# JGRG24

## The 24th Workshop on General Relativity and Gravitation in Japan

**10 (Mon) — 14 (Fri) November 2014**

**KIPMU, University of Tokyo**

**Chiba, Japan**

# Oral presentations: Day 1

# Contents

# Organizing Committees

## Scientific Organizing Committee

Hideki Asada (Hirosaki University)
Takeshi Chiba (Nihon University)
Tomohiro Harada (Rikkyo University)
Kunihito Ioka (KEK)
Hideki Ishihara (Osaka City University)
Masahiro Kawasaki (ICRR, University of Tokyo)
Hideo Kodama (KEK)
Yasufumi Kojima (Hiroshima University)
Kei-ichi Maeda (Waseda University)
Shinji Mukohyama (YITP, Kyoto University)
Takashi Nakamura (Kyoto University)
Ken-ichi Nakao (Osaka City University)
Yasusada Nambu (Nagoya University)
Ken-ichi Oohara (Niigata University)
Misao Sasaki (YITP, Kyoto University)
Masaru Shibata (YITP, Kyoto University)
Tetsuya Shiromizu (Nagoya University)
Jiro Soda (Kobe University)
Naoshi Sugiyama (Nagoya University)
Takahiro Tanaka (YITP, Kyoto University)
Masahide Yamaguchi (Tokyo Institute of Technology)
Jun'ichi Yokoyama (RESCEU, University of Tokyo)

## Local Organizing Committee

Shinji Mukohyama (YITP, Kyoto University; Chair)
Tomohiro Fujita (Kavli IPMU, University of Tokyo)
Ryo Namba (Kavli IPMU, University of Tokyo)
Rio Saitou (YITP, Kyoto University)

# Presentation Award

The JGRG presentation award program was established at the occasion of JGRG22 in 2012. This year, we are pleased to announce the following seven winners of the Outstanding Presentation Award for their excellent presentations at JGRG24. The winners were selected by the selection committee consisting of the JGRG24 SOC based on ballots of the participants.

[Oral Presentation]

Tsuyoshi Houri (Kobe University)
"An upper bound on the number of Killing-Yano tensors"

Ryotaro Kase (Tokyo University of Science)
"Effective field theory approach to modified gravity including Horndeski theory and Horava-Lifshitz gravity”

Kazunari Eda (RESCEU, University of Tokyo)
“Multiple output configuration for a torsion-bar gravitational wave antenna”

Tomohiro Fujita (Kavli IPMU, University of Tokyo)
"Can a Spectator Scalar Field Enhance Inflationary Tensor Modes?”

[Poster Presentation]

Hajime Fukuda (Kavli IPMU, University of Tokyo)
"Leptogenesis during axion inflation”

Daiki Kikuchi (Hirosaki University)
“Relativistic Sagnac effect by CS gravity"

Masato Minamitsuji (IST, University of Lisbon)
“Disformal transformation of cosmological perturbations"

# Programme: Day 1

## Monday 10 November 2014

9:30 Reception desk opens

10:30 Shinji Mukohyama (YITP, Kyoto University)
Opening address [JGRG24(2014)111000]

Morning 1 [Chair: Masahiro Kawasaki]

10:45 Mark Vagins (Kavli IPMU, Super-Kamiokande) [Invited]
"Zen and the Art of Gadolinium-Loaded Water Cherenkov Detectors"
[JGRG24(2014)111001]

11:30 Fuminobu Takahashi (Tohoku)
"Inflation in Axion Landscape" [JGRG24(2014)111002]

11:45 Naoya Kitajima (Tohoku)
"Resonant conversions of QCD axions into hidden axions" [JGRG24(2014)111003]

12:00 Ken'ichi Saikawa (Titech)
"Axion dark matter from topological defects" [JGRG24(2014)111004]

12:15 Taku Hayakawa (ICRR)
"CDM/baryon isocurvature perturbations in a sneutrino curvaton model"
[JGRG24(2014)111005]

12:30 - 14:00 photo & lunch & poster view

Afternoon 1 [Chair: Jiro Soda]

14:00 Steven Gubser (Princeton) [Invited]
"Holographic Fermi surfaces from String Theory" [JGRG24(2014)111006]

14:45 Yoske Sumitomo (KEK)
"De Sitter Vacua from a D-term Generated Racetrack Uplift"
[JGRG24(2014)111007]

15:00 short poster talks (A01 - A19, 1 minute each)

15:30 - 16:00 coffee break & poster view

Afternoon 2 [Chair: Hideki Ishihara]

16:00 Shunichiro Kinoshita (Osaka City)
"Electric field quench in AdS/CFT" [JGRG24(2014)111008]

16:15 Keiju Murata (Keio)
"Turbulent meson condensation in quark deconfinement" [JGRG24(2014)111009]

16:30 Kenji Hotta (Hokkaido)
"Brane-Antibrane and Closed Superstrings at Finite Temperature in the Framework of Thermo Field Dynamics" [JGRG24(2014)111010]

16:45 Tsuyoshi Houri (Kobe)
"An upper bound on the number of Killing-Yano tensors" [JGRG24(2014)111011]

17:00 Laur Jarv (Tartu)
"Invariant quantities in the scalar-tensor theories of gravitation" [JGRG24(2014)111012]

17:15 Atsushi Taruya (YITP, Kyoto)
"Nonlinear mode-coupling of large- scale structure : validity of perturbation theory calculation" [JGRG24(2014)111013]

17:30 - 18:00 poster view

# Opening Address

Shinji Mukohyama (LOC Chair)

[JGRG24(2014)111000]

KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

# The 24th Workshop on General Relativity and Gravitation in Japan (JGRG24)

**November 10-14, 2014**

# JGRG meetings

- JGRG1 (Tokyo Metropolitan University, 4th Dec - 6th Dec 1991)
- JGRG2 (Waseda University, 18th Jan -20th Jan 1993)
- JGRG3 (The University of Tokyo, 17th Jan - 20th Jan 1994)
- JGRG4 (YITP, Kyoto University, 28th Nov - 1st Dec 1994)
- JGRG5 (Nagoya University, 22nd Jan-25th Jan 1996)
- JGRG6 (Tokyo Institute of Technology, 2nd Dec - 5th Dec 1996)
- JGRG7 (YITP, Kyoto University, 27th Oct - 30th Oct 1997)
- JGRG8 (Niigata University, 19th Oct - 22nd Oct 1998)
- JGRG9 (Hiroshima University, 3rd Nov - 6th Nov 1999)
- JGRG10 (Osaka University, 11th Sep - 14th Sep 2000)
- JGRG11 (Waseda University, 9th Jan - 12th Jan 2002)
- JGRG12 (Komaba, The University of Tokyo, 25th Nov - 28th Nov 2002)
- JGRG13 (Osaka City University, 1st Dec - 4th Dec 2003)
- JGRG14 (YITP, Kyoto University, 29th Nov - 3rd Dec 2004)
- JGRG15 (Tokyo Institute of Technology, 28th Nov - 2rd Dec 2005)
- JGRG16 (Niigata Prefectural Center, 27th Nov - 1st Dec 2006)
- JGRG17 (Nagoya University, 3rd Dec - 7th Dec 2007)
- JGRG18 (Hiroshima University, 17th-21st Nov 2008)
- JGRG19 (Rikkyo University, 30th Nov -4th Dec 2009)
- JGRG20 (YITP, Kyoto University, 21st Sep -25th Sep 2010)
- JGRG21 (Tohoku University, 26th Sep -29th Sep 2011)
- JGRG22 (The University of Tokyo, 12-16th Nov 2012)
- JGRG23 (Hirosaki University, 5-8th Nov 2013)
- **JGRG24 @ Kavli IPMU, U of Tokyo, 10-14th Nov 2014**

# JGRG meetings

- The history of JGRG @ http://www-tap.scphys.kyoto-u.ac.jp/jgrg/about.html .
- Professors Maeda and Sasaki wrote "The JGRG workshop series have been supported by active involvement of **young postdocs and graduate students**. In turn, it is hoped that their experience from the JGRG workshop series will help them grow …"

# Presentation Awards

- To encourage **young postdocs and graduate students**, we have awards for outstanding oral/poster presentations.
- Please vote for **the best speaker of the day** (one speaker for each day from Mon to Thu) and for **the best poster** presentation (one poster for the workshop).
- The final decision will be made by JGRG24 SOC. Priority is given to young postdocs and graduate students.

# For Oral

- A voting paper will be handed out during the morning session of each day (Mon-Thu).
- Please write the name of the best speaker (postdoc or graduate student) of the day on it.
- Please consider scientific importance of the talk that you choose.
- Only the participants who attend all the talks on the day has the right to vote.
- Please do NOT vote if you are a partial attendee on the day.
- Ballot boxes will be located near the main entrance for **17:30-18:30, Mon-Thu**.

# For Poster

- A voting paper for the best poster was already handed out at registration.
- Please write the name of the best poster presenter (a postdoc or a graduate student) on it.
- Poster view (Mon-Thu):
  12:30-14:00, 15:30-16:00, 17:30 (or 17:15)-18:00
- Short poster talks (1min each) on Mon & Tue
- Ballot boxes will be located near the main entrance for **17:30-18:30, Mon-Thu.**

## Proceedings in PDF format

- Online proceedings at http://www-tap.scphys.kyoto-u.ac.jp/jgrg/proc/
- For oral presentations, LOC will collect electric files at JGRG24. **PDF format is preferred** while power point and keynote files are also fine.
- For poster presentations, we will announce later.
- Numbering (article ID):

  JGRG24(2014)mmdd** for oral
  JGRG24(2014)P*** for poster

  eg. JGRG24(2014)111001, JGRG24(2014)PA01

## Some logistics

- We will collect presentation files for proceedings
- Awards for excellent talks and posters by young postdocs and graduate students
- Poster view (Mon-Thu):
  12:30-14:00, 15:30-16:00, 17:30 (or 17:15)-18:00
- Short talks (1min each) for posters on Mon & Tue
- All talks will be broadcasted to the satellite room in the 2nd building (1F).
- Group photo taken on Mon before lunch
- Banquet on campus in Wednesday evening
- Two on-campus cafeterias for lunch

# All talks will be broadcasted to the Satellite Room in the 2nd building


**The second building (as well as the main building) will be locked at 18:00.**

# JGRG24 is sponsored by

- Kavli IPMU (WPI)

KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

- MEXT Grant-in-Aid for Scientific Research on Innovative Areas No. 24103006 "Theoretical study for astrophysics through multimessenger observations of gravitational wave sources“ (Tanaka)

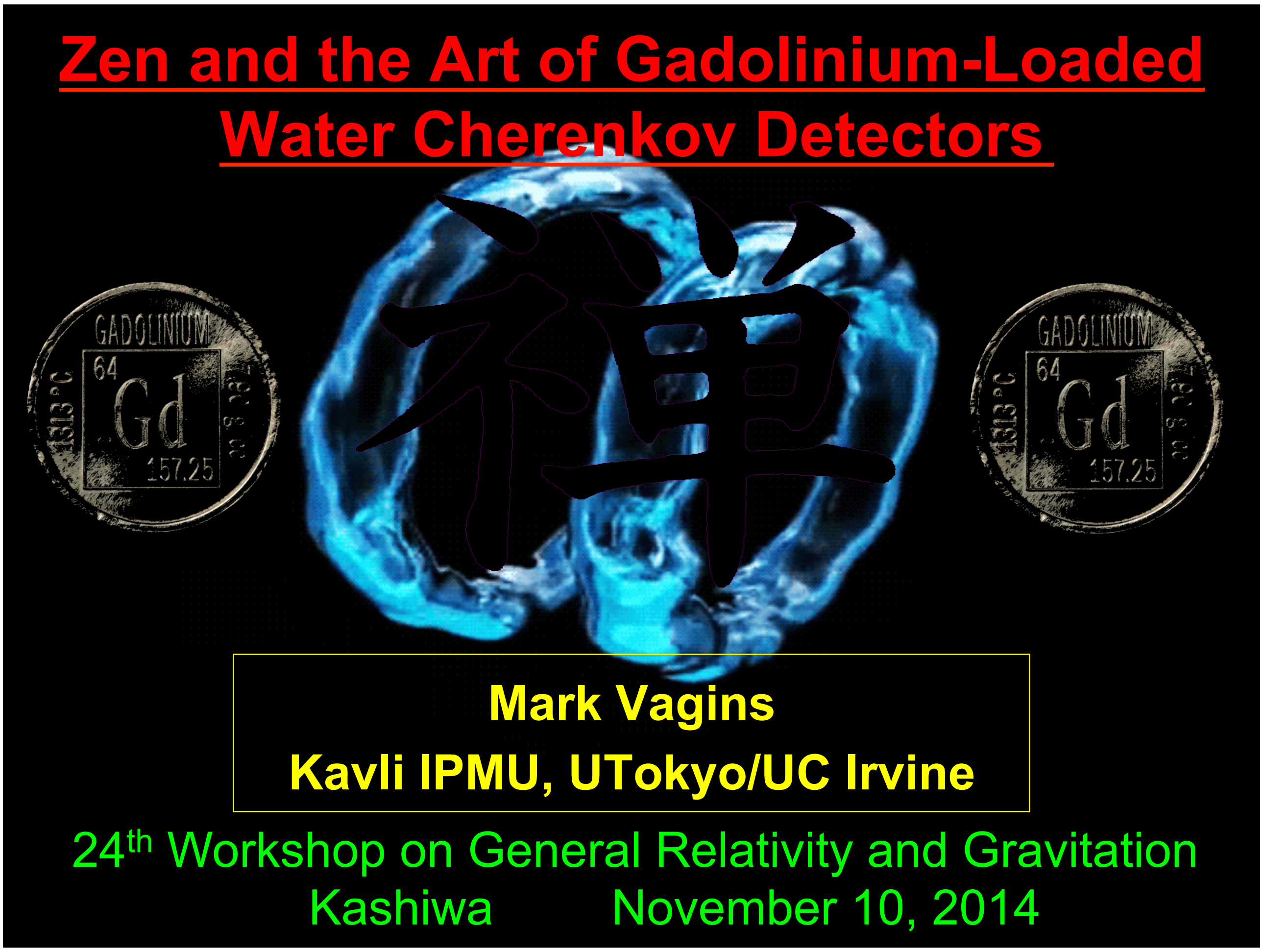
科研費 KAKENHI

Let’s enjoy the workshop!

# “Zen and the Art of Gadolinium-Loaded Water Cherenkov Detectors”

Mark Vagins [Invited]

[JGRG24(2014)111001]


Zen and the Art of Gadolinium-Loaded Water Cherenkov Detectors
禅
GADOLINIUM
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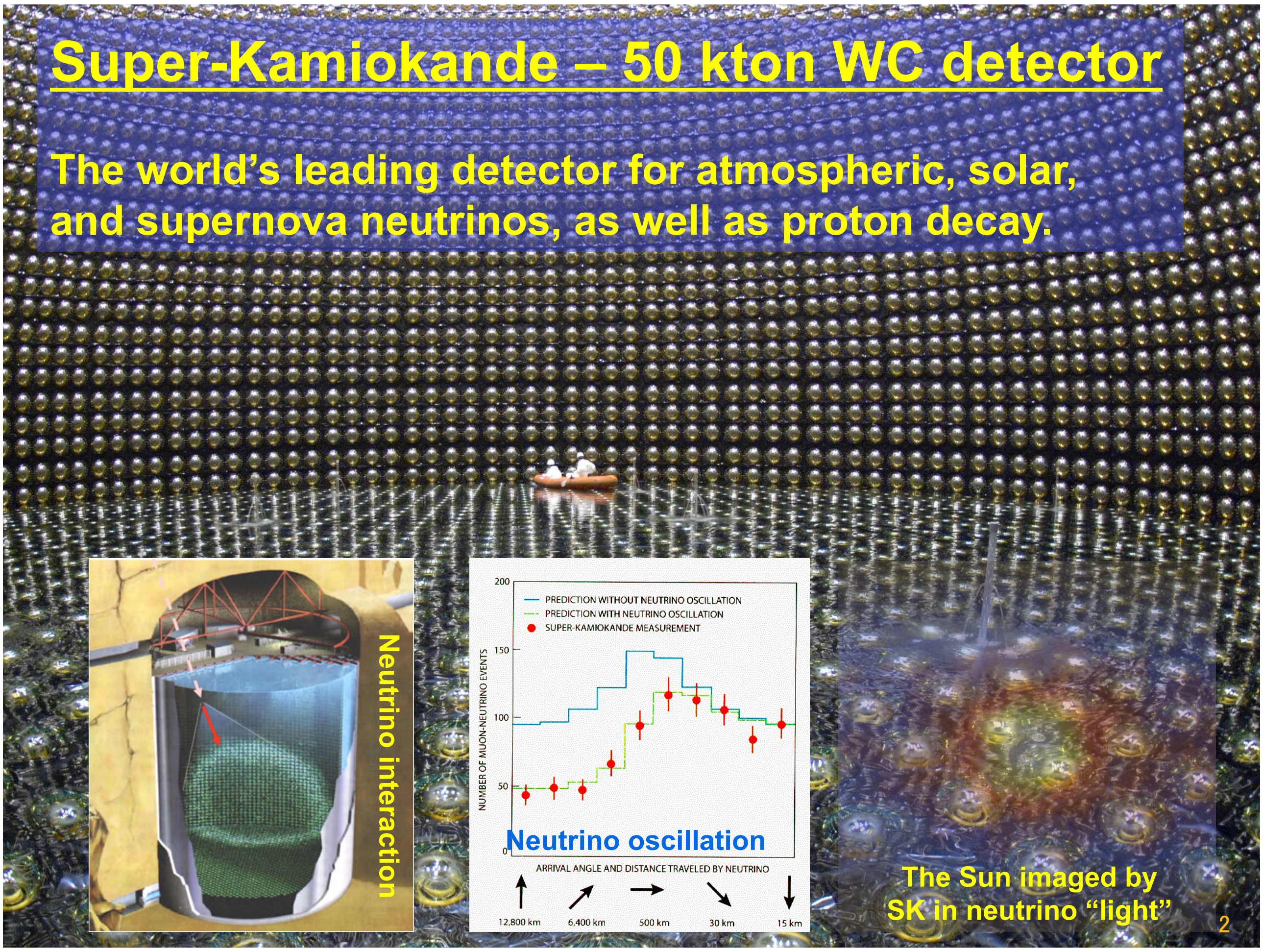
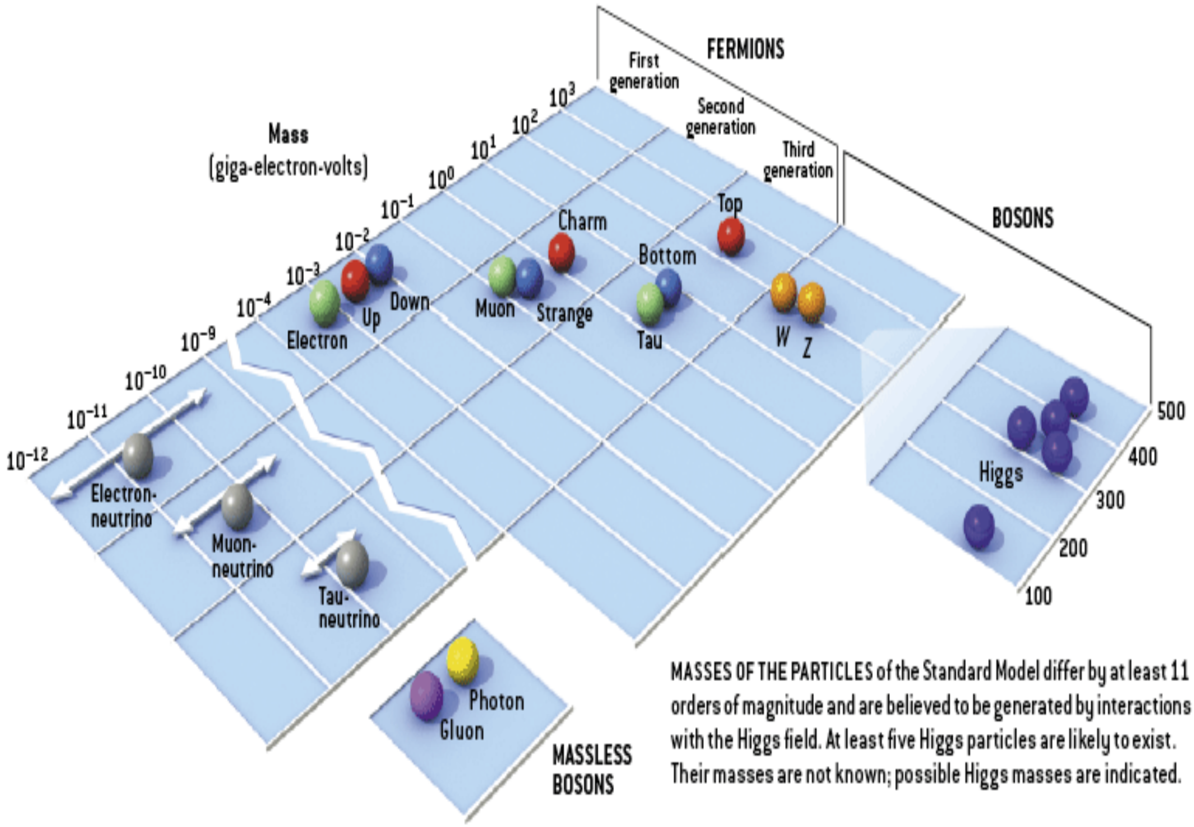
Gd
157.25
Mark Vagins
Kavli IPMU, UTokyo/UC Irvine
24th Workshop on General Relativity and Gravitation
Kashiwa November 10, 2014

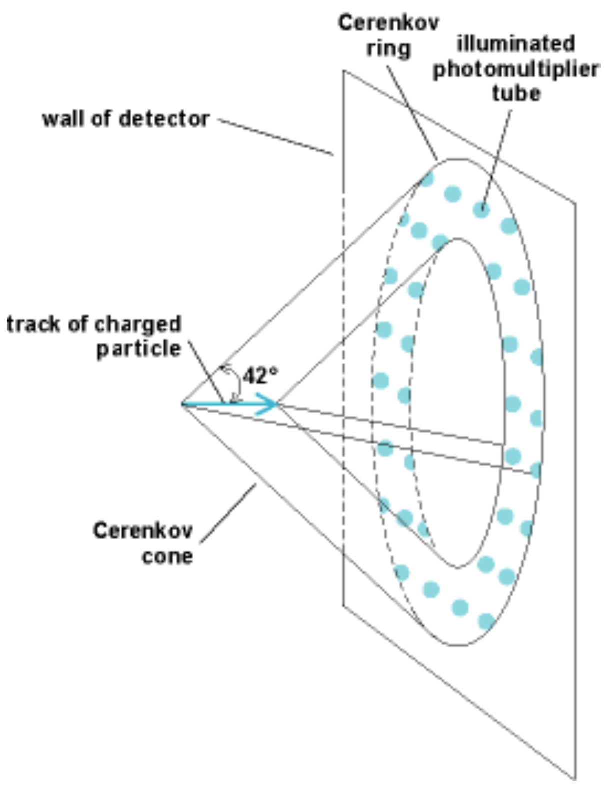

Super-Kamiokande – 50 kton WC detector
The world's leading detector for atmospheric, solar, and supernova neutrinos, as well as proton decay.
Neutrino interaction
PREDICTION WITHOUT NEUTRINO OSCILLATION
PREDICTION WITH NEUTRINO OSCILLATION
SUPER-KAMIOKANDE MEASUREMENT
NUMBER OF MUON-NEUTRINO EVENTS
Neutrino oscillation
ARRIVAL ANGLE AND DISTANCE TRAVELED BY NEUTRINO
12,800 km
6,400 km
500 km
30 km
15 km
The Sun imaged by SK in neutrino "light"
2




MASSES OF THE PARTICLES of the Standard Model differ by at least 11 orders of magnitude and are believed to be generated by interactions with the Higgs field. At least five Higgs particles are likely to exist. Their masses are not known; possible Higgs masses are indicated.

## Ring-Imaging Water Cherenkov Detector

Relativistic charged particles traveling through water make rings of light on the inner wall of the detector. The rings are then imaged by photomultiplier tubes.



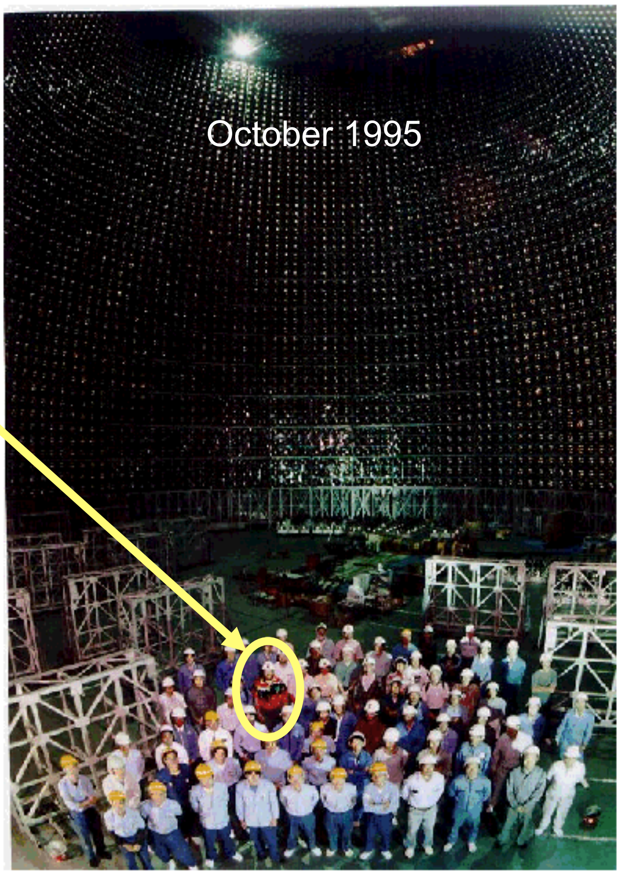
Hamamatsu's incredible 50-cm photomultiplier tube.

Here's a publicity shot from the late 1980's announcing their technological breakthrough..

*Every tube is made out of hand-blown glass.*

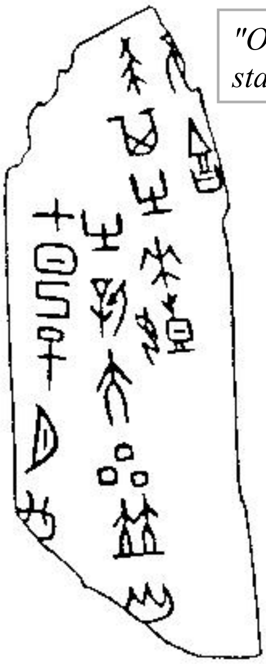
I've been a part of Super-K (and wearing brightly-colored shirts) from its very early days…

January 1996



October 1995

# The appearance of new, temporary stars has long captured the attention of people around the world:



*"On the Jisi day, the 7th day of the month, a big new star appeared in the company of the Ho star."*

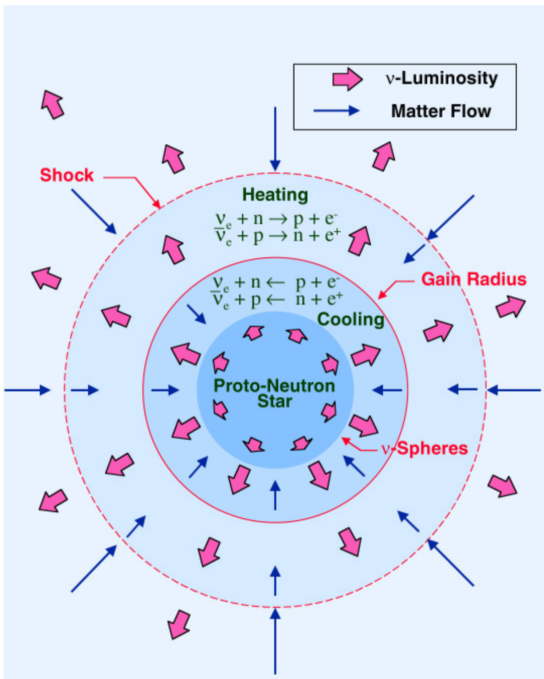
The Ho star is Antares.

