$

$$\Phi := \Delta h + \frac{4\alpha}{3}\bar{R}^{(1)}$$

$$\alpha := \frac{-y_0^+\mathcal{H}^{-1}(0)}{2}$$

treat Φ as an independent variable

by adding $\lambda\left(\Phi - \Delta h - \frac{4\alpha}{3}\bar{R}^{(1)}\right)$

$$S = \frac{M_{pl}^2}{2}\left[\int d^4x\sqrt{-\gamma}\, m_*^2\left\{\Delta h^2 - \Delta h^{\mu\nu}\Delta h_{\mu\nu} - \frac{3}{4}\alpha^2H^2\Phi\left(\Box+4H^2\right)\Phi + \frac{3}{4}\Phi\left(\Phi-2\Delta h\right) - \alpha\Phi\left(R_{(+)}^{(1)} + R_{(-)}^{(1)}\right)\right\}\right.$$
$$\left.+\int d^4x\sqrt{-g_{(+)}}\left(R_{(+)} - \frac{6H^2}{a_+^2}\right) + \chi\int d^4x\sqrt{-g_{(-)}}\left(R_{(-)} - \frac{6H^2}{a_-^2}\right)\right]$$

conformal trsf

…two gravitons interacting through Fierz-Pauli mass term and one scalar whose kinetic term couples to γ …no BD ghost

Equations of motion

$$h^{(i)\,TT}_{\mu\nu} = \frac{-2M_{pl}^{-2}}{a_+^2 + a_-^2\chi}\left[\frac{1}{\Box - 2H^2 - m_i^2}\left\{T^{(i)}_{\mu\nu} - \frac{1}{4}T^{(i)}\gamma_{\mu\nu} + \frac{1}{3\,(m_i^2 - 2H^2)}\left(\nabla_\mu\nabla_\nu - \frac{\Box}{4}\gamma_{\mu\nu}\right)T^{(i)}\right\}\right.$$
$$\left. - \frac{1}{3\,(m_i^2 - 2H^2)}\left(\nabla_\mu\nabla_\nu - \frac{\Box}{4}\gamma_{\mu\nu}\right)\frac{1}{\Box + 4H^2}T^{(i)}\right]$$

$$T^{(0)}_{\mu\nu} := T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu}$$
$$T^{(m)}_{\mu\nu} := \frac{T^{(+)}_{\mu\nu}}{a_+^2} - \frac{T^{(-)}_{\mu\nu}}{a_-^2\chi}$$

Poles at $\Box - 2H^2 = 0,\ m^2$ and $\Box + 4H^2 = 0$

…one massless and one massive gravitons and one scalar (radion)

We find the sign of the coefficient of the pole $\Box + 4H^2 = 0$ flips at

$$2a_\pm^2\chi_\pm r_c^{(+)}\mathcal{H}_\pm - 1 = 0$$

…equivalent to the condition for the ghost-free branch

We succeeded to obtain a ghost-free bigravity+scalar system.

“Perturbations of Cosmological and Black Hole Solutions in Massive Gravity and Bi-Gravity”

by Daisuke Yoshida

[JGRG25(2015)7b6]

PERTURBATIONS OF COSMOLOGICAL AND BLACK HOLE SOLUTIONS IN MASSIVE GRAVITY AND BI-GRAVITY

Daisuke Yoshida (Tokyo Institute of Technology)

Based on arXiv:1509. 02096

Collaborators: **T.Kobayashi** (Rikkyo Univ.), **M.Siino** (Titech), **M.Yamaguchi** (Titech)

JGRJ25 at YITP, 2015.12.10

Motivation

Test of massive/bi gravity by cosmology

Many self-accelerated FLRW solutions of massive/bi-gravity have been found so far, but many solutions suffer from instabilities.

There is a class of cosmological solutions whose stability has not been studied. These solutions have a possibility to avoid unrealistic instabilities.

We investigated the stability of such a class of solutions and found stability of these solutions coincide with corresponding solutions of GR at least up to second order perturbations in EoM.

Contents

1. Brief Review of Massive/Bi gravity
2. Background Solutions
3. Stability

BRIEF REVIEW
OF
MASSIVE/BI GRAVITY

Physical and Fiducial metric

$$g_{\mu\nu}$$

Physical metric,
which coupled with ordinary matter
and describe the space time

$$f_{\mu\nu}$$

Fiducial metric
In massive gravity:
external field which give graviton a mass
In bi-gravity:
dynamical tensor field interacting with $g_{\mu\nu}$

Action and Equation of Motion

$$S = \frac{M_{PL}^2}{2}\int d^4x\sqrt{-g}(R[g_{\mu\nu}] + \mathcal{L}_{matter}[g_{\mu\nu}, \Phi^I])$$
$$+\frac{\kappa^2 M_{PL}^2}{2}\int d^4x\sqrt{-f}(R[f_{\mu\nu}] + \tilde{\mathcal{L}}_{matter}[f_{\mu\nu}, \tilde{\Phi}^I])$$
$$+\frac{M_{PL}^2}{2}\int d^4x\sqrt{-g}2m^2\sum_{i=0}^{4}\beta_i e_i(\gamma)$$

de Rham, Gabadadze, Tolley (2011)
Hassan, Rosen (2011)

$$\gamma^\mu{}_\nu = \sqrt{g^{-1}f}^\mu{}_\nu$$
$$e_n(\gamma) = \gamma^{[\mu_1}{}_{\mu_1}\cdots\gamma^{\mu_n]}{}_{\mu_n}$$

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0 \Longleftrightarrow G^\mu{}_\nu + X[\gamma]^\mu{}_\nu = M_{PL}^{-2}T^\mu{}_\nu$$

$$\frac{\delta S}{\delta f^{\mu\nu}} = 0 \Longleftrightarrow G_f{}^\mu{}_\nu + X_f[\gamma]^\mu{}_\nu = \kappa^{-2}M_{PL}^{-2}T_f{}^\mu{}_\nu$$

$$\begin{aligned} X[\gamma]^\mu{}_\nu \;&=\; m^2\beta_1\gamma^\mu_\nu + m^2\beta_2\left(e_1(\gamma)\gamma^\mu_\nu - (\gamma^2)^\mu{}_\nu\right) \\ &\quad + m^2\beta_3\left(e_2(\gamma)\gamma^\mu_\nu - e_1(\gamma)(\gamma^2)^\mu{}_\nu + (\gamma^3)^\mu{}_\nu\right) - \delta^\mu_\nu\sum_{i=0}^{3} m^2\beta_i e_i(\gamma)\end{aligned}$$

Action and Equation of M

$$
\begin{aligned}
m^2\beta_0 &= -\Lambda^g + m^2(6+4\alpha_3+\alpha_3) \\
m^2\beta_1 &= m^2(-3-3\alpha_3-\alpha_4) \\
m^2\beta_2 &= m^2(1+2\alpha_3+\alpha_4) \\
m^2\beta_3 &= m^2(-\alpha_3-\alpha_4) \\
m^2\beta_4 &= -\Lambda^f + m^2\alpha_4
\end{aligned}
$$

$$S = \frac{M_{PL}^2}{2}\int d^4x\sqrt{-g}(R[g_{\mu\nu}] + \mathcal{L}_{matter}[g_{\mu\nu}, \Phi^I])$$

Key Point

- Correction terms are described by $\gamma^{\mu}{}_{\nu} = \sqrt{g^{-1}f}^{\mu}{}_{\nu}$
- Correction terms have 5 free parameters $m^2\beta_i \Longleftrightarrow m, \alpha_3, \alpha_4, \Lambda^g, \Lambda^f$

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0 \Longleftrightarrow G^{\mu}{}_{\nu} + X[\gamma]^{\mu}{}_{\nu} = M_{PL}^{-2}T^{\mu}{}_{\nu}$$

$$\frac{\delta S}{\delta f^{\mu\nu}} = 0 \Longleftrightarrow G_f{}^{\mu}{}_{\nu} + X_f[\gamma]^{\mu}{}_{\nu} = \kappa^{-2}M_{PL}^{-2}T_f{}^{\mu}{}_{\nu}$$

$$
\begin{aligned}
X[\gamma]^{\mu}{}_{\nu} = {} & m^2\beta_1\gamma^{\mu}{}_{\nu} + m^2\beta_2\left(e_1(\gamma)\gamma^{\mu}{}_{\nu} - (\gamma^2)^{\mu}{}_{\nu}\right) \\
& + m^2\beta_3\left(e_2(\gamma)\gamma^{\mu}{}_{\nu} - e_1(\gamma)(\gamma^2)^{\mu}{}_{\nu} + (\gamma^3)^{\mu}{}_{\nu}\right) - \delta^{\mu}_{\nu}\sum_{i=0}^{3} m^2\beta_i e_i(\gamma)
\end{aligned}
$$

BACKGROUND SOLUTIONS

Metric Ansatz

Our analysis can be extended to spherically symmetric space time.

$$g_{\mu\nu}$$

spherically symmetric space time:

$$\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu &= g_{tt}(t,r)dt^2 + 2g_{tr}(t,r)dtdr + g_{rr}(t,r)dr^2 \\ &\quad + R^2(t,r)(d\theta^2 + \sin^2\theta d\phi^2) \end{aligned}$$

$$f_{\mu\nu}$$

spherically symmetric space time:

$$\begin{aligned} f_{\mu\nu}dx^\mu dx^\nu &= f_{tt}(t,r)dt^2 + 2f_{tr}(t,r)dtdr + f_{rr}(t,r)dr^2 \\ &\quad + A^2(t,r)R^2(t,r)(d\theta^2 + \sin^2\theta d\phi^2) \end{aligned}$$

$$A(t,r) := \sqrt{f_{\theta\theta}/g_{\theta\theta}}$$

We can treat not only cosmological solution but also static spherically symmetric solutions at same time.

Cosmological constant solution

We are interested in the case where
EOMs of $g_{\mu\nu}$ reduce to Einstein equation with a cosmological constant.

$$G^\mu{}_\nu + X[\gamma]^\mu{}_\nu = 8\pi G T^\mu{}_\nu$$

$$X[\gamma]^\mu{}_\nu \approx \text{Constant} \times \delta^\mu{}_\nu$$

Combining with our spherically symmetric metric ansatz,

$$A(t,r) = \sqrt{f_{\theta\theta}/g_{\theta\theta}}$$

$$\gamma^\mu{}_\nu = \sqrt{g^{-1}f}^\mu{}_\nu$$

$$\begin{aligned} X^t{}_r &= -m^2\gamma^t{}_r(3 - 2A + (A-3)(A-1)\alpha_3 + (A-1)^2\alpha_4) = 0 \\ X^r{}_t &= -m^2\gamma^r{}_t(3 - 2A + (A-3)(A-1)\alpha_3 + (A-1)^2\alpha_4) = 0 \end{aligned}$$

We focus on $A(t,r) = const = (2\alpha_3 + \alpha_4 + 1 \pm \sqrt{\alpha_3^2 + \alpha_3 - \alpha_4 + 1})/(\alpha_3 + \alpha_4)$

$$X^t{}_t - X^\theta{}_\theta = m^2 \frac{A^2 - A(\gamma^t{}_t + \gamma^r{}_r) + \gamma^t{}_t\gamma^r{}_r - \gamma^t{}_r\gamma^r{}_t}{1-A}(A - 2 + (A-1)\alpha_3) = 0$$

We focus on $A = (2+\alpha_3)/(1+\alpha_3)$

$$X^\mu{}_\nu = \left(\frac{m^2}{1+\alpha_3} + \Lambda^g\right)\delta^\mu{}_\nu$$

These two requirements can be satisfied when $\alpha_4 = 1 + \alpha_3 + \alpha_3^2$.

