JGRG24

The 24th Workshop on General Relativity and Gravitation in Japan

10 (Mon) — 14 (Fri) November 2014

KIPMU, University of Tokyo

Chiba, Japan

Oral presentations: Day 2

Contents

Programme: Day 2
Tuesday 11 November 2014

Morning 1 [Chair: Tetsuya Shiromizu]

9:30 Claudia de Rham (Case Western) [Invited]
"Cosmology and Massive Gravity" [JGRG24(2014)111101]

10:15 Yasuho Yamashita (YITP, Kyoto)
"Appearance of Boulware-Deser ghost in bigravity with doubly coupled matter" [JGRG24(2014)111102]

10:30 Atsushi Naruko (Titech)
"Cosmology in rotation-invariant massive gravity with non-trivial fiducial metric" [JGRG24(2014)111103]

10:45-11:00 coffee break

Morning 2 [Chair: Ken-ichi Oohara]

11:00 Hayato Motohashi (Chicago)
"Stability of self-accelerating solutions in extended quasidilaton massive gravity" [JGRG24(2014)111104]

11:15 Daisuke Yoshida (Titech)
"Covariant Stueckelberg analysis of dRGT massive gravity with a general fiducial metric" [JGRG24(2014)111105]

11:30 Katsuki Aoki (Waseda)
"Dark matter in ghost-free bigravity theory" [JGRG24(2014)111106]

11:45 Yuki Sakakihara (Kyoto)
"Tensor Spectrum in Bimetric Gravity" [JGRG24(2014)111107]

12:00 Tatsuya Narikawa (Osaka)
"Detectability of bi-gravity with graviton oscillations using gravitational wave observations" [JGRG24(2014)111108]

12:15 Shinsuke Kawai (Sungkyunkwan)
"Improvement of energy-momentum tensor and non-Gaussianities in holographic cosmology" [JGRG24(2014)111109]

12: 30 - 14:00 lunch & poster view

Afternoon 1 [Chair: Tomohiro Harada]

14:00 Andrzej Rostworowski (Jagiellonian) [Invited]
"Current status of the AdS (in)stability" [JGRG24(2014)111110]

14:45 Masashi Kimura (DAMTP)
"Higher-dimensional extremal Reissner- Nordstr¨om black holes are fragile" [JGRG24(2014)111111]

15:00 - 15:30 short poster talks (B01 - B19, 1 minute each)

15:30-16:00 coffee break & poster view

Afternoon 2 [Chair: Ken-ichi Nakao]

16:00 Ryo Namba (Kavli IPMU)
"Toward constructing ghost-free scalar-tensor theories beyond Horndeski" [JGRG24(2014)111112]

16:15 Rio Saitou (YITP, Kyoto)
"Structure of constraints of the theory beyond Horndeski" [JGRG24(2014)111113]

16:30 Xian Gao (Titech)
"Spatially covariant gravity and unifying framework for scalar-tensor theories of gravity" [JGRG24(2014)111114]

16:45 Ryotaro Kase (Tokyo Science)
"Effective field theory approach to modified gravity including Horndeski theory and Horava-Lifshitz gravity" [JGRG24(2014)111115]

17:00 Tomotaka Kitamura (Waseda)
"The Relation Between Tree Unitarity and Renormalizability in Lifshitz Scalar Theory" [JGRG24(2014)111116]

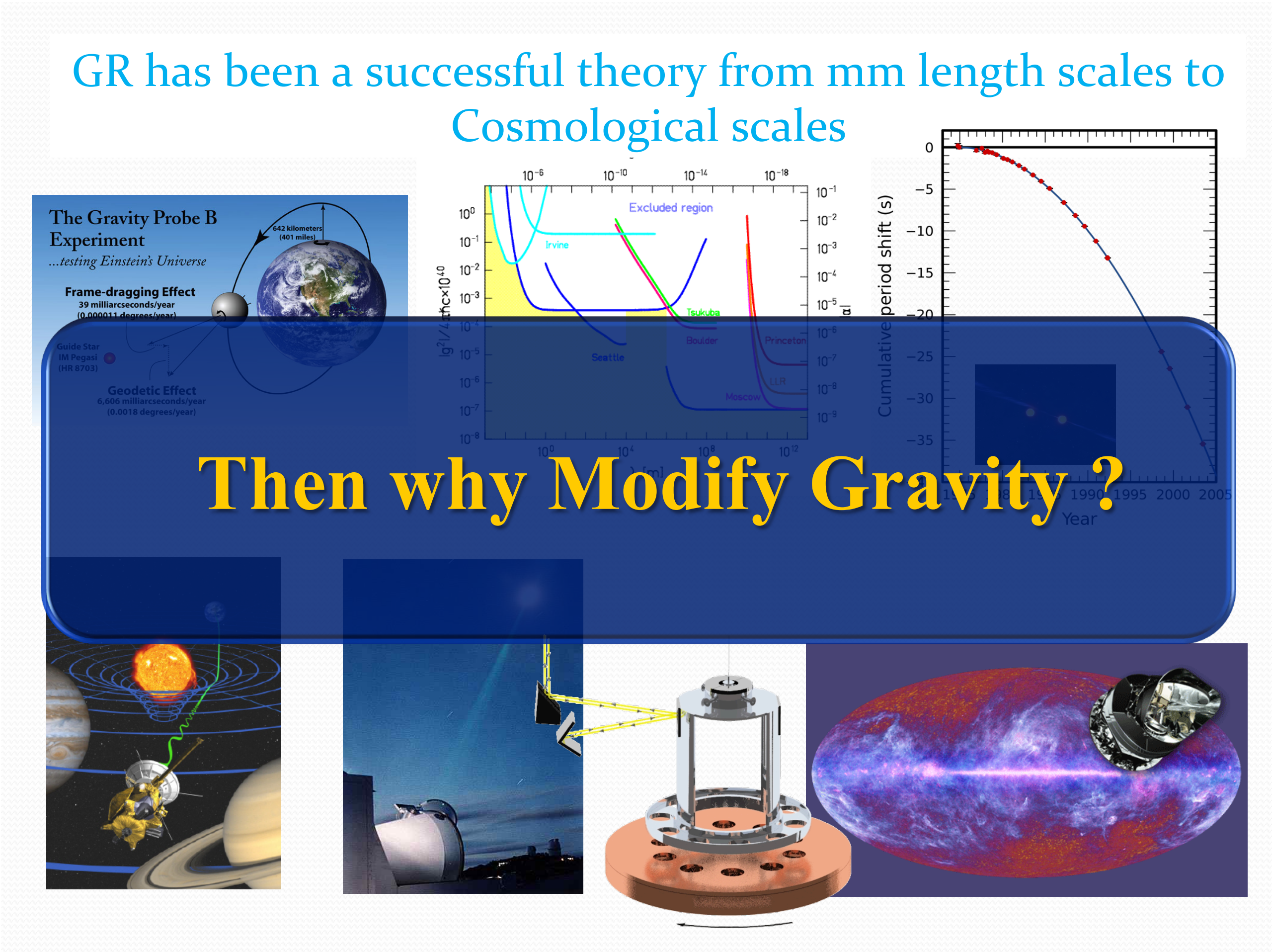
17:15 - 18:00 poster view

“Cosmology and Massive Gravity”

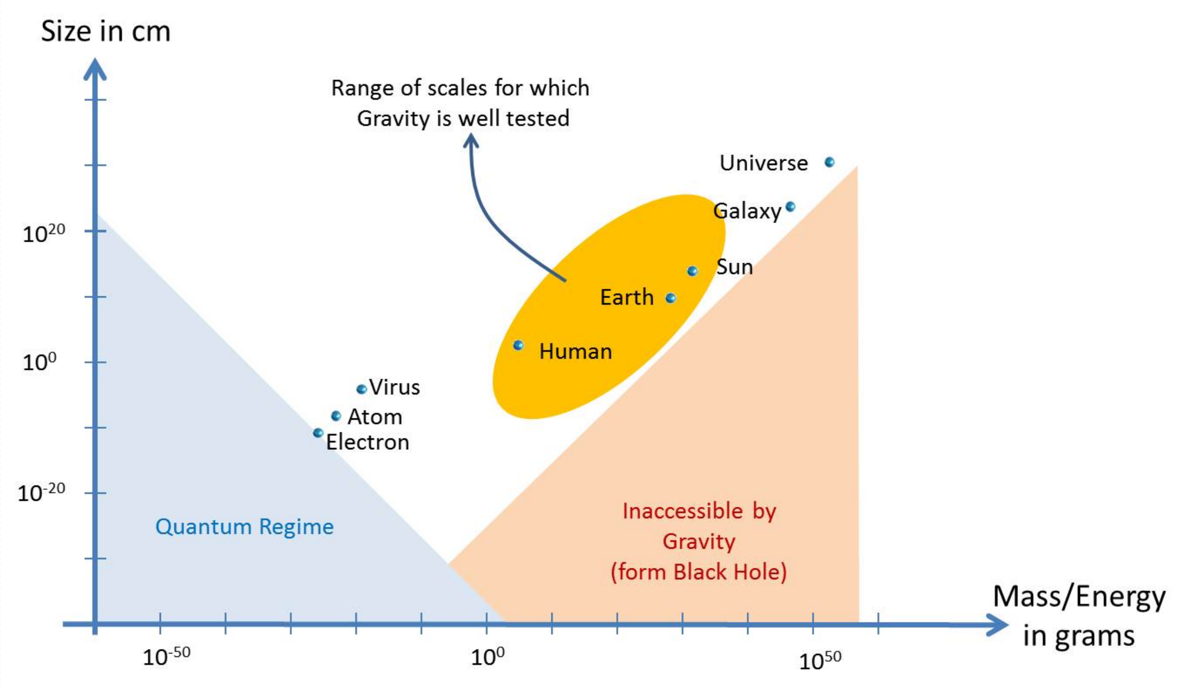
Claudia de Rham [Invited]

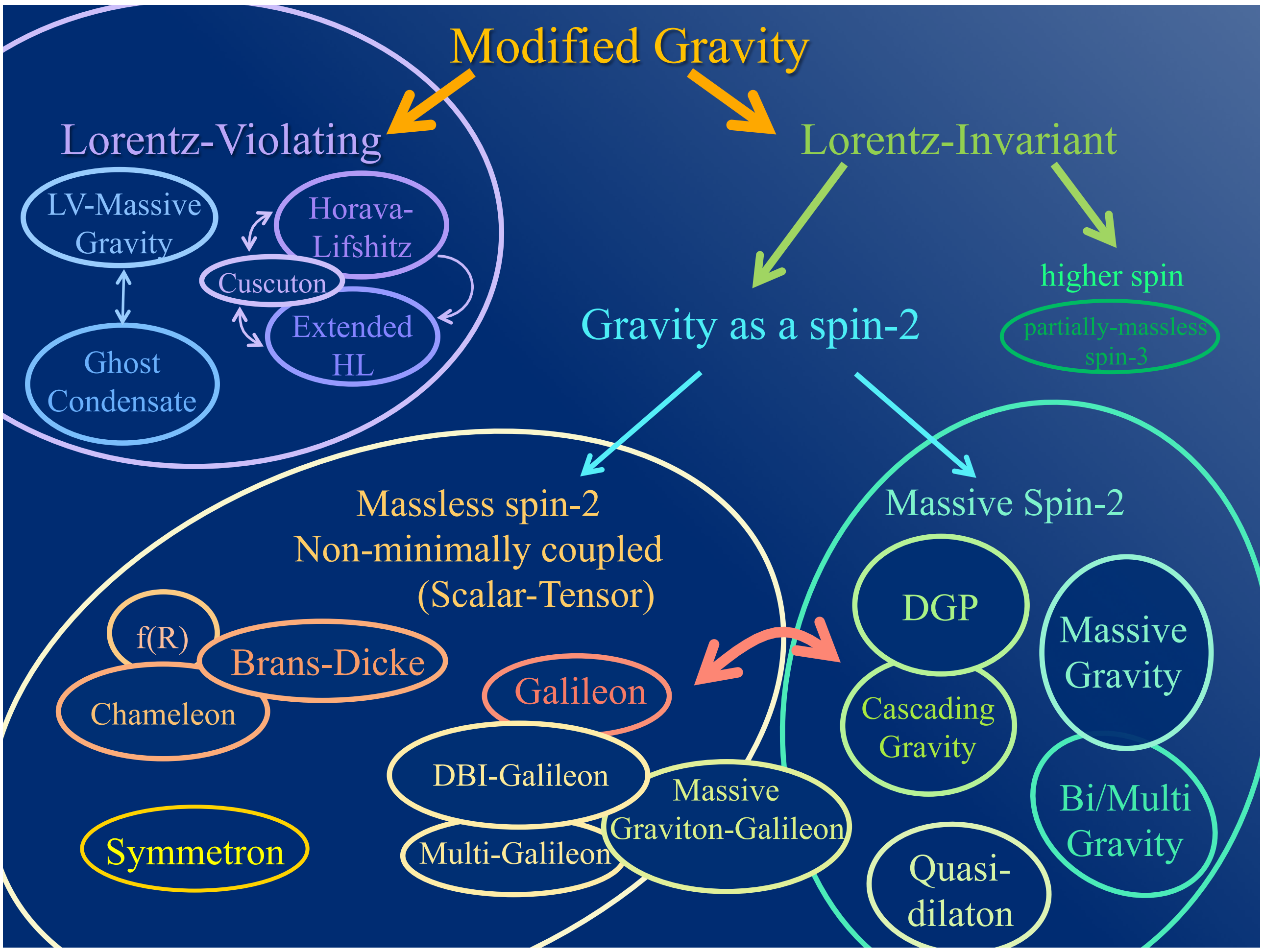
[JGRG24(2014)111101]

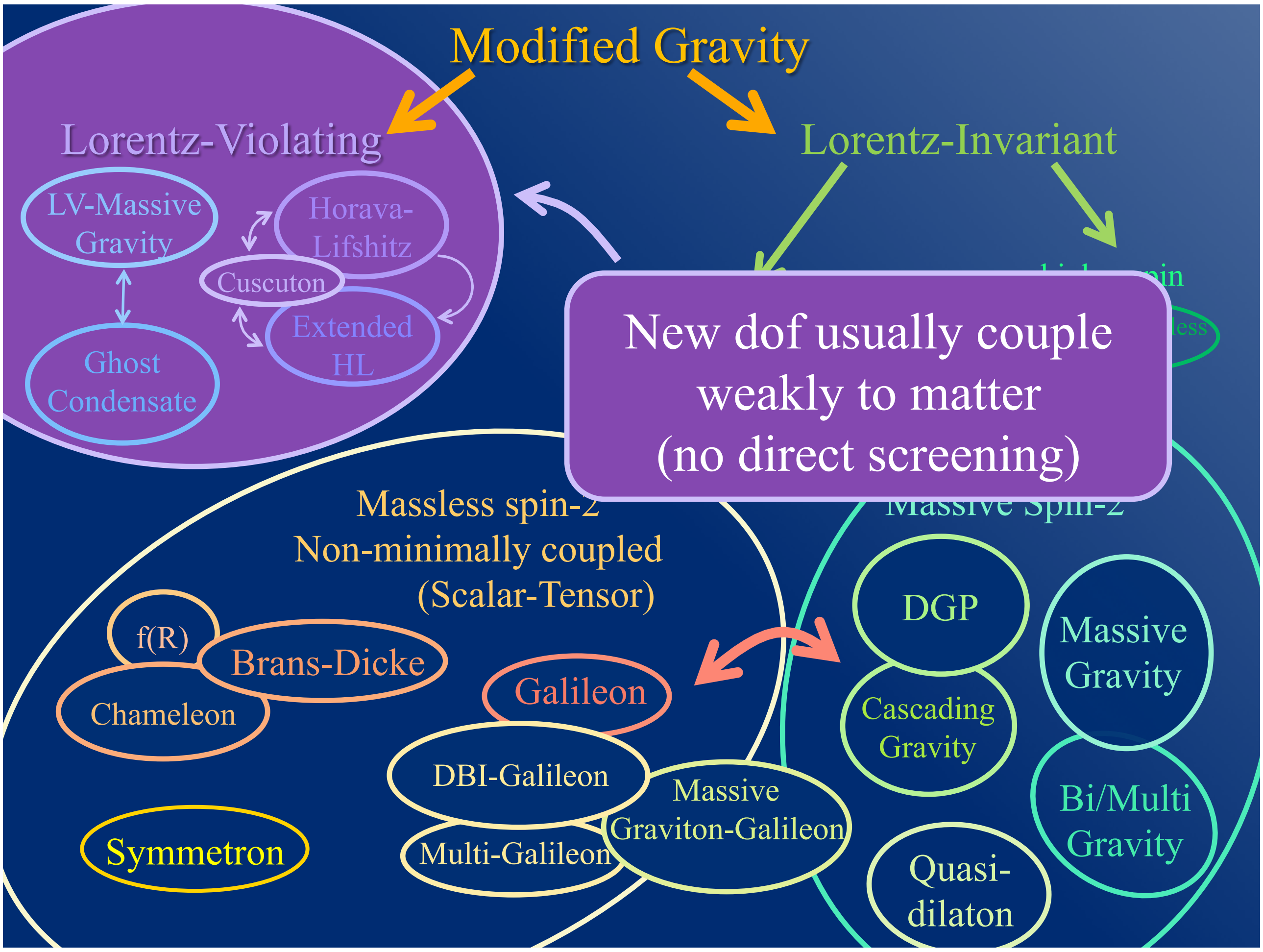

Cosmology
&
Massive Gravity
IPMU - JGRG24
Nov. 11st 2014
Claudia de Rham
IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE
CASE WESTERN RESERVE UNIVERSITY EST. 1826
think beyond the possible

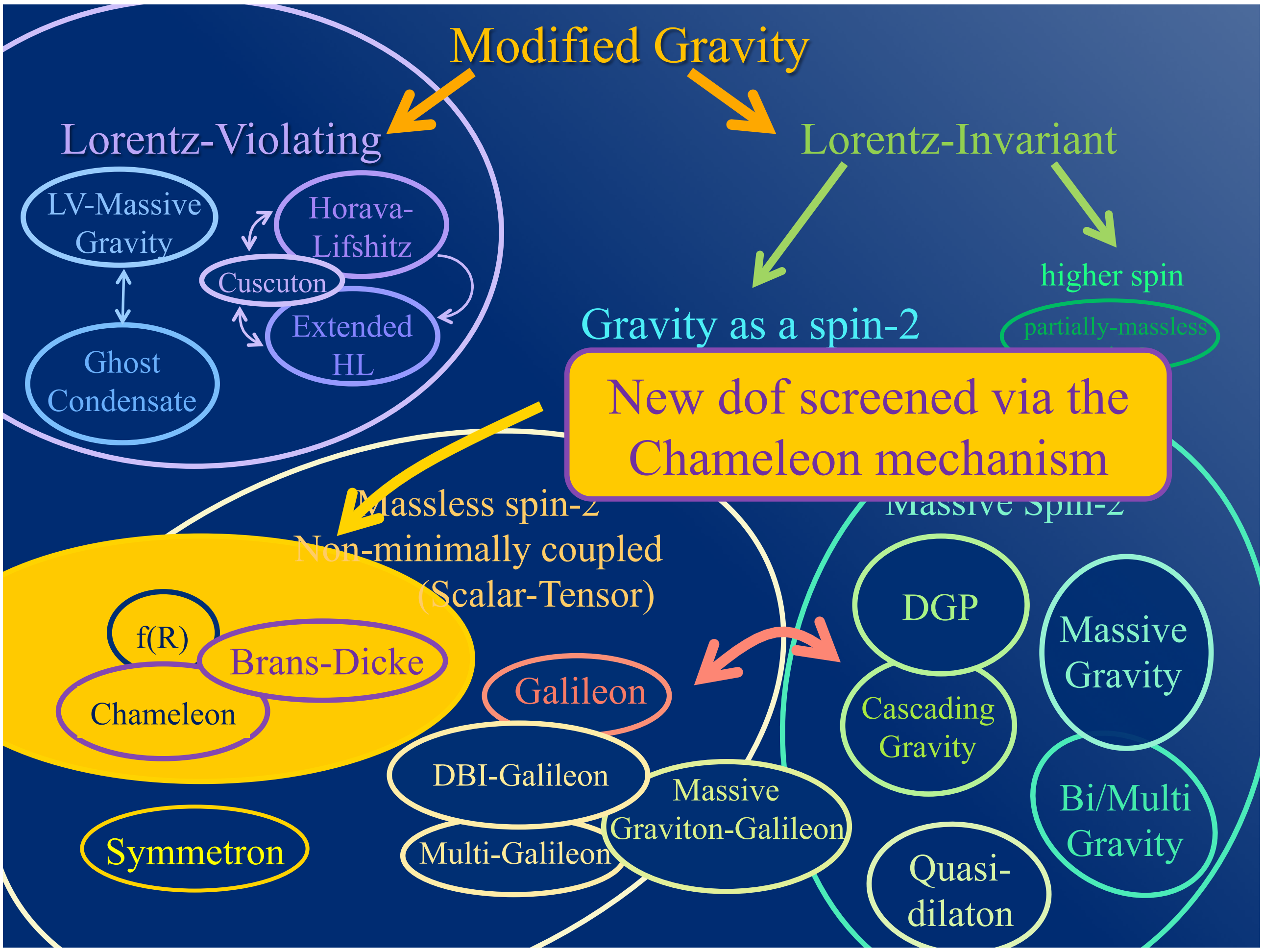

GR has been a successful theory from mm length scales to Cosmological scales
The Gravity Probe B Experiment
...testing Einstein's Universe
Frame-dragging Effect
Geodetic Effect
Excluded region
Cumulative period shift (s)
Year
Then why Modify Gravity ?

Testing gravity requires alternatives theories






Modified Gravity
Lorentz-Violating
Lorentz-Invariant
LV-Massive Gravity
Horava-Lifshitz
Cuscuton
Extended HL
Ghost Condensate
New dof usually couple weakly to matter (no direct screening)
Massless spin-2
Non-minimally coupled (Scalar-Tensor)
f(R)
Brans-Dicke
Chameleon
Galileon
DBI-Galileon
Massive Graviton-Galileon
Multi-Galileon
Symmetron
DGP
Cascading Gravity
Massive Gravity
Bi/Multi Gravity
Quasi-dilaton


Modified Gravity
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Cuscuton
Extended HL
Ghost Condensate
higher spin
partially-massless
Gravity as a spin-2
New dof screened via the Chameleon mechanism
Massless spin-2
Non-minimally coupled (Scalar-Tensor)
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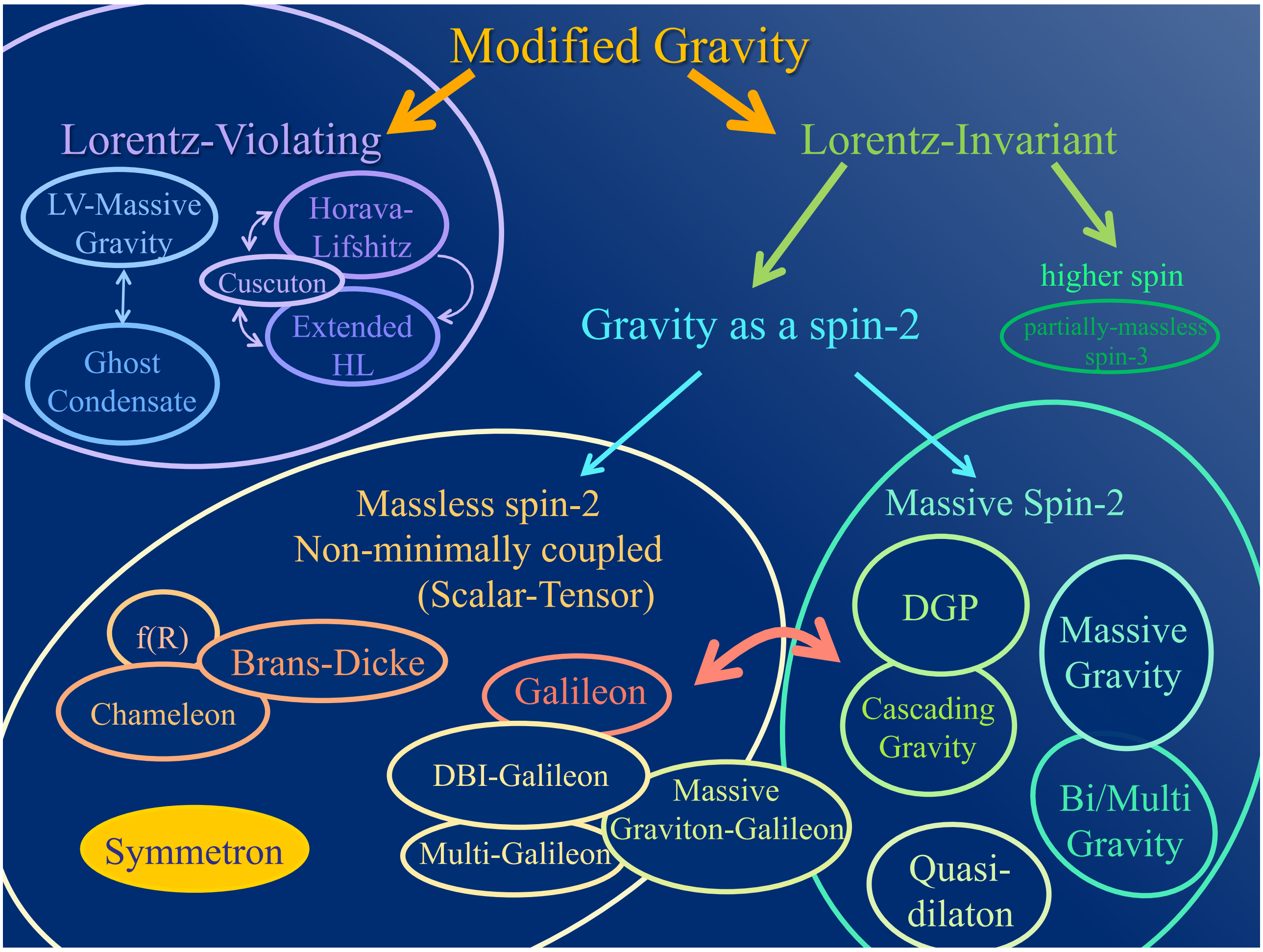
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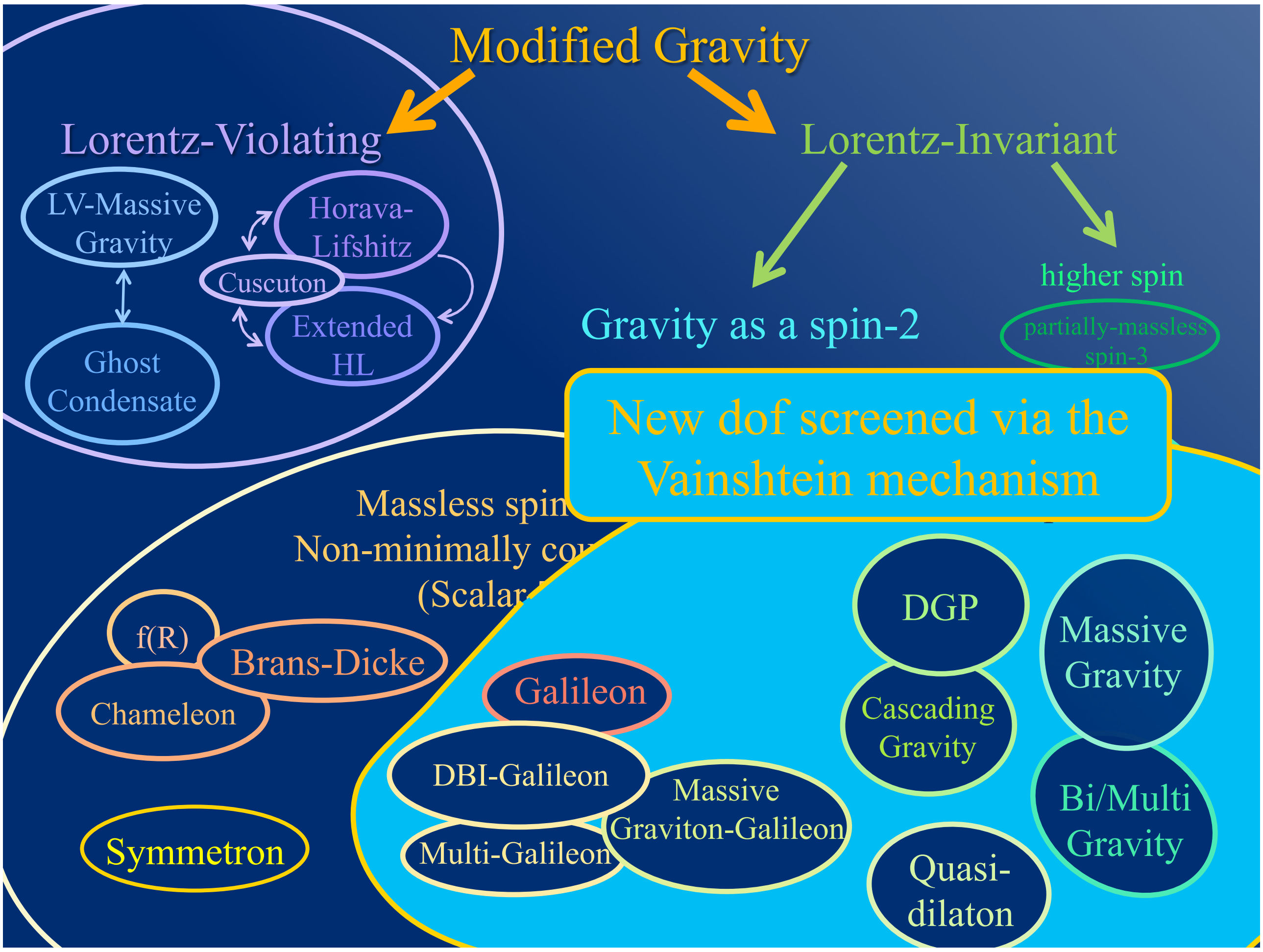
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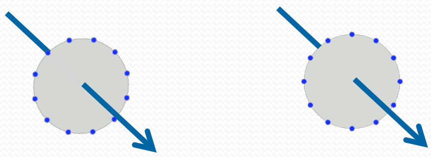
Massive spin-2 fields & Holography

- Spin-2 field may be useful in condensed matter applications of the AdS/CFT correspondence
- ‘realistic’ materials with *momentum relaxation* (lattice) are dual to *massive gravity*
- New dofs in graviton encodes the phonon dynamics

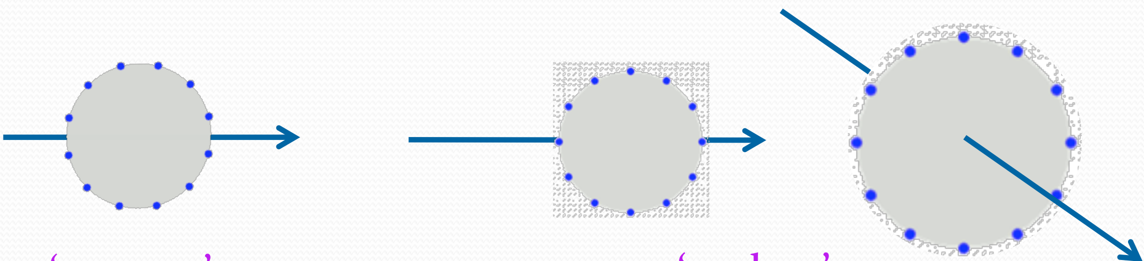
Vegh, arXiv:1301.0537,
Blake, Tong, Vegh, arXiv:1310.3832
Baggioli, Pujolas, arXiv:1411.1003,...

Gravitational Waves

- **GR: 2** polarizations



- In principle GW could have 4 other polarizations



2 ‘vectors’ 2 ‘scalars’

Massive Gravity

- When breaking covariance, GW can in principle propagate up to 6 independent polarizations (in 4d)
- A massive spin-2 field in 4d has 2s+1=5 dofs
- The 6th dof always comes in as a ghost.

$$2 + 4 = 6 = 5 + 1$$

Boulware & Deser, PRD6, 3368 (1972)

Ghost-free Massive Gravity

$$\mathcal{U}_{\rm GF} = \left(\mathcal{K}^2_{\mu\nu} - \mathcal{K}^2\right) + \alpha_3 \left(\mathcal{K}^3 + \cdots\right) + \alpha_4 \left(\mathcal{K}^4 + \cdots\right)$$

- In 4d, there is a 2-parameter family of ghost free theories of Lorentz-invariant massive gravity

$$\mathcal{K}^\mu_\nu[g, \eta] = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\eta_{\alpha\nu}}$$

CdR, Gabadadze, 1007.0443
CdR, Gabadadze, Tolley, 1011.1232

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- Absence of ghost has now been proved fully non-perturbatively in many different languages

CdR, Gabadadze, 1007.0443
CdR, Gabadadze, Tolley, 1011.1232
Hassan & Rosen, 1106.3344
CdR, Gabadadze, Tolley, 1107.3820
CdR, Gabadadze, Tolley, 1108.4521
Hassan & Rosen, 1111.2070
Mirbabayi, 1112.1435
Kluson, 1202.5899
Hassan, Schmidt-May & von Strauss, 1203.5283
Kluson, 1204.2957
Deffayet, Mourad & Zahariade, 1207.6338
Alexandrov, 1308.6586
Kugo, Ohta, 1401.3873
Golovnev, 1401.6343, …

Ghost-free Massive Gravity

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- In 4d, there is a 2-parameter family of ghost free theories of Lorentz-invariant massive gravity
- Absence of ghost has now been proved fully non-perturbatively in many different languages
- As well as around *any reference metric,* be it dynamical → or not **BiGravity !!!**

Hassan, Rosen & Schmidt-May, 1109.3230
Hassan & Rosen, 1109.3515

Degrees of Freedom

Massive Gravity

- 1 massive spin-2

  - 2 helicity-2
  - 2 helicity-1
  - 1 helicity-0

5 dofs

Degrees of Freedom

Massive Gravity

- 1 massive spin-2

  - 2 helicity-2
  - 2 helicity-1
  - 1 helicity-0

5 dofs

- 2 dof in metric
(after gauge fixing)
- 3 Stückelberg fields

Restore diff invariance

$$\sqrt{g^{\mu\alpha}\eta_{\alpha\nu}}$$

$$\eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

$$\sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

$$f_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$$

Degrees of Freedom

Massive Gravity

- 1 massive spin-2
  - 2 helicity-2
  - 2 helicity-1
  - 1 helicity-0

5 dofs

- 2 dof in metric (after gauge fixing)
- 3 Stückelberg fields

Restore diff invariance

Bi-Gravity

- 1 massive & 1 massless spin-2
  - 2x2 helicity-2
  - 2 helicity-1
  - 1 helicity-0

7 dofs

- 2x2 dof in both metrics (after gauge fixing)
- 3 Stückelberg fields

Restore 2nd copy of diff invariance

Gauge Transformation

- Start with Massive Gravity

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left(R - m^2\mathcal{U}[g,f]\right)$$

- With reference metric $f_{\mu\nu} = \eta_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$
- And Stuckelberg fields $\phi^a = x^a + tV^a + s\partial^a \pi$

Gauge Transformation

- Start with Massive Gravity

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left(R - m^2\mathcal{U}[g,f]\right)$$

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- And Stuckelberg fields $\phi^a = x^a + tV^a + s\partial^a\pi$
- Clearly the theory is invariant under a change of gauge

$$s \to s' \qquad\qquad \mathrm{d}^4x\mathcal{L}[s] \equiv \mathrm{d}^4x\mathcal{L}[s']$$

Gauge Transformation

$$\phi^a = x^a + tV^a + s\partial^a\pi$$

- The change of gauge can be viewed as a (field dependent) coordinate transformation,

$$\boxed{\mathcal{D}_{s'}: \begin{cases} x^\mu \longrightarrow \tilde{x}^\mu = x^\mu + s'\partial^\mu\pi(x) \\ \partial_\mu\pi(x) \longrightarrow \tilde{\partial}_\mu\tilde{\pi}(\tilde{x}) = \partial_\mu\pi(x) \\ g_{\mu\nu}(x) \longrightarrow \tilde{g}_{\mu\nu}(\tilde{x}) = M^\alpha_{\ \mu}M^\beta_{\ \nu}g_{\alpha\beta}(x) \end{cases}}$$

With $M^\alpha_{\ \mu} = \dfrac{\partial x^\alpha}{\partial\tilde{x}^\mu} = \left[\mathbb{I} + s'\Pi(x)\right]^{-1} = \left[\mathbb{I} - s'\tilde{\Pi}(\tilde{x})\right]$

$$\Pi_{\mu\nu}(x) = \partial_\mu\partial_\nu\pi(x) \qquad \tilde{\Pi}_{\mu\nu}(\tilde{x}) = \tilde{\partial}_\mu\tilde{\partial}_\nu\tilde{\pi}(\tilde{x})$$

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- The map is invertible and forms a group

$$\mathcal{D}_s^{-1} = \mathcal{D}_{-s} \qquad \mathcal{D}_{s'} \circ \mathcal{D}_s = \mathcal{D}_{s+s'}$$

Trivial invariance under gauge transformation

Galileon Duality

Insight for:
superluminality
(and potentially Quantum Stability and UV completion)

Generalized MG

Insight for:
Cosmology in MG

Limit of Massive (Bi-)Gravity

- In some limit, theory looks like a Galileon

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 + c_3\mathcal{L}_3^{(\mathrm{Gal})}(\pi) + c_4\mathcal{L}_4^{(\mathrm{Gal})}(\pi) + c_5\mathcal{L}_5^{(\mathrm{Gal})}(\pi)$$

- Where π plays the role of the helicity-0 mode

$$\mathcal{L}_n^{(\mathrm{Gal})}(\pi) = \mathcal{E}^{\alpha_1\ldots\alpha_d}\mathcal{E}^{\beta_1\ldots\beta_d}\left(\prod_{j=1}^{n}\partial_{\alpha_j\beta_j}\pi\right)\left(\prod_{k=n+1}^{d}\eta_{\alpha_k\beta_k}\right)$$

Limit of Massive (Bi-)Gravity

- In some limit, theory looks like a Galileon

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 + c_3\mathcal{L}_3^{(\mathrm{Gal})}(\pi) + c_4\mathcal{L}_4^{(\mathrm{Gal})}(\pi) + c_5\mathcal{L}_5^{(\mathrm{Gal})}(\pi)$$

- But there is a “gauge” freedom

$$\mathcal{D}_{s'}: \begin{cases} x^\mu & \longrightarrow \quad \tilde{x}^\mu = x^\mu + s'\partial^\mu\pi(x) \\ \partial_\mu\pi(x) & \longrightarrow \quad \tilde{\partial}_\mu\tilde{\pi}(\tilde{x}) = \partial_\mu\pi(x) \\ g_{\mu\nu}(x) & \longrightarrow \quad \tilde{g}_{\mu\nu}(\tilde{x}) = M^\alpha_{\ \mu}\, M^\beta_{\ \nu}\, g_{\alpha\beta}(x) \end{cases}$$

Dual to a Galileon

- A Galileon theory...

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 + c_3\mathcal{L}_3^{(\mathrm{Gal})}(\pi) + c_4\mathcal{L}_4^{(\mathrm{Gal})}(\pi) + c_5\mathcal{L}_5^{(\mathrm{Gal})}(\pi)$$

Maps to another Galileon theory, with different coefficients

$$p_n = p_n(c_k, s')$$

$$\tilde{\mathcal{L}}_{s'} = -\frac{1}{2}(\partial\tilde{\pi})^2 + p_3\mathcal{L}_3^{(\mathrm{Gal})}(\tilde{\pi}) + p_4\mathcal{L}_4^{(\mathrm{Gal})}(\tilde{\pi}) + p_5\mathcal{L}_5^{(\mathrm{Gal})}(\tilde{\pi})$$

- Maps a theory that exhibits Vainshtein to another one which also exhibits Vainshtein.

Eg.2 Dual to a Free theory

- A Free theory

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2$$

Maps to a specific quintic Galileon theory

$$p_n = \frac{6s'^{n-2}}{(n-2)!(5-n)!}$$

$$\tilde{\mathcal{L}}_{s'} = -\frac{1}{2}(\partial\tilde{\pi})^2 + p_3\mathcal{L}_3^{(\mathrm{Gal})}(\tilde{\pi}) + p_4\mathcal{L}_4^{(\mathrm{Gal})}(\tilde{\pi}) + p_5\mathcal{L}_5^{(\mathrm{Gal})}(\tilde{\pi})$$

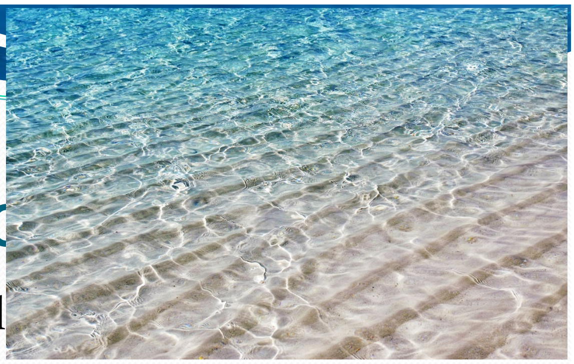
$$\mathcal{L}_{s'=1} = -\frac{1}{2}\det\left(1+\Pi\right)(\partial\pi)^2$$

Dual to a free theory

- The dual theory admits superluminal propagation

$$\mathcal{L}_{s'=1} = -\frac{1}{2}\det\left(1+\Pi\right)\left(\partial\pi\right)^2$$

in the vacuum ! (ie. no matter or other sources)



Dual to a free theo

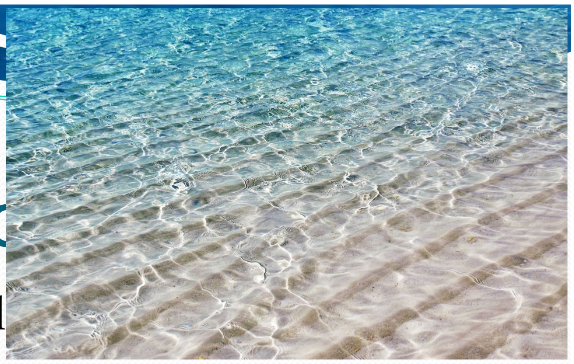
- The dual theory admits superl

$$\mathcal{L}_{s'=1} = -\frac{1}{2}\det\left(1+\Pi\right)\left(\partial\pi\right)^2$$

in the vacuum ! (ie. no matter or other sources)

plane wave solutions $\pi = F(x-t)$

Dual to a free theo



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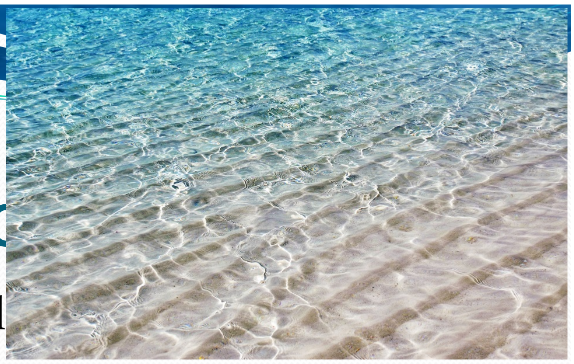
in the vacuum ! (ie. no matter or other sources)

plane wave solutions $\pi = F(x-t)$

- Speed of fluctuations: $c_s = 1$ $\quad c_s = \dfrac{1-F''}{1+F''}$

Superluminal propagation for $F'' < 0$

Dual to a free theo



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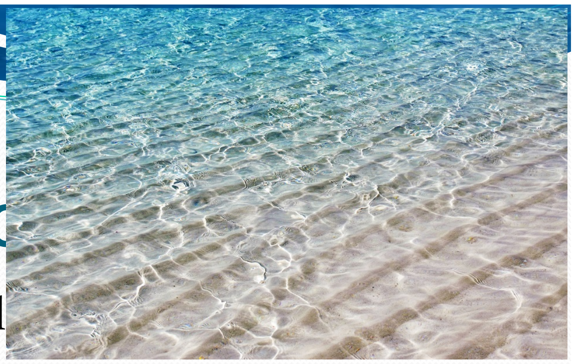
plane wave solutions $\pi = F(x-t)$

- Speed of fluctuations: $c_s = 1$ $\quad c_s = \dfrac{1-F''}{1+F''}$

- FULLY EQUIVALENT to a characteristic analysis

Dual to a free theo



- The dual theory admits superl

$$\mathcal{L}_{s'=1} = -\frac{1}{2}\det\left(1+\Pi\right)(\partial\pi)^2$$

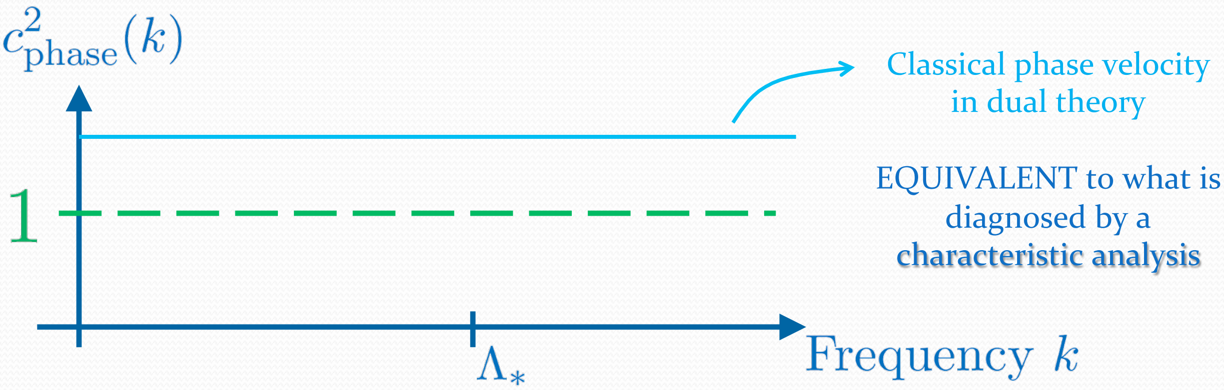
- Yet it maps to a free theory

$$\mathcal{L}_{\rm Free} = -\frac{1}{2}(\partial\pi)^2$$

- Trivially causal, unitary, UV complete,…

Group vs front velocity

- No Paradox here ! Group velocity is:
  - Not invariant
  - Has been observed to be SL in the real world
  - Was computed here classically: Valid till the strong coupling scale



Superluminal phase&group velocities have been observed in real world…

Direct measurement of superluminal group velocity and of signal velocity in an optical fiber

Nicolas Brunner, Valerio Scarani, Mark Wegmüller, Matthieu Legré and Nicolas Gisin
Group of Applied Physics, University of Geneva,
20 rue de l'Ecole-de-Médecine, CH-1211 Geneva 4, Switzerland

(February 1, 2008)
quant-ph/0407155

We present an easy way of observing superluminal group velocities using a birefringent optical fiber and other standard devices. In the theoretical analysis, we show that the optical properties of the setup can be described using the notion of "weak value". The experiment shows that the group velocity can indeed exceed c in the fiber; and we report the first direct observation of the so-called "signal velocity", the speed at which information propagates and that cannot exceed c.

