JGRG24

The 24th Workshop on General Relativity and Gravitation in Japan

10 (Mon) — 14 (Fri) November 2014

KIPMU, University of Tokyo

Chiba, Japan

Oral presentations: Day 4

Contents

Programme: Day 4

Thursday 13 November 2014

Morning 1 [Chair: Jun'ichi Yokoyama]

9:30 Francois Bouchet (IAP, Planck) [Invited]
"Latest results from the Planck collaboration" [JGRG24(2014)111301]

10:15 Daisuke Yamauchi (RESCEU)
"Constraining primordial non-Gaussianity via multi-tracer technique with Euclid and SKA" [JGRG24(2014)111302]

10:30 Ichihiko Hashimoto (YITP, Kyoto)
"Detecting primordial non-Gaussianity from the three-point statistics of halo and weak lensing fields" [JGRG24(2014)111303]

10:45-11:00 coffee break

Morning 2 [Chair: Yasusada Nambu]

11:00 Yuki Watanabe (RESCEU)
"Self-unitarization of New Higgs Inflation" [JGRG24(2014)111304]

11:15 Naoyuki Takeda (ICRR)
"No quasi-stable scalaron lump forms after R^2 inflation" [JGRG24(2014)111305]

11:30 Masaki Yamada (ICRR)
"Gravitational waves as a probe of supersymmetric scale" [JGRG24(2014)111306]

11:45 Tomohiro Nakama (RESCEU)
"Investigating tensor perturbations on small scales from their second-order effects to generate scalar perturbations" [JGRG24(2014)111307]

12:00 Laura Castello Gomar (CSIC)
"A unique Fock quantization for scalar fields in cosmologies with signature change" [JGRG24(2014)111308]

12:15 Sakine Nishi (Rikkyo)
"Generalized Galilean Genesis" [JGRG24(2014)111309]

12:30 - 14:00 lunch & poster view

Afternoon 1 [Chair: Masahide Yamaguchi]

14:00 Leonardo Senatore (Stanford) [Invited]
"The Effective Field Theory of Cosmological Large Scale Structures"
[JGRG24(2014)111310]

14:45 Ippei Obata (Kyoto)
"Chromo - Multi Natural Inflation" [JGRG24(2014)111311]

15:00 Guillem Domenech (Kyoto)
"Conformal frame dependence of Inflation – scalar field with an exponential potential –" [JGRG24(2014)111312]

15:15 Rajeev Kumar Jain (CP3)
"Non-gaussian imprints of primordial magnetic fields from inflation"
[JGRG24(2014)111313]

15:30-16:00 coffee break & poster view

Afternoon 2 [Chair: Takahiro Tanaka]

16:00 Tomohiro Fujita (Kavli IPMU)
"Can a Spectator Scalar Field Enhance Inflationary Tensor Modes?"
[JGRG24(2014)111314]

16:15 Taro Kunimitsu (RESCEU)
"Large tensor mode and sub-Planckian excursion in generalized G-inflation"
[JGRG24(2014)111315]

16:30 Keisuke Harigaya (Kavli IPMU)
"Lower bound on the tensor-to-scalar ratio in a nearly quadratic chaotic inflation model in supergravity" [JGRG24(2014)111316]

16:45 Kohei Kamada (EPFL)
"Cosmic string in the delayed scaling scenario and CMB" [JGRG24(2014)111317]

17:00 Kohji Yajima (Rikkyo)
"Gravitational waves from slow-roll inflation in Lorentz-violating Weyl gravity"
[JGRG24(2014)111318]

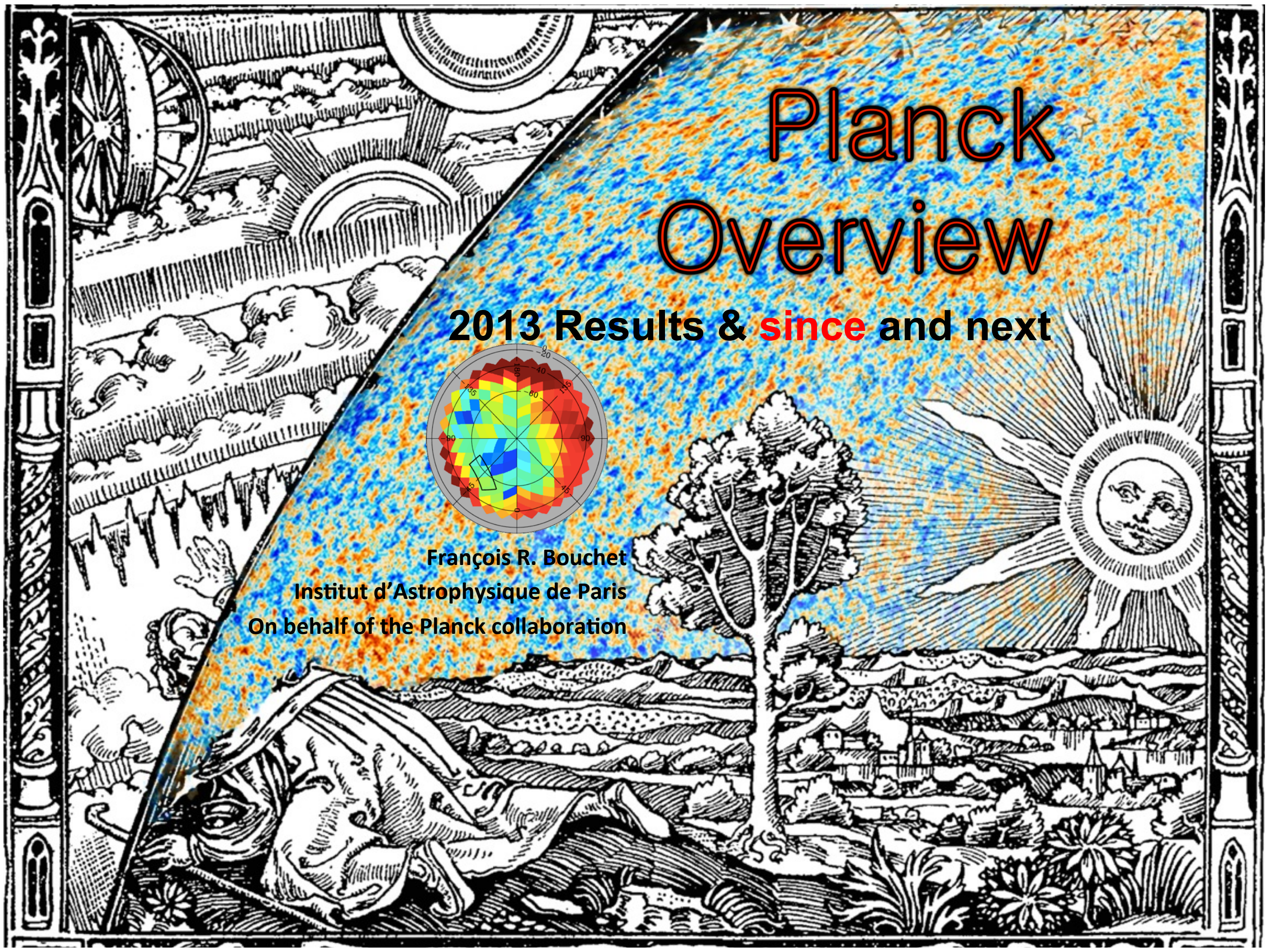
17:15 Tomohiro Harada (Rikkyo)
"Black holes as particle accelerators: a brief review" [JGRG24(2014)111319]

17:30 - 18:00 poster view

“Latest results from the Planck collaboration”

Francois Bouchet [Invited]

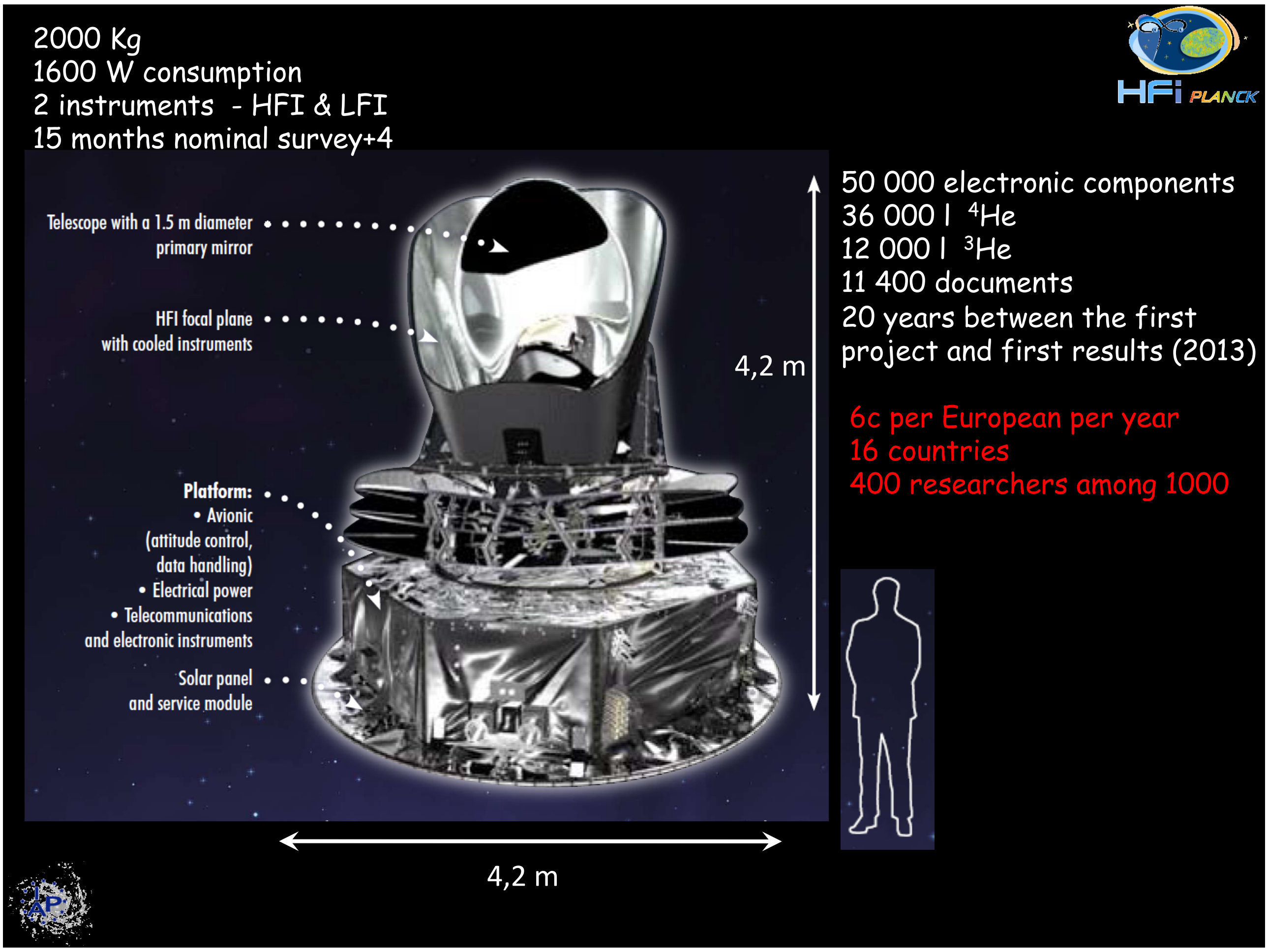
[JGRG24(2014)111301]



Planck Overview

2013 Results & since and next

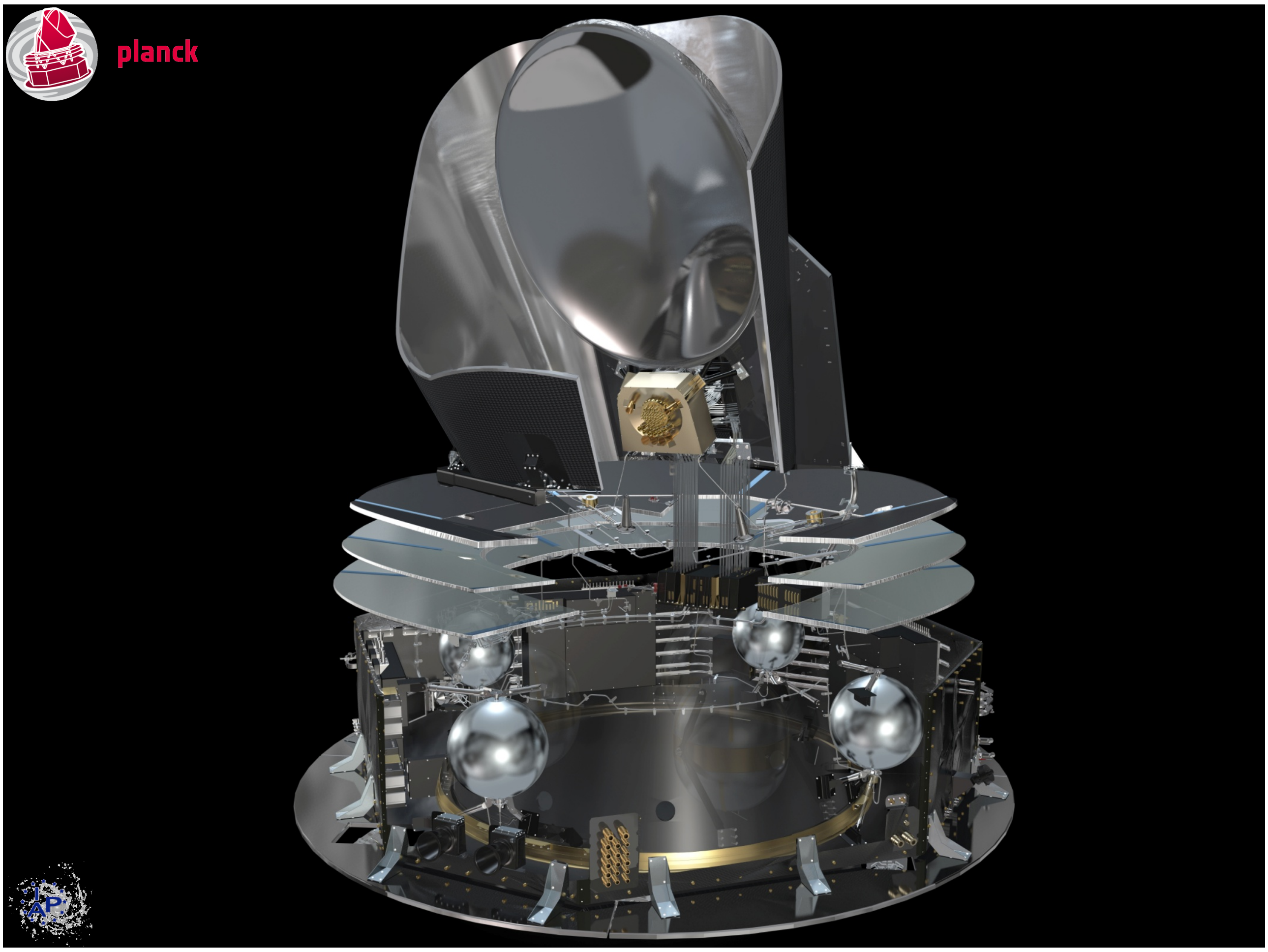
François R. Bouchet
Institut d'Astrophysique de Paris
On behalf of the Planck collaboration

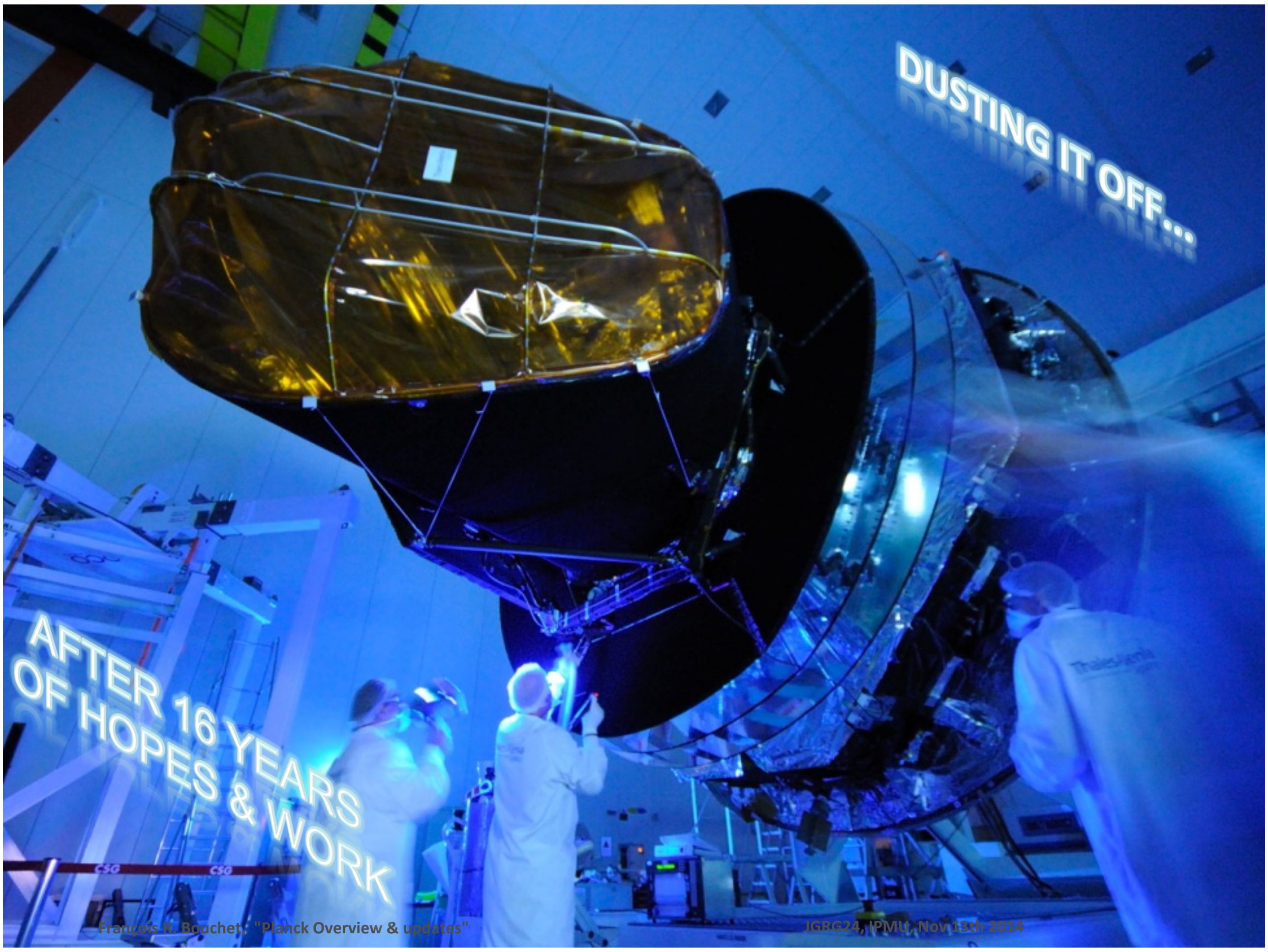


2000 Kg
1600 W consumption
2 instruments - HFI & LFI
15 months nominal survey+4

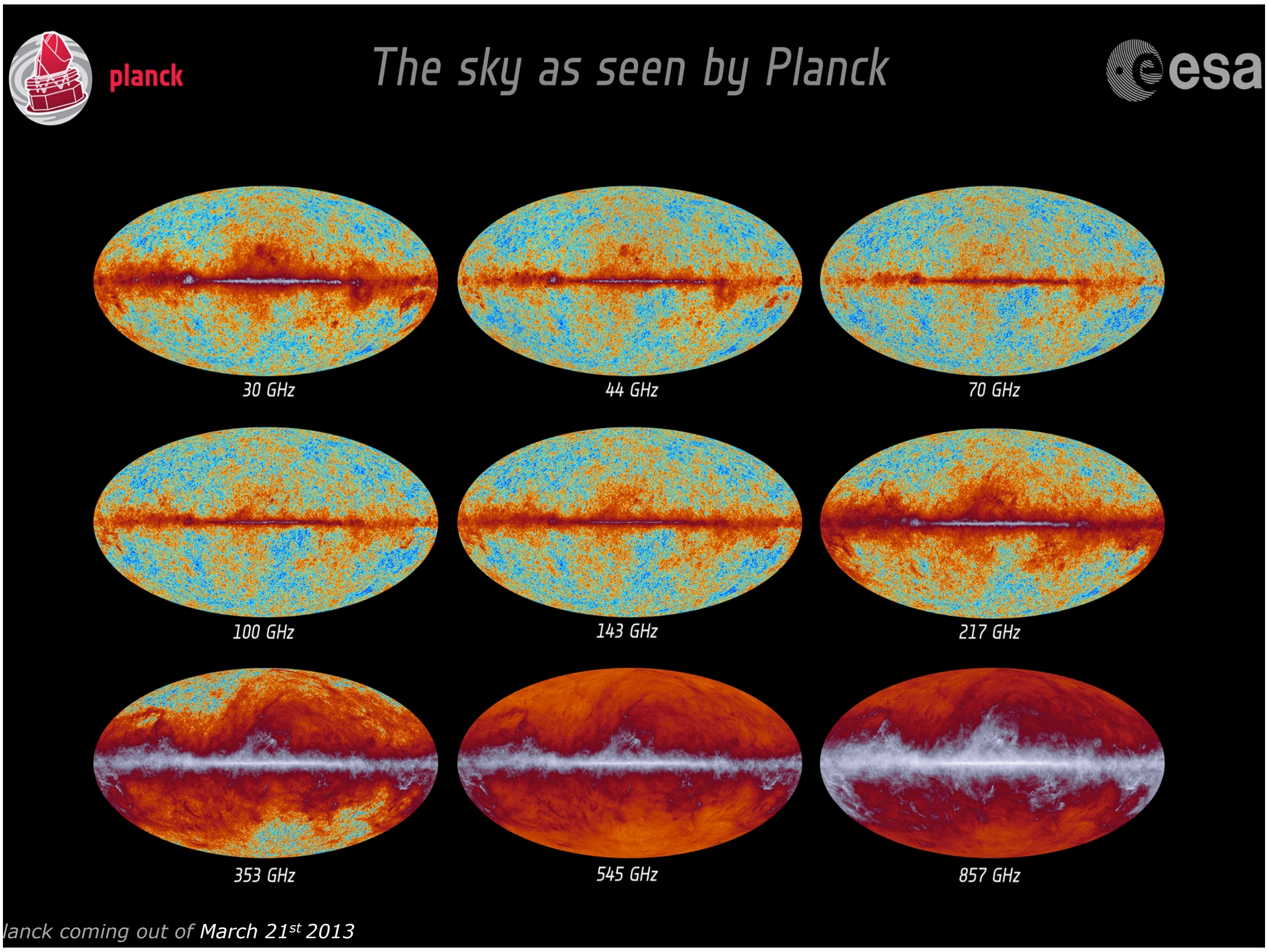
50 000 electronic components
36 000 l ^{4}He
12 000 l ^{3}He
11 400 documents
20 years between the first project and first results (2013)

6c per European per year
16 countries
400 researchers among 1000




DUSTING IT OFF...

AFTER 16 YEARS OF HOPES & WORK

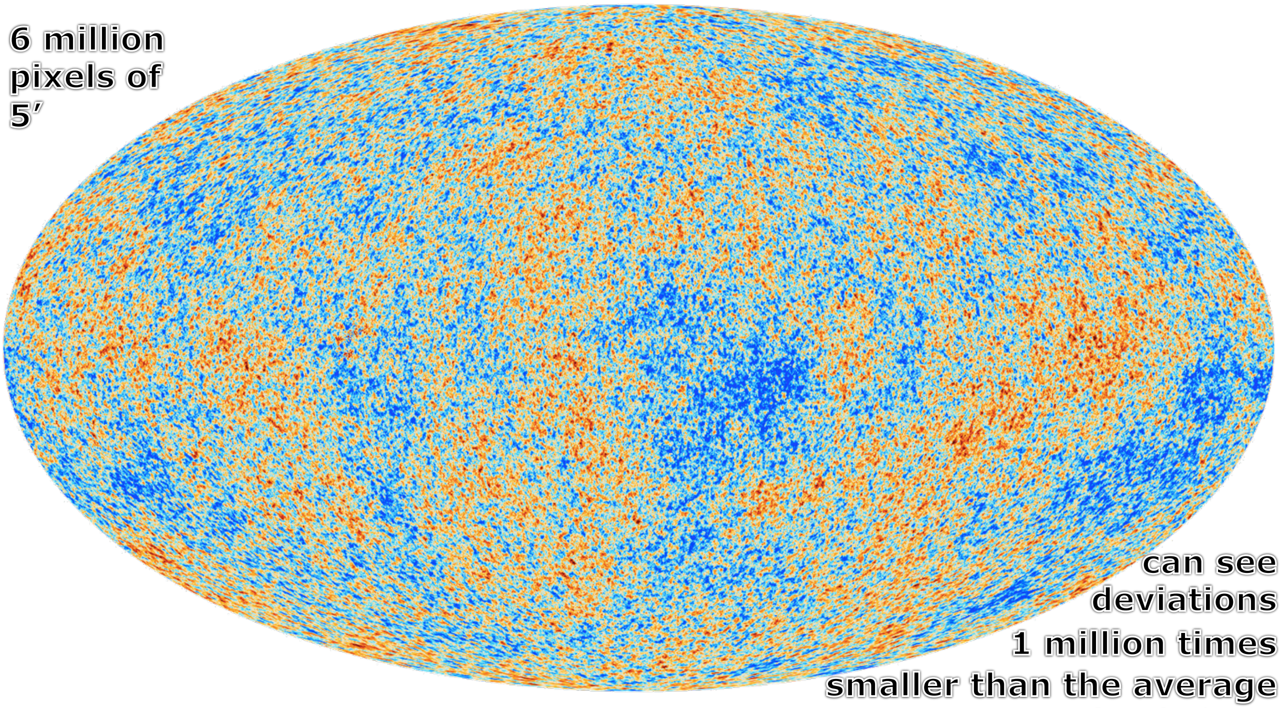


The sky as seen by Planck

lanck coming out of March 21st 2013

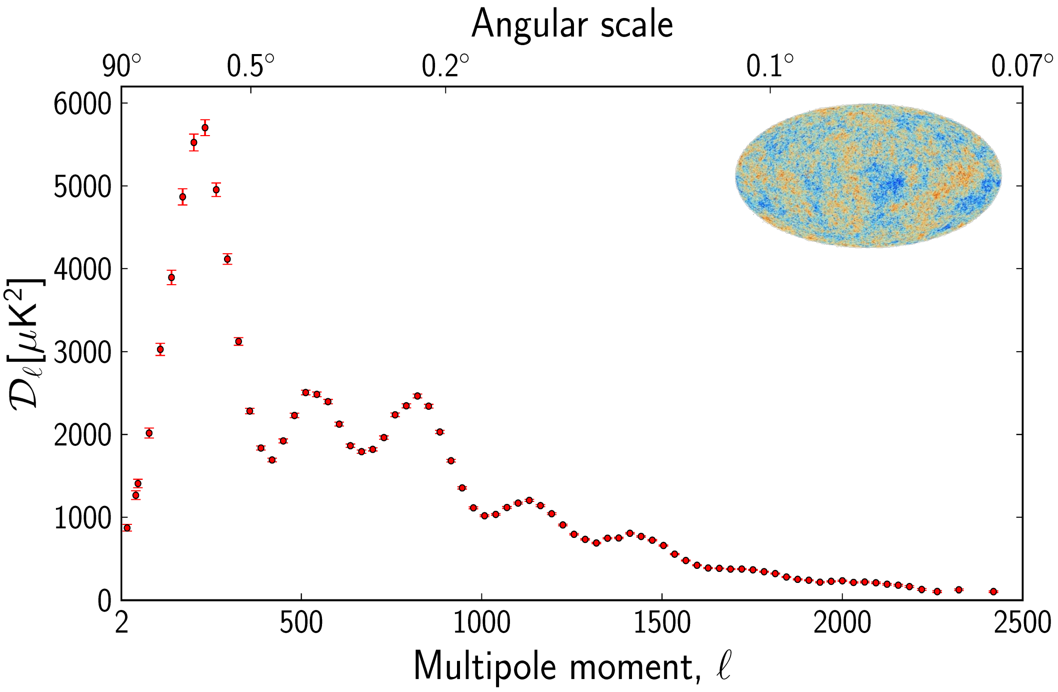
The cosmic microwave background Temperature anisotropies

6 million
pixels of
5′

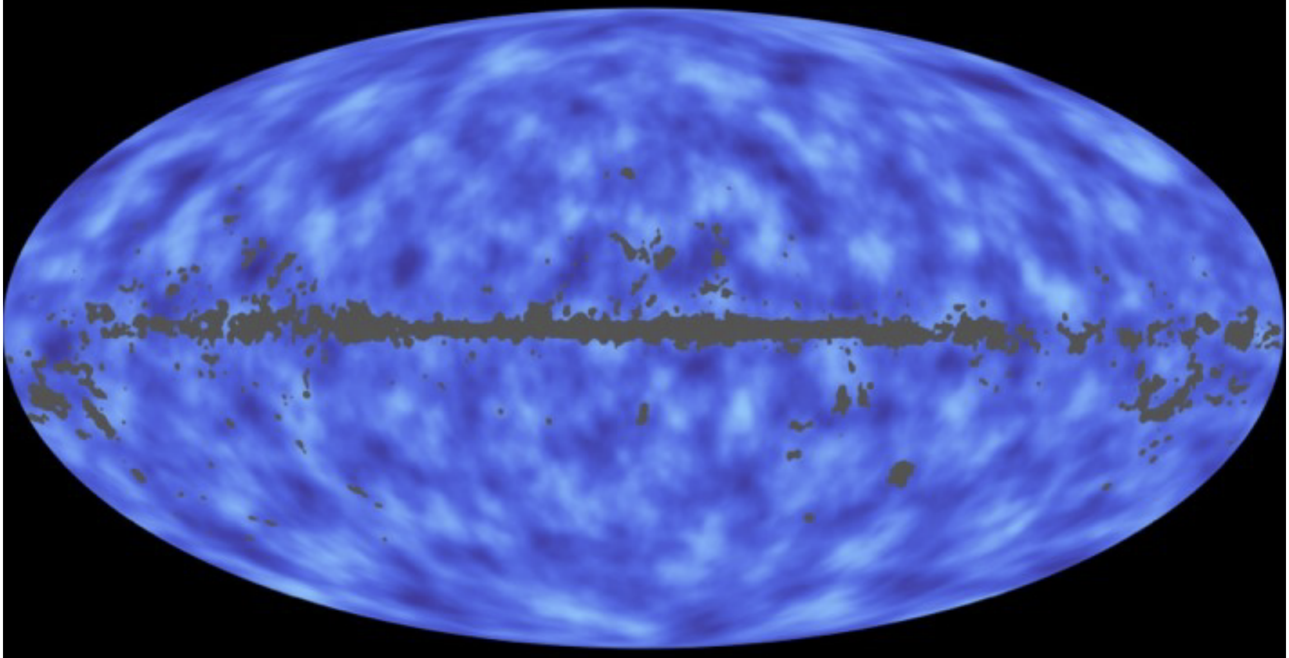


can see
deviations
1 million times
smaller than the average

The Planck power spectrum of Temperature anisotropies

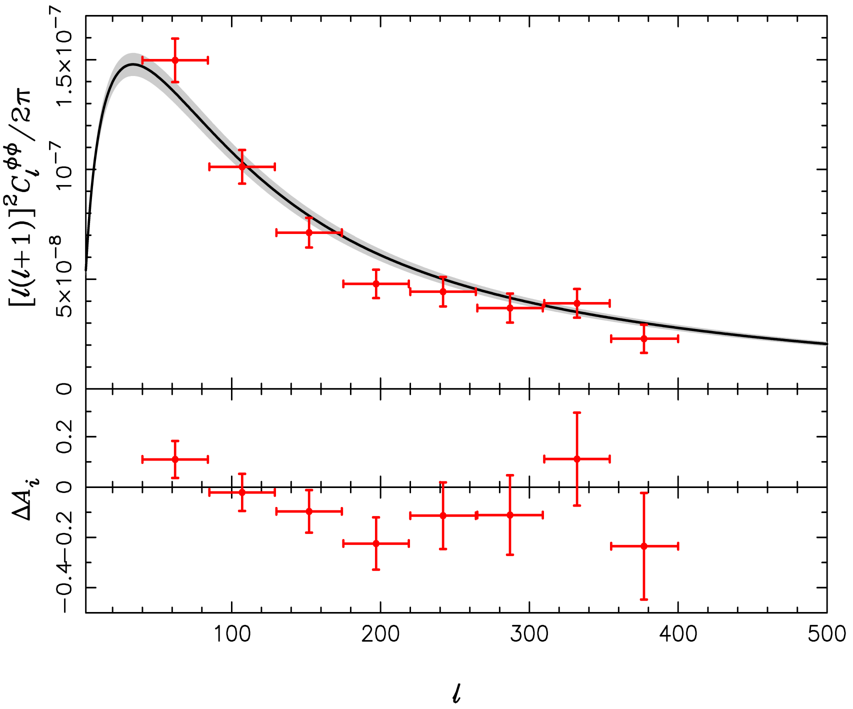


Projected mass map



The (grey) masked area is where foregrounds are too strong to allow an accurate reconstruction

The lensing potential spectrum



(26 detection)

➔ Agrees well with the LCDM prediction from T alone

Base ΛCDM model 6 parameters

Planck alone

	Planck (CMB+lensing)	
Parameter	Best fit	68 % limits
$\Omega_b h^2$	0.022242	0.02217 ± 0.00033
$\Omega_c h^2$	0.11805	0.1186 ± 0.0031
$100\theta_{MC}$	1.04150	1.04141 ± 0.00067
τ	0.0949	0.089 ± 0.032
n_s	0.9675	0.9635 ± 0.0094
$\ln(10^{10} A_s)$	3.098	3.085 ± 0.057

- $\Omega_b h^2$ Baryon density today
- $\Omega_c h^2$ Cold dark matter density today
- Θ Sound horizon size when optical depth τ reaches unity at t ~380 000y)
- τ Optical depth at reionisation, i.e. fraction of the CMB photons re-scattered during it
- A_s Amplitude of the curvature power spectrum
- n_s Scalar power spectrum power law index (n_s-1 measures departure from scale invariance)

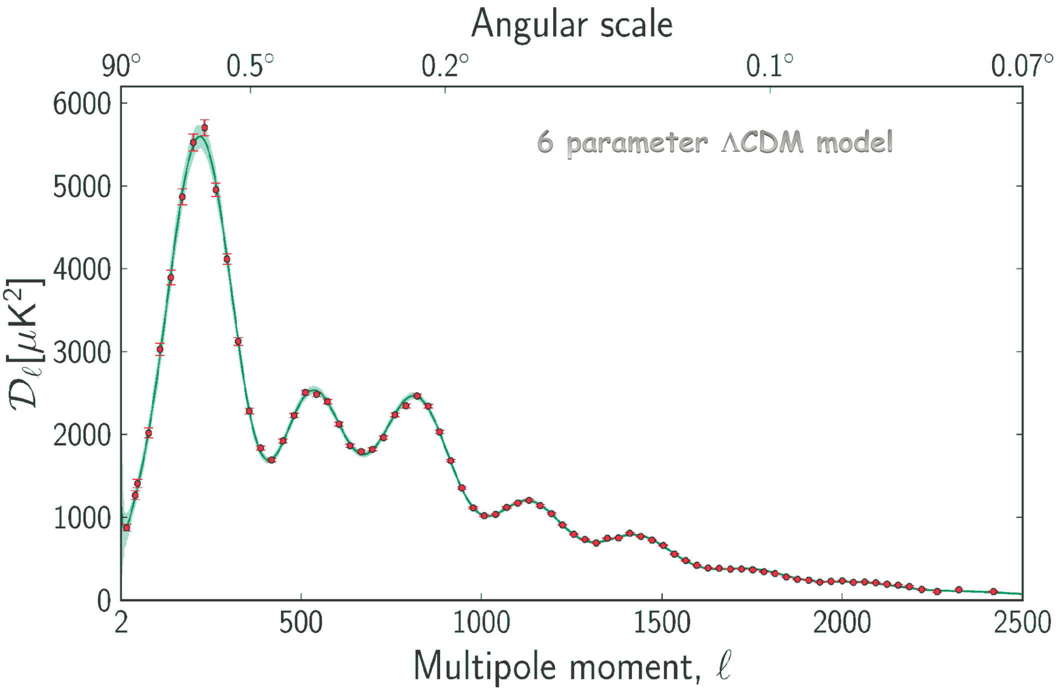
The sound horizon, Θ, determined by the positions of the peaks (7), is now determined with 0.07% precision
(links together $\Omega_b h^2$, $\Omega_c h^2$, H_0 - here as $\Omega_m h^3$)

Exact scale invariance of the primordial fluctuations is ruled out, *at ~4σ*

(as predicted by base inflation models)

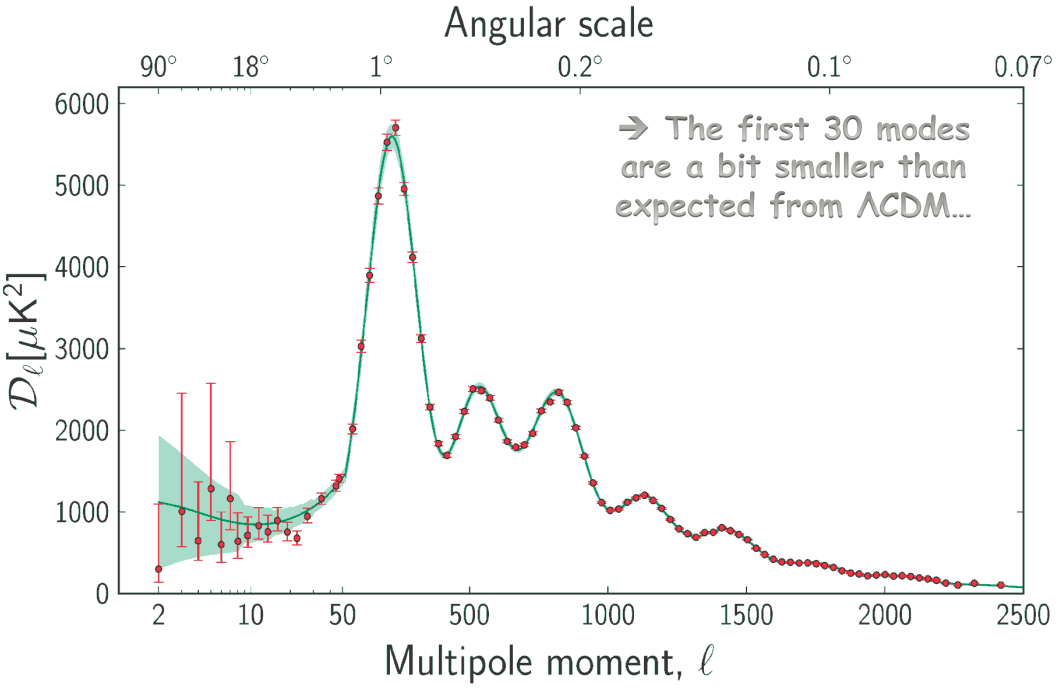
$\theta_* = (1.04148 \pm 0.00066) \times 10^{-2} = 0.596724° \pm 0.00038°$

Theory confronts data



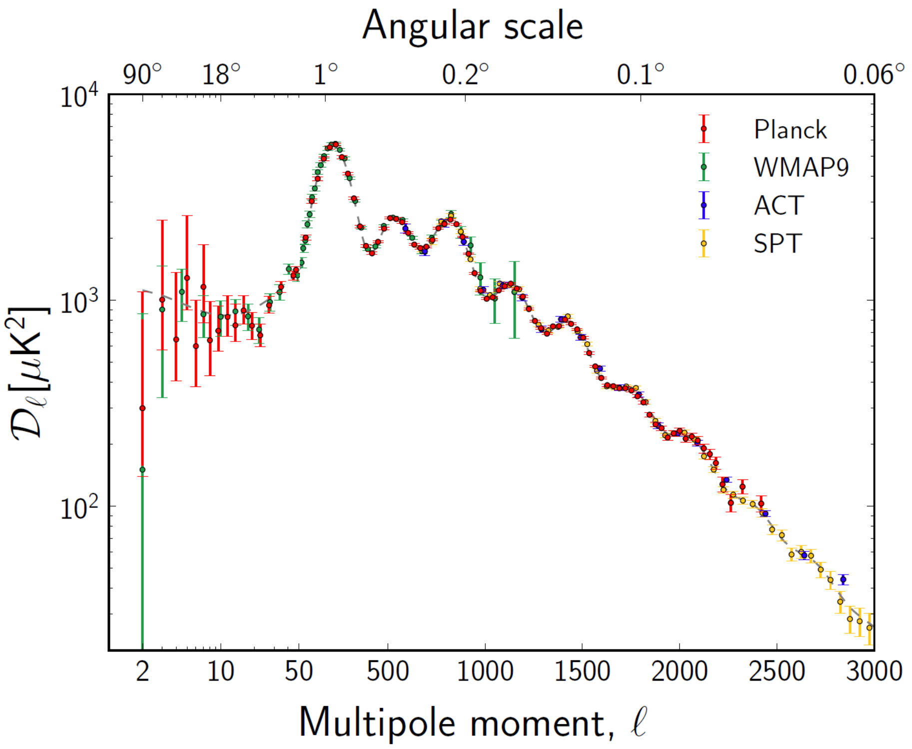
6 parameter ΛCDM model

Zooming on the very largest scales, l<50…



→ The first 30 modes are a bit smaller than expected from ΛCDM…

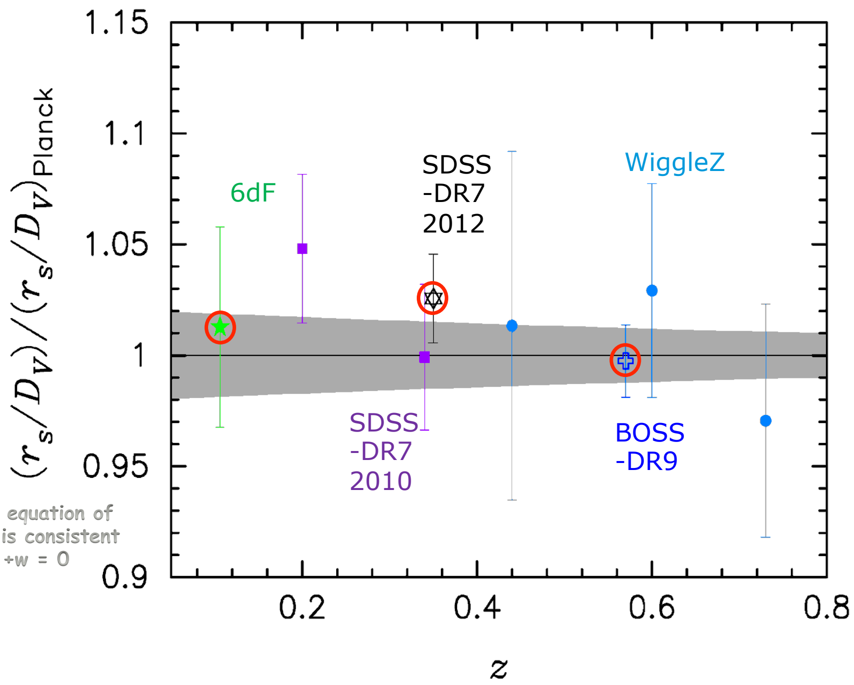
The 2013 CMB temperature landscape

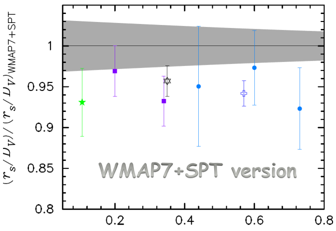


Frai

opean Space Agency

BAO acoustic-scale distance ratio





(see Hu etal12 for explanations by extensions)

Planck Prediction (±1σ shaded area)

→ Planck & BAO are all in quite tight agreement

→ DE equation of state is consistent with 1+w = 0

European Space Agency

François R. Bouchet, "Planck Overview & updates"

Base ΛCDM model 6 parameters

CMB+LSS - 2013

	Planck (CMB+lensing)		*Planck*+WP+highL+BAO	
Parameter	Best fit	68 % limits	Best fit	68 % limits
$\Omega_b h^2$	0.022242	0.02217 ± 0.00033	0.022161	0.02214 ± 0.00024
$\Omega_c h^2$	0.11805	0.1186 ± 0.0031	0.11889	0.1187 ± 0.0017
$100\theta_{MC}$	1.04150	1.04141 ± 0.00067	1.04148	$\boxed{1.04147 \pm 0.00056}$
τ	0.0949	0.089 ± 0.032	0.0952	0.092 ± 0.013
n_s	0.9675	0.9635 ± 0.0094	0.9611	$\boxed{0.9608 \pm 0.0054}$
$\ln(10^{10} A_s)$	3.098	3.085 ± 0.057	3.0973	3.091 ± 0.025

The sound horizon, θ, determined by the positions of the peaks (7), is now determined with 0.05% precision
(links together $\Omega_b h^2$, $\Omega_c h^2$, H_0 - here as $\Omega_m h^3$)

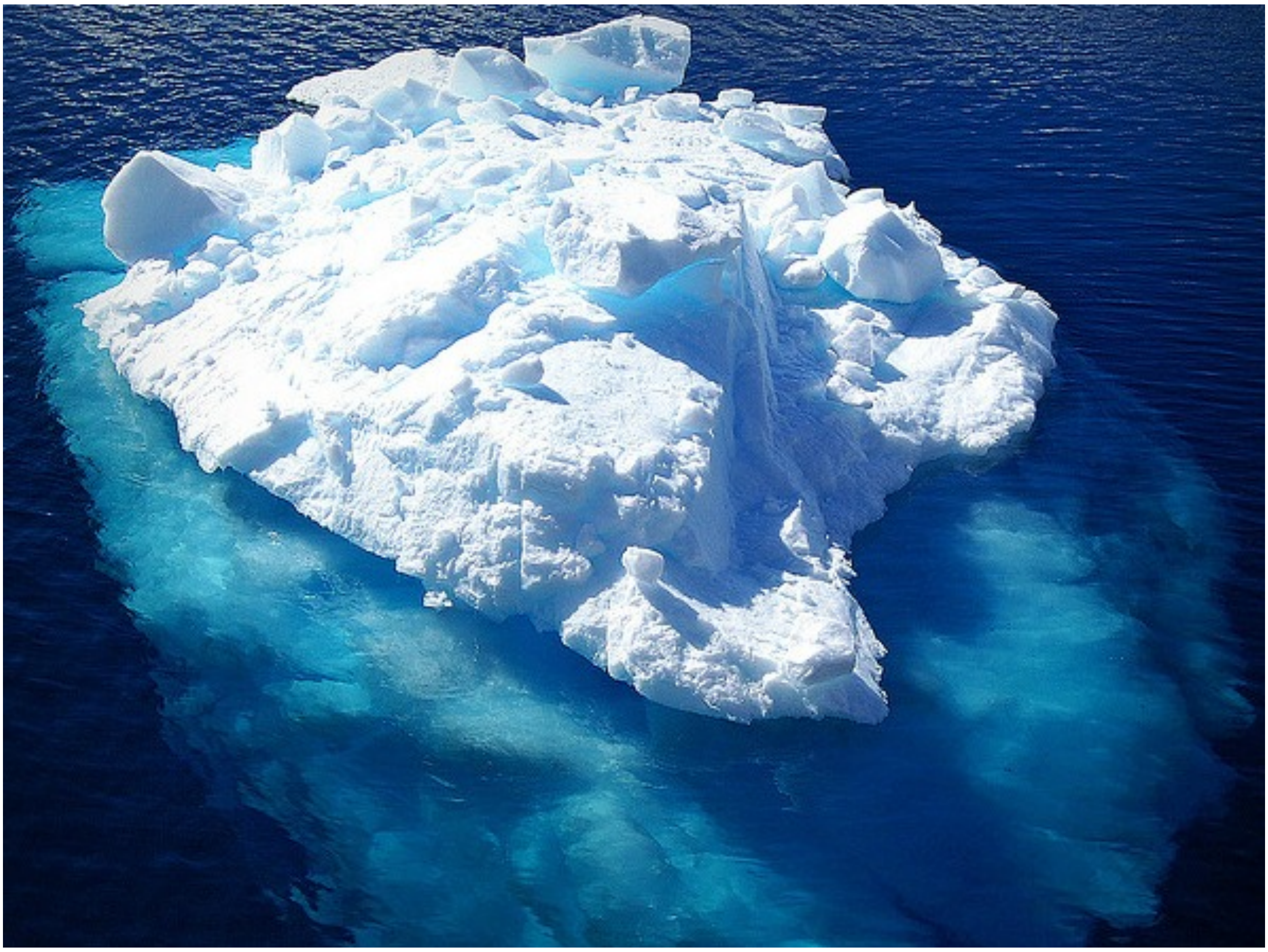
Exact scale invariance of the primordial fluctuations is ruled out, <u>at more than 7σ</u>

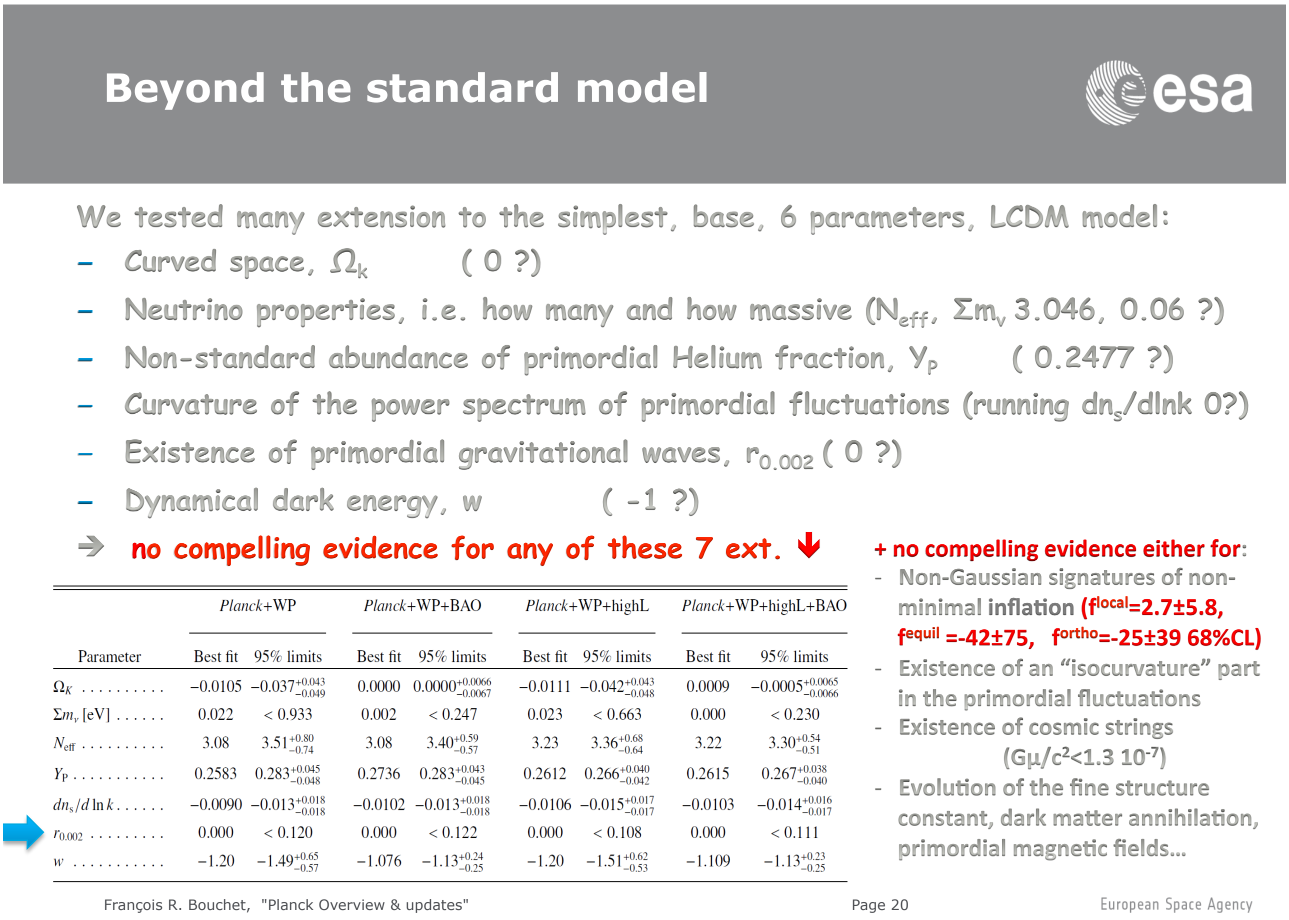
(as predicted by base inflation models)

$$\theta_* = (1.04148 \pm 0.00066) \times 10^{-2} = 0.596724^\circ \pm 0.00038^\circ$$

Summary on base tilted LCDM

- Base LCDM is a very good fit to Planck T spectrum, with parameters (n_s, Ω_b, Ω_c, θ/H_0) accurately determined by Planck alone, with the exception of the (A_s, τ) degeneracy which can be broken by adding WP.
- The model is fully consistent with two other Planck observables, Lensing and Polarization spectra.
- This model is also fully consistent with BAO, and show some tension with direct H_0 determination. The situation regarding Ω_m from SN was unclear at time of release (march 13, but JLA is out now).
- CMB+LSS now exclude scale invariance (n_s=1) at ~7σ





Beyond the standard model

esa

We tested many extension to the simplest, base, 6 parameters, LCDM model:

- Curved space, Ω_k (0 ?)
- Neutrino properties, i.e. how many and how massive (N_{eff}, Σm_ν 3.046, 0.06 ?)
- Non-standard abundance of primordial Helium fraction, Y_P (0.2477 ?)
- Curvature of the power spectrum of primordial fluctuations (running $dn_s/d\ln k$ 0?)
- Existence of primordial gravitational waves, $r_{0.002}$ (0 ?)
- Dynamical dark energy, w (-1 ?)

→ **no compelling evidence for any of these 7 ext.** ↓

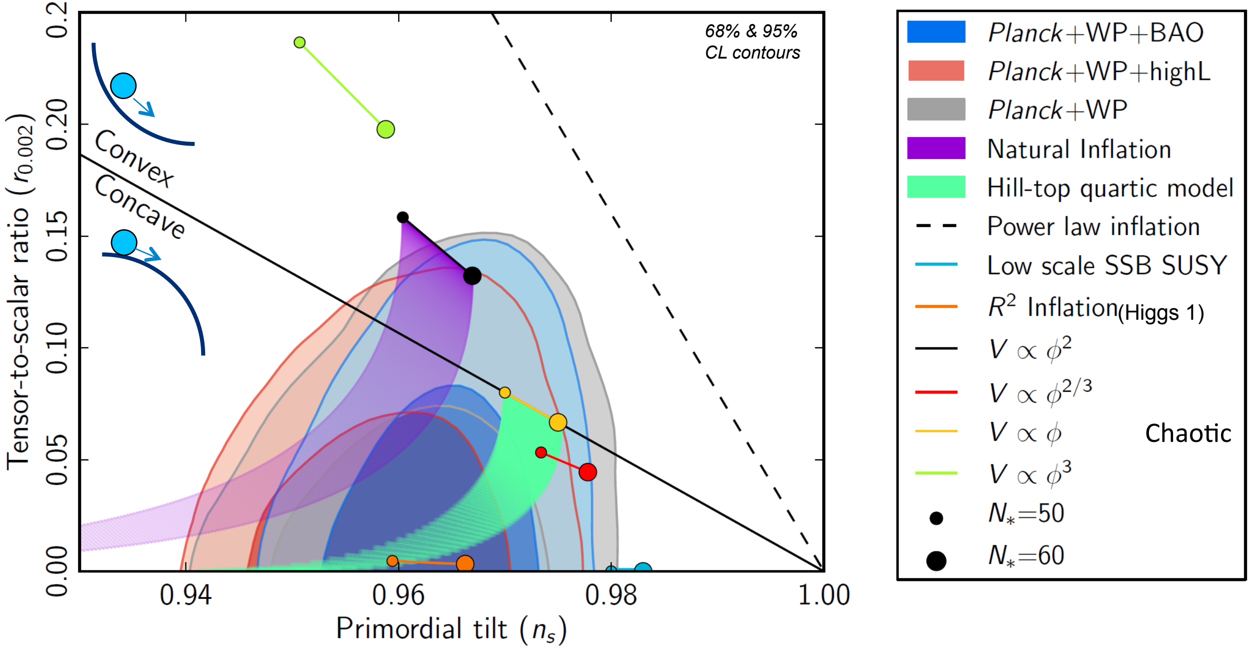
	Planck+WP		*Planck*+WP+BAO		*Planck*+WP+highL		*Planck*+WP+highL+BAO	
Parameter	Best fit	95% limits	Best fit	95% limits	Best fit	95% limits	Best fit	95% limits
Ω_K	-0.0105	$-0.037^{+0.043}_{-0.049}$	0.0000	$0.0000^{+0.0066}_{-0.0067}$	-0.0111	$-0.042^{+0.043}_{-0.048}$	0.0009	$-0.0005^{+0.0065}_{-0.0066}$
Σm_ν [eV]	0.022	< 0.933	0.002	< 0.247	0.023	< 0.663	0.000	< 0.230
$N_{\rm eff}$	3.08	$3.51^{+0.80}_{-0.74}$	3.08	$3.40^{+0.59}_{-0.57}$	3.23	$3.36^{+0.68}_{-0.64}$	3.22	$3.30^{+0.54}_{-0.51}$
$Y_{\rm P}$	0.2583	$0.283^{+0.045}_{-0.048}$	0.2736	$0.283^{+0.043}_{-0.045}$	0.2612	$0.266^{+0.040}_{-0.042}$	0.2615	$0.267^{+0.038}_{-0.040}$
$dn_s/d\ln k$	-0.0090	$-0.013^{+0.018}_{-0.018}$	-0.0102	$-0.013^{+0.018}_{-0.018}$	-0.0106	$-0.015^{+0.017}_{-0.017}$	-0.0103	$-0.014^{+0.016}_{-0.017}$
➡ $r_{0.002}$	0.000	< 0.120	0.000	< 0.122	0.000	< 0.108	0.000	< 0.111
w	-1.20	$-1.49^{+0.65}_{-0.57}$	-1.076	$-1.13^{+0.24}_{-0.25}$	-1.20	$-1.51^{+0.62}_{-0.53}$	-1.109	$-1.13^{+0.23}_{-0.25}$

+ no compelling evidence either for:

- Non-Gaussian signatures of non-minimal **inflation** **(f^{local}=2.7±5.8, f^{equil} =-42±75, f^{ortho}=-25±39 68%CL)**
- Existence of an "isocurvature" part in the primordial fluctuations
- Existence of cosmic strings ($G\mu/c^2<1.3\ 10^{-7}$)
- Evolution of the fine structure constant, dark matter annihilation, primordial magnetic fields…

Constraint on representative Inflation models

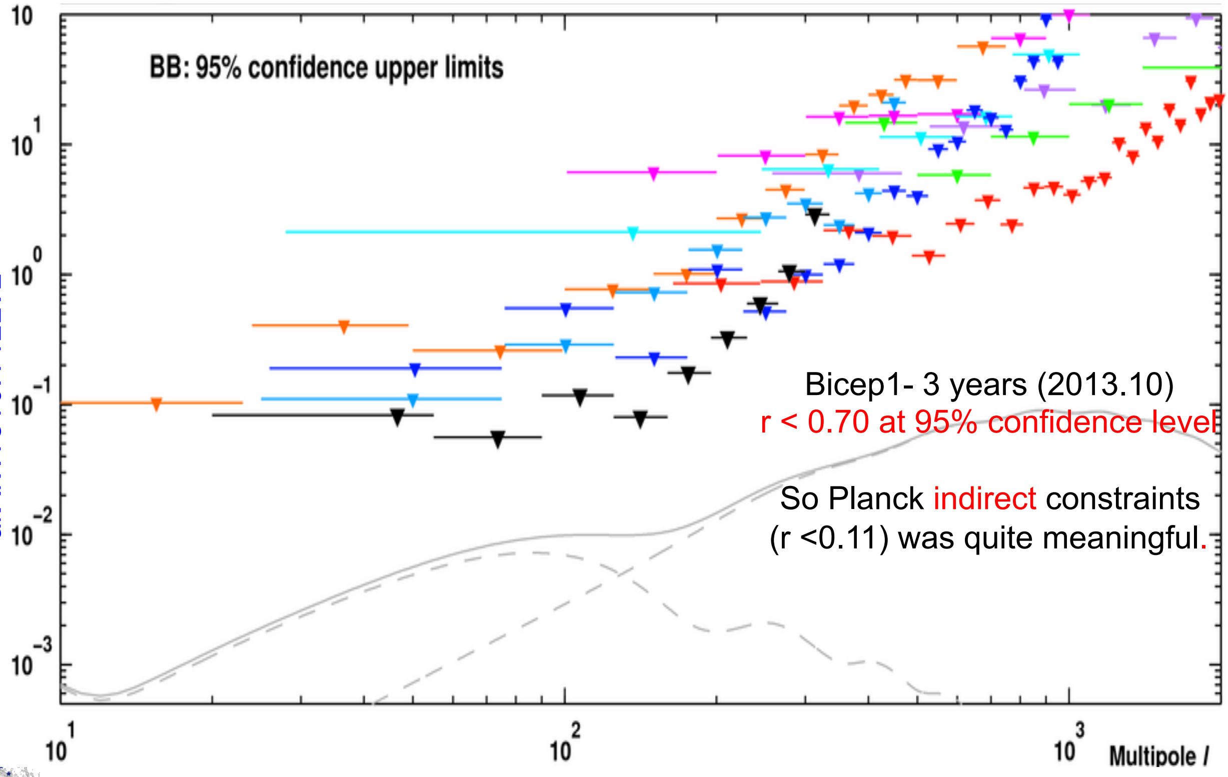
$V_* = (1.9 \times 10^{16}\ \mathrm{GeV})^4\ (r/0.12)$ and $r < 0.11$ @ 95% CL



➔Exponential potential models(power-law inf.), simplest hybrid inflationary models (SB SUSY), monomial potential models of degree n >2 do not provide a good fit to the data.

2013 Status of direct B-modes searches

arXiv:1310.1422v2



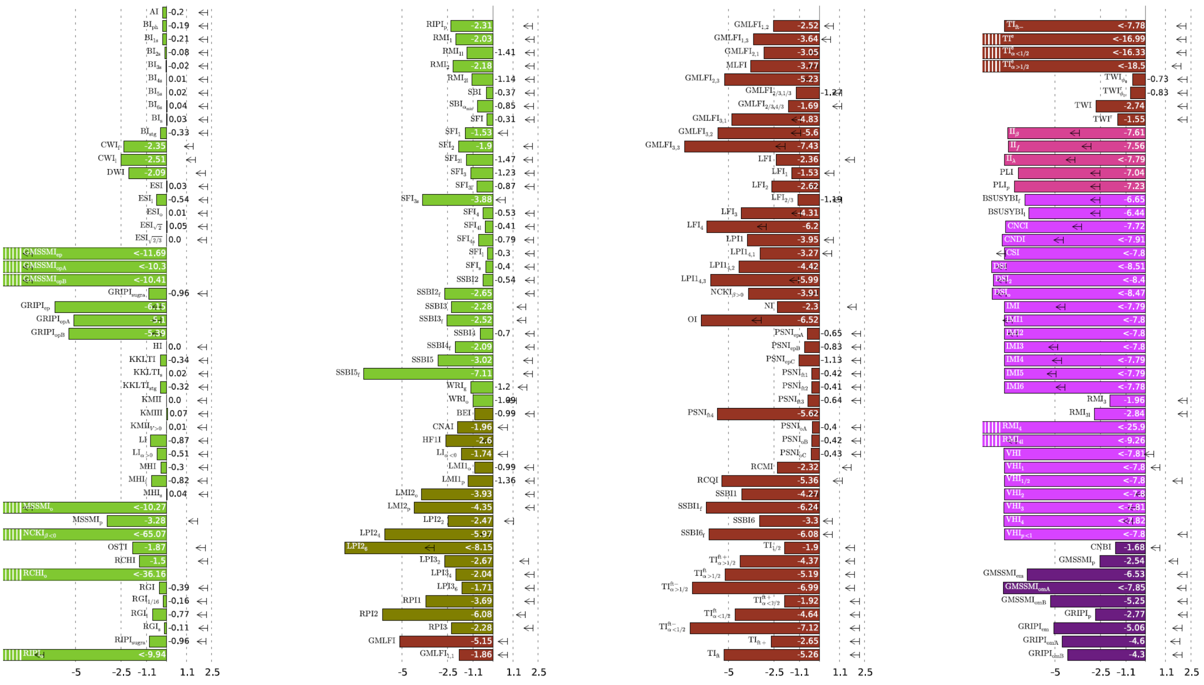
Bicep1- 3 years (2013.10)
r < 0.70 at 95% confidence level

So Planck indirect constraints (r <0.11) was quite meaningful.

Since then…
(>march 2013)

Encyclopædia Inflationaris

Bayesian Evidences $\ln(\mathcal{E}/\mathcal{E}_{\mathrm{HI}})$ and $\ln(\mathcal{L}_{\max}/\mathcal{E}_{\mathrm{HI}})$

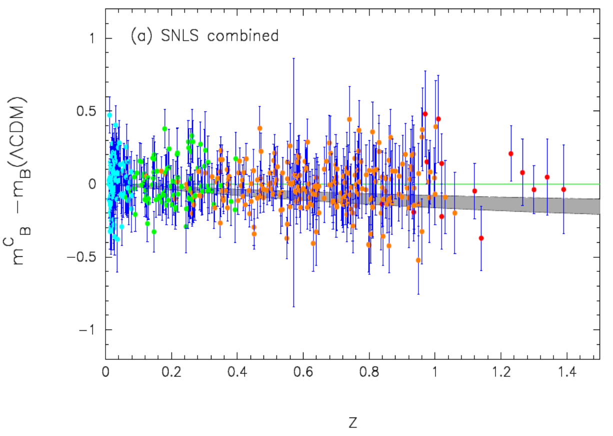


Schwarz-Terrero-Escalante Classification:
🟩 1 🟩 1-2 🟥 2 🟥 2-3 🟪 3 🟪 1-2-3

J.Martin, C.Ringeval, R.Trotta, V.Vennin
ASPIC project

Displayed Evidences: 193

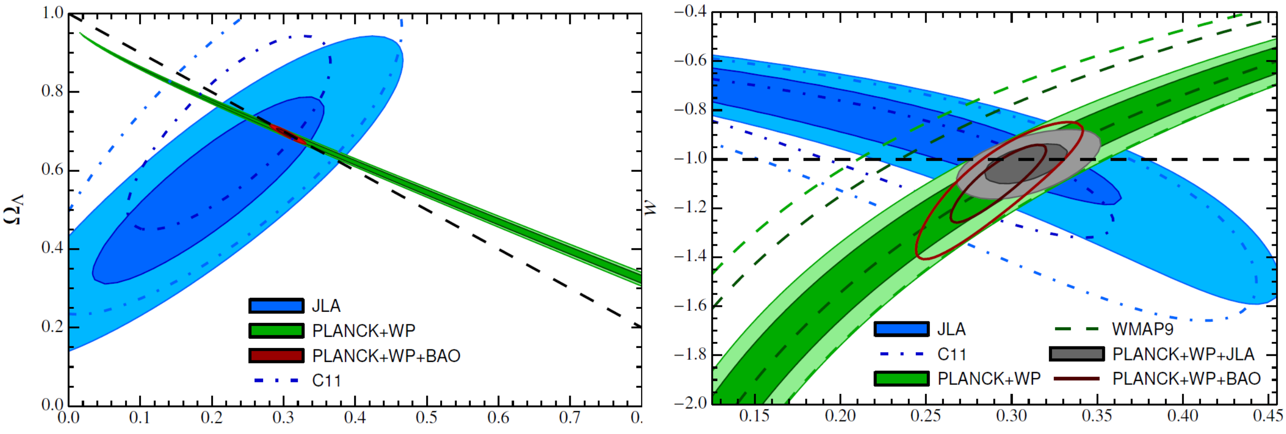
Tension with SNLS results…



Data, BF model and Planck Prediction (±1σ shaded area)

Fig. 18. Magnitude residuals relative to the base ΛCDM model that best fits the SNLS combined sample (left) and the Union2.1 sample (right). The error bars show the 1 σ (diagonal) errors on m_B. The filled grey regions show the residuals between the expected magnitudes and the best-fit to the SNe sample as Ω_m varies across the $\pm 2\,\sigma$ range allowed by *Planck*+WP+highL in the base ΛCDM cosmology. The colour coding of the SNLS samples are as follows: low redshift (blue points); SDSS (green points); SNLS three-year sample (orange points); and *HST* high redshift (red points).

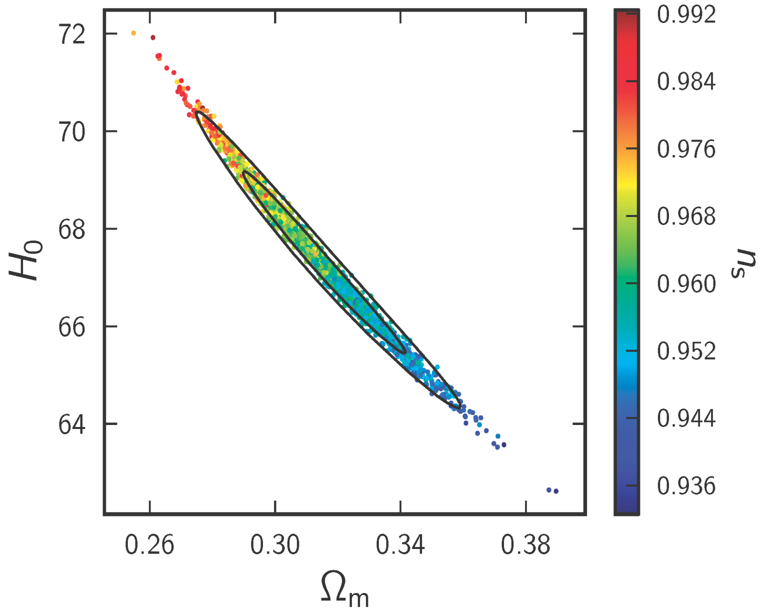
Planck versus JLA (SNLS +SDSS)



Astroph1401.4064 Betoule et al. (JLA)

	Ω_m	w	H_0	$\Omega_b h^2$
Planck+WP+BAO+JLA	0.303 ± 0.012	−1.027 ± 0.055	68.50 ± 1.27	0.0221 ± 0.0003
Planck+WP+BAO	0.295 ± 0.020	−1.075 ± 0.109	69.57 ± 2.54	0.0220 ± 0.0003
Planck+WP+SDSS	0.341 ± 0.039	−0.906 ± 0.123	64.68 ± 3.56	0.0221 ± 0.0003
Planck+WP+SDSS+SNLS	0.314 ± 0.020	−0.994 ± 0.069	67.32 ± 1.98	0.0221 ± 0.0003
Planck+WP+JLA	0.307 ± 0.017	−1.018 ± 0.057	68.07 ± 1.63	0.0221 ± 0.0003
WMAP9+JLA+BAO	0.296 ± 0.012	−0.979 ± 0.063	68.19 ± 1.33	0.0224 ± 0.0005
Planck+WP+C11	0.288 ± 0.021	−1.093 ± 0.078	70.33 ± 2.34	0.0221 ± 0.0003

Sound Horizon



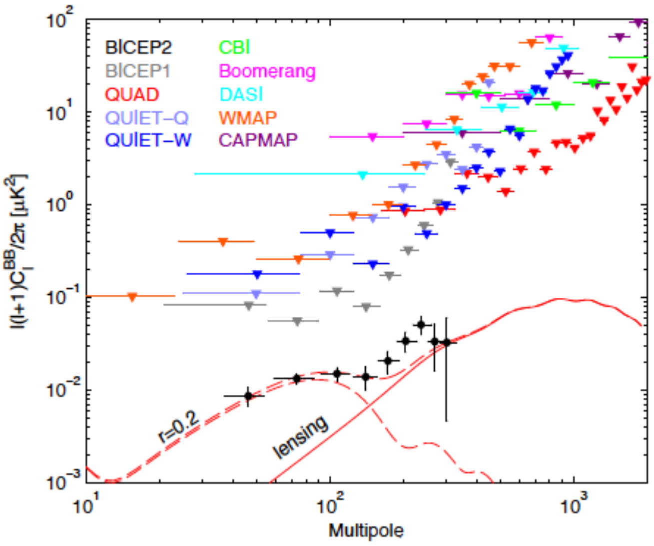
Samples are for Planck only.

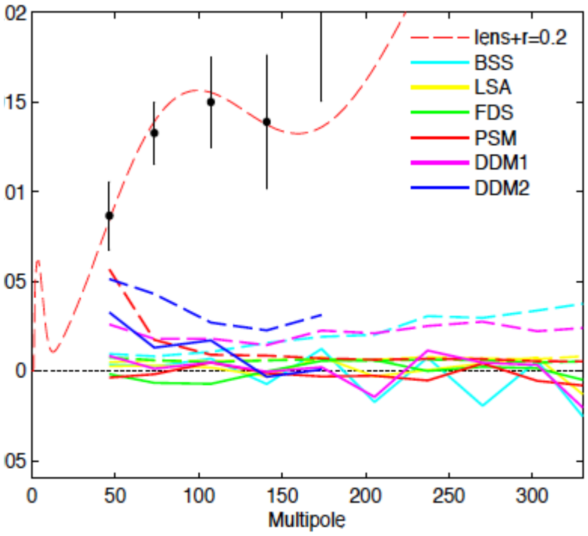
Tighter contours along the degeneracy direction are from Planck +lensing+ WP

r_s is constrained transversally

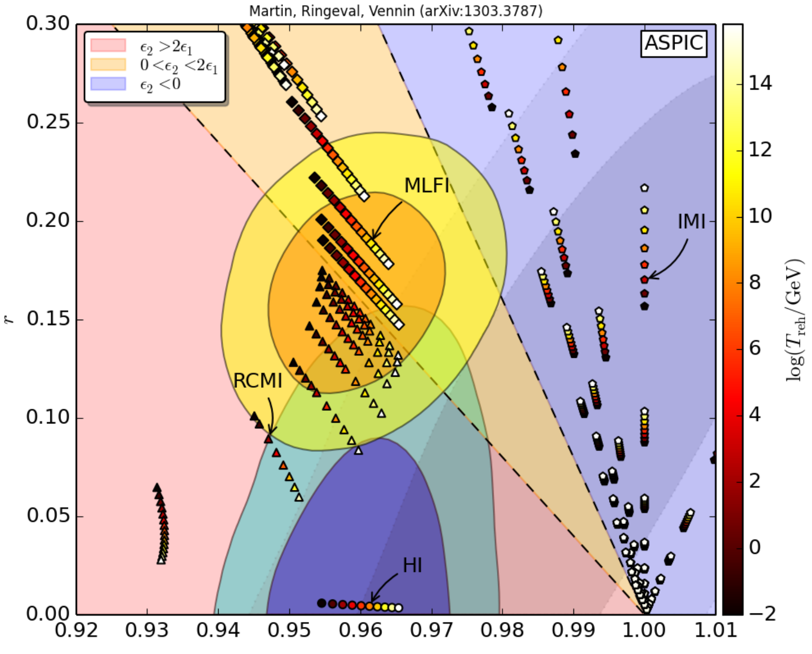
r_s constrains $\Omega_m h^3$ very tightly in LCDM; High Ω_m corresponds to low n_s and H_0

BICEP2, on March 17th 2014





Adding Bicep2 as stated in their paper…

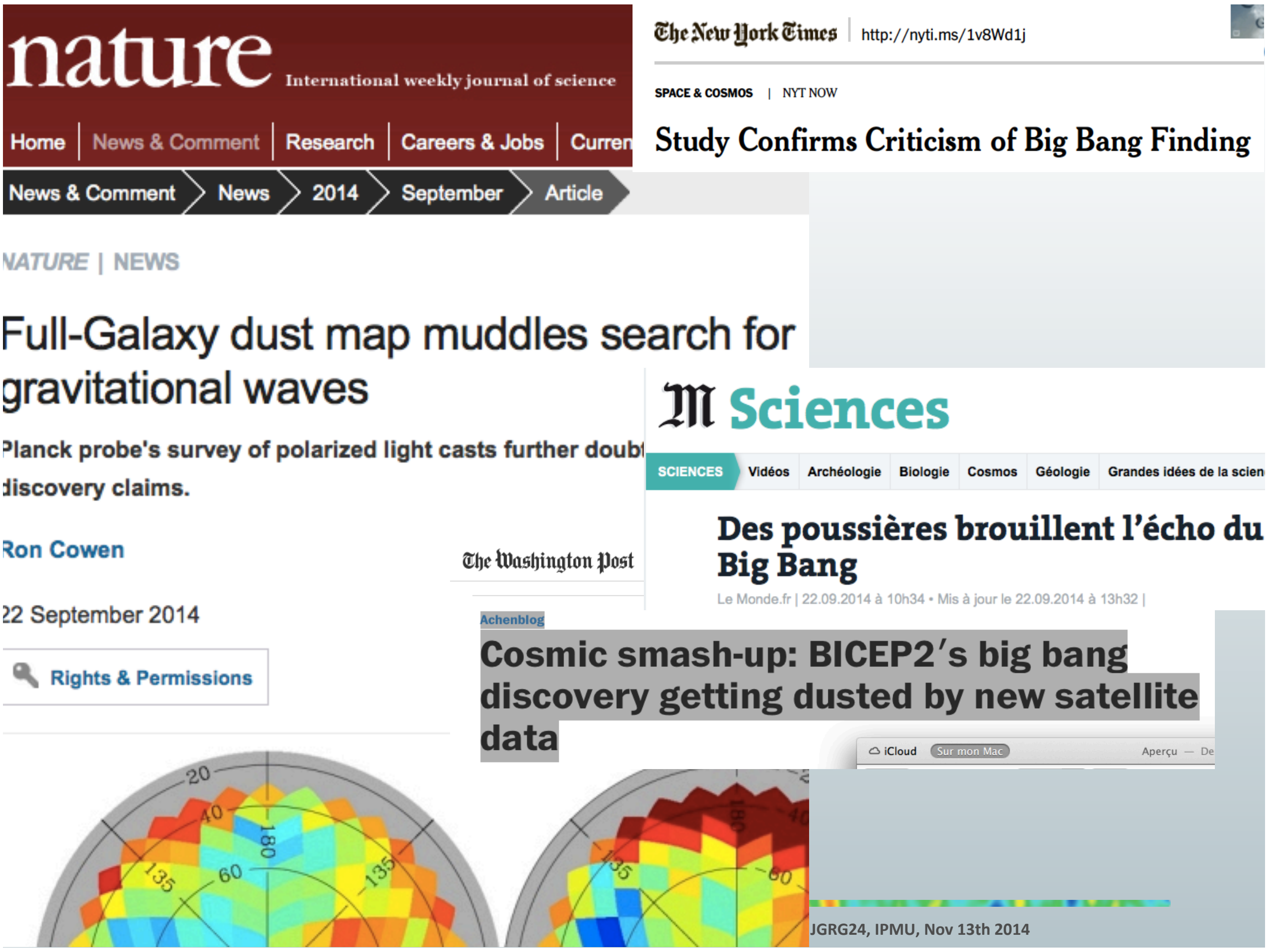


NB: the PRL disfavours a more mundane interpretation, i.e. 100% dust, with 100X150GHz at only 1.7sig! ("The preferred whole sky dust spectrum from Planck [94], …, is also disfavored as an explanation for the excess BB(PTE=0.09,1.7σ)")

---

5 Planck papers on dust polarisation…

- In march 2013, we did not deliver polarisation data, nor performed quantitative analyses of CMB polarisation, due to concerns on that data quality, preventing its general use.
- We still put out preliminary results at ESLAB and in the papers which appeared in May 5th on what we believe can be already extracted safely from the 2013 data (mostly at 353GHz), i.e. on regions of the sky where the signal is strong enough for Galactic studies, purposely excluding the (more demanding) high Galactic sky.
  - *Planck intermediate results. XIX. An overview of the polarized thermal emission from Galactic dust*
  - *Planck intermediate results. XX. Comparison of polarized thermal emission from Galactic dust with simulations of MHD turbulence*
  - *Planck intermediate results. XX. Comparison of polarized thermal emission from Galactic dust with simulations of MHD turbulence*
  - *Planck intermediate results. XXII. Frequency dependence of thermal emission from Galactic dust in intensity and polarization*
- We have kept working on "**the statistical characterisation of dust polarisation at mid&high Galactic latitude"** *which recently appeared* (Sept 22nd) on astroph. The results are based on the 2014 data which we plan to release around the end of the year.