Summary of background solution

$$
\begin{aligned}
g_{\mu\nu} &= g_{tt}(t,r)dt^2 + 2g_{tr}(t,r)dtdr + g_{rr}(t,r)dr^2 + R^2(t,r)(d\theta^2 + \sin^2\theta d\phi) \\
f_{\mu\nu} &= f_{tt}(t,r)dt^2 + 2f_{tr}(t,r)dtdr + f_{rr}(t,r)dr^2 + \left(\frac{2+\alpha_3}{1+\alpha_3}\right)^2 R^2(t,r)(d\theta^2 + \sin^2\theta d\phi) \\
\alpha_4 &= 1 + \alpha_3 + \alpha_3^2
\end{aligned}
$$

$$G^{\mu}{}_{\nu} + \Lambda^g_{eff}\delta^{\mu}{}_{\nu} = 8\pi G T^{\mu}{}_{\nu}$$

$$\Lambda^g_{eff} = m^2/(1+\alpha_3) + \Lambda^g$$

$$G_f{}^{\mu}{}_{\nu} + \Lambda^f_{eff}\delta^{\mu}{}_{\nu} = \kappa^{-2} M_{PL}^{-2} T_f{}^{\mu}{}_{\nu}$$

$$\Lambda^f_{eff} = \kappa^2(-m^2/(2+\alpha_3) + \Lambda^f)$$

Any spherically symmetric solution $g_{\mu\nu}$ of GR can be a solution of massive, bi-gravity.

This class of solutions includes
FLRW solution by Chamseddine,Volkov (2011), Kobayashi,Siino,Yamaguchi,DY (2012)
Schwarzschild de Sitter solution by Nieuwenhuizen (2011), Berezhani et al (2012), Kodama, Arraut (2013)

STABILITY

Perturbations and EoMs

We consider general linear perturbations around our spherically symmetric solutions:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$
$$f_{\mu\nu} = \bar{f}_{\mu\nu} + \delta f_{\mu\nu}$$

$$\begin{aligned}\bar{g}_{\mu\nu} &= \bar{g}_{tt}(t,r)dt^2 + 2\bar{g}_{tr}(t,r)dtdr + \bar{g}_{rr}(t,r)dr^2 + R^2(t,r)(d\theta^2 + \sin^2\theta d\phi) \\ \bar{f}_{\mu\nu} &= \bar{f}_{tt}(t,r)dt^2 + 2\bar{f}_{tr}(t,r)dtdr + \bar{f}_{rr}(t,r)dr^2 + A^2R^2(t,r)(d\theta^2 + \sin^2\theta d\phi)\end{aligned}$$

Equations of motion for perturbations: $A = \frac{2+\alpha_3}{1+\alpha_3}$

$$\delta G^\mu{}_\nu + \delta X^\mu{}_\nu = 8\pi G \delta T^\mu{}_\nu$$

$$\delta X^\mu{}_\nu = m^2 \frac{A^2 - A(\gamma^t{}_t + \gamma^r{}_r) + \gamma^t{}_t\gamma^r{}_r - \gamma^t{}_r\gamma^r{}_t}{(1-A)^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \delta\gamma^\phi{}_\phi & -\delta\gamma^\theta{}_\phi \\ 0 & 0 & -\delta\gamma^\phi{}_\theta & \delta\gamma^\theta{}_\theta \end{pmatrix}$$

$\delta\gamma^I{}_J := \frac{1}{2A}\bar{g}^{IK}(-A^2\delta g_{KJ} + \delta f_{KJ})$ With I, J = θ, ϕ

The correction appears only in θ and ϕ components.

Bianchi Identity

Because of Bianchi identity of G^μ_ν and conservation law of T^μ_ν,

$$\nabla_\mu X^\mu{}_\nu = -\nabla_\mu G^\mu{}_\nu + M_{PL}^{-2}\nabla_\mu T^\mu{}_\nu = 0$$

$$\Rightarrow \bar{\nabla}_\mu \delta X^\mu{}_\nu = 0$$

Combining with $\delta X^\mu{}_\nu = m^2 \frac{A^2 - A(\gamma^t{}_t + \gamma^r{}_r) + \gamma^t{}_t\gamma^r{}_r - \gamma^t{}_r\gamma^r{}_t}{(1-A)^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \delta\gamma^\phi{}_\phi & -\delta\gamma^\theta{}_\phi \\ 0 & 0 & -\delta\gamma^\phi{}_\theta & \delta\gamma^\theta{}_\theta \end{pmatrix}$

$$\Rightarrow \begin{cases} \delta\gamma^\theta{}_\theta + \delta\gamma^\phi{}_\phi = 0 \\ \partial_\theta(\sin\theta\delta\gamma^\theta{}_\phi) = -\partial_\phi(\sin\theta\delta\gamma^\theta{}_\theta) \\ \frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta(\sin^2\theta\delta\gamma^\theta{}_\theta)) + \frac{1}{\sin^2\theta}\partial_\phi\partial_\phi(\sin^2\theta\delta\gamma^\theta{}_\theta) = 0 \end{cases}$$

Laplace equation on sphere $\Rightarrow$ $\sin^2\theta\delta\gamma^\theta{}_\theta = \text{constant over sphere}$

Assuming $\delta\gamma^\theta{}_\theta$ is regular at $\theta = 0, \pi$, solution is $\delta\gamma^\theta{}_\theta = 0$

From remaining equations, $\delta\gamma^\theta{}_\phi = \delta\gamma^\phi{}_\theta = \delta\gamma^\phi{}_\phi = 0$

$\delta\gamma^I{}_J = \frac{1}{2A}\bar{g}^{IK}(-A^2\delta g_{KJ} + \delta f_{KJ}) = 0$
With I, J = θ, ϕ

$$\delta X^\mu{}_\nu = 0$$

Results on Linear Perturbations

Perturbed EoMs reduces to Einstein equations
with 3 constraints between $\delta g_{\mu\nu}$ and $\delta f_{\mu\nu}$.

$$\delta G^{\mu}{}_{\nu} = M_{PL}^{-2}\delta T^{\mu}{}_{\nu}$$

$$\delta f_{IJ} = A^2 \delta g_{IJ} \quad {}_{I,J=\theta,\phi} \quad \Longleftrightarrow \quad \delta X^{\mu}{}_{\nu} = 0$$

$$\delta G_f{}^{\mu}{}_{\nu} = \kappa^{-2} M_{PL}^{-2} \delta T_f{}^{\mu}{}_{\nu}$$

Dynamics of linear perturbations is same as that of GR !

There are only 4 dofs, though bi-gravity has 7 dofs.
3 degrees of freedom disappear at linear order.

?

Do these dofs appear at higher order perturbation?
Is there nonlinear ghost instability at higher order perturbations?

Second order Perturbations

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} + g^{(2)}_{\mu\nu}$$

$$f_{\mu\nu} = \bar{f}_{\mu\nu} + \delta f_{\mu\nu} + f^{(2)}_{\mu\nu}$$

with

$$\begin{cases} \bar{g}_{\mu\nu} = \bar{g}_{tt}(t,r)dt^2 + 2\bar{g}_{tr}(t,r)dtdr + \bar{g}_{rr}(t,r)dr^2 + R^2(t,r)(d\theta^2 + \sin^2\theta d\phi) \\ \bar{f}_{\mu\nu} = \bar{f}_{tt}(t,r)dt^2 + 2\bar{f}_{tr}(t,r)dtdr + \bar{f}_{rr}(t,r)dr^2 + A^2R^2(t,r)(d\theta^2 + \sin^2\theta d\phi) \\ \delta f_{IJ} = A^2 \delta g_{IJ} \end{cases}$$

$$\Longrightarrow \quad \delta X^{(2)\mu}{}_{\nu} \propto \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \delta X^{(2)\theta}{}_{\theta} & \delta X^{(2)\theta}{}_{\phi} \\ 0 & 0 & \delta X^{(2)\phi}{}_{\theta} & \delta X^{(2)\phi}{}_{\phi} \end{pmatrix}$$

The correction appears only in θ and ϕ components.

$$\nabla_\mu X^{\mu}{}_{\nu} = 0 \quad \Longrightarrow \quad \delta X^{(2)\mu}{}_{\nu} = 0 \quad \Longrightarrow$$

EoMs reduce to Einstein equations

This solution is free from non-linear instability !

SUMMARY

Summary

Massive gravity and Bi-gravity have
any spherically symmetric solution of General Relativity.
Its stability is same as GR at least up to second order perturbation.

However this result shows
one cannot distinguish our spherically symmetric solutions of massive and bi-gravity from the corresponding solutions of General Relativity at least up to second order perturbation.

"Scale Invariance at low accelerations and the mass discrepancies in the Universe"

by Mordehai Milgrom (invited)

[JGRG25(2015)I10]

Scale Invariance at low accelerations and the mass discrepancies in the Universe

Moti Milgrom (Weizmann)

Kyoto Relativity Workshop December 2015

(Some) things everyone should know about Dark Matter

- No direct evidence for dark matter, only gravitational anomalies
- Dark matter is needed if we adhere to standard dynamics (Newtonian, GR)
- No known form of matter can be the DM (candidates from BSM lore)
- Another fix to standard dynamics is required – "dark energy"
- Many observations conflicts with DM
- Tens of experiments attempting to detect DM directly and indirectly at all sorts of particle-mass and -type ranges

MOND – synopsis

- MOND hinges on accelerations: These are many orders smaller in galaxies (and cosmology) compared with lab and SS ones.
- Departure at small accelerations $a \lesssim a_0 \approx 1\text{Å}\text{s}^{-2}$.
- Works very well in predicting many properties of galaxies of all types (with practically no free parameters).
- Leaves some discrepancy in galaxy clusters ($\sim$ a factor of 2).
- Not yet a coherent picture for cosmology.
- Strongly connected with cosmology; e.g. $2\pi a_0 \approx H_0 \approx (\Lambda/3)^{1/2}$ $(c = 1)$.
- Several full-fledged effective theories (relativistic and their NR limits). Some based on microscopic arguments. But no "final" theory.