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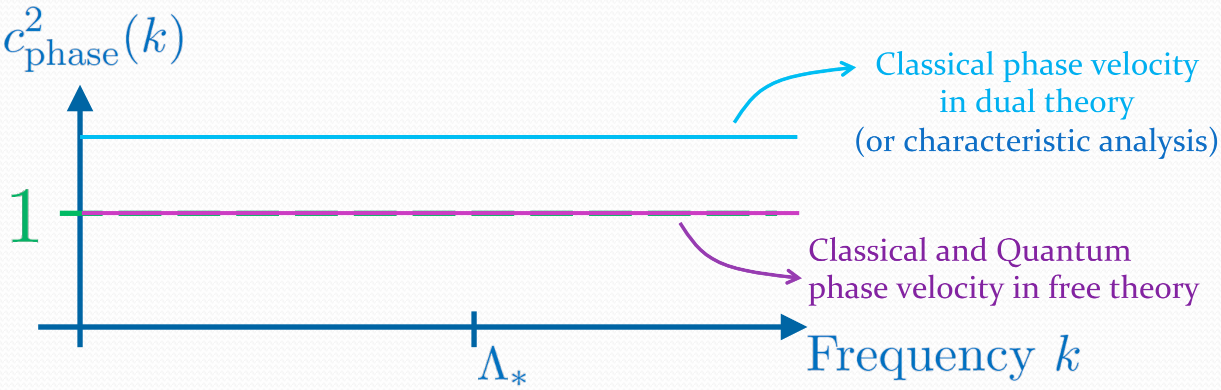

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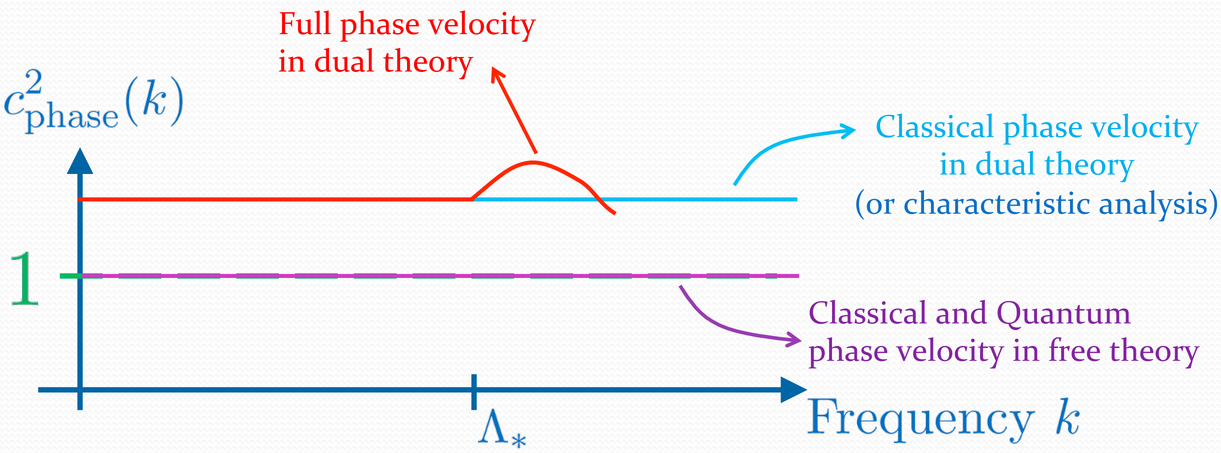

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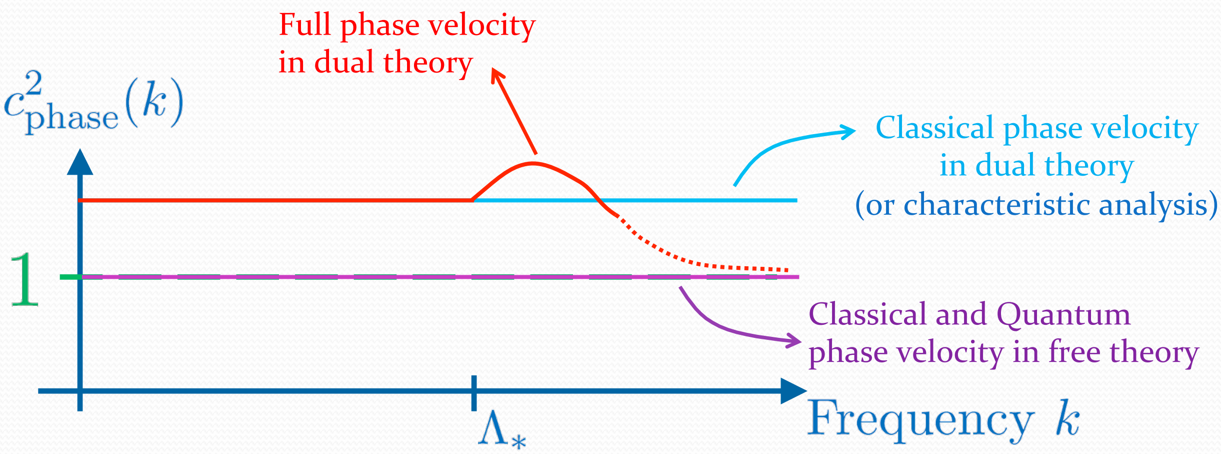


Group vs front velocity

- The classical group and phase velocities may depend on the field representation and may be SL
- The Causal structure is dictated by the **front velocity**
- The front velocity (and therefore the causality) cannot be inferred by a simple classical calculation (**neither by a classical characteristic analysis**)
- If the duality was going through at the **quantum level** one could compute the front velocity in the free theory. Since it is luminal we would infer that the quintic Galileon is actually causal…

Trivial invariance under gauge transformation

Galileon Duality

Insight for:
superluminality
(and potentially Quantum Stability and UV completion)

Generalized MG

Insight for:
Cosmology in MG

Generalized MG

- The framework provides inspiration on how to generalize MG

$$\mathcal{L} = \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \frac{m^2}{2}\sum_{n=2}^{4}\tilde{\alpha}_n(\tilde{\phi}^a\tilde{\phi}_a)\,\mathcal{U}_n[K]\right)$$

Generalized MG

- The framework provides inspiration on how to generalize MG

$$\mathcal{L} = \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \frac{m^2}{2}\sum_{n=2}^{4}\tilde{\alpha}_n(\tilde{\phi}^a\tilde{\phi}_a)\,\mathcal{U}_n[K]\right)$$

- Lorentz invariant !
- Same number of constraints as Ghost-free MG

  $\longrightarrow$ 5 propagating dofs.

Generalized MG

- The framework provides inspiration on how to generalize MG

$$\mathcal{L} = \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \frac{m^2}{2}\sum_{n=2}^{4}\tilde{\alpha}_n(\tilde{\phi}^a\tilde{\phi}_a)\,\mathcal{U}_n[K]\right)$$

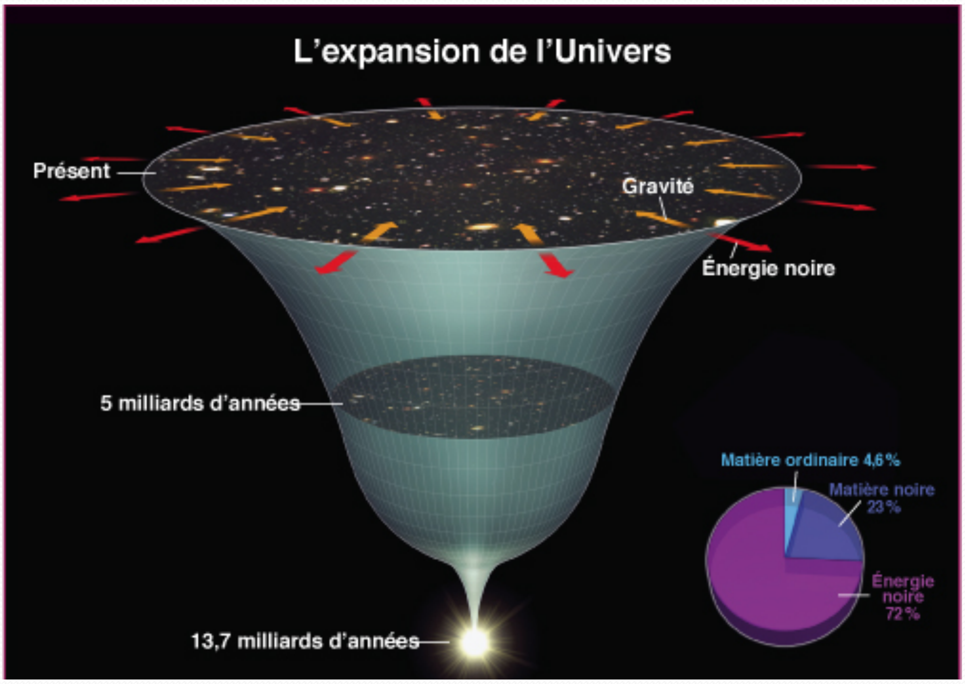
- Lorentz invariant !
- Same number of constraints as Ghost-free MG

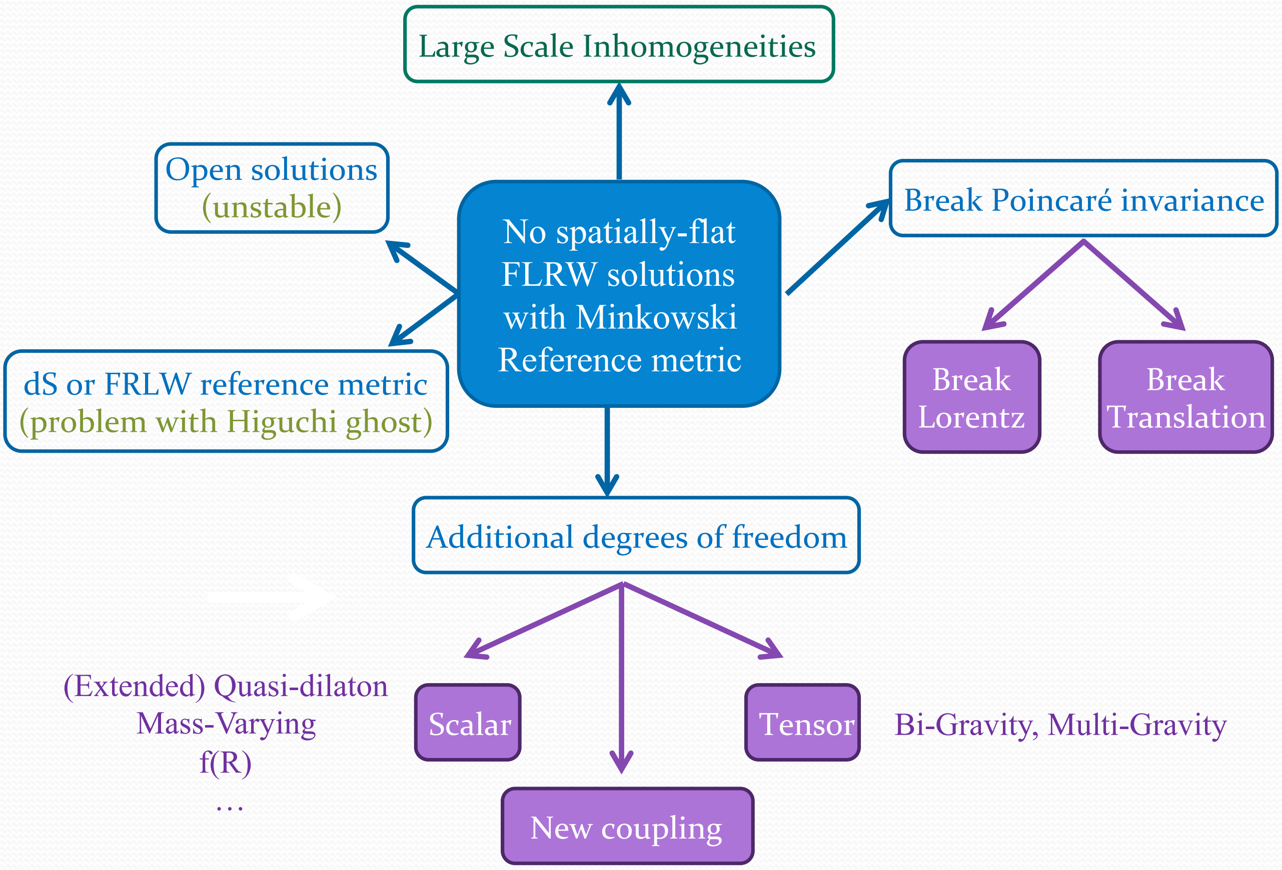
→ 5 propagating dofs.

- In unitary gauge $\tilde{\phi}^a\tilde{\phi}_a = x^2 = x^a x^b \eta_{ab}$

→ translation invariance broken

Consequences for Cosmology





Generalized MG

- The framework provides inspiration on how to generalize MG

$$\mathcal{L} = \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \frac{m^2}{2}\sum_{n=2}^{4}\tilde{\alpha}_n(\tilde{\phi}^a\tilde{\phi}_a)\,\mathcal{U}_n[K]\right)$$

- Allows for exact FLRW solutions

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\sum_n \alpha_n \mathcal{U}_n(a)\right] \simeq 0 \sum_n \alpha'_n(\phi_a\phi^a)\mathcal{U}_n(a) \neq 0$$

From Lorentz invariance to cosmology

- Start with Open Universe (could be thought of as local effect from long wavelength inhomogeneity)

$$\tilde{\phi}^0 = f(t)\sqrt{1+|k|\vec{x}^{\,2}}\,, \quad \tilde{\phi}^i = \sqrt{|k|}f(t)x^i\,.$$

$$f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\dot{f}(t)^2\mathrm{d}t^2 + |k|f^2\,\mathrm{d}\Omega^2_{H^3}$$

$$f(t) = \frac{1}{\sqrt{|k|}} + \chi(t)\,,$$

Gumrukcuoglu, Lin, Mukohyama, arXiv:1109.3845

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$$f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\dot{f}(t)^2\mathrm{d}t^2 + |k|f^2\,\mathrm{d}\Omega^2_{H^3} \quad \longrightarrow \quad -\dot{\chi}(t)^2\mathrm{d}t^2 + (\mathrm{d}\vec{x})^2$$

$$\tilde{\alpha}_n(\tilde{\phi}^a\tilde{\phi}_a) \quad \longrightarrow \quad \alpha_n(\chi(t))$$

$$|k| \to 0$$

$$f(t) = \frac{1}{\sqrt{|k|}} + \chi(t)\,,$$

Gumrukcuoglu, Lin, Mukohyama, arXiv:1109.3845

Exact FLRW solutions

- There are exact (self-accelerating) FLRW solutions

Eg.

$$3M_{\rm Pl}^2 H^2 = \rho_{\rm matter} + m^2 M_{\rm Pl}^2 \left(C_1 + \frac{C_2}{a}\right)\left(C_3 + C_4 \frac{m}{H}\right)$$

CdR, Fasiello, Tolley arXiv:1410.0960

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- Which are stable in the *decoupling limit* where

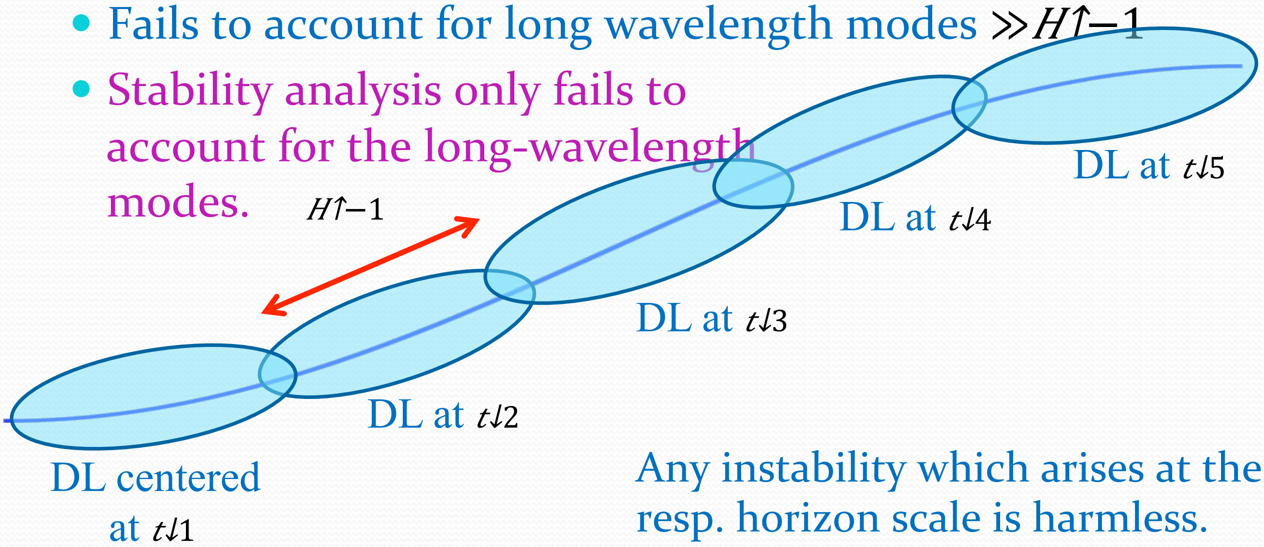
  $m \to 0, \ M_{Pl} \to \infty, \ \Lambda = (m^2 M_{Pl})^{1/3} \to$ fixed

For all the modes (tensors, vectors, scalar, no tachyon, gradient or ghost instability)

CdR, Fasiello, Tolley arXiv:1410.0960

Validity of DL

- Derived a family of DL theories valid for arbitrary time
- Fails to account for long wavelength modes $\gg H^{-1}$
- Stability analysis only fails to account for the long-wavelength modes.



Any instability which arises at the resp. horizon scale is harmless.

Cosmology in the DL

- There are exact (self-accelerating) FLRW solutions

Eg.

$$3M_{\rm Pl}^2 H^2 = \rho_{\rm matter} + m^2 M_{\rm Pl}^2 \left(C_1 + \frac{C_2}{a}\right)\left(C_3 + C_4 \frac{m}{H}\right)$$

- Which are stable in the *decoupling limit*.

→ Full Stability should be explored
→ As well as viability of the resulting Cosmology

CdR, Fasiello, Tolley arXiv:1410.0960

Outlook

- Massive Gravity is a specific framework to study IR modifications of Gravity
- The Vainshtein mechanism comes hand in hand with strong coupling, non-analyticity and superluminalities
- Galileon duality may help understanding these issues
- Theory with these issues is dual to a free and manifestly UV complete theory

Outlook

- One can generalize massive gravity
  while preserving Lorentz invariance
  without ghost
  manifestly 5 degrees of freedom
  but… breaks Poincare invariance
- Generalized massive gravity allows for exact stable FLRW solutions (which can self-accelerate)
- Their full analysis should be explored further

ありがとう

ありがとうございました

IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Causal structure

Coupling to Matter

- If coupling to an *external* source

Eg. $$h_{\mu\nu}(x)T^{\mu\nu}(x) \qquad \text{or} \qquad \pi(x)T(x)$$

$$T(x) \to T(\tilde{x}) = T(x + s'(\partial\pi))$$

- These would map to a non-local coupling.
- The same would happen for GR:
  An external source breaks diffeomorphism invariance

CdR, Fasiello, Tolley 2013
Creminelli, Serone, Trevisan, and Trincherini, 2014

Coupling to Matter

- If coupling to a *dynamical* source

Eg. $$h_{\mu\nu}(x)T^{\mu\nu}(x) \qquad \text{or} \qquad \pi(x)T(x)$$

$$\cancel{T(x) \to T(\tilde{x}) = T(x + s'(\partial\pi))}$$

$$T(x) \to \tilde{T}(\tilde{x})$$

- Dynamical sources preserve diffeomorphism invariance.
- Local dynamical sources map to **local sources**

- This map can be applied to any theory.

CdR, Keltner, Tolley 2014
Kampf & Novotny, 2014

$$T(x) \to \tilde{T}(\tilde{x})$$

Matter transformation

- If coupling to a *dynamical* source
- Matter fields should transform as they would do under a standard coordinate transformation
- Eg. Scalar field $\chi(x) \to \tilde{\chi}(\tilde{x}) = \chi(x)$

CdR, Keltner, Tolley 2014

$$T(x) \to \tilde{T}(\tilde{x})$$

Matter transformation

- If
- M
  st
- E
- A

Local dynamical sources map to **local sources**

Following results are not are artefact of non-locality

$$T_{\mu_1\cdots\mu_\ell}(x) \to \tilde{T}_{\mu_1\cdots\mu_\ell}(\tilde{x}) = M^{\nu_1}_{\mu_1}\cdots M^{\nu_\ell}_{\mu_\ell} T_{\nu_1\cdots\nu_\ell}(x)$$

CdR, Keltner, Tolley 2014

Classical Causal Structure

- Galileon coupled to a scalar field χ with standard kinetic term

$$\mathcal{L} = -\frac{1}{2}\det(1+\Pi)(\partial\pi)^2 - \frac{1}{2}(\partial\chi)^2 + g\chi^2\pi$$

- The field χ propagates **luminaly** independently of the configuration
- There are configurations where the **Galileon is superluminal**
- χ is subluminal compared to the Galileon

$$f_{s'}(\tilde\pi, \partial\tilde\pi) = \left(\tilde\pi - \frac{s'}{2}(\partial\tilde\pi)^2\right)$$

$$M = 1 - s'\tilde\Pi$$

$$Z^{\mu\nu} = (M^{-1})^{\mu}_{\alpha}(M^{-1})^{\nu\alpha}$$

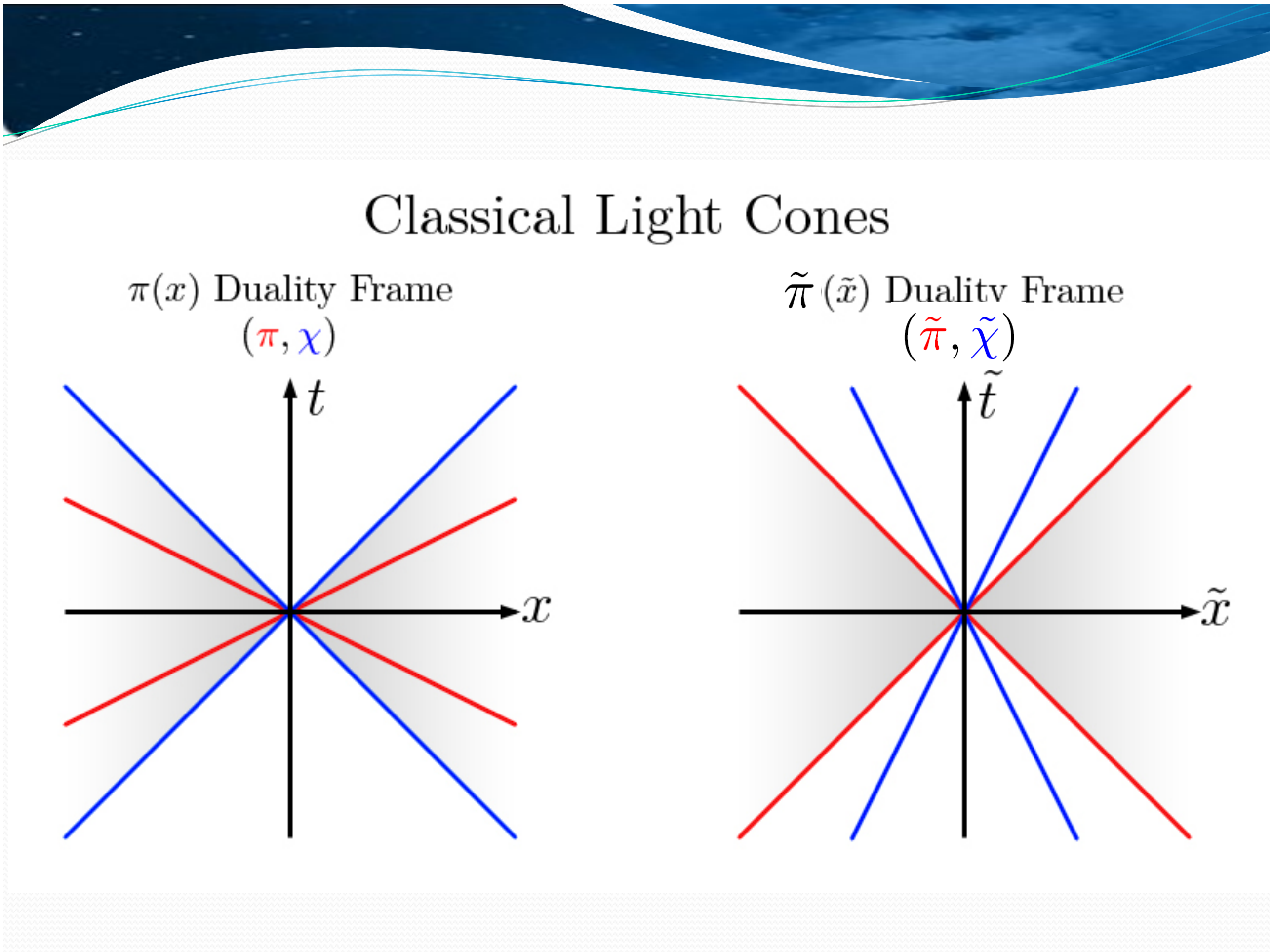
Classical Causal Structure

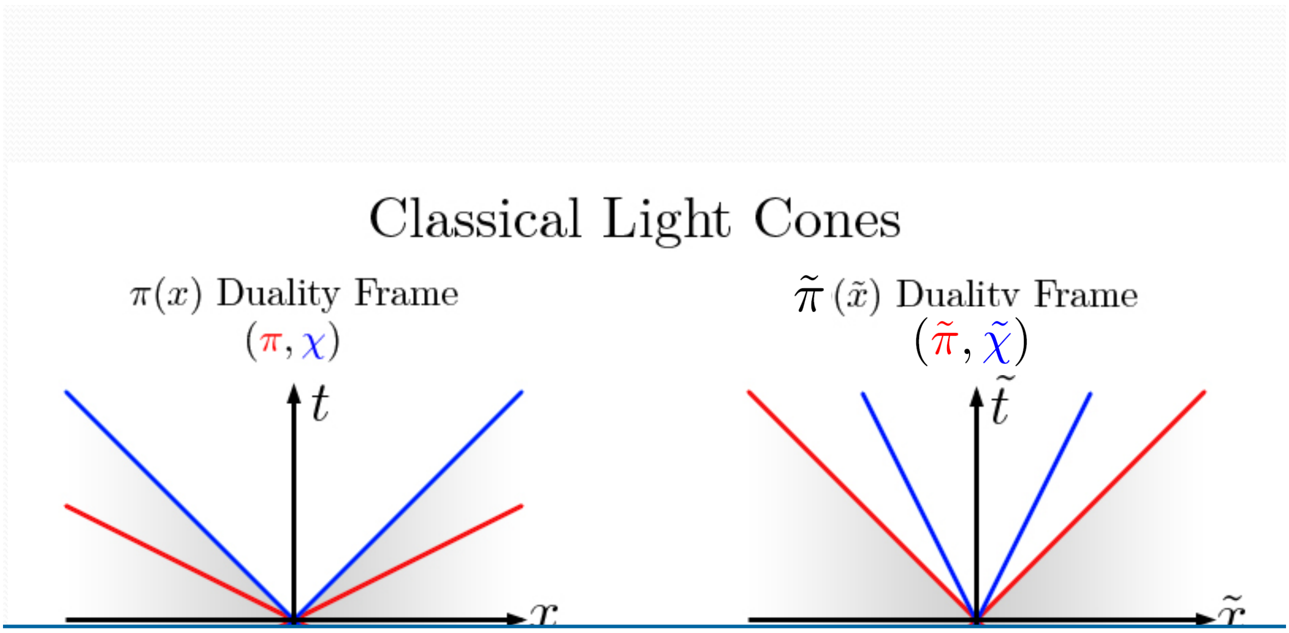
- In the dual picture,

$$\tilde{\mathcal{L}} = -\frac{1}{2}(\partial\tilde\pi)^2 - \frac{1}{2}Z^{\mu\nu}\partial_\mu\tilde\chi\partial_\nu\tilde\chi + g\det M f(\tilde\pi)\tilde\chi^2$$

- $\tilde\pi$ has a standard kinetic term $\longrightarrow$ **propagates luminally**

- $\tilde\chi$ acquires a non-standard kinetic term and propagates **subluminaly**

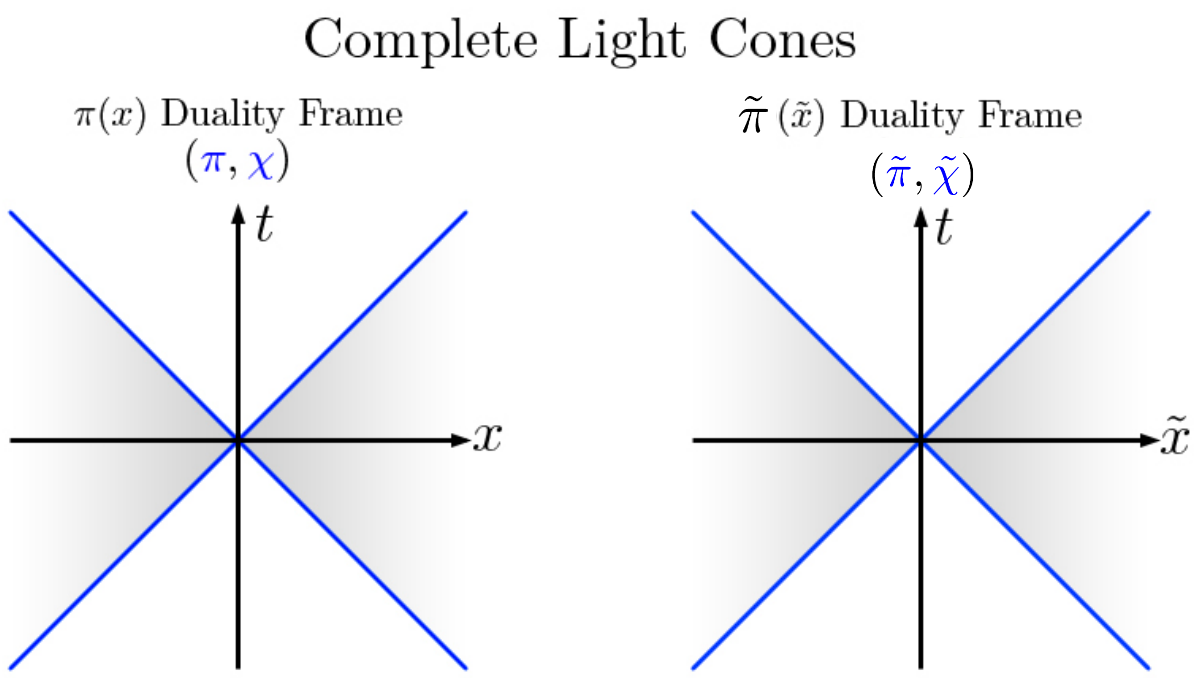




The Classical Causal structure remains preserved

Shown for **any field configuration**,
does not rely on plane waves
does not rely on vacuum

If a UV completion exists then we should have



Quantum Stability ???

Ghost-free Massive Gravity

- Structure of mass term is essential to avoid BD ghost

$$\mathcal{L}_{\mathrm{mGR}} = M_{\mathrm{Pl}}^2 \left(R + m^2 \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right) \right)$$

Boulware & Deser, PRD 6, 3368 (1972)
CdR & Gabadadze, PRD 82, 044020 (2010)
CdR, Gabadadze & Tolley, PRL 106, 231101 (2011)

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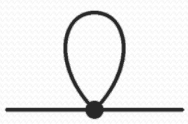
$$\mathcal{L}_{\mathrm{mGR}} = M_{\mathrm{Pl}}^2 \left(R + m^2 \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right) \right)$$

- We expect the structure to detune the potential



$$\frac{m^4}{M_{\mathrm{Pl}}^2} h^2 \longrightarrow \frac{(\partial^2 \pi)^2}{M_{\mathrm{Pl}}^2} \longrightarrow m_{\mathrm{gh}}^2 \sim M_{\mathrm{Pl}}^2$$

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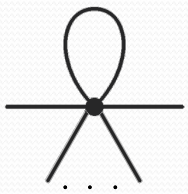

$$\frac{m^4}{M_{\rm Pl}^{n+2}} h^{n+2} \longrightarrow \frac{(\partial^2\pi)^2}{M_{\rm Pl}^2}\left(\frac{\partial^2\pi_0}{\Lambda^3}\right)^n$$

$$m_{\rm gh}^2 \sim M_{\rm Pl}^2 \left(\frac{\partial^2\pi_0}{\Lambda^3}\right)^{-n}$$

CdR, Heisenberg & Ribeiro, 1307.7169

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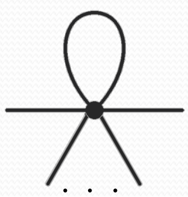

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$$m_{\rm gh}^2 \sim M_{\rm Pl}^2 \underbrace{\left(\frac{\partial^2\pi_0}{\Lambda^3}\right)^{-n}}_{\ll 1}$$

CdR, Heisenberg & Ribeiro, 1307.7169

1-loop Effective Action

- The 1-loop effective action is itself redressed

$$\mathcal{L}_{\rm eff} = \frac{1}{M_{\rm Pl}^2}\frac{1+c_1\frac{\partial^2\pi_0}{\Lambda^3}}{1+c_2\frac{\partial^2\pi_0}{\Lambda^3}}\left(\partial^2\pi\right)^2$$

- The detuning of the potential is never a problem at that level $m_{\rm gh}^2 \gtrsim M_{\rm Pl}^2$

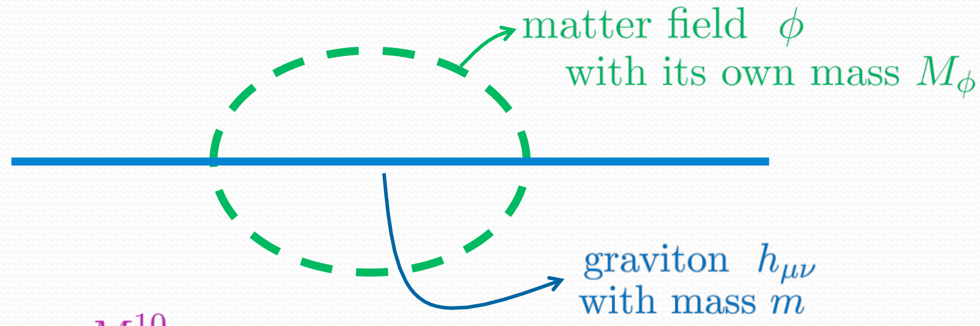
- Even on top of large background configurations

$$\frac{\partial^2\pi_0}{\Lambda^3} \gg 1$$

CdR, Heisenberg & Ribeiro, 1307.7169

Beyond 1-loop…

- At higher order in loops, loops can mix virtual matter fields and graviton fields



Could have a mixing $\frac{M_\phi^{10}}{m^4 M_{\rm Pl}^4}[h^2]$ which could be fatal…

$$\frac{\delta^2 \mathcal{L}}{\delta \phi^2} = Z^{\mu\nu}[\phi]\partial_\nu \partial_\nu$$

Ridding on Irrelevant Operators

- Consider an arbitrary theory

$$\mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + \frac{1}{\Lambda^{n+m}} \partial^{n+2} \phi^{m+2}$$

- The theory exhibits the Vainshtein mechanism if
$$|Z| \sim \left| \frac{\partial^n \phi^m}{\Lambda^{n+m}} \right| \gg 1$$

$$\frac{\delta^2 \mathcal{L}}{\delta \phi^2} = Z^{\mu\nu}[\phi]\partial_\nu \partial_\nu$$

Ridding on Irrelevant Operators

- Consider an arbitrary theory

$$\mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + \frac{1}{\Lambda^{n+m}} \partial^{n+2} \phi^{m+2} + \phi^2 \chi^2$$

- The theory exhibits the Vainshtein mechanism if
$$|Z| \sim \left| \frac{\partial^n \phi^m}{\Lambda^{n+m}} \right| \gg 1$$

- Coupling to heavier fields with $M_\chi \gg \Lambda$ would naively detune the theory…
at least perturbatively

$$\frac{\delta^2\mathcal{L}}{\delta\phi^2} = Z^{\mu\nu}[\phi]\partial_\nu\partial_\nu$$

Ridding on Irrelevant Operators

- Consider the exact Renormalization Group equation

$$\frac{\partial\mathcal{L}_\kappa}{\partial\kappa} = \frac{\hbar}{2}\mathrm{Tr}\left[\frac{\partial_\kappa \hat{R}_\kappa}{\hat{R}_\kappa + \frac{\delta^2\mathcal{L}_\kappa}{\delta\phi^2}}\right]$$

$\hat{R}_\kappa$: regulator operator

$\mathcal{L}_\kappa$: effective average action

κ : IR regulator

Wetterich, 1993

$$\frac{\delta^2\mathcal{L}}{\delta\phi^2} = Z^{\mu\nu}[\phi]\partial_\nu\partial_\nu$$

Ridding on Irrelevant Operators

- Consider the exact Renormalization Group equation

$$\frac{\partial\mathcal{L}_\kappa}{\partial\kappa} = \frac{\hbar}{2}\mathrm{Tr}\left[\frac{\partial_\kappa \hat{R}_\kappa}{\hat{R}_\kappa + Z^{\mu\nu}_\kappa\partial_\mu\partial_\nu}\right]$$

Deep in the Vainshtein Region,

$$|Z| \gg 1 \quad \Rightarrow \quad \frac{\partial\mathcal{L}_\kappa}{\partial\kappa} \to 0$$

Fully Non-perturbatively

CdR & Raquel Ribeiro, arXiv:1405.5213

Ridding on Irrelevant Operators

- Suppressing the loops...

$$\hbar_{\rm eff} = \hbar/Z$$

$$\int \mathcal{D}[\chi]\ e^{-\frac{1}{\hbar}\int \mathrm{d}^4x Z^{\mu\nu}[\phi]\partial_\mu\chi\partial_\nu\chi} \sim \int \mathcal{D}[\chi]\ e^{-\frac{1}{\hbar_{\rm eff}}\int \mathrm{d}^4x\,(\partial\chi)^2}$$

Quantum corrections become irrelevant deep in the Vainshtein regime

$$|Z| \gg 1 \quad \Rightarrow \quad \hbar_{\rm eff} \to 0$$

Fully Non-perturbatively

CdR & Raquel Ribeiro, arXiv:1405.5213

On the Strong Coupling Issue

Q: At what scale does standard perturbativity break down ?

A: The scale is environment-dependent.

In DGP, (really for a cubic Galileon),
At the surface of the Earth from the mass of Earth alone,

$$\Lambda = (1000\mathrm{km})^{-1} \longrightarrow \Lambda_* = (1\mathrm{cm})^{-1}$$

On the Strong Coupling Issue

In massive Gravity, (really in DL or for a quartic Galileon),
Burrage, Kaloper & Padilla tried to answer this question
PRL 111(2013) 021802, arXiv:1211.6001 and found,

$$\Lambda = (1000\mathrm{km})^{-1} \longrightarrow \Lambda_* = (1\mathrm{km})^{-1}$$

From which they conclude that the graviton mass ought to be *bounded...*

$$m \geq \mathcal{O}\,(\mathrm{meV})$$

Do it right !

In arXiv:1211.6001:

1) Looked at a solution which is *not stable* and does not exhibit the Vainshtein mechanism in the first place

Do it right !

In arXiv:1211.6001:

1) Looked at a solution which is *not stable* and does not exhibit the Vainshtein mechanism in the first place

2) Identified the *wrong operator*

Identified the strongly coupled operator dimension-9 operator

$$\bar{h}_{\rm back}\left(\underbrace{[\Pi]^3-3[\Pi][\Pi^2]+2[\Pi^3]}_{\text{Total derivative}}\right)$$

Do it right !

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Identified the strongly coupled operator dimension-9 operator

$$\bar{h}_{\text{back}} \left(\underbrace{[\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]}_{\text{Total derivative}}\right)$$

Instead the first strongly coupled op is dimension-7 operator

$$(\partial^2 \bar{h}_{\text{back}})(\partial\pi)^2 \Box\pi$$

Arise at a higher energy scale !

Do it right !

In arXiv:1211.6001:

1) Looked at a solution which is *not stable* and does not exhibit the Vainshtein mechanism in the first place
2) Identified the *wrong operator*

Same thing when dealt with quartic Galileon

Do it right !

In arXiv:1211.6001:

1) Looked at a solution which is *not stable* and does not exhibit the Vainshtein mechanism in the first place
2) Identified the wrong operator
   Same thing when dealt with quartic Galileon
3) Assumed exact STATIC & spherically symmetric configuration
   Just the dipole from the Earth ~ 10^{-3} radically change their result

Do it right !