NATURE | NEWS

Full-Galaxy dust map muddles search for gravitational waves

Planck probe's survey of polarized light casts further doubt on discovery claims.

Ron Cowen

22 September 2014

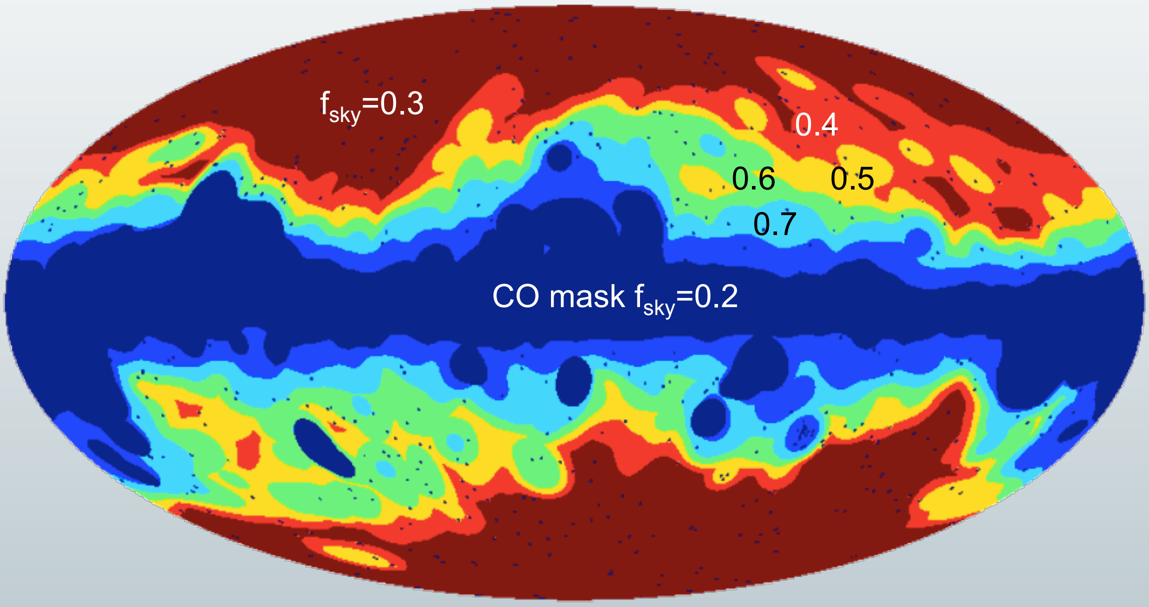
Study Confirms Criticism of Big Bang Finding

Des poussières brouillent l'écho du Big Bang

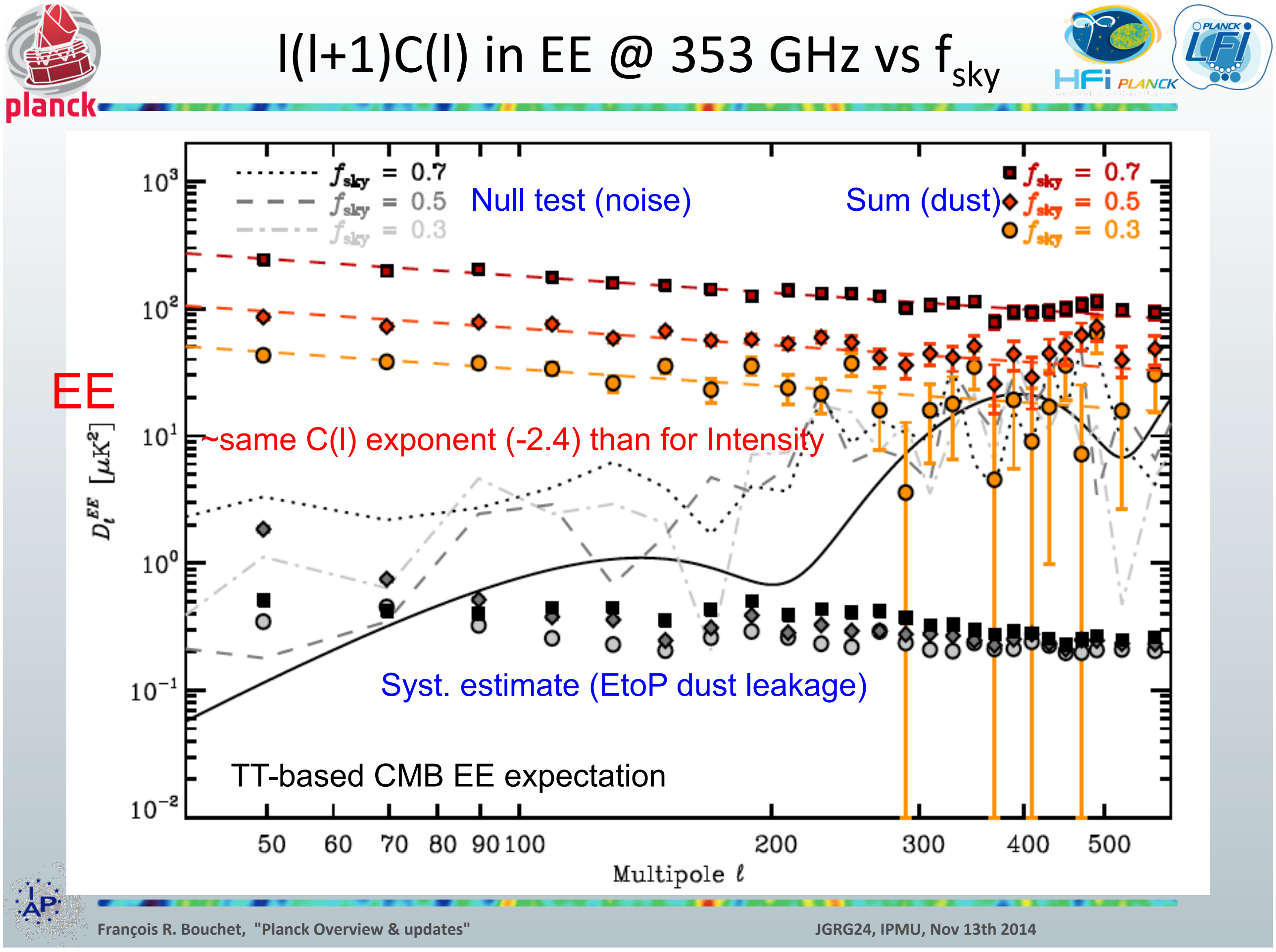
Le Monde.fr | 22.09.2014 à 10h34 • Mis à jour le 22.09.2014 à 13h32 |

Cosmic smash-up: BICEP2's big bang discovery getting dusted by new satellite data

Dust polarisation at high Gal. latitude



From light blue to red, corresponds to 0.7-0.6-0.5(yellow)-0.4-0.3



l(l+1)C(l) in EE @ 353 GHz vs f_{sky}

EE

Null test (noise)

Sum (dust)

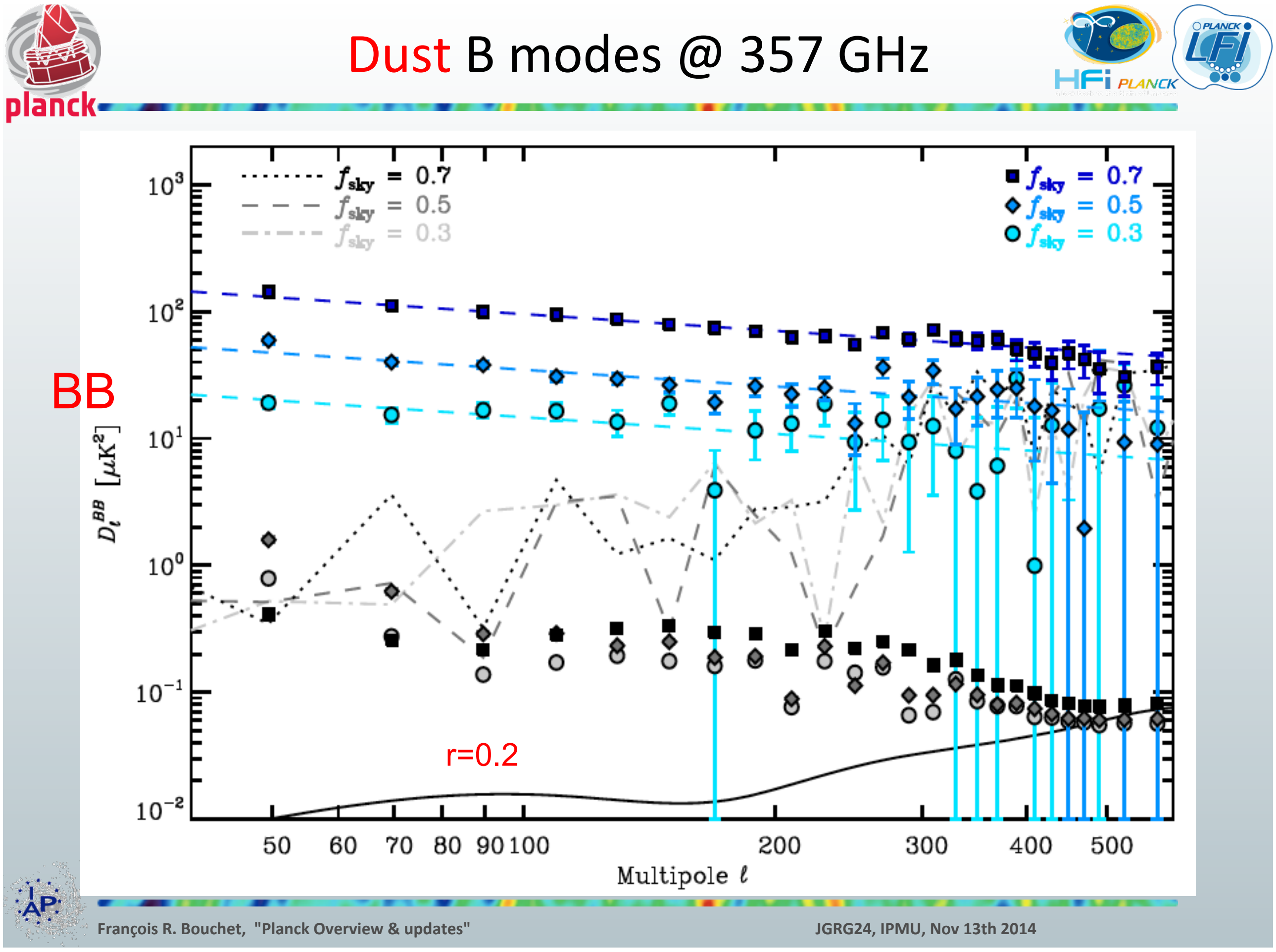
~same C(l) exponent (-2.4) than for Intensity

Syst. estimate (EtoP dust leakage)

TT-based CMB EE expectation

François R. Bouchet, "Planck Overview & updates"

JGRG24, IPMU, Nov 13th 2014



Dust B modes @ 357 GHz

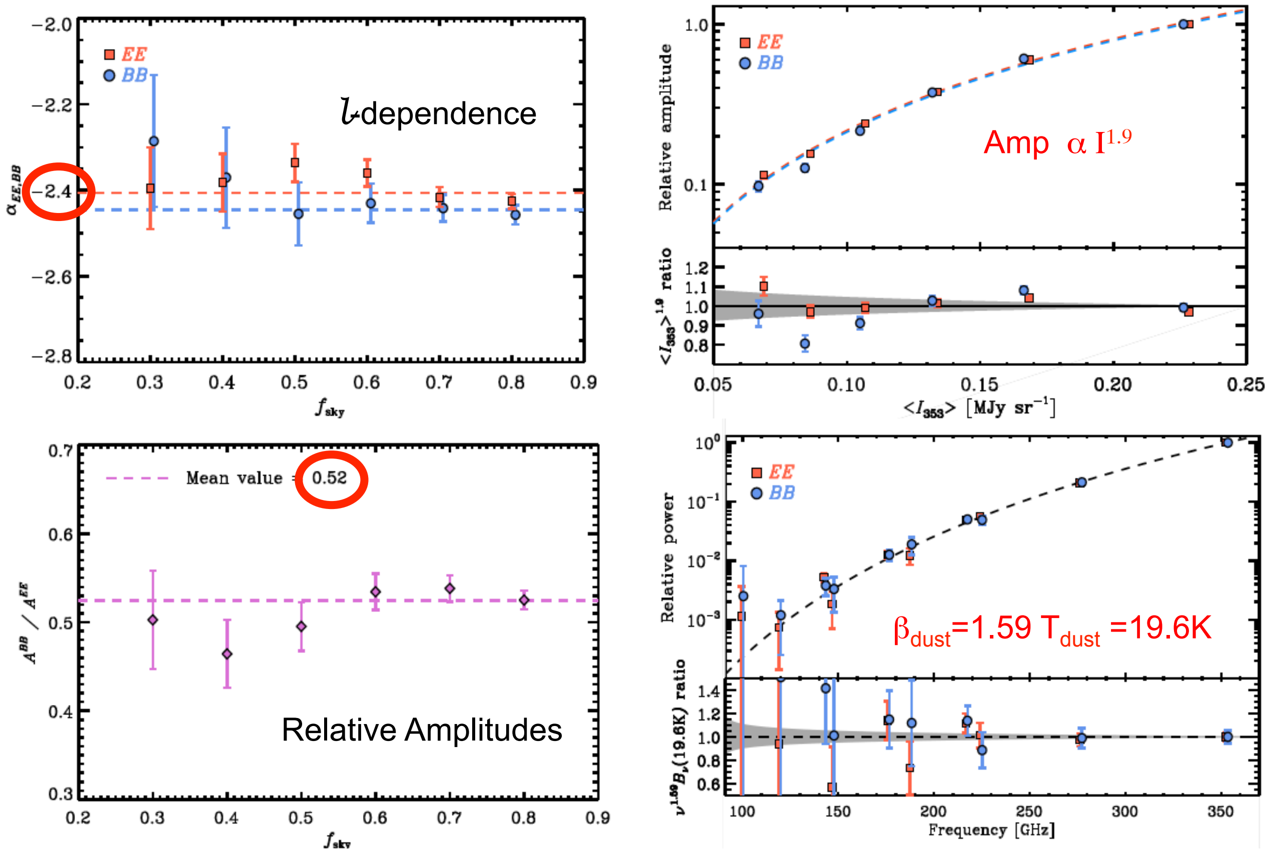
BB

r=0.2

François R. Bouchet, "Planck Overview & updates"

JGRG24, IPMU, Nov 13th 2014

Dust polarisation mean properties



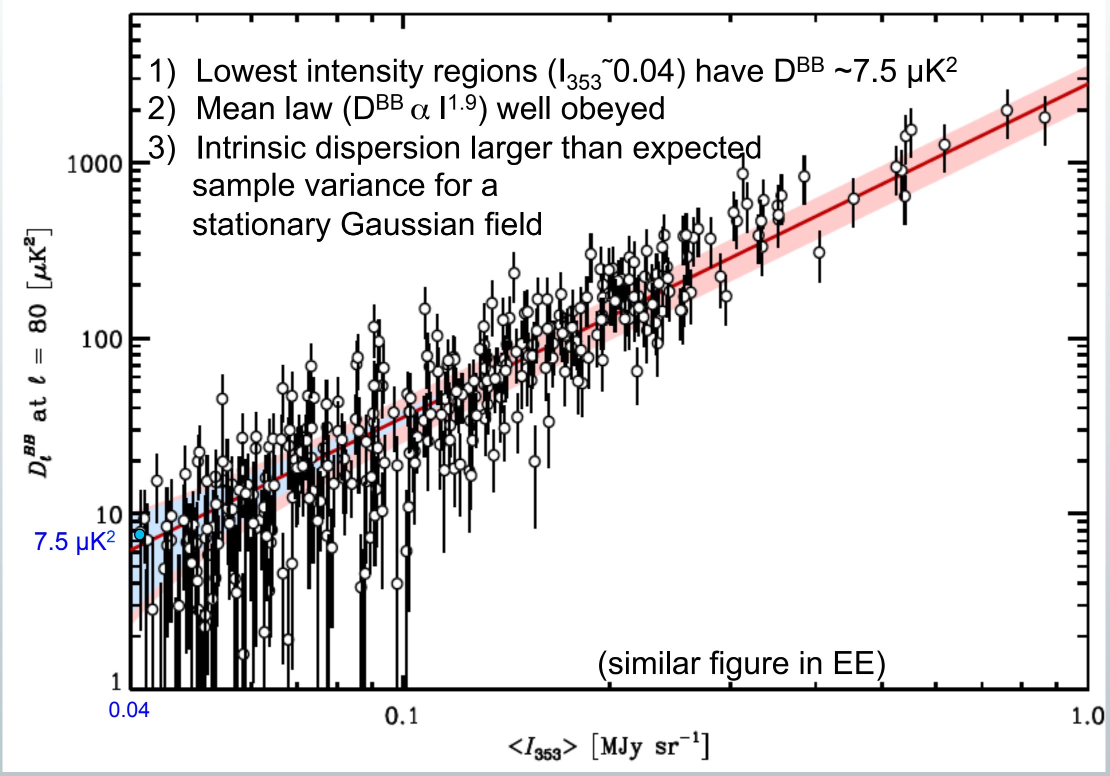
ℓ-dependence

Amp α $I^{1.9}$

Relative Amplitudes

β_{dust}=1.59 T_{dust} =19.6K

353 patches of 400deg^2 (at N_{side}=8 centers)



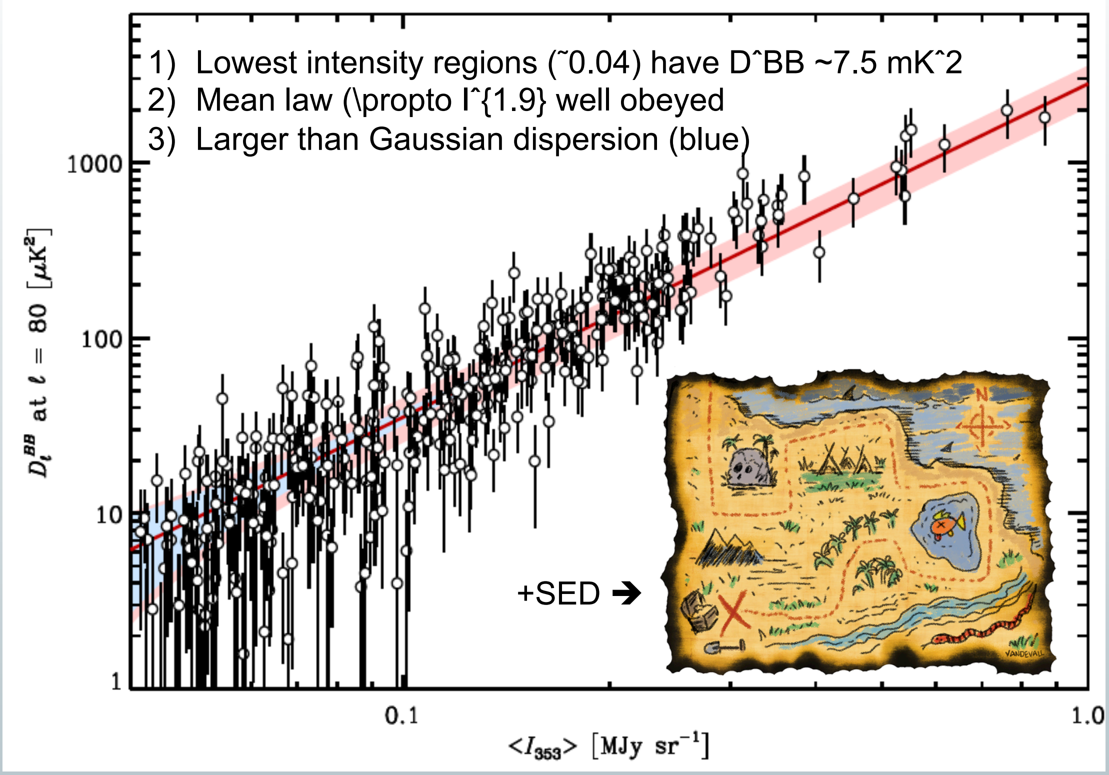
1) Lowest intensity regions (I_{353}˜0.04) have D^{BB} ~7.5 μK^2
2) Mean law (D^{BB} α $I^{1.9}$) well obeyed
3) Intrinsic dispersion larger than expected sample variance for a stationary Gaussian field

7.5 μK^2

0.04

(similar figure in EE)

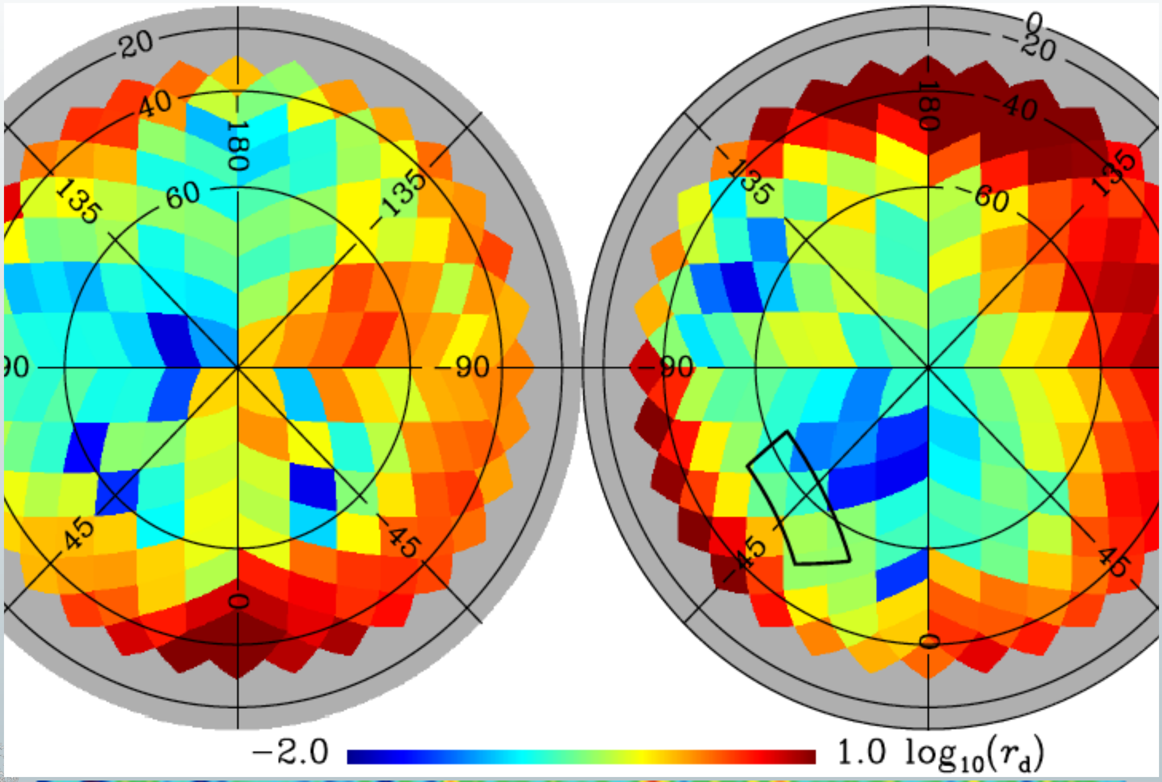
353 patches of 400deg^2 (around Nside=8)



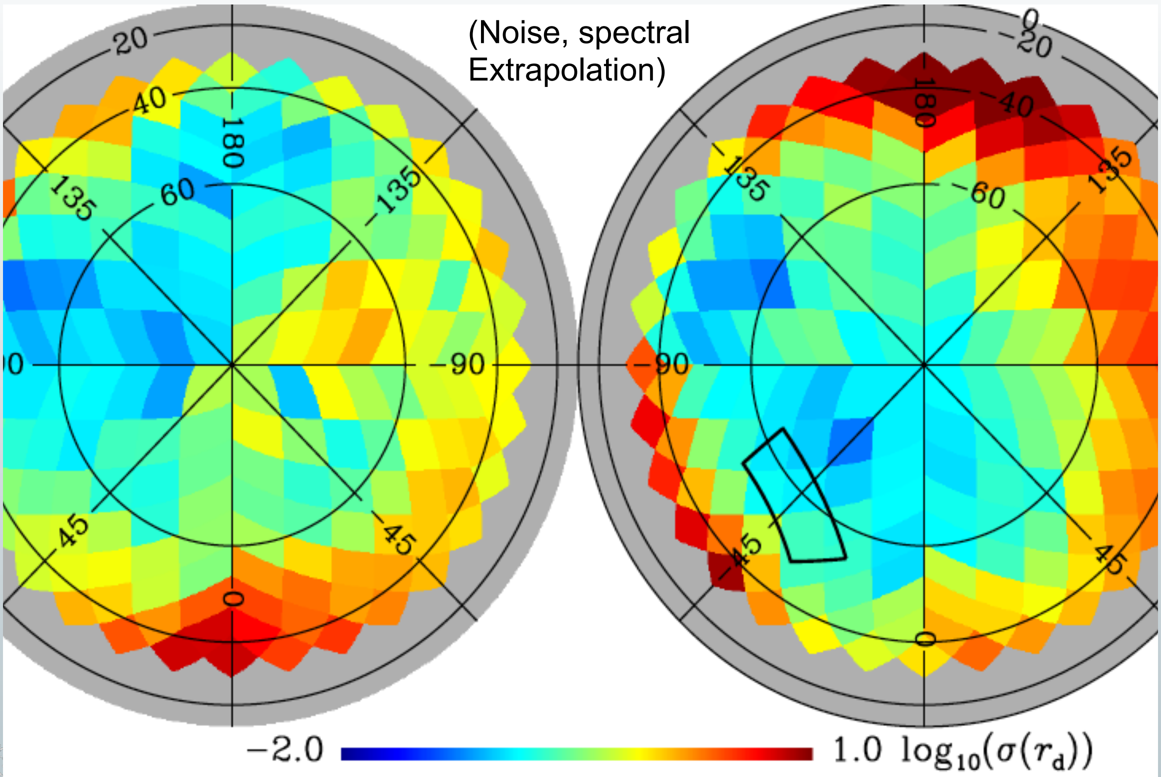
1) Lowest intensity regions (˜0.04) have D^BB ~7.5 mK^2
2) Mean law (\propto I^{1.9} well obeyed
3) Larger than Gaussian dispersion (blue)

+SED ➔

Dust extrapolation at 150GHz in unit of r

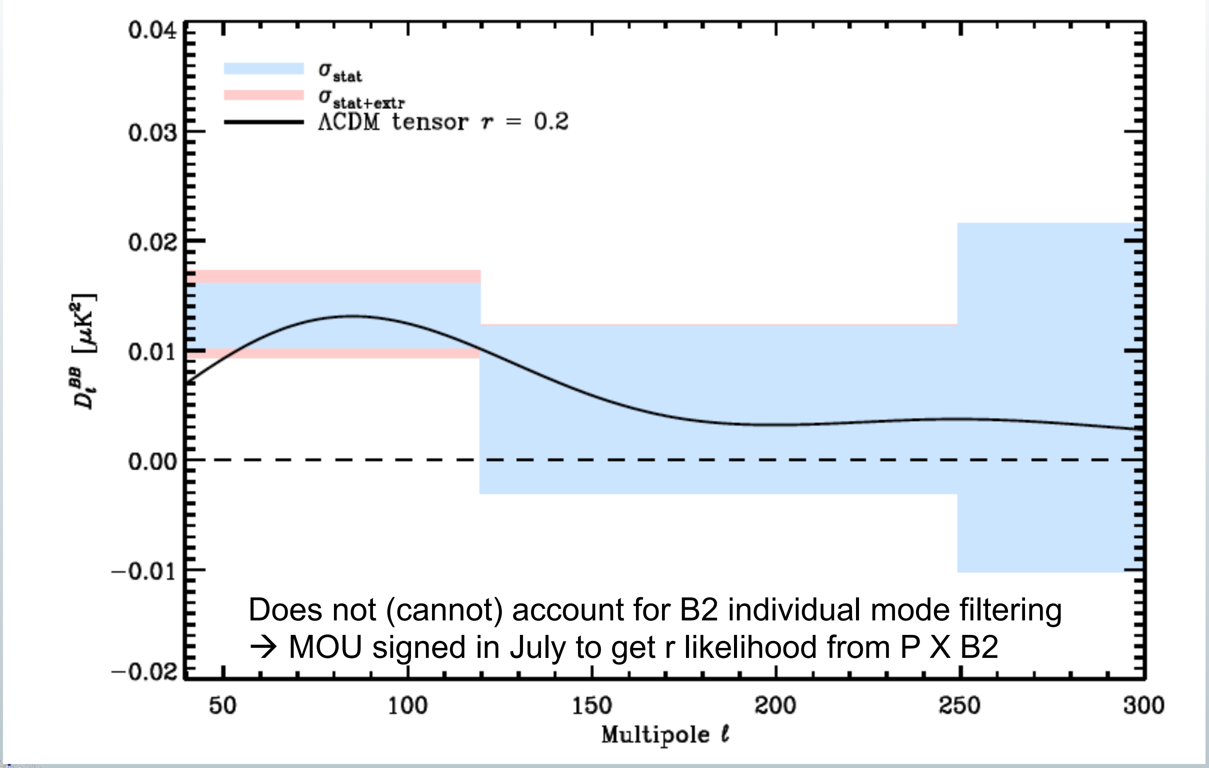


But uncertainties are non negligible



(Noise, spectral Extrapolation)

Best estimate in B2 field wo data access



Does not (cannot) account for B2 individual mode filtering

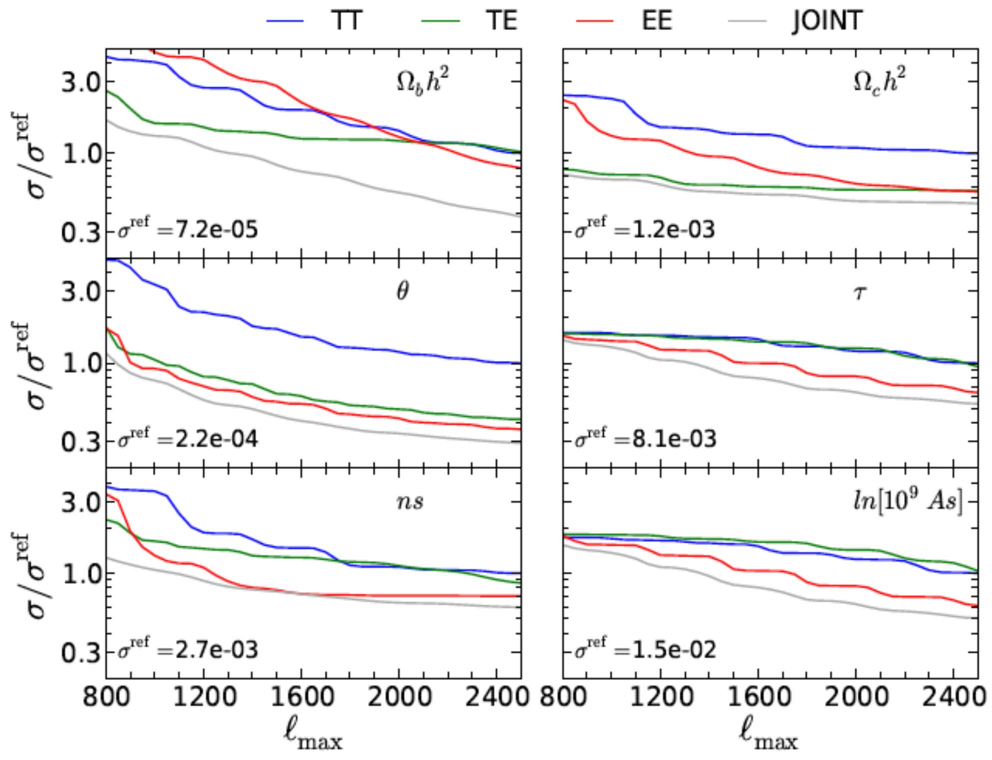
→ MOU signed in July to get r likelihood from P X B2

Consequences

- There appears to be no field to measure the primordial B modes at the degree scale (in the recombination bump) which would be large enough and clean enough for the dust contribution to be neglected.
- The best 30% of the sky have a dust PS TT amplitude only 1.5 larger than the BICEP2 field (covering ˜1% of the sky).
- Conversely, there are fields with I_{353} as low as 0.038 MJy/sr thus better by about a factor of 2 of the (more probable) B2 value.
- *The dispersion is large enough to remove the possibility to choose fields with only* $I_{353.}$

➔ The Planck collaboration has provided a “treasure map", i.e. a sky map of the most promising fields for degree scale work, awaiting for the full “2014” release.

Comparative Information in T & P



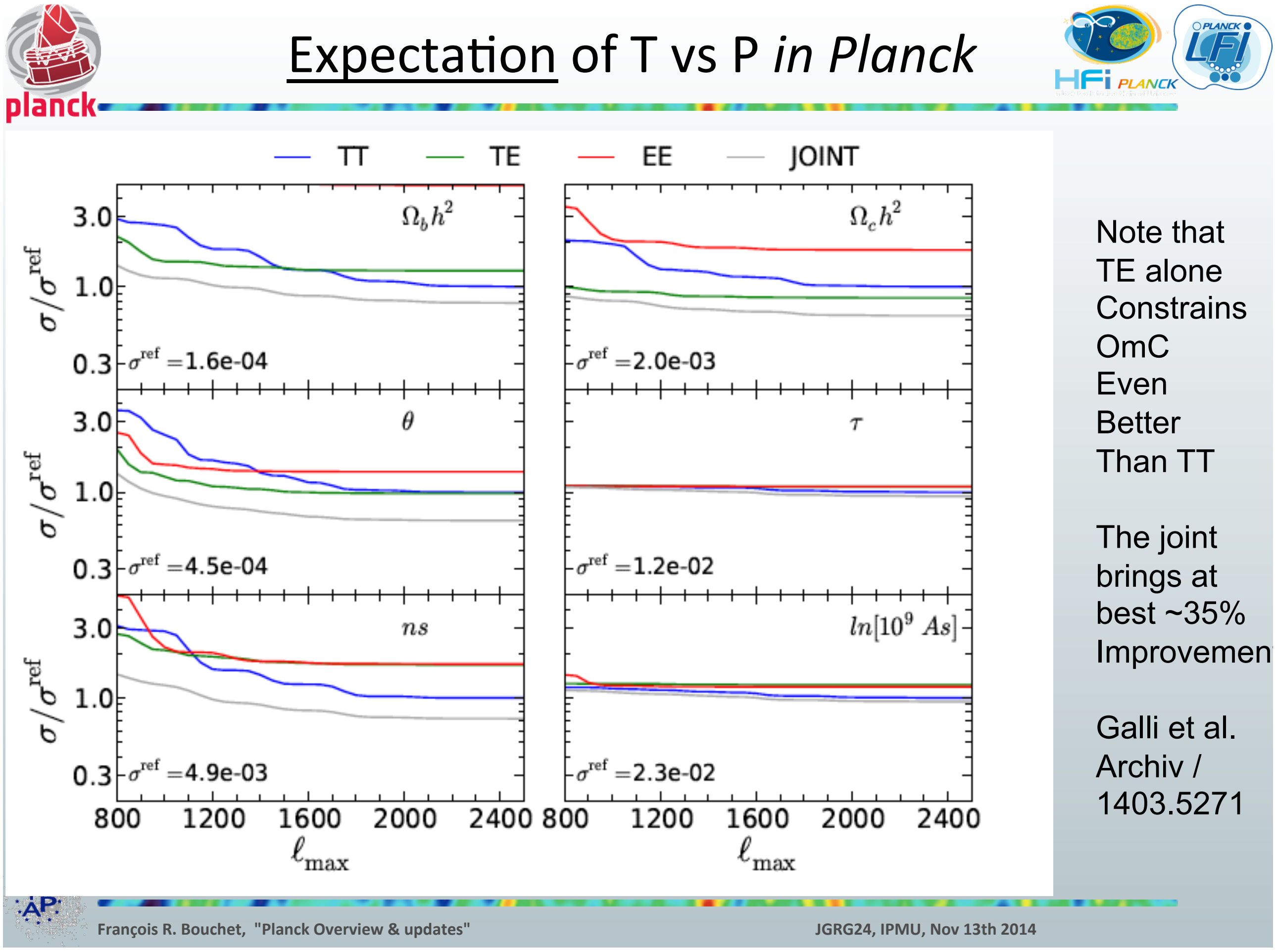
Cosmic Variance Limited expnt: (till l=2500) on LCDM

TE or EE **independently** Constrain parameters Better than TT (up to 2.8 times better)

Yes we can (learn more out of the CMB ☺)

NB: large scale EE is crucial (30 < l < 130)

Galli et al. Archiv/1403.5271



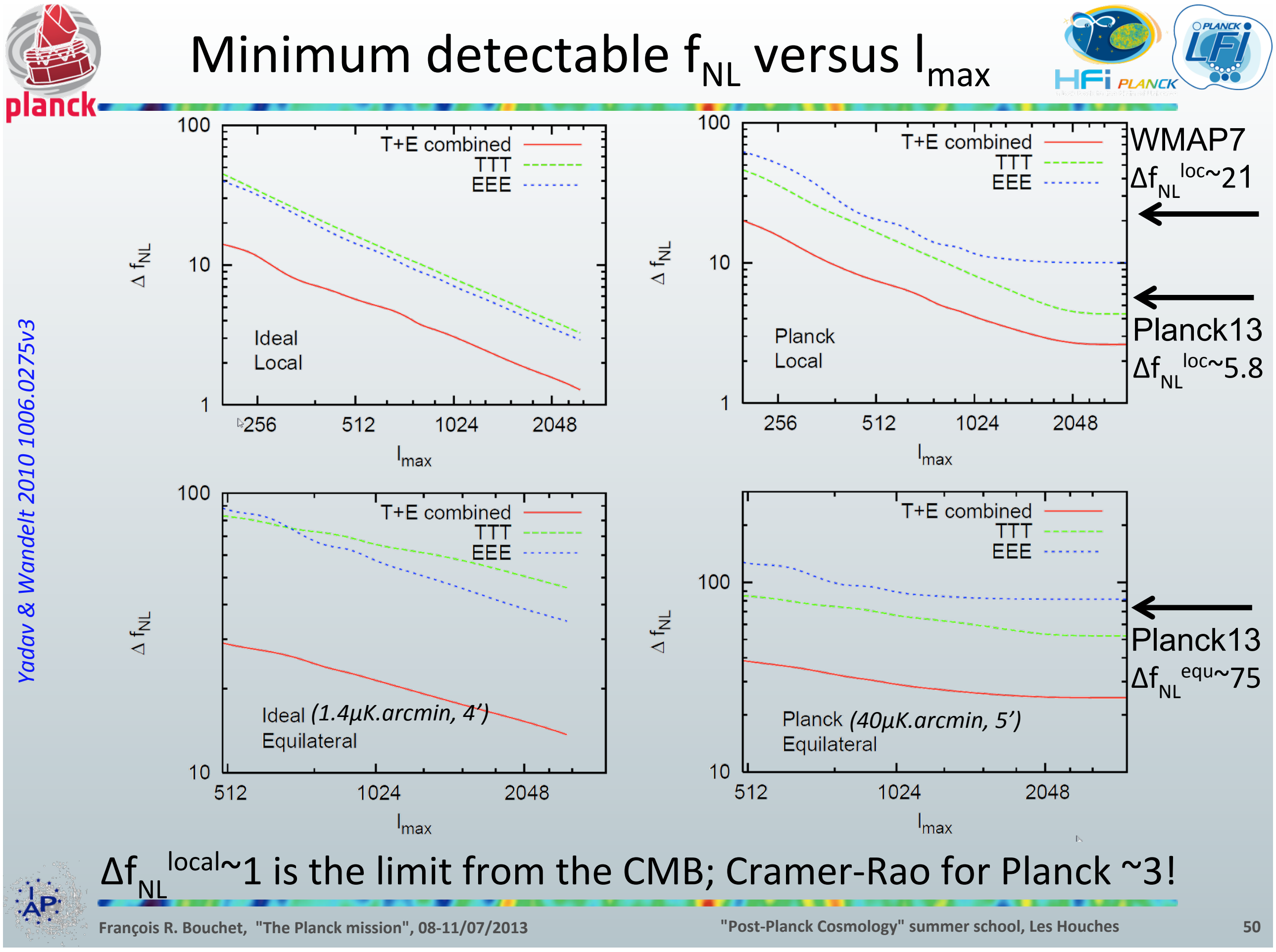
Expectation of T vs P *in Planck*

Note that
TE alone
Constrains
OmC
Even
Better
Than TT

The joint
brings at
best ~35%
Improvemen

Galli et al.
Archiv /
1403.5271

François R. Bouchet, "Planck Overview & updates" JGRG24, IPMU, Nov 13th 2014



Minimum detectable f_{NL} versus l_{max}

Yadav & Wandelt 2010 1006.0275v3

WMAP7
Δf_{NL}^{loc}~21

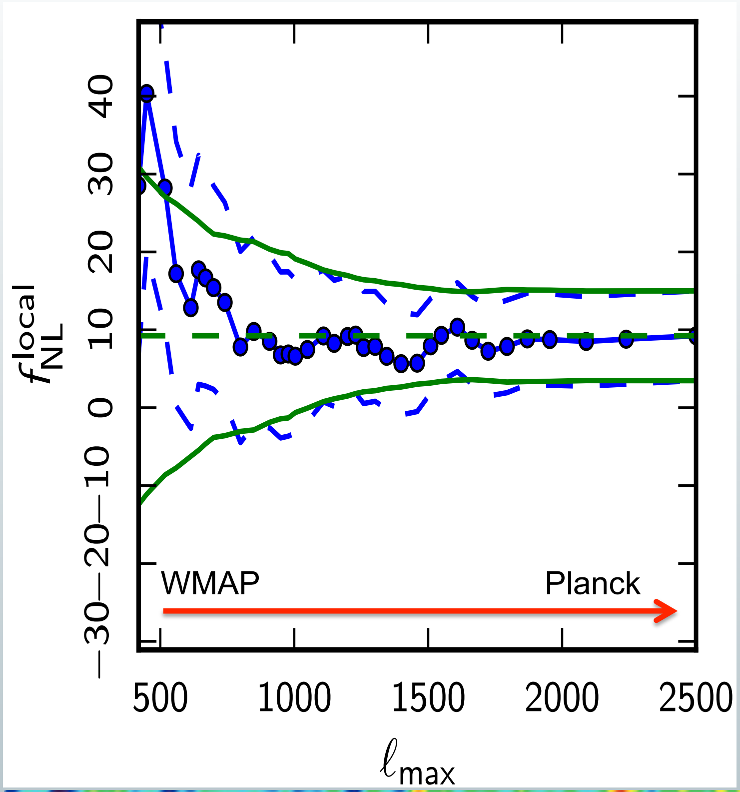
Planck13
Δf_{NL}^{loc}~5.8

Planck13
Δf_{NL}^{equ}~75

Δf_{NL}^{local}~1 is the limit from the CMB; Cramer-Rao for Planck ~3!

François R. Bouchet, "The Planck mission", 08-11/07/2013 "Post-Planck Cosmology" summer school, Les Houches

More modes helps ☺



Limiting the analysis to large scales (low l), we make contact with WMAP9 (which gave f_{NL}^{local}=37.2 ± 20)

Planck now rules out the WMAP central value by ~6 sigma.
(by using 10 times more modes)

NB: this figure is *before* subtraction of ISW X lensing bias, which is clearly visible

François R. Bouchet, "The Planck mission", 08-11/07/2013 "Post-Planck Cosmology" summer school, Les Houches 51

“2014” Release preview

- The 2013 main unidentified HFI systematics have now been identified:
  - *Very long time constants (VLTC between 1 and 10 seconds) with very low amplitudes.*
  - *These VLTC do shift the dipoles (by a few arc min) and create a leakage of the solar dipole into the orbital dipole (& TF variation).*
  - *The current accounting of this allows to calibrate HFI on the orbital dipole with a ~0.1% accuracy! (both intra and inter-bands). This matches LFI. Discrepancy / WMAP understood (inc. 0.6% wrt WMAP dipole).*
  - *The low-ell EE systematics has been reduced by a factor larger than a 100.*
  - *We have also improved the leakage correction and the removal of glitch tails (lower 1/f noise).*
- Does not mean though that there are no troublesome residuals on some (mostly large) scales. Debating how to deal with that at best (and in any case, 2015 legacy)

PLANCK 2014

THE MICROWAVE SKY IN TEMPERATURE AND POLARIZATION

- ➢ Add IAP meeting affiche (e-mail)
- ➢ Get inspiration from Core + tomos 2 slides

1-5 December 2014, Palazzo Costabili, Ferrara, Italy

NEW RESULTS FROM PLANCK AND OTHER EXPERIMENTS ON COSMOLOGY, FUNDAMENTAL PHYSICS, GALACTIC AND EXTRAGALACTIC ASTROPHYSICS, DATA ANALYSIS AND NEXT OBSERVATIONAL CHALLENGES

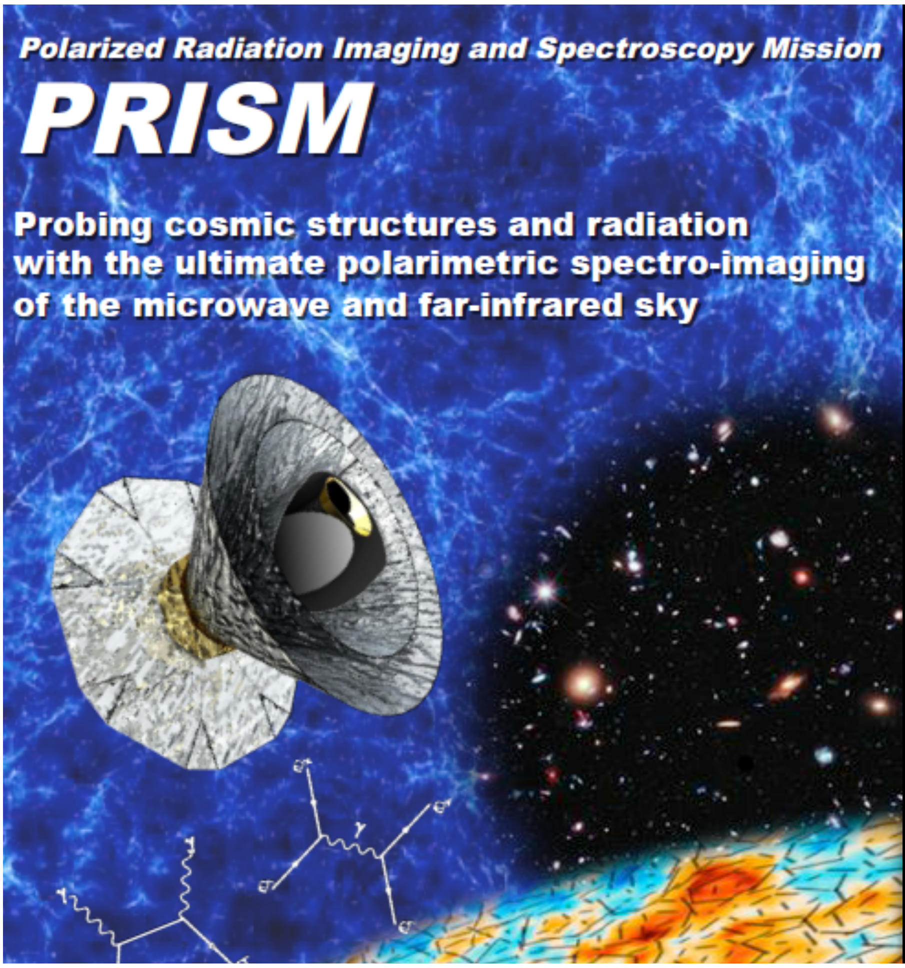
30th Institut d'astrophysique de Paris Colloquium

THE PRIMORDIAL UNIVERSE AFTER PLANCK

From Monday December 15th to Friday December 19th, 2014

The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.



Following the Sampan, BPOL and Core earlier proposals, we proposed PRISM as an L3 mission to ESA; eLisa won the selection (but we were encouraged to apply for an M)

CMB observations from space in Europe

- ESA M4 call for a medium mission. Proposal due Jan. 15th 2015. Budget 450 M€ (ESA) + National contributions for the science payload. Launch 2025.
- Strong interest and support in European countries for such a future CMB mission, e.g. top in France prospective plan for space science.
- **COrE+** minimal concept
  - *CMB B-modes + lensing science for cosmology and fundamental physics.*
  - *6′ resolution, 2.5 μK. arcmin CMB polarisation sensitivity after foreground subtraction. ≈ 1.3m aperture telescope*
  - *Many bands (more than 15) for component separation covering 60-600 GHz; ISM physics.*
  - *budget: ≈550 M€ (450 M€ ESA + 100 M€ European countries)*
- **COrE+** preferred concept
  - *Near-ultimate CMB polarisation space mission*
  - *Extensive astrophysical cosmology (clusters) and extragalactic astrophysics; superior ISM science (with full sky resolution bridging with Herschel in small fields, at highest frequencies)*
  - *≈3 to 4' resolution, ≈1.5 μK. arcmin CMB polarisation sensitivity. ≈ 2m aperture telescope.*
  - *budget: ≈700 to 750 M€ with external partners.*

The journey
Is not over!

“Constraining primordial non-Gaussianity via multi-tracer technique with Euclid and SKA”

Daisuke Yamauchi

[JGRG24(2014)111302]

2014/11/13 JGRG24@IPMU

Constraining primordial non-Gaussianity via multitracer technique with *Euclid* and *SKA*

YAMAUCHI, Daisuke
(RESCEU, U. Tokyo)

DY, K. Takahashi, M. Oguri, PRD90 083520 ,1407.5453

Prof. Bouchet's review

What's Primordial non-Gaussianity?

- Non-Gaussian initial fluctuations arise in several scenarios of inflation.

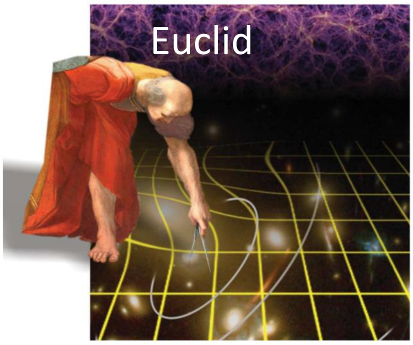
$$\Phi = \phi_{\rm G} + f_{\rm NL}\left(\phi_{\rm G}^2 - \langle \phi_{\rm G}^2 \rangle\right)$$

✓ Even the simplest model predicts small but non-vanishing $f_{\rm NL}$ of *O(0.01).*

- PNG has primarily been constrained from the bispectrum in CMB temperature fluctuations.
  - WMAP : $\sigma(f_{\rm NL}) < 100$ [Bennet+, 2013]
  - Planck : $\sigma(f_{\rm NL}) < 10$ [Planck collaboration, 2013]
  - Ideal : $\sigma(f_{\rm NL}) \sim 3$ [Komatsu+Spergel, 2001]

Main Message

We can test the extremely small primordial non-Gaussianity at the level of $\sigma(f_{\mathrm{NL}})=\mathrm{O}(0.1)$ with Euclid and Square Kilometre Array (SKA).



PNG in Large Scale Structure

- Luminous sources such as galaxies must be most obvious tracers of the large scale structure.
- The galaxy density contrast δ_{gal} is linearly related to the underlying dark matter density contrast δ_{DM} though the bias b_{h}:

$$\delta_{\mathrm{gal}}(M,z,\boldsymbol{k}) = b_{\mathrm{h}}(M,z,k)\delta_{\mathrm{DM}}(z,\boldsymbol{k})$$

✓ In the Gaussian case, the bias is scale-invariant : $b_h=b_h(M,z)$.

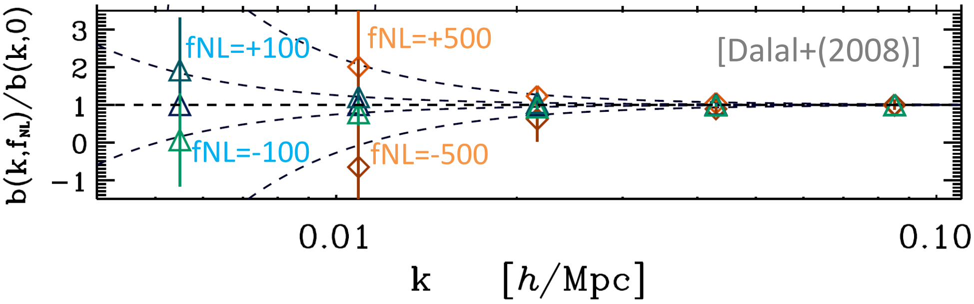
Hashimoto-kun's talk

PNG in Large Scale Structure

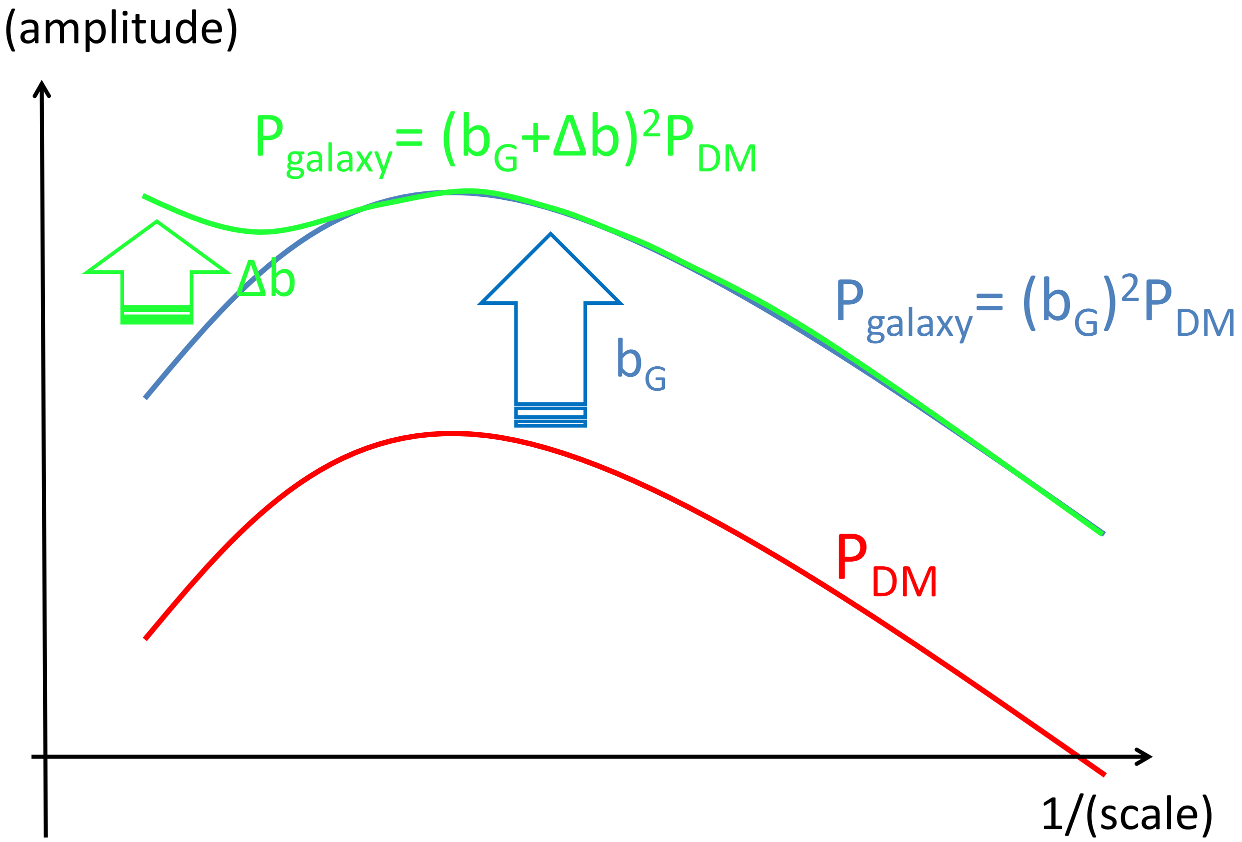
- Primordial non-Gaussianity induces the scale dependent-bias such that the effect dominates at very large scales:

[Dalal+(2008), Desjacques+(2009)]

$$\Delta b = \frac{2 f_{\mathrm{NL}} \delta_{\mathrm{c}}}{\mathcal{M} D_{+}} (b_{\mathrm{L}} - 1) - \frac{1}{\delta_{\mathrm{c}}} \frac{\mathrm{d}}{\mathrm{d}\ln \nu} \left(\frac{\mathrm{d}n/\mathrm{d}M}{\mathrm{d}n_{\mathrm{G}}/\mathrm{d}M} \right)$$



- ✓ Galaxy surveys can effectively constrain $f_{\rm NL}$ to the level comparable to CMB temp. anisotropies.



Accessing ultra-large scales

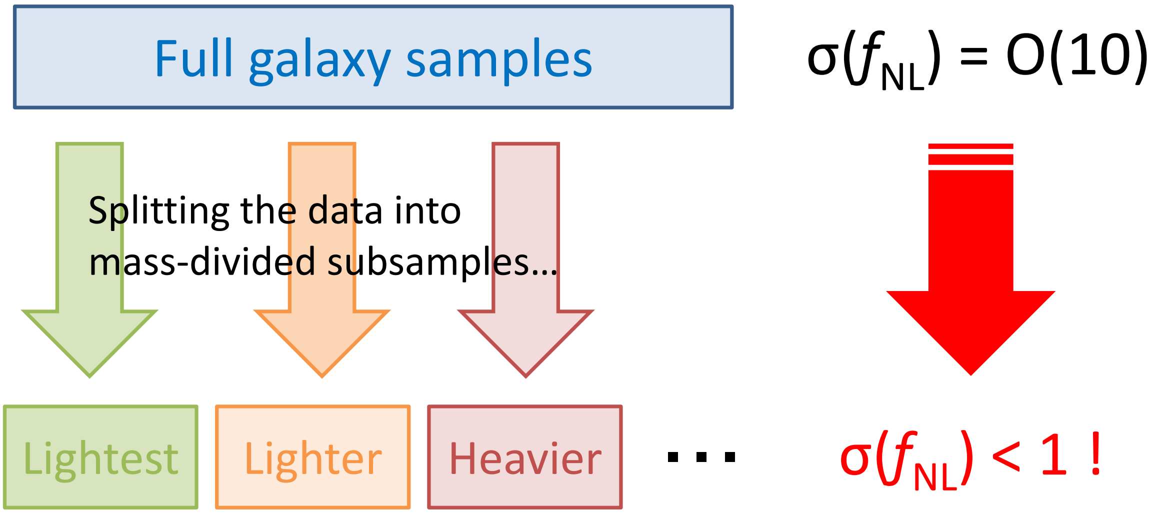
- ➢ Clustering analysis at large scales are limited due to cosmic variance.

MULTITRACER TECHNIQUE [Seljak (2009)]

- a method to reduce the cosmic variance using multiple tracers with different biases.
- The availability of multiple tracers allows significantly improved statistical error in the measurement of f_{NL}.

Multitracer technique [Seljak (2009)]

✓ If we treat the data as the single group, the galaxy survey can constrain f_{NL} to the level comparable to CMB:



Full galaxy samples

$\sigma(f_{\mathrm{NL}}) = O(10)$

Splitting the data into mass-divided subsamples…

Lightest Lighter Heavier ···

$\sigma(f_{\mathrm{NL}}) < 1$!

Multitracer technique

[Seljak (2009)]

✓ Angular power spectra

$$\begin{array}{c|cc} & \text{Lighter} & \text{Heavier} \\ \hline \text{Lighter} & b_1^2 P_\delta + \frac{1}{N_1} & b_1 b_2 P_\delta \\ \text{Heavier} & b_1 b_2 P_\delta & b_2^2 P_\delta + \frac{1}{N_2} \end{array}$$

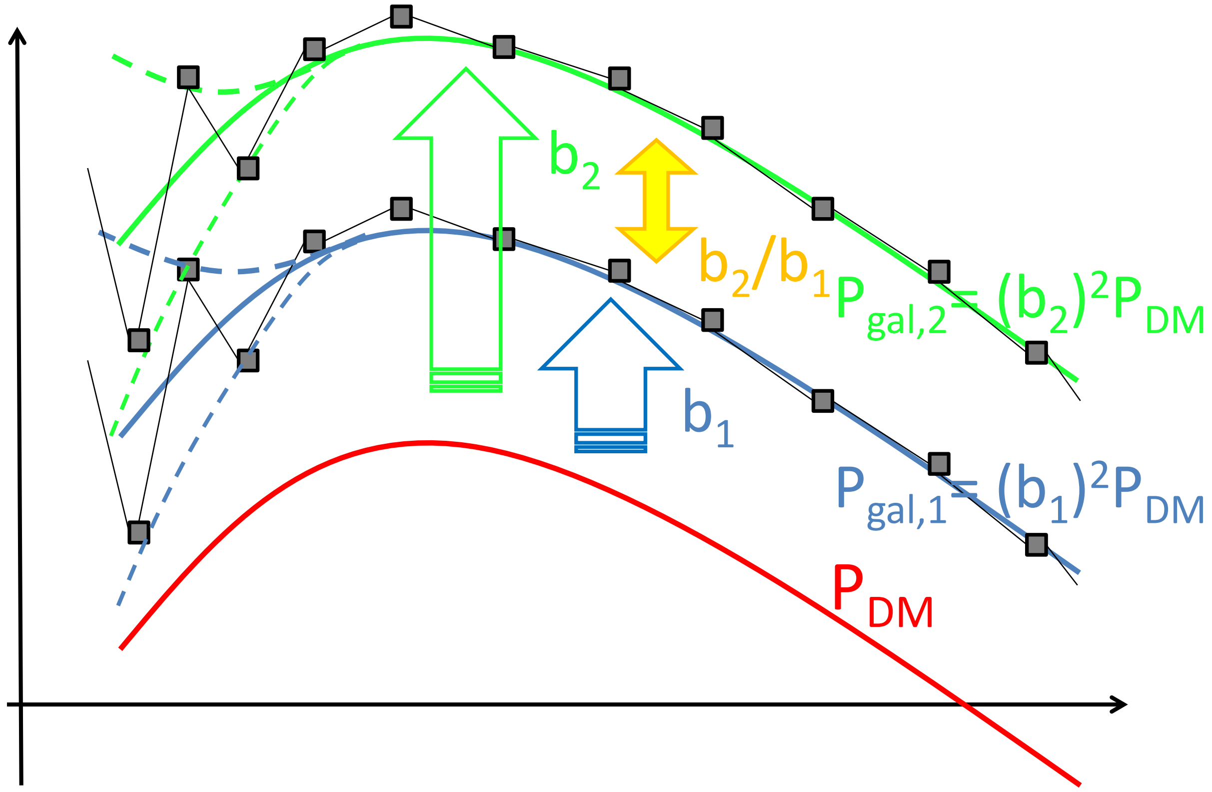
Shot noise

✓ Accuracy for b_2/b_1

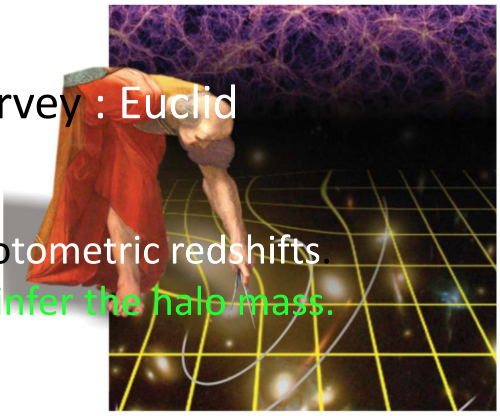
$$\sigma\left(\frac{b_2}{b_1}\right) \propto \sqrt{N_1^{-1} + N_2^{-1}} \quad (\, N_1, N_2 \gg 1\,)$$

We can make a measurement of the ratio of two biases that is only limited by shot noise and hence beats cosmic variance!

The accuracy of the amplitude itself is limited by CV, but for the ratio between the powers there is NO fundamental limit!



Survey design



- Optical/infrared photometric survey : Euclid
  - Covers 15,000 [deg^2].
  - Provides redshift information via photometric redshifts.
  - We use various galaxy properties to infer the halo mass.

- Radio continuum survey : SKA phase-1/2
  - Covers 30,000 [deg^2] out to high-z.
  - The redshift information is not available.
  - Halo mass can be estimated from the galaxy type.
    [Ferramacho+ (2014)]
- SKA+Euclid : 9,000 [deg^2]

Fisher matrix analysis

$$F_{\alpha\beta} = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \sum_{I,J} \frac{\partial C_I(\ell)}{\partial \theta^\alpha} \left[\mathrm{Cov}(\boldsymbol{C}(\ell), \boldsymbol{C}(\ell))\right]^{-1}_{IJ} \frac{\partial C_J(\ell)}{\partial \theta^\beta}$$

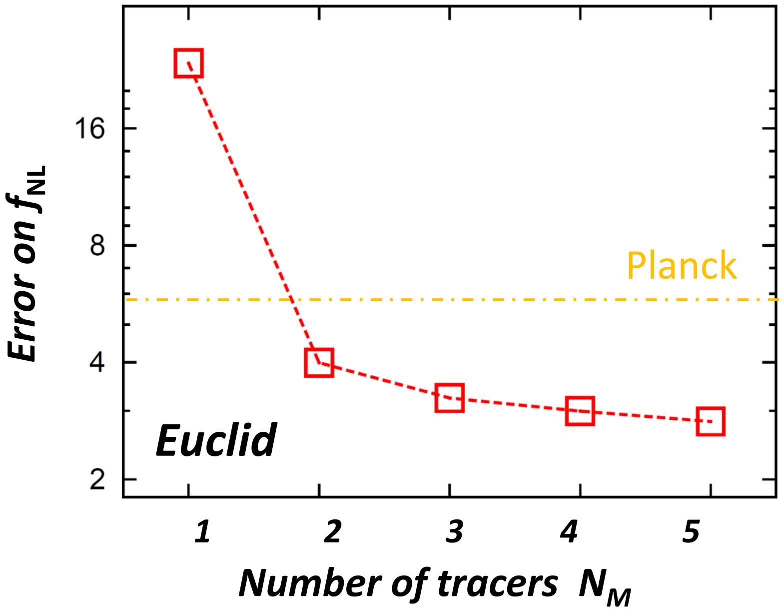
✓ Covariant matrix generalized to multiple tracers with different sky areas with some overlap:

[**DY**+Takahashi+Oguri (2014)]

$$\mathrm{Cov}\left[C_{i(bb')}(\ell), C_{j(\tilde{b}\tilde{b}')}(\ell')\right] = \frac{\delta^{\mathrm{K}}_{ij}\delta^{\mathrm{K}}_{\ell\ell'}}{(2\ell+1)\Delta\ell} \boxed{\frac{4\pi\Omega_w^{(bb'\tilde{b}\tilde{b}')}}{\Omega_w^{(bb')}\Omega_w^{(\tilde{b}\tilde{b}')}}} \left[C_{i(b\tilde{b})}(\ell)C_{i(b'\tilde{b}')}(\ell) + C_{i(b\tilde{b}')}(\ell)C_{i(b'\tilde{b})}(\ell)\right]$$

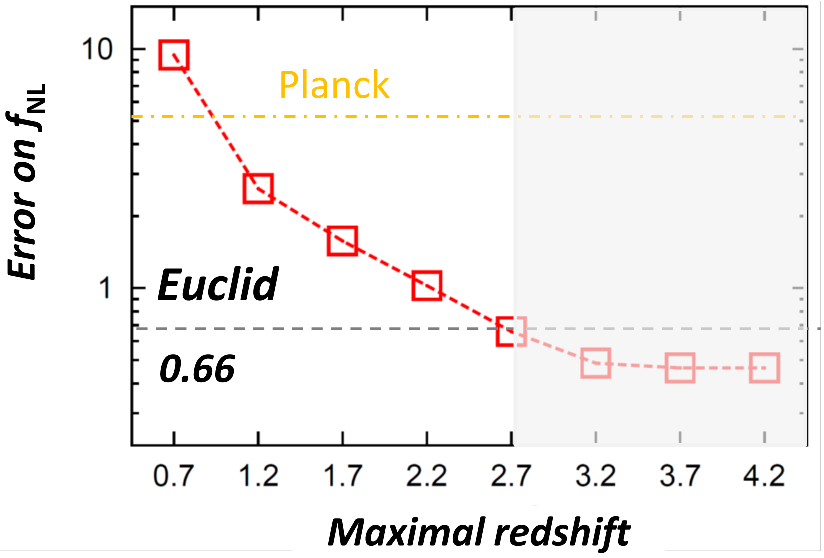
Effect of different sky areas

[**DY**+Takahashi+Oguri (2014)]



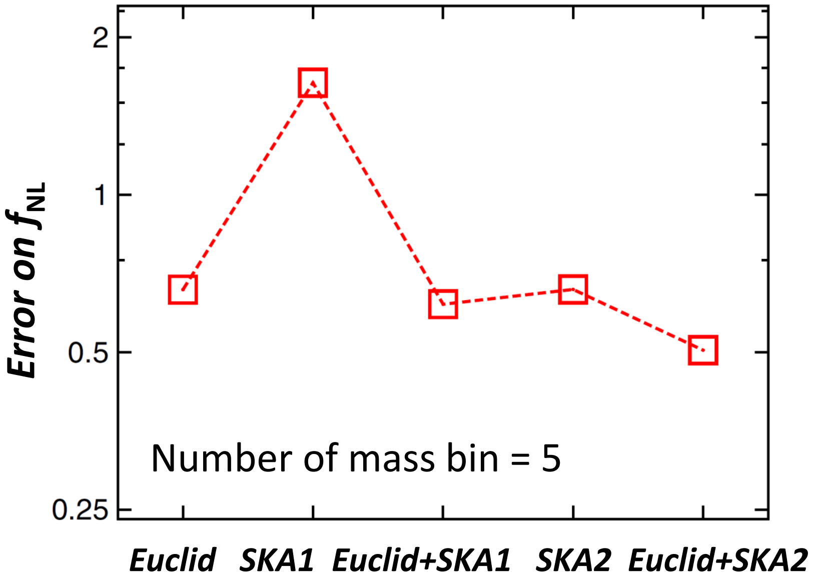
- ✓ The constraining power increases with N_M.
- ✓ Even 2-tracers drastically improve the constraint.

[**DY**+Takahashi+Oguri (2014)]



- ✓ Combining multiple z-bins improves substantially $\sigma(f_{NL})$.
- ✓ Galaxy samples as far as z=3.2 contribute to the constraint.
- ✓ Realistic: $z_{max}=2.7 \rightarrow \sigma(f_{NL})=0.66$

[**DY**+Takahashi+Oguri (2014)]



The constraints of $\sigma(f_{NL})$=O(1) can be obtained even with a single survey. Combining Euclid and SKA, even stronger constraints of $\sigma(f_{NL})$=O(0.1) can be obtained.

Summary

- Splitting the galaxy samples into the subsamples by the inferred halo mass and redshift, constraints on f_{NL} drastically improve.

- The constraints of *$\sigma(f_{NL})=O(1)$* can be obtained even with a single survey. Combining Euclid and SKA, even stronger constraints of *$\sigma(f_{NL})=O(0.1)$* can be obtained.

Thank you!

“Detecting primordial non-Gaussianity from the three-point statistics of halo and weak lensing fields”

Ichihiko Hashimoto

[JGRG24(2014)111303]

Detecting primordial non-Gaussianity from the three-point statistics of halo and weak lensing fields

Ichihiko Hashimoto (YITP)

with Atsushi Taruya(YITP), Shuichiro Yokoyama(Rikkyou-u),
Toshiya Namikawa(Stanford-u),Takahiko Matsubara(Nagoya-u)

2014 11/13 @IPMU

Motivation ~Primordial non-Gaussian~

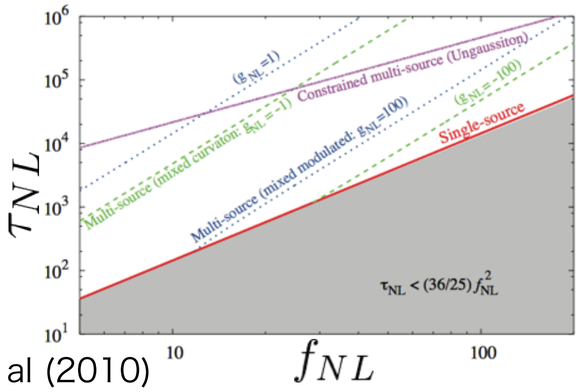
Local-type non-Gaussianity

Gaussian variable

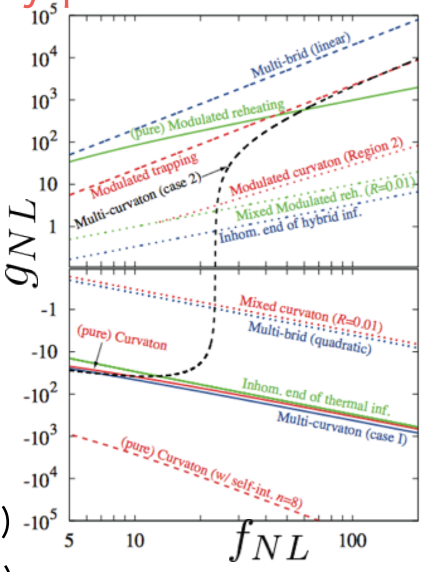
curvature perturbation

$$\Phi(\boldsymbol{x}) = \phi(\boldsymbol{x}) + f_{NL}\{\phi^2(\boldsymbol{x}) - <\phi^2>\} + g_{NL}\phi^3(\boldsymbol{x}) + \cdots$$

$f_{NL}, g_{NL}, \tau_{NL}$: Non-Gaussianity parameters



Suyama et al (2010)



Constraints for CMB CL 95%

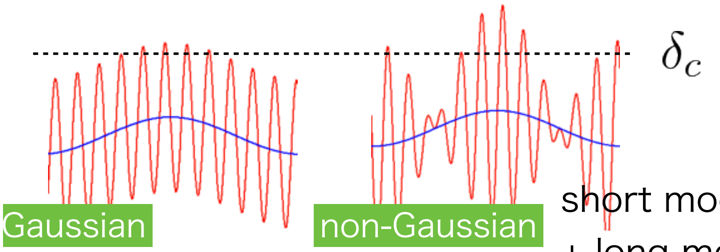
$f_{NL} = 2.7 \pm 11.6$, $\tau_{NL} < 2800$ Planck (2013)

$-7.7 < g_{NL}/10^5 < 1.1$ WMAP (2013)

Large-scale structure

Photometric survey,
like Hyper Suprime-cam(HSC) observe

Galaxy clustering : δ_h
Weak lensing : δ_m



short mode
+ long mode

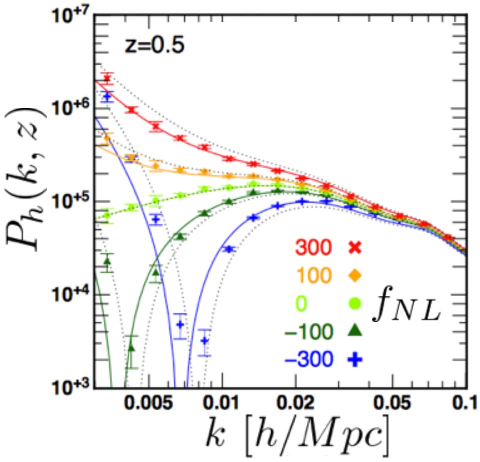
Due to “scale-dependent bias”,

$$\delta_h = (b + \Delta b(\boldsymbol{k}))\delta_m$$
$$\Delta b \propto f_{NL} k^{-2}$$

non-Gaussian effect enhanced.

scale-dependent bias effect arises even power spectrum

$$<\delta_h(\boldsymbol{k})\delta_h(\boldsymbol{k}')> = (2\pi)^3 P_h(k)\delta(\boldsymbol{k}+\boldsymbol{k}')$$



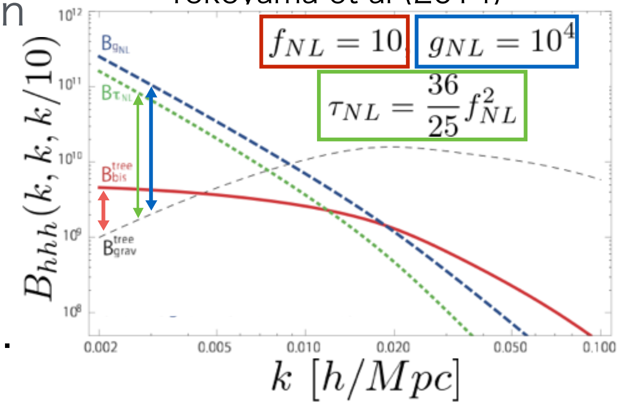
Nishimichi et al. (2010)

Bispectrum

Auto-bispectrum also depend on primordial non-Gaussianity.

$$B = \underline{B^{grav}} + \underline{f_{NL}B_1} + \underline{g_{NL}B_2} + \underline{\tau_{NL}B_3}$$

Yokoyama et al (2014)



We consider cross-bispectra between halo and weak lensing.

$$< \underline{\delta_h(\boldsymbol{l}_1)}\underline{\kappa(\boldsymbol{l}_2)\kappa(\boldsymbol{l}_3)} >= (2\pi)^2 B_{\kappa\kappa h}(l_1, l_2, l_3)\delta_{2D}(\boldsymbol{l}_1 + \boldsymbol{l}_2 + \boldsymbol{l}_3)$$
$$< \underline{\delta_h(\boldsymbol{l}_1)\delta_h(\boldsymbol{l}_2)}\underline{\kappa(\boldsymbol{l}_3)} >= (2\pi)^2 B_{\kappa hh}(l_1, l_2, l_3)\delta_{2D}(\boldsymbol{l}_1 + \boldsymbol{l}_2 + \boldsymbol{l}_3)$$

Question

- How much can we enhance detectability of primordial non-Gaussianity?