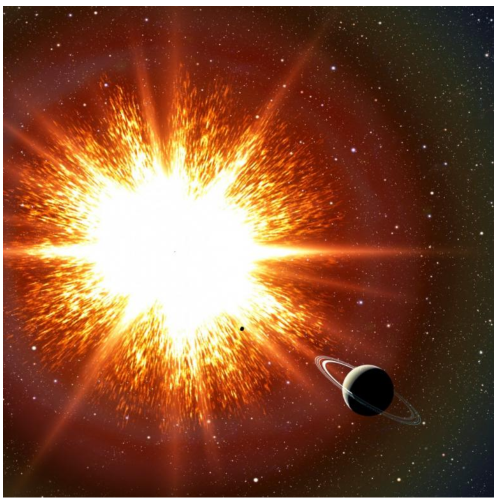
→ This is a 3,500 year-old record of a supernova explosion!

*"On the Xinwei day the new star dwindled."*

A core-collapse supernova is a nearly perfect "neutrino bomb".

Within ten seconds of collapse it releases >98% of its huge energy (equal to a trillion H-bombs/second since the beginning of the universe) as neutrinos.

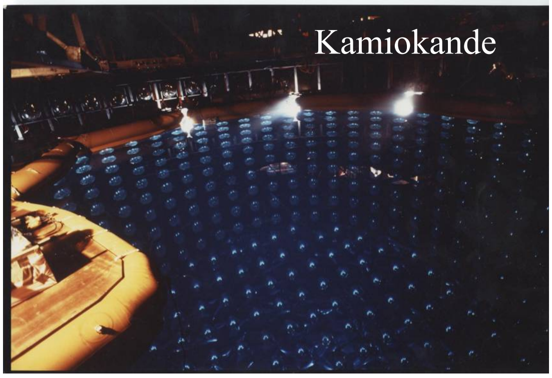


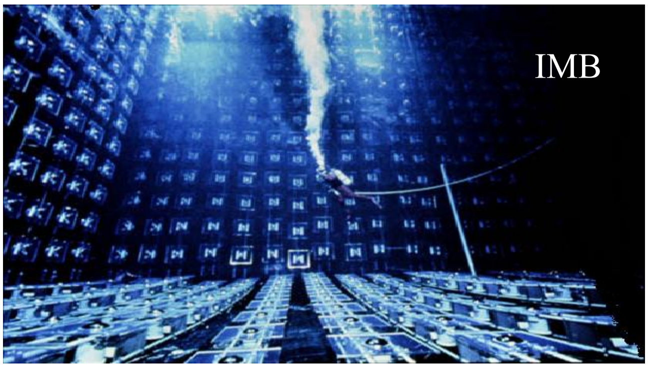



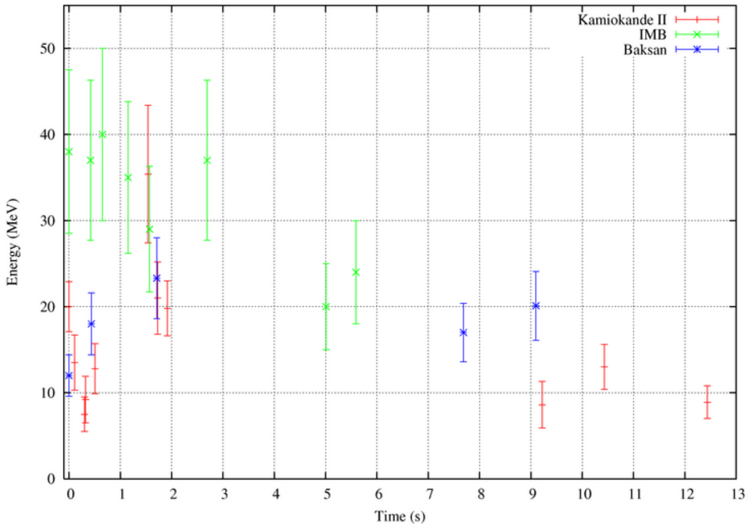
Neutrinos, along with gravitational waves, provide the only possible windows into core collapses' inner dynamics.

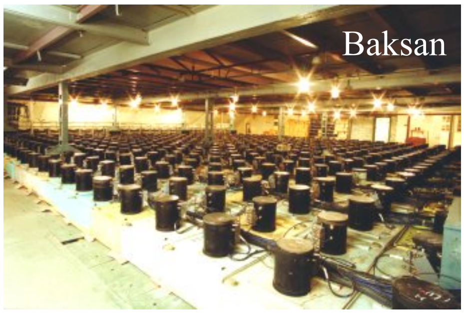
A long time ago, in a (neighbor) galaxy far, far away…

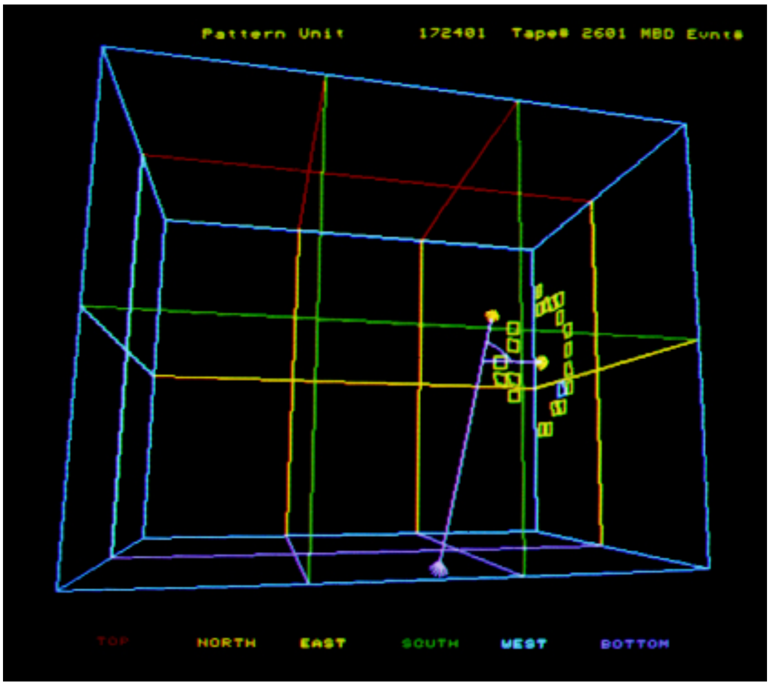

A long time ago, in a (neighbor) galaxy far, far away…



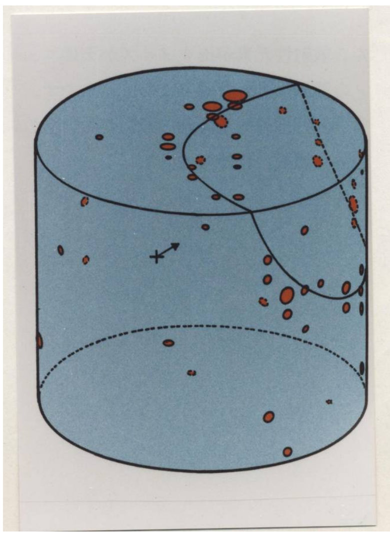




**IMB**
**(in USA)**

**Kamiokande**
**(in Japan)**



## Event Displays of Actual Neutrinos from SN1987A

Sanduleak -69° 202 was gone,
but not forgotten.

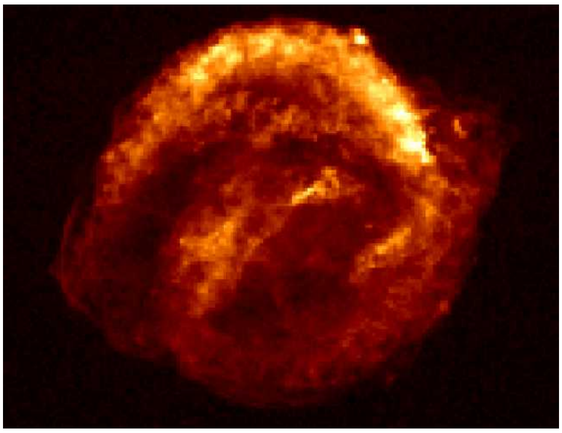
Based on the handful of supernova neutrinos which were detected that day, approximately <u>one theory paper has been published every ten days</u>…

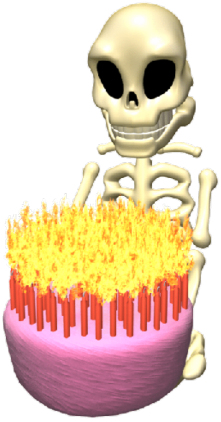
*…for the last <u>twenty-seven years!</u>*

Masatoshi Koshiba ultimately received the Nobel Prize in physics for observing the neutrinos from SN1987A with Kamiokande.

December 10, 2002

We would very much like to collect
some more supernova neutrinos!



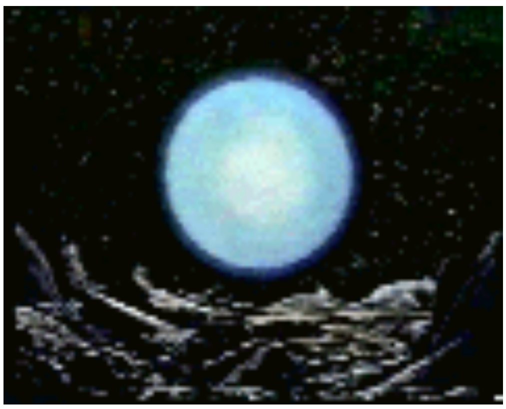


But it has already been over a quarter century since SN1987A, and exactly 410 years and 32 days since a supernova was last definitely observed within our own galaxy.

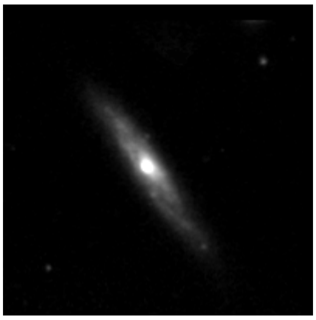
**Yes, it's been a long, cold winter for SN neutrinos…**
**but there is hope!**

So, how can we be certain to see more supernova neutrinos without having to wait too long?

This is not the typical view of a supernova! Which, of course… is good.

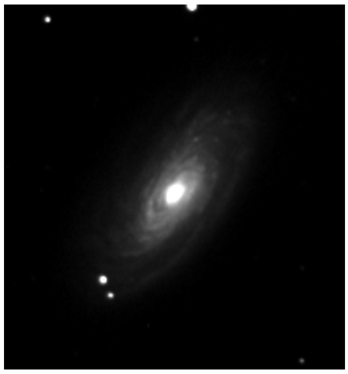


Yes, nearby supernova explosions may be rare, but supernova explosions are extremely common.

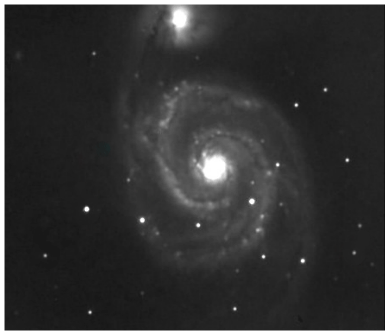


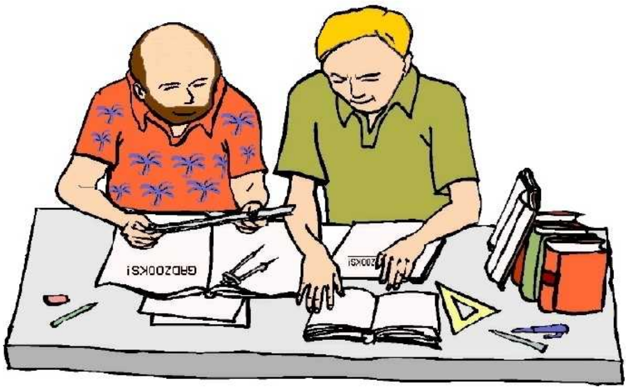
Here's how most of them look to us (video is looped).

There are ***thousands of supernova explosions per hour*** in the universe as a whole!



These produce a diffuse supernova neutrino background [DSNB], also known as the supernova relic neutrinos [SRN].



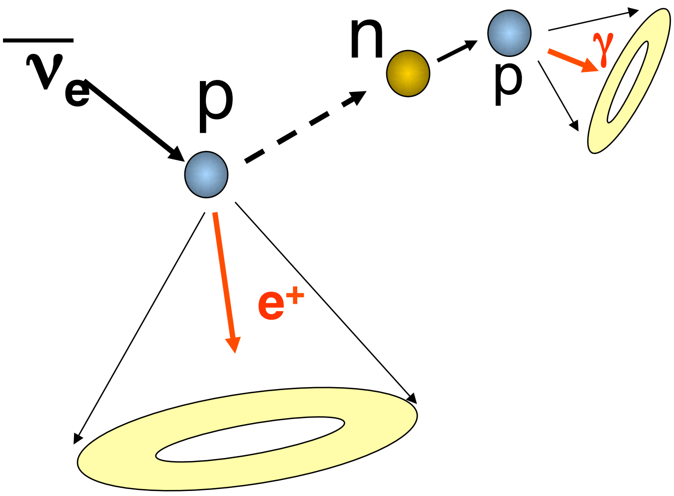



In an attempt to find a way to see the DSNB, theorist John Beacom and I wrote the original GADZOOKS! (Gadolinium Antineutrino Detector Zealously Outperforming Old Kamiokande, Super!) paper.

It proposed loading big WC detectors, specifically Super-K, with water soluble gadolinium, and evaluated the physics potential and backgrounds of a giant antineutrino detector.

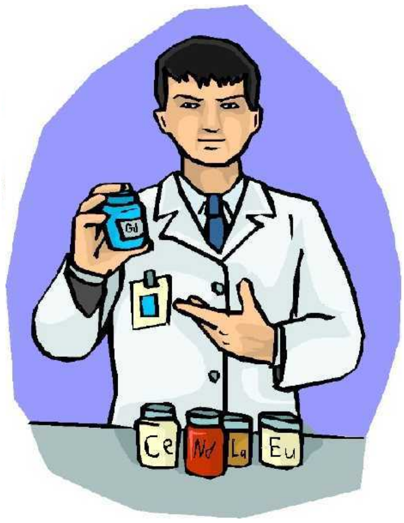
[Beacom and Vagins, *Phys. Rev. Lett.*, **93**:171101, 2004]
(199 citations → one every 19 days for ten years)

Basically, we said, "Let's add 0.2% of a water soluble gadolinium compound to Super-K!"




Positron and gamma ray vertices are within ~50cm.

But, wait… 0.2% of 50 kilotons is 100 *tons!*
What's <u>that</u> going to cost?



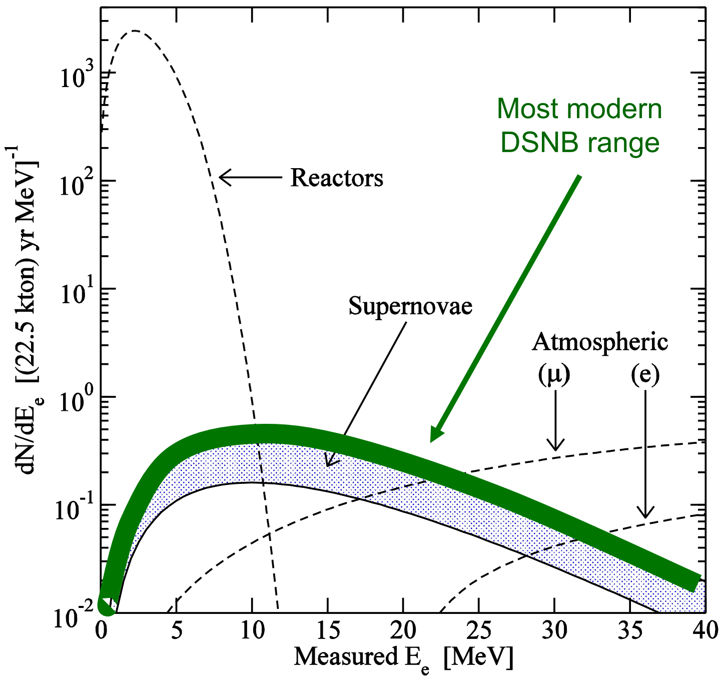

In 1984: $4000/kg –> $400,000,000
In 1993: $485/kg –> $48,500,000
In 1999: $115/kg –> $11,500,000
In 2006: $6/kg –> $600,000

These low, low prices are for real.

Back in 2005, $24,000 bought me 4,000 kg of $GdCl_3$.

*Shipping from Inner Mongolia to Japan was included!*

Here's what the <u>coincident</u> signals in Super-K with $GdCl_3$ or $Gd_2(SO_4)_3$ will look like (energy resolution is applied):




$\bar{\nu}_e$ + p → $e^+$ + n

spatial and temporal separation between prompt $e^+$ Cherenkov light and delayed Gd neutron capture gamma cascade:

$\lambda$=~4cm, $\tau$=~30$\mu$s

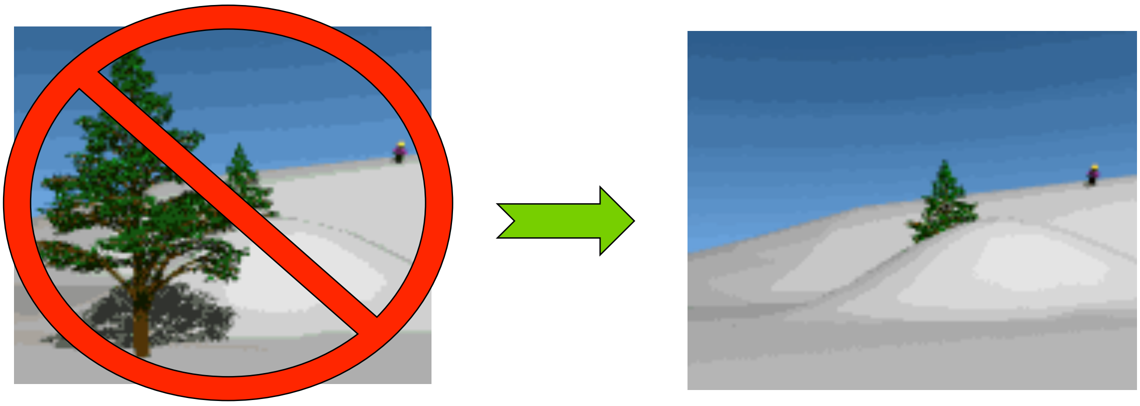
→ A few clean events/yr in Super-K with Gd

Now, Beacom and I never wanted to merely propose a new technique – we wanted to make it work!


[Snowbird photo by A. Kusenko]

Suggesting a major modification of one of the world's leading neutrino detectors may not be the easiest route…

# …and so to avoid wiping out, some careful hardware studies are needed.

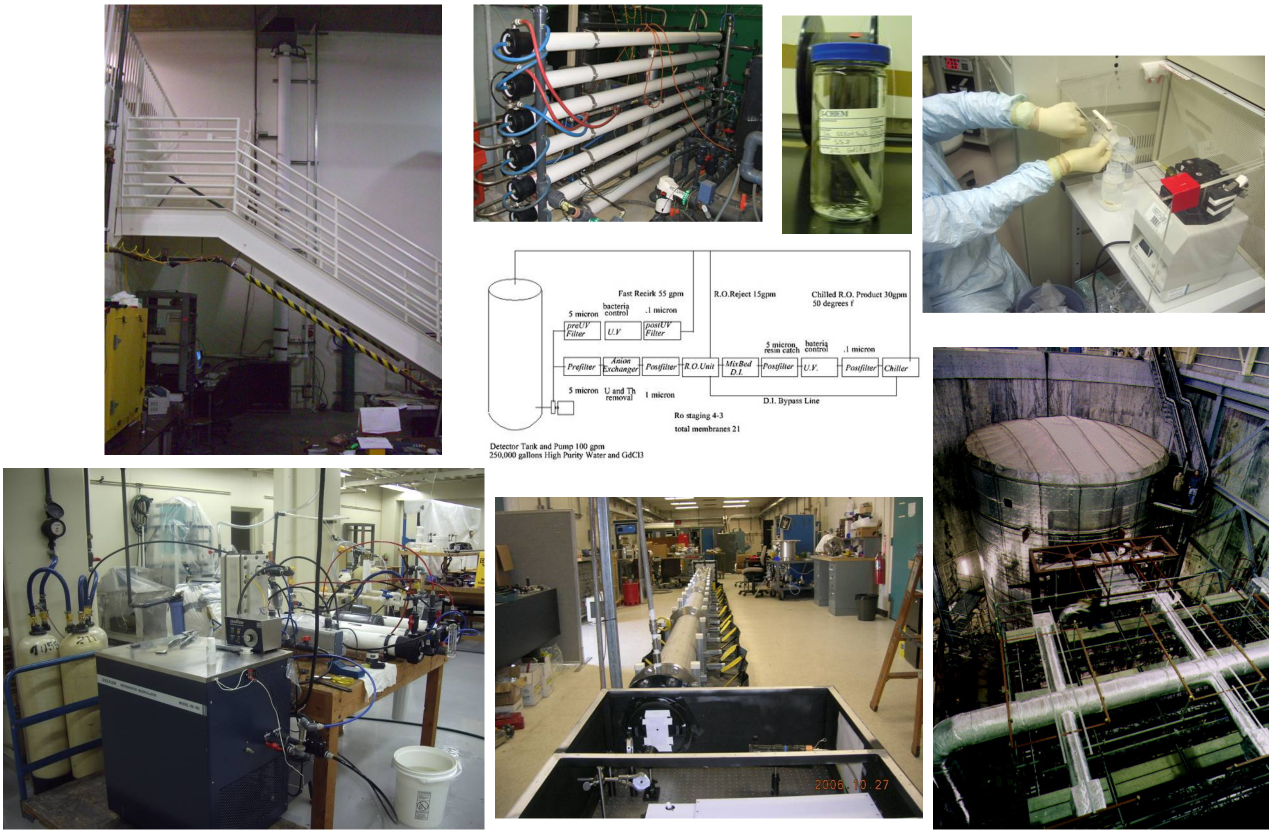


- What does gadolinium do the Super-K tank materials?
- Will the resulting water transparency be acceptable?
- Any strange Gd chemistry we need to know about?
- *How will we filter the SK water but retain dissolved Gd?*

As a matter of fact, I very rapidly made two discoveries regarding $GdCl_3$ while carrying a sample from Los Angeles to Tokyo:

1) $GdCl_3$ is quite opaque to X-rays
2) Airport personnel get very upset when they find a kilogram of white powder in your luggage

Over the last eleven years there have been a large number of Gd-related R&D studies carried out in the US, Japan, and Spain:

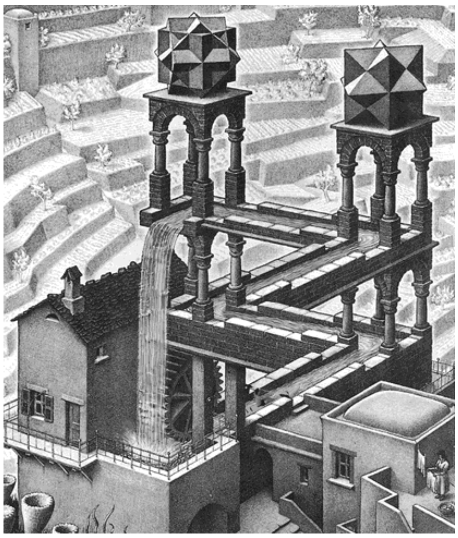



## <u>The Essential Magic Trick</u>

→ We must keep the water in any Gd-loaded detector perfectly clean… *without removing the dissolved Gd.*

→ I've developed a new technology:
"Molecular Band-Pass Filtration"
Staged nanofiltration <u>selectively</u> retains Gd while removing impurities.

Amazingly, the darn thing works!