In arXiv:1211.6001:

1) Looked at a solution which is *not stable* and does not exhibit the Vainshtein mechanism in the first place

2) Identified the *wrong operator*

   Same thing when dealt with quartic Galileon
3) Assumed exact STATIC & spherically symmetric configuration

Correcting for all these errors leads to $\Lambda_* \sim (10\text{cm})^{-1}$ rather than $(1\text{ km})^{-1}$

But even putting these errors aside the reasoning of the paper is unphysical

Do it right !

4) The scale that comes in is always $\frac{\alpha}{m^2 M_{\rm Pl}\sqrt{Z_*}}$

α dimensionless, non-renormalized free parameter

and not m alone $\longrightarrow$ cannot put a bound on the graviton mass itself

Do it right !

4) Cannot identify the mass parameter from that DL
5) Even if all the previous points were correct, one CANNOT use the breaking of perturbativity to put a bound on a physical parameter.

All it means is that in this variable the DL description breaks down

From the Vainshtein mechanism we expect to recover GR better and better the deeper in the SC regime we are

Do it right !

4) Cannot identify the mass parameter from that DL

5) Even if all the previous points were correct, one CANNOT use the breaking of perturbativity to put a bound on a physical parameter.

6) At these scales, one needs to take into account the further screening from the experiment itself (local energy + building, people, etc…)

Do it right !

Finally…

The Galileon Duality suggests of a way (ways) to repackage infinite number of loops such that perturbativity in the new variables is under control up to a much larger energy scale.

“Appearance of Boulware-Deser ghost in bigravity with doubly coupled matter”

Yasuho Yamashita

[JGRG24(2014)111102]

Appearance of Boulware-Deser ghost in bigravity with doubly coupled matter

YITP, Kyoto University
Yasuho Yamashita

in collaboration with A. De Felice and T. Tanaka

bigravity and Boulware-Deser ghost

bigravity : gravity which contains two interacting gravitons

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}\left[R^{(g)} + \underline{2m^2V(g,\,f)}\right] + \cancel{\frac{M_f^2}{2}\int d^4x\sqrt{-f}R^{(f)}}$$

fix f

The interaction term breaks general covariance for g

→ $\underbrace{\text{GR (helicity-2) + 4 gauge breaking (helicity-1, helicity-0,}}_{\text{massive graviton}}$ helicity-0)

This mode's kinetic term has opposite sign!!

Boulware-Deser ghost

Boulware and Deser (1972)

In order to obtain healthy bigravity, we have to tune the interaction form so that the ghost mode is killed by constraints.

ghost-free bigravity

Choosing the form of the interaction as

$$\boxed{V = \sum_{n=0}^{4} c_n \epsilon^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} K^{\nu_1}_{\mu_1} \ldots K^{\nu_n}_{\mu_n} \qquad K^{\nu}_{\mu} = \sqrt{g^{\nu\rho} f_{\rho\mu}}}$$

de Rham, Gabadadze, Tolley (2011)

ADM decomposition
$$N^{-2} = -g^{00}\,, \quad N_i = g_{0i}\,, \quad \gamma_{ij} = g_{ij}\,,$$
$$L^{-2} = -f^{00}\,, \quad L_i = f_{0i}\,, \quad {}^3f_{ij} = f_{ij}\,.$$

define new shift-like vector n^i
and rewrite N^i with n^i

Then Hamiltonian becomes linear in N, L, L^i.

$$H = N\mathcal{C} + L C^L + L^i C^L_i\,.$$

conjugate momentum

$\mathcal{C}$, C^L, C^L_i are functions of $\{\gamma_{ij},\, \pi^{ij},\, {}^3f_{ij},\, p^{ij}\}$

→ One of the Hamiltonian constraints kills BD ghost.

Hassan and Rosen (2012)

Questions in ghost-free bigravity

- What is the hidden metric *f*?
- The form of the interaction is derived technically and artificially.
- The cosmological solutions in ghost-free bigravity do not exist or become unstable at high energies.

  → We want to extend bigravity to more fundamental theory.

We want to embed ghost-free bigravity to higher dimensional gravity.

Correspondence between ghost-free bigravity and DGP 2-brane model with stabilization mechanism

When the two branes are almost flat and its separation is small,
DGP 2-brane model is identical to ghost-free bigravity.

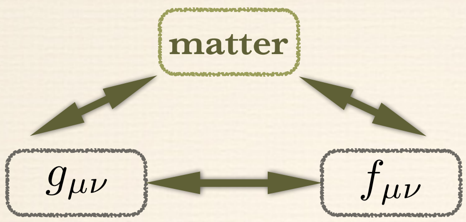
ghost-free bigravity		**DGP 2-brane model**
two metrics	⟷	two metrics induced on the two branes
graviton's mass	⟷	the mass of the lowest massive mode

YY and Tanaka (2014)

It is natural to consider **doubly coupled matter** in ghost-free bigravity by introducing **5-dim matter field** in braneworld model.

doubly coupled matter

However,



coupling through the matter generally detunes the ghost-free structure of the interaction.

→ **BD ghost?**

Consider a free scalar field which couples to both metric:

$$\mathcal{L}_m = \sqrt{-g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) + \sqrt{-f}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right)$$

conjugate momentum $\pi_\phi \sim \left(\frac{1}{N}+\frac{1}{L}\right)\partial_t\phi$

Hamiltonian $\mathcal{H} \ni \frac{NL}{N+L}\pi_\phi^2$ …nonlinear in the lapse fcns → **BD ghost!**

Seeking for models with doubly coupled matter which have no BD ghost

Introduce a k-essence scalar field

$$\mathcal{L}_m = \sqrt{-g}\,P(X,\phi) + \sqrt{-f}\,\tilde{P}(\tilde{X},\phi)$$

$$X = -\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\,, \quad \tilde{X} = -\frac{1}{2}f^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$$

Consider perturbation around FLRW and Bianchi type-1 spacetime

and evaluate the determinant and the eigenvalues of the kinetic matrix A.

When $\det A \neq 0$, an extra d.o.f. exists.

their signs clarify whether the d.o.f. is a ghost mode or not.

Result

$\Rightarrow$ **BD ghost appears unless** $\tilde{P} = \tilde{P}(\phi)$ or $P = P(\phi)$

YY, De Felice and Tanaka (2014)

Seeking for models with doubly coupled matter which have no BD ghost

- another ghost-free model motivated by the quasi-dilaton massive gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_g^2 R^{(g)}}{2} + 2m^2M_{\text{eff}}^2\sum_n c_n e_n\left(\sqrt{g^{\mu\nu}(f_{\mu\nu}+\alpha\partial_\mu\phi\,\partial_\nu\phi)}\right)\right] + \int d^4x\sqrt{-f}\left[\frac{M_f^2R^{(f)}}{2} - \frac{1}{2}f^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right]$$

YY, De Felice and Tanaka (2014)

- matter which couples to an effective metric

$$g_{\mu\nu}^{\text{eff}} = \alpha^2 g_{\mu\nu} + 2\alpha\beta g_{\mu\alpha}\sqrt{g^{\alpha\beta}f_{\beta\nu}} + \beta^2 f_{\mu\nu}$$

This model has BD ghost, but it appears beyond the strong coupling scale.

de Rham, Heisenberg and Rebeiro (2014)

The model of doubly coupled matter is considerably restricted.

… **inconsistent with the intuition in braneworld models.**

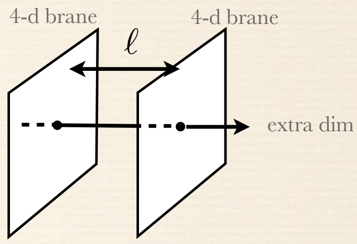
Summary

- We want to derive the ghost-free bigravity from some more fundamental theory which is valid at high energies … **higher dimensional gravity**
- We obtain the ghost-free bigravity as 4-dim effective theory of DGP 2-brane model with stabilization mechanism in the very limited low energy regime.
- This idea suggests that it is natural to consider **doubly coupled matter** in the ghost-free bigravity, however, we found that doubly coupled matter generally brings **BD ghost.**

How?

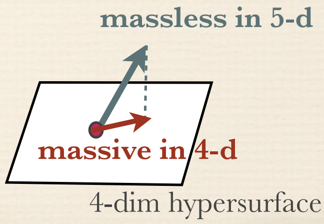
Consider 5-dim braneworld model sandwiched by two branes.

$$S = \frac{M_5^3}{2}\int d^5x\sqrt{-g}\,R \;+\; (\text{boundary term})$$



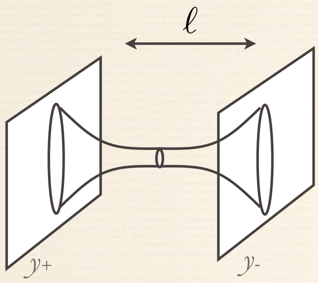
- There is no BD ghost.
- two metrics induced on two branes ⇔ two metrics in bigravity
- 5-dim massless graviton → 4-dim massive graviton on the branes.

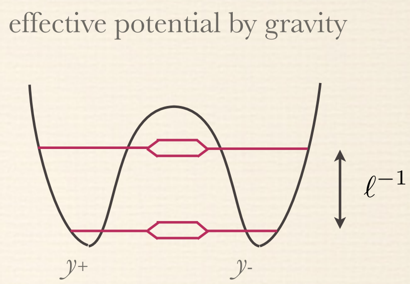
Only one massive mode must have small mass
to reproduce bigravity as a low energy effective theory.



Dvali-Gabadadze-Poratti 2-brane model

4-dim mass spectrum ~ eigenvalue problem in quantum mechanics





high potential barrier

→ nearly degenerate two small mass

However, such thin throat structure is unstable.

$$M_5^3 r_c \int d^4x\sqrt{-g^{(4)}}\,R^{(4)}$$ can take its place

→ **DGP model** $$S = \frac{M_5^3}{2}\int d^5x\sqrt{-g}\left({}^5R + \sum_{\pm} \underline{r_c^{(\pm)}}\,\delta(y-y_\pm)\,{}^4R_{(\pm)}\right)$$

additional length-scale parameter

Stabilization mechanism (Goldberger & Wise)

There is an extra scalar d.o.f. corresponding to the brane separation.

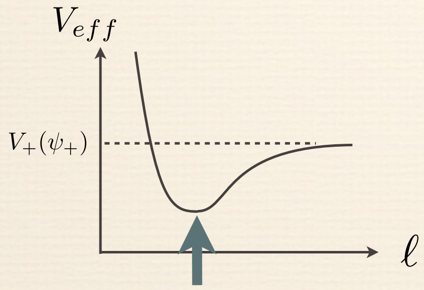
… We should remove it to reproduce bigravity !

We introduce stabilization scalar field to fix the brane separation.

$$S_s = \int d^5x\sqrt{-g}\left(-\frac{1}{2}g^{ab}\psi_{,a}\psi_{,b} - V_B(\psi) - \sum_{\sigma=\pm} \underline{V_{(\sigma)}(\psi)\delta(y-y_\sigma)}\right)$$

$\psi(y_\pm)$: fixed

$\partial_y\psi \to \infty$ as $\ell \to 0$



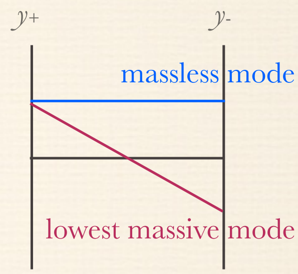
The distance between two branes are stabilized.

graviton's mass spectrum

massless mode always exists

the lowest massive mode

For $\ell \ll r_c$, eigenfunctions become



junction condition:

$$K^{(\pm)}_{\mu\nu} = r_c^{(\pm)}\left(G^{\pm(4)}_{\mu\nu} - \frac{1}{3}G^{\pm(4)}g_{\mu\nu}\right) \xrightarrow{\ell \ll r_c} g_{\mu\nu}/\ell \simeq r_c\Box^{(4)}g_{\mu\nu} = r_c m_1^2 g_{\mu\nu}$$

$$\Rightarrow \quad m_1^2 \simeq \frac{1}{r_c\ell} \ll \frac{1}{\ell^2} \simeq m_2^2$$ **: hierarchy**

mass spectrum (scalar mode)

stabilization mechanism $\rightarrow$ **no massless mode**

If stabilization is weak: $\left|\dfrac{\partial_y \mathcal{H}}{\mathcal{H}^2}\right| \sim \dfrac{(\partial_y \psi)^2}{M_5^3 \mathcal{H}^2} \ll 1$,

the lowest mass becomes $$\mu^2 \approx \frac{2\displaystyle\int_{y_+}^{y_-} \frac{dy}{a^2} + \sum_\sigma \frac{2r_c^{(\sigma)}}{a_\sigma^2} \frac{1}{1-\sigma 2 r_c^{(\sigma)} \mathcal{H}_\sigma}}{\displaystyle\int_{y_+}^{y_-} \frac{dy}{a^4(-\mathcal{H}')}}$$

$\mathcal{H}$: 5-d curvature scale

- stronger stabilization (large $|\mathcal{H}'|$) ➡ **large μ^2**
- $1 \mp 2r_c^{(\pm)} \mathcal{H}_\pm < 0$ make μ^2 negative : tachyonic instability!

➡ corresponds to the **self accelerating branch**

: K.Izumi et al. (2007)

The model which reproduces bigravity

parameters $M_5 = 1.00$

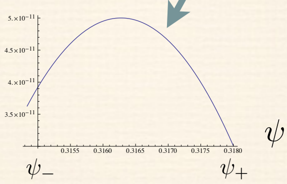
$r_c^{(\pm)} = 1.00 \times 10^5\ , \quad \ell = 1.00$

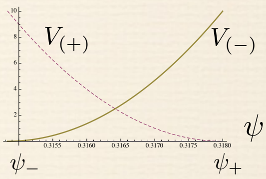
(**brane separation l**)

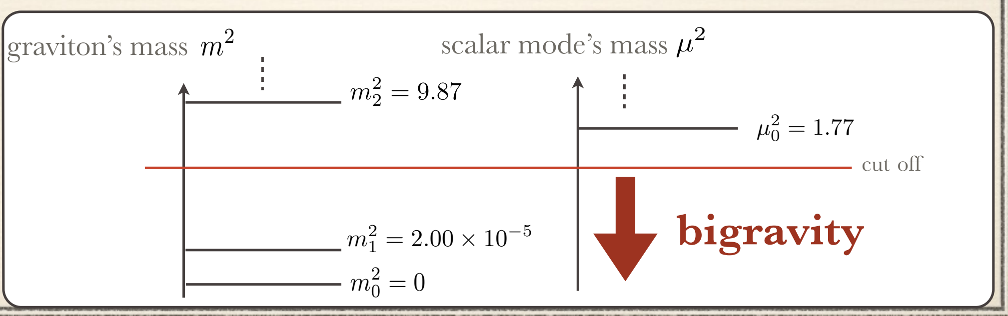
<< (**strength of induced gravity $r_c^{(\pm)}$**)

potential of scalar field

$$S_s = \int d^5x \sqrt{-g} \left(-\frac{1}{2} g^{ab} \psi_{,a} \psi_{,b} - V_B(\psi) - \sum_{\sigma=\pm} V_{(\sigma)}(\psi) \delta(y - y_\sigma) \right)$$







ghost in DGP model

H : 4-dim comoving curvature scale

the regularity on +brane imposes

$$2\left(\sum_i \frac{u_i^2(y_+)}{m_i^2-2H^2}\right)+\frac{1}{H_+^2(2r_c\mathcal{H}_+-1)}\left(\frac{2\kappa^2}{3H_+^2(2r_c\mathcal{H}_+-1)}\left(\sum_i \frac{v_i^2(y_+)}{\mu_i^2+4H^2}\right)+\mathcal{H}_+\right)=0$$

diverges as $m^2 \to 2H^2$: Higuchi bound

diverges as $\mu^2 \to -4H^2$
: critical mass that scalar tachyon appears

$2r_c\mathcal{H}_+-1>0$: self-accelerating branch

$\mu_i^2+4H^2 \to \mp\epsilon$ means $m_i^2-2H^2 \to \pm\epsilon$

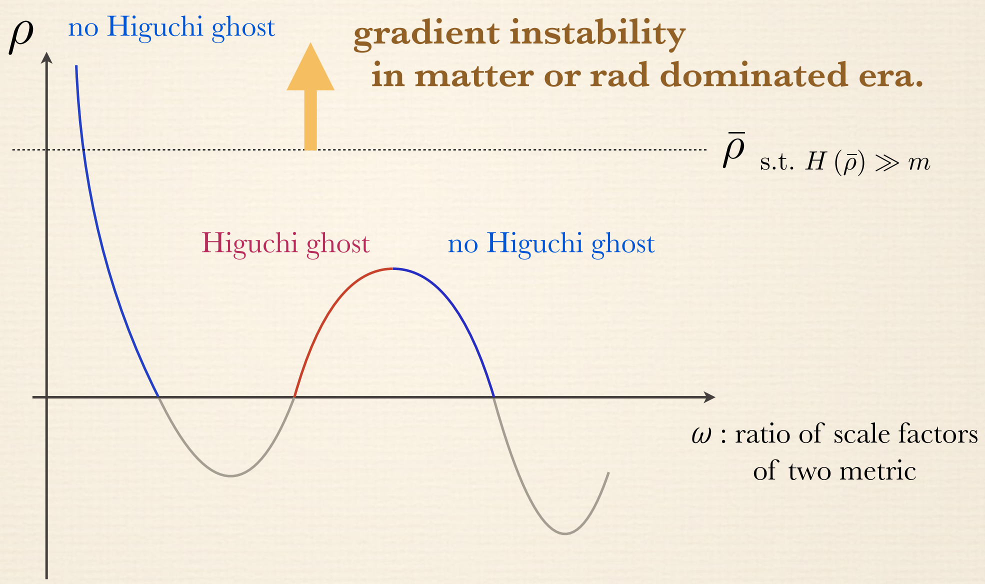
➡ **ghost never disappears** : K.Izumi et. al. (2007)

$2r_c\mathcal{H}_+-1<0$: normal branch

The same identity prohibits m_i^2 & μ_i^2 from crossing their critical masses

➡ **no ghost**

Cosmological solution in ghost-free bigravity



Higuchi ghost in dRGT bigravity

In dRGT model, equation for the de Sitter solution insists

$$\frac{\kappa_4^2}{m^2}\rho_m = \frac{c_1}{\chi\omega} + \left(\frac{6c_2}{\chi} - c_0\right) + \left(\frac{18c_3}{\chi} - 3c_1\right)\omega + \left(\frac{24c_4}{\chi} - 6c_2\right)\omega^2 - 6c_3\omega^3 \equiv f(\omega)$$

ω : ratio of scale factor of two metric

effective mass for massive graviton

$$m_{eff}^2 = m^2(1+(\chi\omega^2)^{-1})\Gamma(\omega) = -\frac{m^2\omega}{3}\underline{f'(\omega)} + 2H^2$$

this sign determines the ghost appearance

$$\Gamma(\omega) \equiv c_1\omega + 4c_2\omega^2 + 6c_3\omega^3$$

For flat vacuum solution, $H \to 0$ as $\omega \to \omega_0$ where $\rho_m(\omega_0) \to 0$,

$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

negative when $\boldsymbol{\Gamma > 0}$ i.e. $\boldsymbol{m_{eff}^2 > 0}$

→ **no Higuchi ghost**

Higuchi ghost in dRGT bigravity

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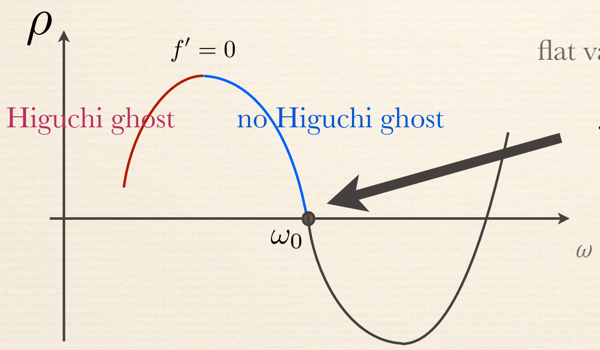


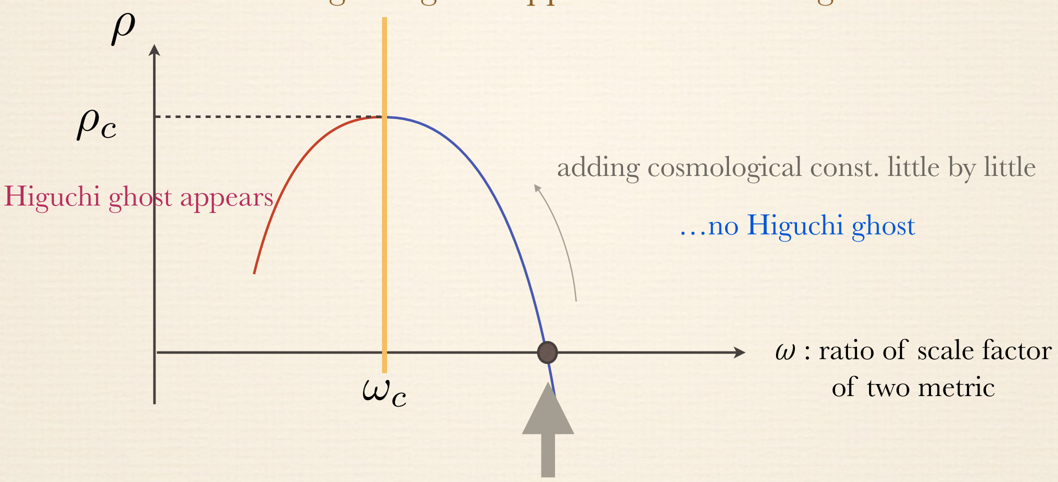
flat vacuum $H = 0, \rho = 0$

$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

...negative when $\Gamma > 0 \Leftrightarrow m_{eff}^2 > 0$

Higuchi ghost in dRGT bigravity

de Sitter solution does not exist above this critical density, and Higuchi ghost appears after crossing the critical ω.



choose the branch connected to the vacuum flat spacetime with positive graviton mass

collapse of the structure in DGP model

junction condition

$$\pm\mathcal{H}_\pm = r_c^{(\pm)} a^{-2} H^2 - \frac{\kappa^2}{6} V_{(\pm)}\left(\psi_\pm\right)$$

consider to add cosmological const. δH on the brane

$$\pm\delta\mathcal{H}_\pm = r_c^{(\pm)} a^{-2} \delta H^2$$

$\delta V_{(\pm)}$ is assumed as very small

$$|\mathcal{H}| \lesssim \frac{1}{r_c^{\pm}}$$ must be satisfied to avoid scalar-mode instability

$\rightarrow$ $$\delta H^2 \gtrsim \frac{1}{r_c^{(\pm)2}}$$ cause instability and break the structure

“Cosmology in rotation-invariant massive gravity with non-trivial fiducial metric”

Atsushi Naruko

[JGRG24(2014)111103]

Cosmology in rotation-invariant massive gravity with non-trivial fiducial metric

Atsushi NARUKO (TiTech)

in collaboration with
David Langlois (APC, Paris)
Shinji Mukohyama (YITP) Ryo Namba (KIPMU)

based on : CQG. 31 (2014), [arXiv : 1405.0358]

Introduction

probably, I can skip this page…

Practical motivation

✓ We would like to consider the dRGT model which is a (the ?) non-linear extension of Firez & Pauli theory.

✓ However, dRGT suffers from several issues :
- no FLAT FLRW solution
- new non-linear ghosts (← vanishing kinetic terms)

◉ abandon isotropy or homogeneity ?

◉ extend the theory ? introduce new d.o.f ??

◉ appropriate (doubly-coupled) matter coupling ???

Lorentz -> SO(3)

✓ The original dRGT model enjoys 4D Lorentz symmetry.

$\delta_{AB}\partial_\mu \Phi^A \partial_\nu \Phi^B$

✓ Universe is expanding !!

→ Lorentz invariance is broken !

→ respect only 3D rotation symmetry !!

✓ It might be natural to consider a massive gravity model which only possesses a 3D maximal symmetry.

✓ Φ^I among [Φ and Φ^I] have SO(3) symmetry :

$$\Phi^I \to \Phi^I + C^I \;\;\&\;\; \Phi^I \to R^I{}_J \Phi^J \quad \Leftrightarrow \quad \delta_{IJ}\partial_\mu \Phi^I \partial_\nu \Phi^J$$

covariant SO(3) gravity

✓ The theory enjoys 4D diffeomorphism invariance

$$\mathcal{L} = \frac{1}{2}R - m^2 V(g_{\mu\nu}\,, \Phi\,, \nabla_\mu\Phi\,, e_{\mu\nu}) \quad e_{\mu\nu} = \delta_{IJ}\nabla_\mu\Phi^I\nabla_\nu\Phi^J$$

✓ Let us introduce 10 scalar functions made of Φ and Φ^I

$$\mathcal{N} \equiv \frac{1}{\sqrt{-g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi}}\,, \qquad \mathcal{N}^I \equiv \mathcal{N} n^\mu \partial_\mu \Phi^I\,,$$

$$\Gamma^{IJ} \equiv (g^{\mu\nu} + n^\mu n^\nu)\partial_\mu\Phi^I\partial_\nu\Phi^J\,, \qquad (n^\mu = \mathcal{N} g^{\mu\nu}\partial_\nu\Phi)$$

which reduce to ADM variables in unitary gauge, $\Phi^A = x^\mu$.

SO(3) gravity w/o BD ghost

✓ No BD condition restricts the form of V as

$$V = \mathcal{U} + \frac{\mathcal{E} - \mathcal{U}_I \mathcal{U}^{IJ} \mathcal{E}_J}{\mathcal{N}}$$

Comelli et al. (2013)

where U and E are free functions of Φ, Γ^{IJ} and ξ^I ($\Leftrightarrow$ n^i).

c.f. $\mathcal{U}(\Gamma^{IJ} - \xi^I\xi^J\,, \delta_{IJ}\,, \Phi)$ and $\mathcal{E}(\Gamma^{IJ}\,, \xi^I\,, \delta_{IJ}\,, \Phi)$

✓ Φ can appear everywhere… c.f. E = $f(\Phi) + g(\Phi)\Gamma^{IJ}\xi_I\xi_J + \ldots$

➡ impose a (dilaton-like) symmetry

$\Phi \to \Phi + C$ & $\Phi^I \to e^{-MC}\Phi^I$ $\Leftrightarrow$ $\mathcal{E}(\Gamma^{IJ}\,, \xi^I\,, b(\Phi)\delta_{IJ})$

($\Phi \to \Phi + C$ $\Leftrightarrow$ $b(\Phi) \to 1$ Comelli et al.)

background

✓ BG cosmology : $ds^2 = - N^2(t)\, dt^2 + a^2(t)\, \delta_{ij}\, dx^i\, dx^j$.

- δ wrt N : $3\, M_{\rm pl}^2\, H^2 = \rho_m + \rho_g(X)\,,$
- δ wrt a : $M_{\rm pl}^2 \left(2\dot{H}/N + 3H^2\right) = -P_m - P_g(X)\,,$
- δ wrt Φ : $2\mathcal{U}'(\dot{b}/b) + H(2\mathcal{E}' - \bar{\mathcal{E}}) = 0$ X = b/a

where $\rho_g = M_{\rm pl}^2\, m^2\, \bar{\mathcal{U}}(X)\,,$ $P_g = M_{\rm pl}^2\, m^2 \left[2\mathcal{U}' - \bar{\mathcal{U}} + (2\mathcal{E}' - \bar{\mathcal{E}})/N\right](X)$

✓ H = 0 or 2 E’ - E = 0 in the case b = 1,

➡ no interesting cosmology or E is constrained…

c.f. Comelli et al. (2013)

perturbations

✓ We have studied 3-types of perturbations in a case without matter where the mass term behaves like c.c. and hence the BG is described by a de-Sitter.

✓ At linear level (quadratic in the action), all types of perturbations have non-vanishing kinetic terms.

⇔ dRGT model (kinetic terms of S and V disappear)

✓ We have derived conditions for healthy perturbations

= no ghost instabilities & no gradient instabilities.

⇒ a broad parameter region those conditions are satisfied

summary

- ✓ investigated a possible extension of the original dRGT model, i.e. SO(3) massive gravity model
- ✓ studied background cosmology where the mass term has a non-trivial time-dependence in general
- ✓ studied perturbations in a case without matter
  - non-vanishing kinetic terms for (S,V,T) perturbations
  - derived conditions for healthy perturbations
- ➡ stability analysis of perturbations in a case with matter

“Stability of self-accelerating solutions in extended quasidilaton massive gravity”

Hayato Motohashi

[JGRG24(2014)111104]

Stability of self-accelerating solutions in extended quasidilaton massive gravity

Hayato Motohashi

Kavli Institute for Cosmological Physics
University of Chicago

HM and W. Hu, PRD90 104008, [arXiv:1408.4813]

The extended quasidilaton massive gravity

Extension of dRGT massive gravity by employing scalar field σ which enjoys the global symmetry

D'Amico et al, 1206.4253
De Felice and Mukohyama, 1306.5502

$$\sigma \to \sigma + \sigma_0, \quad \phi^a \to e^{-\sigma_0/M_{\rm Pl}}\phi^a$$

$$\Rightarrow \tilde{f}_{\mu\nu} \to e^{-2\sigma_0/M_{\rm Pl}}\tilde{f}_{\mu\nu}$$

See also:
De Felice, Gumrukcuoglu and Mukohyama, 1309.3162
Mukohyama, 1410.1996

The extended fiducial metric is dynamical through quasidilaton.

The theory has a flat FLRW solution with an effective cosmological constant induced by graviton mass term.

It was shown that this solution is stable in vacuum.

Is it also stable in the presence of matter?

Setup

Total action $S = S_g + S_m$

Extended quasidilaton massive gravity

$$S_g = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R + 2m_g^2(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4) - \frac{\omega}{M_{\rm Pl}^2}\partial_\mu\sigma\partial^\mu\sigma\right]$$

$$\mathcal{L}_2 = \frac{1}{2}([\mathcal{K}]^2 - [\mathcal{K}^2]),$$

$$\mathcal{L}_3 = \frac{1}{6}([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]),$$

$$\mathcal{L}_4 = \frac{1}{24}([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4])$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - e^{\sigma/M_{\rm Pl}}\left(\sqrt{g^{-1}\tilde{f}}\right)^\mu{}_\nu \qquad \tilde{f}_{\mu\nu} = f_{\mu\nu} - \frac{\alpha_\sigma}{M_{\rm Pl}^2 m_g^2}e^{-2\sigma/M_{\rm Pl}}\partial_\mu\sigma\partial_\nu\sigma,$$

$$f_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b,$$

Matter sector

$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\xi\partial_\nu\xi - V(\xi)\right]$$

Background

Spatially flat FLRW cosmological background

$$ds^2 = -N(t)^2dt^2 + a(t)^2\delta_{ij}dx^i dx^j,$$

$$\phi^0 = \phi^0(t), \quad \phi^i = x^i,$$

$$\sigma = \bar{\sigma}(t), \quad \xi = \bar{\xi}(t).$$

$$X \equiv \frac{e^{\bar{\sigma}/M_{\rm Pl}}}{a}$$

$$r \equiv \frac{n}{N}a.$$

Extended fiducial metric

$$-\tilde{f}_{00} \equiv n(t)^2 = (\dot{\phi}^0)^2 + \frac{\alpha_\sigma}{M_{\rm Pl}^2 m_g^2}e^{-2\bar{\sigma}/M_{\rm Pl}}\dot{\bar{\sigma}}^2,$$

$$\tilde{f}_{ij} = \delta_{ij}.$$

Self-accelerating branch: $J = 0$, $X \equiv e^{\bar{\sigma}/M_{\rm Pl}}/a = \text{const}$

$$\frac{d}{dt}\left[\frac{\dot{\phi}^0}{n}a^4X(X-1)J\right] = 0$$

$$J \equiv 3 + 3(1-X)\alpha_3 + (1-X)^2\alpha_4$$

Background

Friedmann equation with effective cosmological constant

$$3\left(1-\frac{\omega}{6}\right)M_{\rm Pl}^2H^2 = M_{\rm Pl}^2\Lambda_X + \frac{\dot{\xi}^2}{2} + V, \qquad \tilde{M}_{\rm Pl}^2 \equiv M_{\rm Pl}^2\left(1-\frac{\omega}{6}\right)$$

$$-2\left(1-\frac{\omega}{6}\right)M_{\rm Pl}^2\dot{H} = \dot{\xi}^2. \qquad \Lambda_X \equiv m_g^2(X-1)^2[(X-1)\alpha_3 - 3].$$

For positive $\tilde{M}_{\rm Pl}^2$ and Λ_X,

$$\frac{m_g^2}{H_0^2} = \frac{(6-\omega)\Omega_\Lambda}{2(X-1)^2[(X-1)\alpha_3-3]}$$

$$\boxed{\omega < 6} \qquad \boxed{(X-1)\alpha_3 - 3 > 0}$$

Since $\left(\frac{\dot{\phi}^0}{n}\right)^2 = 1 - \frac{\alpha_\sigma e^{-2\bar{\sigma}/M_{\rm Pl}}}{M_{\rm Pl}^2 m_g^2}\frac{\dot{\bar{\sigma}}^2}{n^2} = 1 - \frac{\alpha_\sigma H^2}{m_g^2 X^2 r^2}$

to keep Lorentzian signature for the fiducial metric,

$$\boxed{\alpha_\sigma < \frac{m_g^2 X^2 r^2}{H^2}} \qquad r = 1 + \frac{\omega(3H^2+\dot{H})}{3m_g^2X^2[(X-1)\alpha_3-2]}$$

Scalar perturbations

Working in the unitary gauge

Metric perturbations

$$\delta g_{00} = -2\Phi,$$
$$\delta g_{0i} = a\partial_i B,$$
$$\delta g_{ij} = a^2\left[2\delta_{ij}\Psi + \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\partial_\ell\partial^\ell\right)E\right]$$

Quasidilaton and matter field

$$\sigma = \bar{\sigma} + M_{\rm Pl}\delta\sigma,$$
$$\xi = \bar{\xi} + M_{\rm Pl}\delta\xi,$$

Vacuum case: 2 dof

With matter: 3 dof

Vacuum case

Integrating out nondynamical dof: B , Φ , $\Psi - \delta\sigma$

Two dynamical dof: E , $\Psi + \delta\sigma$

No-ghost condition

$$\det K = \frac{M_{\rm Pl}^4 \omega^2 a^2 H^2 k^6}{r^2(r-1)^2} \frac{2A(r-1)^2(k/aH)^2 + 3(\omega-6)(A-r^2)}{4(A-1)(k/aH)^2 + \omega(6-\omega)} > 0,$$

$$K_{22} = \frac{k^4 M_{\rm Pl}^2}{18} \frac{\omega[2(A-1)(k/aH)^2 + 3(6-\omega)]}{4(A-1)(k/aH)^2 + \omega(6-\omega)} > 0,$$

$$A \equiv \frac{\alpha_\sigma H^2}{m_g^2 X^2}$$

Consequently,

$$\boxed{0 < \omega < 6, \quad 1 < \frac{\alpha_\sigma H^2}{m_g^2 X^2} < r^2.}$$

Note that H and r are constant for vacuum case.

With matter

HM and Hu, 1408.4813

Integrating out nondymnamical dof: B , Φ , ψ_1

Three dynamical dof: (ψ_2, ψ_3, ψ_4)

$(\Psi, \delta\sigma, \delta\xi)$

$\Big\downarrow$ Linear comb.

(ψ_1, ψ_2, ψ_3)

$\psi_4 \equiv E$

For $k/aH \gg 1$

$$K_{44} = \frac{k^4}{72} M_{\rm Pl}^2 a^3 (\omega + \Xi^2) + \cdots,$$

$$\begin{vmatrix} K_{33} & K_{34} \\ K_{34} & K_{44} \end{vmatrix} = \frac{\omega k^4}{144} M_{\rm Pl}^4 a^6 (\Xi v_{32} - 1)^2 + \cdots,$$

$$\det K = \frac{\omega^2 A k^2}{96 r^2 \Xi^2 (A-1)} M_{\rm Pl}^6 a^{11} (3H^2 + \dot{H})(2+\Xi^2)^2 [(1-2\omega)^2 + 2\Xi^2 + \Xi^4] + \cdots.$$

For $k/aH \ll 1$

$$K_{44} = \frac{k^4}{12} M_{\rm Pl}^2 a^3 + \cdots,$$

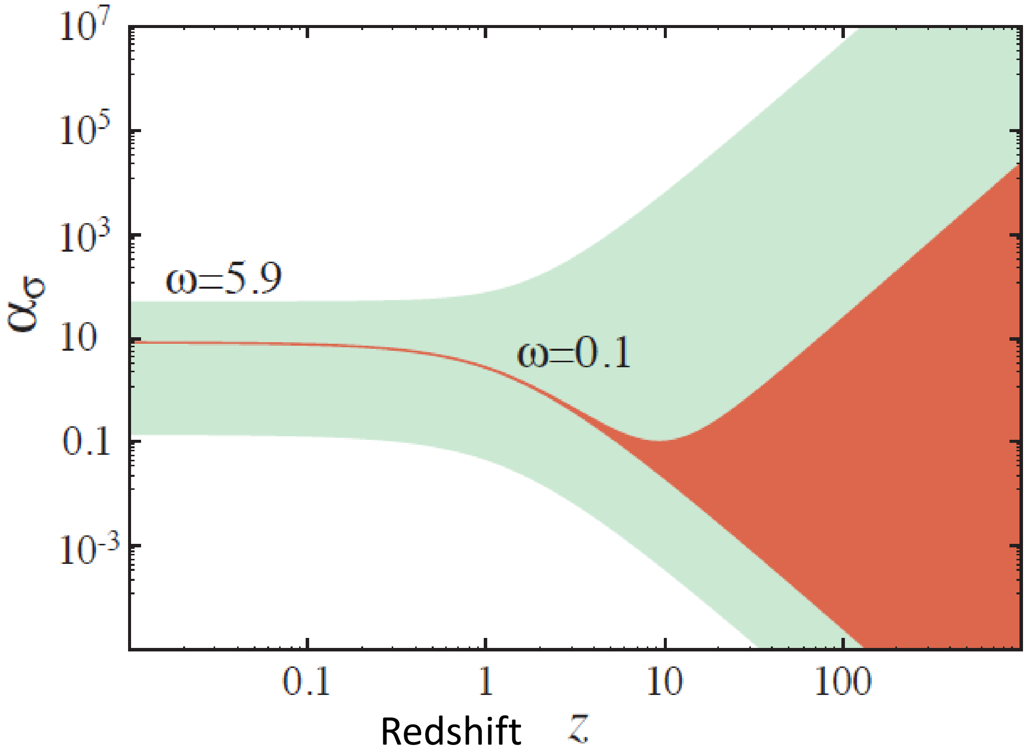
$$\begin{vmatrix} K_{33} & K_{34} \\ K_{34} & K_{44} \end{vmatrix} = \frac{\omega(r^2-A)k^2}{8r^2(r-1)^2} M_{\rm Pl}^4 a^8 (3H^2 + \dot{H})(v_{31} - v_{32})^2 + \cdots,$$

$$\det K = \frac{3\omega(r^2-A)k^2}{16 r^2 (r^2-1)} M_{\rm Pl}^6 a^{11} H^2 [(v_{31} - v_{32})(\Xi v_{21} - 1) - (v_{21} - v_{22})(\Xi v_{31} - 1)]^2 + \cdots.$$

Necessary condition

$$\boxed{0 < \omega < 6, \quad \frac{m_g^2 X^2}{H^2(t)} < \alpha_\sigma < \frac{m_g^2 X^2}{H^2(t)} r^2(t)}$$

ΛCDM expansion history



Stability condition for ΛCDM exp. history

$$\boxed{0 < \omega < 6, \quad \frac{m_g^2 X^2}{H^2(t)} < \alpha_\sigma < \frac{m_g^2 X^2}{H^2(t)} r^2(t)}$$

$$\Downarrow$$

$$\frac{6B}{1+B} < \omega < 6$$

$$\frac{X^2}{2(X-1)^2}\frac{(6-\omega)\,\Omega_\Lambda}{(X-1)\alpha_3 - 3} < \alpha_\sigma < \frac{2\omega}{(X-1)\alpha_3 - 2}\left[1 + \frac{2\omega}{6-\omega}\frac{(X-1)^2}{X^2}\frac{(X-1)\alpha_3 - 3}{(X-1)\alpha_3 - 2}\right]$$

$$B = \frac{X^2}{2(X-1)^2}\frac{(X-1)\alpha_3 - 2}{(X-1)\alpha_3 - 3}(\sqrt{1+\Omega_\Lambda} - 1)$$

Conclusions

- We considered extended quasidilaton with matter and derived necessary conditions for stability:

$$\boxed{0 < \omega < 6, \quad \frac{m_g^2 X^2}{H^2(t)} < \alpha_\sigma < \frac{m_g^2 X^2}{H^2(t)} r^2(t)}$$

- While these appear identical in the form with vacuum case, they provide time-dependent constraint for model parameters.
- There is model parameter region that is initially stable but evolves to an instability.

- More generally, there is nothing intrinsic to the dynamics of the fiducial metric that forbids an evolution from Lorentzian to Euclidian signature. Backgrounds that evolves through such a transition develop a ghost instability.

“Covariant Stueckelberg analysis of dRGT massive gravity

with a general fiducial metric”

Daisuke Yoshida

[JGRG24(2014)111105]

Covariant Stueckelberg Analysis of dRGT massive gravity with a general fiducial metric

Daisuke Yoshida (Tokyo Institute of Technology)

based on arXiv:1409.3074

Collaborators | X.Gao, T.Kobayashi, M.Yamaguchi

Nov.11,2014,JGRG24@IPMU

ABSTRACT

We extend the Stueckelberg analysis of dRGT massive gravity with FLAT fiducial metric to with a GENERAL fiducial metric.

OUTLINE

1. **Introduction of massive gravity**
2. **Stuckelberg analysis with flat fiducial metric**
3. **Extension to GENERAL fiducial metric**

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1. **Introduction of massive gravity**
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3. **Extension to GENERAL fiducial metric**

Massive gravity

The action of nonlinear massive gravity is composed from Einstein-Hilbert action and mass potential of graviton.

$$S = S_{EH}[g] + S_{mass}[g, \bar{g}]$$

Motivation

Theoretically, can graviton have a mass?

Can the graviton mass explain the accelerated universe?

Theoretical feature of massive gravity

Feature 1. fiducial metric

Mass potential is constructed from the graviton $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$

➡ Even in nonlinear level, action include fiducial metric $S_{mass}[g, \bar{g}]$

For simplicity, flat fiducial metric $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$ is often used.
Theoretically, we can use any metric as fiducial metric.

Feature 2. BD ghost

Theoretical feature of massive gravity

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Theoretically, we can use any metric as fiducial metric.

Feature 2. BD ghost Boulware,Deser (1972)

The graviton have 6 d.o.f. in many theory of massive gravity.

massive spin2
5 d.o.f.

+

~~BD ghost~~
~~1 d.o.f.~~
~~unphysical d.o.f.~~

de Rham,Gabadadze,Tolley (2011) dRGT MG

Theoretical feature of massive gravity

Feature 1. fiducial metric

Mass potential is constructed from the graviton $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$

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+

BD ghost
1 d.o.f.

unphysical
d.o.f.

de Rham,Gabadadze,Tolley (2011) dRGT MG

Stueckelberg formalism is very useful to see the presence of ghost.

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu}$$

OUTLINE

1. Introduction of massive gravity
2. Stuckelberg analysis with flat fiducial metric
3. Extension to GENERAL fiducial metric

Stueckelberg Analysis with flat fiducial metric

Stueckelberg analysis occurs in 3step.