Method

Observed bispectrum is defined on
the 2-dimentional celestial sphere.

$$B_{\kappa\kappa h}(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{l}_3)=\int dz\,\underbrace{\left(\frac{W_\kappa(z)}{\chi^2(z)}\right)^2 W_h(z)}_{\text{projection effect}}H^2(z)B_{mmh}\left(\frac{\boldsymbol{l}_1}{\chi(z)},\frac{\boldsymbol{l}_2}{\chi(z)},\frac{\boldsymbol{l}_3}{\chi(z)},z\right)$$

Improved perturbation Theory Matsubara (2011)

power and bispectrum of linear density fluctuation

$$B_{XYZ}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)=\Gamma_X^{(1)}(\boldsymbol{k}_1)\Gamma_Y^{(1)}(\boldsymbol{k}_2)\Gamma_Z^{(1)}(\boldsymbol{k}_3)B_L(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$
$$+\Gamma_X^{(1)}(\boldsymbol{k}_1)\Gamma_Y^{(1)}(\boldsymbol{k}_2)\Gamma_Z^{(2)}(-\boldsymbol{k}_1,-\boldsymbol{k}_2)P_L(k_1)P_L(k_2)\ \cdots$$

h or m

Multi-point propagator contain non-perturbative effect.

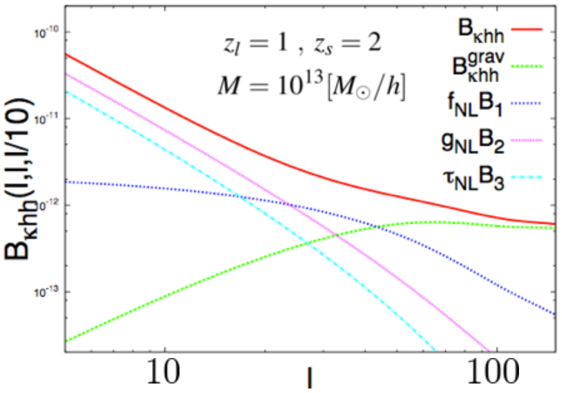
$$(2\pi)^{3-3n}\delta_{3D}(\boldsymbol{k}_1+\boldsymbol{k}_2+\cdots+\boldsymbol{k}_n-\boldsymbol{k})\Gamma_X^{(n)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\cdots,\boldsymbol{k}_n)\equiv\left\langle\frac{\delta^n\delta_X(\boldsymbol{k})}{\delta\delta_L(\boldsymbol{k}_1)\delta\delta_L(\boldsymbol{k}_2)\cdots\delta\delta_L(\boldsymbol{k}_n)}\right\rangle$$

Result ~Scale dependence~

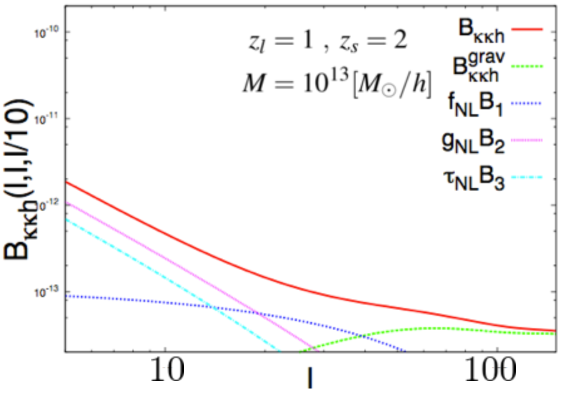
- Cross-bispectra

assume $f_{NL}=10,\ g_{NL}=10^4,\ \tau_{NL}=\frac{36}{25}f_{NL}^2$

- $B_{\kappa hh}$



- $B_{\kappa\kappa h}$



$$\underline{B}=\underbrace{B^{grav}}_{\propto l^2}+\underbrace{f_{NL}B_1}_{\propto l^0}+\underbrace{g_{NL}B_2}_{\propto l^{-2}}+\underbrace{\tau_{NL}B_3}_{\propto l^{-2}}$$

large scale:

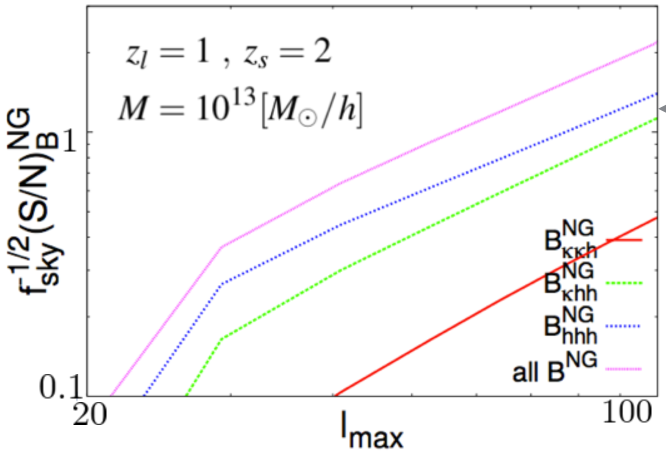
Primordial non-Gaussianity effect enhanced at large scale

Result ~Signal to Noise~

- S/N from primordial non-Gaussianity

$$((S/N)_B^{NG})^2 = \sum_{i,j}(B(l_1^i, l_2^i, l_3^i) - B^{grav})Cov_{ij}^{-1}(B(l_1^j, l_2^j, l_3^j) - B^{grav})$$

$\{i\}$: label of triangle
satisfy $l_i < l_{max}$



S/N of halo-power spectrum
$C_{hh}(l)$ under same assumption

- S/N of cross-bispectra comparable auto-bispectrum
- $1.8 \times (S/N)_{Power}^{NG} \simeq 1.6 \times (S/N)_B^{NG}\ (auto) \simeq (S/N)_B^{NG}\ (auto + cross)$

at $l_{max} = 100$

Summary

- Primordial non-Gaussianity is important to classify inflation.
- Scale-dependent bias enhance signal of primordial non-Gaussianity in LSS.
- By adding cross-bispectra, S/N from primordial non-Gaussianity enhance factor ~1.6 than auto-bispectrum.

Future work

Break degeneracy of $f_{NL}, g_{NL}, \tau_{NL}$

“Self-unitarization of New Higgs Inflation”

Yuki Watanabe

[JGRG24(2014)111304]

Self-unitarization of New Higgs Inflation

Yuki Watanabe
Research Center for the Early Universe (RESCEU), University of Tokyo

arXiv:1403.5766 with C. Germani and N. Wintergerst (LMU Munich)

JGRG24, Kavli IPMU, Kashiwa, Japan
13th November 2014

Higgs boson as the inflaton

- The Standard Model Higgs boson is observed in LHC. In the same experiment, no new particle has been discovered so far.
- The Planck satellite has measured the primordial spectrum of scalar (temperature) perturbations, showing no trace of non-Gaussianity and isocurvature perturbations.
- The BICEP2 has measured the polarization of B-modes in the CMB, thus providing the first evidence for primordial gravitational waves (if they are not from dust).

The Higgs boson can drive inflation with Gravitationally Enhanced Friction (GEF).

$$\phi \sim \sqrt{2\mathcal{H}^\dagger\mathcal{H}} \qquad \mathcal{L}_{\mathcal{H}} = -\mathcal{D}_\mu\mathcal{H}^\dagger\mathcal{D}^\mu\mathcal{H} - \lambda\left(\mathcal{H}^\dagger\mathcal{H} - v^2\right)^2$$

Full action of the GEF inflation

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\Delta^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V\right],$$

where $\Delta^{\alpha\beta} \equiv g^{\alpha\beta} - \frac{G^{\alpha\beta}}{M^2}$.

In a FLRW background, the Friedmann and field eqs read

$$H^2 = \frac{1}{3M_p^2}\left[\frac{\dot\phi^2}{2}\left(1+9\frac{H^2}{M^2}\right) + V\right], \quad \partial_t\left[a^3\dot\phi\left(1+3\frac{H^2}{M^2}\right)\right] = -a^3V' \ .$$

During slow roll in the high friction limit ($H^2/M^2 \gg 1$), the eqs are simplified as

$$\boxed{H^2 \simeq \frac{V}{3M_p^2} \ , \quad \dot\phi \simeq -\frac{V'}{3H}\frac{M^2}{3H^2}}$$

Power of the GEF mechanism

Consistency of the eqs requires the slow roll parameters to be small, i.e.

$$\epsilon \equiv -\frac{\dot H}{H^2} \ll 1 \ , \quad \delta \equiv \frac{\ddot\phi}{H\dot\phi} \ll 1 \ .$$

By explicit calculations, one can show that

$$\epsilon \simeq \frac{V'^2M_p^2}{2V^2}\frac{M^2}{3H^2}, \quad \delta \simeq -\frac{V''M_p^2}{V}\frac{M^2}{3H^2} + 3\epsilon = -\eta + 3\epsilon \ , \quad \eta \equiv \frac{V''M_p^2}{V}\frac{M^2}{3H^2}.$$

We see that, no matter how big the slow roll parameters of GR are

$$\epsilon_{GR} \equiv \frac{V'^2M_p^2}{2V^2} \quad \text{and} \quad \eta_{GR} \equiv \frac{V''M_p^2}{V},$$

there is always a choice of scale $M^2 \ll 3H^2$, during inflation,
such that slow roll parameters are small.

Cosmological perturbations in the GEF inflation

ADM form

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)^2$$

- Use the gauge $\delta\phi = 0$
- then: $h_{ij} = a^2[(1+2\underbrace{\zeta}_{\text{curvature perturbation}})\delta_{ij} + \underbrace{\gamma_{ij}}_{\text{gravitational waves}}]$
- Vary wrt the constraints N, N^i, substitute back into the action and canonically normalize ζ and γ_{ij}
- $N = 1 + \frac{\Gamma}{H}\dot{\zeta}$, $N^i = -\frac{\Gamma}{H}\partial_i\zeta + \frac{\Sigma}{H^2}\partial_i\partial^{-2}\dot{\zeta}$
- $\Gamma(\dot{\phi}, H, M) \simeq 1 + \frac{2}{3}\epsilon$, $\Sigma(\dot{\phi}, H, M) \simeq \frac{\dot{\phi}^2}{2M_p^2}\left[1 + \frac{3H^2}{M^2}\right] \simeq \epsilon H^2$ in the high friction limit $H \gg M$.

Curvature perturbation spectrum

- $\mathcal{L}_{\zeta^2} = \frac{1}{2}[v'^2 - c_s^2(\partial_i v)^2 + \frac{z''}{z}v^2]$ with $c_s^2 = 1 - \mathcal{O}(\epsilon)$
- $\langle\hat{\zeta}_k\hat{\zeta}_{k'}\rangle = (2\pi)^3\delta^{(3)}(k+k')\frac{2\pi^2}{k^3}\mathcal{P}_\zeta$ where $\mathcal{P}_\zeta = \frac{H^2}{8\pi^2\epsilon c_s M_p^2}$
- spectral index: $n_s - 1 = \frac{d\ln\mathcal{P}_\zeta}{d\ln k} \approx -2\epsilon - 2\delta$
- running of the spectral index: $\frac{dn_s}{d\ln k} \approx -6\epsilon\delta - 2\delta\delta' + 2\delta^2$

Matching with the WMAP data, $\mathcal{P}_\zeta = 2\times10^{-9}$, we get a relation

$$\boxed{\frac{M^2}{H^2} = \frac{10^9}{8\pi^2}\frac{V^3}{V'^2 M_p^6}}$$

Note that scalar perturbations are slightly sub-luminal.

Can this lead to observational consequences?

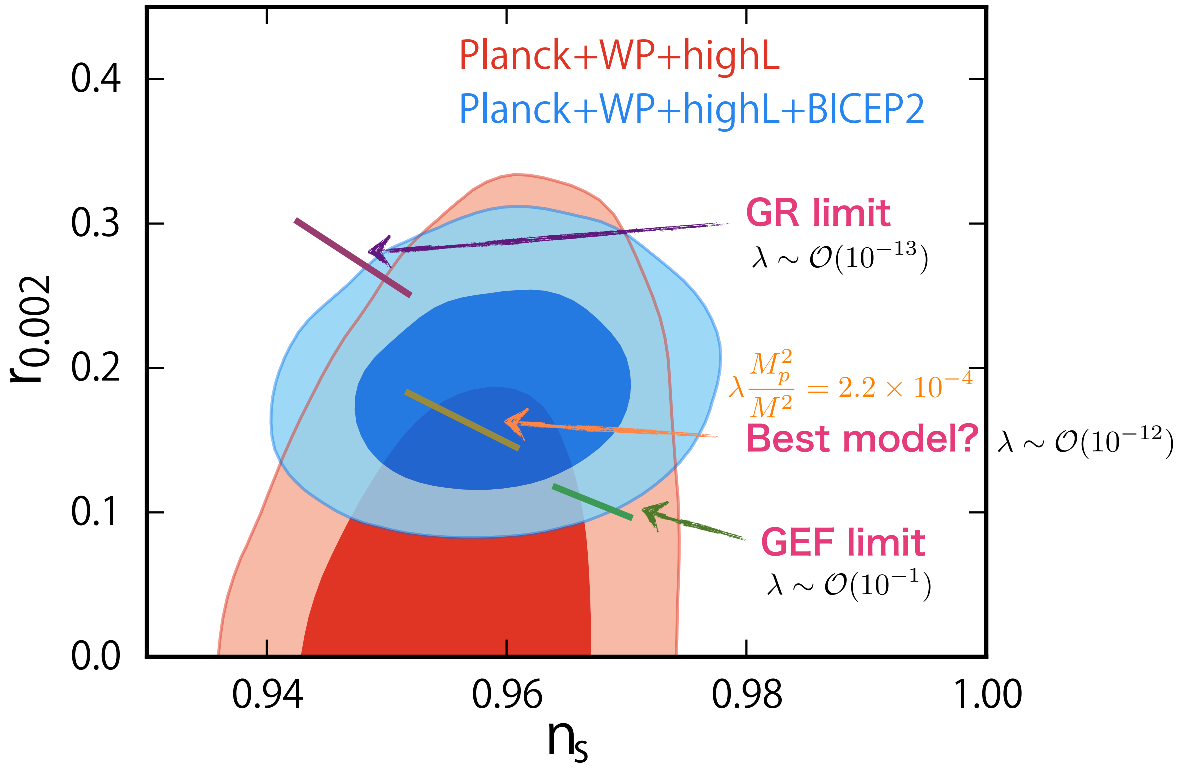
(Any GW or NG due to the new non-linear interaction?)

Gravitational wave spectrum

- $\mathcal{L}_{\gamma^2} = \sum_{\lambda=\pm 2} \frac{1}{2}[v_t'^2 - c_{gw}^2(\partial_i v_t)^2 + \frac{z_t''}{z_t} v_t^2]$ with $c_{gw}^2 = 1 + \mathcal{O}(\epsilon)$
- $\langle \hat{\gamma}_k \hat{\gamma}_{k'} \rangle = (2\pi)^3 \delta^{(3)}(k+k') \frac{2\pi^2}{k^3} \mathcal{P}_\gamma$ where $\mathcal{P}_\gamma = \frac{2H^2}{\pi^2 c_{gw}(1+\epsilon/3) M_p^2}$
- spectral index is red: $n_t = \frac{d\ln \mathcal{P}_\gamma}{d\ln k} \approx -2\epsilon$ テキスト
- tensor to scalar ratio: $r = \frac{\mathcal{P}_\gamma}{\mathcal{P}_\zeta} = 16\epsilon = -8n_t$

Note that GWs are slightly "super-luminal", but this does not mean "acausal" unless a closed timelike curve is formed [Babichev et al 2008].

New constraint on inflation, if BICEP2 is right.



New Higgs Inflation fits BICEP2 and Planck

[Germani & Kehagias ‘10; Germani & YW ‘11; Germani, YW & Wintergerst 1403.5766]

$$\mathcal{L} = \frac{1}{2}M_p^2 R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2}\right)\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4$$

Predictions in GEF limit:
[1106.0502]

$$\boxed{n_s = 0.95, \quad r = 0.16, \quad \frac{dn_s}{d\ln k} = -0.0015,}$$

$$n_s - 1 = -5\epsilon, \quad r = 16\epsilon, \quad \frac{dn_s}{d\ln k} = -15\epsilon^2,$$

$$\frac{\phi_*}{M_p} = 0.037\left(\frac{\mathcal{P}_\zeta}{2\times10^{-9}}\right)^{1/4}\left(\frac{\epsilon}{\lambda}\right)^{1/4}, \quad \frac{M}{M_p} = 9.0\times10^{-6}\left(\frac{\mathcal{P}_\zeta}{2\times10^{-9}}\right)^{3/4}\frac{\epsilon^{5/4}}{\lambda^{1/4}},$$

$$\frac{H}{M_p} = 4.0\times10^{-4}\left(\frac{\mathcal{P}_\zeta}{2\times10^{-9}}\right)^{1/2}\sqrt{\epsilon}, \quad \epsilon = \frac{1}{3N_*+1}, \tag{2.23}$$

GR limit:

$$n_s - 1 = -3\epsilon, \quad \frac{dn_s}{d\ln k} = -3\epsilon^2, \quad \epsilon = \frac{1}{N_*+1}$$

Unitarity issues: inflationary scale

[Germani & YW ‘11; Germani, YW & Wintergerst 1403.5766]

Non-renorm. operator: $\mathcal{L}_{\rm nr} = \frac{1}{2}\frac{G^{\alpha\beta}}{M^2}\partial_\alpha\phi\partial_\beta\phi$

During inflation, and in high friction regime, the perturbed Lagrangian up to cubic order is

$$\mathcal{L}_{\delta\phi,h} = -\frac{1}{2}\bar{h}^{\alpha\beta}\mathcal{E}(\bar{h})_{\alpha\beta} - \frac{1}{2}\partial_\mu\bar{\phi}\partial^\mu\bar{\phi} + \underline{\frac{\mathcal{E}(\bar{h})^{\alpha\beta}}{2H^2M_p}\partial_\alpha\bar{\phi}\partial_\beta\bar{\phi}} + \text{mixings}\ldots$$

$$\bar{h}_{\alpha\beta} = M_p h_{\alpha\beta}$$
$$\bar{\phi} = \frac{\sqrt{3}H}{M}\delta\phi$$

Apparent strong coupling scale: $\Lambda_H \sim \left(H^2M_p\right)^{1/3} \ll M_p$

Unitarity issues: inflationary scale

[Germani & YW '11; Germani, YW & Wintergerst 1403.5766]

Non-renorm. operator: $\mathcal{L}_{\rm nr} = \frac{1}{2}\frac{G^{\alpha\beta}}{M^2}\partial_\alpha\phi\partial_\beta\phi$

During inflation, and in high friction regime, the perturbed Lagrangian up to cubic order is

$$\mathcal{L}_{\delta\phi,h} = -\frac{1}{2}\bar{h}^{\alpha\beta}\mathcal{E}(\bar{h})_{\alpha\beta} - \frac{1}{2}\partial_\mu\bar{\phi}\partial^\mu\bar{\phi} + \underline{\frac{\mathcal{E}(\bar{h})^{\alpha\beta}}{2H^2M_p}\partial_\alpha\bar{\phi}\partial_\beta\bar{\phi}} + \text{mixings}\ldots$$

$$\bar{h}_{\alpha\beta} = M_p h_{\alpha\beta}$$
$$\bar{\phi} = \frac{\sqrt{3}H}{M}\delta\phi$$

Apparent strong coupling scale: $\Lambda_H \sim \left(H^2 M_p\right)^{1/3} \ll M_p$

The apparent scale will be removed by the diagonalizaiton of the scalar-graviton system in the unitary gauge.

$$\mathcal{L}_{\zeta^3} \sim M_p^2\epsilon^2\zeta\dot{\zeta}^2 \sim \frac{\sqrt{\epsilon}}{M_p}\bar{\phi}\dot{\bar{\phi}}^2$$
$$\mathcal{L}_{\gamma\zeta^2} \sim M_p^2\epsilon\gamma_{ij}\partial_i\zeta\partial_j\zeta \sim \frac{1}{M_p}\bar{\gamma}_{ij}\partial_i\bar{\phi}\partial_j\bar{\phi}$$

Strong coupling scale:

$$\Lambda \simeq M_p$$

Unitarity issues: post-inflation

[Germani, YW & Wintergerst 1403.5766]

Let us first consider a model with non-renormalizable potential:

$$\mathscr{H} = \frac{\pi^2}{2} + \frac{1}{2}\partial_i\phi\partial^i\phi + \frac{\phi^6}{\Lambda^2}$$

Suppose we want a region with $\mathcal{H} \sim H^2M_p^2 \gg \Lambda^4$

i.e. a background formed by a large number of particles with very large wavelength. This can be realized by taking $\phi \gg \Lambda.$

$$V_{\text{1-loop}} \sim \frac{\phi^8}{\Lambda^4}\log\frac{\phi}{\mu} + \text{counter-terms}$$

Starting from Minkowski background, a large homogeneous (inflationary) background cannot be obtained without UV-completion, because of **quantum corrections**.

Unitarity issues: post-inflation

[Germani, YW & Wintergerst 1403.5766]

$$\mathcal{L} = \frac{1}{2}M_p^2 R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2}\right)\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4$$

In order to "integrate out" gravity, we take the decoupling limit:

$$M_p \to \infty, \quad \Lambda_M^3 = M^2 M_p < \infty$$

$$\mathcal{L}_{\rm dec} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\left[1 + \frac{(\Box\phi)^2 - \partial_{\mu\nu}\phi\partial^{\mu\nu}\phi}{2\Lambda_M^6}\right] - \frac{\lambda}{4}\phi^4$$

$$\mathcal{H} = \frac{\pi^2}{2(1+3\Delta)} + \frac{1}{2}\partial_i\phi\partial^i\phi(1+\Delta) + \frac{\lambda}{4}\phi^4 \qquad \Delta = \frac{1}{2\Lambda_M^6}[(\partial_i\partial^i\phi)^2 - \partial_{ij}\phi\partial^{ij}\phi]$$

If we consider a homogeneous field $\phi \gg \Lambda_M$ with **small-momentum limit**, quantum corrections are under control thanks to the quartic Galileon interaction. Therefore, the Higgs boson is **unitary** throughout.

Whenever the Hamiltonian density overcomes the scale $M^2M_p^2$, the strong coupling scale will grow with the homogeneous Friedmann background.

Conclusions

- **Data is getting more and more precise, and even a surprise is coming! The detection of inflationary gravitational waves by BICEP2 will be confirmed or falsified by Planck 12/2014.**
- **New Higgs Inflation is compatible with Planck and BICEP2 without having unitarity issues.**

“No quasi-stable scalaron lump forms after R^2 inflation”

Naoyuki Takeda

[JGRG24(2014)111305]

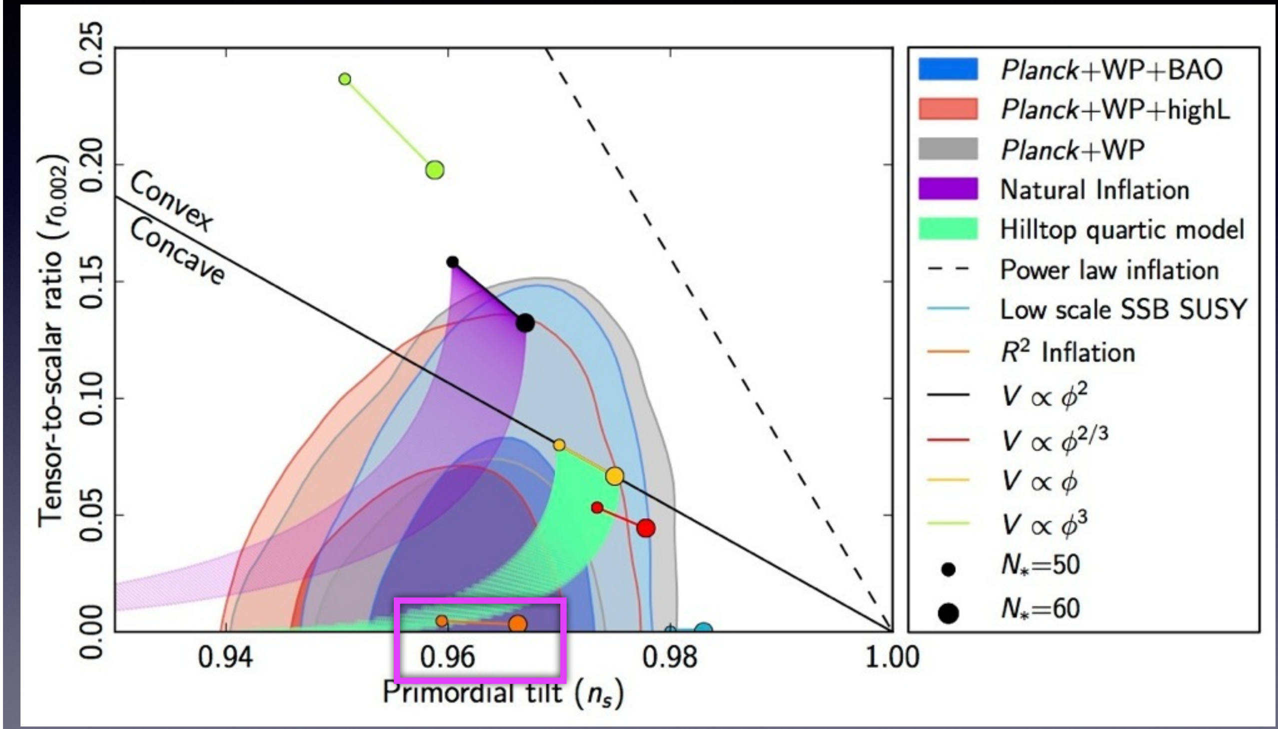
No quasi-stable scalaron lump formas after R2 inflation

Naoyuki Takeda ICRR(Tokyo Uni.)
Yuki Watanabe RESCEU(Tokyo Uni.)

based on
arXiv:1405.3830,
PRD 90, 023519 (2014)

JGRG 2014

intro prediction without N ambiguity



PLANCK 2013

intro

R^2 inflation

$$S_J = \frac{M_p^2}{2}\int d^4x\sqrt{-\hat{g}}(\hat{R} + \boxed{\frac{\hat{R}^2}{6M^2}}) + S_{SM}$$

$g_{\mu\nu} \equiv \hat{g}_{\mu\nu}\exp(\frac{2}{3}\frac{\phi}{M_p})$

1980 Starobinsky

$$\longrightarrow \boxed{\begin{array}{l} S_E = \displaystyle\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - U(\phi)\right] + S_{E,SM} \\ U(\phi) = \displaystyle\frac{3}{4}M^2M_p^2\left[1 - \exp(-\frac{2}{3}\frac{\phi}{M_p})\right]^2 \end{array}}$$

inflation $U(\phi) \simeq \begin{cases} \frac{3}{4}M^2M_p^2 & \phi > M_p \\ \frac{1}{2}M^2\phi^2 & \phi < M_p \end{cases}$

decay $\Gamma_{\rm tot} = \frac{N_\chi}{192\pi}\frac{(M^2+2m_\chi^2)^2}{M_p^2 M}|_{\phi\to\chi\chi} + \frac{N_\psi}{48\pi}\frac{m_\psi^2 M}{M_p^2}|_{\phi\to\bar{\psi}\psi}$ Watanabe 06,10

COBE normal.

$$\boxed{\begin{array}{l} M \simeq 10^{-5}M_p \\ r = 3.9\times 10^{-3},\quad T_R \simeq 10^9\,{\rm GeV},\quad N = 56 + \displaystyle\frac{1}{3}\ln\frac{T_R}{10^9\,{\rm GeV}} \end{array}}$$

---

intro

Preheating and I-ball

preheating

Fluctuation of scalar field exponentially increases during reheating

Kofman, Linde, Starobinsky '94

I-ball

When the potential is shallower than quadratic, the enhanced fluctuation would fragment into I-ball(oscillon).

$V = \frac{1}{2}m^2\phi^2 + \delta V \to \phi \simeq \Phi(r)\sin(mt)$

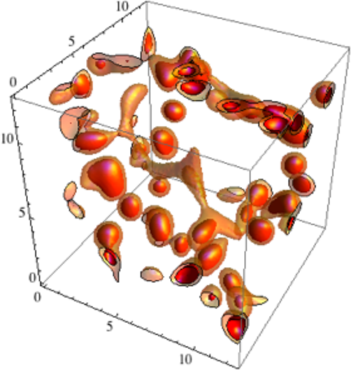
Amin '11

$\longrightarrow I \equiv \frac{1}{2\omega}\int_{\rm volume} dx^3\overline{\dot{\phi}}^2$ quasi-invariant

$\delta V < 0$

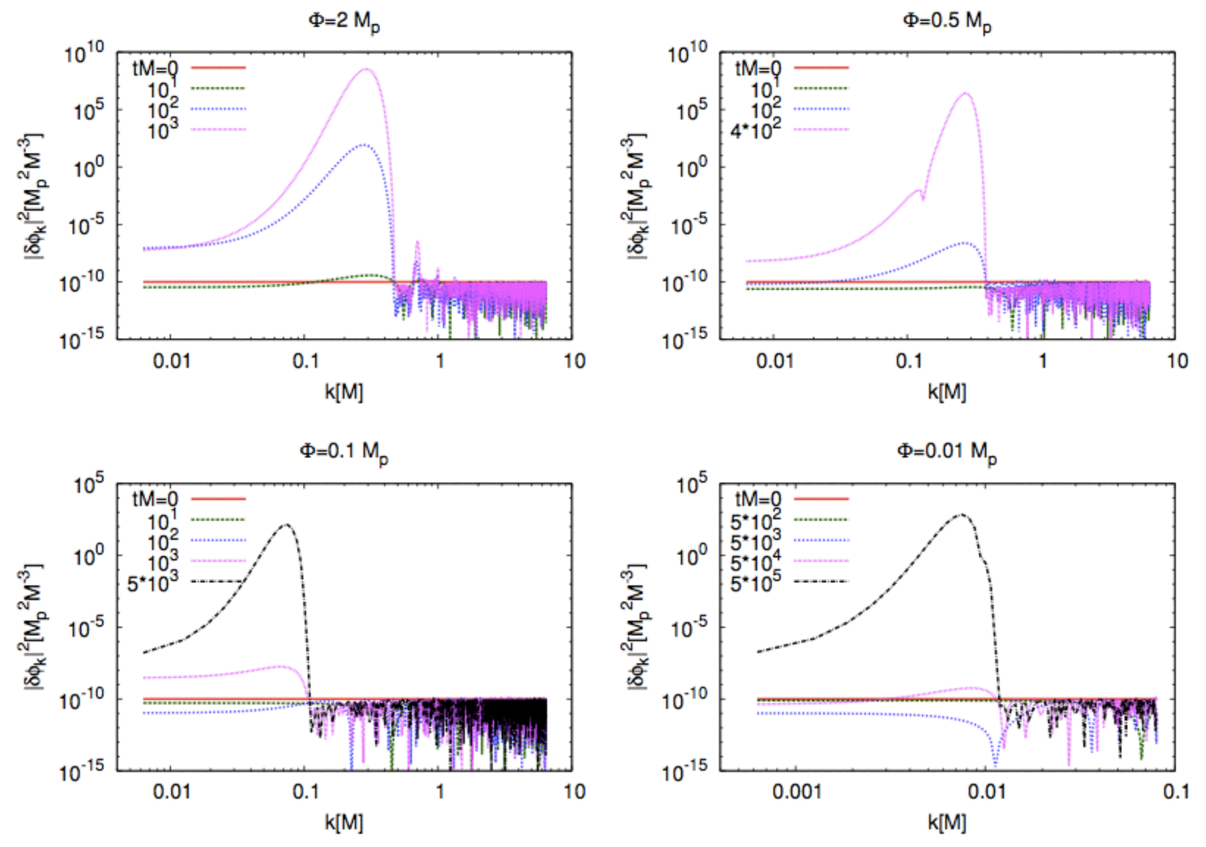
$\to \delta\bar{E}_{\tilde{\omega}}/\delta\Phi = 0$ has the localized solution of $\Phi(r)$

Formation of I-ball would change the decay process of the field



what we did

result: Minkowski



---

what we did

result: Minkowski

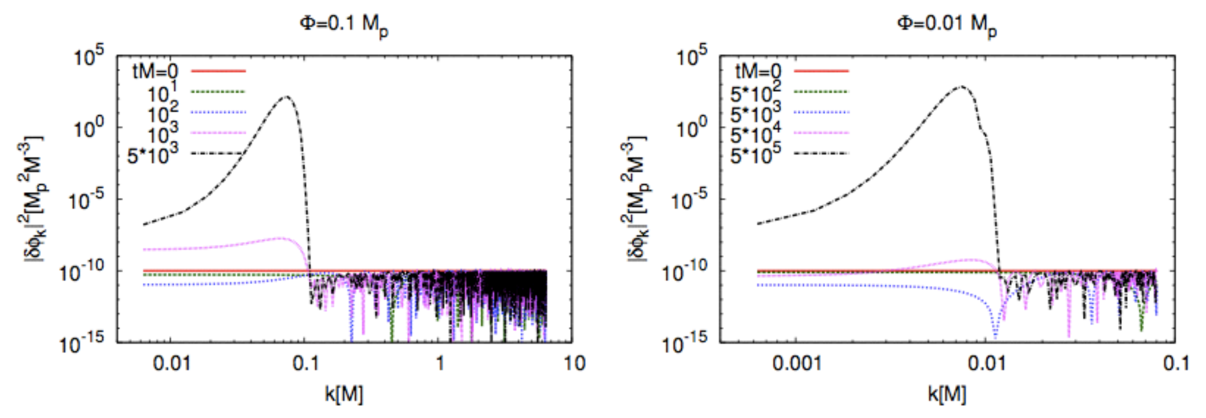
$$\boxed{\frac{d^2}{d(Mt)^2}\delta\phi_k + \left[A_{1k} - 2q_1\cos(2Mt)\right]\delta\phi_k = 0}$$

$$A_{1k} = 4 + 4\left(\frac{k}{M}\right)^2 + \frac{7}{36}q_1^2,\ \ q_1 = 2\sqrt{6}\frac{\Phi}{M_p}$$

$\Phi < 0.2M_p \to q_1 < 1$: narrow resonance

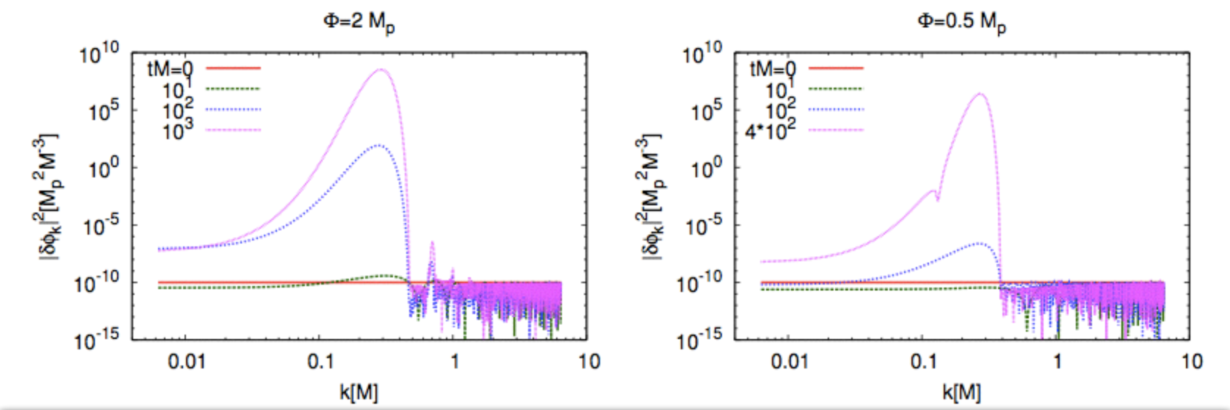
→ $\delta\phi_k \propto \mathrm{e}^{\mu_{k,n}mt}$ at $-\frac{q^n}{n^{n-1}} < A_k - n^2 < \frac{q^n}{n^{n-1}}$

$$\boxed{0 \le \frac{k}{m} \le \frac{q_1}{3\sqrt{2}}\ (n=2)}$$



what we did

result: Minkowski



$$\frac{d^2}{d(Mt)^2}\delta\phi_k + \left[A_{1k} - 2q_1\cos(2Mt)\right]\delta\phi_k = 0$$

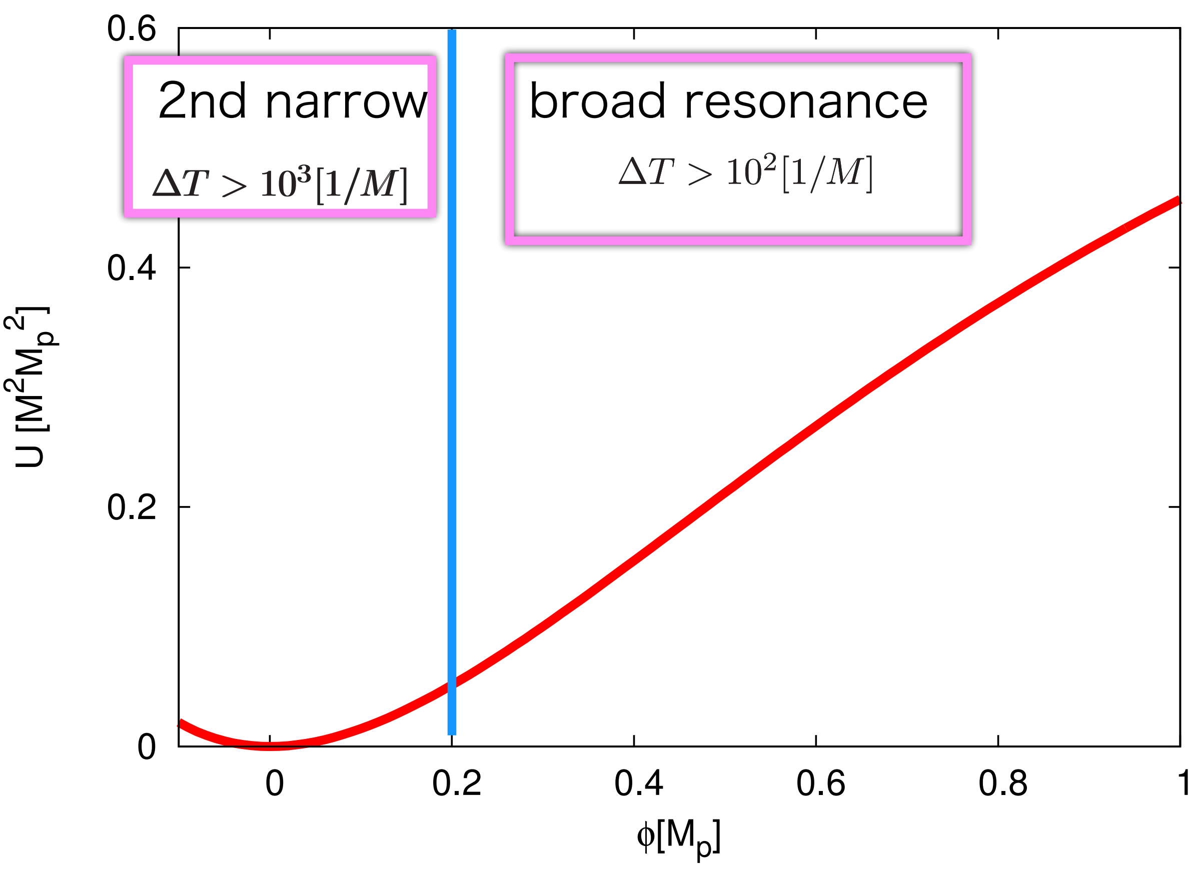
$\Phi > 0.2M_p \rightarrow q_1 > 1$: broad resonace

$\longrightarrow$ $\delta\phi_k \propto \mathrm{e}^{\mu_k mt}$ at $\frac{|\dot{\omega}_k|}{\omega_k^2} > 1$

$$\boxed{\frac{k}{M} \leq -\left[1+\frac{7}{6}\left(\frac{\Phi}{M_p}\right)^2\right] + \sqrt{6}\frac{\Phi}{M_p}\cos(Mt) + \left(\frac{3}{2}\right)^{1/3}\left(\frac{\Phi}{M_p}\right)^{2/3}|\sin(Mt)|^{2/3}}$$

what we did

result: Minkowski

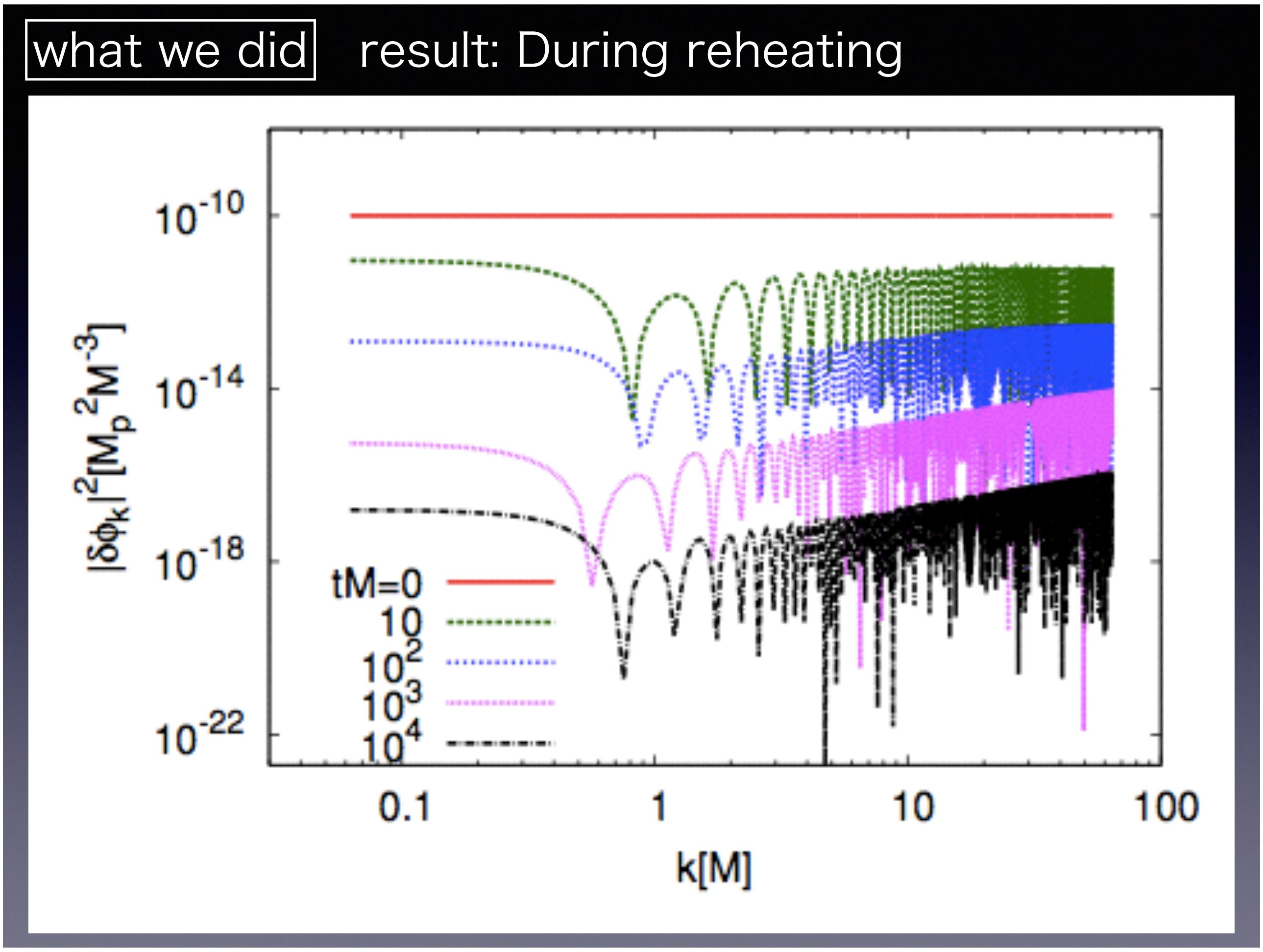


2nd narrow

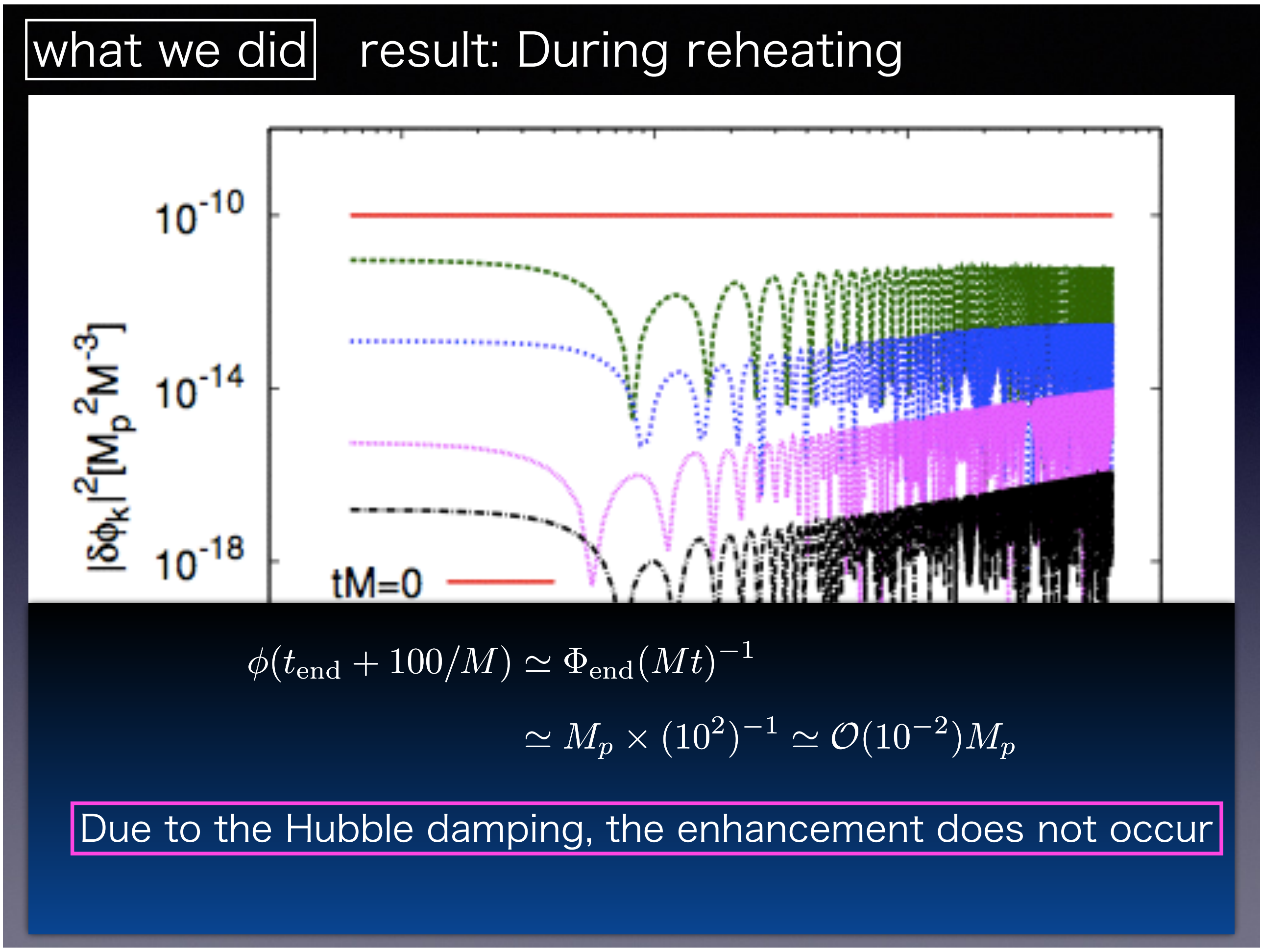
$\Delta T > 10^3[1/M]$

broad resonance

$\Delta T > 10^2[1/M]$



what we did result: During reheating



what we did result: During reheating

$$\phi(t_{\rm end} + 100/M) \simeq \Phi_{\rm end}(Mt)^{-1}$$

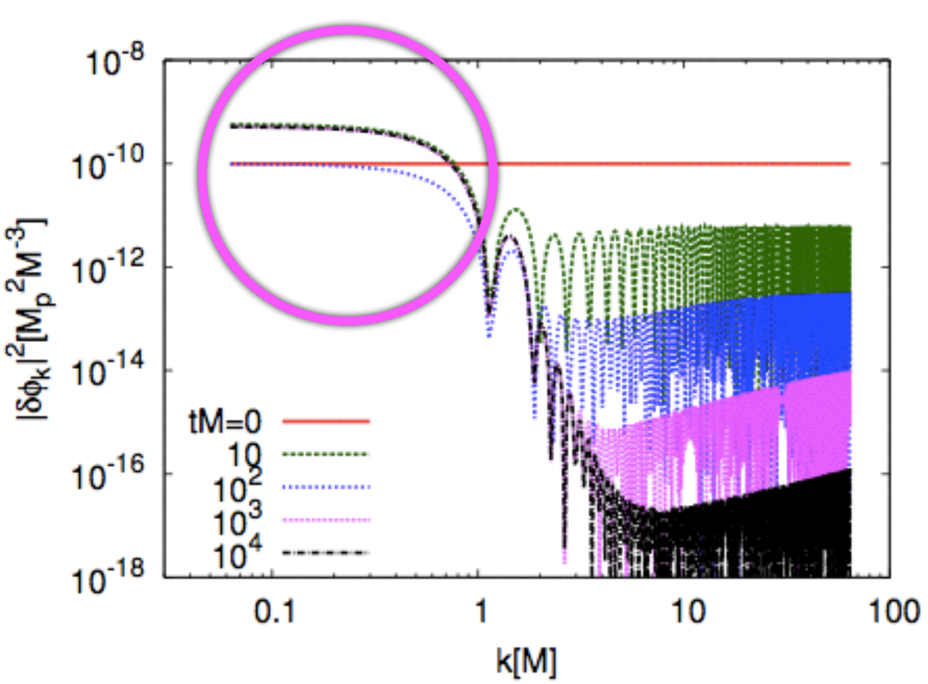
$$\simeq M_p \times (10^2)^{-1} \simeq \mathcal{O}(10^{-2})M_p$$

Due to the Hubble damping, the enhancement does not occur

what we did result: with back reaction

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \omega_k^2\delta\phi_k = 0$$

$$\omega_k^2 = \frac{k^2}{a^2} + U''(\phi_0) + \Delta F,;\ \underline{\Delta F = \frac{2\dot{\phi}_0}{M_p^2 H}U'(\phi_0) + \frac{\dot{\phi}_0^2}{M_p^4 H^2}U(\phi_0)}$$



conclusion

In the case that the potential is shallower than quadratic, there is a possibility that the inflaton fragment into I-ball during the reheating epoch.

In this work, we have investigated the possibility of the formation of I-ball for R2 inflation model.

As a result, we have confirmed that the I-ball is not formed for R2 inflation because the enhancement of fluctuation is suppressed due to the expansion of Universe.

Thus, the perturbative analysis for the reheating of R2 inflation is not modified, and the predictions of n_s, r, N are confirmed.

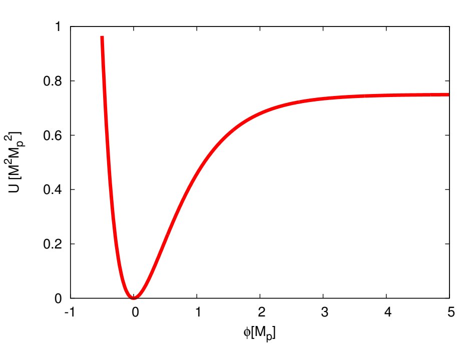
If we include the back reaction of the metric, fluctuation is enhanced at the horizon scale, which is weak to form the I-ball, but has the possibility to form the halo.

intro $\mathcal{P}_\zeta,\ r, n_s, N$ for R^2 inflation

inflation

$$U(\phi) = \frac{3}{4}M^2M_p^2\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2$$

$$\simeq \begin{cases} \frac{3}{4}M^2M_p^2 & \phi > M_p \\ \frac{1}{2}M^2\phi^2 & \phi < M_p \end{cases}$$



decay

$$\mathcal{L}_{E,SM,\mathrm{decay}} = -\frac{m_\chi^2}{2}\mathrm{e}^{-\frac{2}{\sqrt{6}}\frac{\phi}{M_p}}\chi^2 - \mathrm{e}^{-\frac{1}{\sqrt{6}}\frac{\phi}{M_p}}m_\psi\bar{\psi}\psi - \frac{\chi}{\sqrt{6}M_p}\partial_\mu\chi\partial^\mu\phi$$

Komatsu &Watanabe 06, Watanabe 10

$$\longrightarrow \quad \Gamma_{\mathrm{tot}} = \frac{N_\chi}{192\pi}\frac{\left(M^2+2m_\chi^2\right)^2}{M_p^2M}|_{\phi\to\chi\chi} + \frac{N_\psi}{48\pi}\frac{m_\psi^2M}{M_p^2}|_{\phi\to\bar{\psi}\psi}$$

COBE normalisation

$$\boxed{\begin{array}{l} M \simeq 10^{-5}M_p \\ r = 3.9\times10^{-3},\quad T_R \simeq 10^9\mathrm{GeV},\quad N = 56 + \frac{1}{3}\ln\frac{T_R}{10^9\mathrm{GeV}} \end{array}}$$

intro Preheating and I-ball

preheating

Fluctuation of scalar field exponentially increases during inflation

Kofman, Linde, Starobinsky '94

$$\mathcal{L}_{\mathrm{int}} \ni \frac{\lambda'}{4}\phi^4$$

$$\longrightarrow \quad \boxed{\frac{d^2}{d(mt)^2}\delta\phi_k + [A_k + 2q_k\cos(2mt)]\delta\phi_k = 0}$$

$$A_k = 1 + (\frac{k}{m})^2 + \frac{3\lambda}{2}(\frac{\phi_0}{m})^2, \quad q_k = \frac{3}{4}\lambda(\frac{\phi_0}{m})^2$$

$q_k > 1 \to$ broad resonance at $|\dot{\omega}_k|/\omega^2 > 1$

$q_k < 1 \to$ narrow resonance at $-\frac{q_k^n}{n^{n-1}} < A_k - n^2 < \frac{q^n}{n^{n-1}}$

$$\longrightarrow \quad \delta\phi_k \propto \mathrm{e}^{\mu_k mt}$$

Enhanced fluctuation diffuses into other modes

Khlebnikov, Tkachev '96

what we did Numerical simulation

To confirm the evolution of fluctuation, we have executed the numerical simulation and analyze it with Mathieu equation.

$$\boxed{\begin{aligned}&\ddot{\phi}_0 + 3H\dot{\phi}_0 + \frac{\partial}{\partial\phi_0}U(\phi_0) = 0\\ &\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \omega_k^2\delta\phi_k = 0\end{aligned}}$$

$$\phi(t,x) = \phi_0(t) + \delta\phi(t,x),\ \ \omega_k^2 = \tfrac{k^2}{a^2} + \tfrac{\partial^2}{\partial^2\phi}U(\phi_0)$$

We have executed the simulations in 3 situations

$H = 0 \quad \omega_k^2 = k^2 + U''(\phi_0)$: Minkowski

$H \neq 0 \quad \omega_k^2 = \frac{k^2}{a^2} + U''(\phi_0)$: expanding UN.

$H \neq 0 \quad \omega_k^2 = \frac{k^2}{a^2} + U''(\phi_0) + \Delta F$: with back reaction of metric

“Gravitational waves as a probe of supersymmetric scale”

Masaki Yamada

[JGRG24(2014)111306]

Gravitational waves as a probe of supersymmetric scale

Masaki Yamada
ICRR, Univ. of Tokyo

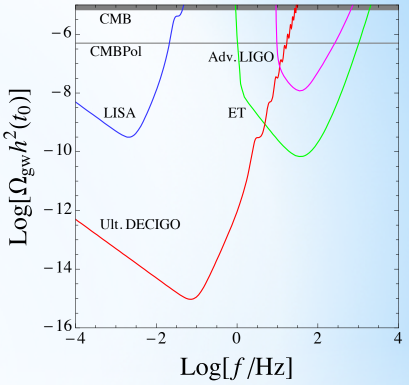
in collaboration with Ayuki Kamada
arXiv:1407.2882 [hep-ph]

JGRG @IPMU
13/November/2014

Introduction: Gravitational waves and new physics

Stochastic gravitational wave signals are predicted by physics beyond the Standard Model:

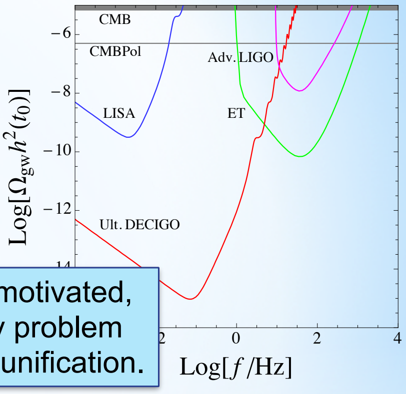
- topological defects (cosmic string, domain wall)
- first order phase transition
- preheating
- quantum fluctuations during inflation



Introduction: Gravitational waves and new physics

Stochastic gravitational wave signals are predicted by physics beyond the Standard Model:

- topological defects (cosmic string, domain wall)
- first order phase transition
- preheating
- quantum fluctuat



Supersymmetric theories are well-motivated, because it addresses the hierarchy problem and also achieves gauge coupling unification.

We have shown that cosmic strings generally form after the end of inflation in supersymmetric theories.

These cosmic strings emit gravitational waves, which give us information of supersymmetric scale!

Flat directions in supersymmetric theories

Affleck, Dine, 85
Dine, Randall, Thomas, 96

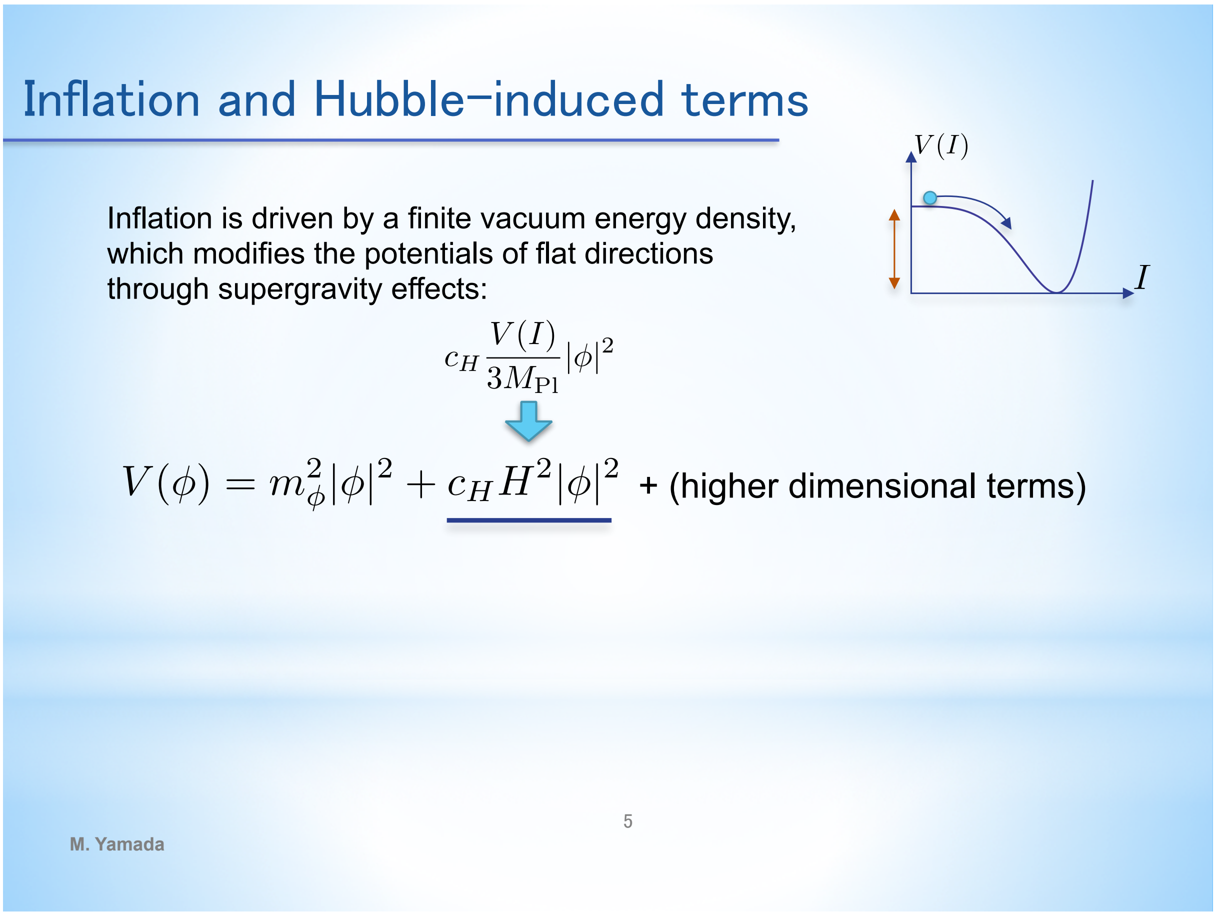
Supersymmetric theories usually predict many complex scalar fields (called flat directions) whose potentials are absent except for soft terms.

$$V(\phi) = m_\phi^2 \, |\phi|^2$$

The dynamics of such flat directions is nontrivial during and after inflation.

flat directions in the MSSM	B-L
LH_u	-1
H_uH_d	0
udd	-1
LLe	-1
QdL	-1
$QQQL$	0
$QuQd$	0
$QuLe$	0
$uude$	0
$ddLL$	-3
$uuee$	1
$QuQue$	1
$QQQQu$	1
$(QQQ)_4LLLe$	-1
$udQdQd$	-1

Gherghetta, Kilda, Martin, 95

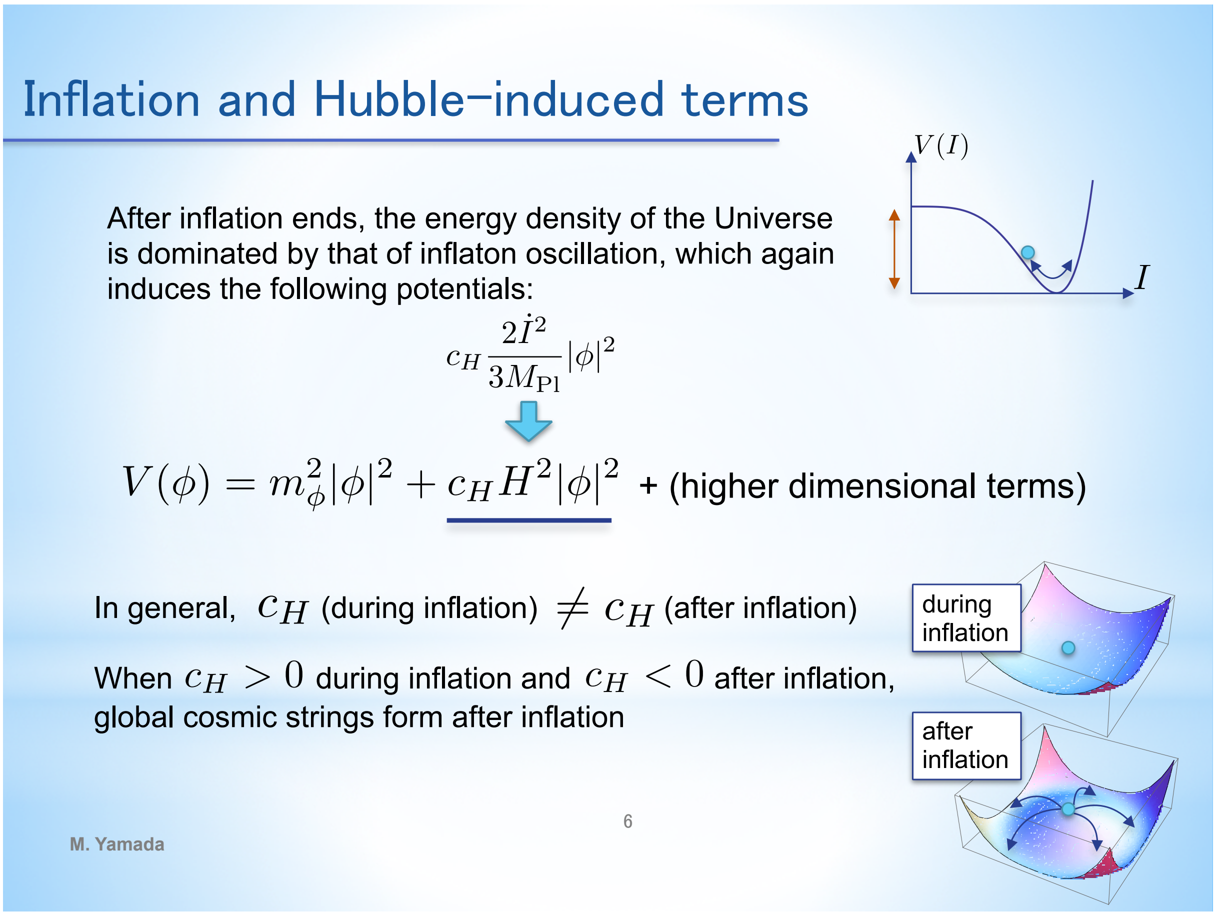


Inflation and Hubble-induced terms

Inflation is driven by a finite vacuum energy density, which modifies the potentials of flat directions through supergravity effects:

$$c_H \frac{V(I)}{3M_{\rm Pl}}|\phi|^2$$

$$V(\phi) = m_\phi^2|\phi|^2 + \underline{c_H H^2|\phi|^2} \text{ + (higher dimensional terms)}$$



Inflation and Hubble-induced terms

After inflation ends, the energy density of the Universe is dominated by that of inflaton oscillation, which again induces the following potentials:

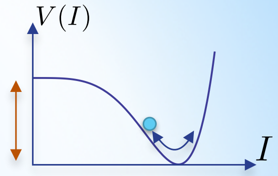
$$c_H \frac{2\dot{I}^2}{3M_{\rm Pl}}|\phi|^2$$

$$V(\phi) = m_\phi^2|\phi|^2 + \underline{c_H H^2|\phi|^2} \text{ + (higher dimensional terms)}$$

In general, c_H (during inflation) $\neq c_H$ (after inflation)

When $c_H > 0$ during inflation and $c_H < 0$ after inflation, global cosmic strings form after inflation

Inflation and Hubble-induced terms

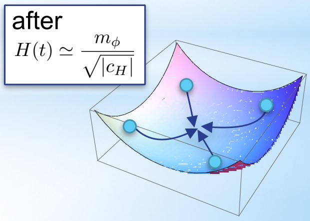


During inflaton oscillation era, the Hubble parameter decreases with time as

$$H^2(t) = \frac{\rho_I(t)}{3M_{\rm Pl}^2} \propto a^{-3}$$

Cosmic strings disappear at the time of $H(t) \simeq m_\phi$.

$$V(\phi) = m_\phi^2|\phi|^2 + \cancel{c_H H^2|\phi|^2}$$ + (higher dimensional terms)



after
$H(t) \simeq \frac{m_\phi}{\sqrt{|c_H|}}$

Properties of cosmic strings

Kamada and M.Y., 14

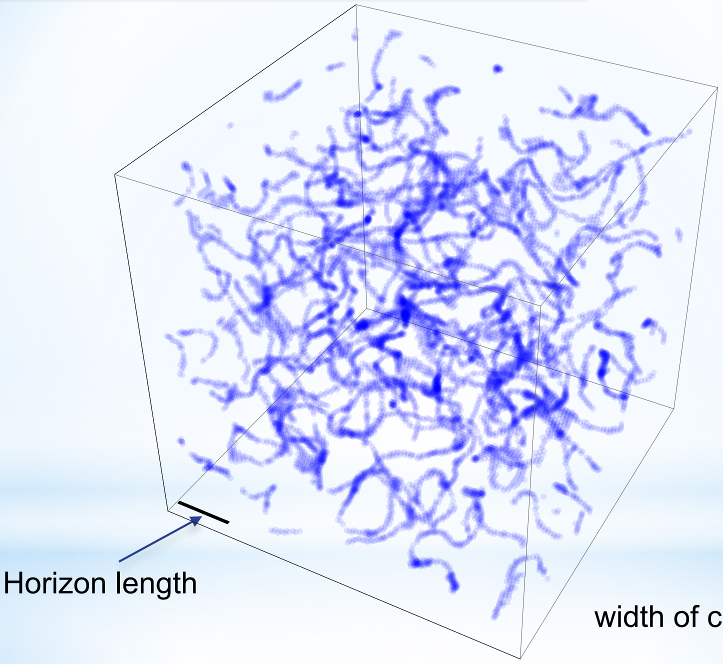
$$V(\phi) = m_\phi^2|\phi|^2 + c_H H^2|\phi|^2$$ + (higher dimensional terms)

$$\frac{\lambda^2}{M_{\rm Pl}^{2n-6}}|\phi|^{2(n-1)}$$

$$c_H > 0 \;\Rightarrow\; c_H < 0$$

- the number of cosmic strings in the Hubble volume $= \mathcal{O}(1)$ (scaling law)
- width of a typical cosmic string $\sim 1/\sqrt{V''} \propto H^{-1}$

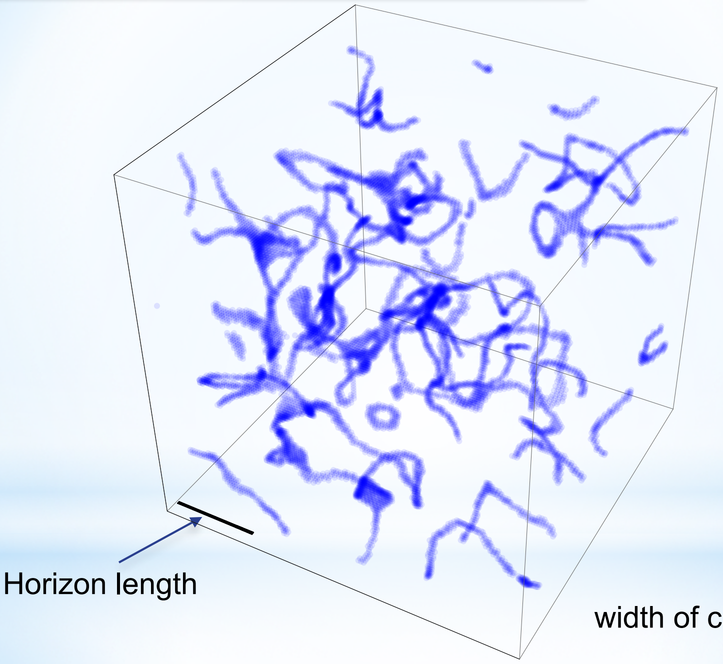
3+1 dim simulation of cosmic string formation



conformal time
τ = 32 H_i^{-1}

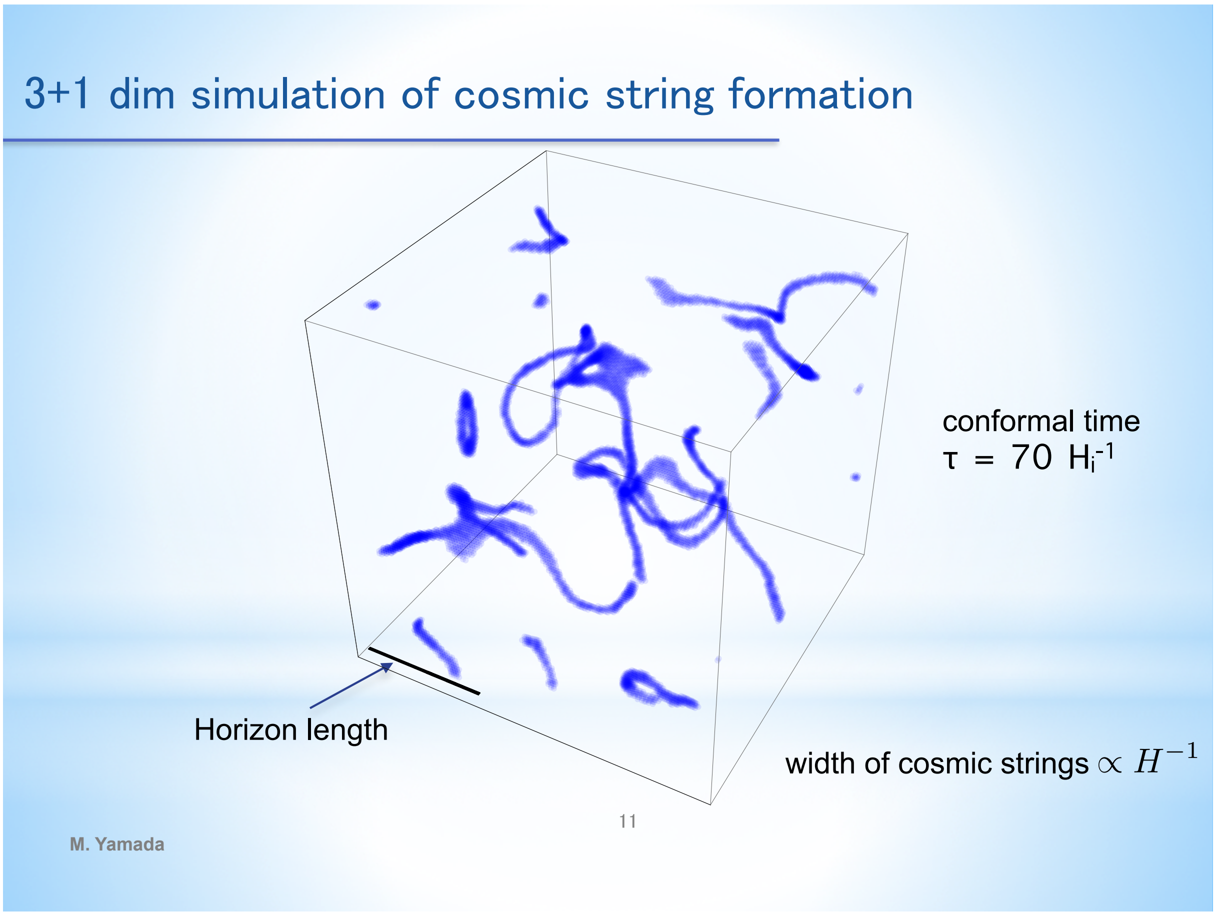
width of cosmic strings $\propto H^{-1}$

3+1 dim simulation of cosmic string formation



conformal time
τ = 48 H_i^{-1}

width of cosmic strings $\propto H^{-1}$

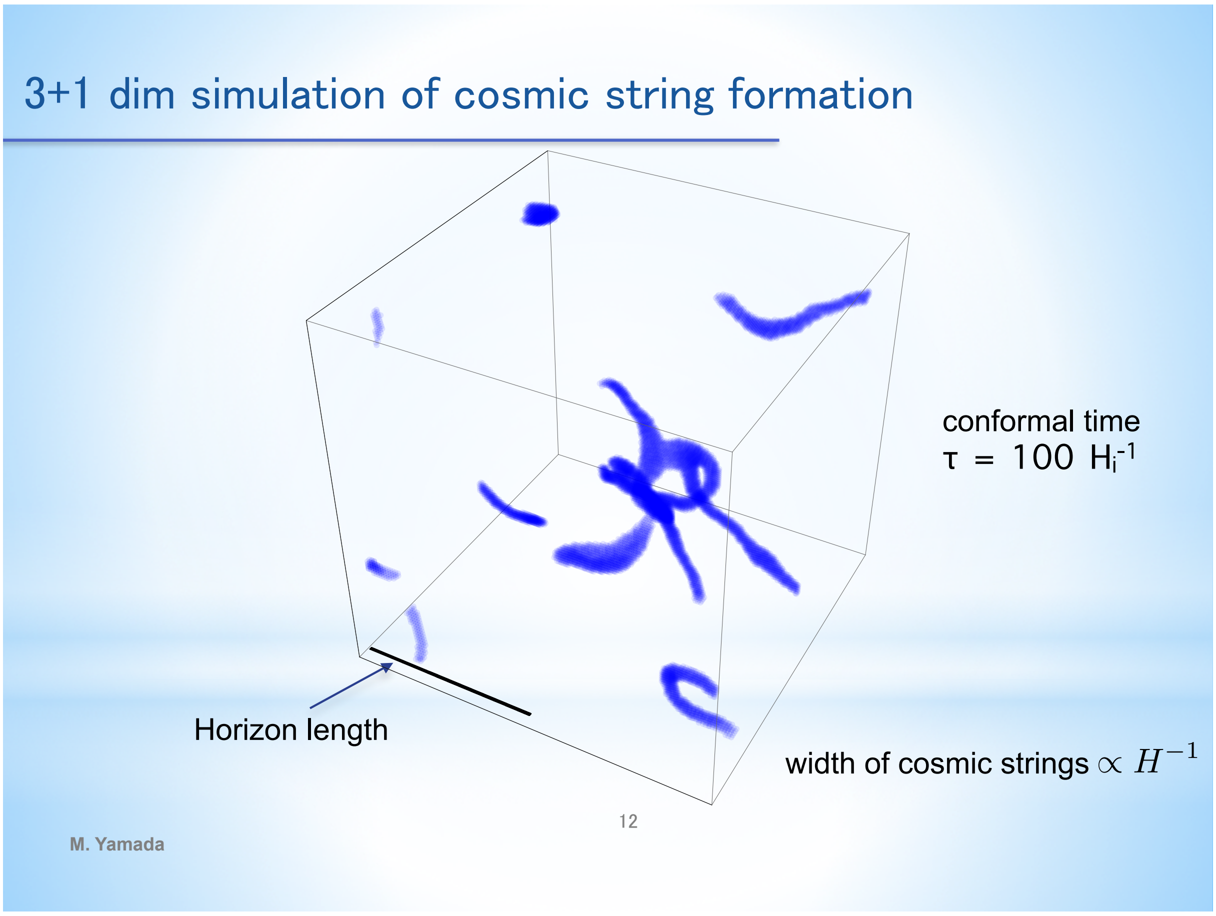


3+1 dim simulation of cosmic string formation

conformal time
τ = 70 H_i^{-1}

Horizon length

width of cosmic strings $\propto H^{-1}$

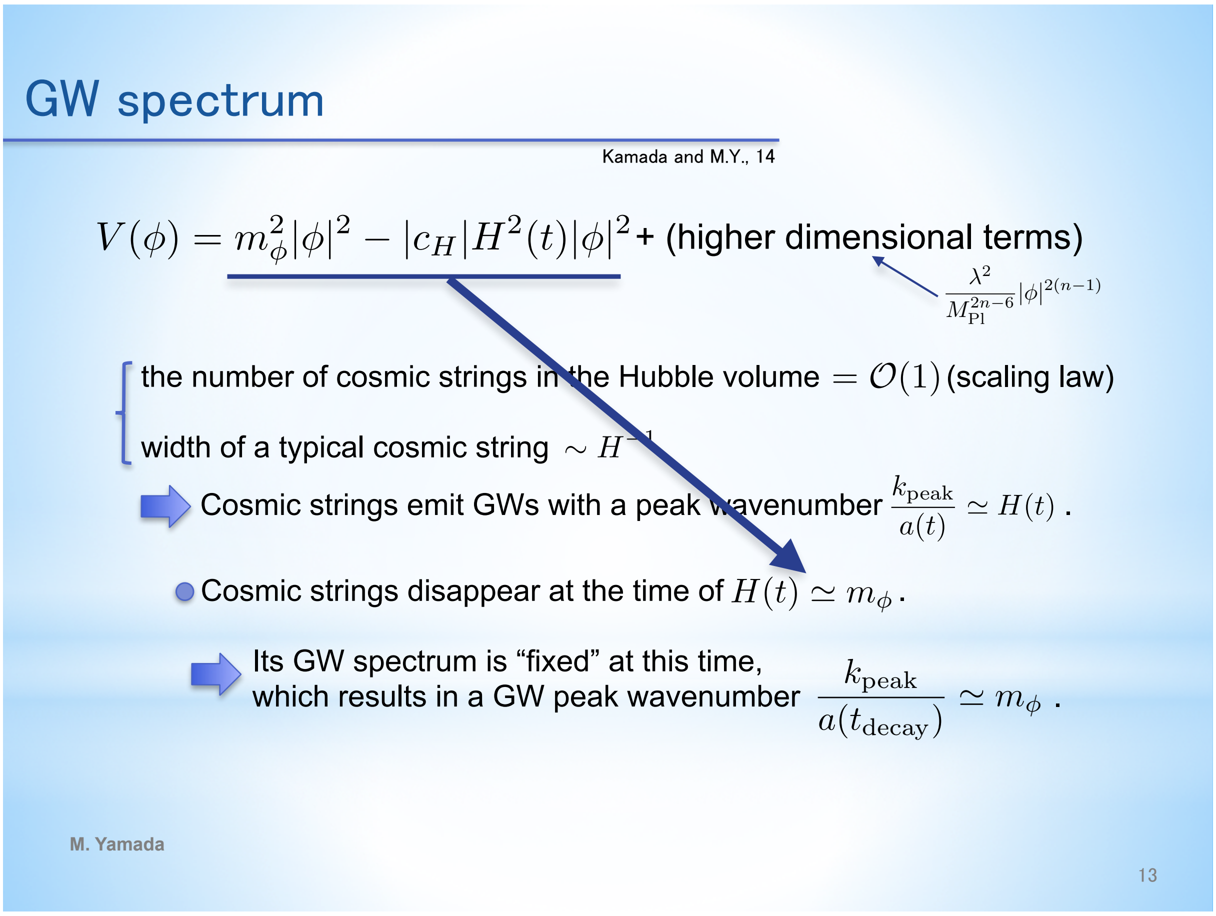


3+1 dim simulation of cosmic string formation

conformal time
τ = 100 H_i^{-1}

Horizon length

width of cosmic strings $\propto H^{-1}$



GW spectrum

Kamada and M.Y., 14

$$V(\phi) = \underline{m_\phi^2|\phi|^2 - |c_H|H^2(t)|\phi|^2} + \text{(higher dimensional terms)}$$

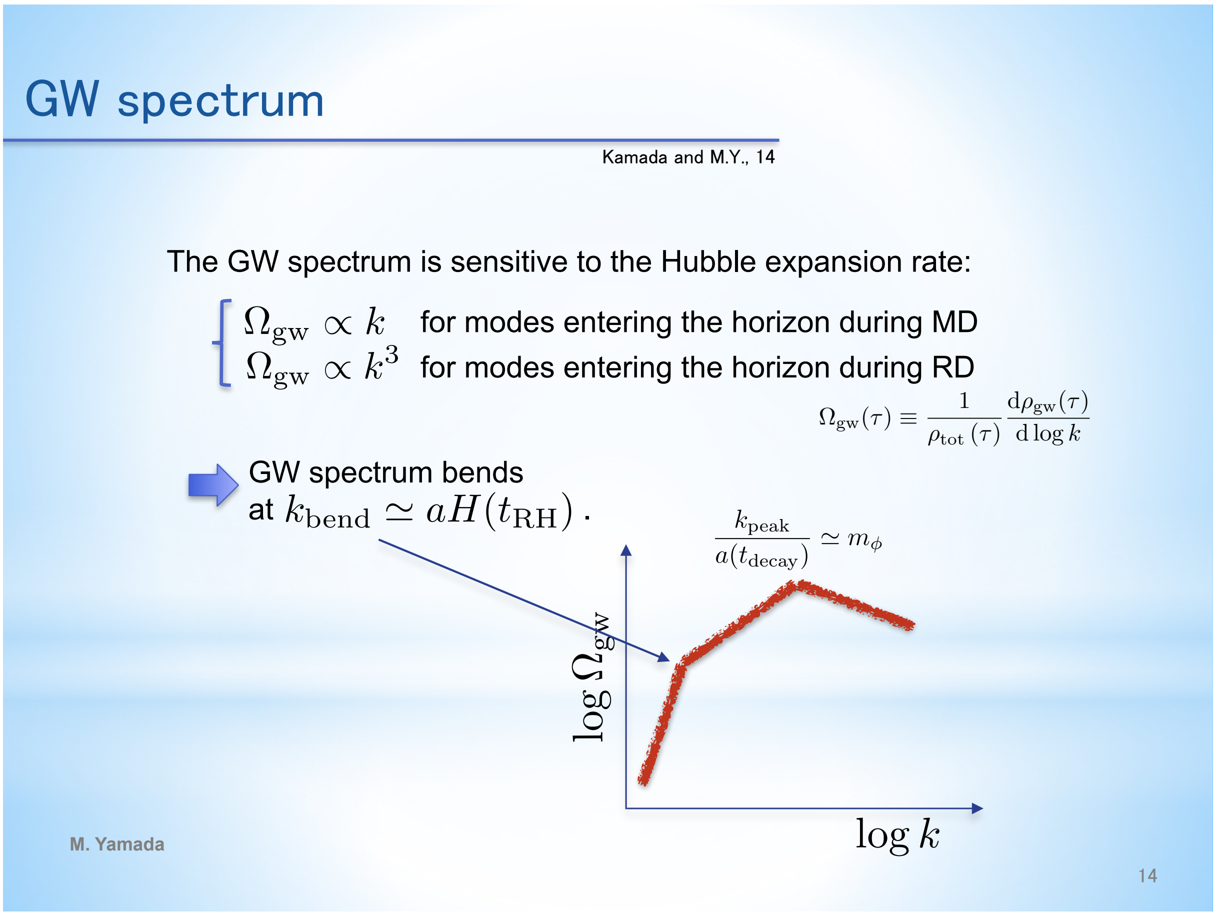
$$\frac{\lambda^2}{M_{\rm Pl}^{2n-6}}|\phi|^{2(n-1)}$$

$\left\{ \begin{array}{l} \text{the number of cosmic strings in the Hubble volume } = \mathcal{O}(1) \text{ (scaling law)} \\ \text{width of a typical cosmic string } \sim H^{-1} \end{array} \right.$

⇒ Cosmic strings emit GWs with a peak wavenumber $\frac{k_{\rm peak}}{a(t)} \simeq H(t)$.