Basic tenets

A theory of dynamics (gravity/inertia) involving a new constant a_0 (beside $G, ...$)

$$\text{Standard limit } (a_0 \rightarrow 0): \quad \text{The Newtonian limit}$$

$$\text{MOND limit}: \ a_0 \rightarrow \infty, \quad G \rightarrow 0, \quad \mathcal{A}_0 \equiv Ga_0 \ fixed:$$

Scale invariance: $(t, \mathbf{r}) \rightarrow \lambda(t, \mathbf{r})$

a_0 is analog to c in relativity or $\hbar$ in QM

An example of a Nonrelativistic theory

Modified gravity:[1]

$$\mathcal{L} = -\frac{a_0^2}{8\pi G}\mathcal{F}[(\vec{\nabla}\phi)^2/a_0^2] - \rho\phi$$

$$\mathbf{a} = -\vec{\nabla}\phi \qquad\qquad \vec{\nabla}\cdot[\mu(\tfrac{|\vec{\nabla}\phi|}{a_0})\vec{\nabla}\phi] = 4\pi G\rho$$

DML: $\mathcal{F}(X) = (2/3)X^{3/2}$ $\qquad \vec{\nabla}\cdot[|\vec{\nabla}\phi|\vec{\nabla}\phi] = 4\pi\mathcal{A}_0\rho$

Conformal invariance

Limit of relativistic theories

[1] ND: $\mathcal{L} = -(8\pi G)^{-1}(\vec{\nabla}\phi)^2 - \rho\phi$ $\qquad \mathbf{a} = -\vec{\nabla}\phi$ $\qquad \vec{\nabla}\cdot[\vec{\nabla}\phi] = 4\pi G\rho$

MOND laws of galactic dynamics

- Essentially follow from only the basic tenets of MOND
- Are independent as phenomenological laws–e.g., if interpreted as effects of DM (just as the BB spectrum, the photo electric effect, H spectrum, superconductivity, etc. are independent in QM)
- Pertain separately to properties of the "DM" alone (e.g., asymptotic flatness, "universal" Σ), of the baryons alone (e.g., $M-\sigma$, maximum Σ), relations between the two (e.g., $M-V$)
- Revolve around a_0 in different roles

Some of the MOND laws

- Asymptotic constancy of orbital velocity: $V(r) \to V_\infty$ (H)
- Light-bending angle becomes asymptotically constant (H)
- The velocity mass relation: $V_\infty^4 = M\mathcal{A}_0$ (H-B)
- DML virial relation: $\sigma^4 \sim M\mathcal{A}_0$
- Discrepancy appears always at $V^2/R = a_0$ (H-B)
- Isothermal spheres have surface densities $\bar{\Sigma} \lesssim a_0/G$ (B)
- The central surface density of "dark halos" is $\approx a_0/2\pi G$ (H)
- Full rotation curves from baryon distribution alone (H-B)

$a_0 =?$

a_0 can be derived in several independent ways:

$$a_0 \approx 1.2 \times 10^{-8} \text{ cm s}^{-2}$$

$$\bar{a}_0 \equiv 2\pi a_0 \approx cH_0 \qquad\qquad \bar{a}_0 \approx c(\Lambda/3)^{1/2}$$

$$\ell_M \equiv c^2/a_0 \approx \ell_U$$

$$a \lessgtr a_0 \qquad \Leftrightarrow \qquad \ell_a \lessgtr \ell_U$$

Why a critical acceleration? MOND length, MOND mass.

No MOND black hole with $R_S \lesssim R_{Hubble}$

Relativistic theories

- Tensor-Vector-Scalar Gravity (TeVeS–Bekenstein 2004, after Sanders 1997) Gravity is described by $g_{\alpha\beta}$, $\mathcal{U}_\alpha$, ϕ: $\tilde{g}_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta}+\mathcal{U}_\alpha\mathcal{U}_\beta)-e^{2\phi}\mathcal{U}_\alpha\mathcal{U}_\beta$

- MOND adaptations of Aether theories (Zlosnik, Ferreira, & Starkman 2007)

$$\mathcal{L}(A, g) = \frac{a_0^2}{16\pi G}\mathcal{F}(\mathcal{K}) + \lambda(A^\mu A_\mu + 1);$$

$$\mathcal{K} = a_0^{-2} A^\gamma{}_{;\alpha} A^\sigma{}_{;\beta}(c_1 g^{\alpha\beta} g_{\gamma\sigma} + c_2\delta^\alpha_\gamma\delta^\beta_\sigma + c_3\delta^\alpha_\sigma\delta^\beta_\gamma + c_4 A^\alpha A^\beta g_{\gamma\sigma}).$$

- Galileon k-mouflage MOND adaptation (Babichev, Deffayet, & Esposito-Farese 2011)

  Also a tensor-vector-scalar theory. Said to improve on TeVeS in various regards (e.g., small enough departures from GR in high-acceleration environments)

- Nonlocal metric MOND theories (Soussa & Woodard 2003; Deffayet, Esposito-Farese, & Woodard 2011, 2014) Pure metric, but highly nonlocal in that they involve $F(\Box)$.

- BIMOBD (Bimetric MOND) (Milgrom 2009-2013)

$$I = -\frac{1}{16\pi G}\int[\beta g^{1/2}R+\alpha\hat{g}^{1/2}\hat{R}-2(g\hat{g})^{1/4}a_0^2\mathcal{M}]d^4x+I_M(g_{\mu\nu},\psi_i)+\hat{I}_M(\hat{g}_{\mu\nu},\chi_i)$$

- Massive bi-gravity plus a polarizable medium (Blanchet & Heisenberg 2015)

"Microscopic" approaches

- DM with novel, unexpected properties, that may behave as dictated by MOND:
  - ▷ Polarized dark medium (Blanchet 2007, Blanchet & Le Tiec 2009, Blanchet & Heisenberg 2015)
  - ▷ Novel baryon-DM interactions (Bruneton & al. 2008)
  - ▷ Dark Fluid (Zhao 2008)
  - ▷ Superfluid (Khoury, Berezhiani & Khoury 2015)

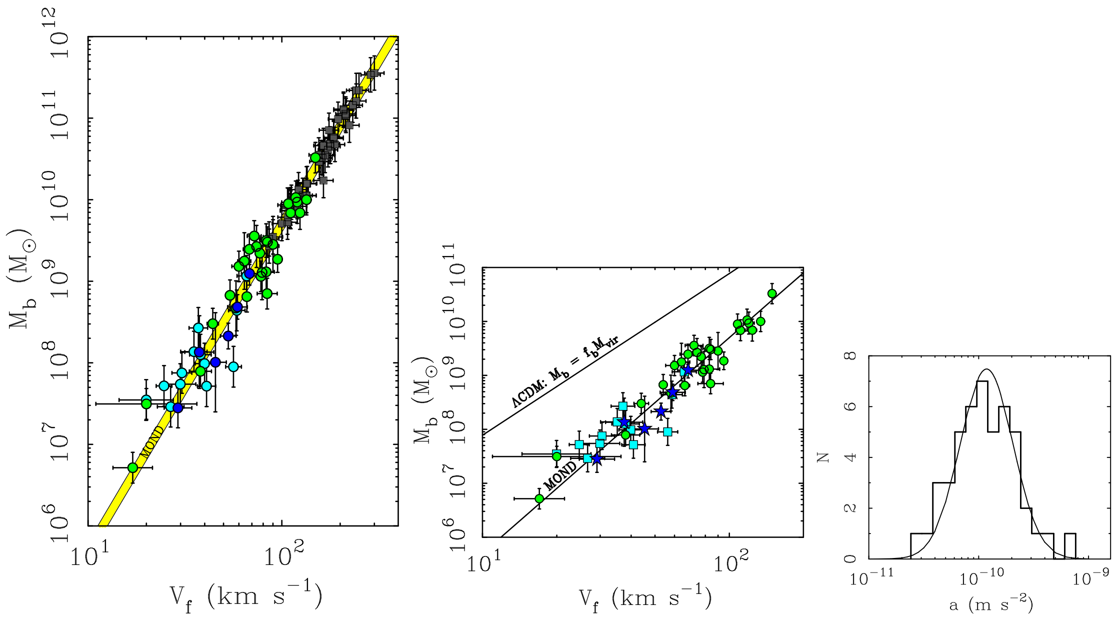
- Entropic effect (Verlinde, Klinkhamer & Kopp 2011, Pikhitsa Ho & al. 2010, Li & Chang 2010), others

- Vacuum effects (Milgrom 1999)

- Membranes with gravitational DoF extra coordinates (Milgrom 2002)

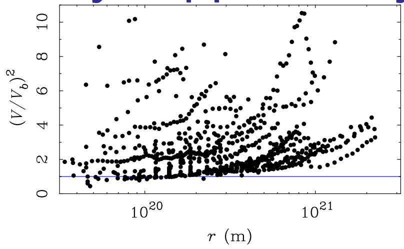
- Horava gravity (Romero & al. 2010), Sanders (2011), Blanchet & Marsat (2011)

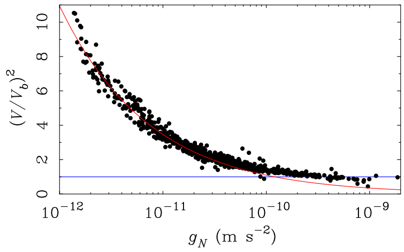
Mass-asymptotic-speed relation



McGaugh (2011)

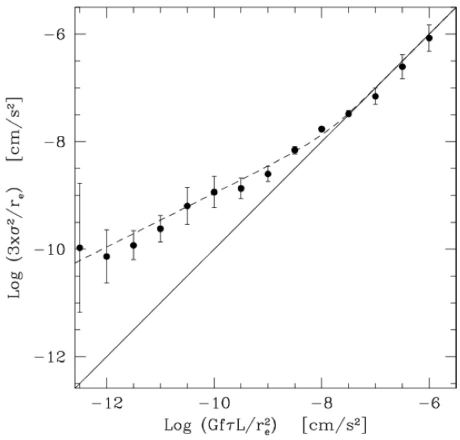
Discrepancy-acceleration correlation for rotationally-supported systems





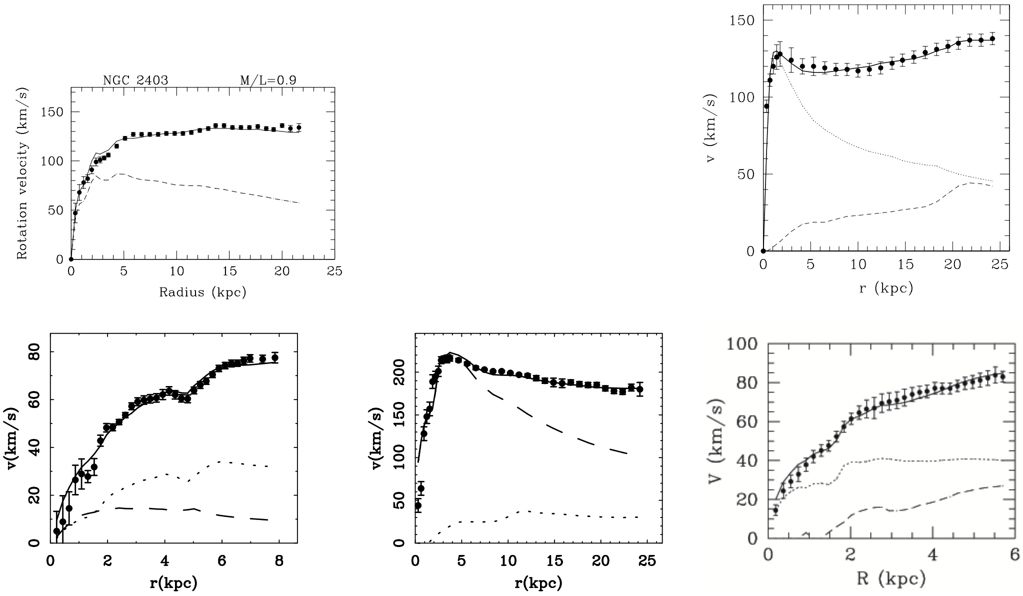
Plotted is g/g_N for 73 disc galaxies (points with $\delta V/V \leq 0.05$) – McGaugh (2015)

