JGRG24

The 24th Workshop on General Relativity and Gravitation in Japan

10 (Mon) — 14 (Fri) November 2014

KIPMU, University of Tokyo

Chiba, Japan

Oral presentations: Day 5

Contents

Programme: Day 5

Friday 14 November 2014

Morning 1 [Chair: Misao Sasaki]

10:00 Hitoshi Murayama (Berkeley/Kavli IPMU, SuMIRe) [Invited]
"soo-mee-ray SUMIRE Subaru Measurements of Images and Redshifts"
[JGRG24(2014)111401]

10:45 Takashi Hiramatsu (YITP, Kyoto)
"Progress of code development: 2nd-order Einstein-Boltzmann solver for CMB anisotropy" [JGRG24(2014)111402]

11:00 Tomo Takahashi (Saga)
"Studying the inflationary Universe with gravitational waves"
[JGRG24(2014)111403]

11:15 - 11:30 coffee break

Morning 2 [Chair: Hideo Kodama]

11:30 Akihiro Ishibashi (Kinki)
"Instability of extremal black holes in higher dimensions" [JGRG24(2014)111404]

11:45 Hajime Sotani (NAOJ)
"Stellar oscillations in Eddington-inspired Born-Infeld gravity"
[JGRG24(2014)111405]

12:00 Kazuharu Bamba (Ochanomizu)
"Inflationary cosmology in R^2 gravity with quantum corrections"
[JGRG24(2014)111406]

12:15 Shuntaro Mizuno (Waseda)
"Combined features in the primordial spectra induced by a sudden turn in two-field DBI inflation" [JGRG24(2014)111407]

12:30 Guillermo A. Mena Marugan (IEM, CSIC)
"Cosmological perturbations in Loop Quantum Cosmology: Mukhanov-Sasaki equations" [JGRG24(2014)111408]

12:45 Yuko Urakawa (Nagoya)
"Inflation from holography" [JGRG24(2014)111409]

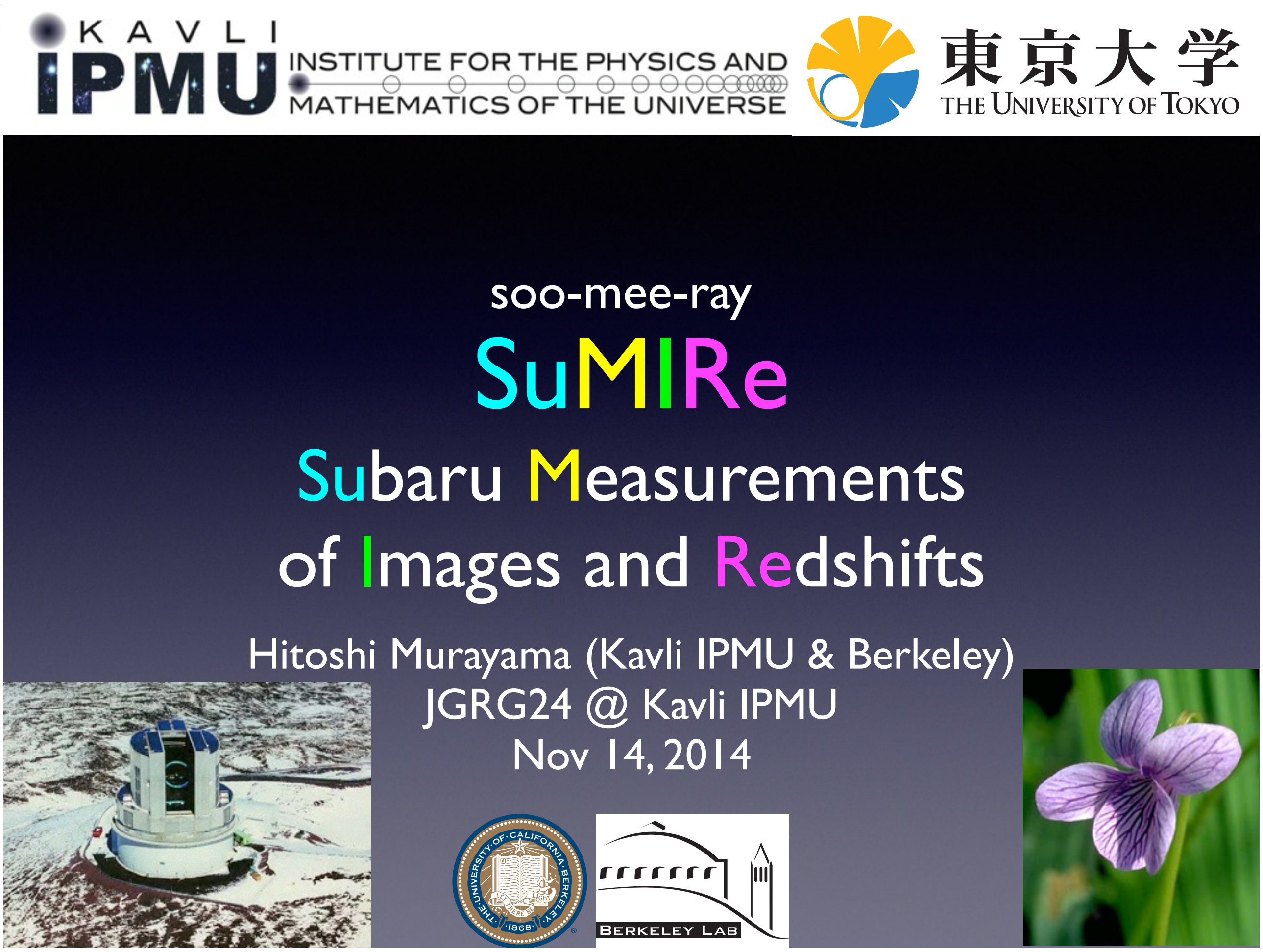
13:00 - 13:15 presentation awards

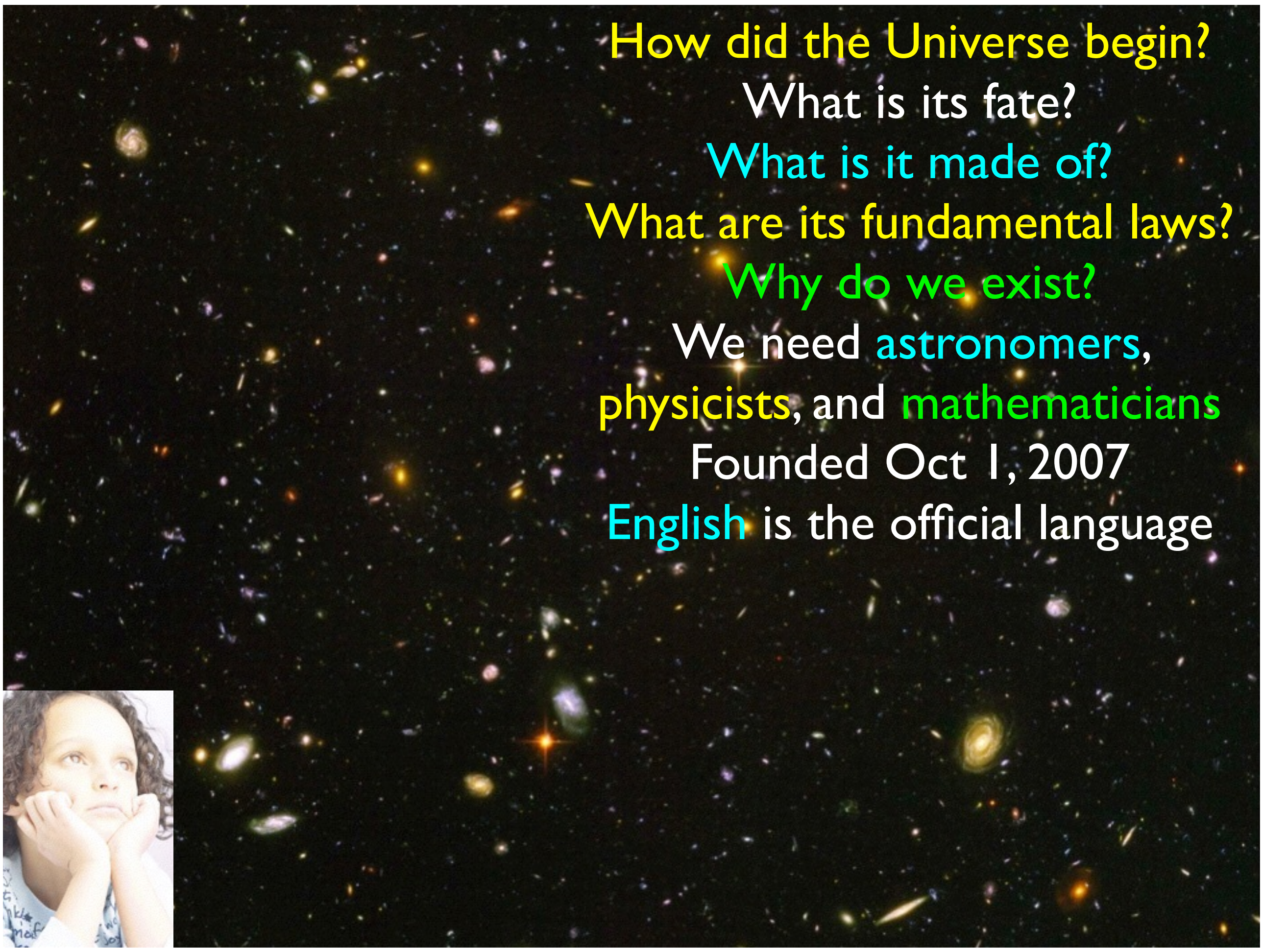
13:15 Kei-ichi Maeda (Waseda)
Closing [JGRG24(2014)111410]

"SuMIRe: Subaru Measurements of Images and Redshifts"

Hitoshi Murayama [Invited]

[JGRG24(2014)111401]

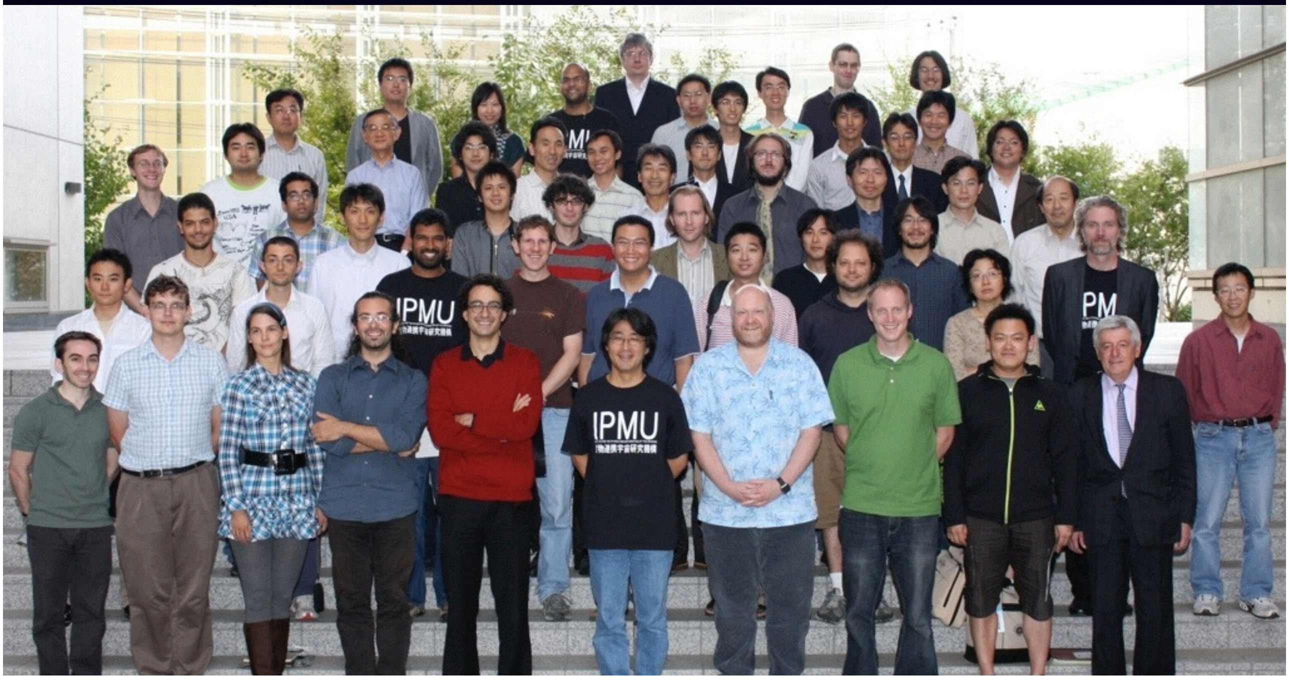

KAVLI IPMU
INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE
東京大学
THE UNIVERSITY OF TOKYO
soo-mee-ray
SuMIRe
Subaru Measurements
of Images and Redshifts
Hitoshi Murayama (Kavli IPMU & Berkeley)
JGRG24 @ Kavli IPMU
Nov 14, 2014
BERKELEY LAB

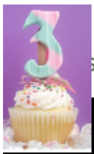

How did the Universe begin?
What is its fate?
What is it made of?
What are its fundamental laws?
Why do we exist?
We need astronomers,
physicists, and mathematicians
Founded Oct 1, 2007
English is the official language


wpi
KAVLI IPMU
Oct 2007

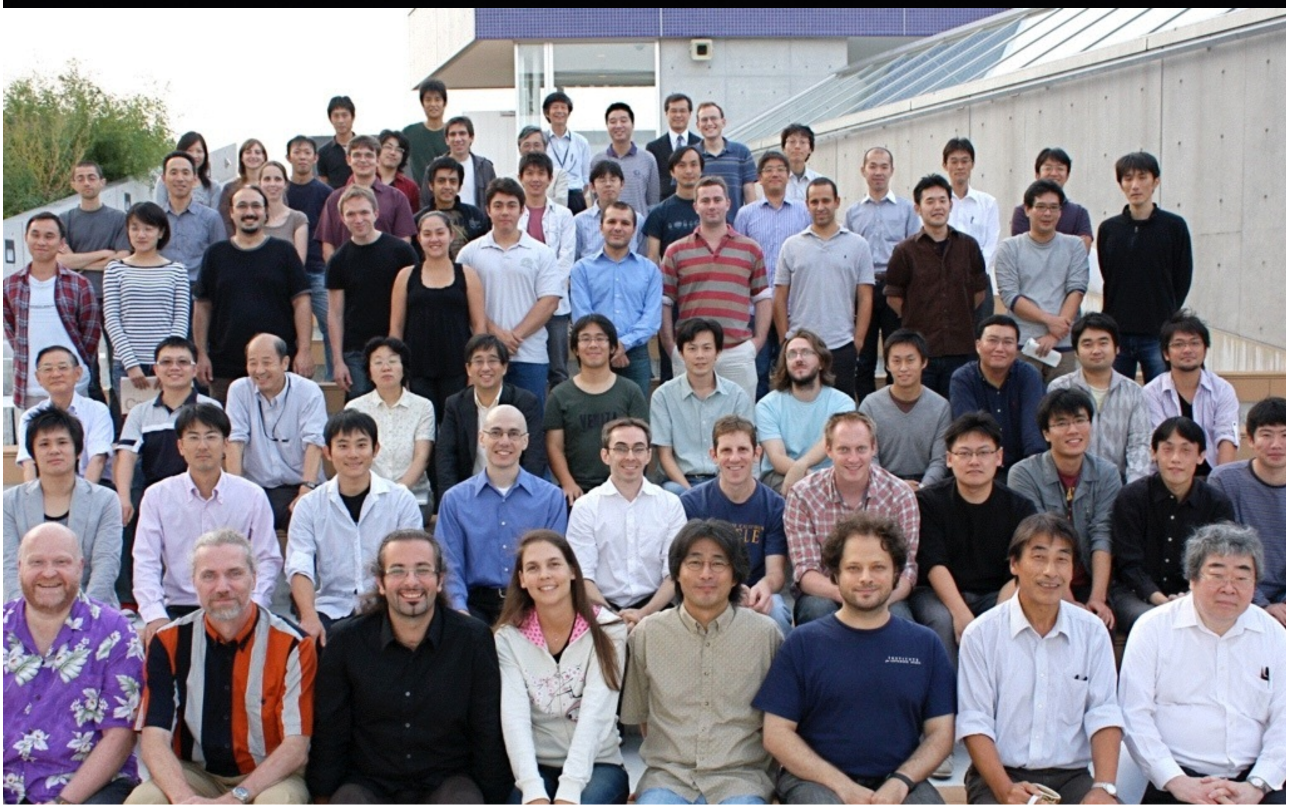

Oct 2008

Oct 2009





Oct 2010



Oct 2011

Oct 2012

Oct 2013

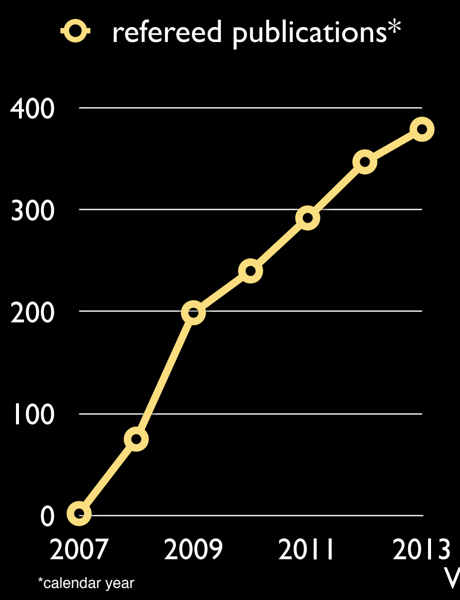
Oct 2014

international members 56%

former IPMU postdocs on faculty (44%!)

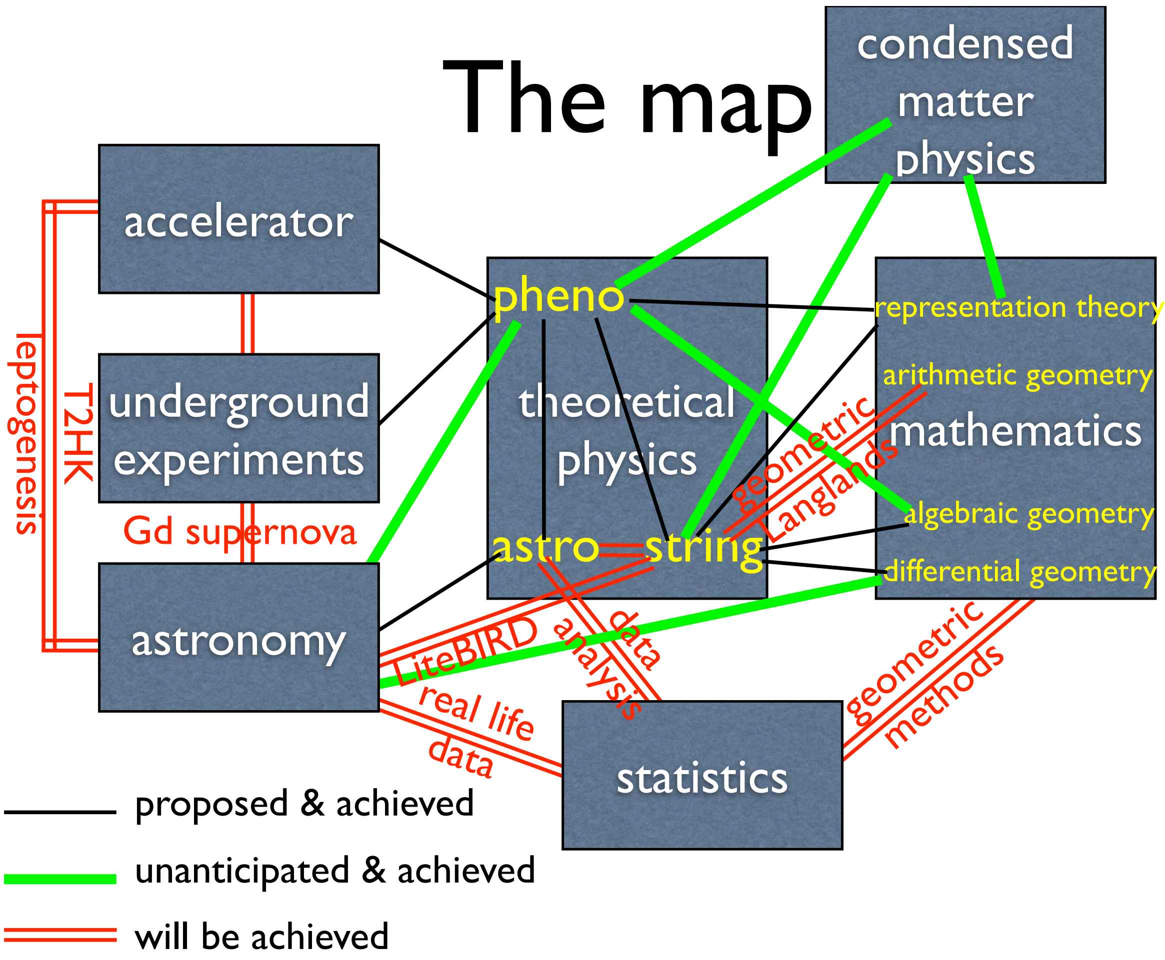
7/1/12	Alexandre Kozlov	Assistant Professor	Kavli IPMU (promoted from postdoc)
9/1/12	Christian Schnell	Assistant Professor	Stony Brook
9/1/12	Johanna Knapp	Assistant Professor	Vienna University of Technology
9/1/12	Minxin Huang	Assistant Professor	Univ of Science & Technology China (Heifei)
11/1/12	Scott Carnahan	Assistant Professor	University of Tsukuba
2/28/14	Surhud Shrikant More	Project Assistant Professor	Kavli IPMU (promoted)
8/21/14	Jing Liu	Assistant Professor	University of South Dakota
8/31/14	Robert Quimby	Associate Professor / Director of Mount Laguna Observatory	San Diego State University
9/30/14	Melina Bersten	Scientific Researcher	CONICET (National Scientific and Technical Research Council - Argentina

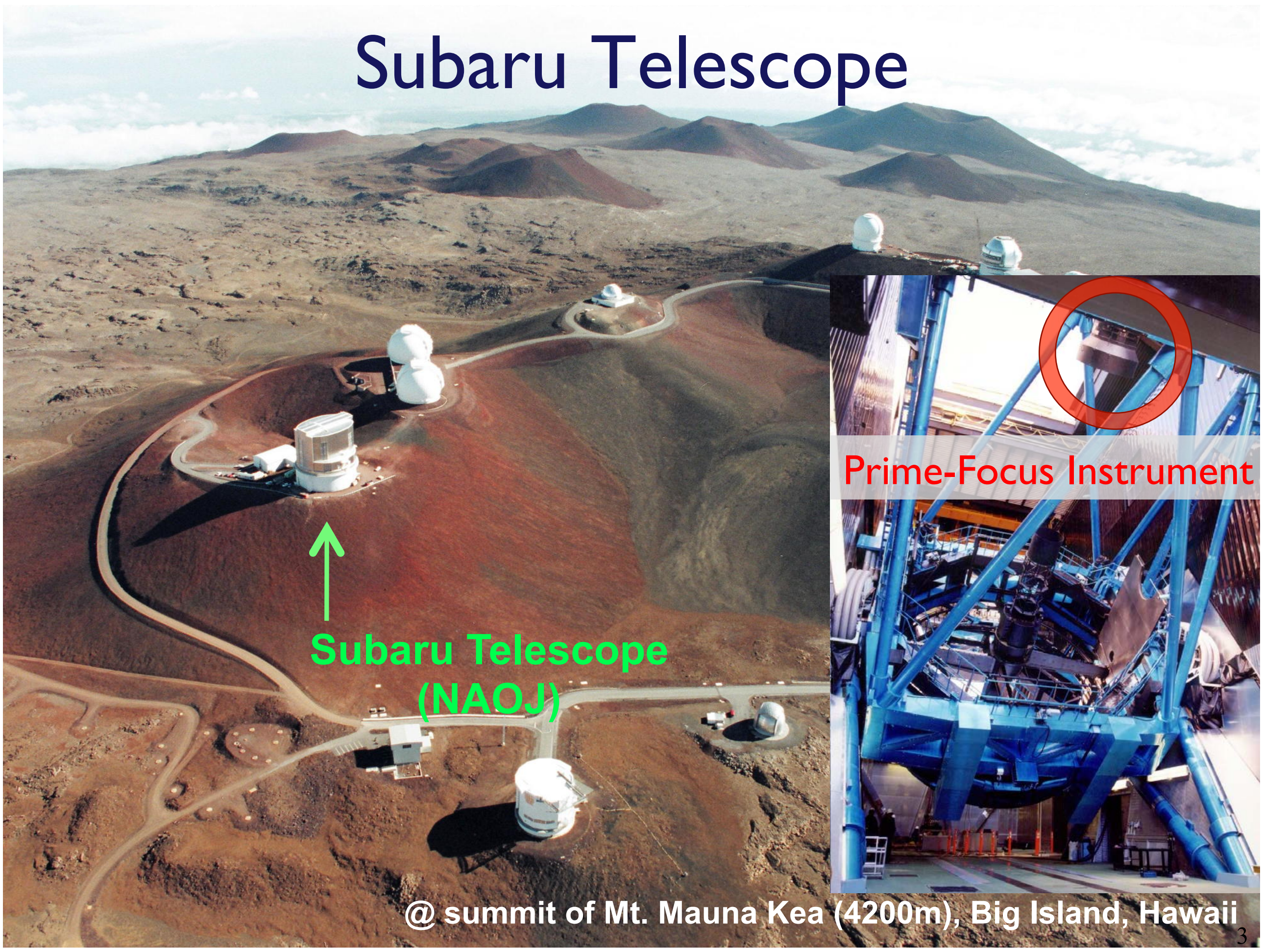
publications per year



institute	citation/ paper	#papers >50 citations
IPMU	16.0	114
IAS	19.7	137
KITP	19.6	49
YITP	11.1	46
Perimeter	13.9	83
ICTP	11.6	72

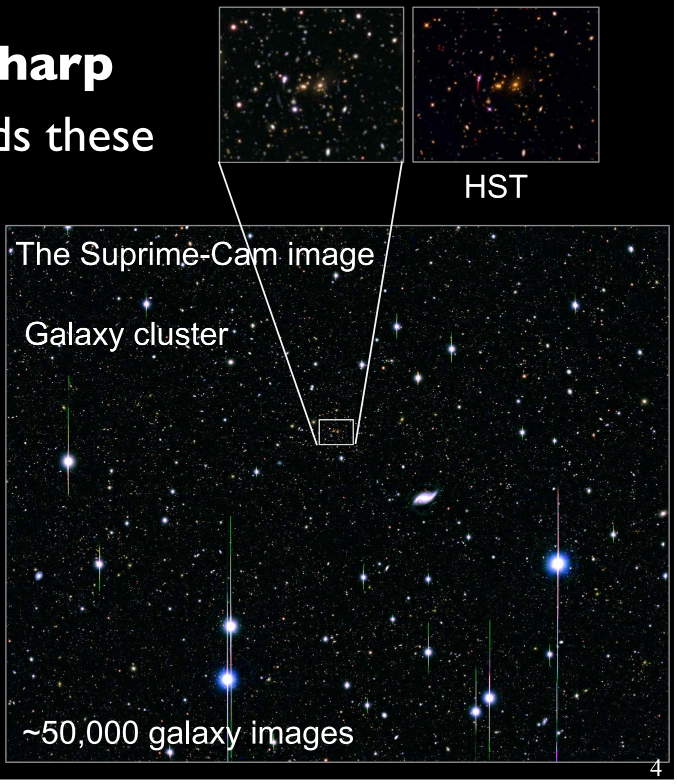
Jan 2008 - Jun. 2014
Web of Science (Thomson Reuters), excluding reviews
fields: astronomy, astrophysics, particle and fields, multidisciplinary physics, mathematics, applied mathematics


The map
condensed matter physics
accelerator
underground experiments
astronomy
pheno
theoretical physics
astro
string
representation theory
arithmetic geometry
mathematics
algebraic geometry
differential geometry
statistics
leptogenesis
T2HK
Gd supernova
geometric Langlands
LiteBIRD
data analysis
real life data
geometric methods
proposed & achieved
unanticipated & achieved
will be achieved


Subaru Telescope
Prime-Focus Instrument
Subaru Telescope (NAOJ)
@ summit of Mt. Mauna Kea (4200m), Big Island, Hawaii

Subaru Telescope:
wide FoV & excellent image quality

- **Fast, Wide, Deep & Sharp**
- a cosmological survey needs these



M. Takada

4

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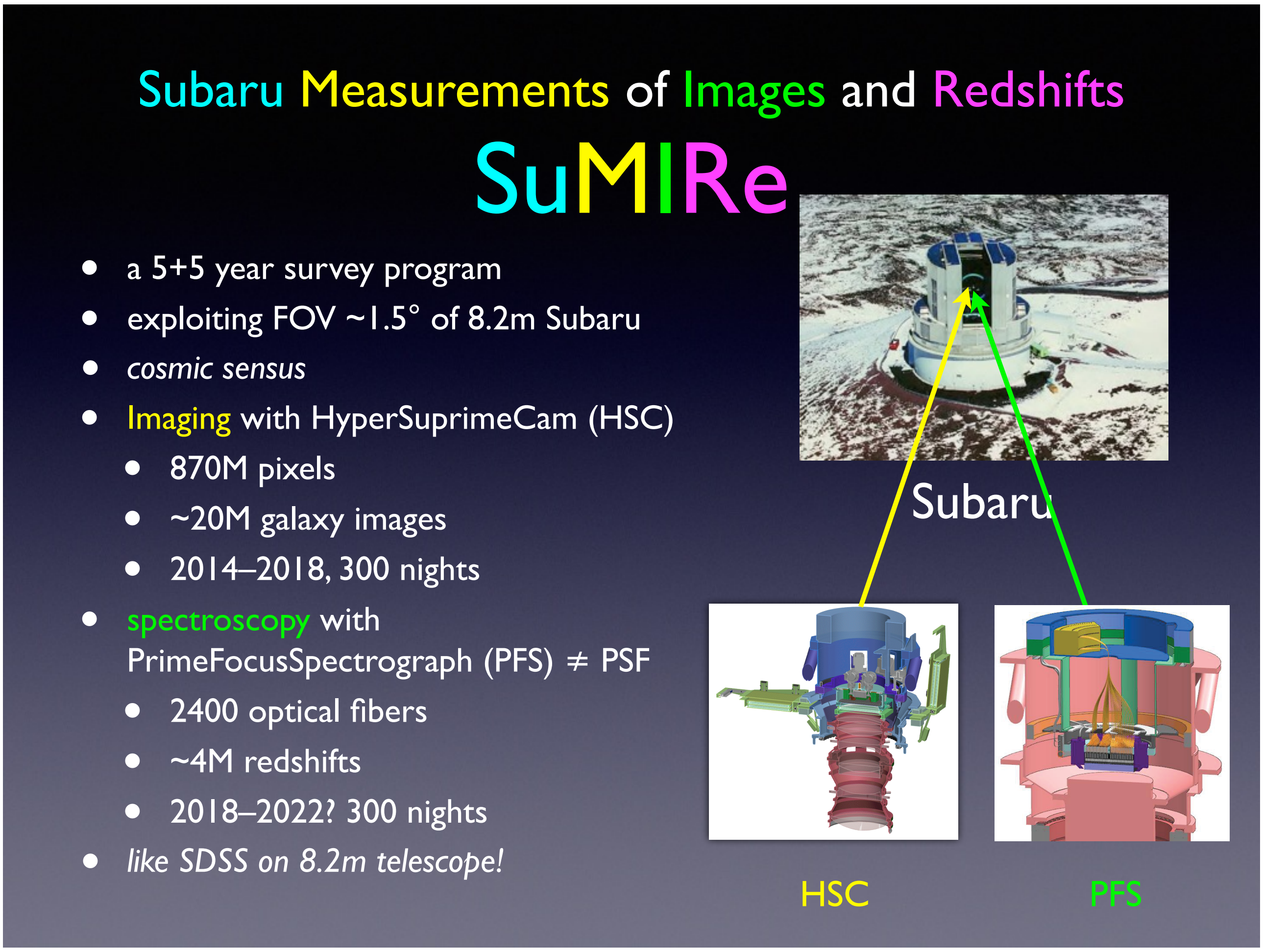
- **Fas**
- a cos

Hyper Suprime-Cam FoV

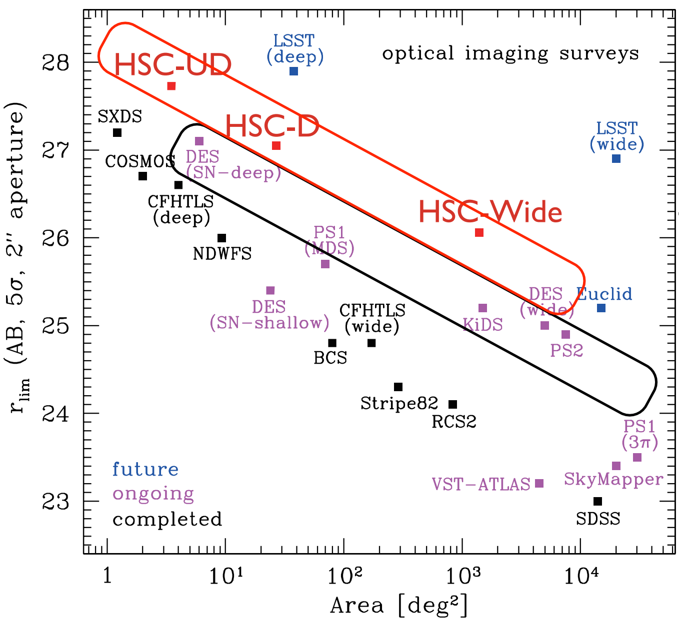
~50,00

M. Takada

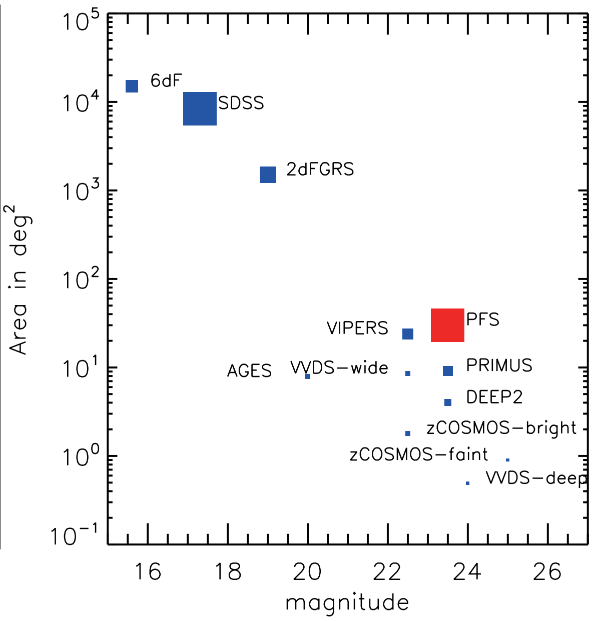
13



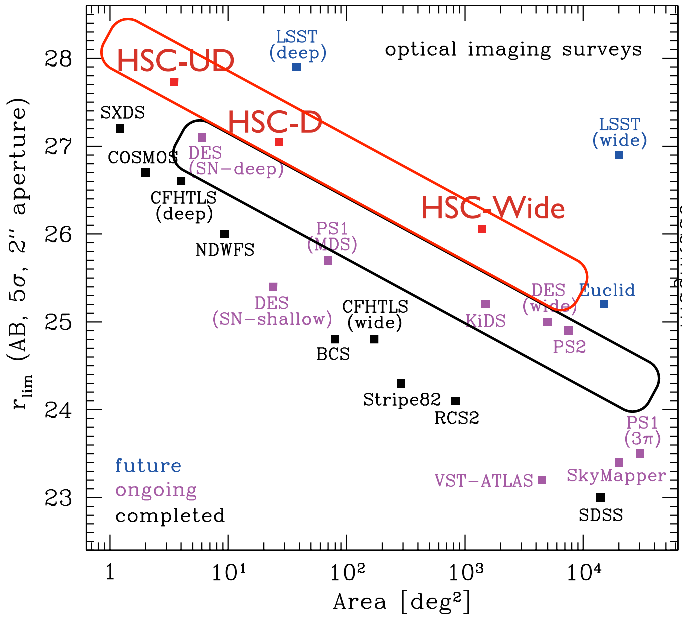
imaging



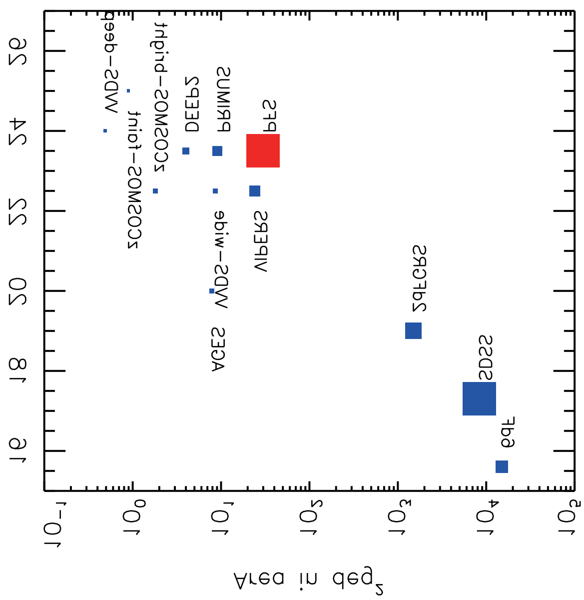
spectroscopy

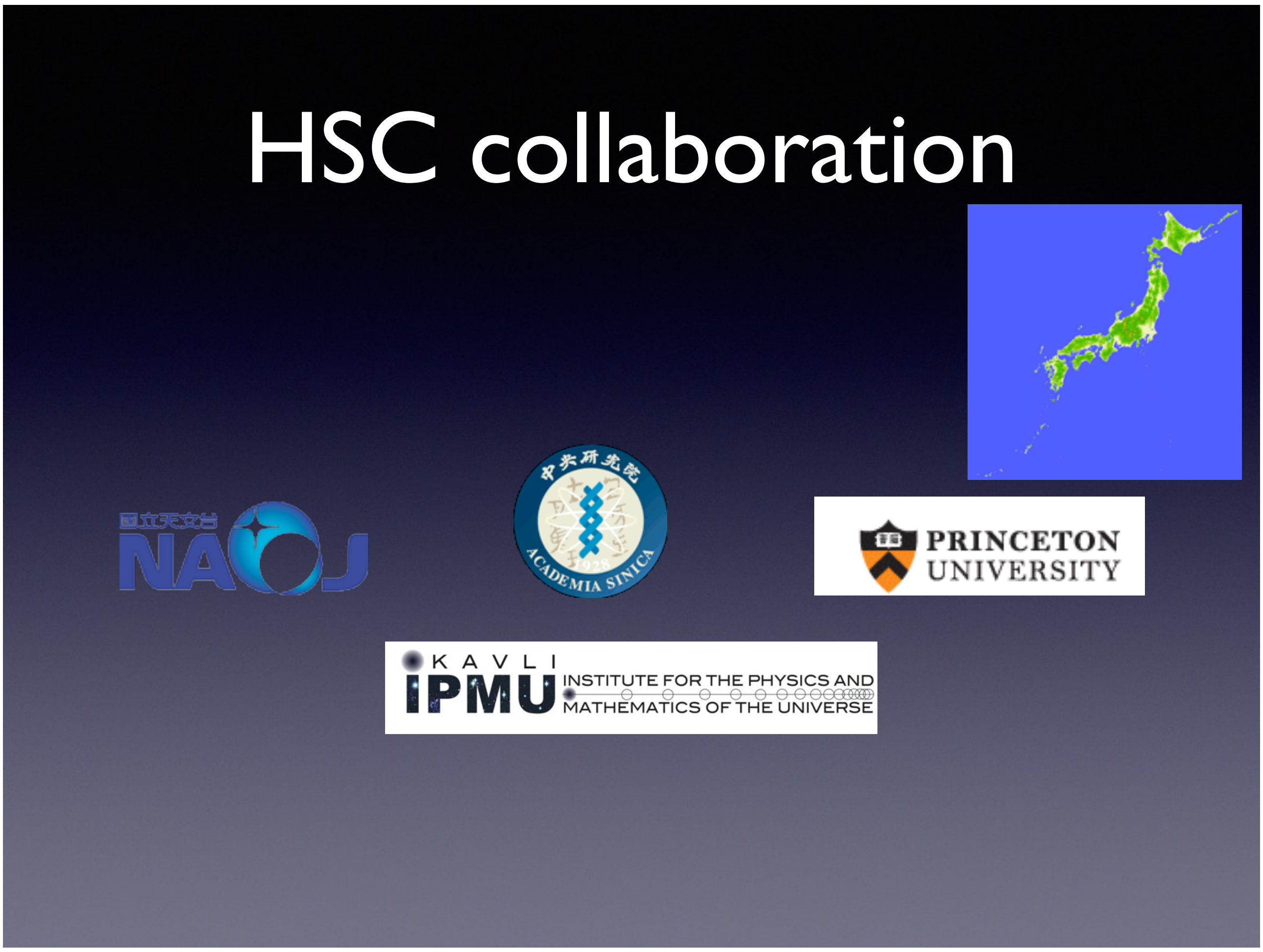


imaging



spectroscopy





PFS collaboration

Caltech

Jet Propulsion Laboratory
California Institute of Technology

NAOJ

Academia Sinica

PRINCETON UNIVERSITY

Max-Planck-Institut für Astrophysik

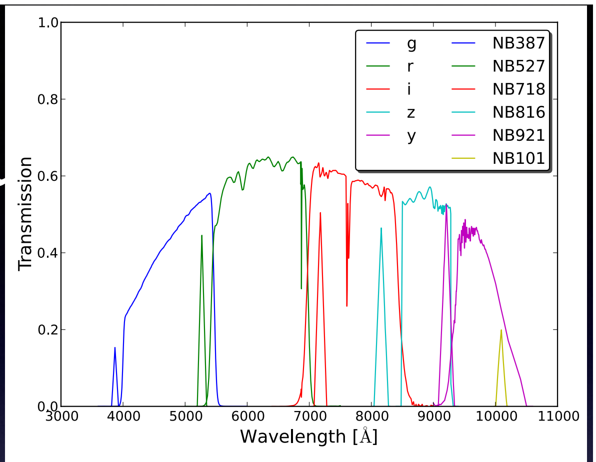
KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

LNA LABORATÓRIO NACIONAL DE ASTROFÍSICA

LAM LABORATOIRE D'ASTROPHYSIQUE DE MARSEILLE

JOHNS HOPKINS UNIVERSITY

HSC para



- FoV: 1.5°, 1.77 deg^2
- 15μm, 870M pixels
- *grizy* + 6 NBs
- three surveys, approved to start Feb 2014

	area (deg	pointings	h	science
Wide	1400	916	4.4 (z<1.5)	WL, galaxies z~1
Deep	28	15	0.5 (1<z<5)	galaxies z<2
Ultra-Deep	3.5	2	0.07 (2<z<7)	LAEs, LBGs, SNeIa

Table 1: **Hyper Suprime-Cam Characteristics**

Instrument weight	3.0 tons (estimate)
Field of View	1.5° diameter, 1.77 deg^2
Vignetting	0 at 0.15°; 26% at edge
Pixel scale	$15\mu m = 0.16''$
Delivered Image Quality	$D_{80} < 0.2''$ in all filters
CCDs	116 2K × 4K Hamamatsu Fully-Depleted
CCD QE	40% at 4000Å, 10,000Å,
CTE	0.999999
Readnoise	4.5 e^-
Data Rate	2.31 GBytes/exposure (16-bit A-to-D)
Focal ratio at Focal Plane	2.25
Overhead between Exposures	29 sec
Wide-Field Corrector	7 optical elements, ADC
Shutter	Roll-Type
Filters	*grizy* + 6 NB; Table 3
Filter Exchanger	6 filters installed at a time
Filter Exchange Time	10 minutes

Table 2: Summary of HSC-Wide, Deep and Ultradeep layers

Layer	Area [deg^2]	# of pointings	Filters & Depth	Volume [$h^{-3}Gpc^3$]	Key Science
Wide	1400	916	*grizy* ($i \simeq 26$)	$\sim 4.4(z < 1.5)$	WL Cosmology, $z \sim 1$ gals, Clusters
Deep	28	15	*grizy*+3NBs ($i \simeq 27$)	$\sim 0.5(1 < z < 5)$	$z \lesssim 2$ gals, SNeIa, WL calib.
Ultradeep	3.5	2	*grizy*+6NBs ($i \simeq 28$)	$\sim 0.07(2 < z < 7)$	high-z gals (LAEs, LBGs), SNeIa