- Cosmic strings disappear at the time of $H(t) \simeq m_\phi$.

⇒ Its GW spectrum is "fixed" at this time, which results in a GW peak wavenumber $\frac{k_{\rm peak}}{a(t_{\rm decay})} \simeq m_\phi$.



GW spectrum

Kamada and M.Y., 14

The GW spectrum is sensitive to the Hubble expansion rate:

$\left\{ \begin{array}{ll} \Omega_{\rm gw} \propto k & \text{for modes entering the horizon during MD} \\ \Omega_{\rm gw} \propto k^3 & \text{for modes entering the horizon during RD} \end{array} \right.$

$$\Omega_{\rm gw}(\tau) \equiv \frac{1}{\rho_{\rm tot}(\tau)} \frac{{\rm d}\rho_{\rm gw}(\tau)}{{\rm d}\log k}$$

⇒ GW spectrum bends at $k_{\rm bend} \simeq aH(t_{\rm RH})$.

GW spectrum

Kamada and M.Y., 14

present peak frequency:

$$f_0 \simeq \left(\frac{g_s(t_0)}{g_s(t_{\rm RH})}\right)^{1/3}\left(\frac{T_0}{T_{\rm RH}}\right)\left(\frac{H_{\rm RH}}{H_{\rm decay}}\right)^{2/3}\frac{k_{\rm peak}}{2\pi a(t_{\rm decay})}$$

$$\sim 10^3\ {\rm Hz}\left(\frac{m_\phi}{10^3\ {\rm GeV}}\right)^{1/3}\left(\frac{T_{\rm RH}}{10^9\ {\rm GeV}}\right)^{1/3}$$

$H_{\rm decay} \simeq m_\phi$

present bend frequency:

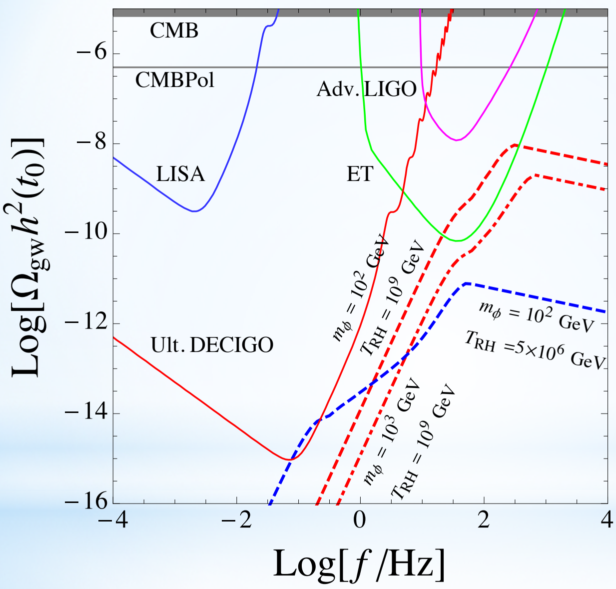
$$f_{\rm bend} = \left(\frac{g_s(t_0)}{g_s(t_{\rm RH})}\right)^{1/3}\left(\frac{T_0}{T_{\rm RH}}\right)\frac{k_{\rm bend}}{2\pi a(t_{\rm RH})}$$

$$\simeq 30\ {\rm Hz}\left(\frac{T_{\rm RH}}{10^9\ {\rm GeV}}\right)$$

We can probe m_ϕ and $T_{\rm RH}$ through GW detection experiments!

GW spectrum

Kamada and M.Y., 14



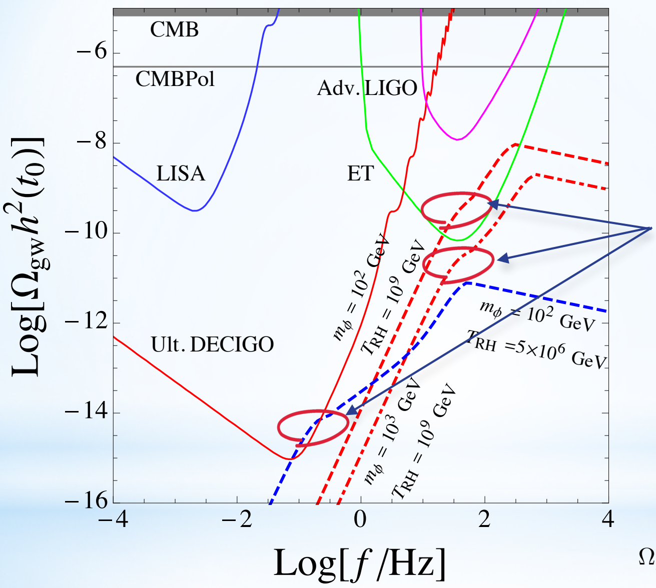
$$\Omega_{\rm gw}(\tau) \equiv \frac{1}{\rho_{\rm tot}(\tau)}\frac{{\rm d}\rho_{\rm gw}(\tau)}{{\rm d}\log k} \sim \left(\frac{\langle\phi\rangle}{M_{\rm Pl}}\right)^4$$

$$\left(\frac{\langle\phi\rangle}{M_{\rm Pl}}\right)^2_{\rm decay} = 0.1$$

$$\Leftrightarrow\ \lambda \sim 10^{-12}$$

GW spectrum

Kamada and M.Y., 14



GW spectrum bends at $k \simeq aH(t_{RH})$

$$\Omega_{gw}(\tau) \equiv \frac{1}{\rho_{tot}(\tau)}\frac{d\rho_{gw}(\tau)}{d\log k} \sim \left(\frac{\langle\phi\rangle}{M_{Pl}}\right)^4$$

$$\left(\frac{\langle\phi\rangle}{M_{Pl}}\right)^2_{decay} = 0.1$$

$$\Leftrightarrow \lambda \sim 10^{-12}$$

Summary

Kamada and M.Y., 14

- We have investigated the dynamics of a flat direction,
  which usually exists in supersymmetric theories,
  and have shown that cosmic strings generally form after inflation.

- These cosmic strings disappear at the time of $H(t) \simeq m_\phi$.

- We can obtain
  the soft mass of the flat direction m_ϕ
  and the reheating temperature of the Universe T_{RH}
  through detection of GWs emitted from these cosmic strings.

“Investigating tensor perturbations on small scales from their second-order effects to generate scalar perturbations”

Tomohiro Nakama

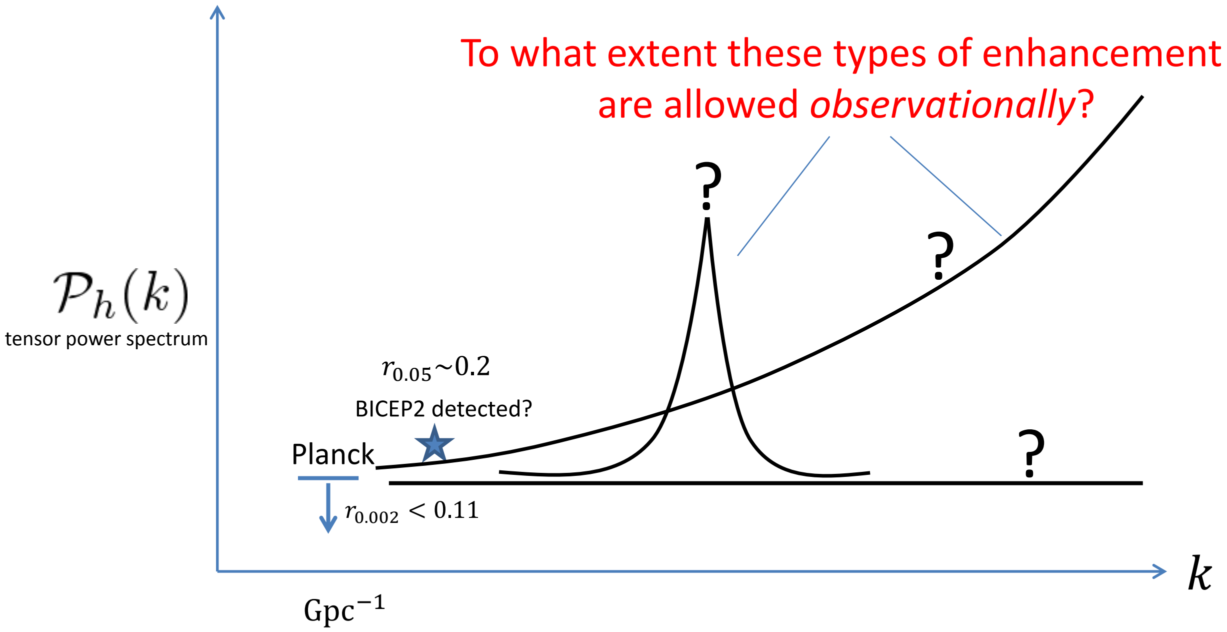
[JGRG24(2014)111307]

Investigating tensor perturbations on small scales from their second-order effects to generate scalar perturbations

Tomohiro Nakama
RESCEU
(JSPS Research Fellow)

in collaboration with
Teruaki Suyama & Jun’ichi Yokoyama

Motivation:
Investigating primordial tensor perturbations on small scales



To what extent these types of enhancement are allowed *observationally*?

Related works: Ota et al. (2014),
Chluba et al. (2014)

Induced scalar perturbations

- Assumption: On small scales, initially (on super-horizon scales),

  tensor pert. >> scalar pert. $h_{ij} \gg \delta_r, \dots$

  $\Downarrow$
- Then scalar perturbations are generated due to the second order effects of tensor pert. $\delta_r, \dots \sim O(h_{ij}{}^2)$

Induced scalar perturbations

- Assumption: On small scales, initially (on super-horizon scales),

  tensor pert. >> scalar pert. $h_{ij} \gg \delta_r, \dots$

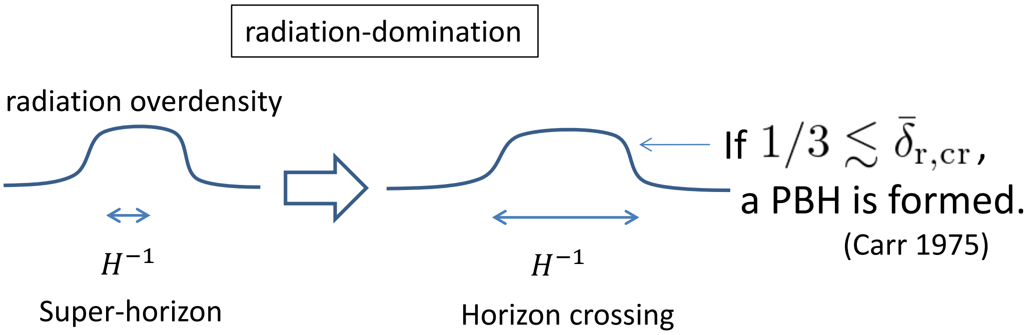
  $\Downarrow$
- Then scalar perturbations are generated due to the second order effects of tensor pert. $\delta_r, \dots \sim O(h_{ij}{}^2)$

  $\Downarrow$
- If tensor pert. is sufficiently large, induced scalar pert. becomes large so that PBHs are overproduced. $\delta_r \sim O(1)$

  →PBH formation
- We can place upper bounds on tensor pert. requiring PBHs are not overproduced.

cf. Saito Yokoyama 2009

Primordial Black Hole (PBH)



radiation-domination

radiation overdensity

H^{-1}

Super-horizon

H^{-1}

Horizon crossing

If $1/3 \lesssim \bar{\delta}_{\mathrm{r,cr}}$,

a PBH is formed.

(Carr 1975)

Various observations have placed upper bounds on the abundance of PBHs on various mass scales.

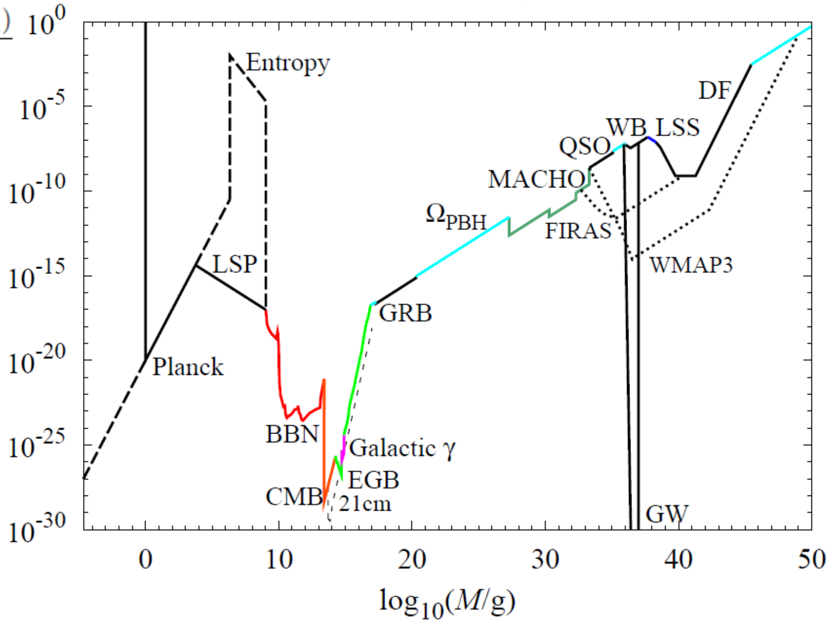
Observational constraints on PBHs of various masses

abundance of PBHs
when they were formed

(Carr, Kohri, Sendouda, Yokoyama, 2010)

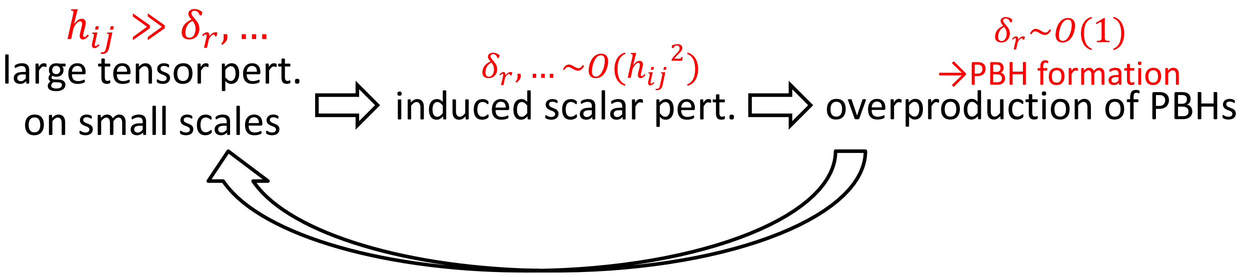
$\sim \beta(M) \equiv \frac{\rho_{\mathrm{PBH}}(t_{\mathrm{i}})}{\rho(t_{\mathrm{i}})}$

abundance



mass

summary of methods to probe tensor fluctuations on small scales



$h_{ij} \gg \delta_r, \ldots$

large tensor pert. on small scales ⇨ $\delta_r, \ldots \sim O({h_{ij}}^2)$ induced scalar pert. ⇨ $\delta_r \sim O(1)$ →PBH formation overproduction of PBHs

obtain upper bounds on initial tensor pert.
to avoid overproduction of PBHs.

Formulation

Metric

$$ds^2 = a^2[-(1+2\Phi)d\eta^2 - 2B_{,i}d\eta dx^i + ((1-2\Psi)\delta_{ij} - 2h_{ij})dx^i dx^j]$$

η: conformal time

The Einstein equations at $O({h_{ij}}^2)$

$$\Delta\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Phi) - \mathcal{H}\Delta B + \overset{\Downarrow}{S_1} = 4\pi G a^2 \delta\rho$$

$$(\Psi' + \mathcal{H}\Phi + \overset{\Downarrow}{S_2})_{,i} = 0$$

$$\Psi'' + \mathcal{H}(2\Psi + \Phi)' + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \frac{1}{2}\Delta(\Phi - \Psi + B' + 2\mathcal{H}B) + \overset{\Downarrow}{S_3} + \overset{\Downarrow}{S_4} = 4\pi G a^2 \delta p$$

$$(\Phi - \Psi + B' + 2\mathcal{H}B - 2\overset{\Downarrow}{S_5})_{,ij} = 0$$

The conservation of energy-momentum tensor

$$\delta\rho' + 3\mathcal{H}(\delta\rho + \delta p) - (\rho + p)\Delta B - 3(\rho + p)\Psi' - 2(\rho + p)h^{ij}h'_{ij} = 0$$

$$\partial_i(\delta p + (\rho + p)\Phi) = 0$$

Source terms

$$S_1 \equiv -\frac{1}{4}h'_{ij}h^{ij\prime} - 2\mathcal{H}h_{ij}h^{ij\prime} + h_{ij}\Delta h^{ij} - \frac{1}{2}\partial_j h_{ik}\partial^k h^{ij} + \frac{3}{4}\partial_k h_{ij}\partial^k h^{ij},$$

$$\Delta S_2 = \partial^i S_i,$$

$$S_i = -h^{jk}\partial_k h'_{ij} + \frac{1}{2}h^{jk\prime}\partial_i h_{jk} + h^{jk}\partial_i h'_{jk}$$

$$S_3 \equiv \frac{3}{4}h'_{ij}h^{ij\prime} + h_{ij}h^{ij\prime\prime} + 2\mathcal{H}h_{ij}h^{ij\prime} - h_{ij}\Delta h^{ij} + \frac{1}{2}\partial_j h_{ik}\partial^k h^{ij} - \frac{3}{4}\partial_k h_{ij}\partial^k h^{ij}$$

$$\Delta S_4 = \frac{1}{2}(\Delta S^i{}_i - \partial^i\partial^j S_{ij}),$$

$$\Delta^2 S_5 = \frac{1}{2}(3\partial^i\partial^j S_{ij} - \Delta S^i_i),$$

$$\begin{aligned} S_{ij} \equiv {} & -h_i{}^{k\prime}h'_{jk} - h_{ik}h_j{}^{k\prime\prime} - 2\mathcal{H}h_i{}^k h'_{jk} + h^{kl}\partial_k\partial_l h_{ij} + h_i{}^k\Delta h_{jk} - h^{kl}\partial_l\partial_i h_{jk} - h^{kl}\partial_l\partial_j h_{ik} \\ & -\partial_k h_{jl}\partial^l h_i{}^k + \partial_l h_{jk}\partial^l h_i{}^k + \frac{1}{2}\partial_i h_{kl}\partial_j h^{kl} + h^{kl}\partial_i\partial_j h_{kl}. \end{aligned}$$

a bit complicated…

Let us focus on one of the eqs.

scalar pert.

$$\Delta\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Phi) - \mathcal{H}\Delta B + \boxed{S_1} = 4\pi G a^2 \delta\rho_r$$

source ~$O({h_{ij}}^2)$

$$\boxed{S_1} \equiv -\frac{1}{4}h'_{ij}h^{ij\prime} - 2\mathcal{H}h_{ij}h^{ij\prime} + h_{ij}\Delta h^{ij} - \frac{1}{2}\partial_j h_{ik}\partial^k h^{ij} + \frac{3}{4}\partial_k h_{ij}\partial^k h^{ij}$$

prime: $\frac{\partial}{\partial\eta}$

Scalar pert. are generated due to the source terms.

Specifying the initial condition

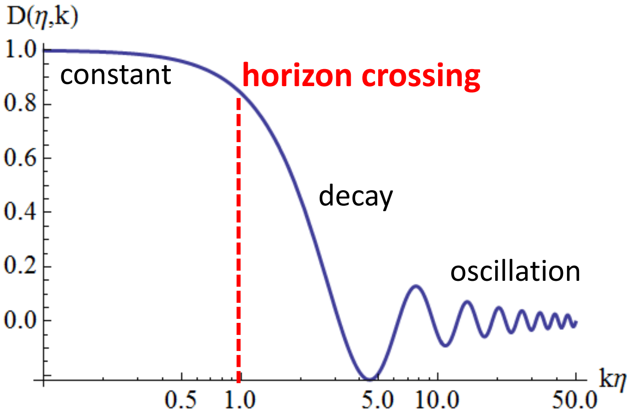
$$h_{ij}(\eta,\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\left(h^{+}(\eta,\boldsymbol{k})e^{+}_{ij}(\boldsymbol{k}) + h^{\times}(\eta,\boldsymbol{k})e^{\times}_{ij}(\boldsymbol{k})\right)$$

+ or ×

initial amplitude

$$h^r(\eta,\boldsymbol{k}) = D(\eta,k)h^r(\boldsymbol{k})$$

Growth factor: $\dfrac{\sin k\eta}{k\eta}$ $(\leftarrow h''_{ij} + 2\mathcal{H}h'_{ij} - \Delta h_{ij} = 0)$



Specifying the initial condition

$$h_{ij}(\eta,\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\left(h^{+}(\eta,\boldsymbol{k})e^{+}_{ij}(\boldsymbol{k}) + h^{\times}(\eta,\boldsymbol{k})e^{\times}_{ij}(\boldsymbol{k})\right)$$

+ or ×

initial amplitude

$$h^r(\eta,\boldsymbol{k}) = D(\eta,k)h^r(\boldsymbol{k})$$

- The definition of the initial power spectrum:

$$\langle h^r(\boldsymbol{k})h^s(\boldsymbol{K})\rangle = \frac{2\pi^2}{k^3}\delta(\boldsymbol{k}+\boldsymbol{K})\delta_{rs}\boxed{\mathcal{P}_h(k)}$$

- As an illustration, we consider a delta-function like power spectrum

$$\mathcal{P}_h(k) = \mathcal{A}^2 k\delta(k-k_p)$$

amplitude

position of spike

Calculation of the power spectrum of the density perturbation

$$\mathcal{P}_h(k) = \boxed{\mathcal{A}^2} k\delta(k-k_p)$$

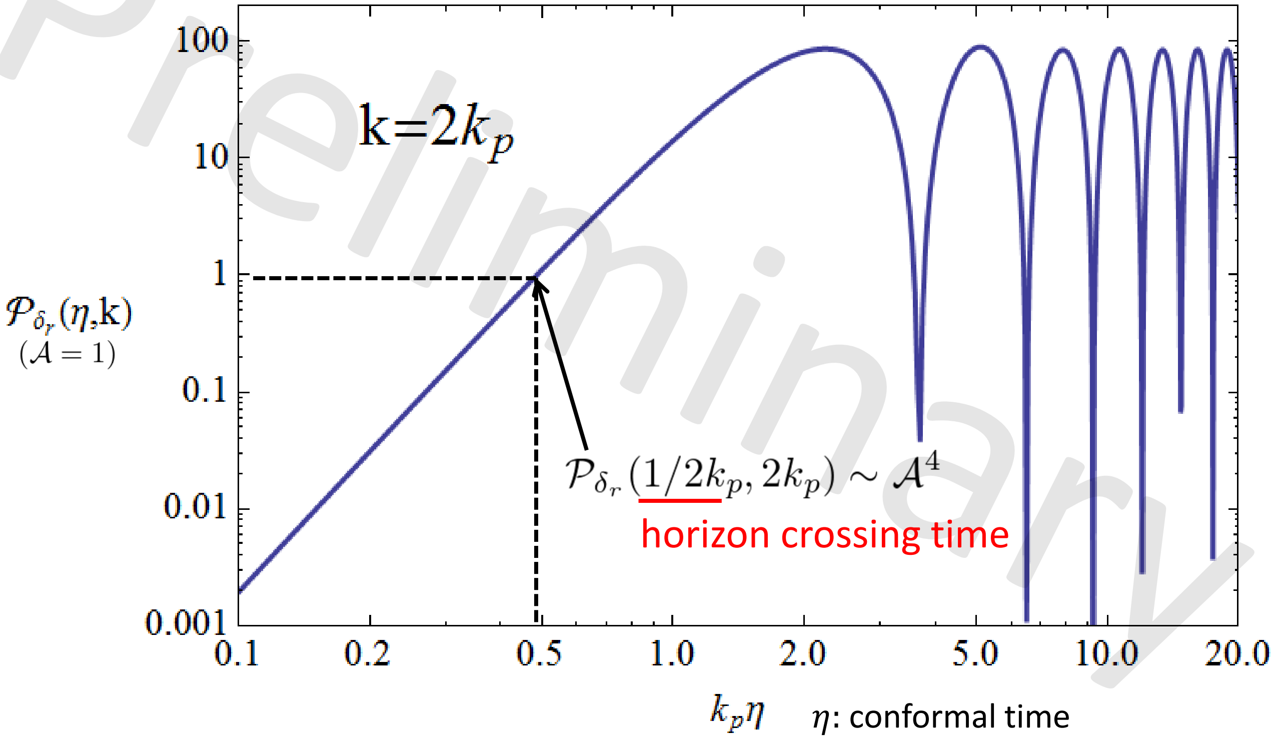
$$\mathcal{P}_{\delta_r}(\eta,k) = \left(\frac{1+c_s^2}{c_s^2}\right)^2 \boxed{\mathcal{A}^4} \left(\frac{k}{k_p}\right)^2 \eta^2 \Theta\left(1-\frac{k}{2k_p}\right) \sum_{rs} F_{rs}\left(\eta,k,k_p,\frac{k}{2k_p}\right)^2$$

⇧

This reflects $\delta_r \sim O({h_{ij}}^2)$

$$F_{rs}(\eta,\boldsymbol{k},\boldsymbol{k}') \equiv \int d\tilde{\eta}(\tilde{\eta}/\eta)A_{rs}(\tilde{\eta},\boldsymbol{k},\boldsymbol{k}')(\partial_\eta-\mathcal{H})g_k(\eta,\tilde{\eta})$$
$$+ D(\eta,k')\left\{-\partial_\eta E_1^{rs} + \left(\frac{1}{2}\overleftarrow{\partial}_\eta+\partial_\eta\right)\left(1-\frac{k'}{k}\mu\right)E_2^{rs}\right\}D(\eta,|\boldsymbol{k}-\boldsymbol{k}'|)$$

The time evolution of the power spectrum



$\mathrm{k}=2k_p$

$\mathcal{P}_{\delta_r}(\underline{1/2k_p}, 2k_p) \sim \mathcal{A}^4$

horizon crossing time

η: conformal time

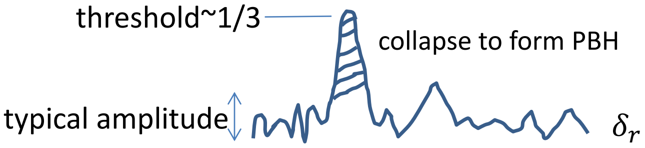
Upper bound on the amplitude of primordial tensor perturbations

PBH formation has to be sufficiently rare

to be consistent with observation

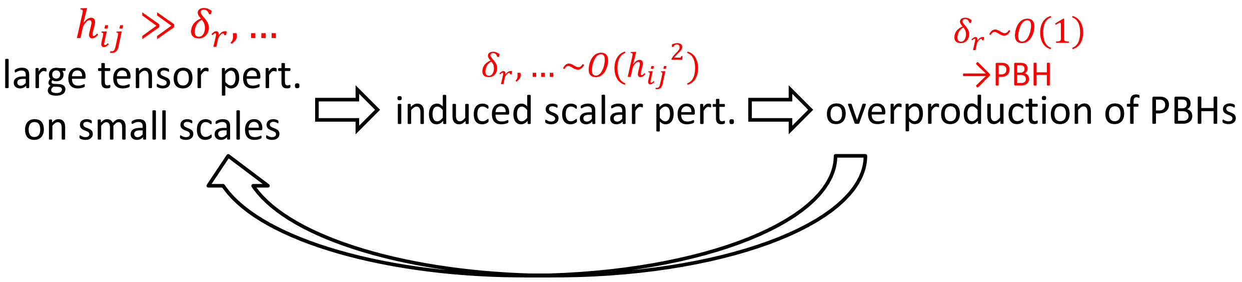
$$10 \lesssim \frac{\text{threshold for PBH formation}}{\text{typical amplitude at crossing}} \sim \frac{1/3}{\mathcal{A}^2}$$

$$\boxed{\rightarrow \mathcal{A}^2 \lesssim 0.03}$$



Summary

1. Method:



$h_{ij} \gg \delta_r, \ldots$

large tensor pert.
on small scales

$\delta_r, \ldots \sim O({h_{ij}}^2)$

induced scalar pert.

$\delta_r \sim O(1)$
→PBH

overproduction of PBHs

obtain upper bounds on initial tensor pert.
to avoid overproduction of PBHs.

2. Result:

$$\mathcal{P}_h(k) = \mathcal{A}^2 k \delta(k - k_p) \rightarrow \boxed{\mathcal{A}^2 \lesssim 0.03}$$

3. Future work:

Other shapes of power spectrum,
upper bounds from ultracompact minihalos,

BBN bound

$$\Omega_{\rm GW}(k) = \frac{1}{6}\mathcal{P}_h(k)$$

Maggiore 2007

$$\mathcal{P}_h(k) = \mathcal{A}^2 k\delta(k-k_p)$$

If GWs give the only extra contribution to N_ν, compared to N_ν=3,

$$\int d(\ln f)\,\Omega_{\rm gw\ at\ nucleosynthesis} \le \frac{\frac{7}{8}(N_\nu-3)}{1+3\times\frac{7}{8}+2\times\frac{7}{8}}\left(\frac{\rho_{\rm rad}}{\rho_{\rm crit}}\right)_{\rm at\ nucleosynthesis}$$

Allen 1996, Maggiore 2000

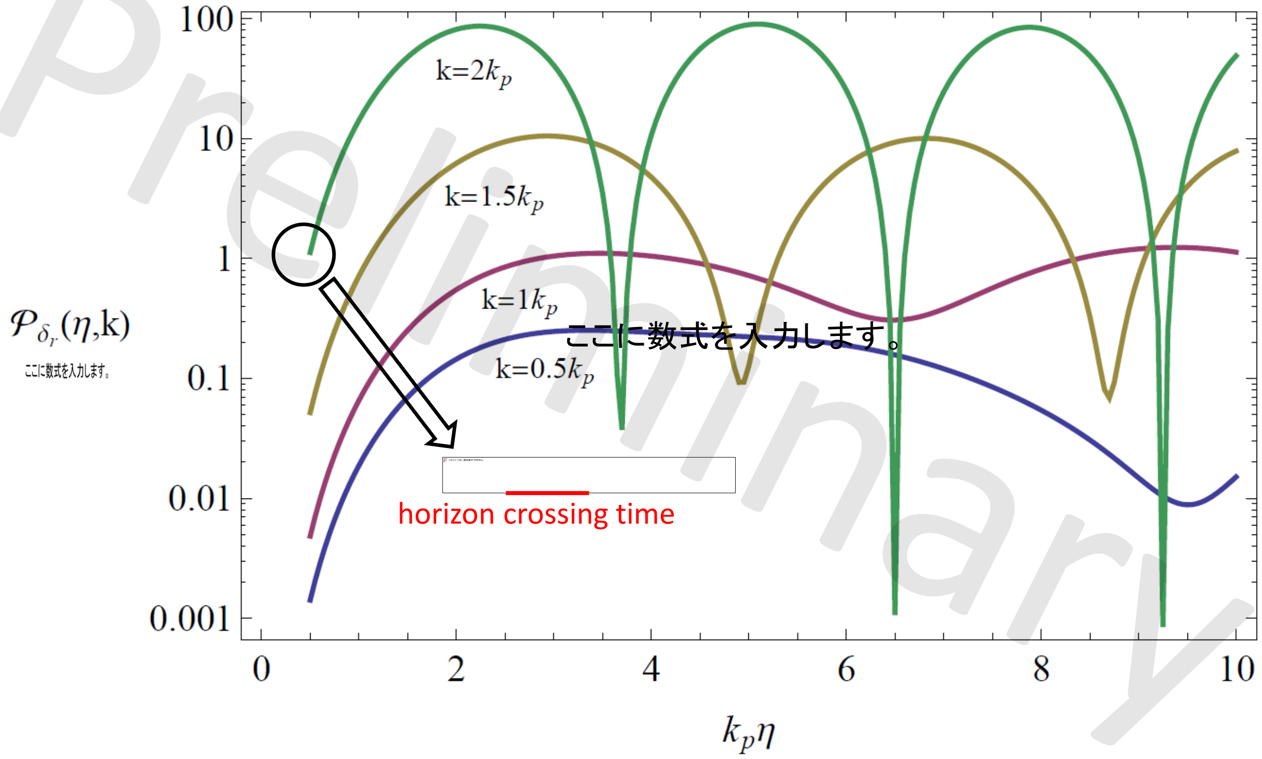
$$\frac{1}{6}\mathcal{A}^2 \quad \le \quad \frac{7}{43}(\underline{3.71+0.47*2}-3)$$

$$\mathrm{N}_{eff} = 3.71^{+0.47}_{-0.45}$$

Steigman 2012

$$\rightarrow \mathcal{A}^2 < 1.6$$

The time evolution of the power spectrum



horizon crossing time

- Combining these equations yields
  the evolution equation for Ψ:

$$\Psi'' + 2\mathcal{H}\Psi' + c_s^2 k^2 \Psi = S,$$

$$S \equiv c_s^2 S_1 - S_3 - \hat{k}^i \hat{k}^j S_{ij} + 2c_s^2 \mathcal{H} h^{ij} h'_{ij}$$

- This can be formally solved as

$$\Psi(\eta, \boldsymbol{k}) = a^{-1}(\eta) \int d\tilde{\eta} \underset{\uparrow}{g_k}(\eta, \tilde{\eta}) a(\tilde{\eta}) S(\tilde{\eta}, \boldsymbol{k})$$

Green’s function

$$g_k'' + \left(c_s^2 k^2 - \frac{a''}{a} \right) g_k = \delta(\eta - \tilde{\eta})$$

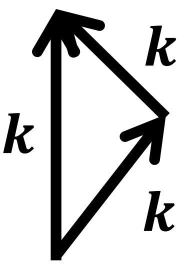
- The energy density perturbation is given by

$$\delta_r = \frac{1 + c_s^2}{c_2^2 \mathcal{H}} (\Psi' + S_2)$$

$$S(\eta,\boldsymbol{k}) = \sum_{rs}\int \frac{d^3\boldsymbol{k}'}{(2\pi)^{3/2}} h^r(\boldsymbol{k}')h^s(\boldsymbol{k}-\boldsymbol{k}')A_{rs}(\eta,\boldsymbol{k},\boldsymbol{k}'),$$

$$A_{rs}(\eta,\boldsymbol{k},\boldsymbol{k}') \equiv f_1(\eta,\boldsymbol{k},\boldsymbol{k}')E_1^{rs} + f_2(\eta,\boldsymbol{k},\boldsymbol{k}')E_2^{rs},$$

$$S \quad \to \quad \left\{\overleftarrow{\partial_\eta}\partial_\eta - \frac{1}{2}(3-c_s^2)k^2 + 3kk'\mu - k'^2\right\}E_1^{rs} +$$
$$\left\{-\frac{1}{4}(3+c_s^2)\overleftarrow{\partial_\eta}\partial_\eta + c_s^2\partial_\eta^2 + 2c_s^2\mathcal{H}\partial_\eta + \frac{1}{8}(1-3c_s^2)k^2 - \frac{1}{2}k'\mu(k-k'\mu) + \frac{3}{4}(1+c_s^2)k'^2\right\}E_2^{rs}.$$



$\boldsymbol{k}-\boldsymbol{k}'$ $(|\boldsymbol{k}-\boldsymbol{k}'| = k_p)$

$(k < 2k_p)$ $\boldsymbol{k}$

$\boldsymbol{k}'$ $(|\boldsymbol{k}'| = k_p)$

$\delta = 10^{-5}$ with mass $10^8 - 10^9 M_\odot$ $\xrightarrow{\text{grows only slowly during R.D.}}$ z_eq 10^{-2} $\xrightarrow{\text{grows in proportion to the scale factor}}$ z~15 ~ 1.68 (collapse)

$\delta = 10^{-3}$ at some small scale $\longrightarrow$ ~ 1.68 (collapse) z~1000

ultracompact minihalos

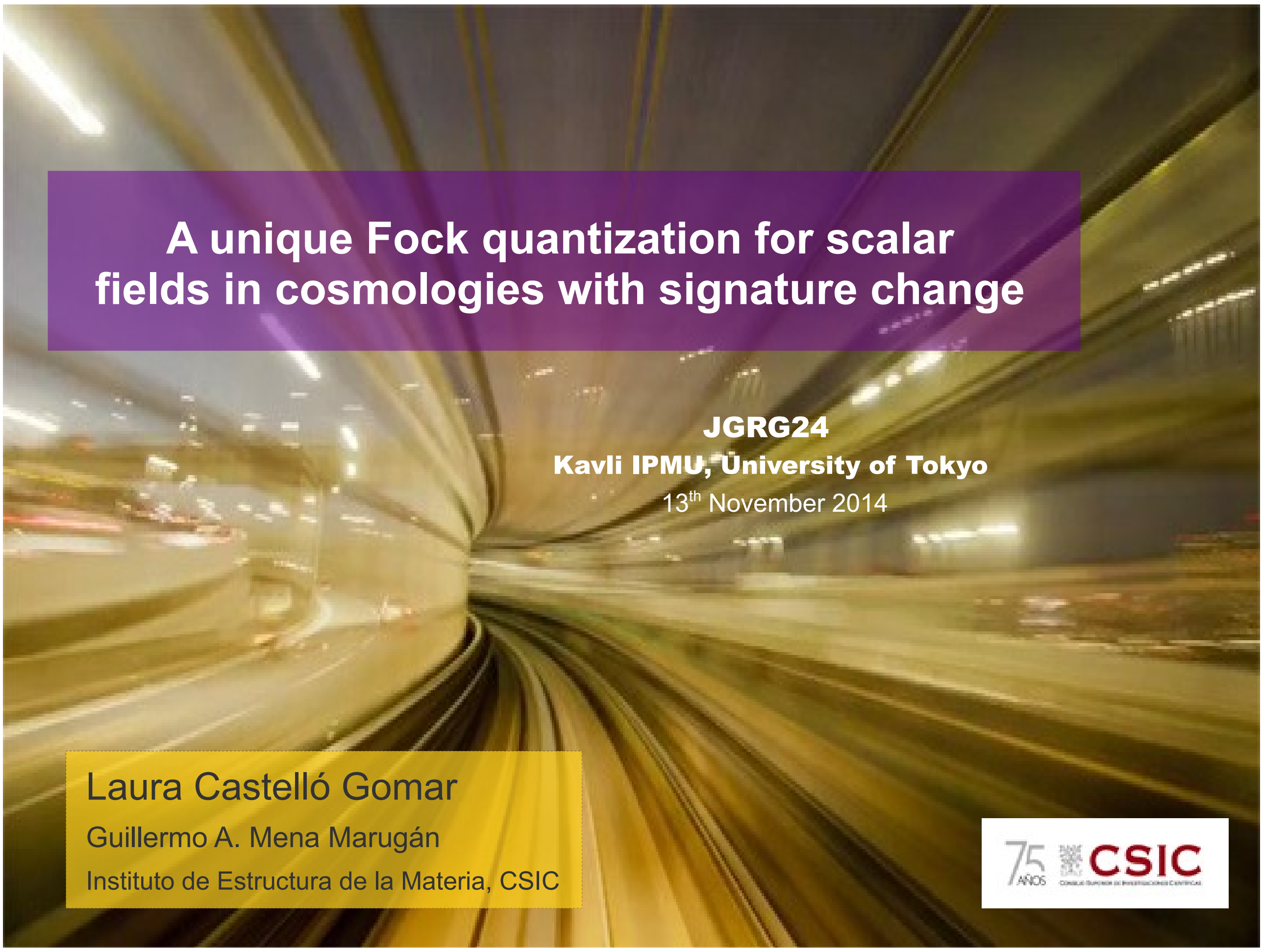
If the initial amplitude is larger,
the overdense region collapses earlier.

for more detailed estimation, see
Bringmann, Scott, Akrami 2012

“A unique Fock quantization for scalar fields in cosmologies with signature change”

Laura Castello Gomar

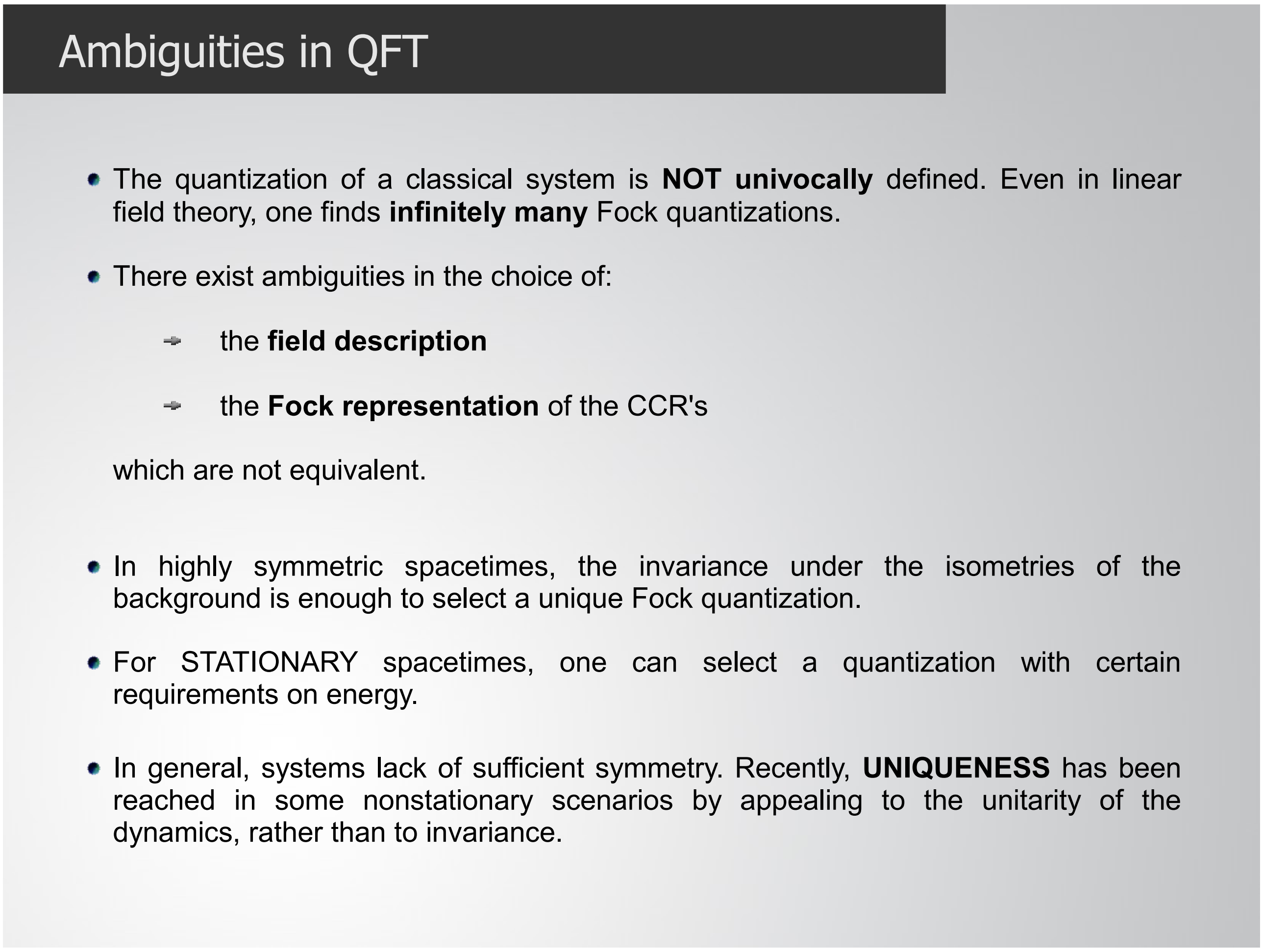
[JGRG24(2014)111308]



A unique Fock quantization for scalar fields in cosmologies with signature change

JGRG24
Kavli IPMU, University of Tokyo
13th November 2014

Laura Castelló Gomar
Guillermo A. Mena Marugán
Instituto de Estructura de la Materia, CSIC

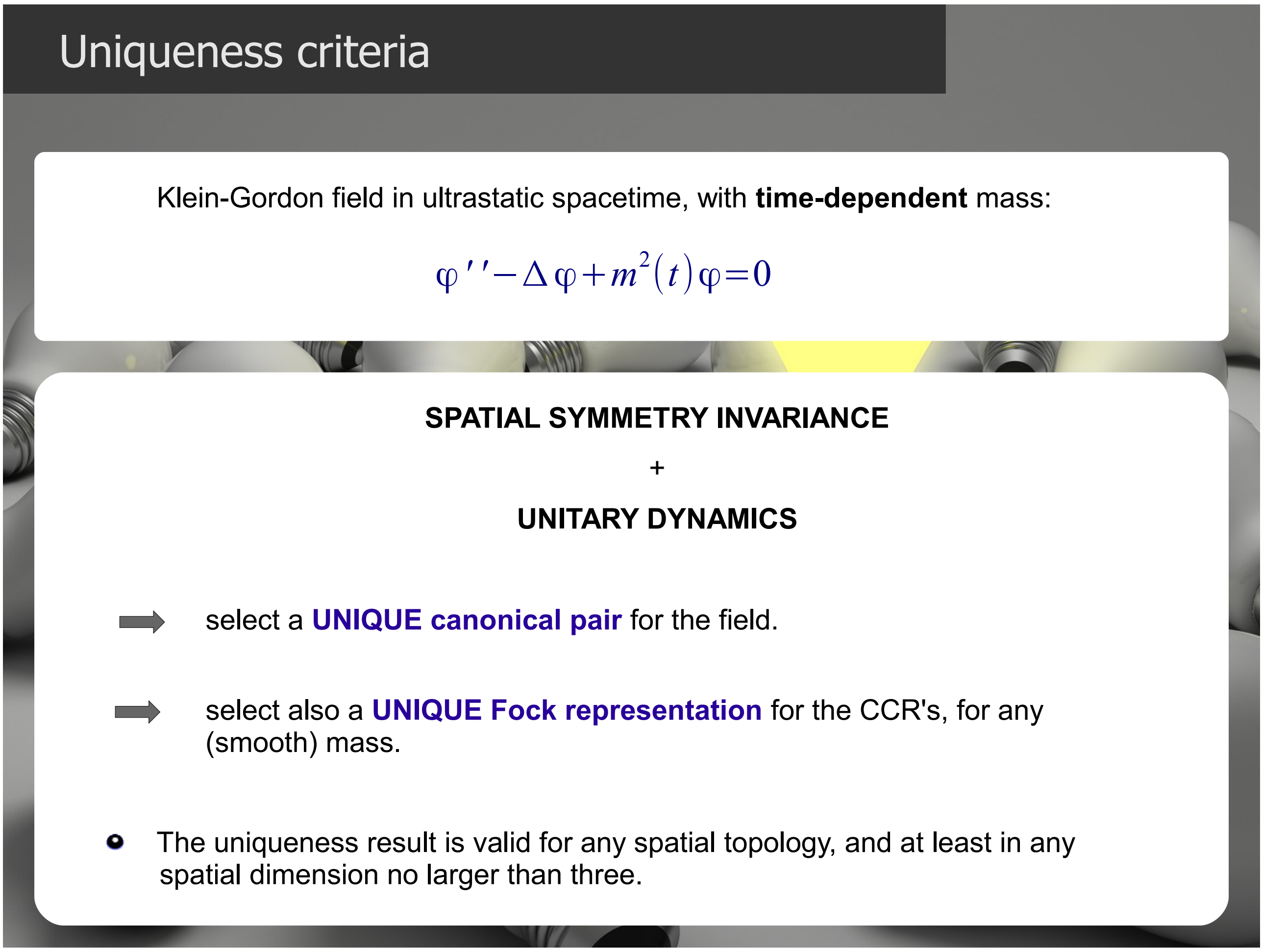


Ambiguities in QFT

- The quantization of a classical system is **NOT univocally** defined. Even in linear field theory, one finds **infinitely many** Fock quantizations.
- There exist ambiguities in the choice of:
  - the **field description**
  - the **Fock representation** of the CCR's

  which are not equivalent.
- In highly symmetric spacetimes, the invariance under the isometries of the background is enough to select a unique Fock quantization.
- For STATIONARY spacetimes, one can select a quantization with certain requirements on energy.
- In general, systems lack of sufficient symmetry. Recently, **UNIQUENESS** has been reached in some nonstationary scenarios by appealing to the unitarity of the dynamics, rather than to invariance.

Uniqueness criteria



Uniqueness criteria

Klein-Gordon field in ultrastatic spacetime, with **time-dependent** mass:

$$\varphi'' - \Delta\varphi + m^2(t)\varphi = 0$$

SPATIAL SYMMETRY INVARIANCE

\+

UNITARY DYNAMICS

➡ select a **UNIQUE canonical pair** for the field.

➡ select also a **UNIQUE Fock representation** for the CCR's, for any (smooth) mass.

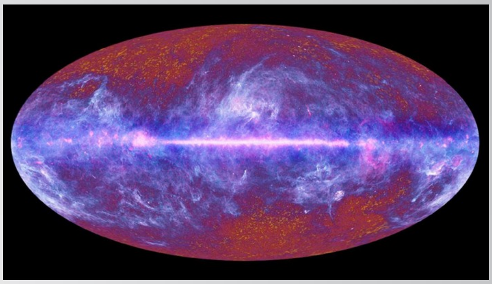
- The uniqueness result is valid for any spatial topology, and at least in any spatial dimension no larger than three.

Motivation: Fields with time dependent mass

$$\varphi'' - \Delta\varphi + m^2(t)\varphi = 0$$

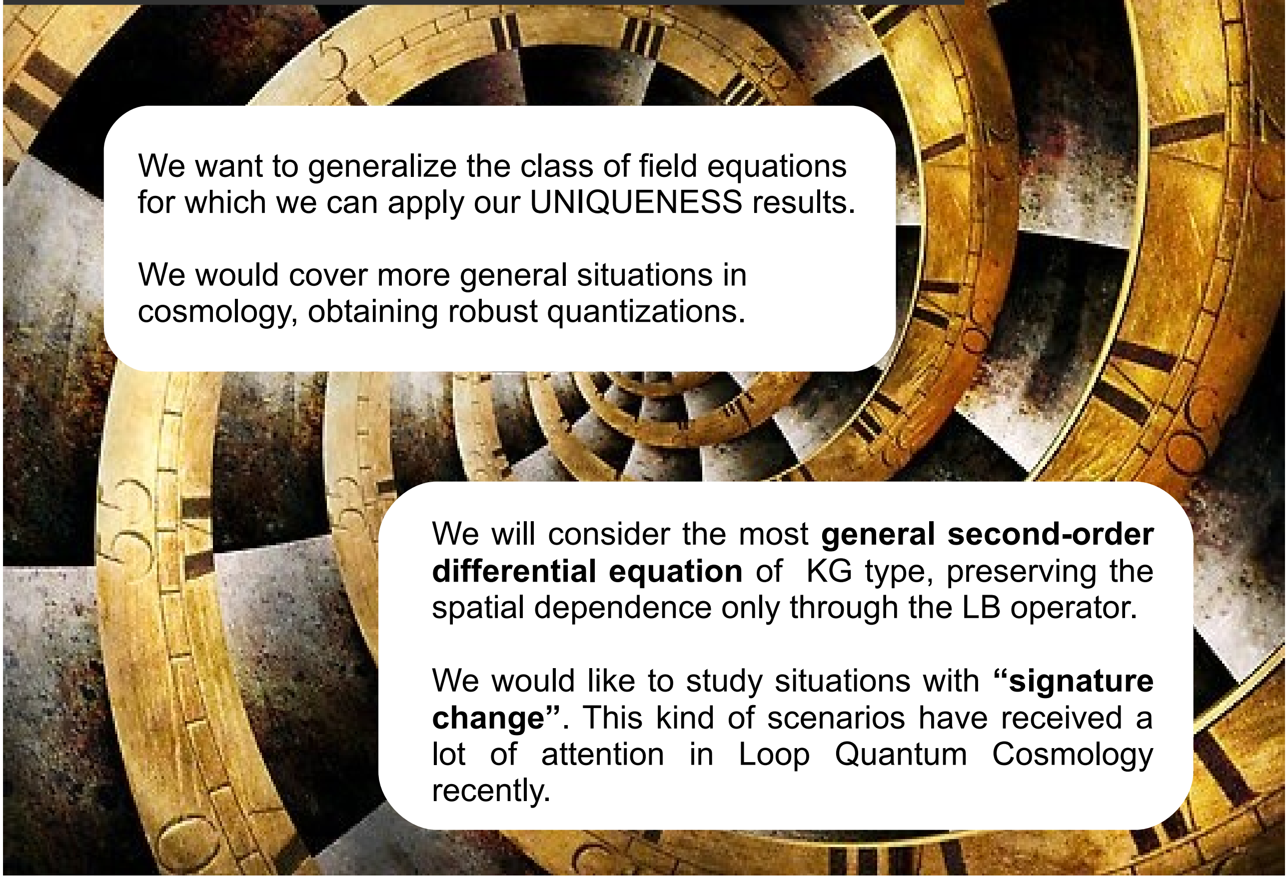
RESCALED FIELDS in **FLAT COSMOLOGIES**
(conformal time)

COSMOLOGICAL PERTURBATIONS



- SCALAR PERTURBATIONS:
  Mukhanov-Sasaki variables (gauge invariant).
- PERTURBATIONS of a MASSIVE FIELD in a suitable gauge: asymptotic behavior.
- TENSORIAL PERTURBATIONS (gravitational waves).

Motivation: Generalized field equations



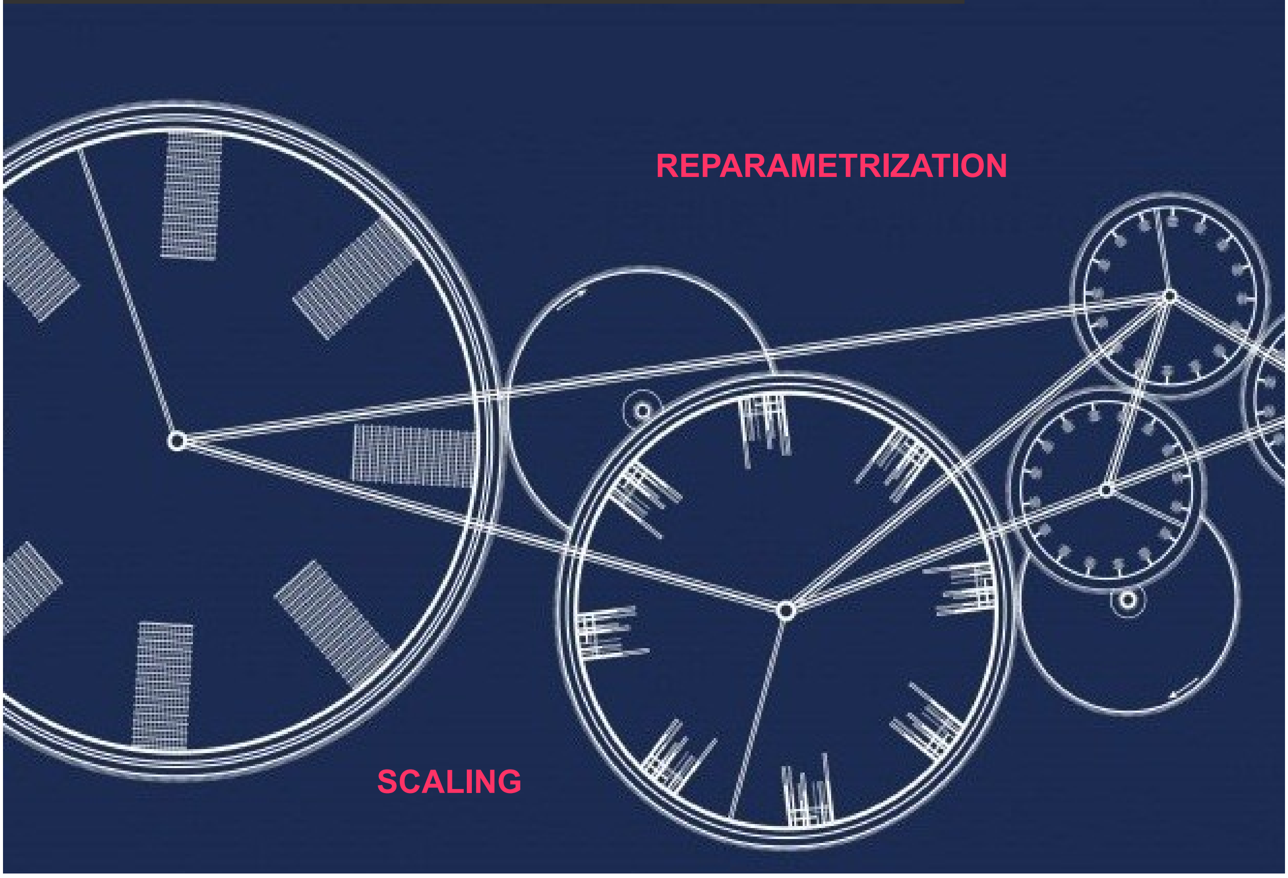
We want to generalize the class of field equations for which we can apply our UNIQUENESS results.

We would cover more general situations in cosmology, obtaining robust quantizations.

We will consider the most **general second-order differential equation** of KG type, preserving the spatial dependence only through the LB operator.

We would like to study situations with **“signature change”**. This kind of scenarios have received a lot of attention in Loop Quantum Cosmology recently.

Generalization of the field equations



Generalization of the field equations

$$\phi''+c(t)\phi'-d(t)\Delta\phi+\tilde{m}^2(t)\phi=0$$

$$\phi(t,\vec{x})=f(t)\varphi(t,\vec{x})$$

SCALING

$$dT=g(t)\,dt\,,\qquad g(t)\neq 0$$

REPARAMETRIZATION

$$\varphi''-\Delta\varphi+m^2(t)\varphi=0$$

Up to time reversal, there is a **bijective correspondance**:

$$f(t)=C\,d(t)^{-1/4}\exp\left[-\frac{1}{2}\int^t c(\bar{t})\,d\bar{t}\right]$$

$$g(t)=s\sqrt{d(t)},\qquad s=\pm$$

Generalization of the field equations

$$\phi''+c(t)\phi'-d(t)\Delta\phi+\tilde{m}^2(t)\phi=0$$

$$\phi(t,\vec{x})=f(t)\varphi(t,\vec{x})$$

SCALING

$$dT=g(t)dt, \qquad g(t)\neq 0$$

REPARAMETRIZATION

$$\varphi''-\Delta\varphi+m^2(t)\varphi=0$$

The new **mass**:

$$m^2(t)=\frac{\tilde{m}^2(t)}{d(t)}-\frac{d''(t)}{4\mathrm{d}^2(t)}+\frac{5(d'(t))^2}{16\ d^3(t)}-\frac{c'(t)}{2\ d(t)}-\frac{c^2(t)}{4\ d(t)}$$

Generalization of the field equations

$$f(t)=C\,d(t)^{-1/4}\exp\left[-\frac{1}{2}\int^t c(\bar{t})d\,\bar{t}\right]$$

SCALING

$$g(t)=s\sqrt{d(t)}, \qquad s=\pm$$

REPARAMETRIZATION

$$m^2(t)=\frac{\tilde{m}^2(t)}{d(t)}-\frac{d''(t)}{4\mathrm{d}^2(t)}+\frac{5(d'(t))^2}{16\ d^3(t)}-\frac{c'(t)}{2\ d(t)}-\frac{c^2(t)}{4\ d(t)}$$

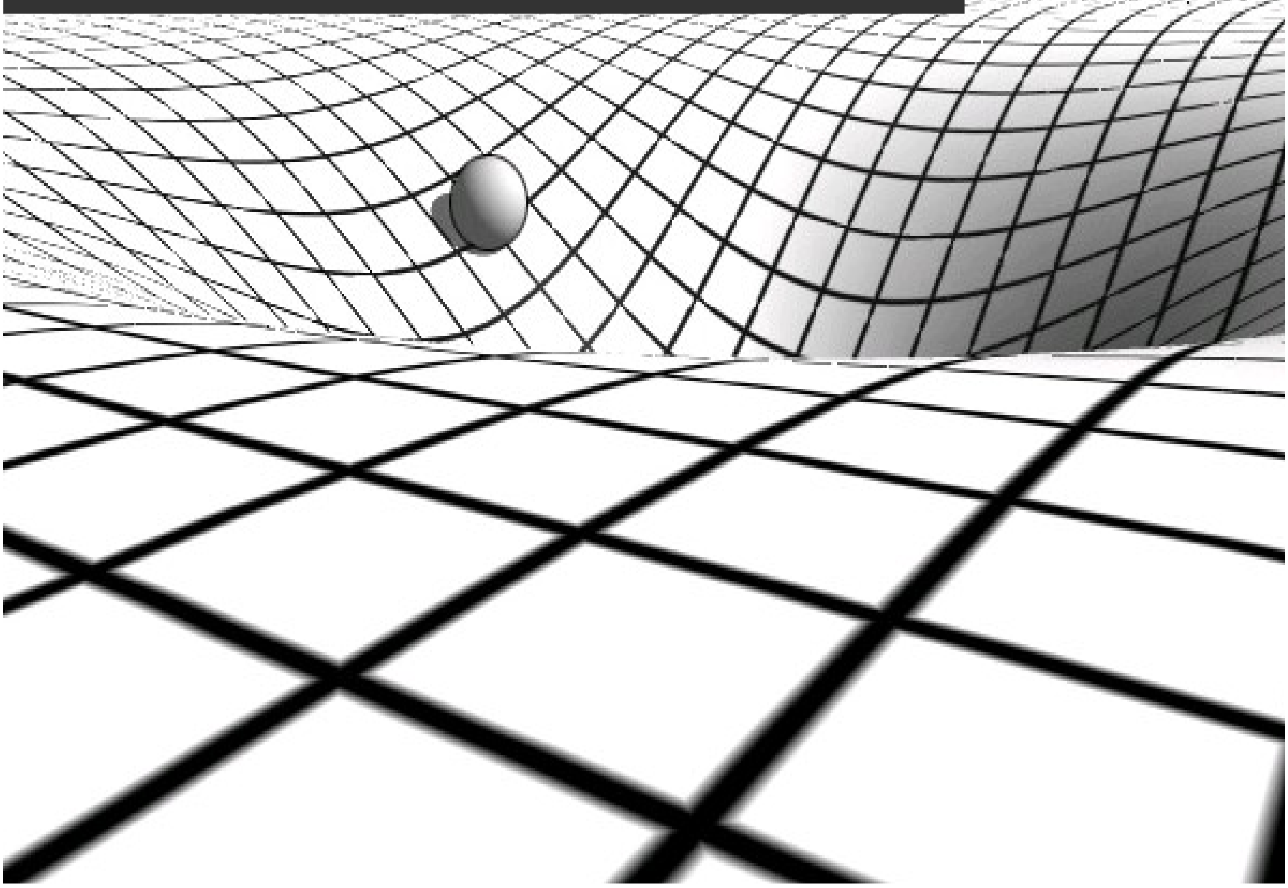
MASS

❗ When the function $d(t)$ vanishes:

- → The mass $m(t)$ explodes, in general.
- → The scaling and the reparametrization are ill defined.

❗ If it becomes negative, the new time parametrization turns imaginary.

Space-time Interpretation



Space-time Interpretation

Let us consider a **conformally ultrastatic spacetime**, with normal spatial sections:

$$ds^2 = -N^2(t)\,dt^2 + a^2(t)\,h_{ij}(x)\,dx^i\,dx^j$$

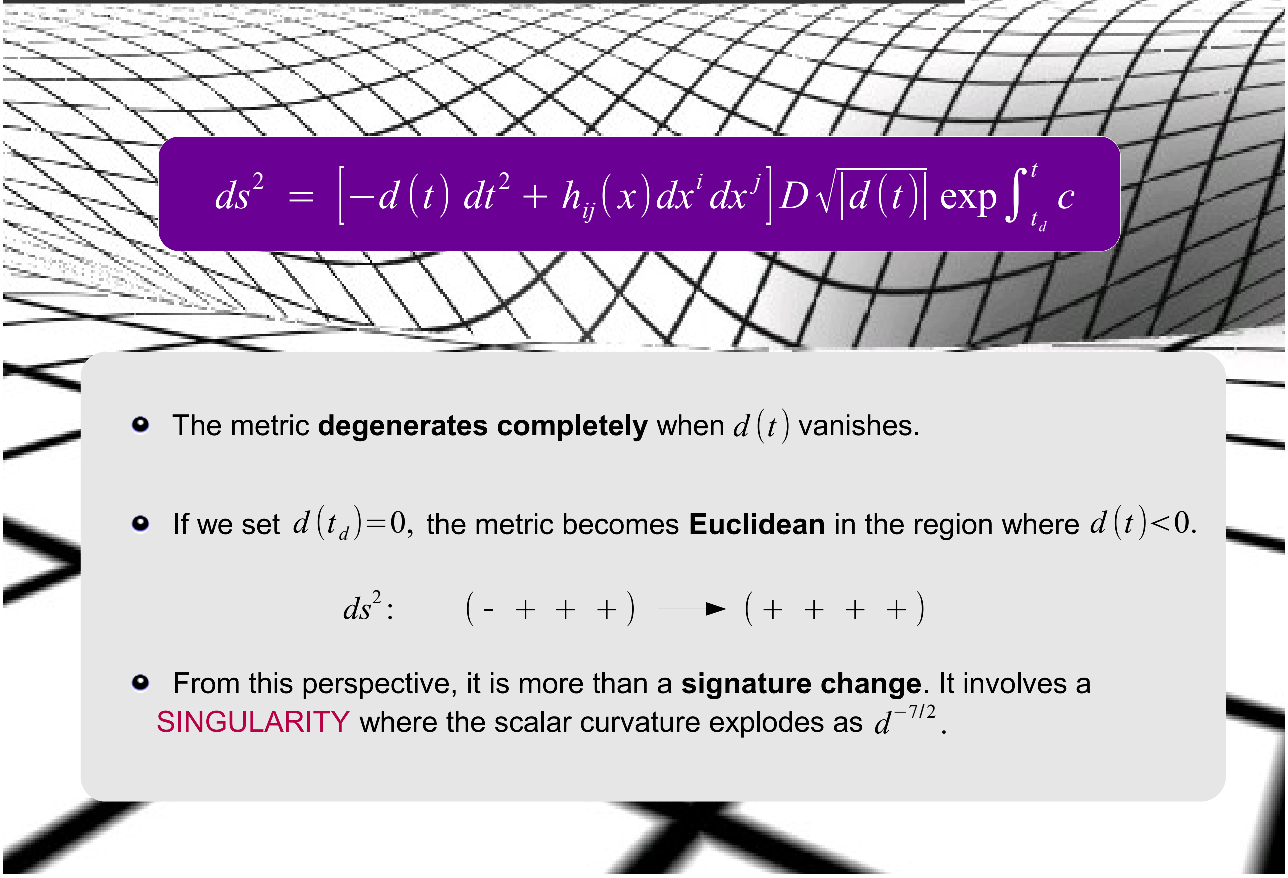
The considered field equations are the corresponding Klein-Gordon equations
(of mass $\bar{m}(t)$) under the **univocal correspondence:**

$$a^4(t) = d(t)\exp\left[\int^t 2c(\bar{t})\,d\bar{t}\right] \qquad N^4(t) = d^3(t)\exp\left[\int^t 2c(\bar{t})\,d\bar{t}\right]$$

$$\boxed{\phi'' + c(t)\phi' - d(t)\Delta\phi + \tilde{m}^2(t)\phi = 0}$$

Where: $\tilde{m}(t) = N(t)\,\bar{m}(t)$

Space-time Interpretation



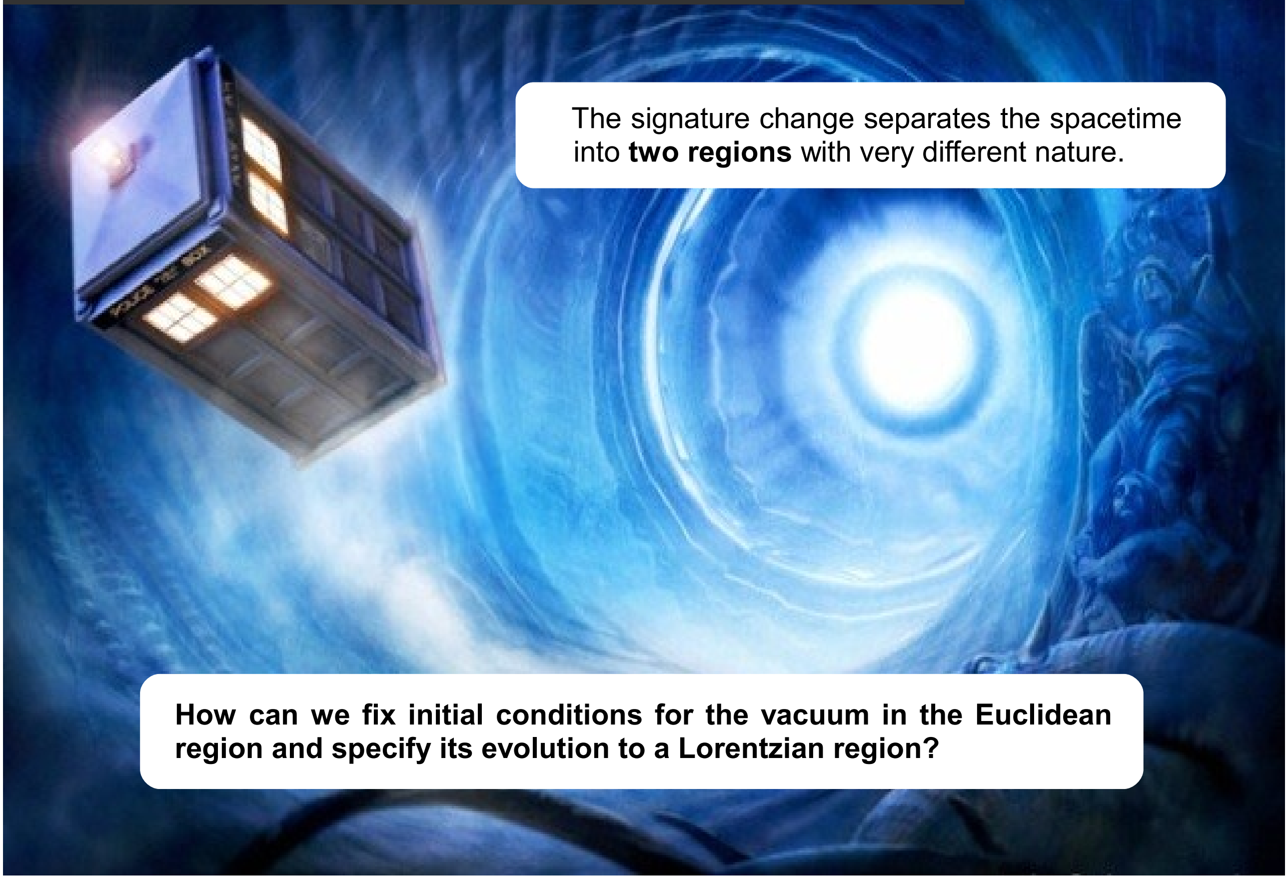
$$ds^2 = \left[-d(t)\,dt^2 + h_{ij}(x)\,dx^i\,dx^j\right] D\sqrt{|d(t)|}\,\exp\int_{t_d}^{t} c$$

- The metric **degenerates completely** when $d(t)$ vanishes.
- If we set $d(t_d)=0$, the metric becomes **Euclidean** in the region where $d(t)<0$.

$$ds^2: \qquad (-\ +\ +\ +) \longrightarrow (+\ +\ +\ +)$$

- From this perspective, it is more than a **signature change**. It involves a SINGULARITY where the scalar curvature explodes as $d^{-7/2}$.