Discrepancy-acceleration correlation for pressure-supported systems

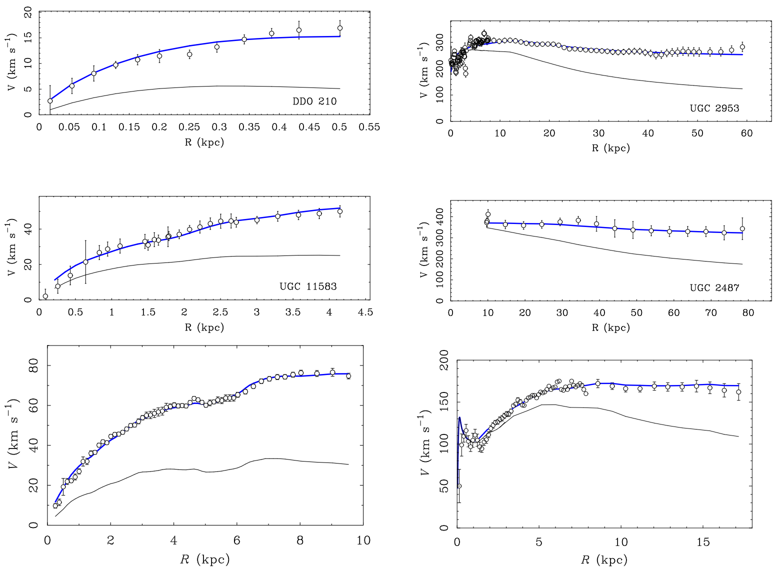


Scarpa (2006)

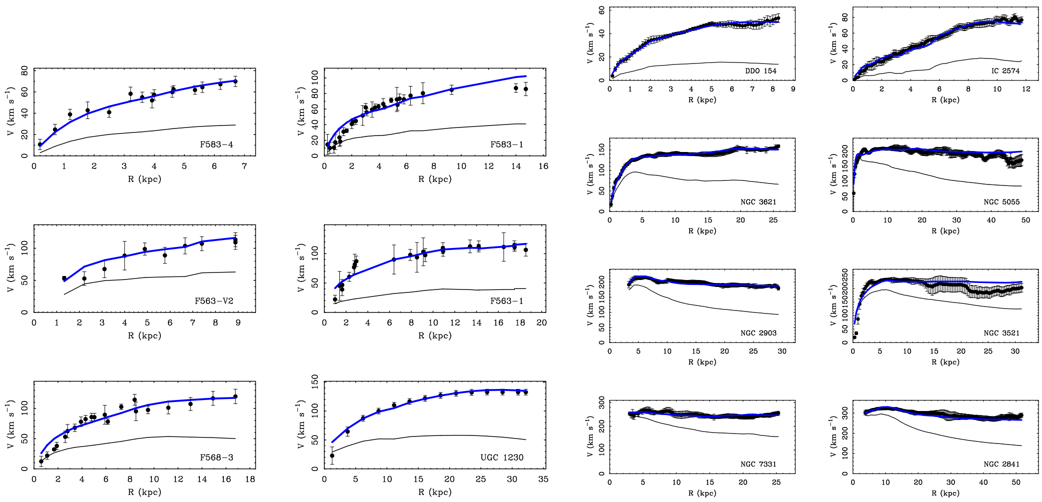
Rotation Curves of Disc Galaxies



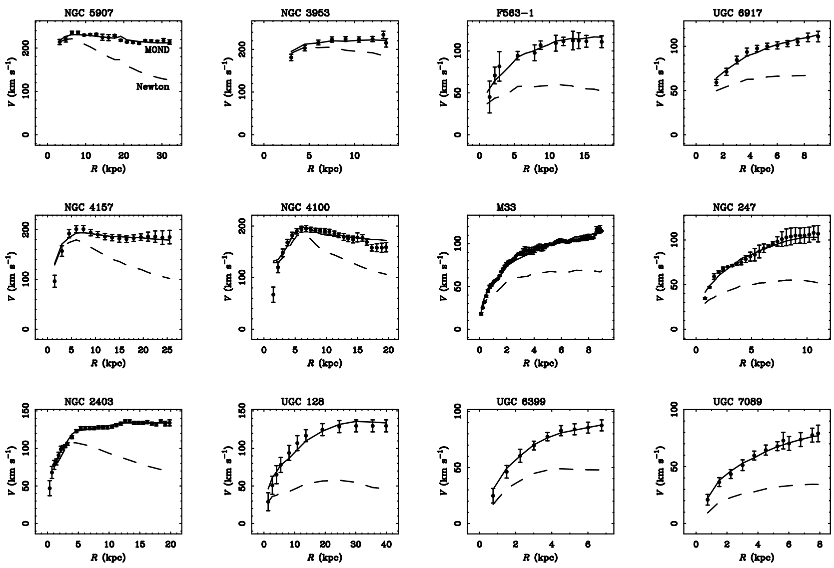
Sanders 2005 and Sanders and McGaugh 2002



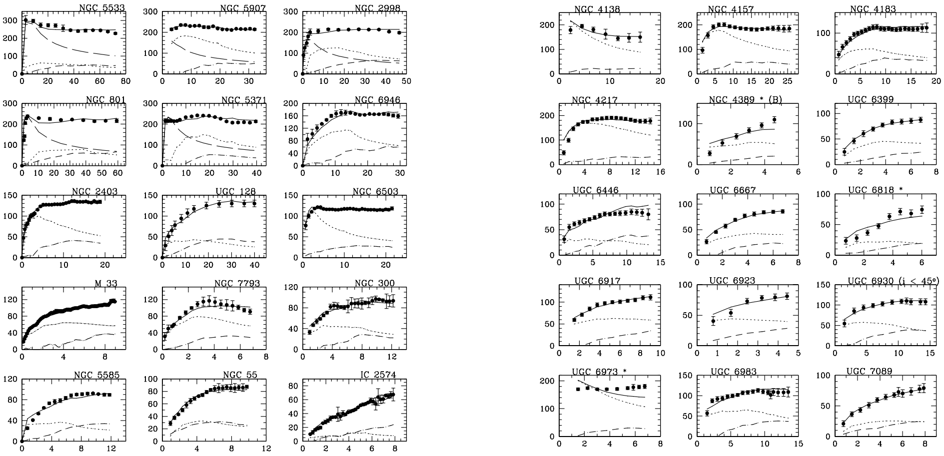
Famaey and McGaugh (2012)



Famaey and McGaugh (2012)

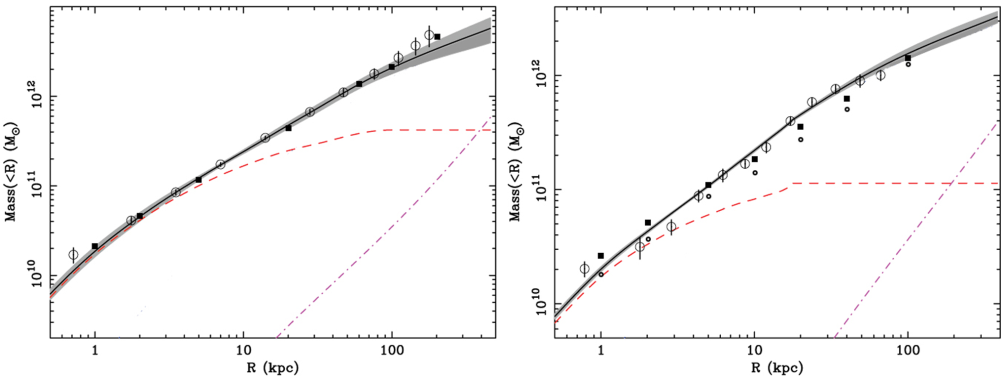


McGaugh



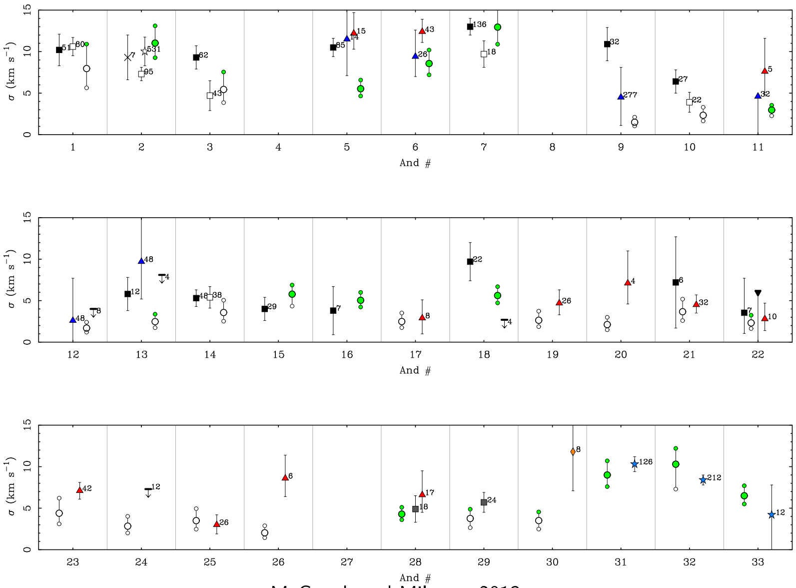
from Sanders and McGaugh 2002

x-ray Ellipticals, tested over an acceleration range $\sim 10a_0 - 0.1a_0$



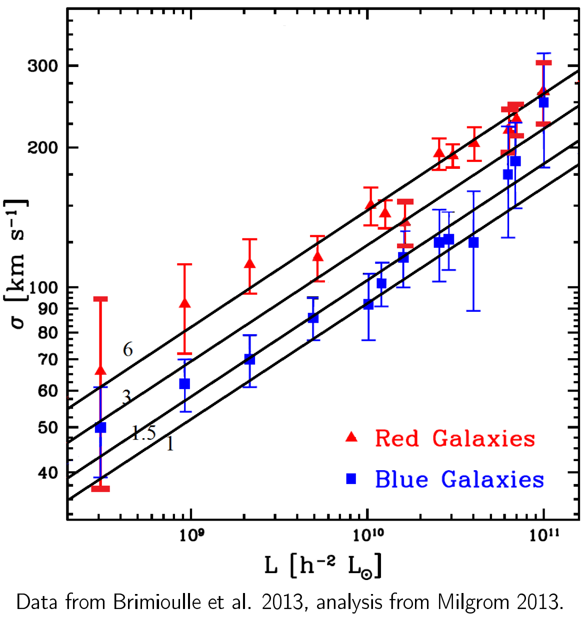
Baryon (dashed) and dynamical masses (grey band and large circles) from Humphrey et al. 2011,2012; MOND points (squares and small rings) from Milgrom (2012)

Andromeda satellites–internal dynamics



McGaugh and Milgrom 2013.