Subaru Strategic Program

Wide-field imaging with Hyper Suprime-Cam: Cosmology and Galaxy Evolution

A Strategic Survey Proposal for the Subaru Telescope

~170 collaborators

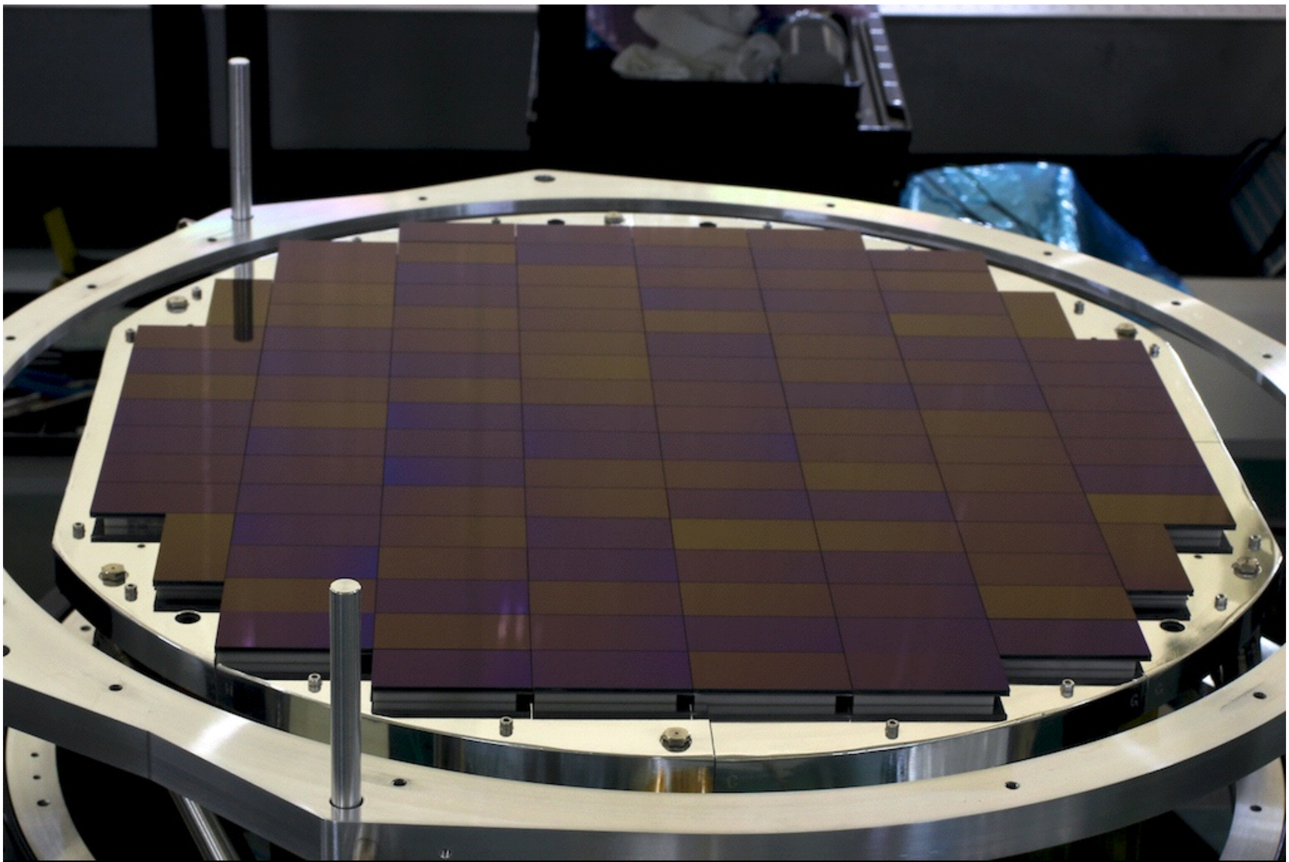
PI: Satoshi Miyazaki (NAOJ)

survey chair: Masahiro Takada

Co-PI: [illegible] Iwata (NAOJ)

unprecedented 300 nights approved

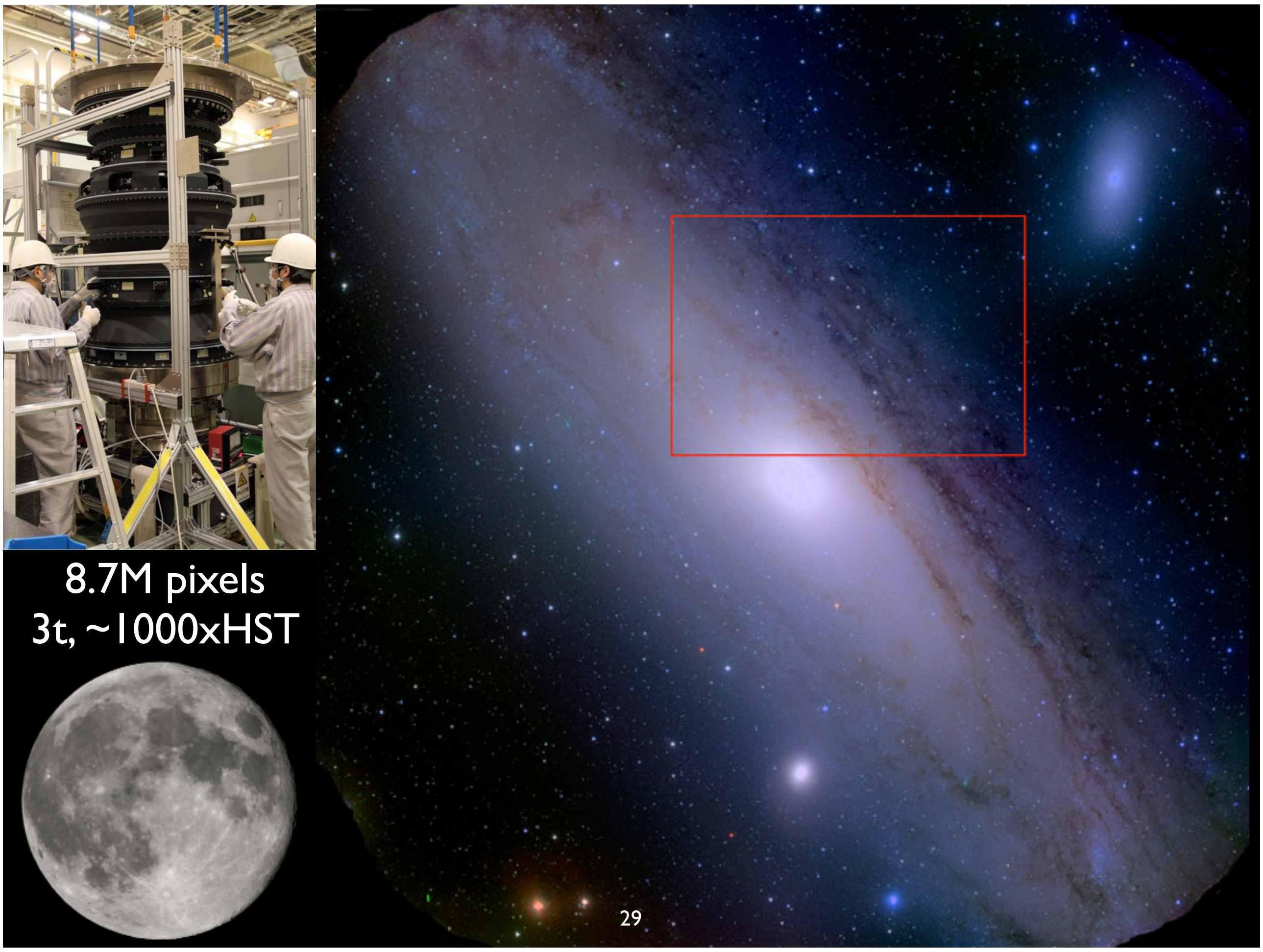
collaboration team[1]: S. Abe[1], H. Aihara*[2,3], M. Akiyama[4], K. Aoki[5], N. Arimoto*[5], N. A. Bahcall[6], [illegible]on[3], J. Bosch[6], K. Bundy†[3], C. W. Chen[7], M. Chiba†[4], T. Chiba[8], N. E. Chisari[6], J. Coupon[7], M. [illegible]noki[9] S. Foucaud[10], M. Fukugita[3], H. Furusawa†[5], T. Futamase[4], [illegible], T. Goto[11], J. E. Greene[6], [illegible][6], T. Hamana†[5], T. Hashimoto[2], M. Hayashi[5], Y. Higuchi[2,5], [illegible], J. C. Hill[6], P. T. P. Ho*[7], [illegible][7], K. Y. Huang†[7], H. Ikeda[13], M. Imanishi[5], N. Inada[14], [illegible]noue[15], W.-H. Ip[1], T. Ito[5], K. [illegible] M. Iye[5], H. Y. Jian[17], Y. Kakazu[18], H. Karoji[3], [illegible], [illegible] Katayama[3], T. Kawaguchi[19], S. [illegible][5], I. Kayo[20], T. Kitayama[20], G. R. Knapp[6], [illegible], K. Kohno[2], M. Koike[5], E. Kokubo[5], M. [illegible]Y. Komiyama[5], A. Konno[2], Y. Koyama[5], C. N. [illegible], D. Lang[6], A. Leauthaud†[3], M. J. Lehner[7], K.-[illegible] Lin[7], Y.-T. Lin†[7], C. P. Loomis[6], R. H. [illegible], P. S. Lykawka[21], K. Maeda[3], R. Mandelbaum†[22], Y. [illegible] K. Matsuoka[13,23], Y. Matsuoka[12], [illegible][2], T. Minezaki[2], H. Miyatake[6], R. Momose[2], A. More[3], S. [illegible]J. Moriya[3], T. Morokuma†[2], [illegible]rayama*[3], K. Nag[illegible]ine[24], T. Nagao†[23], S. Nagataki[23], Y. Naito[2], K. [illegible] F. Nakata[5], H. Nakaya[illegible], [illegible]mikawa[2], C.-C. Ngeow[1], T. Nishimichi[3], H. Nishioka[7], A. J. Nishizawa†[3], [illegible], M. Oguri†[3], [illegible], N. Okabe[7], S. [illegible], S. Okamura[26], J. Okumura[23], S. Okumura[27], Y. Okura[5], Y. Ono[2] M. Onodera[28], K. Ota[23], M. [illegible], S. Oyabu[12], P. A. Price[6], R. Quimby[3], C. E. Rusu[2,5], S. Saito[29], T. Saito[3], [illegible][30], M. Sato[12], [illegible]buya[5], K. Shimasaku†[2], A. Shimono[3], S. Shinogi[2], M. Shirasaki[2], J. D. Silverman[illegible], [illegible]N. Spergel*[6,3], M. [illegible]auss†[6], H. Sugai[3], N. Sugiyama[12,3], D. Suto[2], Y. Suto*[2], K. Tadaki[2], M. Takada†[illegible], [illegible]Takahashi[31], S. Takah[illegible], T. Takata[5], T. T. Takeuchi[12], N. Tamura[3], M. Tanaka[5], M. Tanaka†[3], M. Tana[illegible], Y. Taniguchi[13], A. Taruya[2], T. Terai[5], Y. Terashima[13], N. Tominaga[32], J. Toshikawa[30], T. Totani[23], M. Tsai[1], E. L. Turner*[6], Y. Ueda[23], K. Umetsu[7], Y. Urata†[1], Y. Utsumi[5], B. Vulcani[3], K. Wada[33], S.-Y. Wang[7], W.-H. Wang[7], T. Yamada[4], Y. Yamada[5], K. Yamamoto[34], H. Yamanoi[5], C.-H. Yan[7], N. Yasuda†[3], A. Yonehara[35], F. Yoshida†[5], N. Yoshida[2], M. Yoshikawa[36], S. Yuma[2] (1) NCU, Taiwan (2) Tokyo (3) Kavli IPMU (4) Tohoku (5) NAOJ (6) Princeton (7) ASIAA (8) Nihon (9) Tokyo Keizai (10) NTNU, Taiwan (11) DARK, Copenhagen (12) Nagoya (13) Ehime (14) NNCT (15) Osaka Sangyo (16) Barcelona (17) NTU, Taiwan (18) Chicago (19) Tsukuba (20) Toho (21) Kinki (22) CMU (23) Kyoto (24) Las Vegas (25) KIAA, China (26) Hosei (27) JSGA (28) ETH (29) Berkeley (30) GUAS (31) Hirosaki (32) Konan (33) Kagoshima (34) Hiroshima (35) Kyoto Sangyo (36) JAXA

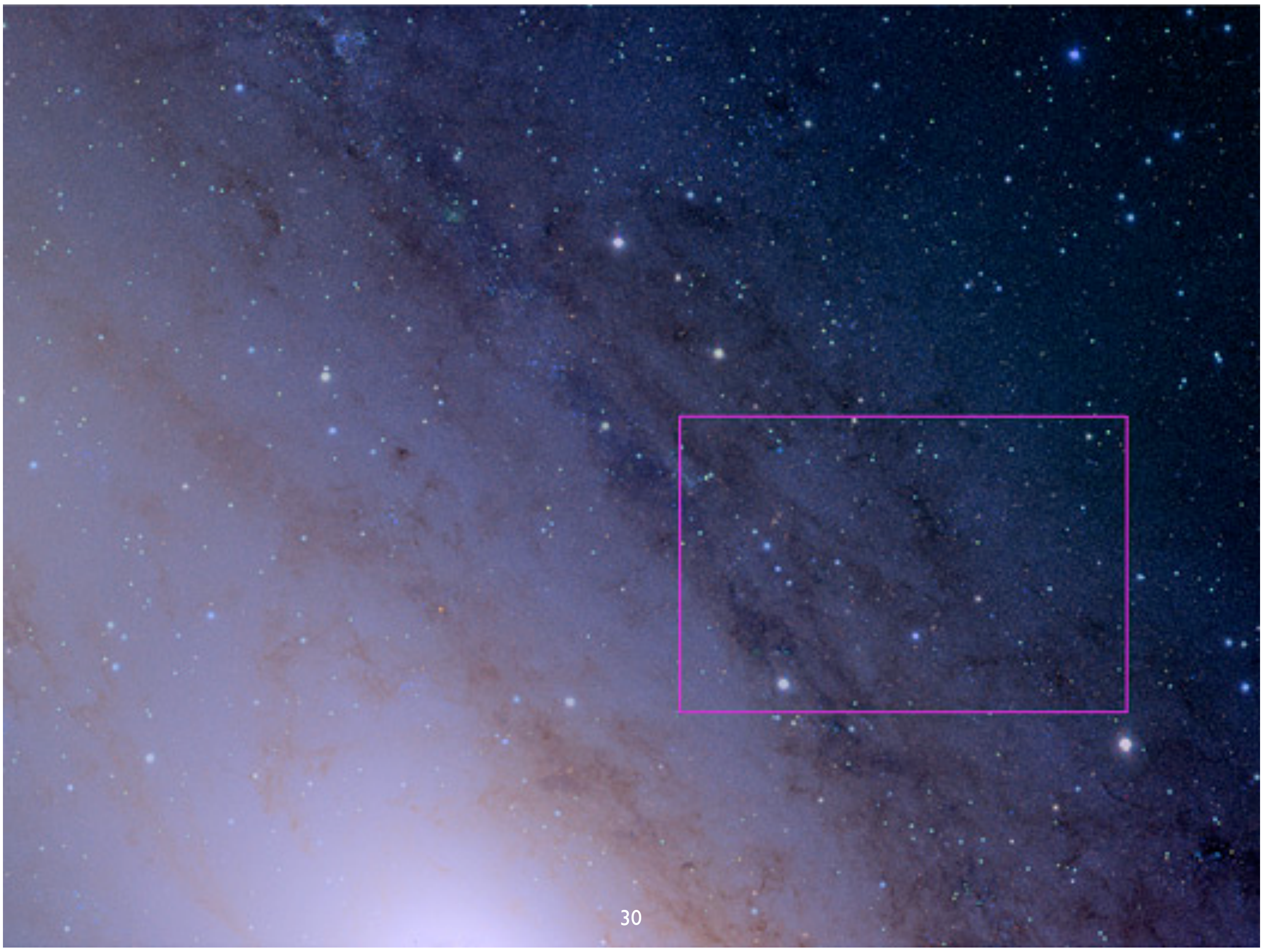


すばる望遠鏡に搭載された Hyper Suprime-Cam

2012年8月16日撮影 (180倍速)

Installing Hyper Suprime-Cam on the Subaru Telescope


8.7M pixels
3t, ~1000xHST
29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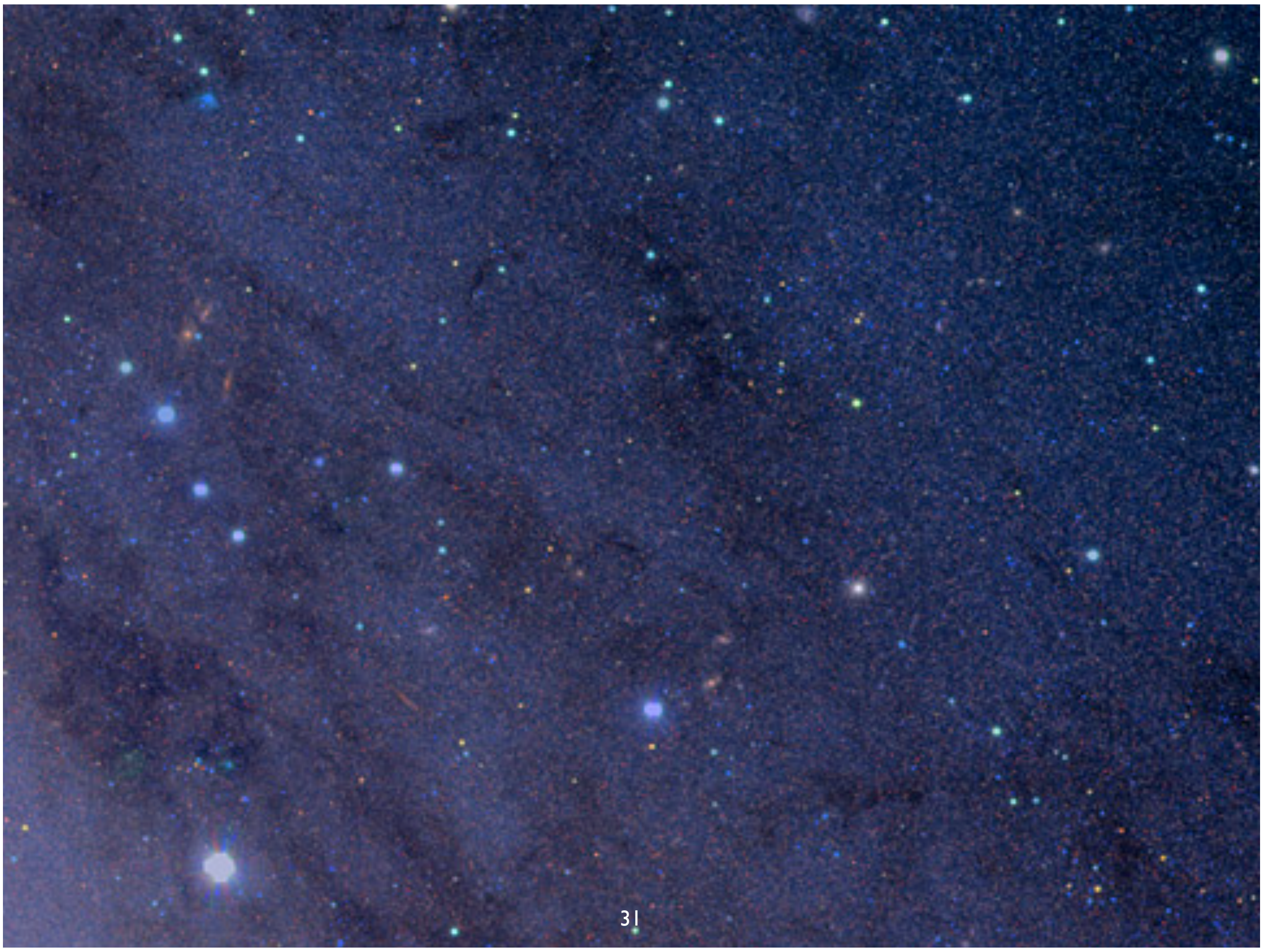

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すばる望遠鏡に「世界最強」カメラ

暗黒宇宙 謎解明挑む

アンドロメダ くっきり

ハワイ・すばる望遠鏡 巨大カメラ交換

視野 世界最大

アンドロメダ銀河「一望」

すばる望遠鏡の新カメラで撮影したアンドロメダ銀河。一度にほぼ全体を撮影でき、一つ一つの星を判別できる ＝国立天文台提供

世界最高級の性能を誇る米ハワイ島のすばる望遠鏡で、目に相当する巨大なカメラを交換し、大幅な性能アップを達成したと国立天文台が31日、発表した。すばる望遠鏡の特長だった広い視野をさらに7倍近くに広げ、一度に観測できる範囲はこのクラスの望遠鏡で世界最大となった。

望遠鏡の中心的装置のカメラ交換は、1999（平成11）年の観測開始から初めて。新カメラを使って過去に例のない規模の宇宙の探査に乗り出し、宇宙最大の謎である暗黒物質や暗黒エネルギーの解明に挑戦する。

天文台は新カメラの初画像として、アンドロメダ銀河を公開。これまで部分的に撮影して合成していたが、ほぼ全体を一度に捉えた。

新たに搭載したのは全長3㍍、重さ3㌧で8億7千万画素の巨大デジタルカメラ。従来の8千万画素のカメラでは満月よりやや大きい範囲が見えていたが新カメラは満月9個分の広さを撮影できる。

天文台は昨年8月、天体の光を集める主鏡から約15㍍上に新カメラを設置。

宇宙「暗黒の謎」に挑む

すばる望遠鏡、新カメラで威力

アンドロメダ銀河 鮮明

世界最高級の性能を誇る米ハワイ島のすばる望遠鏡で、巨大なカメラを交換し、大幅な性能アップを達成したと国立天文台が31日、発表した。すばる望遠鏡の特長だった広い視野をさらに7倍近く広げ、一度に観測できる範囲はこのクラスの望遠鏡で世界最大。望遠鏡の中心的な装置となるカメラの交換は、1999年の観測開始から初めてとなる。

世界最大の視野

ほぼ全体捉える

天文台は新カメラの初画像として、ハワイから最も大きく見える銀河のアンドロメダ銀河の姿を公開。これまで部分的に撮影して合成していたが、ほぼ全体を一度に捉えた。極めて鮮明な画像で、一つ一つの星を判別できるという。

新カメラを使って過去に例のない規模の宇宙の探査に乗り出し、宇宙最大の謎である暗黒物質や暗黒エネルギーの解明に挑戦する。

すばる望遠鏡の構造

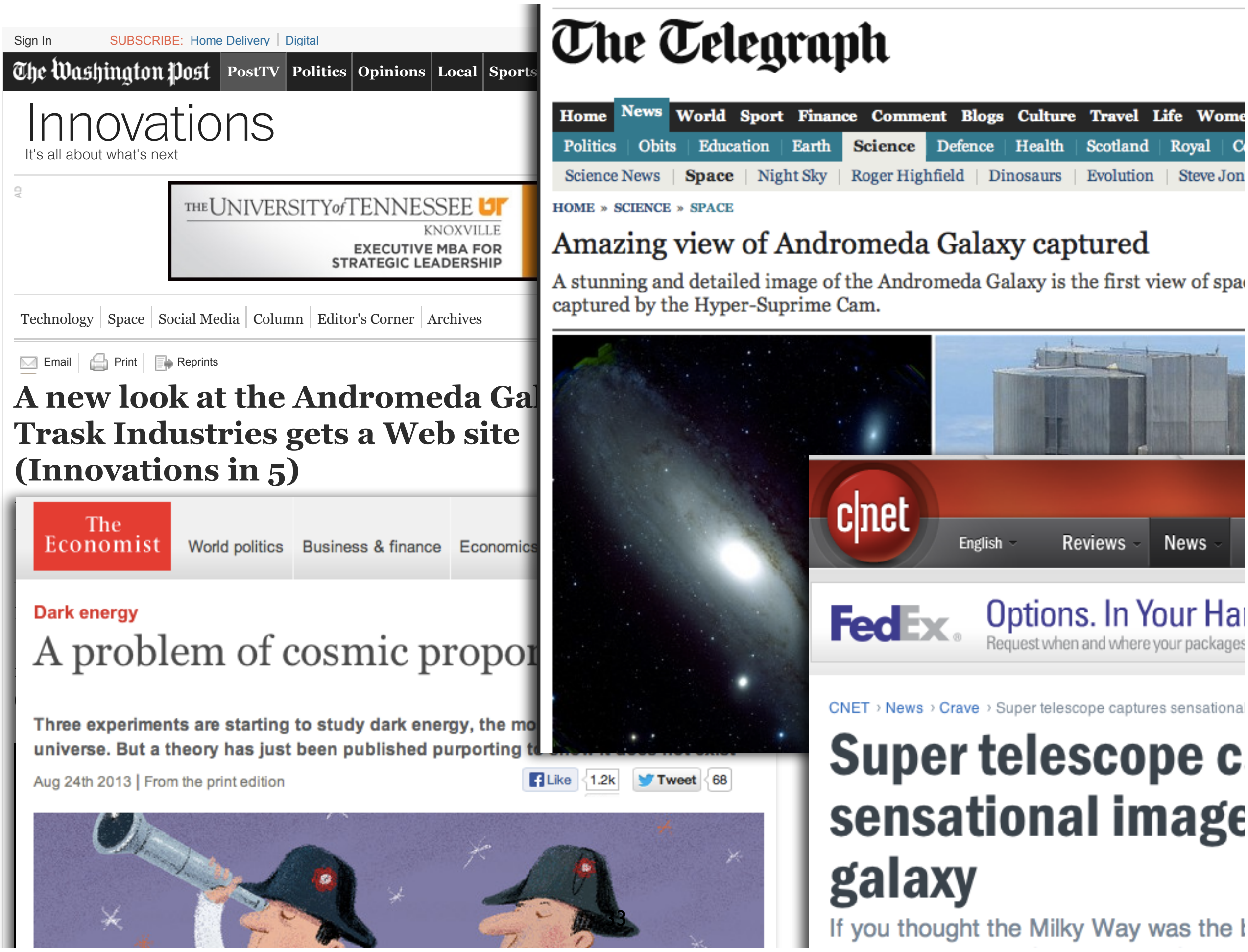
新カメラ（全長約3m、重さ約3トン）

▼すばる望遠鏡 国立天文台 8・2㍍で世界最大級だ。

「宇宙国勢調査」多くの銀河観測

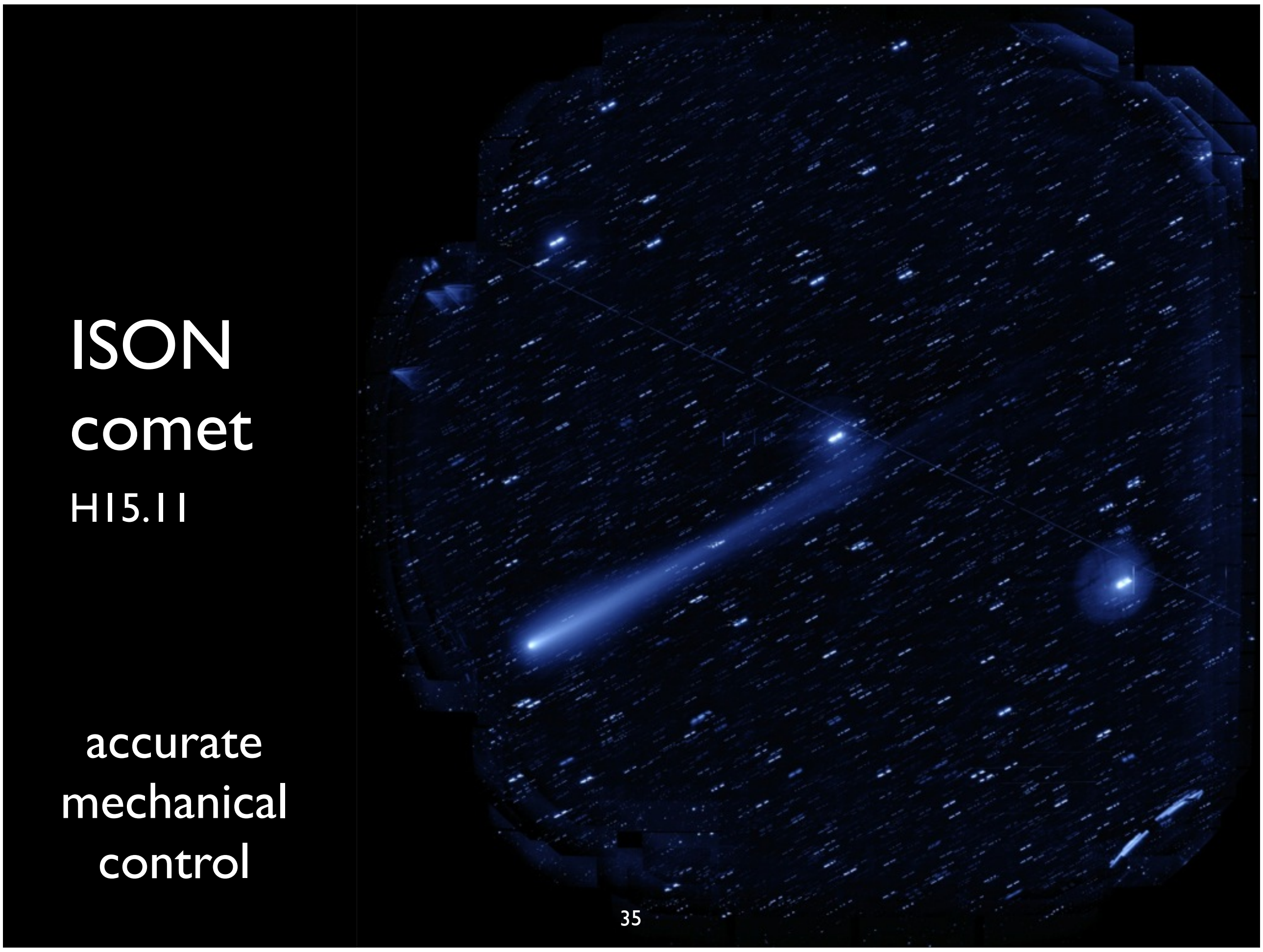
性能カメラ撮影

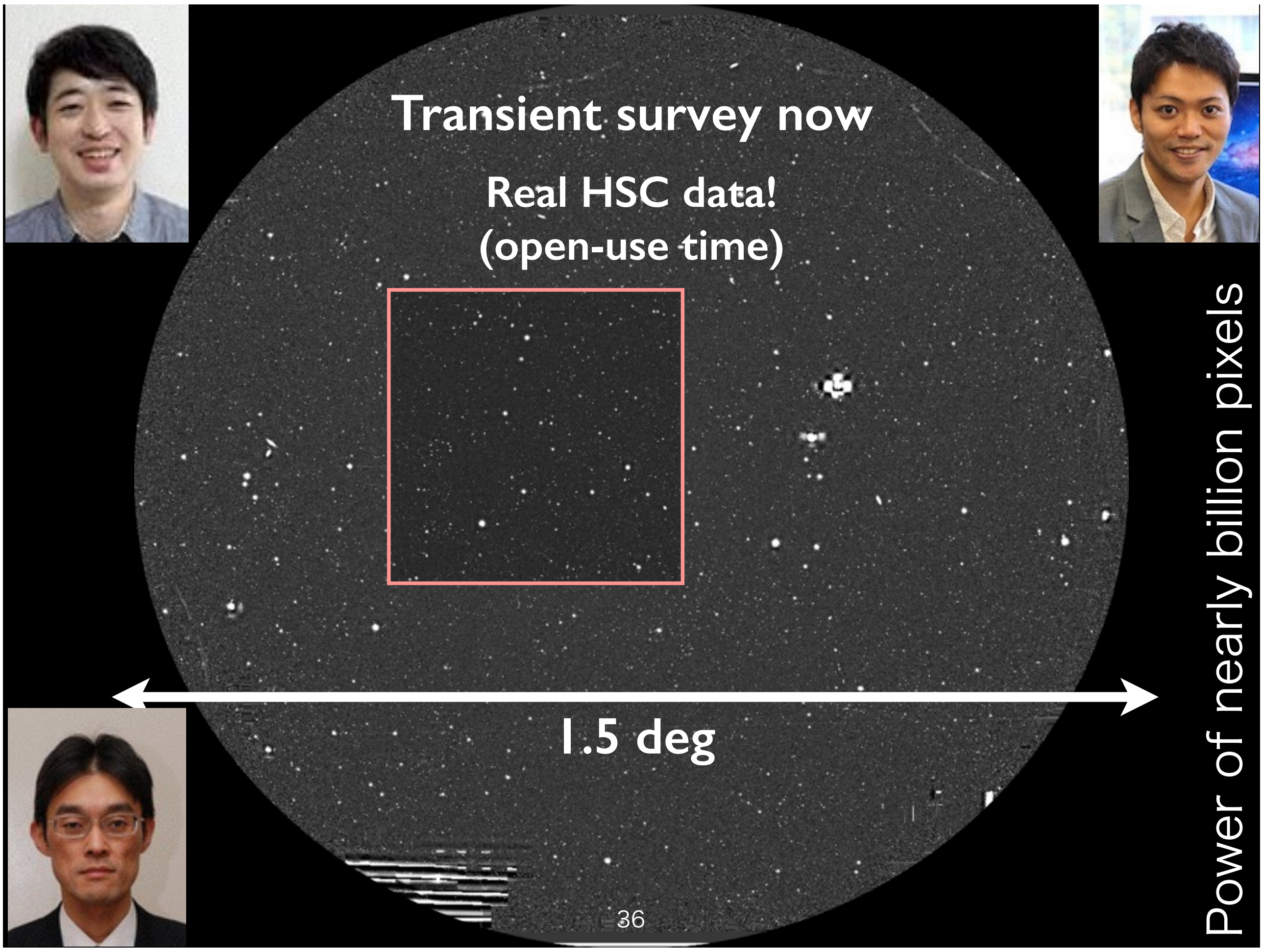
32

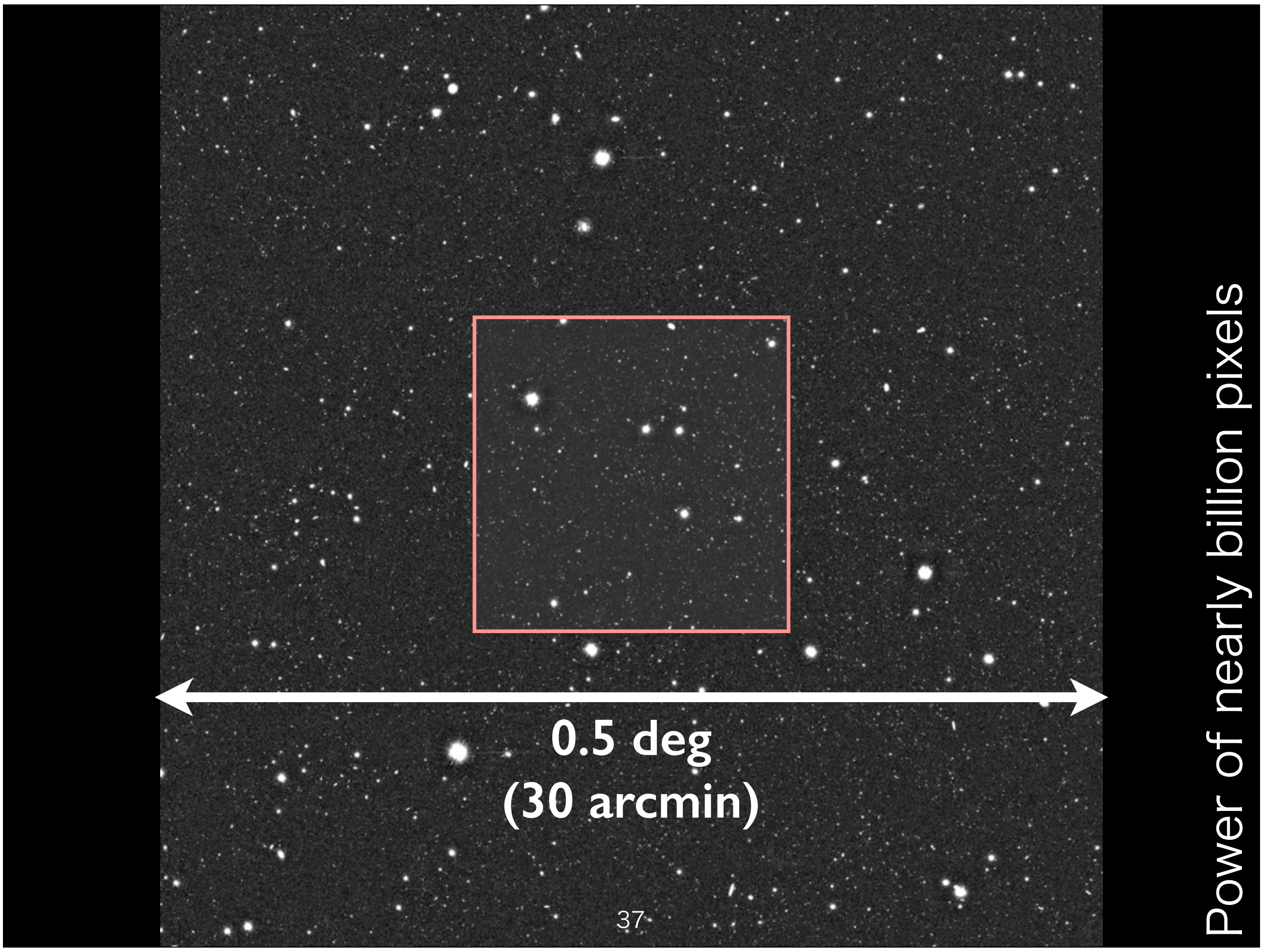


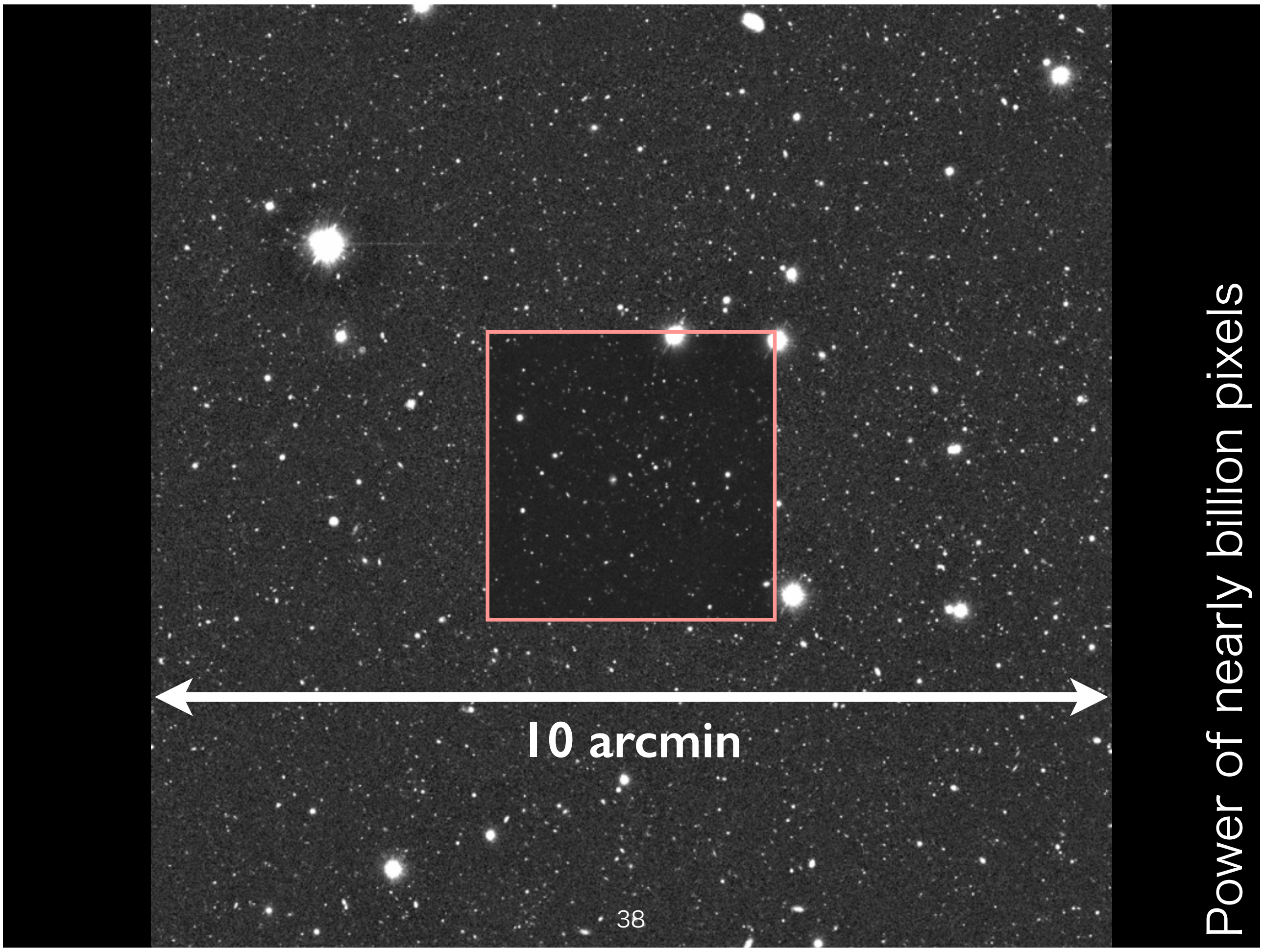
superb performance

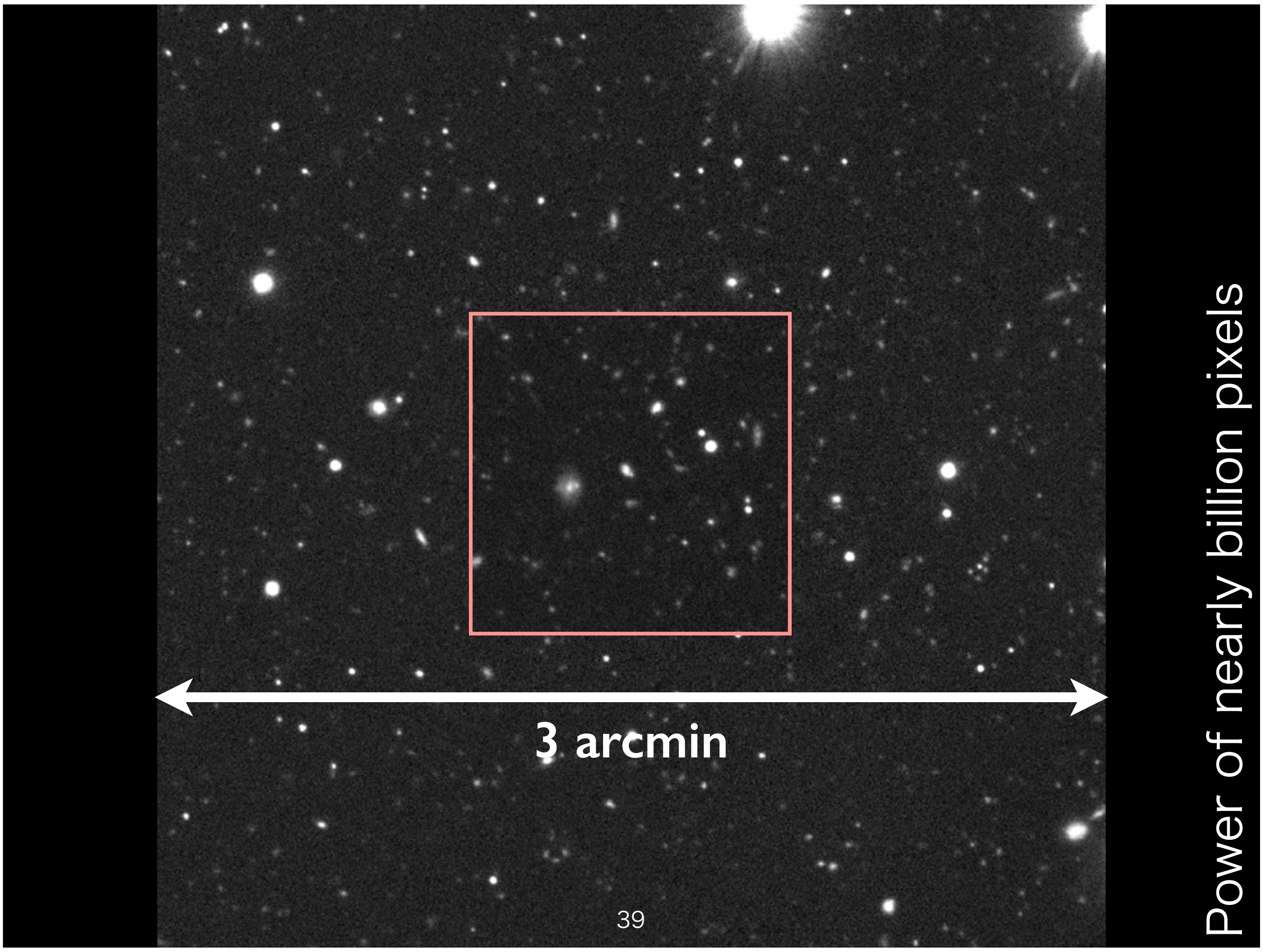
HSC: 3 colors in 2.5hours HST: 1 color in 500 hours