Vacuum dynamics with signature change



The signature change separates the spacetime into **two regions** with very different nature.

How can we fix initial conditions for the vacuum in the Euclidean region and specify its evolution to a Lorentzian region?

Vacuum dynamics with signature change

We study the evolution of a fixed vacuum state in the **Euclidean** region:

i. We choose a complete set of solutions in the Lorentzian region $\left\{\varphi_n^{\pm}(T)\,\psi_n(\vec{x})\right\}$.

ii. Scaling by the invers of the scale factor and reparametrizating in terms of the time τ corresponding to the lapse $N^2 = \epsilon a^6$, $\epsilon = \pm$, we find the set of **solutions** $\left\{\phi_n^{\pm}(\tau)\,\psi_n(\vec{x})\right\}$

$$\ddot{\phi} = -\epsilon\left[a^4\Delta\phi + a^6\bar{m}^2\phi\right]$$

iii. **Wick rotation** of the modes in the Euclidean regime $\phi_n^{\pm(E)} = \lim_{\tilde{\tau}\to i\tau}\phi_n^{\pm}(\tilde{\tau})$.

iv. The solutions can be expressed as a linear combination of these modes with coefficients $c_n^{\pm(E)}$ and $c_n^{\pm}$, respectively, for the Euclidean and Lorentzian regions.

v. We set the initial conditions at τ_0. We require **continuity** conditions of the field and its time derivative at the signature change instant, in which the **metric degenerates**.

Vacuum dynamics with signature change

- Imposing the continuity conditions, we obtain a linear system for each mode that relates the coefficients of the Euclidean and Lorentzian regions:

$$\begin{pmatrix} c_n^{+} \\ c_n^{-} \end{pmatrix} = \begin{pmatrix} -I_n^{(+-)} & -I_n^{(--)} \\ I_n^{(++)} & I_n^{(-+)} \end{pmatrix} \begin{pmatrix} c_n^{+(E)} \\ c_n^{-(E)} \end{pmatrix}$$

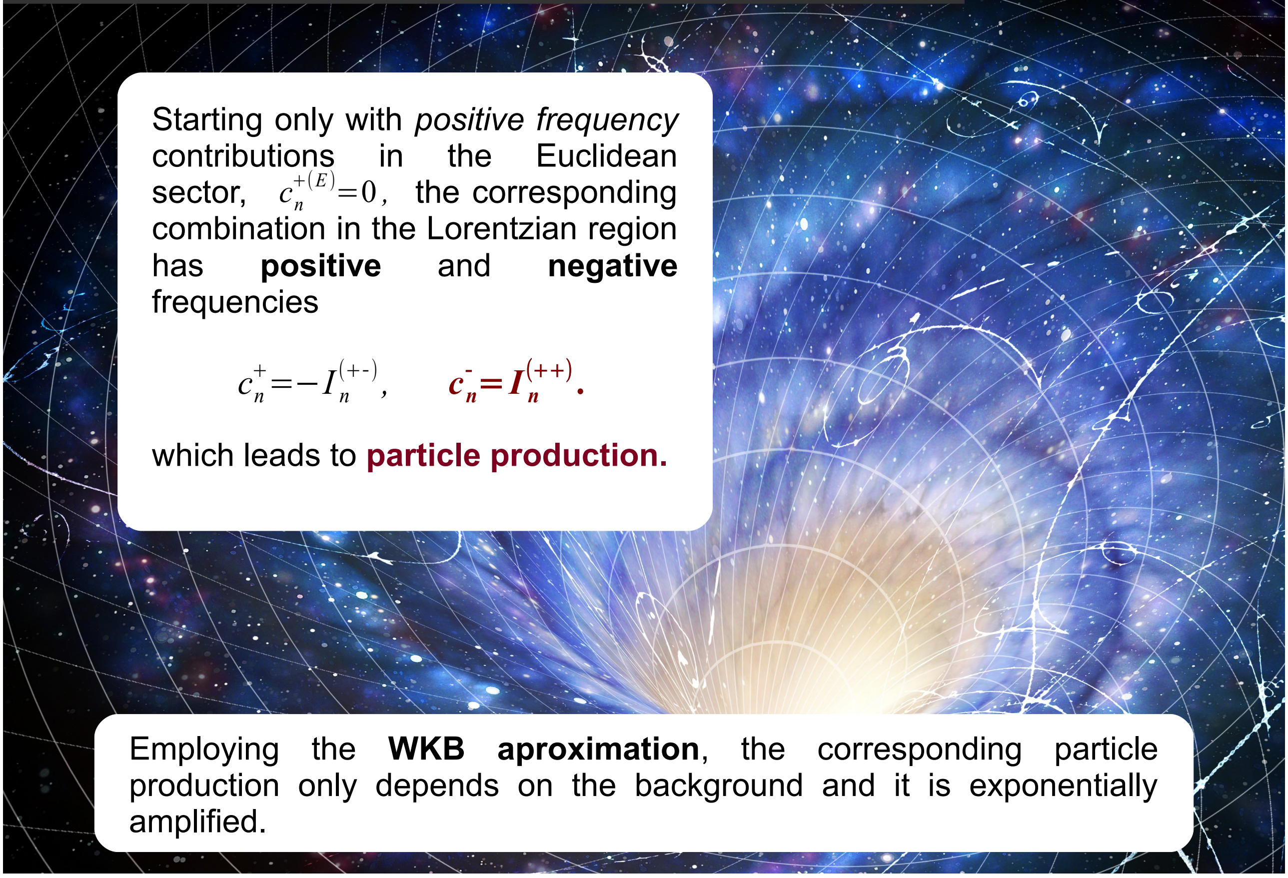
where $I_n^{(rs)} = \lim_{\tau\to 0}\langle\phi_n^{r(E)}(\tau),\phi_n^{s}(\tau)\rangle,\quad r,s = +$ ó $-$.

Using that the modes are orthonormal under the KG-type product.

- The field φ with unitary evolution in the Lorentzian region:

$$\varphi = a(T)\sum_n\left(c_n^{+}\phi_n^{+}[\tau(T)] + c_n^{-}\phi_n^{-}[\tau(T)]\right)\psi_n(\vec{x}).$$

Vacuum dynamics with signature change



Starting only with *positive frequency* contributions in the Euclidean sector, $c_n^{+(E)}=0$, the corresponding combination in the Lorentzian region has **positive** and **negative** frequencies

$$c_n^{+}=-I_n^{(+-)}, \qquad \boldsymbol{c_n^{-}=I_n^{(++)}}.$$

which leads to **particle production.**

Employing the **WKB aproximation**, the corresponding particle production only depends on the background and it is exponentially amplified.

Conclusions

- A set of criteria to SELECT a preferred **UNIQUE CLASS** of Fock quantizations for scalar fields in a variety of nonstationary spacetimes with compact spatial topology
- Removing the ambiguities provides physical predictions with great robustness.
- **Generalization** to all the second order equations of motion, through the combination of a scaled field configuration and a time reparametrization, univocally determined.
- **Space-time interpretation** of the considered equation of motion, as fields propagating in conformally ultrastatic spacetimes.
- **Signature change** ⟶ elliptic rather than hyperbolic partial differential equations for physical modes.

  ⟶ space-time **singularity**: there exists a point where the metric is totally degenerated and the scalar invariant curvature becomes infinity.
- Evolution of a vacuum state from a Euclidean to a Lorentzian region.
- Generally, there exists an exponentially amplified **“particle production”**.


THANK YOU
ありがとう

“Generalized Galilean Genesis”

Sakine Nishi

[JGRG24(2014)111309]

Generalized Galilean Genesis

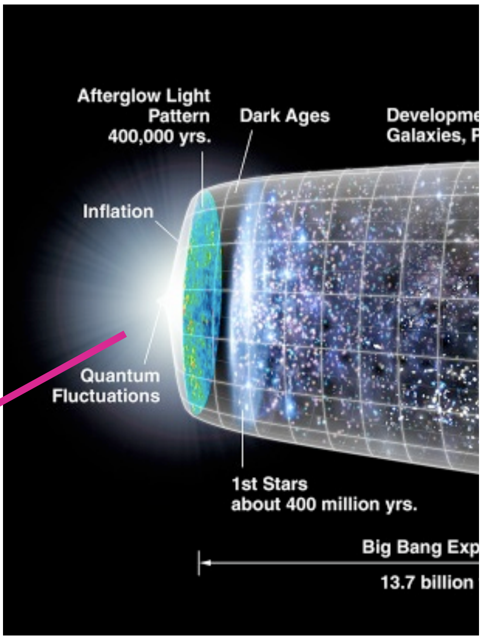
JGRG24@IPMU

Sakine Nishi (Rikkyo University)
in collaboration with Tsutomu Kobayashi (Rikkyo University)
In preparation.

Outline

- Introduction
- Genesis (Previous study -> Generalization)
- Background
- Perturbations (tensor, scalar -> curvaton)
- Conclusion

Introduction



- There are many kinds of models which explain the early universe.
- Galilean Genesis
  - alternative to inflation
  - originally constructed in galileon theory.

    -> Horndeski theory in our study

Introduction

- Horndeski theory

  $$X := -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

  - action $$S_{\rm Hor} = \int d^4x\sqrt{-g}\Big\{G_2(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi,X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$
  - the most general scalar-tensor theory
  - field eqs. have no 3rd and higher derivative terms

[G. W. Horndeski, Int. J. Theor. Phys. 10 (1974)]

[T. Kobayashi, M. Yamaguchi and J. Yokoyama, Prog. Theor. Phys. **126**, 511 (2011)]

Introduction

- Motivation

Only inflation can explain the early universe?

1. Background , Problems (flatness e.t.c.)
2. Perturbations (tensor, scalar) Check!

-> compare genesis to other inflation models
and discuss observational implications

Galilean Genesis

- alternative to inflation model
- Previous study
  - action

$$\boxed{\mathcal{S}=\int \mathrm{d}x^4\sqrt{-g}\left[f^2e^{2\phi}(\partial\phi)^2+\frac{f^3}{\Lambda^3}(\partial\phi)^2\Box\phi+\frac{f^3}{2\Lambda^3}(\partial\phi)^4\right]}$$

$$G_2=f^2e^{2\phi}(\partial\phi)^2+\frac{f^3}{2\Lambda^3}(\partial\phi)^4,\quad G_3=\frac{f^3}{\Lambda^3}(\partial\phi)^2\Box\phi,\quad G_4=G_5=0$$

-> subclass of Horndeski action

  - solutions

$$t\to-\infty\quad:\quad a(t)\simeq 1\ ,\ H(t)\simeq-\frac{f^2}{3M_{Pl}^2}\frac{1}{H_0^2t^3}$$

$$t\to t_0\ :\ a(t)=\exp\left[\frac{8f^2}{3H_0^2M_{Pl}^2}\frac{1}{(t_0-t)^2}\right],\quad H(t)\simeq\frac{16f^2}{3M_{Pl}^2}\frac{1}{H_0^2(t_0-t)^3}$$

[P. Creminelli, A. Nicolis and E. Trincherini, JCAP **1011**, 021 (2010)]

Galilean Genesis

- Generalization

introduce a parameter α

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y), \quad G_3 = e^{2\alpha\lambda\phi} g_3(Y),$$

$$G_4 = \frac{M_{\rm Pl}^2}{2} + e^{2\alpha\lambda\phi} g_4(Y), \quad G_5 = e^{-2\lambda\phi} g_5(Y). \quad Y := e^{-2\lambda\phi} X$$

$g_i(Y)$ are arbitrary functions

- include the various models of Genesis

α=1 ->

[P. Creminelli, A. Nicolis and E. Trincherini, JCAP **1011**, 021 (2010)]
[P. Creminelli, K. Hinterbichler, J. Khoury, A. Nicolis, E. Trincherini, [arXiv:1209.3768 [hep-th]]]
[D. Pirtskhalava, L. Santoni, E. Trincherini, P. Uttayarat [arXiv:1410.0882 [hep-th]]]

- solutions

$(-\infty < t < 0)$

$$a(t) \simeq 1 + \frac{1}{2\alpha}\frac{h_0}{(-t)^{2\alpha}} \;, \quad H(t) \propto \frac{1}{(-t)^{2\alpha+1}},$$

Background

- Inflation

$$a(t) = a(t_i)e^{H_{inf}(t-t_i)}$$

Exponentially expansion

- Friedman eq.

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3c^2}\rho$$

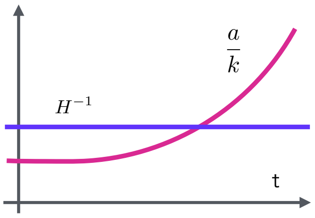
- Genesis

$$a(t) \simeq 1 + \frac{1}{2\alpha}\frac{h_0}{(-t)^{2\alpha}} \quad (-\infty < t < 0)$$

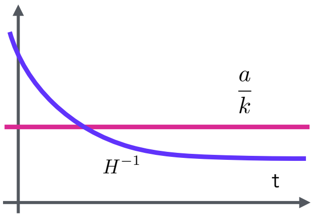
started from the Minkowski spacetime

- Friedman eq.

$$\mathcal{E} \simeq e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y_0) + \frac{3K}{a^2}M_{Pl}^2 = 0$$



solve the flatness problem in the same way



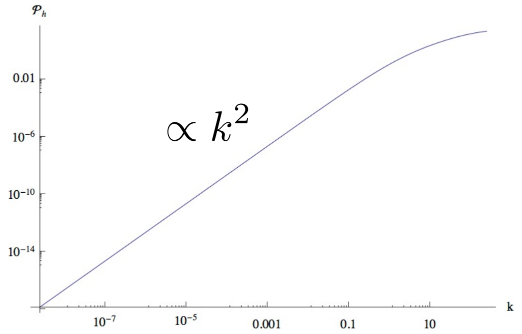
Perturbation (tensor)

- Wave eq. $\ddot{h}_{ij} + (3H + \frac{\dot{\mathcal{G}}_T}{\mathcal{G}_T})\dot{h}_{ij} - \frac{\mathcal{F}_T}{a^2\mathcal{G}_T}\nabla^2 h_{ij} = 0$
- Powerspectrum

Action

$$\mathcal{S}_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla^2 h_{ij})^2\right]$$

$$\simeq 1$$



-> in Minkowski spacetime fluctuation do not grow

This is too small to detect.

Perturbation (scalar)

- Action $\mathcal{L}_\zeta = \mathcal{A}(Y_0)(-t)^{2\alpha}\left[\dot{\zeta}_k^2 - k^2 c_s^2 \zeta_k^2\right]$
- Wave eq. $\ddot{\zeta}_k - \frac{2\alpha}{(-t)}\dot{\zeta}_k + k^2 c_s^2 \zeta_k = 0$
- solution

$$\zeta_k = \frac{1}{2}\sqrt{\frac{\pi}{\mathcal{A}(Y_0)}}(-t)^{\nu} H_{\nu}^{(1)}(\omega_k(-t)), \quad \nu = \frac{1}{2} - \alpha$$

  - $0 < \alpha < \frac{1}{2}$: decaying mode + const.
  - $\alpha > \frac{1}{2}$: growing mode + const.

Perturbation (scalar)

- $0 < \alpha < \frac{1}{2}$

$$\mathcal{P}_\zeta(k) = \frac{c_s^{-2\nu} 2^{2\nu-2} \Gamma(\nu)^2}{\pi^3 \mathcal{A}(Y_0)} k^{3-2\nu}$$

$$n_s = 2\alpha + 3$$

- $\alpha > \frac{1}{2}$

$$\mathcal{P}_\zeta(k) = \frac{c_s^{2\nu} 2^{-2\nu-3} \Gamma(\nu)^2 (-t_{end})^4}{\pi^3 \mathcal{A}(Y_0)} k^{3+2\nu}$$

genesis phase ends at t_end

$$\boxed{n_s = 5 - 2\alpha}$$

  - $\alpha = 2$: flat spectrum
  - $\alpha \neq 2$: introducing the curvaton field

Curvaton

- introduce the conformal metric $(\beta \simeq 1)$

$$\hat{g}^{\mu\nu} = e^{2\beta\lambda\phi} g_{\mu\nu}$$

- Lagrangian $\mathcal{L}_\sigma = -\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}m^2\sigma^2$

[P. Creminelli, A. Nicolis and E. Trincherini, JCAP **1011**, 021 (2010)]

- p and ρ of curvaton have to be subdominant.

$$(t \to 0)$$

$$p_\sigma, \rho_\sigma \propto (-t)^{-2\beta}, \quad p_\phi, \rho_\phi \propto (-t)^{-2(\alpha+1)}$$

-> $\alpha > 2\beta - 1 \simeq 1 \quad (\beta \simeq 1)$

Curvaton

- Power spectrum of curvaton fluctuation

$$\mathcal{P}_{\delta\sigma}(k) = \frac{2^{3\beta-1}Y_0^{\beta}\lambda^{2\beta}\Gamma(\beta-\frac{1}{2})^2}{\pi^3}k^{2-2\beta}$$

$$n_s = 3 - 2\beta \simeq 1 \quad (\beta \simeq 1)$$

-> we get a flat power spectrum.

- this is only in the case of $\alpha>1$

- For $0<\alpha<1$ curvaton mechanism does not work.

Conclusions

- Galilean Genesis and it's generalization

- background and perturbations in Galilean Genesis

- make the scale invariant power spectrum

-> $\alpha = 2$

“The Effective Field Theory of Cosmological Large Scale Structures”

Leonardo Senatore [Invited]

[JGRG24(2014)111310]

Leonardo Senatore (Stanford)

The Effective Field Theory of Large Scale Structure

the way to go for inflation

How do we probe inflation

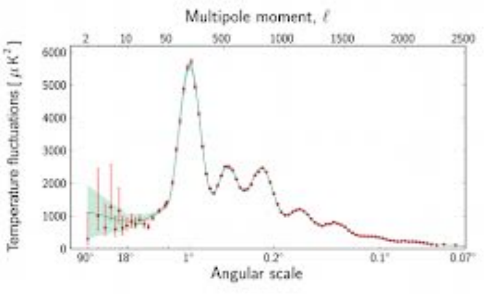
- The only observable we are testing from the background solution is

  $\Omega_K \lesssim 3\times 10^{-3}$

- All the rest, comes from the fluctuations
- For the fluctuations
  - they are primordial
  - they are scale invariant
  - they have a tilt $n_s - 1 \simeq -0.04 \sim \mathcal{O}\left(\frac{1}{N_e}\right)$
  - they are quite gaussian

$$\mathrm{NG} \sim \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^{3/2}} \lesssim 10^{-3}$$

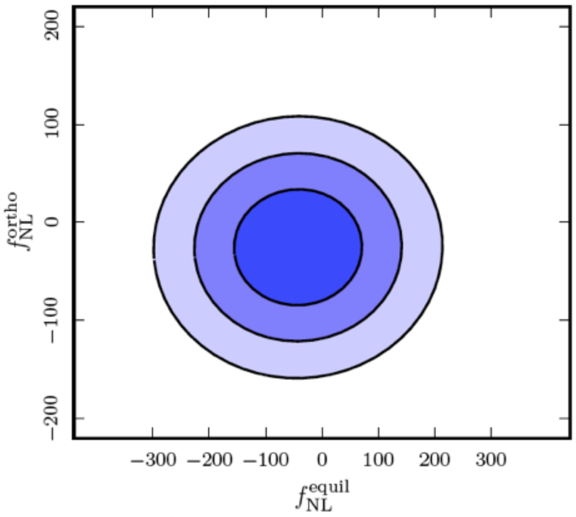
  - both scalar and maybe tensors

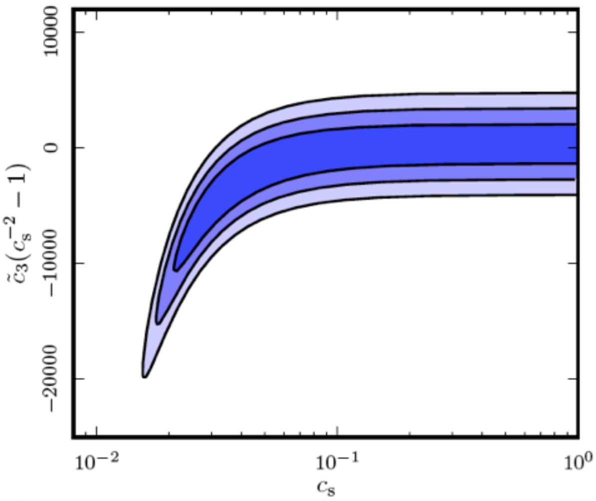


Limits in terms of parameters of a Lagrangian

- $$S=\int d^4x\,\sqrt{-g}\left[-\frac{M_{\rm Pl}^2\dot H}{c_s^2}\left(\dot\pi^2-c_s^2\frac{(\partial_i\pi)^2}{a^2}\right)+(M_{\rm Pl}^2\dot H)\frac{1-c_s^2}{c_s^2}\left(\frac{\dot\pi(\partial_i\pi)^2}{a^2}+\frac{A}{c_s^2}\dot\pi^3\right)+\cdots\right]$$

with C. Cheung, P. Creminelli, L. Fitzpatrick, J. Kaplan **JHEP 2008**





- these are limits on the cutoff of the theory $\sim \dfrac{\dot\pi^3}{\Lambda^2}$

with Smith and Zaldarriaga, **JCAP2010**
Planck Collaboration **2013**

What has Planck done to theory?

- Planck improve limits wrt WMAP by a factor of ~3.
- Since $$\mathrm{NG}\sim\frac{H^2}{\Lambda_U^2}\quad\Rightarrow\quad\Lambda_U^{\rm min,\,Planck}\simeq 2\,\Lambda_U^{\rm min,\,WMAP}$$
- Given the absence of known or nearby threshold, this is not much.
- Planck was great
- but Planck was not good enough
  - – not Plank’s fault, but Nature’s faults
    - Please complain with Nature
- Planck was an opportunity for a detection, not much an opportunity to change the theory in absence of detection (luckily WMAP had a tilt a 2.5 σ , so we got to $6\ \sigma$)
- On theory side, little changes
  - – contrary for example to LHC, which was crossing thresholds
    - Any result from LHC is changing the theory

Cosmology is going to change in a few months

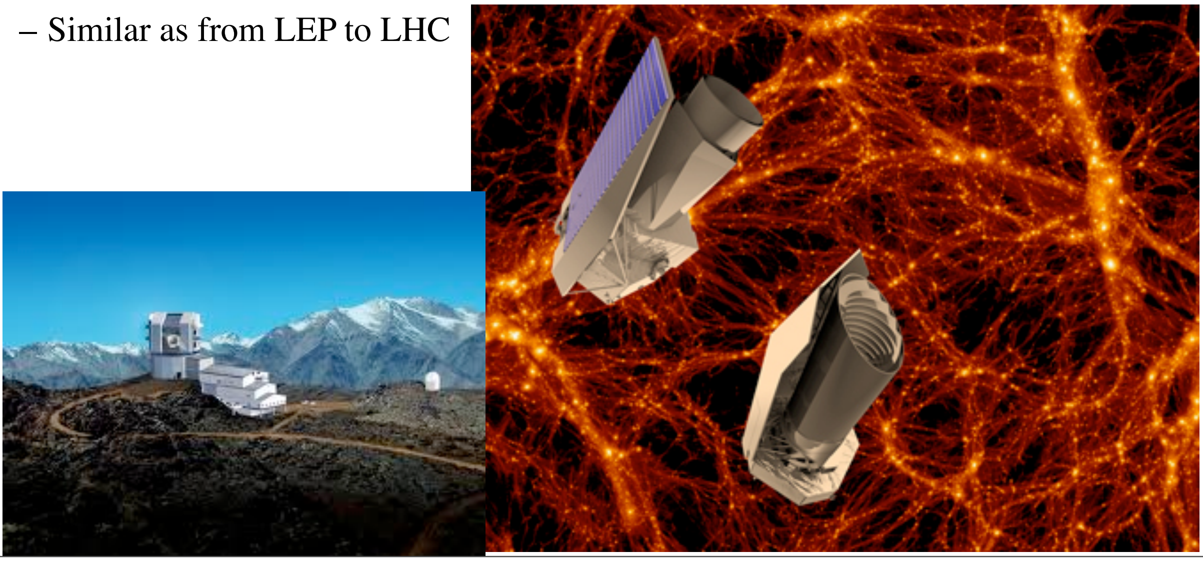
- Tremendous progress has been made through observation of the primordial fluctuations
- In order to increase our knowledge of Inflation, we need more modes
- Planck will soon have observed all the modes from the CMB
- and then what?
- I will assume we are not lucky
  - no B-mode detection
  - no signs from the beginning of inflation
- Unless we find a way to get more modes, the game is over
- Large Scale Structures offer the only medium-term place for hunting for more modes
  - but we are compelled to understand them
    - I do not think, so far, we understand them well enough

What is next?

- Euclid and LSST like: this is our only next chance
  - we need to understand how many modes are available

$$\text{Number of modes} \sim \left(\frac{k_{\rm max}}{k_{\rm min}}\right)^3$$

  - Need to understand short distances
  - Similar as from LEP to LHC

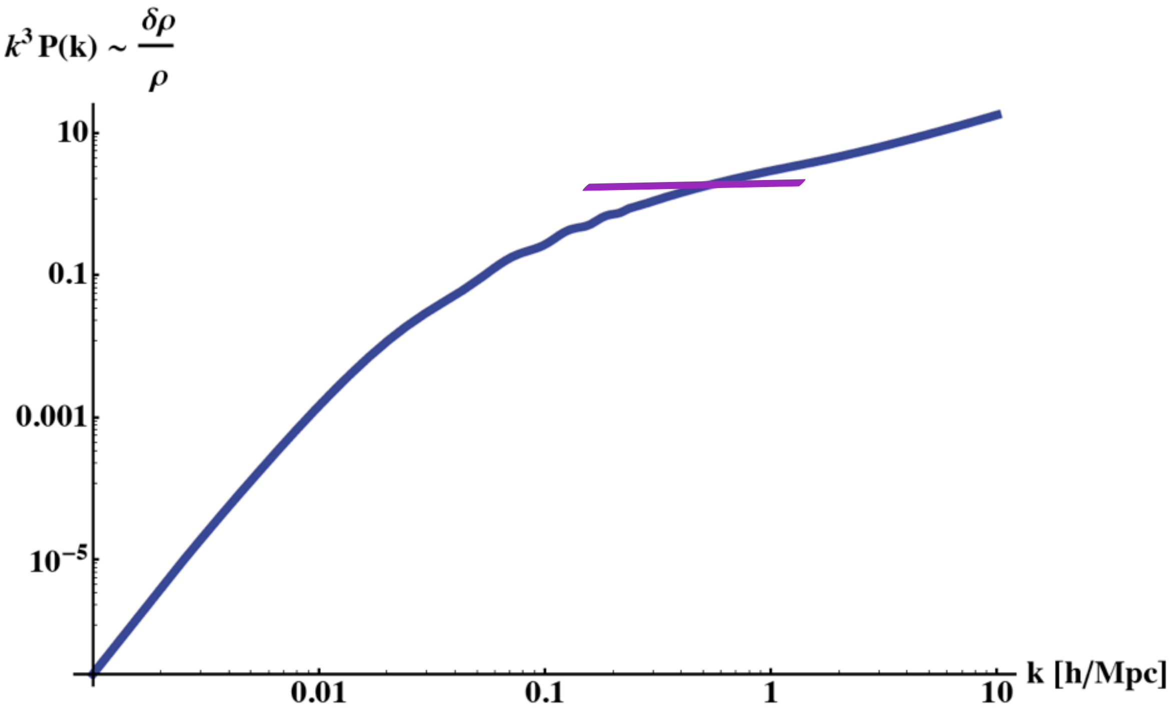


The Effective Field Theory of Cosmological Large Scale Structures

Redshift Space distortions in the EFTofLSS	with Zaldarriaga **1409**
Bias in the EFTofLSS	me alone **1406**
The one-loop bispectrum in the EFTofLSS	with Angulo, Foreman, Schmittful **1406** see also Baldauf, Mirbabayi, Mercolli,Pajer **1406**
The IR-resummed EFTofLSS	with Zaldarriaga **1404**
The Lagrangian-space EFTofLSS	with Porto and Zaldarriaga **JCAP1405**
The EFTofLSS at 2-loops	with Carrasco, Foreman and Green **JCAP1407**
The 2-loop power spectrum and the IR safe integrand	with Carrasco, Foreman and Green **JCAP1407**
The Effective Theory of Large Scale Structure (EFTofLSS)	with Carrasco and Hertzberg **JHEP 2012**
Cosmological Non-linearities as an Effective Fluid	with Baumann, Nicolis and Zaldarriaga **JCAP 2012**

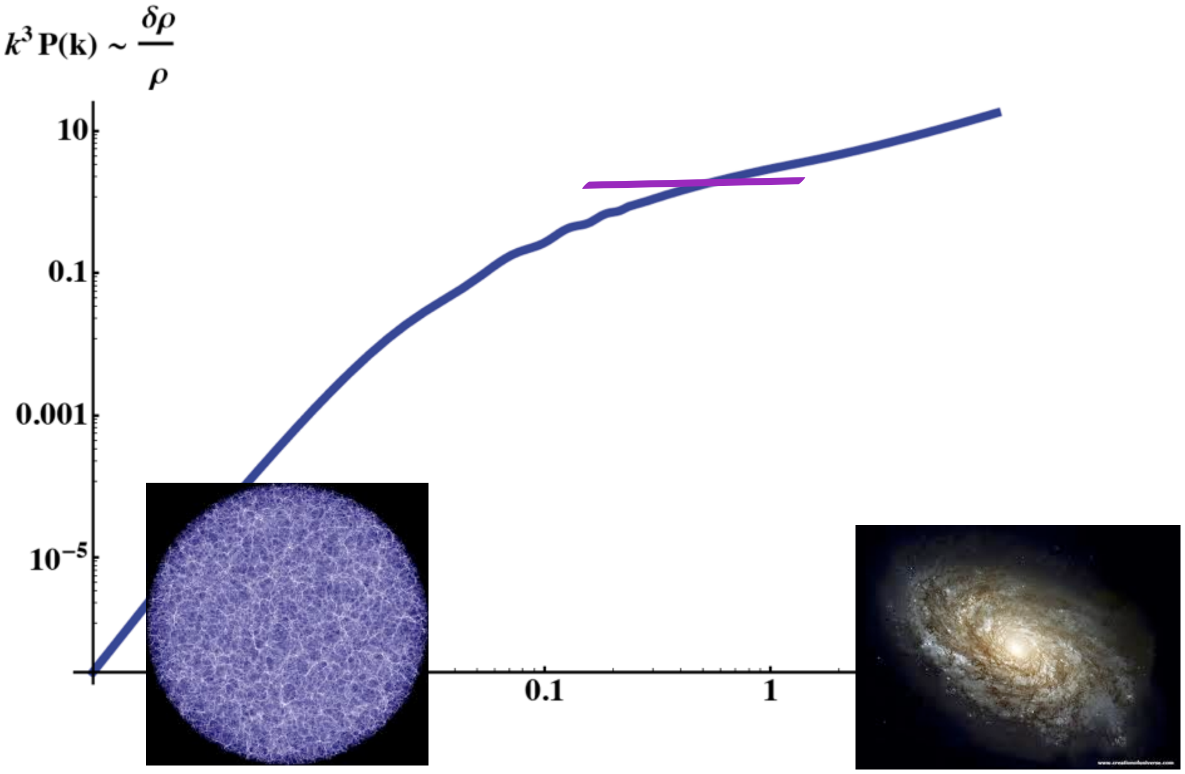
A well defined perturbation theory

- Non-linearities at short scale



A well defined perturbation theory

- Non-linearities at short scale



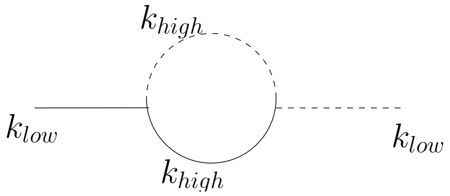
A well defined perturbation theory

- Standard perturbation theory is not well defined
- Standard techniques
  - – perfect fluid $\dot{\rho} + \partial_i \left(\rho v^i\right) = 0$,
  - – expand in $\delta \sim \frac{\delta\rho}{\rho}$ and solve iteratively

$$\delta^{(n)} \sim \int \text{GreenFunction} \times \text{Source}^{(n)} \left[\delta^{(1)}, \delta^{(2)}, \ldots, \delta^{(n-1)}\right]$$

$$\Rightarrow \quad \langle \delta_k^{(2)} \delta_k^{(2)} \rangle \sim \int d^3k' \, \langle \delta_{k-k'}^{(1)} \delta_{k-k'}^{(1)} \rangle \, \langle \delta_{k'}^{(1)} \delta_{k'}^{(1)} \rangle$$

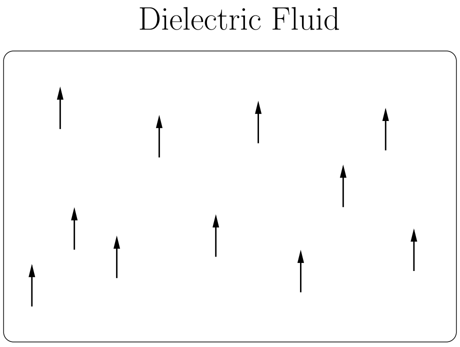
- Perturbative equations break in the UV
  - – $\delta \sim \frac{k}{k_{NL}} \gg 1 \quad \text{for} \quad k \gg k_{NL}$
  - – no perfect fluid if we truncate



Idea of the
Effective Field Theory

Consider a dielectric material

- Very complicated on atomic scales d_{atomic}
- On long distances $d \gg d_{\text{atomic}}$
  - we can describe atoms with their gross characteristics
    - polarizability $\vec{d}_{\text{dipole}} \sim \alpha\, \vec{E}_{\text{electric}}$: average response to electric field
  - we are led to a uniform, smooth material, with just some macroscopic properties
    - we simply solve Maxwell dielectric equations, we do not solve for each atom.
- The universe looks like a dielectric

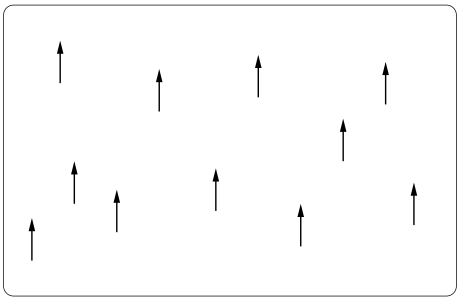


Consider a dielectric material

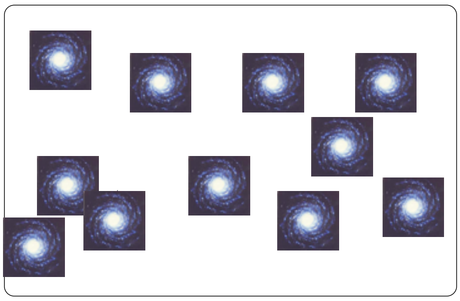
- Very complicated on atomic scales d_{atomic}
- On long distances $d \gg d_{\text{atomic}}$
  - – we can describe atoms with their gross characteristics
    - polarizability $\vec{d}_{\text{dipole}} \sim \alpha\, \vec{E}_{\text{electric}}$: average response to electric field
  - – we are led to a uniform, smooth material, with just some macroscopic properties
    - we simply solve Maxwell dielectric equations, we do not solve for each atom.
- The universe looks like a dielectric

EM → GR

Dielectric Fluid

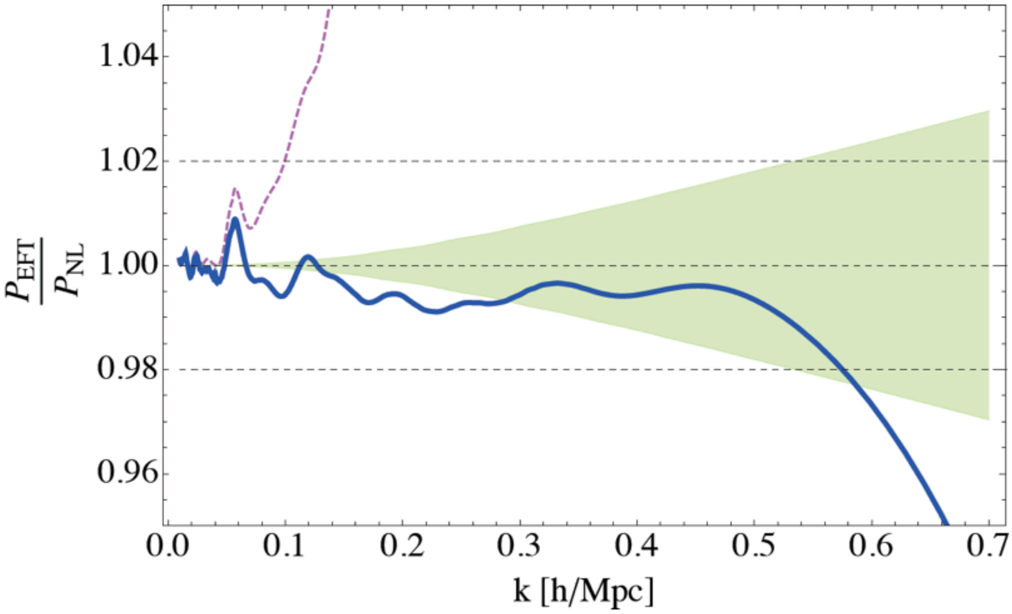


Dielectric Fluid



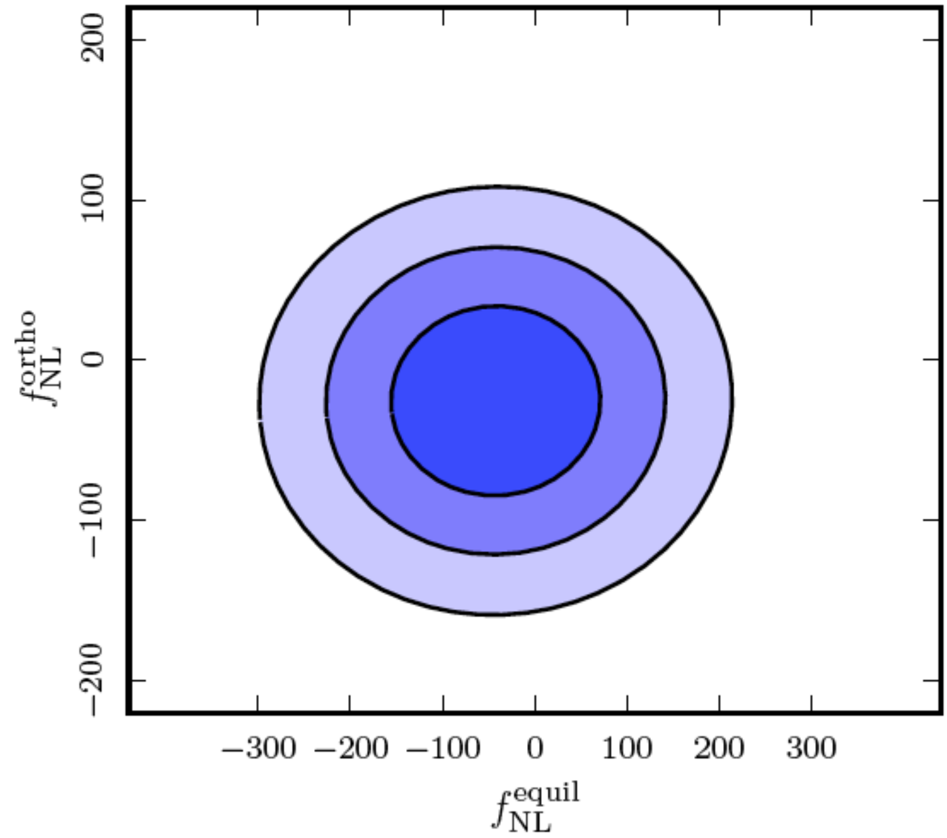
Bottom line result

- A well defined perturbation theory
- 2-loop in the EFT, with IR resummation

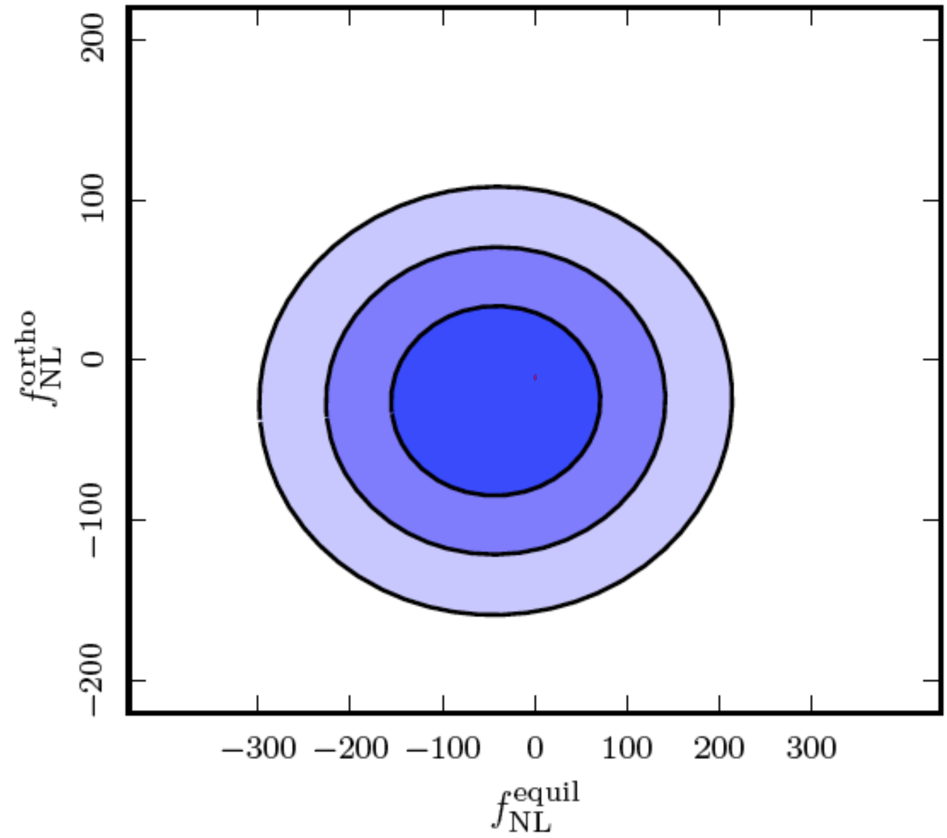


- Data go as k^3_{max} : naively factor of 200 more modes than before

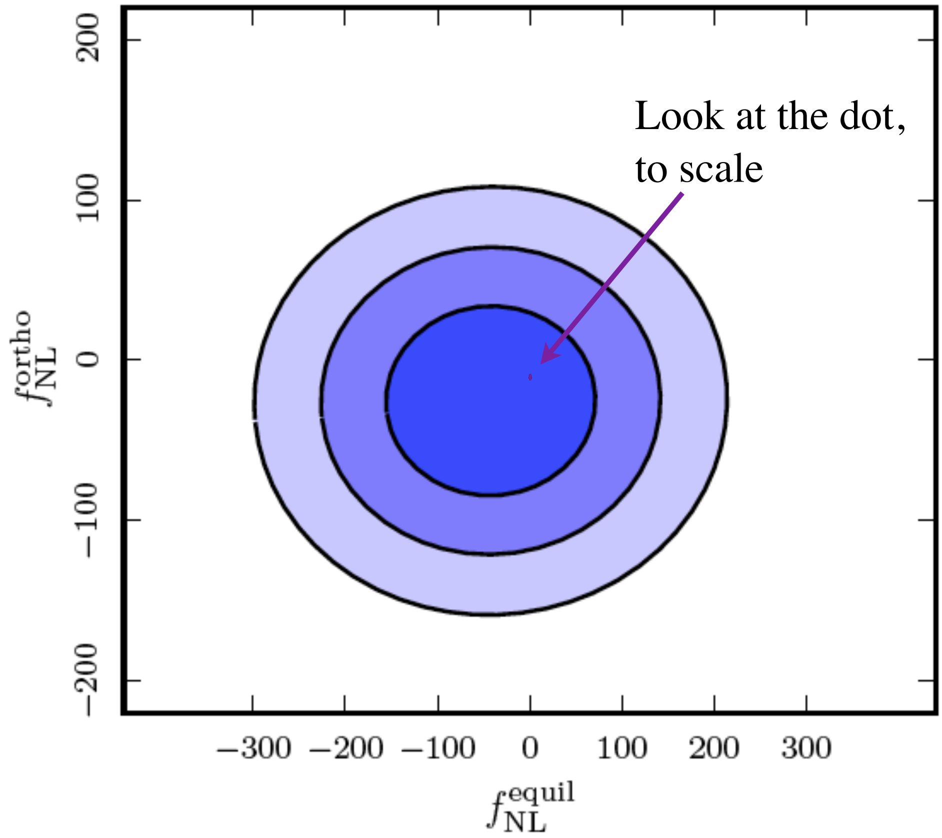
With this



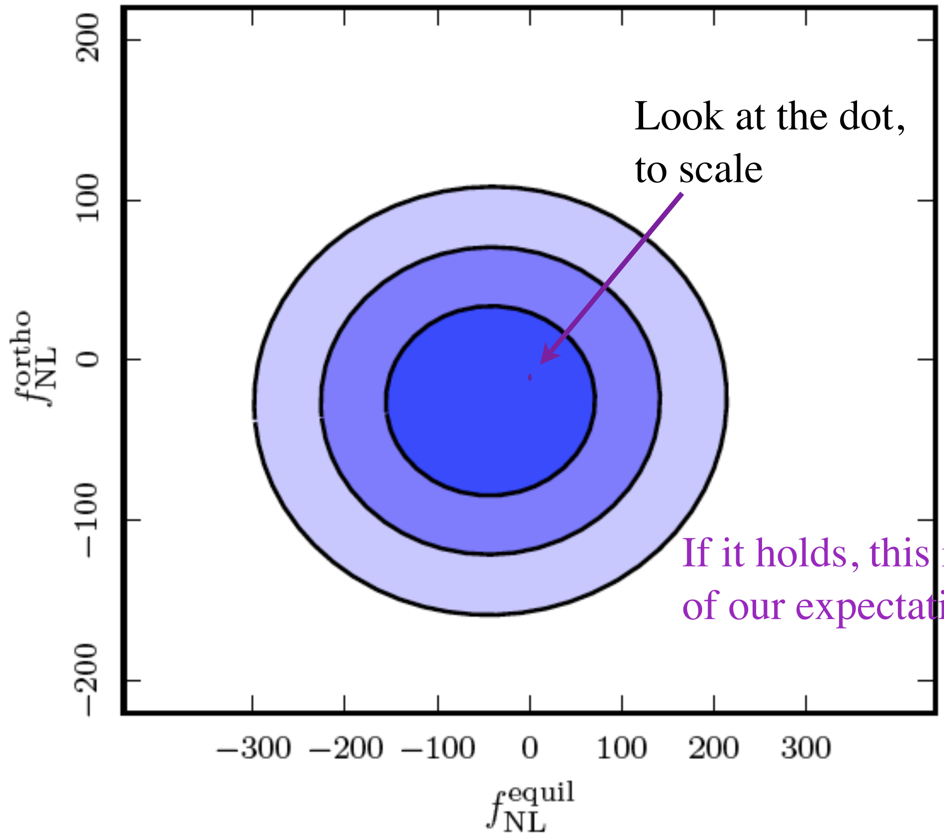
With this



With this



With this



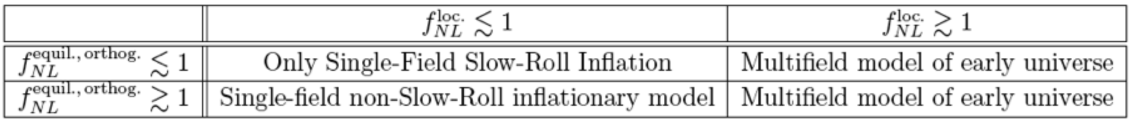
If it holds, this is a revolution of our expectations

What would we get?

- If we push

$$f_{NL} \lesssim 1$$

  - then we rule out all theories of early universe but
    - Single-Field Slow-Roll Inflation
    - As all other theories are more interacting that this
      - all interactions are so small that we are perturbatively close to slow roll inflation
- Huge discovery without a detection



	$f_{NL}^{\rm loc.} \lesssim 1$	$f_{NL}^{\rm loc.} \gtrsim 1$
$f_{NL}^{\rm equil., orthog.} \lesssim 1$	Only Single-Field Slow-Roll Inflation	Multifield model of early universe
$f_{NL}^{\rm equil., orthog.} \gtrsim 1$	Single-field non-Slow-Roll inflationary model	Multifield model of early universe

---

Construction of the Effective Field Theory

Point-like Particle versus Extended Objects

- On short distances, we have point-like particles
  - they move

$$\frac{d^2\vec{z}(\vec{q},\eta)}{d\eta^2}+\mathcal{H}\frac{d\vec{z}(\vec{q},\eta)}{d\eta}=-\vec{\partial}_x\Phi[\vec{z}(\vec{q},\eta)]$$

  - induce overdensities

$$1+\delta(\vec{x},\eta)=\int d^3q\;\delta^{(3)}(\vec{x}-\vec{z}(\vec{q},\eta))$$

  - Source gravity

$$\partial^2\Phi(\vec{x})=\mathcal{H}^2\delta(\vec{x})$$

Point-like Particle versus Extended Objects

- But we cannot describe point-like particles: we need to focus on long distances.
  - We deal with Extended objects
    - they move differently:

$$\frac{d^2\vec{z}(\vec{q},\eta)}{d\eta^2}+\mathcal{H}\frac{d\vec{z}(\vec{q},\eta)}{d\eta}=-\vec{\partial}_x\Phi[\vec{z}(\vec{q},\eta)]$$

Point-like Particle versus Extended Objects

- But we cannot describe point-like particles: we need to focus on long distances.
  - We deal with Extended objects
    - they move differently:

$$\frac{d^2\vec{z}_L(\vec{q},\eta)}{d\eta^2}+\mathcal{H}\frac{d\vec{z}_L(\vec{q},\eta)}{d\eta} = -\vec{\partial}_x\left[\Phi_L[\vec{z}_L(\vec{q},\eta)]+\frac{1}{2}Q^{ij}(\vec{q},\eta)\partial_i\partial_j\Phi_L[\vec{z}_L(\vec{q},\eta)]+\cdots\right]+\vec{a}_S(\vec{q},\eta)$$

Point-like Particle versus Extended Objects

- They induce number over-densities and real-space multipole moments

$$1+\delta_{n,L}(\vec{x},\eta) \equiv \int d^3\vec{q}\;\delta^3(\vec{x}-\vec{z}_L(\vec{q},\eta))\ ,$$

$$\mathcal{Q}^{i_1\ldots i_p}(\vec{x},\eta) \equiv \int d^3\vec{q}\;Q^{i_1\ldots i_p}(\vec{q},\eta)\delta^3(\vec{x}-\vec{z}_L(\vec{q},\eta))$$

- they source gravity with the `overall' mass

$$\partial_x^2\Phi_L=\frac{3}{2}\mathcal{H}^2\Omega_m\left(\delta_{n,L}(\vec{x},\eta)+\frac{1}{2}\partial_i\partial_j\mathcal{Q}^{ij}(\vec{x},\eta)-\frac{1}{6}\partial_i\partial_j\partial_k\mathcal{Q}^{ijk}(\vec{x},\eta)+\cdots\right)\equiv\frac{3}{2}\mathcal{H}^2\Omega_m\delta_{m,L}(\vec{x},\eta)$$

$$\sim \text{Energy}_{\text{electrostatic}} = q\,V+\vec{d}\cdot\vec{E}+\ldots$$

- These equations can be derived from smoothing the point-particle equations
  - but actually these are the assumption-less equations

How do we treat the new terms?

- Similar to treatment of material polarizability: $\vec{d}_{\text{dipole}} \sim \vec{d}_{\text{intrinsic}} + \alpha\,\vec{E}$
- Take moments:

$$Q^{ij} = \langle Q^{ij} \rangle_S + Q^{ij}_{\mathcal{S}} + Q^{ij}_{\mathcal{R}}$$

- Expectation value $\langle Q^{ij} \rangle_S = l_S^2(\eta)\delta_{ij}$
- Response (non-local in time) $Q_{ij,\mathcal{R}} \sim l_1(\eta)^2\, \partial_i\partial_j \Phi_L(\vec{z}_L(\vec{q},\eta))$
- Stochastic noise

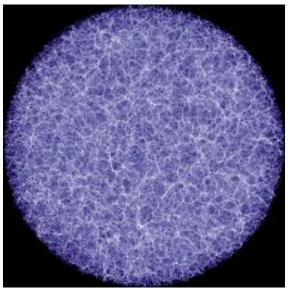
$$\langle Q_{\mathcal{S}} \rangle = 0 \qquad \langle Q_{\mathcal{S}} Q_{\mathcal{S}} \ldots \rangle \neq 0$$

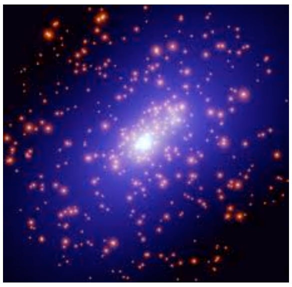
- Overall $Q_{ij}(\vec{x},t) = l_0^2(t)\,\delta_{ij} + l_1^2(t)\,\partial_i\partial_j\Phi(\vec{x},t) + \ldots$
- In summary: we obtain an expression just in terms of long-wavelength variables

$$\frac{\partial^2}{H^2}\Phi(\vec{x},t) = \delta(\vec{x},t) + \partial_i\partial_j Q_{ij}\left(\delta(\vec{x},t),\ldots\right) + \ldots$$

This EFT is non-local in time

- For local EFT, we need hierarchy of scales.
  - In space we are ok





  - In time we are not ok: all modes evolve with time-scale of order Hubble

with Carrasco, Foreman and Green **1310**

Carroll, Leichenauer, Pollak **1310**

- $\Longrightarrow$ The EFT is local in space, non-local in time
  - Technically it does not affect much because the linear propagator is local in space

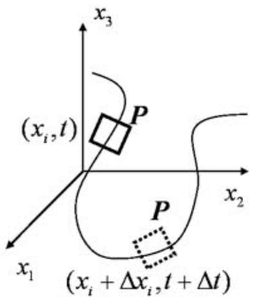
When do we stop?

- Similar to treatment for material polarizability: $\vec{d}_{\rm dipole} \sim \alpha\, \vec{E}_{\rm electric}$, $Q_{ij}^{\rm electric} = c\, E_i E_j$, …
- Short distance physics is taken into account by expectation value, response, and noise
- Poisson equation breaks when $\delta_{n,L}(\vec{x},\eta) \sim \partial_i\partial_j \mathcal{Q}^{ij}(\vec{x},\eta)$
  - gravitational potential from quadrupole moment ~ the one from center of mass
- By dimensional analysis, this happens for distances shorter than a critical length
  - the non-linear scale $k \gtrsim k_{\rm NL}$
  - on long distances, $k \ll k_{\rm NL}$, write as many terms as precision requires.
    - Manifestly convergent expansion in $\left(\frac{k}{k_{\rm NL}}\right) \ll 1$

Connecting with the Eulerian Treatment

- In the universe, finite-size particles move

$$\vec{z}(\vec{q},t) = \vec{q} + \vec{s}(\vec{q},t)$$



- In Lagrangian space, we do not expand in $\vec{s}(\vec{q},t)$
- In Eulerian, we do: we describe particles from a fixed position
  - Expand in $k\, s \ll 1$
- There are three expansion parameters for a given wavenumber

$$\epsilon_{s>} = k^2 \int_k^\infty \frac{d^3k'}{(2\pi)^3}\frac{P_{11}(k')}{k'^2} ,$$

Effect of Short Displacements

$$\epsilon_{\delta<} = \int_0^k \frac{d^3k'}{(2\pi)^3} P_{11}(k') ,$$

Effect of Long Overdensities

$$\epsilon_{s<} = k^2 \int_0^k \frac{d^3k'}{(2\pi)^3}\frac{P_{11}(k')}{k'^2} ,$$

Effect of Long Displacements:
Lagrangian does not expands in this

Connecting with the Eulerian Treatment

- Expand in all parameters (Eulerian treatment)
- The resulting equations are equivalent to Eulerian fluid-like equations

$$\nabla^2\phi = H^2\frac{\delta\rho}{\rho}$$

$$\partial_t\rho + H\rho + \partial_i(\rho v^i) = 0$$

$$\dot{v}^i + Hv^i + v^j\partial_j v^i = \frac{1}{\rho}\partial_j\tau^{ij}$$

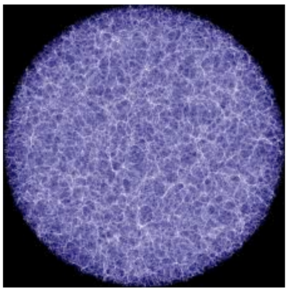
  – here it appears a non trivial stress tensor for the long-distance fluid

$$\tau_{ij} = p_0\,\delta_{ij} + c_s^2\,\delta_{ij}\,\partial^2\delta\rho + \ldots$$

Perturbation Theory
with the EFT

A non-renormalization theorem

- Can the short distance non-linearities change completely the overall expansion rate of the universe, possibly leading to acceleration without Λ ?



- In terms of the short distance perturbation, the effective stress tensor reads

$$\rho_L = \rho_S \left(1 + v_S^2 + \Phi_S\right)$$
$$p_L = \rho_S \left(2 v_S^2 + \Phi_L\right)$$

- when objects virialize, the induced pressure vanish
  – ultraviolet modes do not contribute (like in SUSY)
- The backreaction is dominated by modes at the virialization scale

$$\Rightarrow \quad w_{\text{induced}} \sim 10^{-5}$$

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**

Perturbation Theory within the EFT

- In the EFT we can solve iteratively (loop expansion) $\delta_\ell, v_\ell, \Phi_\ell \ll 1$

$$\nabla^2\phi = H^2 \frac{\delta\rho}{\rho}$$
$$\partial_t \rho + H\rho + \partial_i(\rho v^i) = 0$$
$$\dot{v}^i + H v^i + v^j \partial_j v^i = \frac{1}{\rho}\partial_j \tau^{ij}$$

$$\tau_{ij} = p_0\, \delta_{ij} + c_s^2\, \delta_{ij}\, \partial^2 \delta\rho$$

Perturbation Theory within the EFT

- Regularization and renormalization of loops (scaling universe)
  - evaluate with cutoff. By dim analysis:

$$P_{1-\text{loop}} = c_0^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right)^2 \left(\frac{k}{k_{\rm NL}}\right) P_{11} + c_1^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right) \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}$$

$$+c_2^\Lambda \log\left(\frac{\Lambda}{k_{\rm NL}}\right) \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + c_1^{\rm finite} \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}}$$

Perturbation Theory within the EFT

- Regularization and renormalization of loops (scaling universe)
  - evaluate with cutoff. By dim analysis:

$$P_{1-\text{loop}} = c_0^\Lambda \cancel{\left(\frac{\Lambda}{k_{\rm NL}}\right)^2 \left(\frac{k}{k_{\rm NL}}\right)} P_{11} + c_1^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right) \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}$$

$$+c_2^\Lambda \cancel{\log\left(\frac{\Lambda}{k_{\rm NL}}\right) \left(\frac{k}{k_{\rm NL}}\right)^3} P_{11} + c_1^{\rm finite} \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}}$$

  - absence of counterterm $\tau_{ij} = p_0\,\delta_{ij} + c_s^2\,\delta_{ij}\,\partial^2\delta\rho$

Perturbation Theory within the EFT

- Regularization and renormalization of loops (scaling universe)
  - evaluate with cutoff. By dim analysis:

$$P_{1-\text{loop}} = \cancel{c_0^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right)^2 \left(\frac{k}{k_{\rm NL}}\right)} P_{11} + c_1^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right) \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}$$
$$+ c_2^\Lambda \log \cancel{\left(\frac{\Lambda}{k_{\rm NL}}\right) \left(\frac{k}{k_{\rm NL}}\right)^3} P_{11} + c_1^{\rm finite} \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}} \; \frac{k}{{}^{,}{\rm NL}}$$

  - absence of counterterm $\quad \tau_{ij} = p_0\, \delta_{ij} + c_s^2\, \delta_{ij}\, \partial^2 \delta\rho$

$$\Longrightarrow \qquad P_{1-\text{loop},\ \text{counter}} = c^\Lambda_{\rm counter} \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}$$

$$\Longrightarrow \qquad c^\Lambda_{\rm counter} = -c_1^\Lambda + \delta c_{\rm counter} \left(\frac{k_{\rm NL}}{\Lambda}\right)$$

$$\Longrightarrow$$
$$P_{1-\text{loop}} + P_{1-\text{loop},\ \text{counter}} = \delta c_{\rm counter} \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\rm finite} \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}$$

Calculable terms in the EFT

- Has everything being lost?

$$P_{1-\text{loop}} + P_{1-\text{loop},\ \text{counter}} = \underset{\nearrow}{\delta c_{\rm counter}} \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\rm finite} \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}$$

  - to make result finite, we need to add a counterterm with finite part
    - need to fit to data (like a coupling constant), but cannot fit the k-shape

Calculable terms in the EFT

- Has everything being lost?

$$P_{1-\text{loop}} + P_{1-\text{loop, counter}} = \delta c_{\text{counter}} \left(\frac{k}{k_{\text{NL}}}\right)^2 P_{11} + c_1^{\text{finite}} \left(\frac{k}{k_{\text{NL}}}\right)^3 P_{11}$$

  – to make result finite, we need to add a counterterm with finite part

  - need to fit to data (like a coupling constant), but cannot fit the k-shape

  – the subleading finite term is not degenerate with a counterterm.

  - it cannot be changed
  - it is calculable by the EFT

    – so it predicts an observation $c_1^{\text{finite}} = 0.044$

Lesson from Renormalization

- Each loop-order L contributed a finite, calculable term of order

$$P_{\text{L}-\text{loops}} \sim \left(\frac{k}{k_{\text{NL}}}\right)^L$$

  – each higher-loop is smaller and smaller

- This happens after canceling the divergencies with counterterms

$$P_{\text{L}-\text{loops; without counterterms}} = \left(\frac{\Lambda}{k_{\text{NL}}}\right)^L \frac{k^2}{k_{\text{NL}}^2} P(k)$$

  - each loop contributes the same

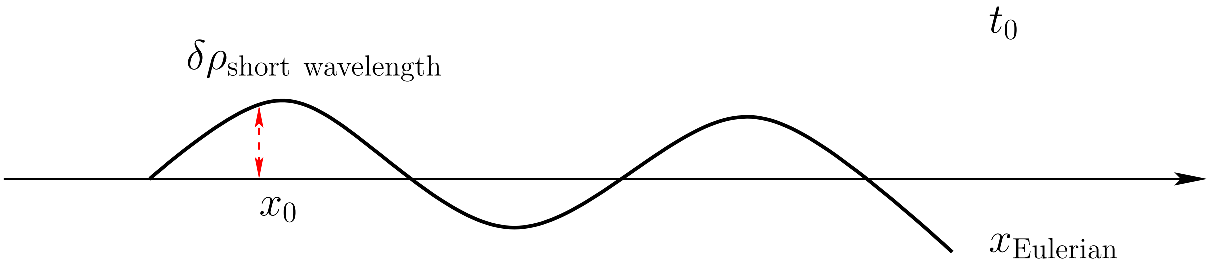
- Up to 2-loops, we need only the 1-loop counterterm

IR-resummation

with Zaldarriaga **1404**

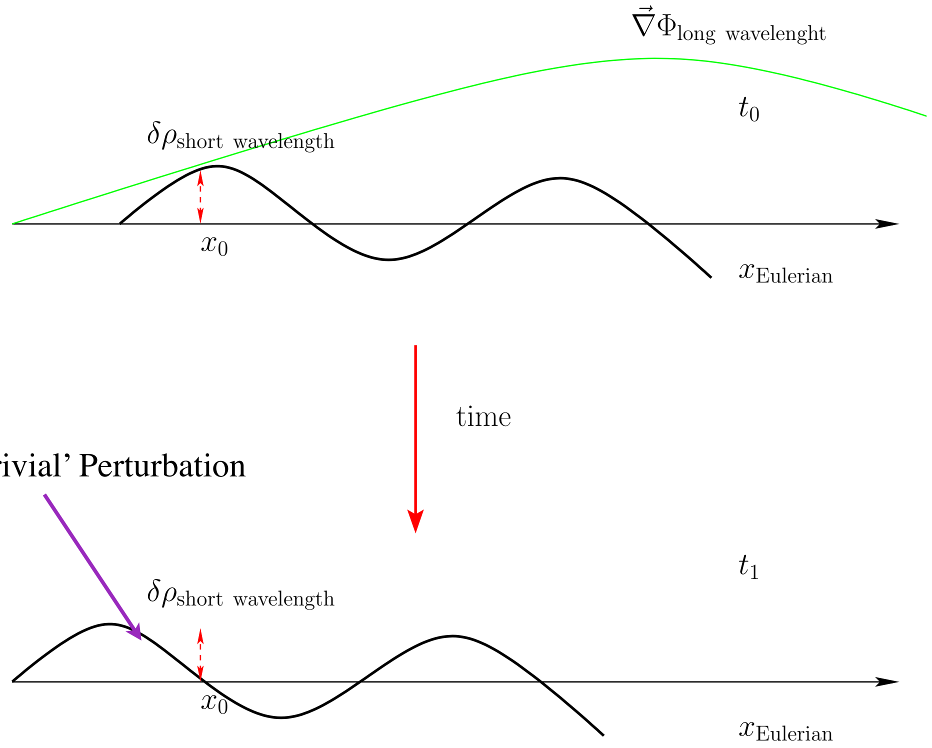
The Effect of Long-modes on Shorter ones

- In Eulerian treatment



The Effect of Long-modes

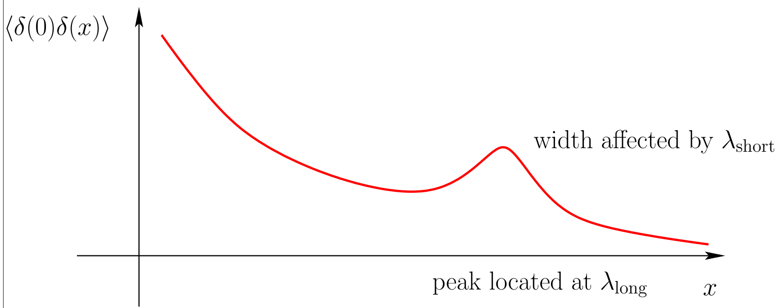
- Add a long `trivial' force (trivial by GR)
- This tells you that one can resum the IR modes: this is the Lagrangian treatment

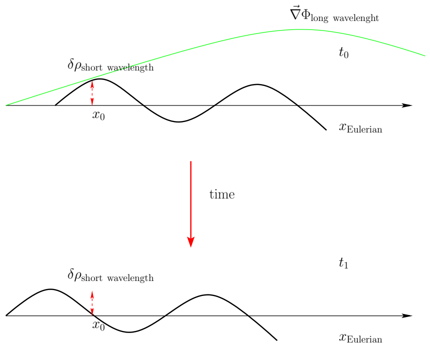


Big `trivial' Perturbation

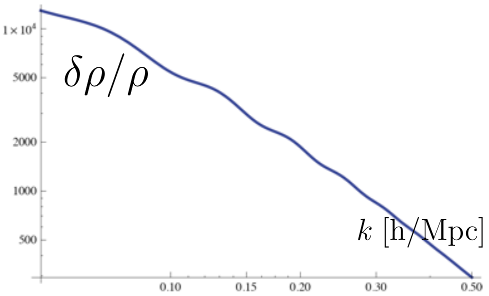
The Effect of Long-modes

with Zaldarriaga **1304**



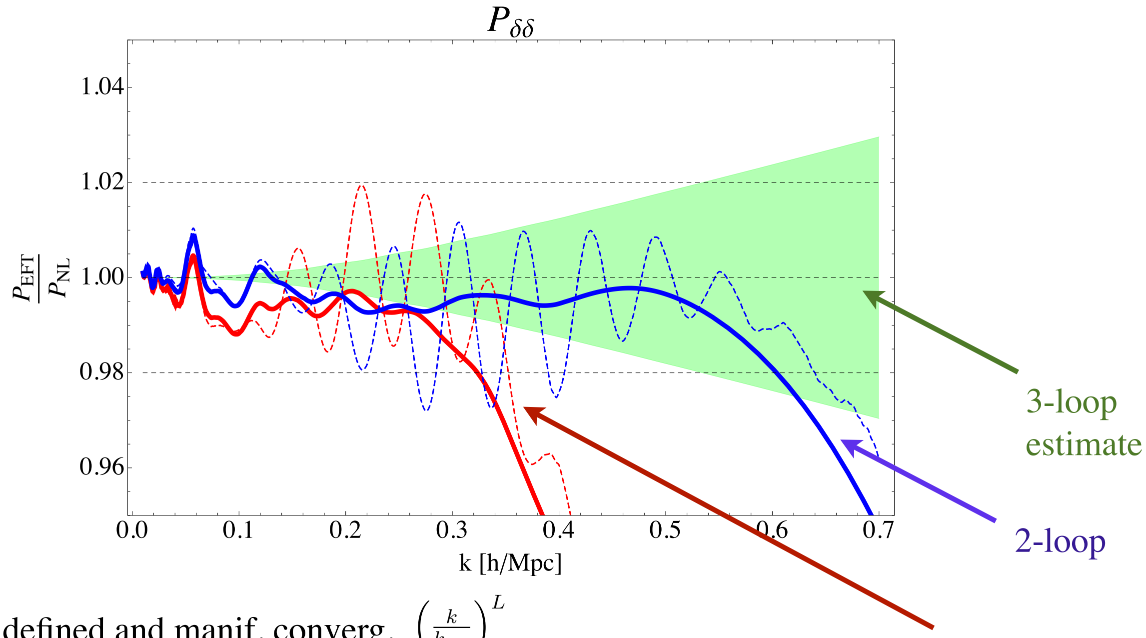


- For equal time matter correlators, naively no effect
- But the universe has features!
- Even on equal time correlators, IR modes of order the BAO scale do not cancel!
  - In Fourier space these are the wiggles
- To compute the width, IR-BAO modes are relevant
- But they just do kinematics, so we can resum them!