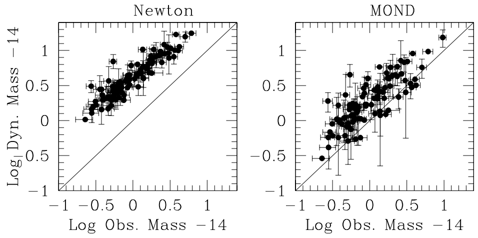
Galaxy-galaxy lensing



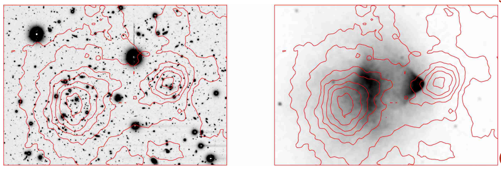
Data from Brimioulle et al. 2013, analysis from Milgrom 2013.

Galaxy Clusters

- Galaxy clusters



Sanders 1999



Clowe et al. 2006

Cosmological mass discrepancies

- We know the baryon fraction from nucleosynthesis $\sim 4\%$ (today)
- Cosmological "dark matter" $\sim 5 \times \mathrm{baryons}$
- "Dark energy" $(\Lambda) \sim 3 \times \mathrm{matter}$ (today)
- $a_0 \sim \Lambda^{1/2}$
- Expansion-history-"DM" requires $DM/baryons \approx 2\pi\ G \rightarrow 2\pi G$?

What is behind the phenomenological success of MOND?

- Truly new dynamics with direct links between Universe at large and local dynamics?
- "A phenomenological device that accounts well for the phenomena, but is not really new dynamics?"
  - ▷ "...For these hypotheses need not be true nor even probable. On the contrary, if they provide a calculus consistent with the observations, that alone is enough ... " Copernicus (Osiander)
  - ▷ Reality of atoms and molecules:
    "The principle point of debate among chemists was whether atoms were real objects or only mnemonic devices for coding chemical regularities and laws." (Pais)

DM?

- ▷ DM distribution is determined from that of the baryons.
- ▷ But DM to baryon ratio varies greatly and also differs from cosmological value.
- ▷ It is inconceivable that CDM will ever explain MOND: for individual galaxies the outcome depends on the unknown history of formation, interactions/mergers, ejection of most baryons, etc..

Summary

- MOND is a paradigm still under construction that replaces DM with new physics at accelerations below $a_0 \sim cH_0 \sim c\Lambda^{1/2}$.
- Anchored in symmetry
- Several theoretical directions
- It achieves a lot, and does it very well.
- Some important things that it was not yet shown with certainty to do
- Rather unlikely that MOND phenomenology can be explained as some organizing principle for CDM.

"Beyond Inflation and Beyond Horndeski Theory"

by Masahide Yamaguchi (invited)

[JGRG25(2015)I11]

Beyond Inflation and Beyond Horndeski theory

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T. Kobayashi, MY, J. Yokoyama

$c = \hbar = 1, \quad M_G = 1/\sqrt{8\pi G} \sim 2.4 \times 10^{18}\mathrm{GeV}.$

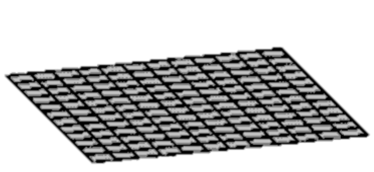
Inflation

Inflation, characterized as quasi De Sitter expansion, can naturally solve the problems of the standard big bang cosmology.

- **The horizon problem**
- **The flatness problem**
- **The origin of density fluctuations**
- **The monopole problem**
- …

Generic predictions of inflation

- **Spatially flat universe**

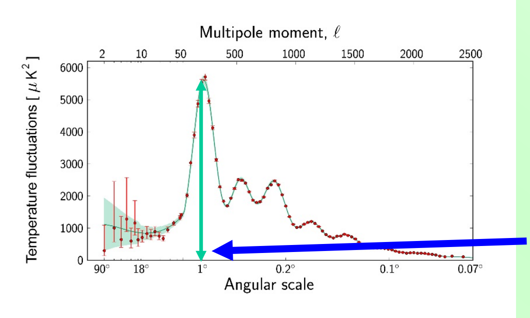


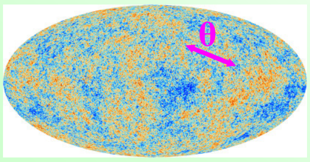
- **Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations**
- **Almost scale invariant and Gaussian primordial tensor fluctuations**

→ **Generates anisotropy of CMBR.**

Observations of CMB anisotropies

Planck TT correlation :





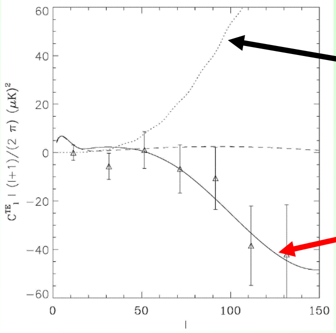
Green line : prediction by inflation
Red points : observation by PLANCK

Angle θ ~ 180° / 1

Total energy density ↔ Geometry of our Universe

Our Universe is spatially flat !!

WMAP TE correlation :



Causal seed models

Superhorizon models
(adiabatic perturbations)

Unfortunately, primordial tensor perturbations have not yet been observed.

What happened before inflation ?

and/or

How did the Universe begin ?

Contents

Introduction

What happened before inflation ?

and/or

How did the Universe begin ?

Look back to the past of the Universe

It is often claimed that, if cosmic time goes back to the past, the energy density gets larger and larger, and it eventually reaches the Planck energy density.

So, unless one completes quantum gravity theory, one cannot discuss the state at the extremely early stage (or even at the onset) of the Universe.

For a perfect fluid : $T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - g_{\mu\nu} p$

The homogeneous and isotropic (Friedmann) Universe:

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij} dx^i dx^j$$

$$\Rightarrow \quad \dot{\rho} = -3H(\rho + p).$$

As long as $\rho + p \geq 0$ (and H > 0 for the expanding Universe)

$$\Rightarrow \quad \dot{\rho} \leq 0.$$

We do not consider bouncing (contracting) Universe.

Null energy condition (NEC)

$$T_{\mu\nu}\xi^\mu\xi^\nu \geq 0$$

for any null vector ξ^μ. $(g_{\mu\nu}\xi^\mu\xi^\nu = 0)$

This is the weakest among all of the local classical energy conditions.

For a perfect fluid : $T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - g_{\mu\nu} p$

$$\Rightarrow \quad \text{NEC} \Leftrightarrow \rho + p \geq 0$$

As long as the NEC is conserved, the Universe cannot start from a low energy state in the expanding Universe.

N.B. Borde & Vilenkin showed with NEC (plus some conditions) that a future-eternal inflationary model cannot be globally extended into the infinite past; i.e., it is not geodesically complete in the past direction.

How robust is the NEC ?

- **Canonical kinetic term with potential:**

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi).$$

$$\Rightarrow \begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases} \Rightarrow \rho + p = \dot{\phi}^2 \geq 0.$$

(NEC is conserved)

- **How about k-inflation ?** **(Armendariz-Picon, Damour, Mukhanov 1999)**

$$\mathcal{L} = K(\phi, X), \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$$

$$\Rightarrow \begin{cases} \rho = 2XK_X - X \\ p = K \end{cases} \quad (K_X \equiv \partial K/\partial X) \Rightarrow \rho + p = 2XK_X.$$

Apparently, it looks that, if Kx < 0, it can violate the NEC.
But, this is not the case.

Primordial density fluctuations

Garriga & Mukhanov 1999

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2 e^{2\zeta} d\boldsymbol{x}^2$$

Comoving gauge :

$$\phi = \phi(t), \qquad \delta\phi = 0.$$

Prescription:

- **Expand the action up to the second order**
- **Eliminate α and β by use of the constraint equations**
- **Obtain quadratic action for ζ**

$$\Rightarrow S_S^{(2)} = \int dt d^3x\, a^3\, M_G^2 \frac{\epsilon}{c_s^2}\left(\dot{\zeta}^2 - \frac{c_s^2}{a^2}\zeta_{,k}\zeta_{,k}\right)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{XK_X}{M_{\rm pl}^2 H^2}, \qquad c_s^2 = \frac{K_X}{K_X + 2XK_{XX}}$$

(sound velocities of curvature perturbations)

In order to avoid the ghost and gradient instabilities, ε > 0 & cs^2 > 0.

(Hsu et al. 2004)
(See also Dubovsky et al. 2006)

$$\Rightarrow \rho + p = 2XK_X > 0.$$

Stable violation of NEC is impossible within k-inflation

It is impossible to break the NEC stably within k-inflation.

- Background solutions can break NEC apparently.
- But, the perturbations around them always become unstable for such background solutions.

This is quite reasonable in some sense because violation of NEC must pay some price. (see Sawicki & Vikman 2013, Easson, Sawicki, Vikman 2013)
e.g. An observer with almost speed of light observes arbitrary negative energy.

$$T_{\mu\nu}\xi^\mu\xi^\nu \geq 0 \underset{??}{\Longrightarrow} R_{\mu\nu}\xi^\mu\xi^\nu \geq 0$$

N.B. k-inflation is the most general action coming from phi and its first derivatives. $\left(\mathcal{L} = K(\phi, X), \quad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.\right)$

$\Rightarrow$ One may wonder how about introducing higher derivative terms.

$\Rightarrow$ Ostrogradski's instabilities

Theories with higher derivatives

and/or

What is the most general scalar-tensor theory (without ghost instabilities) ?

Lagrangian

Why does Lagrangian generally depend on only a position q and its velocity dot{q} ?

Newton recognized that an acceleration, which is given by the second time derivative of a position, is related to the Force :

$$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = \boldsymbol{F}\left(\boldsymbol{x}, \dot{\boldsymbol{x}}\right).$$

The Euler-Lagrange equation gives an equation of motion up to the second time derivative if a Lagrangian is given by L = L(q,dot{q},t).

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0, \quad \Longrightarrow \quad \ddot{q} = \ddot{q}\left(\dot{q}, q\right) \quad \Longrightarrow \quad q(t) = Q\left(\dot{q}_0, q_0, t\right).$$

(if $p := \frac{\partial L}{\partial \dot{q}}$ depends on dot{q} ⇔ non-degenerate condition.)

What happens if Lagrangian depends on higher derivative terms ?