STEP1. Stueckelberg trick

Introduce the Stueckelberg fields

$$\eta_{\mu\nu} \to f_{\mu\nu} = \partial_\mu \phi^\alpha \partial_\nu \phi^\beta \eta_{\alpha\beta}$$

$S_{mass}[g_{\mu\nu}, \eta_{\mu\nu}]$

gauge fixing $\phi^\mu = x^\mu$

$S_{mass}[g_{\mu\nu}, f_{\mu\nu}]$

STEP2. helicity decomposition

$$\phi^a = x^a - \pi^a$$

$$\pi^a = A^a + \partial^a \pi$$

helicity-2 $h_{\mu\nu}$:2 d.o.f.

helicity-1 A_μ :2 d.o.f.

helicity-0 π :1+1 d.o.f

Additional d.o.f. appear when the e.o.m. have higher time derivative.

STEP3. decoupling limit

Omit the interaction term beyond the cut off scale.

In dRGT theory cut off scale is $\Lambda_3 = (m^2 M_{PL})^{1/3}$

dRGT massive gravity

de Rham, Gabadadze, Tolley (2011)

dRGT mass potential

$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{g^{-1}f}^\mu{}_\nu$

$(Tr\mathcal{K})^2 + \cdots$

$$S_{dRGT} = \frac{M_{PL}^2}{2} m^2 \int d^4x \sqrt{-g}(\mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4)$$

$$S_{EH} + S_{dRGT} \to \int dx^4 \left[-\frac{3}{4} \partial_\mu \hat{\pi} \partial^\mu \hat{\pi} - \frac{3(1+\alpha_3)}{4\Lambda_3^3} \mathcal{L}^{(3)}_{Gal}(\hat{\pi}) + \cdots \right]$$

decoupling limit

unmixing, normalization

$\Lambda_3 = (m^2 M_{PL})^{1/3}$

Equation of motion include only 2nd time derivative.

π does not have additional d.o.f,
then theory is BD ghost free.

$$\eta_{\mu\nu} \to \bar{g}_{\mu\nu}$$

X.Gao, T.Kobayashi, M.Yamaguchi, D.Y. arXiv:1409.3074

OUTLINE

1. Introduction of massive gravity
2. Stuckelberg analysis with flat fiducial metric
3. **Extension to GENERAL fiducial metric**

dRGT massive gravity with general fiducial metric

$$S_{dRGT}[g,\eta] \quad \Rightarrow \quad S_{dRGT}[g,\bar{g}]$$

- Hamiltonian analysis shows this theory have 5 d.o.f.

⇒ BD ghost free

Hassan,Rosen (2012)
Hassan,Rosen,Schmidt-May(2012)

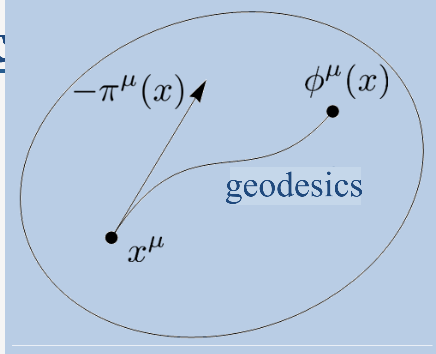
- However Stueckelberg Analysis have not been constructed in general fiducial case.

In the case of de Sitter fiducial metrirc — de Rham,Renaux-Petel (2013)
In the case of FLRW fiducial metric — Fasiello,Tolley(2013)

Our Purpose

- To construct Stueckelberg formalism
- To confirm BD ghost free by Stueckelberg formalism

Modification from the flat fiduc



STEP1. Stueckelberg trick

$$\bar g_{\mu\nu} \to f_{\mu\nu} = \partial_\mu \phi^\alpha \partial_\nu \phi^\beta \, \bar g_{\alpha\beta}$$

STEP2. Definition of Stueckelberg field

$$\phi^\mu(x) = x^\mu - \pi^\mu - \frac{1}{2}\bar\Gamma^\mu_{\nu\rho}\pi^\nu\pi^\rho + \mathcal{O}(\pi^3) \qquad \pi^a = A^a + \partial^a \pi$$

π^μ is covariant vector !

STEP 3. decoupling limit

Omit the interaction beyond the cut off scale

$$\Lambda_3 = (m^2 M_{PL})^{1/3}$$

$\bar R_{\mu\nu\rho\sigma} \lesssim m^2$ → Cut off scale remain Λ_3

Result: Action in Stueckelberg Language

$$S_{EH} + S_{dRGT} = \int d^4x\sqrt{-\bar g}(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \cdots)$$

$$\mathcal{L}_2 = -\frac{1}{4}\hat h_{\mu\nu}\mathcal{E}^{\mu\nu,\rho\sigma}\hat h_{\rho\sigma} - \frac{1}{2}\left(\frac{3}{2}\bar g_{\mu\nu} - \frac{\bar R_{\mu\nu}}{m^2}\right)\bar\nabla^\mu\hat\pi\bar\nabla^\nu\hat\pi$$

$$\mathcal{L}_3 = -\frac{3(1+3\alpha_3)}{4\Lambda_3^3}(\bar\nabla\hat\pi)^2\bar\Box\hat\pi + \frac{1}{2\Lambda_3^3}\mathcal{A}_{\mu\nu\rho\sigma}\bar\nabla^\mu\hat\pi\bar\nabla^\nu\hat\pi\bar\nabla^\rho\bar\nabla^\sigma\hat\pi$$

$$\mathcal{L}_4 = -\frac{1+8\alpha_3+9\alpha_3^2+8\alpha_4}{4\Lambda_3^6}(\bar\nabla\hat\pi)^2\left((\bar\Box\hat\pi)^2 - \bar\nabla_\rho\bar\nabla_\sigma\hat\pi\bar\nabla^\rho\bar\nabla^\sigma\hat\pi\right) + \frac{1}{4\Lambda_3^6}(\alpha_3+4\alpha_4)\hat h^{\mu\nu}X^{(3)}_{\mu\nu}(\hat\pi)$$
$$+\frac{1}{2\Lambda_3^6}\left(\mathcal{B}_{\mu\nu\rho\sigma\rho'\sigma'}\bar\nabla^{\rho'}\bar\nabla^{\sigma'}\hat\pi - \frac{1}{3}\mathcal{C}_{\lambda\mu\nu\rho\sigma}\bar\nabla^\lambda\hat\pi\right)\bar\nabla^\mu\hat\pi\bar\nabla^\nu\hat\pi\bar\nabla^\rho\bar\nabla^\sigma\hat\pi$$

$$\mathcal{A}_{\mu\nu\rho\sigma} \equiv \frac{1}{m^2}\left[(1+2\alpha_3)\left(\bar R_{\mu\nu}\bar g_{\rho\sigma} + \bar R_{\rho(\mu\nu)\sigma}\right) - \alpha_3\left(\bar g_{\rho(\mu}\bar R_{\nu)\sigma} + \bar g_{\sigma(\mu}\bar R_{\nu)\rho}\right)\right],$$

$$\mathcal{B}_{\mu\nu\rho\sigma\rho'\sigma'} \equiv \frac{1}{m^2}\left[\frac{3}{2}(\alpha_3+2\alpha_4)\bar R_{\mu\nu}\left(2\bar g_{\rho[\sigma}\bar g_{\sigma']\rho'}\right) + 12\alpha_4\bar R_{\mu[\rho}\bar g_{\rho'][\sigma}\bar g_{\sigma']\nu}\right.$$
$$\left. -\frac{1}{3}(1+9\alpha_3+18\alpha_4)\left(\bar R_{\mu\rho\nu[\sigma}\bar g_{\sigma']\rho'} - \bar R_{\mu\rho'\nu[\sigma}\bar g_{\sigma']\rho}\right) - 6\alpha_4\bar g_{\mu[\rho}\bar R_{\rho']\nu\sigma\sigma'}\right],$$

$$\mathcal{C}_{\lambda\mu\nu\rho\sigma} \equiv \frac{1}{m^2}\left[\bar g_{\rho\sigma}\bar\nabla_{(\lambda}\bar R_{\mu\nu)} + \frac{1}{3}\left(\bar\nabla_\lambda\bar R_{\mu(\rho\sigma)\nu} + \bar\nabla_\mu\bar R_{\lambda(\rho\sigma)\nu} + \bar\nabla_\nu\bar R_{\lambda(\rho\sigma)\mu}\right)\right].$$

There are new curvature correction terms !

$$
\begin{aligned}
X^{(3)}_{\mu\nu}(\hat{\pi}) &\equiv \bar{g}_{\mu\nu}\left(\left(\bar{\Box}\hat{\pi}\right)^3 - 3\bar{\Box}\hat{\pi}\bar{\nabla}_\rho\bar{\nabla}_\sigma\hat{\pi}\bar{\nabla}^\rho\bar{\nabla}^\sigma\hat{\pi} + 2\bar{\nabla}^\rho\bar{\nabla}_\sigma\hat{\pi}\bar{\nabla}^\sigma\bar{\nabla}_\lambda\hat{\pi}\bar{\nabla}^\lambda\bar{\nabla}_\rho\hat{\pi}\right) \\
&\quad +3\bar{\nabla}_\mu\bar{\nabla}_\nu\hat{\pi}\left(\bar{\nabla}_\rho\bar{\nabla}_\sigma\hat{\pi}\bar{\nabla}^\rho\bar{\nabla}^\sigma\hat{\pi} - \left(\bar{\Box}\hat{\pi}\right)^2\right) + 6\bar{\nabla}^\rho\bar{\nabla}_\mu\hat{\pi}\left(\bar{\nabla}_\nu\bar{\nabla}_\rho\hat{\pi}\bar{\Box}\hat{\pi} - \bar{\nabla}_\nu\bar{\nabla}^\sigma\hat{\pi}\bar{\nabla}_\rho\bar{\nabla}_\sigma\hat{\pi}\right), \\
\mathcal{A}_{\mu\nu\rho\sigma} &\equiv \frac{1}{m^2}\left[(1+2\alpha_3)\left(\bar{R}_{\mu\nu}\bar{g}_{\rho\sigma} + \bar{R}_{\rho(\mu\nu)\sigma}\right) - \alpha_3\left(\bar{g}_{\rho(\mu}\bar{R}_{\nu)\sigma} + \bar{g}_{\sigma(\mu}\bar{R}_{\nu)\rho}\right)\right], \\
\mathcal{B}_{\mu\nu\rho\sigma\rho'\sigma'} &\equiv \frac{1}{m^2}\left[\frac{3}{2}(\alpha_3+2\alpha_4)\bar{R}_{\mu\nu}\left(2\bar{g}_{\rho[\sigma}\bar{g}_{\sigma']\rho'}\right) + 12\alpha_4\bar{R}_{\mu[\rho}\bar{g}_{\rho'][\sigma}\bar{g}_{\sigma']\nu}\right. \\
&\qquad \left. -\frac{1}{3}(1+9\alpha_3+18\alpha_4)\left(\bar{R}_{\mu\rho\nu[\sigma}\bar{g}_{\sigma']\rho'} - \bar{R}_{\mu\rho'\nu[\sigma}\bar{g}_{\sigma']\rho}\right) - 6\alpha_4\bar{g}_{\mu[\rho}\bar{R}_{\rho']\nu\sigma\sigma'}\right], \\
\mathcal{C}_{\lambda\mu\nu\rho\sigma} &\equiv \frac{1}{m^2}\left[\bar{g}_{\rho\sigma}\bar{\nabla}_{(\lambda}\bar{R}_{\mu\nu)} + \frac{1}{3}\left(\bar{\nabla}_\lambda\bar{R}_{\mu(\rho\sigma)\nu} + \bar{\nabla}_\mu\bar{R}_{\lambda(\rho\sigma)\nu} + \bar{\nabla}_\nu\bar{R}_{\lambda(\rho\sigma)\mu}\right)\right].
\end{aligned}
$$

Application 1: Confirmation of BD ghost free

$$\frac{1}{2}\frac{\bar{R}_{\mu\nu}}{m^2}\bar{\nabla}^\mu\hat{\pi}\bar{\nabla}^\nu\hat{\pi}$$

$$\frac{1}{2\Lambda_3^3}\mathcal{A}_{\mu\nu\rho\sigma}\bar{\nabla}^\mu\hat{\pi}\bar{\nabla}^\nu\hat{\pi}\bar{\nabla}^\rho\bar{\nabla}^\sigma\hat{\pi}$$

$$\frac{1}{2\Lambda_3^6}\left(\mathcal{B}_{\mu\nu\rho\sigma\rho'\sigma'}\bar{\nabla}^{\rho'}\bar{\nabla}^{\sigma'}\hat{\pi} - \frac{1}{3}\mathcal{C}_{\lambda\mu\nu\rho\sigma}\bar{\nabla}^\lambda\hat{\pi}\right)\bar{\nabla}^\mu\hat{\pi}\bar{\nabla}^\nu\hat{\pi}\bar{\nabla}^\rho\bar{\nabla}^\sigma\hat{\pi}$$

These curvature correction produce at most 2nd derivative term in equation of motion.
All higher derivative term are canceled due to the symmetric property

$$
\begin{aligned}
&\mathcal{A}_{\mu\nu\rho\sigma} = \mathcal{A}_{(\mu\nu)\rho\sigma} = \mathcal{A}_{\mu\nu(\rho\sigma)}, \\
&\mathcal{B}_{\mu\nu\rho\sigma\rho'\sigma'} = \mathcal{B}_{(\mu\nu)\rho\sigma\rho'\sigma'} = -\mathcal{B}_{\mu\nu\rho'\sigma\rho\sigma'} = -\mathcal{B}_{\mu\nu\rho\sigma'\rho'\sigma}, \\
&\mathcal{C}_{\lambda\mu\nu\rho\sigma} = \mathcal{C}_{(\lambda\mu\nu)\rho\sigma} = \mathcal{C}_{\lambda\mu\nu(\rho\sigma)}.
\end{aligned}
$$

We have confirmed this theory is BD ghost free !

Application 2: Generalized Higuchi bound

$$\mathcal{L}_2 = -\frac{1}{4}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\rho\sigma}\hat{h}_{\rho\sigma} - \frac{1}{2}\boxed{\left(\frac{3}{2}\bar{g}_{\mu\nu} - \frac{\bar{R}_{\mu\nu}}{m^2}\right)}\bar{\nabla}^\mu\hat{\pi}\bar{\nabla}^\nu\hat{\pi}$$

$$\boxed{\left(\frac{3}{2}\bar{g}_{\mu\nu} - \frac{\bar{R}_{\mu\nu}}{m^2}\right) < 0}$$

π itself become ghost!

$\underbrace{1}_{\text{ghost}} + \cancel{1}$ d.o.f

In order to avoid such a ghost instability, curvature scale of fiducial metric is constraint by graviton mass scale.

$$\boxed{\left(\frac{3}{2}\bar{g}_{\mu\nu} - \frac{\bar{R}_{\mu\nu}}{m^2}\right) > 0}$$

Generalized Higuchi bound

Summary

We extend the Stueckelberg analysis of dRGT massive gravity with FLAT fiducial metric to with GENERAL fiducial metric

Especially, by

modification 1 **defining the Stueckelberg fields covariantly,**
modification 2 **generalizing decoupling limit,**

we can

Result **write down the action in Stueckelberg language up to 4th order.**

As an application of this formalism, we succeed to

App. 1 **confirm the theory is free from BD ghost,**
App. 2 **generalize the Higuchi bound.**

Stueckelberg Analysis

Original theory

$S_{mass}[g_{\mu\nu}, \eta_{\mu\nu}]$

$$\phi^a = x^a$$
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

massive spin-2 $h_{\mu\nu}$: 5+1 d.o.f.

Introduce the Stuckelberg fields ϕ^a

Equivalent theory in different gauge

Stueckelberg formalism

$S_{mass}[g_{\mu\nu}, f_{\mu\nu}]$

$$f_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}$$

$$\phi^a = x^a - \pi^a$$
$$\pi^a = A^a + \partial^a \pi$$
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

massless spin-2 $h_{\mu\nu}$:2 d.o.f.
massless spin-1 A_μ :2 d.o.f.
massless spin-0 π :1+1 d.o.f

The existence of BD ghost is related ONLY π

Modification of Stueckelberg Analysis

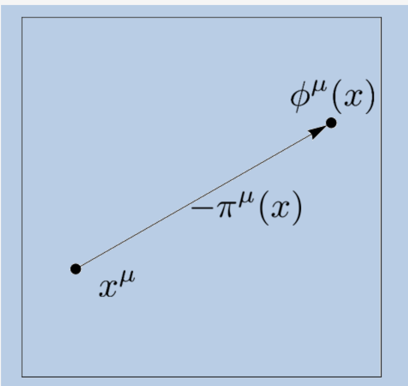
STEP2. Definition of Stueckelberg field

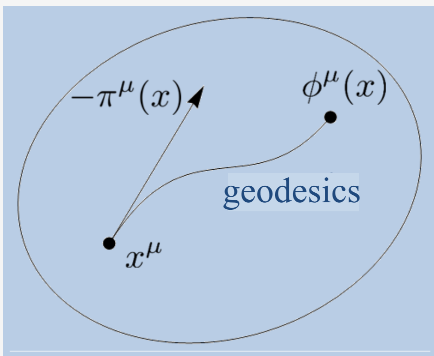
Flat case definition

$$\phi^a = x^a - \pi^a$$

new definition

$$\phi^\mu(x) = x^\mu - \pi^\mu - \frac{1}{2}\bar{\Gamma}^\mu_{\nu\rho}\pi^\nu\pi^\rho + \mathcal{O}(\pi^3)$$





“Dark matter in ghost-free bigravity theory”

Katsuki Aoki

[JGRG24(2014)111106]

Dark matter in ghost-free bigravity theory

JGRG24
Nov., 11th , 2014@Kavli IPMU

Waseda University,
Katsuki Aoki

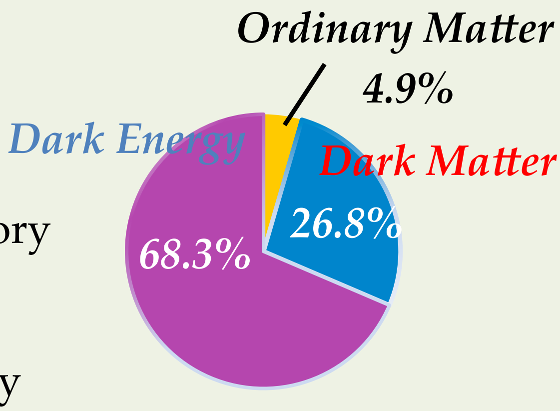
Based on

KA and K. Maeda, PRD 89, 064051 (2014).

KA and K. Maeda, arXiv: 1409. 0202.

Massless graviton or massive graviton?

- ✓ General Relativity
  Massless spin-2 field
- ✓ dRGT Massive gravity theory
  Massive spin-2 field
- ✓ HR Bigravity gravity theory
  Massless spin-2 and Massive spin-2 fields



Can the modification of gravity explain the biggest problem in modern cosmology?

$m \sim 10^{-33}\,\mathrm{eV} \sim Gpc^{-1} \Rightarrow$ Dark energy

$m \sim 10^{-27}\,\mathrm{eV} \sim kpc^{-1} \Rightarrow$ Dark matter?

Hassan-Rosen bigravity theory

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\mathcal{R}(f)$$
$$- \frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4} b_i \mathscr{U}_i(g,f) + S^{[\mathrm{m}]}$$
$$\kappa^2 = \kappa_g^2 + \kappa_f^2$$

Bigravity theory contains two metrics.

Physical matter Dark matter? Reappearance of ghost?

$$S^{[\mathrm{m}]} = \boxed{S_g^{[\mathrm{m}]}(g,\phi_g) + S_f^{[\mathrm{m}]}(f,\psi_f)} + S^{[\mathrm{m}]}(g,f,\psi_{\mathrm{double}})$$

Twin matters Doubly coupled matter

Can we interpret another matter field as a dark matter?

Homothetic solution

If two metrics are proportional, the equation of motion
is exactly same as GR with a cosmological constant.

$f_{\mu\nu} = K^2 g_{\mu\nu}$, $K = const \Rightarrow$ GR solution

$$G_{\mu\nu}(g) + \Lambda_g\, g_{\mu\nu} = \kappa_g^2 T^{[\mathrm{m}]}{}_{\mu\nu}\,,\qquad \Lambda_g(K) = K^2\Lambda_f(K)\,,$$
$$\mathcal{G}_{\mu\nu}(f) + \Lambda_f\, f_{\mu\nu} = \kappa_f^2 \mathcal{T}^{[\mathrm{m}]}{}_{\mu\nu} \qquad \text{with} \qquad \kappa_f^2\mathcal{T}^{[\mathrm{m}]}{}_{\mu\nu} = \kappa_g^2 T^{[\mathrm{m}]}{}_{\mu\nu}$$

$$\Lambda_g(K) = m^2\frac{\kappa_g^2}{\kappa^2}\left(b_0 + 3b_1K + 3b_2K^2 + b_3K^3\right)\,,$$
$$\Lambda_f(K) = m^2\frac{\kappa_f^2}{\kappa^2}\left(b_4 + 3b_3K^{-1} + 3b_2K^{-2} + b_1K^{-3}\right)$$

Minkowski, de Sitter and Anti-de Sitter spacetimes are also
vacuum solutions as homothetic solutions in bigravity.

Perturbation around homothetic background

The homothetic solution is obtained as an attractor in the context of cosmology (KA and K. Maeda 14′).

$$f_{\mu\nu} \neq K^2 g_{\mu\nu} \;\to\; f_{\mu\nu} \approx K^2 g_{\mu\nu}$$

The linear perturbation around homothetic background can be decomposed to massless and massive graviton modes.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h^{[g]}_{\mu\nu}, \qquad f_{\mu\nu} = \bar{f}_{\mu\nu} + K^2 h^{[f]}_{\mu\nu} \quad \Longrightarrow \quad h^{[-]}_{\mu\nu} = h^{[g]}_{\mu\nu} - h^{[f]}_{\mu\nu}, \qquad h^{[+]}_{\mu\nu} = \frac{m_f^2}{m_{\rm eff}^2} h^{[g]}_{\mu\nu} + \frac{m_g^2}{m_{\rm eff}^2} h^{[f]}_{\mu\nu}$$

Effective mass

$$m_{\rm eff}^2 = m_g^2 + m_f^2 \qquad m_g^2 := \frac{m^2 \kappa_g^2}{\kappa^2}(b_1 K + 2b_2 K^2 + b_3 K^3), \qquad m_f^2 := \frac{m^2 \kappa_f^2}{K^2 \kappa^2}(b_1 K + 2b_2 K^2 + b_3 K^3)$$

Basic idea

$$h^{[-]}_{\mu\nu} = h^{[g]}_{\mu\nu} - h^{[f]}_{\mu\nu}, \quad \Longleftarrow \text{ Massive mode = FP theory}$$

$$h^{[+]}_{\mu\nu} = \frac{m_f^2}{m_{\rm eff}^2} h^{[g]}_{\mu\nu} + \frac{m_g^2}{m_{\rm eff}^2} h^{[f]}_{\mu\nu} \quad \Longleftarrow \text{ Massless mode = GR}$$

Massless and massive modes couple to both twin matters.

Our spacetime is given by both massive and massless modes.

$$h^{[g]}_{\mu\nu} = h^{[+]}_{\mu\nu} + \frac{m_g^2}{m_{\rm eff}^2} h^{[-]}_{\mu\nu} \qquad\qquad h^{[g]}_{\mu\nu} \approx h^{[+]}_{\mu\nu}$$

Both massive and massless modes survive. | The massive mode decays. Only the massless mode survives.

$m_{\rm eff}^{-1}$ → scale

Gravitational potential on flat background

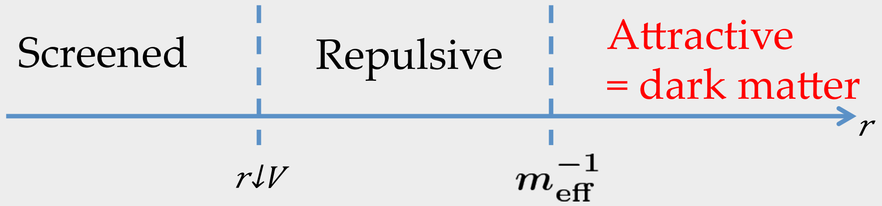
The gravitational potential is induced by f-matter field as well as g-matter field through the interaction terms.

✓ Outside Vainshtein radius

$$\Phi_g = -\frac{GM_g}{r}\left(\frac{m_f^2}{m_{\rm eff}^2} + \frac{4m_g^2}{3m_{\rm eff}^2}e^{-m_{\rm eff}\,r}\right) - \frac{m_g^2}{m_{\rm eff}^2}\frac{K^2\mathcal{G}\mathcal{M}_f}{r}\left(\underline{1-\frac{4}{3}e^{-m_{\rm eff}\,r}}\right)$$

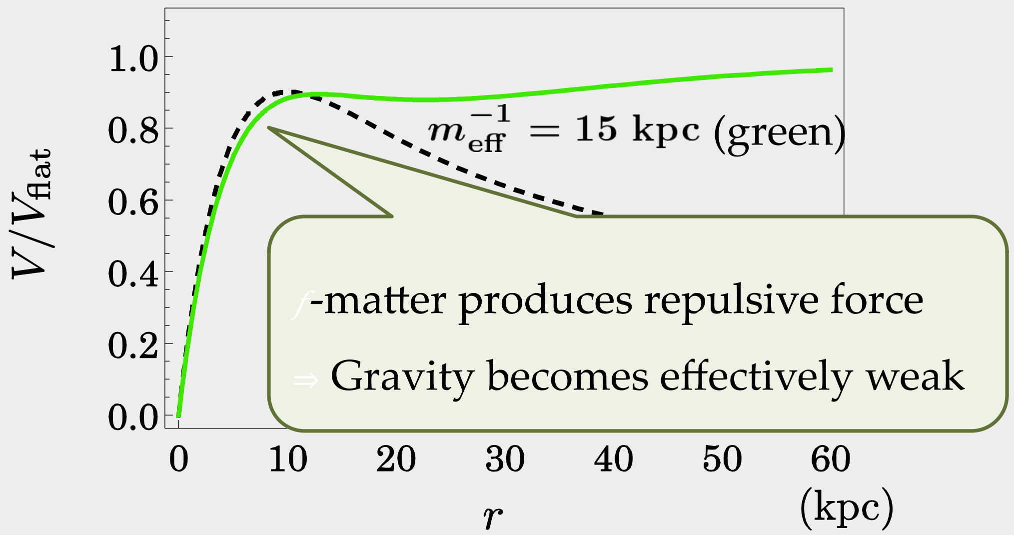
vDVZ discontinuity

repulsive force in $m_{\rm eff}\,r \ll 1$



$$r_V := \left(\frac{|GM_g - K^2\mathcal{G}\mathcal{M}_f|}{m_{\rm eff}^2}\right)^{1/3}$$

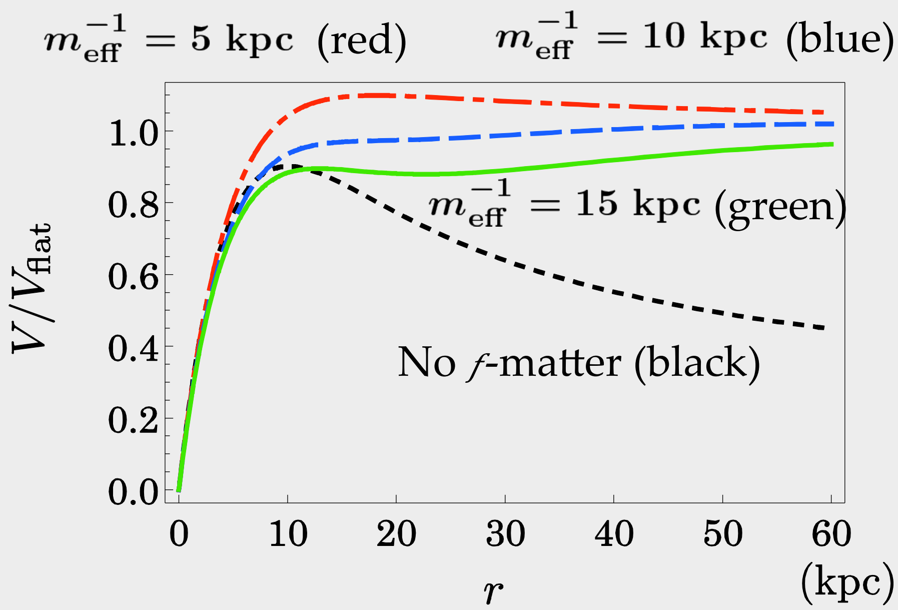
Rotation curve in galaxy



Vainshtein radius: $r_V \lesssim 0.1$ kpc

$$\rho_g \propto \exp[-r/r_{\rm gal}], \quad \rho_f \propto \frac{1}{1+(r/r_{\rm halo})^2} \quad (r_{\rm gal} = r_{\rm halo} = 3 \text{ kpc})$$

Rotation curve in galaxy



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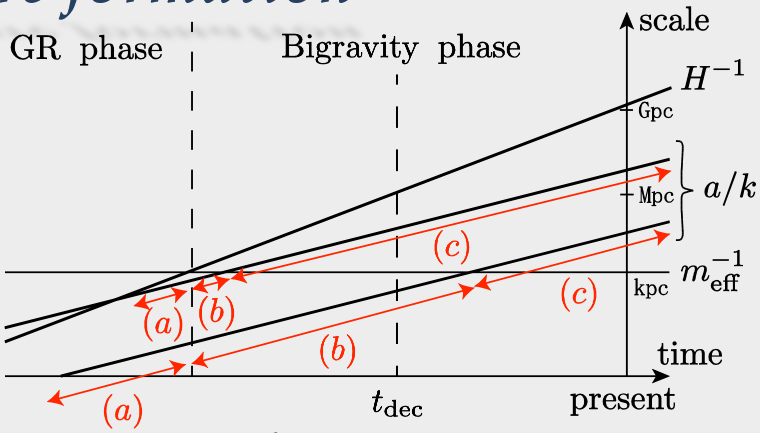
$$\rho_g \propto \exp[-r/r_{\rm gal}], \quad \rho_f \propto \frac{1}{1+(r/r_{\rm halo})^2} \quad (r_{\rm gal} = r_{\rm halo} = 3 \text{ kpc})$$

Structure formation



$(a)\ a/k \ll H^{-1} \ll m_{\rm eff}^{-1}$

The evolutions restore to GR like Vainshtein screening

$(b)\ a/k \ll m_{\rm eff}^{-1} \ll H^{-1}$

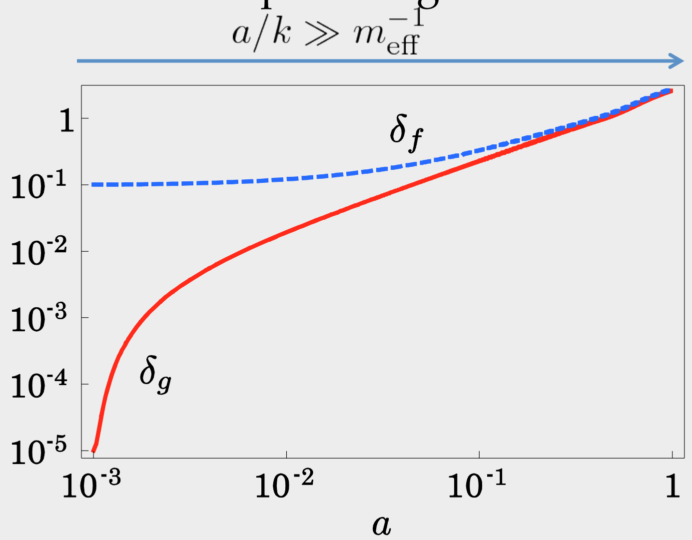
The f-matter produces repulsive force

$(c)\ m_{\rm eff}^{-1} \ll a/k \ll H^{-1}$

The f-matter acts as ordinary dark matter

Growth history of large-scale structure

outside Compton length



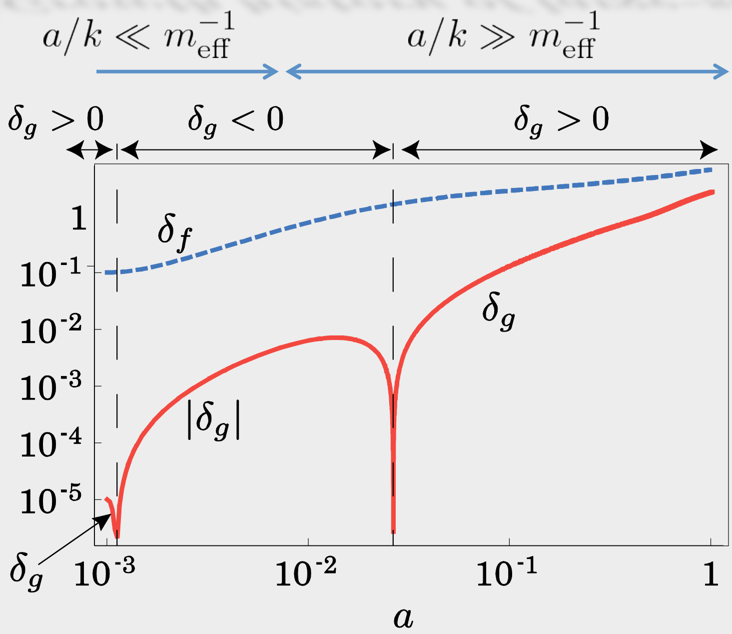
$$k^{-1} = 10 \text{ Mpc}$$

$$\rho_g = \bar{\rho}_g(1+\delta_g),$$
$$\rho_f = \bar{\rho}_f(1+\delta_f)$$

Background:
dust dominant universe

For large scale perturbation, the evolution of physical matter perturbation is similar to CDM model in GR.

Growth history of large-scale structure

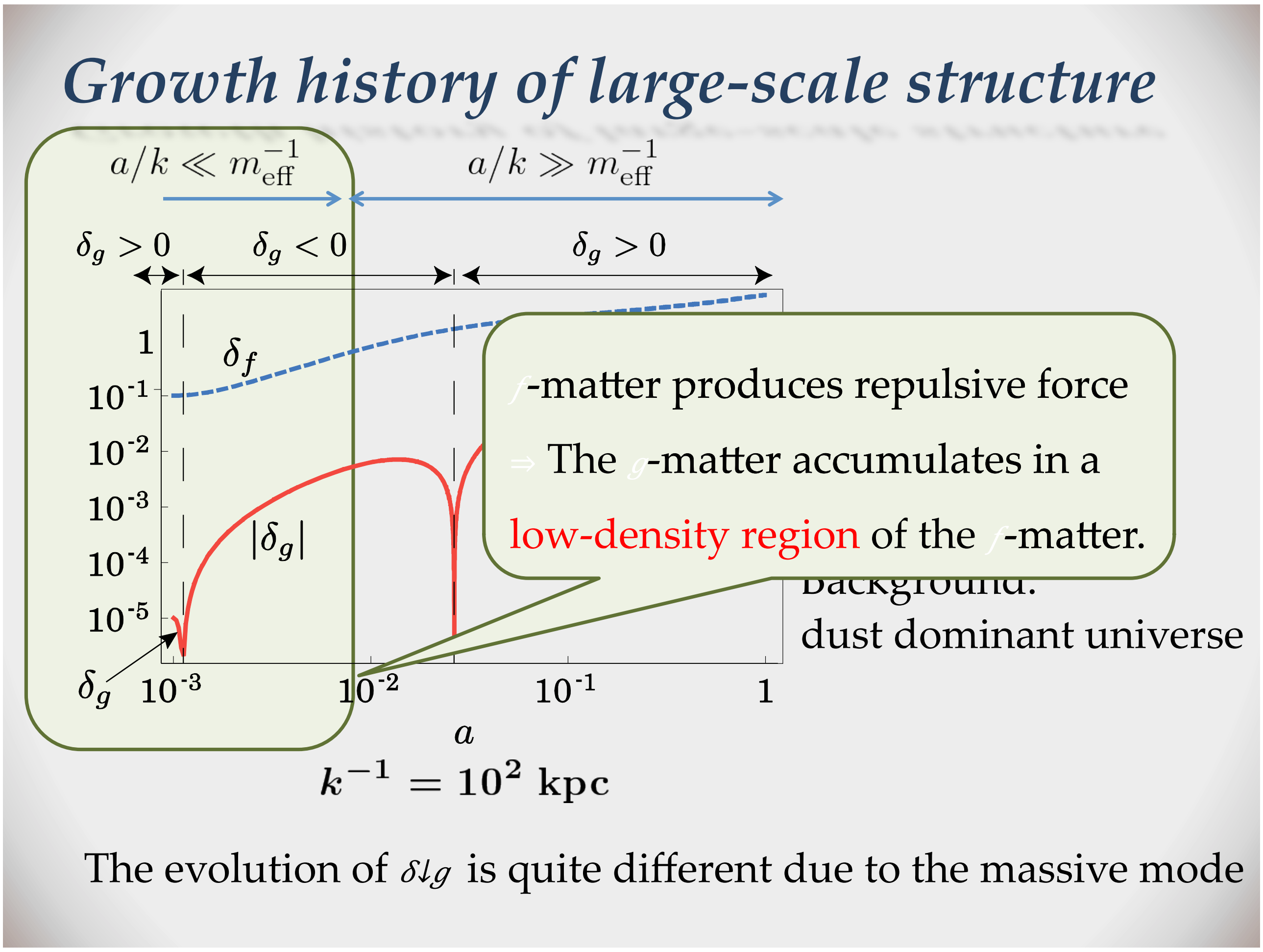


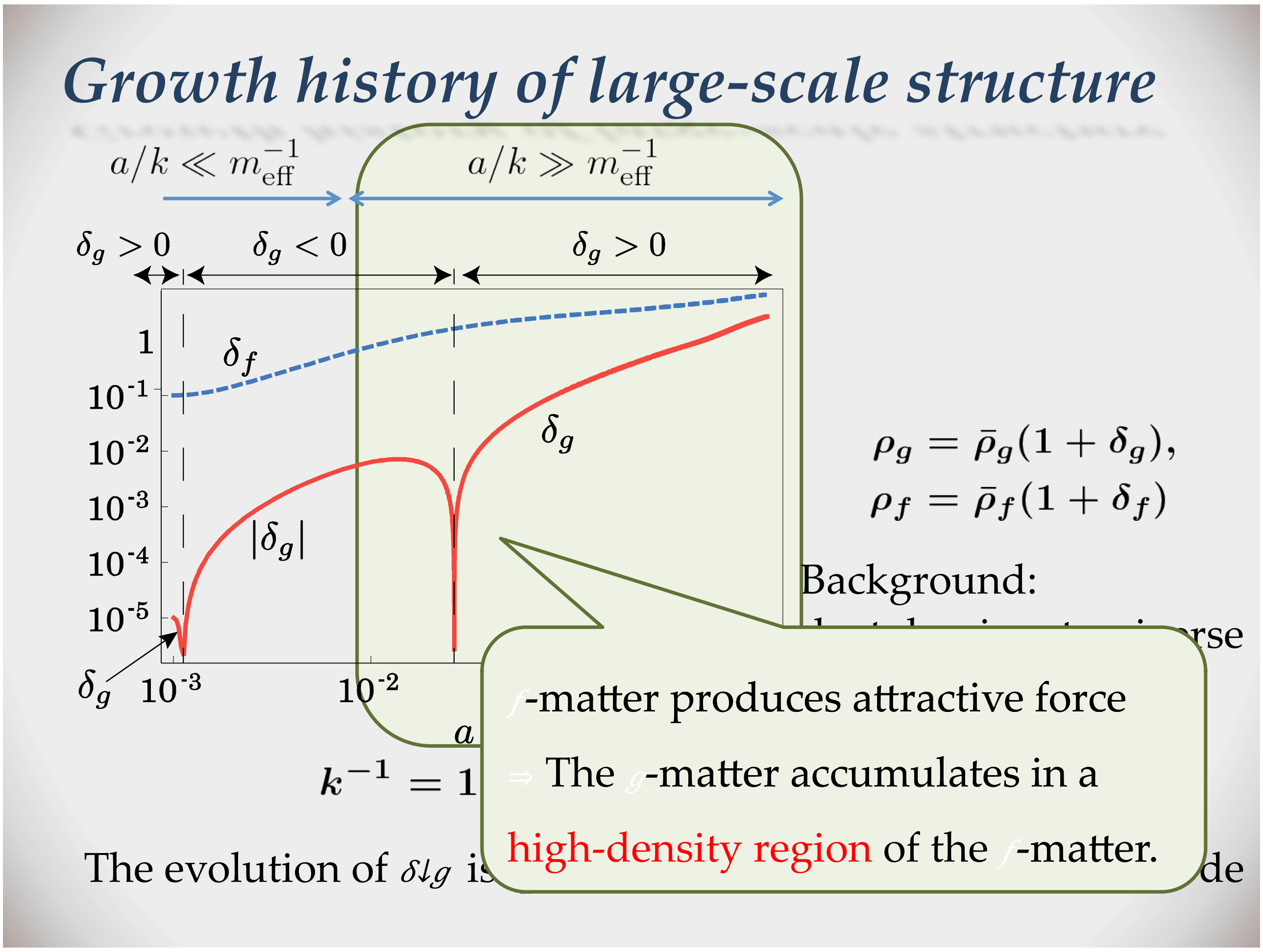
$$k^{-1} = 10^2 \text{ kpc}$$

$$\rho_g = \bar{\rho}_g(1+\delta_g),$$
$$\rho_f = \bar{\rho}_f(1+\delta_f)$$

Background:
dust dominant universe

The evolution of δ_g is quite different due to the massive mode


Growth history of large-scale structure
$a/k \ll m_{\rm eff}^{-1}$
$a/k \gg m_{\rm eff}^{-1}$
$\delta_g > 0$
$\delta_g < 0$
$\delta_g > 0$
δ_f
$|\delta_g|$
δ_g
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-3}
10^{-2}
10^{-1}
1
a
$k^{-1} = 10^2$ kpc
f-matter produces repulsive force
$\Rightarrow$ The g-matter accumulates in a
low-density region of the f-matter.
Background:
dust dominant universe
The evolution of δ_g is quite different due to the massive mode


Growth history of large-scale structure
$a/k \ll m_{\rm eff}^{-1}$
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$\delta_g > 0$
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δ_f
$|\delta_g|$
δ_g
δ_g
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-3}
10^{-2}
a
$k^{-1} = 1$
$\rho_g = \bar{\rho}_g(1+\delta_g),$
$\rho_f = \bar{\rho}_f(1+\delta_f)$
Background:
f-matter produces attractive force
$\Rightarrow$ The g-matter accumulates in a
high-density region of the f-matter.
The evolution of δ_g is

Summary

- ✓ Another one of twin matters can be candidate of dark matter

$$m \gtrsim 10^{-27}\,\mathrm{eV} \sim kpc^{-1} \Rightarrow \text{Dark matter}$$

- ✓ There are two important scales:
  Compton wavelength and Screening scale

small

The phenomena are restored to GR and we can not see the effect of f-matter.

$\boxed{\text{Screening scale}}$

There are some changes from GR.

$\boxed{\text{Compton wavelength}}$

The phenomena of bigravity with twin matters are similar to GR with CDM.

large

Can graviton have a mass?