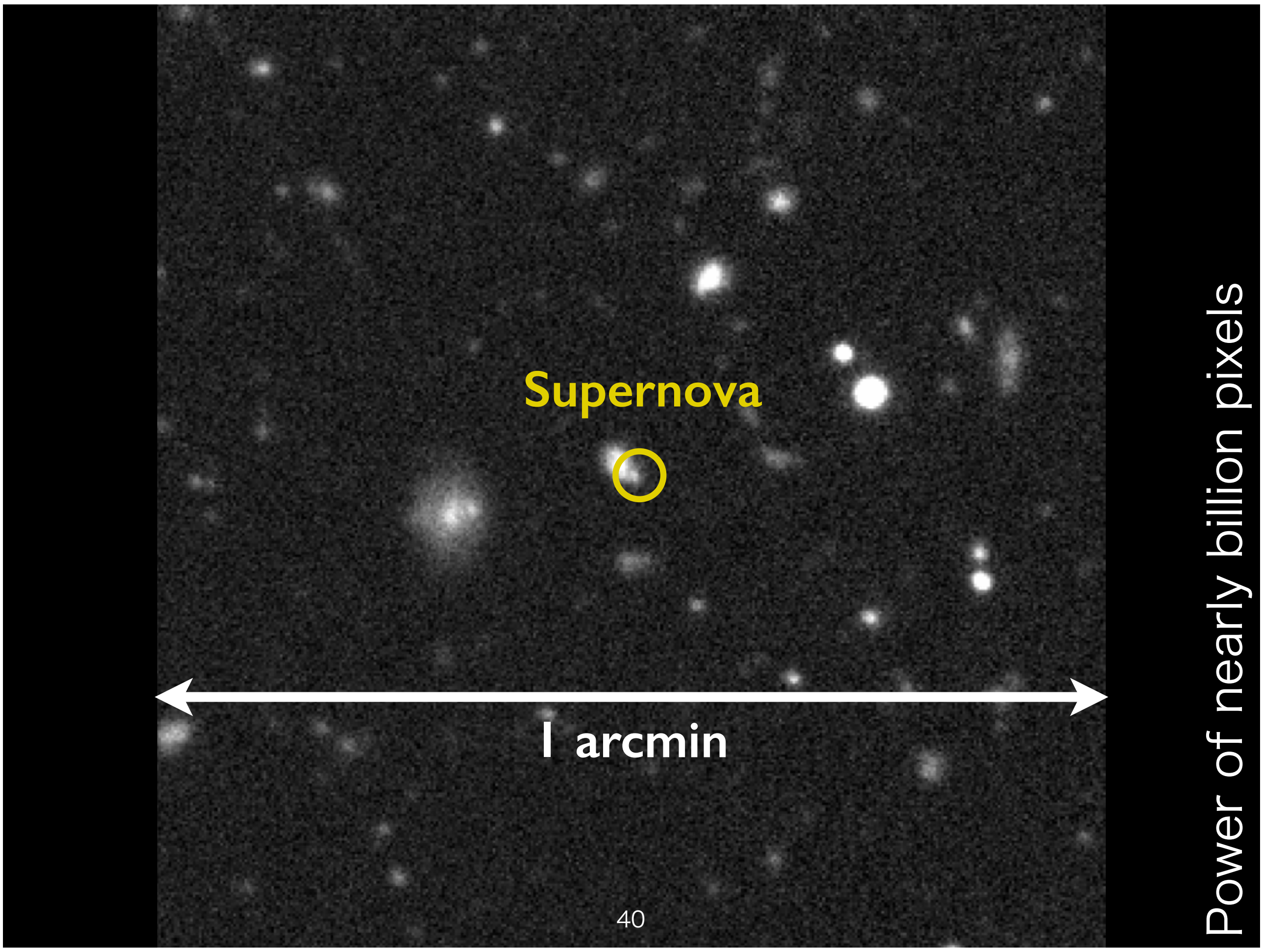

ISON
comet
H15.11
accurate
mechanical
control
35

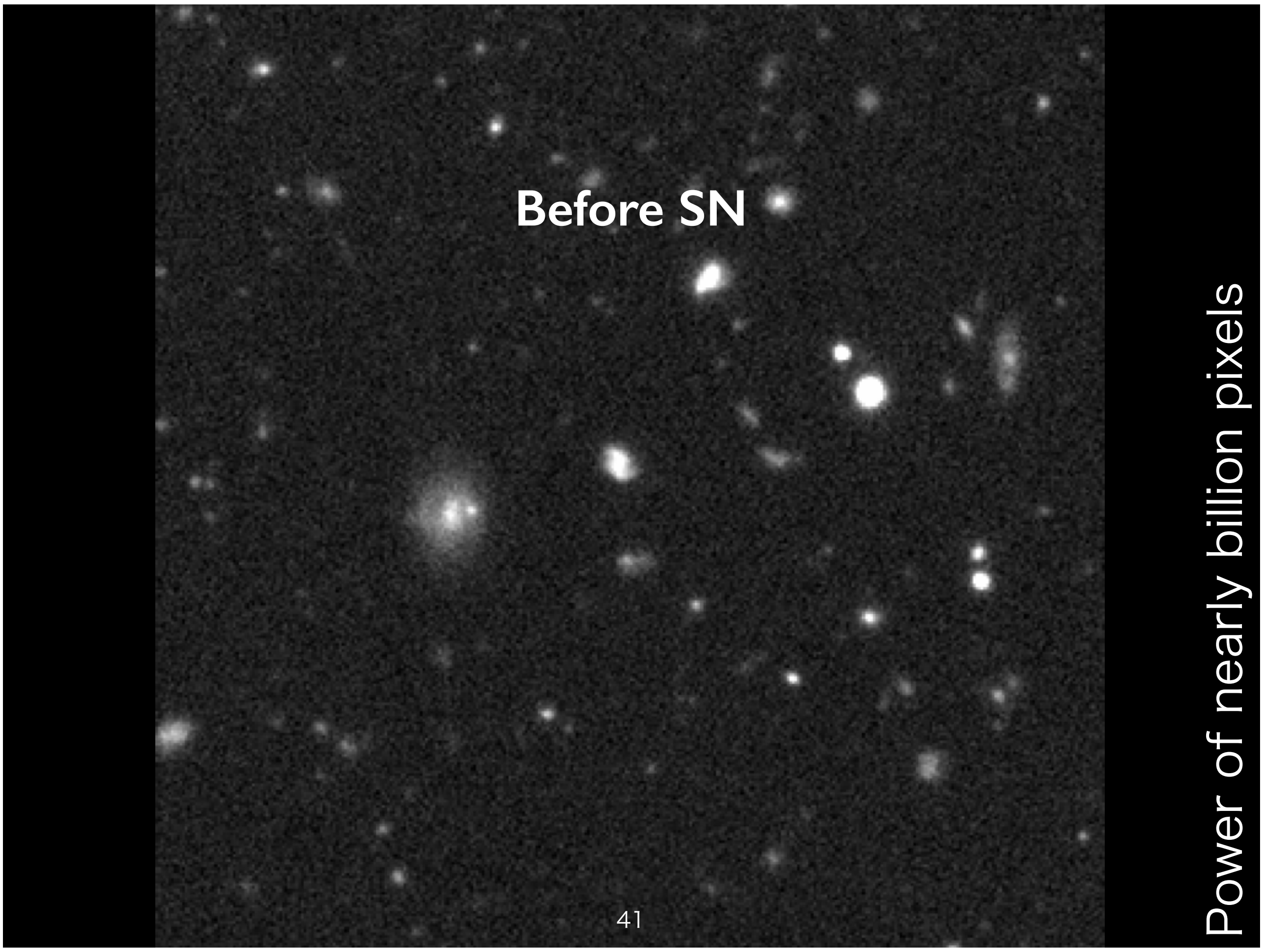

Transient survey now
Real HSC data!
(open-use time)
1.5 deg
Power of nearly billion pixels
36

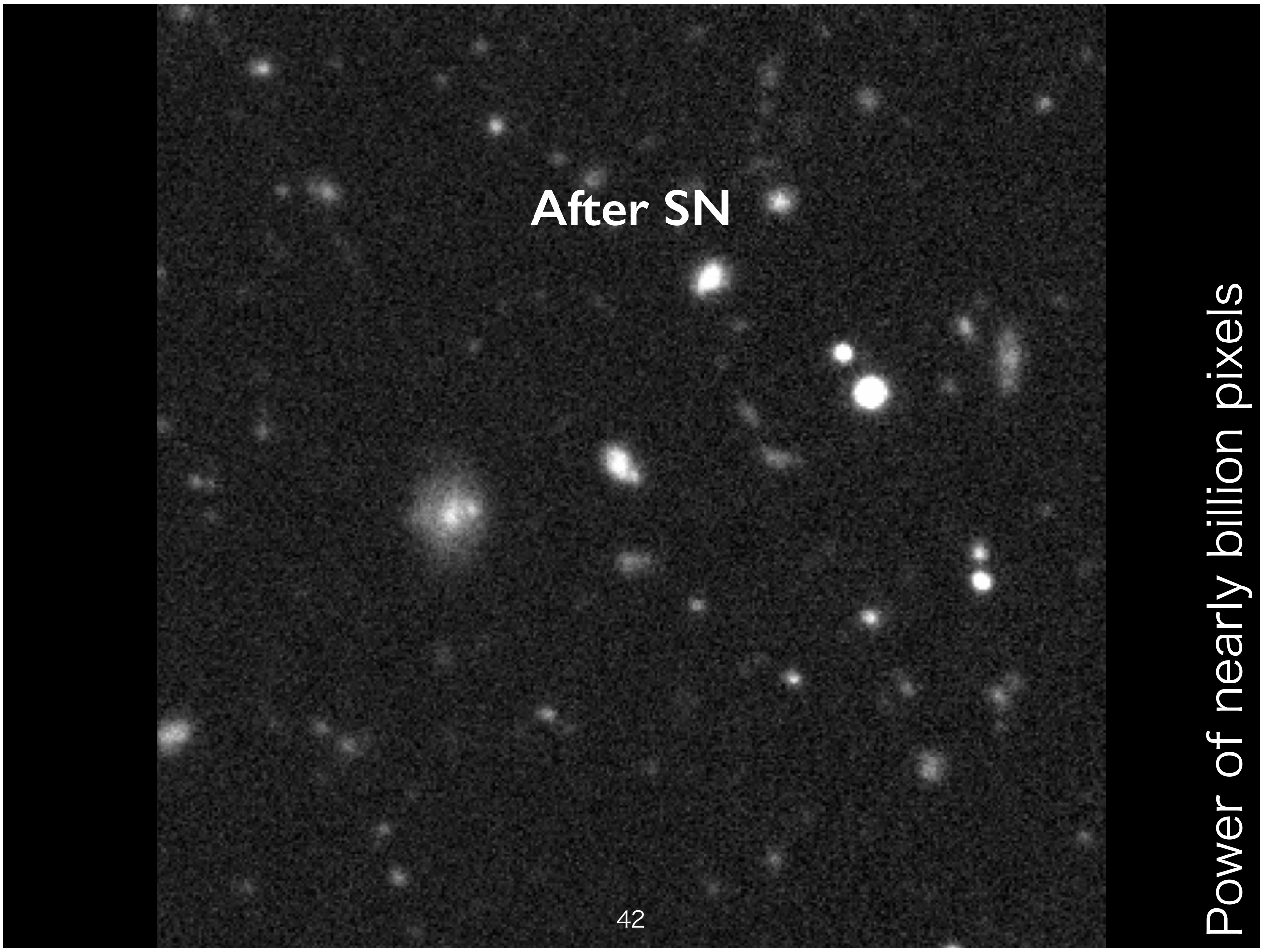

0.5 deg
(30 arcmin)
37
Power of nearly billion pixels

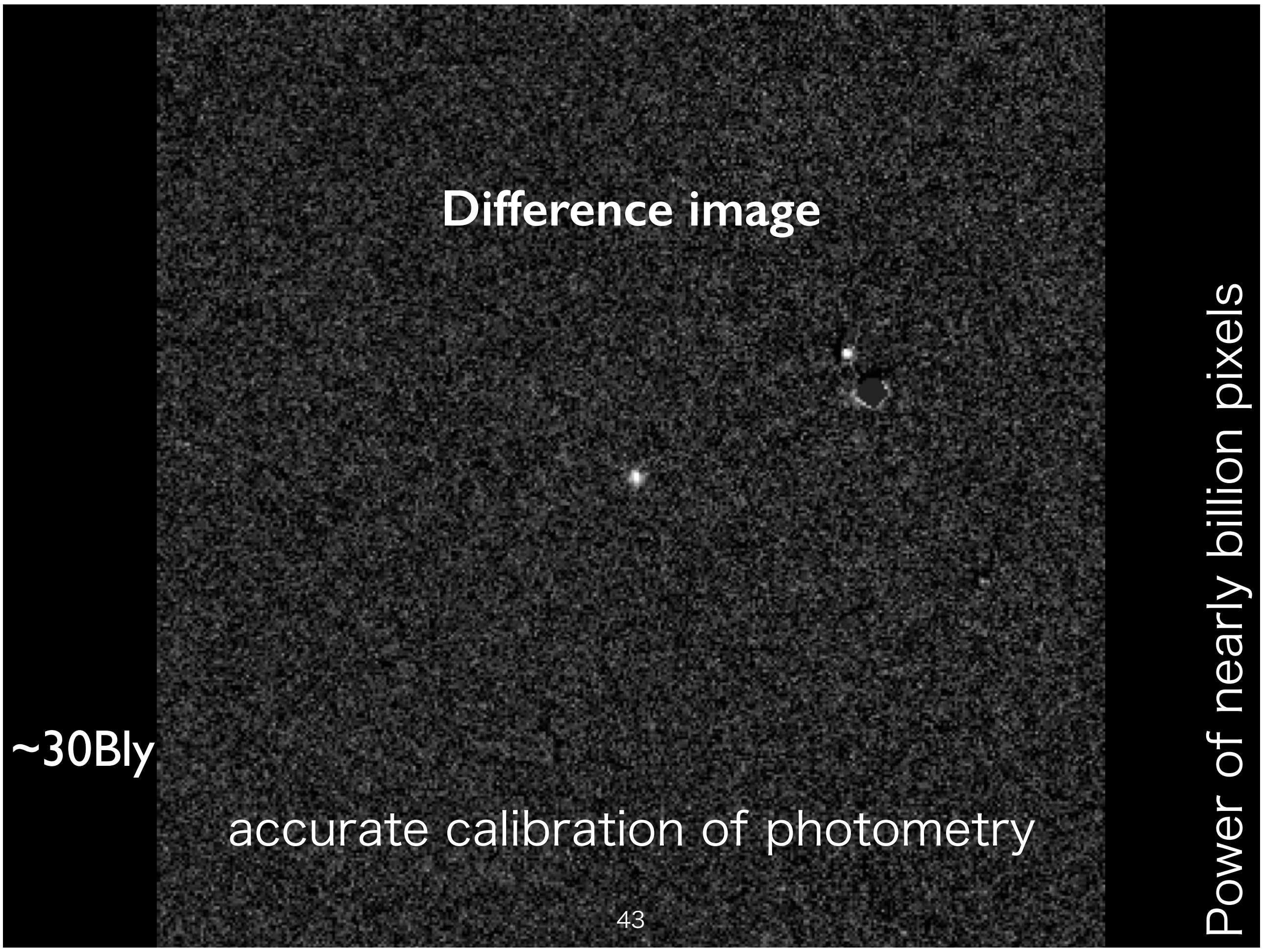

10 arcmin
38
Power of nearly billion pixels


3 arcmin
Power of nearly billion pixels
39

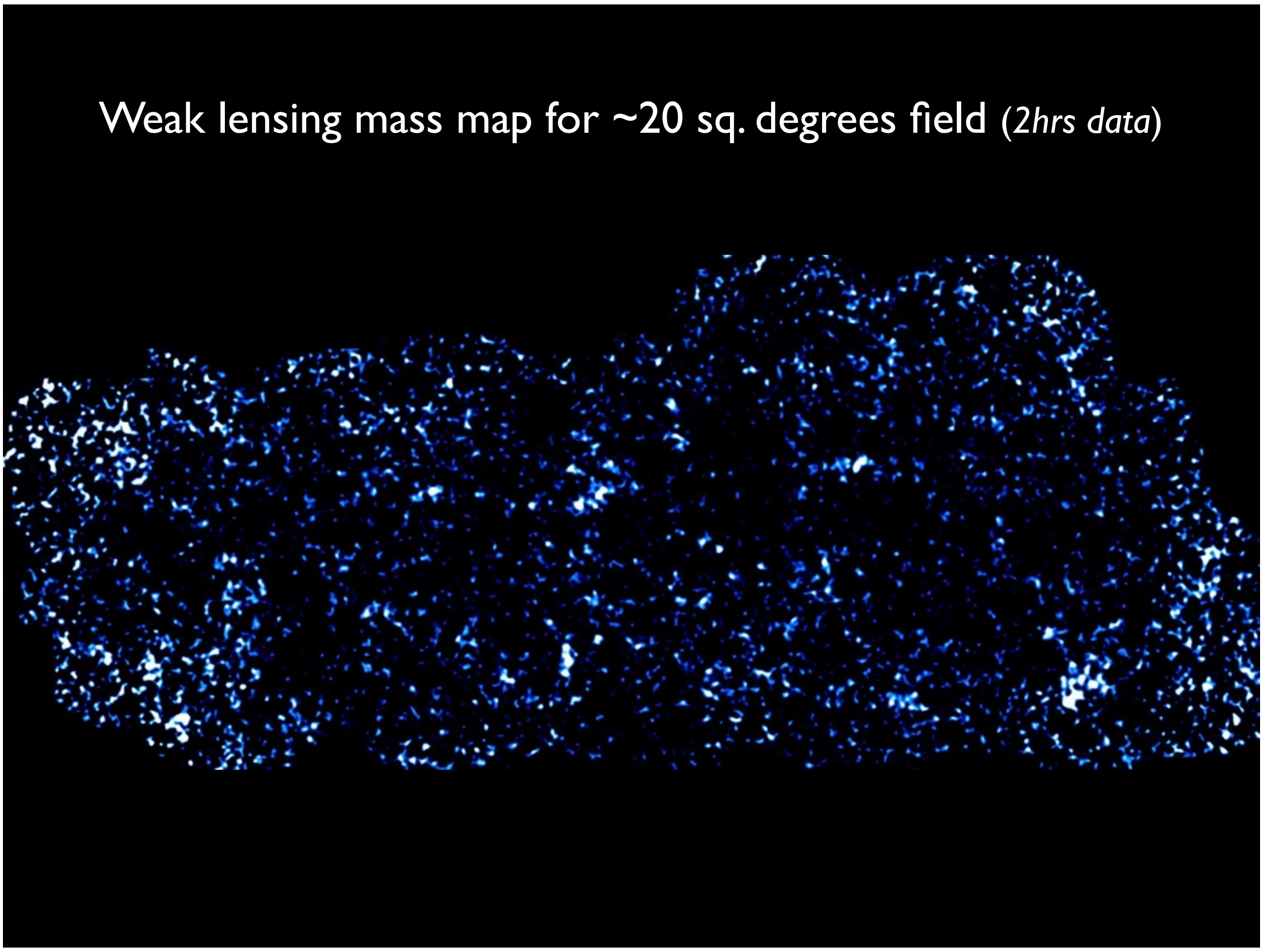

Supernova
1 arcmin
Power of nearly billion pixels
40

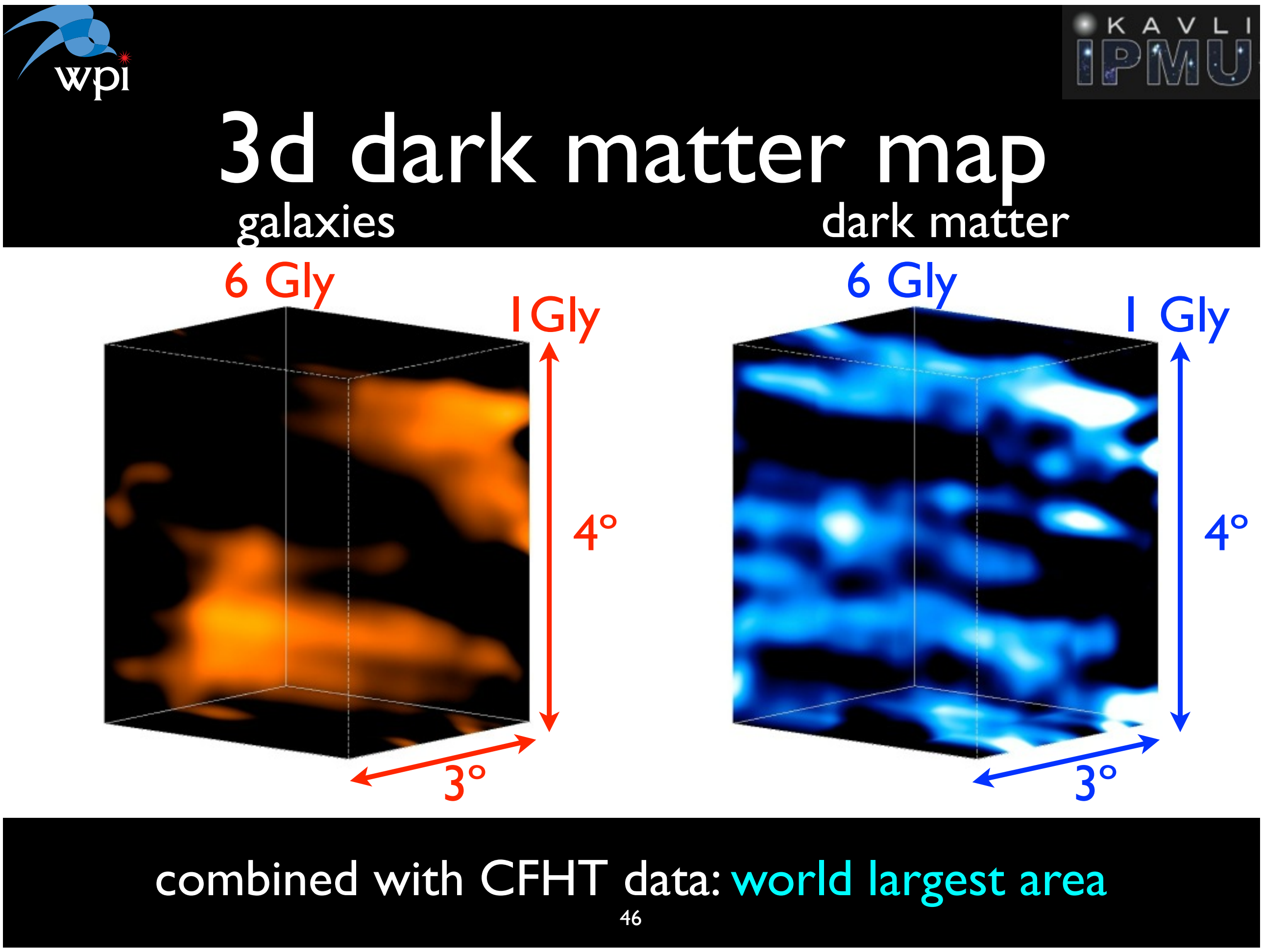

Before SN
Power of nearly billion pixels
41

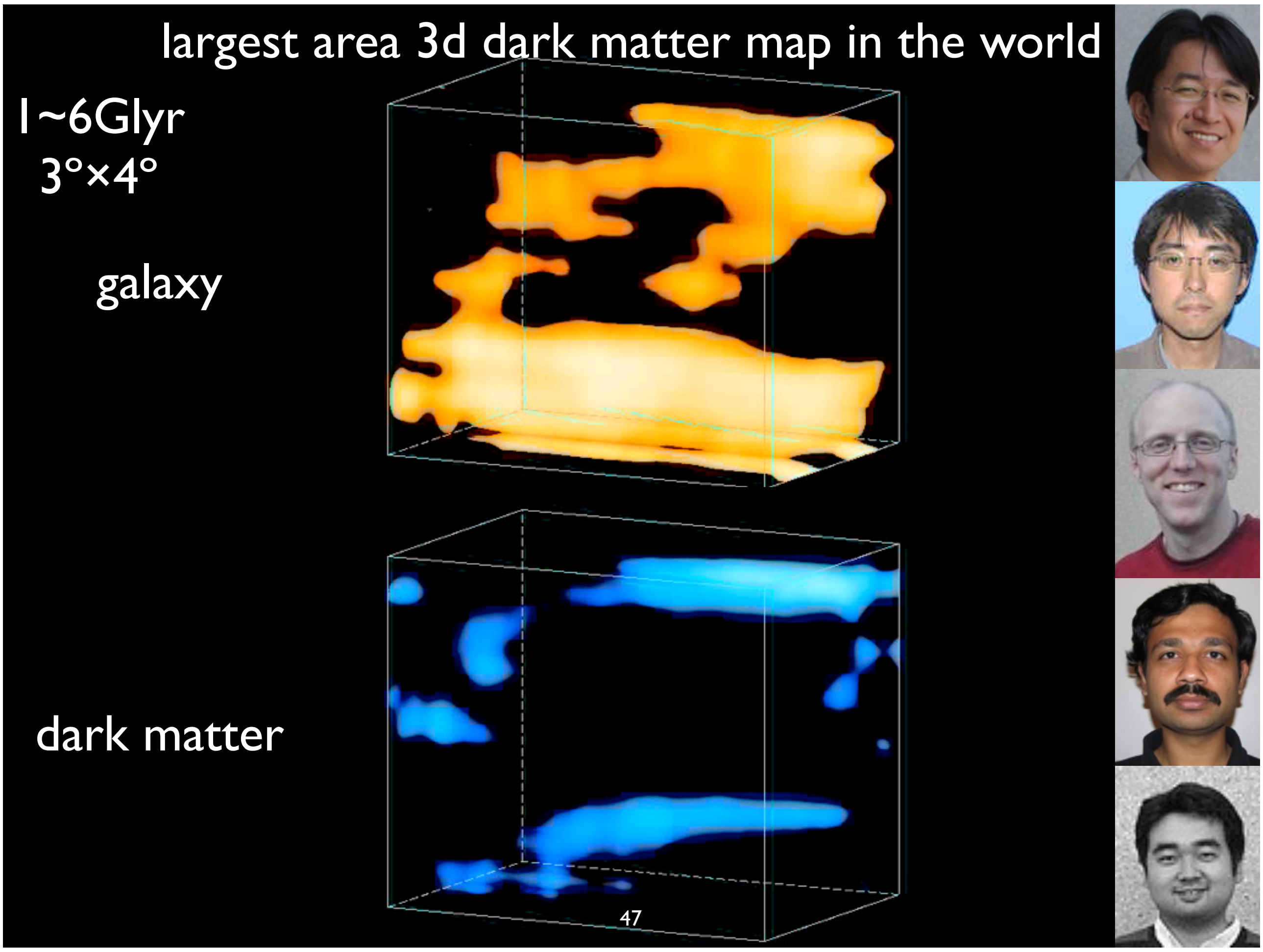

After SN
Power of nearly billion pixels
42


Difference image
~30Bly
accurate calibration of photometry
43
Power of nearly billion pixels

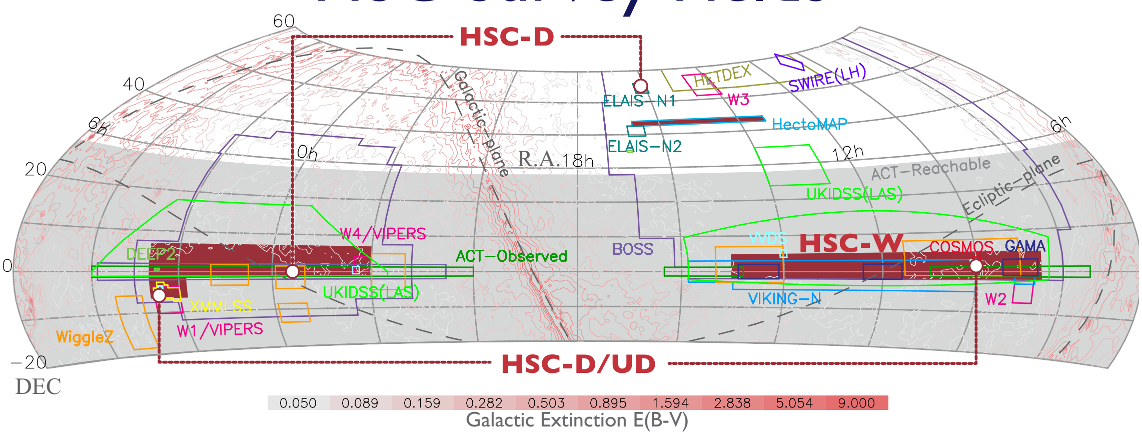

riz
44


Weak lensing mass map for ~20 sq. degrees field (2hrs data)


wpi
KAVLI
IPMU
3d dark matter map
galaxies
dark matter
6 Gly
1Gly
4°
3°
6 Gly
1 Gly
4°
3°
combined with CFHT data: world largest area
46

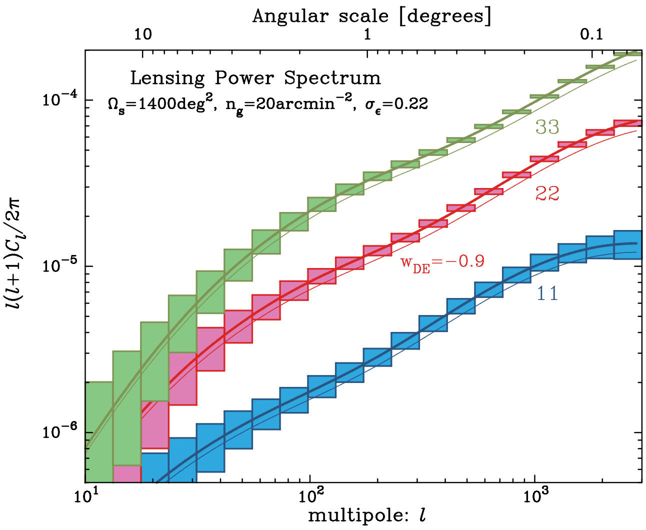


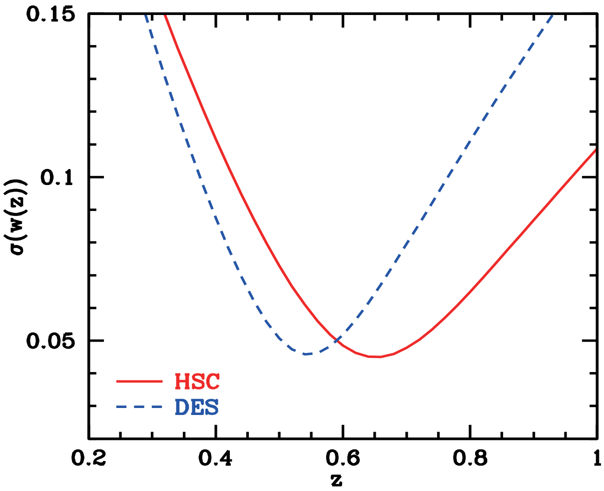
HSC Survey Fields



- HSC Survey Fields selected based on
  - Overlap with SDSS regions, and overlap with other interesting datasets (ACT CMB, NIR, spectroscopic surveys, …)
  - Low dust extinction
  - Spread in RA

Cosmic Shear





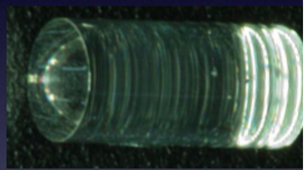
10 *Wide-field imaging with Hyper Suprime-Cam*

Data	$w_{\rm pivot}$	w_a	FoM	γ_g	$m_{\nu,\rm tot}$[eV]	$f_{\rm NL}$	n_s	α_s
BOSS-*BAO*	0.064	1.04	15	–	–	–	0.018	0.0057
HSC(WL)-*B* (baseline)	0.080	0.86	15	0.15	0.16	30	0.014	0.0041
HSC(WL)-*O* (optimistic)	0.068	0.66	22	0.083	0.082	18	0.013	0.0040
HSC(WL+SN)-*B*	0.043	0.60	39	0.15	0.16	30	0.014	0.0041
HSC(WL+SN)-*O*	0.041	0.45	54	0.081	0.081	18	0.013	0.0040
HSC-*O*+[BOSS-$P(k)$]	0.028	0.36	99	0.038	0.076	17	0.011	0.0029
HSC-*O*+[BOSS+PFS]	0.027	0.19	196	0.035	0.07	17	0.009	0.0022

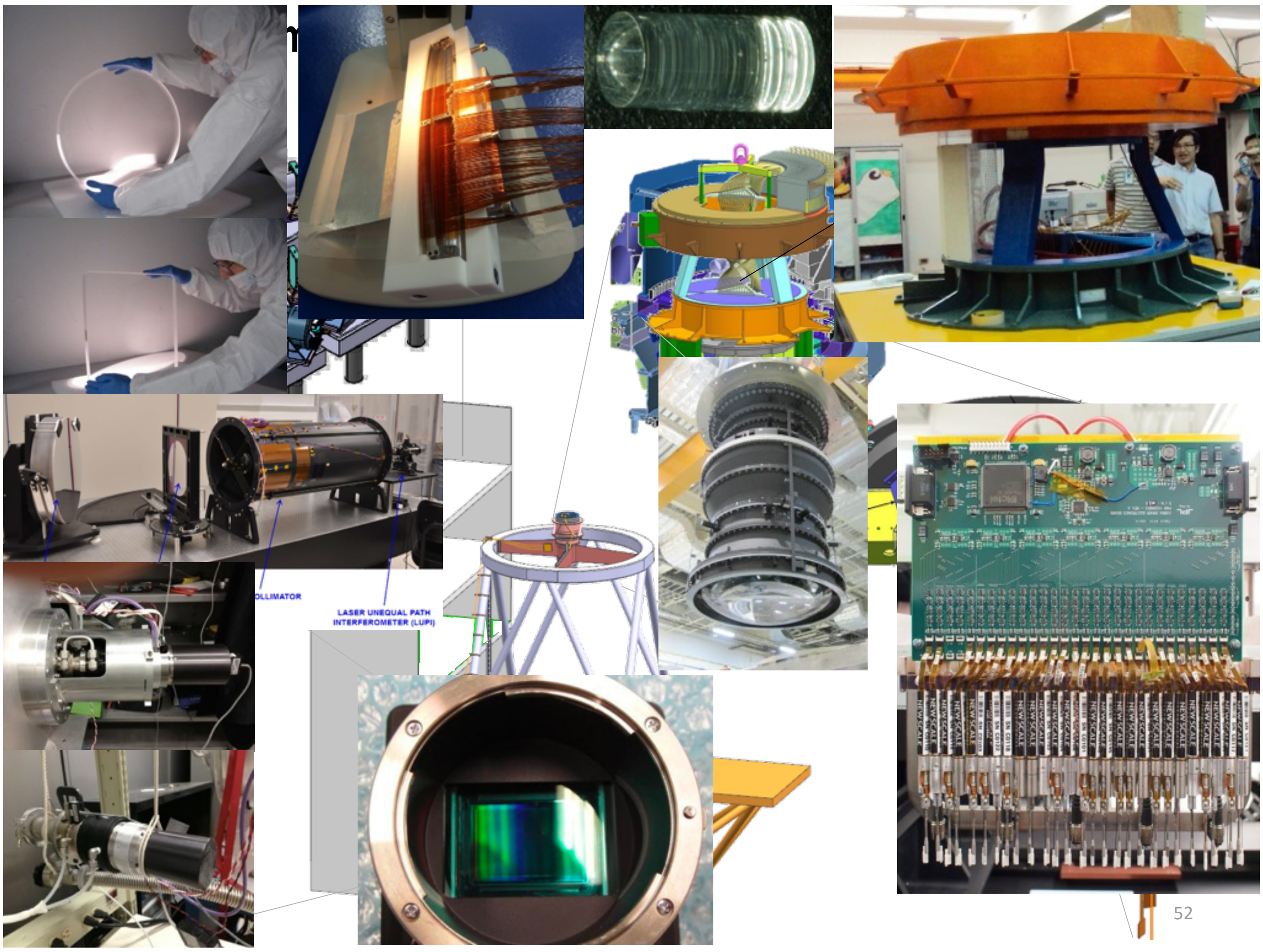
Table 3: Expected parameter accuracies for HSC cosmology using the Oguri & Takada (2011) shear method: The "Baseline" case ("HSC(WL)-*B*") uses clusters with $z < 1$ and masses $M_{\rm halo} > 10^{14}h^{-1}M_\odot$, and without priors on nuisance parameters, whereas the "Optimistic" case ("HSC(WL)-*O*"), uses clusters to $z = 1.4$, with some *conservative* priors on nuisance parameters. The DE constraints listed in this table are also conservative in the sense that the errors include marginalization over non-standard cosmological parameters such as γ_g, $m_{\nu,\rm tot}$, and $f_{\rm NL}$. The rows denoted "WL+SN" include the above HSC-WL and SNeIa measurements. The last two rows show the expected constraints when we combine the HSC observables with spectroscopic surveys, BOSS and PFS (see Ellis et al. 2012 regarding the planned PFS survey). The joint constraints assume that the HSC-WL observables can remove the spectroscopic galaxy bias uncertainty, by comparing the galaxy clustering with the dark matter distributions reconstructed from the HSC-WL observables. This analysis does *not* include constraints from cosmic shear, which is largely independent, with different systematics, and serves as a valuable cross-check.