Results

EFT of Large Scale Structures



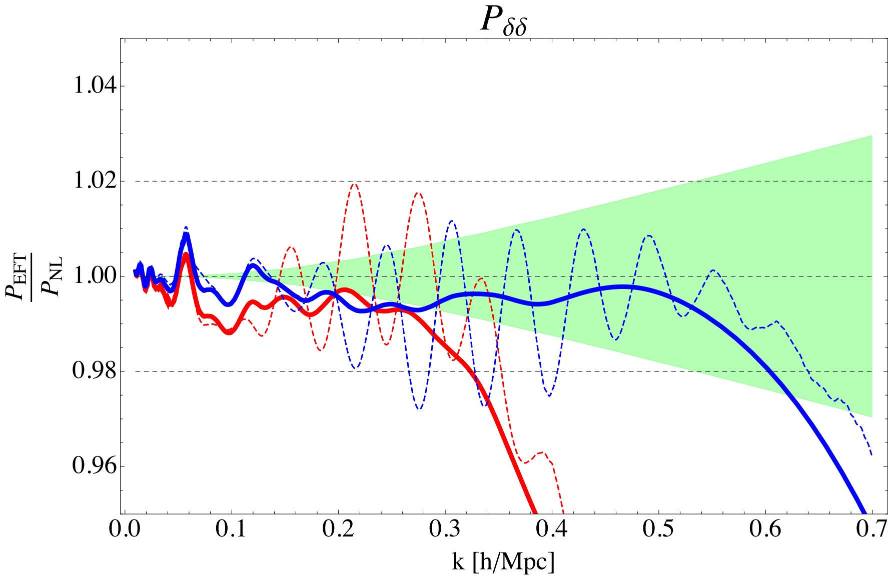
3-loop estimate

2-loop

1-loop

- Well defined and manif. converg. $\left(\frac{k}{k_{\rm NL}}\right)^L$
- Every perturbative order improves the agreement as it should
- We know when we should fail, and we fail when we should

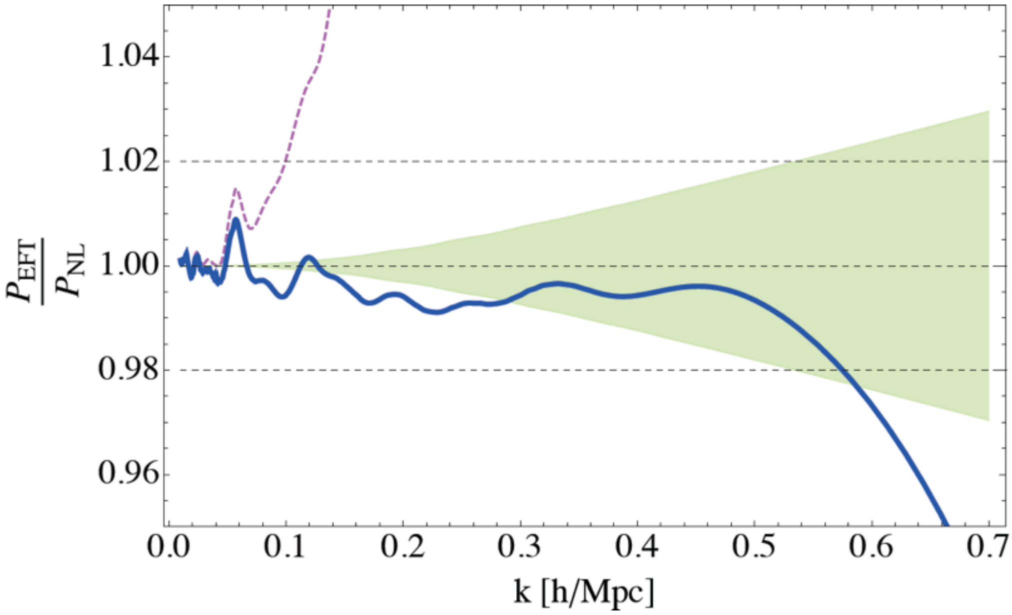
EFT of Large Scale Structures



- The lines with oscillations are obtained without resummation in the IR
  - Getting the BAO peak wrong

with Carrasco, Foreman and Green **1310**

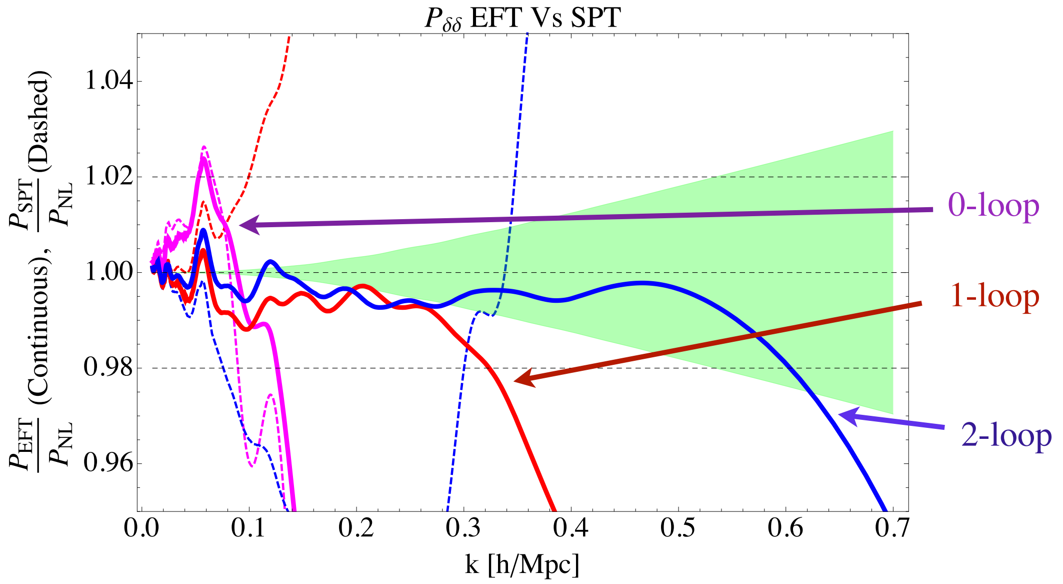
EFT of Large Scale Structures



- we fit until $k_{\rm max} \simeq 0.6\, h\, {\rm Mpc}^{-1}$, as where we should stop fitting
  - there are 200 more quasi linear modes than previously believed!

with Zaldarriaga **1404**

EFT of Large Scale Structures



- Comparison with Standard Treatment (feel free to ask about RPT)
- For the EFT, change from 1-loop to 2-loop predicted

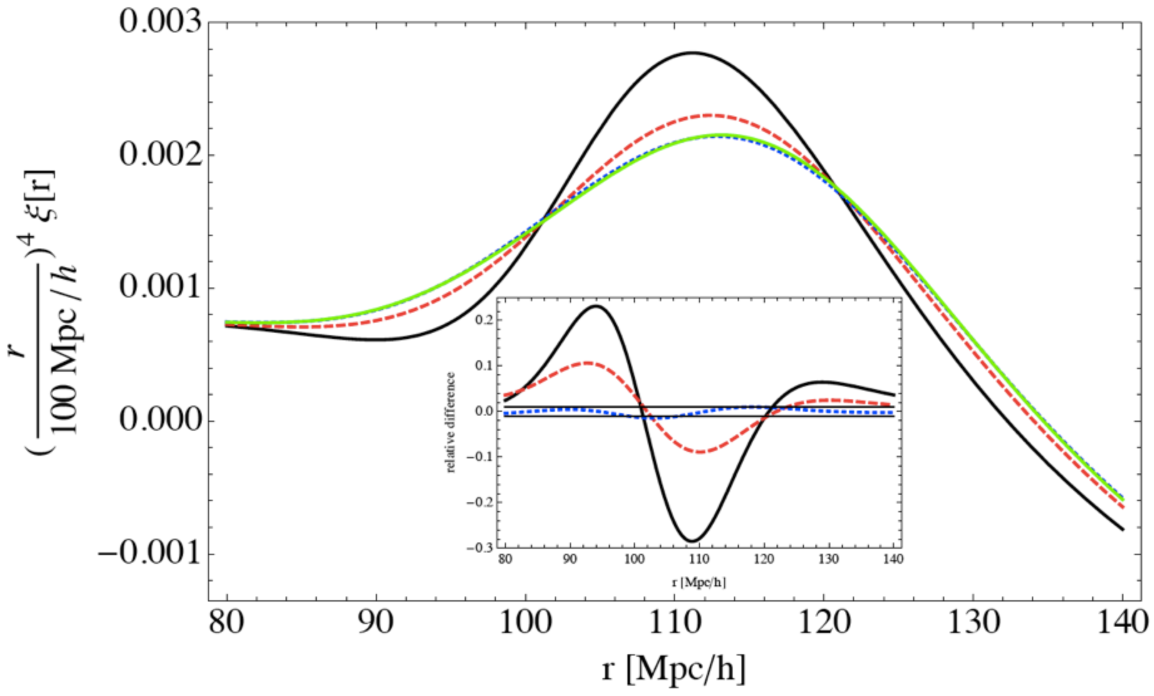
$$P_{\text{EFT-2-loop}} = P_{11} + P_{\text{1-loop}} + P_{\text{2-loop}} - 2\,(2\pi)(c^2_{s(1)} + c^2_{s(2)})\frac{k^2}{k^2_{\rm NL}}P_{11} + (2\pi)c^2_{s(1)}P^{(c_s,p)}_{\text{1-loop}} + (2\pi)^2 c^4_{s(1)}\frac{k^4}{k^4_{\rm NL}}P_{11}$$

  - the other new terms are clearly important
  - they `conspire' to the right answer

The BAO peak in `5 minutes'

- The IR-resummation is crucial to get the BAO peak right.
  - we can do this very quickly.

with Zaldarriaga **1404**

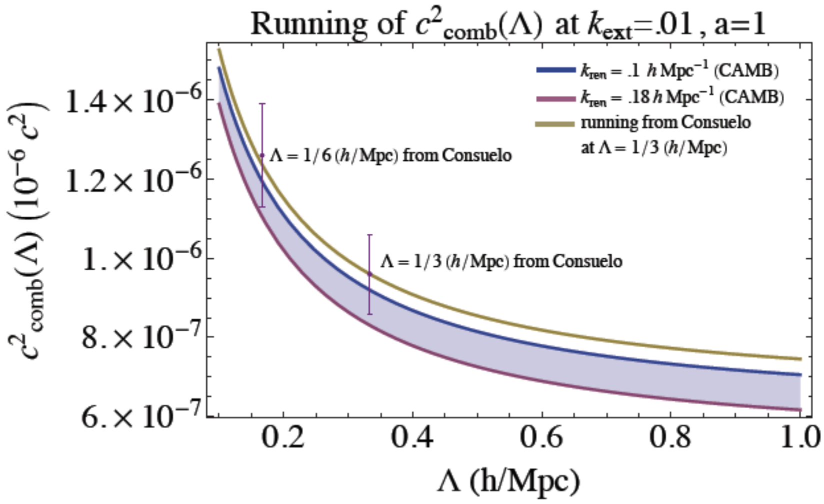


Measuring Parameters from small N-body Simulations

Measuring parameters from N-body sims.

- The EFT parameters can be measured from small N-body simulations
  - similar to what happens in QCD: lattice sims
- As you change smoothing scale, the result changes

$$\frac{d\,c_s}{d\Lambda} = \frac{d}{d\,\Lambda}\int^{\Lambda} d^3k\; P_{13}(k)$$



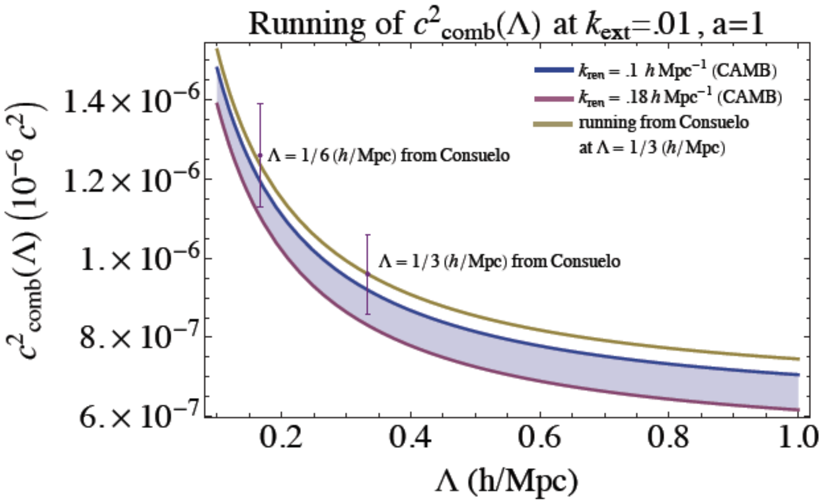
- Perfect agreement with fitting at low energies
  - like measuring F_π from lattice sims and $\pi\pi$ scattering

with Carrasco and Hertzberg **JHEP 2012**

Measuring parameters from N-body sims.

- The EFT parameters can be measured from small N-body simulations
  - similar to what happens in QCD: lattice sims
- As you change smoothing scale, the result changes

$$\frac{d\,c_s}{d\Lambda} = \frac{d}{d\,\Lambda}\int^{\Lambda} d^3k\; P_{13}(k)$$



- Perfect agreement with fitting at low energies
  - like measuring F_π from lattice sims and $\pi\pi$ scattering
  - UV dof

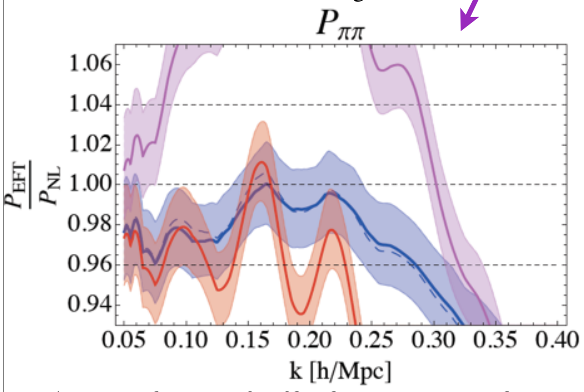
$$[\partial_i\partial_j v_k](\vec r) = [\partial_i\partial_j\pi_k](\vec r)/[\rho](\vec r) - [\partial_i\pi_k](\vec r)[\partial_j\rho]/([\rho](\vec r))^2 - [\partial_j\pi_k](\vec r)[\partial_i\rho]/([\rho](\vec r))^2$$
$$- [\pi_k](\vec r)[\partial_i\partial_j\rho](\vec r)/([\rho](\vec r))^2 + 2[\pi_k](\vec r)[\partial_i\rho](\vec r)[\partial_j\rho](\vec r)/([\rho](\vec r))^3$$

arrasco and Hertzberg **JHEP 2012**

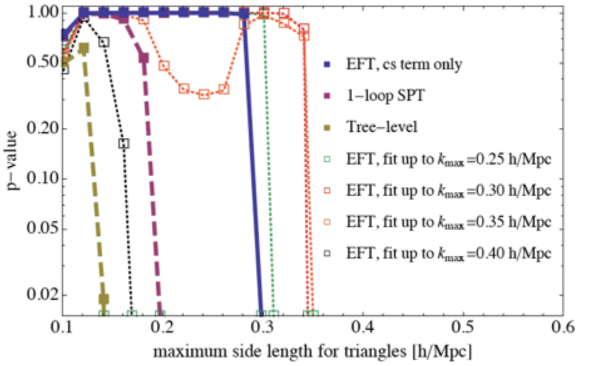
Other Observables

Momentum and Bispectrum

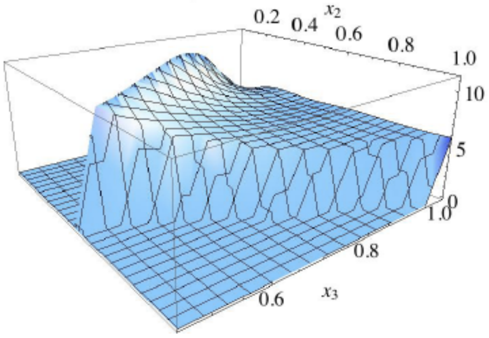
with Zaldarriaga **1404**



with Angulo, Foreman and Schmittful **1406**



- At one-loop, similarly great results
  - with no additional parameter
  - as good as they should
  - very non-trivial functional forms



- Similar formulas just worked out for Bias

Senatore **1406** See also (McDoland and Roy **0902)**

- and Redshfit space distortions

with Zaldarriaga **1409**

Velocity field

- Momentum is a natural quantity, as connected to density by conservation law
- Velocity is not a natural quantity $\vec{v}(\vec{x}) = \dfrac{\vec{\pi}(\vec{x})}{\rho(\vec{x})}$
- It is a local composite operator: needs its own new counterterms:

$$v_{l,R}(\vec{x},t) = v_l(\vec{x},t) - e_1 \partial\delta(\vec{x},t) + \cdots$$

with Carrasco, Foreman and Green **1310**

  - no new counterterm for the equations

- Because of this, and because it is a viscous fluid, we generate vorticity

$$\langle \omega_k^2 \rangle \sim \alpha_1 \left(\frac{k}{k_{\rm implement.}} \right)^2 + \alpha_2 \left(\frac{k}{k_{\rm NL}} \right)^{\sim 3}$$

  - from local counterterm
  - from viscosity

- Predicted result seems to be verified in sims

Velocity field

- Momentum is a natural quantity, as connected to density by conservation law
- Velocity is not a natural quantity $\vec{v}(\vec{x})=\dfrac{\vec{\pi}(\vec{x})}{\rho(\vec{x})}$
- It is a local composite operator: needs its own new counterterms:

$$v_{l,R}(\vec{x},t)=v_l(\vec{x},t)-e_1\partial\delta(\vec{x},t)+\cdots$$

with Carrasco, Foreman and Green **1310**

  – no new counterterm for the equations

- Because of this, and because it is a viscous fluid, we generate vorticity

$$\langle\omega_k^2\rangle\sim\alpha_1\left(\frac{k}{k_{\text{implement.}}}\right)^2+\alpha_2\left(\frac{k}{k_{\text{NL}}}\right)^{\sim 3}$$

  – from local counterterm

  – from viscosity

- Predicted result seems to be verified in sims

Velocity field

- Momentum is a natural quantity, as connected to density by conservation law
- Velocity is not a natural quantity $\vec{v}(\vec{x})=\dfrac{\vec{\pi}(\vec{x})}{\rho(\vec{x})}$
- It is a local composite operator: needs its own new counterterms:

$$v_{l,R}(\vec{x},t)=v_l(\vec{x},t)-e_1\partial\delta(\vec{x},t)+\cdots$$

with Carrasco, Foreman and Green **1310**

  – no new counterterm for the equations

- Because of this, and because it is a viscous fluid, we generate vorticity

$$\langle\omega_k^2\rangle\sim\alpha_1\left(\frac{k}{k_{\text{implement.}}}\right)^2+\alpha_2\left(\frac{k}{k_{\text{NL}}}\right)^{\sim 3}$$

  – from local counterterm

  – from viscosity

- Predicted result seems to be verified in sims

Velocity field

- Momentum is a natural quantity, as connected to density by conservation law
- Velocity is not a natural quantity $\vec{v}(\vec{x}) = \dfrac{\vec{\pi}(\vec{x})}{\rho(\vec{x})}$
- It is a local composite operator: needs its own new counterterms:

$$v_{l,R}(\vec{x},t) = v_l(\vec{x},t) - e_1 \partial\delta(\vec{x},t) + \cdots$$

with Carrasco, Foreman and Green **1310**

  – no new counterterm for the equations

- Because of this, and because it is a viscous fluid, we generate vorticity

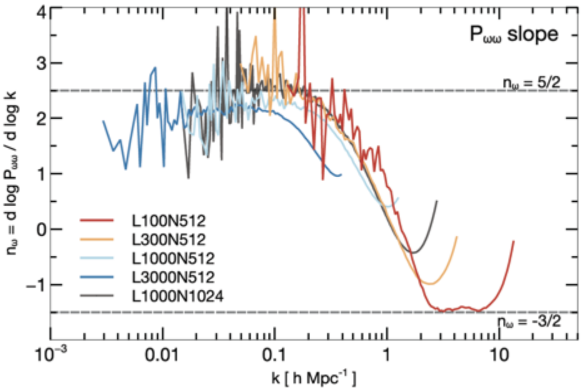
$$\langle \omega_k^2 \rangle \sim \alpha_1 \left(\frac{k}{k_{\rm implement.}}\right)^2 + \alpha_2 \left(\frac{k}{k_{\rm NL}}\right)^{\sim 3}$$

  – from local counterterm

  – from viscosity

Hahn, Angulo, Abel, **to appear**

see also Pueblas and Scoccimarro **08**



- Predicted result seems to be verified in sims

Velocity field

- Momentum is a natural quantity, as connected to density by conservation law
- Velocity is not a natural quantity $\vec{v}(\vec{x}) = \dfrac{\vec{\pi}(\vec{x})}{\rho(\vec{x})}$
- It is a local composite operator: needs its own new counterterms:

$$v_{l,R}(\vec{x},t) = v_l(\vec{x},t) - e_1 \partial\delta(\vec{x},t) + \cdots$$

with Carrasco, Foreman and Green **1310**

  – no new counterterm for the equations

- Because of this, and because it is a viscous fluid, we generate vorticity

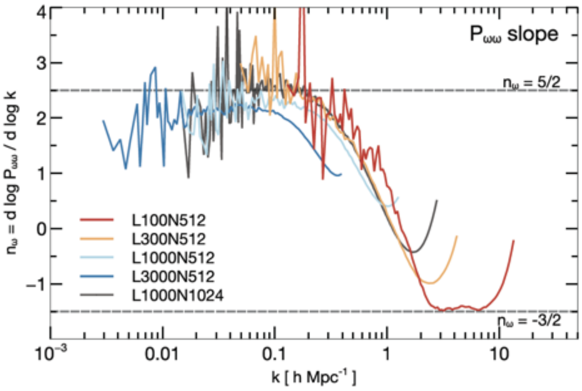
$$\langle \omega_k^2 \rangle \sim \alpha_1 \left(\frac{k}{k_{\rm implement.}}\right)^2 + \alpha_2 \left(\frac{k}{k_{\rm NL}}\right)^{\sim 3}$$

  – from local counterterm

  – from viscosity

Hahn, Angulo, Abel, **to appear**

see also Pueblas and Scoccimarro **08**



- Predicted result seems to be verified in sims

- Former analytic techniques got zero

End to SPT-like resummations

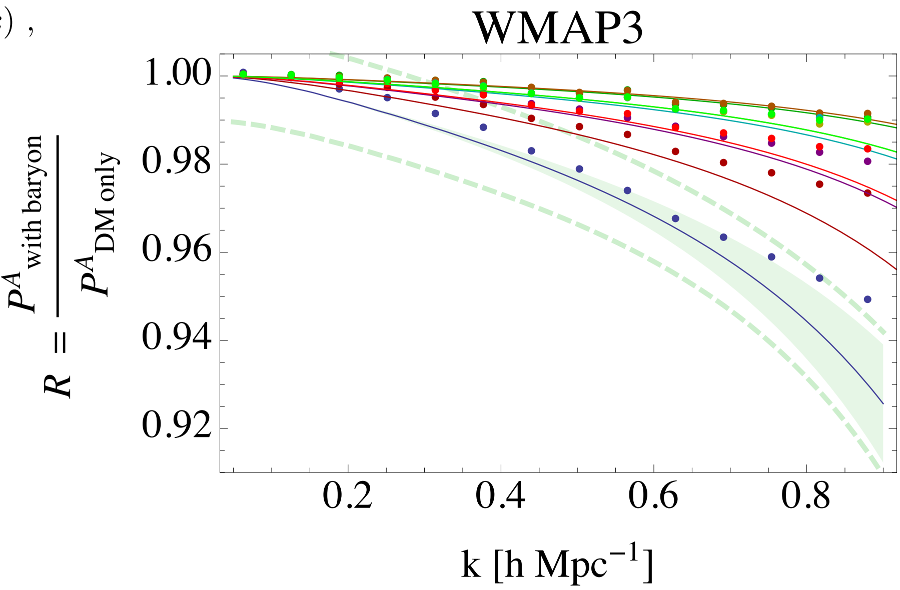
Baryonic Effects

with Lewandoski and Perko **to appear**

Baryons

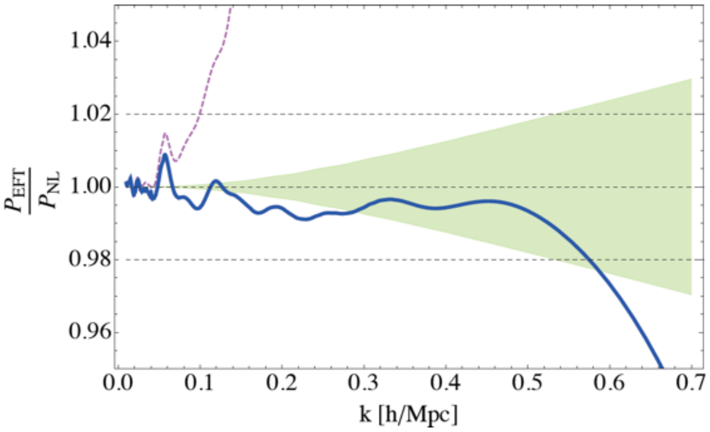
- Baryons heat, but do not move $\Longrightarrow$ they can be described as extended objects
  - Universe with CDM+Baryons $\Longrightarrow$ EFTofLSS with 2 species
  - The functional form is predicted by the EFTofLSS

$$\Delta P_b(k) \propto \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}^A(k) \ ,$$



- Awesome!

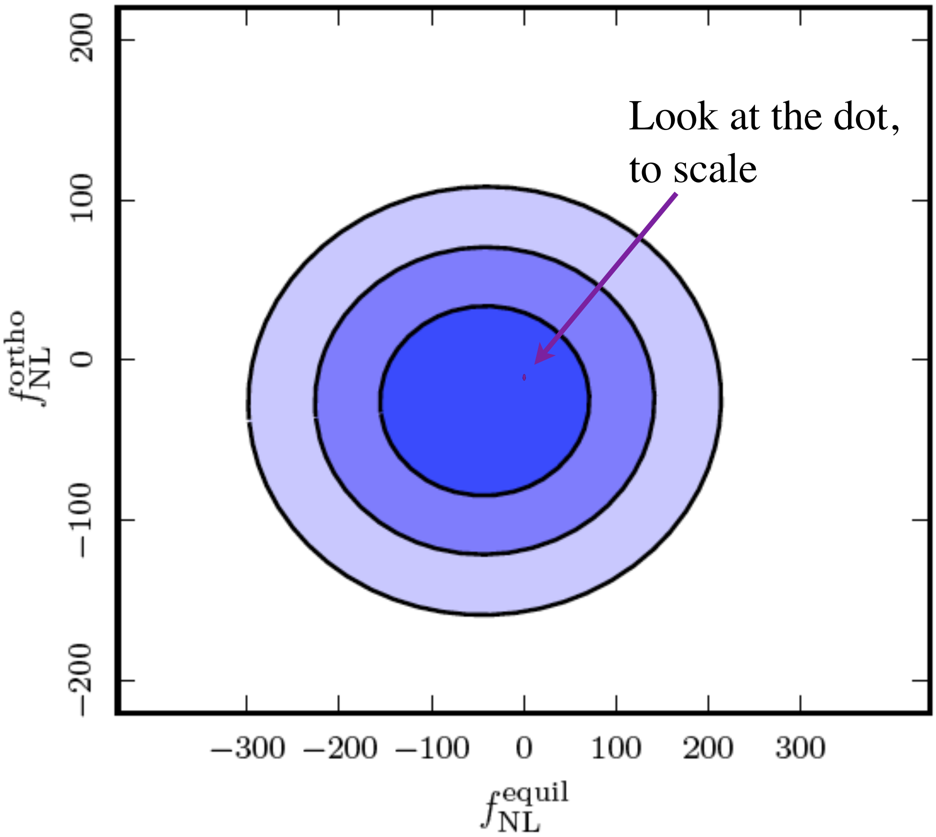
EFT of Large Scale Structures



- A manifestly convergent perturbation theory $\left(\frac{k}{k_{\rm NL}}\right)^L$
- we fit until $k_{\rm max} \simeq 0.6\, h\,{\rm Mpc}^{-1}$, as where we should stop fitting
  - – there are 200 more quasi linear modes than previously believed!
  - – huge impact on possibilities, for ex: $f_{\rm NL}^{\rm equil.,\,orthog.} \lesssim 1$
- Can all of us handle it?! This is an huge opportunity and a challenge for us

---

With this



Look at the dot, to scale

Conclusions

- Many (most?) of the features of QFT appear in the EFT of LSS:
  - – Loops, divergencies, counterterms and renormalization
  - – non-renormalization theorems
  - – Calculable and non-calculable terms
  - – Measurements in lattice and lattice-running
  - – IR-divergencies
- Results seem to be amazing, many calculations and verifications to do:
  - – like if we just learned perturbative QCD, and LHC was soon turning on
    - higher n-point functions
    - Validation with simulation
  - – With a growing number of groups (Caltech, Princeton, IAS, Cambridge, CEA, Zurich..., just after 2-loop result, a workshop was organized by Princeton)
- If this works, the 10-yr future of Early Cosmology is good, even with no luck

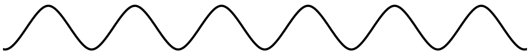
---

Make Peace and no War

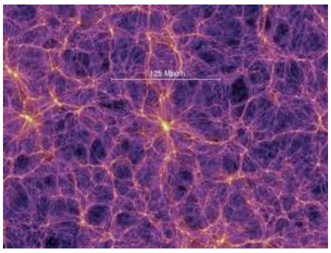
- Let us not fight between Simulations and Perturbation Theory

Perturbation Theory *and* Simulations

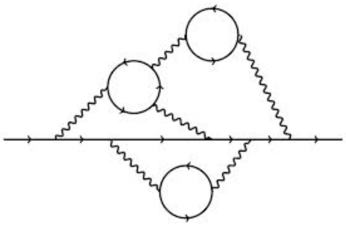
- There is room for everybody: the two approaches are *complementary*



Short Wavelengths:
Simulations



Long Wavelengths:
Perturbation Theory



“Chromo — Multi Natural Inflation”

Ippei Obata

[JGRG24(2014)111311]

Chromo – Multi Natural Inflation

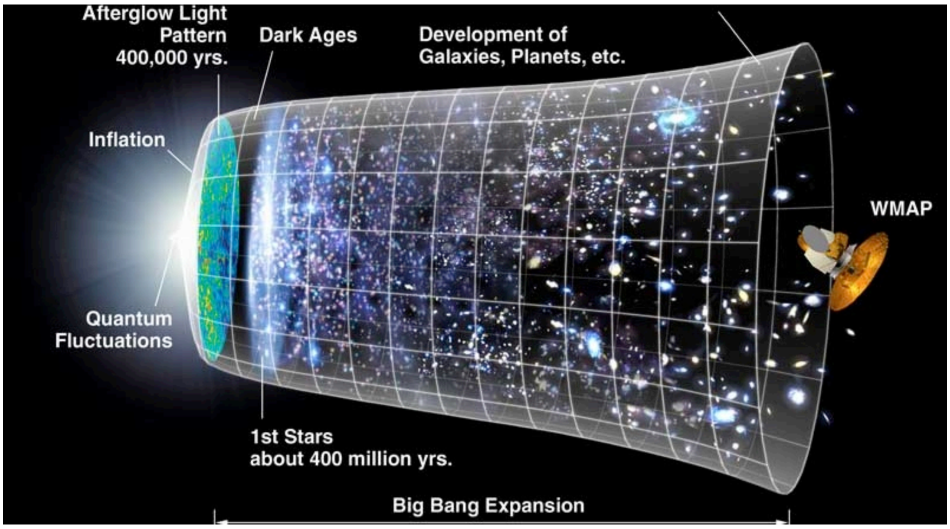
Ippei Obata (Kyoto univ. M2)
Collaborators: Takashi Miura and Jiro Soda
(Kobe univ.)

Table of contents

1. Introduction
2. Chromo - Natural Inflation
3. Chromo - Multi Natural Inflation
4. Summary and Outlook

Table of contents

Inflationary paradigm



$$\text{volume} : a(t)^3 \simeq e^{3Ht}, \ H \equiv \dot{a}/a \simeq \text{const}$$

The Inflationary mechanism

- A scalar particle “Inflaton” occurs exponential expansion :

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}\partial^\alpha\varphi\partial_\alpha\varphi - V(\varphi)\right]$$

φ : inflaton

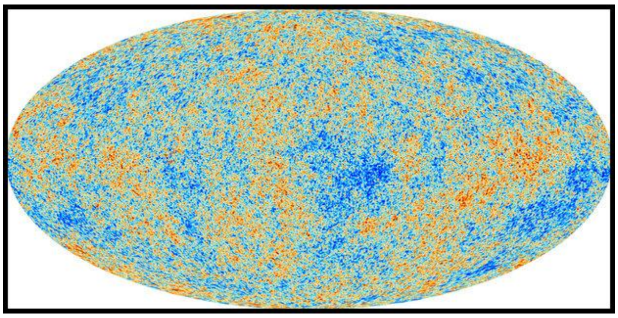
- It rolls very slowly on the slope of its potential :

$$\cancel{\ddot{\varphi}} + 3H\dot{\varphi} + V'(\varphi) = 0$$

$$\epsilon_V \equiv \frac{M_{pl}^2}{2}\left(\frac{V_\varphi}{V}\right)^2 \ll 1 \qquad \eta_V \equiv M_{pl}^2\frac{V_{\varphi\varphi}}{V} \ll 1$$

“Naturalness” of the potential parameters

The potential form is constrained by CMB obserbation.



Ex)

$$V(\varphi) = \frac{1}{2}m^2\varphi^2,\ \frac{1}{4!}\lambda\varphi^4$$

Observation

$$\frac{\delta T}{T} \sim 10^{-5} \longrightarrow m \sim 10^{13}\,\mathrm{GeV}, \quad \lambda \sim 10^{-12}$$

$$\ll \delta m \sim \Lambda_{\mathrm{UV}} \qquad \ll 1$$

"Naturalness" ← "Symmetry"

Natural Inflation

K. Freese, J. A. Frieman, and A.V. Olinto, PRL. **65**, 3234 (1990)

Use the shift symmetry of the axion !

$$\varphi \to \varphi + \mathrm{const.}$$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}\partial^\alpha\varphi\partial_\alpha\varphi - \mu^4(1-\cos(\frac{\varphi}{f}))\right]$$

φ : axion(inflaton)

We can generate small parameters dynamically:

$$m \sim \frac{\mu^2}{f}, \quad \lambda \sim \frac{\mu^4}{f^4}$$

However…

$$f \gtrsim M_{pl}$$

Table of contents

Chromo-Natural Inflation

P. Adshead and M. Wyman, PRL **108**, 261302 (2012)

Action :

$$S=\int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R-\frac{1}{2}\partial^\alpha\varphi\partial_\alpha\varphi-\mu^4\left(1-\cos\left(\frac{\varphi}{f}\right)\right)-\frac{1}{4}F^{a\alpha\beta}F^a_{\alpha\beta}-\lambda\frac{\varphi}{4f}\tilde{F}^{a\alpha\beta}F^a_{\alpha\beta}\right]$$

$$\boxed{\lambda \gg 1}$$

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\epsilon^{abc}A^b_\mu A^c_\nu$$

$$A^a{}_0=0,\;\; A^a{}_i=a(t)\phi(t)\delta^a{}_i : \mathrm{SU(2)\ gauge\ field}$$

Slow-roll parameters :

$$\epsilon_H\approx\frac{f}{\lambda}\frac{1+m_\phi^2}{m_\phi}\frac{V_\varphi}{V}\qquad \eta_H\approx\frac{f}{\lambda}\frac{1+m_\phi^2}{m_\phi}\left(\frac{2V_\varphi}{V}-\frac{V_{\varphi\varphi}}{V_\varphi}\right)$$

$$m_\phi\equiv\frac{g\phi}{H}$$

$\longrightarrow$ We can make $f \ll M_{pl}$

Remarkable prediction

P. Adshead, E. Martinec and M. Wyman, PRD**88**, no.2, 021302 (2013)

Considering tensor fluctuations...

$$ds^2=a(\tau)^2[-d\tau^2+(\delta_{ij}+h_{ij})dx^idx^j]\qquad A^a_i=a\phi\delta^a_i+t^a_i$$

interacts metric perturbation

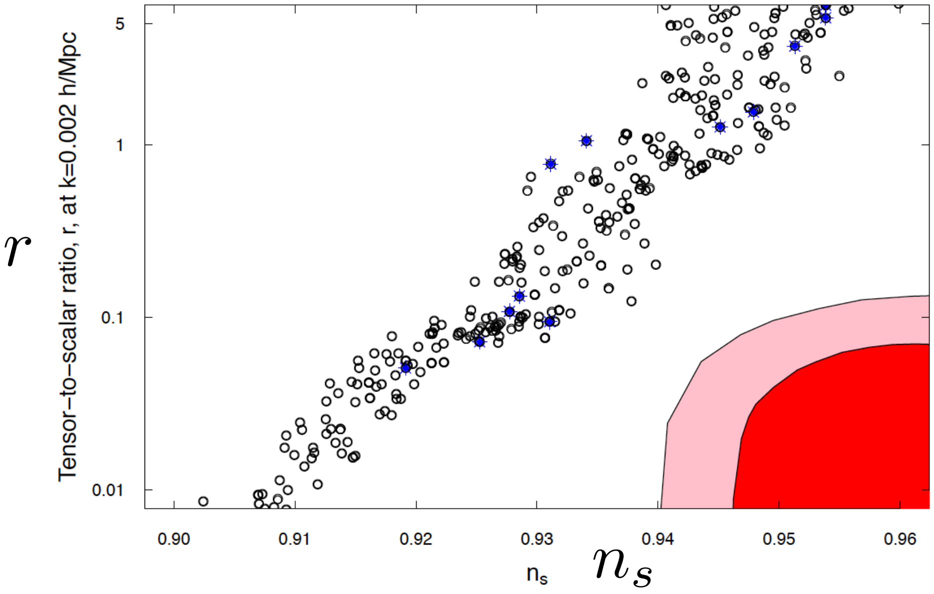
Chern-Simons term in gauge sector can produce a chiral spectrum of gravitational waves:

$$\lambda\frac{\varphi}{4f}\tilde{F}^{a\alpha\beta}F^a_{\alpha\beta}\longrightarrow\boxed{\Delta^2_{h+}(k)\ll\Delta^2_{h-}(k)}$$

This amplitude depends on mass parameter: $m_\phi=\dfrac{g\phi}{H}$

However…

CMB observational constraint :



P. Adshead, E. Martinec and M. Wyman, PRD**88**, no.2, 021302 (2013)

Table of contents

1. Introduction
2. Chromo - Natural Inflation
3. Chromo - Multi Natural Inflation
4. Summary and Outlook

Chromo - Multi Natural Inflation

- Action: $(M_{pl} = 1)$

$$S = \int dx^4 \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}(\partial_\mu \omega)^2 - V(\chi,\omega) - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\left(\lambda_\chi \frac{\chi}{f} + \lambda_\omega \frac{\omega}{h}\right)\tilde{F}^{a\mu\nu}F^a_{\mu\nu}\right]$$

- Potential:

$$V(\chi,\omega) = \mu^4(1-\cos\left(\frac{\chi}{f}\right)) + \mu^4(1-\cos\left(\frac{\omega}{h}\right))$$
$$= \tilde{V}(\chi) + \tilde{V}(\omega)$$

$$\lambda_\chi \ll 1 \ll \lambda_\omega$$

- SU(2)Gauge:

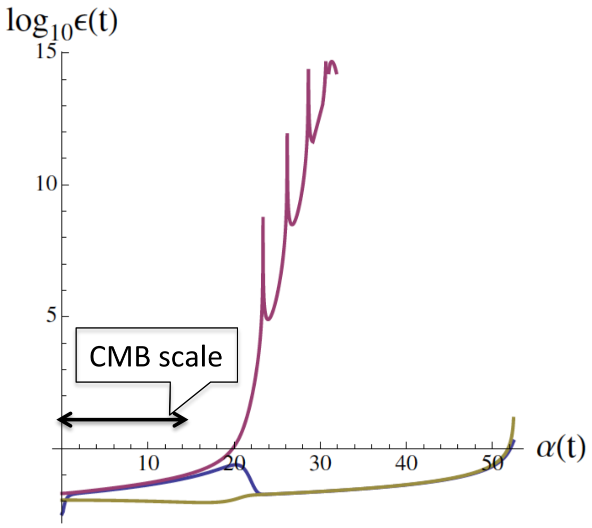
$$A^a_i = a(t)\phi(t)\delta^a_i, \; A^a_0 = 0$$

χ : Natural inflaton

ω : Chromo-Natural inflaton

Inflationary dynamics

Slow-roll parameter:



🟦 $\epsilon_H = -\dot{H}/H^2$

🟥 $\epsilon_n = \frac{1}{2}\left(\frac{\tilde{V}_\chi}{\tilde{V}(\chi)}\right)$

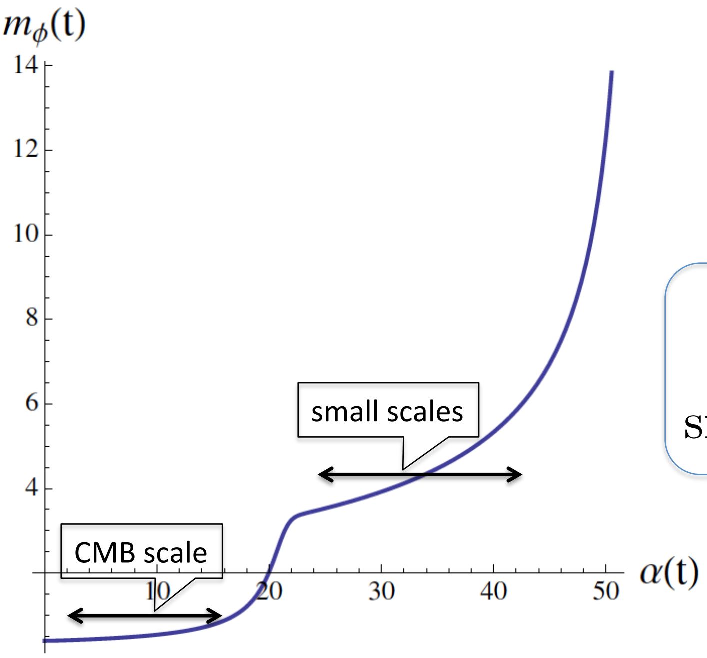
🟨 $\epsilon_{ch} = \frac{h}{\lambda_\omega}\frac{1+m_\phi^2}{m_\phi}\frac{\tilde{V}_\omega}{\tilde{V}(\omega)}$

e-folds: 0 〜 20 ... Natural Inflation

e-folds: 20 〜 50 ... Chromo-Natural

$$(f, h, \mu, g, \lambda_\omega) = (\; 5,\; 5\times 10^{-4}, 10^{-2},\; 10^{-3},\; 1.5\times 10^{3})$$

Dynamics of mass parameter



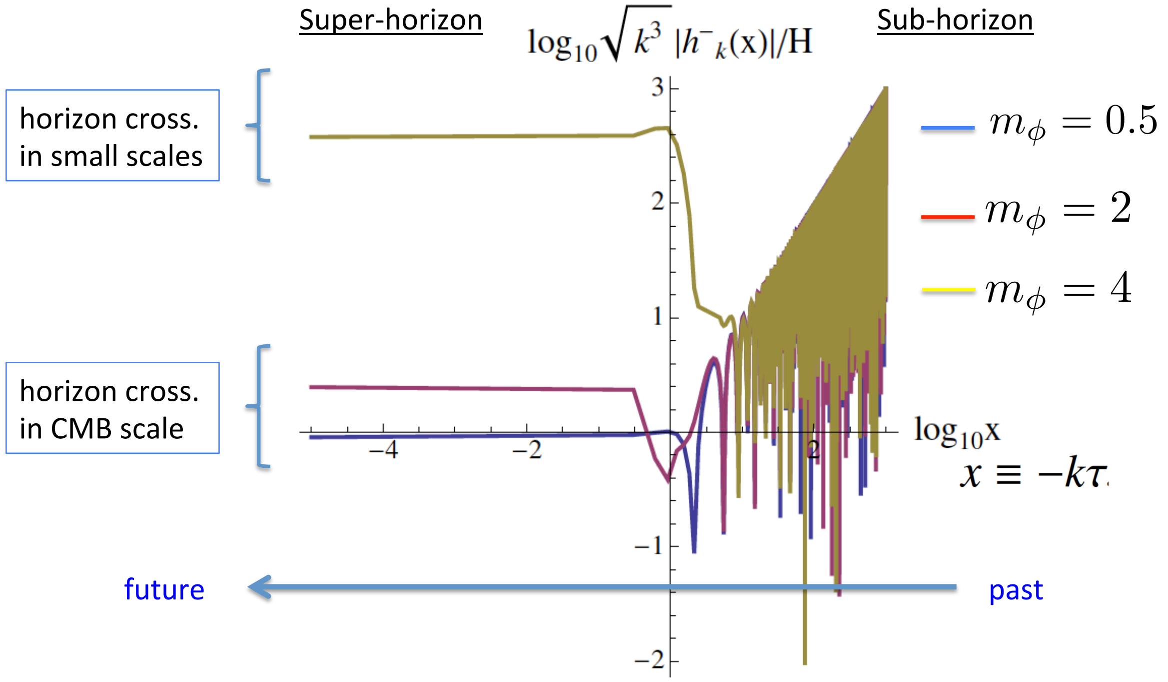
small scales

CMB scale

CMB scale : $m_\phi \sim 0.5$

small scales : $m_\phi \gtrsim 4$

Chiral gravitational waves



Super-horizon

Sub-horizon

horizon cross. in small scales

horizon cross. in CMB scale

$x \equiv -k\tau$

future

past

Table of contents

Summary and Outlook

- Chromo–Natural Inflation predicts chirally-enhanced gravitational waves. However, it is hard to satisfy CMB observational constraints.

- Our new scenario might avoid to overproduce chiral gravitational waves in the CMB scale and generate sizable chiral power spectrum in smaller scales.

- Is it possible? We leave more detailed analyses for future work.

Appendix.

The problem of Natural Inflation

In order to occur inflation…

$$V(\varphi) = \mu^4 (1 - \cos(\frac{\varphi}{f}))$$

$$\epsilon_V \equiv \frac{M_{pl}^2}{2} \left(\frac{V'}{V} \right)^2 \sim \frac{M_{pl}^2}{f^2} \ll 1 \qquad \eta_V \equiv M_{pl}^2 \frac{V''}{V} \sim \frac{M_{pl}^2}{f^2} \ll 1$$

The axion decay constant is required to have super-Planckian :

$$f \gtrsim M_{pl}$$

Chromo-Natural Inflation

- EOMs and constraint :

$$3H^2 = \frac{1}{2}\dot{\varphi}^2 + \frac{3}{2}(\dot{\phi} + H\phi)^2 + \frac{3}{2}g^2\phi^4 + V(\varphi)$$
$$\dot{H} = -\frac{1}{2}\dot{\varphi}^2 - (\dot{\phi} + H\phi)^2 - g^2\phi^4$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = -3\frac{\lambda}{f}g\phi^2(\dot{\phi} + H\phi)$$
$$\ddot{\phi} + 3H\dot{\phi} + (\dot{H} + 2H^2)\phi + 2g^2\phi^3 = \frac{\lambda}{f}g\phi^2\dot{\varphi}$$

$$\epsilon_H \approx \frac{f}{\lambda}\frac{1+m_\phi^2}{m_\phi}\frac{V'}{V} \qquad \eta_H \approx \frac{f}{\lambda}\frac{1+m_\phi^2}{m_\phi}\left(\frac{2V'}{V} - \frac{V''}{V'}\right)$$

Remarkable prediction

Considering tensor perturbations… ($M_{pl} = 1$)

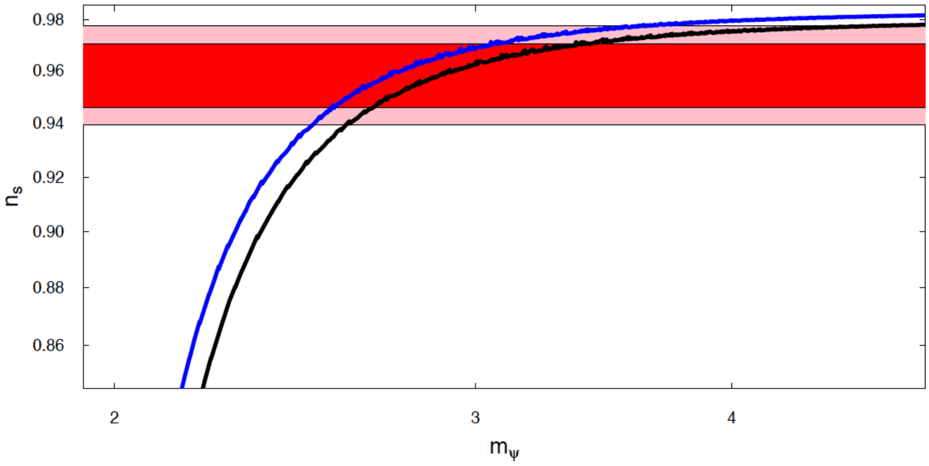
$$ds^2 = a(\tau)^2[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$
$$A_i^a = a\phi\delta_i^a + t_i^a$$
$$\psi_{ij}(\boldsymbol{x}, \tau) = 2\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\Sigma_A e_{ij}^A(\boldsymbol{k})\psi_{\boldsymbol{k}}^A(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,, \qquad \psi_{ij} \equiv a(\tau)h_{ij}$$
$$t_{ij}(\boldsymbol{x}, \tau) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\Sigma_A e_{ij}^A(\boldsymbol{k})t_{\boldsymbol{k}}^A(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,,$$

$$\frac{d^2\psi_{\boldsymbol{k}}^A}{dx^2} + \left(1 - \frac{2}{x^2} - \frac{2}{x^2}(1 - m_\phi^2)\phi^2\right)\psi_{\boldsymbol{k}}^A \approx 2\frac{\phi}{x}\frac{dt_{\boldsymbol{k}}^A}{dx} + 2m_\phi(m_\phi \pm x)\frac{\phi}{x^2}t_{\boldsymbol{k}}^A\,,$$
$$\frac{d^2t_{\boldsymbol{k}}^A}{dx^2} + \left(1 + \frac{m}{x^2} \pm \frac{m_t}{x}\right)t_{\boldsymbol{k}}^A \approx -2\phi\frac{d}{dx}\left(\frac{\psi_{\boldsymbol{k}}^A}{x}\right) + 2m_\phi(m_\phi \pm x)\frac{\phi}{x^2}\psi_{\boldsymbol{k}}^A\,,$$

Observational constraints

- Spectral index :

$$n_s - 1 \simeq -2\epsilon_H + \eta_H + 2\frac{d\log\varphi(0)}{dN}$$ ← depends on $m_\psi = \frac{g\psi}{H}$



Friedman and EOM

- Friedman equation :

$$3H^2 = \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}\dot{\omega}^2 + \frac{3}{2}(\dot{a\phi})^2 a^{-2} + \frac{3}{2}g^2\phi^4 + V$$

- EOM :

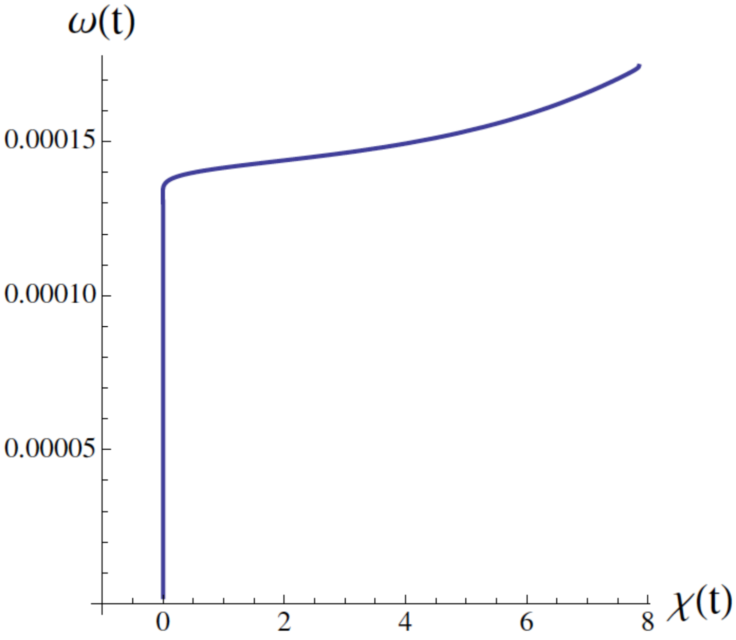
$$\ddot{\chi} + 3H\dot{\chi} + V_\chi = 0,$$

$$\ddot{\omega} + 3H\dot{\omega} + V_\omega = -\frac{g\lambda_\omega}{h}(\dot{a\phi})^3 a^{-3},$$

$$\ddot{\phi} + 3H\dot{\phi} + (\dot{H} + 2H^2)\phi + 2g^2\phi^3 = g\frac{\lambda_\omega}{h}\phi^2\dot{\omega}$$

$$\dot{H} = -\frac{1}{2}\dot{\chi}^2 - \frac{1}{2}\dot{\omega}^2 - (\dot{\phi} + H\phi)^2 - g^2\phi^4$$

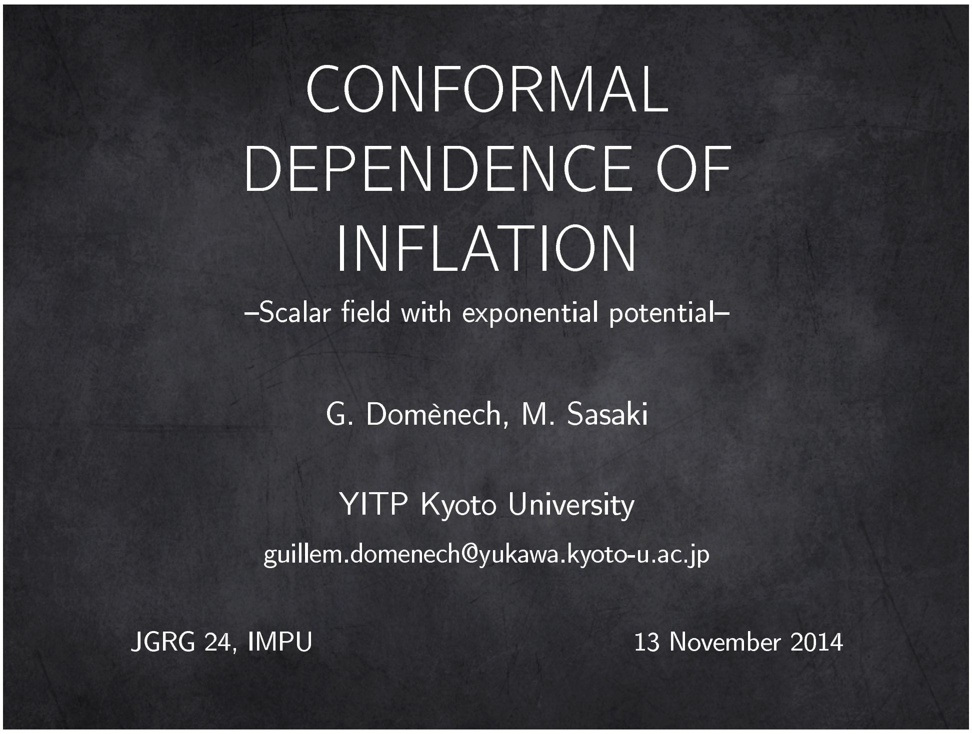
Inflationary trajectory



“Conformal dependence of inflation – scalar field with an exponential potential –”

Guillem Domenech

[JGRG24(2014)111312]



CONFORMAL DEPENDENCE OF INFLATION

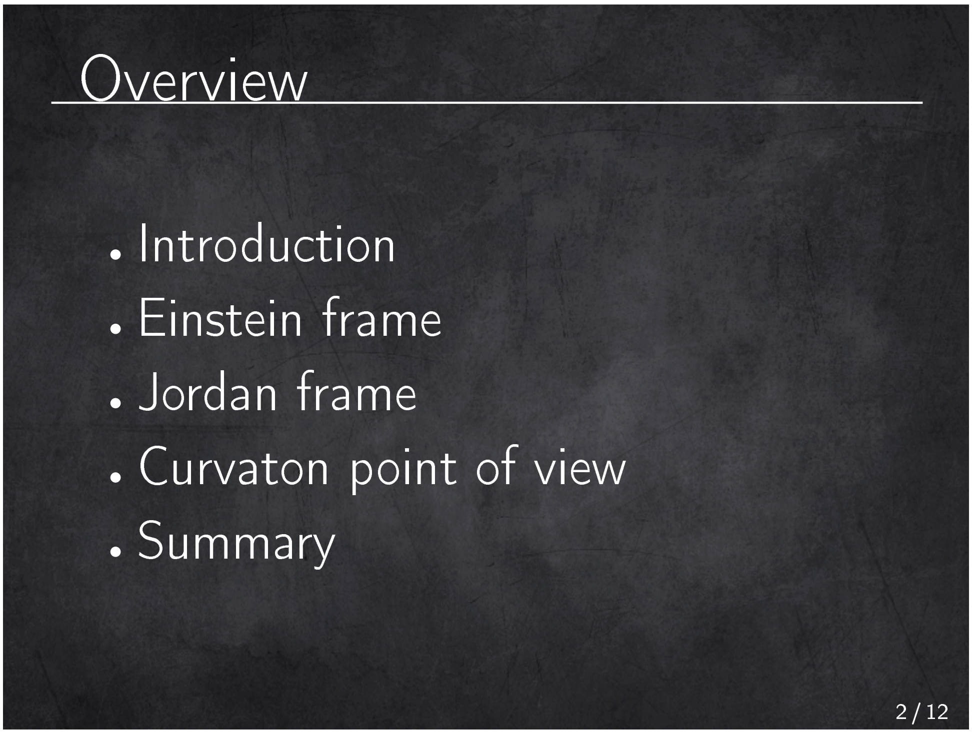
–Scalar field with exponential potential–

G. Domènech, M. Sasaki

YITP Kyoto University

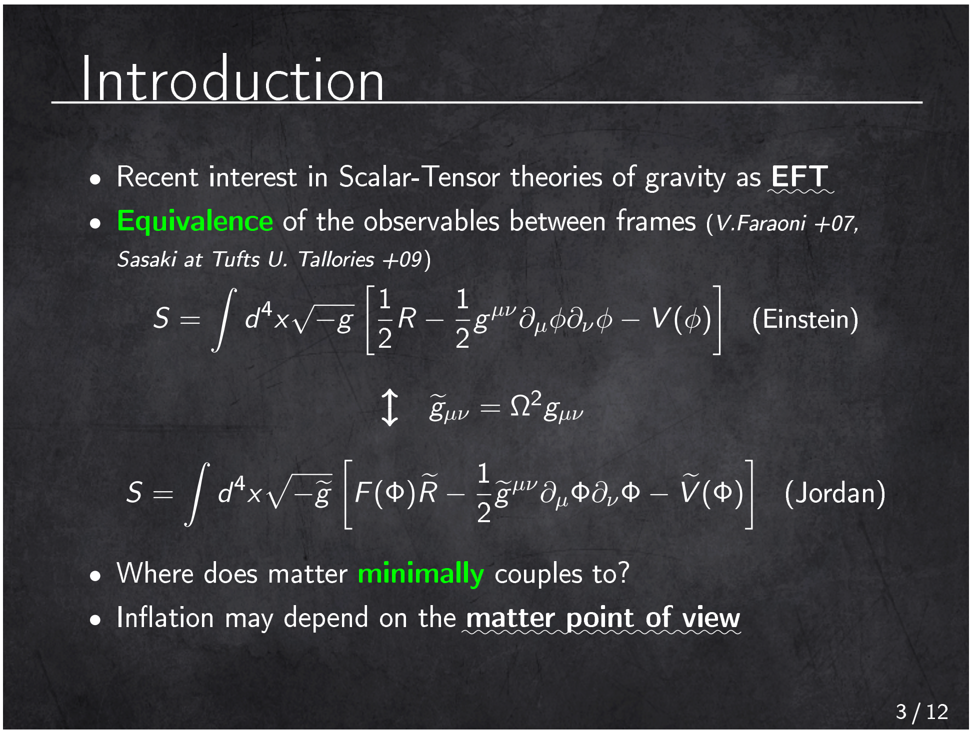
guillem.domenech@yukawa.kyoto-u.ac.jp

JGRG 24, IMPU 13 November 2014



Overview

- Introduction
- Einstein frame
- Jordan frame
- Curvaton point of view
- Summary



Introduction

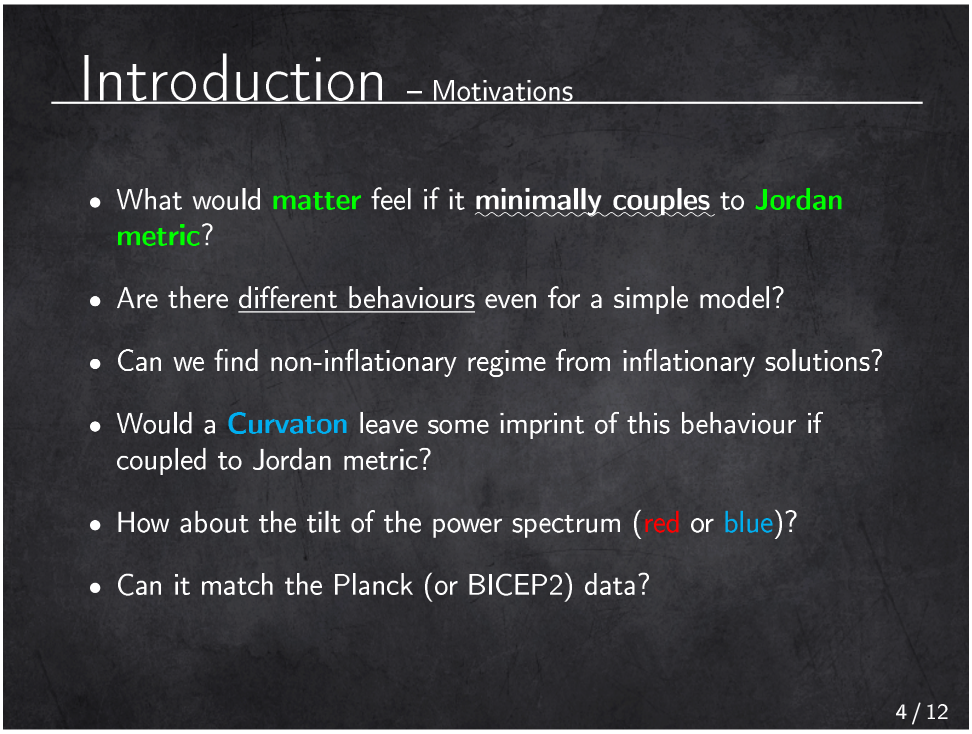
- Recent interest in Scalar-Tensor theories of gravity as **EFT**
- **Equivalence** of the observables between frames (*V.Faraoni +07, Sasaki at Tufts U. Tallories +09*)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \quad \text{(Einstein)}$$

$$\updownarrow \quad \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

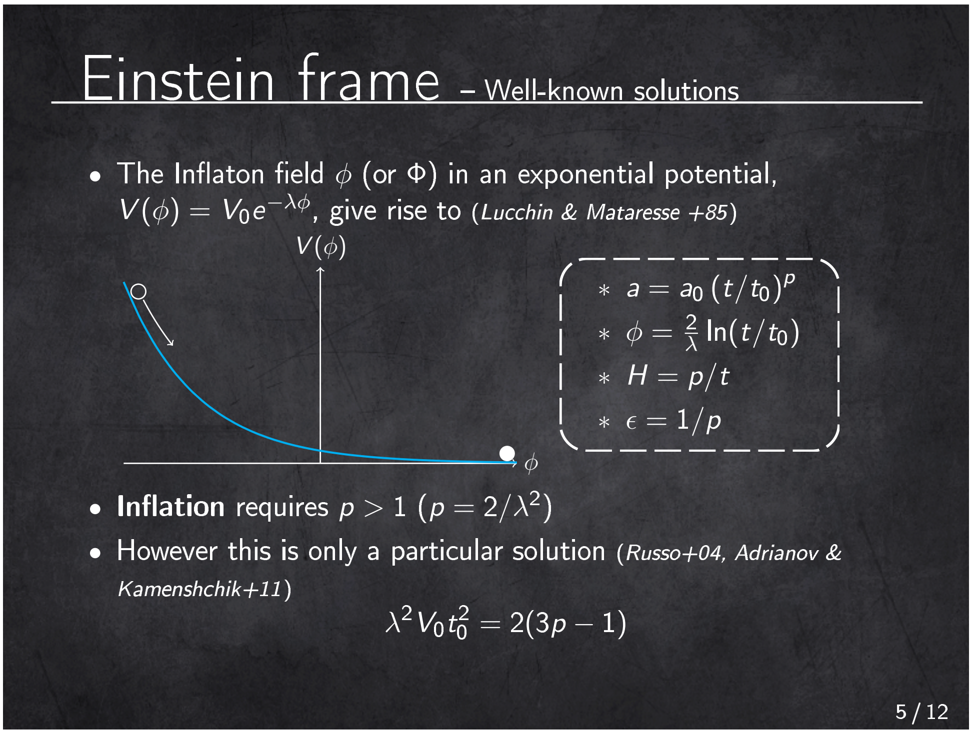
$$S = \int d^4x\sqrt{-\tilde{g}}\left[F(\Phi)\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \tilde{V}(\Phi)\right] \quad \text{(Jordan)}$$

- Where does matter **minimally** couples to?
- Inflation may depend on the **matter point of view**



Introduction – Motivations

- What would **matter** feel if it **minimally couples** to **Jordan metric**?
- Are there different behaviours even for a simple model?
- Can we find non-inflationary regime from inflationary solutions?
- Would a **Curvaton** leave some imprint of this behaviour if coupled to Jordan metric?
- How about the tilt of the power spectrum (red or blue)?
- Can it match the Planck (or BICEP2) data?



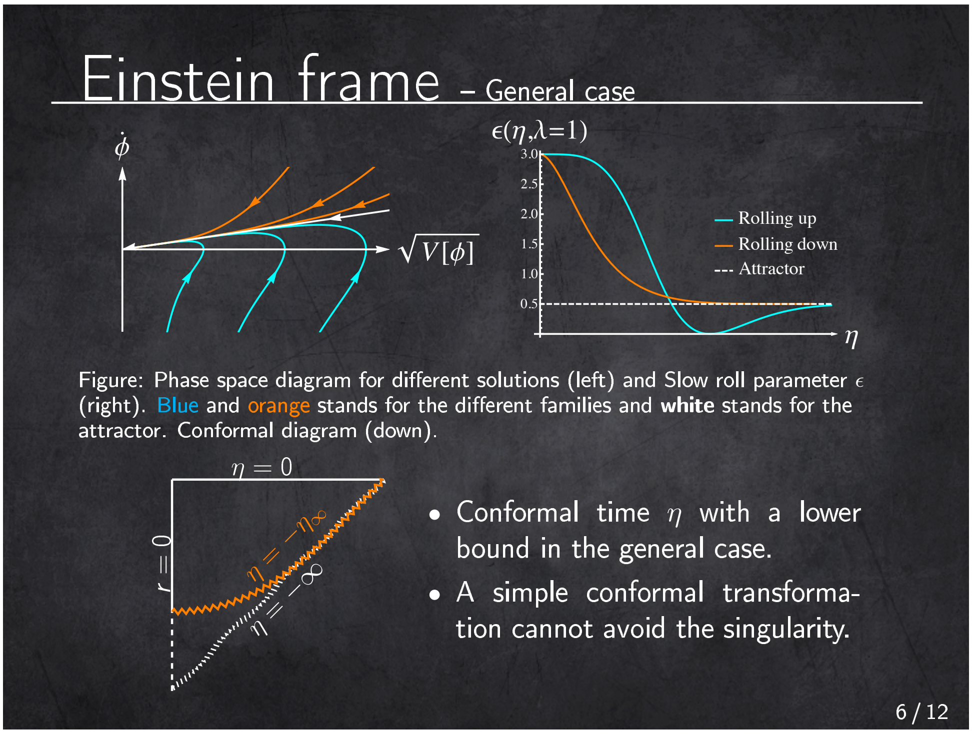
Einstein frame – Well-known solutions

- The Inflaton field ϕ (or Φ) in an exponential potential, $V(\phi) = V_0 e^{-\lambda\phi}$, give rise to (*Lucchin & Mataresse +85*)

∗ $a = a_0\,(t/t_0)^p$
∗ $\phi = \frac{2}{\lambda}\ln(t/t_0)$
∗ $H = p/t$
∗ $\epsilon = 1/p$

- **Inflation** requires $p > 1$ ($p = 2/\lambda^2$)
- However this is only a particular solution (*Russo+04, Adrianov & Kamenshchik+11*)

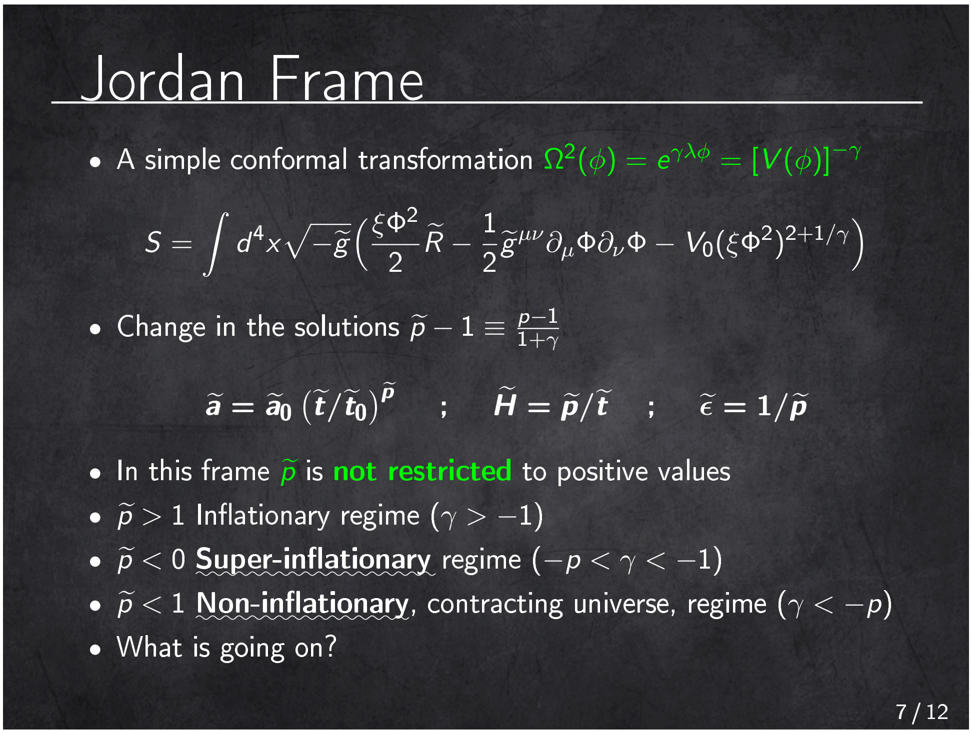
$$\lambda^2 V_0 t_0^2 = 2(3p-1)$$



Einstein frame – General case

Figure: Phase space diagram for different solutions (left) and Slow roll parameter ϵ (right). Blue and orange stands for the different families and **white** stands for the attractor. Conformal diagram (down).

- Conformal time η with a lower bound in the general case.
- A simple conformal transformation cannot avoid the singularity.



Jordan Frame

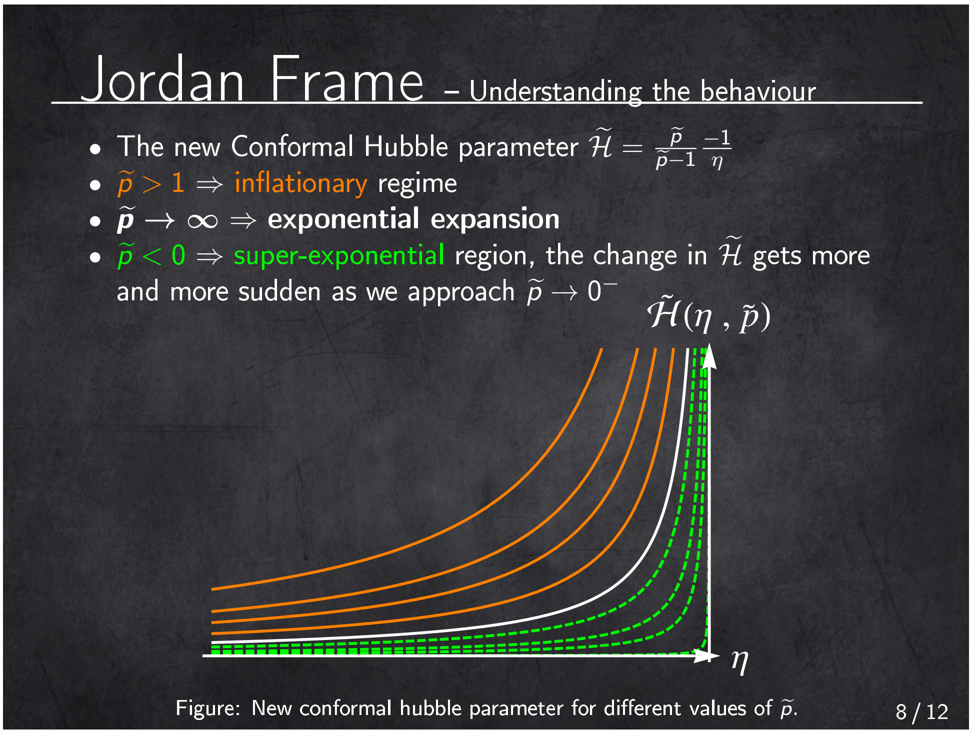
- A simple conformal transformation $\Omega^2(\phi) = e^{\gamma\lambda\phi} = [V(\phi)]^{-\gamma}$

$$S = \int d^4x\sqrt{-\tilde{g}}\left(\frac{\xi\Phi^2}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V_0(\xi\Phi^2)^{2+1/\gamma}\right)$$

- Change in the solutions $\tilde{p} - 1 \equiv \frac{p-1}{1+\gamma}$

$$\boldsymbol{\tilde{a} = \tilde{a}_0\,(\tilde{t}/\tilde{t}_0)^{\tilde{p}} \quad ; \quad \tilde{H} = \tilde{p}/\tilde{t} \quad ; \quad \tilde{\epsilon} = 1/\tilde{p}}$$

- In this frame $\tilde{p}$ is **not restricted** to positive values
- $\tilde{p} > 1$ Inflationary regime $(\gamma > -1)$
- $\tilde{p} < 0$ **Super-inflationary** regime $(-p < \gamma < -1)$
- $\tilde{p} < 1$ **Non-inflationary**, contracting universe, regime $(\gamma < -p)$
- What is going on?



Jordan Frame – Understanding the behaviour

- The new Conformal Hubble parameter $\tilde{\mathcal{H}} = \frac{\tilde{p}}{\tilde{p}-1}\frac{-1}{\eta}$
- $\tilde{p} > 1 \Rightarrow$ inflationary regime
- $\boldsymbol{\tilde{p} \to \infty \Rightarrow}$ **exponential expansion**
- $\tilde{p} < 0 \Rightarrow$ super-exponential region, the change in $\tilde{\mathcal{H}}$ gets more and more sudden as we approach $\tilde{p} \to 0^-$

Figure: New conformal hubble parameter for different values of $\tilde{p}$.

Adding a Curvaton – Power Spectrum

- Well established **frame independent** results (*Gong & Sasaki +11*) in slow-roll regime (more exact in *Garriga & Mukhanov +08*)

$$\tilde{P}_{\tilde{\mathcal{R}}}(\kappa) = P_{\mathcal{R}}(\kappa) = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 = \frac{H_0^2}{(2\pi)^2\lambda^2}\left(\frac{\kappa}{\kappa_0}\right)^{-\frac{2}{p-1}}$$

$$\boldsymbol{n_s - 1 = -\frac{2}{p-1}} \quad ; \quad \boldsymbol{r = \frac{16}{p}}$$

- The usual power-law inflation **cannot satisfy Planck** constraints ($r_{plack} < 0.1$; $n_{s,Planck} = 0.96$; $n_s > 0.987$). Although it is slightly more consistent with BICEP2 results ($r_{BICEP2} < 0.2$; $n_s > 0.974$).
- Tension with Planck can be alleviated by recalling a spectator **Curvaton** (*Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi +01*). Or by considering a non-canonical scalar field (*Unnikrishnan+13*).

Adding a Curvaton – Power spectrum

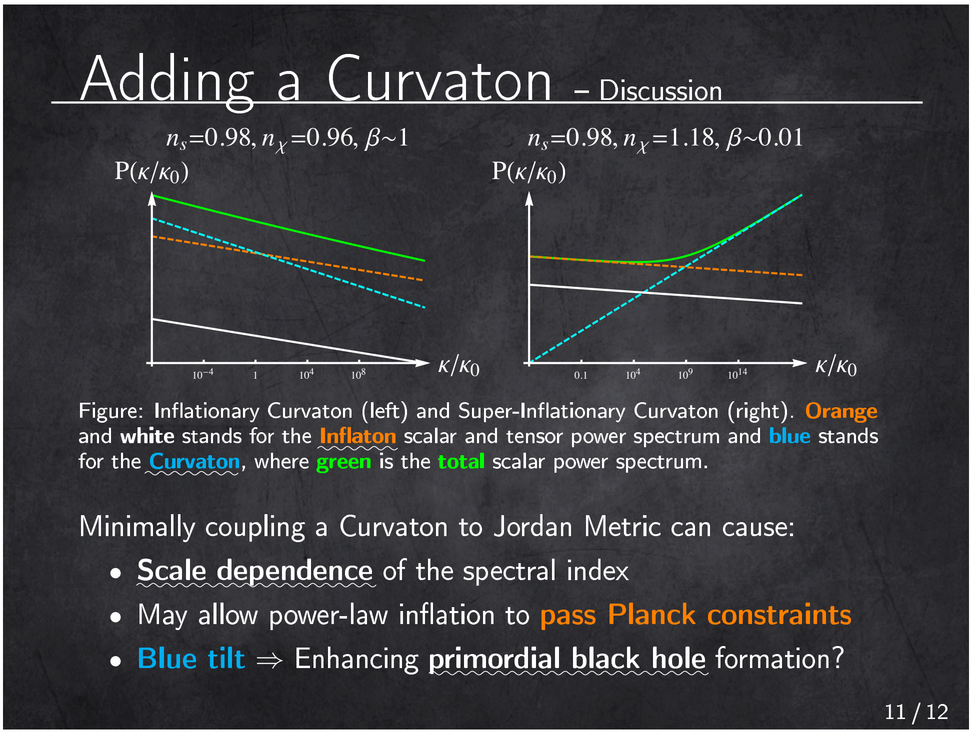
- Massless Curvaton χ minimally coupled to a frame

$$\tilde{P}_\chi(\kappa) = \frac{4\pi\kappa^3}{(2\pi)^3}|\chi_\kappa|^2 = \frac{\tilde{H}_0^2}{(2\pi)^2}\left(\frac{\tilde{\kappa}}{\tilde{\kappa}_0}\right)^{-\frac{2}{\tilde{p}-1}} \qquad \boxed{\boldsymbol{n_\chi - 1 = -\frac{2}{\tilde{p}-1}}}$$

- **Super-inflationary** Curvaton $\Rightarrow n_\chi > 1$ (**blue tilt**)
- The total Power spectrum is $P_{tot} = P_{\mathcal{R}} + \sigma P_\chi$

$$n_{tot} - 1 = \frac{d\ln(P_{\mathcal{R}} + \sigma P_\chi)}{d\ln k} \quad ; \quad r_{tot} = \frac{r}{1 + \sigma\frac{P_\chi}{P_{\mathcal{R}}}}$$

- If β is the amplitude of $\sigma\frac{P_\chi}{P_{\mathcal{R}}}$ then $r_{tot} = \frac{r}{1+\beta\kappa^{n_\chi - n_s}}$ and at the pivot scale $n_{tot} = \frac{n_s + \beta n_\chi}{1+\beta}$

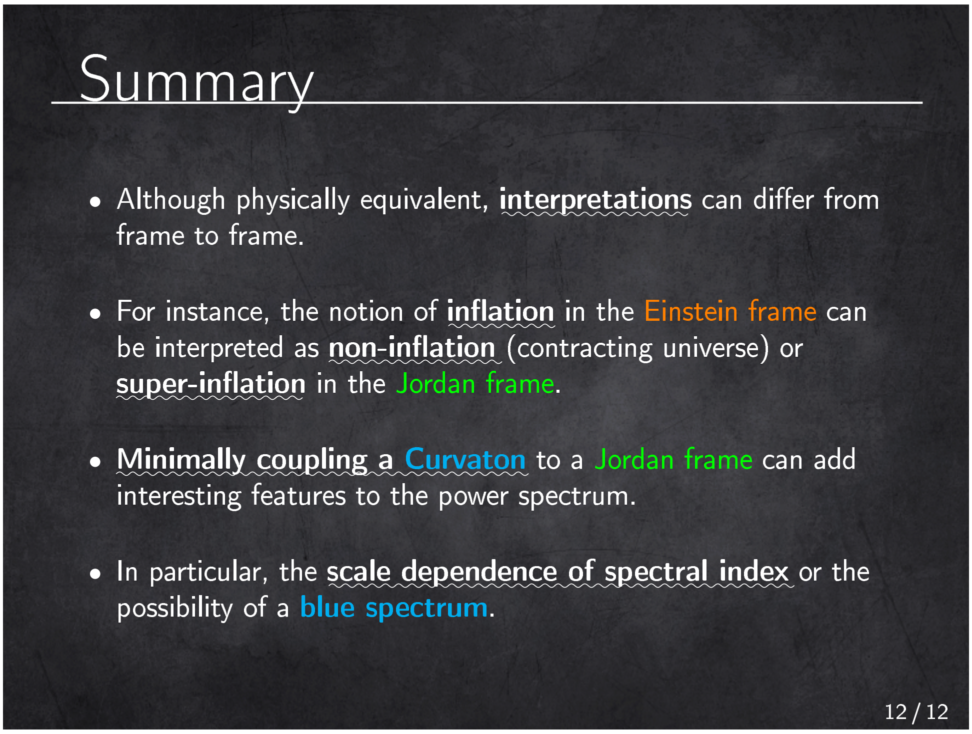


Adding a Curvaton – Discussion

Figure: Inflationary Curvaton (left) and Super-Inflationary Curvaton (right). **Orange** and **white** stands for the **Inflaton** scalar and tensor power spectrum and **blue** stands for the **Curvaton**, where **green** is the **total** scalar power spectrum.