Fierz-Pauli massive gravity (1939)

⇨ physical metric $g_{\mu\nu}$ = background $\eta_{\mu\nu}$ + perturbation $h_{\mu\nu}$

$S_{gravity}=S_{EH}(g)+S_{FP}(\eta,h)$

de Rham-Gabadadze-Tolley massive gravity (2011)

⇨ physical metric $g_{\mu\nu}$ & fiducial metric $f_{\mu\nu}$

$S_{gravity}=S_{EH}(g)+S_{NL}(g,f)$

Hassan-Rosen Bigravity (2011)

⇨ physical metric $g_{\mu\nu}$ & another dynamical metric $f_{\mu\nu}$

$S_{gravity}=S_{EH}(g)+S_{EH}(f)+S_{NL}(g,f)$

Small cosmological constant and large mass

$$\Lambda_g(K) = m^2\frac{\kappa_g^2}{\kappa^2}\left(b_0+3b_1K+3b_2K^2+b_3K^3\right),$$

$$\Lambda_f(K) = m^2\frac{\kappa_f^2}{\kappa^2}\left(b_4+3b_3K^{-1}+3b_2K^{-2}+b_1K^{-3}\right)$$

with $\Lambda_g(K) = K^2\Lambda_f(K)$,

$$m_{\rm eff}^2 = m_g^2 + m_f^2 \qquad m_g^2 := \frac{m^2\kappa_g^2}{\kappa^2}(b_1K+2b_2K^2+b_3K^3),$$

$$m_f^2 := \frac{m^2\kappa_f^2}{K^2\kappa^2}(b_1K+2b_2K^2+b_3K^3)$$

κ_g^2/κ_f^2	$2c_3^2+3c_4$	$K_{\rm dS}$	$\Lambda_g/m_{\rm eff}^2$
1	1	5.08	0.0815
10^{-12}	1	8.85	5.11×10^{-11}
1	10^{-12}	4.00	9.34×10^{-14}
10^{-6}	10^{-6}	4.00	8.10×10^{-11}

Scale dependence of the another matter effect

small scale

The gravity is produced by only physical matter

$r_V \lesssim \mathbf{0.1\ kpc}$

The gravitational force becomes effectively weak

$m_{\rm eff}^{-1} \sim \mathbf{kpc}$

The f-matter behaves like ordinary dark matter

Galactic disk

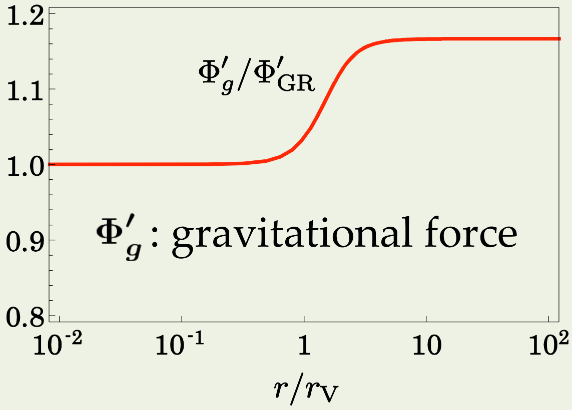
large scale

Gravitational potential on flat background

T

a

✓



The effect of f-matter is screened inside Vainshtein radius.

ity

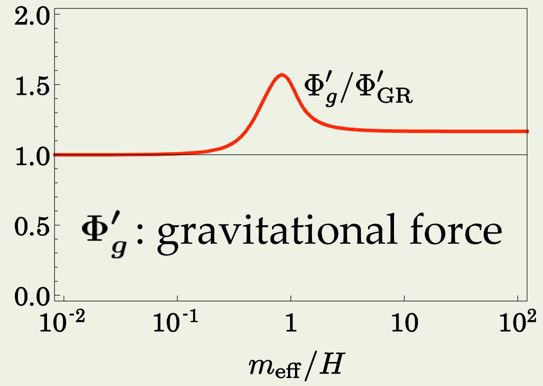
$$r_V := \left(\frac{|GM_g - K^2\mathcal{G}\mathcal{M}_f|}{m_{\rm eff}^2}\right)^{1/3} \ll 1$$

Screened | Repulsive | Attractive = dark matter

r

$r{\downarrow}V$ $m_{\rm eff}^{-1}$

Structure formation



The effect of f-matter is screened in early universe.

$(a)\ a/k \ll H^{-1} \ll m_{\rm eff}^{-1}$

The evolutions restore to GR like Vainshtein screening

$(b)\ a/k \ll m_{\rm eff}^{-1} \ll H^{-1}$

The f-matter produces repulsive force

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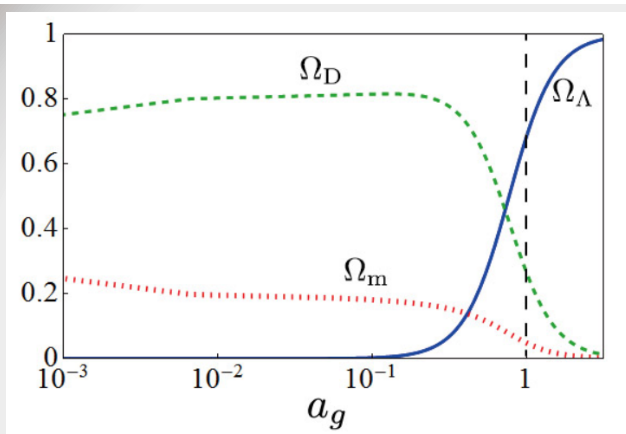
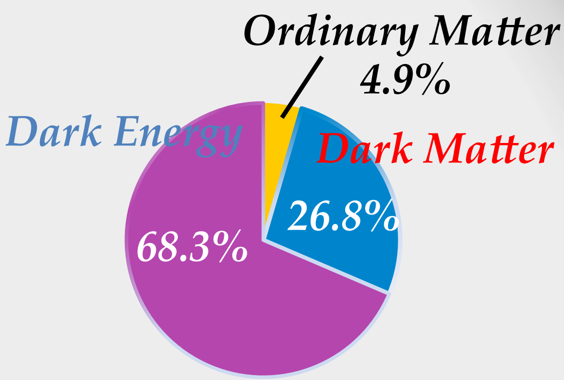


m↓eff≫H↓g↑



$$H_g^2 + \frac{K}{a_g^2} = \frac{\Lambda_g}{3} + \frac{\kappa_{\rm eff}^2}{3}\left[\rho_g + \rho_{\rm D}\right]$$

$$\kappa_{\rm eff}^2 = \kappa_g^2\left[1 - \frac{3m_g^2}{3m_{\rm eff}^2 - 2\Lambda_g}\right] \qquad \rho_{\rm D} = \frac{3m_f^2}{3m_f^2 - 2\Lambda_g} K^4 \rho_f$$

$$\rho_f \propto a_f^{-3} \propto a_g^{-3} + \mathcal{O}(a_g^{-6})$$

The f matter behaves like dark matter component on $g↓\mu\nu$.

“Tensor Spectrum in Bimetric Gravity”

Yuki Sakakihara

[JGRG24(2014)111107]

2014/11/11 JGRG24 @IPMU 11:45-12:00

Tensor Spectrum in Bimetrc Gravity

Yuki Sakakihara (Kyoto University)

This research is collaborated with Jiro Soda (Kobe University)

Massive graviton

Does the graviton have its mass? How many species does it have?
We know few about the graviton...

Suppose there are two (or more) gravitons...

- In order to realize 1/r gravitational force,
  at least, one of them should be sufficiently light.

~~2 interacting massless graviton~~

1 massless graviton
+ 1 massive graviton

We can realize such a theory
with two metrics interacting with each other.

Bimetric Gravity

(de Rham et. al., 2011, Hassan and Rosen, 2012)

two metrics
- $g_{\mu\nu}$: physical metric
- $f_{\mu\nu}$: the other metric

In order that the theory has stable solutions, the form of the interaction terms are restricted. (They include five theoretical parameters.)

minimal bimetric model

$$m^2 M_e^2 \int d^4x \, \frac{1}{2}\sqrt{-g}\left(L^\mu_\nu L^\nu_\mu - (L^\mu_\mu)^2\right)$$

$$L^\mu_\nu := \delta^\mu_\nu - \sqrt{(g^{\mu\lambda} f_{\lambda\nu})}$$

$$\frac{1}{M_e^2} = \frac{1}{M_g^2} + \frac{1}{M_f^2}$$

Inflation in bimetric gravity

If the other metric exists,
do some problems happen?
How can we see the effects on observations?

For example, about inflation

- Can we construct inflating solutions with a inflaton as in the case of GR? → Yes, we can.
- Are they stable solutions? → One branch of the solutions is guaranteed to be stable. (YS et al 2013)
- What is the feature of the gravitational waves generated during inflation?

Inflation in bimetric gravity

If the other metric exists,
do some problems happen?
How can we see the effects on observations?

For example, about inflation

- Can we construct inflating solutions with a inflaton as in the case of GR? → Yes, we can.
- Are they stable solutions? → One branch of the solutions is guaranteed to be stable. (YS et al 2013)
- What is the feature of the gravitational wave generated during inflation?

Today's topic

Bimetric gravity (+ inflaton) action

$$S = \underbrace{\frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g_{\mu\nu}]}_{\text{kinetic terms of physical metric}} + \underbrace{\int d^4x\sqrt{-g}\Big(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\varphi - V[\varphi]\Big)}_{\text{scalar field (inflaton)}}$$

$$+\underbrace{\frac{M_f^2}{2}\int d^4x\sqrt{-f}R[f_{\mu\nu}]}_{\text{kinetic terms of the other metric}} + \underbrace{m^2 M_e^2\int d^4x\,\frac{1}{2}\sqrt{-g}\big(L^\mu_\nu L^\nu_\mu - (L^\mu_\mu)^2\big)}_{\text{interaction terms of the metrics}}$$

$$L^\mu_\nu := \delta^\mu_\nu - \sqrt{(g^{\mu\lambda}f_{\lambda\nu})}$$

$$\frac{1}{M_e^2} = \frac{1}{M_g^2} + \frac{1}{M_f^2}$$

Homogeneous isotropic solutions

(1) Substitute the homogeneous isotropic ansatz into the action

$$g_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + e^{2\alpha(t)}(dx^2+dy^2+dz^2)$$
$$f_{\mu\nu}dx^\mu dx^\nu = -M^2(t)dt^2 + e^{2\beta(t)}(dx^2+dy^2+dz^2)$$
$$\varphi = \varphi(t)$$

(2) Variational principle $\rightarrow$ 3 equations of motion and 2 constrains.

The time derivative of these constraints gives a relation between the lapse functions.

$$\Rightarrow M = \zeta\epsilon N$$

$\zeta := \frac{\mathrm{d}\beta}{\mathrm{d}\alpha}$: the ratio of expansion rates

$\epsilon := e^{\beta-\alpha}$: the ratio of scale factors

(3) We obtain several branches of the solutions

$\rightarrow$ The only one branch is stable, in which epsilon has the value from 0 to 1 .

Slow-roll approximation

$H := \dot{\alpha}$

$\zeta := \frac{\mathrm{d}\beta}{\mathrm{d}\alpha}$

$\epsilon := e^{\beta-\alpha}$

(1) Slow-roll limit (de Sitter)

- H = const.
- ϵ = const.
- $\zeta = 1$

(2) The first order of slow-roll approximation

Slow-roll parameter $s := -\frac{\dot{H}}{H^2}$ $s \ll 1$ We neglect $\mathcal{O}(s^2)$, $\dot{s}$

- $\dot{\varphi}^2 = 2M_g^2 sH^2\left(1 + \underline{\frac{M_f\epsilon^2(3-2\epsilon)}{M_f}}\right)$
- $\delta\zeta := \zeta - 1 = \mathcal{O}(s)$ difference from GR
- $\dot{\epsilon} = \epsilon H\delta\zeta = \mathcal{O}(s)$ ϵ is time dependent in this order.

Tensor perturbation

$\delta g_{ij} = q_{ij}$, $\delta f_{ij} = p_{ij}$ satisfy TT conditions: $q^i{}_{j|i} = 0$, $q^i{}_i = 0$, $p^i{}_{j|i} = 0$, $p^i{}_i = 0$

Flavor eigen state (g and f) $\delta^2 \mathcal{L}_{\rm int} \propto (p-q)^2$ do not vanish in the slow-roll limit.

Rotation

$$\begin{pmatrix} q \\ p \end{pmatrix} = \frac{1}{(\kappa^2+\epsilon^2)^{1/2}} \begin{pmatrix} \kappa & -\epsilon \\ \kappa & \kappa^2/\epsilon \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

where $\kappa := \sqrt{\zeta M_g / M_f}$

Mass eigen state (x and y) Cross terms vanish in the slow-roll limit.

$\delta^2 \mathcal{L} =$ massless part (x) + massive part (y) + order s cross terms

x: orthogonal to y   y: proportional to (p-q)

We can obtain analytic solutions in the slow-roll limit
and construct higher order solutions order by order.

Tensor Spectra in the mass eigen state

Subscripts 0 mean the values in the slow-roll limit.

In the first order of the slow-roll parameter, …

- $$\langle xx \rangle = \left(\frac{H_0}{\pi M_g}\right)^2 (-\eta)^{3+2s-2\nu_X} \left(\frac{k}{2}\right)^{3-2\nu_X} \left(\frac{\Gamma(\nu_X)}{\Gamma(\frac{3}{2})}\right)^2 \left(1 + \frac{4s\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)^{-\nu_X} H_0^{2s}$$

$\boxed{\sim \text{const.}}$

where $\nu_X = \frac{3}{2} + s\left(1 + \frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)$

- $\langle xy \rangle \propto \boxed{\frac{1}{e^{3\alpha}} \to 0}$
- $\langle yy \rangle \propto \boxed{\frac{1}{e^{3\alpha}} \to 0}$

They are negligible compared with $\langle xx \rangle$

Tensor Spectra in the flavor eigen state

From the relation $\begin{pmatrix} q \\ p \end{pmatrix} = \frac{1}{(\kappa^2+\epsilon^2)^{1/2}} \begin{pmatrix} \kappa & -\epsilon \\ \kappa & \kappa^2/\epsilon \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, where $\kappa := \sqrt{\zeta M_g/M_f}$

$$\langle qq \rangle = \langle qp \rangle = \langle pp \rangle = \frac{\kappa^2}{\kappa^2+\epsilon^2}\langle xx \rangle + \mathcal{O}(e^{-3\alpha})$$

$$= \frac{\kappa^2}{\kappa^2+\epsilon^2}\left(\frac{H_0}{\pi M_g}\right)^2 (-H_0\eta)^{-2s\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}} \left(\frac{k}{2H_0}\right)^{-2s\left(1+\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)} \left(\frac{\Gamma(\nu_X)}{\Gamma(\frac{3}{2})}\right)^2 \left(1 - \frac{6s\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)$$

Tensor Spectra in the flavor eigen state

From the relation $\begin{pmatrix} q \\ p \end{pmatrix} = \frac{1}{(\kappa^2+\epsilon^2)^{1/2}} \begin{pmatrix} \kappa & -\epsilon \\ \kappa & \kappa^2/\epsilon \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, where $\kappa := \sqrt{\zeta M_g/M_f}$

$$\langle qq \rangle = \langle qp \rangle = \langle pp \rangle = \frac{\kappa^2}{\kappa^2+\epsilon^2}\langle xx \rangle + \mathcal{O}(e^{-3\alpha})$$

$$= \frac{\kappa^2}{\kappa^2+\epsilon^2}\left(\frac{H_0}{\pi M_g}\right)^2 (-H_0\eta)^{-2s\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}} \left(\frac{k}{2H_0}\right)^{-2s\left(1+\frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)} \left(\frac{\Gamma(\nu_X)}{\Gamma(\frac{3}{2})}\right)^2 \left(1 - \frac{6s\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)$$

Features

- Tensor amplitudes are suppressed due to the mixing in the flavor eigen state.

  $\langle qq \rangle = \langle qp \rangle = \langle pp \rangle = \frac{\kappa_0^2}{\kappa_0^2+\epsilon_0^2}\left(\frac{H_0}{\pi M_g}\right)^2$ (in the lowest order)

- The amplitudes are conserved in the first order of slow-roll approx.. $\frac{d\log\langle q^2 \rangle}{d\eta} = 0$

- spectral index $n_T = -2s\left(1 + \frac{2\epsilon_0^2(1-\epsilon_0)}{\kappa_0^2+\epsilon_0^2}\right)$

Future work

- Relation to observational values …How about the scalar tensor ratio?

  ➡ Calculation of scalar perturbations

- If we consider $m^2 \ll V/3M_g^2$ situation, (de Felice et al, 2014)
  this solution will suffer gradient instability in the radiation dominant era.

  ➡ Since we have thought only about a minimal bimetric model,
  the extension to more general model may circumvent this instability.

- The tensor perturbations of the other metric couple to the scalar field through ϵ and ζ .

$$\zeta = 1 + \frac{\dot{\varphi}^2}{M_g^2[m_{\rm eff}^2 - 2H^2]}$$

  ➡ Parametric resonance may happen in the preheating era.

  ➡ Enhancement of the physical tensor amplitude
  though the mixing terms

“Detectability of bi-gravity with graviton oscillations using gravitational wave observations”

Tatsuya Narikawa

[JGRG24(2014)111108]

JGRG24, Nov 11, 2014

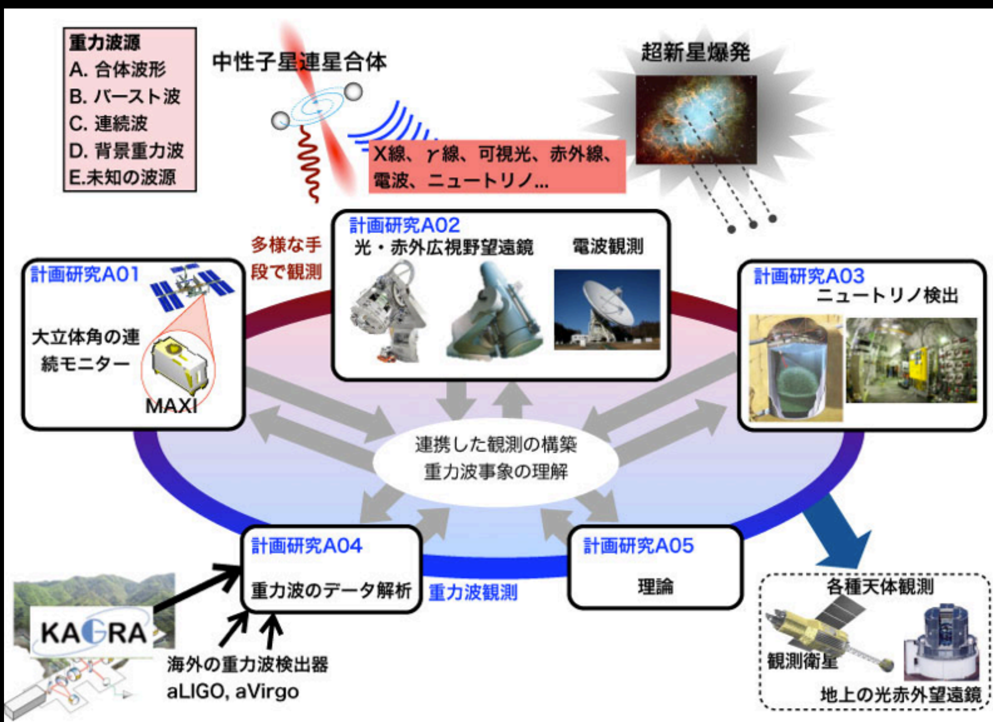
Detectability of bi-gravity with graviton oscillations using gravitational wave observations

Tatsuya Narikawa (Osaka U)
with
K. Ueno, H. Tagoshi, T. Tanaka, N. Kanda, T. Nakamura

Outline
- I) Graviton oscillations
- II) Bayesian model selection for GW
- III) A detectable region of bi-gravity

1

``Data Analysis'' sub-group in ``Grant-in-Aid for Scientific Research on Innovative Area - New Developments in Astrophysics Through Multi-Messenger Observation of Gravitational Waves Sources-''



2

Gravitational waves will be detected within a few years.

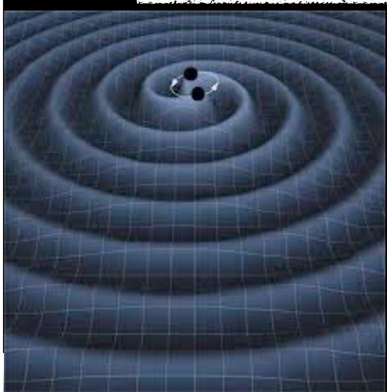
(1) The advanced ground-based laser interferometers, such as aLIGO, aVirgo, KAGRA will be full operation.

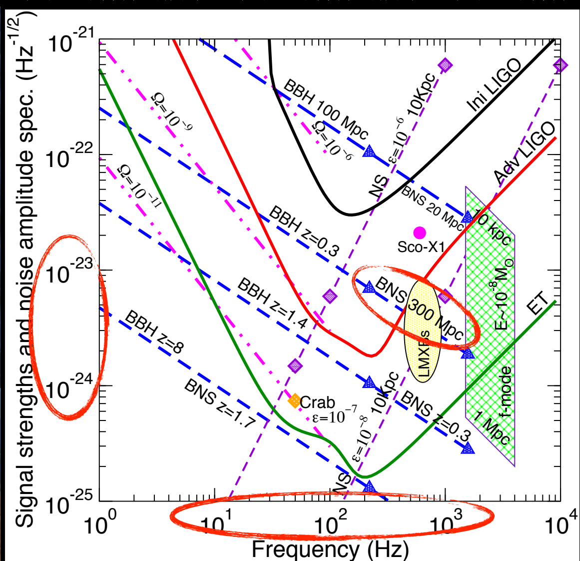
[Flaminio will review it.]

http://www.nature.com/news/physics-wave-of-the-future-1.15561

Gravitational waves will be detected within a few years.

(2) Predicted event rate of GWs from compact binaries is ~10 events a year, within ~ a few 100Mpc.





[Sathyaprakash & Schutz (2009)]

Sensitivity band
10Hz<f<1000Hz

Parameter estimation and Model selection

Once a detection candidate of GW will be identified, the next step is to extract full information of the source parameters.

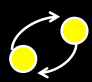
[mass, distance, time, sky location, spin, ...]

Testing gravity is also one of important themes.

[model selection: Modified gravity (MG) vs GR]

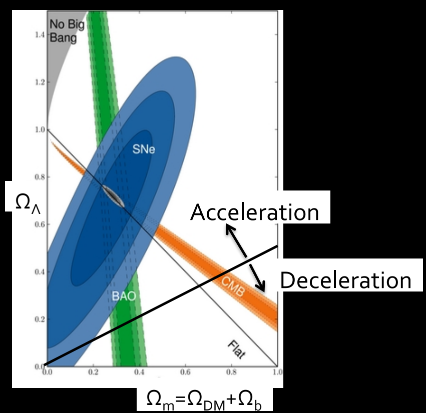
GWs will be powerful probes of strong-field, dynamical aspect of gravity.

$$\epsilon \equiv \frac{v^2}{c^2} = \frac{2GM}{Rc^2} \sim 1 \qquad v/c \sim 1$$

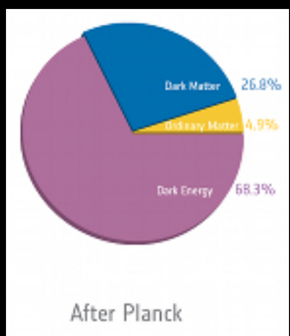


Why alternative theories of gravity?

as an alternative to dark energy



[Suzuki *et al.*, 1105.3470]



$$\ddot{a} > 0$$

$H_0 \sim 10^{-33}$eV

Observations of the SNe, the CMB, and the BAO consistently suggest the current cosmic acceleration.
However, the origin is unknown.

Why Bi-gravity?

Massive gravity

[de Rham's review]
[Mukohyama-san's review JGRG22]

Can graviton have mass? $m_g \sim H_0$?
May lead to acceleration without dark energy

Consistent theory found in 2010 [dRGT] but does not have a suitable FLRW background solution.

In the case of bi-gravity, we have two gravitons.

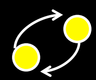
assuming matter interacts only with g [Hassan & Rosen 2011]

The double spatially flat FLRW background [Comelli, et al. 2011]

Owing to the Vainshtein screening, almost the same prediction as GR in the weak filed.

However, the gravitational waveforms differ from those of GR, due to graviton oscillations.

Motivation



To investigate the detectability of the corrections to gravitational waveforms from compact binaries due to graviton oscillations.

Propagation of the GWs in bi-gravity

Short wavelength approximation (k>>m>>H)

$$\begin{cases} \ddot{h} - \Delta h + m^2\Gamma_c(h - \tilde{h}) = 0 \\ \ddot{\tilde{h}} - \tilde{c}^2\Delta\tilde{h} + \dfrac{m^2\Gamma_c\tilde{c}}{\kappa\xi_c^2}(\tilde{h} - h) = 0 \end{cases}$$

[Comelli, et al. 2012]

Propagation modes: h_1 and h_2.

The observed signal in the frequency-domain

Inspiral waveform

[De Falice, Nakamura, Tanaka, 2014]

$$h(f) = A(f)e^{i\Phi(f)}\left[B_1 e^{i\delta\Phi_1(f)} + B_2 e^{i\delta\Phi_2(f)}\right]$$

h and ¥tilde h interfere during propagation.: Graviton oscillations

In this talk, we do not consider the relation between c-1 and μ.

9

Effect of Bi-gravity on GW [De Felice, Nakamura, Tanaka, 1304.3920]

The GWs differ from those of GR, due to graviton oscillations.

Waveforms:

$$h(f) = h_{\mathrm{GR}}(f)\left[B_1 e^{i\delta\Phi_1(f)} + B_2 e^{i\delta\Phi_2(f)}\right]$$

where Phase corrections:

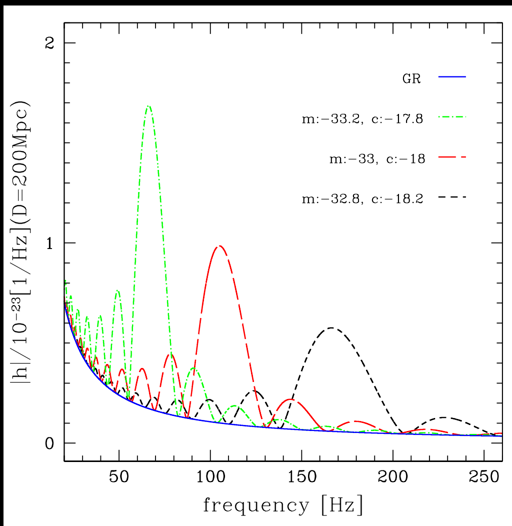
$$\delta\Phi_{1,2} = -\frac{\mu D\sqrt{\tilde{c}-1}}{2\sqrt{2x}}\left(1 + x \mp \sqrt{1 + x^2 + 2x\frac{1-\kappa\xi_c^2}{1+\kappa\xi_c^2}}\right)$$

Degrees of mixing:

$$\begin{aligned} B_1 &= \cos\theta_g(\cos\theta_g + \sqrt{\kappa}\xi_c\sin\theta_g) \\ B_2 &= \sin\theta_g(\sin\theta_g - \sqrt{\kappa}\xi_c\cos\theta_g) \end{aligned}$$

with $\theta_g = \frac{1}{2}\cot^{-1}\left(\frac{1+\kappa\xi_c^2}{2\sqrt{\kappa}\xi_c}x + \frac{1-\kappa\xi_c^2}{2\sqrt{\kappa}\xi_c}\right)$

$$x \equiv \frac{2(2\pi f)^2(\tilde{c}-1)}{\mu^2}$$



x~1 @peaks

|h| is enhanced at x~1.

μ: effective graviton mass

$\tilde{c}$-1: speed of $\tilde{h}$

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Parameter Estimation of GWs using Bayesian statistics

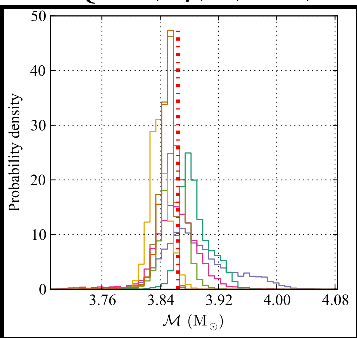
Bayes' theorem   posterior ∝ prior × likelihood

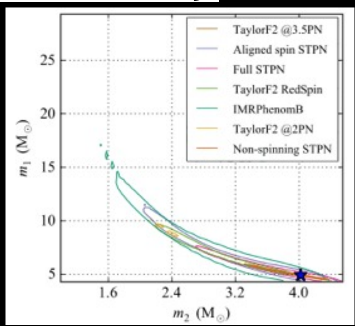
$$p(\theta|d,H) = \frac{p(\theta|H)p(d|\theta,H)}{p(d|H)}$$

H: hypothesis (GW signal embedded in data)
d: data (d=h+n)

GW waveform of non-spinning CBC with 9 parameters

$$\vec{\theta} = \{\mathcal{M}, \eta, \iota, d_L, t_c, \phi_c, \alpha, \delta, \psi\}$$





Results using LALInference

[LIGO-Virgo, PRD88 (2013) 062001 [arXiv: 1304.1775]]

Bayesian model selection

Which model better describes the data?

The odds ratio and the Bayes factor are useful for model selection.

$$\mathcal{O}_{\mathrm{MG,GR}} = \frac{p(\mathrm{MG}|d)}{p(\mathrm{GR}|d)} = \frac{p(\mathrm{MG})}{p(\mathrm{GR})}\frac{p(d|\mathrm{MG})}{p(d|\mathrm{GR})} \equiv \frac{p(\mathrm{MG})}{p(\mathrm{GR})} B_{\mathrm{MG,GR}}$$

The Bayes factor is the ratio of marginalized likelihoods of hypotheses.

The marginalized likelihood:

$$p(d|H) \equiv \int d\theta\, p(d|\theta,H)\, p(\theta|H)$$

is computationally expensive

In GW data analysis, the integrand is the noise weighted integral of the data and the model waveform given θ

$$p(d|\theta,H) \propto \exp[-(d-h(\theta)|d-h(\theta))/2]$$

"confidence" levels of B_{XY}

B_{XY}	$2\log B_{XY}$	Evidence for model X
<1	<0	Negative (supports model Y)
1 to 3	0 to 2	Not worth more than a bare mention
3 to 12	2 to 5	Positive
12 to 150	5 to 10	Strong
>150	>10	Very Strong

[Cornish & Littenberg 0704.1808]

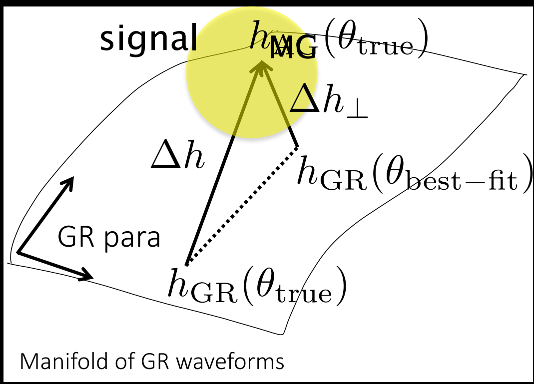
A unified model-selection based on Bayesian inference

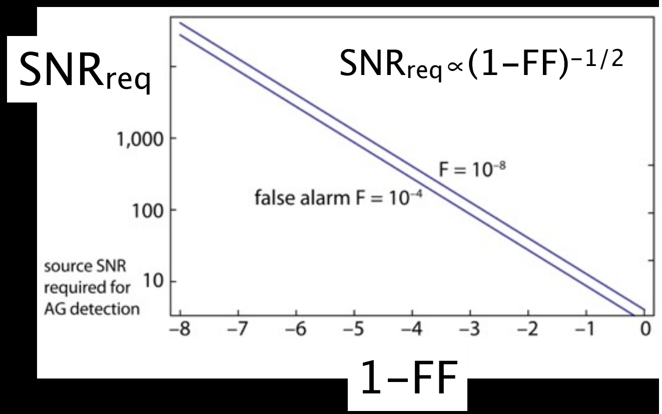
[Cornish+, 1105.2088; Vallisneri, 1207.4759; Del Pozzo+, 1408.2356]

Assumptions: large SNR, FF~1,...

$$\log \mathrm{BF} \sim \frac{1}{2}(1-\mathrm{FF}^2)\mathrm{SNR}^2 + \mathcal{O}[(1-\mathrm{FF}^2)^2]$$

$O_{AG,GR} > O_{thr}$ for a FAP → SNR_{req} for MG detection.





$$\mathrm{FF}(\theta_{\mathrm{MG}}) = \max_{\theta_{\mathrm{GR}}} \frac{(h_{\mathrm{GR}}(\theta_{\mathrm{GR}})|h_{\mathrm{MG}}(\theta_{\mathrm{MG}}))}{|h_{\mathrm{GR}}(\theta_{\mathrm{GR}})||h_{\mathrm{MG}}(\theta_{\mathrm{MG}})|}$$

$SNR_{res} = SNR(1-FF)^{1/2}$

SNRreq : required SNR for detection of deviation from GR [Vallisneri, 2012]

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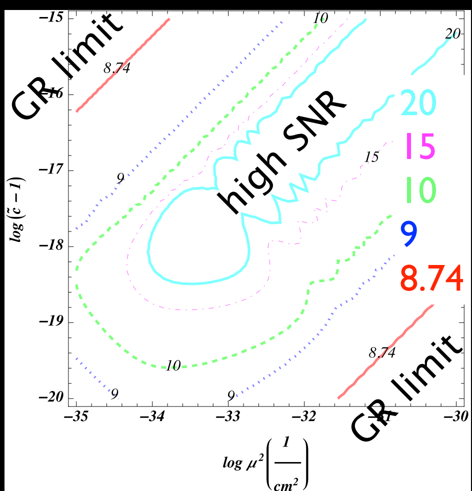
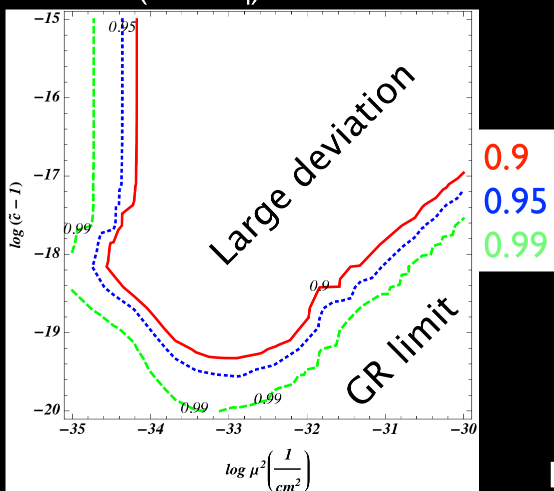
SNR contour vs FF (SNR_{req})

Point: f_{peak} within detector bandwidth, 10Hz<f<1000Hz

SNR contour



FF (SNR_{req}) contour



Source: BNS, 200Mpc
$\kappa\xi_c^2=100$

$$\mathrm{SNR} = |h| = \sqrt{(h|h)}.$$

$$(h_{\mathrm{A}}|h_{\mathrm{B}}) \equiv 4\mathrm{Re}\int_{f_{\min}}^{f_{\max}} \frac{h_{\mathrm{A}}(f)h_{\mathrm{B}}(f)}{S_n(f)}df$$

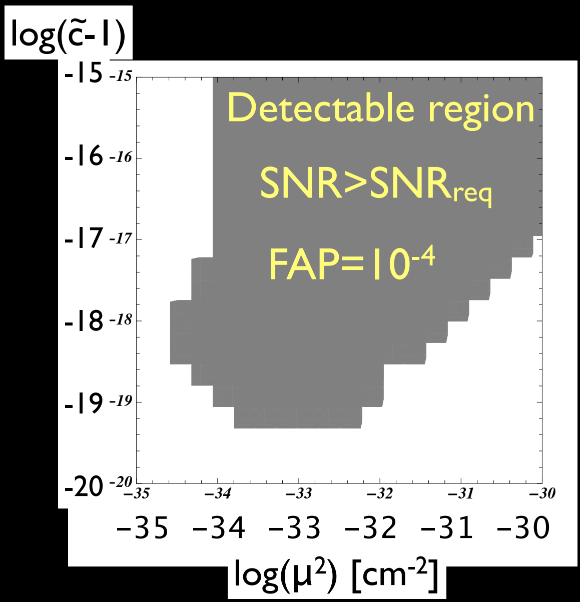
$$\mathrm{FF}(\theta_{\mathrm{MG}}) = \max_{\theta_{\mathrm{GR}}} \frac{(h_{\mathrm{GR}}(\theta_{\mathrm{GR}})|h_{\mathrm{MG}}(\theta_{\mathrm{MG}}))}{|h_{\mathrm{GR}}(\theta_{\mathrm{GR}})||h_{\mathrm{MG}}(\theta_{\mathrm{MG}})|}$$

deviation of MG from GR

FAP=10^{-4}

FF	SNR_{req}
0.9	8.699
0.95	12.3
0.99	27.5

Detectable region of the Bi-gravity corrections to the GR waveforms



[TN, Ueno, Tagoshi, Tanaka, Kanda, Nakamura, in prep.]

There is a detectable region!

effective mass: $\mu > 10^{-17}$cm^{-1}
propagation speed: $\tilde{c} - 1 > 10^{-19}$

Source:
NS-NS (1.4Msun-1.4Msun)
d_L=200Mpc
sensitivity curve: aLIGO, ZDHP
$\kappa\xi_c^2$=100

Conclusion

- GW will be detected soon.
- Testing gravity theory with GW
- Investigate the detectability of bi-gravity with graviton oscillations with KAGRA
- Bayesian model selection for GW
- There is a detectabile region: $\mu > 10^{-17}$cm^{-1}, $\tilde{c} - 1 > 10^{-19}$
- GWs can be powerful probe of bi-gravity.

“Improvement of energy-momentum tensor and non-Gaussianities in holographic cosmology”

Shinsuke Kawai

[JGRG24(2014)111109]

Improvement of energy-momentum tensor and non-Gaussianities in holographic cosmology

Shinsuke Kawai (SKKU, South Korea)

Based on arXiv:1403.6220
with Yu Nakayama

JGRG24 @IPMU, 11 November 2014

Overview

- Inflation is good. Maybe too good.
- UV theory? — holography: inflationary spacetime ⇔ 3d QFT
- Holographic description of inflation — immature
- What is the dual 3d QFT?
- Universality of CFT — model independent feature: $T_{\mu\nu}$
- Conformal invariant or scale invariant?
- Our results: Breaking of conf. invariance ⇔ non-Gaussianity

Holographic cosmology

- dS/CFT proposal [Witten] [Strominger]
- Inflation as dS holography with RG flow [Larsen, van der Schaar, Leigh (2002)] [Maldacena (2002)] [many others]
- Power spectrum and bispectrum, assuming particular field content in the 3d QFT [McFadden, Skenderis]
- Power spectrum and bispectrum, including effects of RG flow [Bzowski, McFadden, Skenderis, Garriga, Urakawa, others]

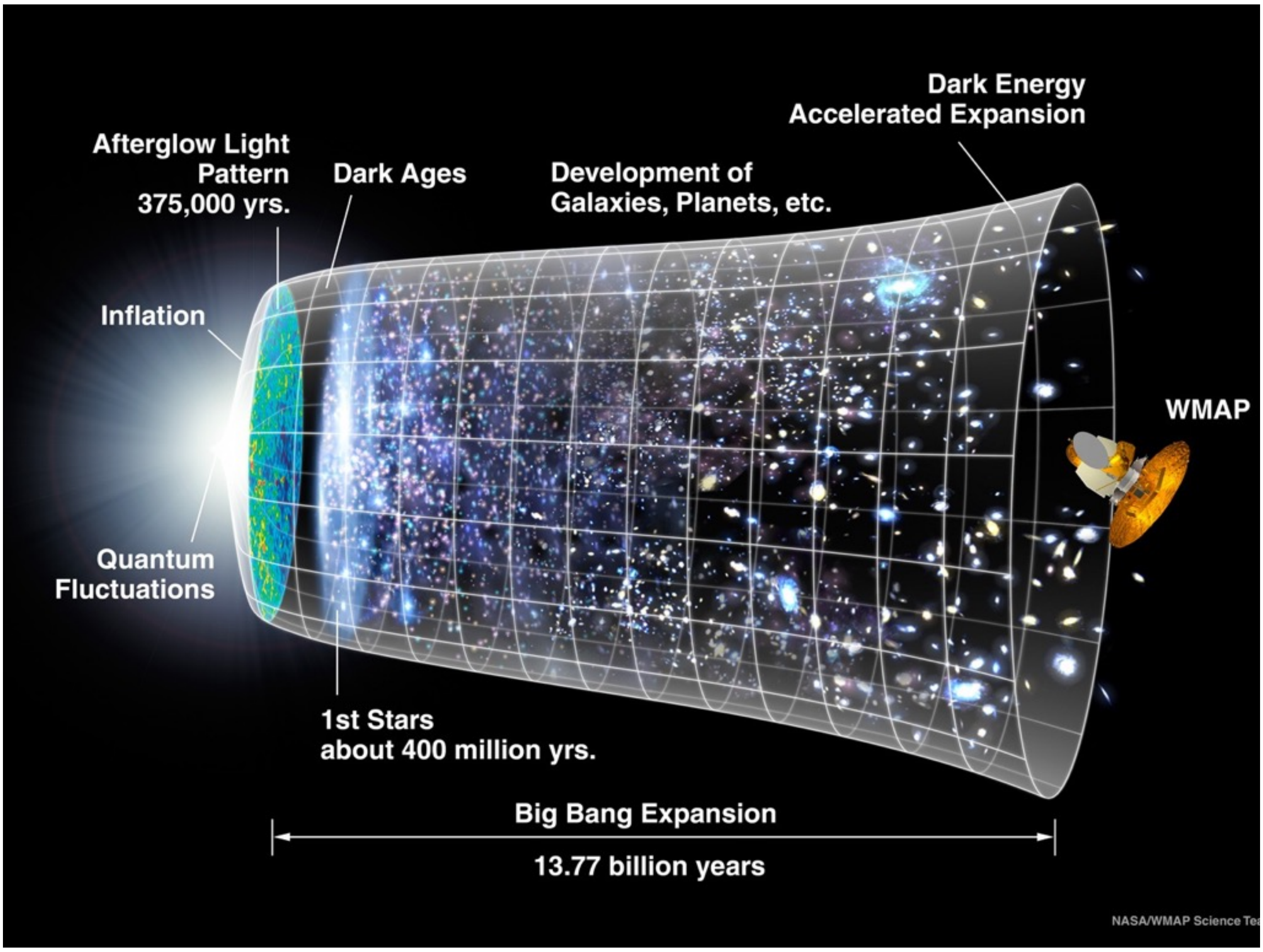
(A)dS/CFT

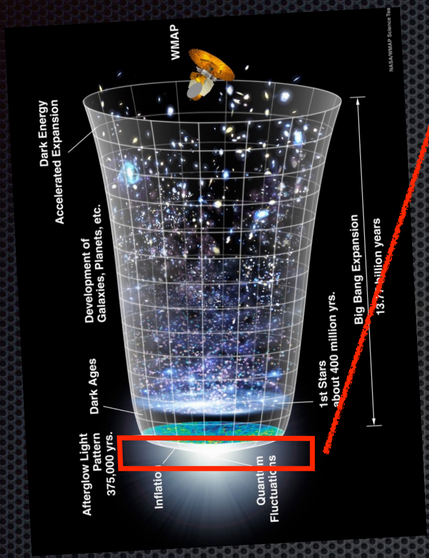
- Strongly coupled/weakly coupled duality
- A tool to compute strongly coupled dynamics using Einstein gravity, or quantum gravitational dynamics using perturbative QFT

$$\Psi_{\text{dS}}[\underbrace{g_{ij}(x), g^I(x)}_{\text{boundary conditions}}] = Z_{\text{CFT}}[\underbrace{g_{ij}(x), g^I(x)}_{\text{sources of } T^{ij}(x), \mathcal{O}_I(x)}]$$

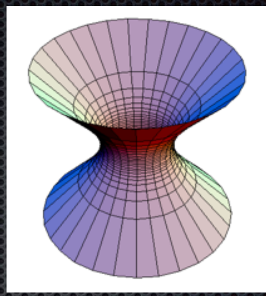
- Dictionary: boundary value of metric = source of EM tensor in the boundary theory
- Metric fluctuations ⟺ correlators of the boundary EM tensor

Is the CMB conformal invariant,
or just scale invariant?





4d (approximate)
de Sitter spacetime



De Sitter group is *SO*(4,1):
rotations in the ambient spacetime.