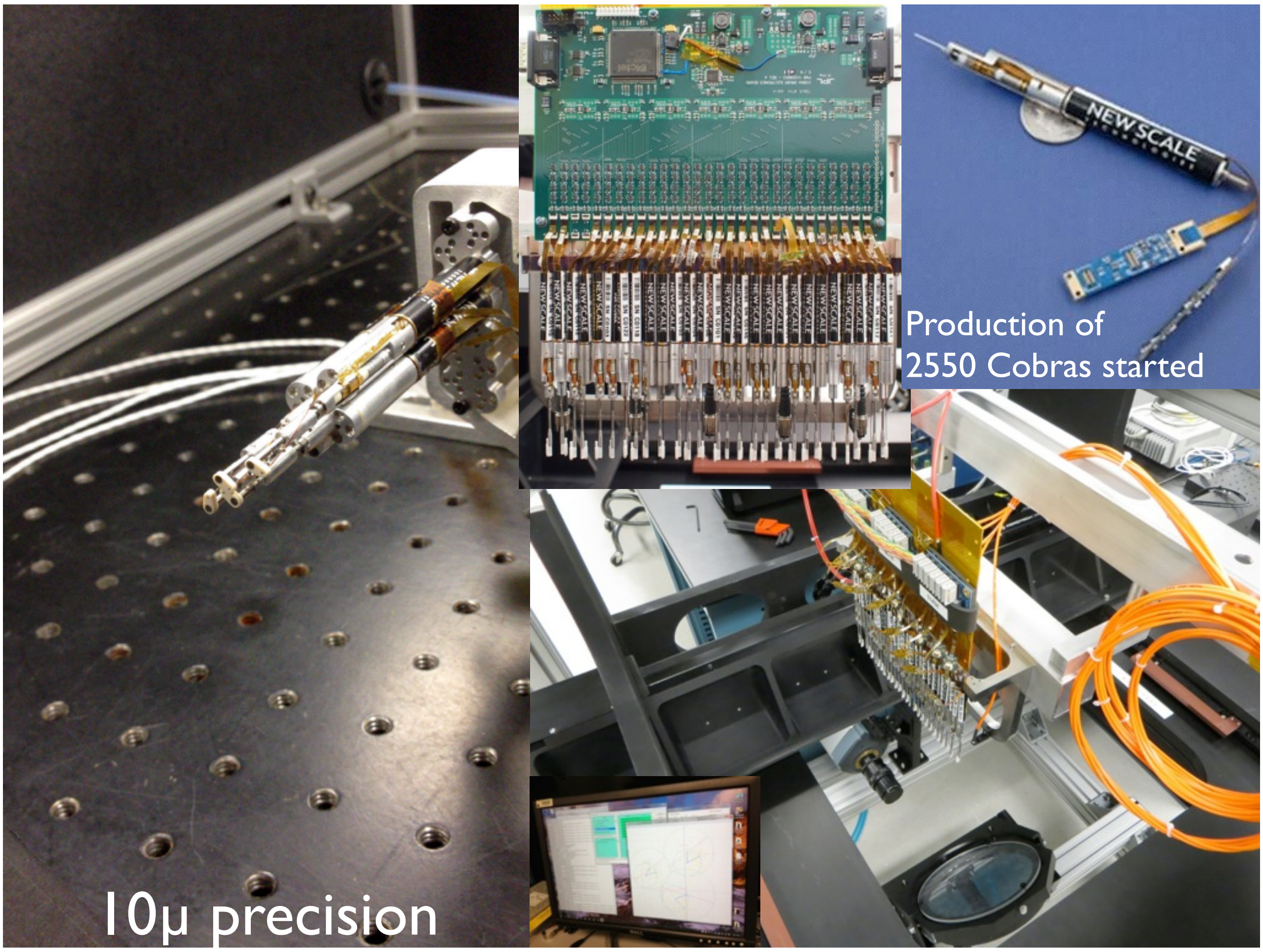
PFS parameters

- 2400 fibers, 128μm, microlens f/2.2→f/2.8
- FoV: 1.3 degrees
- share WFC with HSC
- 4 spectrographs for 600 fibers each
- λ=0.38–1.26μm with three arms

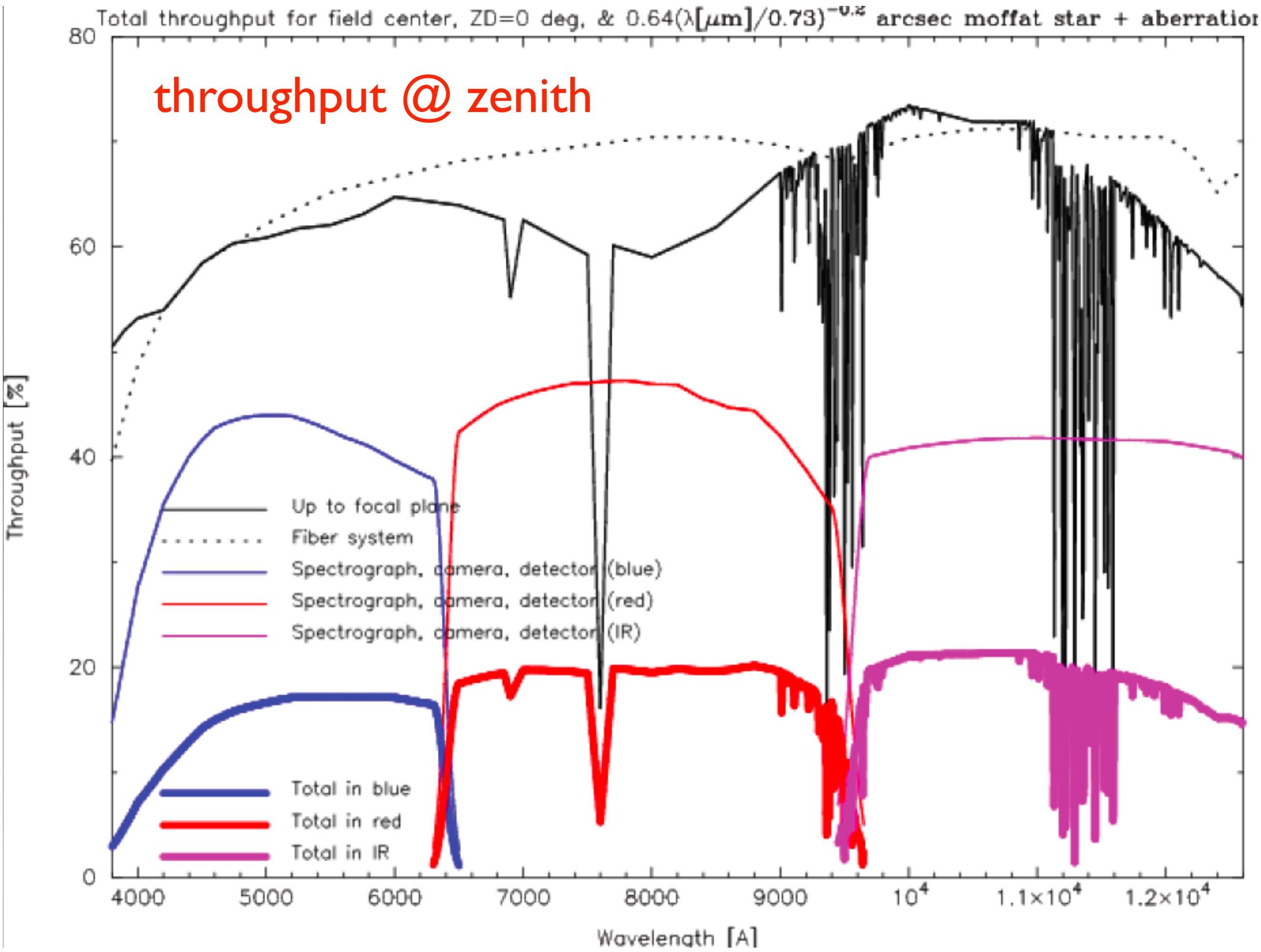


blue	2800–6400 nm	R~2500	Hamamatsu (special coating)
red	6400–9550 nm	R~3200, 5000	Hamamatsu (same as HSC)
near IR	9550–12600 nm	R~4500	Teledyne HgCdTe (<1.7μ)





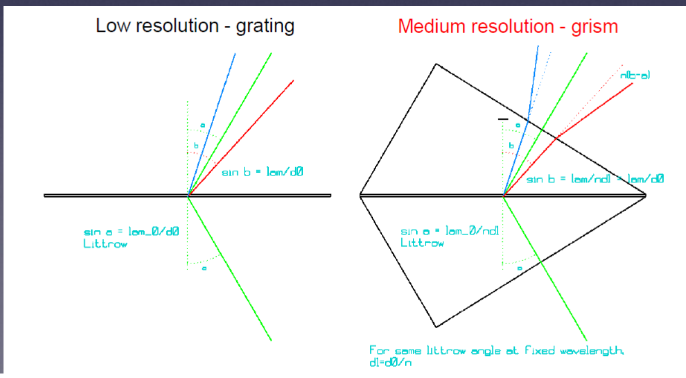
Production of
2550 Cobras started
10μ precision

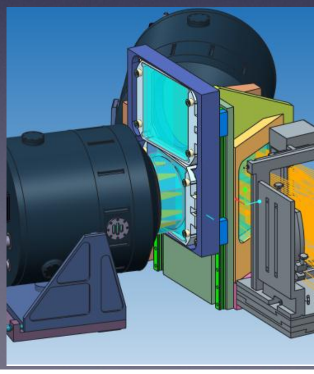


Total throughput for field center, ZD=0 deg, & 0.64(λ[μm]/0.73)^-0.2 arcsec moffat star + aberration
throughput @ zenith
Throughput [%]
Wavelength [A]
Up to focal plane
Fiber system
Spectrograph, camera, detector (blue)
Spectrograph, camera, detector (red)
Spectrograph, camera, detector (IR)
Total in blue
Total in red
Total in IR

Medium Resolution

- re-evaluation of galactic archaeology: dynamics study found very exciting together with the GAIA data, now with medium resolution option R~5000 for red
- simple design with little risk





Publ. Astron. Soc. Jpn (2014) 66 (1), R1 (1–51)
doi: 10.1093/pasj/pst019
Advance Access Publication Date: 2014 February 17
Review

Review

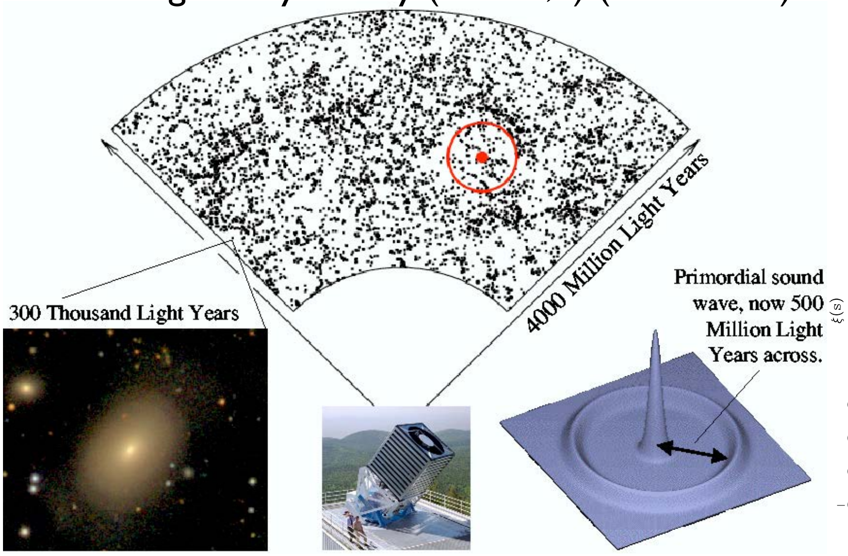
Extragalactic science, cosmology, and Galactic archaeology with the Subaru Prime Focus Spectrograph

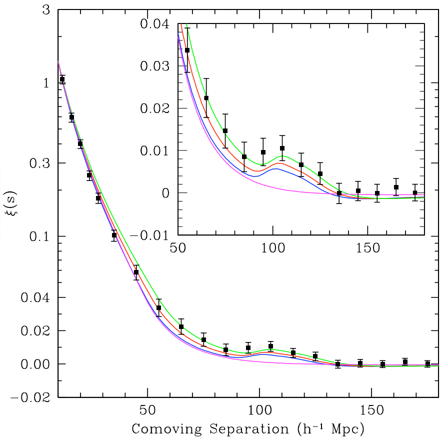
Masahiro Takada,[1,*] Richard S. Ellis,[2] Masashi Chiba,[3] Jenny E. Greene,[4] Hiroaki Aihara,[1,5] Nobuo Arimoto,[6] Kevin Bundy,[1] Judith Cohen,[2] Olivier Doré,[2,7] Genevieve Graves,[4] James E. Gunn,[4] Timothy Heckman,[8] Christopher M. Hirata,[2] Paul Ho,[9] Jean-Paul Kneib,[10] Olivier Le Fèvre,[10] Lihwai Lin,[9] Surhud More,[1] Hitoshi Murayama,[1,11] Tohru Nagao,[12] Masami Ouchi,[13] Michael Seiffert,[2,7] John D. Silverman,[1] Laerte Sodré, Jr.,[14] David N. Spergel,[1,4] Michael A. Strauss,[4] Hajime Sugai,[1] Yasushi Suto,[5] Hideki Takami,[6] and Rosemary Wyse[8]

M. Takada

BAO: standard ruler

Sloan Digital Sky Survey (SDSS-I,II) (2000-2008)





Eisenstein et al. (05)

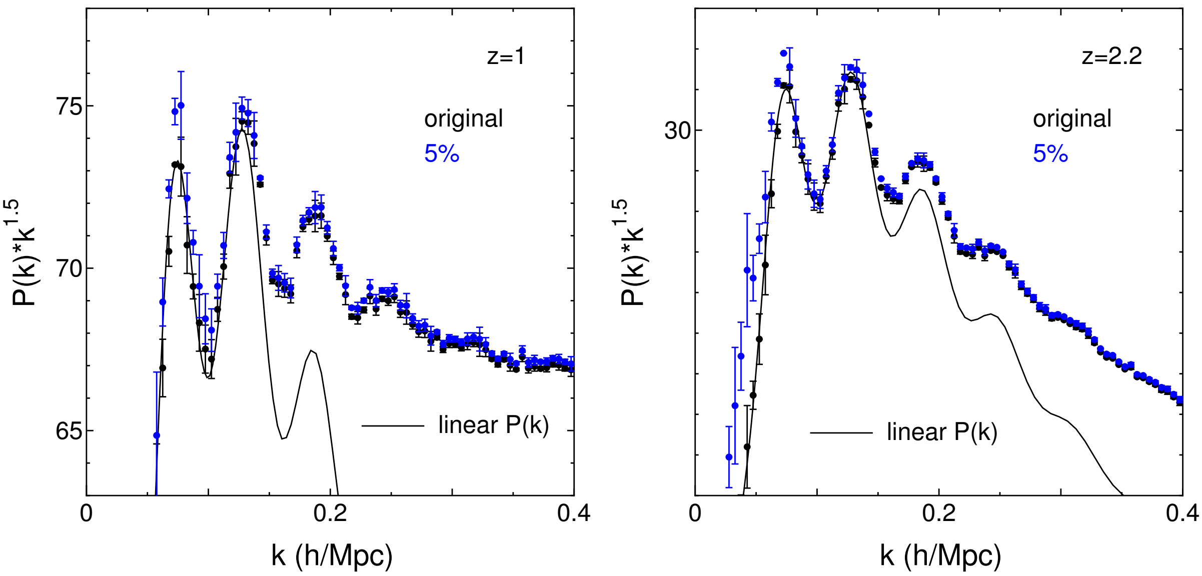
$$r_{\rm BAO} = D_A(z)\Delta\theta_{\rm obs} \qquad r_{\rm BAO} = \frac{\Delta z_{\rm obs}}{H(z_{\rm survey})}$$

Dark Energy Task Force Report (DETF)

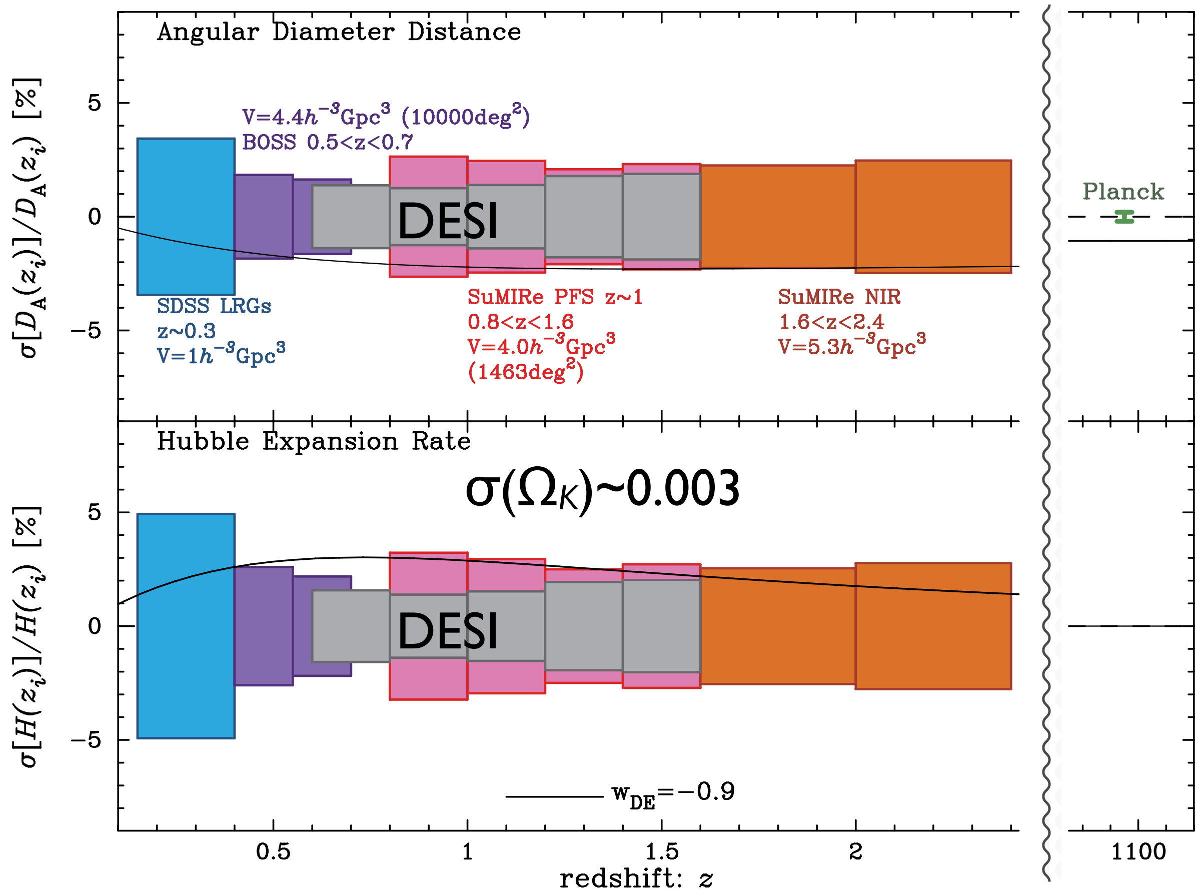
a. The **BAO** technique has only recently been established. It is less affected by astrophysical uncertainties than other techniques.

39

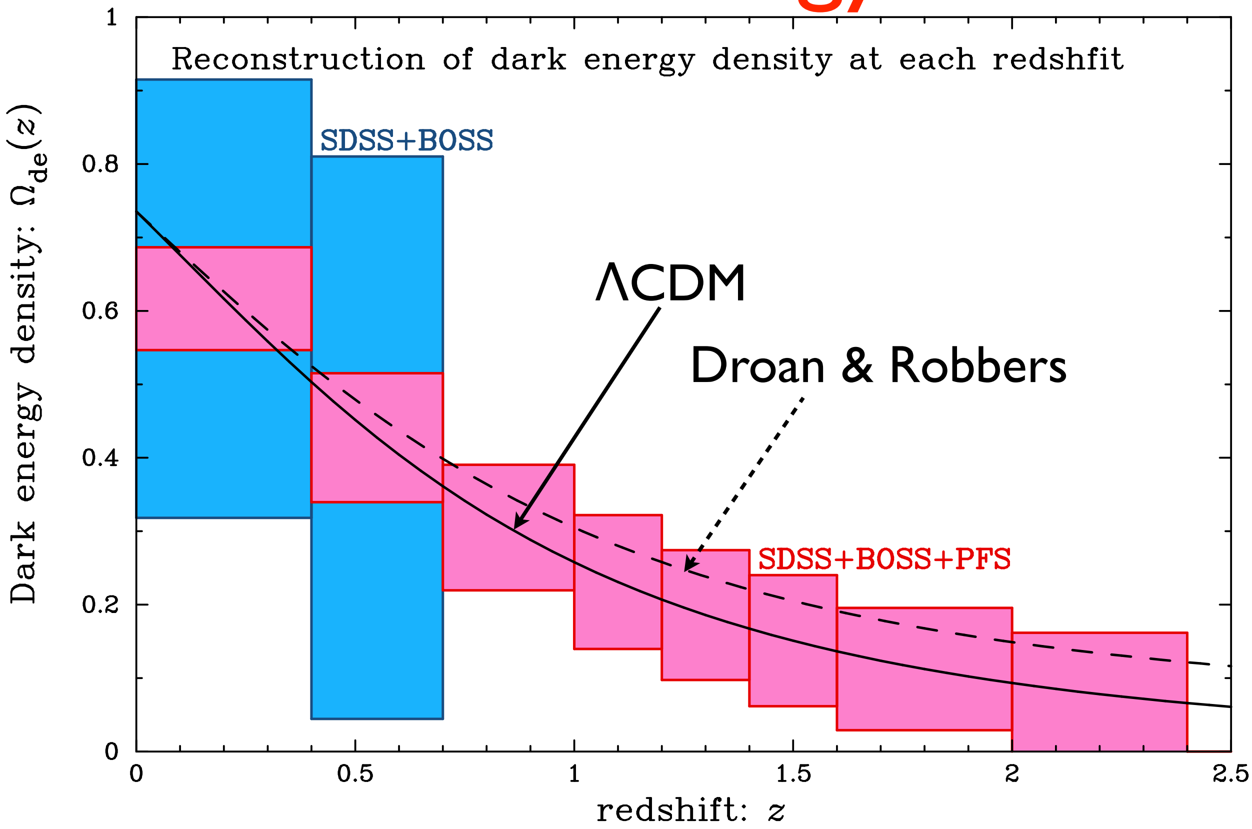
Ripples, Ripples, Ripples

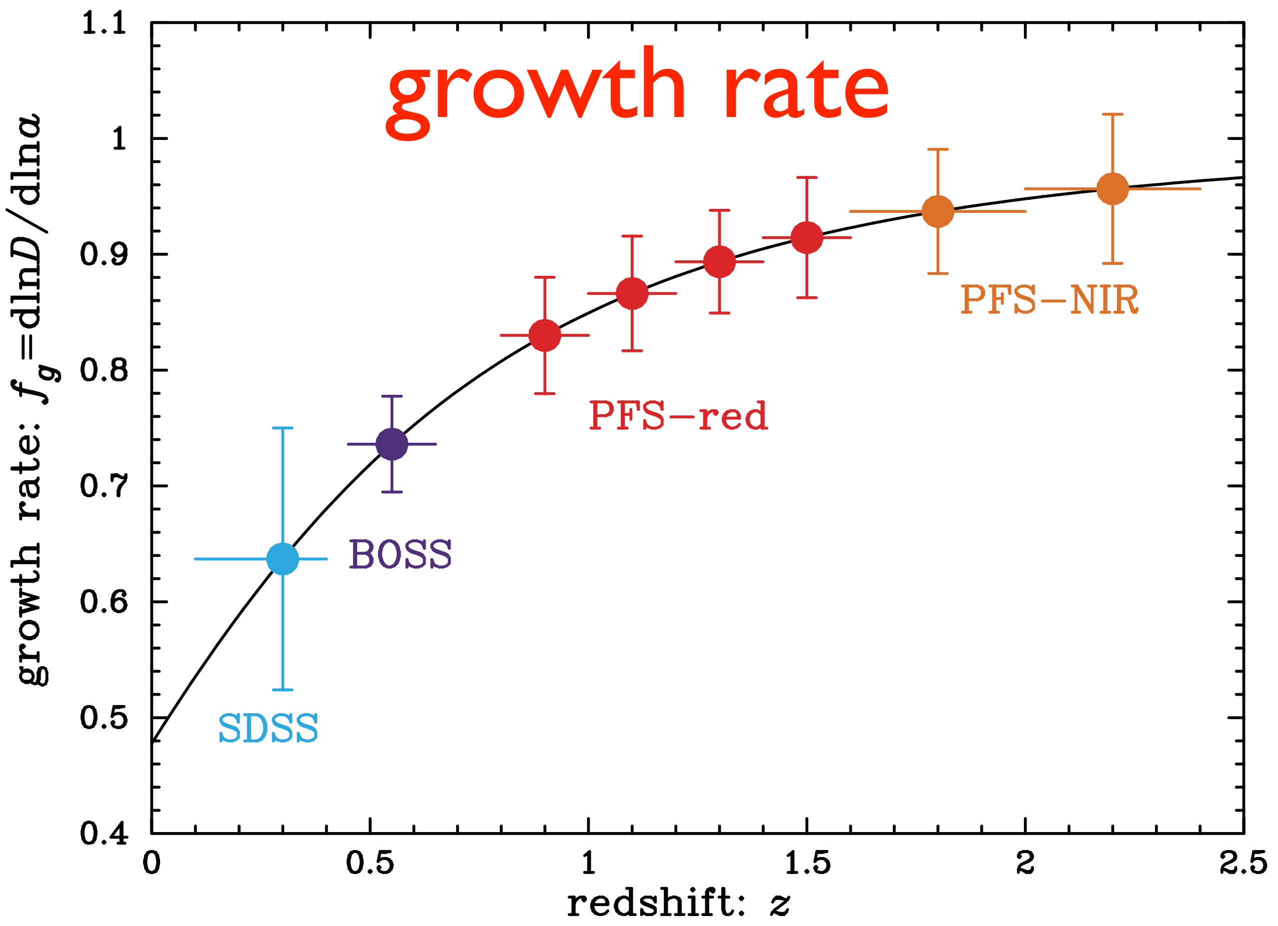


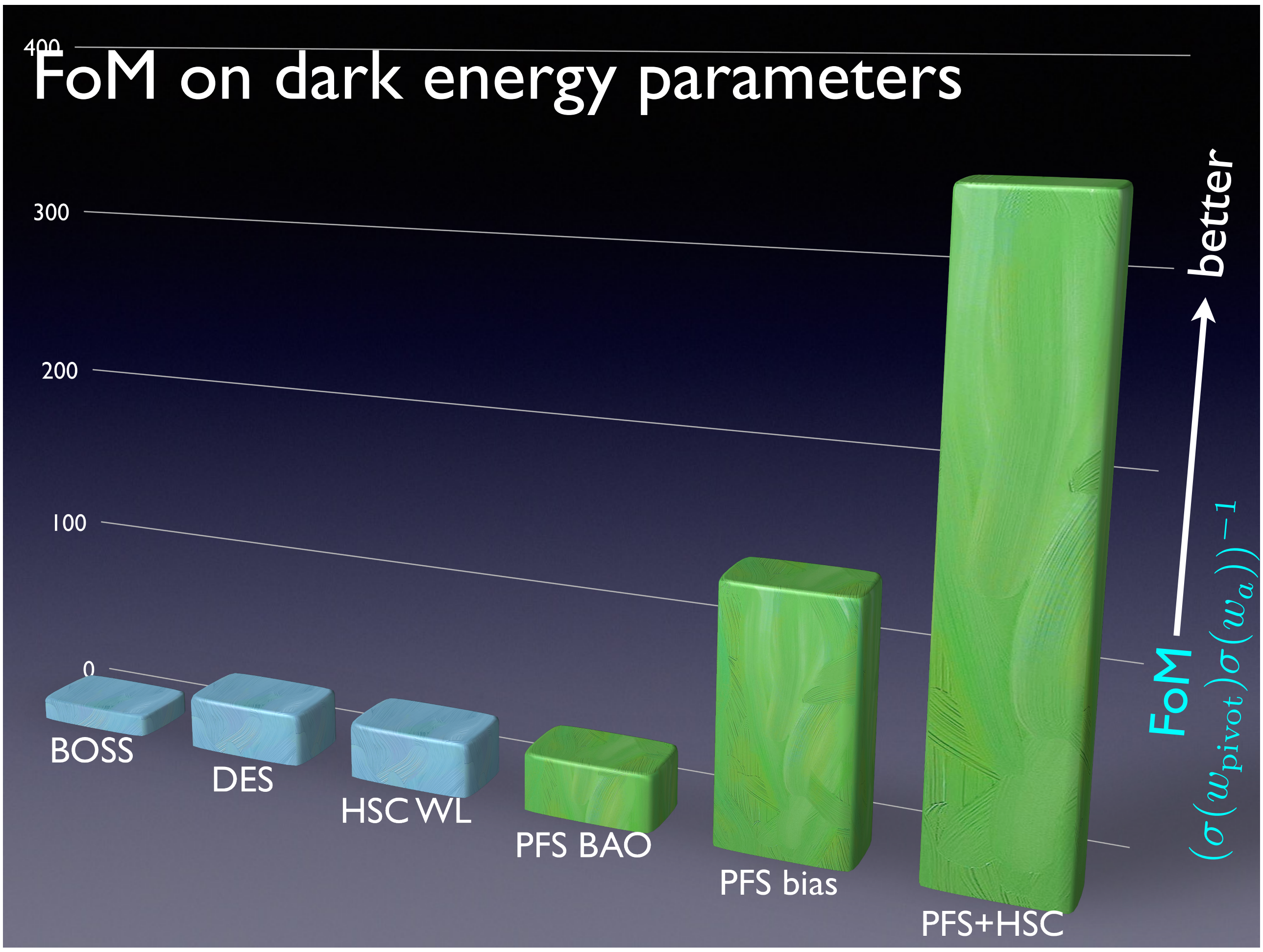
ELGs [OII] > 8.5σ, 15 min exposure

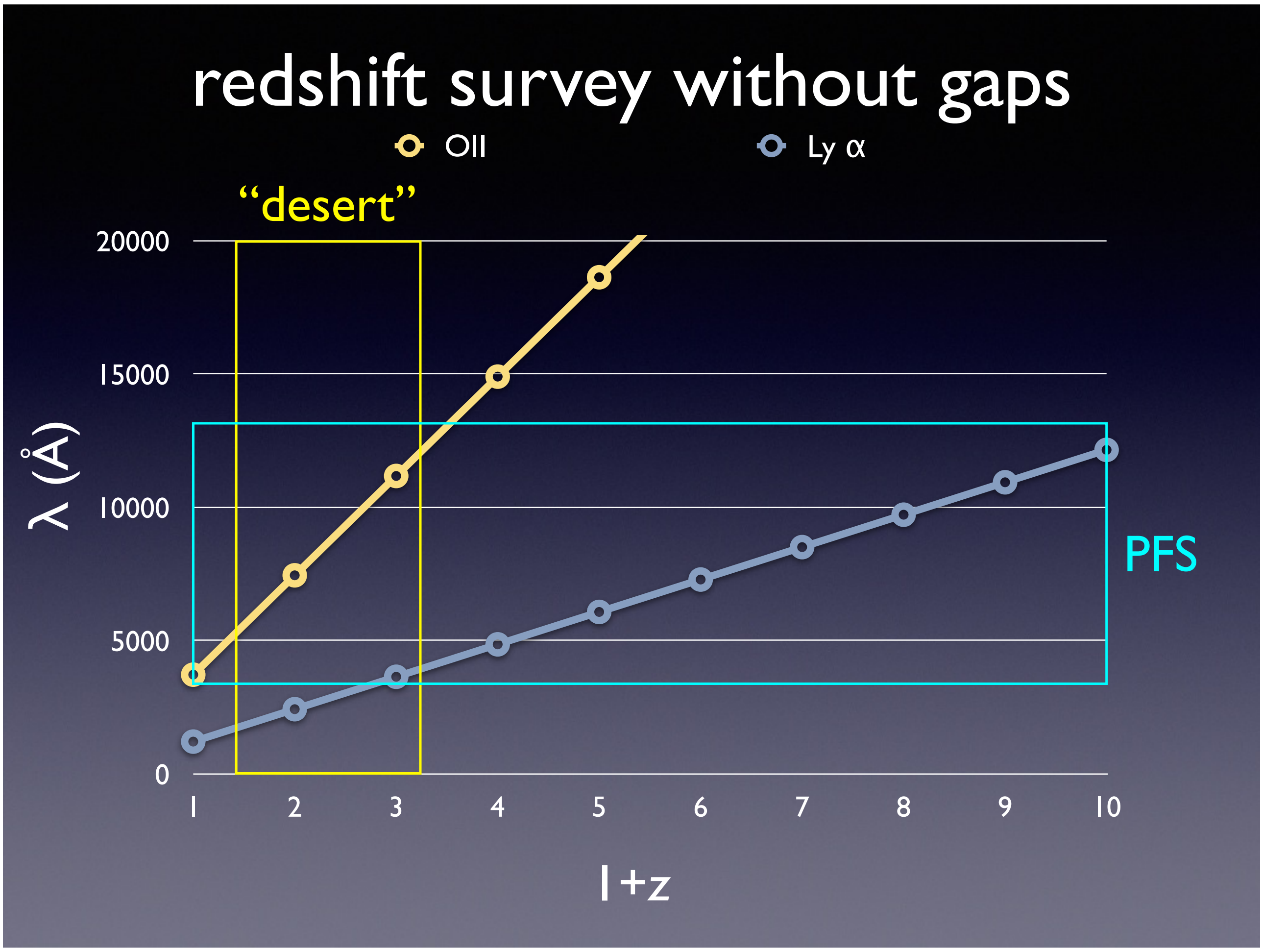

Angular Diameter Distance
V=4.4h^-3Gpc^3 (10000deg^2)
BOSS 0.5<z<0.7
DESI
SDSS LRGs
z~0.3
V=1h^-3Gpc^3
SuMIRe PFS z~1
0.8<z<1.6
V=4.0h^-3Gpc^3
(1463deg^2)
SuMIRe NIR
1.6<z<2.4
V=5.3h^-3Gpc^3
Planck
σ[D_A(z_i)]/D_A(z_i) [%]
Hubble Expansion Rate
σ(Ω_K)~0.003
DESI
w_DE=−0.9
σ[H(z_i)]/H(z_i) [%]
redshift: z
1100

Dark Energy

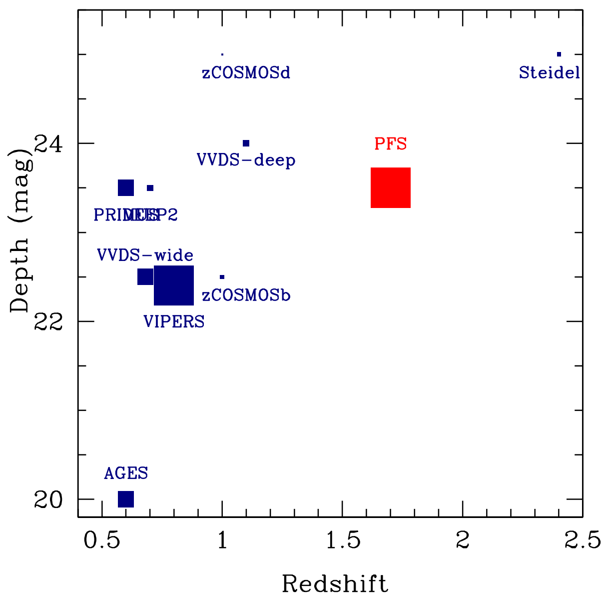

Reconstruction of dark energy density at each redshfit
SDSS+BOSS
ΛCDM
Droan & Robbers
SDSS+BOSS+PFS
Dark energy density: Ω_de(z)
redshift: z

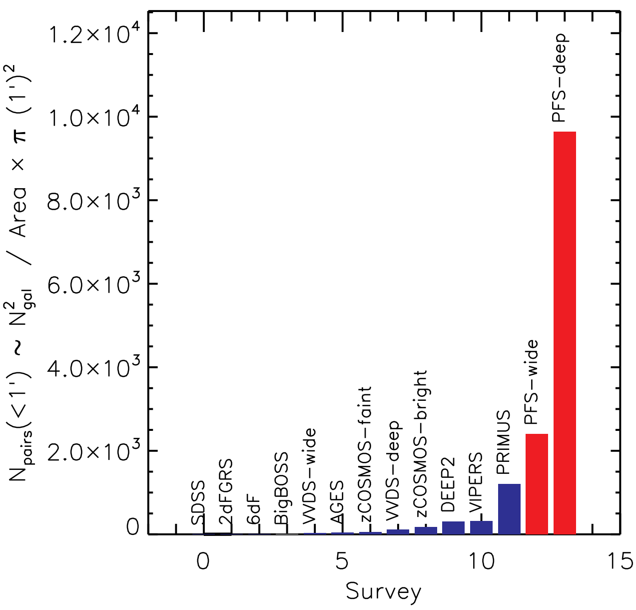

growth rate
growth rate: f_g=dln$D$/dln$a$
redshift: z
SDSS
BOSS
PFS−red
PFS−NIR
0
0.5
1
1.5
2
2.5
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1


FoM on dark energy parameters
400
300
200
100
0
BOSS
DES
HSC WL
PFS BAO
PFS bias
PFS+HSC
better
FoM
$(\sigma(w_{\rm pivot})\sigma(w_a))^{-1}$



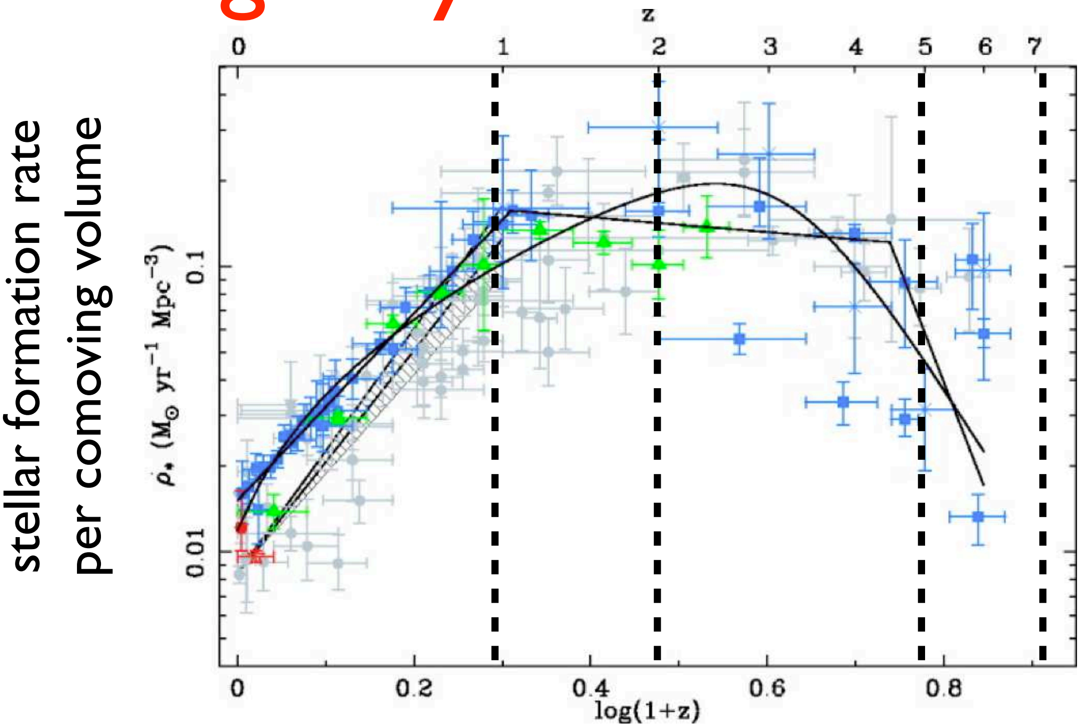
galaxy evolution





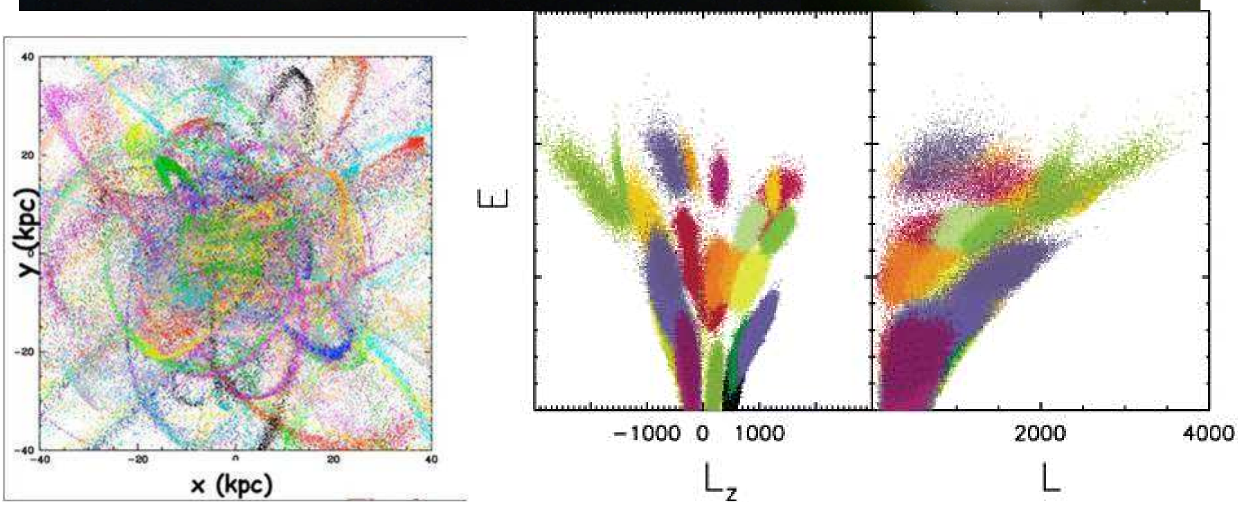
with coverage from 380–1260nm,
unbiased survey with no “*redshift desert*”

galaxy evolution

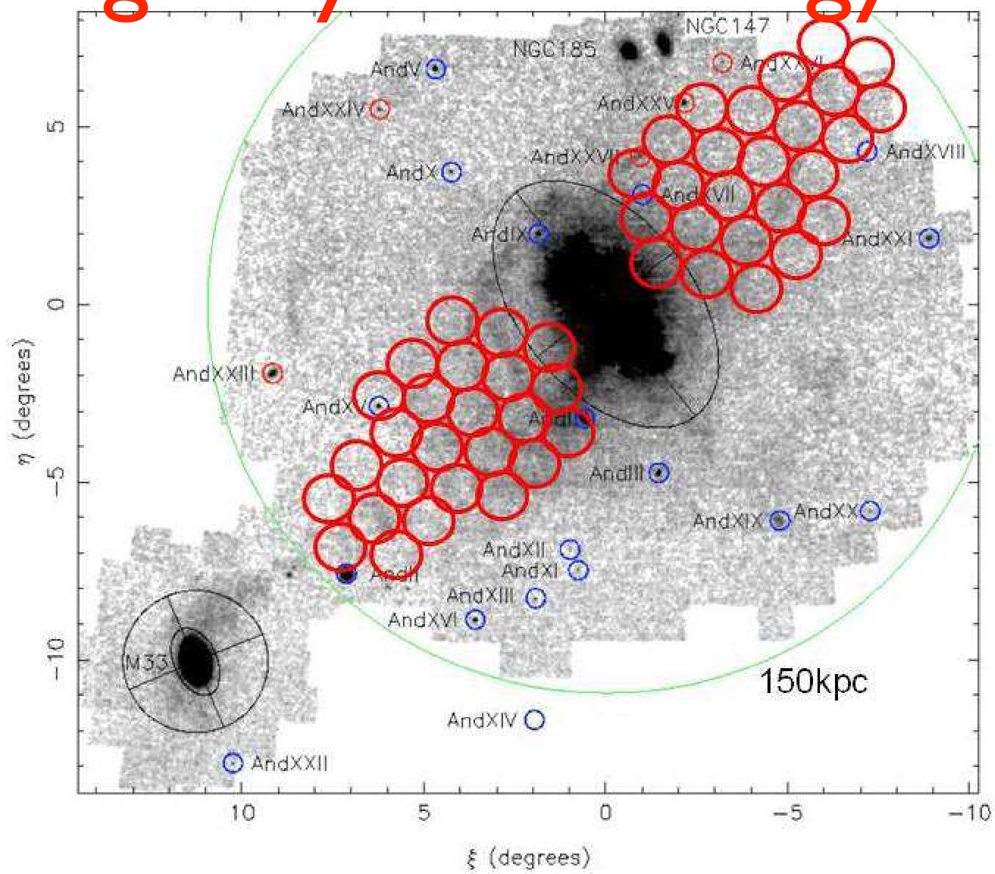


with coverage from 380–1260nm,
unbiased survey with no "*redshift desert*"