This technology will support a variety of applications, such as:

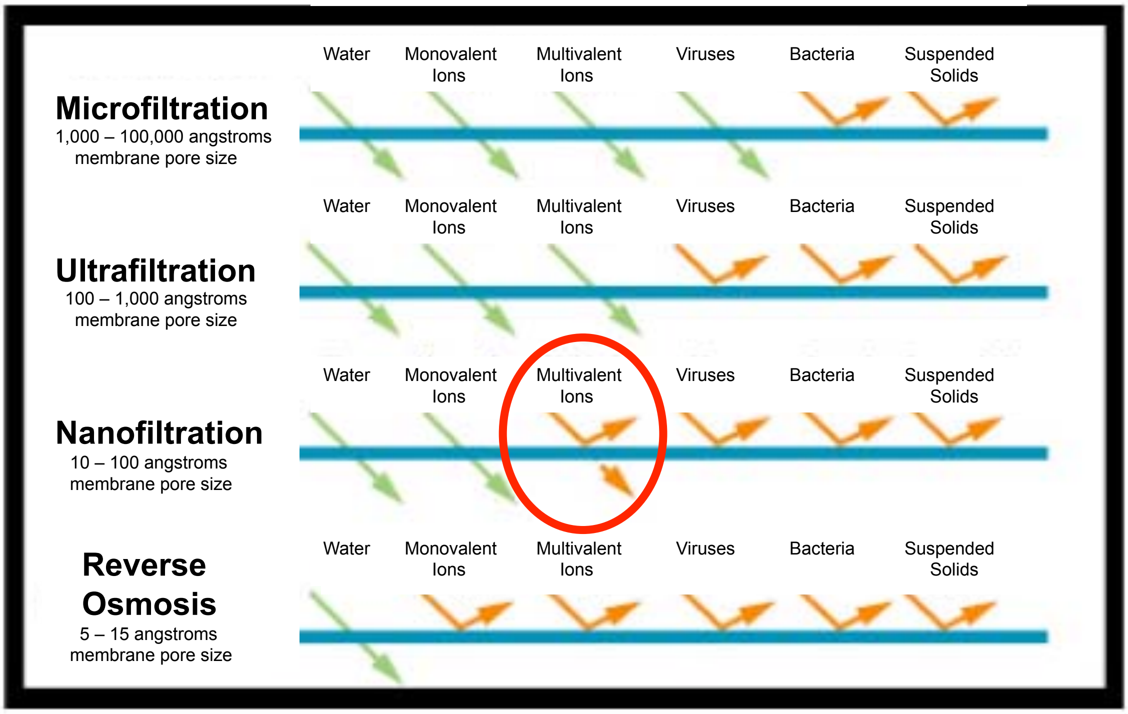
→ Supernova neutrino and proton decay searches

→ Remote detection of clandestine fissile material production

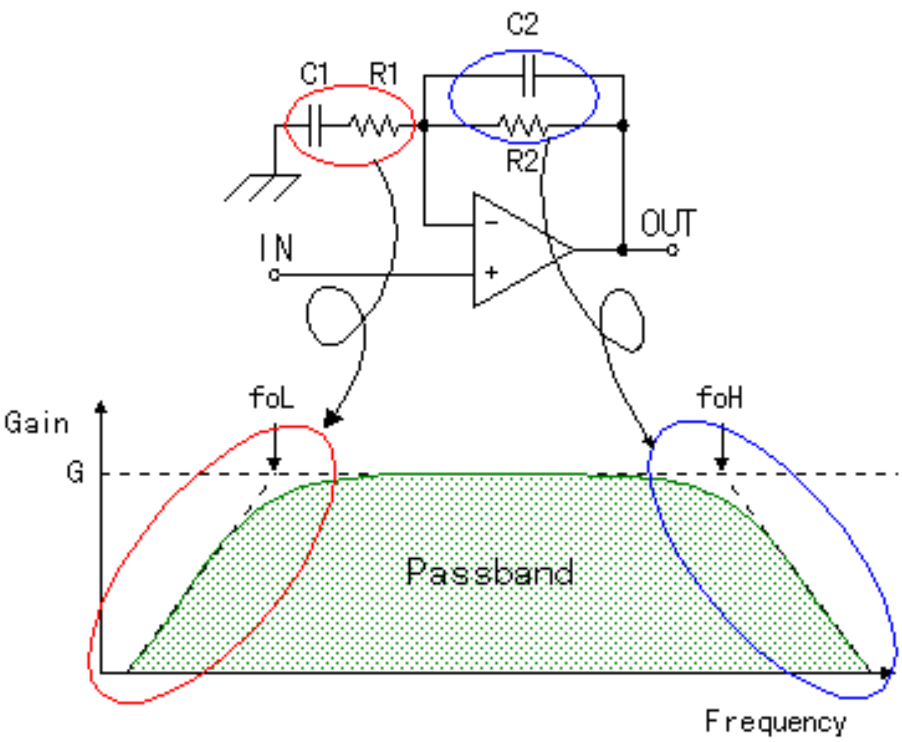
→ Efficient generation of clean drinking water without electricity

# Membrane-based Filtering Technologies

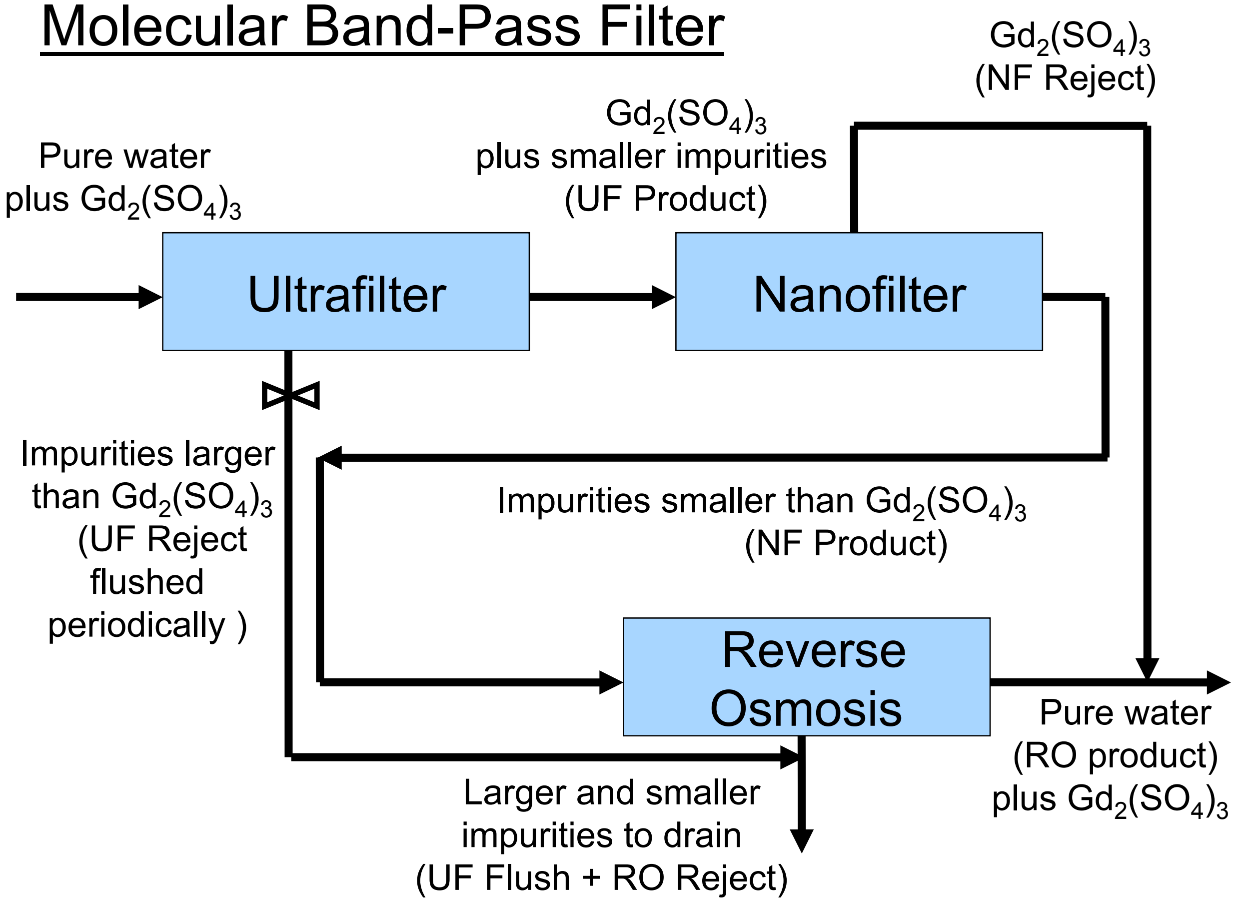
$Gd_2(SO_4)_3 \rightarrow 2\ Gd^{3+} + 3\ (SO_4)^{2-}$



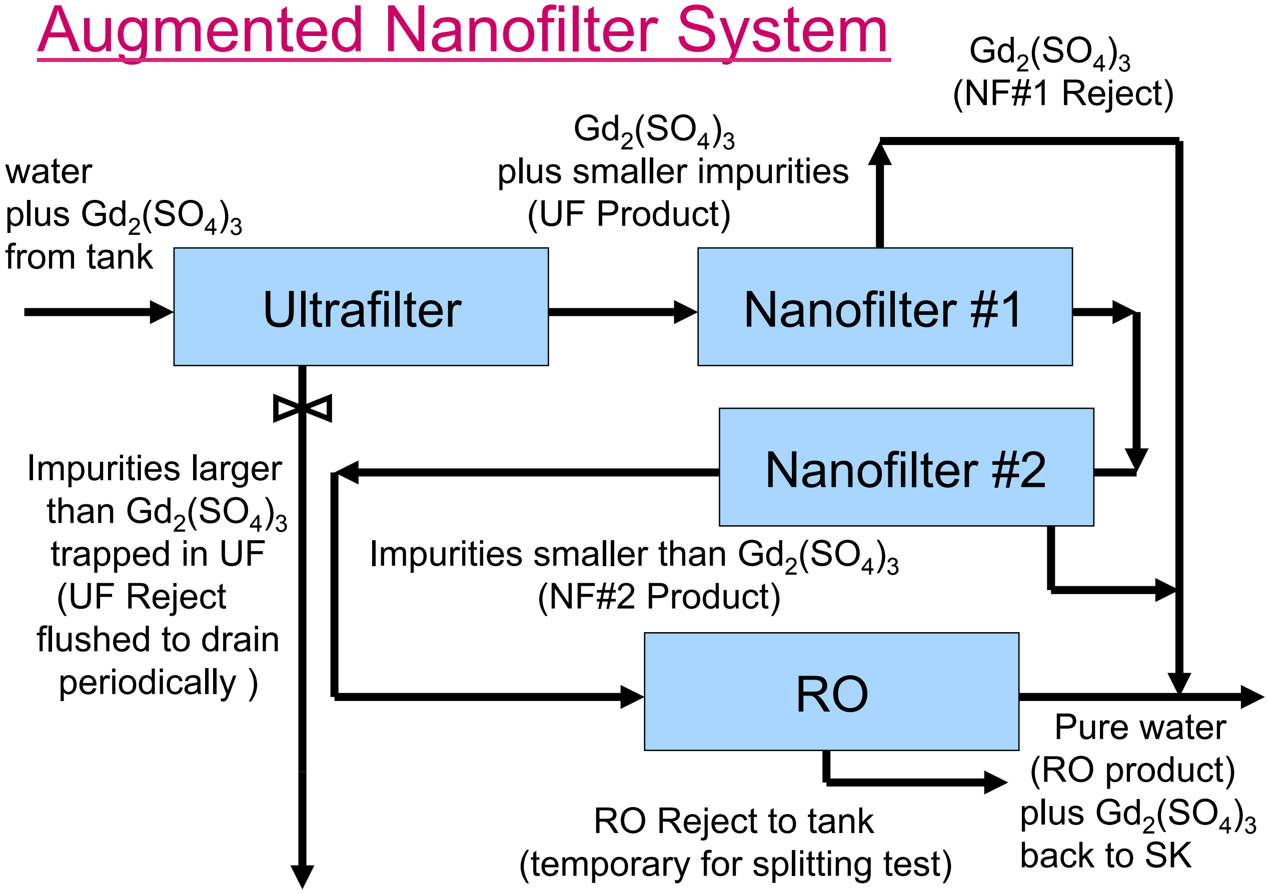

# Electrical Band-Pass Filter



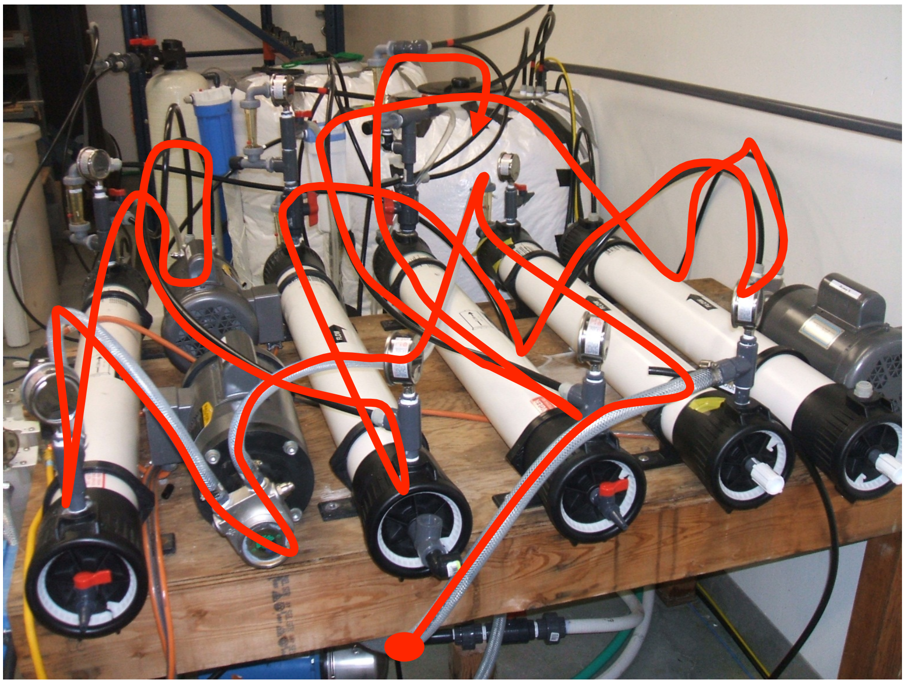
## Molecular Band-Pass Filter



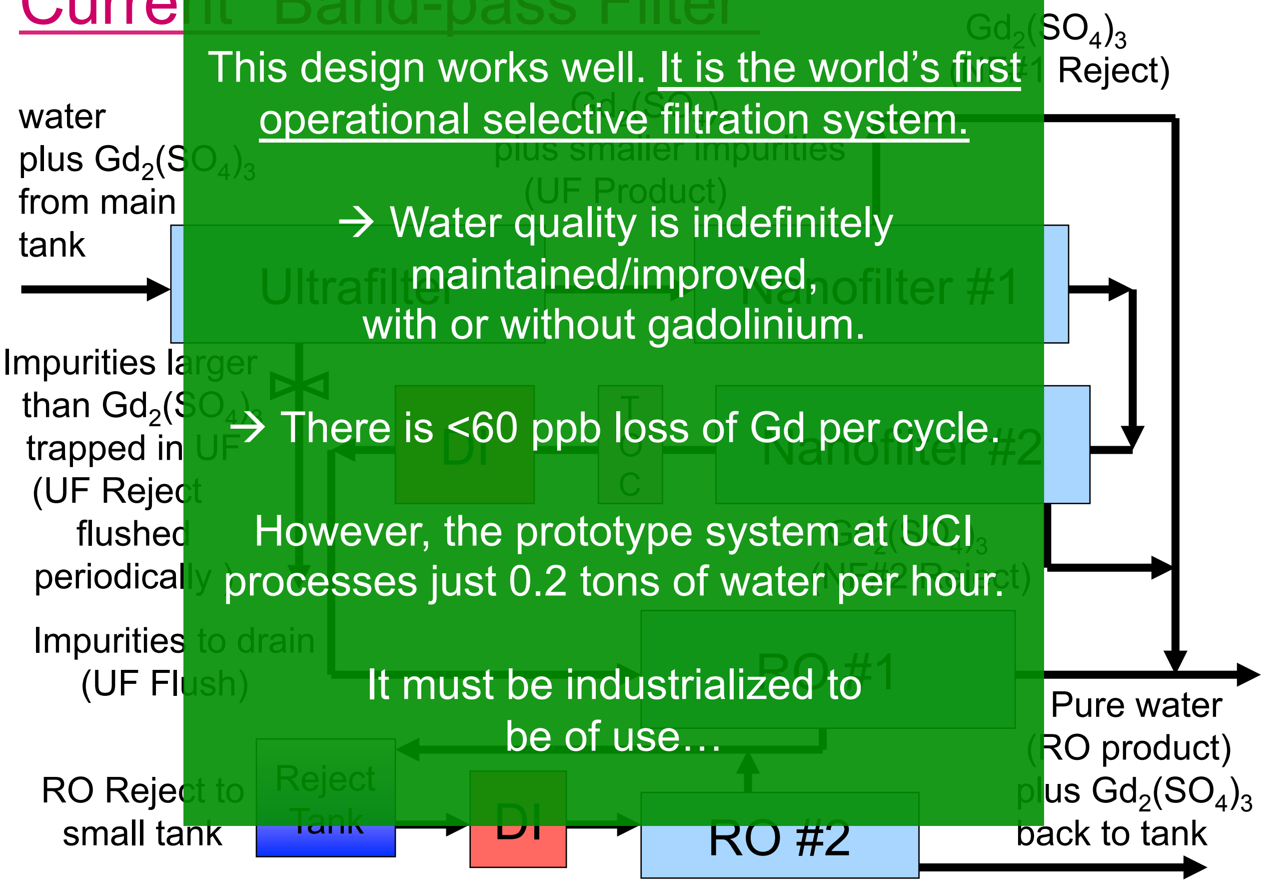

## Augmented Nanofilter System



# Current Selective Filtration Setup @ UCI




## Current "Band-pass Filter"




This design works well. <u>It is the world's first operational selective filtration system.</u>

→ Water quality is indefinitely maintained/improved, with or without gadolinium.

→ There is <60 ppb loss of Gd per cycle.

However, the prototype system at UCI processes just 0.2 tons of water per hour.

It must be industrialized to be of use…

In 2008 I underwent a significant transformation…

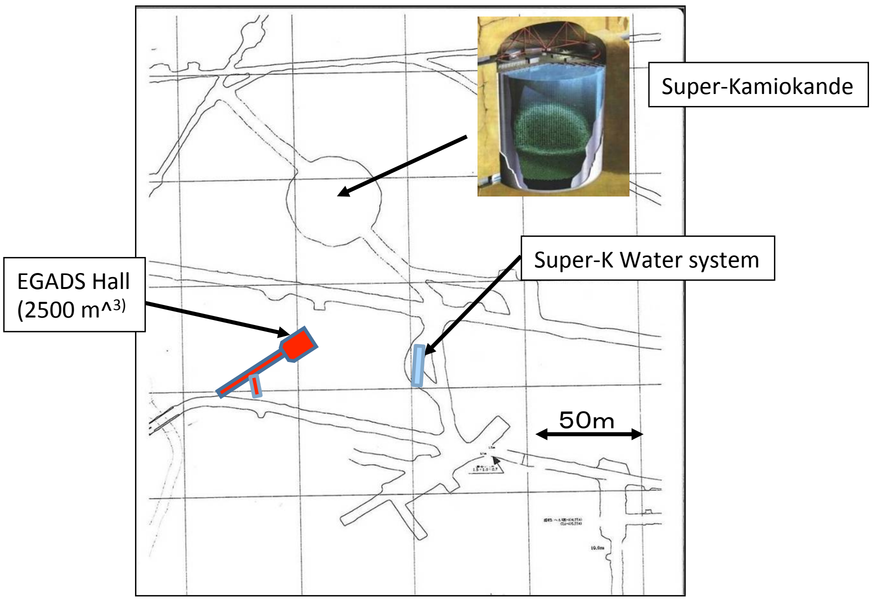
I joined UTokyo's newly-formed IPMU as their first full-time *gaijin* professor, though I still retain a "without salary" position at UCI and will continue Gd studies there.

I was explicitly hired to make gadolinium work in water!

IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

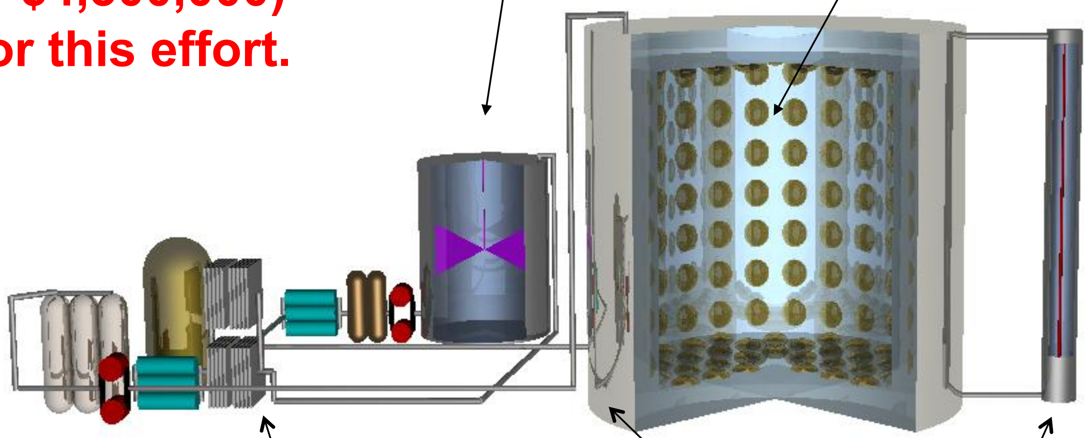
A dedicated Gd test facility has been built in the Kamioka mine, complete with its own water filtration system, 50-cm PMT's, and DAQ electronics.

**This 200 ton-scale R&D project is called EGADS – Evaluating Gadolinium's Action on Detector Systems.**



# EGADS Facility

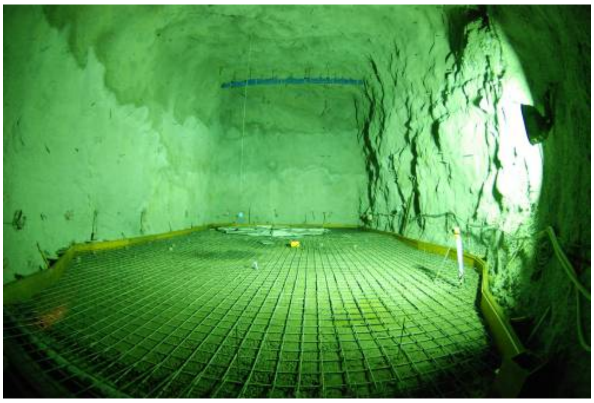
**In June of 2009 we received full funding (~$4,300,000) for this effort.**




[graphic by A. Kibayashi]

## Hall E and EGADS

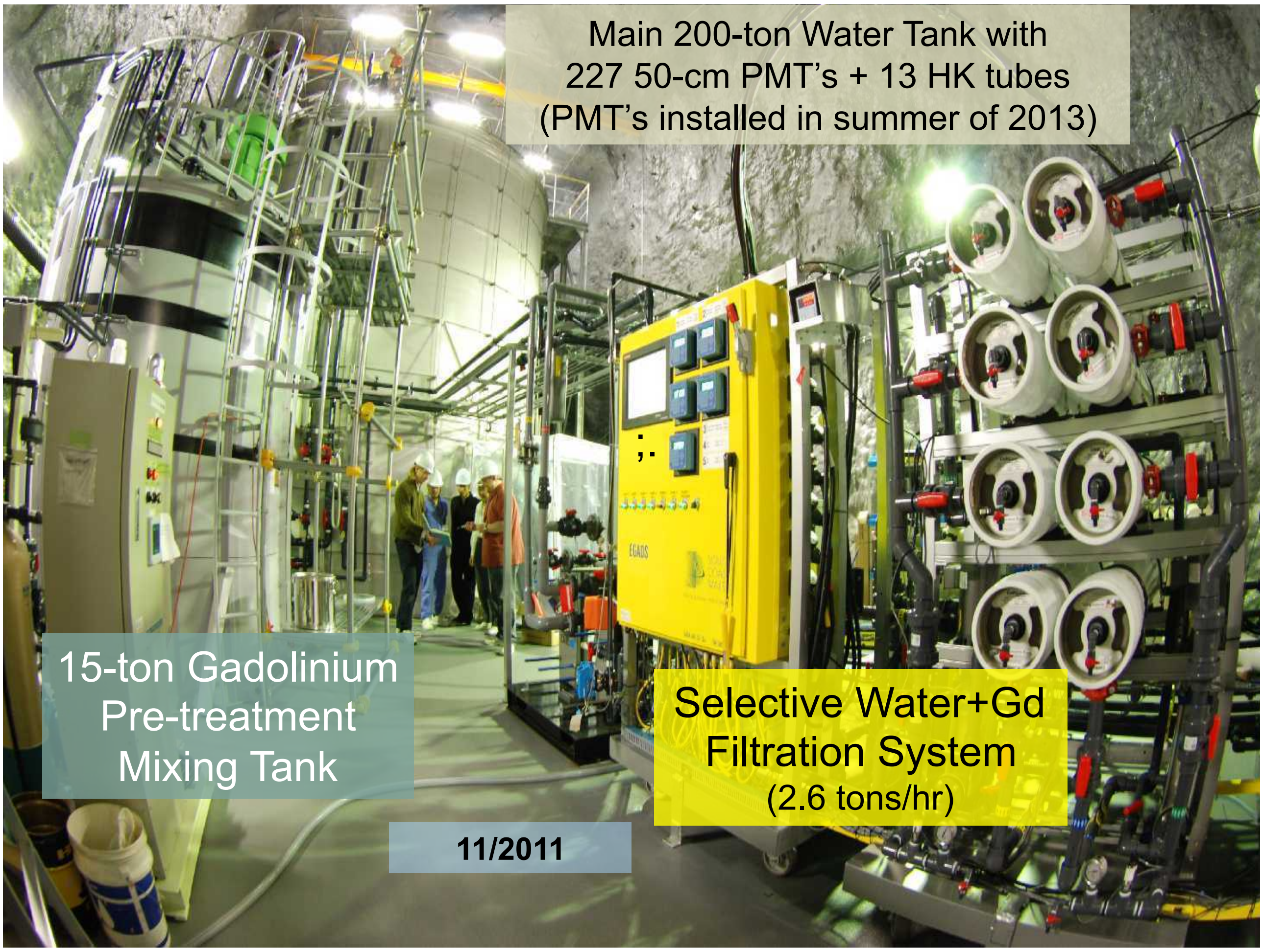
12/2009

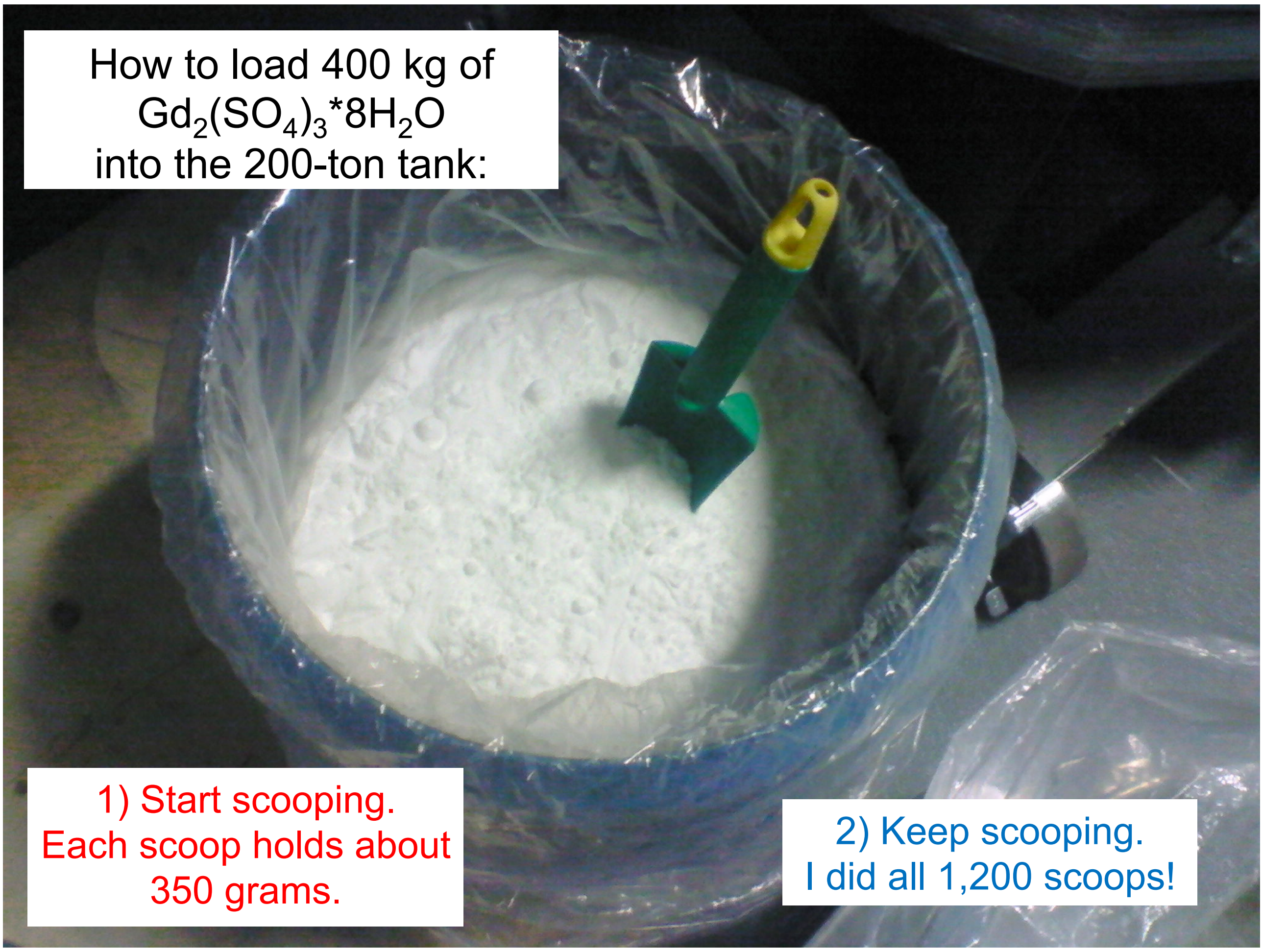


2/2010

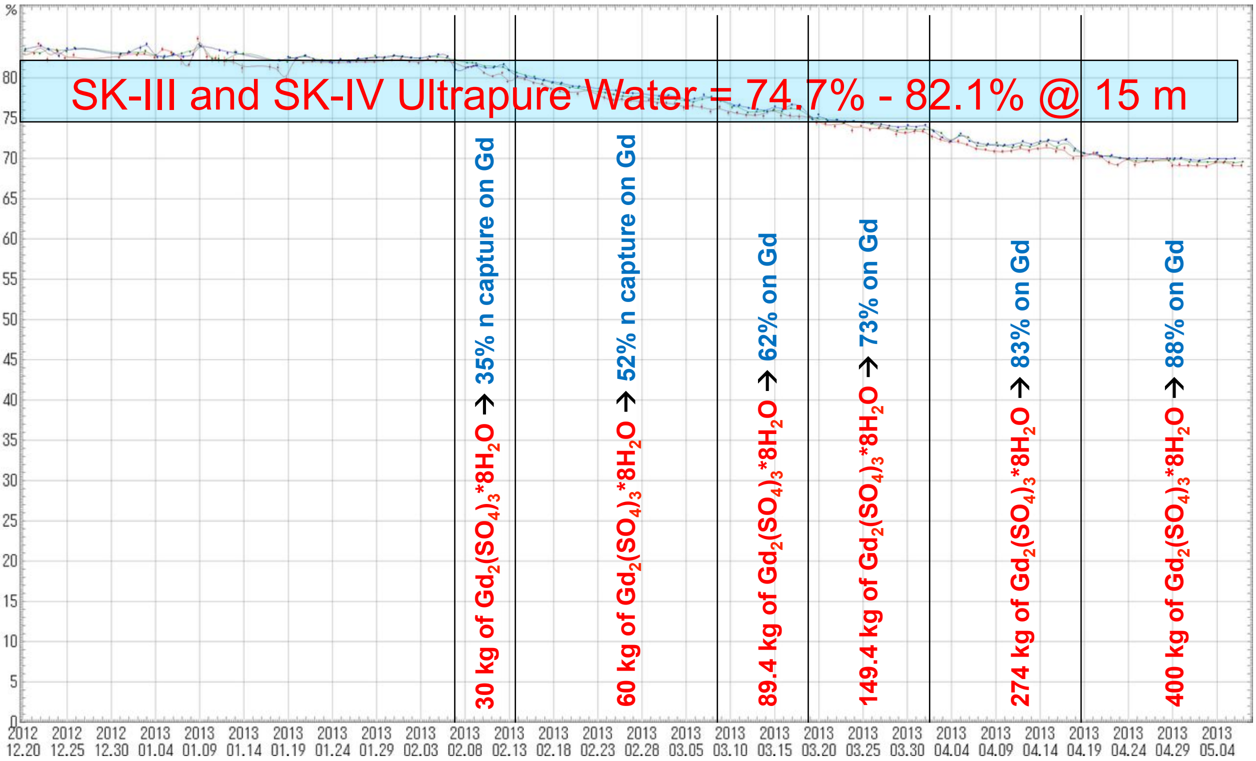
6/2010

12/2010


Main 200-ton Water Tank with
227 50-cm PMT's + 13 HK tubes
(PMT's installed in summer of 2013)
15-ton Gadolinium
Pre-treatment
Mixing Tank
11/2011
Selective Water+Gd
Filtration System
(2.6 tons/hr)