Isomorphic to *3d conformal group*

Conformal transformation on inflationary spacetime

- FRW metric: $ds^2 = \dfrac{-d\tau^2 + (dx^i)^2}{H^2\tau^2}, \quad -\infty < \tau < 0$
  - P_i: translation in 3d space (homogeneity)
  - M_{ij}: rotation in 3d space (isotropy)
  - D: simultaneous scaling $\tau \to \lambda\tau, \quad x^i \to \lambda x^i$
    (➔ scale invariance)
  - K_i: nonlinear transformation $\tau \to \tau + 2(\boldsymbol{b}\cdot\boldsymbol{x})\tau,$

$$x^i \to x^i + (\boldsymbol{\tau}^2 - \boldsymbol{x}^2)b^i + 2(\boldsymbol{b}\cdot\boldsymbol{x})x^i$$

(➔ ?)

Observables

- Scale invariance: 7 parameters (P_i, M_{ij}, D)
- Conformal invariance: 10 parameters (P_i, M_{ij}, D, K_i)
  - Conformal invariance impose strong constraints on correlation functions (power spectrum, bispectrum, trispectrum, etc.) of primordial fluctuations
  - Gravitons and curvatons: conformal
    [Maldacena Pimentel 2011] [Creminelli 2011]
  - Inflaton fluctuations: only scale invariant

Correlation functions

for quasi-primary field of dimension Δ

$$\phi(x) \to \phi'(x') = \left|\frac{\partial x'}{\partial x}\right|^{-\Delta/d} \phi(x)$$

Poincaré + scaling	Conformal

2pt correlators

Poincaré + scaling:

$$\langle \phi_1(x_1)\phi_2(x_2)\rangle = \frac{C_{12}}{|x_1 - x_2|^{\Delta_1+\Delta_2}}$$

Conformal:

$$\langle \phi_1(x_1)\phi_2(x_2)\rangle = \frac{C_{12}}{|x_1 - x_2|^{2\Delta_1}} \quad (\Delta_1 = \Delta_2)$$

$$= 0 \quad (\Delta_1 \neq \Delta_2)$$

3pt correlators

Poincaré + scaling:

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle = \frac{C_{123}}{x_{12}^a x_{23}^b x_{31}^c}$$

$$x_{ij} = |x_i - x_j|, \quad a + b + c = \Delta_1 + \Delta_2 + \Delta_3$$

Conformal:

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle$$

$$= \frac{C_{123}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3} x_{23}^{\Delta_2+\Delta_3-\Delta_1} x_{31}^{\Delta_3+\Delta_1-\Delta_2}}$$

Energy-momentum tensor

- EM tensor: conserved current of translation
- In presence of rotation (or Lorentz) symmetry, EM tensor can be made symmetric (Belinfante tensor)
- If in addition scaling current conserved and Virial $V^j = \partial_i L^{ij}$ exists, EM tensor can be made traceless
- Traceless EM tensor ➔ classical conformal symmetry (invariance of the action) $x^i \to x^i + \epsilon^i$ $\quad \delta g_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i = \frac{2}{d} g_{ij} \partial_k \epsilon^k$

$$\delta S = \int d^d x T^{ij} \partial_i \epsilon_j = \frac{1}{2} \int d^d x T^{ij} \underline{(\partial_i \epsilon_j + \partial_j \epsilon_i)} = \frac{1}{d} \int d^d x T^i{}_i \partial_j \epsilon^j$$

EM tensor and symmetries

- Poincaré = translation + rotation

conserved current

improvement term

$$T^{ij} = T_c^{ij} + \partial_k B^{kij} + \boxed{\frac{1}{2} \partial_k \partial_\ell X^{k\ell ij}}$$

Symmetric and traceless EM tensor

- Scaling symmetry + virial $\quad V^j = \partial_i L^{ij}$

Trace identity (local Callan-Symanzik equation):

$$T^i{}_i = \beta^I \mathcal{O}_I + \partial_i J^i + \kappa^\alpha \Box \mathcal{O}_\alpha$$

Our work

- Holographic cosmology with EM tensor improvement
- Recall the trace identity: $T^i{}_i = \underbrace{\beta^I \mathcal{O}_I + \partial_i J^i}_{=0 \text{ in exact dS}} + \underbrace{\kappa^\alpha \Box \mathcal{O}_\alpha}_{\text{improvement term}}$
- Action: $S = \frac{1}{2}\int d^3x \sqrt{g}\left(g^{ij}\partial_i\phi^I\partial_j\phi^I + \underbrace{\xi R(\phi^I)^2}\right)$

  $I = 1, 2, \cdots, N_\xi$

  ξ=0: minimal coupling; ξ=⅛: conformal coupling
- The improvement term affects the observables
- Computed power spectrum and bispectrum including the improvement term in exact dS

Power spectra

- Holographic computation
  - Scalar power spectrum $\Delta^2_{\rm S}(k) = \frac{k^3}{2\pi^2}\langle\langle\zeta(k)\zeta(-k)\rangle\rangle = \frac{16}{\pi^2 N_\xi (1-8\xi)^2}$
  - Tensor power spectrum $\Delta^2_{\rm T}(k) = \frac{k^3}{2\pi^2}\langle\langle\gamma^*_{ij}(k)\gamma^{ij}(-k)\rangle\rangle = \frac{512}{\pi^2 N_\xi}$
  - Tensor/scalar ratio $r \equiv \frac{\Delta^2_{\rm T}(k)}{\Delta^2_{\rm S}(k)} = 32(1-8\xi)^2$
- Observation
  - [Planck (2013)] $\Delta^2_{\rm S}(k_0) = 2.215 \times 10^{-9}$ $\quad k_0 = 0.05\ \mathrm{Mpc}^{-1}$
  - [BICEP2 (2014)] $r = 0.20^{+0.07}_{-0.05}$ $\quad N_\xi \gg 1, \quad \left|\xi - \frac{1}{8}\right| \approx 10^{-2}$

Central charge of the holographic universe [Larsen Strominger]: $C_T = \frac{3}{32}\frac{N_\xi}{\pi^2} \approx 10^9.$

Non-Gaussianities

$$\langle\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle\rangle = f_{\rm NL}^{\rm local}B_\zeta^{\rm local} + f_{\rm NL}^{\rm equil}B_\zeta^{\rm equil} + f_{\rm NL}^{\rm ortho}B_\zeta^{\rm ortho}$$

- Holographic computation with improvement term

$$f_{\rm NL}^{\rm local} = 0, \quad f_{\rm NL}^{\rm equil} = \frac{5}{36}(1+24\xi), \quad f_{\rm NL}^{\rm ortho} = -\frac{10}{9}\xi$$

- Observation [Planck 2013]

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8, \quad f_{\rm NL}^{\rm equil} = -42 \pm 75, \quad f_{\rm NL}^{\rm ortho} = -25 \pm 39$$

  - Holographic computation is consistent with observational constraint (but hopeless to detect in near future)
  - Similar to in-in formalism sub-horizon computation of NG

[Bzowski, McFadden, Skenderis (2009-2013)]: $f_{\rm NL}^{\rm local} = f_{\rm NL}^{\rm ortho} = 0, \quad f_{\rm NL}^{\rm equil} = \frac{5}{36}$

bosons, fermions, gauge fields (ξ = 0, ⅛)

Summary

- Holography may help us understand the primordial fluctuations better.
- Improvement of EM tensor — scale invariant but not necessarily conformal invariant density fluctuations
- Equilateral and orthogonal type non-Gaussianities of O(1) predicted (but no local type)

Thank you for your attention.

“Current status of the AdS (in)stability”

Andrzej Rostworowski [Invited]

[JGRG24(2014)111110]

Current status of the AdS (in)stability

Andrzej Rostworowski

Jagiellonian University

joint work with Piotr Bizoń, Joanna Jałmużna and Maciej Maliborski

JGRG24, 11th Nov. 2014

Anti-de Sitter spacetime in $d+1$ dimensions

Anti-de Sitter spacetime is the maximally symmetric solution of the vacuum Einstein equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = 0\,,$$

with negative cosmological constant $\Lambda < 0$.

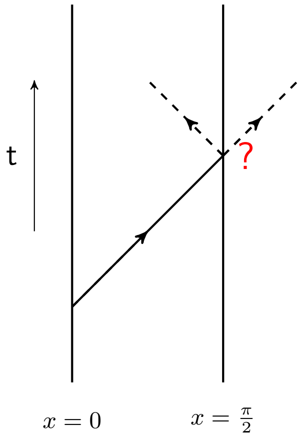
Peculiar causal structure of AdS

$$ds^2 = \frac{\ell^2}{(\cos x)^2}\left[-dt^2 + dx^2 + (\sin x)^2\, d\Omega^2_{S^{d-1}}\right], \quad -\infty < t < \infty,\ 0 \le x < \frac{\pi}{2}$$

solves $R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = 0$ for $\Lambda = -d(d-1)/(2\ell^2)$

Conformal infinity $x = \pi/2$ is the timelike hypersurface $\mathcal{I} = \mathbb{R} \times S^{d-1}$ with the boundary metric $ds^2_{\mathcal{I}} = -dt^2 + d\Omega^2_{S^{d-1}}$

- Null geodesics get to infinity in finite time (but infinite affine length)
- AdS is **not globally hyperbolic** - to make sense of evolution one needs to choose boundary conditions at $\mathcal{I}$
- Asymptotically AdS spacetimes by definition have the same conformal boundary as AdS



Is AdS stable?

- By the positive energy theorem AdS space is the ground state among asymptotically AdS spacetimes (much as Minkowski space is the ground state among asymptotically flat spacetimes)
- Minkowski spacetime was proved to be asymptotically stable by [Christodoulou&Klainerman, 1993]
- Key difference between Minkowski and AdS: the mechanism of stability of Minkowski - **dissipation of energy by dispersion** - is absent in AdS (for no-flux boundary conditions $\mathcal{I}$ acts as a mirror)
- The problem of stability of AdS has not been explored until recently; notable exceptions: proof of local well-posedness by [Friedrich, 1995], proof of rigidity of AdS [Anderson, 2006]

Two kinds of stability

- Consider a nonlinear evolution equation $\frac{du}{dt} = A(u)$ and its equilibrium solution ϕ (that is $A(\phi) = 0$). Let $u = \phi + w$.
  The equilibrium ϕ is (nonlinearly) **stable** if

$$\|w(0)\|_1 \text{ is small } \Rightarrow \|w(t)\|_2 \text{ is small for all } t > 0$$

- Consider the linear equation $\frac{dv}{dt} = Lv$, where $L = A'(\phi)$.
  The equilibrium ϕ is **linearly stable** if

$$\|v(0)\|_1 \text{ is small } \Rightarrow \|v(t)\|_2 \text{ is small for all } t > 0$$

- Key idea of linearization: as long as $w(t)$ remains small, the nonlinear part in $A(u) = Lw + N(w)$ is negligible.
- Linear stability does not imply stability!
- The equilibrium ϕ is **unstable/linearly unstable** if it is not stable/linearly stable.
- In case of instability there arises a question: what happens as $t \to \infty$?

Model for nonlinear dynamics

- The problem seems tractable only in $1 + 1$ dimensions
  $\Rightarrow$ spherical symmetry $\Rightarrow$ need matter to generate dynamics
- Simple matter model: massless scalar field ϕ in d+1 dimensions

$$G_{\alpha\beta} + \Lambda\, g_{\alpha\beta} = 8\pi G \left(\partial_\alpha \phi\, \partial_\beta \phi - \frac{1}{2} g_{\alpha\beta} \partial_\mu \phi \partial^\mu \phi \right), \; \Lambda = -d(d-1)/(2\ell^2),$$

$$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \phi = 0$$

- In the asymptotically flat case ($\Lambda = 0$) this model has led to important insights (proof of the weak cosmic censorship by [Christodoulou, 1986-1999] and the discovery of critical phenomena at the threshold for black hole formation by [Choptuik, 1993])
- Remark: For even $d \geq 4$ there is a way to bypass Birkhoff's theorem (cohomogeneity-two Bianchi IX ansatz, [Bizoń,Chmaj&Schmidt, 2005])

Model

- The line element for asymptotically AdS spacetimes at spherical symmetry

$$ds^2 = \frac{\ell^2}{\cos^2 x}\left(-Ae^{-2\delta}dt^2 + A^{-1}dx^2 + \sin^2 x\, d\Omega^2_{S^{d-1}}\right) ,$$

$(t, x) \in \mathbb{R} \times [0, \pi/2)$.

- Field equations (units $8\pi G = d - 1$)

$$\delta' = -\frac{\sin 2x}{2}\left(\Phi^2 + \Pi^2\right) , \qquad A' = 2(1-A)\frac{d-1-\cos 2x}{\sin 2x} - A\delta' ,$$
$$\dot{\Pi} = \frac{1}{\tan^{d-1} x}\left(\tan^{d-1} x A e^{-\delta}\Phi\right)' , \qquad \dot{\Phi} = \left(Ae^{-\delta}\Pi\right)' , \qquad .$$

- Auxiliary variables ($' = \partial_x$, $\dot{} = \partial_t$): $\Pi = A^{-1}e^{\delta}\dot{\phi}$ and $\Phi = \phi'$.

- AdS space: $\phi \equiv 0$, $A \equiv 1$, $\delta \equiv \text{const.}$

Boundary conditions

- Smoothness at the center enforces parity conditions on the fields at $x = 0$ (where Λ is irrelevant)
- Mass function and asymptotic mass:

$$m(t, x) = (1 - A(t, x)) \sec^2 x \tan^{d-2} x$$

$$M = \lim_{x\to\pi/2} m(t, x) = \int_0^{\pi/2} \left(A\Phi^2 + A\Pi^2\right)(\tan x)^{d-1}\, dx$$

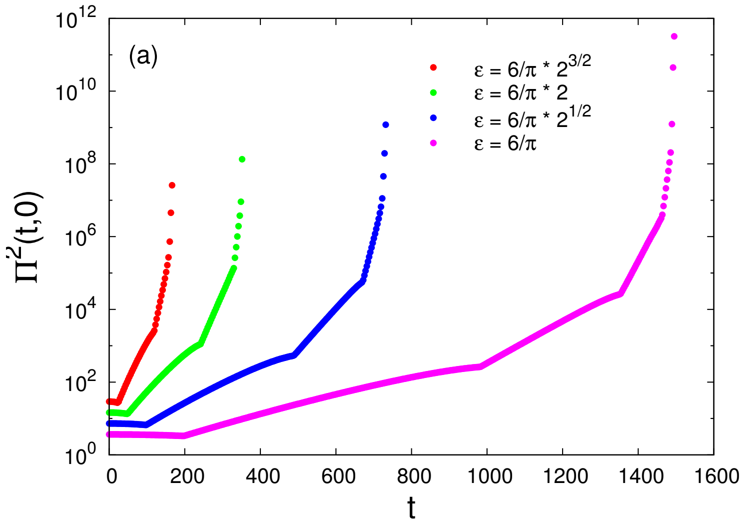
- Smoothness at spatial infinity and the demand for the total mass M to be finite put reflecting boundary conditions on ϕ at $x = \pi/2$, in particular (using $z = \pi/2 - x$)

$$\phi(t, x) = f_\infty(t)\, z^d + \mathcal{O}\left(z^{d+2}\right) ,$$
$$A(t, x) = 1 - Mz^d + \mathcal{O}\left(z^{d+2}\right) , \quad \delta'(t, x) = \mathcal{O}\left(z^{2d-1}\right) .$$

  For this model there is no freedom in prescribing boundary data
- The problem is locally well-posed [Friedrich, 1995], [Holzegel&Smulevici, 2011]

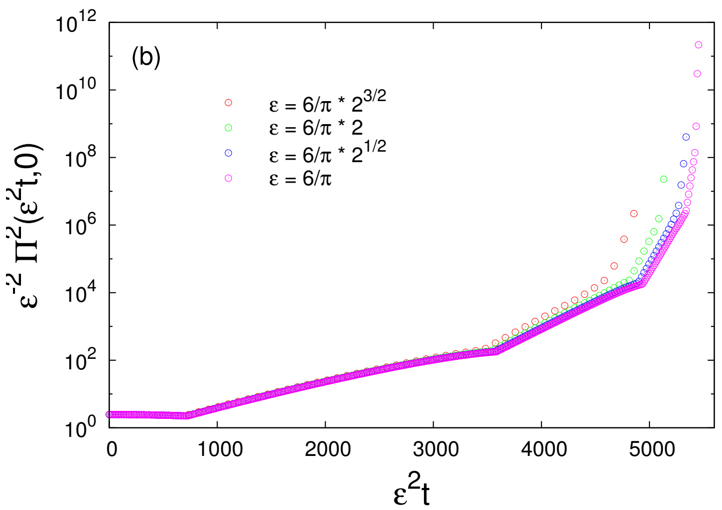
Animation

Key evidence for instability



Ricci scalar $R = 2\left(\Phi^2 - \Pi^2\right)/\ell^2 - 12/\ell^2$

Key evidence for instability



Onset of instability at time $t = \mathcal{O}(\varepsilon^{-2})$

Spectral properties

- Linearized equation [Ishibashi&Wald, 2004]

$$\ddot{\phi} + L\phi = 0\,, \quad L = -\frac{1}{\tan^{d-1}x}\,\partial_x\left(\tan^{d-1}x\,\partial_x\right)\,,$$

  With the above boundary conditions L is essentially self-adjoint on $L^2([0,\pi/2];\tan^{d-1}x\,dx)$

- Eigenvalues and eigenvectors (oscillons) of L read $(j = 0, 1, \ldots)$

$$\omega_j^2 = (d+2j)^2, \quad e_j(x) = N_j\,\cos^d x\,P_j^{(d/2-1,d/2)}(\cos 2x)\,,$$

- It follows that AdS is linearly stable, linear solution

$$\phi(t,x) = \sum_{j\geq 0} \alpha_j \cos(\omega_j t + \beta_j)\,e_j(x)\,,$$

  with amplitudes α_j and phases β_j determined by the initial data.
- The spectrum is fully resonant and nondispersive (!): $d\omega_j/dj = \pm 2$

Energy spectrum in $3+1$ dimensions

- Spectral decomposition of the total energy

$$M = \int_0^{\pi/2} \left(A\Phi^2 + A\Pi^2\right)\tan^2 x\,dx = \sum_{j=0}^{\infty} E_j(t)$$

  where $E_j := (e_j\,, \sqrt{A}\,\Pi)^2 + \omega_j^{-2}(e'_j\,, \sqrt{A}\,\Phi)^2$
- Energy spectrum (E_j as a function of j) is an important characteristic of turbulent dynamics
  Animation

- Just before collapse $E_j \sim j^{-\alpha}$ with $\alpha \approx 1.2\ (6/5??)$

Remarks

- Weakly turbulent behavior seems to be common for (non-integrable) nonlinear wave equations on bounded domains (e.g. NLS on torus, [Colliander&Keel, 2008], [Staffilani,Takaoka&Tao, 2008], [Carles&Faou, 2010]) and our work shows that Einstein's equations are not an exception.

- For Einstein's equations the transfer of energy to high frequencies cannot proceed forever because concentration of energy on smaller and smaller scales inevitably leads to the formation of a black hole.

- The role of negative cosmological constant seems to be purely kinematical, that is the only role of Λ is to confine the evolution in an effectively bounded domain. Similar turbulent dynamics has been observed for small perturbations of Minkowski in a box [Maliborski, 2012]

- Generalizations: different matter models (complex scalar field [Buchel,Lehner&Liebling, 2012], Yang-Mills [Maliborski, PhD Thesis 2014]), relaxing symmetry (pure gravity [Dias,Horowitz&Santos, 2011], [Bantilan,Pretorius&Gubser, 2012]), instability of AdS_{2+1} [Bizoń&Jałmużna, 2013]

Regular, stable asymptotically AdS solutions

- *Anti-de Sitter space is unstable against the formation of a black hole under a large class of arbitrarily small generic perturbations...*
  (also in higher dimensions [Jałmużna,R&Bizoń, 2011], [Buchel,Lehner&Liebling, 2012])
- *... but there are also initial data that may stay close to AdS solution; Einstein-scalar-AdS equations may admit time-quasiperiodic solutions* [Bizoń&R, 2011]
- Analogous conjecture for vacuum Einstein's equations – existence of geons [Dias,Horowitz&Santos, 2011], [Dias,Horowitz,Marolf&Santos, 2012].
- aAdS time-periodic solutions with scalar field (massless: [Maliborski&R, 2013], massive: [Kim, arXiv:1411.1633])
- Boson stars (standing waves) in AdS [Buchel,Liebling&Lehner, 2013]

Time-periodic asymptotically AdS solutions. Perturbative construction.

- We search for solutions of the form

$$\phi = \varepsilon \cos(\omega_\gamma t) e_\gamma(x)/e_\gamma(0) + \mathcal{O}(\varepsilon^3)\,,$$

with one *dominant* mode, ε (the amplitude $\phi(0,0)$) is a small parameter.

- We rescale the time variable

$$\tau = \Omega_\gamma t, \quad \Omega_\gamma = \omega_\gamma + \sum_{\text{even } \lambda \geq 2} \varepsilon^\lambda \omega_{\gamma,\lambda}$$

and expand the fields perturbatively ε

$$\phi = \varepsilon \cos(\tau) e_\gamma(x) + \sum_{\text{odd } \lambda \geq 3} \varepsilon^\lambda \phi_\lambda(\tau, x),$$

$$\delta = \sum_{\text{even } \lambda \geq 2} \varepsilon^\lambda \delta_\lambda(\tau, x), \qquad 1 - A = \sum_{\text{even } \lambda \geq 2} \varepsilon^\lambda A_\lambda(\tau, x),$$

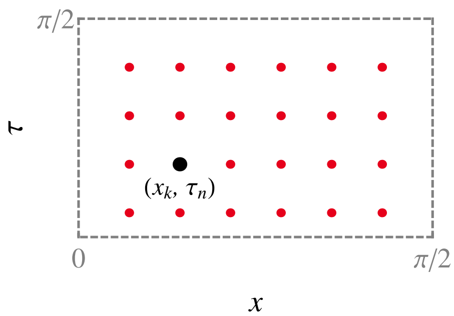
Time-periodic asymptotically AdS solutions. Numerical construction.

$$\phi = \sum_{0 \leq j < K} f_j(\tau) e_j(x) = \sum_{0 \leq i < N} \sum_{0 \leq j < K} f_{i,j} \cos((2i+1)\tau) e_j(x)\,,$$

$$\Pi = \sum_{0 \leq j < K} p_j(\tau) e_j(x) = \sum_{0 \leq i < N} \sum_{0 \leq j < K} p_{i,j} \sin((2i+1)\tau) e_j(x)\,.$$

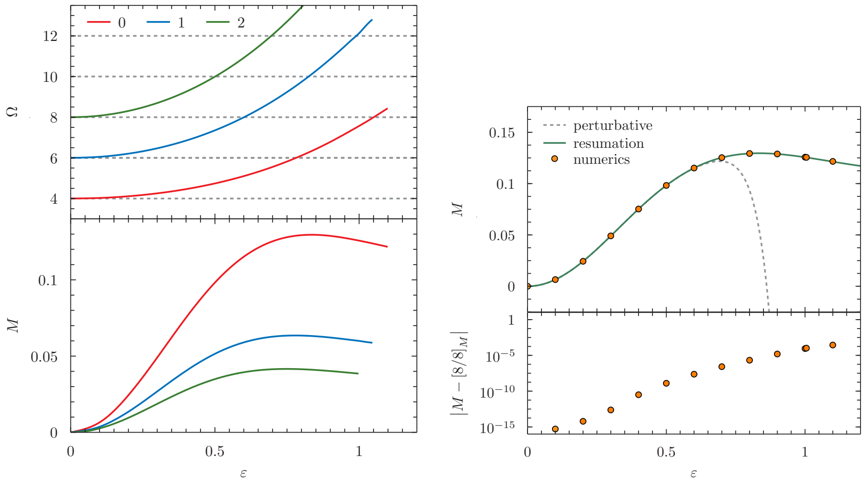
- Find the solution by determining $2 \times K \times N + 1$ numbers
- Set the equations on a numerical grid of $K \times N$ collocation points
- Add one equation for the normalization condition

$$\sum_{0 \leq i < N} \sum_{0 \leq j < K} f_{i,j} e_j(0) = \varepsilon$$



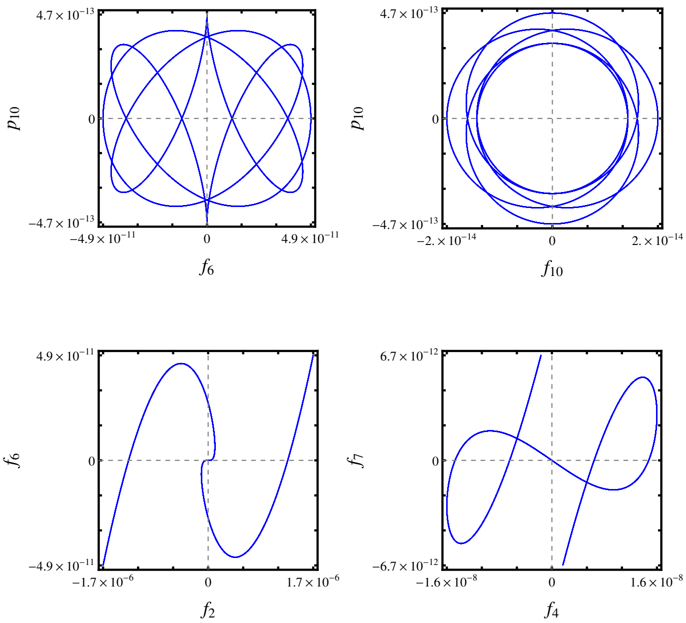
Highly nonlinear system solved with the Newton-Raphson algorithm.

Time-periodic asymptotically AdS solutions (d=4). Results & consistency.



From [Maliborski, PhD Thesis 2014]

Non-linear stability ($d=4$, $\gamma=0$, $\varepsilon=0.01$)



Closed curves on the slices of phase space – strong **evidence** for the non-linear **stability**. Sections of the phase space spanned by the set of Fourier coefficients

$$\{f_j(t),\, p_k(t)\}\,,$$

$$\phi = \sum_{0\le j<K} f_j(t)e_j(x)$$

$$\Pi = \sum_{0\le j<K} p_j(t)e_j(x)$$

[Animation (from M. Maliborski)]

Remarks

- There exist (non-linearly) stable time-periodic solutions in Einstein AdS–massless scalar field system.
- Cosmological constant confines the evolution in an effectively bounded domain – the possibility of the existence of time-periodic solutions (in contrast to asymptotically flat case)
- Time-periodic solutions in pure vacuum case
  - in the cohomogeneity – two Bianchi IX ansatz ([Bizoń,Chmaj&Schmidt, 2005]): [Maliborski, PhD Thesis 2014]
  - with helical Killing field [Horowitz&Santos, 2014]
- The existence of time-periodic solutions of (non-linear) wave equations on compact domains seems to be common [Maliborski, PhD Thesis 2014]

How to bypass Birkhoff in five dimensions to study the vacuum case

- Odd-dimensional spheres admit non-round homogeneous metrics
- Homogeneous metric on S^3

$$g_{S^3} = e^{2B}\sigma_1^2 + e^{2C}\sigma_2^2 + e^{2D}\sigma_3^2,$$

  where σ_k are left-invariant one-forms on $SU(2)$

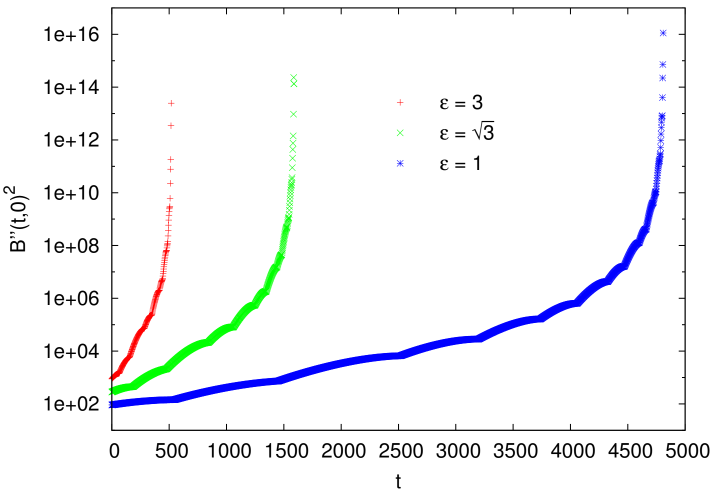
$$\sigma_1 + i\,\sigma_2 = e^{i\psi}(\cos\theta\, d\phi + i\, d\theta), \quad \sigma_3 = d\psi - \sin\theta\, d\phi.$$

  - $B = C = D$: round metric with $SO(4)$ symmetry
  - $B \neq C \neq D$: anisotropic metric with $SU(2)$ symmetry (squashed S^3)
- [Bizoń,Chmaj&Schmidt, 2005]: use g_{S^3} as an angular part of the five dimensional metric (cohomogeneity-two triaxial Bianchi IX ansatz). For AdS_{4+1}, with $B = C$ (the biaxial case):

$$ds^2 = \frac{\ell^2}{\cos^2 x}\left(-Ae^{-2\delta}dt^2 + A^{-1}dx^2 + \frac{1}{4}\sin^2 x\left(e^{2B}(\sigma_1^2 + \sigma_2^2) + e^{-4B}\sigma_3^2\right)\right),$$

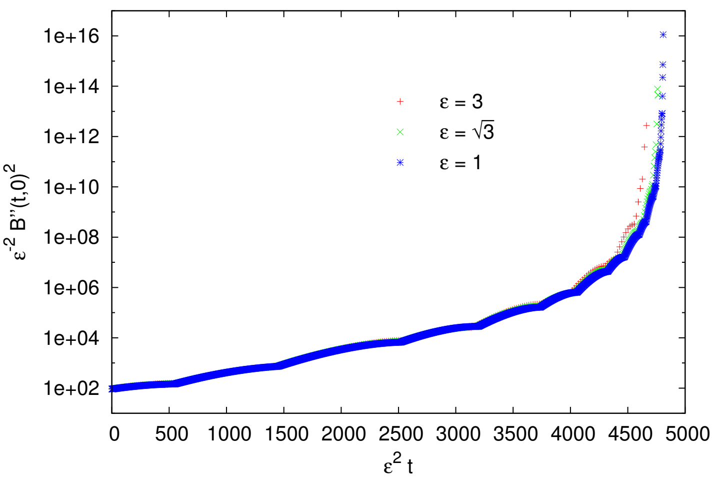
  where A, δ, B are functions of (t, x).

Blowup of the Kretschmann scalar



$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}(t,0) = 40 + 864\, B''(t,0)^2$$

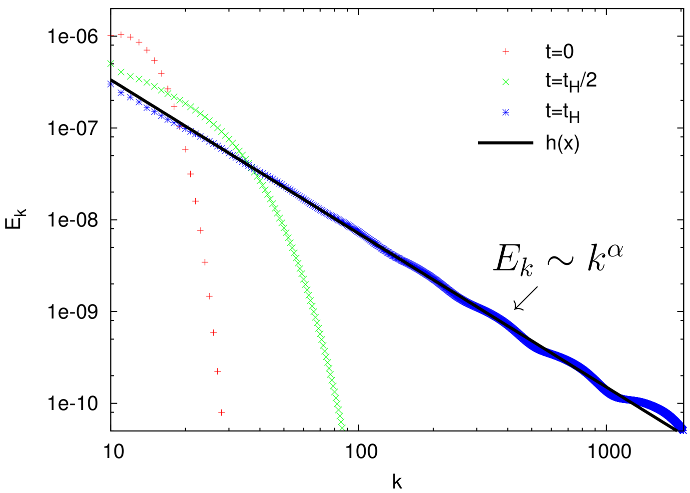
Key evidence for instability



Conjecture

Within the cohomogeneity-two Bianchi IX ansatz AdS_5 is unstable against black hole formation under arbitrarily small gravitational perturbations

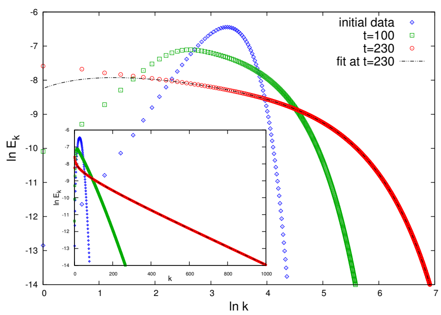
Spectrum of energy

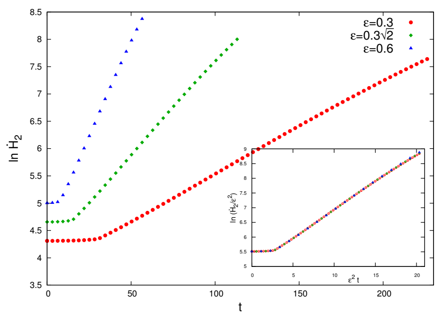


Universal power–law exponent: $\alpha \approx -1.67$ (-5/3?)

Weak turbulent instability of AdS_{2+1}

In $2+1$ dimension there is a mass-gap for a black hole formation: if $M < 1$ black hole can not form. Two options for the end state of evolution for small initial data $0 < M << 1$: naked (conical) singularity or global-in-time regularity [Bizoń&Jałmużna, 2013], [Jałmużna, 2014].





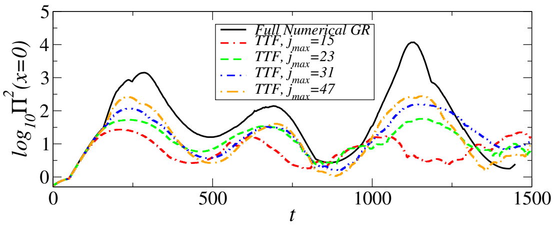
Analyticity strip method [Sulem,Sulem&Frisch, 1983]:

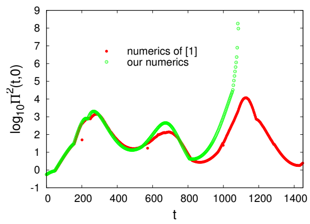
$$E_k(t) = C(t)\, k^{-\beta(t)}\, e^{-2\rho(t) k}$$

Fit: $\rho(t) = \rho_0 e^{-t/T}$, with $\rho_0 \sim \mathcal{O}\left(\epsilon^0\right),\ T \sim \mathcal{O}\left(\epsilon^{-2}\right)$

V. Balasubramanian et al., *Holographic Thermalization, stability of AdS, and the Fermi-Pasta-Ulam-Tsingou paradox*, PRL113, 071601 (2014)

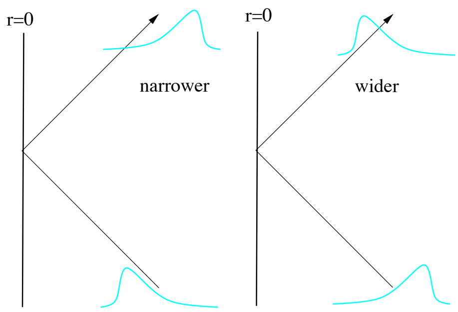
Two-modes initial data and the inverse cascade.

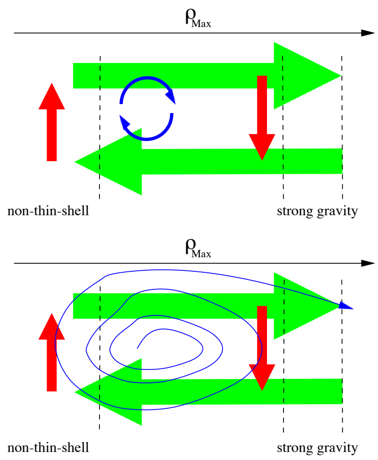




F.V. Dimitrakopoulos et al., *Instability corners in AdS space*, arXiv:1410.1880

Hierarchy of scales: $\ell >> x_0 >> w >> \epsilon^2$





N. Deppe et al., *Stability of AdS in Einstein Gauss Bonnet Gravity*, arXiv:1410.1869

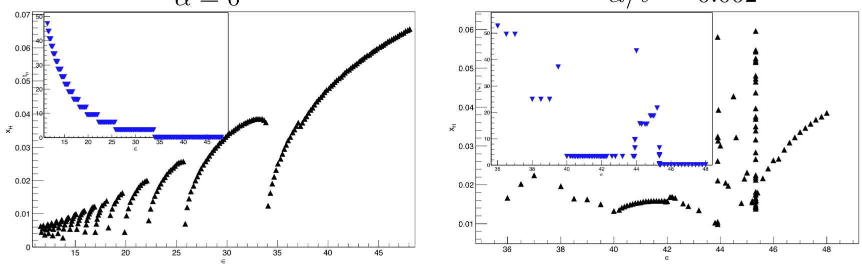
Including Gauss–Bonnet term (in 4+1):

$$S = \int d^5x\sqrt{-g}\left\{\frac{1}{2\kappa}\left[R - 2\Lambda + \frac{\alpha}{2}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right)\right] - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi\right\}$$

with $\Lambda = -(6/\ell^2)(1 - \alpha/\ell^2)$

Threshold for a black hole formation: $\alpha/2$

$\alpha = 0$ $\qquad\qquad$ $\alpha/\ell^2 = 0.002$



Conclusions

- Dynamics of asymptotically AdS spacetimes is an exceptional meeting point of fundamental problems in general relativity, PDE theory, theory of turbulence, and high energy physics. Understanding of these connections is at its infancy.
- From numerical explorations of Einstein's equations there can grow understanding, conjectures, and roads to proofs and phenomena that would not have been imaginable in the pre-computer era. The role of computation in general relativity seems destined to expand in future.

“Higher-dimensional extremal Reissner-Nordström black holes are fragile”

Masashi Kimura

[JGRG24(2014)111111]

Higher-dimensional extremal Reissner-Nordström BHs are fragile

Masashi Kimura
(DAMTP, University of Cambridge)

w/ K.Tanabe (KEK) in preparation

11th Nov 2014 JGRG 2014

Introduction and Summary

Introduction

Why extremal RN black hole?

- Supersymmetric BHs

- Construction of toy models

  e.g. multi-BHs, coalescing BHs, Kaluza-Klein BHs, etc…

Summary

Stationary perturbation around extremal RN BHs behaves

$$\sim (r - r_h)^{\ell/(D-3)}$$

$D = 4$: integer power

$D \geq 5$: fractional power

$\Longrightarrow$ smoothness is broken

$\ell = 2$ modes cause curvature singularities if $D \geq 6$

Details

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Reissner-Nordström BHs

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2_{\mathrm{S}^{\mathrm{D}-2}}$$

$$A_\mu dx^\mu = \sqrt{\frac{2(D-2)}{D-3}} \frac{Q}{r^{D-3}} dt$$

$$f = 1 - \frac{2M}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}$$

horizon radius: $r_h = \left(M + \sqrt{M^2 - Q^2}\right)^{1/D-3}$

$|Q| = M \implies$ extremal horizon

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Perturbation around RN BHs

By using Ishibashi & Kodama formalism, we can separately discuss tensor/vector/scalar perturbation around RN BHs

Hereafter, we mainly focus on tensor perturbations for simplicity

Vector/scalar modes have qualitatively same features

Master eq for stationary perturbation

$$\delta g_{\mu\nu}dx^\mu dx^\nu = r^2 \boxed{h^{(T)}(r)} \mathbb{T}_{ij}dx^i dx^j$$

$$\begin{pmatrix} \triangle_{S^{D-2}}\mathbb{T}_{jk} = -[\ell(\ell+D-3)-2]\mathbb{T}_{jk} \\ D^i\mathbb{T}_{ij}=0, \quad \mathbb{T}^i_i = 0 \qquad \ell \geq 2 \end{pmatrix}$$

$$\frac{d^2}{dr_*^2}h^{(T)} = \frac{f}{r}Vh^{(T)} \qquad \left(\frac{d}{dr_*} = f\frac{d}{dr}\right)$$

$$V = \frac{(D-4)(D-2)}{4} + \frac{(D-2)^2}{4}\frac{2M}{r^{D-3}} - \frac{(D-2)(3D-8)}{4}\frac{Q^2}{r^{2(D-3)}} + \ell(\ell+D-3)$$

solutions for Master eqs

$$h^{(T)} = C_1 g(y)^{1+\ell/(D-3)} {}_2F_1(a_T, a_T, 2a_T; g(y)) + C_2 g(y)^{-\ell/(D-3)} {}_2F_1(b_T, b_T, 2b_T; g(y))$$

$$g(y) = \frac{2y\sqrt{1-(Q/M)^2}}{2-y+y\sqrt{1-(Q/M)^2}}$$

$$y := 2M/r^{D-3} \qquad a_T = 1+\ell/(D-3)$$

$$b_T = -\ell/(D-3)$$

$$0 < y < y_h \left(:= \frac{2}{(1+\sqrt{1-(Q/M)^2})} \right)$$

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Near horizon behavior

non-extremal case

$$h^{(T)} \sim \tilde{C}_1 + \tilde{C}_2 \ln(y_h - y)$$

extremal case

$$h^{(T)} \sim \tilde{C}_1 (y_h - y)^{\ell/(D-3)} + \tilde{C}_2 (y_h - y)^{-1-\ell/(D-3)}$$

This is due to the difference
of the boundary condition at the horizon

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$$h^{(T)} \sim \tilde{C}_1 (y_h - y)^{\ell/(D-3)}$$

▪perturbed metric vanishes at the horizon
horizon is locally spherically symmetric

▪If $D \geq 5$, the power can be fractional
horizon is not smooth

$\ell = 2$ modes cause p.p.
curvature singularities if $D \geq 6$

However they are relatively mild

easy to be broken or not

▪If a generic stationary perturbation always causes ill-behaved curvature singularity, we should say "not easy to be broken "
However, it is not the case now

▪Now, our solutions are physically acceptable
horizon (smoothness) is "easy to be broken" against stationary perturbations

Summary

- Horizon is not smooth for generic stationary perturbations around higher dim extremal RN BHs

- vector/scalar modes and AdS/dS cases have qualitatively same features

Discussions

- Near horizon geometry
- Physical interpretation in AdS/CFT context
- non existence of “regular” multi BHs in $D \geq 6$
- BF bound and instability

Thank you

“Toward constructing ghost-free scalar-tensor theories beyond Horndeski”

Ryo Namba

[JGRG24(2014)111112]

Toward constructing ghost-free scalar-tensor theories beyond Horndeski

Ryo Namba

Kavli IPMU

The 24th Workshop on General Relativity and Gravitation (JGRG24)
November 11, 2014

C. Lin, S. Mukohyama, RN and R. Saitou, JCAP 10(2014)071, [arXiv:1408.0670]
S. Mukohyama, RN and R. Saitou, *in progress*

Introduction

Q: What is the most general healthy scalar-tensor theory?