Ostrogradski's theorem

Assume that $L = L(q, \dot{q}, \ddot{q})$ **and** $\frac{\partial L}{\partial \ddot{q}}$ **depends on** $\ddot{q}$ **:**

(Non-degeneracy)

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\mathrm{d}^2}{\mathrm{d}^2 t}\left(\frac{\partial L}{\partial \ddot{q}}\right) = 0, \quad \Longrightarrow \quad q^{(4)} = q^{(4)}\left(q^{(3)}, \ddot{q}, \dot{q}, q\right).$$

Canonical variables :

$$\begin{cases} Q_1 := q, & P_1 := \dfrac{\partial L}{\partial \dot{q}} - \dfrac{\mathrm{d}}{\mathrm{d}t}\dfrac{\partial L}{\partial \ddot{q}}, \\ Q_2 := \dot{q}, & P_2 := \dfrac{\partial L}{\partial \ddot{q}}. \end{cases}$$

Non-degeneracy ⇔ $\ddot{q} = \ddot{q}\left(q, \dot{q}, \frac{\partial L}{\partial \ddot{q}}\right)$ ⇔ $\ddot{q} = \ddot{q}\left(Q_1, Q_2, P_2\right)$

Hamiltonian:

$$\begin{aligned} H(Q_1, Q_2, P_1, P_2) &:= P_1\dot{q} + P_2\ddot{q} - L \\ &= P_1 Q_2 + P_2\ddot{q}(Q_1, Q_2, P_2) - L(Q_1, Q_2, \ddot{q}(Q_1, Q_2, P_2)). \end{aligned}$$

These canonical variables really satisfy the canonical EOM : $\dot{Q}_i = \frac{\partial H}{\partial P_i}, \quad \dot{P}_i = -\frac{\partial H}{\partial Q_i}.$

P1 depends linearly on H so that no system of this form can be stable !!

N.B. $\frac{\partial L}{\partial \phi} - \partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu \phi)}\right) + \partial_\mu\partial_\nu\left(\frac{\partial L}{\partial(\partial_\mu\partial_\nu \phi)}\right) = 0.$ $\frac{i}{(p^2+m_1^2)(p^2+m_2^2)} = \frac{1}{m_2^2 - m_1^2}\left(\frac{i}{p^2+m_1^2} - \frac{i}{p^2+m_2^2}\right).$

(propagators)

Loophole of Ostrogradski's theorem

We can break the non-degeneracy condition which requires that $\frac{\partial L}{\partial \ddot{q}}$ depends on ddot{q}.

In case Lagrangian depends on only a position q and its velocity dot{q}, degeneracy implies that EOM is first order, which represents not the dynamics but the constraint.

$\Updownarrow$

In case Lagrangian depends on q, dot{q}, ddot{q}, degeneracy implies that EOM is second order, which can represent the dynamics.

Galileon field

Nicolis et al. 2009
Deffayet et al. 2009

The theory has Galilean shift symmetry in flat space :

$$\partial_\mu \phi \longrightarrow \partial_\mu \phi + b_\mu$$

$$\begin{cases} \mathcal{L}_1 = \phi \\ \mathcal{L}_2 = (\partial\phi)^2 \\ \mathcal{L}_3 = (\partial\phi)^2 \, \Box\phi \\ \mathcal{L}_4 = (\partial\phi)^2 \left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2 \right] \\ \mathcal{L}_5 = (\partial\phi)^2 \left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\partial_\mu\partial_\nu\phi)^2 + 2\,(\partial_\mu\partial_\nu\phi)^3 \right] \end{cases}$$

$$(\partial_\mu\partial_\nu\phi)^2 = \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi,$$
$$(\partial_\mu\partial_\nu\phi)^3 = \partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi$$

Lagrangian has higher order derivatives, but EOM is second order.

What is the most general scalar-tensor theory whose equations of motion are up to second order ?

Generalized Galileon

Deffayet et al. 2009, 2011

$$\begin{cases} \mathcal{L}_2 = \boxed{K(\phi, X),} \\ \mathcal{L}_3 = -\boxed{G_3(\phi, X)}\Box\phi, \\ \mathcal{L}_4 = \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = \boxed{G_5(\phi, X)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$X = -\frac{1}{2}(\nabla\phi)^2, \qquad G_{iX} \equiv \partial G_i/\partial X.$$

Covariantization of the flat Galileon theory.
Is this the most general scalar tensor theory whose EOMs are up to second order ?

NB :
- **$G_4 = M_G^2 / 2$ yields the Einstein-Hilbert action**
- **$G_4 = f(\varphi)$ yields a non-minimal coupling of the form $f(\varphi)R$**
- **The new Higgs inflation with $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ comes from $G_5 \propto \varphi$ after integration by parts.**

Horndeski's theorem

Horndeski 1974

In 1974, Horndeski presented the most general action (in four dimensions) constructed from the metric g, the scalar field φ, and their derivatives, $\partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \partial^3 g_{\mu\nu}, \cdots, \partial\phi, \partial^2\phi, \partial^3\phi, \cdots$ still having second-order equations.

$$\begin{aligned}\mathcal{L}_H = & \ \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\left[\kappa_1\nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi + \kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\right] \\ & +\delta^{\alpha\beta}_{\mu\nu}\left[(F+2W)R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\right] - 6\left(F_\phi + 2W_\phi - X\kappa_8\right)\Box\phi + \kappa_9.\end{aligned}$$

κ1, κ3, κ8, κ9, F : functions of φ & X with $F_X = 2(\kappa_3 + 2X\kappa_{3X} - \kappa_{1\phi})$.

W = W(φ)

$\delta^{\alpha_1\alpha_2\ldots\alpha_n}_{\mu_1\mu_2\ldots\mu_n} = n!\delta^{[\alpha_1}_{\mu_1}\delta^{\alpha_2}_{\mu_2}\ldots\delta^{\alpha_n]}_{\mu_n}$.

What is the relation between Generalized Galileon and Horndeski's models ?

⇒ **Both models are completely equivalent :** Kobayashi, MY, Yokoyama 2011

$$\left\{\begin{aligned} K &= \kappa_9 + 4X\int^X dX'\left(\kappa_{8\phi} - 2\kappa_{3\phi\phi}\right), \\ G_3 &= 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X dX'(\kappa_8 - 2\kappa_{3\phi}), \\ G_4 &= 2F - 4X\kappa_3, \\ G_5 &= -4\kappa_1, \end{aligned}\right.$$

$$\left\{\begin{aligned} \mathcal{L}_2 &= K(\phi, X), \\ \mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & \quad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{aligned}\right.$$

Beyond Horndeski theory

Horndeski theory

Horndeski 1974
Deffayet et al. 2011
Kobayashi et al. 2011

Horndeski theory (= Generalized Galileon) :

$$\left\{\begin{array}{lcl}\mathcal{L}_2 & = & \boxed{K(\phi, X)}, \\ \mathcal{L}_3 & = & -\boxed{G_3(\phi, X)}\Box\phi, \\ \mathcal{L}_4 & = & \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 & = & \boxed{G_5(\phi, X)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & & -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].\end{array}\right.$$

$$X = -\frac{1}{2}(\nabla\phi)^2, \qquad G_{iX} \equiv \partial G_i/\partial X.$$

This is the most general (single) scalar-tensor theory which yields second-order (scalar and gravitational) equations of motion.

But, in order to avoid the Ostrogradski instabilities, this requirement can be too strong. For this purpose, only time derivatives should be second order while spacial ones can be higher.

(c.f. Another direction is to include infinite higher derivatives.)

Beyond Horndeski theory

Gleyzes et al. 2014
Gao 2014

ADM decomposition:
(φ= const surfaces)

$$ds^2 = -N^2dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

$$\Rightarrow \quad \phi = \phi(t), \quad X = \dot\phi^2(t)/(2N^2)$$

(φ and X are functions of t and N, and vice versa.)

$$F(\phi, X) \leftrightarrow F(t, N)$$

Unit normal vector : $n_\mu = \alpha\nabla_\mu\phi, \quad \alpha = \frac{1}{\sqrt{2X}}.$

$$\Rightarrow \quad \nabla_\mu\nabla_\nu\phi = \alpha^{-1}(K_{\mu\nu} - n_\mu a_\nu - n_\nu a_\mu) - \alpha n^\lambda\nabla_\lambda X\, n_\mu n_\nu$$

$$\left\{\begin{array}{l} K_{\mu\nu} = h^\sigma_\mu\nabla_\sigma n_\nu, \qquad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \\ a_\mu = \dot n_\mu = n^\nu\nabla_\nu n_\mu \end{array}\right.$$

$$R = R^{(3)} - \left(K^2 - K_{\mu\nu}K^{\mu\nu}\right) + 2\nabla_\mu\left(Kn^\mu - a^\mu\right)$$

...

(ADM => phi)

$$\Leftarrow \quad K_{\mu\nu} = -\alpha\left[\nabla_\mu\nabla_\nu\phi - \alpha^4\nabla_\mu\phi\nabla_\nu\phi\nabla_\sigma\phi\nabla^\sigma X - \alpha^2(\nabla_\mu\phi\nabla_\nu X + \nabla_\nu\phi\nabla_\mu X)\right]$$

...

Beyond Horndeski theory II

Gleyzes et al. 2014
Gao 2014

Horndeski theory (= Generalized Galileon) :

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \quad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$L = \sqrt{\gamma} N \sum_a \mathcal{L}_a,$$

$$\begin{cases} \mathcal{L}_2 = A_2(t, N), \\ \mathcal{L}_3 = A_3(t, N)K, \\ \mathcal{L}_4 = A_4(t, N)\left(K^2 - K_{ij}^2\right) + B_4(t, N)R^{(3)}, \\ \mathcal{L}_5 = A_5(t, N)\left(K^3 - 3KK_{ij}^2 + 2K_{ij}^3\right) + B_5(t, N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right). \end{cases}$$

with $A_4 = -B_4 - N\frac{\partial B_4}{\partial N}, \quad A_5 = \frac{N}{6}\frac{\partial B_5}{\partial N}.$

Kij : extrinsic curvature
Rij(3) : intrinsic curvature

Gleyzes et al. (GLPV) pointed out that, even if the above two relations are absent, the number of the propagating degrees of freedom remains unchanged. Gao showed that further extension is possible.

Further extention (our later setup)

Gao 2014

$$\mathcal{L} = \left(a_0 + a_1 R^{(3)} + a_3 R^{(3)2} + a_4 R_{ij}^{(3)} R^{(3)ij} + a_5 a_i a^i\right) K$$
$$+ \left[\left(a_2 + a_6 R^{(3)}\right) R^{(3)ij} + a_7 R^{(3)i}{}_k R^{(3)jk} + a_8 a^i a^j\right] K_{ij} + \cdots$$

$(a_i = a_i(t, N))$

$$L = \sqrt{\gamma} N \sum_a \mathcal{L}_a$$

$$\begin{cases} \mathcal{L}_2 = A_2(t, N), \\ \mathcal{L}_3 = A_3(t, N)K, \\ \mathcal{L}_4 = A_4(t, N)\left(\lambda_1 K^2 - K_{ij}^2\right) + B_4(t, N)R^{(3)}, \\ \mathcal{L}_5 = A_5(t, N)\left(\lambda_2 K^3 - 3\lambda_3 KK_{ij}^2 + 2K_{ij}^3\right) \\ \qquad + B_5(t, N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right). \end{cases}$$

(The GLPV theory corresponds to the case with $\lambda_1 = \lambda_2 = \lambda_3 = 1$.)

Stable violation of NEC with higher derivative terms

Is it possible to violate the NEC stably if one includes higher derivative terms ?

Galilean Genesis

Creminelli et al. 2010
Nicolis et al. 2009

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_G^2 R + f^2 e^{2\phi} (\partial\phi)^2 + \frac{f^3}{\Lambda^3} (\partial\phi)^2 \Box\phi + \frac{f^3}{2\Lambda^3} (\partial\phi)^4 \right]$$

(In the flat spacetime limit, this theory has conformal symmetry SO(4,2).)