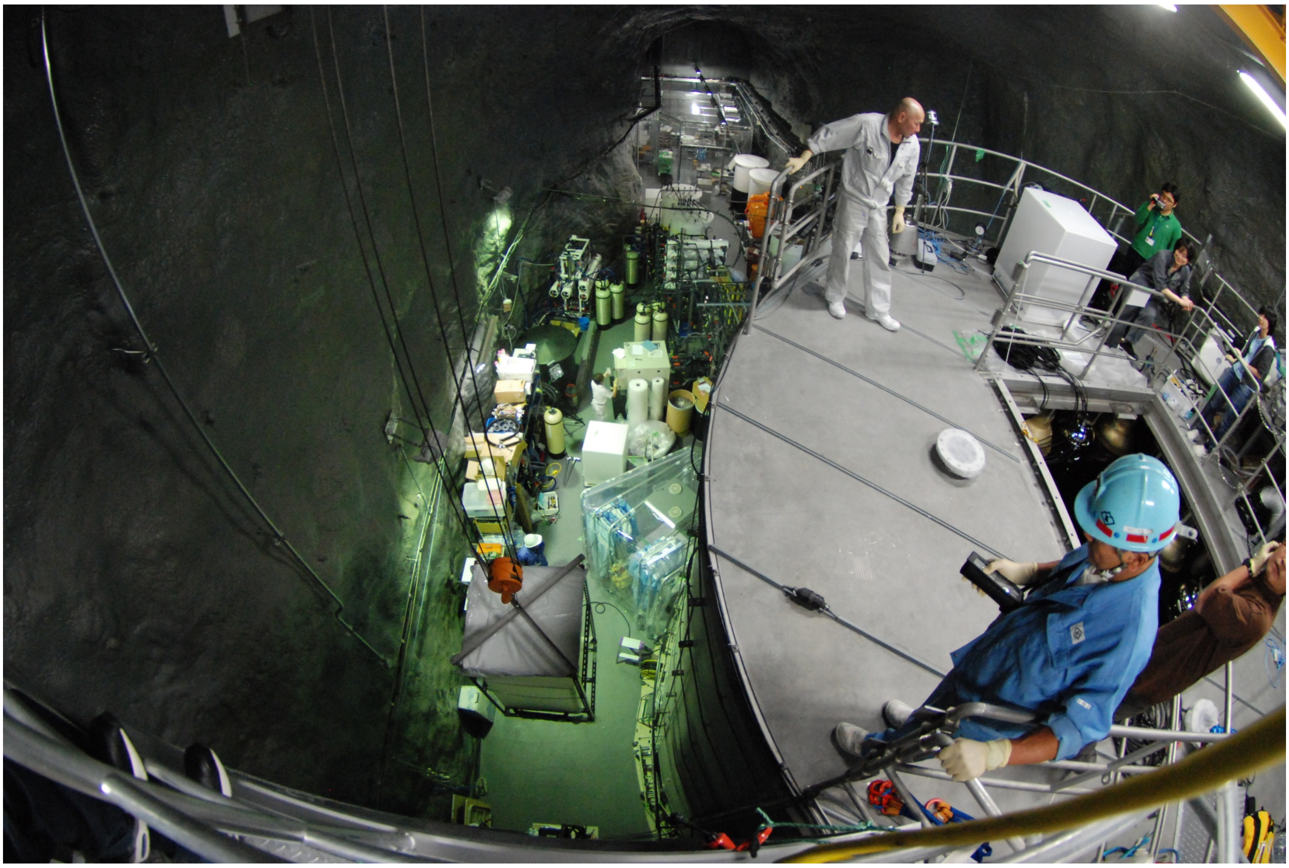

How to load 400 kg of
$Gd_2(SO_4)_3*8H_2O$
into the 200-ton tank:
1) Start scooping.
Each scoop holds about
350 grams.
2) Keep scooping.
I did all 1,200 scoops!

## Percentage of light remaining after 15 meters of travel

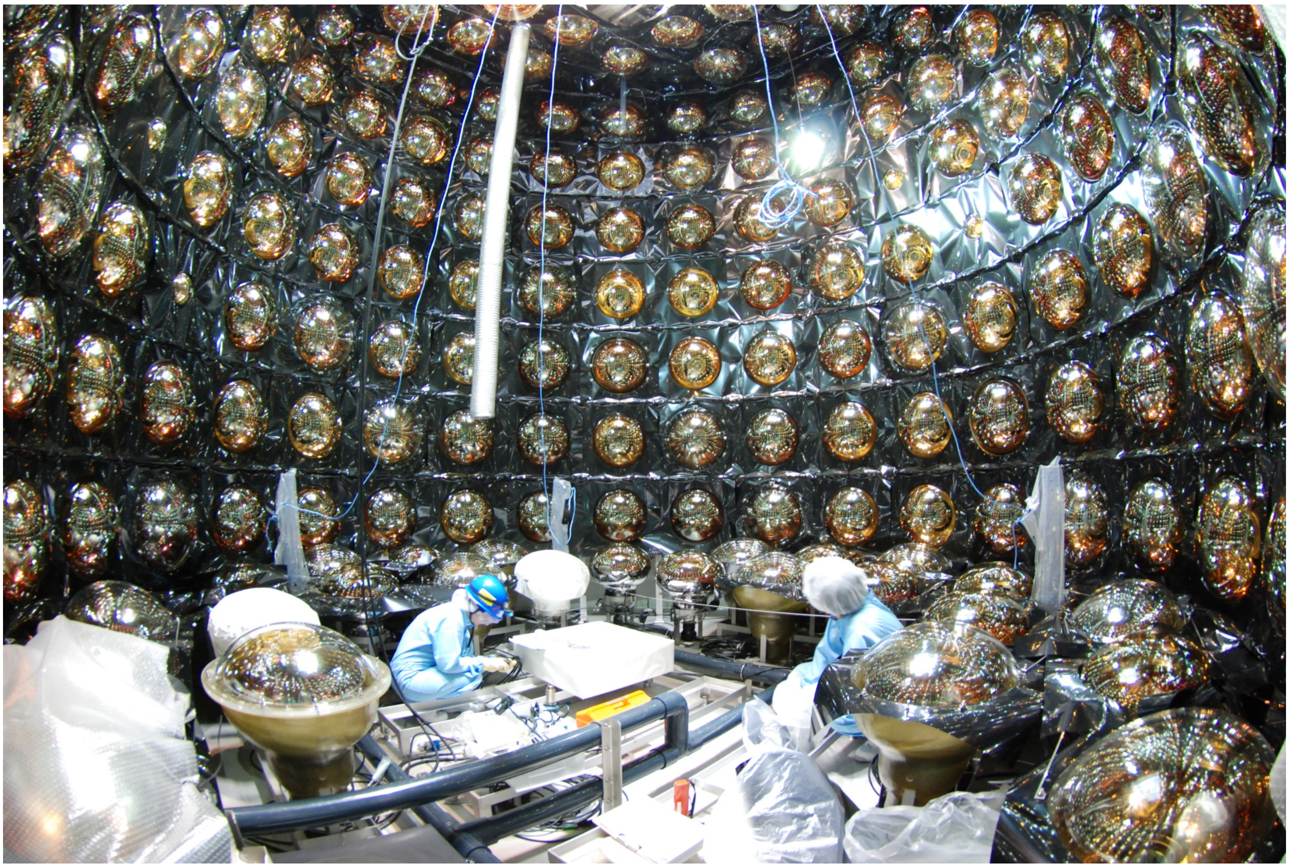



Water circulated continuously at 2.5 tons/hr.

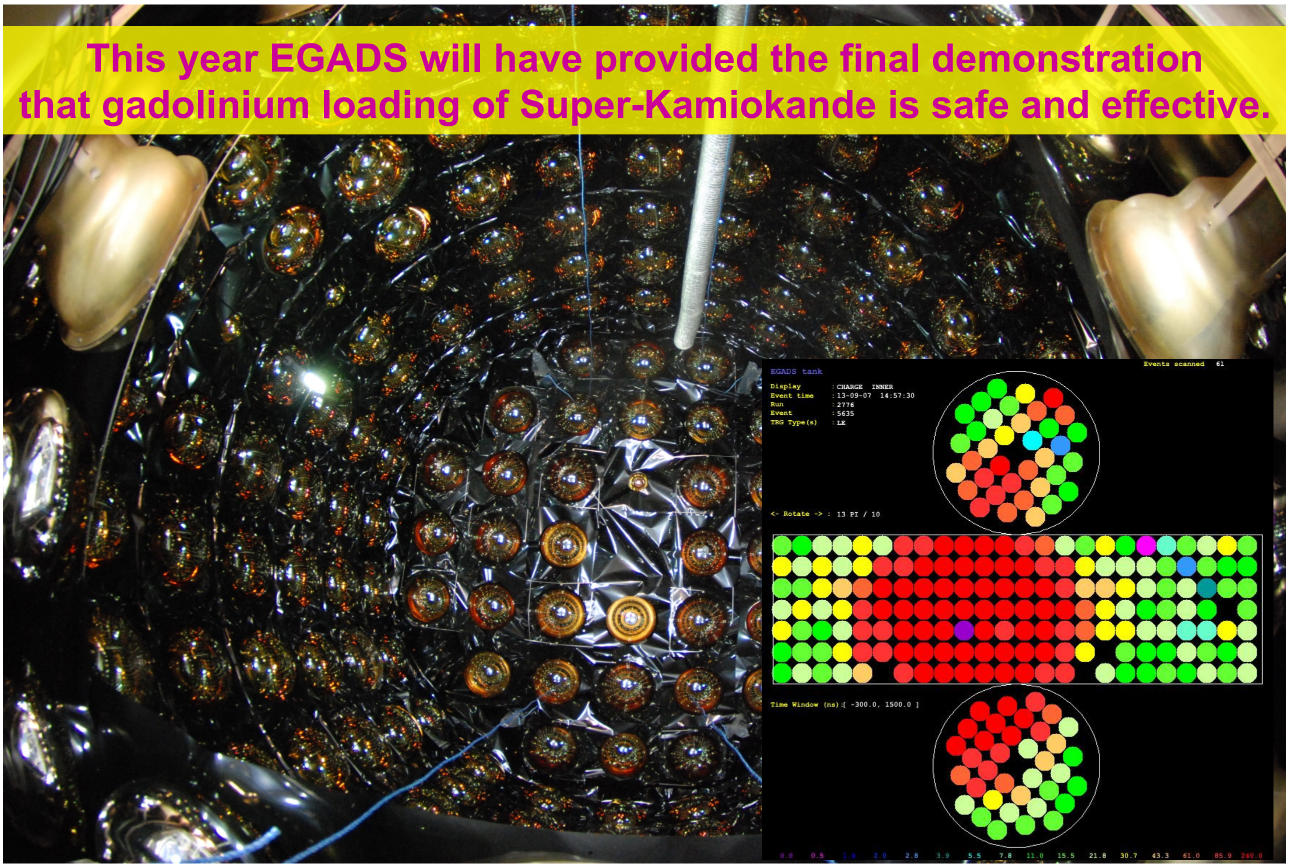
No detectable loss of gadolinium after months of operation!



EGADS PMT installation; August 2013



Working Inside the EGADS Tank; August 2013




**Looking Down Into the Completed EGADS Detector**

**Insert: Event Display of a Downward-Going Cosmic Ray Muon**

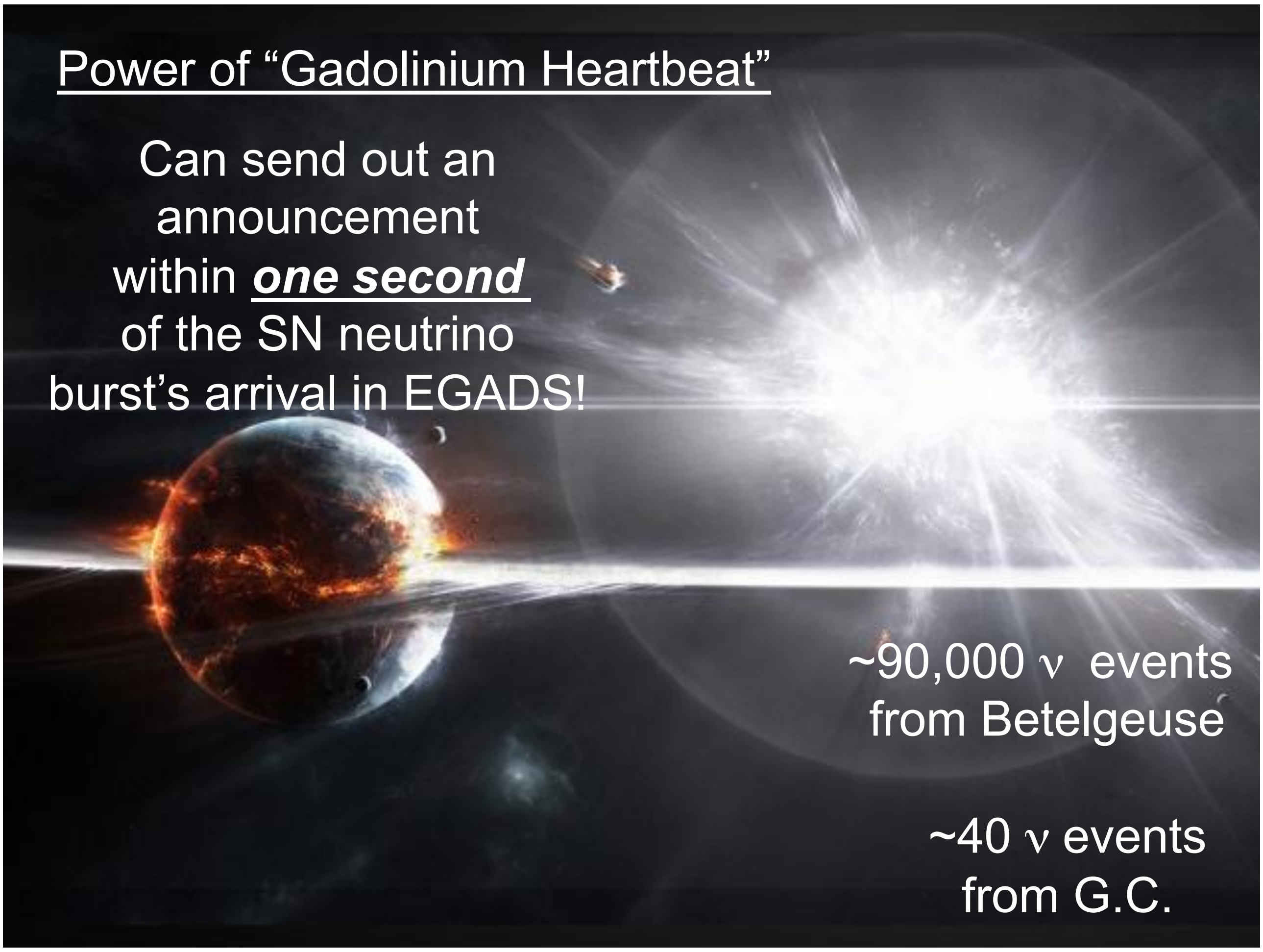
Since we expect all Gd R&D to be completed soon,
what happens to the valuable EGADS facility after that?

E valuating
G adolinium's
A ction on
D etector
S ystems

## Recently Funded: Multi-messenger Supernova Astronomy

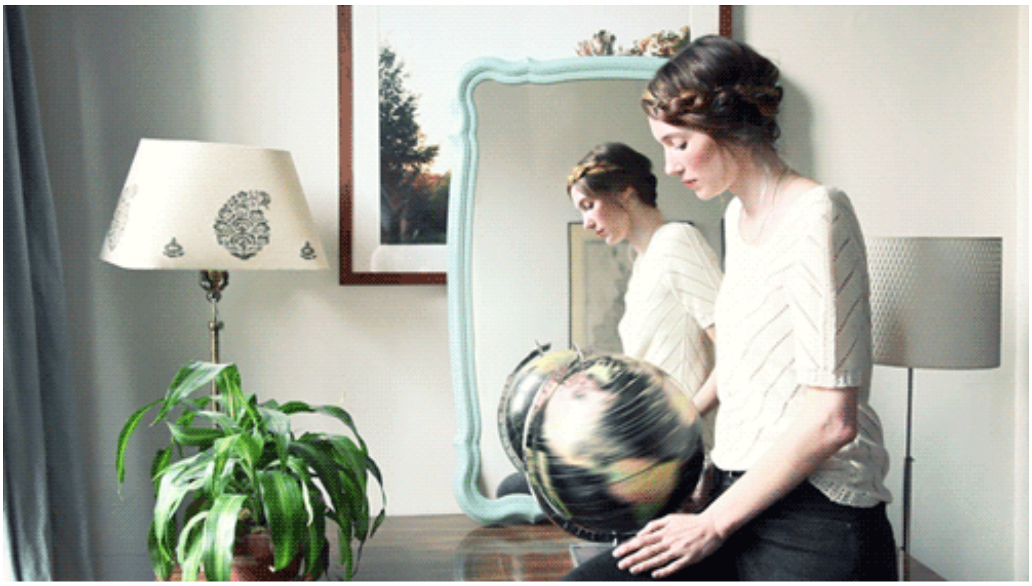

### Special features of SN neutrinos and GW's

- Provide image of core collapse itself (identical t=0)
- Only supernova messengers which travel without attenuation to Earth (dust does not affect signal)
- Guaranteed full-galaxy coverage




In 2015 we expect to be ready to detect supernova neutrinos with EGADS from anywhere in our galaxy, and send independent, immediate alerts to the world.

→ **No politics!** ←



By 2017 it is very likely we will be adding Gd in Super-K.

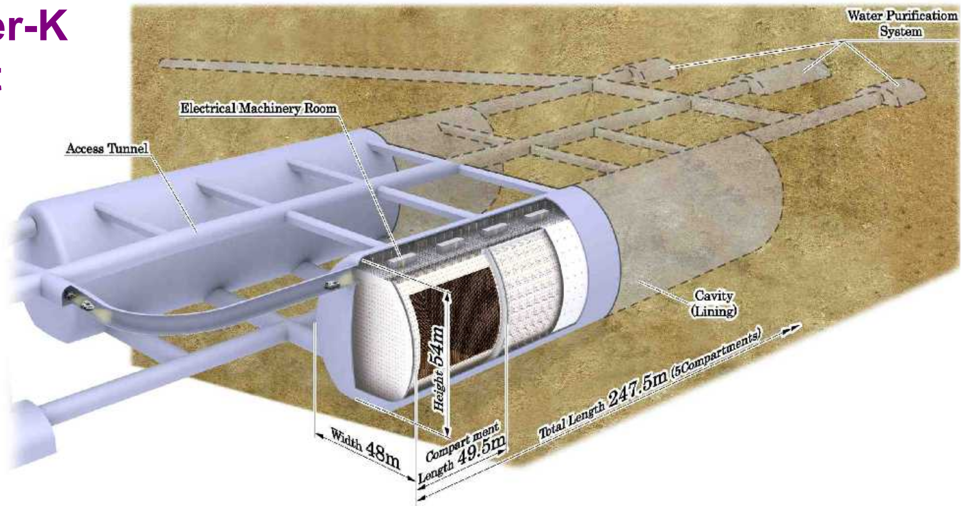
Gadolinium loading is part of the executive summary!

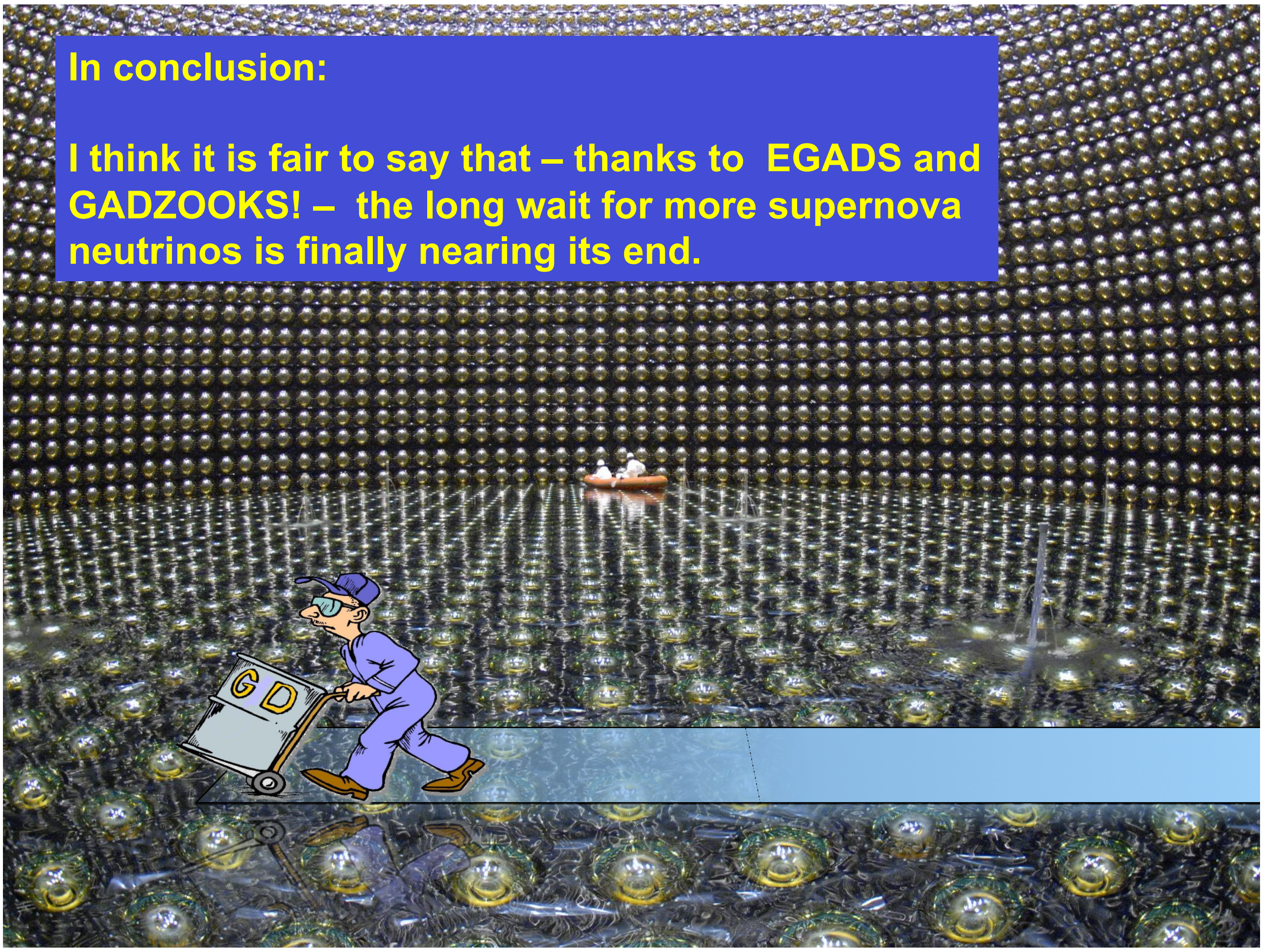
*In 2011, the official Hyper-Kamiokande Letter of Intent appeared on the arXiv:1109.3262*

1.0 Mton total water volume

0.56 Mton fiducial volume
(25 X Super-K)

**With Gd, Hyper-K should collect SN1987A-like numbers of supernova neutrinos... *every month!***



“Inflation in Axion Landscape”

Fuminobu Takahashi

[JGRG24(2014)111002]

# Inflation in Axion Landscape

10th Nov. 2014
JGRG24 @ IPMU

Fuminobu Takahashi
(Tohoku)

# Observation vs Theory

Scalar mode

$$P_{\mathcal{R}} = A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

Tensor mode

$$P_t = A_t \left(\frac{k}{k_0}\right)^{n_t}$$

$$A_s = \frac{V^3}{2\sqrt{3}V'^2},$$

$$n_s = 1 + 2\frac{V''}{V} - 3\left(\frac{V'}{V}\right)^2,$$

$$r = 8\left(\frac{V'}{V}\right)^2$$

$$A_s,\ n_s,\ r \equiv \frac{A_t}{A_s} \Longleftrightarrow V,\ V', V''$$

$V$: the inflaton potential

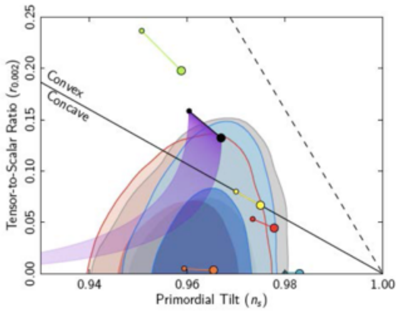
# Natural and Multi-Natural Inflation

## - Natural inflation

Freese, Frieman, Olinto `90

$$V = \Lambda^4 \left(1 - \cos\left(\frac{\phi}{f}\right)\right)$$

Only large-field inflation is possible, and f is bounded below: $f \gtrsim 5M_P$



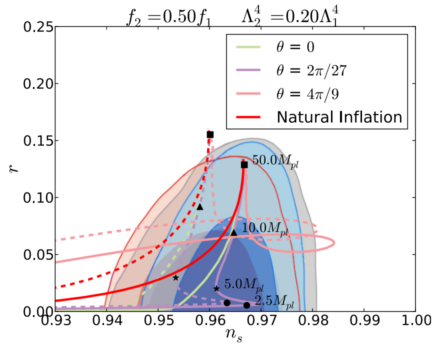

## - Multi-Natural inflation

Czerny, FT 1401.5212, Czerny, Higaki, FT 1403.0410, 1403.5883

$$V = \sum_{i=1}^{N_{\rm source}} \Lambda_i^4 \cos\left(\frac{\phi}{f_i} + \theta_i\right) + {\rm const.}$$

For $N_{\rm source}$ = 2, various values of ($n_s$,r) are possible as in the polynomial chaotic inf.