- Cosmological applications: accelerating expansion of the universe
- Adding one scalar is a minimal extension of GR
- Testing GR $\sim$ modifying GR

Horndeski (generalized Galileon) theory

Horndeski '74, Nicolis et al & Deffayet et al '09

- **Most general scalar-tensor theory with 2nd-order field equations**
  - Higher-order equations would increase the dimension of phase space
  - *Ostrogradski's theorem*:
    A linear instability in the system with a Lagrangian which genuinely depends on more than one time derivative
- Rather fine-tuned combination of coupling constants
  - in general de-tuned by quantum loops

GLPV Theory

Gleyzes, Langlois, Piazza & Vernizzi '14

- A class of theories larger than the Horndeski
- Higher-order equations of motion
- Still, the true propagating d.o.f. are **3**, the same as Horndeski
- Their analysis of Hamiltonian structure was not complete

Bottom line of the talk

1. There are scalar-tensor theories beyond the Horndeski
   - Still only 3 physical d.o.f.
   - Ghost free
2. Non-trivial constraint structure
   - Eliminates potentially dangerous extra d.o.f.
3. An explicit example by studying the GLPV theory

The GLPV Action

$$S = \int d^4x \sqrt{-g} \sum_{n=2}^{5} \mathcal{L}_n$$

$$\mathcal{L}_2 = P\,, \quad \mathcal{L}_3 = -G_3\,\Box\phi$$

$$\mathcal{L}_4 = G_4\,R^{(4)} + [G_{4X} + XF_4]\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]$$
$$+F_4\,\phi^\mu\phi^\nu\,(\Box\phi\,\phi_{\mu\nu} - \phi_{\mu\rho}\phi^\rho{}_\nu)$$

$$\mathcal{L}_5 = G_5\,G^{(4)}_{\mu\nu}\phi^{\mu\nu} + \frac{1}{6}\,(4XF_5 - G_{5X})\left[(\Box\phi)^3 - 3\Box\phi\,\phi^\mu{}_\nu\phi^\nu{}_\mu + 2\phi^\mu{}_\nu\phi^\nu{}_\rho\phi^\rho{}_\mu\right]$$
$$+F_5\,\phi^\mu\phi^\nu\left[(\Box\phi)^2\,\phi_{\mu\nu} - \phi^\rho{}_\sigma\phi^\sigma{}_\rho\phi_{\mu\nu} + 2\,\phi^\sigma{}_\mu\phi^\rho{}_\sigma\phi_{\nu\rho} - 2\Box\phi\,\phi^\rho{}_\mu\phi_{\nu\rho}\right]$$

- $\phi_\mu \equiv \nabla_\mu\phi,\ \phi_{\mu\nu} \equiv \nabla_\nu\nabla_\mu\phi,\ \Box\phi \equiv \nabla^\mu\nabla_\mu\phi\,, \ldots, X \equiv -\frac{1}{2}\nabla^\mu\phi\,\nabla_\mu\phi$
- Arbitrary functions: $P = P(\phi, X)$, $G_n = G_n(\phi, X)$, $F_n = F_n(\phi, X)$
- **Contains the Horndeski theory**
  - For $F_4 = F_5 = 0$
- **Contains the original Galileon theory**
  - For $G_4 = G_5 = 0$, $F_4 \neq 0$, $F_5 \neq 0$

The GLPV Action in the Unitary Gauge

Unitary gauge: $\phi = t\,, \quad X = 1/(2N^2)$

ADM decomposition: $ds^2 = -N^2dt^2 + h_{ij}\,(dx^i + N^i dt)\,(dx^j + N^j dt)$

The GLPV action reduces to

$$S = \int dt\,d^3x\,N\sqrt{h}\sum_{n=2}^{5} L_n\,,$$

$$L_2 = A_2\,, \quad L_3 = A_3\,K\,, \quad L_4 = A_4\left(K^2 - K^i_jK^j_i\right) + B_4R^{(3)}\,,$$

$$L_5 = A_5\left(K^3 - 3KK^{ij}K_{ij} + 2K^i_jK^j_kK^k_i\right) + B_5\,K^{ij}G^{(3)}_{ij}\,,$$

- Extrinsic curvature: $K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - {}^{(3)}D_iN_j - {}^{(3)}D_jN_i\right)$, $K \equiv K^i_i$
- Arbitrary functions: $A_n = A_n(t, N)$, $B_n = B_n(t, N)$
- **Broken time diffeomorphism, preserved spatial diffeomorphism**

Hamiltonian structure

Goal: to understand the Hamiltonian structure and count the d.o.f.

20-dimensional phase space:

$$(N, \pi_N)\ , \quad (N^i, \pi_i)\ , \quad (h_{ij}, \pi^{ij})$$

No $\dot{N}$ or $\dot{N}^i$ in the Lagrangian

$$\Longrightarrow\ \pi_N = 0\ , \quad \pi_i = 0$$

Hamiltonian takes the form

$$\begin{aligned} H &\equiv \int d^3x \left[\pi^{ij}\dot{h}_{ij} - N\sqrt{h}\sum_{n=2}^{5} L_n\right] \\ &= \int d^3x \left[\mathcal{H}(t, N, h_{ij}, \pi^{ij}) + N^i\, \mathcal{H}_i(h_{ij}, \pi^{ij})\right] \end{aligned}$$

Poisson brackets:

$$\{F, G\}_{\rm P} \equiv \frac{\delta F}{\delta N}\frac{\delta G}{\delta \pi_N} - \frac{\delta F}{\delta \pi_N}\frac{\delta G}{\delta N} + \frac{\delta F}{\delta N^i}\frac{\delta G}{\delta \pi_i} - \frac{\delta F}{\delta \pi_i}\frac{\delta G}{\delta N^i} + \frac{\delta F}{\delta h_{ij}}\frac{\delta G}{\delta \pi^{ij}} - \frac{\delta F}{\delta \pi^{ij}}\frac{\delta G}{\delta h_{ij}}$$

Nature of constraints

Constraints

Primary constraints:

$$\pi_N \approx 0\ , \quad \pi_i \approx 0$$

Secondary constraints:

$$\frac{d}{dt}\pi_N \approx \{\pi_N\,, H\}_{\rm P} \approx -\frac{\partial \mathcal{H}}{\partial N} \equiv \mathcal{C} \approx 0\ , \quad \frac{d}{dt}\pi_i \approx \{\pi_i\,, H\}_{\rm P} \approx -\mathcal{H}_i \approx 0$$

- Weak equality "≈" holds on the constraint surface
- All the Poisson brackets with π_i vanish:

$$\Longrightarrow\ \{\pi_i\,,\pi_j\}_{\rm P} \approx \{\pi_i\,,\pi_N\}_{\rm P} \approx \{\pi_i\,,\mathcal{H}_j\}_{\rm P} \approx \{\pi_i\,,\mathcal{C}\}_{\rm P} \approx 0$$

- Spatial diffeomorphism is in fact reflected by the linear combination

$$\tilde{\mathcal{H}}_i \equiv \mathcal{H}_i + \pi_N \partial_i N$$

$$\Longrightarrow\ \{\tilde{\mathcal{H}}_i\,,\pi_j\}_{\rm P} \approx \{\tilde{\mathcal{H}}_i\,,\pi_N\}_{\rm P} \approx \{\tilde{\mathcal{H}}_i\,,\tilde{\mathcal{H}}_j\}_{\rm P} \approx \{\tilde{\mathcal{H}}_i\,,\mathcal{C}\ {}_{\rm P} \approx 0$$

Nature of constraints

First-class constraints:

$$\pi_i \approx 0\,, \quad \tilde{\mathcal{H}}_i \equiv \mathcal{H}_i + \pi_N\,\partial_i N \approx 0$$

Second-class constraints:

$$\pi_N \approx 0\,, \quad \mathcal{C} \approx 0$$

The "total" Hamiltonian takes the form

$$H_{\rm tot} = \int d^3x \left[\mathcal{H} + N^i \mathcal{H}_i + \lambda^i \pi_i + \lambda_N \pi_N + n^i \tilde{\mathcal{H}}_i + \lambda_{\mathcal{C}} \mathcal{C}\right]$$

Lagrange multipliers

Second-class constraints $\Longrightarrow$ $\lambda_{\mathcal{C}}$ & λ_N are determined by the consistency

$$\frac{d}{dt}\pi_N \approx \{\pi_N\,, H_{\rm tot}\}_{\rm P} \approx -\frac{\partial \mathcal{C}}{\partial N}\lambda_{\mathcal{C}} \approx 0$$

$$\frac{d}{dt}\mathcal{C} \approx \{\mathcal{C}, H_{\rm tot}\}_{\rm P} \approx \{\mathcal{C}, \mathcal{H}\}_{\rm P} + N^i \{\mathcal{C}, \mathcal{H}_i\}_{\rm P} + \{\mathcal{C}, \pi_N\}_{\rm P}\, \lambda_N \approx 0$$

Gauge fixing

First-class constraints $\Longrightarrow$ λ^i & n^i are yet to be determined.

Introduce gauge-fixing conditions

$$\mathcal{F}^i \approx 0\,, \quad \mathcal{G}^i \approx 0$$

◇ Require: $\det \begin{pmatrix} \{\mathcal{F}^i, \pi_j\}_{\rm P} & \{\mathcal{G}^i, \pi_j\}_{\rm P} \\ \{\mathcal{F}^i, \tilde{\mathcal{H}}_j\}_{\rm P} & \{\mathcal{G}^i, \tilde{\mathcal{H}}_j\}_{\rm P} \end{pmatrix} \not\approx 0$

Hamiltonian with gauge-fixing terms

$$H'_{\rm tot} = \int d^3x \left[\mathcal{H} + N^i \mathcal{H}_i + \lambda^i \pi_i + \lambda_N \pi_N + n^i \tilde{\mathcal{H}}_i + \lambda_{\mathcal{C}} \mathcal{C} + \lambda_i^{\mathcal{F}} \mathcal{F}^i + \lambda_i^{\mathcal{G}} \mathcal{G}^i\right]$$

◇ Total 14 second-class constraints

$$\pi_i \approx 0\,, \quad \pi_N \approx 0\,, \quad \tilde{\mathcal{H}}_i \approx 0\,, \quad \mathcal{C} \approx 0\,, \quad \mathcal{F}^i \approx 0\,, \quad \mathcal{G}^i \approx 0$$

◇ $(20 - 14) =$ 6-dimensional phase space $=$ **3 degrees of freedom!**

Concluding Remarks

Scalar-tensor theories beyond Horndeski

- An example: GLPV theory
- We performed the Hamiltonian analysis in the unitary gauge

Constraint structure is essential

- Reduces the dimension of the phase space
- Eliminates the ghost-like d.o.f.
- The Horndeski theories do not take such constraints into account

Remaining questions:

- Understanding of the GLPV theory in the general gauge
  - Discussions on this issue come next by Rio Saitou
- ***General framework to remove pathological d.o.f.***

“Structure of constraints of the theory beyond Horndeski”

Rio Saitou

[JGRG24(2014)111113]

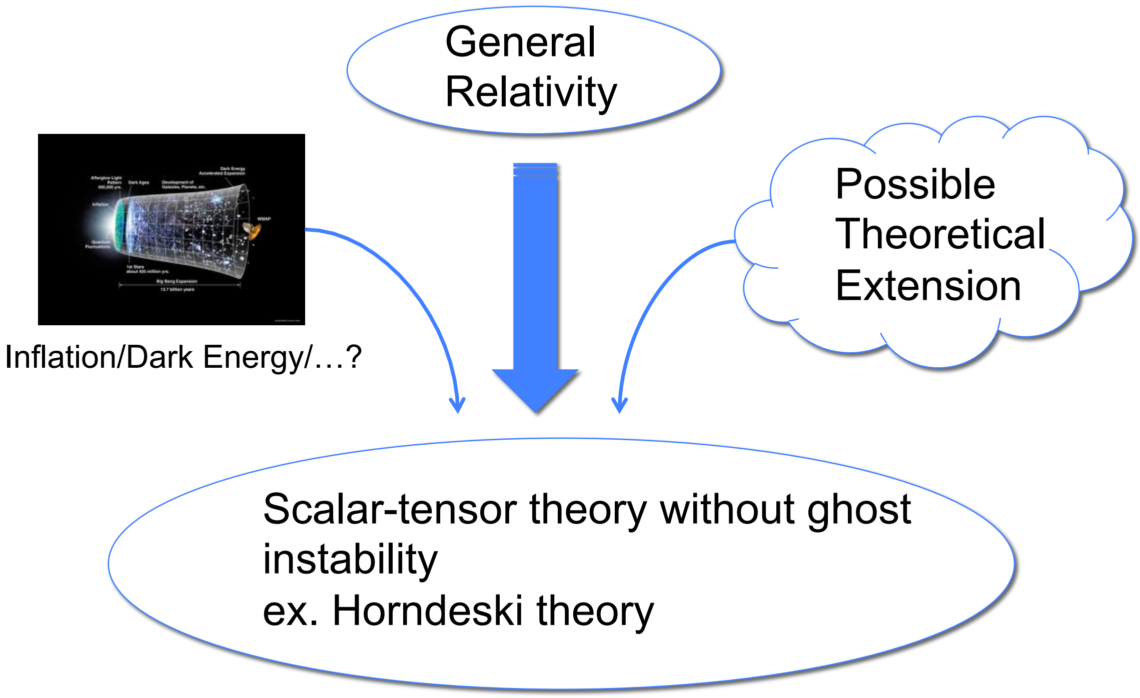
Structure of Constraints for the Theory *Beyond* Horndeski

Rio Saitou
(YITP, Kyoto Univ. / KIPMU, Tokyo Univ.)

Collaboration with Chunshan Lin, Shinji Mukohyama and Ryo Namba

Based on the work in progress and JCAP10(2014)071
JGRG24@KIPMU 2014/11/11

Scalar-Tensor theory for Gravitation



The GLPV theory *beyond* Horndeski

Gleyzes, Langlois, Piazza and Vernizzi (2014)
Lin, Mukohyama, Namba, RS (2014)

- UNITARY GAUGE (Φ = t) Action in ADM form

$$I = \int dx^4 \sqrt{-g}\,[L_2 + L_3 + L_4 + L_5]$$

$$L_2 = P(\phi, X),$$

$$L_3 = -G_3(\phi, X)\Box\phi$$

NO EXTRA DEGREES OF FREEDOM !

$$L_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} - \frac{1}{6}\left[G_{5X}(\phi, X) - 4XF_5(\phi, X)\right]\left[(\Box\phi)^3\right.$$

$$K = -3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi^{\mu}_{\nu}\phi^{\nu}_{\rho}\phi^{\rho}_{\mu}\Big] + F_5(\phi, X)\left[(\Box\phi)^2\phi_{\mu\nu} - 2\Box\phi\phi_{\mu\rho}\phi^{\rho}_{\nu}\right.$$

$$K_2 = -\phi^{\rho\sigma}\phi_{\rho\sigma}\phi_{\mu\nu} + 2\phi_{\mu\rho}\phi^{\rho\sigma}\phi_{\sigma\nu}\big]\,\phi^{\mu}\phi^{\nu}$$

$$K_3 = K^3 - 3KK^{ij}K_{ij} + 2K^i{}_j K^j{}_k K^k{}_i$$

The GLPV theory *beyond* Horndeski

Gleyzes, Langlois, Piazza and Vernizzi (2014)
Lin, Mukohyama, Namba, RS (2014)

- GENERAL GAUGE Action in Covariant form

$$I = \int dx^4 \sqrt{-g}\,[L_2 + L_3 + L_4 + L_5]$$

$$L_2 = P(\phi, X),$$

NO EXTRA DEGREES OF FREEDOM ??

$$+F_4(\phi, X)\left[\Box\phi\phi_{\mu\nu} - \phi_{\mu\rho}\phi^{\rho}_{\nu}\right]\phi^{\mu}\phi^{\nu}$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} - \frac{1}{6}\left[G_{5X}(\phi, X) - 4XF_5(\phi, X)\right]\left[(\Box\phi)^3\right.$$

$$\left. - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi^{\mu}_{\nu}\phi^{\nu}_{\rho}\phi^{\rho}_{\mu}\right] + F_5(\phi, X)\left[(\Box\phi)^2\phi_{\mu\nu} - 2\Box\phi\phi_{\mu\rho}\phi^{\rho}_{\nu}\right.$$

$$\left. -\phi^{\rho\sigma}\phi_{\rho\sigma}\phi_{\mu\nu} + 2\phi_{\mu\rho}\phi^{\rho\sigma}\phi_{\sigma\nu}\right]\phi^{\mu}\phi^{\nu}$$

The GLPV theory in GENERAL GAUGE

- The degrees of freedom (dof) should be the same as in the unitary gauge, that is, 6 dof.
- How do constraints enter in the theory?

UNITARY GAUGE

Spatial diffeomorphism

Two 2nd class constraints

GENERAL GAUGE

Full diffeomorphism

???

- Studying general gauge tells us richer information of the theory, which we can not get from the unitary gauge.

ex. Static case $\phi = \phi(\vec{x})$, Decoupling limit (Minkowski limit) and etc..

The GLPV theory in GENERAL GAUGE

- The degrees of freedom (dof) should be the same as in the unitary gauge, that is, 6 dof.
- How do constraints enter in the theory?

Let’s study general gauge!

- Studying general gauge tells us richer information of the theory, which we can not get from the unitary gauge.

ex. Static case $\phi = \phi(\vec{x})$, Decoupling limit (Minkowski limit) and etc..

1. Introduction
2. Convenient form of the Lagrangian
3. Hamiltonian in general gauge
4. Minkowski limit and flat FLRW case
5. Summary

- I omit the remaining section because it's a preliminary result. Thank you.

“Spatially covariant gravity and unifying framework for scalar-tensor theories of gravity”

Xian Gao

[JGRG24(2014)111114]

Spatially covariant gravity and unifying framework for scalar-tensor theories

Xian Gao (高 顯)
Tokyo Institute of Technology

November 11, 2014
Kavli IPMU, the University of Tokyo
JGRG 24

X. Gao, *Phys.Rev. D* ***90*** *(2014) 081501(R), [arXiv:1406.0822]*
X. Gao*, Phys.Rev. D* ***90*** *(2014) in press, [arXiv:1409.6708]*
X. Gao*, [arXiv:141x.xxxx]*

Scalar-tensor theory?

Inflation, dark energy and dark matter have been strong motivations for alternative gravity theories beyond Einstein's general relativity.

→ Scalar-tensor theory:
scalar modes in addition to the tensor modes of GR.

→ How to introduce these extra degrees of freedom?

From *k*-essence to Horndeski

The most straightforward way:
to add gravity with extra scalar field(s), covariantly.

***k*-essence:** $\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} + K(\phi, X)\right]$ $\quad X = -\frac{1}{2}(\nabla\phi)^2$

Over the years, *k*-essence was studied as the most general theory for a single scalar field, which involves at most **first** derivatives of the field in the Lagrangian.

Higher derivatives $\rightarrow$ Extra unwantted mode(s)?

From *k*-essence to Horndeski

The most general single scalar-tensor theory:

- of which the Lagrangian involves **second** derivatives,

$$\mathcal{L}(\phi, \nabla\phi, \nabla\nabla\phi)$$

- the equations of motion stay at the **second** order in derivatives
  $\rightarrow$ only **one** scalar degree of freedom beyond GR

[G. W. Horndeski, Int.J.Theor.Phys. 10, 363 (1974)]
*[C. Deffayet, **X. Gao**, D. Steer, and G. Zahariade, Phys.Rev.D84, 064039 (2011)]*

$$\mathcal{L}_2 = G_2(X, \phi),$$

$$\mathcal{L}_3 = G_3(X, \phi)\,\Box\phi,$$

*[**D**vali, **G**abadadze and **P**orrati, Phys.Lett.B485, 208(2000)]*

$$\mathcal{L}_4 = G_4(X, \phi)\,R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$

$$\mathcal{L}_5 = G_5(X, \phi)\,G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right].$$

Beyond Horndeski?

- Even higher (≧3) order derivatives?
- Degrees of freedom unchanged (2 tensor + 1 scalar)?

This "straightforward and covariant" approach can only bring us so far...

Alternative approach?

Additional degree(s) of freedom may arise when symmetries are reduced:

- Massive gravity: 2t + 2v + 1s — breaks spacetime diff.
- Massive vector: 2v + 1s — breaks $U(1)$
- **Scalar-tensor theory**: 2t + 1s — spacetime diff. $\Downarrow$ **spatial** diff.

Example 1: EFT of inflation

Cosmological backgrounds breaks the full spacetime symmetries by choosing a preferred time direction or preferred spatial sclices.

The Lagrangian respects unbroken spatial diffs of the FRW background.

The basic ingredients are just perturbative ADM variables:

$$\delta N, \qquad \delta K_{\mu\nu}$$

lapse function — extrinsic curvature

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \Lambda(t) + f_1(t)\,\delta N + f_2(t)\,\delta N^2 + \cdots \right.$$
$$\left. + g_1(t)\,\delta K^\mu_\mu + g_2(t)\left(\delta K^\mu_\mu\right)^2 + g_3(t)\,\delta K_{\mu\nu}\delta K^{\mu\nu} + \cdots\right]$$

[Cheung, Creminelli, Fitzpatrick, Kaplan, and Senatore, JHEP 0803, 014 (2008)]

Example 2: Hořava gravity

Hořava gravity:

$$S^{(\text{Horava})} = \frac{1}{2}\int d^4x N\sqrt{h}\left(K_{ij}K^{ij} - \lambda K^2 + \mathcal{V}\left[h_{ij}, {}^{(3)}R_{ij}, D_i\right]\right)$$

[P. Horava, Phys.Rev. D79, 084008 (2009)]

Healthy extensions:

$$S^{(\text{Healthy Ext.})} = \frac{1}{2}\int d^4x N\sqrt{h}\left(c_1 a_i a^i + c_2\left(a_i a^i\right)^2 + c_3 R_{ij}a^i a^j + \cdots\right)$$

$$a_i = \partial_i \ln N$$

[Blas, Pujolas & Sibiryakov, JHEP 0910, 029 (2009)]

→ *N* enters the Hamiltonian "nonlinearly"!

Example 3: Horndeski in ADM form

Fixing the unitary (uniform scalar field) gauge: $\phi(t,\vec{x}) \equiv \phi_0(t) \equiv t$

$$\nabla_\mu \phi = -\frac{1}{N}\delta^0_\mu$$

$$\nabla_\mu\nabla_\nu\phi = -\delta^0_\mu\delta^0_\nu\frac{1}{N^2}\left(\partial_t \ln N - N^i\nabla_i \ln N\right) + \frac{2}{N}\delta^0_{(\mu}\delta^i_{\nu)}\partial_i \ln N - \frac{1}{N}\delta^i_\mu\delta^j_\nu K_{ij}$$

Horndeski in the ADM form:

[Gleyzes, Langlois, Piazza & Vernizzi, arXiv:1304.4840]

$$\begin{aligned}\mathcal{L}^{\text{Horndeski}} \simeq\ & G_2 + \frac{1}{N^2}\frac{\partial F_3}{\partial\phi} \\ & + \left[G_4 - \frac{1}{2N^2}\frac{\partial\left(G_5 - F_5\right)}{\partial\phi}\right]{}^{(3)}R \\ & + \left[\left(\frac{\partial F_3}{\partial N} - 2\frac{1}{N}\frac{\partial G_4}{\partial\phi}\right)h_{ij} - \frac{1}{N}F_5\,{}^{(3)}G_{ij}\right]K^{ij} \\ & - \left(\frac{\partial\left(NG_4\right)}{\partial N} + \frac{1}{2N^2}\frac{\partial G_5}{\partial\phi}\right)\left(K^2 - K_{ij}K^{ij}\right) \\ & - \frac{1}{6}\frac{\partial G_5}{\partial N}\left(K^3 - 3KK_{ij}K^{ij} + 2K^i_jK^j_kK^k_i\right)\end{aligned}$$

Example 3: Horndeski in ADM form

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functions of (t, N)

Beyond Horndeski

Fixing the unitary (uniform scalar field) gauge: $\phi\left(t,\vec{x}\right)\equiv\phi_0\left(t\right)\equiv t$

$$\nabla_\mu\phi=-\frac{1}{N}\delta^0_\mu$$

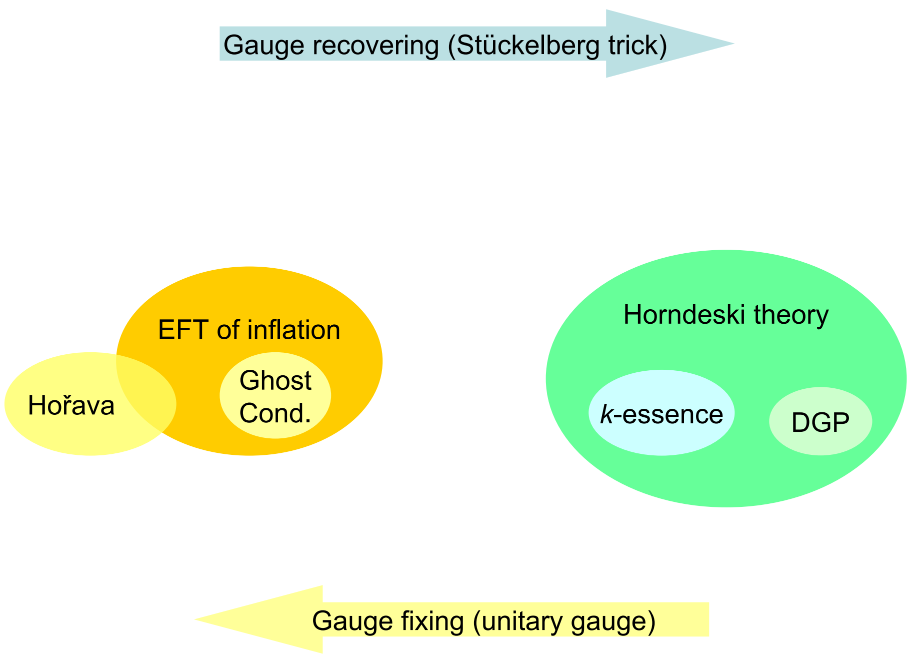
$$\nabla_\mu\nabla_\nu\phi=-\delta^0_\mu\delta^0_\nu\frac{1}{N^2}\left(\partial_t\ln N-N^i\nabla_i\ln N\right)+\frac{2}{N}\delta^0_{(\mu}\delta^i_{\nu)}\partial_i\ln N-\frac{1}{N}\delta^i_\mu\delta^j_\nu K_{ij}$$

GLPV model (deformed Horndeski):

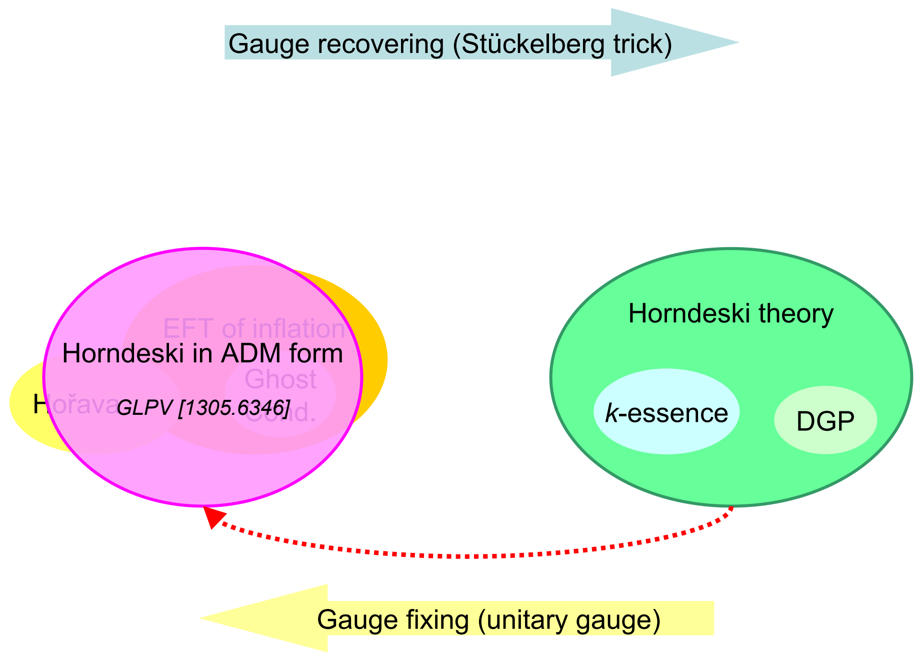
[Gleyzes, Langlois, Piazza & Vernizzi, arXiv:1404.6495]

$$\begin{aligned}\mathcal{L}^{\mathrm{GLPV}}=\;& A_2\left(t,N\right)\\ &+\left[\quad B_4\left(t,N\right)\quad\right]{}^{(3)}R\\ &+\left[\left(\quad A_3\left(t,N\right)\quad\right)h_{ij}+B_5\left(t,N\right){}^{(3)}G_{ij}\right]K^{ij}\\ &+\left(\quad A_4\left(t,N\right)\quad\right)\left(K^2-K_{ij}K^{ij}\right)\\ &+A_5\left(t,N\right)\left(K^3-3KK_{ij}K^{ij}+2K^i_jK^j_kK^k_i\right)\end{aligned}$$

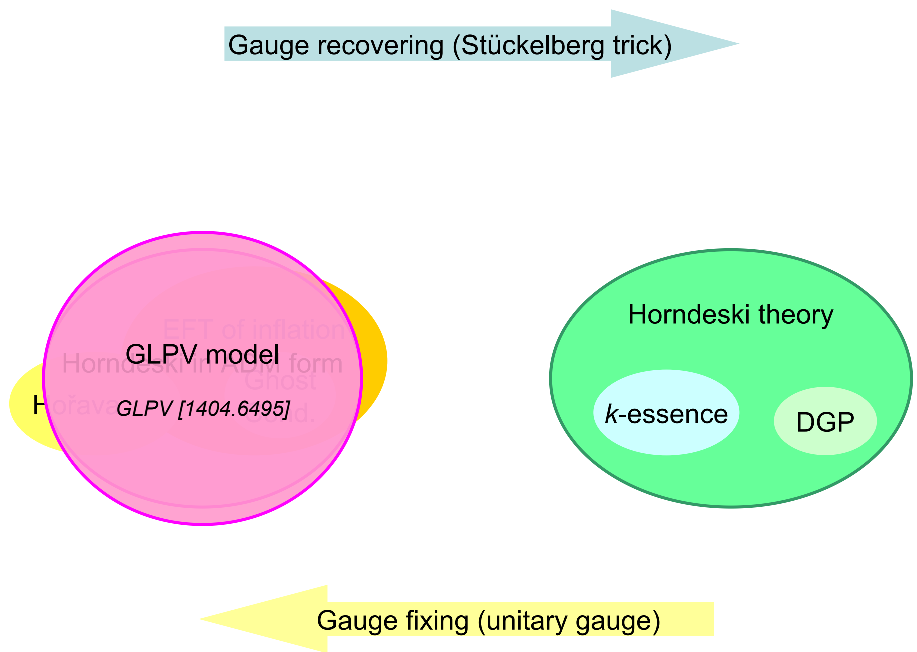
Landscape of theories



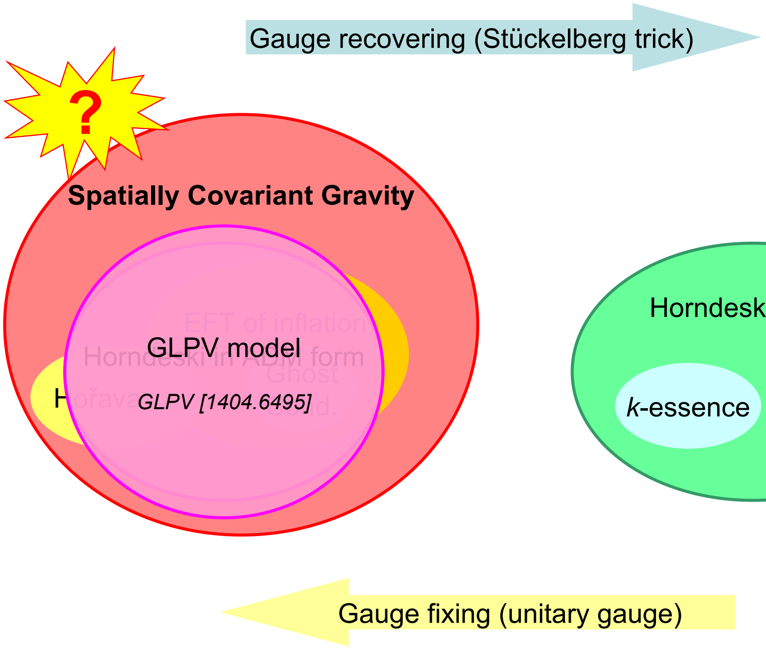
Landscape of theories



Landscape of theories



Landscape of theories



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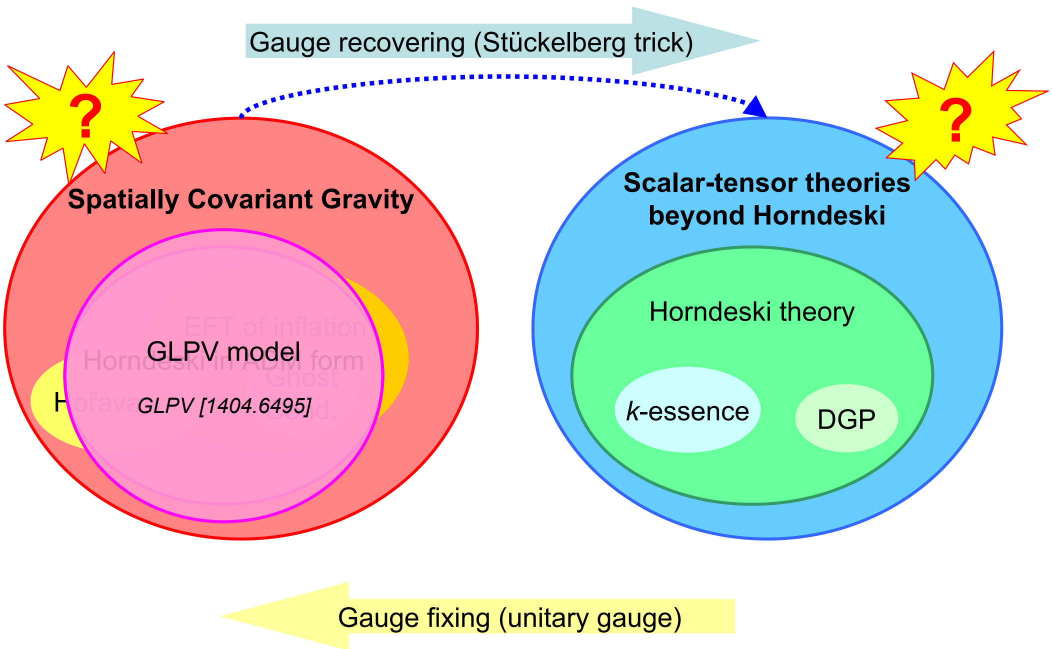


Spatially covariant gravity

A general class of Lagrangians that respect the spatial diffeomorphism:

$$\sqrt{-g}\mathcal{L} = N\sqrt{h}\left(\sum_{n=1}\underbrace{\mathcal{G}_{(n)}^{i_1j_1,\cdots,i_nj_n} K_{i_1j_1}\cdots K_{i_nj_n}}_{\text{"kinetic terms"}} + \underbrace{\mathcal{V}}_{\text{"potential terms"}}\right)$$

[X. Gao, Phys.Rev. D 90 (2014) 081501]

where $\mathcal{V}, \mathcal{G}_{(n)}$'s are functions of

$$\left(t, N, {}^{(3)}h_{ij}, {}^{(3)}R_{ij}, \nabla_i\right)$$

"Translating" to the covariant language (Stueckelberg trick)

$$N \to N = 1/\sqrt{2X}, \qquad h_{ij} \to h_{\mu\nu} = g_{\mu\nu} + \frac{1}{2X}\partial_\mu\phi\partial_\nu\phi, \qquad X \equiv -(\partial\phi)^2/2,$$

$$K_{ij} \to K_{\mu\nu} = -\frac{1}{\sqrt{2X}}\left[\nabla_\mu\nabla_\nu\phi - \frac{1}{4X}\nabla_\mu\phi\nabla_\nu\phi\nabla_\rho\phi\nabla^\rho\ln X - \nabla_{(\mu}\phi\nabla_{\nu)}\ln X\right],$$

All terms can be written covariantly in terms of ϕ and its derivatives.

→ A more general class of scalar-tensor theories beyond Horndeski, which propagate **2 tensor + 1 scalar** dofs, although the equations of motion are generally higher order.

Constraint analysis

4 primary constraints:

$$\pi_N \equiv \frac{\partial\left(N\sqrt{h}\mathcal{L}\right)}{\partial\dot{N}} \approx 0, \qquad \pi_i \equiv \frac{\partial\left(N\sqrt{h}\mathcal{L}\right)}{\partial\dot{N}^i} \approx 0,$$

Extended Hamiltonian: $H_{\rm ex} = \int d^3x\left(N\tilde{\mathcal{C}} + N_i\mathcal{C}^i + \lambda^N\pi_N + \lambda^i\pi_i\right)$

$$\tilde{\mathcal{C}} = \tilde{\mathcal{C}}\left(t, N, h_{ij}, R_{ij}, \nabla_i, \pi^{ij}\right), \qquad \mathcal{C}^i \equiv -2\sqrt{h}\nabla_j\left(\frac{\pi^{ij}}{\sqrt{h}}\right)$$

N appears nonlinearly in the Hamiltonian, as the space-dependent time reparametrization invariance is broken.

4 secondary constraints:

$$\frac{\mathrm{d}}{\mathrm{d}t}\pi_N \approx \{\pi_N, H_{\rm ex}\}_{\rm P} = -\mathcal{C} = -\mathcal{C}\left(t, N, h_{ij}, R_{ij}, \nabla_i, \pi^{ij}\right),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\pi_i \approx \{\pi_i, H_{\rm ex}\}_{\rm P} = -\mathcal{C}_i.$$

Degrees of freedom

Poisson brackets among all 8 constraints:

X. Gao, [arXiv:1409.6708]

$\{\cdot,\cdot\}_{\mathrm{P}}$	π_N	π_j	$\mathcal{C}$	$\mathcal{C}_j$
π_N	0	0	$-\frac{\delta\mathcal{C}}{\delta N}$	0
π_i	0	0	0	0
$\mathcal{C}$	$\frac{\delta\mathcal{C}}{\delta N}$	0	0	$-\frac{\delta\mathcal{C}}{\delta N}\nabla_j N$
$\mathcal{C}_i$	0	0	$\frac{\delta\mathcal{C}}{\delta N}\nabla_i N$	0

Eigenvalues: 6 zero, 2 non-zero: $\pm\left|\frac{\delta\mathcal{C}}{\delta N}\right|\sqrt{1+(\nabla_i N)^2}$

→ Among (linearly independent combinations of) 8 constraints:
6 are first class, 2 are second class

→ Number of degrees of freedom:

$$
\begin{aligned}
\text{number of d.o.f.} &= \frac{1}{2}\big(2\times\text{number of canonical variables} - 2\times\text{number of first class constraints} \\
&\quad -\text{number of second class constraints}\big) \\
&= \frac{1}{2}(2\times 10 - 2\times 6 - 2) = 3.
\end{aligned}
$$

Main message

• Single-field scalar-tensor theories can be written as theories of spatially covariant gravity.

• We propose a very general framework for the spatially covariant gravity theories.

• When restoring general covariance, such spatially covariant gravity theories yield single-field scalar-tensor theories with higher order equations of motion.

Thank you for your attention!

“Effective field theory approach to modified gravity including Horndeski theory and Horava-Lifshitz gravity”

Ryotaro Kase

[JGRG24(2014)111115]

“JGRG24,” IPMU in Tokyo, 11th Nov. 2014.

Effective field theory approach to modified gravity including Horndeski theory and Horava–Lifshitz gravity

R. Kase and S. Tsujikawa, arXiv 1409.1984

Tokyo University of Science
Ryotaro Kase

1. Introduction

- **Discovery of late-time cosmic acceleration**

In 1998, the discovery of late-time cosmic acceleration based on Type Ia supernovae is reported. The source for this acceleration is named dark energy.

The equation of state defined below characterizes dark energy.

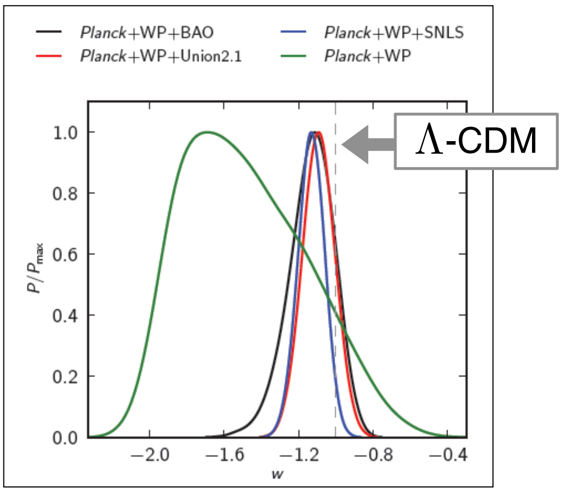
$$w \equiv P/\rho$$

Condition for acceleration :

$$w < -1/3$$

- Planck+WP+SNLS

$$w = -1.13^{+0.13}_{-0.14}\ (95\%\mathrm{CL})$$



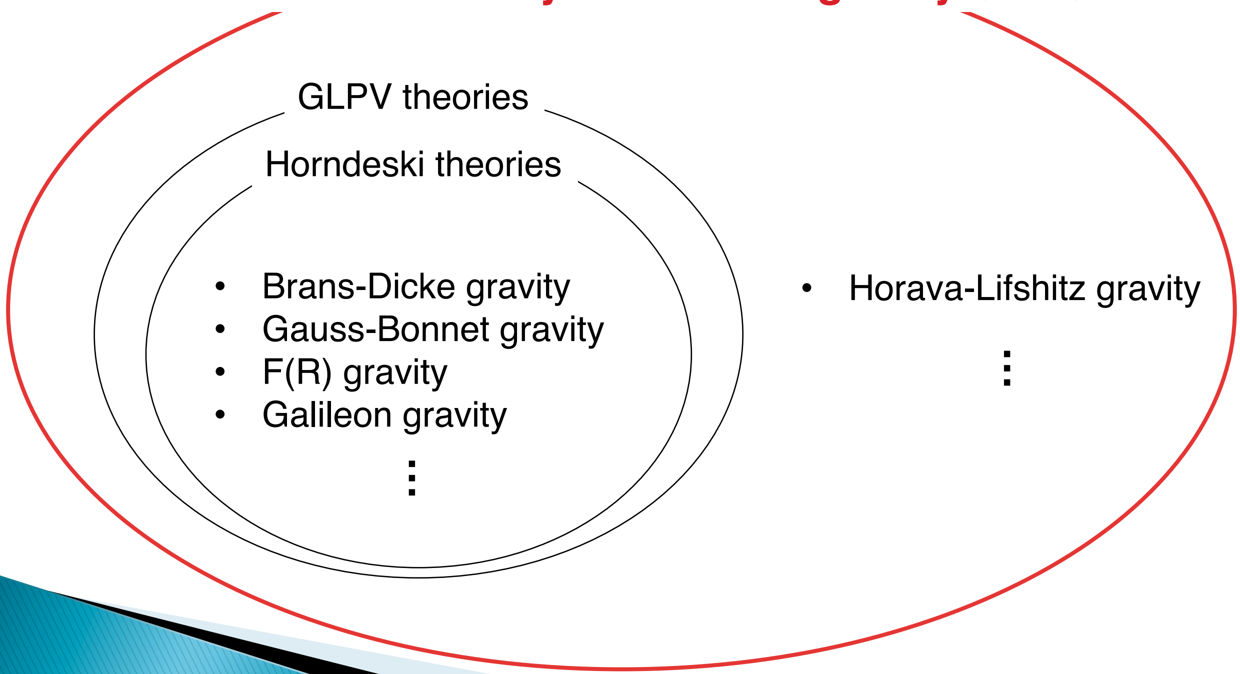
Planck collaboration arXiv:1303.5076 [astro-ph.CO]

Dark energy problem may imply some modification of gravity on large scales.

1. Introduction

- **Models based on Modified gravity**

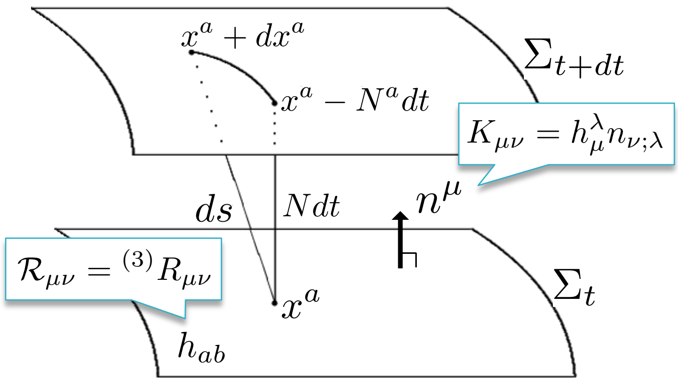
Effective field theory of modified gravity（EFT）



1. Introduction

- **EFT on the cosmological background**

J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, JCAP 1308, 025 (2013)



$$ds^2 = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

Under the unitary gauge $(\delta\phi = 0)$,

$$\phi = \phi(t)$$

constant time hypersurfaces

$\Updownarrow$

uniform ϕ hypersurfaces

$$n^\mu = -\phi_;{}^\mu / \sqrt{-X}$$

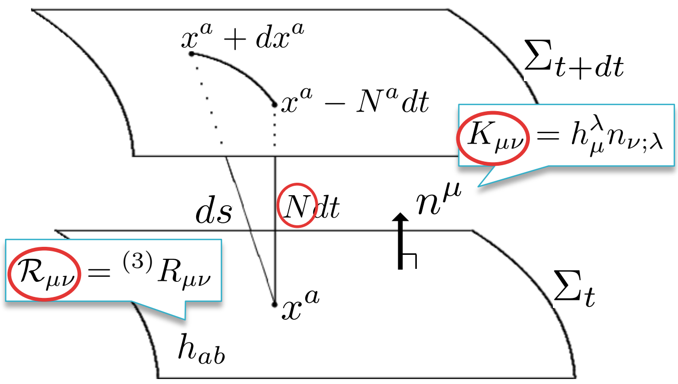
$X = \phi_;{}^\mu \phi_{;\mu}$

A scalar field ϕ associated with the modification of gravity is absorbed into the constant time hypersurfaces.