- **Energy-momentum tensor :**

$$\left\{ \begin{aligned} \rho &= -f^2 \left(e^{2\phi}\dot{\phi}^2 - \frac{3}{2}\frac{f}{\Lambda^3}\dot{\phi}^4 - 6H\frac{f}{\Lambda^3}\dot{\phi}^3 \right), \\ p &= -f^2 \left(e^{2\phi}\dot{\phi}^2 - \frac{1}{2}\frac{f}{\Lambda^3}\dot{\phi}^4 + 2\frac{f}{\Lambda^3}\dot{\phi}^2\ddot{\phi} \right). \end{aligned} \right.$$

- **A background solution, (t : -∞ -> 0) :** **Starts from Minkowski in infinite past.**

$$e^{\phi} \simeq \frac{1}{\sqrt{2Y_0}} \frac{1}{(-t)}, \quad H \simeq \frac{h_0}{(-t)^3}, \quad \left(a(t) \simeq 1 + \frac{h_0}{2(-t)^2} \right).$$

$$\left(Y_0 \equiv \frac{\Lambda^3}{3f}, \quad h_0 \equiv \frac{1}{2M_G^2}\frac{f^3}{\Lambda^3} \right)$$

$$\Rightarrow \quad \rho + p \simeq -\frac{f^3}{\Lambda^3}\frac{4}{(-t)^4} < 0.$$

(Actually, you can verify that H increases.)

(The NEC is violated !!)

Primordial density fluctuations

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2 e^{2\zeta} d\boldsymbol{x}^2$$

Comoving gauge :

$$\phi = \phi(t), \quad \delta\phi = 0.$$

$$\Rightarrow \quad S_S^{(2)} = \int dt d^3x\, a^3 \left(\mathcal{G}_s \dot{\zeta}^2 - \frac{\mathcal{F}_s}{a^2}\zeta_{,k}\zeta_{,k}\right)$$

In order to avoid the ghost and gradient instabilities, Gs > 0 & Fs > 0.

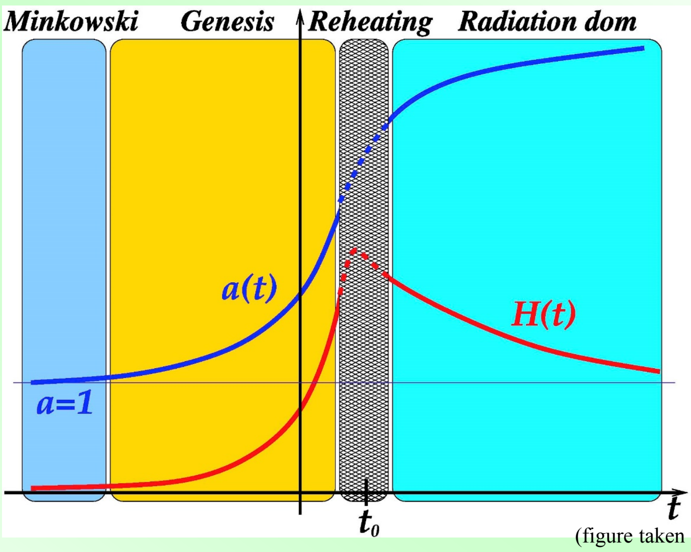
$$\mathcal{G}_s = \mathcal{F}_s \simeq 6M_G^4 \frac{\lambda^3}{f^3}(-t)^2 > 0.$$

(The NEC is violated stably !!)

N.B.
- **A spectator field like curvaton is responsible for primordial density perturbations because the genesis field predicts too blue (ns ~3) perturbations in this simple model.**
- **Primordial tensor perturbations are not generated at first order.**

Galilean Genesis II

Creminelli et al. 2010
Nicolis et al. 2009



(figure taken from Creminelli et al. 1007.0027)

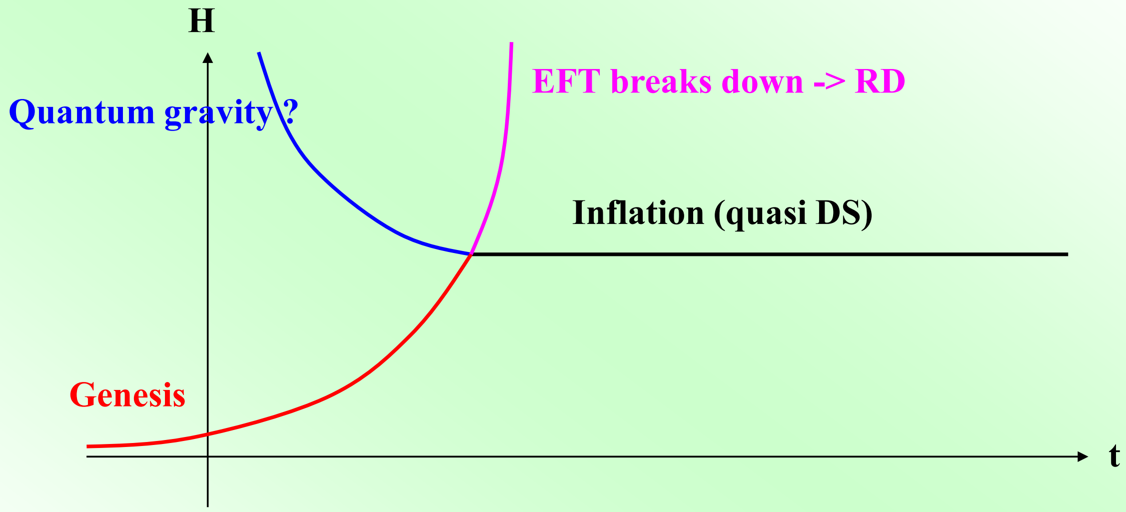
- **In this scenario, the effective theory breaks around $t \sim t_0 = 0$. So, it is assumed that the energy density of the genesis field is converted to radiation, in which hot Universe starts.**
- **Of course, this is not necessarily a fault of this scenario. A more fundamental theory will be able to describe the transition adequately.**

(See 1401.4024 written by Rubakov for good review)

From Genesis to inflation

From Genesis to inflation

Pirtskhalava et al. 2014



- As a epoch before inflation (and the onset of the Universe), use of Galilean Genesis is proposed by Pirtskhalava et al.
- Unfortunately, in their concrete construction, the gradient instabilities appear during the transition from Genesis to inflation. They are dangerous for large k modes even during short period because of $\propto e^{\mathrm{Im}(c_s)kt}$.

We try to construct a concrete workable example, in which the Universe starts from Minkowski spacetime in the infinite past, and is smoothly connected to inflation, followed by reheating (graceful exit).

Our setup

Gao 2014

$$L = \sqrt{\gamma} N \sum_a \mathcal{L}_a$$

$$\begin{cases} \mathcal{L}_2 = A_2(t,N), \\ \mathcal{L}_3 = A_3(t,N)K, \\ \mathcal{L}_4 = A_4(t,N)\left(\lambda_1 K^2 - K_{ij}^2\right) + B_4(t,N)R^{(3)}, \\ \mathcal{L}_5 = A_5(t,N)\left(\lambda_2 K^3 - 3\lambda_3 K K_{ij}^2 + 2K_{ij}^3\right) \\ \qquad +B_5(t,N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right). \end{cases}$$

(The **GLPV theory** corresponds to the case with $\lambda_1 = \lambda_2 = \lambda_3 = 1$.)

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

$$\begin{cases} N = \overline{N}(t)\,(1+\delta n)\,, \\ N_i = \overline{N}(t)\partial_i \chi, \\ \gamma_{ij} = a^2(t)e^{2\zeta}\left(e^h\right)_{ij}. \end{cases}$$

curvature perturbations

tensor perturbations

$\left(h_{ii} = h_{ij,j} = 0\right)$

Perturbations

● **Tensor perturbations :**

$$\mathcal{L}_T^{(2)} = \frac{\overline{N}a^3}{8}\left[\frac{\mathcal{G}_T}{\overline{N}^2}\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial h_{ij})^2\right]$$

$$\begin{cases} \mathcal{G}_T := -2A_4 - 6(3\lambda_3 - 2)A_5 H, \\ \mathcal{F}_T := 2B_4 + \frac{1}{\overline{N}}\frac{dB_5}{dt}. \end{cases}$$

$$\left(H := \frac{\dot{a}}{(\overline{N}a)}\right)$$

● **Curvature perturbations : (spatial higher derivative appears !!)**

$$\mathcal{L}_S^{(2)} = \overline{N}a^3\left[\mathcal{G}_S\frac{\dot{\zeta}^2}{\overline{N}^2} + \zeta\left(\mathcal{F}_S\frac{\partial^2}{a^2} - \mathcal{H}_S\frac{\partial^4}{a^4}\right)\zeta\right] \Rightarrow \omega^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S}k^2 + \frac{\mathcal{H}_S}{\mathcal{G}_S}\frac{k^4}{a^2}.$$

$$\frac{\mathcal{F}_S}{\mathcal{G}_S} = c_s^2$$

$$\begin{cases} \mathcal{G}_S := \dfrac{\Sigma\mathcal{G}_T^2}{\Theta^2 + \Sigma\mathcal{C}} + 3\mathcal{G}_T, \\ \mathcal{F}_S := \dfrac{1}{\overline{N}a}\dfrac{d}{dt}\left(\dfrac{a\Theta\mathcal{G}_B\mathcal{G}_T}{\Theta^2 + \Sigma\mathcal{C}}\right) - \mathcal{F}_T, \\ \mathcal{H}_S := \dfrac{\mathcal{G}_B^2\mathcal{C}}{\Theta^2 + \Sigma\mathcal{C}}. \end{cases}$$

$$\begin{cases} \Sigma := \overline{N}A_2' + \frac{1}{2}\overline{N}^2 A_2'' + \frac{3}{2}\overline{N}^2 A_3'' H \\ \qquad +3\eta_4\left(2A_4 - 2\overline{N}A_4' + \overline{N}^2 A_4''\right)H^2 \\ \qquad +3\eta_5\left(6A_5 - 4\overline{N}A_5' + \overline{N}^2 A_5''\right)H^3, \\ \Theta := \frac{\overline{N}A_3'}{2} - 2\eta_4\left(A_4 - \overline{N}A_4'\right)H \\ \qquad -3\eta_5\left(2A_5 - \overline{N}A_5'\right)H^2, \\ \mathcal{G}_A := -2\eta_4 A_4 - 6\eta_5 A_5 H, \\ \mathcal{G}_B := 2\left(B_4 + \overline{N}B_4'\right) - H\overline{N}B_5', \\ \mathcal{C} := (1-\lambda_1)A_4 - (6 + 9\lambda_2 - 15\lambda_3)A_5 H. \end{cases}$$

$$(\eta_4 := (3\lambda_1 - 1)/2,\ \eta_5 := (9\lambda_2 - 9\lambda_3 + 2)/2)$$

N.B. ● **C = 0 for λ1 = λ2 = λ3 = 1.**

● **Even if Fs < 0 (with Gs > 0), the curvature perturbations with large k are stabilized for Hs > 0.**