**No lower bound on the decay constants.**




# Aligned Natural Inflation

Kim, Nilles, Peloso, hep-ph/0409138

Czerny, Higaki, FT 1403.5883, Harigaya and Ibe 1404.3511, Choi, Kim, Yun, 1404.6209, Higaki, FT, 1404.6923, Tye, Won, 1404.6988, Kappl, Krippendorf, Nilles, 1404.7127, Bachlechner et al, 1404.7496, Ben-Dayan, Pedro, Westphal,1404.7773 ,Long, McAllister, McGuirk 1404.7852, Choi, Kim, Kyae 1410.1762

**The effectively large decay constant can be realized by the alignment of two (or more) axion potentials.**

- Two axions: $\phi_1 \to \phi_1 + 2\pi f_1 \quad \phi_2 \to \phi_2 + 2\pi f_2$

$$V(\phi_i) = \Lambda_1^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(m_1 \frac{\phi_1}{f_1} + m_2 \frac{\phi_2}{f_2}\right)\right]$$

Let us focus on the first term. Then there is a flat direction orthogonal to the combination in the cosine function.

# Aligned Natural Inflation

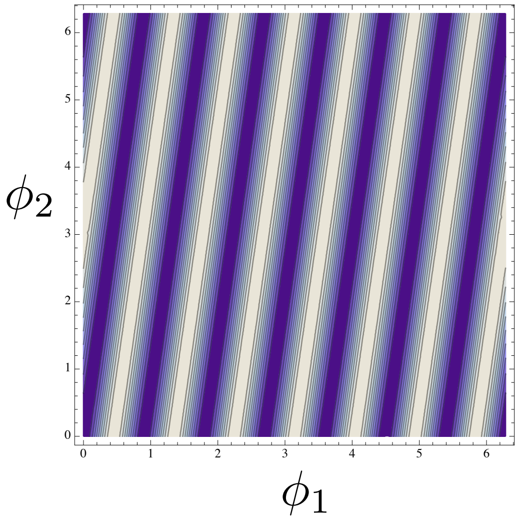
Kim, Nilles, Peloso, hep-ph/0409138

$$V(\phi_i) = \Lambda_1^4\left[1-\cos\left(n_1\frac{\phi_1}{f_1}+n_2\frac{\phi_2}{f_2}\right)\right]+\Lambda_2^4\left[1-\cos\left(m_1\frac{\phi_1}{f_1}+m_2\frac{\phi_2}{f_2}\right)\right]$$

Flat direction extends over more than the Planck scale, if $n_1 \gg n_2$, even for $f_1$, $f_2 < M_P$.

If $n_1/n_2 \approx m_1/m_2$, the effectively large decay constant $f_{\rm eff} > M_P$ is realized.

$$f_{\rm eff} = \frac{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}{|n_1 m_2 - n_2 m_1|}$$




# Aligned Natural Inflation

- Multiple axions: $\phi_i \equiv \phi_i + 2\pi f_i \quad (i=1,\cdots,N)$

$$V(\phi_i) = \sum_{i=1}^{N}\Lambda_i^4\left[1-\cos\left(\sum_{j=1}^{N}\frac{n_{ij}\phi_j}{f_j}\right)\right]$$

For a moderately large N (> 5 or so), the effective decay constant can be enhanced w/o hierarchy among the anomaly coefficients.

Choi, Kim, Yun, 1404.6209

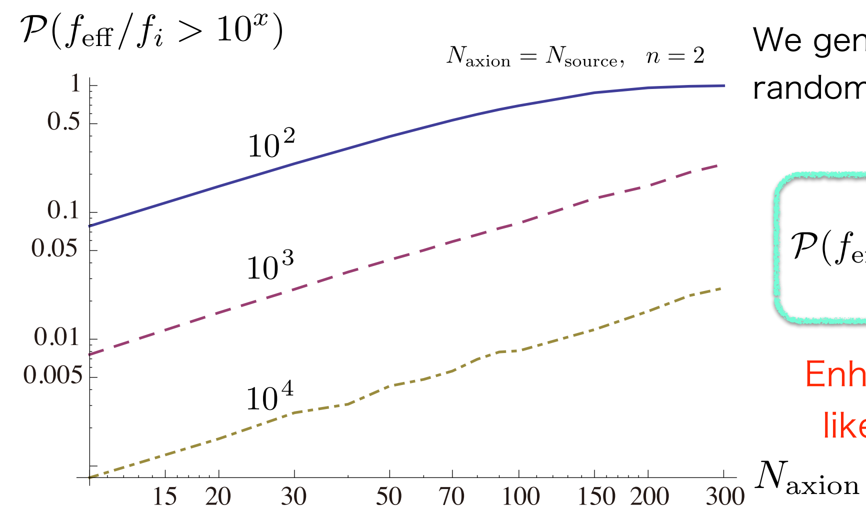
Prob. dist. was studied in detail for various cases including the cases of $N_{\rm source} \neq N_{\rm axion}$ and of no hierarchies among $\Lambda_i$ .

Higaki, FT, 1404.6923

# Aligned Natural Inflation

$$V(\phi_\alpha) = \sum_{i=1}^{N_{\rm source}} \Lambda_i^4 \left(1 - \cos\left(\sum_{\alpha=1}^{N_{\rm axion}} n_{i\alpha}\frac{\phi_\alpha}{f_\alpha} + \theta_i\right)\right) + C$$

- Prob dist for the enhancement of the decay constant




We generated integer-valued random matrix $-2 \le n_{i\alpha} \le 2$

$$\mathcal{P}(f_{\rm eff}/f_i) \sim N_{\rm axion}\left(\frac{f_i}{f_{\rm eff}}\right)$$

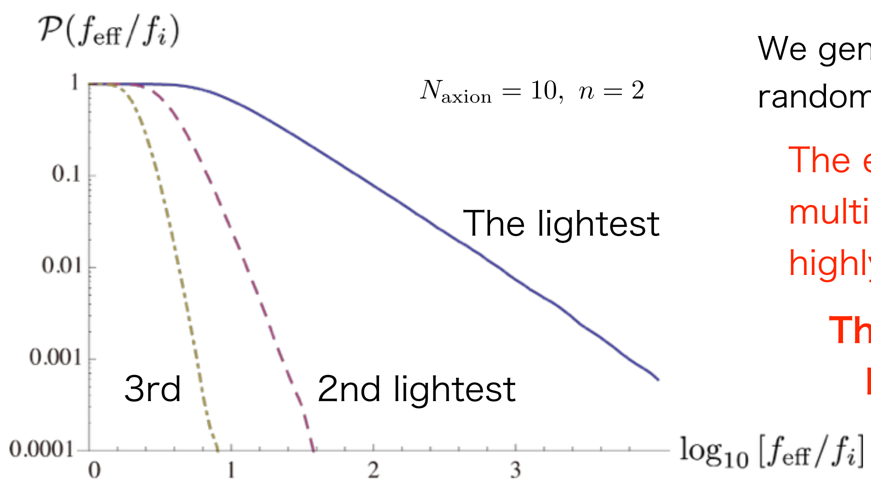
Enhancement becomes likely for larger $N_{\rm axion}$.

Higaki, FT, 1404.6923

# Aligned Natural Inflation

$$V(\phi_i) = \sum_{i=1}^{N_{\rm source}} \Lambda_i^4 \cos\left(\sum_{j=1}^{N_{\rm axion}} a_{ij}\frac{\phi_j}{f_j} + \theta_i\right) + V_0$$

- Prob dist for the enhancement of the decay constant




We generated integer-valued random matrix $-n \le a_{ij} \le n$

The enhancement along multiple directions is highly unlikely.

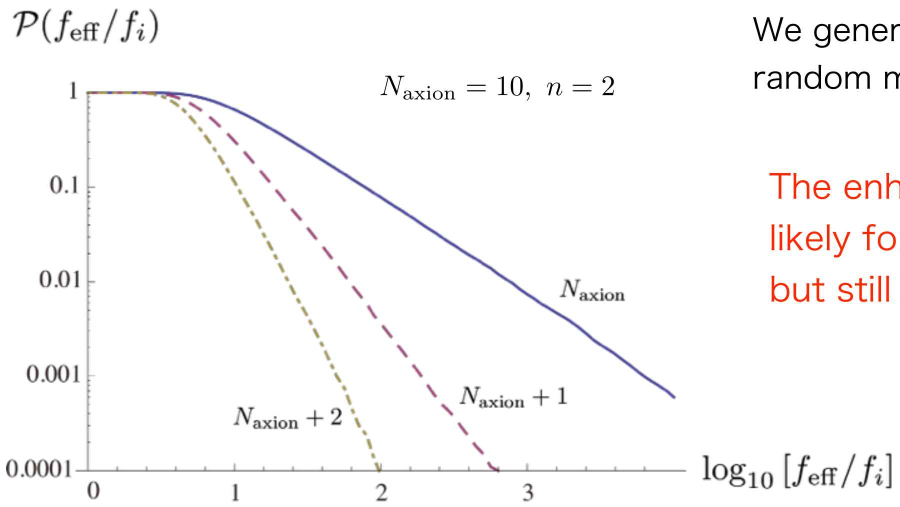
**The inflaton is the lightest axion!**

Higaki, FT, 1404.6923

# Aligned Natural Inflation

$$V(\phi_i) = \sum_{i=1}^{N_{\rm source}} \Lambda_i^4 \cos\left(\sum_{j=1}^{N_{\rm axion}} a_{ij}\frac{\phi_j}{f_j} + \theta_i\right) + V_0$$

- Prob dist for the enhancement of the decay constant




We generated integer-valued random matrix $-n \leq a_{ij} \leq n$

The enhancement is less likely for larger $N_{\rm source}$, but still possible.

Higaki, FT, 1404.6923

# Axion Landscape

Higaki, FT 1404.6923, 1409.8409

$$V(\phi_\alpha) = \sum_{i=1}^{N_{\rm source}} \Lambda_i^4 \left(1 - \cos\left(\sum_{\alpha=1}^{N_{\rm axion}} n_{i\alpha}\frac{\phi_\alpha}{f_\alpha} + \theta_i\right)\right) + C$$

Discrete and degenerate minima if $N_{\rm source} = N_{\rm axion}$.

Many local minima with different energy for $N_{\rm source} > N_{\rm axion}$.

"Axion Landscape"

# Axion Landscape

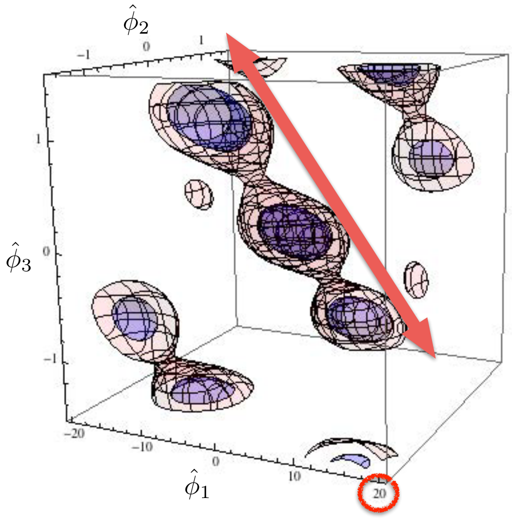
Higaki, FT 1404.6923, 1409.8409

$$V(\phi_\alpha) = \sum_{i=1}^{N_{\rm source}} \Lambda_i^4 \left(1 - \cos\left(\sum_{\alpha=1}^{N_{\rm axion}} n_{i\alpha}\frac{\phi_\alpha}{f_\alpha} + \theta_i\right)\right) + C$$

Many local minima with different energy for $N_{\rm source} > N_{\rm axion}$.

There appears a very flat direction with $f_{\rm eff} > M_P$, if the KNP mechanism works.

Multiple axions create numerous local minima as well as a very flat direction.





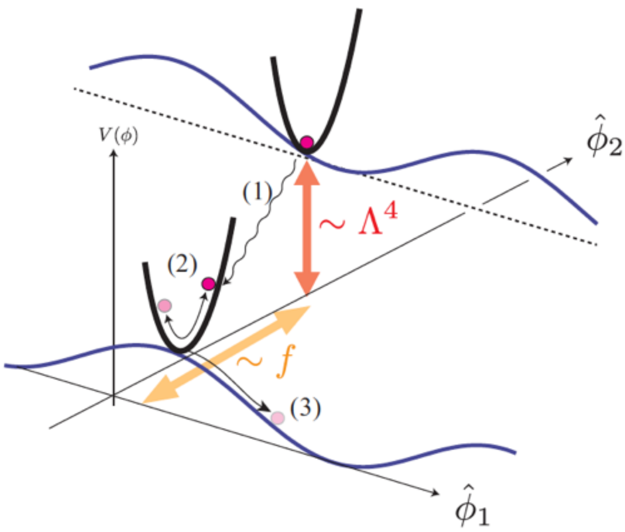
# Axion Landscape


Eternal inflation takes place in one of the local minima.

(1)Tunneling from excited states

(2)Heavy axions fast roll and oscillate.

(3)Slow-roll inflation takes place along the very light direction, which appears due to the KNP.




Higaki, FT 1404.6923, 1409.8409

✓ **Eternal inflation and slow-roll inflation after bubble nucleation are realized in a unified manner.**

✓ **Also there may be a pressure toward smaller $N_e$.**

# Implications of Axion Landscape

Higaki, FT 1404.6923, 1409.8409

(1)($n_s$,r): **Natural or multi-natural inflation**. If there is a pressure toward smaller $N_e$, deviation from the quadratic chaotic inflation is expected.

(2)**Negative spatial curvature** if the total e-folding $N_e$ is just 50-60, due to the pressure toward smaller $N_e$ in the axion landscape.

Linde `95, Freivogel et al `05, Yamauchi et al `11, Bousso et al `13


# Implications of Axion Landscape

Higaki, FT 1404.6923, 1409.8409

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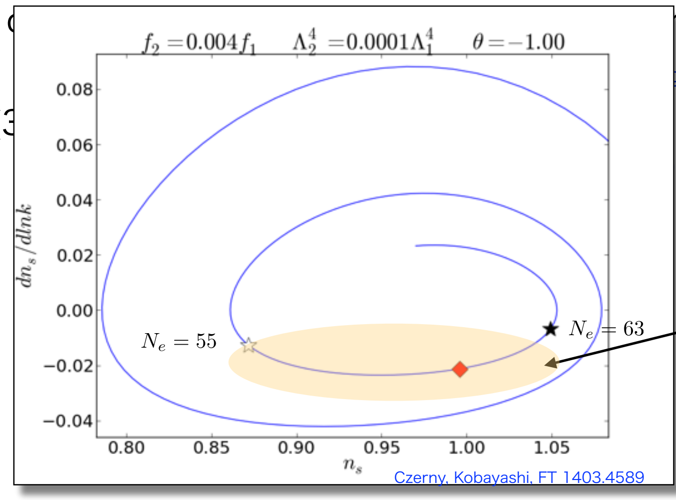



the axion landscape.

gel et al `05, Yamauchi et al `11, Bousso et al `13

(3 ... modulations.

11.3988, Czerny, Kobayashi, FT 1403.4589

Running is almost constant over CMB scales.

# Implications of Axion Landscape

Higaki, FT 1404.6923, 1409.8409

(1)($n_s$,r): **Natural or multi-natural inflation**. If there is a pressure toward smaller $N_e$, deviation from the quadratic potential is expected.

(2)**Negative spatial curvature** if the total e-folding $N_e$ is just 50-60, due to the pressure toward smaller $N_e$ in the axion landscape.

Linde `95, Freivogel et al `05, Yamauchi et al `11, Bousso et al `13

(3)**Running spectral index** due to small modulations.

Kobayashi, FT 1011.3988, Czerny, Kobayashi, FT 1403.4589

(4)**Non-Gaussianity** due to possible couplings to gauge fields.

Barnarby, Peloso, 1011.1500, Barnaby, Namba, Peloso, 1102.4333.

$$\mathcal{L} = \frac{\phi_\alpha}{f_\alpha} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Current constraint reads $f_\alpha \gtrsim 6 \times 10^{16}\,\mathrm{GeV}\sqrt{\frac{\epsilon_\alpha}{0.01}}$

# Conclusions

- **Large-field inflation** realized by shift symmetry.
    - Polynomial chaotic/multi-natural inflation lead to various values of ($n_s$,r).
- **Axion landscape**
    - Multiple axions lead to a flat direction with an effectively super-Planckian decay constant.
    - Multiple axions also form a landscape.
    - Eternal inflation and subsequent (multi-)natural inflation realized in a unified manner.
    - Negative spatial curvature, running spectral index, non-Gaussianity.

“Resonant conversions of QCD axions into hidden axions”

Naoya Kitajima

[JGRG24(2014)111003]

# Resonant conversion of QCD axions into hidden axions (and suppressed isocurvature perturbations)

**Naoya Kitajima**
Tohoku University

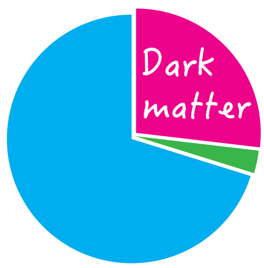
arXiv:1411.XXXX (today!)

In collaboration with **Fuminobu Takahashi**
(Tohoku University)

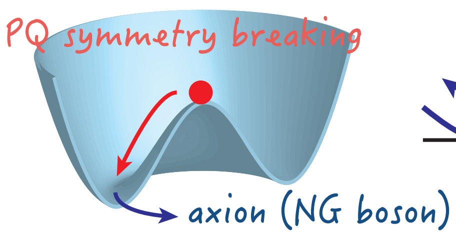
JGRG24, 10 Nov. 2014, Kavli IPMU

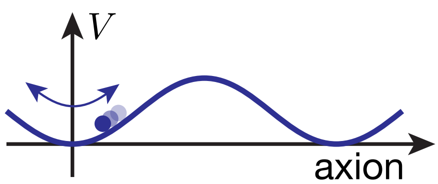


## 1. Introduction




- **Our Universe is filled with the Dark matter**
- **Axion may exist** <= strong CP problem



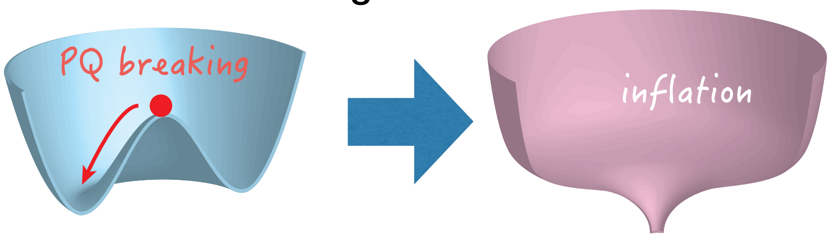




$$\Omega_a h^2 \simeq 0.195\theta_i^2 f(\theta_i)\left(\frac{F_a}{10^{12}\ \mathrm{GeV}}\right)^{1.184}$$

**Axion $\overset{?}{=}$ Dark matter**

## Isocurvature problem

**PQ breaking => inflation**

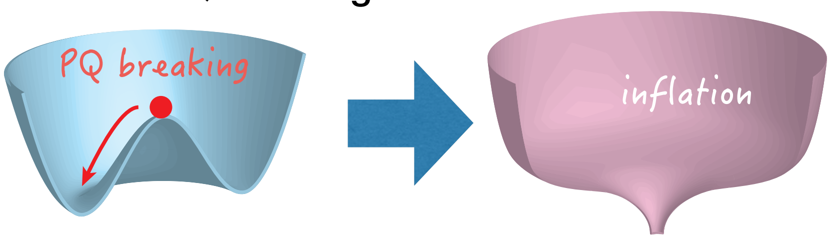



**! Isocurvature perturbation**



## Isocurvature problem

**PQ breaking => inflation**




**! Isocurvature perturbation**

### Axion CDM isocurvature perturbation

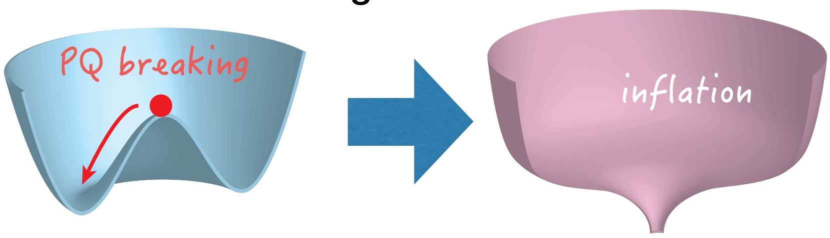
$$\Delta_{\mathcal{S},\mathrm{CDM}} = \left(\frac{\Omega_a}{\Omega_{\mathrm{CDM}}}\right)\Delta_{\mathcal{S},a} \quad \text{and} \quad \Delta_{\mathcal{S},a} = \frac{\partial \ln \Omega_a}{\partial \theta_i}\frac{H_{\mathrm{inf}}}{2\pi F_a} \sim \frac{\delta\Omega_a}{\Omega_a}$$

Current constraint : $\Delta^2_{\mathcal{S},\mathrm{CDM}} < \frac{\beta}{1-\beta}\Delta^2_{\mathcal{R}}$ with $\beta = 0.039$

Planck collaboration:1303.5076,1303.5082

## Isocurvature problem

**PQ breaking => inflation**




**! Isocurvature perturbation**

Axion CDM isocurvature perturbation

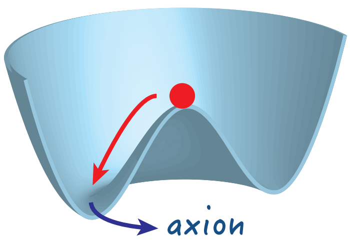
$$\Delta_{\mathcal{S},\mathrm{CDM}} = \left(\frac{\Omega_a}{\Omega_{\mathrm{CDM}}}\right)\Delta_{\mathcal{S},a} \quad \text{and} \quad \Delta_{\mathcal{S},a} = \frac{\partial \ln \Omega_a}{\partial \theta_i}\frac{H_{\mathrm{inf}}}{2\pi F_a} \sim \frac{\delta\Omega_a}{\Omega_a}$$

Tightly constrained

Current constraint : $\Delta^2_{\mathcal{S},\mathrm{CDM}} < \frac{\beta}{1-\beta}\Delta^2_{\mathcal{R}}$ with $\beta = 0.039$

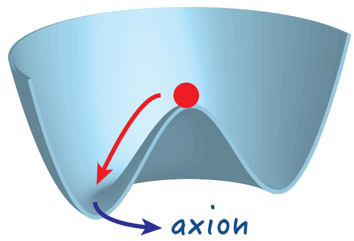
Planck collaboration:1303.5076,1303.5082

5



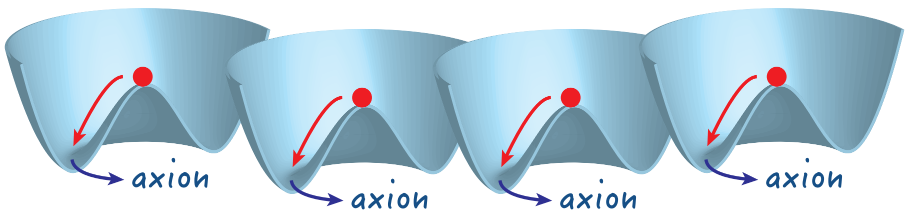

**Single axion** → $F_a,\ \theta_i$

6




Single axion → $F_a, \ \theta_i$

## Multi-axion






QCD axion + *Hidden axion*
with mass mixing

MSW-like resonance

Abundance is suppressed

Hill and Ross, Nucl.Phys.(1988)

QCD axion + *Hidden axion*
with mass mixing

↓

MSW-like resonance

↓

Abundance is suppressed

Hill and Ross, Nucl.Phys.(1988)

**Our work**

- Precise numerical calculations
- Implication for isocurvature perturbations

***New suppression mechanism***
***for isocurvature perturbations***

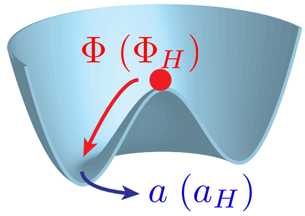


## 2. Model

PQ field & hidden PQ field with heavy quarks

$$\mathcal{L} = \kappa \Phi Q \bar{Q} + \frac{\lambda}{M_P} \Phi \Phi_H Q_H \bar{Q}_H$$

$\mathrm{U}(1)_{\mathrm{PQ}}$ & $\mathrm{U}(1)_{\mathrm{H}}$




↓

Potential for QCD and hidden axions

$$V(a, a_H) = m_a^2(T) F_a^2 \left[1 - \cos\left(\frac{a}{F_a}\right)\right] + m_H^2 F_H^2 \left[1 - \cos\left(\frac{a_H}{F_H} + \frac{a}{F_a}\right)\right]$$