Minimally coupling a Curvaton to Jordan Metric can cause:

- **Scale dependence** of the spectral index
- May allow power-law inflation to **pass Planck constraints**
- **Blue tilt** ⇒ Enhancing **primordial black hole** formation?



Summary

- Although physically equivalent, **interpretations** can differ from frame to frame.
- For instance, the notion of **inflation** in the Einstein frame can be interpreted as **non-inflation** (contracting universe) or **super-inflation** in the Jordan frame.
- **Minimally coupling a Curvaton** to a Jordan frame can add interesting features to the power spectrum.
- In particular, the **scale dependence of spectral index** or the possibility of a **blue spectrum**.

THANK YOU!

“Non-gaussian imprints of primordial magnetic fields from inflation”

Rajeev Kumar Jain

[JGRG24(2014)111313]

Non-gaussian imprints of primordial magnetic fields from inflation

Rajeev Kumar Jain

CP3 Origins
Cosmology & Particle Physics

University of Southern Denmark

JGRG24, Kavli IPMU
Nov. 10-14, 2014

Plan of the talk

- Cosmic magnetic fields: Brief introduction and generation from inflation
- Magnetic non-Gaussianity: Cross-correlations with primordial curvature perturbations
- A *new* magnetic consistency relation
- The full in-in calculation
- Conclusions

Our universe is magnetized!

- Large scale magnetic fields are present everywhere in the universe e.g. in our solar system, in stars, in galaxies, in clusters, in galaxies at high redshifts and also in the intergalactic medium.
- Galaxies: B ~ 1 – 10 μG with coherence length as large as 10 kpc.

  Clusters: B ~ 0.1 – 1 μG, coherent on scales up to 100 kpc.

  Filaments: B ~ 10^{-7} – 10^{-8} G, coherent on scales up to 1 Mpc (Kronberg 2010).

  Intergalactic medium: B > 10^{-16} G, coherent on Mpc scales, the lower bound arises due to the absence of extended secondary GeV emission around TeV blazars (Neronov and Vovk, 2010), or even more robust limits of B > 10^{-19}G (Takahashi et al. 2011).

Primordial magnetic fields from inflation

- Standard EM action is conformally invariant - the EM fluctuations do not grow in any conformally flat background like FRW - need to break it to generate magnetic fields.
- Various possible couplings:
  - Kinetic coupling: $\lambda(\phi, \mathcal{R}) F_{\mu\nu} F^{\mu\nu}$
  - Axial coupling: $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$
  - Mass term: $M^2(\phi, \mathcal{R}) A_\mu A^\mu$

Magnetic non-Gaussianity

- If magnetic fields are produced during inflation, they are likely to be correlated with the primordial curvature perturbations.
- Such cross-correlations are non-Gaussian in nature and it is very interesting to compute them in different models of inflationary magnetogenesis.
- We consider the following correlation here:

$$\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(k_3) \rangle$$

(Ordinary) non-Gaussianity

- The primordial perturbations are encoded in the two-point function or the power spectrum

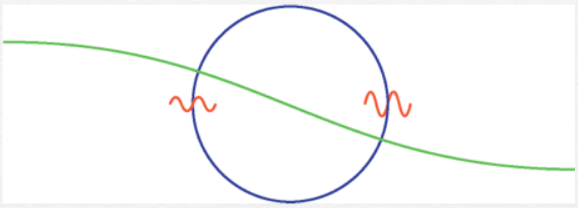
$$\langle \zeta_k \zeta_{k'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') P_\zeta(k)$$

- A non-vanishing three-point function $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ is a signal of NG.
- Introduce f_{NL} as a measure of NG.

$$f_{NL} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle / P_\zeta(k_1) P_\zeta(k_2) \ + \ perm.$$

(semi)Classical estimate

(for squeezed limit)



- Consider $\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle$ in the squeezed limit i.e.
  - The long wavelength mode rescales the background for short wavelength modes

$$ds^2 = -dt^2 + a^2(t)\, e^{2\zeta(t,\mathbf{x})} d\mathbf{x}^2$$

  - Taylor expand in the rescaled background

$$\langle\zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1} = \langle\zeta_{k_2}\zeta_{k_3}\rangle + \zeta_1 \frac{\partial}{\partial\zeta_1}\langle\zeta_{k_2}\zeta_{k_3}\rangle + \dots$$

$$\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1} \approx \left\langle\zeta_{k_1}\,\langle\zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1}\right\rangle \sim \langle\zeta_{k_1}\zeta_{k_1}\rangle\, k\frac{d}{dk}\langle\zeta_{k_2}\zeta_{k_3}\rangle$$

$$\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle \sim -(n_s-1)\,\langle\zeta_{k_1}\zeta_{k_1}\rangle\,\langle\zeta_{k_2}\zeta_{k_3}\rangle$$

(Maldacena, 2002)

Non-gaussian cross-correlation

- Define the cross-correlation bispectrum of the curvature perturbation with magnetic fields as

$$\langle\zeta(\mathbf{k}_1)\mathbf{B}(\mathbf{k}_2)\cdot\mathbf{B}(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- Introduce the magnetic non-linearity parameter

$$B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv b_{NL}P_\zeta(k_1)P_B(k_2)$$

- *Local* resemblance between f_{NL} and b_{NL}

$$\zeta = \zeta^{(G)} + \frac{3}{5}f_{NL}^{local}\left(\zeta^{(G)}\right)^2$$

$$\mathbf{B} = \mathbf{B}^{(G)} + \frac{1}{2}b_{NL}^{local}\zeta^{(G)}\mathbf{B}^{(G)}$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- Use the same semi-classical argument to derive the consistency relation.
- Consider $\langle \zeta(\tau_I, \mathbf{k}_1) A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\rangle$ in the squeezed limit.
- The effect of the long wavelength mode is to shift the background of the short wavelength mode.

$$\lim_{k_1 \to 0} \langle \zeta(\tau_I, \mathbf{k}_1) A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\rangle = \left\langle \zeta(\tau_I, \mathbf{k}_1) \left\langle A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\right\rangle_B \right\rangle$$

- Since the vector field only feels the background through the coupling, all the effects of the long wavelength mode is indeed captured by

$$\lambda_B = \lambda_0 + \frac{d\lambda_0}{d\ln a}\delta \ln a + \cdots = \lambda_0 + \frac{d\lambda_0}{d\ln a}\zeta_B + \ldots \,.$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- First compute the two point function of the vector field in the modified background

$$\begin{aligned}\langle A_i(\tau, \mathbf{x}_2) A_j(\tau, \mathbf{x}_3)\rangle_B &= \left\langle \frac{1}{\lambda_B} v_i(\tau, \mathbf{x}_2) v_j(\tau, \mathbf{x}_3)\right\rangle \\ &\simeq \frac{1}{\lambda_0}\langle v_i(\tau, \mathbf{x}_2) v_j(\tau, \mathbf{x}_3)\rangle - \frac{1}{\lambda_0^2}\frac{d\lambda}{d\ln a}\zeta_B \langle v_i(\tau, \mathbf{x}_2) v_j(\tau, \mathbf{x}_3)\rangle\end{aligned}$$

where $v_i = \sqrt{\lambda}\, A_i$ is the linear canonical vector field.

- One finally finds

$$\begin{aligned}&\lim_{k_1 \to 0} \langle \zeta(\tau_I, \mathbf{k}_1) A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\rangle \\ &\simeq -\frac{1}{H}\frac{\dot{\lambda}}{\lambda}\langle \zeta(\tau_I, \mathbf{k}_1)\zeta(\tau_I, -\mathbf{k}_1)\rangle_0 \langle A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3)\rangle_0\end{aligned}$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- In terms of magnetic fields, the correlation becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle$$
$$= -\frac{1}{H} \frac{\dot{\lambda}}{\lambda} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

- With the coupling $\lambda(\phi(\tau)) = \lambda_I(\tau/\tau_I)^{-2n}$, we obtain

$$\boxed{b_{NL} = n_B - 4}$$

- For scale-invariant magnetic field spectrum, $n_B = 0$ and therefore, $b_{NL} = -4$
- Not so small......compared to $b_{NL} \sim \mathcal{O}(\epsilon, \eta)$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- In the squeezed limit $k_1 \ll k_2, k_3 = k$, we obtain a new *magnetic consistency relation*

$$\boxed{\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(\mathbf{k_3}) \rangle = (n_B - 4)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k)}$$

with $b_{NL}^{\text{local}} = (n_B - 4)$

- Compare with Maldacena's consistency relation

$$\boxed{\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = -(n_s - 1)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_\zeta(k)}$$

with $f_{NL}^{\text{local}} = -(n_s - 1)$

The full in-in calculation

- One has to cross-check the consistency relation by doing the full in-in calculation

$$\langle\Omega|\,\mathcal{O}(\tau_I)\,|\Omega\rangle = \langle 0|\,\bar{\mathrm{T}}\left(e^{i\int_{-\infty}^{\tau_I} d\tau H_{\rm int}}\right)\mathcal{O}(\tau_I)\mathrm{T}\left(e^{-i\int_{-\infty}^{\tau_I} d\tau H_{\rm int}}\right)|0\rangle$$

- The result is

$$\begin{aligned}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle &= \frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)|\zeta^{(0)}_{k_1}(\tau_I)|^2|A^{(0)}_{k_2}(\tau_I)||A^{(0)}_{k_3}(\tau_I)| \\ &\times\left[\left(\delta_{il}-\frac{k_{2,i}k_{2,l}}{k_2^2}\right)\left(\delta_{lj}-\frac{k_{3,l}k_{3,j}}{k_3^2}\right)\left(k_2k_3\,\tilde{\mathcal{I}}^{(1)}_n+\mathbf{k}_2\cdot\mathbf{k}_3\,\tilde{\mathcal{I}}^{(2)}_n\right)\right. \\ &\left.-\left(\delta_{il}-\frac{k_{2,i}k_{2,l}}{k_2^2}\right)k_{3,l}\left(\delta_{jm}-\frac{k_{3,j}k_{3,m}}{k_3^2}\right)k_{2,m}\,\tilde{\mathcal{I}}^{(2)}_n\right]\end{aligned}$$

A generic result

RKJ & Sloth, 2013

and the integrals.......

- The two integrals are

$$\begin{aligned}\tilde{\mathcal{I}}^{(1)}_n &= \frac{\pi^3}{2}\frac{2^{-2n-1}}{\Gamma^2(n+1/2)}(-k_2\tau_I)^{n+1/2}(-k_3\tau_I)^{n+1/2} \\ &\times\,\mathrm{Im}\Big[(1+ik_1\tau_I)e^{-ik_1\tau_I}H^{(1)}_{n+1/2}(-k_2\tau_I)H^{(1)}_{n+1/2}(-k_3\tau_I) \\ &\times\int^{\tau_I}d\tau\tau(1-ik_1\tau)e^{ik_1\tau}H^{(2)}_{n-1/2}(-k_2\tau)H^{(2)}_{n-1/2}(-k_3\tau)\Big]\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{I}}^{(2)}_n &= \frac{\pi^3}{2}\frac{2^{-2n-1}}{\Gamma^2(n+1/2)}(-k_2\tau_I)^{n+1/2}(-k_3\tau_I)^{n+1/2} \\ &\times\,\mathrm{Im}\Big[(1+ik_1\tau_I)e^{-ik_1\tau_I}H^{(1)}_{n+1/2}(-k_2\tau_I)H^{(1)}_{n+1/2}(-k_3\tau_I) \\ &\times\int^{\tau_I}d\tau\tau(1-ik_1\tau)e^{ik_1\tau}H^{(2)}_{n+1/2}(-k_2\tau)H^{(2)}_{n+1/2}(-k_3\tau)\Big]\end{aligned}$$

The cross-correlation with magnetic fields...

- Using this relation

$$\langle\zeta(\tau_I,\mathbf{k}_1)\mathbf{B}(\tau_I,\mathbf{k}_2)\cdot\mathbf{B}(\tau_I,\mathbf{k}_3)\rangle = -\frac{1}{a_0^4}\left(\delta_{ij}\mathbf{k}_2\cdot\mathbf{k}_3-\mathbf{k}_{2,i}\mathbf{k}_{3,j}\right)\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$$

- The cross-correlation with magnetic fields is

$$\begin{aligned}\langle\zeta(\tau_I,\mathbf{k}_1)\mathbf{B}(\tau_I,\mathbf{k}_2)\cdot\mathbf{B}(\tau_I,\mathbf{k}_3)\rangle = &-\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)|\zeta^{(0)}_{k_1}(\tau_I)|^2|A^{(0)}_{k_2}(\tau_I)||A^{(0)}_{k_3}(\tau_I)|\\ &\times\left[\left(\mathbf{k}_2\cdot\mathbf{k}_3+\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)^3}{k_2^2k_3^2}\right)k_2k_3\tilde{\mathcal{I}}^{(1)}_n+2(\mathbf{k}_2\cdot\mathbf{k}_3)^2\tilde{\mathcal{I}}^{(2)}_n\right]\ .\end{aligned}$$

- The two integrals can be solved exactly for different values of n.

RKJ & Sloth, 2013

The flattened shape

- In this limit, $k_1 = 2k_2 = 2k_3$, the second integral dominates

$$\tilde{\mathcal{I}}^{(2)}_2 \simeq -\frac{3k_1^3}{(k_2k_3)^{5/2}}\ln(-k_t\tau_I)$$

- The cross-correlation thus becomes

$$\langle\zeta(\tau_I,\mathbf{k}_1)\mathbf{B}(\tau_I,\mathbf{k}_2)\cdot\mathbf{B}(\tau_I,\mathbf{k}_3)\rangle \simeq 96\ln(-k_t\tau_I)P_\zeta(k_1)P_B(k_2)$$

- For the largest observable scale today, $\ln(-k_t\tau_I)\sim -60$,

$$\left|b_{NL}^{flat}\right| \sim 5760$$

RKJ & Sloth, 2013

The squeezed limit

- In this limit, the integrals are

$$\tilde{\mathcal{I}}_n^{(1)} = \pi \int^{\tau_I} d\tau\, \tau J_{n-1/2}(-k\tau) Y_{n-1/2}(-k\tau)$$

$$\tilde{\mathcal{I}}_n^{(2)} = \tilde{\mathcal{I}}_{n+1}^{(1)}\ .$$

- The cross-correlation now becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

with $b_{NL} = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I} = n_B - 4$ in agreement with the magnetic consistency relation.

RKJ & Sloth, 2013

Conclusions

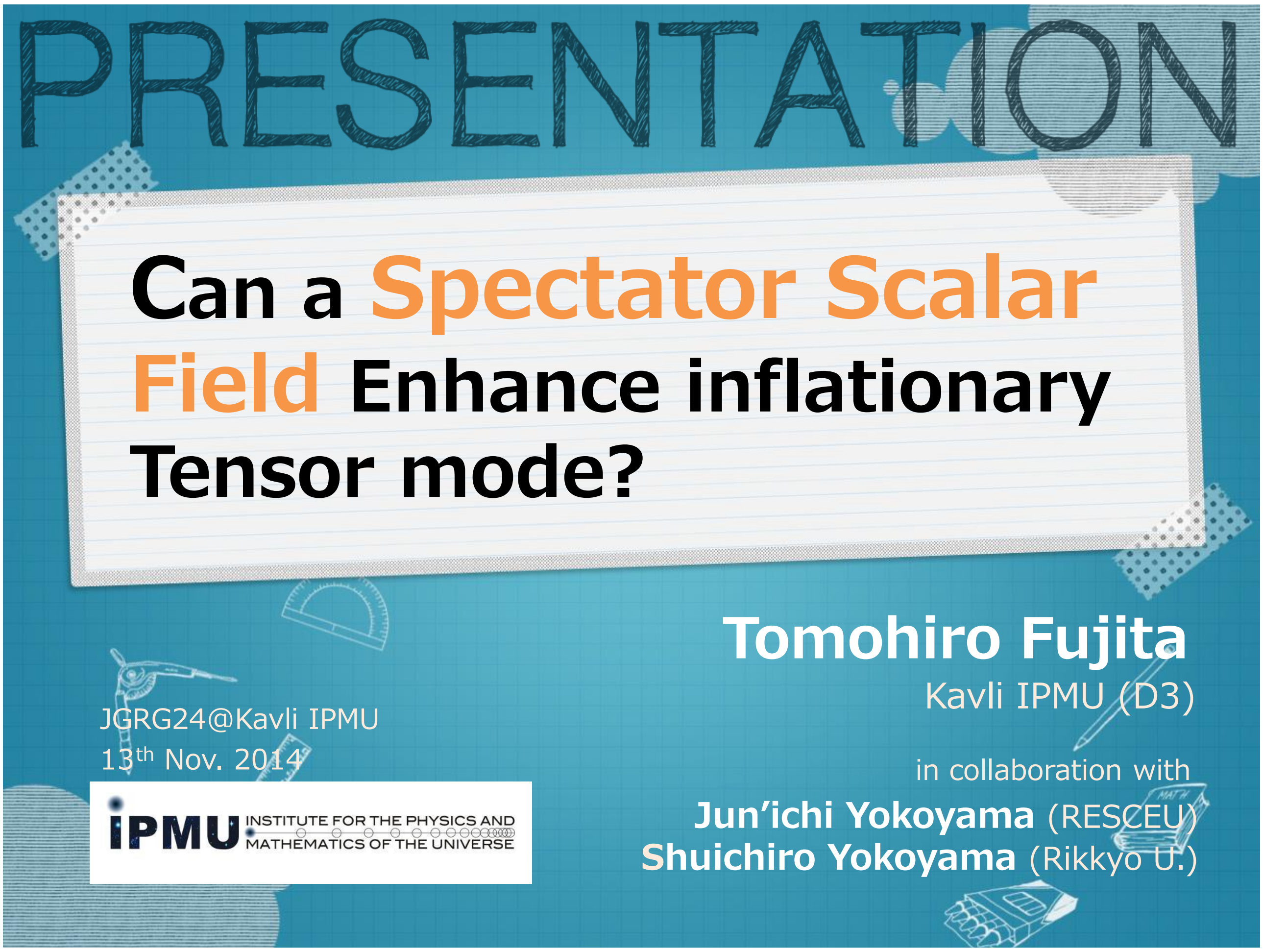
- Primordial non-Gaussianities induced by magnetic fields are very interesting.
- The consistency relation is an important theoretical tool to cross-check the full in-in calculations.
- If the consistency relation is violated, it will rule out an important class of models for inflationary magnetogenesis.
- The magnetic non-Gaussianity parameter is quite large in the flattened limit and can have interesting phenomenological consequences.

Thank you for your attention

“Can a Spectator Scalar Field Enhance Inflationary Tensor Modes?”

Tomohiro Fujita

[JGRG24(2014)111314]



PRESENTATION

Can a Spectator Scalar Field Enhance inflationary Tensor mode?

JGRG24@Kavli IPMU
13th Nov. 2014

Tomohiro Fujita
Kavli IPMU (D3)

in collaboration with
Jun'ichi Yokoyama (RESCEU)
Shuichiro Yokoyama (Rikkyo U.)

1

THE THEME
OF CHAPTER IS...

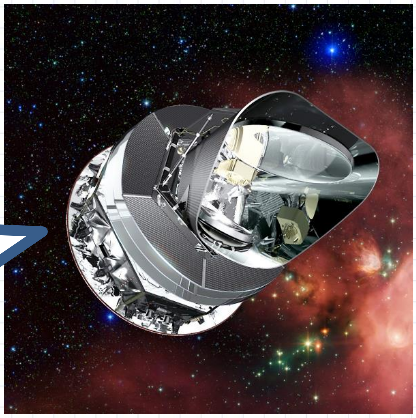
Introduction

1 Introduction

PRESENTATION

What if....

We detect
$r = 0.1$



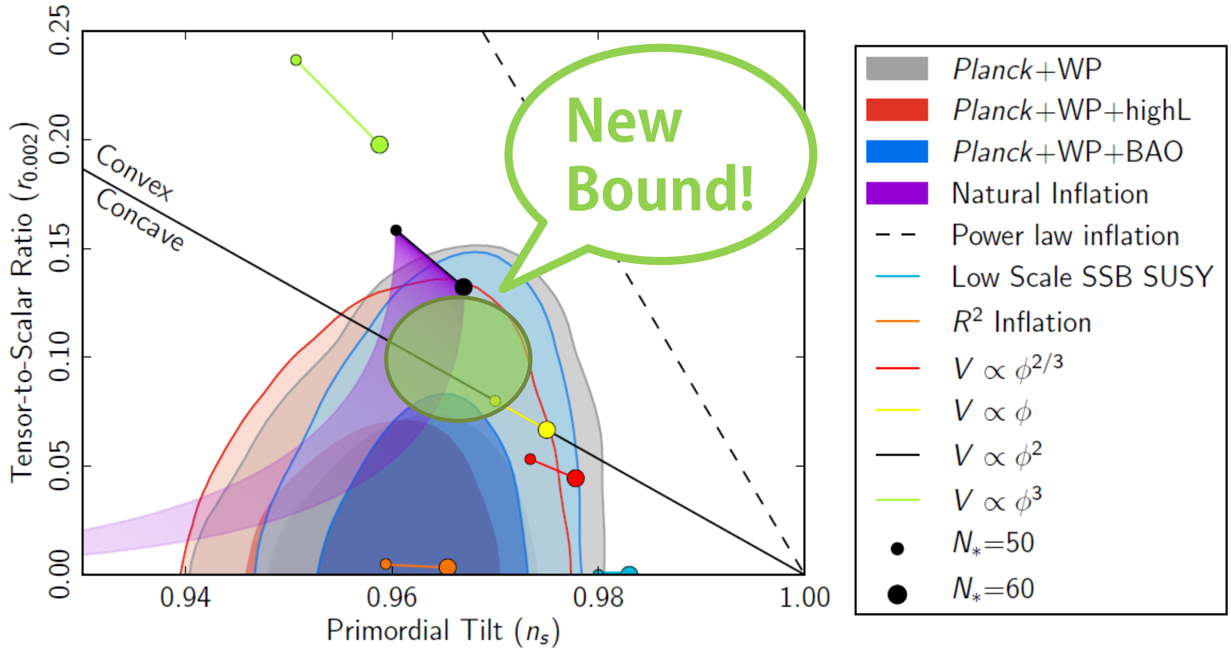
Planck satellite

---

1 Introduction

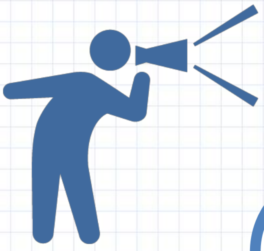
PRESENTATION

What if $r = 0.1$



1 Introduction

PRESENTATION



Wait!

It is perhaps too early to say we know ρ_{inf} even if we observe r.

---

1 Introduction

PRESENTATION

Slow-roll inflation predicts

$$\mathcal{P}_h^{\text{vac}} = \frac{2H^2}{\pi M_{\text{Pl}}^2} \propto \rho_{\text{inf}}$$

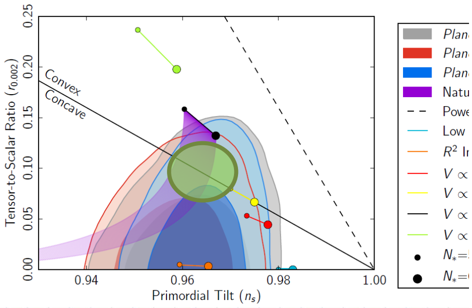
It does not necessarily mean

Observed GW $\mathcal{P}_h^{\text{obs}} \propto \rho_{inf}$.

BCS, other $\mathcal{P}_h$ may exist.

1 Introduction

PRESENTATION



- **Big Assumption**

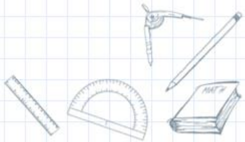
  $\mathcal{P}_h^{\text{vac}}$ is dominant in $\mathcal{P}_h^{\text{obs}}$.

  All other GWs are sub-dominant.

- **Key Question**

  Can an alternative source

  produce dominant GWs?

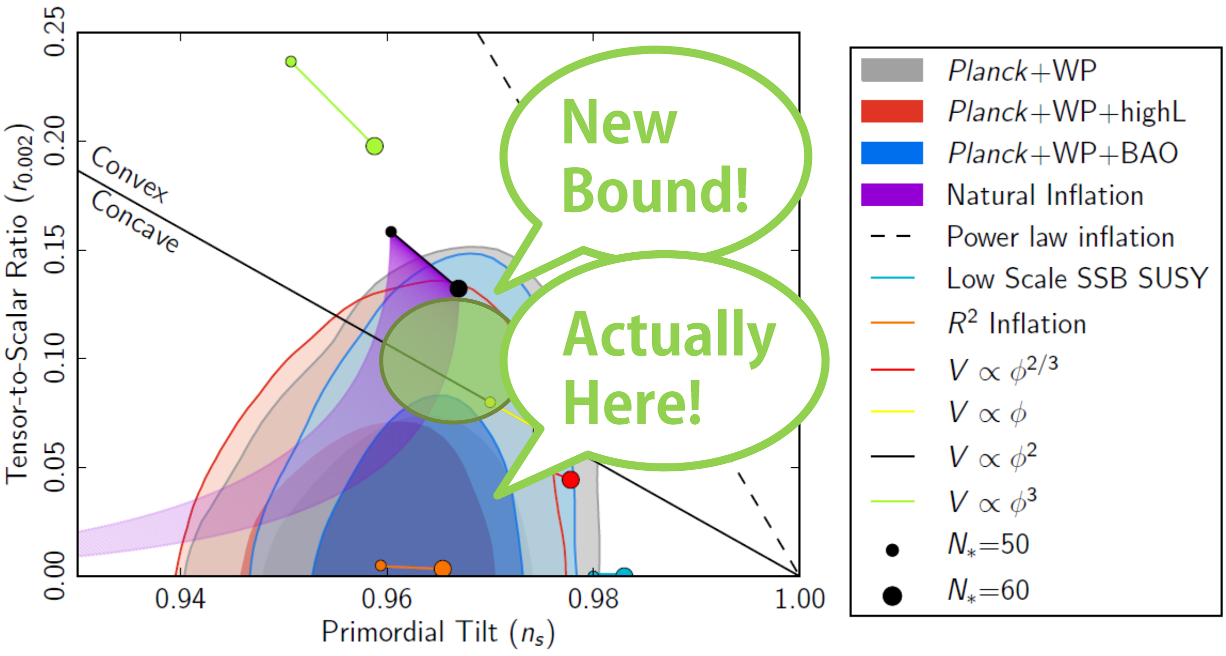


---

1 Introduction

PRESENTATION

What if $r = 0.1$



1 Introduction

PRESENTATION

$\mathcal{P}_h^{\mathrm{alt}}$ can drastically change Observational consequence

→ **Important to study $\mathcal{P}_h^{\mathrm{alt}}$**

---

1 Introduction

PRESENTATION

§ 1 Quick Summary

"Can alternative source produce dominant GWs?"

2

THE THEME
OF CHAPTER IS...

Model

---

2 Model

PRESENTATION

Previous works

- Senatore, Silverstein & Zaldarriaga(2011)
  - Particle creation, etc.
- Mukohyama et al.(2014), Ferreira & Sloth(2014)
  - Vector field 2nd order perturbation.
- Creimnelli et al.(2014), Cannone et al.(2014)
  - Small sound speed of graviton.
- Biagetti et al.(2013), Biagetti et al.(Today!)
  - Scalar field 2nd order perturbation.

2 Model

PRESENTATION

Biagetti, Fasiello & Riotto (2013) consider

$$S_{\delta\sigma} = \int \mathrm{d}^3x\,\mathrm{d}\tau\, a^4 \left[\frac{1}{2a^2} \left(\delta\sigma'^2 - c_s^2 (\nabla\delta\sigma)^2 \right) - V_{(2)} \right]$$

---

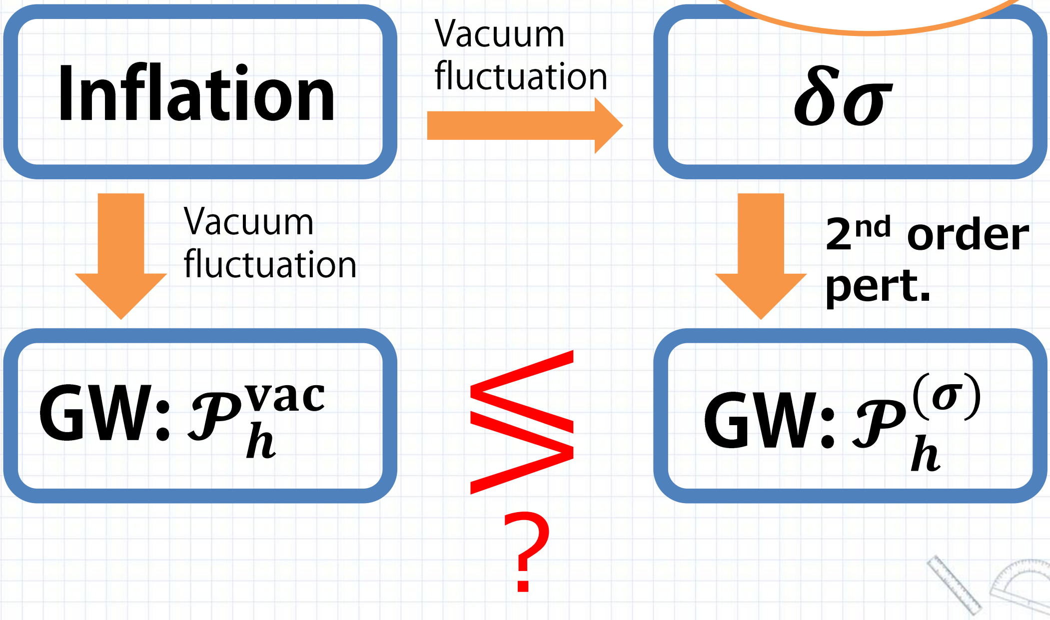
2 Model

PRESENTATION

Prof. Yokoyama

Biagetti, Fasiello & Riotto (2013)

Let's call it "Tensoron".



Inflation

Vacuum fluctuation

$\boldsymbol{\delta\sigma}$

Vacuum fluctuation

2nd order pert.

GW: $\mathcal{P}_h^{\mathrm{vac}}$

$\lessgtr$?

GW: $\mathcal{P}_h^{(\sigma)}$

2 Model

PRESENTATION

Biagetti, Fasiello & Riotto (2013) consider

$$\boxed{S_{\delta\sigma} = \int \mathrm{d}^3x\,\mathrm{d}\tau\, a^4 \left[\frac{1}{2a^2}\left(\delta\sigma'^2 - c_s^2(\nabla\delta\sigma)^2\right) - V_{(2)}\right]}$$

Small sound speed ($c_s \ll 1$) amplifies $\delta\sigma_k$

$$\Rightarrow \quad \mathcal{P}_h^{(\sigma)} \sim \frac{H^4}{c_s^{18/5} M_{\rm Pl}^4}$$

They claim that it can be larger than $\mathcal{P}_h^{\rm vac}$.

---

2 Model

PRESENTATION

Fujita, Yokoyama & Yokoyama [1411.XXXX] consider

$$\boxed{\mathcal{L} = \frac{1}{2}M_{\rm Pl}^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + P(X,\sigma),}$$

GR

Inflaton

Tensoron

$X \equiv \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$

2 Model

PRESENTATION

We perturbed the action up to 3rd order,

$$\begin{aligned} NP(X,\sigma) &= (1+\delta N)P(X,\sigma) \\ &= P^{(0)} \quad \text{(0th order)} \\ &\quad + P^{(0)}\delta N + P_X^{(0)}\left(\dot\sigma_0\dot{\delta\sigma} - \dot\sigma_0^2\delta N\right) + P_\sigma^{(0)}\delta\sigma \quad \text{(1st order)} \\ &\quad + \frac{1}{2}P_X^{(0)}\left[\dot{\delta\sigma}^2 - 2\dot\sigma_0 a^{-2}\partial_i\psi\partial_i\delta\sigma - 2\dot\sigma_0\dot{\delta\sigma}\delta N + \dot\sigma_0^2\delta N^2 - a^{-2}(\partial_i\delta\sigma)^2\right] \\ &\quad + \frac{1}{2}P_{XX}^{(0)}\left(\dot\sigma_0\dot{\delta\sigma} - \dot\sigma_0^2\delta N\right)^2 + \frac{1}{2}P_{\sigma\sigma}^{(0)}\delta\sigma^2 + P_\sigma^{(0)}\delta\sigma\delta N \quad \text{(2nd order)} \\ &\quad + \frac{1}{2}P_{XX}^{(0)}\dot\sigma_0\dot{\delta\sigma}^3 - \left(\frac{1}{2}P_X^{(0)} - 2P_{XX}^{(0)}\dot\sigma_0^2\right)\dot{\delta\sigma}^2\delta N + \left(P_X^{(0)} + \frac{5}{2}P_{XX}^{(0)}\dot\sigma_0^2\right)\dot\sigma_0\dot{\delta\sigma}\delta N^2 \\ &\quad - \left(\frac{1}{2}P_X^{(0)} + P_{XX}^{(0)}\dot\sigma_0^2\right)\dot\sigma_0^2\delta N^3 + \left(P_X^{(0)} + P_{XX}^{(0)}\dot\sigma_0^2\right)\left(\dot\sigma_0\delta N - \dot{\delta\sigma}\right)a^{-2}\partial_i\psi\partial_i\delta\sigma \\ &\quad - \frac{1}{2}P_{XX}^{(0)}\dot\sigma_0\dot{\delta\sigma}a^{-2}(\partial_i\delta\sigma)^2 - \frac{1}{2}\left(P_X^{(0)} - P_{XX}^{(0)}\dot\sigma_0^2\right)\delta N a^{-2}(\partial_i\delta\sigma)^2 \\ &\quad + \frac{1}{2}P_{\sigma\sigma}^{(0)}\delta\sigma^2\delta N + \frac{1}{2}P_X^{(0)}h_{ij}a^{-2}\partial_i\delta\sigma\partial_j\delta\sigma \quad \text{(3rd order)} \ + \mathcal{O}(\delta\sigma^4). \qquad \text{(A.2)} \end{aligned}$$

+ inflaton sector and gravity sector

2 Model

PRESENTATION

Fujita, Yokoyama & Yokoyama [1411.XXXX] consider

$$\boxed{\mathcal{L} = \frac{1}{2}M_{\rm Pl}^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + P(X,\sigma),} \qquad X \equiv \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$$

The second order action includes

$$\Rightarrow \quad \mathcal{L}_\sigma^{(2)} \supset \underline{\left(P_{XX}\dot\sigma_0{}^2 + P_X\right)}\delta\dot\sigma^2 - a^{-2}\underline{P_X}(\partial_i\delta\sigma)^2$$

$P_X \equiv \partial P/\partial X$
$P_{XX} \equiv \partial^2 P/\partial X^2$

Sound Speed: $c_s^2 \equiv \dfrac{P_X}{P_{XX}\dot\sigma_0{}^2 + P_X}$

Small c_s ⟺ Time KT ≫ Spatial KT

2 Model

PRESENTATION

We found tensoron induces large curvature perturbation.

The EoMs of $\mathcal{R}$ and h_{ij} are

Curv. Pert. $$\mathcal{R}'' + 2\mathcal{H}\mathcal{R}' - \partial_i^2\mathcal{R} = -\frac{P_{XX}\dot{\sigma}_0^2}{4M_{\rm Pl}^2}\partial_i\delta\sigma\partial_i\delta\sigma,$$

GW $$h''_{ij} + 2\mathcal{H}h'_{ij} - \partial_k^2 h_{ij} = \frac{2P_X}{M_{\rm Pl}^2}\tilde{T}_{ij}^{lm}\partial_l\delta\sigma\partial_m\delta\sigma.$$

⇒ **The coupling of GW is suppressed** $$\left|\frac{h\delta\sigma^2 \text{ coupling}}{\mathcal{R}\delta\sigma^2 \text{ coupling}}\right| \simeq 8c_s^2$$

Small c_s leads to $\mathcal{P}_R^{(\sigma)} \gg \mathcal{P}_h^{(\sigma)}$

2 Model

PRESENTATION

We obtain

$$\mathcal{P}_h^{(\sigma)} \simeq \frac{H^4}{c_s^{\textcircled{3}}M_{\rm Pl}^4}, \quad \overset{\text{Small } c_s}{\ll} \quad \mathcal{P}_\mathcal{R}^{(\sigma)} \simeq \frac{H^4}{c_s^{\textcircled{7}}M_{\rm Pl}^4} \quad \overset{\text{CMB}}{\leq} \quad \mathcal{P}_\mathcal{R}^{\rm obs}$$

Since $\mathcal{P}_\mathcal{R}^{(\sigma)} < \mathcal{P}_\mathcal{R}^{\rm obs}$, a lower bound on c_s is derived.

⇒ $$\frac{\mathcal{P}_h^{(\sigma)}}{\mathcal{P}_h^{\rm vac}} \lesssim 10^{-5}\left(\frac{H}{10^{14}\mathrm{GeV}}\right)^{2/7}$$

GW induced by Tensoron cannot be dominant!

2 Model

PRESENTATION

Even if we **extend** the Tensoron action,

Galileon theory: $\mathcal{L}_\sigma = P(X,\sigma) - G(X,\sigma)\Box\sigma,$

We get $\mathcal{P}_h^{(\sigma)} \ll \mathcal{P}_\mathcal{R}^{(\sigma)}$, and hence $\mathcal{P}_h^{(\sigma)} \ll \mathcal{P}_h^{\mathrm{vac}}$ in the **same way**.

Single spectator field with small c_s can't produce dominant GW.

---

2 Model

PRESENTATION

§ 2 Quick Summary

"Single spectator field can't produce dominant GW"

3 New model (Preliminary result)

PRESENTATION

Future Work

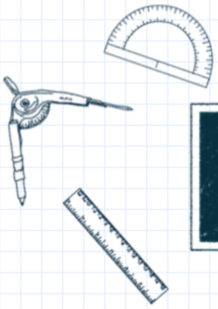
① A bit more study on this model. Show it is **consistent**.

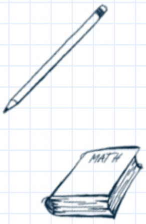
ex) Full $\mathcal{P}_h^{(\sigma)}$, Back reaction on χ, other constraint(?)

② Seek for **another** natural model.

③ Find a way to **distinguish** $\mathcal{P}_h^{\mathrm{vac}}$ from $\mathcal{P}_h^{\mathrm{alt}}$

ex) consistency relation $r = -8n_T$, Non-gaussianity of GWs.





Fin

THE THEME
OF CHAPTER IS...

Thank you!

“Large tensor mode and sub-Planckian excursion in generalized G-inflation”

Taro Kunimitsu

[JGRG24(2014)111315]

Large tensor mode and sub-Planckian excursion in Generalized G-inflation

Taro Kunimitsu (RESCEU, UTokyo)

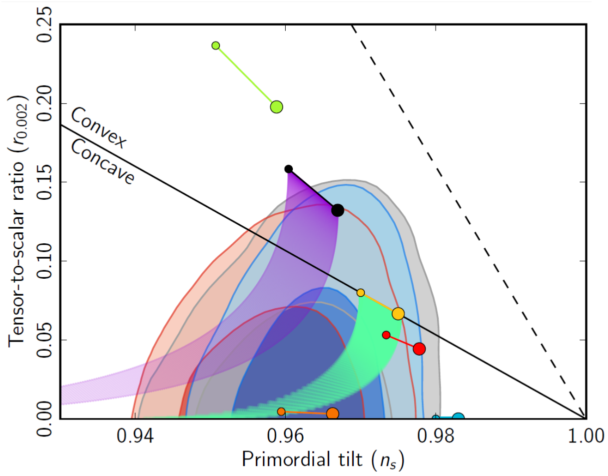
In collaboration with Teruaki Suyama, Yuki Watanabe, Jun'ichi Yokoyama

arXiv:1411.xxxx (hopefully…)

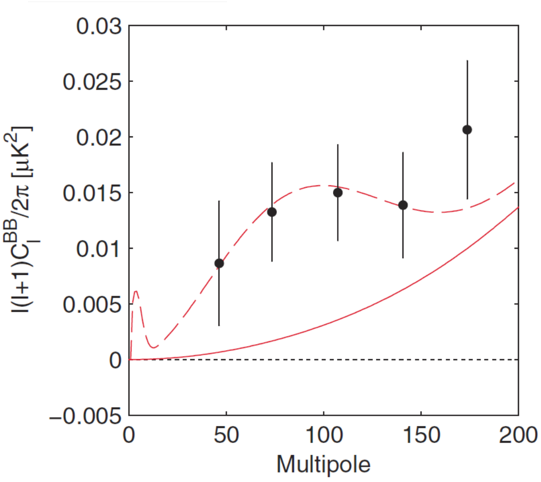
Disclaimer
(What this talk is NOT about)

- This is NOT a direct evasion of the Lyth bound.
- Assumptions we make are not necessarily general.
- Still, we feel that what we are doing could have possible applications.

Inflation and tensor-modes



Planck (2013)
r<0.11 (2σ C.L.)



BICEP2 (2014)
r=0.20+0.07-0.05

Observable tensor-to-scalar

- Lyth bound (Lyth 1997)

Observable tensor mode
→ super-Planckian excursion of the Inflaton

$$\Delta\phi \gtrsim 3\sqrt{\frac{r}{0.01}}M_P \quad \text{for} \quad N \sim 50-60$$

↑ For a single field canonical scalar field
→<u>what are the models that can evade this</u>?

Is super-Planckian excursion a problem?

- Without assumptions, no.
  - Explicit UV models – e.g. SUGRA
  - New d.o.f. at the Planck scale

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \lambda\phi^2\chi^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M_P^2\chi^2$$

$$\rightarrow \quad \sim \frac{1}{M_P^2}\phi^6$$

Avoiding super-Planckian excursion

1.Rescale the inflaton (trivial)

$$\phi \rightarrow c\phi$$

2.Change the kinetic structure of the inflaton

$$X = -\frac{1}{2}(\partial\phi)^2 \rightarrow P(\phi, X)$$

3.**Generalized G-inflation**

Generalized G-inflation (aka Horndeski theory)

Most general action with e.o.m. of at most second order derivatives

Kobayashi, Yamaguchi, Yokoyama (2011)

$$S=\sum_{i=2}^{5}\int d^4x\ \sqrt{-g}\mathcal{L}_i$$

$$\begin{aligned}\mathcal{L}_2 &= K(\phi,X),\\ \mathcal{L}_3 &= -G_3(\phi,X)\Box\phi,\\ \mathcal{L}_4 &= G_4(\phi,X)R+G_{4X}\left[(\Box\phi)^2-(\nabla_\mu\nabla_\nu\phi)^2\right],\\ \mathcal{L}_5 &= G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi-\frac{1}{6}G_{5X}\left[(\Box\phi)^3-3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2+2(\nabla_\mu\nabla_\nu\phi)^3\right],\end{aligned}$$

Generalized G-inflation (aka Horndeski theory)

Most general action with e.o.m. of at most second order derivatives

Kobayashi, Yamaguchi, Yokoyama (2011)

$$S=\sum_{i=2}^{5}\int d^4x\ \sqrt{-g}\mathcal{L}_i$$

$$\begin{aligned}\mathcal{L}_2 &= K(\phi,X),\\ \mathcal{L}_3 &= -G_3(\phi,X)\Box\phi,\\ \mathcal{L}_4 &= G_4(\phi,X)R+G_{4X}\left[(\Box\phi)^2-(\nabla_\mu\nabla_\nu\phi)^2\right],\\ \mathcal{L}_5 &= G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi-\frac{1}{6}G_{5X}\left[(\Box\phi)^3-3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2+2(\nabla_\mu\nabla_\nu\phi)^3\right],\end{aligned}$$

- We will consider **Potential driven models**.
- For the nontrivial models, the canonical kinetic term is dominated over by the newly introduced terms

Field excursion in G^2-inflation

$$N = \int H dt = \int \frac{H}{\dot{\phi}} d\phi \quad \rightarrow \quad \Delta\phi \geq N \left(\frac{\dot{\phi}}{H}\right)_{\min}$$

- expand the free functions in terms of X

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \frac{1}{2}h_2(\phi)X^2, \qquad G_i(\phi, X) = g_i(\phi) + h_i(\phi)X$$

Field excursion in G^2-inflation

$$N = \int H dt = \int \frac{H}{\dot{\phi}} d\phi \quad \rightarrow \quad \Delta\phi \geq N \left(\frac{\dot{\phi}}{H}\right)_{\min}$$

- expand the free functions in terms of X

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \frac{1}{2}h_2(\phi)X^2, \qquad G_i(\phi, X) = g_i(\phi) + h_i(\phi)X$$

$$\frac{\dot{\phi}}{H} = \sqrt{\frac{g_4 r}{8Y}} \left(\mathcal{K} + h_2 X + 6H^2 h_4 + 4H\dot{\phi}(h_3 + H^2 h_5)\right)^{-1/2}$$

$$Y = \frac{\mathcal{K} + h_2 X + 6H^2 h_4 + 4H\dot{\phi}(h_3 + H^2 h_5)}{\mathcal{K} + 3h_2 X + 6H^2 h_4 + 6H\dot{\phi}(h_3 + H^2 h_5)} \sim \mathcal{O}(1)$$

Field excursion in G^2-inflation

$$\Delta\phi \geq \frac{N}{4}(r^{1/2}q)M_P \simeq 0.6\left(\frac{N}{7}\right)\left(\frac{r}{0.1}\right)^{1/2} qM_P,$$

$$q \equiv \left[\frac{2g_4}{YM_P^2\left(\mathcal{K}+h_2X+6H^2h_4+4H\dot{\phi}(h_3+H^2h_5)\right)}\right]^{1/2}$$

Field excursion in G^2-inflation

$$\Delta\phi \geq \frac{N}{4}(r^{1/2}q)M_P \simeq 0.6\left(\frac{N}{7}\right)\left(\frac{r}{0.1}\right)^{1/2} qM_P,$$

$$q \equiv \left[\frac{2g_4}{\boxed{YM_P^2\left(\mathcal{K}+h_2X+6H^2h_4+4H\dot{\phi}(h_3+H^2h_5)\right)}}\right]^{1/2}$$

Making this part large leads to sub-Planckian field excursion!

Example: Potential driven G-inflation

$$S = \int d^4x \sqrt{-g} \left[X + \frac{1}{M^3} X \Box \phi - V(\phi) \right] \qquad \left(h_3 = -\frac{1}{M^3} \right)$$

Example: Potential driven G-inflation

$$S = \int d^4x \sqrt{-g} \left[\cancel{X} + \underset{\sim \frac{H\dot{\phi}}{M^3} X \gg X}{\frac{1}{M^3} X \Box \phi} - V(\phi) \right] \qquad \left(h_3 = -\frac{1}{M^3} \right)$$

Example: Potential driven G-inflation

$$S=\int d^4x\sqrt{-g}\left[\cancel{X}+\frac{1}{M^3}X\Box\phi-V(\phi)\right]\qquad\left(h_3=-\frac{1}{M^3}\right)$$

$$\sim\frac{H\dot{\phi}}{M^3}X\gg X$$

$N=\int Hdt=\int\frac{H}{\dot{\phi}}d\phi$, slow roll equation of motion $\dot{\phi}=-\sqrt{\frac{M^3V_\phi}{9H^2}}$

$$\phi_*=[5N+2]^{\frac{2}{5}}\left(\frac{M^{\frac{3}{2}}M_P^2}{m}\right)^{\frac{2}{5}}\quad\text{for}\quad V(\phi)=\frac{1}{2}m^2\phi^2$$

$$\Delta\phi\lesssim\phi_*=2.6\times10^{-3}\left(\frac{M}{10^{12}\mathrm{GeV}}\right)M_P\quad\text{for}\quad N=60$$

Example: Potential driven G-inflation

$$S=\int d^4x\sqrt{-g}\left[\cancel{X}+\frac{1}{M^3}X\Box\phi-V(\phi)\right]\qquad\left(h_3=-\frac{1}{M^3}\right)$$

$$\sim\frac{H\dot{\phi}}{M^3}X\gg X$$

$N=\int Hdt=\int\frac{H}{\dot{\phi}}d\phi$, slow roll equation of motion $\dot{\phi}=-\sqrt{\frac{M^3V_\phi}{9H^2}}$

$$\phi_*=[5N+2]^{\frac{2}{5}}\left(\frac{M^{\frac{3}{2}}M_P^2}{m}\right)^{\frac{2}{5}}\quad\text{for}\quad V(\phi)=\frac{1}{2}m^2\phi^2$$

$$\Delta\phi\lesssim\phi_*=2.6\times10^{-3}\left(\frac{M}{10^{12}\mathrm{GeV}}\right)M_P\quad\text{for}\quad N=60$$

$n_s=0.970,\ r=0.11$ Observable tensor modes!

Consistency of the models

We introduced new mass scales

$$\frac{1}{M^3} X \Box \phi \to \frac{1}{M^2} \phi^6 \quad ?$$

We want to check that the model is not destroyed by quantum corrections (a new "Lyth bound").

Quantum corrections

We calculate quantum corrections by modifying the second order action into an effective canonical form (de Rham, Ribeiro (2014))

$$S_2 = \int d^4x \sqrt{-g_{\rm eff}} \left[-\frac{1}{2} g^{\mu\nu}_{\rm eff} \partial_\mu \delta\phi \partial_\nu \delta\phi - \frac{1}{2} \widetilde{V}'' \delta\phi^2 \right]$$

↓ where $g^{\mu\nu}_{\rm eff}(\phi, g_{\mu\nu})$

$$\Delta S \sim \frac{1}{32\pi^2} \int d^4x \sqrt{-g_{\rm eff}} \left[\frac{1}{2} \widetilde{V}''^2 - \frac{1}{6} \widetilde{V}'' R_{\rm eff} + \frac{1}{120} R^2_{\rm eff} + \frac{1}{60} R^{\rm eff}_{\mu\nu} R^{\mu\nu}_{\rm eff} \right]$$

Barvinsky, Vilkovisky (1990)

Quantum corrections

We calculate quantum corrections by modifying the second order action into an effective canonical form (de Rham, Ribeiro (2014))

$$S_2 = \int d^4x\sqrt{-g_{\mathrm{eff}}}\left[-\frac{1}{2}g^{\mu\nu}_{\mathrm{eff}}\partial_\mu\delta\phi\partial_\nu\delta\phi - \frac{1}{2}\widetilde{V}''\delta\phi^2\right]$$

↓ where $g^{\mu\nu}_{\mathrm{eff}}(\phi, g_{\mu\nu})$

$$\Delta S \sim \frac{1}{32\pi^2}\int d^4x\sqrt{-g_{\mathrm{eff}}}\left[\boxed{\frac{1}{2}\widetilde{V}''^2} - \frac{1}{6}\widetilde{V}''R_{\mathrm{eff}} + \frac{1}{120}R^2_{\mathrm{eff}} + \frac{1}{60}R^{\mathrm{eff}}_{\mu\nu}R^{\mu\nu}_{\mathrm{eff}}\right]$$

Barvinsky, Vilkovisky (1990)

$$\widetilde{V}'' = \frac{\sqrt{-g}}{\sqrt{-g_{\mathrm{eff}}}}V'' \ll \sqrt{V}$$

Can be ignored!

Example: Potential driven G-inflation

$$S = \int d^4x\sqrt{-g}\left[X + \frac{1}{M^3}X\Box\phi - V(\phi)\right]$$

Second order action (de Sitter background)

$$S_2 = \int d^4x a^3\left[\left(\frac{1}{2} - \frac{3H\dot{\phi}}{M^3}\right)\dot{\delta\phi}^2 - \frac{1}{a^2}\left(\frac{1}{2} - \frac{\ddot{\phi}}{M^3} - \frac{2H\dot{\phi}}{M^3}\right)(\partial_i\delta\phi)^2 - \frac{1}{2}V''(\phi)\delta\phi^2\right]$$

Example: Potential driven G-inflation

$$S=\int d^4x\sqrt{-g}\left[X+\frac{1}{M^3}X\Box\phi-V(\phi)\right]$$

Second order action (de Sitter background)

$$S_2=\int d^4x a^3\left[\left(\frac{1}{2}-\frac{3H\dot{\phi}}{M^3}\right)\dot{\delta\phi}^2-\frac{1}{a^2}\left(\frac{1}{2}-\frac{\ddot{\phi}}{M^3}-\frac{2H\dot{\phi}}{M^3}\right)(\partial_i\delta\phi)^2-\frac{1}{2}V''(\phi)\delta\phi^2\right]$$

$$\rightarrow \quad S_2=\int d^4x\sqrt{-g_{\rm eff}}\left[-\frac{1}{2}g^{\mu\nu}_{\rm eff}\partial_\mu\delta\phi\partial_\nu\delta\phi-\frac{1}{2}\widetilde{V}''\delta\phi^2\right]$$

with $g^{\rm eff}_{\mu\nu}(\phi_0)=\mathrm{diag}(A,B,B,B)$

$$A=\left(1-\frac{2\ddot{\phi}}{M^3}-\frac{4H\dot{\phi}}{M^3}\right)^{\frac{3}{2}}\left(1-\frac{6H\dot{\phi}}{M^3}\right)^{-\frac{1}{2}} \qquad B=a^2\left(1-\frac{2\ddot{\phi}}{M^3}-\frac{4H\dot{\phi}}{M^3}\right)^{\frac{1}{2}}\left(1-\frac{6H\dot{\phi}}{M^3}\right)^{\frac{1}{2}}$$

Example: Potential driven G-inflation

$$S=\int d^4x\sqrt{-g}\left[X+\frac{1}{M^3}X\Box\phi-V(\phi)\right]$$

$$\Delta S\sim\frac{1}{32\pi^2}\int d^4x\sqrt{-g_{\rm eff}}\left[\frac{1}{2}\widetilde{V}''^2-\frac{1}{6}\widetilde{V}''R_{\rm eff}+\frac{1}{120}R_{\rm eff}^2+\frac{1}{60}R^{\rm eff}_{\mu\nu}R^{\mu\nu}_{\rm eff}\right]$$

$$R_{\rm eff}\sim H^2\ll\sqrt{V} \qquad \widetilde{V}''\sim\frac{\sqrt{\lambda}}{D^2}M_PH\ll\sqrt{V}$$

↗

$$D=\frac{H\dot{\phi}}{M^3}\gg 1$$

cf. $\frac{1}{M^3}X\Box\phi\sim\frac{H\dot{\phi}}{M^3}X\gg X$

Quantum corrections can be ignored!

Conclusions

- Sub-Planckian excursion with large tensor modes is possible in the framework of Generalized G-inflation.
- We demonstrated this using explicit models.
- We showed the internal consistency of these models. (they are not destroyed by quantum corrections)

Generalized G-inflation

$$H^2 \simeq \frac{V}{6g_4}, \qquad 3HJ \simeq -V_\phi + 12H^2 g_{4\phi}$$

$$J \simeq (\mathcal{K} + h_2 X)\dot{\phi} + 6H(h_3 X + H h_4 \dot{\phi} + H^2 h_5 X)$$

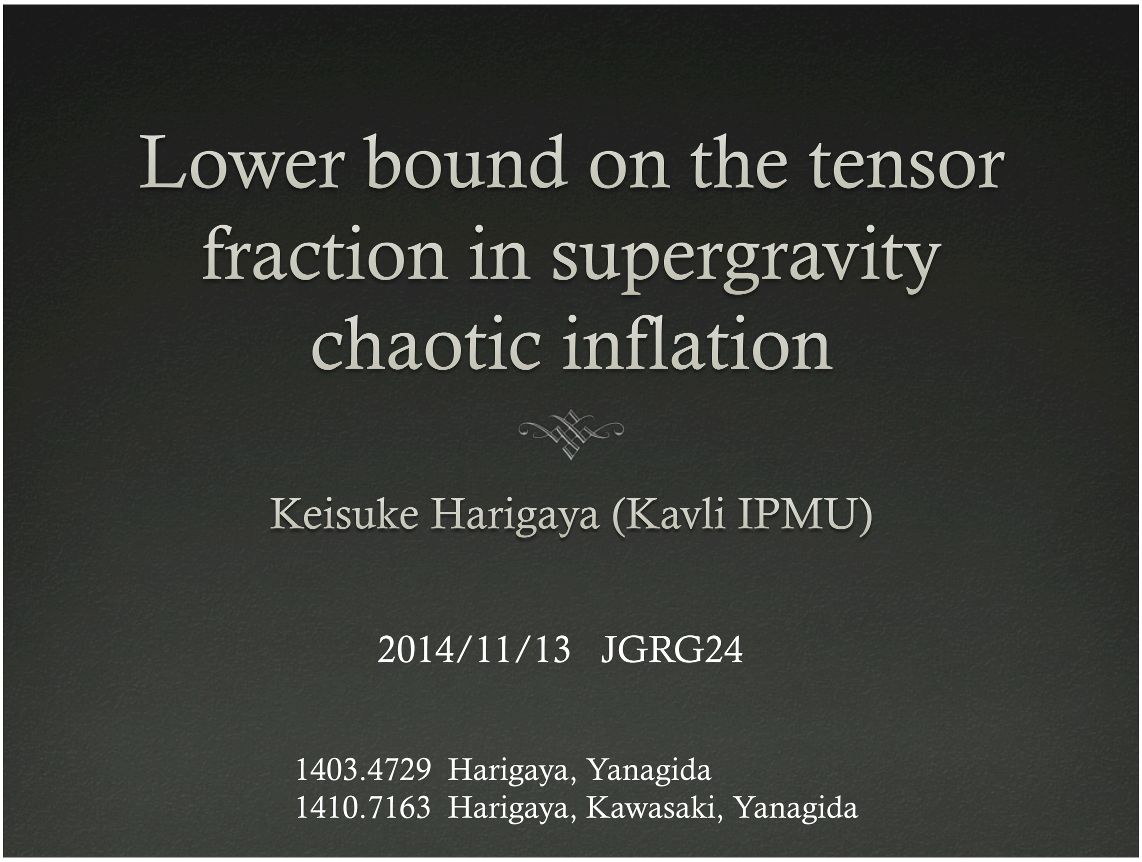
$$r = 16\left(\frac{\mathcal{F}_S}{\mathcal{F}_T}\right)^{3/2}\left(\frac{\mathcal{G}_S}{\mathcal{G}_T}\right)^{-1/2}$$

$$\mathcal{F}_S \simeq \frac{X}{H^2}(\mathcal{K} + h_2 X + 6H^2 h_4) + \frac{4\dot{\phi}X}{H}(h_3 + H^2 h_5), \quad \mathcal{F}_T \simeq 2g_4,$$
$$\mathcal{G}_S \simeq \frac{X}{H^2}(\mathcal{K} + 3h_2 X + 6H^2 h_4) + \frac{6\dot{\phi}X}{H}(h_3 + H^2 h_5), \quad \mathcal{G}_T \simeq 2g_4.$$

“Lower bound on the tensor fraction in supergravity chaotic inflation”

Keisuke Harigaya

[JGRG24(2014)111316]

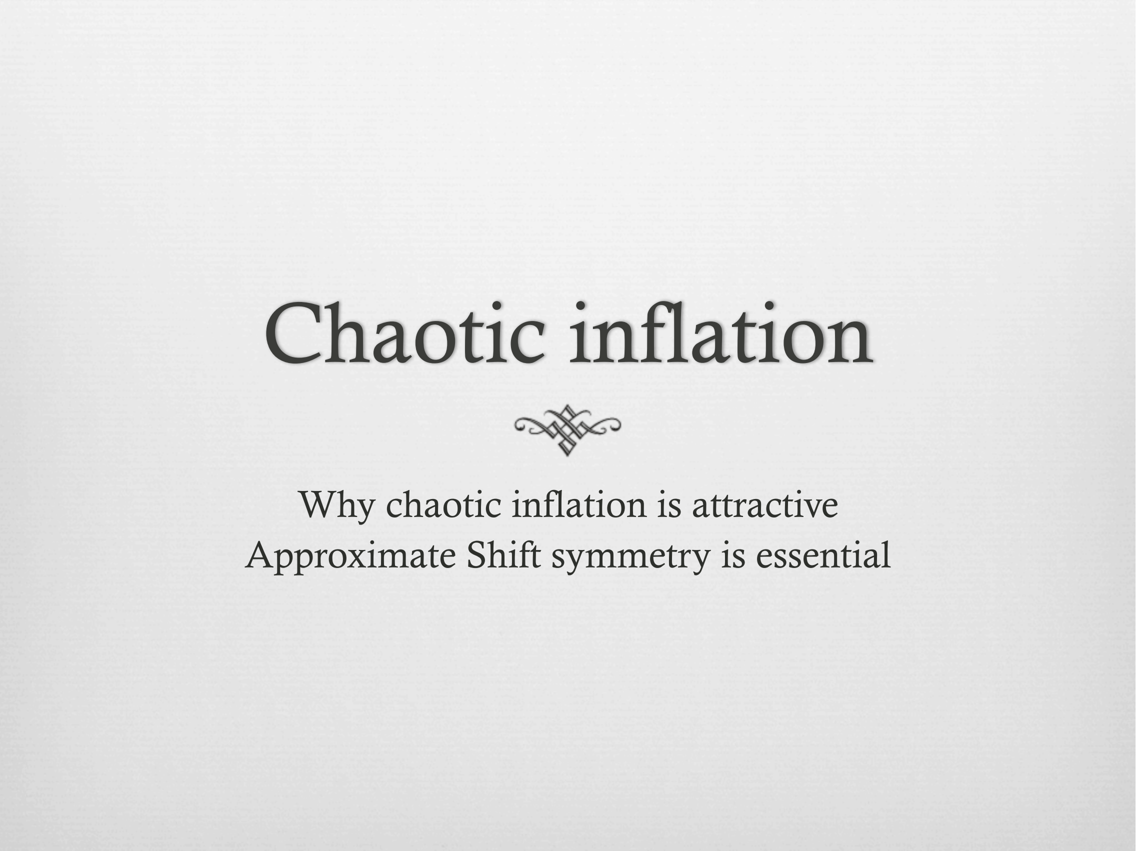


Lower bound on the tensor fraction in supergravity chaotic inflation

Keisuke Harigaya (Kavli IPMU)

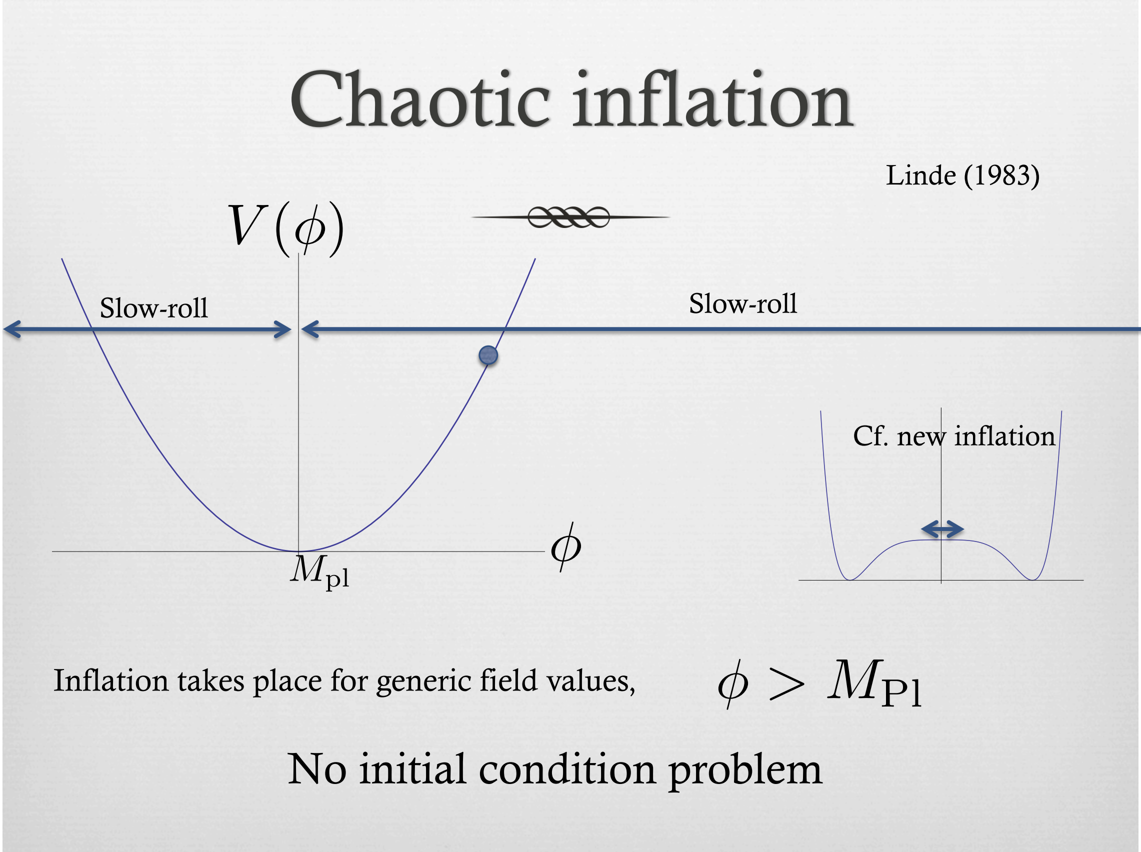
2014/11/13 JGRG24

1403.4729 Harigaya, Yanagida
1410.7163 Harigaya, Kawasaki, Yanagida



Chaotic inflation

Why chaotic inflation is attractive
Approximate Shift symmetry is essential

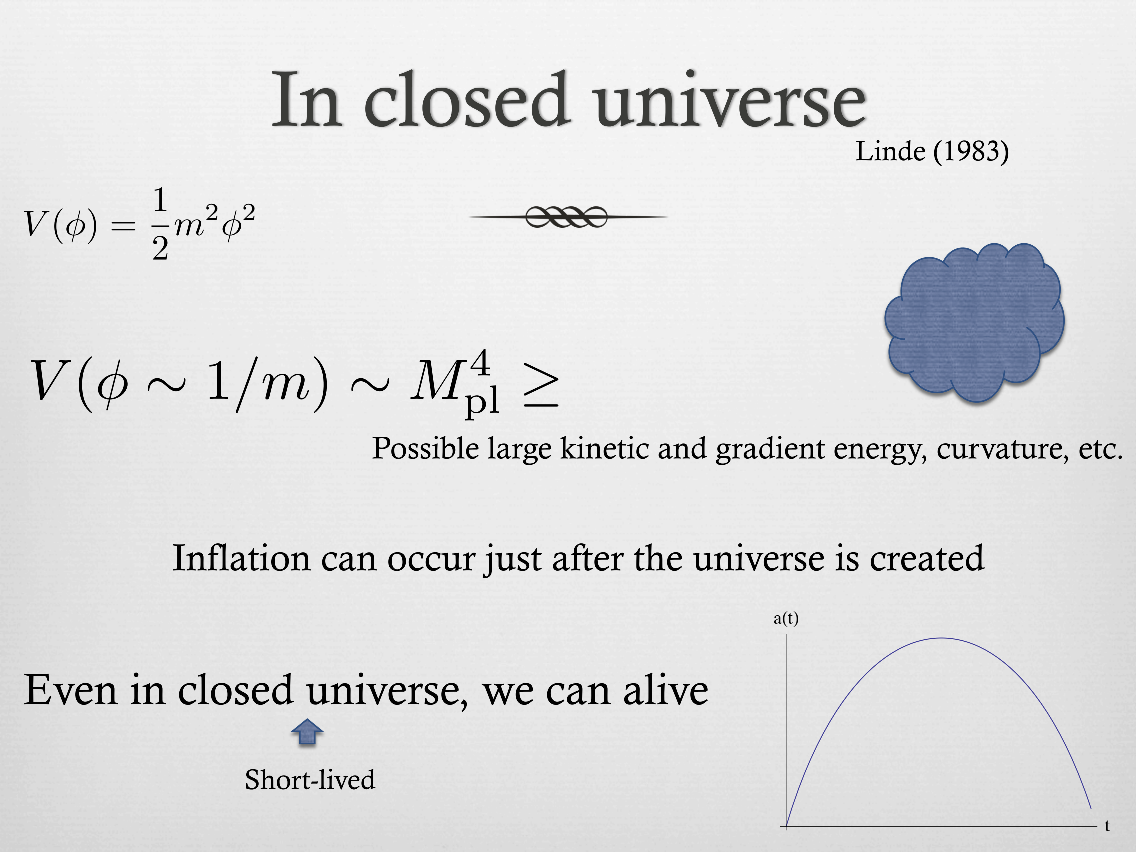


Chaotic inflation

Linde (1983)

Inflation takes place for generic field values, $\phi > M_{\rm Pl}$

No initial condition problem



In closed universe

Linde (1983)

$V(\phi) = \frac{1}{2}m^2\phi^2$

$$V(\phi \sim 1/m) \sim M_{\rm pl}^4 \geq$$

Possible large kinetic and gradient energy, curvature, etc.

Inflation can occur just after the universe is created

Even in closed universe, we can alive

Short-lived

Shift symmetry

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \cdots$$

$$m^2 < 10^{-10}, \lambda < 10^{-13}, \cdots$$

$$M_{\rm Pl} \equiv 1$$

Suggest approximate shift symmetry

$$\phi \to \phi + C$$

Softly broken by a parameter m : $\mathcal{L}_{\rm shift\ \ breaking}(m\phi)$

Guarantee stability against quantum corrections

SUGRA chaotic inflation

Two sources of shift symmetry breaking

SUSY

- Coupling unification
- Dark matter candidate
- Well-controlled quantum field theory
- Relax the hierarchy problem

SUGRA potential

Boson ⇔Fermion symmetry
Weyl fermion have 2 d.o.f

→ Complex scalars ϕ^i

Kahler potential $K(\phi^i, \phi^{*\bar{i}}) = \phi^i \phi^{*\bar{i}} + \cdots$

$\left(\mathcal{L}_{\rm kin} = K_{i\bar{i}} \partial\phi^i \partial\phi^{*\bar{i}}\right)$

Super potential $W(\phi^i)$

$$V = e^K \left[K^{\bar{i}i} D_i W D_{\bar{i}} W^* - 3|W|^2 \right]$$

$$D_i W \equiv W_i + K_i W$$

SUGRA potential

$$K(\phi^i, \phi^{*\bar{i}}), \quad W(\phi^i)$$

$$V = e^{K}\left[K^{\bar{i}i} D_i W D_{\bar{i}} W^* - 3|W|^2\right]$$

$$D_i W \equiv W_i + K_i W$$

Obstacle to the slow-roll inflation

Shift symmetry

$$\Phi \to \Phi + iC$$

Kawasaki, Yamaguchi and Yanagida (2000)

$$K = K(\Phi + \Phi^\dagger) = c(\Phi + \Phi^\dagger) + (\Phi + \Phi^\dagger)^2/2 + \cdots$$

$$\Phi = (\sigma + i\phi)/\sqrt{2}$$

ϕ : inflaton

$$K(\cancel{\phi})$$

Shift symmetry breaking in super potential

$$W = mX\Phi$$

$$|W_X|^2 \to \frac{1}{2}m^2\phi^2$$

$$m \sim 10^{-5}$$

Breaking in K

Li, Zi, Nanopoulos (2013)
Harigaya, Yanagida (2014)

A very special feature of SUSY

$K(\phi^i, \phi^{*\bar{i}})$ Renormalized

$W(\phi^i)$ Not renormalized

By quantum corrections

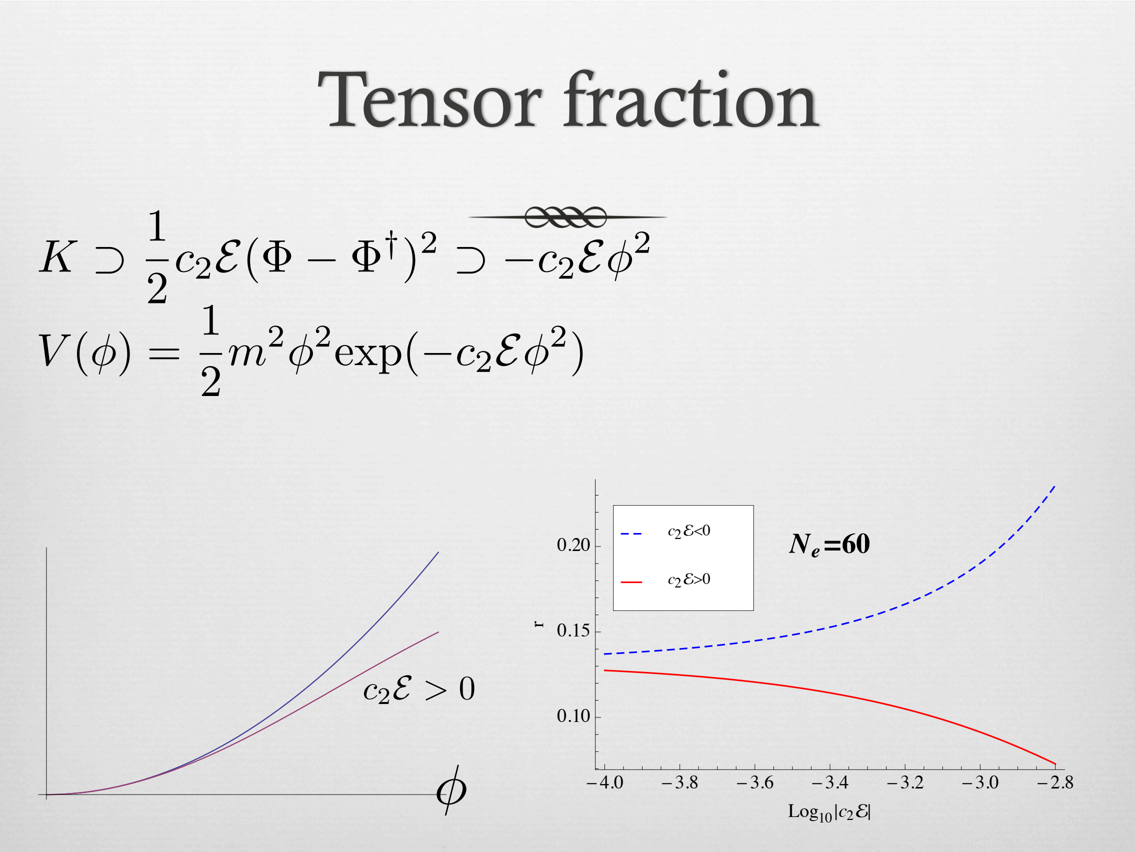
(perturvatibely)

$$K \supset F(\mathcal{E}(\Phi - \Phi^\dagger)^2)$$

$$W = mX\Phi$$

(Assume parity for simplicity)
$\Phi \to -\Phi$

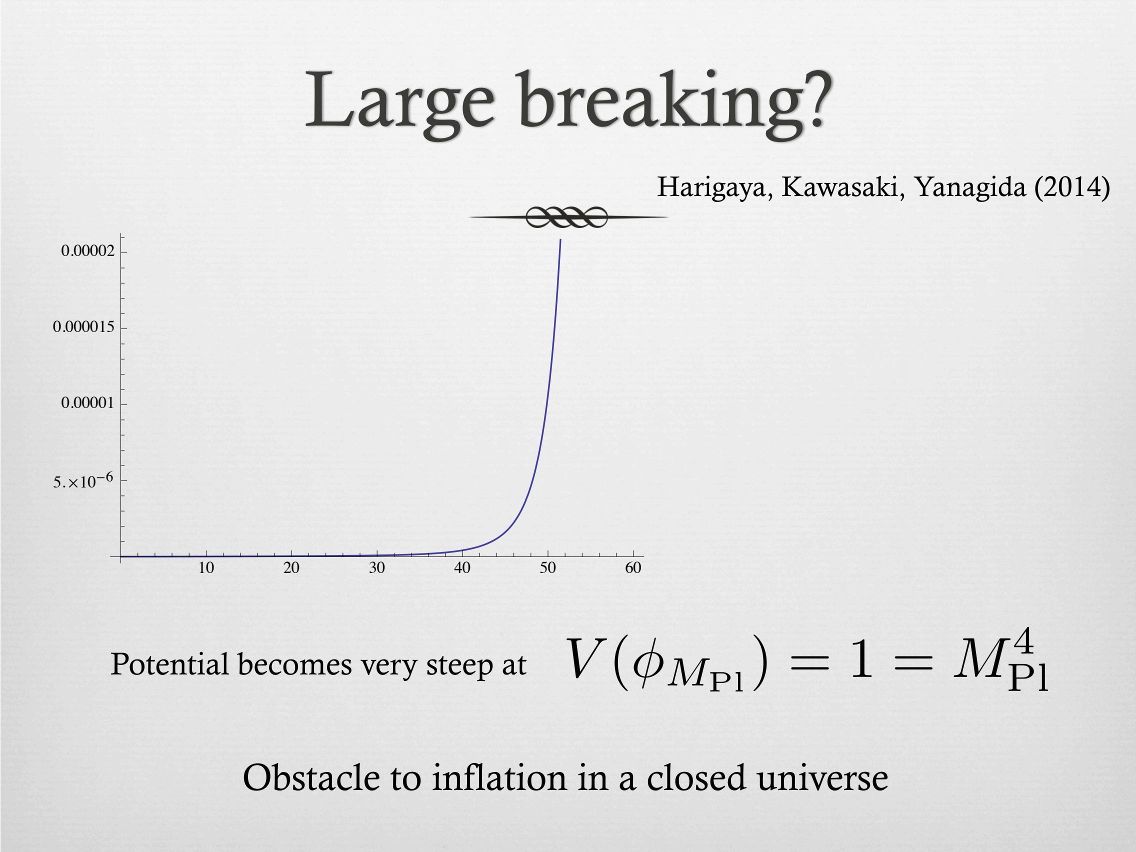
$|\mathcal{E}| \gg m$ is stable against quantum corrections



Tensor fraction

$$K \supset \frac{1}{2}c_2\mathcal{E}(\Phi - \Phi^\dagger)^2 \supset -c_2\mathcal{E}\phi^2$$

$$V(\phi) = \frac{1}{2}m^2\phi^2\exp(-c_2\mathcal{E}\phi^2)$$

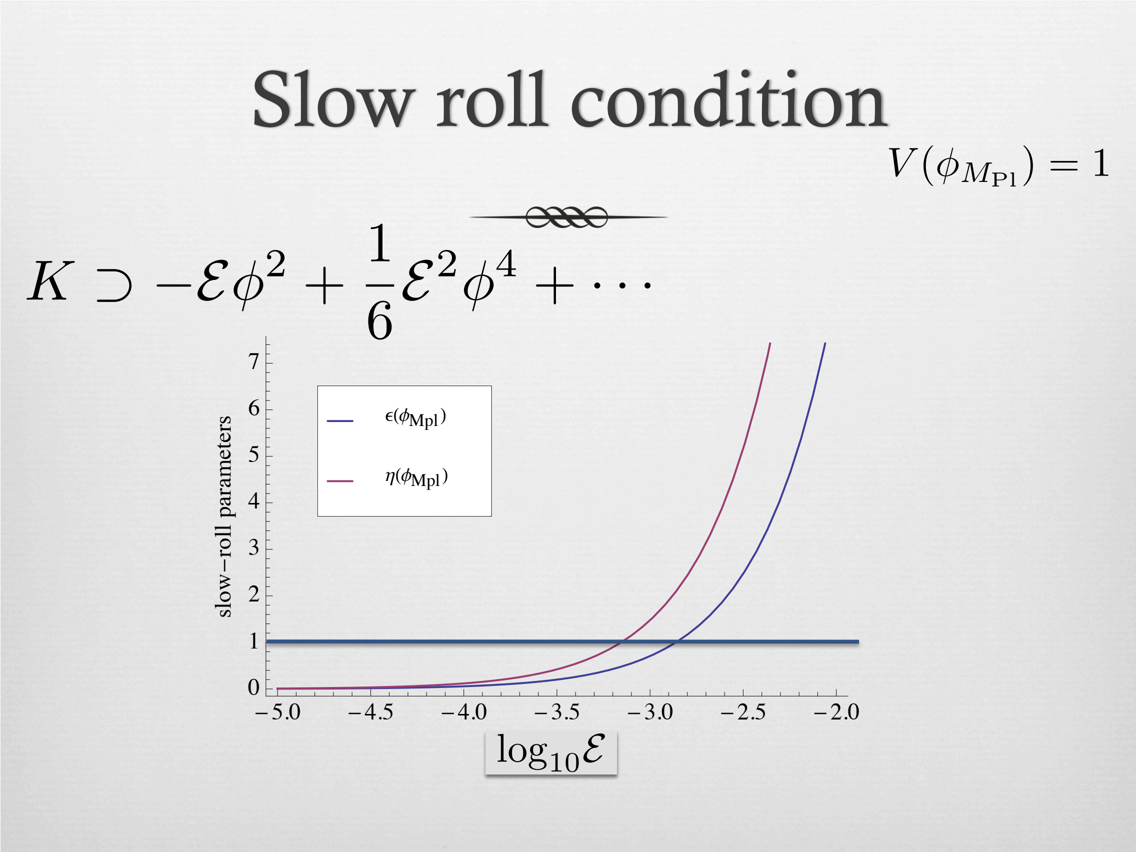


Large breaking?