Concrete example

$$\begin{cases} A_2 = M_2^4 f^{-2(\alpha+1)}(t)a_2(N), \\ A_3 = M_3^3 f^{-(2\alpha+1)}(t)a_3(N), \\ A_4 = -\dfrac{M_G^2}{2} + M_4^2 f^{-2\alpha}(t)a_4(N), \\ A_5 = M_5 f(t)a_5(N), \qquad (\alpha > 0) \end{cases}$$

(φ, X) ⇔ (t, N)

$$\begin{cases} \mathcal{L}_2 = A_2(t,N), \\ \mathcal{L}_3 = A_3(t,N)K, \\ \mathcal{L}_4 = A_4(t,N)\left(\lambda_1 K^2 - K_{ij}^2\right) + B_4(t,N)R^{(3)}, \\ \mathcal{L}_5 = A_5(t,N)\left(\lambda_2 K^3 - 3\lambda_3 K K_{ij}^2 + 2K_{ij}^3\right) \\ \qquad +B_5(t,N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right). \end{cases}$$

Background dynamics : $\mathcal{L}^{(0)} = \overline{N}a^3\left(A_2 + 3A_3 H + 6\eta_4 A_4 H^2 + 6\eta_5 A_5 H^3\right).$

$$\begin{cases} -\mathcal{E} := (\overline{N}A_2)' + 3\overline{N}A_3' H + 6\eta_4\overline{N}^2(\overline{N}^{-1}A_4)'H^2 + 6\eta_5\overline{N}^3(\overline{N}^{-2}A_5)'H^3 = 0, \\ \mathcal{P} := A_2 - 6\eta_4 A_4 H^2 - 12\eta_5 A_5 H^3 - \frac{1}{\overline{N}}\frac{d}{dt}\left(A_3 + 4\eta_4 A_4 H + 6\eta_5 A_5 H^2\right) = 0. \end{cases}$$

$$\left(' := d/d\overline{N}\right)$$

● **Genesis phase (t < t0) :** $f(t) \simeq \dot{f}_0 t \quad (\dot{f}_0 = \text{const} < 0)$

$$\Rightarrow \begin{cases} \overline{N} \simeq N_0\,(= \text{const}) \quad \text{with} \quad a_2(N_0) + N_0 a_2'(N_0) = 0. \\ H = -\dfrac{\hat{p}}{2(2\alpha+1)\eta_4 M_G^2}\dfrac{N_0}{|\dot{f}_0|}f^{-(2\alpha+1)} \sim \dfrac{1}{(-t)^{2\alpha+1}}; \\ a = 1 - \dfrac{\hat{p}}{4\alpha(2\alpha+1)\eta_4 M_G^2}\dfrac{N_0^2}{\dot{f}_0^2}f^{-2\alpha}. \end{cases}$$

$$\left(\hat{p} = M_2^4 a_2(N_0) + (2\alpha+1)M_3^3 a_3(N_0)\frac{\dot{f}_0}{N_0}\right)$$

(The background dynamics for α = 1 coincides with that of the original Genesis model.)

Concrete example II

● **Inflationary phase (tend > t > t0) :** $f(t) \simeq f_1\,(=\text{const})$

$$\Rightarrow \begin{cases} \overline{N} \simeq N_{\rm inf}\,(=\text{const}), \\ H \simeq H_{\rm inf}\,(=\text{const}). \end{cases}$$

with
$$\begin{cases} -\mathcal{E} = (N_{\rm inf}A_2)' + 3N_{\rm inf}A_3' H_{\rm inf} + 6\eta_4 N_{\rm inf}^2 (N_{\rm inf}^{-1}A_4)' H_{\rm inf}^2 \\ \qquad +6\eta_5 N_{\rm inf}^3 (N_{\rm inf}^{-2}A_5)' H_{\rm inf}^3 = 0, \\ \mathcal{P} = A_2 - 6\eta_4 A_4 H_{\rm inf}^2 - 12\eta_5 A_5 H_{\rm inf}^3 = 0. \end{cases}$$

N.B. A weak time dependence of f(t) yields slight deviation from exact DS.

● **Graceful exit (t > tend) :** $f(t) \sim t^{1/(\alpha+1)}$

If m > 4, gravitational reheating can work.

$$\Rightarrow \begin{cases} \overline{N} \simeq N_{\rm e}\,(=\text{const}), \\ H^2 \sim 1/t^2 \sim f^{-2(\alpha+1)} \propto 1/a^m \end{cases} \quad \left(m := 3N_{\rm e}a_2'/(N_{\rm e}a_2)' > 0\right).$$

with
$$\begin{cases} -\mathcal{E} = (N_{\rm e}A_2)' + 3\eta_4 M_G^2 H^2 + \mathcal{O}(f^{-(3\alpha+2)}) = 0, \\ \mathcal{P} = A_2 + 3\eta_4 M_G^2 H^2 + \dfrac{2\eta_4 M_G^2}{N_{\rm e}}\dfrac{dH}{dt} + \mathcal{O}(f^{-(3\alpha+2)}) = 0. \end{cases}$$

Perturbations

● **Tensor perturbations :**

$$\mathcal{L}_T^{(2)} = \frac{\overline{N}a^3}{8}\left[\frac{\mathcal{G}_T}{\overline{N}^2}\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial h_{ij})^2\right] \qquad \begin{cases} \mathcal{G}_T := -2A_4 - 6(3\lambda_3-2)A_5 H, \\ \mathcal{F}_T := 2B_4 + \dfrac{1}{\overline{N}}\dfrac{dB_5}{dt}. \end{cases}$$

$$\left(H := \frac{\dot{a}}{(\overline{N}a)}\right)$$

● **Curvature perturbations : (spatial higher derivative appears !!)**

$$\mathcal{L}_S^{(2)} = \overline{N}a^3\left[\mathcal{G}_S\frac{\dot{\zeta}^2}{\overline{N}^2} + \zeta\left(\mathcal{F}_S\frac{\partial^2}{a^2} - \mathcal{H}_S\frac{\partial^4}{a^4}\right)\zeta\right] \Rightarrow \omega^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S}k^2 + \frac{\mathcal{H}_S}{\mathcal{G}_S}\frac{k^4}{a^2}.$$

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$$\begin{cases} \Sigma := \overline{N}A_2' + \frac{1}{2}\overline{N}^2A_2'' + \frac{3}{2}\overline{N}^2A_3''H \\ \qquad +3\eta_4\left(2A_4 - 2\overline{N}A_4' + \overline{N}^2A_4''\right)H^2 \\ \qquad +3\eta_5\left(6A_5 - 4\overline{N}A_5' + \overline{N}^2A_5''\right)H^3, \\ \Theta := \frac{\overline{N}A_3'}{2} - 2\eta_4\left(A_4 - \overline{N}A_4'\right)H \\ \qquad -3\eta_5\left(2A_5 - \overline{N}A_5'\right)H^2, \\ \mathcal{G}_A := -2\eta_4A_4 - 6\eta_5A_5H, \\ \mathcal{G}_B := 2\left(B_4 + \overline{N}B_4'\right) - H\overline{N}B_5', \\ \mathcal{C} := (1-\lambda_1)A_4 - (6+9\lambda_2-15\lambda_3)A_5H. \end{cases}$$

$$(\eta_4 := (3\lambda_1-1)/2,\ \eta_5 := (9\lambda_2-9\lambda_3+2)/2)$$

N.B. ● **C = 0 for λ1 = λ2 = λ3 = 1.**

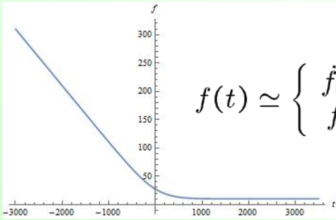
● **Even if Fs < 0 (with Gs > 0), the curvature perturbations with large k are stabilized for Hs > 0.**

Numerical calculations

From Genesis to inflation :

$$f(t) = \frac{\dot f_0}{2}\left\{t - \frac{\ln[2\cosh(st)]}{s}\right\} + f_1,$$

$$\dot f_0 = -10^{-1},\ f_1 = 10,\ s = 2\times 10^{-3} \simeq t_0^{-1}$$



$$f(t) \simeq \begin{cases} \dot f_0 t & \text{for } t \ll t_0 \\ f_1 & \text{for } t \gg t_0 \end{cases}$$

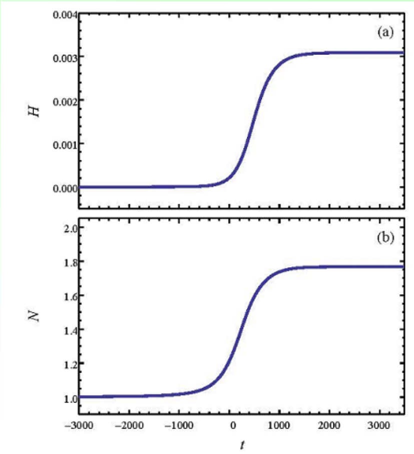


FIG. 2: The background evolution of the Hubble parameter H (a) and the lapse function N (b) around the genesis-de Sitter transition.

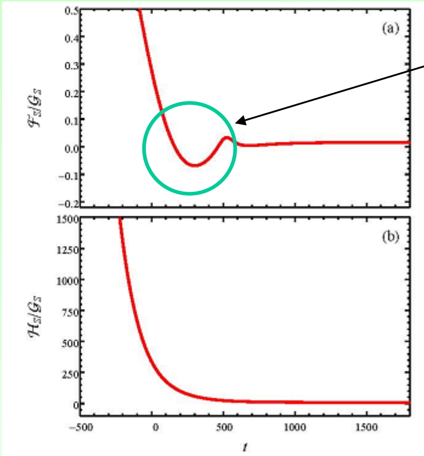


FIG. 3: The sound speed squared $\mathcal{F}_S/\mathcal{G}_S$ (a) and the coefficient of k^4 (divided by $\mathcal{G}_S$) (b) around the genesis-de Sitter transition.

During short period, Fs becomes negative. But, the perturbations for large k are stabilized thanks to the k^4 terms.

The perturbations for small k grow during short period, but growth is mild and finite.

The situation is similar to the transition from inflation to RD.

Conclusions and discussions

- We constructed a concrete example from Galilean Genesis to inflationary phase followed by graceful exit, based on the recent development beyond the Horndeski theory.
- The sound velocities squared (or Fs) during transitions from Genesis to inflation and from inflation to RD become negative for a short period.
- But thanks to a non-trivial dispersion relation coming from the fourth order derivative term in the quadratic action, modes with higher k are completely stable and the growth of perturbations with smaller k is finite and controllable.
- Our model can describe a Genesis scenario with graceful exit (even without inflationary phase), in which no (first order) primordial tensor perturbations are produced. The detection or non-detection of primordial tensor perturbations may discriminate Genesis scenarios with or without inflation.

Conclusions and discussions II

- Unfortunately, modes which exit horizon during genesis phase are still superhorizon if inflation lasts long enough.
- I hope young people to consider a theory beyond inflation (an epoch before inflation) and to invent a novel method to probe such currently superhorion perturbations.