$m_a, m_H$ : masses
$F_a, F_H$ : decay constants

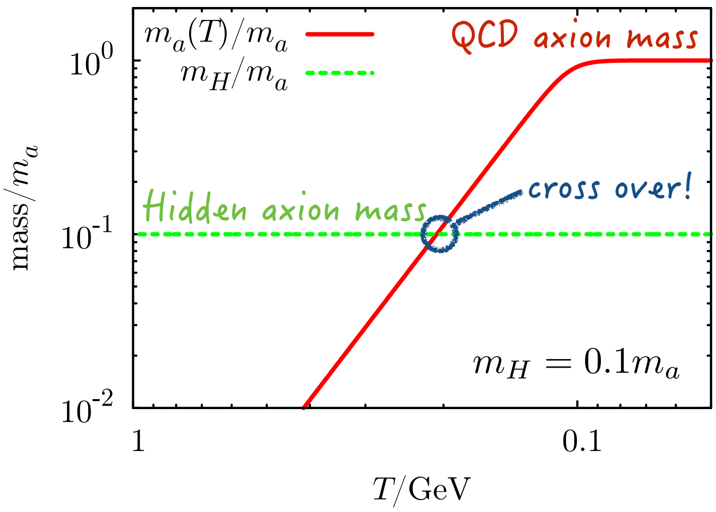
**Charge assignments**

| | $\Phi$ | $\Phi_H$ | $Q$ | $\bar{Q}$ | $Q_H$ | $\bar{Q}_H$ |
|---|---|---|---|---|---|---|
| $\mathrm{U}(1)_{\mathrm{PQ}}$ | 1 | 0 | 1/2 | −1/2 | 1/2 | −1/2 |
| $\mathrm{U}(1)_{\mathrm{H}}$ | 0 | 1 | 0 | 0 | 1/2 | −1/2 |

## Temperature dependent QCD axion mass

$$m_a(T) = \begin{cases} 4.05 \times 10^{-4} \dfrac{\Lambda_{\rm QCD}^2}{F_a} \left(\dfrac{T}{\Lambda_{\rm QCD}}\right)^{-3.34} & \text{for } T > 0.26\Lambda_{\rm QCD} \\ 3.82 \times 10^{-2} \dfrac{\Lambda_{\rm QCD}^2}{F_a} & \text{for } T < 0.26\Lambda_{\rm QCD}, \end{cases}$$

**If zero temp. QCD axion mass >> hidden axion mass…**



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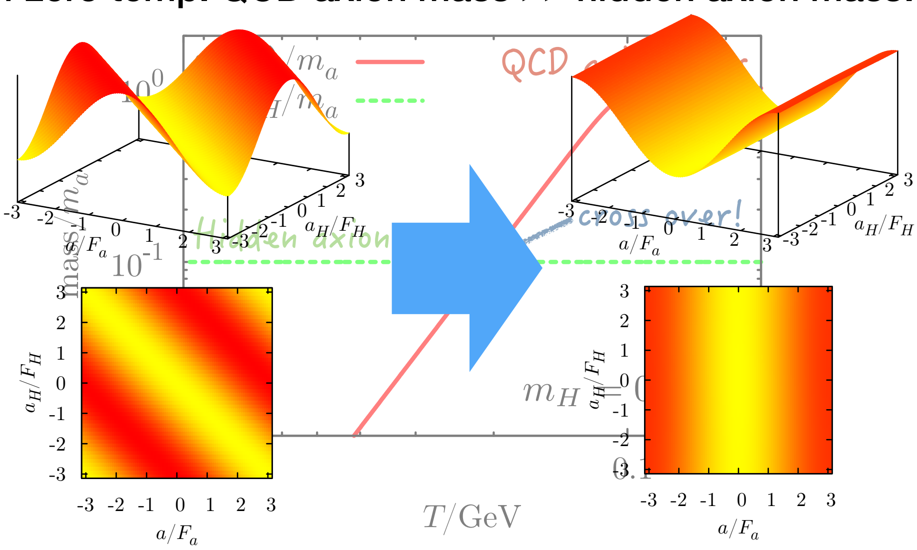

## Linearized equation of motion

$$\ddot{A} + 3H\dot{A} + M^2 A = 0$$

with

$$A = \begin{pmatrix} a \\ a_H \end{pmatrix} \quad \text{and} \quad M^2 = \begin{pmatrix} m_a^2(T) + (F_a/F_H)^2 m_H^2 & (F_H/F_a) m_H^2 \\ (F_H/F_a) m_H^2 & m_H^2 \end{pmatrix}$$

Mass eigenstates

$$\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} = O^T M^2 O \quad \text{and} \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = O^T A$$

$a_1$ : Light axion

$a_2$ : Heavy axion

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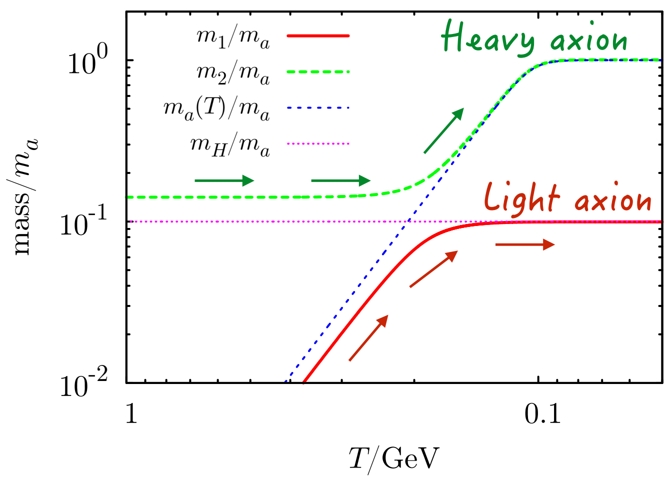
## Evolution of mass eigenvalues




hidden axion —> QCD axion

QCD axion —> hidden axion

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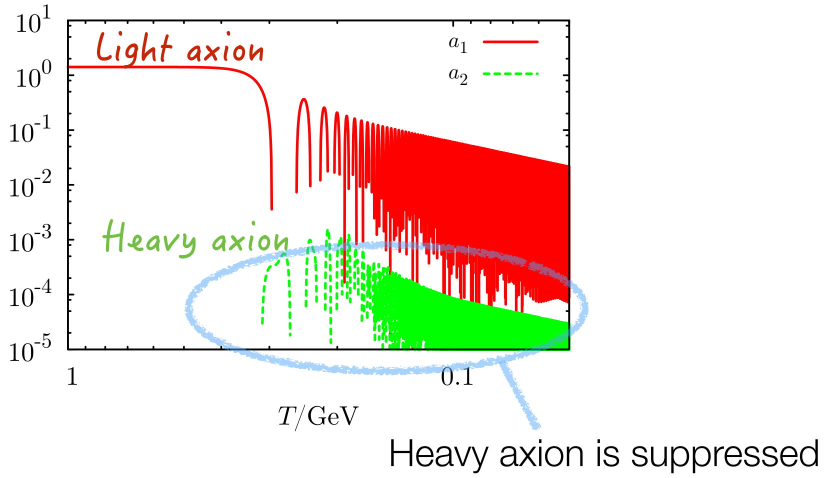
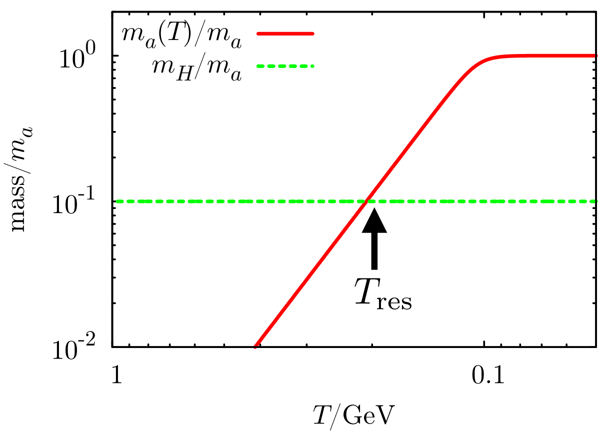
## Heavy axion is suppressed, but…






## Adiabaticity, Anharmonicity

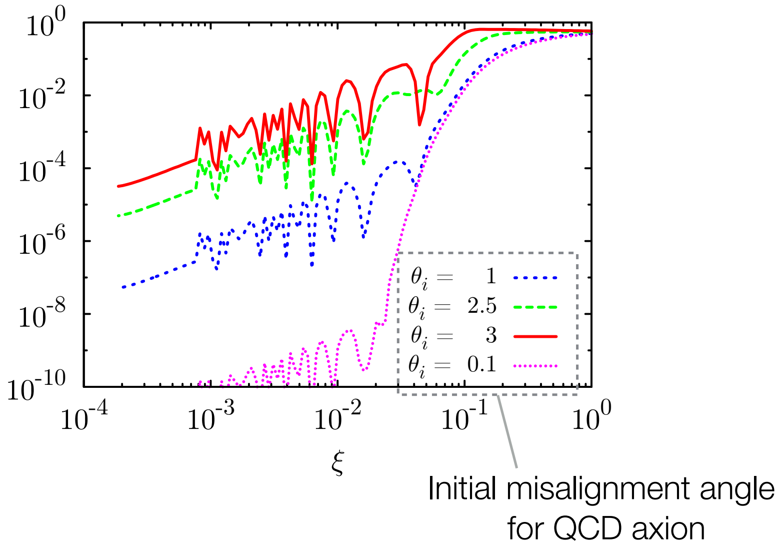
Adiabaticity parameter : $\xi = \dfrac{H(T_{\rm res})}{m_H} = \dfrac{H(T_{\rm res})}{m_a(T_{\rm res})}$



# Adiabaticity, Anharmonicity

Adiabaticity parameter : $\xi = \frac{H(T_{\rm res})}{m_H} = \frac{H(T_{\rm res})}{m_a(T_{\rm res})}$

$\frac{\text{\# of heavy axion}}{\text{\# of total axion}}$



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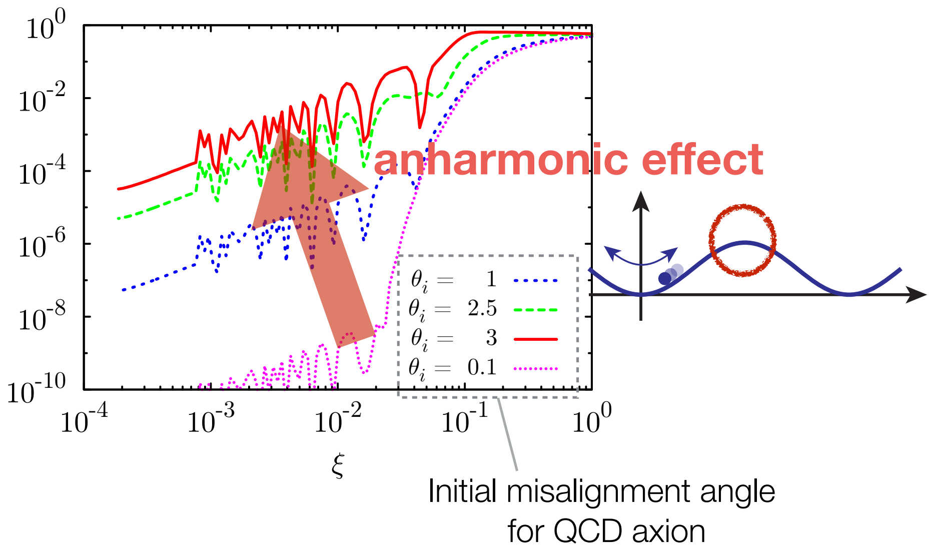
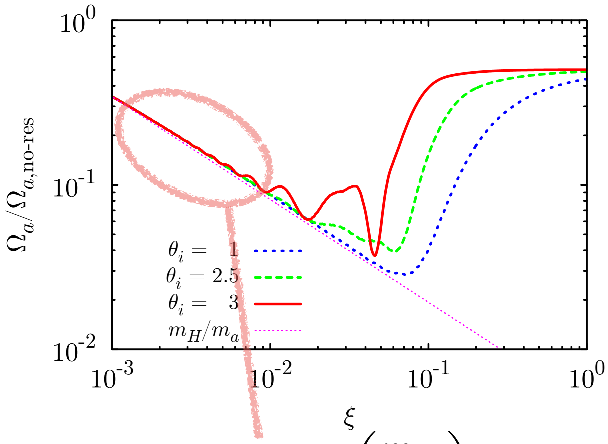


# 3. Cosmological implications

## (i) Abundance

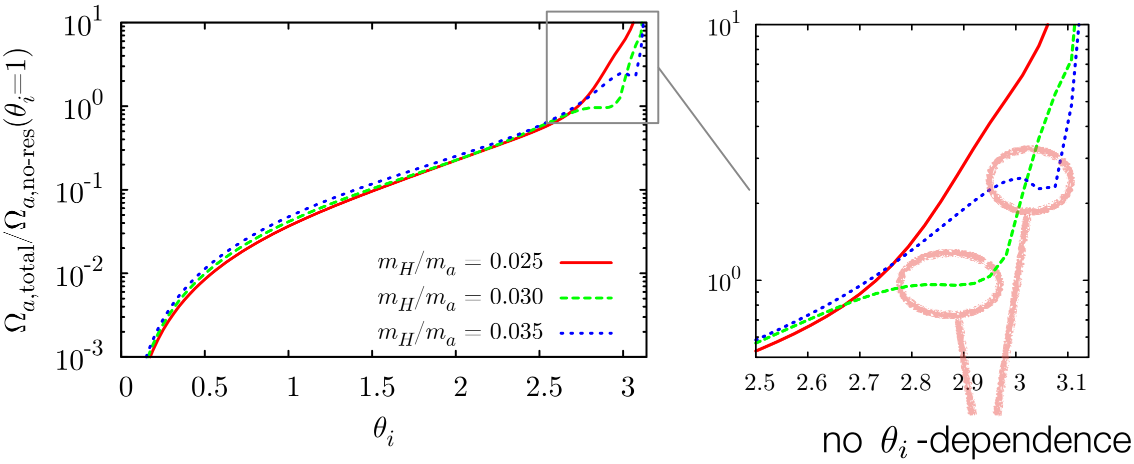



$$\Omega_a \to \left(\frac{m_H}{m_a}\right)\Omega_a$$

$n_a$ is fixed and $m_a \to m_H$ ➜ $\rho_a = m_H n_a$

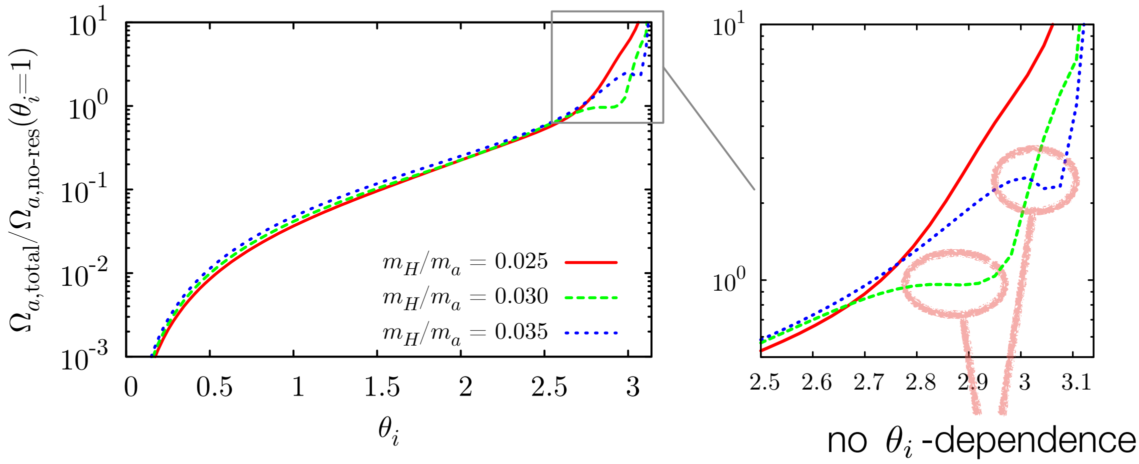


## (ii) Isocurvature perturbations

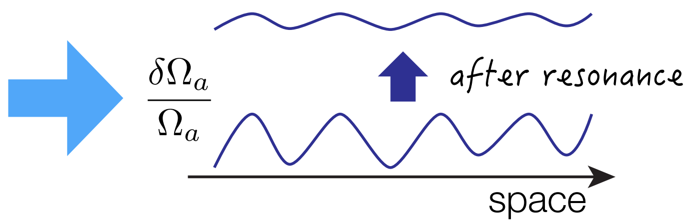



no $\theta_i$ -dependence

# (ii) Isocurvature perturbations




Density perturbation of the axion

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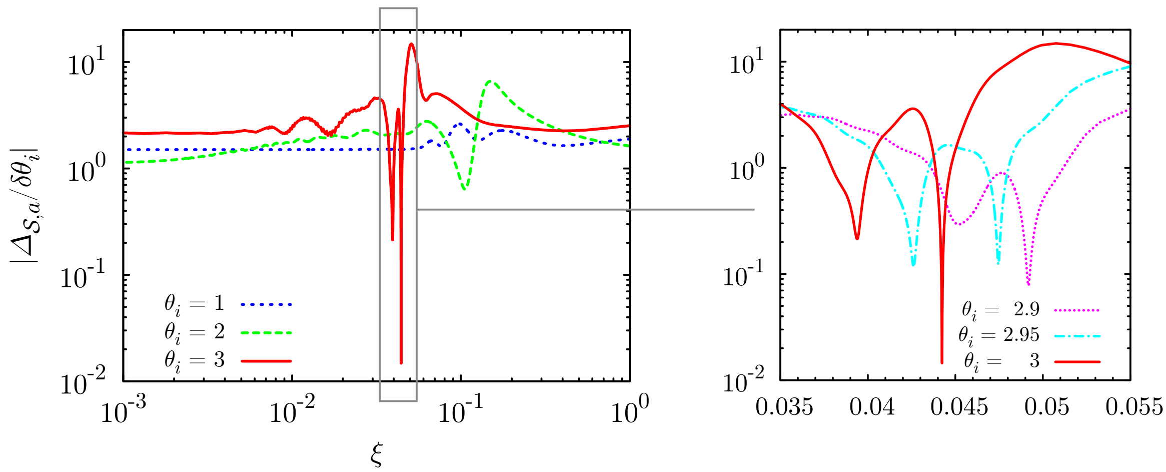
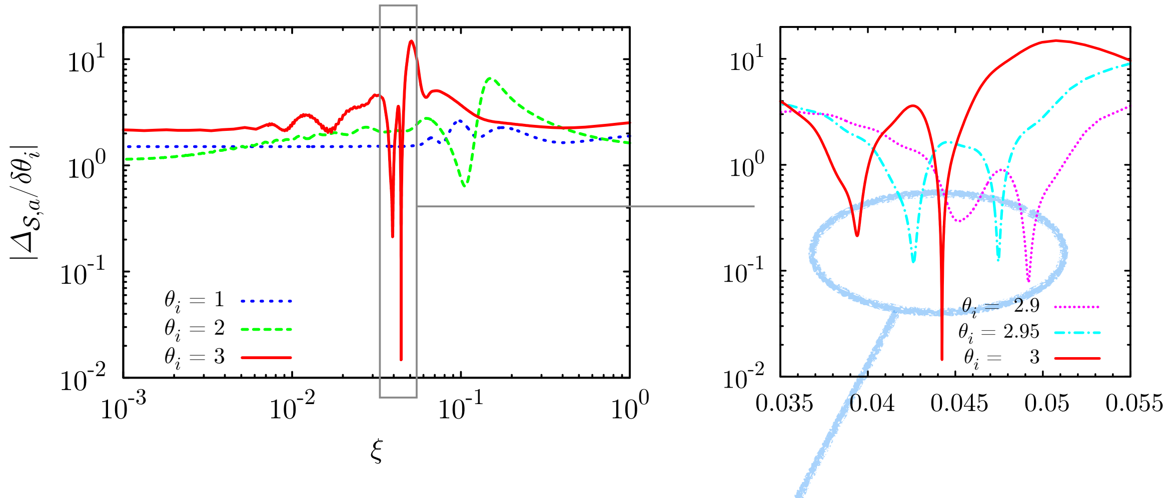
# (ii) Isocurvature perturbations



## (ii) Isocurvature perturbations




**Isocurvature perturbation can be suppressed!**



## 4. Conclusions

- We considered QCD axion & hidden axion with mass mixing
- MSW-like resonance takes place
  and QCD axion is converted into hidden axions
- Abundance can be suppressed
- Isocurvature perturbation can also be suppressed!

**We proposed a completely new mechanism**
**to suppress isocurvature perturbations**

Thank you!

# “Axion dark matter from topological defects”

Ken’ichi Saikawa

[JGRG24(2014)111004]

# Axion dark matter from topological defects

Ken'ichi Saikawa
Tokyo Institute of Technology

Collaborate with T. Hiramatsu (YITP), M. Kawasaki (ICRR) and T. Sekiguchi (Helsinki)

Based on:
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi (work in progress)
and also on:
T. Hiramatsu, M. Kawasaki, KS, astro-ph.CO/1012.4558. [JCAP08 (2011) 030]
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, hep-ph/1202.5851. [PRD85, 105020 (2012)]
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, hep-ph/1207.3166. [JCAP01 (2013) 001]



# Abstract

- Numerical simulation of topological defects (strings & domain walls) which arise in axion models
- Calculate axion CDM abundance produced from defects
- Observational constraints

# QCD axion as dark matter candidate

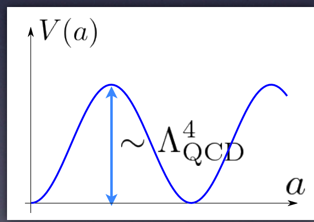
- Motivated by Pecccei-Quinn mechanism Peccei and Quinn (1977) as a solution of the strong CP problem
- Spontaneous breaking of continuous Peccei-Quinn symmetry at

$$T \simeq F_a \simeq 10^{8-11}\mathrm{GeV}$$ "axion decay constant"

  - Nambu-Goldstone theorem
    → emergence of the (massless) particle ≡ axion
    Weinberg(1978), Wilczek(1978)
- Axion has a small mass (QCD effect)
  → pseudo-Nambu-Golstone boson

$$m_a \sim \frac{\Lambda_{\mathrm{QCD}}^2}{F_a} \sim 6 \times 10^{-5}\mathrm{eV}\left(\frac{10^{11}\mathrm{GeV}}{F_a}\right)$$

$$\Lambda_{\mathrm{QCD}} \simeq \mathcal{O}(100)\mathrm{MeV}$$



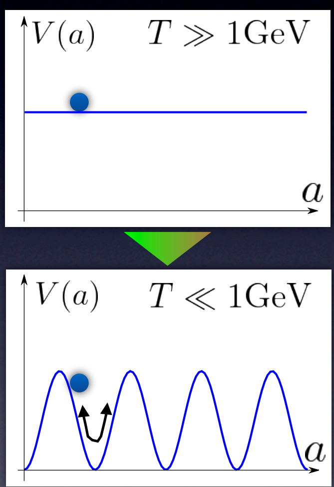

- Tiny coupling with matter + non-thermal production
  → good candidate of cold dark matter



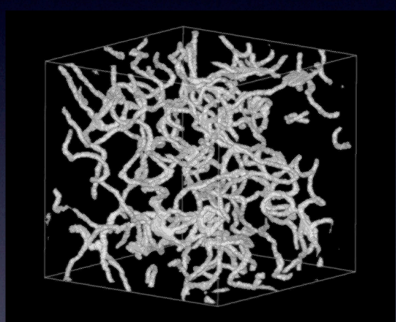
# How axions are produced ?

## Three mechanisms

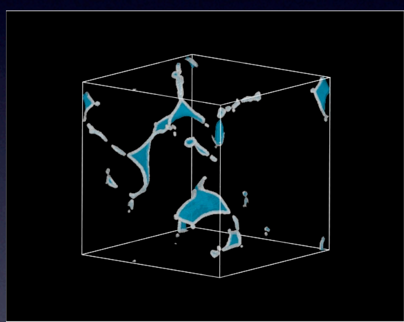
(1) misalignment mechanism




(2) radiation from strings



(3) radiation from string-wall systems



If Peccei-Quinn symmetry is broken after inflation, additional contributions (2) & (3) become relevant

# Axionic string and axionic domain wall

Peccei-Quinn field (complex scalar field)

$\Phi = |\Phi| e^{ia(x)/\eta}$  $a(x)$: axion field

$F_a = \eta / N_{\mathrm{DW}}$

String formation $T \lesssim F_a$

Spontaneous breaking of U(1)PQ

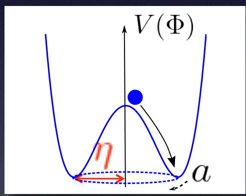
$V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - \eta^2)^2$

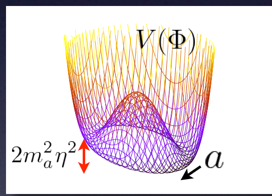
Domain wall formation $T \lesssim 1\mathrm{GeV}$

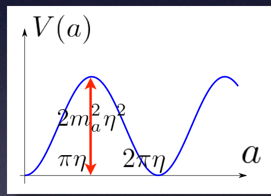
QCD effect

$V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - \eta^2)^2 + m_a^2 \eta^2 (1 - \cos(a/\eta))$

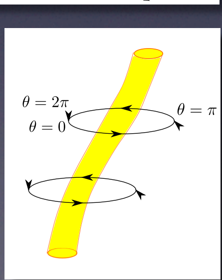
field space

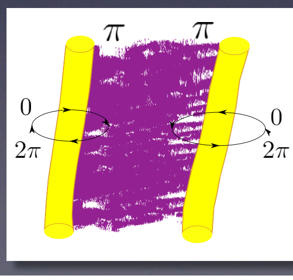



coordinate space




strings attached by domain walls

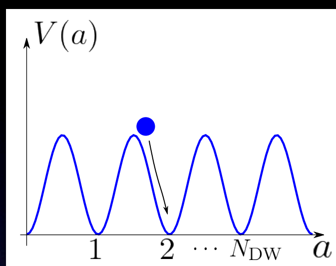


# Domain wall problem

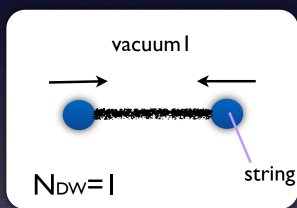
- Domain wall number $N_{\mathrm{DW}}$
  - $N_{\mathrm{DW}}$ degenerate vacua

$$V(a) = \frac{m_a^2 \eta^2}{N_{\mathrm{DW}}^2}(1 - \cos(N_{\mathrm{DW}} a/\eta))$$

$N_{\mathrm{DW}}$ : integer determined by QCD anomaly




- If $N_{\mathrm{DW}} = 1$, string-wall systems are unstable
  - Decay soon after the formation

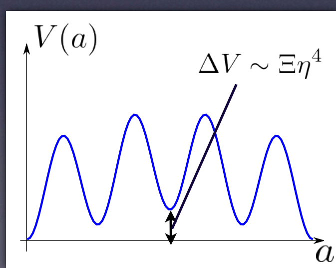



- If $N_{\mathrm{DW}} > 1$, string-wall systems are stable
  - come to overclose the universe

Zel'dovich, Kobzarev and Okun, JETP 40, 1 (1975)

  - We may avoid this problem by introducing an explicit symmetry breaking term (walls become unstable)  Sikivie, PRL 48, 1156 (1982)

$$V(\Phi) = \frac{m_a^2 \eta^2}{N_{\mathrm{DW}}^2}(1 - \cos(N_{\mathrm{DW}} a/\eta)) - \Xi \eta^3 (\Phi e^{-i\delta} + \mathrm{h.c.})$$