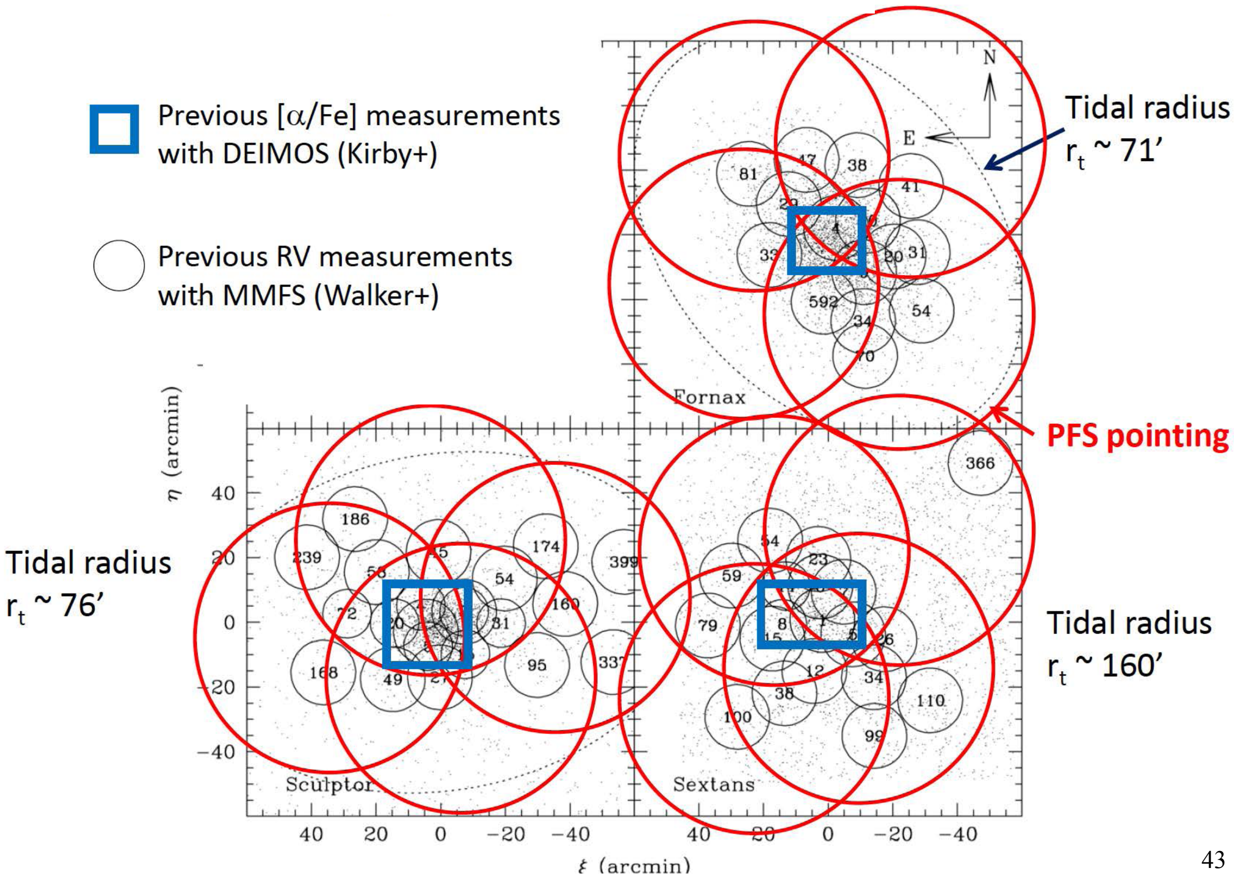
galaxy archaeology

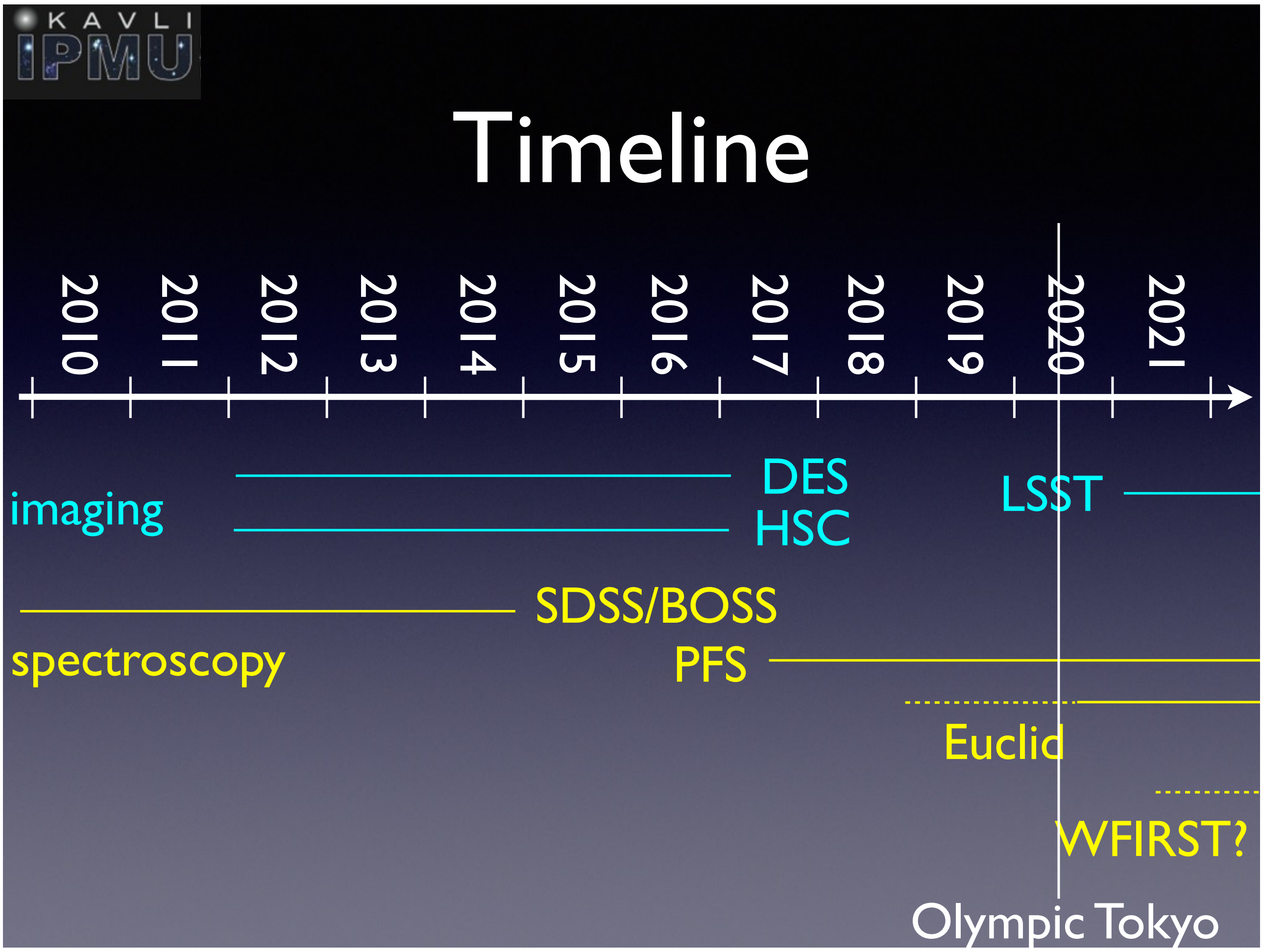


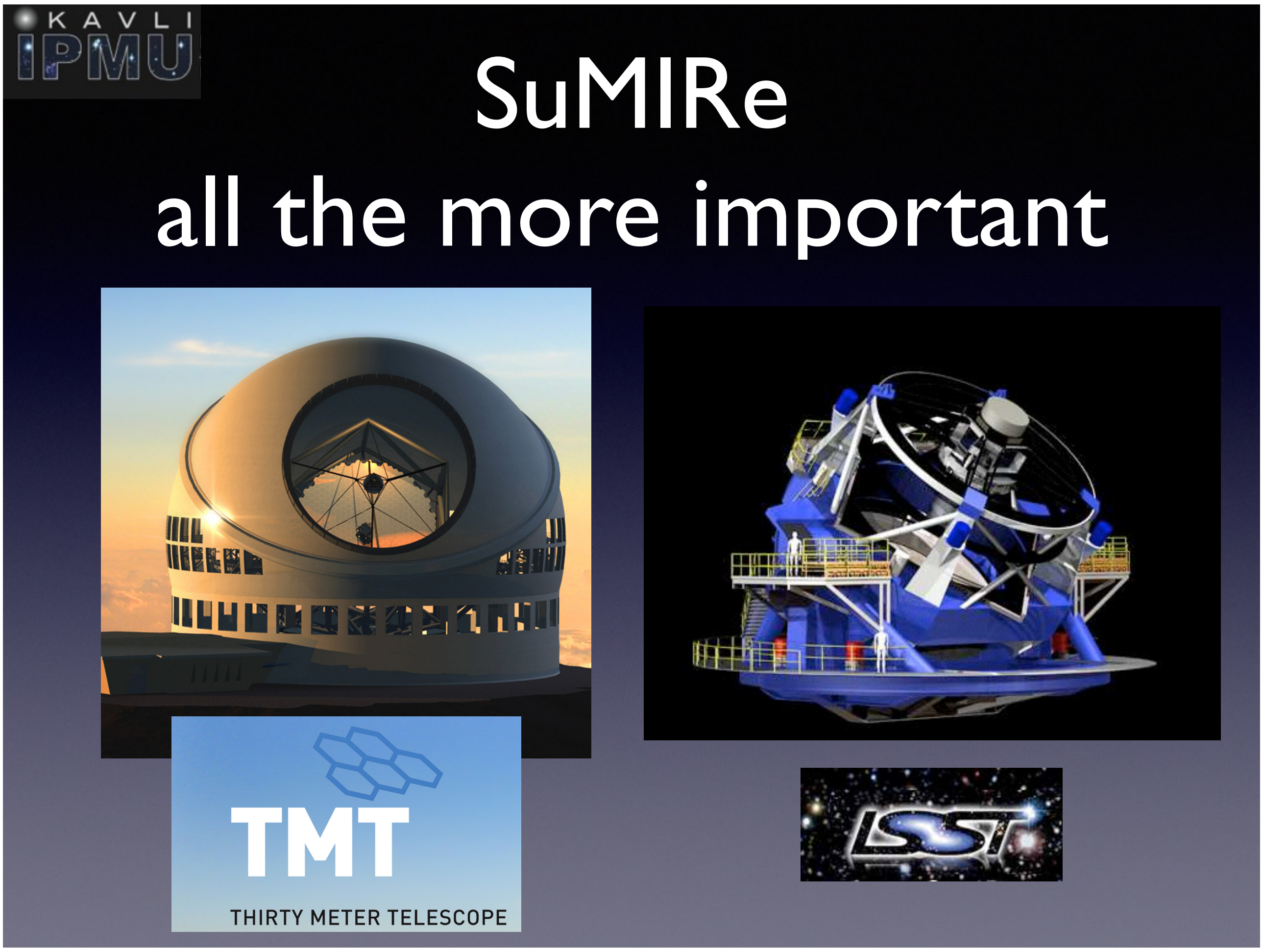
galaxy archaeology

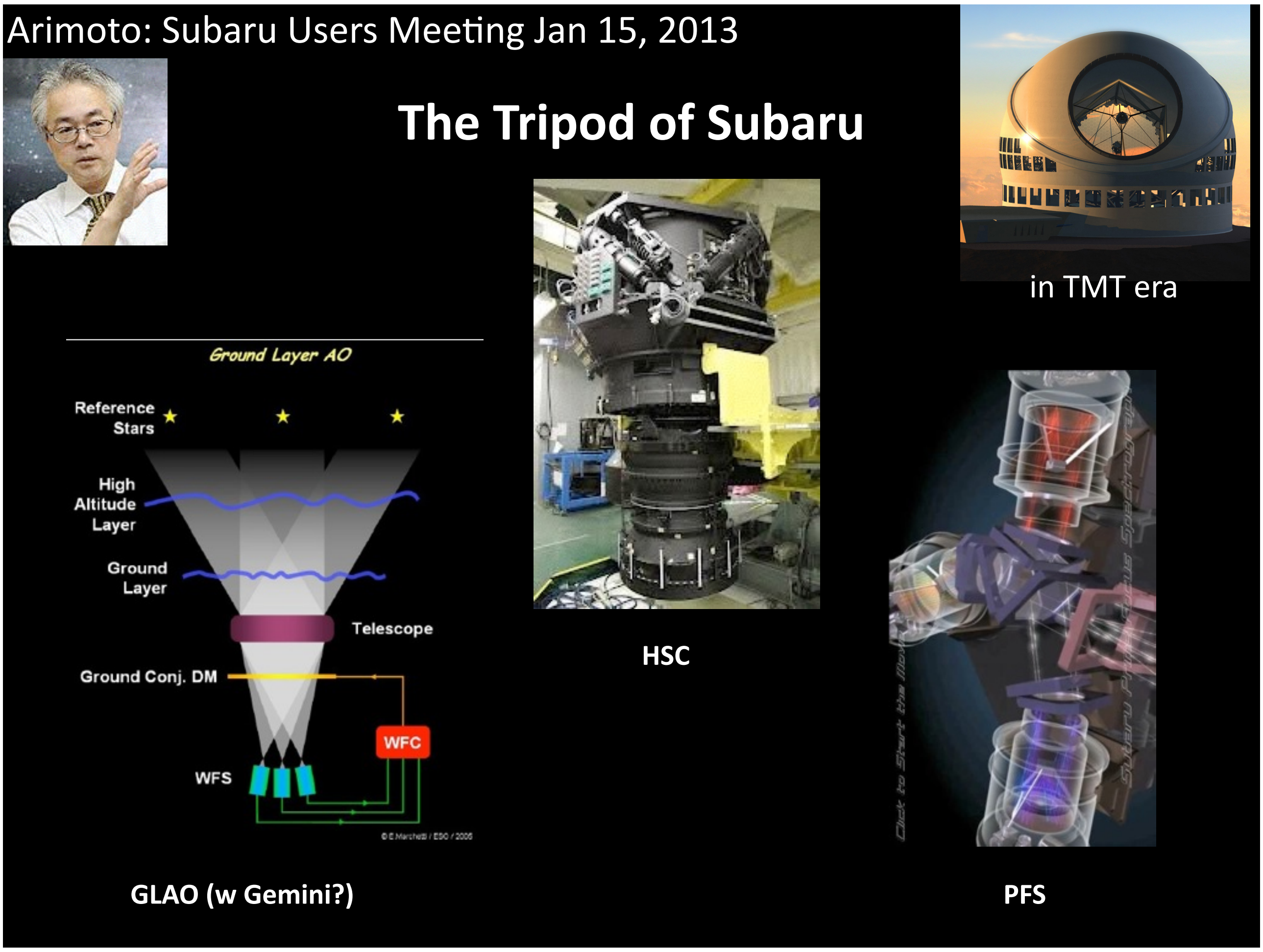


Proposed PFS pointings

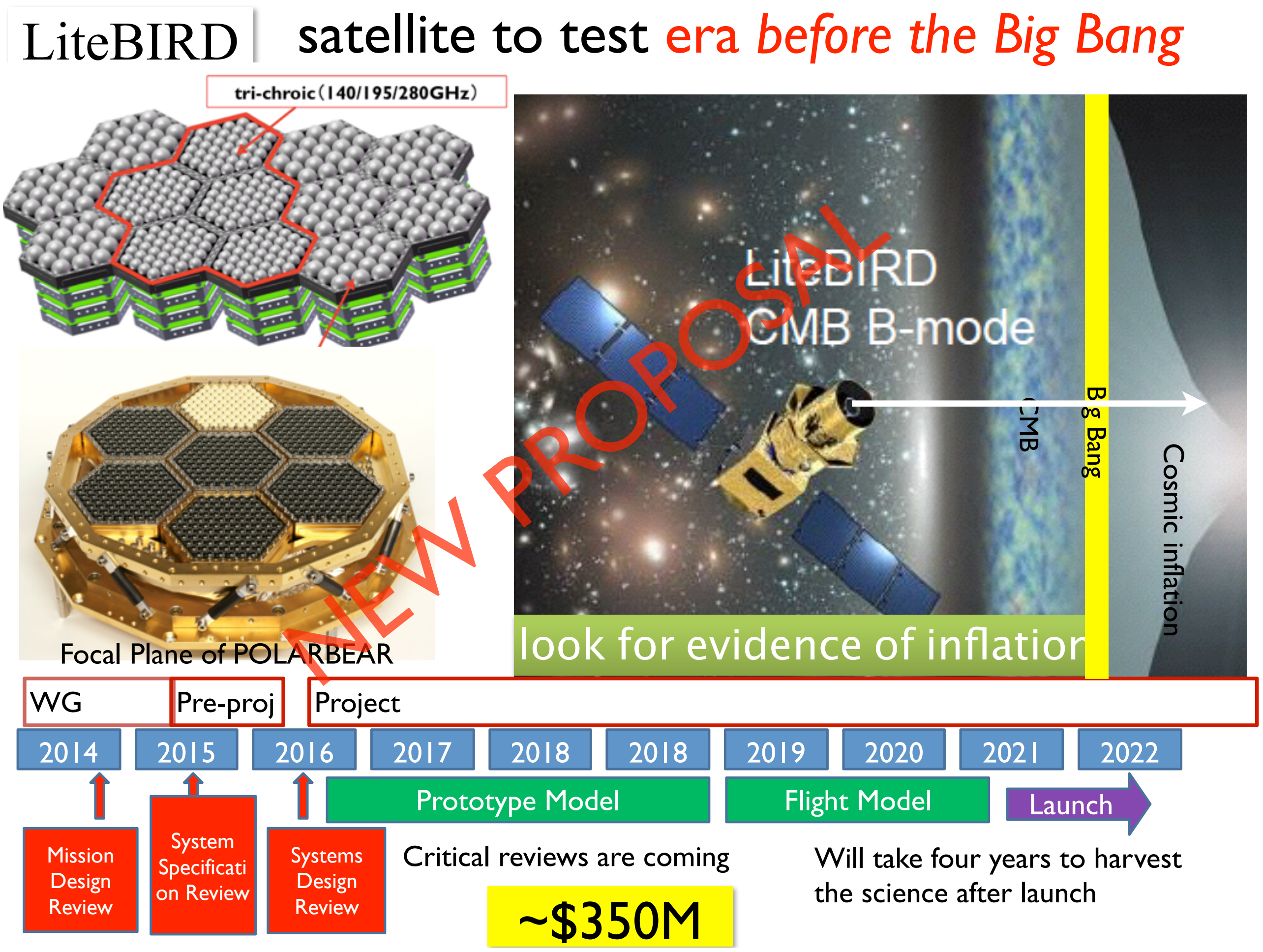


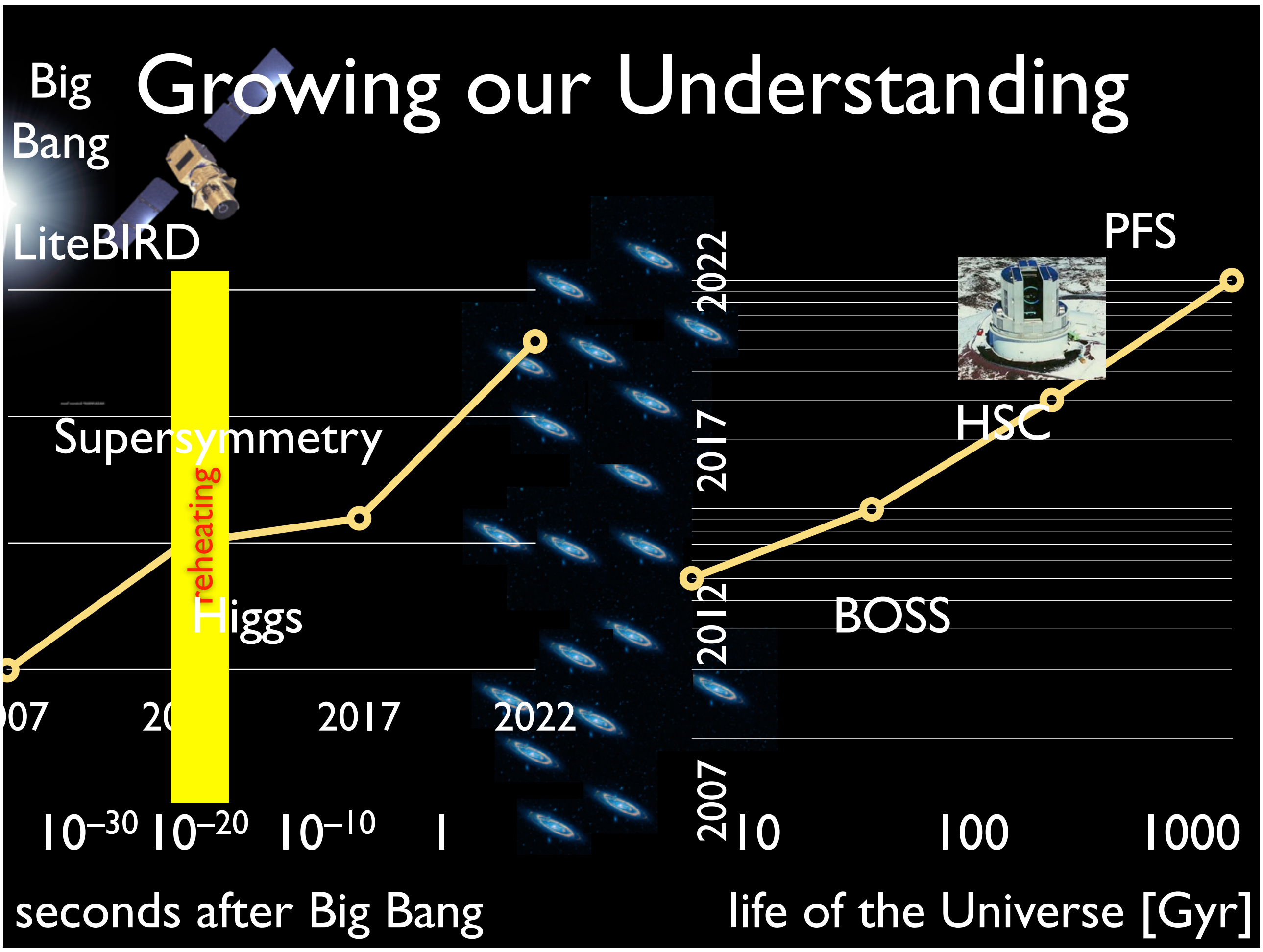

KAVLI IPMU
Timeline
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
imaging
DES
HSC
LSST
SDSS/BOSS
spectroscopy
PFS
Euclid
WFIRST?
Olympic Tokyo


KAVLI IPMU
SuMIRe
all the more important
TMT
THIRTY METER TELESCOPE
LSST


Arimoto: Subaru Users Meeting Jan 15, 2013
The Tripod of Subaru
in TMT era
Ground Layer AO
Reference Stars
High Altitude Layer
Ground Layer
Telescope
Ground Conj. DM
WFC
WFS
HSC
GLAO (w Gemini?)
PFS


PFS Rocks!
Subaru Prime Focus Spectrograph
http://sumire.ipmu.jp/pfs/intro.html


LiteBIRD
satellite to test era before the Big Bang
tri-chroic (140/195/280GHz)
Focal Plane of POLARBEAR
NEW PROPOSAL
LiteBIRD
CMB B-mode
CMB
Big Bang
Cosmic inflation
look for evidence of inflation
WG
Pre-proj
Project
2014
2015
2016
2017
2018
2018
2019
2020
2021
2022
Prototype Model
Flight Model
Launch
Mission Design Review
System Specification Review
Systems Design Review
Critical reviews are coming
~$350M
Will take four years to harvest the science after launch


Growing our Understanding
Big Bang
LiteBIRD
Supersymmetry
reheating
Higgs
07
20
2017
2022
10^-30 10^-20 10^-10 1
seconds after Big Bang
2022
2017
2012
2007
PFS
HSC
BOSS
10
100
1000
life of the Universe [Gyr]

“Progress of code development: 2nd-order Einstein-Boltzmann solver for CMB anisotropy”

Takashi Hiramatsu

[JGRG24(2014)111402]

10-14 Nov 2014 @ Kavli IPMU

Progress of code development

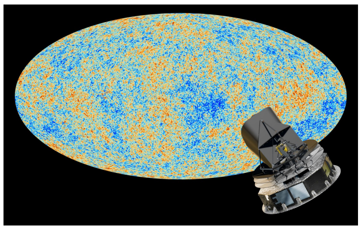
2nd-order Einstein-Boltzmann solver for CMB anisotropy

Takashi Hiramatsu

Yukawa Institute for Theoretical Physics (YITP)
Kyoto University

Collaboration with Ryo Saito (APC), Atsushi Naruko (TITech), Misao Sasaki (YITP)

Introduction : Non-Gaussianity

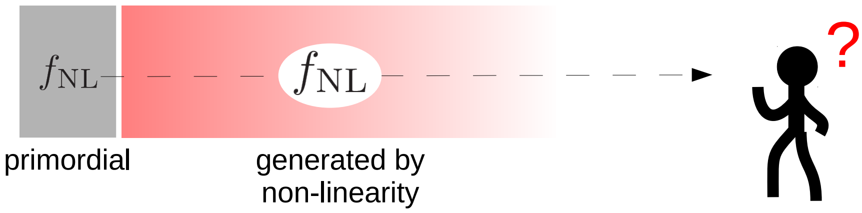


Focus on the statistical property of fluctuations...

Non-gaussianity can distinguish between inlation models (slow-roll, multi-field, DBI inflation, etc.)

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3 B_\zeta(k_1,k_2,k_3)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

... parameterised by $f_{\rm NL}$



To evaluate the total amount of $f_{\rm NL}$ from non-linearity is an important task.

3 Boltzmann solvers for 2nd-order perturbations are available, but the resultant $f_{\rm NL}$ is not converged... Do it ourselves !

Introduction : current status of my code

Takashi Hiramatsu

1st-order perturbations | *2nd-order perturbations*

Implementing basic equations
History of electron density
line-of-sight integral
+angular power spectrum
Qualitative check
Quantitative check with CAMB

Implementing basic equations 1

NOW → 2nd-order line-of-sight formula
R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]
NOW → Bispectrum estimator

Implementing basic equations 2
Speed-up + Optimisation

1st-order 98%

2nd-order 35%

1st-order perturbation equations

Takashi Hiramatsu

Photon temperature

$$\begin{cases} \dot\Theta^{\rm T}_0 = -k\Theta^{\rm T}_1 - \dot\Phi \\ \dot\Theta^{\rm T}_1 = \frac{1}{3}k(-2\Theta^{\rm T}_2 + \Theta^{\rm T}_0) + \dot\tau\left(\Theta^{\rm T}_1 + \frac{1}{3}v_b\right) + \frac{1}{3}k\Psi \\ \dot\Theta^{\rm T}_2 = \frac{1}{5}k(-3\Theta^{\rm T}_3 + 2\Theta^{\rm T}_1) + \dot\tau\left(\Theta^{\rm T}_2 - \frac{1}{10}\Pi\right) \\ \dot\Theta^{\rm T}_\ell = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta^{\rm T}_{\ell+1} + \ell\Theta^{\rm T}_{\ell-1}\right] + \dot\tau\Theta^{\rm T}_\ell \end{cases}$$

Photon polarisation

$$\begin{cases} \dot\Theta^{\rm P}_0 = -k\Theta^{\rm P}_1 + \dot\tau\left(\Theta^{\rm P}_0 - \frac{1}{2}\Pi\right) \\ \dot\Theta^{\rm P}_1 = \frac{1}{3}k(-2\Theta^{\rm P}_2 + \Theta^{\rm P}_0) + \dot\tau\Theta^{\rm P}_1 \\ \dot\Theta^{\rm P}_2 = \frac{1}{5}k(-3\Theta^{\rm P}_3 + 2\Theta^{\rm P}_1) + \dot\tau\left(\Theta^{\rm P}_2 - \frac{1}{10}\Pi\right) \\ \dot\Theta^{\rm P}_\ell = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta^{\rm P}_{\ell+1} + \ell\Theta^{\rm P}_{\ell-1}\right] + \dot\tau\Theta^{\rm P}_\ell \end{cases}$$

Massless neutrino temperature

$$\begin{cases} \dot\Theta_{\rm N0} = -k\Theta_{\rm N1} - \dot\Phi \\ \dot\Theta^{\rm N}_1 = \frac{1}{3}k(-2\Theta^{\rm N}_2 + \Theta^{\rm N}_0) + \frac{1}{3}k\Psi \\ \dot\Theta^{\rm N}_2 = \frac{1}{5}k(-3\Theta^{\rm N}_3 + 2\Theta^{\rm N}_1) \\ \dot\Theta^{\rm N}_\ell = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta^{\rm N}_{\ell+1} + \ell\Theta^{\rm N}_{\ell-1}\right] \end{cases}$$

CDM, baryon

$$\begin{cases} \dot\delta_{\rm c} = -ikv_{\rm c} - 3\dot\Phi \\ \dot\delta_{\rm b} = -ikv_{\rm b} - 3\dot\Phi \\ \dot v_{\rm c} = -\mathcal{H}v_{\rm c} - ik\Psi \\ \dot v_{\rm b} = -\mathcal{H}v_{\rm b} - ik\Psi + \frac{\dot\tau}{R}(v_{\rm b} + 3i\Theta_{\rm T1}) \end{cases}$$

Gravity
(conformal Newtonian gauge)

$$\dot\Phi = -\frac{k^2}{3\mathcal{H}}\Phi + \mathcal{H}\Psi + \frac{\mathcal{H}_0^2}{2\mathcal{H}}\delta\Omega_0$$

Line-of-sight integral

Takashi Hiramatsu

Large Boltzmann hierarchy, say $\ell \lesssim 2000$, is required, but it is too hard to calculate...

Up to last-scattering surface,

$$\Theta^{\mathrm{T,P}}_{\ell+1} \sim \frac{k\eta}{2\tau}\Theta^{\mathrm{T,P}}_{\ell} \ll \Theta^{\mathrm{T,P}}_{\ell}$$

$$\Theta^{\mathrm{N}}_{\ell+1} \sim \frac{k\eta}{2}\Theta^{\mathrm{N}}_{\ell} \ll \Theta^{\mathrm{N}}_{\ell}$$

superhorizon scale

$$\frac{k}{\mathcal{H}} \approx k\eta \ll 1$$

tight-coupling

$$\mathcal{H} \ll \dot{\tau}$$

So, to guarantee the accuracy of $\Theta_{\ell\leq 2}, \Phi, \delta_b, \delta_c, v_b, v_c$

$$\ell_{\max} \sim 15$$

After LSS, we use the integral representation, *line-of-sight formula*.

Seljak, Zaldarriaga, APJ 469 (1996) 437

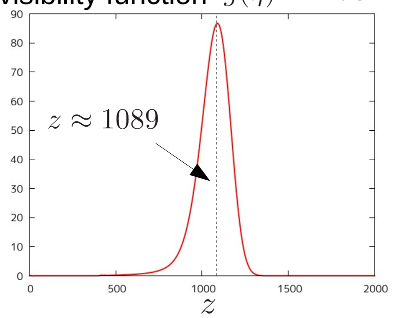
Line-of-sight formula

Takashi Hiramatsu

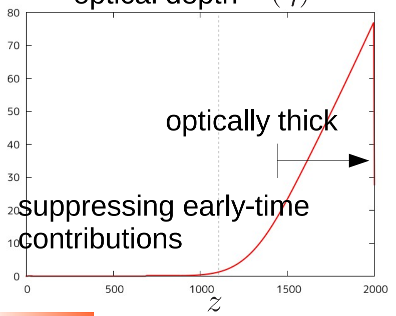
$$\Theta_\ell(k,\eta_0) = \int_0^{\eta_0} d\eta\, S(k,\eta) j_\ell[k(\eta_0-\eta)]$$

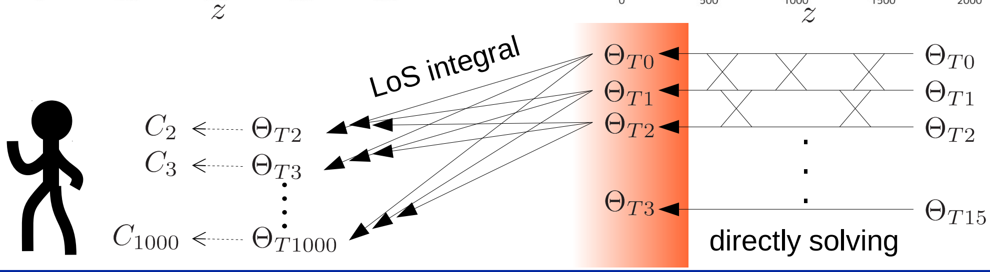
$$S(k,\eta) = g(\eta)\left[\Psi + \left(\Theta_0 + \frac{1}{4}\Pi\right)\right] + \frac{i}{k}\frac{d}{d\eta}[g(\eta)v_b(k,\eta)] + \frac{3}{4k^2}\frac{d^2}{d\eta^2}[g(\eta)\Pi]$$
$$+e^{-\tau}\left[\dot{\Psi}(k,\eta) - \dot{\Phi}(k,\eta)\right]$$

visibility function $g(\eta) = -\dot{\tau}e^{-\tau}$



optical depth $\tau(\eta)$

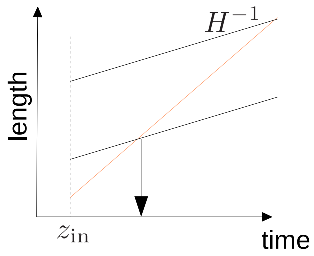




Initial conditions

YITP
Takashi Hiramatsu

Deep in radiation dominant epoch where all modes are larger than horizon scale. We set $z_{\rm in} = 1.44 \times 10^6$



Adiabatic initial condition

unchanged potential, superhorizon, similarly fluctuated radiation dominant

$$\delta_c = \delta_b = 3\Theta_{T0} = 3\Theta_{N0} = \zeta$$
$$v_b = v_c = -3\Theta_{T1} = -3\Theta_{N1} = \frac{k}{3\mathcal{H}}\zeta$$
$$\Phi = \frac{2}{3}\left(1 + \frac{2}{5}f_\nu\right)\zeta$$
$$\Psi = -\frac{10}{15 + 4f_\nu}\zeta$$

massless neutrino fraction : $f_\nu = \rho_\nu/\rho_R$

Separating primordial (quantum) curvature perturbation, we focus on the *transfer functions*,

$$\Phi(k,\eta) = \mathcal{T}_\Phi(k,\eta)\zeta(k,\eta_{\rm in})$$

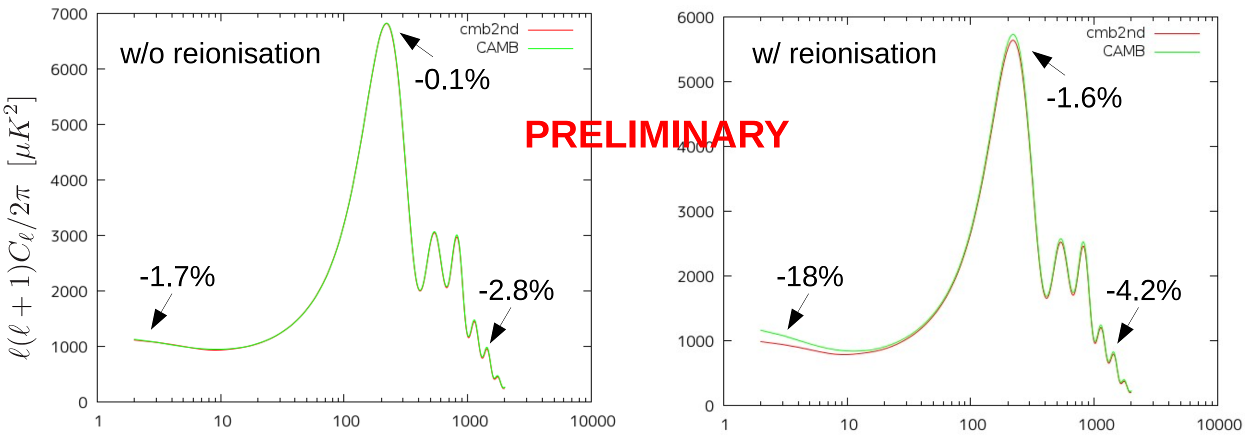
quantum
classical

Angular power spectrum (1st-order)

YITP
Takashi Hiramatsu

Angular power spectrum

$$C_\ell = \frac{2}{\pi}\int_0^\infty dk\, k^2 \mathcal{T}_{\Theta_\ell}(k,\eta)^2 P_\zeta(k,\eta_{\rm in})$$



existing codes

CMBFAST : Seljak, Zaldarriaga, APJ469 (1996) 437
CAMB : Lewis, Challinor, APJ538 (2000) 473
CLASS II : Blas, Lesgourgues, Tram, JCAP 1107 (2011) 034
CosmoLib : Huang, JCAP 1206 (2012) 012

$\Delta^2_\zeta(k_{\rm pivot}) = 2.46 \times 10^{-9}$ $\quad h^2\Omega_{\rm CDM} = 0.114$

$k_{\rm pivot} = 0.002\ {\rm Mpc}^{-1}$ $\quad h^2\Omega_{\rm B} = 0.0226$

$n_s = 0.96$ $\quad h = 0.7$

Go to 2nd-order

Takashi Hiramatsu

2nd-order contributions appear in...

- Einstein-Boltzmann equations for 2nd-order quantities sourced by [1st-order]^2

  CDM+Baryon+Gravity have been implemented,
  but Baryon-Photon/Gravity-Photon couplings are not considered yet.

  ...skip this topic in today's talk...

  `CMBquick` CMBquick : Creminelli, Pitrou, Vernizzi, arXiv:1109.1822
  `SONG` SONG : Pettinari, arXiv:1405.2280 (thesis)
  `CosmoLib2nd` CosmoLib2nd : Huang, Vernizzi, arXiv:1212.3573

- Line-of-sight formula sourced by [1st-order]^2

  Formulations of “curve”-of-sight have been completed by ...
  R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]
  (cf. Fidler, Koyama, Pettinari, arXiv:1409.2461)

2nd-order line(curve)-of-sight formula

Takashi Hiramatsu

R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]

$$\delta I^{(\mathrm{II})} = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \mathcal{T}^{(\mathrm{II})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{n}_{\mathrm{obs}}) \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)$$

$$\mathcal{T}^{(\mathrm{II})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{n}_{\mathrm{obs}}) = F(\mathbf{n}_{\mathrm{obs}}) \int_0^{\eta_0} d\eta' \, F_S(\hat{k}_1) S(k_1, \eta') e^{i\mathbf{k}_1 \cdot \mathbf{n}_{\mathrm{obs}}(\eta_0 - \eta')} \times \int d\eta_1 \, F_T(\hat{k}_2) T(k_2, \eta_1, \eta') e^{i\mathbf{k}_2 \cdot \mathbf{n}_{\mathrm{obs}}(\eta_0 - \eta_1)}$$

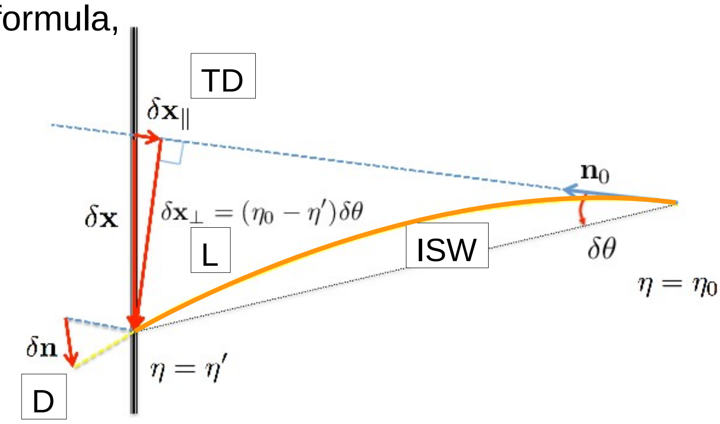
We found 7 combinations in this formula,

[Fluc. on LSS] x [gravitational]

- Source x ISW
- Source x Lensing
- Source x Time-delay
- Source x Deflection

[gravitational] x [gravitational]

- ISW x ISW
- ISW x Lensing
- ISW x Time-delay



2nd-order line(curve)-of-sight formula

Takashi Hiramatsu

Source x Lensing

Spergel, Goldberg, PRD59(1999)103001 [astro-ph/9811252]
Goldberg, Spergel, PRD59(1999)103002 [astro-ph/9811251]
Seljak, Zaldarriaga, PRD60(1999)043504 [astro-ph/9811123]
Planck collaboration, A&A 571(2014) A24 [arXiv:1303.5084]

$$S(k_1,\eta') = 4k_1 g(\eta')\left[\Theta_{T0}+\Psi\right] + 4\frac{d}{d\eta'}\left(\frac{g(\eta')v_b}{k_1}\right) + 4\mathcal{P}_2\left(\frac{1}{ik_1}\frac{d}{d\eta'}\right)\left[g(\eta')\Pi\right]$$

$$T(k_2,\eta_1,\eta') = k_2(\eta_1-\eta')\left[\Psi(k_2,\eta_1)-\Phi(k_2,\eta_1)\right]$$

$$F(\mathbf{n}_{\rm obs})F_S(\hat{k}_1)F_T(\hat{k}_2) = -\sum_{\lambda=\pm}(i\epsilon^\lambda\cdot\hat{k}_1)(i\epsilon^\lambda\cdot\hat{k}_2)$$

Bispectrum

Spin-weighted Gaunt integral

$$B^{m_1m_2m_3}_{\ell_1\ell_2\ell_3} = 2[1+(-1)^{\ell_1+\ell_2+\ell_3}]\mathcal{G}^{m_1m_2m_3;(+1)(-1)0}_{\ell_1\ell_2\ell_3}\int_0^{\eta_0} d\eta'\, b^S_{\ell_1}(\eta')b^T_{\ell_2}(\eta') + 5 \text{ perms.}$$

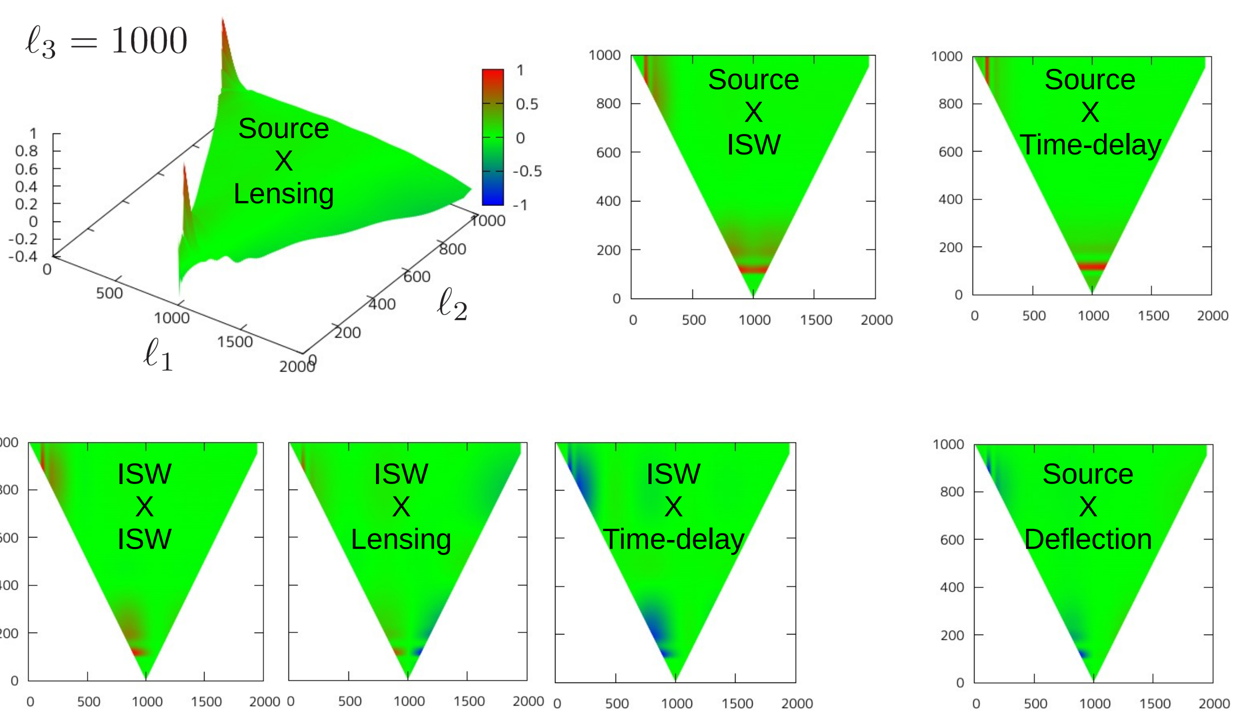
$$b^S_{\ell_1}(\eta') = \frac{2}{\pi}\sqrt{\frac{\ell_1(\ell_1+1)}{2}}\int dk_1\, k_1^2 P_\zeta(k_1)\mathcal{T}_{\Theta_{\ell_1}}(k_1)\frac{S(k_1,\eta')}{k_1(\eta_0-\eta')}j_{\ell_1}[k_1(\eta_0-\eta')]$$

$$b^T_{\ell_2}(\eta') = \frac{2}{\pi}\sqrt{\frac{\ell_2(\ell_2+1)}{2}}\int_{\eta'}^{\eta_0} d\eta_1\int dk_2\, k_2^2 P_\zeta(k_2)\mathcal{T}_{\Theta_{\ell_2}}(k_1)\frac{T(k_2,\eta_1,\eta')}{k_2(\eta_0-\eta_1)}j_{\ell_2}[k_2(\eta_0-\eta_1)]$$

All 7 combinations have been implemented in my code.

Shape of bispectra

Takashi Hiramatsu



Bispectrum templates

Takashi Hiramatsu

Bispectrum

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3 B_\zeta(k_1,k_2,k_3)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

Bartolo et al., Phys.Rep.402(2004)103 [arXiv:astro-ph/0406398]

Amplitude and shape of bispectrum

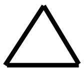
Amplitude parametrised by $f_{\rm NL}$

local-type

$$\zeta(\mathbf{x}) = \zeta_L(\mathbf{x}) + f_{\rm NL}(\zeta_L^2 - \langle\zeta_L^2\rangle)$$

$$\longrightarrow B_\zeta^{\rm local} = 2 f_{\rm NL}^{\rm local}\left[P_\zeta(k_1)P_\zeta(k_2) + 2\ {\rm perms}\right] \quad \left(P_\zeta(k) \propto \frac{1}{k^{4-n_s}}\right)$$

equilateral-type



$$B_\zeta^{\rm equil} = 6 f_{\rm NL}^{\rm equil}\left[-P_\zeta(k_1)P_\zeta(k_2) + 2\ {\rm perms} + \left\{P_\zeta(k_1)^{1/3}P_\zeta(k_2)^{2/3}P_\zeta(k_3) + 5\ {\rm perms}\right\}\right]$$

orthogonal-type

$$B_\zeta^{\rm ortho} = 6 f_{\rm NL}^{\rm ortho}\left[-3P_\zeta(k_1)P_\zeta(k_2) + 2\ {\rm perms} + 3\left\{P_\zeta(k_1)^{1/3}P_\zeta(k_2)^{2/3}P_\zeta(k_3) + 5\ {\rm perms}\right\}\right]$$

+ a variety of non-separable types

Planck collaboration, A&A 571(2014) A24 [arXiv:1303.5084]

Fitting bispectrum to templates

Takashi Hiramatsu

Using the least-square method, we determine the fitting parameter $f_{\rm NL}^{(i)}$ so that

$$\chi^2 \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3}^{\ell_{\rm max}} \frac{\left(B_{\ell_1\ell_2\ell_3} - \sum_i f_{\rm NL}^{(i)} B_{\ell_1\ell_2\ell_3}^{(i)}\right)^2}{\sigma_{\ell_1\ell_2\ell_3}^2}$$

is minimised.

local-type
equilateral-type
orthogonal-type

$$F^{ij} \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3} \frac{B_{\ell_1\ell_2\ell_3}^{(i)} B_{\ell_1\ell_2\ell_3}^{(j)}}{\sigma_{\ell_1\ell_2\ell_3}^2}$$

$$G^j \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3} \frac{B_{\ell_1\ell_2\ell_3} B_{\ell_1\ell_2\ell_3}^{(j)}}{\sigma_{\ell_1\ell_2\ell_3}^2}$$

$$f_{\rm NL}^{(i)} = (F^{-1})^{ij} G^j$$

Komatsu, Spergel, PRD63 (2001) 063002

$f_{\rm NL}$

PRELIMINARY

(The following values would chang through bug-fixing)

	Local	Equilateral	Orthogonal
Source x ISW	-0.082	1.7	-0.37
Source x Lensing	3.4	220	-170
Source x Time-delay	21	-49	79
Source x Deflection	0.019	-3.2	2.0
ISW x ISW	0.000030	0.68	0.20
ISW x Lensing	0.029	3.5	-2.2
ISW x Time-delay	-0.052	-0.43	-0.043

Planck's analysis : $f_{\rm NL}^{\rm lensing} \sim 7 \longrightarrow f_{\rm NL} \sim 2.7 \pm 5.8$

"Source x Time-delay" seems to be larger than expected. Many bugs still stay in my code ... ?

Here is the frontier of my code development..

Summary : overview of my code

- Full scratch development, completely independent of existing codes
- C++
- Parallelised by OpenMP
- Time evolution : 1-stage 2nd-order implicit Runge-Kutta (Gauss-Legendre) method (implementing up to 4th-order schemes)
- Line-of-sight Integration : Trapezoidal/Simpson's rule
- Interpolation scheme : Polynomial approximation (up to $\mathcal{O}(h^5)$)
- Ready for implementing a variety of recombination/reionisation simulators
- Fast evaluation of spherical Bessel functions, and (specific) Gaunt integrals

Summary : current status

- 1st-order looks fine ! (except for reionisation)

- We implemented 2nd-order Boltzmann equations only for gravity and matter. (skipped today)

- We implemented “curve”-of-sight formulas (2nd-order line-of-sight) for scalar contributions of temperature fluctuations.