1. Introduction

EFT on the cosmological background

J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, JCAP 1308, 025 (2013)



$$ds^2 = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

Under the unitary gauge $(\delta\phi = 0)$,

$$\phi = \phi(t)$$

constant time hypersurfaces

$\Updownarrow$

uniform ϕ hypersurfaces

$$n^\mu = -\phi_;{}^\mu / \sqrt{-X}$$

$$\boxed{S = \int d^4x \sqrt{-g} L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}; t)}$$

$$K \equiv K^\mu{}_\mu\,, \quad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu}\,, \quad \mathcal{R} \equiv \mathcal{R}^\mu{}_\mu\,, \quad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\,, \quad \mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}\,.$$

1. Introduction

Horndeski Lagrangians in the EFT language

$$G_2(\phi, X) \to G_2(N, t) \quad \left(X = -\dot{\phi}^2/N^2\right),$$

$$\boxed{n^\mu = -\phi_;{}^\mu / \sqrt{-X}}$$

$$G_3(\phi, X)\Box\phi \to 2(-X)^{3/2} F_{3,X} K - X F_{3,\phi} \quad (G_3 = F_3 + 2XF_{3,X}),$$

$$\vdots \qquad\qquad \vdots$$

$$L = A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - \mathcal{S}) + B_4(N,t)\mathcal{R}$$
$$+ A_5(N,t)K_3 + B_5(N,t)\left(\mathcal{U} - K\mathcal{R}/2\right),$$

with $A_4 = 2XB_{4,X} - B_4 \quad A_5 = -XB_{5,X}/3$

$(K_3 = 3H(2H^2 - 2KH + K^2 - \mathcal{S}) + O(3))$

The Horndeski theory is a subclass of the EFT of modified gravity.

$$\boxed{S = \int d^4x \sqrt{-g} L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}; t)}$$

$$K \equiv K^\mu{}_\mu\,, \quad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu}\,, \quad \mathcal{R} \equiv \mathcal{R}^\mu{}_\mu\,, \quad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\,, \quad \mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}\,.$$

1. Introduction

- **Horava gravity in the EFT language**

In order to include the Horava gravity and its extension in the EFT framework, we need to add extra terms to the EFT Lagrangian.

Gao's talk! X. Gao Phys. Rev. D90 (2014) 081501

- **Projectable Horava-Lifshitz gravity** $(\delta N = 0)$

∇_i : 3D covariant derivative

$$L = \frac{M_{\rm pl}^2}{2}\left[\mathcal{S} - \lambda K^2 + \mathcal{R} - M_{\rm pl}^{-2}\left(g_2\mathcal{R}^2 + g_3\mathcal{Z}\right) - M_{\rm pl}^{-4}\left(g_4\mathcal{Z}_1 + g_5\mathcal{Z}_2\right)\right]$$

$$\left(\mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\,, \quad \mathcal{Z}_1 \equiv \nabla_i\mathcal{R}\nabla^i\mathcal{R}\,, \quad \mathcal{Z}_2 \equiv \nabla_i\mathcal{R}_{jk}\nabla^i\mathcal{R}^{jk}\right)$$

The terms $\mathcal{Z}_1$, $\mathcal{Z}_2$ allow the $z = 3$ scaling characterized by the transformation $t \to c^z t$ and $x^i \to c x^i$.

The theory is power-counting renormalizable.

However, in this theory, the no-ghost condition and the condition to avoid a Laplacian instability cannot be satisfied at the same time. Moreover there is the strong coupling problem in the deep IR regime.

1. Introduction

- **Non-projectable Horava-Lifshitz gravity** $(\delta N \neq 0)$

D. Blas, O. Pujolas and S. Sibiryakov, (2010)

In the non-projectable extended version of the Horava gravity, the acceleration vector $a_\nu = n^\lambda n_{\nu;\lambda} = \nabla_\nu \ln N$ does not vanish. In this case one can consider the Lagrangian

$$L_{\mathcal{V}_3} = -\frac{1}{2M_{\rm pl}^2}\left(g_4\mathcal{Z}_1 + g_5\mathcal{Z}_2 + \eta_4\alpha_4 + \eta_5\alpha_5 + \cdots\right)\,,$$

$$L_{\mathcal{V}_2} = -\frac{1}{2}\left(g_2\mathcal{R}^2 + g_3\mathcal{Z} + \eta_2\alpha_2 + \eta_3\alpha_3 + \cdots\right)\,,$$

$$L_{\mathcal{V}_1} = \frac{M_{\rm pl}^2}{2}\left(\mathcal{R} + \eta_1\alpha_1\right)\,, \qquad \alpha_1 \equiv a_i a^i\,, \quad \alpha_2 \equiv a_i\Delta a^i\,, \quad \alpha_3 \equiv \mathcal{R}\nabla_i a^i\,, \quad \alpha_4 \equiv a_i\Delta^2 a^i\,, \quad \alpha_5 \equiv \Delta\mathcal{R}\nabla_i a^i\,,$$

$L_{\mathcal{V}_3}$, $L_{\mathcal{V}_2}$, $L_{\mathcal{V}_1}$ are invariant under $z = 3,\, 2,\, 1$ rescaling, respectively.

1. Introduction

- **EFT Lagrangian including Horndeski theories and Horava gravity**

$$S = \int d^4x \sqrt{-g}\, L\,(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}, \mathcal{Z}_1, \mathcal{Z}_2, \alpha_1, \cdots, \alpha_5; t)$$

RK and S. Tsujikawa, arXiv:1409.1984

$$K \equiv K^{\mu}{}_{\mu}\,, \quad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu}\,, \quad \mathcal{R} \equiv \mathcal{R}^{\mu}{}_{\mu}\,, \quad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\,,$$
$$\mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}\,, \mathcal{Z}_1 \equiv \nabla_i\mathcal{R}\nabla^i\mathcal{R}\,, \quad \mathcal{Z}_2 \equiv \nabla_i\mathcal{R}_{jk}\nabla^i\mathcal{R}^{jk}\,,$$
$$\alpha_1 \equiv a_i a^i\,, \quad \alpha_2 \equiv a_i \Delta a^i\,, \quad \alpha_3 \equiv \mathcal{R}\nabla_i a^i\,, \quad \alpha_4 \equiv a_i \Delta^2 a^i\,, \quad \alpha_5 \equiv \Delta\mathcal{R}\nabla_i a^i\,,$$

Other terms such, e.g. $\mathcal{R}_i^j\mathcal{R}_j^k\mathcal{R}_k^i$, can be taken into account, but they are irrelevant to scalar linear perturbations on the flat FLRW background.

2. Background Equations

$$S = \int d^4x \sqrt{-g}\, L\,(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}, \mathcal{Z}_1, \mathcal{Z}_2, \alpha_1, \cdots, \alpha_5; t)$$
$$ds^2 = -(1+2\delta N)dt^2 + 2\nabla_i\psi dx^i dt + a^2(t)(1+2\zeta)\delta_{ij}dx^i dx^j\,,$$

Expanding the Lagrangian up to linear order as

e.g. $L_{,N} = \partial L/\partial N$

$$L = \bar{L} + L_{,N}\delta N + L_{,K}\delta K + L_{,\mathcal{S}}\delta\mathcal{S} + L_{,\mathcal{R}}\delta\mathcal{R} + L_{,\mathcal{Z}}\delta\mathcal{Z} + L_{,\mathcal{U}}\delta\mathcal{U} + O(2)\,,$$

expressing ADM variables in terms of metric variables, e.g. $K_{ij} = (\partial_t h_{ij} - \nabla_i N_j - \nabla_j N_i)/(2N)$, we obtain the following background equations of motion.

$$S_1 = \int d^4x \sqrt{-\bar{g}}\left[\mathcal{E}^N\delta N + \mathcal{E}^h\delta\sqrt{h}\right]\,,$$
$$\mathcal{E}^N = \bar{L} + L_{,N} - 3H\mathcal{F} = 0\,,$$
$$\mathcal{E}^h = \bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F} = 0\,.$$
$$(\mathcal{F} \equiv L_{,K} + 2HL_{,\mathcal{S}})$$

3. Second order perturbations

$$S=\int d^4x\sqrt{-g}\,L\left(N,K,\mathcal{S},\mathcal{R},\mathcal{Z},\mathcal{U},\mathcal{Z}_1,\mathcal{Z}_2,\alpha_1,\cdots,\alpha_5;t\right)$$

$$ds^2=-(1+2\delta N)dt^2+2\nabla_i\psi dx^i dt+a^2(t)(1+2\zeta)\delta_{ij}dx^i dx^j\,,$$

Expanding the Lagrangian up to second order as

$$\begin{aligned}L=&\;\bar{L}-\dot{\mathcal{F}}-3H\mathcal{F}+(\dot{\mathcal{F}}+L_{,N})\delta N+\mathcal{E}\delta_1\mathcal{R}\\&+\left(\frac{1}{2}L_{,NN}-\dot{\mathcal{F}}\right)\delta N^2+\frac{1}{2}\mathcal{A}\delta K^2+\mathcal{B}\delta K\delta N+\mathcal{C}\delta K\delta_1\mathcal{R}+\mathcal{D}\delta N\delta_1\mathcal{R}+\mathcal{E}\delta_2\mathcal{R}+\frac{1}{2}\mathcal{G}\delta_1\mathcal{R}^2\\&+L_{,\mathcal{S}}\delta K^\mu_\nu\delta K^\nu_\mu+L_{,\mathcal{Z}}\delta\mathcal{R}^\mu_\nu\delta\mathcal{R}^\nu_\mu+\sum_{i=1}^{2}L_{,\mathcal{Z}_i}\delta\mathcal{Z}_i+\sum_{i=1}^{5}L_{,\alpha_i}\delta\alpha_i+O(3)\,,\end{aligned}$$

e.g. $\mathcal{A}=L_{,KK}+4HL_{,\mathcal{S}K}+4H^2L_{,\mathcal{S}\mathcal{S}}\,,$

Varying with respect to δN and $\Delta\psi$, we obtain evolution equations as

$$(2L_{,N}+\dots)\,\delta N-2L_{,\alpha_1}\Delta\delta N-2L_{,\alpha_2}\Delta^2\delta N-2L_{,\alpha_4}\Delta^3\delta N-\mathcal{W}\Delta\psi=3\mathcal{W}\dot{\zeta}+\dots\,,$$
$$\mathcal{W}\delta N-(\mathcal{A}+2L_{,\mathcal{S}})\Delta\psi=-(3\mathcal{A}+2L_{,\mathcal{S}})\dot{\zeta}+4\mathcal{C}\Delta\zeta$$

Using these equations the second order Lagrangian is expressed in terms of a single variable ζ.

3. Second order perturbations

$$S=\int d^4x\sqrt{-g}\,L\left(N,K,\mathcal{S},\mathcal{R},\mathcal{Z},\mathcal{U},\mathcal{Z}_1,\mathcal{Z}_2,\alpha_1,\cdots,\alpha_5;t\right)$$

$$ds^2=-(1+2\delta N)dt^2+2\nabla_i\psi dx^i dt+a^2(t)(1+2\zeta)\delta_{ij}dx^i dx^j\,,$$

- **In the absence of higher order spatial derivatives**

$(\mathcal{C}=0\,,\;4\mathcal{G}+3L_{,\mathcal{Z}}=0\,,\;\mathcal{A}+2L_{,\mathcal{S}}=0\,,\;8L_{,\mathcal{Z}_1}+3L_{,\mathcal{Z}_2}=0\,,\;L_{,\alpha_1}=L_{,\alpha_2}=\cdots=L_{,\alpha_5}=0\,.)$

$$\mathcal{L}_2=a^3Q_s\left[\dot{\zeta}^2-\frac{c_s^2}{a^2}(\partial\zeta)^2\right]\,.$$

$$Q_s\equiv\frac{2L_{,\mathcal{S}}}{\mathcal{W}^2}\left[3\mathcal{W}^2+4L_{,\mathcal{S}}(2L_{,N}+L_{,NN}-6H\mathcal{W}+12H^2L_{,\mathcal{S}})\right]\,,$$
$$c_s^2\equiv\frac{2}{Q_s}\left(\dot{\mathcal{M}}+H\mathcal{M}-\mathcal{E}\right)\,,$$
$$\mathcal{M}\equiv\frac{4L_{,\mathcal{S}}}{\mathcal{W}}\left(L_{,\mathcal{R}}+L_{,N\mathcal{R}}+HL_{,N\mathcal{U}}+\frac{3}{2}HL_{,\mathcal{U}}\right)\,,$$
$$\mathcal{W}\equiv L_{,KN}+H\left(2L_{,N\mathcal{S}}-3L_{KK}-2L_{,\mathcal{S}}\right)-12H^2L_{,K\mathcal{S}}-12H^3L_{,\mathcal{S}\mathcal{S}}\,.$$

Stability conditions

$$Q_s>0\text{ and }c_s^2>0\,.$$

4. Application to Horndeski and GLPV

$$L = A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - \mathcal{S}) + B_4(N,t)\mathcal{R} + A_5(N,t)K_3 + B_5(N,t)\left(\mathcal{U} - K\mathcal{R}/2\right) \,,$$

J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, arXiv:1404.6495

Horndeski theories correspond to

$$A_4 = 2XB_{4,X} - B_4 \qquad A_5 = -XB_{5,X}/3$$

- **Stability conditions**

Condition to avoid ghost instability

$$Q_s > 0 \leftrightarrow 9\mathcal{W}^2 + 8L_{,\mathcal{S}}w > 0$$

$$\mathcal{W} = A_{3,N} + 4HA_{4,N} + 6H^2A_{5,N} - 4HA_4 - 12H^2A_5 \,,$$
$$w = 18H^2(A_4 + 3HA_5) + 3(A_{2,N} - 6H^2A_{4,N} - 12H^3A_{5,N}) + 2(A_{2,NN} + 3HA_{3,NN} + 6H^2A_{4,NN} + 6H^3A_{5,NN})/3 \,.$$

4. Application to Horndeski and GLPV

$$L = A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - \mathcal{S}) + B_4(N,t)\mathcal{R} + A_5(N,t)K_3 + B_5(N,t)\left(\mathcal{U} - K\mathcal{R}/2\right) \,,$$

J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, arXiv:1404.6495

Horndeski theories correspond to

$$A_4 = 2XB_{4,X} - B_4 \qquad A_5 = -XB_{5,X}/3$$

- **Stability conditions**

Condition to avoid Laplacian instability

$$c_s^2 > 0 \leftrightarrow \dot{\mathcal{M}} + H\mathcal{M} - \mathcal{E} > 0$$

$$\mathcal{M} = -\frac{4(A_4 + 3HA_5)(B_4 + B_{4,N} - HB_{5,N}/2)}{A_{3,N} + 4HA_{4,N} + 6H^2A_{5,N} - 4HA_4 - 12H^2A_5} \,,$$
$$\mathcal{E} = B_4 + \dot{B}_5/2 \,.$$

4. Application to Horndeski and GLPV

- **Dark energy in the presence of matter**

$$S = \int d^4x\sqrt{-g}\,[L(N,K,\mathcal{S},\mathcal{R},\mathcal{U};t) + P(\varphi,Y)]\,. \quad (Y = \varphi_{;}{}^{\mu}\varphi_{;\mu})$$

radiation	non-relativistic matter R. J. Scherrer (2004)
$P(\varphi,Y) = b_1 Y^2$	$P(\varphi,Y) = b_2(Y-Y_0)^2$
$w = 1/3$	$w = \frac{Y-Y_0}{3Y-Y_0} \simeq 0$ (when $Y \simeq Y_0$)

Sound speeds squared

$$\left(c_s^2 - c_{sH1}^2\right)\left(c_s^2 - c_{sH2}^2\right) = \frac{16L_{,\mathcal{S}}^2}{Q_s\mathcal{W}^2}\boxed{\left(\frac{\mathcal{MW}}{4L_{,\mathcal{S}}^2}-1\right)}\dot{\varphi}^2 P_{,Y}\left[2c_s^2 - c_{sH2}^2\left(\frac{\mathcal{MW}}{4L_{,\mathcal{S}}^2}+1\right)\right].$$

In the Horndeski limit this term vanishes and we obtain

Dark energy: $c_{sH1}^2 = \frac{1}{Q_s}\left[2(\dot{\mathcal{M}} + H\mathcal{M} - \mathcal{E}) + \left(\frac{4L_{,\mathcal{S}}\dot{\varphi}}{\mathcal{W}}\right)^2 P_{,Y}\right],$

Matter: $c_{sH2}^2 = \frac{P_{,Y}}{P_{,Y} - 2\dot{\varphi}^2 P_{,YY}}\,.$

However, outside the Horndeski domain, both sound speeds should be modified.

4. Application to Horndeski and GLPV

- **Dark energy in the presence of matter**

$$S = \int d^4x\sqrt{-g}\,[L(N,K,\mathcal{S},\mathcal{R},\mathcal{U};t) + P(\varphi,Y)]\,. \quad (Y = \varphi_{;}{}^{\mu}\varphi_{;\mu})$$

Please see also
RK and S. Tsujikawa, Phys. Rev. D90 (2014) 044073

- detailed calculation
- evolution of sound speeds during the cosmological history

radiation	non-relativistic matter R. J. Scherrer (2004)
$P(\varphi,Y) = b_1 Y^2$	$P(\varphi,Y) = b_2(Y-Y_0)^2$
$w = 1/3$	$w = \frac{Y-Y_0}{3Y-Y_0} \simeq 0$ (when $Y \simeq Y_0$)

Sound speeds squared

$$\left(c_s^2 - c_{sH1}^2\right)\left(c_s^2 - c_{sH2}^2\right) = \frac{16L_{,\mathcal{S}}^2}{Q_s\mathcal{W}^2}\boxed{\left(\frac{\mathcal{MW}}{4L_{,\mathcal{S}}^2}-1\right)}\dot{\varphi}^2 P_{,Y}\left[2c_s^2 - c_{sH2}^2\left(\frac{\mathcal{MW}}{4L_{,\mathcal{S}}^2}+1\right)\right].$$

In the Horndeski limit this term vanishes and we obtain

Dark energy: $c_{sH1}^2 = \frac{1}{Q_s}\left[2(\dot{\mathcal{M}} + H\mathcal{M} - \mathcal{E}) + \left(\frac{4L_{,\mathcal{S}}\dot{\varphi}}{\mathcal{W}}\right)^2 P_{,Y}\right],$

Matter: $c_{sH2}^2 = \frac{P_{,Y}}{P_{,Y} - 2\dot{\varphi}^2 P_{,YY}}\,.$

However, outside the Horndeski domain, both sound speeds should be modified.

5. Application to Horava gravity

- **Projectable Horava-Lifshitz gravity** $(\delta N = 0)$

$$L = \frac{M_{\rm pl}^2}{2}\left[\mathcal{S} - \lambda K^2 + \mathcal{R} - M_{\rm pl}^{-2}\left(g_2\mathcal{R}^2 + g_3\mathcal{Z}\right) - M_{\rm pl}^{-4}\left(g_4\mathcal{Z}_1 + g_5\mathcal{Z}_2\right)\right]$$

$$\mathcal{L}_2 = M_{\rm pl}^2 a^3\left(\frac{3\lambda-1}{\lambda-1}\dot{\zeta}^2 - \zeta\mathcal{O}\zeta\right)$$

$$\mathcal{O} \equiv \Delta + \frac{\Delta^2}{M_2^2} - \frac{\Delta^3}{M_3^4}\,, \quad \Delta \equiv \nabla^i\nabla_i\,, \quad M_2^2 \equiv M_{\rm pl}^2(8g_2+3g_3)^{-1}\,, \quad M_3^4 \equiv M_{\rm pl}^4(8g_4+3g_5)^{-1}\,.$$

Conditions to avoid ghost and Laplacian instability
can not be satisfied at the same time.

which coincides with the results in
K. Koyama and F. Arroja, JHEP 1003, 061 (2010),
S. Mukohyama, Class. Quant. Grav. 27, 223101 (2010).

5. Application to Horava gravity

- **Non-projectable Horava-Lifshitz gravity** $(\delta N \neq 0)$

$$L = \frac{M_{\rm pl}^2}{2}\Big[\mathcal{S} - \lambda K^2 + \mathcal{R} + \eta_1\alpha_1 - M_{\rm pl}^{-2}\left(g_2\mathcal{R}^2 + g_3\mathcal{Z} + \eta_2\alpha_2 + \eta_3\alpha_3\right) - M_{\rm pl}^{-4}\left(g_4\mathcal{Z}_1 + g_5\mathcal{Z}_2 + \eta_4\alpha_4 + \eta_5\alpha_5\right)\Big]\,.$$

In the IR regime, on the Minkowski BG,

$$\mathcal{L}_2 = M_{\rm pl}^2\frac{3\lambda-1}{\lambda-1}\left[\dot{\zeta}^2 - c_s^2(\partial\zeta)^2\right] \qquad \left(c_s^2 = \frac{\lambda-1}{3\lambda-1}\frac{2-\eta_1}{\eta_1}\right)$$

which coincides with the results in
D. Blas, O. Pujolas and S. Sibiryakov,
Phys. Rev. Lett. 104, 181302 (2010)

6. Conclusions

- We studied the EFT approach to modified gravity including Horndeski theories and Horava-Lifshitz gravity on the flat isotropic cosmological BG.
- Expanding the action up to second order, we derived the background equations of motion, equations of motion for linear perturbations and stability conditions.
- We applied our general results to Horndeski theories, its generalization (GLPV theories), Horava gravity and its healthy extension.
- In the presence of matter components, sound speeds squared are nontrivially modified in GLPV theories. We showed that Horndeski theories and GLPV theories can be distinguished from each other by the scalar propagation speeds c_s^2.
- We showed that our general results conveniently recover stability conditions of Horava gravity and its healthy extension already derived in the literature.

4. Application to Horndeski and GLPV

$$
\begin{aligned}
L = & A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - \mathcal{S}) + B_4(N,t)\mathcal{R} \\
& + A_5(N,t)K_3 + B_5(N,t)\left(\mathcal{U} - K\mathcal{R}/2\right) , \\
& \qquad \left(K_3 = 3H(2H^2 - 2KH + K^2 - \mathcal{S}) + O(3)\right)
\end{aligned}
$$

Horndeski theories correspond to

$$A_4 = 2XB_{4,X} - B_4 \qquad A_5 = -XB_{5,X}/3$$

- **Tensor perturbations** $h_{ij} = a^2(t)(\delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{ik}\gamma_{kj})$

$$S_2^{(h)} = \int d^4x \, \frac{a^3}{4}\left[L_{,\mathcal{S}}\dot{\gamma}_{ij}^2 - \frac{\mathcal{E}}{a^2}(\partial_k\gamma_{ij})^2\right] .$$

Stability conditions

$$L_{,\mathcal{S}} = -A_4 - 3HA_5 > 0 ,$$

$$\mathcal{E} = B_4 + \dot{B}_5/2 > 0 .$$

4. Application to Horndeski and GLPV

The inflationary power spectra of curvature and tensor perturbations

In the case where slow-roll parameters $\epsilon \equiv -\dot{H}/H^2\,,\ \delta_{Q_s} \equiv \dot{Q}_s/(HQ_s)\,,\ \delta_{c_s} \equiv \dot{c}_s/(Hc_s)$ are much smaller than unity,

- Scalar perturbation $\mathcal{L}_2 = a^3 Q_s \left[\dot{\zeta}^2 - \frac{c_s^2}{a^2}(\partial\zeta)^2\right]\,.$

$$\boxed{n_s - 1 \simeq -2\epsilon - \delta_{Q_s} - 3\delta_{c_s}\,.}$$

- Tensor perturbation $S_2^{(h)} = \sum_{\lambda=+,\times} \int d^4x\, a^3 Q_t \left[\dot{h}_\lambda^2 - \frac{c_t^2}{a^2}(\partial h_\lambda)^2\right]\,,$

$$r = 4\frac{Q_s c_s^3}{Q_t c_t^3}\,,$$

$$Q_t \equiv \frac{L_{,\mathcal{S}}}{2}\,,\quad c_t^2 \equiv \frac{\mathcal{E}}{L_{,\mathcal{S}}}\,.$$

4. Application to Horndeski and GLPV

Covariantized Galileon

Covariant Galileon : Covariantized Minkowski Galileon + Gravitational counter term

A. Nicolis, et al. (2009)
C. Deffayet, et al. (2009)

Since EOMs remain second order in general BG, it is inside the Horndeski domain.

$$A_2 = \frac{c_2}{2}X\,,\quad A_3 = \frac{c_3}{3M^3}(-X)^{3/2}\,,\quad A_4 = -\frac{M_{\rm pl}^2}{2} - \frac{3c_4}{4M^6}X^2\,,\quad A_5 = \frac{c_5}{2M^9}(-X)^{5/2}\,,$$
$$B_4 = \frac{M_{\rm pl}^2}{2} - \frac{c_4}{4M^6}X^2\,,\quad B_5 = -\frac{3c_5}{5M^9}(-X)^{5/2}\,.$$

***Covariantized* Galileon** : Covariantized Minkowski Galileon

Higher order derivatives may appear in general BG. Thus it is outside the Horndeski domain.

However, due to the symmetry of the FRW space-time, BG EOMs in two models become same. At the level of second order perturbations differences appear.

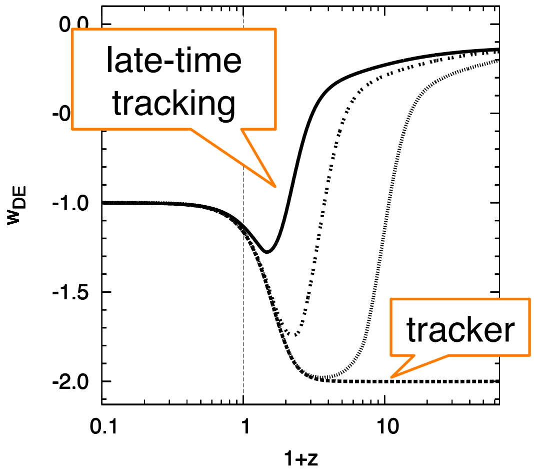
$$A_2 = \frac{c_2}{2}X\,,\quad A_3 = \frac{c_3}{3M^3}(-X)^{3/2}\,,\quad A_4 = -\frac{M_{\rm pl}^2}{2} - \frac{3c_4}{4M^6}X^2\,,\quad A_5 = \frac{c_5}{2M^9}(-X)^{5/2}\,,$$
$$B_4 = \frac{M_{\rm pl}^2}{2}\,,\quad B_5 = 0\,.$$

4. Application to Horndeski and GLPV

BG evolution

$$r_1 \equiv \frac{\dot{\chi}_{\rm dS} H_{\rm dS}}{\dot{\chi} H}\,, \quad r_2 \equiv \frac{H}{H_{\rm dS}} \left(\frac{\dot{\chi}}{\dot{\chi}_{\rm dS}}\right)^5\,,$$

- There is the dS point at $r_1 = r_2 = 1$.
- The tracker solution $(r_1 = 1)$ is in tension with the observational data.
- The late-time tracking solution $(r_1^{\rm ini} \ll 1)$ is consistent with the observational data.



S. Nesseris, A. De Felice and S. Tsujikawa, Phys. Rev. D 82, 124054 (2010)

4. Application to Horndeski and GLPV

Evolution of the propagation speed along the late-time tracking

(A) Covariant Galileon

$$c_{s1}^2 = \begin{cases} \dfrac{1}{40}(\Omega_r + 1) & [({\rm i})\ r_1 \ll 1,\ r_2 \ll 1]\,, \\ \dfrac{8 + 10\alpha - 9\beta + \Omega_r(2 + 3\alpha - 3\beta)}{3(2 - 3\alpha + 6\beta)} & [({\rm ii})\ r_1 = 1,\ r_2 \ll 1]\,, \\ \dfrac{(\alpha - 2\beta)(4 + 15\alpha^2 - 48\alpha\beta + 36\beta^2)}{2(2 + 3\alpha - 6\beta)(2 - 3\alpha + 6\beta)} & [({\rm iii})\ r_1 = 1,\ r_2 = 1]\,. \end{cases}$$

Under the no-ghost conditions,

$$L_S > 0, \quad g_2 > 0 \quad \Rightarrow \quad \beta > 0, \quad -2 < 3(\alpha - 2\beta) < 2$$

$$c_2 x_{\rm dS}^2 = 6 + 9\alpha - 12\beta$$
$$c_3 x_{\rm dS}^3 = 2 + 9\alpha - 9\beta$$

the above propagation speed of sound is positive in any regime.

A. De Felice and S. Tsujikawa, Phys. Rev. Lett. 105, 111301 (2010)

4. Application to Horndeski and GLPV

- **Evolution of the propagation speed along the late-time tracking**

(B) *Covariantized* Galileon

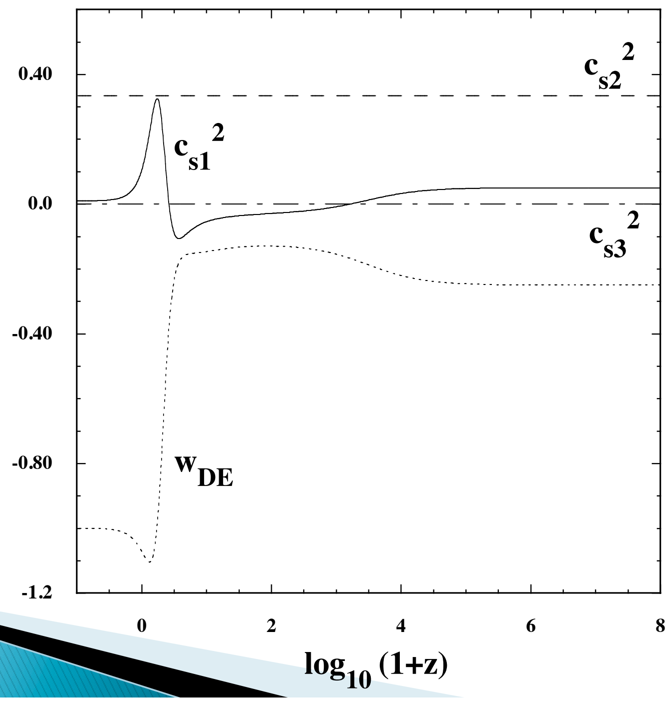
$$c_{s1}^2 = \begin{cases} \dfrac{1}{40}(3\Omega_r - 1) & [\text{(i)}\ r_1 \ll 1,\ r_2 \ll 1]\,, \\ \dfrac{16 - 15(\alpha - 2\beta) + \Omega_r(4 - 3\alpha + 6\beta)}{6(2 - 3\alpha + 6\beta)} & [\text{(ii)}\ r_1 = 1,\ r_2 \ll 1]\,, \\ \dfrac{\alpha - 2\beta}{2 + 3\alpha - 6\beta} & [\text{(iii)}\ r_1 = 1,\ r_2 = 1]\,. \end{cases}$$

In the regime (i), the propagation speed become negative during the matter dominated epoch!!

4. Application to Horndeski and GLPV

- **Evolution of the propagation speed along the late-time tracking**

(B) *Covariantized* Galileon



“The Relation Between Tree Unitarity and

Renormalizability in Lifshitz Scalar Theory”

Tomotaka Kitamura

[JGRG24(2014)111116]

The Relation between Renormalizability and Tree Unitarity in Lifshitz Scalar Theory

Tomotaka Kitamura (Waseda U)

in collaboration with Takeo Inami (National Taiwan U)
Keisuke Izumi (Le CosPA)

Purpose

Our final goal is to check the renormalizability of Horava-Lifshtiz gravity via tree unitarity
but
We have faced some problems in HL gravity

⇩

In this work, we try to check the relation between renormalizability and tree unitarity in Lifshtiz scalar theory as a toy model for understanding the problems of HL gravity

Contents

1.Introduction

Important problem in Hořava gravity

z=3 (1+3) dim **P.Hořava '09**

$$\mathcal{S}_{\mathrm{HL}} = \int dt d^3x \sqrt{g} N \{ \frac{M_p}{2} \left(K_{ij}K^{ij} - \lambda K^2 \right)$$
$$+ \left(\alpha_1 \nabla_i R_{jk} \nabla^i R^{jk} + \alpha_2 \nabla_i R \nabla^i R + \cdots \right) \} \qquad (i,j,k = 1,2,3)$$

Horava proposed Power-counting renormalizable gravity theory for solving non-renormalizable problem in Einstein gravity

But no proof of renormalizability in HL gravity

then, we are trying to check the renormalizability of HL gravity using the equivalence between renormalizability and tree unitarity

1.Introduction

Suggestion by Llewellyn Smith

C.H.Llewellyn Smith '73

the equivalence between renomalizability and tree unitarity

tree unitarity $\simeq$ renormalizability

(e.g) Yang-Mills theory
Einstein gravity
Weinberg-Salam model

Tree unitarity

an scattering amplitude does not grow as $E \longrightarrow \infty$

$$\mathcal{M} \sim E^{\varepsilon} \ (\varepsilon \leq 0) \qquad E \longrightarrow \infty$$

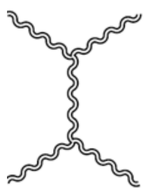
$\mathcal{M}$ amplitude E Energy in center of mass

if $\epsilon \leq 0$, theory has tree unitarity

1.Introduction

Differentiation of high-energy behavior between Einstein gravity and HL gravity

Einstein gravity



$$\mathcal{M} \sim k^{2}$$

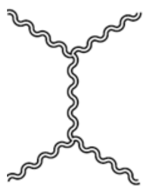
Higher spatial derivative in Horava gravity improves UV behavior

$$\mathcal{P} \sim \frac{1}{k^2} \qquad \mathcal{P}_{HL} \sim \frac{1}{\omega^2 - k^6}$$

but

High-energy behavior of scattering amplitude is more worse ??

Horava gravity



$$\mathcal{M} \sim \mathrm{k}^{6}$$

How should we interpret this worse behavior ??

1.Introduction

Questions about the work in tree unitarity of HL gravity

1) Is the equivalence held for Lifshitz-type model?
anisotropic scaling of space and time

2) How should we take high-energy limit because of Lorentz violation ?

In relativistic theory, any system → CM system tanks to Lorentz sym

In non-rela theory, all systems are independent

ω is from time derivative $\mathbf{k}$ is from spacial derivative

Which should we take high energy limit ??

3) How should we interpret the power of scattering amplitude in HL gravity? $\sim \mathbf{k}^6$??

1.Introduction

Lifshitz scalar theory

Lifshitz scaling $[x] = -1 \quad [t] = -z$ **in mass dim**

$\vec{x} \mapsto b\vec{x}$ $\quad b$ **arbitrary number**

$t \mapsto b^z t$ $\quad z$ **dynamical critical exponent**

z **degree of anisotropy between space and time**

z=3 (1+3)dim **with shift sym** $\phi \to \phi + c \quad c = const$

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int} \qquad \mathcal{L}_{free} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\phi\triangle^3\phi$$

$$\mathcal{L}_{int} = \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6$$

$$\mathcal{L}_3 = \alpha_1(\triangle^2\phi)(\partial_i\phi)^2 + \alpha_2(\triangle\phi)^3$$
$$\vdots$$

this Lifshtz scalar is constructed of most general 6th derivative term with shift sym

We try to check renormalizability and tree unitarity in Lifshtiz scalar theory for answering the questions of HL gravity

2. Unitarity and Optical theorem

For answering the Questions, we check the origin of tree unitary

Optical theorem is derivated from Unitarity of S-matrix

(a)Unitarity of S-matrix $\mathrm{S}^{\dagger}\mathrm{S} = 1$

(b)Optical theorem $\mathrm{Im}\mathcal{M}_{\mathrm{nn}} = -\pi \underbrace{\Sigma_{\mathrm{n}'} \mid \mathcal{M}_{\mathrm{n}'\mathrm{n}} \mid^2}_{\text{cross section}}$

Remark; "n" is some information of external line

this theorem limits scattering amplitude using a value

(1) $\mid \mathcal{M}_{\mathrm{nn}} \mid \geq \; Im \mid \mathcal{M}_{\mathrm{nn}} \mid \geq \; \pi \mid \mathcal{M}_{\mathrm{nn}} \mid^2 \;\; \rightarrow \;\; \mid \mathcal{M}_{\mathrm{nn}} \mid \leq \frac{1}{\pi}$

(2) $$\mathrm{Im}\langle \mathrm{f} \mid \mathrm{T} \mid \mathrm{i} \rangle = \Sigma \int \frac{\mathrm{d}^3\mathrm{k}_1}{\omega_1} \cdots \frac{\mathrm{d}^3\mathrm{k}_\mathrm{n}}{\omega_\mathrm{n}} \delta^4(\Sigma_\mathrm{i}\mathrm{k}_\mathrm{i} - \mathbf{p})$$
$$\times \langle \mathrm{k}_1 \cdots \mathrm{k}_\mathrm{n} \mid \mathrm{T} \mid \mathrm{i} \rangle^* \langle \mathrm{k}_1 \cdots \mathrm{k}_\mathrm{n} \mid \mathrm{T} \mid \mathrm{i} \rangle$$

(1) &(2) determine a power of energy in scattering amplitude of high-energy

2. Unitarity and Optical theorem

Optical theorem **In the case of Lifshitz type theory** $\omega^2 = \gamma \mathbf{k}^6$

more detail to how to determine the value

$$\mathrm{Im}\langle \mathrm{f} \mid \mathrm{T} \mid \mathrm{i} \rangle = \Sigma \int \frac{\mathrm{d}^3\mathrm{k}_1}{\omega_1} \cdots \frac{\mathrm{d}^3\mathrm{k}_\mathrm{n}}{\omega_\mathrm{n}} \delta^4(\Sigma_\mathrm{i}\mathrm{k}_\mathrm{i} - \mathbf{p})$$
$$\times \langle \mathrm{k}_1 \cdots \mathrm{k}_\mathrm{n} \mid \mathrm{T} \mid \mathrm{i} \rangle^* \langle \mathrm{k}_1 \cdots \mathrm{k}_\mathrm{n} \mid \mathrm{T} \mid \mathrm{i} \rangle$$

$$[\mathrm{Im}\langle 2 \mid \mathrm{T} \mid 2 \rangle] = \mathrm{k}^{\beta_2}$$

$[\frac{d^3k_n}{\omega_n}] = k^0$ **Dimension of RHS and LHS lead to the following inequality**

$[\delta^3(\Sigma_i k_i - \mathbf{p})] = \mathbf{k}^{-3} \quad \rightarrow \quad \beta_2 \leq 6$

$$[\delta(\Sigma_i \omega_i - E)] = k^{-3}$$

$$[\langle k_1 \cdots k_n \mid T \mid i \rangle] = k^{\beta_n}$$

2. Unitarity and Optical theorem

(i) Theory with Lorentz symmetry

$$\omega^2 = \kappa \mathbf{k}^3$$

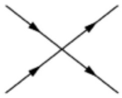
(a) 2-n scattering

amplitude $\langle n \mid T \mid 2 \rangle \sim k^{\beta_n}$ $\beta_n \leq 2 - n$ tree unitarity ○

(b) 2-2 scattering

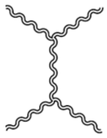
amplitude $\langle 2 \mid T \mid 2 \rangle \sim k^{\beta_2}$ $\beta_2 \leq 0$ tree unitarity ○

(e.g 1) ϕ^4 theory



$\mathcal{M} \sim \lambda \ (\sim k^0)$ tree unitarity ○

(e.g 2) Einstein gravity



$\mathcal{M} \sim k^2$ tree unitarity ×

2. Unitarity and Optical theorem

(ii) Theory without Lorentz symmetry

$$\omega^2 = \gamma \mathbf{k}^6$$

(a) 2-n scattering

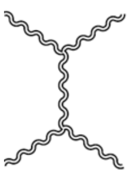
amplitude $\langle n \mid T \mid 2 \rangle \sim k^{\beta_n}$ $\beta_n \leq 6$ tree unitarity ○

(b) 2-n scattering

amplitude $\langle 2 \mid T \mid 2 \rangle \sim k^{\beta_2}$ $\beta_2 \leq 6$ tree unitarity ○

Origin of differentiation is dispersion relation $\omega^2 \sim k^6$

(e.g) Horava gravity (a part of diagram)



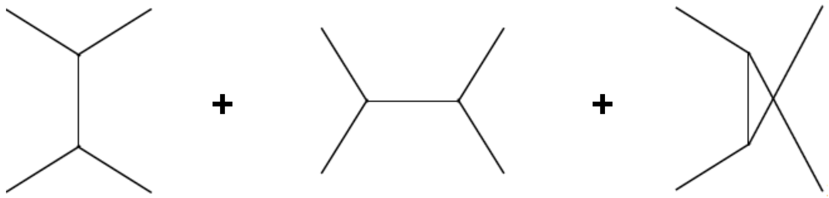
$\mathcal{M} \sim \mathbf{k}^6$ tree unitarity is ○ ??

At least, we can understand the behavior of the power in high-energy scattering amplitude

3. Tree Unitarity in Lifshitz scalar theory

(e.g)4-point function

$\mathcal{M} =$



$\mathcal{M} \sim \mathbf{k}^6 \qquad \mathbf{k} \to \infty$

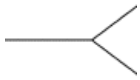
tree unitarity ○

(cf)

Propagator

$$\frac{i}{\omega^2 - \mathbf{k}^6}$$

Vertex $\alpha_1(\triangle^2\phi)(\partial_i\phi)^2$



$-\alpha_1[(\mathbf{k_2}\cdot\mathbf{k_3})(\mathbf{k_1})^4 + \cdots]$

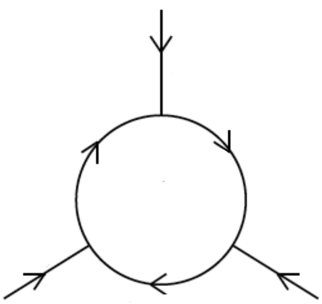
4. One loop calculation in Lifshitz scalar

(e.g)

Vertex $\alpha_1(\triangle^2\phi)(\partial_i\phi)^2$

extracting the property of leading order, we find the following property

One loop graph



$$\mathcal{M} = \int d\omega d^3\mathbf{k}\ \Sigma\alpha_n \frac{\mathbf{k}^a\mathbf{p}^b}{(\omega^2+\mathbf{k}^6)^n}$$

$$\mathcal{M} \sim \left(\int d\mathrm{E}\ \mathrm{E}^{1-\frac{b}{3}}\right)\mathbf{p}^b$$

if $b \geq 6$, **no divergence in** $\mathrm{E} \to \infty$

even if $b \leq 6$, **there are divergence**

but we can renormalize using counter term

this Lifshitz scalar is finite!! and b=6 is critical value!!

b=6 is same value of the power of high-energy limit !!

Summary

1. checking the power of the high energy limit in Lifshitz scalar

the power is independent on the way to take the high-energy limit

(e.g)

we can take the high energy limit
$\omega, \mathbf{k_1}, \mathbf{k_2}, \text{and}, \mathbf{k_3}, \text{or}, \text{the combination of them}$ **O.K.**

2. We almostly confirmed the equivalence between renormalizability and tree unitarity in Lifshitz scalar theory

We can use the equivalence for checking renormalizability of Horava gravity!!

Thank you!