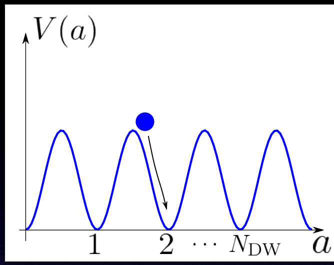


# Domain wall problem

- Domain wall number $N_{DW}$
  - $N_{DW}$ degenerate vacua

$$V(a) = \frac{m_a^2 \eta^2}{N_{\rm DW}^2}(1 - \cos(N_{\rm DW} a/\eta))$$

$N_{\rm DW}$ : integer determined by QCD anomaly

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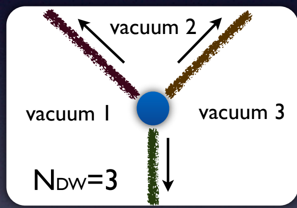


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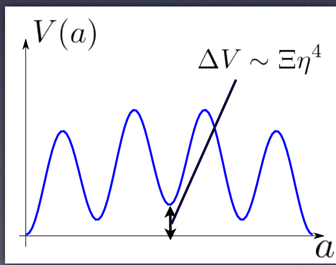


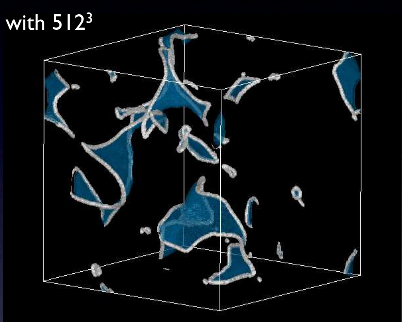


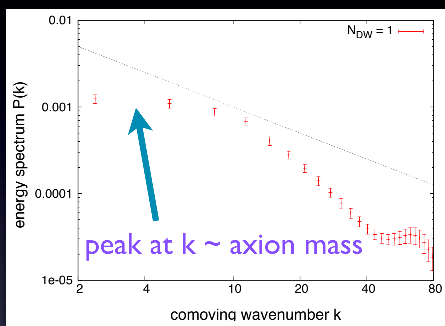

# $N_{DW}$ = 1: short-lived domain walls

Hiramatsu, Kawasaki, KS, Sekiguchi (2012)

- Numerical simulation of domain walls bounded by strings
  → estimate energy spectrum of radiated axions

3D lattice with $512^3$






- Axion density from decay of string wall systems $\Omega_{a,\rm dec}$ is comparable to axion densities from other sources

$$\Omega_{a,\rm dec} \sim \Omega_{a,\rm mis} \sim \Omega_{a,\rm string}$$

- Constraint on the Peccei-Quinn scale

$$\Omega_{a,\rm tot} \leq \Omega_{\rm CDM}$$

$$\Omega_{a,\rm tot} = \Omega_{a,\rm mis} + \Omega_{a,\rm string} + \Omega_{a,\rm dec}$$

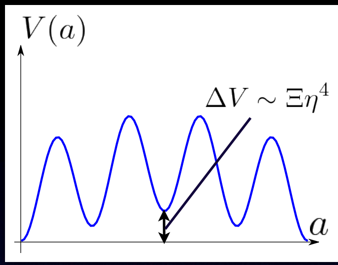
$$F_a \lesssim \mathcal{O}(10^{10})\mathrm{GeV}$$

$$m_a \gtrsim \mathcal{O}(10^{-4})\mathrm{eV}$$

# $N_{DW} > 1$: long-lived domain walls

- Domain walls are long-lived and decay due to the bias term

$$V_{\rm bias}(\Phi) = -\Xi\eta^3(\Phi e^{-i\delta} + {\rm h.c.})$$




- For small bias

  Long-lived domain walls emit a lot of axions which might exceed the observed matter density

  Cosmology → large bias is favored

- For large bias

  Bias term shifts the minimum of the potential and might spoil the original Peccei-Quinn solution to the strong CP problem

$$\bar\theta = \frac{2\Xi N_{\rm DW}^3 F_a^2 \sin\delta}{m_a^2 + 2\Xi N_{\rm DW}^2 F_a^2 \cos\delta} < 7\times 10^{-12}$$

$\delta$ : phase of bias term

  CP → small bias is favored

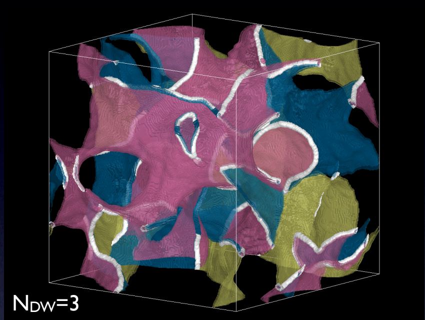
- Consistent parameters ?

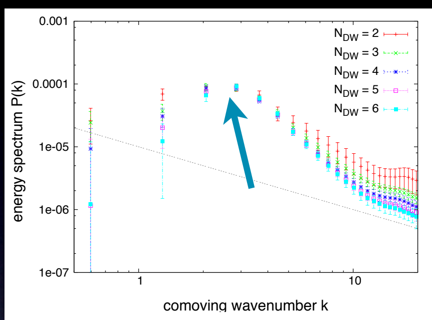


# Numerical simulations

- 3D lattice with $512^3$ → spectrum of radiated axions

  Hiramatsu, Kawasaki, KS, Sekiguchi (2012)

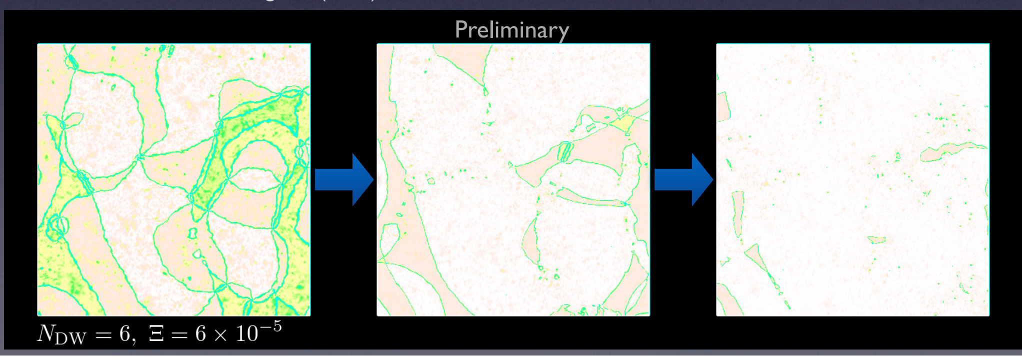



Peak at k ~ axion mass

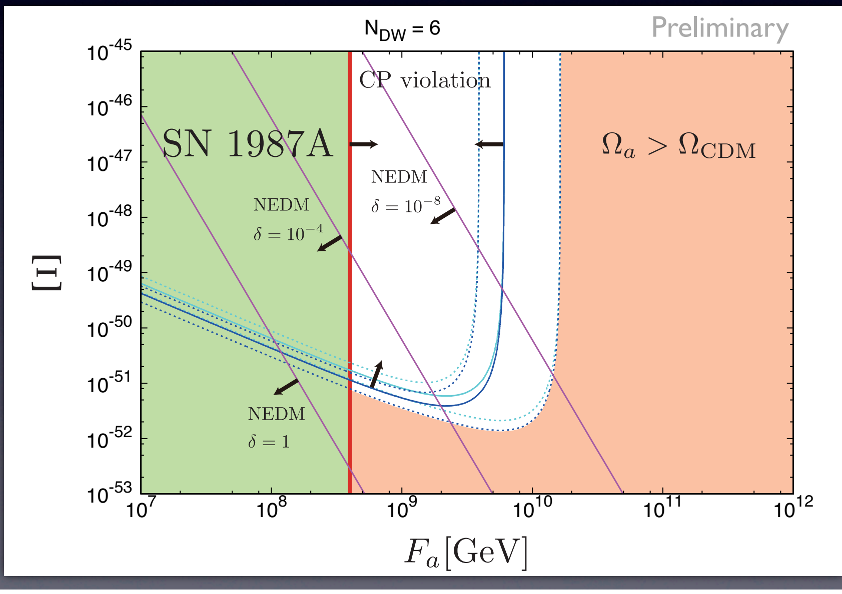
- 2D lattice with $8192^2$ $16384^2$ $32768^2$ → decay time of domain walls

  Hiramatsu, Kawasaki, KS, Sekiguchi (2014)



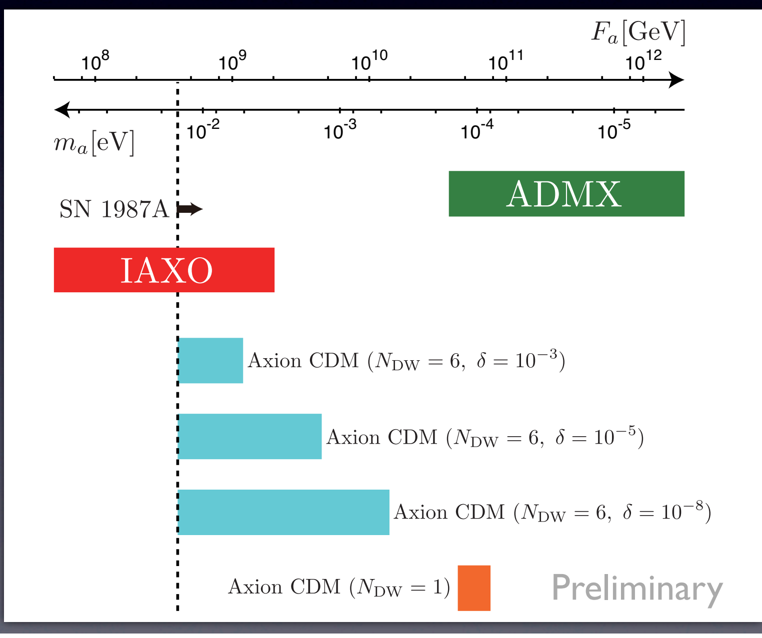
# Constraints

- Axion density $\Omega_{a,\mathrm{mis}} + \Omega_{a,\mathrm{string}} + \Omega_{a,\mathrm{dec}} \leq \Omega_{\mathrm{CDM}}$
- Astrophysical constraint (SN1987A) $F_a > 4 \times 10^8 \mathrm{GeV}$
- Neutron electric dipole moment (NEDM) $\bar{\theta} < 0.7 \times 10^{-11}$




10/12

- Additional contribution from string-wall systems
  → axions can be CDM at low $F_a$ (high $m_a$)
- Prediction of models with $N_{DW}$>1 strongly depends on the degree of tuning in $\delta$
- It can be probed in the next generation experiments

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# Summary

- We investigated the scenario where PQ symmetry is broken after inflation
- Radiation from string-wall systems gives additional contribution to the CDM abundance
- Axion can be dominant component of dark matter if

$$F_a \simeq \mathcal{O}(10^{10})\mathrm{GeV}$$
$$m_a \simeq \mathcal{O}(10^{-4})\mathrm{eV}$$
for $N_{DW} = 1$

$$F_a \simeq \mathcal{O}(10^{8}\text{–}10^{10})\mathrm{GeV}$$
$$m_a \simeq \mathcal{O}(10^{-4}\text{–}10^{-2})\mathrm{eV}$$
for $N_{DW} > 1$

- Mass ranges can be probed in the future experiments

“CDM/baryon isocurvature perturbations in a sneutrino curvaton model”

Taku Hayakawa

[JGRG24(2014)111005]

# CDM/baryon isocurvature perturbations in a sneutrino curvaton model

JCAP10(2014)068
arXiv:1409.1669 [hep-ph]
K. Harigaya, TH, M. Kawasaki, S. Yokoyama

Taku Hayakawa (ICRR, Univ of Tokyo)

## Introduction

- Inflation
  - In the simplest case, only the inflaton is the source of density perturbations.
  - But it is possible that another scalar field produces perturbations.

  → Curvaton model

  Enqvist, Sloth
  Lyth, Wands
  Moroi ,Takahashi (2001)

- Curvaton model
  - Matter isocurvature perturabtions may also be produced.
  - However, matter isocurvature perturbaitons are strictly constrained from CMB observations.

**We must avoid the stringent observational constraint.**

# Introduction

◆ Sneutrino curvaton model

- Both CDM/baryon isocurvature perturbations are generated and cancel each other.

➡ Cancellation of isocurvature perturbations

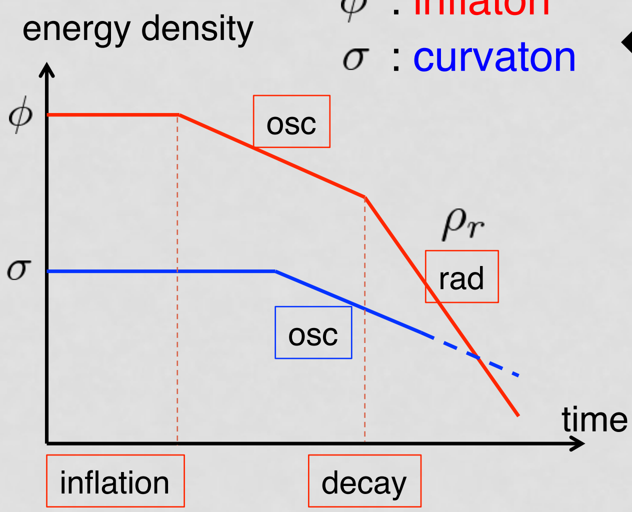
- Our model can suppress CMB fluctuations on large scales.
- Suppression may solve the tension between the Planck data and the BICEP2 detection.
- It leads to improved agreement with the Planck data. K.M. Smith et al. (2014)

➡ Compensation for tensor contribution to CMB fluctuations

# Curvaton model

$\phi$ : inflaton

$\sigma$ : curvaton




Curvature perturbations evolve on super-horizon scales after the inflaton decay.

◆ Curvature perturbations

$$\zeta = \zeta_{\rm inf} + \frac{f_{\rm dec}}{3} S_\sigma$$

$\zeta$ : curvature perturbation on the uniform density slice

$S_\sigma = 3(\zeta_\sigma - \zeta_{\rm inf})$ : curvaton isocurvature perturbation

$$f_{\rm dec} = \left. \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma} \right|_{\rm curvaton\ decay}$$

## Right-handed sneutrino

◆ Superpotential

$$W = \frac{1}{2} M_i N_i N_i + \lambda_{ij} N_i L_j H_u$$

$N_i$ : right-handed neutrino
$L_i$ : lepton doublets
$H_u$: up-type Higgs

- We assume $M_1 \sim 10^{12}\,\mathrm{GeV}$. $M_1 < M_2 < M_3$
- In our model, the lightest right-handed sneutrino $\widetilde{N}_1$ plays the role of the curvaton.

inflaton → (reheating) → gravitino production → (stable) → gravitino DM
gravitino production → (unstable) → LSP DM

$\widetilde{N}_1$ curvaton → (decay) → lepton asymmetry → (sphaleron process) → baryon asymmetry

# Isocurvature perturbations

◆ Matter isocurvature perturbations

$$S_m = \frac{\Omega_{\rm CDM}}{\Omega_m} S_{\rm CDM} + \frac{\Omega_b}{\Omega_m} S_b \qquad S_{{\rm CDM}/b} \equiv 3(\zeta_{{\rm CDM}/b} - \zeta)$$

- CDM is produced from decay products of the inflaton and decouples from thermal bath before the curvaton decay.

$$\zeta_{\rm CDM} = \zeta_{\rm inf} \longrightarrow S_{\rm CDM} = -f_{\rm dec} S_\sigma < 0$$ negative

- Baryon number is non-thermally produced from the curvaton.

$$\zeta_b = \zeta_\sigma \longrightarrow S_b = (1 - f_{\rm dec}) S_\sigma > 0$$ positive

## Cancellation of isocurvature perturbations

◆ Matter isocurvature perturbations

$$S_m = \left[ \frac{\Omega_{\rm CDM}}{\Omega_m}(-f_{\rm dec}) + \frac{\Omega_b}{\Omega_m}(1 - f_{\rm dec}) \right] S_\sigma$$

- If we appropriately take $f_{\rm dec}$ , $S_{\rm CDM}$ and $S_b$ cancel each other.
- CMB observations are sensitive only to total matter isocurvature perturbations. If $S_m = 0$, we can avoid the stringent constraint.

Condition on $f_{\rm dec}$

$$f_{\rm dec} \simeq 0.16$$

→ Cancellation of isocurvature perturbations

## Compensation for tensor contribution to CMB

- On large scales, CMB fluctuations originate from the Sachs-Walfe effect.

$$\left\langle \left( \frac{\Delta T}{T} \right)^2_{\rm SW} \right\rangle \simeq \frac{1}{25} \left[ \mathcal{P}_\zeta + 4\mathcal{P}_{S_m} + 4\mathcal{P}_{\zeta S_m} + \frac{5}{6}\mathcal{P}_T \right]$$

- The curvaton produces both $\zeta$ and $S_m$ . $\mathcal{P}_{\zeta S_m} \neq 0$
- If $\mathcal{P}_{\zeta S_m} < 0$, compensation for tensor contribution can be realized.

$$4\mathcal{P}_{S_m} + 4\mathcal{P}_{\zeta S_m} + \frac{5}{6}\mathcal{P}_T = 0$$

Condition on $\frac{\mathcal{P}_T}{\mathcal{P}_{S_\sigma}}$ and $f_{\rm dec}$

$$4\left( \frac{\Omega_b}{\Omega_m} - f_{\rm dec} \right)\left( \frac{\Omega_b}{\Omega_m} - \frac{2}{3} f_{\rm dec} \right) + \frac{5}{6}\frac{\mathcal{P}_T}{\mathcal{P}_{S_\sigma}} = 0$$

↓

Compensation for tensor contribution to CMB

◆ We will express the conditions by model parameters.

- $\frac{\mathcal{P}_T}{\mathcal{P}_{S_\sigma}}$ $$\frac{\mathcal{P}_T}{\mathcal{P}_{S_\sigma}} = 2\left(\frac{\sigma_*}{M_{\rm pl}}\right)^2$$ $\sigma_*$ : curvaton field value during inflation; $M_{\rm pl}$ : the reduced Planck mass
- $f_{\rm dec} = \left.\frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}\right|_{\sigma{\rm decay}}$

$$\left.\frac{\rho_\sigma}{\rho_r}\right|_{\sigma{\rm decay}} = \frac{1}{6}\left(\frac{\sigma_*}{M_{\rm pl}}\right)^2 \frac{T_{\rm reh}}{T_{\rm dec}}$$

$T_{\rm reh}$ : reheating temperature
$T_{\rm dec}$ : curvaton decay temperature

◆ Baryon asymmetry

$$\frac{n_B}{s} \simeq 1.5\times10^{-11}\left(\frac{\sigma_*}{10^{17}\,{\rm GeV}}\right)^2\left(\frac{T_{\rm reh}}{10^9\,{\rm GeV}}\right)\delta_{CP}$$

◆ Dark matter abundance

$$\Omega_{\rm CDM}h^2 \simeq 3.8\times10^{-2}\left(\frac{m_{\rm LSP}}{1\,{\rm TeV}}\right)\left(\frac{T_{\rm reh}}{10^9\,{\rm GeV}}\right)$$

for $T_{\rm reh} \sim 10^9\,{\rm GeV}$

# Cancellation of isocurvature perturbations

◆ Results

- Cancellation requires $f_{\rm dec}$ to be 0.16. $f_{\rm dec} \longleftrightarrow \sigma_*^2\frac{T_{\rm reh}}{T_{\rm dec}}$
- To produce the observed baryon asymmetry,

$$T_{\rm dec} \simeq 7\times10^6\,{\rm GeV}$$

B asymmetry $\longleftrightarrow$ $\sigma_*^2 T_{\rm reh}$

- To produce the observed dark matter abundance,

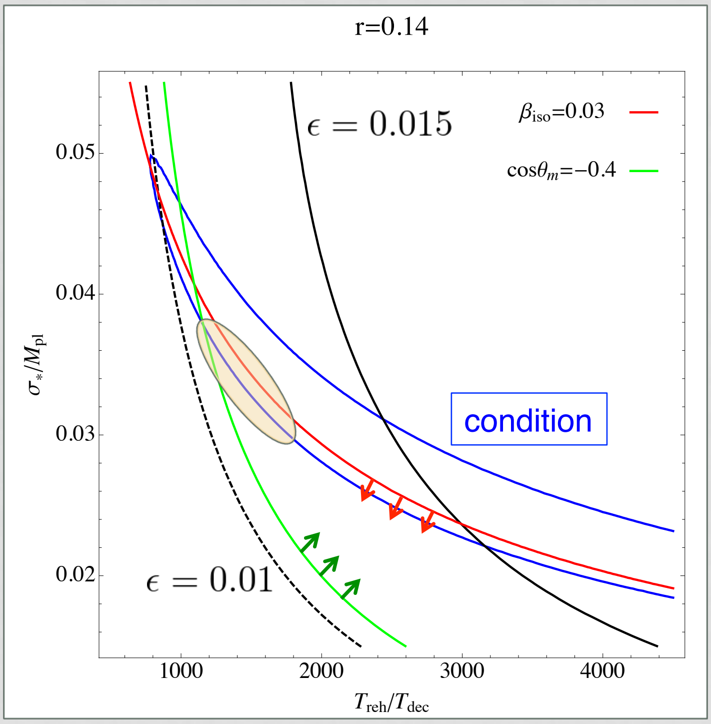
$$T_{\rm reh} \simeq 10^9\,{\rm GeV}\left(\frac{m_{\rm LSP}}{1\,{\rm TeV}}\right)^{-1}$$
$$\sigma_* \simeq 10^{17}\,{\rm GeV}\left(\frac{m_{\rm LSP}}{1\,{\rm TeV}}\right)^{1/2}$$

Dark matter abundance $\longleftrightarrow$ $T_{\rm reh}, m_{\rm LSP}$

## Compensation for tensor contribution to CMB

◆ **Results**

The condition for compensation can be rewritten in terms of $\sigma_*$ and $T_{\rm reh}/T_{\rm dec}$.




isocurvature fraction

$$\beta_{\rm iso} \equiv \frac{\mathcal{P}_{S_m}}{\mathcal{P}_\zeta + \mathcal{P}_{S_m}} \lesssim 0.03$$

correlation

$$\cos\theta_m \equiv \frac{\mathcal{P}_{\zeta S_m}}{\sqrt{\mathcal{P}_\zeta \mathcal{P}_{S_m}}} \lesssim -0.4$$

Kawasaki, Sekiguchi, Takahashi, Yokoyama (2014)

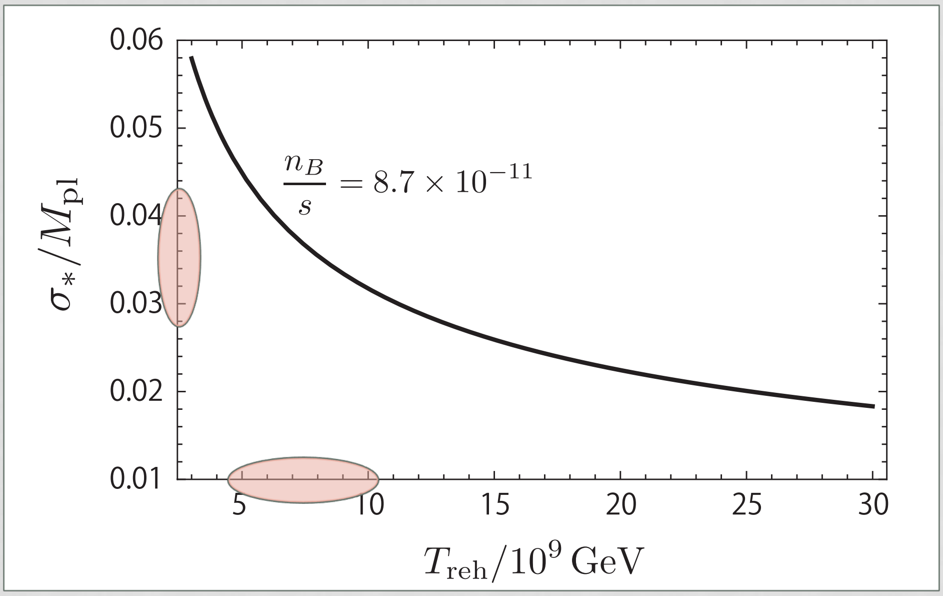
slow-roll parameter

$$\epsilon \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2 \sim 0.01$$

$$\sigma_*/M_{\rm pl} \simeq 0.03\text{-}0.04$$
$$T_{\rm reh}/T_{\rm dec} \simeq \mathcal{O}(10^3)$$

● **Baryon asymmetry**

B asymmetry $\longleftrightarrow$ $\sigma_*^2 T_{\rm reh}$




$$\sigma_*/M_{\rm pl} \simeq 0.03\text{-}0.04$$
$$T_{\rm reh}/T_{\rm dec} \simeq \mathcal{O}(10^3)$$

$\Rightarrow$

$$T_{\rm reh} \simeq 10^{9\text{-}10}\ {\rm GeV}$$
$$T_{\rm dec} \simeq 10^{6\text{-}7}\ {\rm GeV}$$

◆ Both scenarios require $T_{\rm reh} \sim 10^9\,{\rm GeV}$.

- Dark matter abundance

$$\Omega_{\rm CDM} h^2 \simeq 3.8 \times 10^{-2} \left(\frac{m_{\rm LSP}}{1\,{\rm TeV}}\right) \left(\frac{T_{\rm reh}}{10^9\,{\rm GeV}}\right)$$

- Unstable gravitino
  - Decay of gravitinos may destroy the success of the BBN. To aboid it, $m_{3/2} \gtrsim 10\,{\rm TeV}$.
  - $m_{\rm LSP} \sim \mathcal{O}(0.1\text{-}1)\,{\rm TeV}$
  - The mass hierarchy $(m_{\rm LSP} \ll m_{3/2})$ is naturally explained if the gaugino mass is generated only by the anomaly mediation.
- Stable gravitino
  - $m_{3/2({\rm LSP})} \sim \mathcal{O}(0.1\text{-}1)\,{\rm TeV}$
  - Decay of Next-to-LSP may destroy the success of the BBN. To avoid it, NLSP must be left-handed sneutrino.

## ◆ Conclusions

- We have constructed the sneutrino curvaton model which can not only avoid the stringent constraints but also suppress CMB fluctuations on large scales.
- Baryon asymmetry is explained by the non-thermal leptogenesis from the sneutrino curvaton.
- Origin of dark matters is gravitino production during reheating.
- Successful model requires $T_{\rm reh} \simeq 10^{9\text{-}10}\,{\rm GeV}$, $T_{\rm dec} \simeq 10^{6\text{-}7}\,{\rm GeV}$ and $\sigma_* \simeq 10^{17}\,{\rm GeV}$.
- If the gravitino is unstable, $m_{3/2} \gtrsim 10\,{\rm TeV}$ and $m_{\rm LSP} \sim \mathcal{O}(0.1\text{-}1)\,{\rm TeV}$.
  If the gravitino is stable, $m_{3/2} \sim \mathcal{O}(0.1\text{-}1)\,{\rm TeV}$.
  The NLSP must be the left-handed sneutrino.

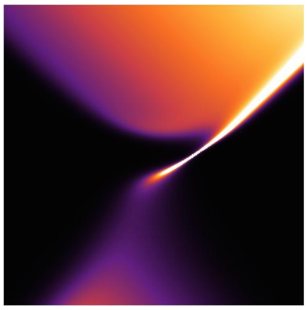
# “Holographic Fermi surfaces from String Theory”

Steven Gubser [Invited]

[JGRG24(2014)111006]

# Holographic Fermi surfaces from top-down constructions

Steve Gubser



Based on 1112.3036, 1207.3352, 1312.7347 with O. DeWolfe and C. Rosen
and 1411.nnnn with C. Cosnier-Horeau

*Kavli IPMU, JGRG24, November 10, 2014*

## Contents

# 1. Charged black holes in anti-de Sitter space

We're looking for Fermion normal modes in charged black hole backgrounds in $AdS_4$ and $AdS_5$.