- Bispectrum estimator has been implemented, and preliminary results are obtained. But, many bugs still stay in my code ... ?

Summary : to-do and application ?

To-do

- Check the reionisation models used in Boltzmann solver

- Implement pure 2nd-order Boltzmann equations for radiation

- Bug-fixing of bispectrum estimator

Applications ?

- 2nd-order gravitational waves, magnetic field from $[\text{1st-order}]^2$

- curve-of-sight for polarisation

- curve-of-sight for [Scalar] x [Tensor] & [Tensor] x [Tensor]

- y-distortion to photon's distribution function ?

“Studying the inflationary Universe with gravitational waves”

Tomo Takahashi

[JGRG24(2014)111403]

Studying the inflationary Universe with gravitational waves

Tomo Takahashi
(Saga University)

JGRG24, Kavli IPMU
14 November, 2014

Ref: Ryusuke Jinno, Takeo Moroi, TT 1406.1666, JCAP

What we want to know about the inflationary Universe

- What is the inflaton?
  - Shape of the potential?
  - Structure of the kinetic term?
  - Number of fields?
- What is the origin of density fluctuations?
  - Inflaton?
  - Some other field (e.g., curvaton)?
- What is the thermal history after inflation?
  - Reheating temperature?

⋮

Probes of the inflationary Universe

- Primordial scalar fluctuations

  -- Power spectrum (amplitude, scale-dependence)

  -- Non-Gaussianity (bispectrum, trispectrum)

- Primordial tensor fluctuations (Gravitational waves)

$$ds^2 = -dt^2 + a(t)^2 \left[1 + h_{ij}\right] dx^i dx^j$$

$$h_{ij} = \sqrt{8\pi G} \sum_{A=+,\times} \int d^3k \; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} h_k(t) e^A_{ij}(\boldsymbol{n})$$

$$\rightarrow \langle h_{k_1} h_{k_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \mathcal{P}_T(k_1)$$

Probes of the inflationary Universe

- Primordial scalar fluctuations

  -- Power spectrum (amplitude, scale-dependence)

  -- Non-Gaussianity (bispectrum, trispectrum)

- Primordial tensor fluctuations (Gravitational waves)

  -- Tensor power spectrum (amplitude, scale-dependence)

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}}\right)^{n_T}$$

Tensor power spectrum

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}}\right)^{n_T}$$

- Tensor-to-scalar ratio $r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta}$

Planck:

$$r < 0.11 \ (95\ \%\ \text{C.L.})$$

[Planck, Ade et al.1303.5082]

BICEP2:

$$r = 0.20^{+0.07}_{-0.05} \quad (68\%\ \text{C.L.})$$

[Ade et al, BICEP2 1403.3985]

Tensor power spectrum

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}}\right)^{n_T}$$

- Tensor spectral index

BICEP2 alone:

$$n_T = 1.36 \pm 0.83 \quad (68\%\ \text{C.L.})$$

BICEP2 +TT prior:

$$n_T = 1.67 \pm 0.53 \quad (68\%\ \text{C.L.})$$

[Gerbino et al. 1403.5732]

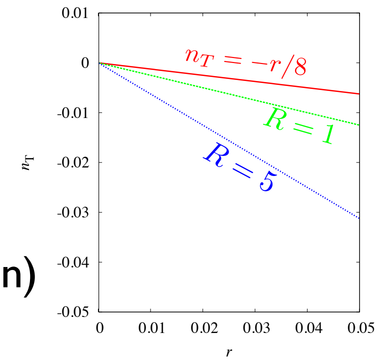
Observables of gravitational waves

- Amplitude
  - → probes the energy scale of inflation
- Spectral index n_T
  - → probes the dynamics of inflation
  - → checks the consistency relation

Consistency test of inflation

- Standard inflation model

$$n_T = -2\epsilon$$
$$r = 16\epsilon$$
$$\rightarrow \quad n_T = -\frac{r}{8}$$



- Curvaton model (inflaton+curvaton)

$$n_T = -2\epsilon$$
$$r = \frac{16\epsilon}{1+R}$$
$$\rightarrow \quad n_T = -(1+R)\frac{r}{8}$$

$$R \equiv \frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{(\phi)}}$$

$R \to 0$: (pure) inflaton case

$R \to \infty$: (pure) curvaton case

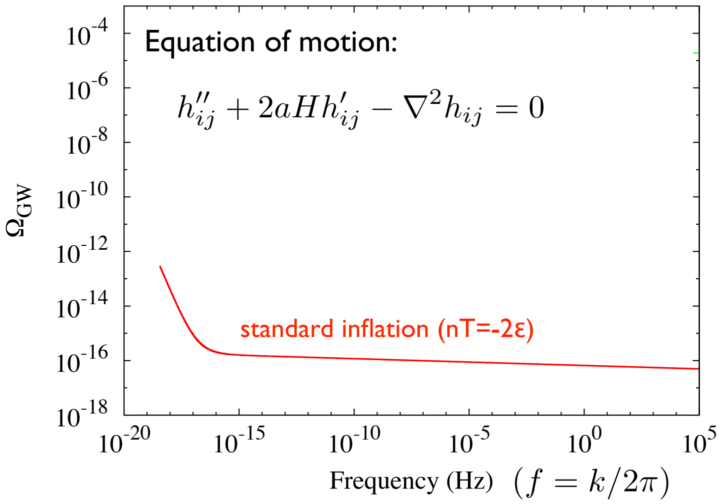
→ Tensor scale-dependence should give important test of the inflationary Universe

Observables of gravitational waves

- Amplitude
  - → probes the energy scale of inflation
- Spectral index n_T
  - → probes the dynamics of inflation
  - → checks the consistency relation
- Reheating temperature T_R

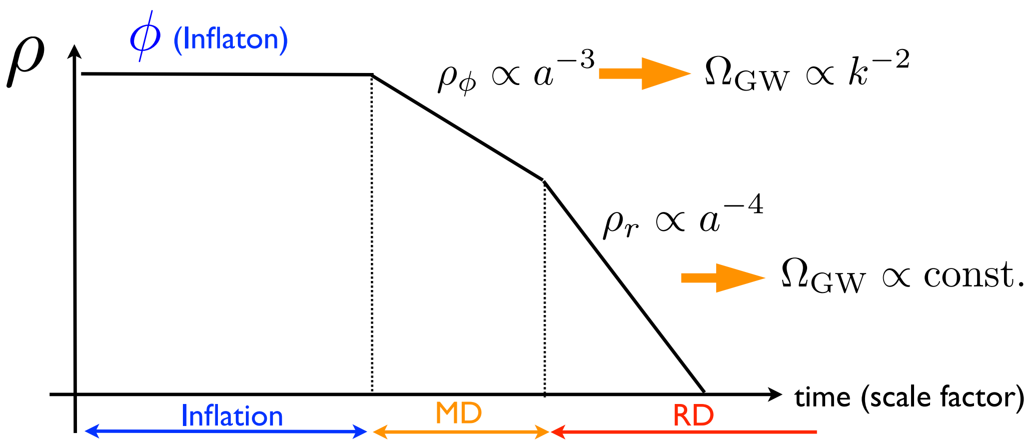
GW spectrum: $\Omega_{\rm GW}(k) \equiv \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}}{d\ln k}$

$$\rho_{\rm GW} = \frac{1}{64\pi G a^2}\left\langle (\partial_\tau h_{ij})^2 + (\nabla h_{ij})^2 \right\rangle$$



GW spectrum and thermal history

- Taking into account the thermal history (e.g., reheating after inflation), GW spectrum changes.



GW spectrum and thermal history

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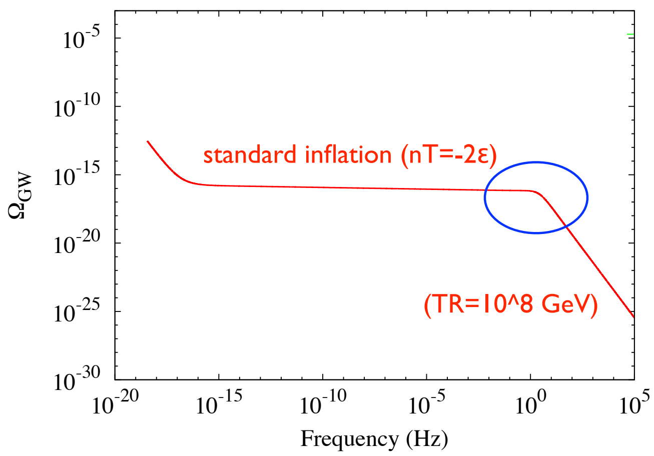
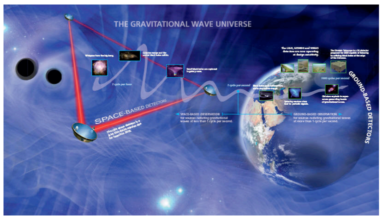


How can we observe GWs?

- We consider here future space-based interferometer direct detection experiments such as DECIGO.
  - Future space-based exp. are sensitive at f ~ 1 Hz.
  - We may be able to probe nT very precisely.
  - We may be able to probe T_R.

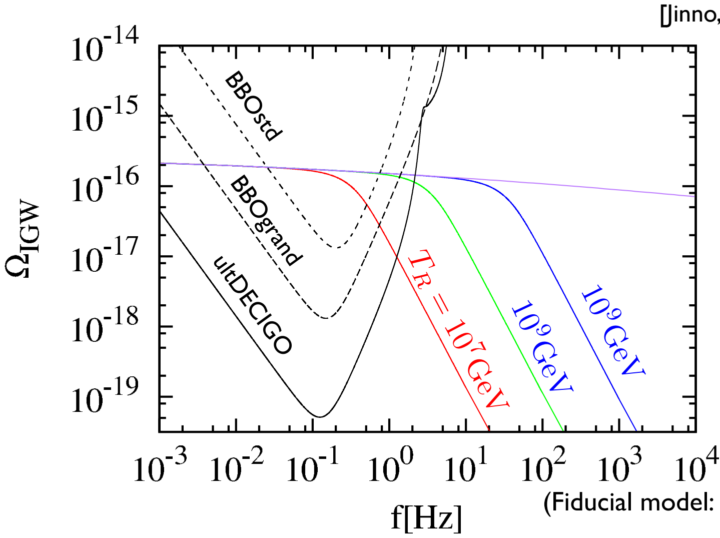


[http://www.personal.soton.ac.uk/nils/rsweb/thefacts.htm]

Probing inflation with future direct detection of GWs

- Future space-based exp. are sensitive at f ~ 1 Hz.

[Jinno, Moroi, TT 1406.1666]



(Fiducial model: quadratic chaotic inflation)

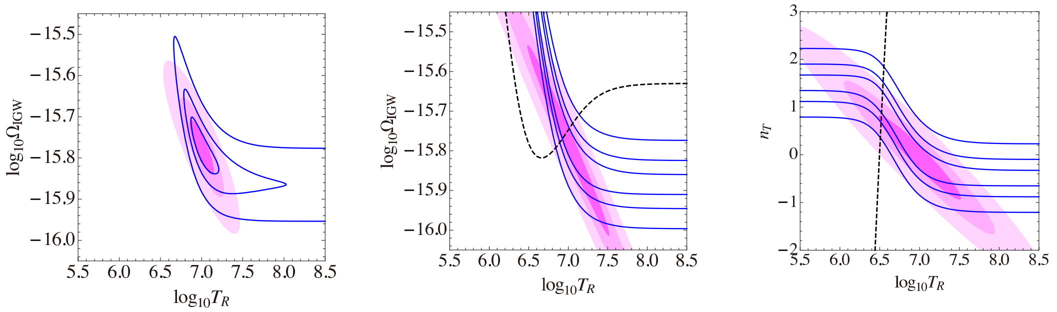
Expected constraints

[Jinno, Moroi, TT 1406.1666]

(See also Kuroyanagi, Nakayama, Yokoyama 1410.6618)

- Fiducial model: TR = 10^7 GeV

(BBOstd is assumed.)



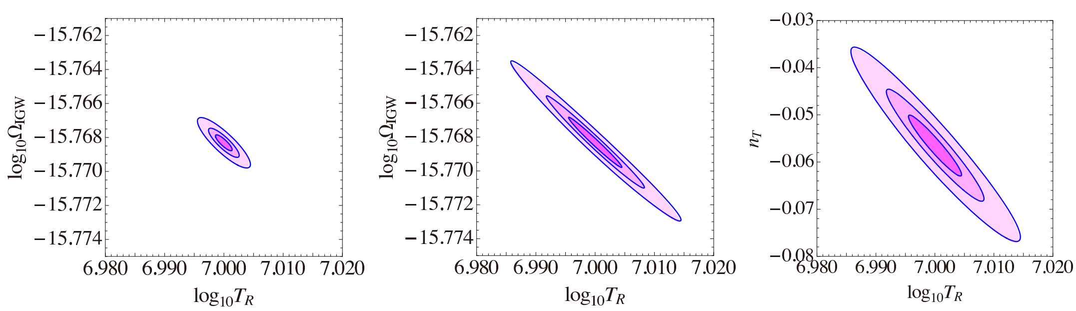
(nT fixed) (nT marginalized) ($\Omega_{\rm IGW}$ marginalized)

(Blue: $\Delta\chi$^2=5.99, $T_{\rm obs}$ =1,3,10 yrs)

(Fiducial model: quadratic chaotic inflation)

Expected constraints from ultDECIGO

- Fiducial model: TR = 10^7 GeV



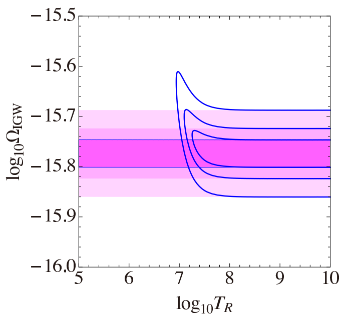
(nT fixed) (nT marginalized) ($\Omega_{\rm IGW}$ marginalized)

(Fiducial model: quadratic chaotic inflation)

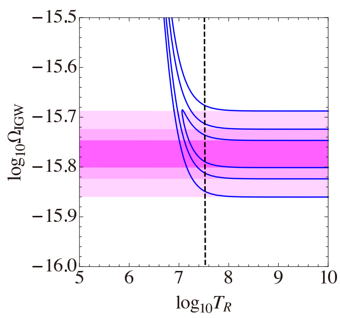
Expected constraints

- Fiducial model: TR = 10^9 GeV

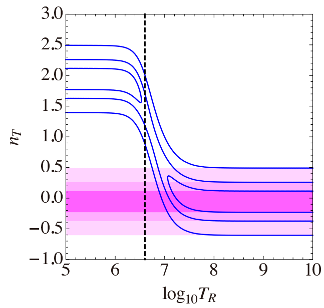
(BBOstd is assumed.)



(nT fixed)



(nT marginalized)

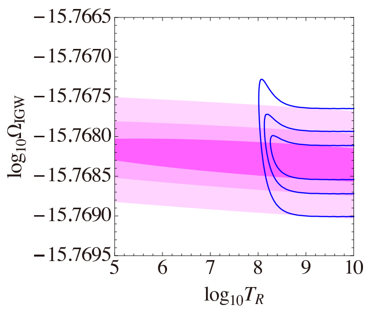


(Ω_{IGW} marginalized)

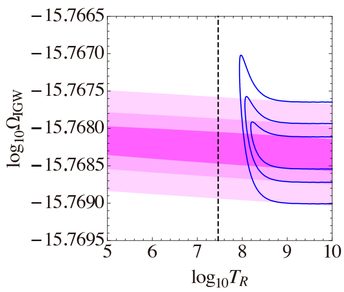
(Fiducial model: quadratic chaotic inflation)

Expected constraints from ultDECIGO

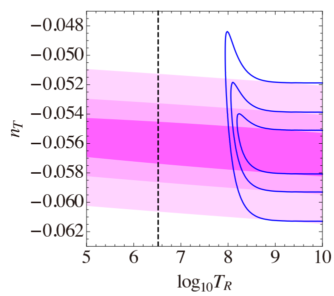
- Fiducial model: TR = 10^9 GeV



(nT fixed)



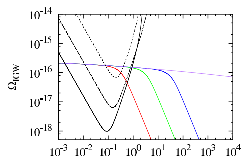
(nT marginalized)



(Ω_{IGW} marginalized)

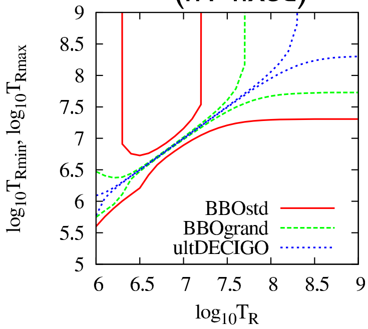
(Fiducial model: quadratic chaotic inflation)

Reheating temperature

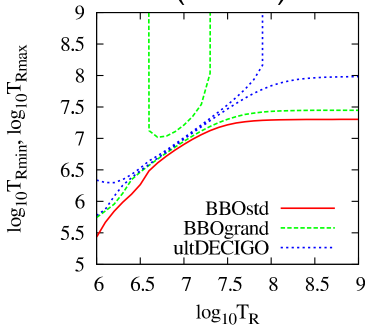


- We may be able to obtain upper/lower bound for T_R

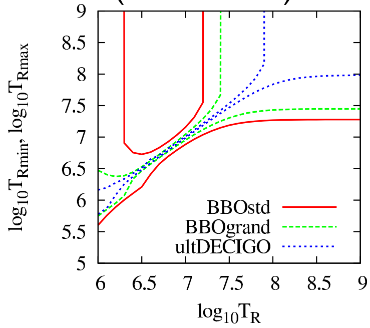
(nT fixed)



(nT < 0)



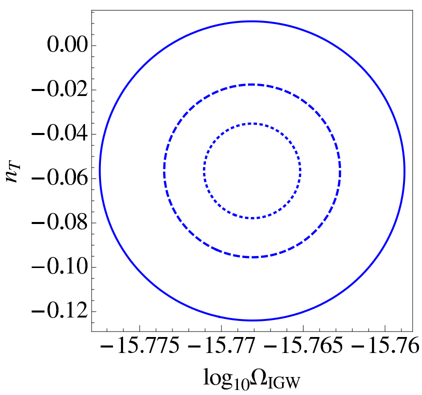
(-1 < nT < 0)



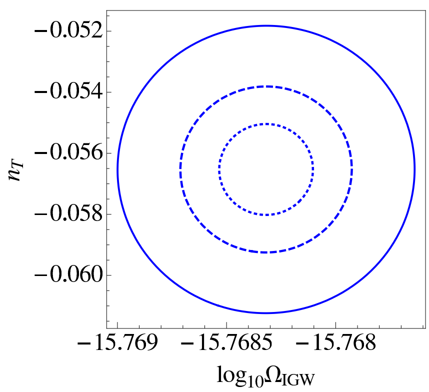
(Fiducial model: quadratic chaotic inflation)

Expected sensitivity: Amp. and nT

BBOstd



ultDECIGO



1 yr

3 yrs

10 yrs

(Fiducial model: quadratic chaotic inflation)

(We have chosen the pivot scale such that correlation between Omega_GW and nT vanishes.)

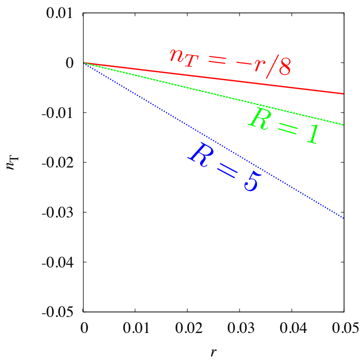
Consistency test of inflation: yet another test?

- Standard inflation model

$$n_T = -2\epsilon$$
$$r = 16\epsilon$$
$\rightarrow$ $n_T = -\frac{r}{8}$

- Curvaton model

$$n_T = -2\epsilon$$
$$r = \frac{16\epsilon}{1+R}$$
$\rightarrow$ $n_T = -(1+R)\frac{r}{8}$



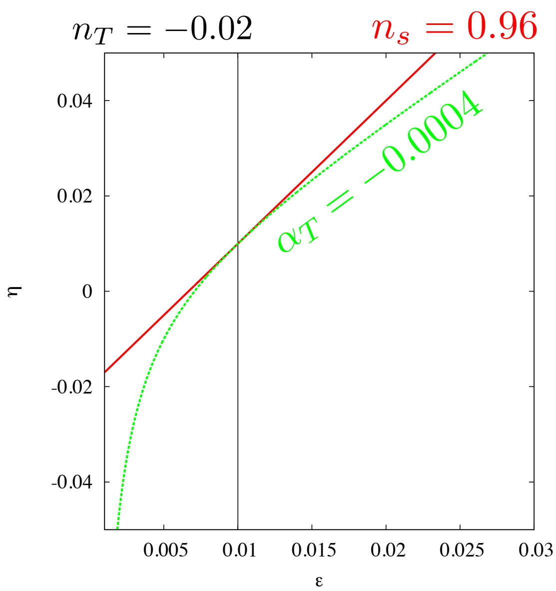
$$R \equiv \frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{(\phi)}}$$

R→0: (pure) inflaton case

R→∞: (pure) curvaton case

→ Tensor scale-dependence should give important test of the inflationary Universe

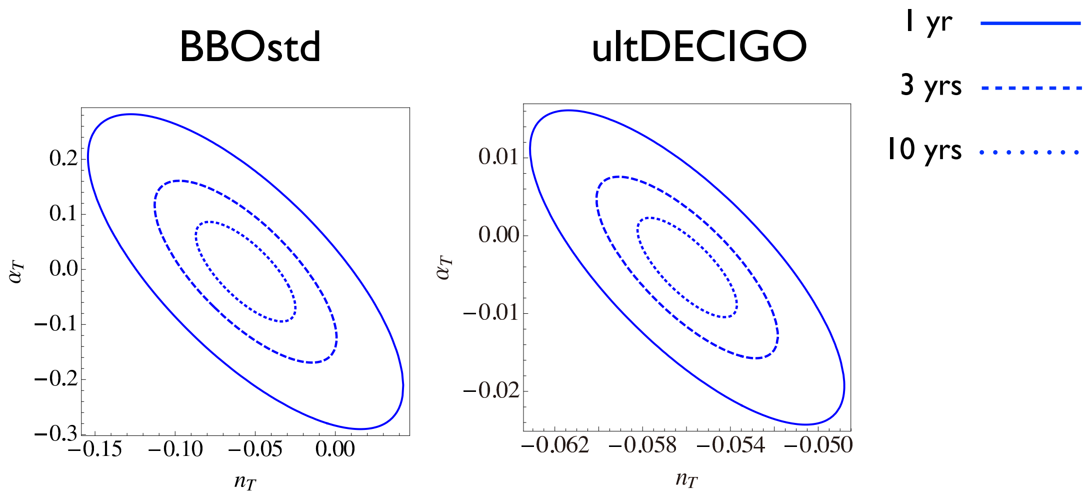
Yet another consistency test?



$$n_s = 1 - 6\epsilon + 2\eta$$
$$n_T = -2\epsilon$$
$$\alpha_T = -4\epsilon(2\epsilon - \eta)$$

If three lines cross at one point, we can check the consistency of the predictions.

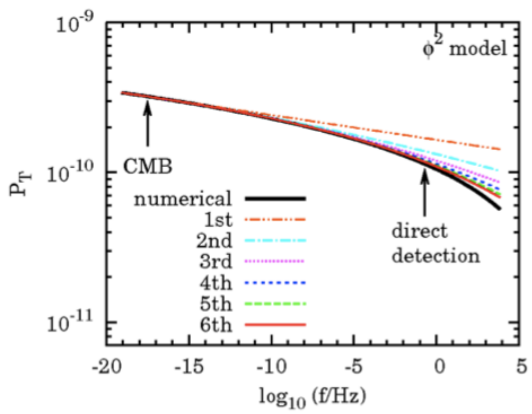
Expected sensitivity: nT and αT



(Fiducial model: quadratic chaotic inflation)

CMB+spaced-based GW exp.?

- CMB and future space-based GW scales are so different. Need some care when we use both CMB and direct detection GW obs.



[Kuroyanagi, TT 2011]

Summary

- Gravitational waves would be very useful to probe the inflationary Universe.

  (If confirmed to be sizable amplitude in any observations.)

- Probing the tensor spectral index would give crucial consistency test of the inflationary Universe

- Future direct interferometer experiments may probe the rehearing temperature.

- Even higher order scale-dependence of GW may be probed and it would give an important test of the inflationary prediction.

“Instabilities of extremal black holes in higher dimensions”

Akihiro Ishibashi

[JGRG24(2014)111404]

Instabilities of extremal black holes in higher dimensions

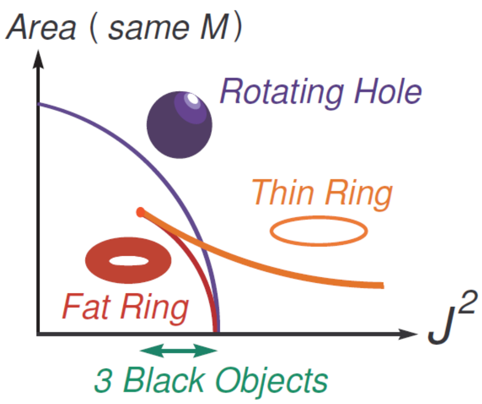
Akihiro Ishibashi
(Kinki University)

JGRG-14 Nov. 2014 at IPMU

based on arXiv:1408.0801 w/ S. Hollands

BH Classification problem in Higher Dimensions

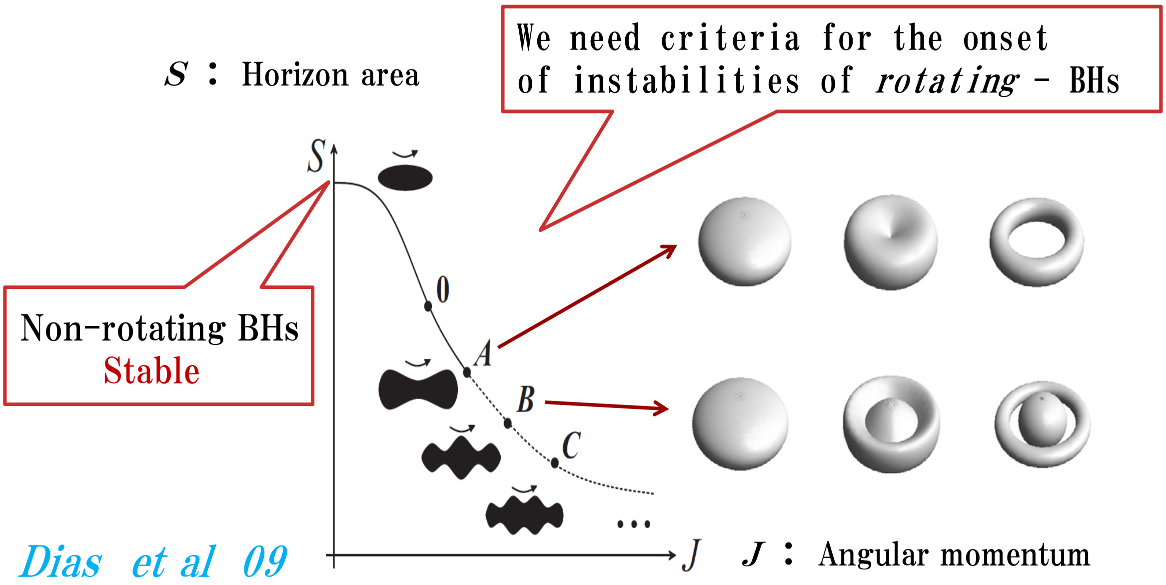
- *No uniquness* in D>4 GR
- Classification of them is yet under way



To classify ⇨ need to study their stability

Instabilities are signals of bifurcation to something different, implying more variety of solutions.

Phase space of Higher Dimensional BHs

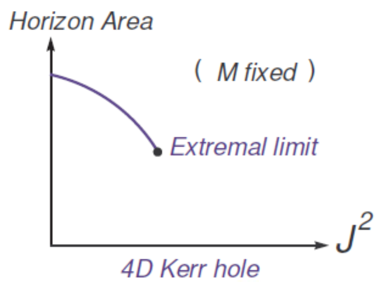


Instability ⇨ Bifurcation: New branch of solutions

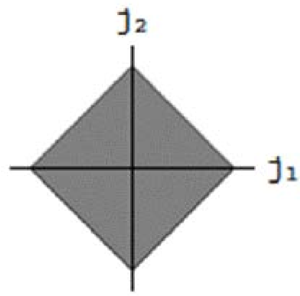
However, the phase space is so large…

Start wih classifying *Extremal* black holes

- Limit of zero Hawking temperature
  $T_H \to 0$



- Play an important role in various contexts
  e.g. Supergravity
  Entropy counting
  Kerr/CFT



5D Myers-Perry hole

- *Boundary* of the space of all BHs

Classify "boundaries of the solution space" from the stability view point

Stability analyses

- Linear perturbation analysis
- Nice to have master equations,

  e.g., Teukolsky equations in 4D

- Unfortunately there is *no* Teukolsky type master equation for higher dimensional (extremal/non-extremal) black holes

To classify extremal black holes …

- Helpful to study *"near-horizon geometries"* which

  * arise as a scaling limit of extremal black hole
  * satisfy the same dynamics
  * possess more symmetries
  * admit Teukolsky type master equations

Near Horizon Geometry (NHG)

$$ds^2 = 2dud\rho - \rho^2\alpha du^2 - 2\rho du\beta_A dx^A + \mu_{AB}dx^A dx^B$$

- Diffeomorphism

$$(u, \rho, x^A) \mapsto \left(\frac{u}{\epsilon}, \epsilon\rho, x^A\right)$$

- Scalimg limit $\epsilon \to 0$

α, β_A, μ_{AB} become functions of x^A

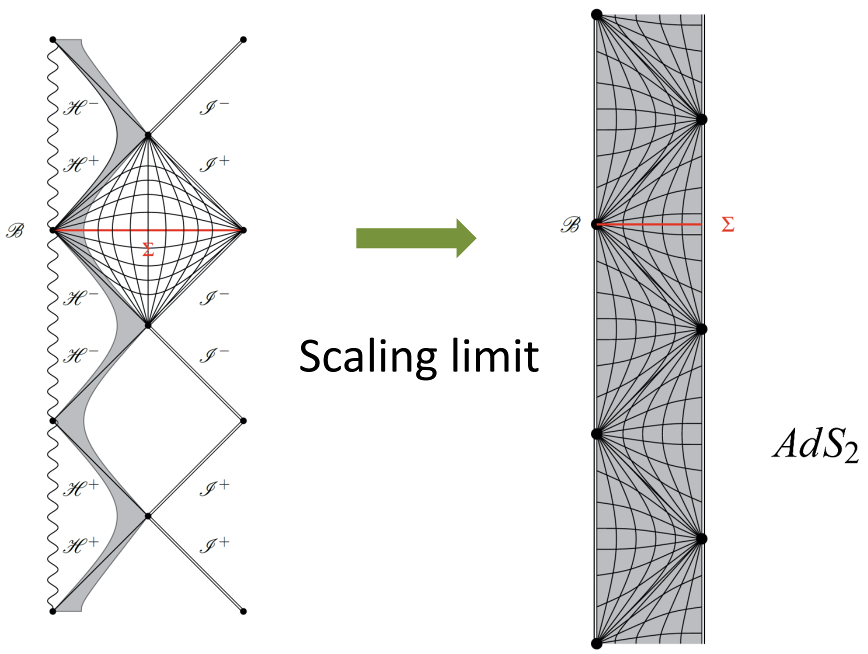
$$\mathrm{d}s^2 = L^2\,\mathrm{d}\hat{s}^2 + g_{IJ}(\mathrm{d}\phi^I + k^I\,\hat{A})(\mathrm{d}\phi^J + k^J\,\hat{A}) + \mathrm{d}\sigma^2_{d-n-2}$$

AdS_2 $\mathrm{d}\hat{s}^2 = -R^2\mathrm{d}T^2 + \frac{\mathrm{d}R^2}{R^2}$

$\hat{A} = -R\mathrm{d}T$

Near-Horizon scaling

Carter 72



A horizon neighborhood of Extreme black hole

Near-Horizon Geometry

Durkee & Reall conjectured that

When axi-symmetric perturbations on the NHG violate AdS_2 -BF-bound on the NHG, then the original extremal BH is unstable

… based on numerical results:

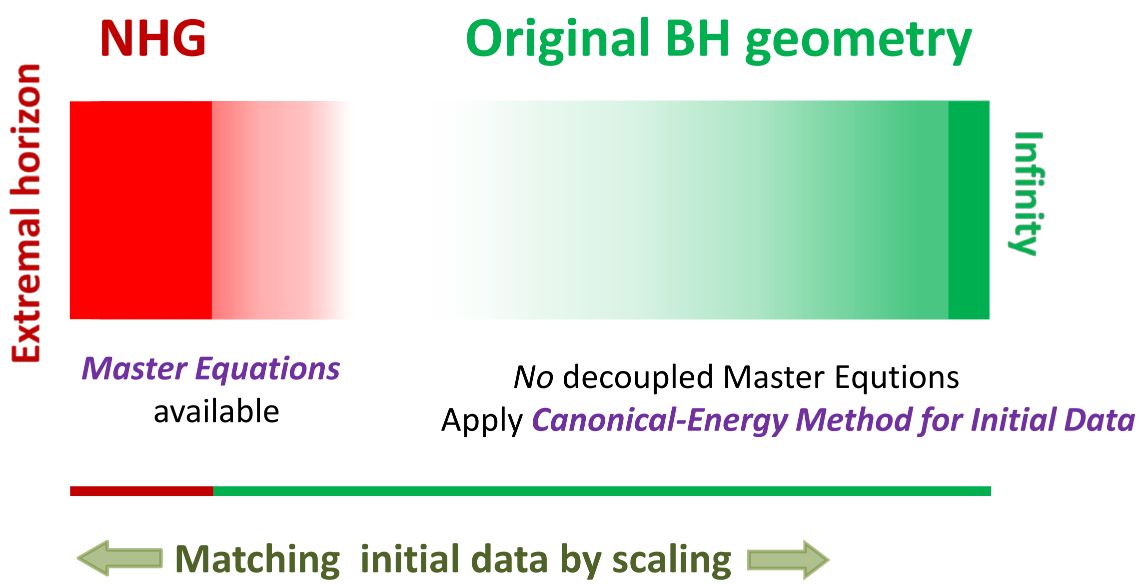
Durkee-Reall 11

Purpose

We show this conjecture by using

- Hertz-potential
- Canonical energy method
- Initial data correction

Strategy for proving DR Conjecture



Attempt to construct ***negative*** energy initial data **on NHG**

Construct ***negative*** canonical Energy for **the BH initial data**

Decoupled Master equations on NHG

Thanks to high symmetry of NHG, metric perturbations γ_{ab} is written in terms of Hertz potential

$$U^{AB} = \psi \cdot Y^{AB} \cdot \exp(i\underline{m} \cdot \underline{\phi})$$

that obeys AdS_2 Klein-Gordon equation

$$-\frac{1}{R^2}\frac{\partial^2 \psi}{\partial T^2} + \frac{\partial}{\partial R}\left(R^2 \frac{\partial \psi}{\partial R}\right) - \frac{2iq}{R}\frac{\partial \psi}{\partial T} - \lambda \psi = 0$$

$$\mathscr{A} Y = \lambda Y \,, \qquad \pounds_{\partial/\partial \phi^I} Y = 0$$

where $\mathscr{A}$ is 2nd-order operator on the horizon section

We show the following theorem:

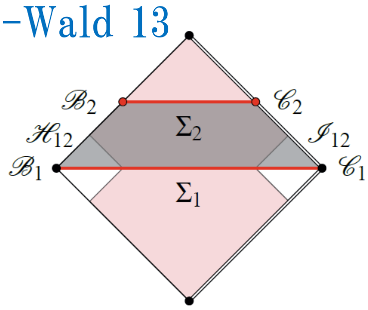
If the eigenvalue of $\mathscr{A}$ violates the effective BF-bound

$$\lambda < -\frac{1}{4}$$

then the original extremal black hole is unstable

Canonical energy for initial data

Hollands-Wald 13



Symplectic current

$$w^a = \frac{1}{16\pi} P^{abcdef}(\gamma_{2bc}\nabla_d\gamma_{1ef} - \gamma_{1bc}\nabla_d\gamma_{2ef})$$

Symplectic form $W(\Sigma;\gamma_1,\gamma_2) \equiv \int_\Sigma \star w(g;\gamma_1,\gamma_2)$

Canonical energy $\mathscr{E}(\Sigma,\gamma) \equiv W(\Sigma;\gamma,\pounds_K\gamma) - B(\mathscr{B},\gamma) - C(\mathscr{C},\gamma)$

1) $\mathscr{E}$ is gauge invariant

$$B(\mathscr{B},\gamma) = \frac{1}{32\pi}\int_{\mathscr{B}} \gamma^{ab}\delta\sigma_{ab}$$

2) $\mathscr{E}$ is monotonically decreasing
for any axi-symmetric perturbation

$$C(\mathscr{C},\gamma) = -\frac{1}{32\pi}\int_{\mathscr{C}} \tilde{\gamma}^{ab}\delta\tilde{N}_{ab}$$

Construction of a perturbaton with negative canonical energy

- For initial data: $(f_0, f_1) \equiv \left(\psi\big|_{T=0}, \frac{\partial}{\partial T}\psi\big|_{T=0}\right)$

$$f_0(R) = \frac{R^N}{(R+\varepsilon)^{N+1/2}(1+R^N e^{1/(1-R)})}\,, \qquad f_1(R) = 0$$

for $0 < R < 1$ and $f_0(R) = 0$ for $R \geq 1$

$$\mathscr{E} = \frac{1}{8\pi}\underline{(\lambda + \tfrac{1}{4})}(\lambda^2 + 2a^2\lambda + a^4 - 9a^2 + \tfrac{7}{2})\ \log\varepsilon^{-1} + O(1)$$

where $a = \underline{k}\cdot\underline{m}$

If $a = 0$ $\mathscr{E} < 0$ for $\lambda < -\frac{1}{4}$ and sufficiently small $\varepsilon > 0$

This energy expression holds only on NHG,
not on the original BH geometry.

One can *correct it to hold on the original BH geometry* by using Corvino-Schoen's method.

Corvino-Schoen 03

Summary

- We have proven Durkee-Reall conjecture that *extremal black holes are unstable when the eigenvalue* λ *of the operator* $\mathscr{A}$ *is less than the effective BF bound* $-1/4$

  The stability analysis is thus reduced to an *analysis on the horizon cross-section*, which is a *much simpler* problem than analyzing the perturbed Einstein equations.

- Our proof uses
  (i) *Canonical energy method*
  (ii) *Symmetry of the NHG and Hertz potential*
  (iii) *Structure of the constraint equations*

- Our method is applicable to rotating *extremal AdS* BHs and also for *near-extremal* BHs

"Stellar oscillations in Eddington-inspired Born-Infeld gravity"

Hajime Sotani

[JGRG24(2014)111405]

Stellar oscillations in Eddington-inspired Born-Infeld gravity

Hajime SOTANI (NAOJ)

alternative theory & EiBI

- general relativity has been successful in explaining the phenomena and experiments in weak field-regime
  - the tests of general relativity in strong field-regime are still quite poor
  - several modified gravitational theories are proposed
- Eddington-inspired Born-Infeld (EiBI) gravity (Banados & Ferreira 2010)
  - avoid the big bang singularity
  - based on
    the Eddington action & the nonlinear electrodynamics of Born-Infeld
  - the metric and the connection are considered as independent fields, as in the Palatini-type approach to GR
  - EiBI can deviate from GR only when the matter exists

field eqs. in EiBI

- action

$$S = \frac{1}{16\pi}\frac{2}{\kappa}\int d^4x\left(\sqrt{|g_{\mu\nu}+\kappa R_{\mu\nu}|} - \lambda\sqrt{-g}\right) + S_{\rm M}[g, \Psi_{\rm M}]$$

  - the limit of S_M=0 or κ=0 reduces to the Einstein-Hilbert action →EiBI in such limits coincides with general relativity

- parameters in theory
  - λ: Λ=(λ-1)/κ
    → λ=1 to focus on the relativistic stars with asymptotically flatness
  - κ: the dimension of length squared

- field equations

an auxiliary metric

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}q^{\mu\sigma}(q_{\sigma\alpha,\beta} + q_{\sigma\beta,\alpha} - q_{\alpha\beta,\sigma}),$$

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu},$$

$$\sqrt{-q}q^{\mu\nu} = \sqrt{-g}g^{\mu\nu} - 8\pi\kappa\sqrt{-g}T^{\mu\nu}$$

standard energy-momentum tensor with indices raised with $g_{\mu\nu}$

spherically symmetric objects

- metric

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu}dt^2 + e^{\lambda}dr^2 + f(d\theta^2 + \sin^2\theta d\phi^2),$$

$$q_{\mu\nu}dx^\mu dx^\nu = -e^{\beta}dt^2 + e^{\alpha}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

- assume the perfect fluid

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu}$$

- TOV eqs. in EiBI

$$m' = \frac{r^2}{4\kappa}\left[\frac{a}{b^3} - \frac{3}{ab} + 2\right],$$

$$e^{-\alpha} = 1 - \frac{2m}{r}, \quad \nu' = -\frac{2p'}{\epsilon + p}$$

+ equation of state (EOS)
→ one can determine the stellar models in EiBI

$$p' = -e^{\alpha}\left[\frac{2m}{r^2} + \frac{r}{2\kappa}\left(\frac{a}{b^3} + \frac{1}{ab} - 2\right)\right]\left[\frac{2}{\epsilon + p} + 4\pi\kappa\left(\frac{3}{b^2} + \frac{1}{a^2c_s^2}\right)\right]^{-1}$$

  - where $a \equiv \sqrt{1 + 8\pi\kappa\epsilon}$ and $b \equiv \sqrt{1 - 8\pi\kappa p}$

constraint on κ

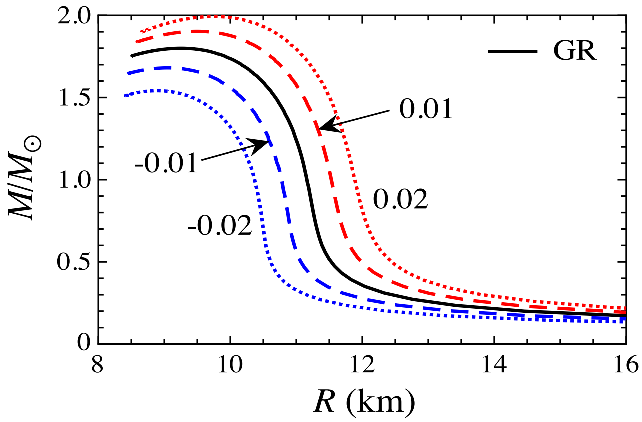
- the constraints on κ are discussed in term of the solar observations, big bang nucleosynthesis, and the existence of neutron stars

$$|\kappa| \lesssim 1 \ \mathrm{m^5 kg^{-1} s^{-2}}$$ (Pani et al 2011)

- we adopt $8\pi\varepsilon_\circ\kappa$ as a normalized constant
  → $8\pi\varepsilon_\circ|\kappa| \lesssim 75$

stellar models in EiBI

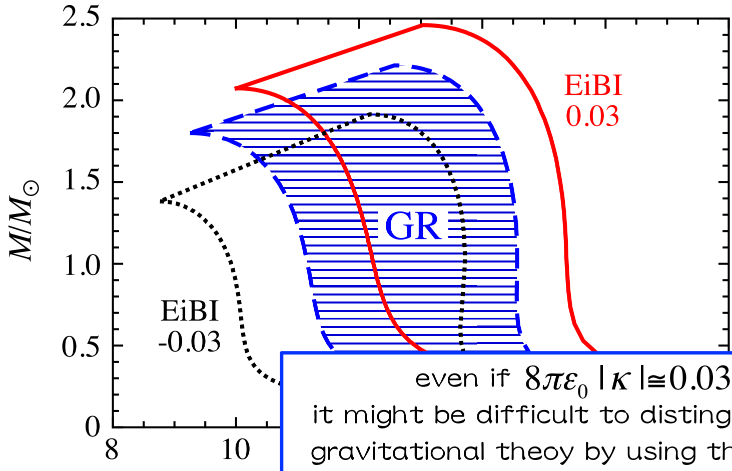
- with FPS EOS



  – one can observe the obvious deviation from the predictions in GR, if EOS is known

uncertainty in MR due to EOS

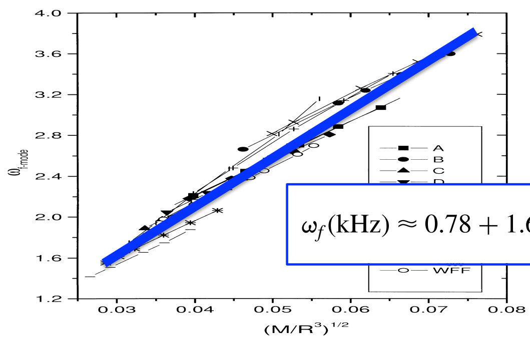
- expected MR relation for EOS with stiffness between FPS and Shen EOSs



even if $8\pi\varepsilon_0|\kappa|\cong 0.03$,
it might be difficult to distinguish the gravitational theoy by using the direct mesasurements of M & R

how to distinguish

- we consider the possibility by using the stellar oscillation spectra (HS 2014b)
  - in GR, it is known that the frequencies of f-mode oscillation are almost independent of EOS, which depend only on the stellar average density $(M/R^3)^{1/2}$

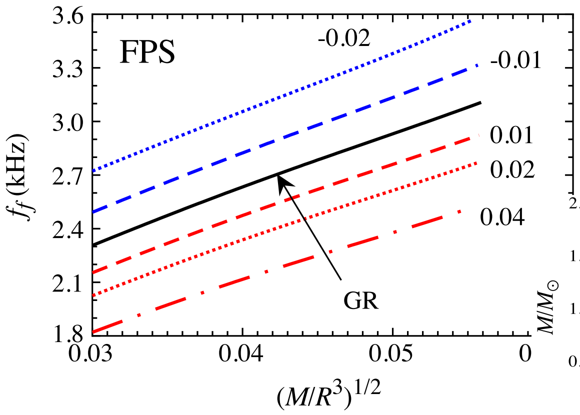


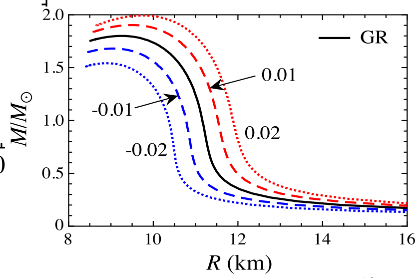
$$\omega_f(\text{kHz}) \approx 0.78 + 1.635\left(\frac{\bar{M}}{\bar{R}^3}\right)^{1/2}$$

Andersson & Kokkotas (1998)

frequencies of f-mode

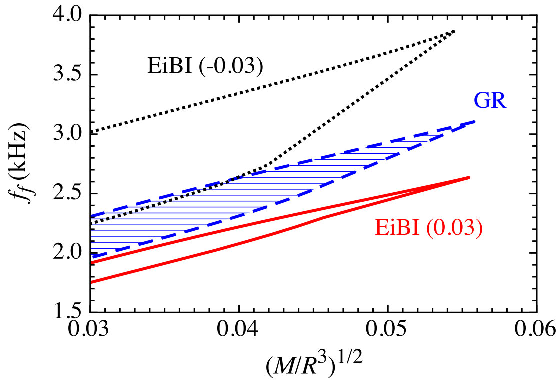
- f-mode frequencies in EiBI are expected,
  - positive κ : lower average density → lower frequencies
  - negative κ : larger average density → larger frequencies





uncertainties due to EOS in fundamental oscillations of f-mode

- expected f-mode frequencies of NSs constructed with EOS with stiffness between FPS and Shen EOSs



- EiBI with $8\pi\varepsilon_0 |\kappa| \geq 0.03$ could be distinguished from GR independently of EOS for NS matter.

conclusion

- we focus on the oscillations of relativistic stars in EiBI.
- we find that the frequencies of stellar oscillations strongly depend on the coupling constant in EiBI.
- we show the possibility to distinguish EiBI from GR via the observation of stellar oscillation spectra (or GWs).
  - EiBI with $8\pi\varepsilon_0 |\kappa| \geq 0.03$ could be distinguished from GR independently of EOS for NS matter.
- with the further constraints on EOS, one might distinguish EiBI even with $8\pi\varepsilon_0 |\kappa| \leq 0.03$ from GR.