Harigaya, Kawasaki, Yanagida (2014)

Potential becomes very steep at $V(\phi_{M_{\rm Pl}}) = 1 = M_{\rm Pl}^4$

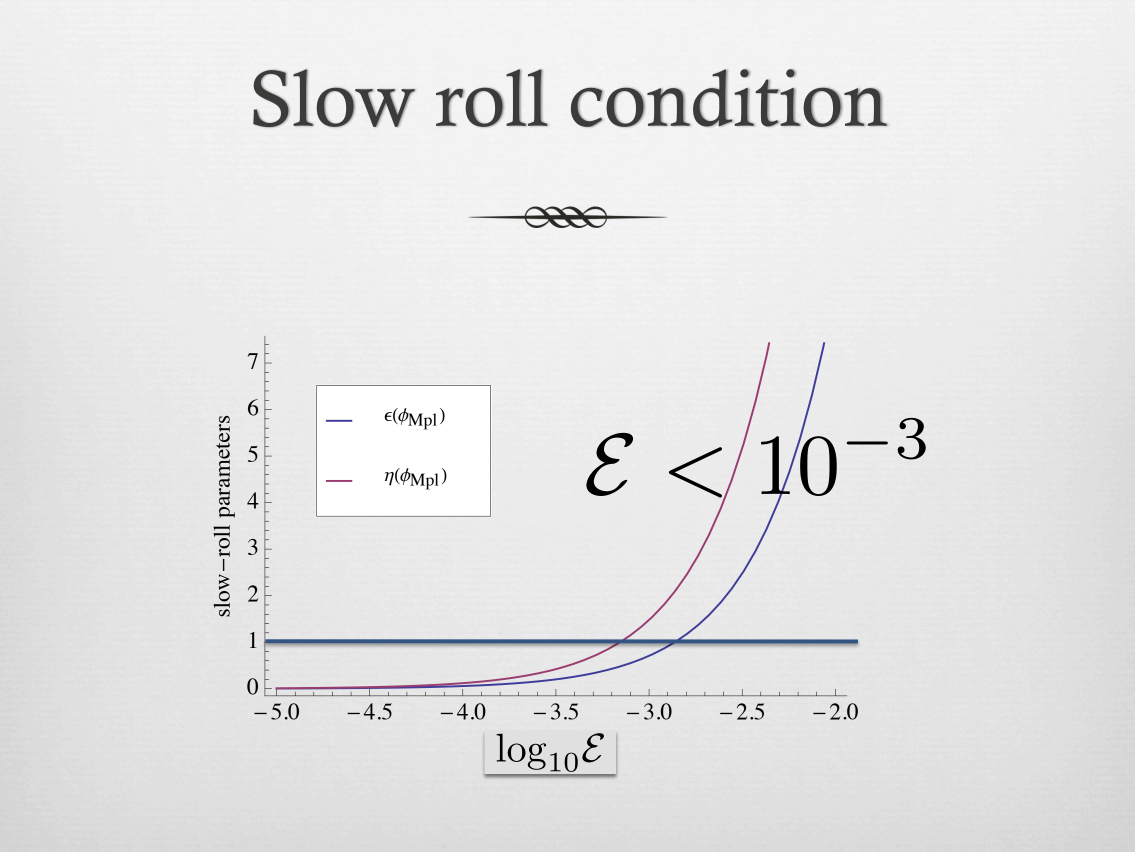
Obstacle to inflation in a closed universe



Slow roll condition

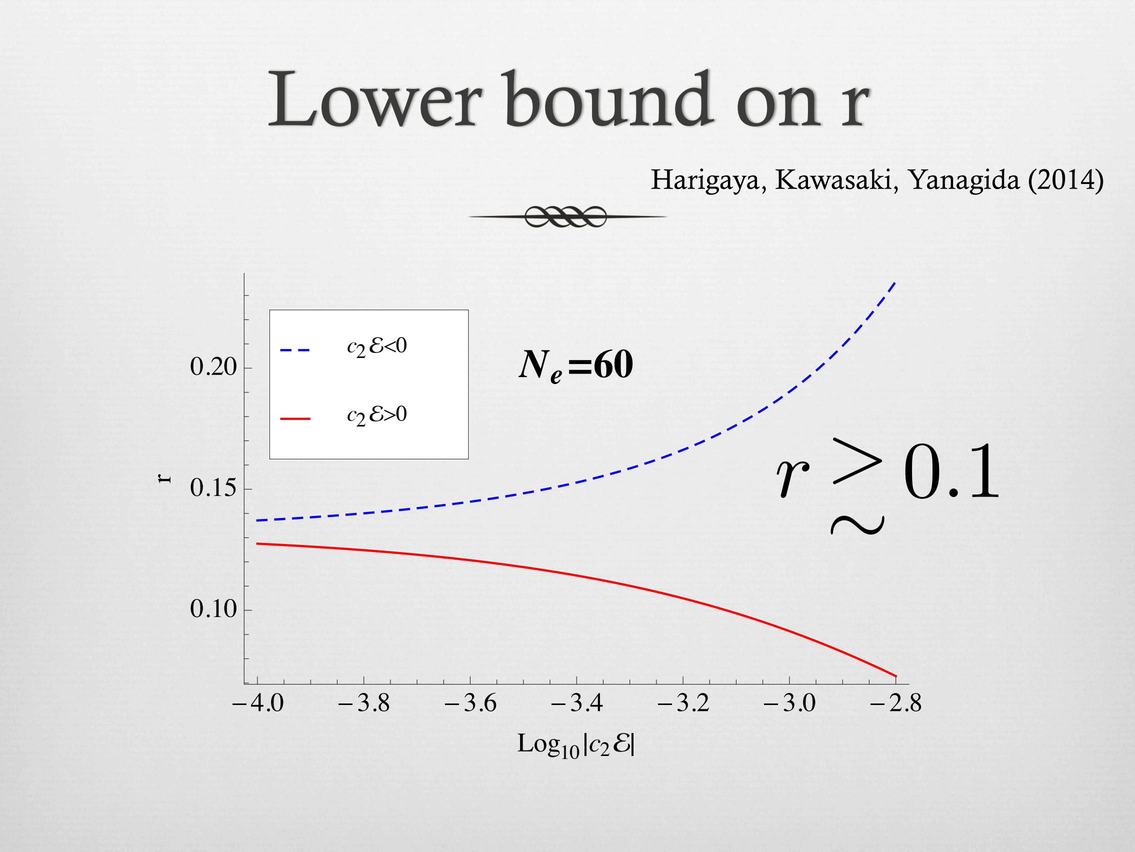
$V(\phi_{M_{\rm Pl}}) = 1$

$$K \supset -\mathcal{E}\phi^2 + \frac{1}{6}\mathcal{E}^2\phi^4 + \cdots$$



Slow roll condition

$$\mathcal{E} < 10^{-3}$$



Lower bound on r

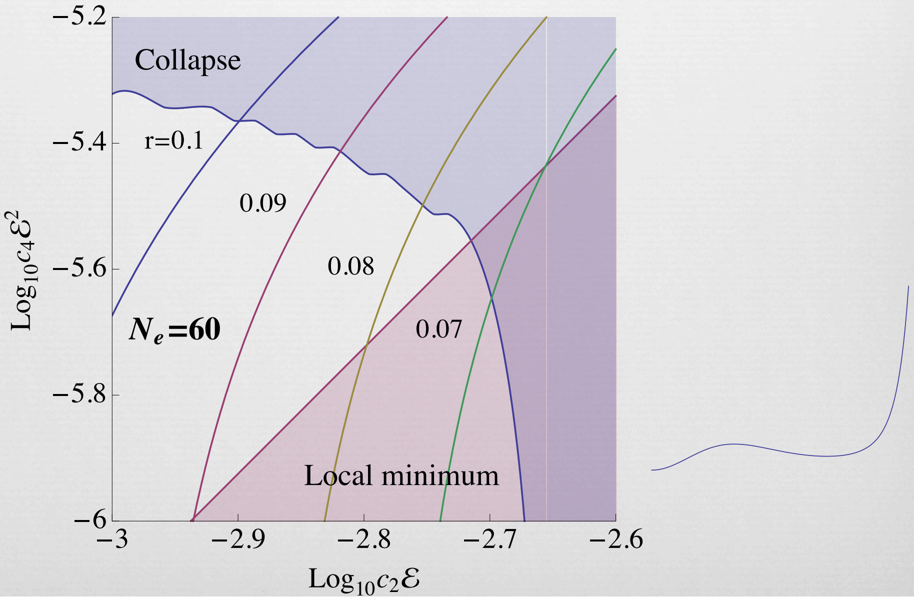
Harigaya, Kawasaki, Yanagida (2014)

N_e=60

$$r \gtrsim 0.1$$

More detail

$$K \supset -c_2\mathcal{E}\phi^2 + \frac{1}{6}c_4\mathcal{E}^2\phi^4 + \cdots$$



Summary

- Chaotic inflation is free from the initial condition problem
- Shift symmetry breaking in the Kahler potential lower tensor fraction r
- In a closed universe, $r \gtrsim 0.1$

1403.4729 Harigaya, Yanagida
1410.7163 Harigaya, Kawasaki, Yanagida

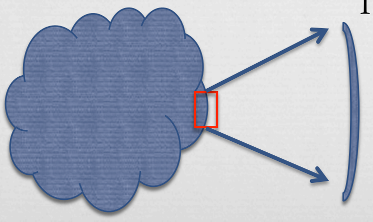
Back up

Cosmic inflation

Guth (1981)

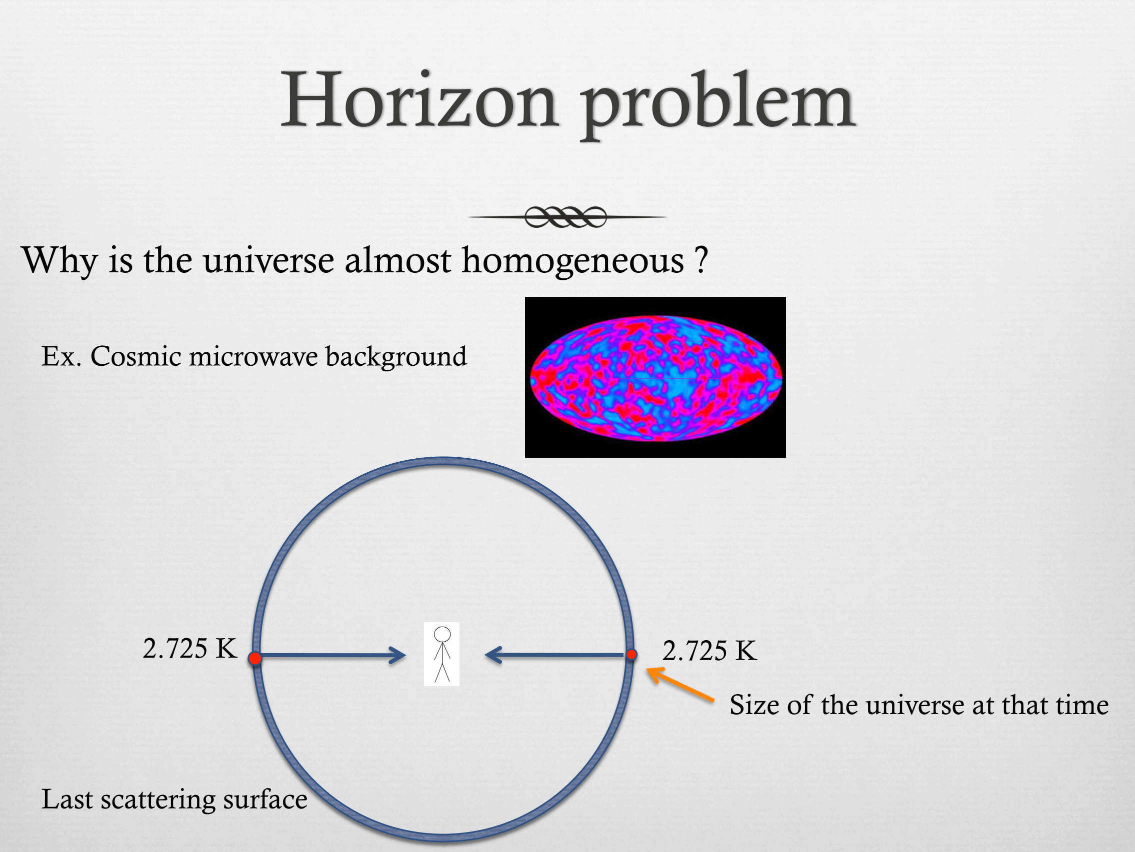
Quasi- exponential expansion of the universe at the very early stage

- Solve the Horizon & Flatness problem
- Generate the cosmic perturbation



The universe we observe

Flat and homogeneous !



Horizon problem

Why is the universe almost homogeneous ?

Ex. Cosmic microwave background

Flatness problem

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + x^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right]$$

Observation :

$$K < 10^{-2}H_0^2 \quad (a_0 = 1)$$

$$K/a^2 \simeq -H^2 + \rho/3$$

In the early universe, $a << 1$ $\quad |K/a^2| \ll H^2$

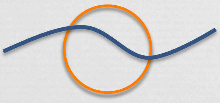
The energy density must be extremely tuned

Can ordinary expansion explain the flatness or the horizon problem?

No.

Physical size $\propto a$

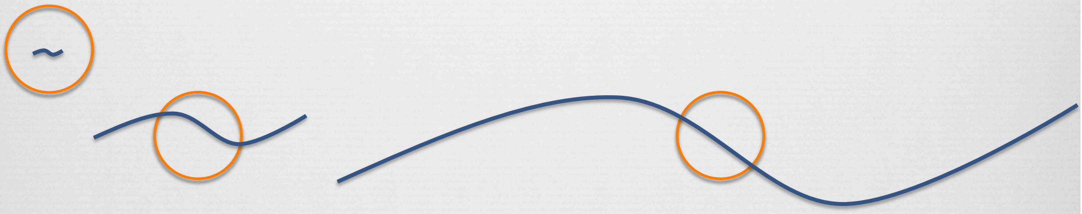
Hubble horizon $\propto a^2(\mathrm{RD})$,
$a^{3/2}(\mathrm{MD})$,
$a(\text{negative curvature})$



For a given scale (e.g. CMB scale),
the horizon used to be relatively smaller

Constant energy!

Hubble horizon = constant



The horizon used to be relatively larger

All the scale we observe used be within a Hubble radius

Closed universe

In FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + x^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right]$$ K > 0

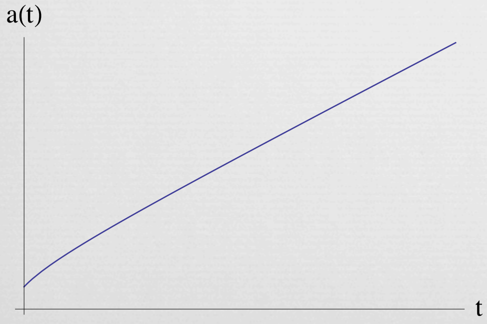
$$\int_0^{1/\sqrt{K}} \frac{dr}{\sqrt{1-Kr^2}} = \frac{\pi}{2\sqrt{K}}$$ Finite universe

Finite Life time

Open universe

In FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + x^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right]$$ K < 0



Infinite size,
infinite life time

“Cosmic string in the delayed scaling scenario and CMB”

Kohei Kamada

[JGRG24(2014)111317]

Cosmic string in the delayed scaling scenario and CMB

based on: KK, Y. Miyamoto, D. Yamauchi & J. Yokoyama, PRD90 (2014) 083502 (arXiv:1407.2951)

Kohei Kamada
(EPF Lausanne, JSPS fellow)

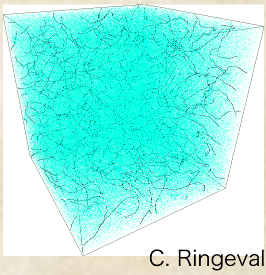
JGRG24, 13/11/2014, Kavli IPMU

Courtesy H.Oide

Introduction

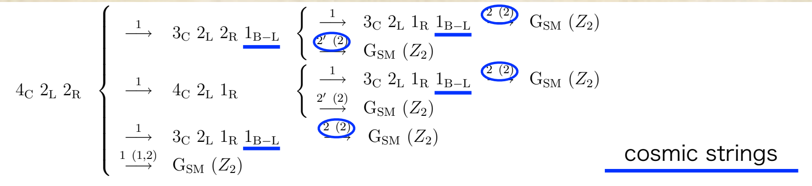
Cosmic string...

- Line-like topological defect associated with symmetry breaking.
- Almost unavoidably produced when GUT breaks down to the Standard Model gauge group.



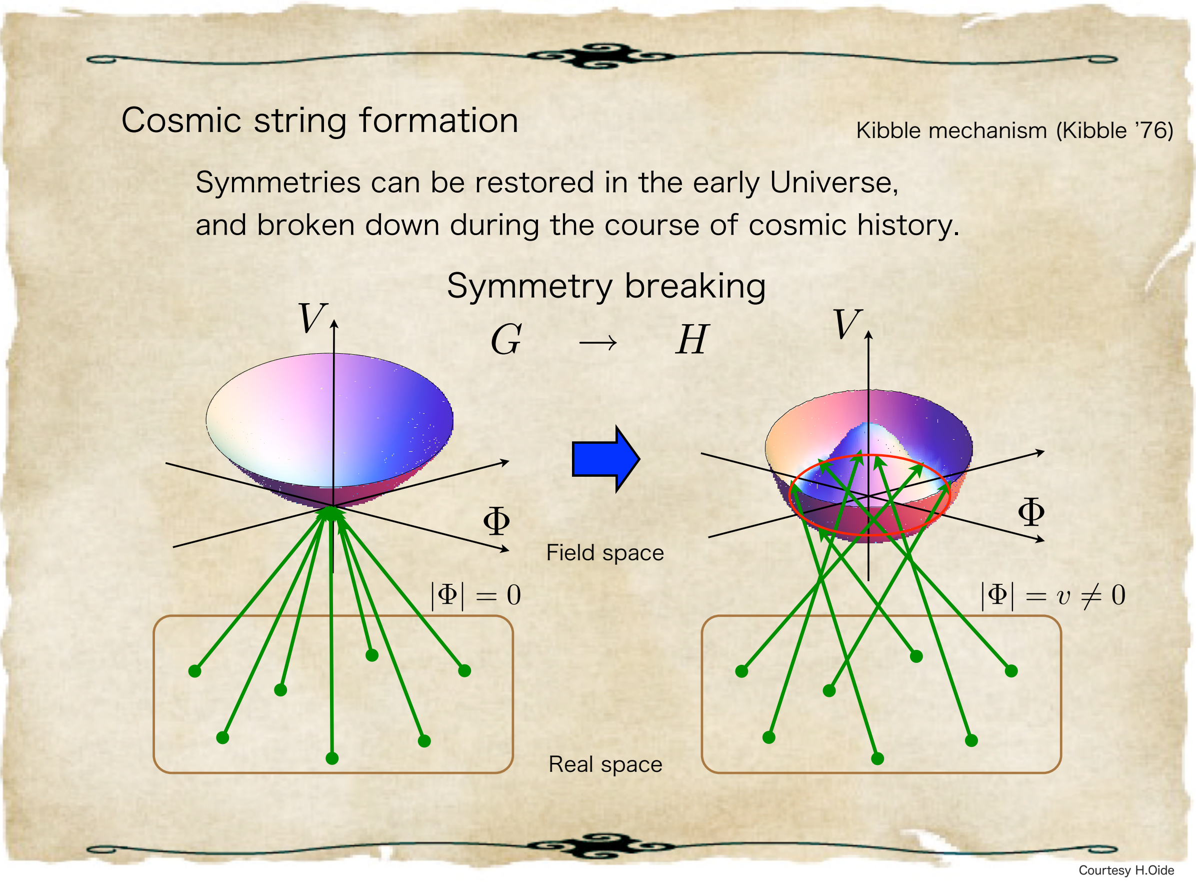
C. Ringeval+ ('07)

e.g.) R. Jeannerot+ ('03)



Study of cosmic string can lead to the understanding of the nature of the Standard Model and beyond.

Courtesy H.Oide



Cosmic string formation

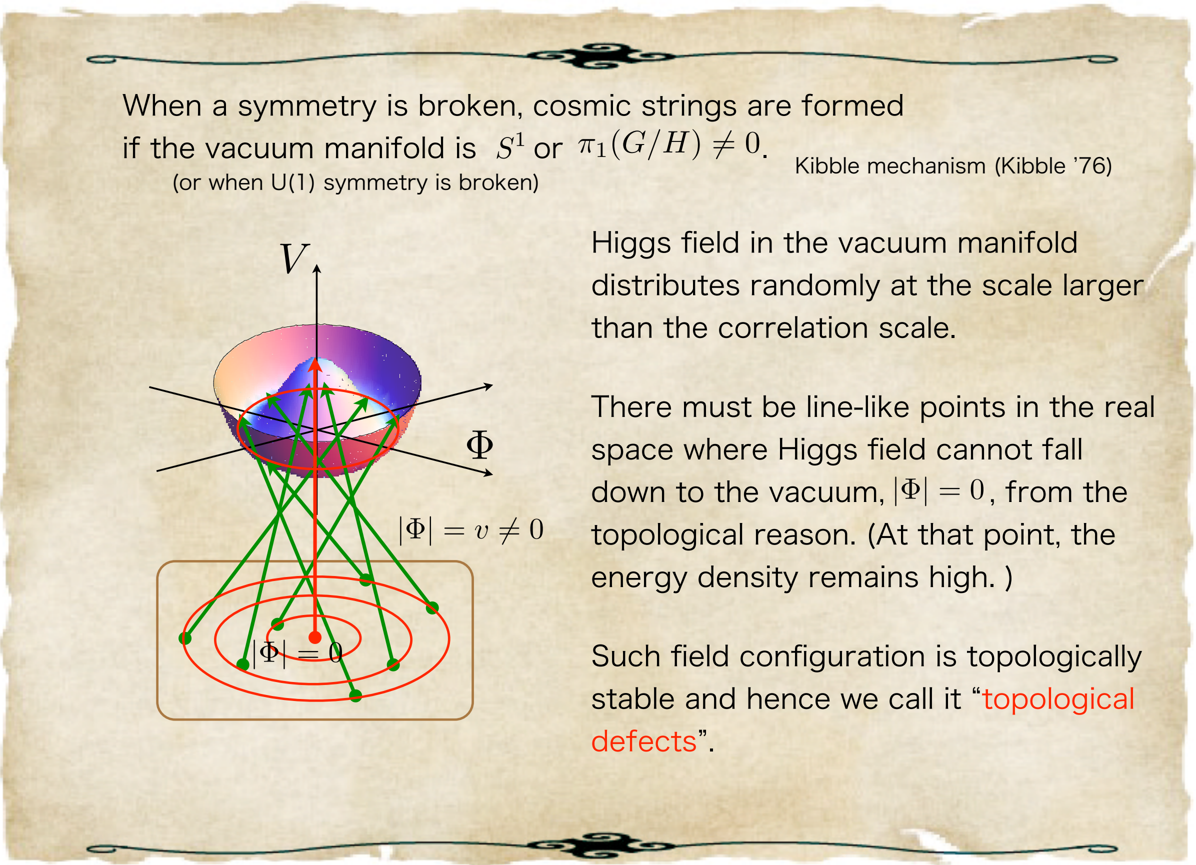
Kibble mechanism (Kibble '76)

Symmetries can be restored in the early Universe,
and broken down during the course of cosmic history.

Symmetry breaking

$G \quad \rightarrow \quad H$

Courtesy H.Oide



When a symmetry is broken, cosmic strings are formed
if the vacuum manifold is S^1 or $\pi_1(G/H) \neq 0$.
(or when U(1) symmetry is broken)

Kibble mechanism (Kibble '76)

Higgs field in the vacuum manifold distributes randomly at the scale larger than the correlation scale.

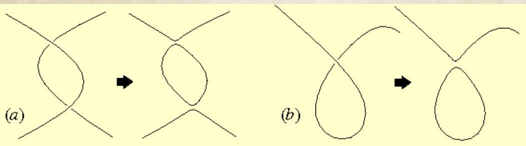
There must be line-like points in the real space where Higgs field cannot fall down to the vacuum, $|\Phi| = 0$, from the topological reason. (At that point, the energy density remains high.)

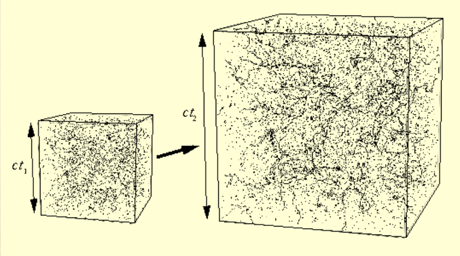
Such field configuration is topologically stable and hence we call it "topological defects".

Courtesy H.Oide

Scaling behavior of the cosmic string network (Kibble '85)

The energy density of cosmic strings decays as a^{-2} and hence they may overclose the Universe if they are produced in the early Universe...





However, cosmic string network forms loops when they intersect, and hence its characteristic scale remains constant relative to the Hubble length.

-> They do not overclose the Universe!

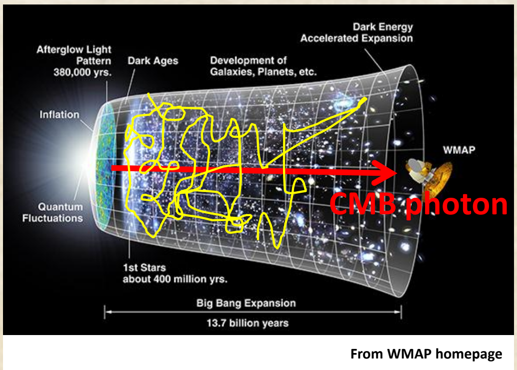
=> They are still in our Universe, and it is possible to observe their traces in CMB, GWB, or cosmic rays.

Courtesy H.Oide

Traces of cosmic strings in CMB (Albrecht+ '97; Seljak+ '97)

cosmic strings between the last scattering surface and us generates the fluctuation of CMB temperature/polarization.

From slide of T.Suyama



From WMAP homepage

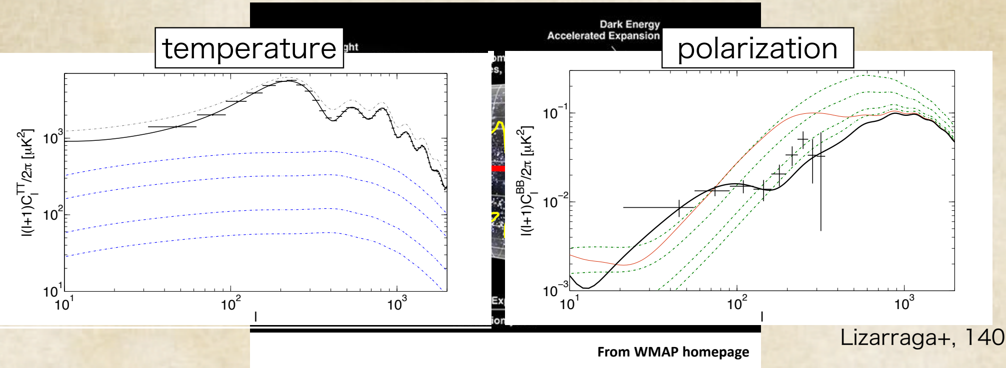
Courtesy H.Oide

Traces of cosmic strings in CMB

(Albrecht+ '97; Seljak+ '97)

cosmic strings between the last scattering surface and us generates the fluctuation of CMB temperature/polarization.

From slide of T.Suyama



Lizarraga+, 1403.4924

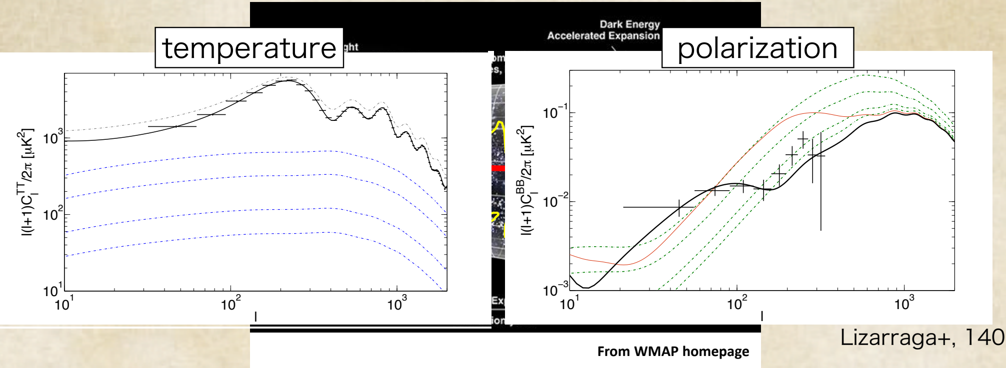
Courtesy H.Oide

Traces of cosmic strings in CMB

(Albrecht+ '97; Seljak+ '97)

cosmic strings between the last scattering surface and us generates the fluctuation of CMB temperature/polarization.

From slide of T.Suyama



Lizarraga+, 1403.4924

Planck temperature observation gives the strong constraint on the cosmic string tension; $G\mu \lesssim (1-3)\times 10^{-7}$ (Planck collaboration, 1303.5085)

Related to the symmetry breaking scale.
#CMB can see them only through gravity.

There are uncertainties in the model of cosmic string.

Courtesy H.Oide

Delayed scaling scenario

(Lazarides+ '84; Vishniac+ '87; Yokoyama, '88; KK+ '12)

The discussion for the effect on CMB is based on the assumption that the cosmic string entered the scaling regime well before recombination.

-> Observational predictions are very generic.

It is true for the case of hybrid inflation or thermal-mass triggered phase transition.

Courtesy H.Oide

Delayed scaling scenario

(Lazarides+ '84; Vishniac+ '87; Yokoyama, '88; KK+ '12)

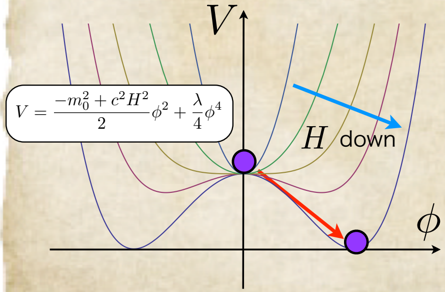
The discussion for the effect on CMB is based on the assumption that the cosmic string entered the scaling regime well before recombination.

-> Observational predictions are very generic.

It is true for the case of hybrid inflation or thermal-mass triggered phase transition.

However, it is possible for the phase transition to take place DURING inflation, since the symmetry is naturally restored during inflation due to the "Hubble-induced" mass, $c^2H^2\phi^2$ coming from

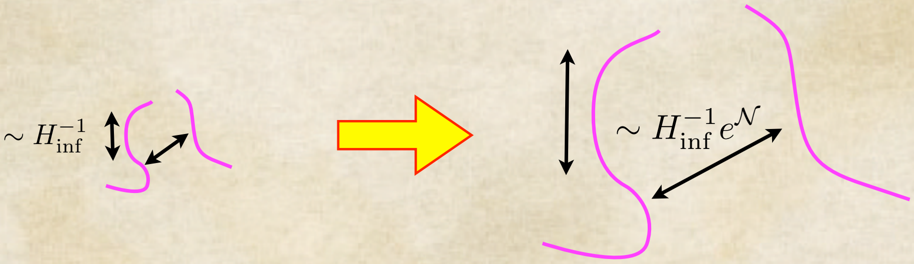
- non minimal coupling to gravity: $\xi\phi^2 R$
- direct coupling between inflaton and Higgs: $\kappa\phi_{\rm inf}^2\phi^2$
- gravitational coupling in SUSY F-term inflation: $e^{|\phi|^2/M_{\rm Pl}^2}V_{\rm inf}$
- and so on...



If the Hubble-induced mass and zero-temp. mass are comparable and Hubble parameter decreases relatively largely, cosmic string can be formed during inflation.

Courtesy H.Oide

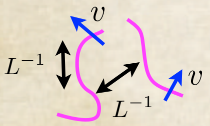
The characteristic length, which would be the Hubble length at CS formation, gets exponentially long at the end of inflation.



At the end of inflation, CSs are distributed at the superhorizon scales, and characteristic length evolves just $\propto a$ after that.

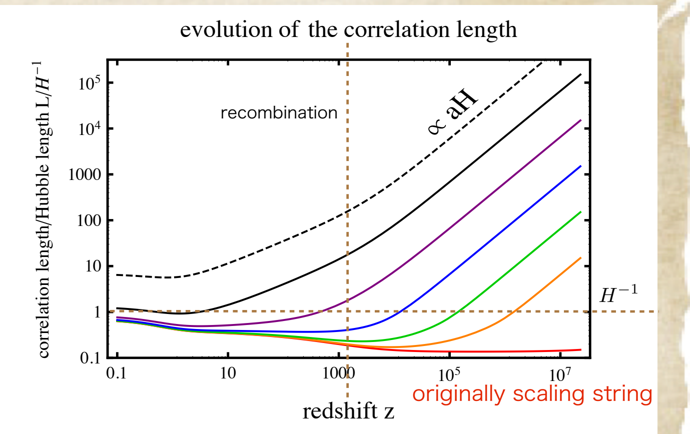
Courtesy H.Oide

Adopting velocity-dependent one-scale model (approximation), we find the typical evolution of the correlation length of CS network and how the system would approach the scaling regime. (Martins+ '96, '00)



$$\frac{dL}{dt} = (1+v^2)HL + \frac{1}{2}\tilde{c}v$$

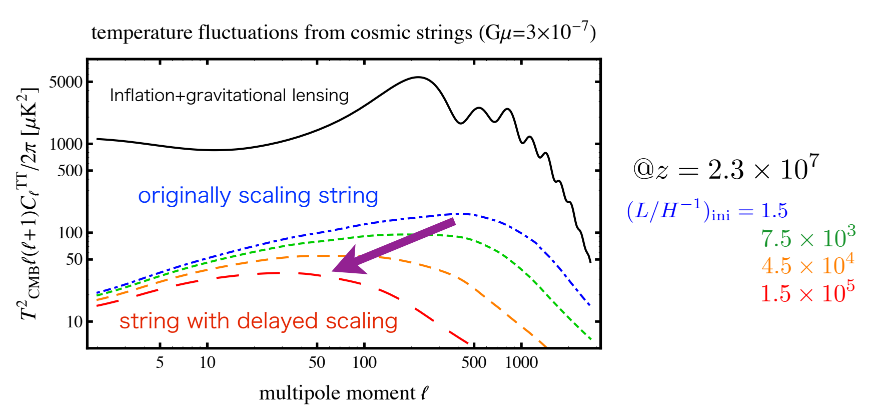
$$\frac{dv}{dt} = (1-v^2)\left(\frac{\tilde{k}(v)}{L} - 2Hv\right)$$



It takes a few orders of redshift for the system to enter the scaling regime after the characteristic length comes to subhorizon scales.

Courtesy H.Oide

String-induced CMB temperature fluctuations

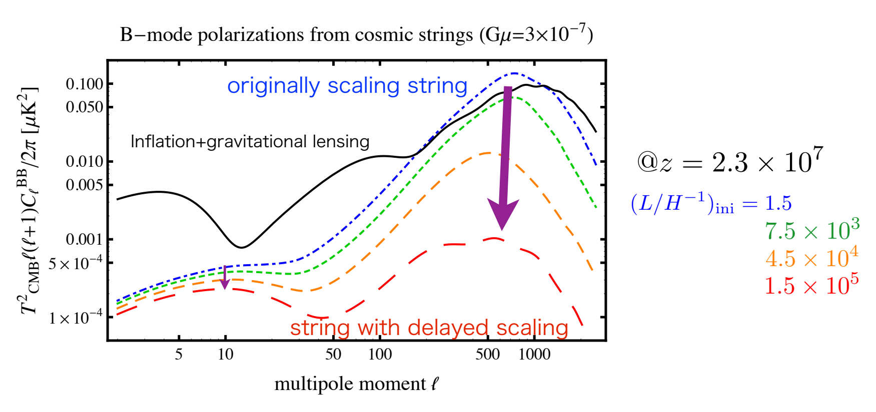


used CMBACT[v4] ('99 Pogosian+Moss)

The position of the peak is determined by the time when the network enters the scaling regime.

Courtesy H.Oide

String-induced CMB polarization fluctuations

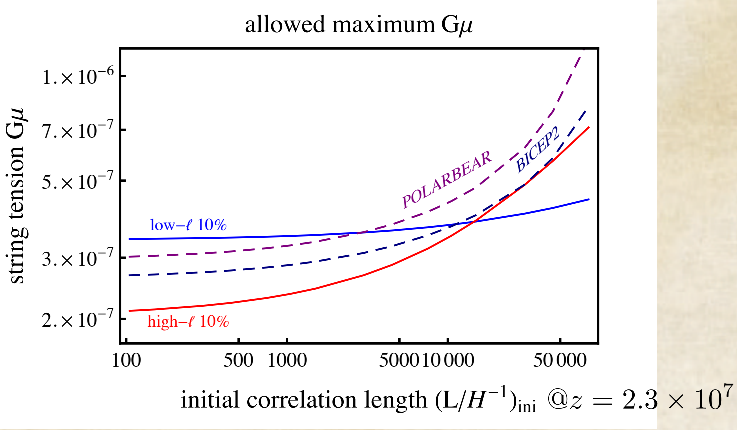


used CMBACT[v4] ('99 Pogosian+Moss)

The position of the peak is determined by recombination and reionization. Their amplitude is determined by the number of strings at that time.

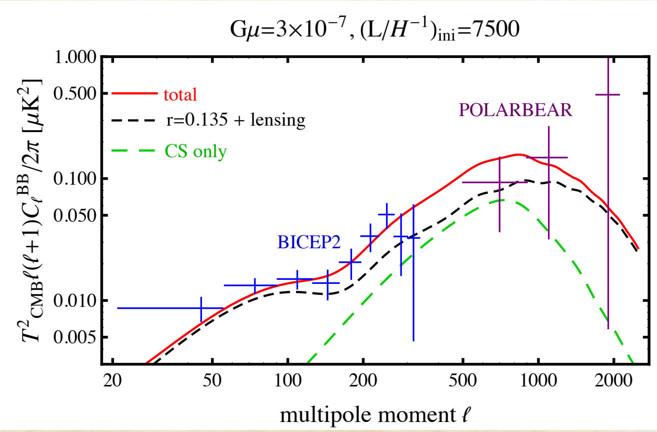
Courtesy H.Oide

Constraint on the string tension



Courtesy H.Oide

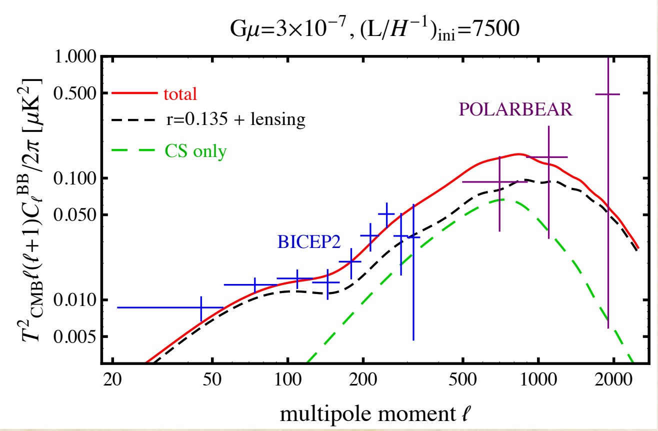
BICEP2?



$\Delta\chi^2 \sim -9$ for GW (r=0.2)+GL vs GW (r=0.135)+CS+GL

Courtesy H.Oide

BICEP2?



$\Delta\chi^2 \sim -9$ for GW (r=0.2)+GL vs GW (r=0.135)+CS+GL

But they are most likley dust...? (1409.5738, Planck collaboration)

Courtesy H.Oide

Summary

- Cosmic strings are key ingredients for both cosmology and high energy physics.
- Their formation during inflation is an interesting possibility.
- The string network enters scaling regime later in this case, which can reduce the high multipole moment of both CMB temperature and polarization fluctuations.

Open issues

- We assumed several idealization, such as one-scale model.
  -> need numerical simulations.
- We gave just qualitative constraints.
  -> Combined analysis of Planck temperature/polarization data and other experiments (including BICEP2) is needed to give a precise constraint.

Courtesy H.Oide

“Gravitational waves from slow-roll inflation in Lorentz-violating Weyl gravity”

Kohji Yajima

[JGRG24(2014)111318]

Gravitational waves from slow-roll inflation in Lorentz-violating Weyl gravity

Kohji Yajima (D1 / Rikkyo University)
Tsutomu Kobayashi (Rikkyo University)

In preparation

In the very early universe

- We don't know quantum gravity.
- Quantum corrections may be important.
- We put the higher orders of curvature invariants into the Einstein-Hilbert action.

$$c = 1$$
$$\hbar = 1$$
$$\kappa = 8\pi G$$

But, in general

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left(R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \lambda R_{\mu\nu\rho\gamma}R^{\mu\nu\rho\gamma} + ...\right)$$

often generates ghost degrees of freedom.

Weyl gravity

N. Deruelle, M. Sasaki, Y. Sendouda and A. Youssef, JHEP **09**, 009 (2012)

$$S[g_{ab}] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left(R - \cancel{\frac{\gamma}{2}C_{abcd}C^{abcd}}\right) \longrightarrow \text{ghosts}$$

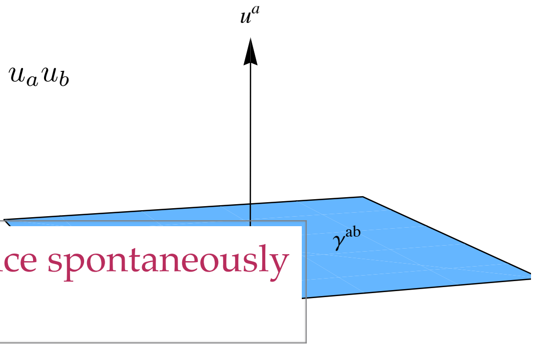
$$[\gamma] = L^2$$

$$S[g_{ab},\chi] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left(R + 2\gamma C_{abcd}C_{efgh}\gamma^{ae}\gamma^{bf}\gamma^{cg}u^d u^h\right) + S_\chi[g_{ab},\chi]$$

Assumption : $\partial_a\chi$ is everywhere timelike and future-directed.

$$u_a \equiv \frac{\partial_a\chi}{\sqrt{-\partial_b\chi\partial^b\chi}} \qquad \text{and} \qquad \gamma_{ab} \equiv g_{ab} + u_a u_b$$

u^a determines a preferred time direction.



This theory breaks local Lorentz-invariance spontaneously but ghost-free!!

Gravitational waves in Weyl gravity

Action

$$S[g_{ab},\chi] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left(R + 2\gamma C_{abcd}C_{efgh}\gamma^{ae}\gamma^{bf}\gamma^{cg}u^d u^h\right) + S_\chi[g_{ab},\chi]$$

Gravitational waves

$$g_{\alpha\beta}dx^\alpha dx^\beta = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

The perturbations from the Weyl-squared term:

$$\begin{aligned}{}^{(2)}S_{W2}[h_{ij}] &= \int d\eta d^3x\sqrt{-g}\,{}^{(1)}C_{abcd}{}^{(1)}C_{efgh}\gamma^{ae}\gamma^{bf}\gamma^{cg}u^d u^h \\ &= \int d\eta d^3x\,{}^{(1)}C_{ijk0}{}^{(1)}C^{ijk}{}_0\Big|_{a(\eta)=1} \\ &= \frac{1}{2}\int d\eta d^3x\,\partial_k h'_{ij}\partial^k h'^{ij}\end{aligned}$$

this contains first order time derivatives

Gravitational waves in Weyl gravity

The total action for tensor perturbations is

$$S_T[h_{ij}] = \frac{1}{8\kappa}\int d\eta d^3x \left[\underbrace{a^2(h'_{ij}h'^{ij} - \partial_k h_{ij}\partial^k h^{ij})}_{\text{from Einstein-Hilbert's action}} + \underbrace{4\gamma\partial_k h'_{ij}\partial^k h'^{ij}}_{\text{from Weyl-squared term}}\right]$$

the momentum conjugate to h_{ij}

$$\pi^{ij} = \frac{\partial\mathcal{L}}{\partial h'_{ij}} = \frac{1}{4\kappa}(a^2 h'^{ij} - 4\gamma\triangle h'^{ij})$$

canonical quantization:

$$[\hat{h}_{ij}(\eta,\vec{x}_1), \hat{\pi}^{ij}(\eta,\vec{x}_2)] = 2i\delta(\vec{x}_1 - \vec{x}_2)$$

all other commutators are zero.

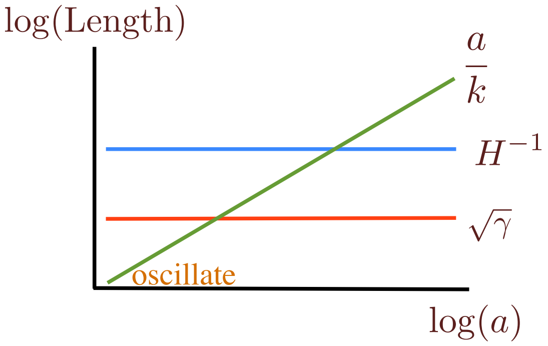
Quantization

N. Deruelle, M. Sasaki, Y. Sendouda and A. Youssef, JHEP **09**, 009 (2012)

In the de-Sitter background,
the early time behavior of the mode

$0 < \sqrt{\gamma}H < 1$: oscillate

$\sqrt{\gamma}H > 1$: never oscillate



We quantize the tensor perturbations by making the mode coincide with the positive frequency mode in Minkowski space-time at early time.

So quantization is carried out in $0 < \sqrt{\gamma}H < 1$.

Quantization

N. Deruelle et al. JHEP **09**, 009 (2012)

$$\hat{h}_{ij}(\eta,\vec{x}) = \sum_{\lambda=1,2}\int \frac{d^3k}{(2\pi)^{3/2}}\left[e^{\lambda}_{ij}(\vec{k})\,\hat{a}^{\lambda}_{\vec{k}}\,h_k(\eta)\,e^{i\vec{k}\cdot\vec{x}} + \mathrm{h.c.}\right]$$

where

$$h_k = -i\sqrt{\frac{\kappa\nu}{8k^3\epsilon^3}}H\left\{\frac{\sqrt{\pi}\Gamma(-i\nu/2)}{\Gamma^2(5/4-i\nu/4)}h_{(g)} - \frac{\sqrt{\pi}\Gamma(-i\nu/2)}{\Gamma^2(-1/4-i\nu/4)}h_{(d)}\right\}$$

$$h_{(g)}(y) = \frac{1}{2}F\left(\frac{-1-i\nu}{4},\frac{-1+i\nu}{4},-\frac{1}{2};-4y^2\right)$$

$$h_{(d)}(y) = \frac{32}{3}y^3F\left(\frac{5+i\nu}{4},\frac{5-i\nu}{4},\frac{5}{2};-4y^2\right)$$

$$\nu \equiv \begin{cases} \sqrt{1/\gamma H^2-1} & \text{if} \quad 0<\sqrt{\gamma}H<1 \\ i\sqrt{1-1/\gamma H^2} & \text{if} \quad \sqrt{\gamma}H>1 \end{cases}$$

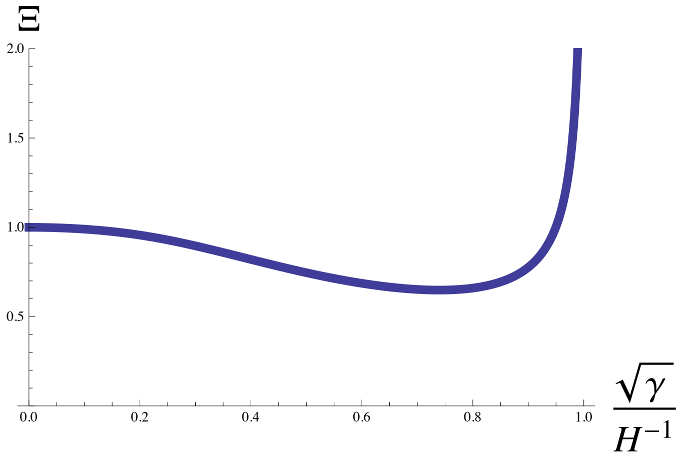
$$y \equiv -k\eta\sqrt{\gamma}H$$

Power spectrum of gravitational waves in de Sitter expansion

N. Deruelle et al. JHEP **09**, 009 (2012)

Comparing the amplitude of gravitational waves in Weyl gravity with GR.

$$\Xi \equiv \frac{\text{amplitude of power spectrum of GWs in Weyl gravity}}{\text{amplitude of power spectrum of GWs in GR}}$$



Slow-roll inflation in Weyl gravity

Action in cosmic time

$$S_T = \frac{1}{8\kappa}\int dt d^3x \left[a(a^2+4\gamma k^2)\dot{h}_{\vec{k}}^2 - ak^2 h_{\vec{k}}^2\right]$$

E.O.M

$$\ddot{f}_{\vec{k}} + \omega_k^2 f_{\vec{k}} = 0,$$

$$f_{\vec{k}} := a^{3/2}\sqrt{1+4\gamma k^2/a^2}\, h_{\vec{k}}$$

$$\omega_k^2 = -\frac{1}{4}\left(H^2+2\dot{H}\right) + \frac{k^2/a^2 - 2H^2 - \dot{H}}{1+4\gamma k^2/a^2} - \frac{4H^2\gamma k^2/a^2}{(1+4\gamma k^2/a^2)^2}$$

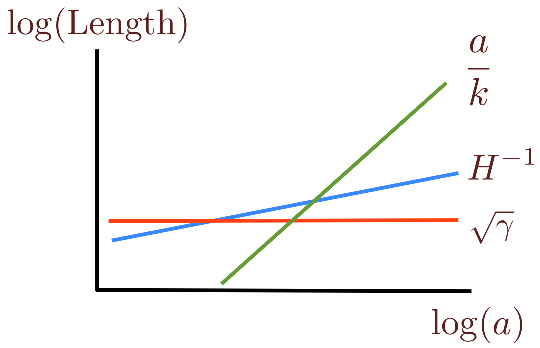
Slow-roll parameter $\epsilon \equiv -\frac{\dot{H}}{H^2}$

we assume this parameter is nearly constant

Short wavelength mode $\frac{a^2}{k^2} \ll \gamma < H^{-2}$

$$\omega_k^2 \simeq \frac{1}{4\gamma} + \frac{1}{t^2}\left(\frac{1}{4}-\mu^2\right),$$

$$\mu = \frac{1-\epsilon}{2\epsilon}$$



the positive frequency mode

$$f_{\vec{k}} = (\pi t)^{1/2} H_\mu^{(2)}\left(\frac{t}{2\sqrt{\gamma}}\right)$$

E.O.M

$$\ddot{f}_{\vec{k}} + \omega_k^2 f_{\vec{k}} = 0,$$

$$f_{\vec{k}} := a^{3/2}\sqrt{1 + 4\gamma k^2/a^2}\, h_{\vec{k}}$$

$$\omega_k^2 = -\frac{1}{4}\left(H^2 + 2\dot{H}\right) + \frac{k^2/a^2 - 2H^2 - \dot{H}}{1 + 4\gamma k^2/a^2} - \frac{4H^2\gamma k^2/a^2}{(1 + 4\gamma k^2/a^2)^2}$$

with the initial condition

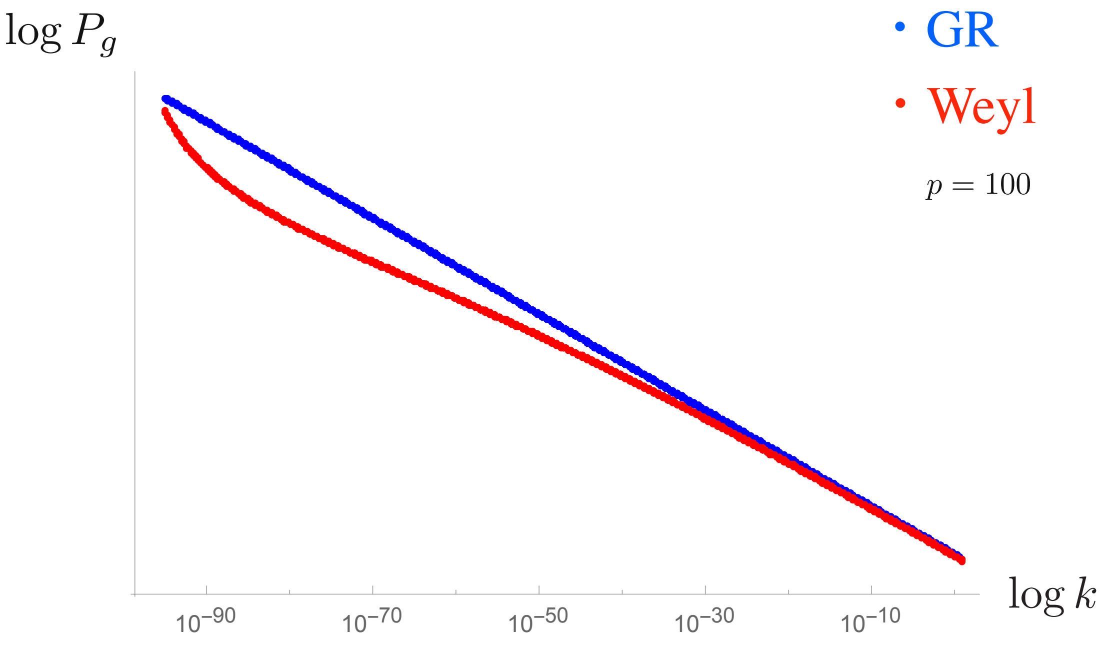
$$f_{\vec{k}} = (\pi t)^{1/2} H_\mu^{(2)}\left(\frac{t}{2\sqrt{\gamma}}\right)$$

$$\mu = \frac{1-\epsilon}{2\epsilon}$$

We solve the equation, numerically, in power-law inflation: $a(t) \propto t^p$

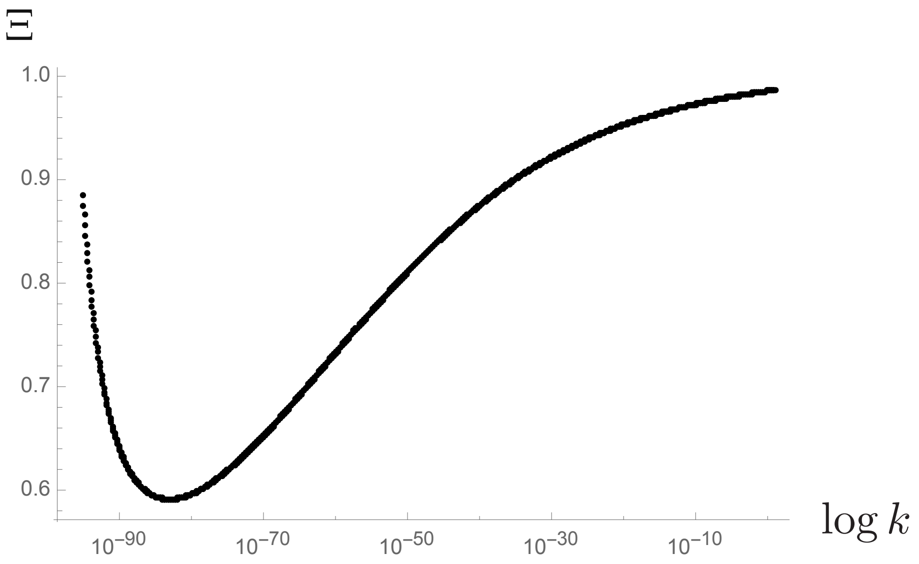
$(p > 2)$

power spectrum of gravitational waves



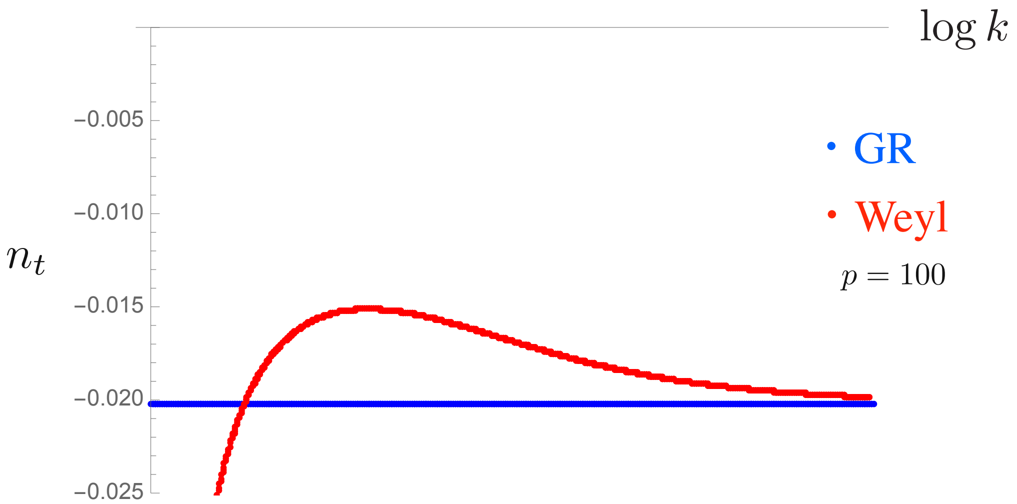
power spectrum of gravitational waves

$$\Xi \equiv \frac{\text{amplitude of power spectrum of GWs in Weyl gravity}}{\text{amplitude of power spectrum of GWs in GR}}$$



spectral index of gravitational waves

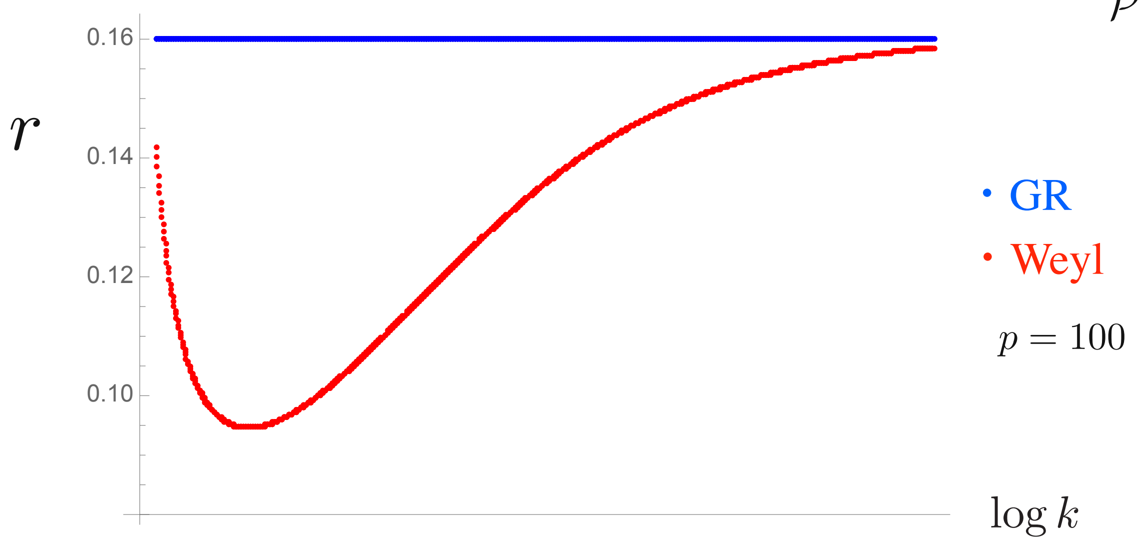
we calculate the spectral index of gravitational waves



$$n_t \equiv \left.\frac{d\log \mathcal{P}_g}{d\log k}\right|_{k=aH}$$

tensor to scalar ratio

$$r \equiv \frac{\mathcal{P}_g}{\mathcal{P}_R}$$

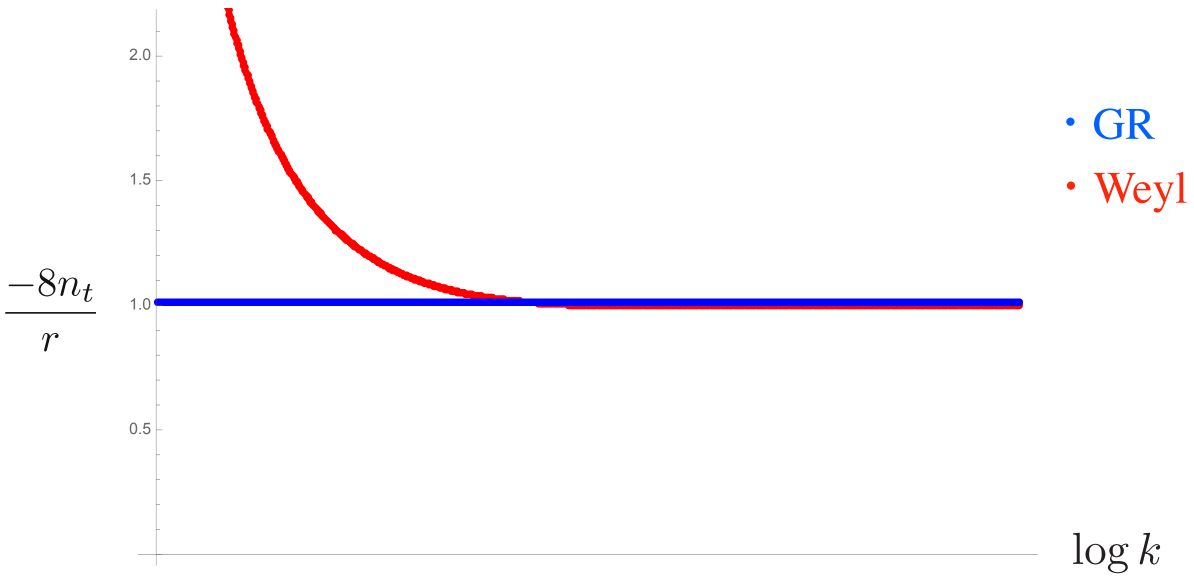


The power spectrum of scalar perturbations in Weyl gravity is the same in GR.

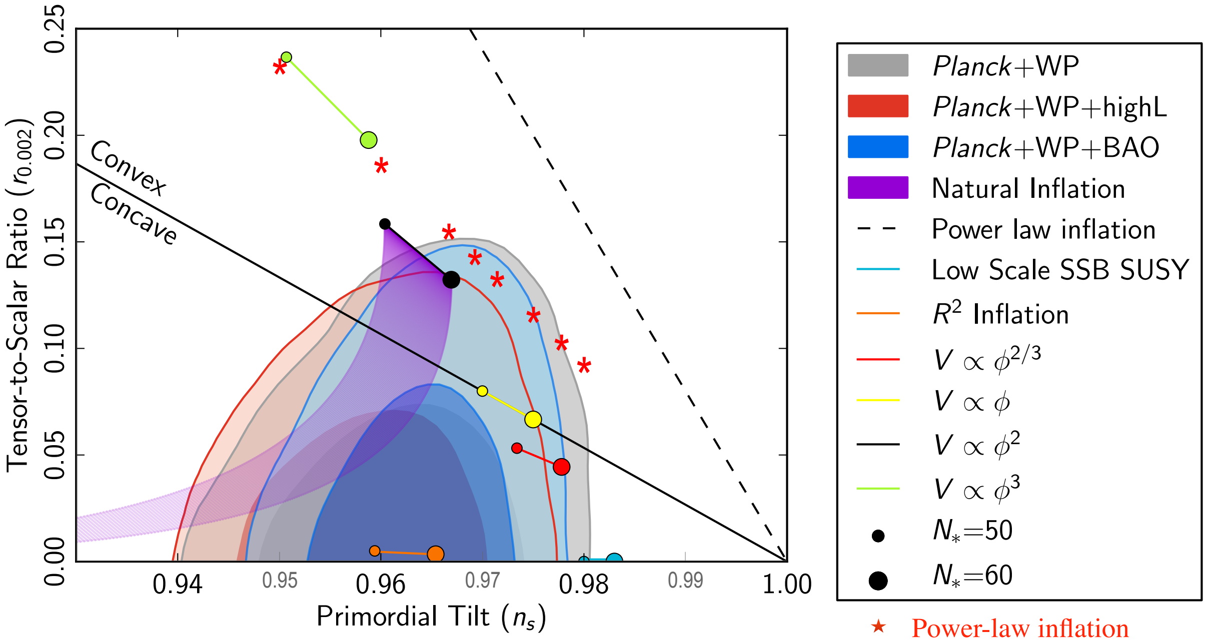
$$\mathcal{P}_R\,(\text{in Weyl}) = \mathcal{P}_R\,(\text{in GR}) \qquad n_s\,(\text{in Weyl}) = n_s\,(\text{in GR})$$

Consistency relation

The consistency relation: $\underline{r = -8n_t}$



Planck 2013 results. XXII. Constraints on inflation [arXiv:1303.5082]



★ Power-law inflation
in Weyl gravity

Summary

- We calculate the power spectrum of gravitational waves from slow-roll inflation in Weyl gravity.
- This theory decreases the power spectrum of gravitational waves from GR.
- The consistency relation is violated by quantum corrections.
- In small scale, the tensor to scalar ratio is almost the same as GR, but it decreases in large scale.

“Black holes as particle accelerators: a brief review”

Tomohiro Harada

[JGRG24(2014)111319]

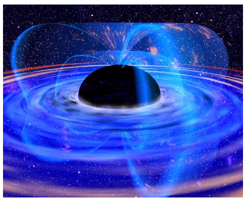
Black holes as particle accelerators: a brief review

Tomohiro Harada

Department of Physics, Rikkyo University

13/11/2014 JGRG24 @ IPMU
Based on arXiv:1409.7502 with Masashi Kimura (Cambridge)

Rotating BHs as particle accelerators



as

- Kerr BHs act as particle accelerators.
  (Bañados, Silk and West 2009, Piran, Shaham and Katz 1975)
  The CM energy of colliding particles can be unboundedly high near the horizon.
- Not only microscopic particles but also macroscopic objects, such as BHs and compact stars, are accelerated.
- This short talk is only an extract of the brief review.

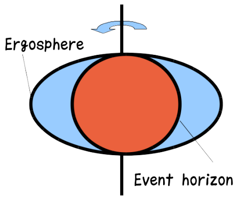
Kerr BHs

- Kerr metric

$$ds^2 = -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar\sin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \left(r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2,$$

where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$.

- Nondimensional spin: $a_* = a/M$
- Horizon: $r_H = r_+ = M + \sqrt{M^2 - a^2}$
- Ergosphere: $r_E = M + \sqrt{M^2 - a^2\cos^2\theta}$
- Angular velocity: $\Omega_H = a/(r_H^2 + a^2)$
- Extremal: $a_* = 1$



Formal divergence in CM energy of colliding particles

- Total energy observed in the centre-of-mass frame

$$p^a_{\rm tot} = p^a_1 + p^a_2, \quad E^2_{\rm cm} = -p^a_{\rm tot} p_{{\rm tot}a}.$$

- $E_{\rm cm}$ for near-horizon collision in the equatorial plane is formally given by

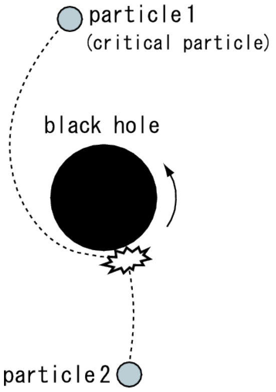
$$E^2_{\rm cm} = \frac{m_1^2 r_H^2 + (L_1 - aE_1)^2}{r_H^2}\frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1} + (1 \leftrightarrow 2) + \cdots ,$$

where both particles are assumed to be infalling.

- **Divergent if $E - \Omega_H L = 0$ for either of the particles.**
- We call particles with $E - \Omega_H L = 0$ *critical particles*.

The orbit of the critical particle

- **The critical particle can reach the horizon from infinity, if and only if the Kerr BH is extremal**, for which $\Omega_H = 1/(2M)$ and $L = 2ME$.
- It rotates infinitely many times around the BH and takes infinitely long proper time to reach the horizon.



CM energy in finite time

- Suppose particles 1 (critical) and 2 (noncritical) be released at rest at infinity.

$$\frac{E_{\rm cm}}{2m} \approx \sqrt{\frac{(2-\sqrt{2})(2-l_2)M}{2(r_{\rm col}-M)}},$$

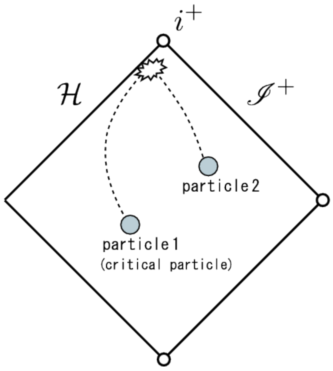
  where $l := L/(mM)$.
- The Killing time T for particle 1 to reach $r = r_{\rm col}$

$$T = -\int_{r_i}^{r_{\rm col}} dr \frac{\sqrt{r}(r^2+Mr+2M^2)}{\sqrt{2M}(r-M)^2} \simeq \frac{2\sqrt{2}M^2}{r_{\rm col}-M}.$$

- We then obtain

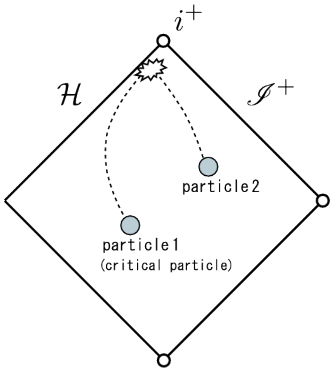
$$\begin{aligned} E_{\rm cm} &\approx m\sqrt{(\sqrt{2}-1)(2-l_2)\frac{T}{M}} \\ &\simeq 2.5\times 10^{20}{\rm eV}\left(\frac{T}{10\ {\rm Gyr}}\right)^{1/2}\left(\frac{M}{M_\odot}\right)^{-1/2}\left(\frac{m}{1\ {\rm GeV}}\right). \end{aligned}$$

The critical particle is accelerated unboundedly.



The critical particle approaches the event horizon, which is a null hypersurface. This implies that **the critical particle is accelerated to the speed of light** with infinite time.

The infalling particle is accelerated unboundedly.



The infalling particle is accelerated to the light speed. If the observer can stay at a constant radius near the horizon, he or she will see the particle falls with almost the speed of light. (cf. Zaslavskii 2011)

The structure of circular orbits around a BH

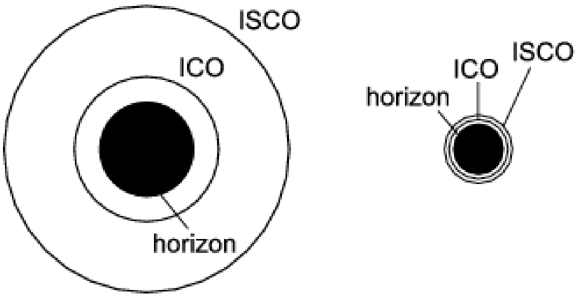
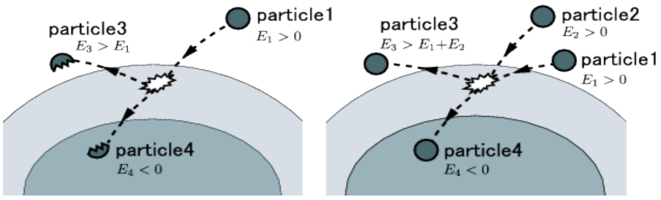


Figure: The Schwarzschild and near-extremal Kerr BHs.

The observer can stay at a constant radius near the horizon only for a near-extremal Kerr BH, where both the Innermost Stable Circular Orbit (ISCO) and Innermost Circular Orbit (ICO) are close to the horizon.

Collisional Penrose process



- The high energy collision may produce superheavy and/or superenergetic particles.
- Energy efficiency: $\eta = E_3/(E_1 + E_2)$
- **The efficiency is** $\lesssim 1.5$ **for the original BSW collision.** Hence, the ejecta can only be modestly more energetic than the incident particles. (Bejger et al. 2012, Harada, Nemoto and Miyamoto 2012)

Too low flux to be observed by a distant observer

- Observable effects are discussed. (Bañados et al. 2011, Williams 2011, Gariel, Santos and Silk 2014)
- **The flux of the ejecta particles from the BSW collision is too low for the Fermi satelite to detect, due to strong redshift and diminished escape fraction** (McWilliams 2013).

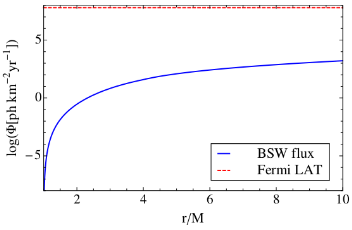
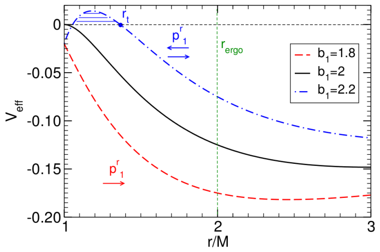


FIG. 1 (color online). Integrated flux Φ reaching an observer at $D_L = 10$ kpc from inside radius r (solid line), compared to the flux sensitivity of the Fermi LAT for a one year exposure (dashed line).

Revision of the upper limit and super-Penrose process

- **If we allow one of the colliding particles to be supercritical ($L > 2mM$) for an extremal Kerr BH, the efficiency can be as large as 14 for a variant of BSW collision.** (Schnittman arXiv:1410.6446)
- **If we allow one of the colliding particles to be that must be created inside the ergosphere, the efficiency can be arbitrarily high**, for which high $E_{\rm cm}$ is not essential. (Berti, Brito and Cardoso arXiv:1410.8534)

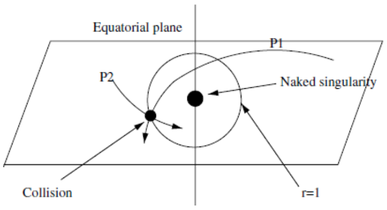


High energy collision in non-Kerr BHs

- Neutral particle accelerators
  - Kerr BHs (Bañados, Silk and West 2009, ...), KN family (Wei et al. 2010, Liu, Chen and Jing 2011), Accelerating and rotating BHs (Yao et al. 2011)
  - Dirty BHs (Zaslavskii 2010, 2012), Sen BHs (Wei et al. 2010), ...
- Charged particle accelerator
  - Reissner-Nordström BHs (Zaslavskii 2010)
  - General stationary charged BHs (Zhu et al. 2011), ...
- Higher-dimensions
  - Myers-Perry BHs (Abdujabbarov et al. 2013, Tsukamoto, Kimura and Harada 2014): Fine-tuning is still needed.

High energy collision in non-BH spacetimes

- **High energy collision occurs in a deep potential well. If there is no horizon, head-on collision is also physically motivated and hence fine-tuning is relaxed.**
  - Overspinning Kerr/Superspinar (Patil and Joshi 2011, Stuchlík and Schee 2012, 2013), JNW spacetimes (Patil and Joshi 2012), Overcharged RN (Patil et al. 2012)
  - **Naked singularity is not essential**: Bardeen magnetic monopoles (Patil and Joshi 2012), Rotating wormholes (Bambi and Tsukamoto)



(Patil and Joshi 2011)

Summary

- **Particle acceleration by near-exteremal Kerr BHs is founded on the basic properties of geodesic orbits.**
- **The achievable energy is subjected to several physical effects, such as finite acceleration time.**
- **Although the ejecta from the original BSW collision will not be directly observed, the observability of high energy particles is still tantalizing.**
- **Particle acceleration without horizon is advantageous to observation, if there is an extremely deep potential.**