“Inflationary cosmology in R^2 gravity with quantum corrections”

Kazuharu Bamba

[JGRG24(2014)111406]

Inflationary cosmology in R^2 gravity with quantum corrections

Main reference: Phys. Rev. D 90, 023525 (2014) [arXiv:1404.4311 [gr-qc]]

The 24th Workshop on General Relativity and Gravitation in Japan (JGRG24)
Kavli IPMU, the University of Tokyo

14th November, 2014

Presenter: **Kazuharu Bamba** (***LGSPC, Ochanomizu University***)

Collaborators: **Guido Cognola** (***Dep. of Phys., Trento University***)
Sergei D. Odintsov (***ICE/CSIC-IEEC and ICREA***)
Sergio Zerbini (***Dep. of Phys., Trento University***)

I. Introduction

Planck satellite

[Ade *et al.* [Planck Collaboration], arXiv:1303.5076; arXiv:1303.5082]

- **Spectral index of power spectrum of the curvature perturbations**

$$n_{\rm s} = 0.9603 \pm 0.0073 \,(95\%\ {\rm CL})$$

- **Tensor-to-scalar ratio**

$$r < 0.11 \,(95\%\ {\rm CL})$$

⇒ R^2 **Inflation**

Motivation

- **Various modified gravity theories have recently been proposed to explain cosmic acceleration.**

Inflation ⟵ R^2 **gravity**

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

Dark Energy problem ⟵ $f(R)$ **gravity**

[Capozziello, Carloni and Troisi, Recent Res. Dev. Astron. Astrophys. 1, 625 (2003)]

[Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]

[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]

Purpose

- **We compare the nature of classical expressions of modified gravity with that with quantum corrections.**

⇩

We investigate a generalized model whose Lagrangian is described by a function of $f(R, K, \phi)$.

R : Scalar curvature ϕ : Scalar field

K : Kinetic term of ϕ

Purpose (2)

- **We show that in the Jordan and Einstein frames, $f(R)$ gravity is equivalent in the quatum level.**

- **We discuss the stability of the de Sitter solutions and explore the influence of the one-loop quantum correction on inflation in R^2 gravity with the quantum correction.**

Cf. [Cognola, Elizalde, Nojiri, Odintsov and Zerbini, JCAP 0502, 010 (2005)]

II. Model

Action $$I = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}} f(\tilde{R}, \tilde{K}, \tilde{\phi})$$ $$\kappa^2 = 8\pi G$$

$$\tilde{K} = (1/2)\, \tilde{g}^{ij} \tilde{\nabla}_i \tilde{\phi} \tilde{\nabla}_j \tilde{\phi}$$

G : Gravitatiotanl constant

$\tilde{\nabla}_i$: Covariant derivative

$\tilde{\Delta}$: Laplacian

* The tilde denotes the quantities in the Einstein frame.

Gravitational field equation

$$f_{\tilde{R}}\, \tilde{R}_{ij} - \frac{1}{2} f\, \tilde{g}_{ij} + \left(\tilde{g}_{ij} \tilde{\Delta} - \tilde{\nabla}_i \tilde{\nabla}_j \right) f_{\tilde{R}} + \frac{1}{2} f_{\tilde{K}}\, \tilde{\nabla}_i \tilde{\phi} \tilde{\nabla}_j \tilde{\phi} = 0$$

Equation of motion for $\tilde{\phi}$

$$\tilde{g}^{ij} \tilde{\nabla}_i \left(f_{\tilde{K}}\, \tilde{\phi}\, \tilde{\nabla}_j \phi \right) = f_{\tilde{\phi}}$$

$$f_{\tilde{R}} \equiv \frac{\partial f}{\partial \tilde{R}}$$

Solutions for the equations of motion

- There is a constant curvature solution:

$$\tilde{R} = R$$

- For $\tilde{\phi} = \phi = \text{constant}$,

$$\longrightarrow \quad f_R \, R_{ij} - \frac{1}{2} f_0 \, g_{ij} = 0 \, , \qquad f_\phi = 0$$

$$f_0 = f(R, K, \phi)$$

⇨ The set of background fields (constant curvature, constant scalar field) is a solution of the following equations:

$$R \, f_R - 2 f_0 = 0 \, , \qquad f_\phi = 0$$

III. Quantum equivalence

- Modified gravity: Described in the Jordan frame

$\longrightarrow$ $f(R)$ gravity: Can also be described in the Einstein frame

These are equivalent in the classical level.

[Maeda, Phys. Rev. D 39, 3159 (1989)]

[Fujii and Maeda, *The Scalar-Tensor Theory of Gravitation* (2003)]

⇨ We show the on-shell quantum equivalence of $f(R)$ gravity.

[Buchbinder, Odintsov and Shapiro, *Effective action in quantum gravity* (1992)]

[Fradkin and Tseytlin, Nucl. Phys. B234, 472 (1984)]

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Quantum equivalence (2)

$$I_{\text{Jord}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}\, f(R)$$: **Jordan frame**

$$\tilde{g}_{ij} = e^{\sigma} g_{ij}$$: **Conformal transformation**

$$I_{\text{Eins}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}}\, \tilde{f}(\tilde{R}, K, \sigma)$$: **Einstein frame**

$$\tilde{f}(\tilde{R}, \tilde{K}, \sigma) = \tilde{R} - \frac{3}{2} \tilde{g}^{ij}\, \partial_i \sigma \partial_j \sigma - V(\sigma)$$

$$e^{\sigma} = f'(R), \qquad R = \Phi(e^{\sigma})\,, \qquad \Phi \circ f' = 1$$

$$V(\sigma) \equiv e^{-\sigma} \Phi(e^{\sigma}) - e^{-2\sigma} f(\Phi(e^{\sigma})) \qquad f' \equiv \frac{\partial f}{\partial R}$$

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Quantum equivalence (3)

Jordan frame

Contribution of scalars to the effective action

$$\Gamma^{\text{Jord}}_{\text{on-shell}} = \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-f_{RR} \left(\Delta_0 + \frac{R}{3} \right) + \frac{f_R}{3} \right) \right]$$

$$+\text{classical and higher spin contributions}$$

Δ_0 : Laplacian acting on scalars

μ^2 : Renormalization parameter

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Quantum equivalence (4)

Einstein frame

$$V''(\sigma) \equiv \frac{\partial^2 V(\sigma)}{\partial \sigma^2}$$

$$\Gamma^{\rm Eins}_{\rm on-shell} = \frac{1}{2} \ln \det \left[\frac{1}{\tilde{\mu}^2} \left(3\tilde{\Delta}_0 - V''(\sigma) \right) \right]$$

+classical and higher spin contributions

$$= \frac{1}{2} \ln \det \left[\frac{1}{\tilde{\mu}^2} \left(\frac{3\Delta_0}{f_R} + \frac{R}{f_R} - \frac{1}{f_{RR}} \right) \right]$$

+classical and higher spin contributions

By the redefinition of $\tilde{\mu}$, this can become equivalent to $\Gamma^{\rm Jord}_{\rm on-shell}$.

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R^2 gravity

Jordan frame

$$f(R) = R + \frac{R^2}{6M^2}$$

M^2 : Mass parameter

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

$$\Gamma^{\rm Jord}_{\rm on-shell} = \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_0 + M^2 \right) \right]$$

+classical and higher spin contributions

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R^2 gravity (2)

Einstein frame

$$\tilde{f}(\tilde{R}, \tilde{K}, \sigma) = \tilde{R} - \frac{3}{2}\,\tilde{g}^{ij}\partial_i\sigma\partial_j\sigma - \frac{3}{2}\,M^2\left(1 - e^{-\sigma}\right)^2$$
$$= \tilde{R} - 3\tilde{K} - \frac{3}{2}\,M^2\left(1 - e^{-\sigma}\right)^2$$

(Cf. $\sigma \to \sqrt{2/3}\phi/M_{\rm P}$)

$$\Gamma^{(1)}_{\rm on-shell} = \frac{\mathcal{V}}{2}\,M^4\left(\ln\frac{M^2}{\mu^2} - \frac{3}{2}\right)$$

$\mathcal{V}$: Volume

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R^2 inflation

Jordan frame

$M^2/R \ll 1$ **: During inflation**

$$L(R) = \frac{1}{2}\,M_{\rm P}^2\left[R + \frac{R^2}{6M^2} + \frac{R^2}{384\pi^2 M_{\rm P}^2}\left(C_1\,\ln\frac{R}{\mu^2} + C_2\right)\right] + O\left(\frac{M^2}{R}\right)$$

$C_1 = O(1), \qquad C_2 \sim 300$

↓

Conformal transformation

$$\tilde{g}_{ij} = e^{\sigma} g_{ij}$$

$M_{\rm P}$: Reduced Planck mass

R^2 inflation (2)

Einstein frame

$$I_{\text{Eins}} = \frac{1}{\kappa^2} \int d^4x \sqrt{-\tilde{g}} \left(\tilde{R} - \frac{3}{2} \tilde{g}^{ij} \partial_i \sigma \partial_j \sigma - V(\sigma) \right)$$

$$V(\sigma) = (1 - e^{-\sigma})^2 \ \frac{a + 2b \left\{ 1 + \log|e^\sigma - 1| - \log\left[4|b\mu| W\left(\frac{|e^\sigma - 1| e^{(a+b)/2b}}{4|b\mu|} \right) \right] \right\}}{\left[4bW\left(\frac{|e^\sigma - 1|] e^{(1+b)/2b}}{4|b\mu|} \right) \right]^2}$$

W : Lambert function

$$a = \frac{1}{6M^2} + \frac{C_2}{384\pi^2 M_P^2} \ , \qquad b = \frac{C_1}{384\pi^2 M_P^2}$$

R^2 inflation (3)

Jordan frame

$M^2/R \gg 1$ **: At the end of inflation**

$$L(R) \ = \ \frac{1}{2} M_P^2 \left[R + \frac{R^2}{6M^2} - \frac{M^4}{32\pi^2 M_{\text{P}}^2} \left(\ln \frac{M^2}{\mu^2} - \frac{3}{2} \right) + O\left(R^2 \ln \frac{R}{M_{\text{P}}^2 \mu^2} \right) \right]$$

$$\Lambda(\mu) = \frac{M^4}{16\pi^2 M_{\text{P}}^2} \left(\ln \frac{M^2}{\mu^2} - \frac{3}{2} \right)$$

: Effective cosmological constant

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R^2 inflation (4)

Einstein frame: R + Scalar field theory for ϕ

ϕ : Inflaton field

$$V(\phi) = \frac{1}{2} M_{\rm P}^2 \left[\frac{3M^2}{2} \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm P}}} \right)^2 + 2\Lambda(\mu)\, e^{-2\sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm P}}} \right]$$

$V(\phi)$ **becomes the minimum at** $\phi = \phi_*$.

$\phi/M_{\rm P} \gg 1$ **:** R^2 **Inflation**

$$\Rightarrow \quad V(\phi_*) = \frac{3M_{\rm P}^2 M^2 \Lambda(\mu)}{3m^2 + 4\Lambda(\mu)}$$

Contribution of the quantum correction : $\mathcal{O}((M/M_{\rm P})^2)$

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IV. Stability issue

The one-loop on-shell effective action

$$\Gamma^{(1)}_{\rm on-shell} = \underline{\frac{1}{2} \ln\det \left[\frac{1}{\mu^4} \left(a_2 \Delta_0^2 + a_1 \Delta_0 + a_0 \right) \right]}$$

$$- \frac{1}{2} \ln\det \left[\frac{1}{\mu^2} \left(-\Delta_1 - \frac{R}{4} \right) \right] + \frac{1}{2} \ln\det \left[\frac{1}{\mu^2} \left(-\Delta_2 + \frac{R}{6} \right) \right]$$

$$\begin{aligned} a_0 &= f_R \left[R f_{R\phi}^2 + f_{\phi\phi}(f_R - R f_{RR}) \right] \\ a_1 &= f_R \left[f_K (R f_{RR} - f_R) + f_{R\phi}^2 - 3 f_{\phi\phi} f_{RR} \right] \\ a_2 &= 3 f_K f_R f_{RR} \end{aligned}$$

Δ_1 : Laplacian acting on transverse-traceless vectors

Δ_2 : Laplacian acting on transverse-traceless tensors

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Stability issue (2)

Stability condition:

All of the eigen values for the operators in $\Gamma^{(1)}_{\rm on-shell}$ are not negative.

Minimum eigen values of $\Delta_0, \ \Delta_1, \ \Delta_2$:

$\longrightarrow$ **The minimum eigen value of Δ_0 is the smallest one.**

⇨ **The first term of $\Gamma^{(1)}_{\rm on-shell}$ including Δ_0 is related to the stability.**

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Stability issue (3)

$$f(R, K, \phi) = F(R) - \frac{1}{2} g^{ij} \, \partial_i \phi \partial_j \phi = F(R) - K$$

$$M_P^2/2 = 1$$

Stability condition

a_1/a_2 **:Non-negative value**

$$\Longrightarrow \quad \frac{F_R}{F_{RR}} - R \geq 0$$

V. Conclusions

- We have studied a generalized model whose Lagrangian is described by a function of $f(R, K, \phi)$.
- We have shown the on-shell quantum equivalence of $f(R)$ gravity in the Jordan and Einstein frames.
- We have examined the stability of the de Sitter solutions and the one-loop quantum correction to inflation in quantum-corrected R^2 gravity.

Back up slides

Stability issue (4)

$$f(R, K, \phi) = R - \frac{1}{2} g^{ij}\, \partial_i \phi \partial_j \phi - \frac{1}{2} m^2 \phi^2 + \xi R \phi^2$$

$m, \ \xi$: Constant

- **Background solution:** $R = \dfrac{m^2}{2\xi}\,, \quad \phi = \pm \dfrac{1}{\sqrt{\xi}}$

$\longrightarrow$ **For** $\xi \neq -1/6$ **,**

Stability condition

$$-\frac{a_0}{a_1} = -\frac{m^2}{1+6\xi} \geq 0 \quad \Longrightarrow \quad \xi < -\frac{1}{6}$$

Inflation

$$\varepsilon = \left(\frac{1}{V}\frac{dV}{d\phi}\right)^2 = \frac{1}{3}\left(\frac{V'(\sigma)}{V(\sigma)}\right)^2, \qquad \eta = \frac{2}{V}\frac{d^2 V}{d\phi^2} = \frac{2}{3}\frac{V''(\sigma)}{V(\sigma)}$$

$$n_{\rm s} = 1 - 6\epsilon + 2\eta\,, \quad r = 16\epsilon$$

$$M \sim 0.1\, M_P\,, \qquad \mu \sim M$$

$$\phi_k \equiv \phi(\tilde{t}_k) \sim 7.756 M_P \qquad \sigma = \sqrt{2/3}\phi/M_P$$

⇨ $n_{\rm S} \sim 0.968$

$r = 0.0028$

$\tilde{t}_k$: 1st horizon crossing time for mode *k*

Inflation (2)

$$f(R) = R + \alpha R^2$$ [Starobinsky, Phys. Lett. B 91, 99 (1980)]

$$V(\Psi) = \frac{1}{8\alpha}\left(1 - e^{-\sqrt{2/3}\Psi}\right)^2$$

$$n_S \simeq 1 - \frac{2}{N}, \qquad r = \frac{12}{N^2}$$

$$\Psi = \sqrt{\frac{3}{2}}\ln(1 + 2\alpha R)$$

$$8\pi G = 1$$

- $N = 50 \Rightarrow n_S = 0.960$, $r = 0.00480$
- $N = 60 \Rightarrow n_S = 0.967$, $r = 0.00333$

Cf. [Hinshaw *et al.*, Astrophys. J. Suppl. 208, 19 (2013)]

Planck satellite

- **Spectral index of power spectrum of the curvature perturbations**

$$n_s = 0.9603 \pm 0.0073 \ (95\% \ \mathrm{CL})$$

- **Running of the spectral index**

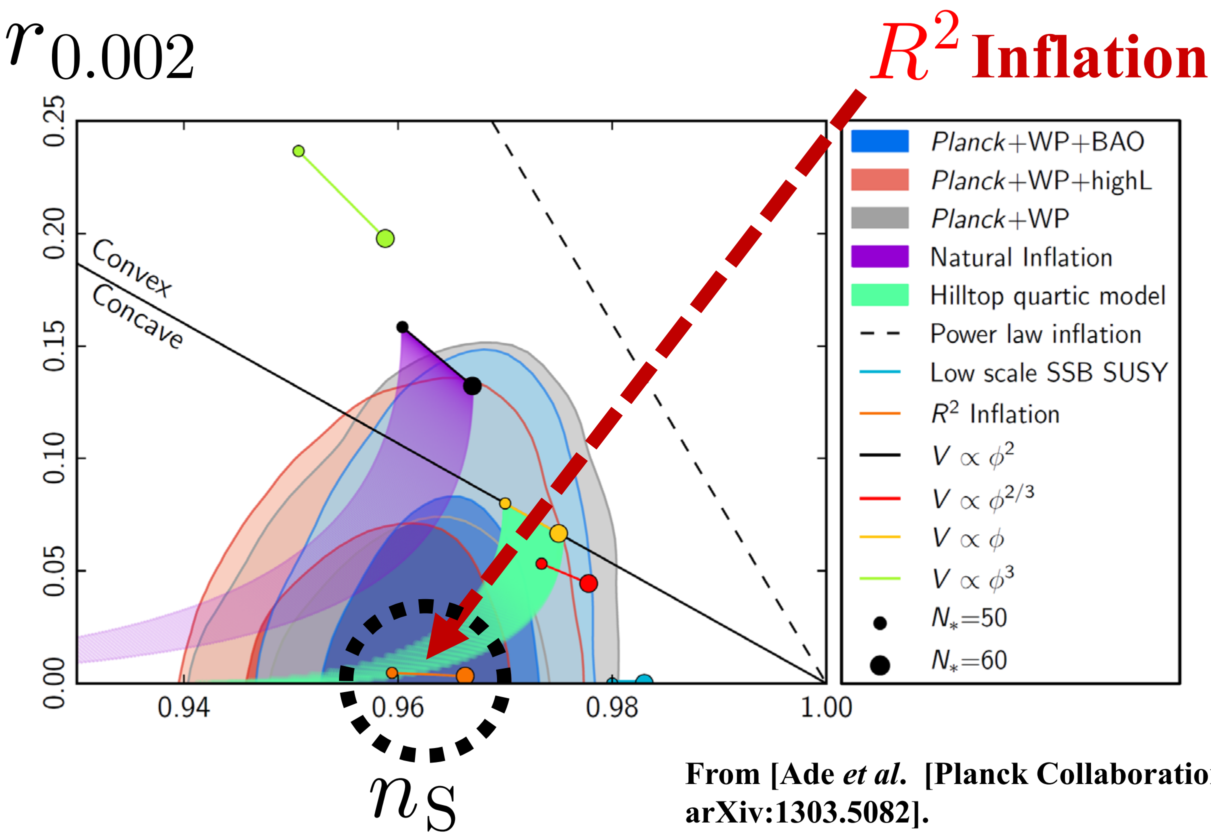
$$\alpha_s = -0.0134 \pm 0.0090 \ (68\% \ \mathrm{CL})$$

- **Tensor-to-scalar ratio**

$$r < 0.11 \ (95\% \ \mathrm{CL})$$

[Ade *et al.* [Planck Collaboration], arXiv:1303.5076; arXiv:1303.5082]

Planck results



From [Ade *et al.* [Planck Collaboration], arXiv:1303.5082].

BICEP2 experiment

$$r = 0.20^{+0.07}_{-0.05} \; (68\% \; \mathrm{CL})$$

[Ade *et al.* [BICEP2 Collaboration], Phys. Rev. Lett. 112, 241101 (2014)]

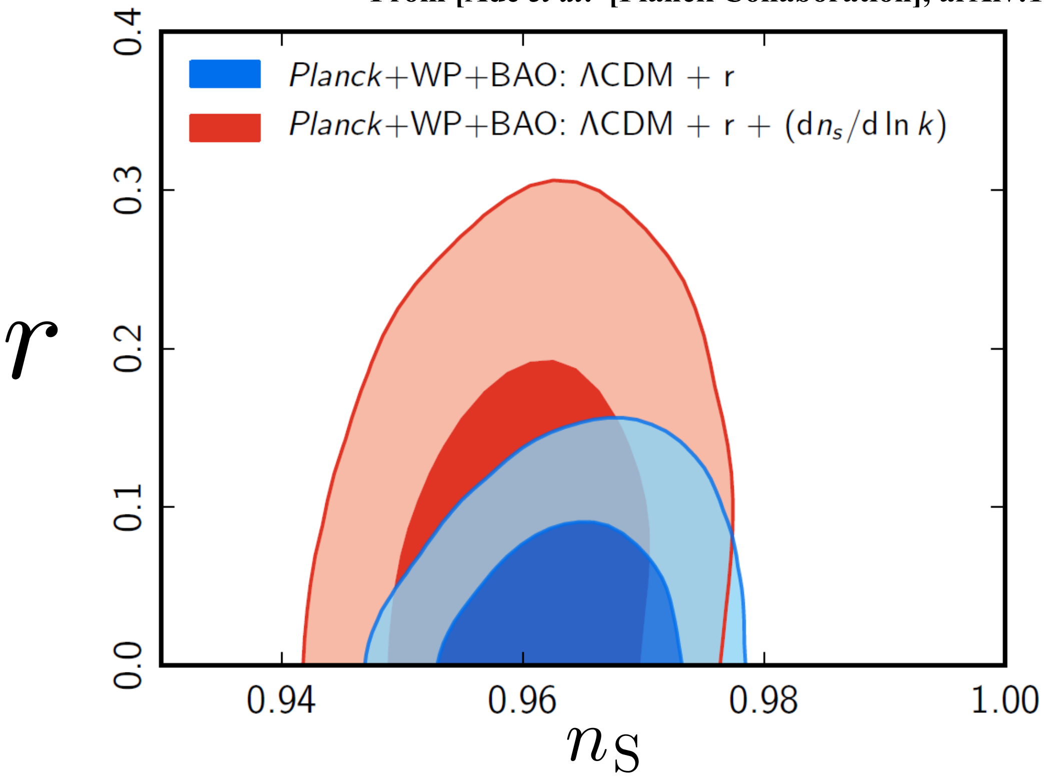
Cf. [Ade *et al.* [Planck Collaboration], arXiv:1405.0871 [astro-ph.GA]]

[Ade *et al.* [Planck Collaboration], arXiv:1405.0874 [astro-ph.GA]]

[Adam *et al.* [Planck Collaboration], arXiv:1409.5738 [astro-ph.CO]]

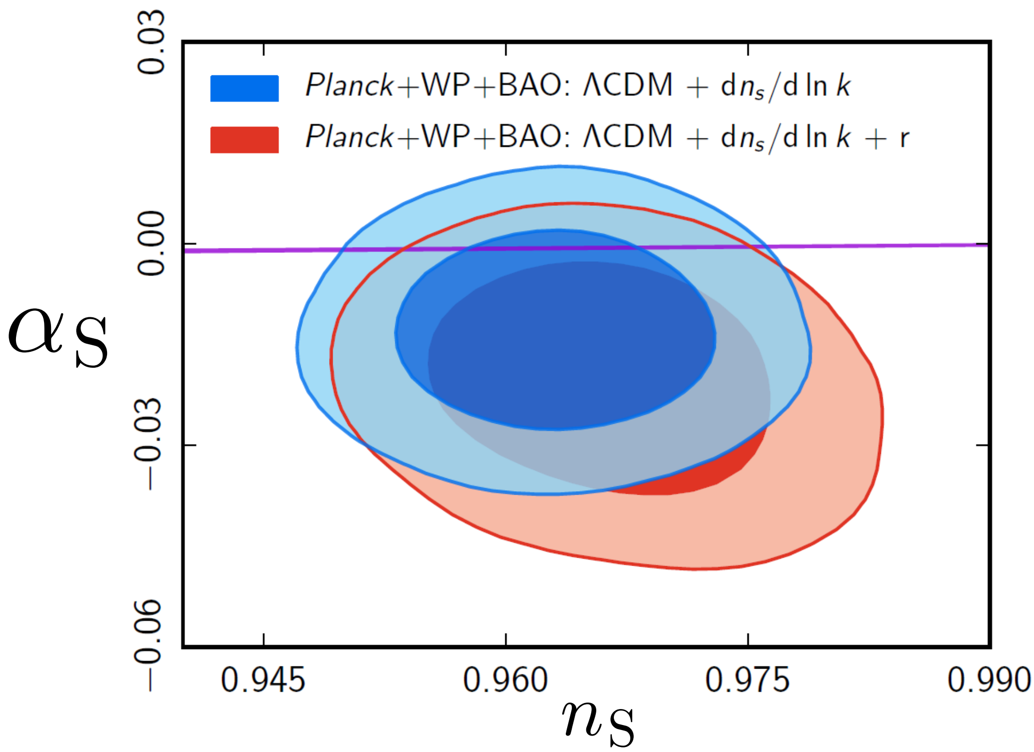
Planck satellite (2)

From [Ade *et al.* [Planck Collaboration], arXiv:1303.5082].



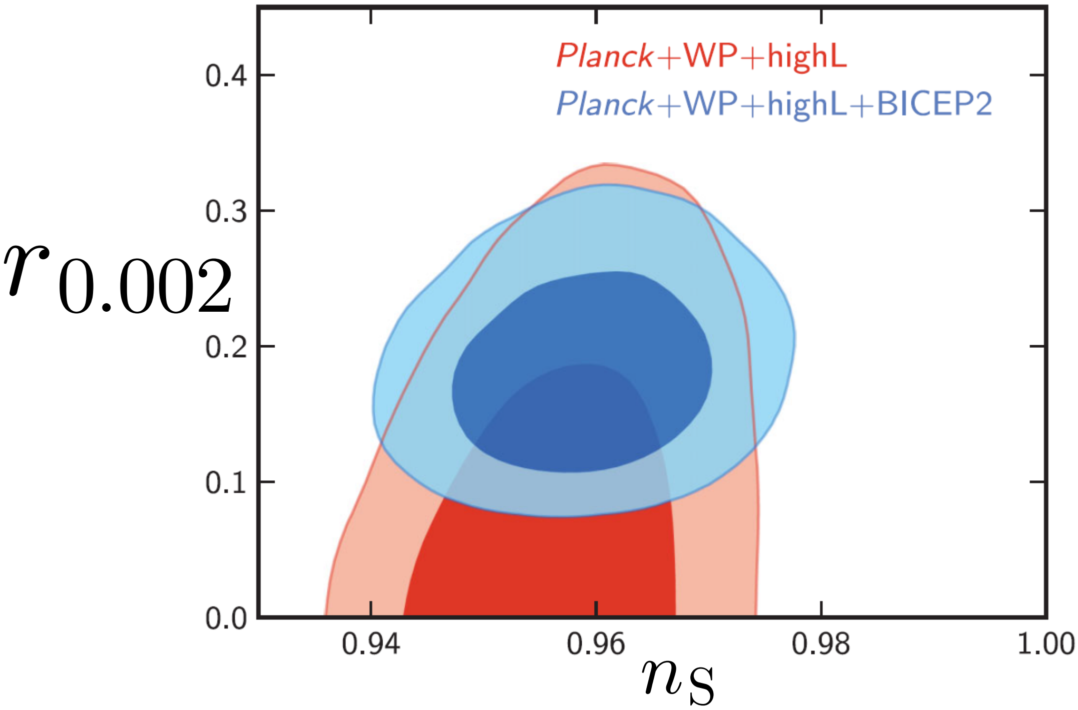
Planck satellite (3)

From [Ade *et al.* [Planck Collaboration], arXiv:1303.5082].



BICEP2 experiment (2)

[Ade *et al*. [BICEP2 Collaboration], Phys. Rev. Lett. 112, 241101 (2014)]



Quantum Correction

$$
\begin{aligned}
\Gamma^{(1)}_{\text{Landau}} &= \frac{1}{2}\mathcal{V}\, M_{\text{P}}^2\left(R+\frac{R^2}{6M^2}\right)+\frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\frac{1}{2}\Delta_{\,0}-\frac{R}{2}\right)\right] \\
&\quad +\frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_{\,1}-\frac{R}{4}\right)\right]-\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_{\,0}-\frac{R}{2}+M^2\right)\right] \\
&\quad -\frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_{\,2}+\frac{R(R+12M^2)}{6(R+3M^2)}\right)\right]
\end{aligned}
$$

$$
\mathcal{V}=\frac{384\pi^2}{R^2}
$$

Stability issue

$$\Gamma_{\rm on-shell} = \frac{24\pi f_0}{GR^2} + \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_0 + X_1\right)\right] + \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_0 + X_2\right)\right]$$
$$-\frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_1 - \frac{R}{4}\right)\right] + \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_2 + \frac{R}{6}\right)\right]$$

$$X_{1,2} = \frac{1}{2}\left(-\frac{a_1}{a_2} \pm \sqrt{\frac{a_1^2}{a_2^2} - \frac{4a_0}{a_2}}\right)$$

Condition to obtain two positive solution:

$$\frac{a_1}{a_2} < 0\ , \qquad \left(\frac{a_1}{a_2}\right)^2 \geq \frac{4a_0}{a_2} \geq 0$$

Quantizatin of the maximally symmetric (de Sitter) space

Euclidean action

$$I_{\rm E}[\tilde{g}, \tilde{\phi}] = -\frac{1}{16\pi G}\int d^4x\, \sqrt{-\tilde{g}}\, f(\tilde{R}, \tilde{K}, \tilde{\phi})$$

$$R_{ijrs} = \frac{R}{12}\left(g_{ir}g_{js} - g_{is}g_{jr}\right)\,, \quad R_{ij} = \frac{R}{4}g_{ij}\ , \qquad R = constant$$

g_{ij} : Metric of the maximally symmetric space

Fluctuations around the constant curvature solution

$$\begin{cases} \tilde{g}_{ij} = g_{ij} + h_{ij}\ , & |h_{ij}| \ll 1 \\ \tilde{g}^{ij} = g^{ij} - h^{ij} + h^{ik}h^j_k + \mathcal{O}(h^3)\,, & h = g^{ij}h_{ij} \\ \tilde{\phi} = \phi + \varphi\,, & |\varphi| \ll 1 \end{cases}$$

Euclidean action

Around the background fields $\{g_{ij}, \phi\}$, we expand $\sqrt{-\tilde{g}} f(\tilde{R}, \tilde{K}, \tilde{\phi})$.

$$I_{\rm E}[g,\phi] \sim -\frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\;[f_0 + \mathcal{L}_1 + \underline{\mathcal{L}_2}\,]$$

Quantum correction

$$f_0 = f(R, K, \phi)$$

$$\mathcal{L}_1 \;=\; \frac{1}{4}X\,h + f_\phi \phi$$

$$X = 2f_0 - Rf_R$$

Euclidean action (2)

$$\begin{aligned}\mathcal{L}_2 \;=\; & -\frac{1}{2}f_R h^i_k \nabla_i\nabla_j h^{jk} + \frac{1}{4}f_R h_{ij}\Delta h^{ij} - \frac{1}{24}Rf_R h_{ij}h^{ij} + \frac{1}{2}f_R h\nabla_i\nabla_j h^{ij} \\ & + f_{R\phi}\varphi\nabla_i\nabla_j h^{ij} + \frac{1}{2}f_{RR}\nabla_i\nabla_j h^{ij}\nabla_r\nabla_s h^{rs} - f_{RR}\Delta h\nabla_i\nabla_j h^{ij} - \frac{1}{4}Rf_{RR}h\nabla_i\nabla_j h^{ij} \\ & -\frac{1}{48}Rf_R h^2 - \frac{1}{2}f_K\varphi\Delta\varphi - \frac{1}{4}f_R h\Delta h - f_{R\phi}h\Delta\varphi + \frac{1}{4}Rf_{RR}h\Delta h + \frac{1}{2}f_{\phi\phi}\varphi^2 \\ & -\frac{1}{4}Rf_{R\phi}h\varphi + \frac{1}{32}R^2 f_{RR}h^2 + \frac{X}{16}(h^2 - 2h_{ij}h^{ij}) + \frac{1}{2}f_\phi h\varphi\end{aligned}$$

On-shell Lagrangian density: $X \to 0\,,\; f_\phi \to 0$

Expansion of h_{ij}

$$h_{ij} = \hat{h}_{ij} + \nabla_i \xi_j + \nabla_j \xi_i + \nabla_i \nabla_j \sigma + \frac{1}{4} g_{ij} (h - \Delta_0 \sigma)$$

σ : Scalar component

ξ_i : Vector component

$\hat{h}_{ij}$: Tensor component

$$\nabla_i \xi^i = 0 , \qquad \nabla_i \hat{h}^i_j = 0 , \qquad \hat{h}^i_i = 0$$

Expression of $\mathcal{L}_2$

$$\begin{aligned}
\mathcal{L}_2 \;=\; & \frac{1}{32} \sigma \left(9 f_{RR} \Delta\Delta\Delta\Delta - 3 f_R \Delta\Delta\Delta + 6 f_{RR} R \Delta\Delta\Delta \right. \\
& \left. - f_R R \Delta\Delta + f_{RR} R^2 \Delta\Delta - 3X\Delta\Delta - RX\Delta \right) \sigma \\
& + \frac{1}{32} h \left(9 f_{RR} \Delta\Delta - 3 f_R \Delta + 6 f_{RR} R \Delta - {f_R}^2 R + f_{RR} R^2 + X\right) h \\
& + \frac{1}{16} h \left(-9 f_{RR} \Delta\Delta\Delta + 9 f_R \Delta\Delta - 6 f_R \Delta\Delta - 6 f_{RR} R \Delta\Delta + f_R R \Delta - f_{RR} R^2 \Delta\right) \sigma \\
& + \frac{1}{2} \varphi \left(-f_K \Delta + f_{\phi\phi}\right) \varphi + \frac{1}{4} h \left(-3 f_{R\phi} \Delta + 2 f_\phi - f_{R\phi} R\right) \varphi \\
& + \frac{1}{4} \sigma \left(+3 f_{R\phi} \Delta\Delta + f_{R\phi} R \Delta\right) \varphi \\
& + \frac{1}{16} \xi_i \left(4X\Delta + 4RX\right) \xi^i + \frac{1}{24} \hat{h}_{ij} \left(6 f_R \Delta - f_R R - 3X\right) \hat{h}^{ij}
\end{aligned}$$

Lagrangian

Gauge condition

$$\chi_k = \nabla_j h^j_k - \frac{1+\rho}{4}\nabla_k h$$

ρ : Real parameter

Gauge fixing

$$\mathcal{L}_{gf} = \frac{1}{2}\chi^i G_{ij}\chi^j$$

$$G_{ij} = \gamma\, g_{ij} + \beta\, g_{ij}\Delta$$

$\gamma,\ \beta$: Constants

Lagrangian (2)

Ghost Lagrangian

[Buchbinder, Odintsov, Shapiro, *Effective action in quantum gravity* (1992)]

$$\mathcal{L}_{gh} = B^i G_{ik}\frac{\delta\chi^k}{\delta\varepsilon^j}C^j$$

C_k : Ghost vector

B_k : Anti ghost vector

$$\frac{\delta\chi^i}{\delta\varepsilon^j} = g_{ij}\Delta + R_{ij} + \frac{1-\rho}{2}\nabla_i\nabla_j \quad \longleftarrow \quad \delta h_{ij} = \nabla_i\varepsilon_j + \nabla_j\varepsilon_i$$

$$\Longrightarrow \quad \mathcal{L}_{gh} = B^i\left(\gamma H_{ij} + \beta\Delta H_{ij}\right)C^j$$

$$H_{ij} = g_{ij}\left(\Delta + \frac{R_0}{4}\right) + \frac{1-\rho}{2}\nabla_i\nabla_j$$

Lagrangian (3)

$$
\begin{aligned}
\mathcal{L}_{gf} &= \frac{\gamma}{2}\left[\xi^k\left(\Delta_1+\frac{R_0}{4}\right)^2\xi_k+\frac{3\rho}{8}h\left(\Delta_0+\frac{R_0}{3}\right)\Delta_0\sigma\right.\\
&\qquad\left.-\frac{\rho^2}{16}h\,\Delta_0\,h-\frac{9}{16}\sigma\left(\Delta_0+\frac{R_0}{3}\right)^2\Delta_0\sigma\right]\\
&\quad+\frac{\beta}{2}\left[\xi^k\left(\Delta_1+\frac{R_0}{4}\right)^2\Delta_1\xi_k+\frac{3\rho}{8}h\left(\Delta_0+\frac{R}{4}\right)\left(\Delta_0+\frac{R}{3}\right)\Delta_0\sigma\right.\\
&\quad\left.-\frac{\rho^2}{16}h\left(\Delta_0+\frac{R_0}{4}\right)\Delta_0 h-\frac{9}{16}\sigma\left(\Delta_0+\frac{R_0}{4}\right)\left(\Delta_0+\frac{R_0}{3}\right)^2\Delta_0\sigma\right]
\end{aligned}
$$

Δ_0 : Laplacian acting on scalars

Δ_1 : Laplacian acting on transverse-traceless vectors

Δ_2 : Laplacian acting on transverse-traceless tensors

Lagrangian (4)

$$
\begin{aligned}
\mathcal{L}_{gh} &= \gamma\left\{\hat{B}^i\left(\Delta_1+\frac{R_0}{4}\right)\hat{C}^j+\frac{\rho-3}{2}b\left(\Delta_0-\frac{R_0}{\rho-3}\right)\Delta_0 c\right\}\\
&+\beta\left\{\hat{B}^i\left(\Delta_1+\frac{R_0}{4}\right)\Delta_1\hat{C}^j+\frac{\rho-3}{2}b\left(\Delta_0+\frac{R_0}{4}\right)\left(\Delta_0-\frac{R_0}{\rho-3}\right)\Delta_0 c\right\}
\end{aligned}
$$

$$
C_k \equiv \hat{C}_k+\nabla_k c\,, \qquad \nabla_k\hat{C}^k=0
$$

$$
B_k \equiv \hat{B}_k+\nabla_k b\,, \qquad \nabla_k\hat{B}^k=0
$$

Lagrangian (5)

Total Lagrangian: $\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$

$$Z^{(1)} = e^{-\Gamma^{(1)}} = (\det G_{ij})^{-1/2} \int D[h_{ij}]D[C_k]D[B^k] \exp\left(-\int d^4x \sqrt{g}\,\mathcal{L}\right)$$

$$= (\det G_{ij})^{-1/2} \det J_1^{-1} \det J_2^{1/2}$$

$$\times \int D[h]D[\hat{h}_{ij}]D[\xi^j]D[\sigma]D[\hat{C}_k]D[\hat{B}^k]D[c]D[b] \exp\left(-\int d^4x \sqrt{g}\,\mathcal{L}\right)$$

$$J_1 = \Delta_0\,, \qquad J_2 = \left(\Delta_1 + \frac{R_0}{4}\right)\left(\Delta_0 + \frac{R_0}{3}\right)\Delta_0$$

$$\det G_{ij} = \text{const} \ \det\left(\Delta_1 + \frac{\gamma}{\beta}\right) \det\left(\Delta_0 + \frac{R_0}{4} + \frac{\gamma}{\beta}\right)$$

[Buchbinder, Odintsov and Shapiro, *Effective action in quantum gravity* (1992)]
[Fradkin and Tseytlin, Nucl. Phys. B234, 472 (1984)]

Effective action

- $\rho = 1, \quad \beta = 0$:

$$\Gamma = I_{\rm E}[g,\phi] + \Gamma^{(1)}\,, \qquad I_{\rm E}[g,\phi] = \frac{24\pi f_0}{GR^2}$$

$$\Gamma^{(1)} = \frac{1}{2}\ln\det\left(b_4\Delta_0^4 + b_3\Delta_0^3 + b_2\Delta_0^2 + b_1\Delta_0 + b_0\right) - \ln\det\left(-\Delta_0 - \frac{R}{2}\right)$$

$$+ \frac{1}{2}\ln\det\left(-\Delta_1 - \frac{R}{4} - \frac{X}{2\gamma}\right) - \ln\det\left(-\Delta_1 - \frac{R}{4}\right)$$

$$+ \frac{1}{2}\ln\det\left(-\Delta_2 + \frac{R}{6} + \frac{X}{2f_R}\right)$$

$b_k \ (k = 0, \cdots 4)$ consists of $f(R, K, \phi)$ and its derivatives.

Effective action (2)

- $X \to 0\,,\ f_\phi \to 0$:

$$\Gamma^{(1)}_{\rm on-shell} = \frac{1}{2}\ln\det\left[\frac{1}{\mu^4}\left(a_2\Delta_0^2 + a_1\Delta_0 + a_0\right)\right]$$

$$-\frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_1 - \frac{R}{4}\right)\right] + \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_2 + \frac{R}{6}\right)\right]$$

$a_k\ (k = 0, 1, 2)$ consists of $f(R, K, \phi)$ and its derivatives.

μ^2 : Renormalized parameter

Effective action (3)

- $\rho = 1,\ \beta = 0,\ \gamma = \infty$:

$$\Gamma^{(1)}_{\rm Landau} = \frac{1}{2}\ln\det\left[\frac{1}{\mu^8}\left(c_4\Delta_0^4 + c_3\Delta_0^3 + c_2\Delta_0^2 + c_1\Delta_0 + c_0\right)\right]$$

$$-\frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_1 - \frac{R}{4}\right)\right] - \ln\det\left[\frac{1}{\mu^2}\left(-\Delta_0 - \frac{R}{2}\right)\right]$$

$$+\frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_2 - \frac{R}{3} + \frac{f_0}{f_R}\right)\right]$$

$c_k\ (k = 0, \cdots, 4)$ consists of $f(R, K, \phi)$ and its derivatives.

Expression of coefficients

$$
\begin{aligned}
a_0 &= f_R\left[Rf_{R\phi}^2 + f_{\phi\phi}(f_R - Rf_{RR})\right] \\
a_1 &= f_R\left[f_K(Rf_{RR} - f_R) + f_{R\phi}^2 - 3f_{\phi\phi}f_{RR}\right] \\
a_2 &= 3f_K f_R f_{RR}
\end{aligned}
$$

$$
\begin{aligned}
b_0 = {} & \frac{4{f_\phi}^2 X}{\gamma} + 4{f_\phi}^2 R - \frac{4f_\phi f_{R\phi} RX}{\gamma} - 4f_\phi f_{R\phi} R^2 + \frac{f_{\phi\phi} f_R RX}{\gamma} \\
& + f_{\phi\phi} f_R R^2 - \frac{f_{\phi\phi} f_{RR} R^2 X}{\gamma} - f_{\phi\phi} f_{RR} R^3 - \frac{f_{\phi\phi} X^2}{\gamma} - f_{\phi\phi} RX \\
& + \frac{{f_{R\phi}}^2 R^2 X}{\gamma} + {f_{R\phi}}^2 R^3
\end{aligned}
$$

Expression of coefficients (2)

$$
\begin{aligned}
b_1 = {} & -\frac{f_K f_R RX}{\gamma} - f_K f_R R^2 + \frac{f_K f_{RR} R^2 X}{\gamma} + f_K f_{RR} R^3 \\
& + \frac{f_K X^2}{\gamma} + f_K RX + \frac{4{f_\phi}^2 f_R}{\gamma} - \frac{4{f_\phi}^2 f_{RR} R}{\gamma} \\
& + 12{f_\phi}^2 - \frac{12 f_\phi f_{R\phi} X}{\gamma} - 20 f_\phi f_{R\phi} R + \frac{2 f_{\phi\phi} f_R X}{\gamma} + 4 f_{\phi\phi} f_R R \\
& - \frac{5 f_{\phi\phi} f_{RR} RX}{\gamma} - 7 f_{\phi\phi} f_{RR} R^2 - 2 f_{\phi\phi} X + \frac{5{f_{R\phi}}^2 RX}{\gamma} + 7{f_{R\phi}}^2 R^2 \\
b_2 = {} & -\frac{2 f_K f_R X}{\gamma} - 4 f_K f_R R + \frac{5 f_K f_{RR} RX}{\gamma} + 7 f_K f_{RR} R^2 \\
& + 2 f_K X - \frac{12{f_\phi}^2 f_{RR}}{\gamma} - 24 f_\phi f_{R\phi} + 4 f_{\phi\phi} f_R - \frac{6 f_{\phi\phi} f_{RR} X}{\gamma} \\
& - 16 f_{\phi\phi} f_{RR} R + \frac{6{f_{R\phi}}^2 X}{\gamma} + 16{f_{R\phi}}^2 R
\end{aligned}
$$

Expression of coefficients (3)

$$b_3 = -4f_K f_R + \frac{6 f_K f_{RR} X}{\gamma} + 16 f_K f_{RR} R - 12 f_{\phi\phi} f_{RR} + 12 {f_{R\phi}}^2$$

$$b_4 = 12 f_K f_{RR}$$

$$\begin{aligned}
c_0 &= R\left[f_{\phi\phi}(2Rf_R - 2f_0 - R^2 f_{RR}) + (2f_\phi - Rf_{R\phi})^2\right] \\
c_1 &= f_0\,(2f_K R - 4f_{\phi\phi}) + 12{f_\phi}^2 - 20 f_\phi f_{R\phi} R \\
&\quad + R\left(-2f_K f_R R + f_K f_{RR} R^2 + 6 f_{\phi\phi} f_R - 7 f_{\phi\phi} f_{RR} R + 7{f_{R\phi}}^2 R\right) \\
c_2 &= 4 f_0 f_K - 6 f_K f_R R + 7 f_K f_{RR} R^2 - 24 f_\phi f_{R\phi} \\
&\quad + 4 f_{\phi\phi}\,(f_R - 4 f_{RR} R) + 16 {f_{R\phi}}^2 R \\
c_3 &= -4\left(f_K\,(f_R - 4f_{RR}R) + 3 f_{\phi\phi} f_{RR} - 3{f_{R\phi}}^2\right) \\
c_4 &= 12 f_K f_{RR}
\end{aligned}$$

$f(R)$ gravity

$$\begin{aligned}
\Gamma^{(1)}_{\text{on-shell}} &= \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-f_{RR}\left(\Delta_0 + \frac{R}{3}\right) + \frac{f_R}{3}\right)\right] \\
&\quad - \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_1 - \frac{R}{4}\right)\right] + \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_2 + \frac{R}{6}\right)\right]
\end{aligned}$$

$$\begin{aligned}
\Gamma^{(1)}_{\text{Landau}} &= \frac{1}{2}\ln\det\left[\frac{1}{\mu^4}\left(f_{RR}(6\Delta_0^2 + 5R\Delta_0 - 2f_R(\Delta_0 + R) + 2f_0\right)\right] \\
&\quad - \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_1 - \frac{R}{4}\right)\right] - \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_0 - \frac{R}{2}\right)\right] \\
&\quad + \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_2 - \frac{R}{3} + \frac{f_0}{f_R}\right)\right]
\end{aligned}$$

“Combined features in the primordial spectra induced by a sudden turn in two-field DBI inflation”

Shuntaro Mizuno

[JGRG24(2014)111407]

JGRG24 @ Kavli IPMU 2014. 11. 14

Combined features in the primordial spectra induced by a sudden turn in two-field DBI inflation

Shuntaro Mizuno (Waseda)

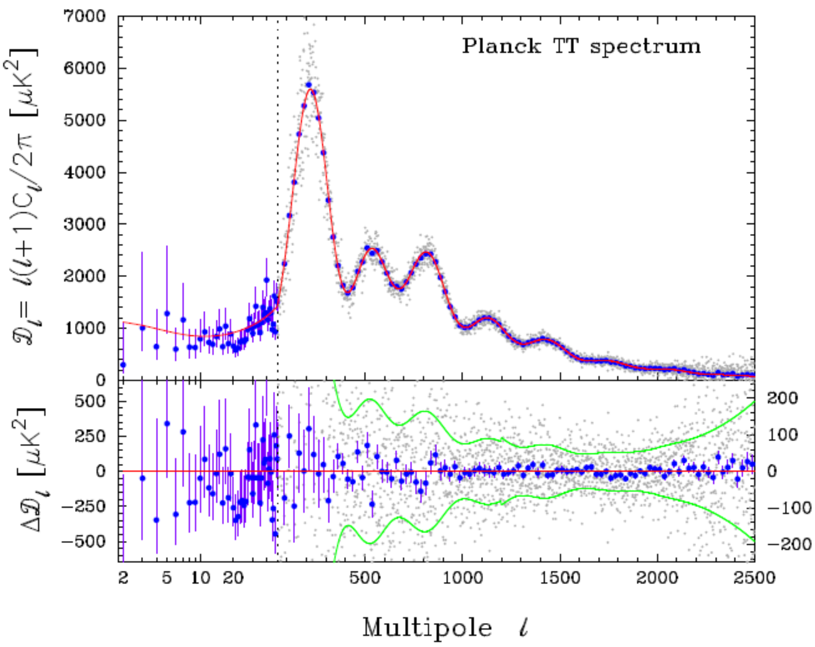
WIAS

SM, R.Saito, D.Langlois, arXiv:1405.4257, to appear in JCAP

X.Gao, D.Langlois, SM, JCAP 1210 040

X.Gao, D.Langlois, SM, JCAP 1310 023

Features in primordial power spectrum



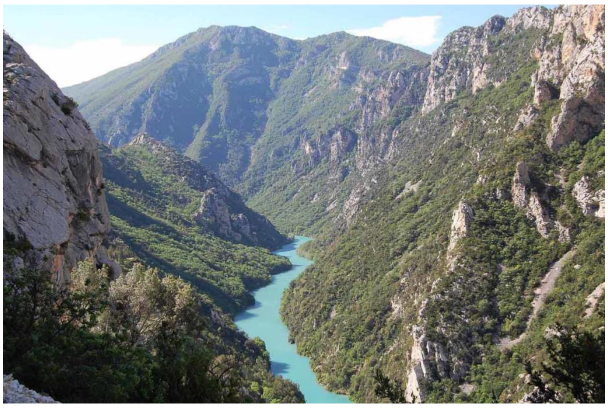
There might be Features at $\ell \sim 30$, $\ell \sim 750$ and $\ell \sim 1800$

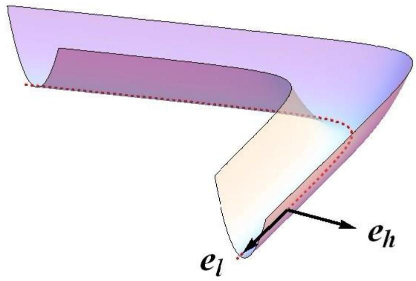
Heavy field in inflation

- Conventional wisdom
  - Perturbations from heavy fields (m >> H) are suppressed
  - Only perturbations from light fields are relevant to observables
- Recent progress

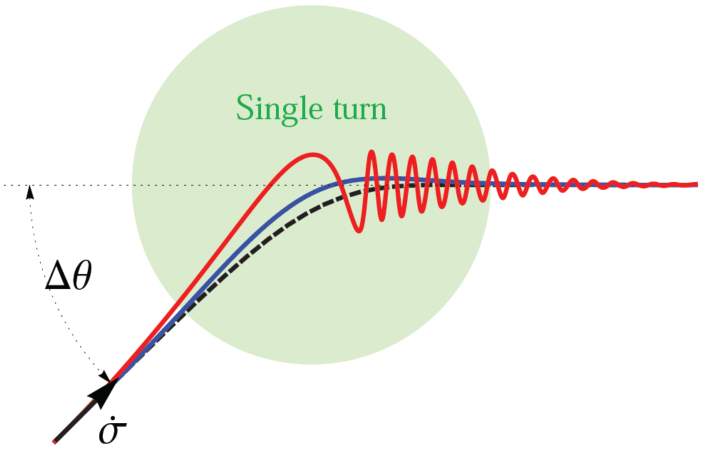
  Turn in the inflaton trajectory (change of light/heavy directions)

  ⇒ features in the primordial spectrum





Schematic picture of single sudden turn



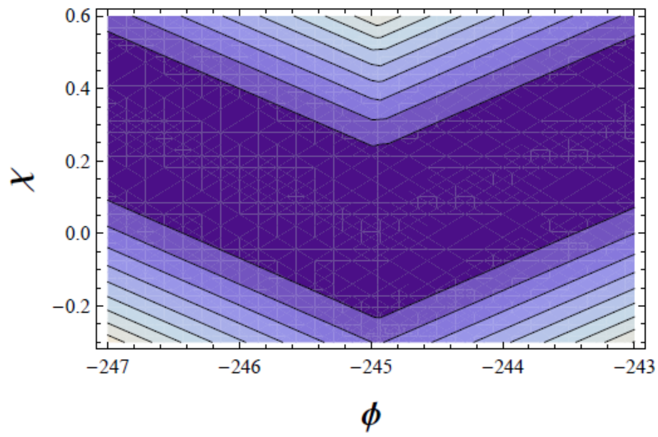
- During the turn, the trajectory deviates from the potential minimum
- For soft turn, it smoothly relaxes to the minimum of the potential
- For sharp turn, it relaxes to the minimum via oscillations

Modeling of the turn (canonical field)

Gao, Langlois, SM, `12

- θ_p (light-heavy basis) is useful for the sharp turn
- For a simple description, one can take

$$\dot{\theta}_p(t) = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2} \qquad \Delta t_{\rm turn} \sim \mu^{-1}$$



Going down to ϕ direction

Potential valley is given by

$$\chi = \pm \tan\frac{\Delta\theta}{2}(\phi - \phi_0)$$

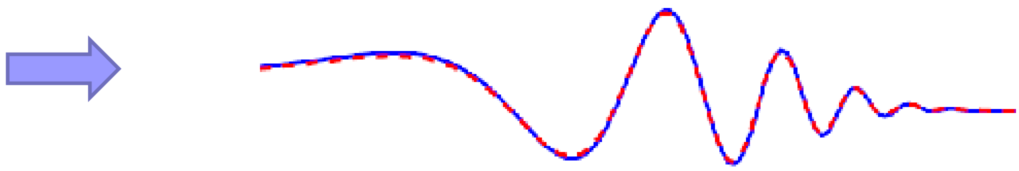
Features by a soft turn

- For $\mu < m_h$ the turn is soft
- Effective single light field description with a smaller sound speed

$$\frac{1}{c_s^2} = 1 + 4\frac{\dot{\theta}_p^2}{m_h^2}$$

Tolley & Wyman `09,

Achucarro, Gong, Hardeman, Palma, Patil `10



Oscillatory features $k \sim (aH)_{\rm turn}$

- Fitting to CMB data

Achucarro, Atal, Ortiz, Torrado `14

Features by a sharp turn

Gao, Langlois, SM `12, `13 (See also Noumi, Yamaguchi `13)

- For $\mu > m_h$ light field effective theory breaks down

- Two features in different scales

Mixing effect

$$(\Delta\theta)^2 \frac{m_h}{H} \frac{\cos x}{x}$$

$$x \equiv \frac{k}{(aH)_{\rm turn}}$$

Resonance effect

$$\bar{\epsilon}\,(\Delta\theta)^2 \left(\frac{m_h}{H}\right)^{-\frac{3}{2}} \frac{\cos(\ln y)}{y^3}$$

$$y \equiv \frac{k}{(am_h)_{\rm turn}}$$

negligible for canonical models !!

Resonance in a model with derivative couplings

Saito, Nakashima, Takamizu, Yokoyama `12, Saito, Takamizu `13

- Action

$$S_m \equiv -\int {\rm d}x^4\sqrt{-g}\left[\underbrace{\frac{1}{2}(\partial\phi)^2 + V(\phi)}_{\text{Inflaton}} + \underbrace{\frac{1}{2}(\partial\chi)^2 + \frac{m^2}{2}\chi^2}_{\text{Heavy field}} + \underbrace{K_n + K_d}_{\text{Coupling}}\right]$$

with $K_n \equiv \frac{\lambda_n}{2\Lambda_n}\chi(\partial\phi)^2 \quad K_d \equiv \frac{\lambda_{d1}}{4\Lambda_d^4}(\partial\chi)^2(\partial\phi)^2 + \frac{\lambda_{d2}}{4\Lambda_d^4}(\partial\chi\cdot\partial\phi)^2$

- Evolution equation for inflaton perturbations

$$\ddot{v}_k + m^2\left[\left(\frac{k}{am}\right)^2 - \frac{q_n}{2}\cos(mt) - 2q_d\cos(2mt)\right]v_k = 0$$

$$q_n \equiv \lambda_n\frac{\hat{\chi}_0}{\Lambda_n} \quad q_d \equiv -(\lambda_{d1} - 2\lambda_{d2})\frac{m^2\hat{\chi}_0^2}{8\Lambda_d^4}\left(\frac{k}{am}\right)^2 + (\lambda_{d1} + 2\lambda_{d2})\frac{m^2\hat{\chi}_0^2}{4\Lambda_d^4}$$

Mathieu equation which describes parametric resonance !!

DBI inflation with a turning trajectory

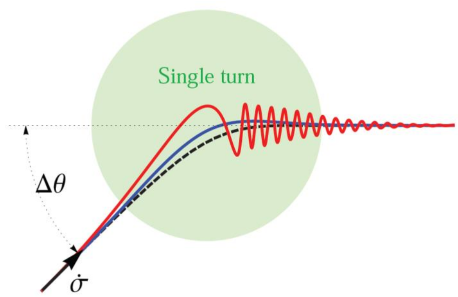
- Model

$$P(X^{IJ}, \phi^K) = -\frac{1}{f(\phi^I)}\left(\sqrt{\mathcal{D}} - 1\right) - V(\phi^I)$$

with $\mathcal{D} = \det(\delta^\mu_\nu + f\delta_{IJ}\partial^\mu\phi^I\partial_\nu\phi^J)$ $\quad X^{IJ} \equiv -\frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^J \quad I = l, h$

Derivative coupling

Potential $V(\phi^I)$ includes a sudden turn as before.



$$\dot{\theta}_p(t) = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2}$$

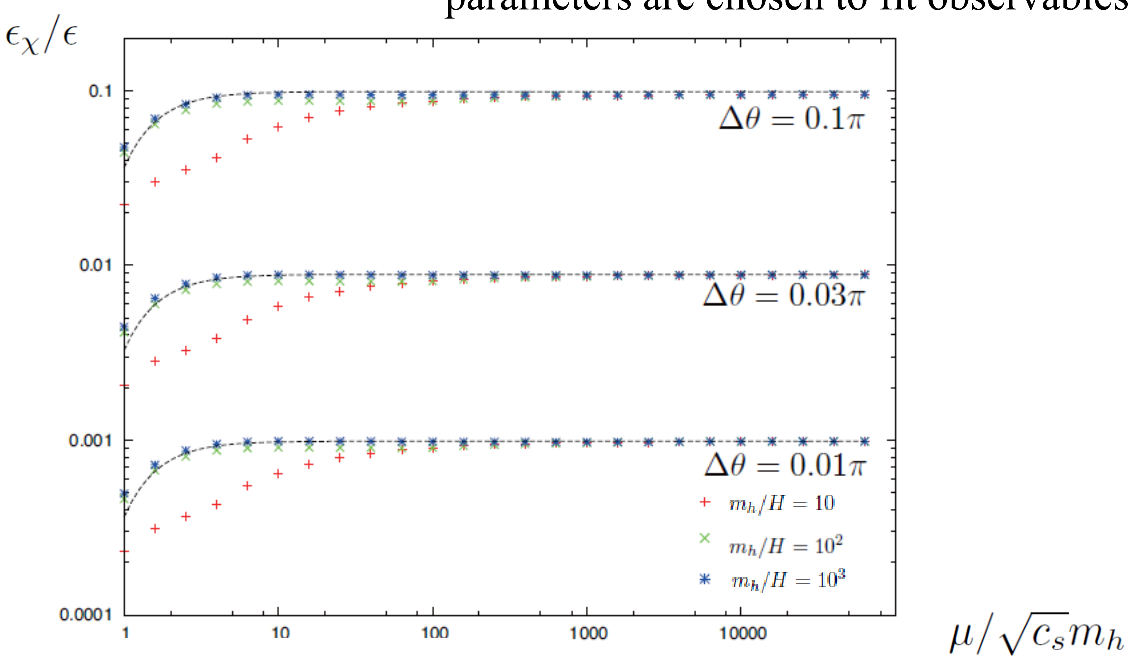
Overall angle

μ ~ 1/ Δt

For consistency, we restrict c_s is not much smaller than 1

Efficiency of the heavy field excitation

parameters are chosen to fit observables



$\epsilon_\chi/\epsilon \to (\Delta\theta)^2$ in the sharp turn limit

(coincide with the simple analytic estimation)

Relation between the features

▪Features by the resonance effect from the derivative coupling

$$\frac{\Delta\mathcal{P}_\zeta}{\mathcal{P}_\zeta} = \mathcal{O}(0.1)(1-c_s^2)\left(\frac{\epsilon_\chi/\epsilon}{0.1}\right)\left(\frac{m_h/H}{10}\right)^{1/2}$$

DBI inflation with a sharp turn $\epsilon_\chi/\epsilon \to (\Delta\theta)^2$

▪Relation between the amplitudes of two features

$$\frac{\Delta\mathcal{P}_{\zeta,\mathrm{res}}}{\mathcal{P}_\zeta} \sim (1-c_s^2)\left(\frac{m_h}{H}\right)^{-1/2}\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta} \qquad \Delta f_{\mathrm{NL,res}} \sim (1-c_s^2)^2\left(\frac{m_h}{H}\right)^{3/2}\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta}$$

Cf. Only Gravitational coupling

$$\epsilon\left(\frac{m_h}{H}\right)^{-\frac{5}{2}}\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta} \qquad \epsilon^2\left(\frac{m_h}{H}\right)^{\frac{1}{2}}\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta}$$

Conclusions

- Possibility of multiple features in primordial spectra
- Influence of heavy modes from a sharp turn
  - Oscillatory feature by mixing effect $k_{\mathrm{mix}} \sim (aH)_{\mathrm{turn}}$
  - Oscillatory feature by resonance effect $k_{\mathrm{res}} \sim (am_h)_{\mathrm{turn}}$

    appears for a sharp turn, but negligible for canonical models
- DBI inflation with a sharp turn (perturbative region)

$$\frac{\Delta\mathcal{P}_{\zeta,\mathrm{res}}}{\mathcal{P}_\zeta} \sim (1-c_s^2)\left(\frac{m_h}{H}\right)^{-1/2}\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta} \qquad \Delta f_{\mathrm{NL,res}} \sim (1-c_s^2)^2\left(\frac{m_h}{H}\right)^{3/2}\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta}$$

- Need to analyze the cases with small sound speed

“Cosmological perturbations in Loop Quantum Cosmology:

Mukhanov-Sasaki equations”

Guillermo A. Mena Marugan

[JGRG24(2014)111408]

Cosmological Perturbations in LQC: Mukhanov-Sasaki Equations

Guillermo A. Mena Marugán

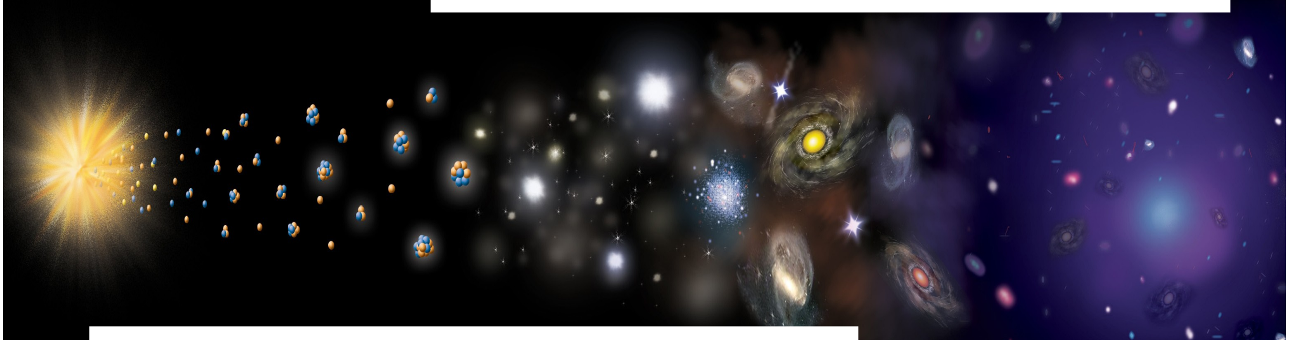
IEM-CSIC (L. Castelló Gomar, M. Fernández-Méndez & J. Olmedo)

75 AÑOS CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

JGRG24, 14 November 2014

The model

- We consider **perturbed** FRW universes with a massive, minimally coupled scalar field, in **LQC**. The model can generate inflation.



- The most interesting case is flat topology. We assume **compact** spatial sections.

Perturbations of **compact** (flat) FRW with a scalar field

Approximation: Truncation at **quadratic** perturbative order in the action. (Halliwell & Hawking)

- Uses the **modes** of the Laplace-Beltrami operator of the FRW spatial sections.
- Perturbations have **no zero modes**.
- Corrections to the action are **quadratic**.

LQC approaches

- In LQC, two types of approaches have been considered :

☆ **Effective** equations from the closure of the constraint algebra.

☆ Direct quantization:

→ ***Dressed metric***, with perturbations propagating on it as test fields, and no genuine perturbed quantum metric.

→ **Hybrid** approach.

Hybrid approach

Approximation: Effects of quantum geometry are only accounted for in the background

- Loop quantum corrections on matter **d.o.f.** and perturbations are ignored.

Uniqueness of the Fock description

- The **ambiguity** in selecting a **Fock representation** in QFT is removed by:

  - appealing to *background* spatial **symmetries**.
  - demanding the **UNITARITY** of the quantum evolution.

- There is additional ambiguity in the **separation of the background** and the matter field. This introduces time-dependent canonical field transformations.

- Our proposal selects a **UNIQUE canonical pair** and an **EQUIVALENCE CLASS** of invariant **Fock representations** for their CCR's.

Classical system: FRW

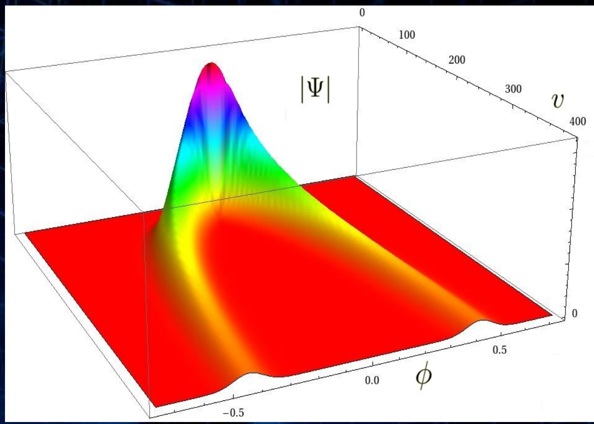
- **Massive scalar field ϕ coupled to a compact, flat FRW universe.**

Geometry:

$$\{c, p\} = 8\pi G\gamma/3. \quad V = |p|^{3/2} = a^3.$$

Hamiltonian constraint:

$$C_0 = -\frac{3}{4\pi G\gamma^2} p^2 c^2 + \pi_\phi^2 + m^2 V^2 \phi^2.$$



γ: Immirzi parameter.

- Specific **LQC proposal** such that it is optimal for numerical computation.

Classical system: FRW + Inhomogeneities

- Expand inhomogeneities in a **Fourier basis** of sines (-) and cosines (+), with frequency $\omega_n^2 = \vec{n}\cdot\vec{n}$.
- Consider **scalar perturbations, excluding zero modes**
- Call $g_{\vec{n},\pm}(t)$ and $k_{\vec{n},\pm}(t)$ the (properly scaled) **Fourier coefficients** of the lapse and shift.
- At **quadratic** order: $H = \frac{N_0 \sigma}{16\pi G} C_0 + \sum \left(N_0 H_2^{\vec{n},\pm} + N_0 g_{\vec{n},\pm} H_1^{\vec{n},\pm} + k_{\vec{n},\pm} \tilde{H}_1^{\vec{n},\pm} \right).$

Mukhanov-Sasaki variables

- We change variables to these **gauge invariants** for the perturbations.
- Their field equations match criteria for the choice of a unique Fock quantization.
- The change can be extended to a canonical set which includes the perturbative constraints.
- The homogeneous variables get quadratic corrections.

Mukhanov-Sasaki

- After this canonical transformation, the **Hamiltonian constraint** (at our perturbative order) amounts to:

$$H=\frac{N_0\sigma}{2V}\left[\boldsymbol{C}_0+\sum C_2^{\vec{n},\pm}\right].$$

$$\boldsymbol{C}_0=\pi_\phi^2-H_0^2(FRW,\phi).$$

$$H_0^2=\frac{3}{4\pi G\gamma^2}p^2c^2-m^2V^2\phi^2.$$

- The quadratic perturbative Hamiltonian is just the **Mukhanov-Sasaki Hamiltonian** in the rescaled variables.
- At this order, it is linear in the homogeneous field momentum

$$\boldsymbol{C}_2^{\vec{n},\pm}=-\Theta_e^{\vec{n},\pm}-\Theta_o^{\vec{n},\pm}\pi_\phi.$$

Born-Oppenheimer ansatz

- Consider states whose evolution in the inhomogeneities and FRW geometry **split**, with *positive frequency* in the homogeneous sector:

$$\Psi=\chi_0(V,\phi)\psi(\{N\},\phi), \qquad \chi_0(V,\phi)=\boldsymbol{P}\left[\exp\left(i\int_{\phi_0}^{\phi} d\tilde{\phi}\ \hat{H}_0(\tilde{\phi})\right)\right]\chi_0(V).$$

The FRW state is peaked (*semiclassical*).

- Disregard **nondiagonal** elements for the FRW geometry sector in the constraint and call:

$$\boldsymbol{d}_\phi\hat{O}=\partial_\phi\hat{O}-i\ [\hat{H}_0,\hat{O}].$$

Born-Oppenheimer ansatz

- The **diagonal** FRW-geometry part of the constraint gives:

$$-\partial_\phi^2\psi-i(2\langle\hat{H}_0\rangle_\chi-\langle\hat{\Theta}_o\rangle_\chi)\partial_\phi\psi=\left[\langle\hat{\Theta}_e+(\hat{\Theta}_o\hat{H}_0)_{sym}\rangle_\chi+i\langle\boldsymbol{d}_\phi\hat{H}_0-\frac{1}{2}\boldsymbol{d}_\phi\hat{\Theta}_o\rangle_\chi\right]\psi.$$

- The term in cyan can be ignored if $\langle\hat{H}_0\rangle_\chi$ is **not negligible small**.

- Besides, if we can **neglect**: a) The second derivative of ψ, b) The total ϕ-derivative of $2\hat{H}_0-\hat{\Theta}_o$,

$$-i\partial_\phi\psi=\frac{\langle\hat{\Theta}_e+(\hat{\Theta}_o\hat{H}_0)_{sym}\rangle_\chi}{2\langle\hat{H}_0\rangle_\chi}\psi.$$

Schrödinger-like equation.

Effective Mukhanov-Sasaki equations

- Starting from the **Born-Oppenheimer** form of the constraint and assuming a direct **effective** counterpart for the **inhomogeneities**:

$$d^2_{\eta_\chi} v_{\vec{n},\pm} = -v_{\vec{n},\pm}\left[4\pi^2\omega_n^2 + \langle \hat{\theta}_{e,(v)} + \hat{\theta}_{o,(v)} \rangle_\chi \right],$$

$$\langle \hat{\theta}_{e,(v)} + \hat{\theta}_{o,(v)} \rangle_\chi v^2_{\vec{n},\pm} = -\frac{\langle 2\hat{\Theta}_e + 2(\hat{\Theta}_o \hat{H}_0)_{sym} - i\, \boldsymbol{d}_\phi \hat{\Theta}_o \rangle_\chi}{2\langle [1/\hat{V}]^{-2/3} \rangle_\chi} - 4\pi^2 \omega_n^2 v^2_{\vec{n},\pm} - \pi^2_{v_{\vec{n},\pm}}.$$

where we have introduced a **state-dependent conformal time.**

- The effective equations are of harmonic **oscillator** type, with no dissipative term, and **hyperbolic in the ultraviolet** regime.

Conclusions

- We have considered the **hybrid quantization** of a FRW universe with a massive scalar field perturbed at **quadratic** order in the action.
- The model has been described in terms of **Mukhanov-Sasaki** variables.
- A **Born-Oppenheimer** ansatz leads to a Schrödinger equation for the inhomogeneities.
- We have derived effective **Mukhanov-Sasaki equations**. The ultra-violet regime is **hyperbolic**.

“Inflation from holography”

Yuko Urakawa

[JGRG24(2014)111409]

Inflation from holography

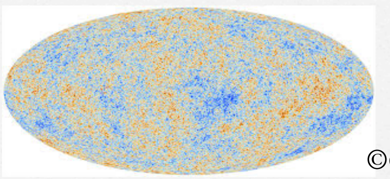
Yuko Urakawa (IAR, Nagoya U.)

with J.Garriga, K. Skenderis, F. Vernizzi

Y.U. & J.G. arXiv:1303.5997, JCAP 1307, 033
arXiv:1403.5497, JHEP 1406, 086
in progress
Y.U., J.G.&K.S. arXiv:1410.3290
Y.U., J.G.&F.V. in progress

Inflation from holography

WMAP, PLANCK, BICEP2(?), ...



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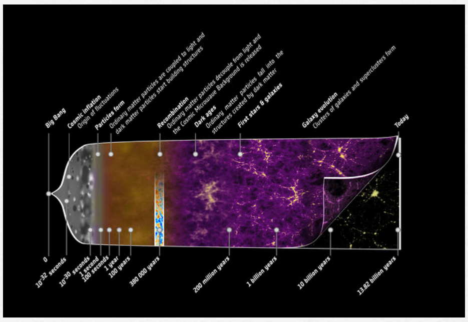
Inflation is now quite compelling.

- Can we describe inflation holographically?

If YES, what's the prediction?

If NO, what's the obstacle?

UV sensitivity of inflation



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energy ⟵ ⟶ time

- Perturbation with controlled radiative corrections

(ex) $$P(k) \propto \frac{1}{\varepsilon}\left(\frac{H}{M_{\rm pl}}\right)^2 \frac{1}{k^3}$$

- High energy scale near Plank scale
- Large excursion of the inflaton (?)

Geometry of AdS and dS

Anti de Sitter (AdS)

Vacuum with $\Lambda < 0$

in $R^{2,3}$ (-,-, +, +, +) SO(2,3)

$$-X_0^2 - X_1^2 + \sum_{a=2,3,4} X_a^2 = -A^2$$

$$ds^2 = l_{\rm AdS}^2\left(\frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2}\right)$$

de Sitter (dS)

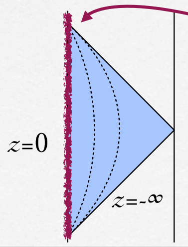
Vacuum with $\Lambda > 0$

in $R^{1,4}$ (-,+, +, +, +) SO(1,4)

$$-X_0^2 + X_1^2 + \sum_{a=2,3,4} X_a^2 = A^2$$

$$ds^2 = l_{\rm dS}^2\left(\frac{-d\eta^2 + dx^2 + dy^2 + dw^2}{\eta^2}\right)$$

Boundary

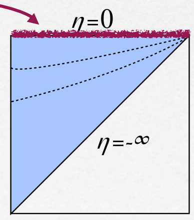


z:const, R^3

$l_{\rm AdS} \to i l_{\rm dS}$

$z \to i\eta$

$t \to iw$

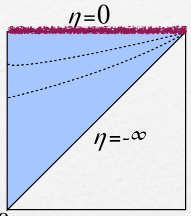


η:const, R^3

dS/CFT

Strominger(01), Witten(01)

- CFT lives on the spacelike boundary at the future infinity of dS.



Maldacena(02)

- Wave function for bulk gravity $\Psi_{dS}[g]=Z_{CFT}$
  Probability distribution $P_{dS}[g]=|Z_{CFT}|^2$

  - Euclidean AdS $S_{AdS\ \mathrm{ren}} = -\frac{1}{\kappa^2}\int \frac{d^3\boldsymbol{k}}{2\pi^3}\frac{1}{2}R_{AdS}^2 k^3 \phi_0(-k)\phi_0(k)$

$\xrightarrow{Z\sim e^{-S}}$ $\langle O(k)O(-k)\rangle = -\frac{\delta^2 S_{AdS\ \mathrm{ren}}}{\delta\phi_0(k)\delta\phi_0(-k)} = \frac{1}{2}\frac{R_{AdS}^2}{\kappa^2}k^3$

  - dS $\langle\phi(k)\phi(-k)\rangle = -\frac{1}{2\mathrm{Re}\langle O(k)O(-k)\rangle}\Big|_{R_{AdS}=-iR_{dS}} = \frac{\kappa^2}{R_{dS}^2}\frac{1}{k^3}$

Analytic cont. connects dual boundaries of dS and AdS

Challenges of dS/CFT

- Holographic direction is time like.
  Dual boundary theories to dS are non-unitary.
  Good property?
- Poor understanding on analytic continuation.
  Extendable to a non-perturbative example in 1/N?
- Lack of a concrete example.
  First concrete example of dS/CFT

  Anninos, Hartman, &Strominger(11)

  Vasiliev gravity in dS_4 $\longleftrightarrow$ Sp(N) CFT_3 living at $\mathscr{I}^+$

  $\Lambda \to -\Lambda$ N → - N

Breaking symmetry

de Sitter space

4D hyperboloid:

$$ds_4^2 = \{\eta_{\mu\nu} X^\mu X^\nu = H^{-2}\}$$

in 5D flat spacetime

SO(1,4) ⟷

CFT on R^3

- Poincare T.
- Dilatation
- Special C.T.

Cosmological const. Λ
+ inflaton φ
Breaking dS sym.

↓ Inflation

CFT
+ φO (ex)mass
Breaking conf. sym.

↓ Deformed CFT

Holographic inflation

4D bulk

Inflation
= dS + modulation

←

3D boundary

QFT
CFT+ deformation

$$\Psi_{\text{bulk}}[\varphi] = Z_{\text{QFT}}[g]$$

$$Z_{\text{QFT}} = \int D\chi \exp\left[-S_{\text{CFT}} - \int g\mathcal{O}[\chi]\right]$$

deformation (the term $\int g\mathcal{O}[\chi]$)

Necessary building blocks:
- φ & g relation?
- t & μ relation?

Summary of the status ('14)

Holographic inflation: Boundary to Bulk   Consistency check

Single-field slow-roll inflation

- Time evolution of ϕ, ζ ← Solving RG flow in boundary

  Bzowski et al. (12), Garriga & Y.U. (14),+

  Consistent with bulk prediction via CPT

- Conservation of ζ at large scales if $a(t) \propto \mu^C$ C : const

  Garriga & Y.U. (14, in progress)

- Consistency relation $f_{\rm NL} \sim (1-n_s)$

  Bzowski et al. (12), Schalm et al. (12)

Multi-field slow-roll inflation

- Time evolution of ϕ, ζ, δs ← Solving RG flow in boundary

  Consistent with bulk prediction via δN formalism

  Garriga, Skenderis, & Y.U. (14), Garriga, Y.U., & Vernizzi (in progress)

Background evolution

Reconstruction of potential

KG equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

↓

$$\frac{d\phi}{d\ln a} = -\frac{2}{\kappa^2}\frac{1}{W(\phi)}\frac{\partial W(\phi)}{\partial \phi}$$

$$V(\phi) = \frac{8}{\kappa^2}\left[\frac{3}{2}W^2(\phi) - \frac{1}{\kappa^2}\left(\frac{\partial W(\phi)}{\partial \phi}\right)^2\right]$$

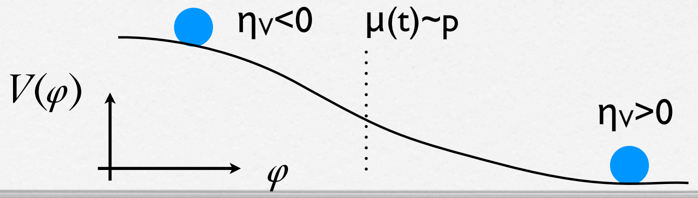
RG equation

$$\frac{dg}{d\ln\mu} = \lambda g + \frac{\tilde{C}}{2}g^2 + O(g^3)$$

Klebanov et al. (11)

$g(\mu, \mathbf{x}) = \kappa\phi(t(\mu), \mathbf{x})$

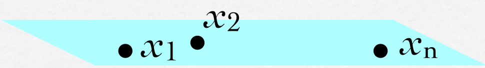
$d\ln a = C\, d\ln\mu$



Primordial perturbations

ζ Correlators

in cosmology (bulk) | in boundary QFT



flat space R=0

$\langle \delta\phi(x_1)\delta\phi(x_2)\cdots\delta\phi(x_n)\rangle$ ← $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_n)\rangle$

$$\Psi_{\rm bulk}[\varphi] = Z_{\rm QFT}[g]$$

J.G.&Y.U.(13)

$$\zeta = -\frac{H}{\dot\phi}\delta\phi + \frac{\varepsilon_2}{4}\left(\frac{H}{\dot\phi}\right)^2 \delta\phi^2 + \cdots$$

at large scales

$$\varepsilon_1 \equiv \frac{1}{2}\frac{\dot\phi^2}{H^2}$$

$$\varepsilon_2 \equiv \frac{\mathrm{d}\ln\varepsilon_2}{\mathrm{d}\ln a}$$

↓

$\langle \zeta(x_1)\zeta(x_2)\cdots\zeta(x_n)\rangle$

Conservation of ζ

Power spectrum

J.G.&Y.U.(14)

$$\langle\zeta(\boldsymbol{x}_1)\zeta(\boldsymbol{x}_2)\rangle_{\rm conn} = W^{(2)\,-1}(\boldsymbol{x}_1,\,\boldsymbol{x}_2)$$

$$B_1(\mu) = -\frac{\partial\delta\phi}{\partial\zeta}\bigg|_{\zeta=0}$$

$$W^{(2)}(\boldsymbol{x}_1,\,\boldsymbol{x}_2) = -2{\rm Re}\left[B_1^2(\mu)\langle\mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\rangle_\mu\right]$$

Conservation ⟶ $\frac{\mathrm{d}}{\mathrm{d}\mu}\left[B_1(\mu)\sqrt{Z(\mu)}\right] = 0$

Gauge transformation $B_1 = \frac{\dot\phi}{H} = \frac{\mathrm{d}\phi}{\mathrm{d}\ln a}$

RG flow $\sqrt{Z(\mu)} = 4p^{-\lambda}\frac{\beta(p)}{\beta(\mu)}$

⟶ Identification between t & μ

$$\ln(\mu/\mu_0) = \mathcal{C}\ln(a/a_0)$$

$\mathcal{C}$: const

$\mathcal{C}$=1+O(ε)

Conserved Power spectrum

$$P_\zeta(k) \simeq -\frac{1}{c\beta^2(p)}\frac{1}{k^3}\left(\frac{k}{p}\right)^{-2\lambda}\left[1+\left(\frac{k}{p}\right)^{\lambda}\right]^4 \simeq -\frac{1}{c\beta^2(k)}\frac{1}{k^3}$$

cf Agrees with the result of Bzowski+ (12) in $\mu \to \infty$

Remarks

1.Amplitude

$$\beta = \frac{dg}{d\ln\mu} \sim \frac{d(\phi/M_{pl})}{d\ln a} = \sqrt{2\varepsilon}$$

$-c \simeq (M_{\rm pl}/H_{\rm dS})^2$ Strominger(01) $\longrightarrow$ $-\frac{1}{c\beta^2} \sim \frac{1}{\varepsilon}\left(\frac{H}{M_{pl}}\right)^2$

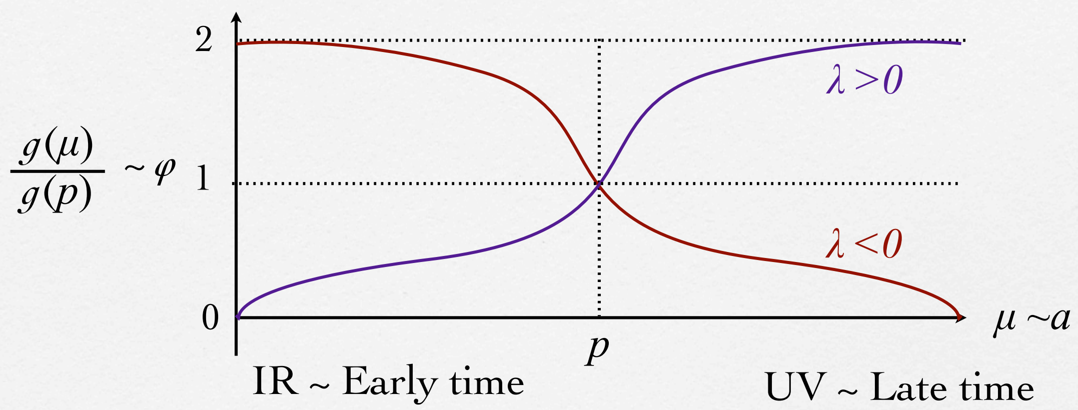
Maldacena(02)

2. Spectral index

For $k >> p$ n_s-1=2$|\lambda|$ Blue-tilted

For $k << p$ n_s-1=-2$|\lambda|$ Red-tilted

Evolution of "inflaton"



N.B. n_s - 1=-6ε+2η

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