- Simplest such background is extremal RN$AdS_4$:

$$ds^2 = \frac{r^2}{L^2}(f dt^2 - d\vec{x}^2) - \frac{L^2}{r^2}\frac{dr^2}{f} \qquad A_\mu dx^\mu = \mu\left(1 - \frac{r_H}{r}\right)$$
$$f = 1 - 4\left(\frac{r_H}{r}\right)^3 + 3\left(\frac{r_H}{r}\right)^4 \tag{1}$$

- Simplest fermion to consider obeys massless charged Dirac equation:

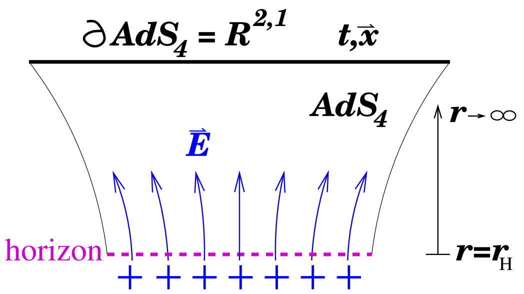
$$\gamma^\mu(\nabla_\mu - iqA_\mu)\chi = 0\,. \tag{2}$$

- Supergravity gives relations $g = \frac{1}{\sqrt{2}L}$, $\mu = \frac{\sqrt{6}r_H}{L}$, and $q = g$. Generally we'll choose $L = 1$. If also $r_H = 1$, then one finds a normal mode at

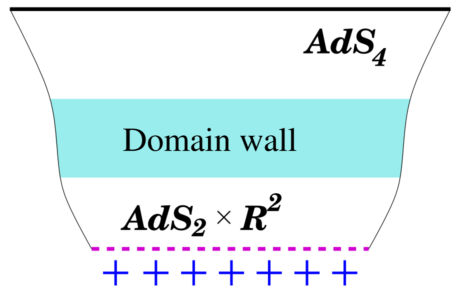
$$\omega = 0 \qquad k = k_F \equiv 0.9185\,. \tag{3}$$



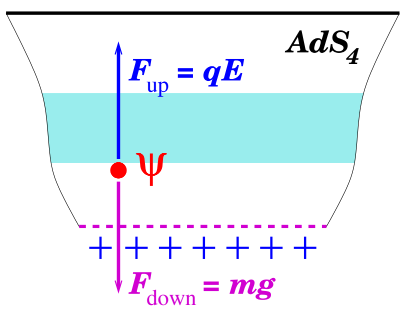
Electric field comes from charge behind horizon.



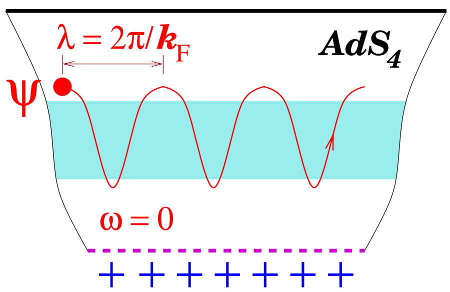

Back-reacted geometry is a domain wall from $AdS_4$ to $AdS_2 \times \mathbf{R}^2$.




Gravitational attraction and electrostatic repulsion compete to determine behavior of test particles.




Normalizable fermions at $\omega = 0$ and $k = k_F \neq 0$ stay above the horizon and below the boundary.



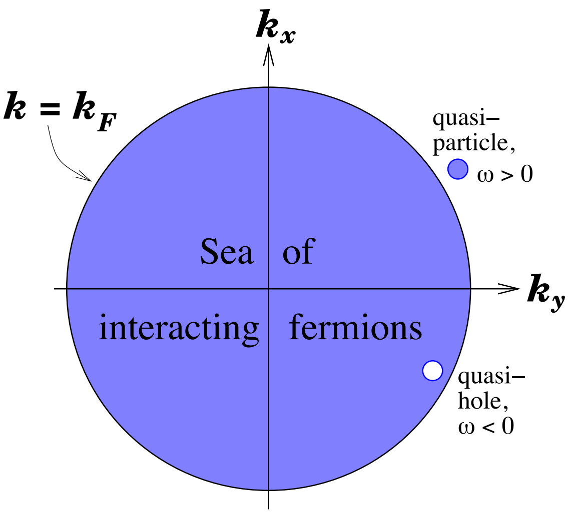
# 2. Fermionic Green's functions

In a strongly interacting system with a finite density of fermions, it's convenient to define the Fermi surface in terms of a Green's function:

$$G(\omega, k) = \left\langle \mathcal{O}_\chi(\omega, \vec{k}) \mathcal{O}_\chi^\dagger(-\omega, -\vec{k}) \right\rangle \approx \frac{h_1}{(k - k_F) - \frac{1}{v_F}\omega - h_2 e^{i\gamma} \omega^{2\nu_F}}$$

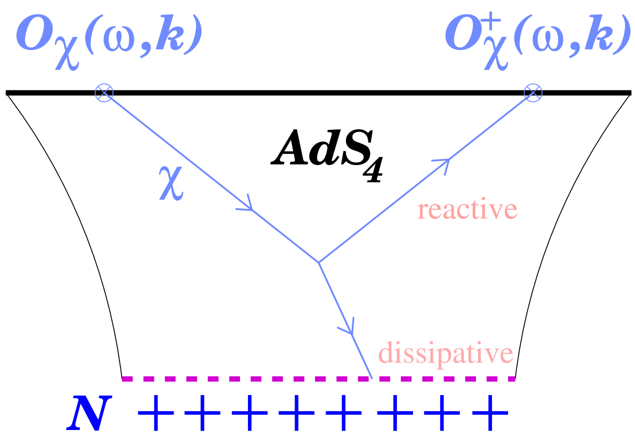
when $k \approx k_F$ and $\omega \approx 0$.

- A singularity in $G(\omega, k)$ at $\omega = 0$ and finite $k = k_F$ *defines* the presence of a Fermi surface.
- $v_F$ is Fermi velocity.
- Assuming $\nu_F > 1/2$, low-energy dispersion relation is $\omega \approx v_F(k - k_F)$.
- If $\nu_F > 1/2$ or if $e^{i\gamma}$ is nearly real, quasi-particles' width is much smaller than their energy.






The simplest charged black holes in $AdS$ are purely bosonic backgrounds. To "see" the fermions in the dual description, bounce a test fermion off the black hole and look for Green's function singularities:




- Equation solved is a variant of Dirac equation for $\chi$.
- At arbitrary $(\omega, k)$, read off $\langle \mathcal{O}_\chi \mathcal{O}_\chi^\dagger \rangle$ from solution of fermion wave eq in black hole background.
- As $\omega \to 0$ and $k \to k_F$, overlap with fermion normal mode causes $\langle \mathcal{O}_\chi \mathcal{O}_\chi^\dagger \rangle$ to diverge.

**Fermi surfaces in boundary theory correspond to fermion normal modes in the bulk.**

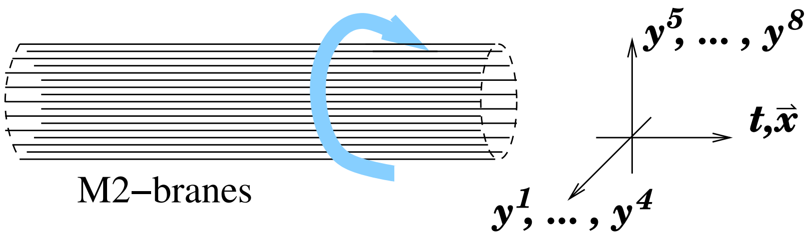
# 3. Problems and their solutions

- Previous calculations focus on ad hoc lagrangians, e.g. [Liu-McGreevy-Vegh '09, Cubrovic-Zaanen-Schalm '09].
  - Instead, let's work with fermions of maximal gauged supergravity in $D = 4$ and $D = 5$: reductions / truncations of M-theory on $S^7$ and type IIB on $S^5$.
- AdS-Reissner-Nordstrom black holes have non-zero entropy at $T = 0$, which is hard to understand in field theory.
  - Work with variants of RN$AdS_4$ which can be embedded in M-theory or type IIB and have no zero-point entropy.
- Field theory interpretation, e.g. in $\mathcal{N} = 4$ super-Yang-Mills theory, has been obscure.
  - Formulate "boson rule" and "fermion rule" which capture results of many supergravity calculations in terms of field theory quantities.
- Supergravity calculations are hard work!
  - Find some strong collaborators.



# 4. Supergravity backgrounds and spinning branes

Charged black holes in $AdS_4$ come from spinning D3-branes;
Charged black holes in $AdS_4$ come from spinning M2-branes.




$D = 4$, $\mathcal{N} = 8$ supergravity [de Wit and Nicolai, 1982] has $SO(8)$ gauge symmetry associated with the $S^7$ directions coming from $y^1 \ldots y^8$.

- A semi-pedagogical introduction can be found in [de Wit, hep-th/0212245].
- Field content is: graviton $g_{\mu\nu}$, $8$ gravitini $\psi^i_\mu$, $28$ gauge fields $A^{ij}_\mu$, $56$ Majorana spinors $\chi^{ijk}$, and $70$ real scalars $\phi^{ijkl}$.
- Eight-valued indices $i, j, \ldots$ characterize either the internal symmetry group $SU(8)$ or the gauge group $SO(8)$ (in a spinorial rep wrt $S^7$).

Here are the main equations for setting up fermions in RN$AdS_4$ with only $A_\mu = A_\mu^{12}$ non-zero and round $S^7$:

$$D_\mu \chi_{ijk} \equiv \nabla_\mu \chi_{ijk} + 3gA_\mu{}^m{}_{[i}\,\chi_{jk]m} \qquad \text{(even for more general gauge fields)}$$

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} + 6g^2 + \mathcal{L}_{1/2} \qquad \text{(specialized to round } S^7\text{)}$$

$$\mathcal{L}_{1/2} = -\frac{1}{12}\bar\chi^{ijk}(\gamma^\mu D_\mu - \overleftarrow{D}_\mu \gamma^\mu)\chi_{ijk} \qquad \text{(The Dirac kinetic term for } \chi_{ijk}\text{)}$$

$$- \frac{1}{2}\left(F^+_{\mu\nu ij} O^{+\mu\nu ij} + \text{h.c.}\right) \qquad \text{(Eventually can ignore this } F^+O^+ \text{ bit...}$$

$$O^{+\mu\nu ij} \equiv -\frac{\sqrt{2}}{144}\epsilon^{ijklmnpq}\bar\chi_{klm}\sigma^{\mu\nu}\chi_{npq} \qquad \text{...which looks like Pauli couplings...}$$

$$- \frac{1}{2}\bar\psi_{\rho k}\sigma^{\mu\nu}\gamma^\rho\chi^{ijk} + (\psi_\rho^2 \text{ term}) \qquad \text{...and } \chi\psi \text{ mixing)}\,.$$

To see that you can drop $F^+O^+$, note that $ij = 12$, so none of $klm$ or $npq$ are $1$ or $2$: thus $\chi_{klm}$, $\chi_{npq}$, and also $\chi^{ijk} = \chi^{12k}$, are all *uncharged*.

The upshot: Form $\chi = \chi_{1jk} + i\chi_{2jk}$ and find simple massless Dirac equation,

$$\gamma^\mu \left(\nabla_\mu - \frac{i}{\sqrt{2}L}A_\mu\right)\chi = 0\,.$$



A more general case was worked out recently in [DeWolfe-Henriksson-Rosen '14], based on arbitrary combinations of charges in $U(1)^4 \subset SO(8)$:

$$\begin{pmatrix} A_\mu^{12} \\ A_\mu^{34} \\ A_\mu^{56} \\ A_\mu^{78} \end{pmatrix} = \frac{1}{\sqrt{8}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} A_\mu^a \\ A_\mu^b \\ A_\mu^c \\ A_\mu^d \end{pmatrix} \tag{4}$$

With $A_\mu^a \neq A_\mu^b \neq A_\mu^c \neq A_\mu^d$, one must turn on three of the $70$ scalars to find consistent solutions. Relevant part of $\mathcal{D} = 4$, $\mathcal{N} = 8$ action is

$$\mathcal{L} = R - \frac{1}{2}(\partial\vec\phi)^2 + \frac{2}{L^2}(\cosh\phi_1 + \cosh\phi_2 + \cosh\phi_3) - \frac{1}{4}\sum_{i=a,b,c,d} e^{-\lambda_i}(F^i_{\mu\nu})^2 \tag{5}$$

where

$$\begin{pmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{pmatrix} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \tag{6}$$

These scalars parametrize oblateness / prolateness of the $S^7$.

The general charged black brane solution we want to consider is

$$ds_4^2 = e^{2A(r)}\left[-h(r)dt^2 + d\vec{x}^2\right] + \frac{e^{2B(r)}}{h(r)}dr^2 \qquad A^i = \Phi_i(r)dt \qquad \phi_A = \phi_A(r)$$

where

$$\begin{aligned} A = -B &= \log\frac{r}{L} + \frac{1}{4}\sum_i \log H_i \\ h &= 1 - \frac{r}{r_H}\prod_i \frac{r_H + Q_a}{r + Q_a} \\ \lambda_i &= -2\log H_i + \frac{1}{2}\sum_j \log H_j \\ \Phi_i &= \frac{1}{L}\sqrt{\frac{Q_i}{r_H}}\frac{\sqrt{\prod_j (r_H + Q_j)}}{r_H + Q_i}\left(1 - \frac{r_H + Q_i}{r + Q_i}\right) . \end{aligned} \tag{7}$$

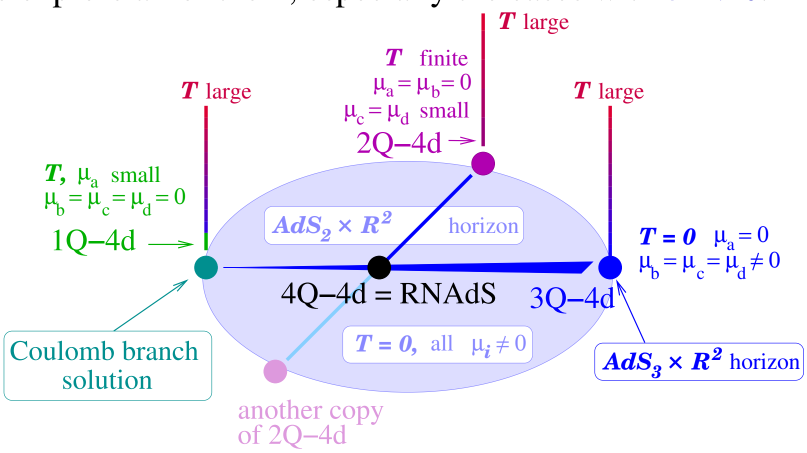
and one can show

$$s = \frac{1}{4GL^2}\sqrt{\prod_j (r_H + Q_j)} \tag{8}$$

with $s \to 0$ as $r_H \to 0$ provided at least one of the $Q_j = 0$.



There are several qualitatively different behaviors for these charged black branes, and we aim to explore all of them, especially the cases with $s \to 0$.




- 1Q-4d, 2Q-4d, 3Q-4d are the main cases we'll consider; 4Q-4d was the simplest case, already discussed.
- $r_H \to 0$ limit is singular for 1Q-4d, 2Q-4d, 3Q-4d.
- To make sure that supergravity is applicable, we'll turn on small non-zero $r_H$.
- Order of limits gets subtle: For example, 2Q-4d is a $r_H \to 0$ limit with $\mu_a = \mu_b = 0$, not the same as a $\mu_a = \mu_b \to 0$ limit with $T = 0$.

# 5. Fermion equations of motion

$D = 4, \mathcal{N} = 8$ supergravity lagrangian is schematically

$$\mathcal{L} = \mathcal{L}_b + \frac{1}{2}\bar{\chi} D_\chi \chi + \bar{\psi}_\mu O_{\text{mix}} \chi + \frac{1}{2}\bar{\psi}_\mu D_{\text{Rarita-Schwinger}} \psi_\mu + \mathcal{O}(\text{fermion}^4) \qquad (9)$$

Our main task is to decouple the quadratic fermion action and solve resulting linear equations to get two-point functions $\langle \mathcal{O}_\chi \mathcal{O}_\chi^\dagger \rangle$.

- Some of the **56** fermions $\chi_{ijk}$ can mix with the **8** gravitini $\psi_\mu^i$, giving them a mass (super-Higgs). We don't want these.
- Because bosonic background has no charged fields under $U(1)^4$, we know that $\chi_{ijk}$ can't couple with $\psi_\mu^i$ if it has an $SO(8)$ weight not in the **8**. There are $32$ such $\chi_{ijk}$, and dual operators are schematically $\text{tr}\,\lambda Z$.
- Of the $24$ remaining $\chi_{ijk}$, there are $16$ which don't couple to the $\psi_\mu^i$, and $8$ that do, but we haven't worked out which are which. So ignore them all and focus on the special $32$.
- Similar results are available from [Gubser-DeWolfe-Rosen '13] in the case of $D = 5$, $\mathcal{N} = 8$ supergravity; fields of interest are dual to operators $\text{tr}\,\lambda Z$.



In 4-dim: The fermion equations of motion we want to study take the form

$$\left[ i\gamma^\mu \nabla_\mu + \gamma^\mu A_\mu^j \mathbf{Q}_j + \sigma^{\mu\nu} F_{\mu\nu}^j \mathbf{P}_j + \mathbf{M} \right] \vec{\chi} = 0\,. \qquad (10)$$

$\vec{\chi}$ is a $32$-component vector, and the matrices $\mathbf{Q}_j$, $\mathbf{P}_j$, and $\mathbf{M}$ all commute (!).

Simultaneous eigenvectors satisfy

$$\left[ i\gamma^\mu \nabla_\mu + \frac{1}{4} \sum_j \left( q_j \gamma^\mu A_\mu^j + \frac{i}{2} p_j e^{-\lambda_j/2} \sigma^{\mu\nu} F_{\mu\nu}^j + m_j e^{\lambda_j/2} \right) \right] \chi = 0\,. \qquad (11)$$

$\nabla_\mu$ includes spin connection but not gauge connections

Gauge couplings and Pauli couplings.

Spatially variable mass term, $m \to 0$ at $\partial AdS_4$.

and we can tabulate the parameters $(q_j, p_j, m_j)$.

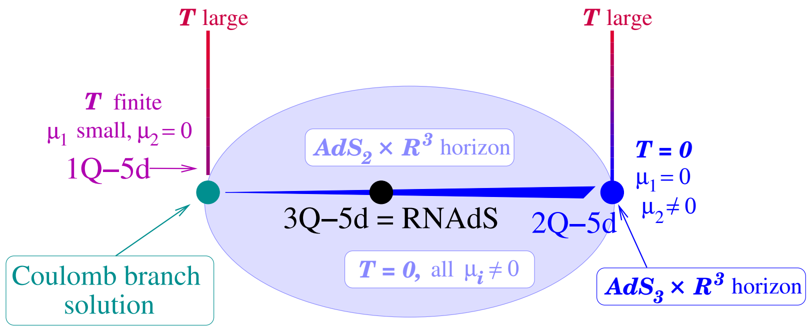
Dual operators follow from values of $q_j$: E.g. $q_j = (3, 1, 1, -1)$ corresponds to $\text{tr}\,\lambda Z$ where

$$[\lambda]_{SO(8)} = (\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}) \qquad [Z]_{SO(8)} = (1, 0, 0, 0) \quad \text{i.e.} \quad Z = X_1 + iX_2 \qquad (12)$$

We'll denote $Z_j = X_{2j-1} + iX_{2j}$ for $j = 1, 2, 3, 4$.

**In 5-dim:** Gauge group is $SO(6) \supset U(1)^3$, but we restricted to the case

$$\begin{aligned} A^a_\mu &\equiv A^{12}_\mu = a_\mu & \qquad A^b_\mu &\equiv A^{34}_\mu = A^c_\mu \equiv A^{56}_\mu = A_\mu \\ \mu_a &= \mu_1 & \qquad \mu_b &= \mu_c = \mu_2/\sqrt{2} \end{aligned} \tag{13}$$




Only one scalar in supergravity is active, $\phi$ in the $\mathbf{20}'$ of $SO(6)$; it is dual to $\mathcal{O}_\phi = \mathrm{tr}(2|Z_1|^2 - |Z_2|^2 - |Z_3|^2)$, where $Z_j = X_{2j-1} + iX_{2j}$.

$24$ of the $\mathbf{48}$ fermions $\chi_{abc}$ are dual to $\mathrm{tr}\,\lambda Z$ and obey equations of the form

$$\Big[i\gamma^\mu\nabla_\mu + 2q_1\gamma^\mu a_\mu + 2q_2\gamma^\mu A_\mu + ip_1 e^{-2\phi/\sqrt{6}}\gamma^{\mu\nu}f_{\mu\nu} + ip_2 e^{\phi/\sqrt{6}}\gamma^{\mu\nu}F_{\mu\nu} \\ - 2(m_1 e^{-\phi/\sqrt{6}} + m_2 e^{2\phi/\sqrt{6}})\Big]\chi = 0 \tag{14}$$



# 6. Extracting Green's functions

Two tricks simplify Dirac equations significantly:

1. Separation of variables with a nice prefactor:

$$\chi(t,\vec{x},r) = \frac{1}{\sqrt[4]{-\det g_{mn}}} e^{-i\omega t + ikx^1}\psi(r) \qquad \text{where} \qquad m,n = t,1,2 \tag{15}$$

2. Choice of basis for gamma matrices and spinors:

$$\gamma^t = \begin{pmatrix}\sigma_1 & 0 \\ 0 & \sigma_1\end{pmatrix} \quad \gamma^1 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2\end{pmatrix} \quad \gamma^r = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & i\sigma_3\end{pmatrix}, \quad \psi = \begin{pmatrix}\psi_{1-} \\ \psi_{1+} \\ \psi_{2-} \\ \psi_{2+}\end{pmatrix}$$

results in

$$(\partial_r + X\sigma_3 + Yi\sigma_2 + Z\sigma_1)\begin{pmatrix}\psi_{\alpha-} \\ \psi_{\alpha+}\end{pmatrix} = 0 \tag{16}$$

$$X = -\frac{e^B}{4\sqrt{h}}\sum_j m_j e^{\lambda_j/2}, \qquad Y = -\frac{e^{-A+B}}{h}\left[\omega + \frac{1}{4}\sum_j q_j\Phi_j\right], \qquad Z = -\frac{e^{-A+B}}{\sqrt{h}}\left[(-1)^\alpha k - \frac{e^{-B}}{4}\sum_j p_j e^{-\lambda_j/2}\partial_r\Phi_j\right]$$

Solving Dirac equation is now straightforward in principle:

- Infalling solution at the horizon is $\psi_- = i\psi_+ = \frac{i}{2}(r - r_H)^{-\frac{i\omega}{4\pi T}}$.
- Numerically solve equation for two-component spinor from $r_i = r_H + \delta r$ up to some large $r_f$.
- Fit to asymptotic forms at large $r$ obtained from solving in pure $AdS_{d+1}$:

$$\psi_{\alpha+} = A_\psi r^{m-\frac{d}{2}} + B_\psi r^{-m-1-\frac{d}{2}} \qquad \psi_{\alpha-} = C_\psi r^{m-1-\frac{d}{2}} + D_\psi r^{-m-\frac{d}{2}} \tag{17}$$

- The Green's function in $d = D - 1$ dimensions is

$$G_R(t, \vec{x}) = -i\theta(t)\langle[\mathcal{O}_\chi(t,x), \mathcal{O}_\chi^\dagger(0,0)]\rangle = \int d^d x \, e^{-i\omega t + i\vec{k}\cdot\vec{x}} G_R(\omega, \vec{k}) \tag{18}$$

- Appropriate AdS/CFT prescription gives $G_R(\omega, k) = D_\psi / A_\psi$.
- $A_\psi = 0$ makes fermion wave-function normalizable at boundary.
- Dissipationless modes are possible at $\omega = 0$: Fermion normal mode if also $A_\psi = 0$. Thus a Fermi surface ($G_R = \infty$) corresponds to a normal mode.



# 7. Examples

Thanks to a relation $G_{11}(\omega, k) = G_{22}(\omega, -k)$, we can get all information from $G_{22}$. Cases examined in 5-d were the following:

| # | Dual operator | $m_1$ | $m_2$ | $q_1$ | $q_2$ | $p_1$ | $p_2$ | 1Q-5d | 2Q-5d |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $\lambda_1 Z_1$ | $-\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{2}$ | $1$ | $-\frac{1}{4}$ | $\frac{1}{2}$ | $\mathrm{Y}^{1A}$ | $\mathrm{N}^{1D}$ |
| 2 | $\lambda_2 Z_1$ | $-\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{2}$ | $-1$ | $-\frac{1}{4}$ | $-\frac{1}{2}$ | $\mathrm{Y}^{1A}$ | $\mathrm{N}^{1E}$ |
| 3 | $\bar\lambda_3 Z_1, \bar\lambda_4 Z_1$ | $-\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{2}$ | $0$ | $-\frac{1}{4}$ | $0$ | $\mathrm{Y}^{1A}$ | $\mathrm{N}^{1F}$ |
| 4 | $\lambda_1 Z_2, \lambda_1 Z_3$ | $\frac{1}{2}$ | $-\frac{1}{4}$ | $\frac{1}{2}$ | $2$ | $\frac{1}{4}$ | $0$ | $\mathrm{N}^{1B}$ | $\mathrm{Y}^{1G}$ |
| 5 | $\bar\lambda_2 Z_2, \bar\lambda_2 Z_3$ | $\frac{1}{2}$ | $-\frac{1}{4}$ | $-\frac{1}{2}$ | $2$ | $-\frac{1}{4}$ | $0$ | $\mathrm{N}^{1C}$ | $\mathrm{Y}^{1G}$ |
| 6 | $\lambda_3 Z_2, \lambda_4 Z_3$ | $\frac{1}{2}$ | $-\frac{1}{4}$ | $-\frac{1}{2}$ | $1$ | $-\frac{1}{4}$ | $-\frac{1}{2}$ | $\mathrm{N}^{1C}$ | $\mathrm{Y}^{1H}$ |
| 7 | $\bar\lambda_3 Z_3, \bar\lambda_4 Z_2$ | $\frac{1}{2}$ | $-\frac{1}{4}$ | $\frac{1}{2}$ | $1$ | $\frac{1}{4}$ | $-\frac{1}{2}$ | $\mathrm{N}^{1B}$ | $\mathrm{Y}^{1H}$ |

"Boson Rule:" You get a Fermi surface for $\mathrm{tr}\,\lambda Z$ iff $Z$ has an expectation value.

- 1Q-5d has $\langle \mathrm{tr}(2|Z_1|^2 - |Z_2|^2 - |Z_3|^2)\rangle > 0$, so $\langle \mathrm{tr}\,|Z_1|^2\rangle > 0$.
- 2Q-5d has $\langle \mathrm{tr}(2|Z_1|^2 - |Z_2|^2 - |Z_3|^2)\rangle < 0$, so $\langle \mathrm{tr}\,|Z_2|^2\rangle > 0$, $\langle \mathrm{tr}\,|Z_3|^2\rangle > 0$.

4-d cases are a bit more intricate:

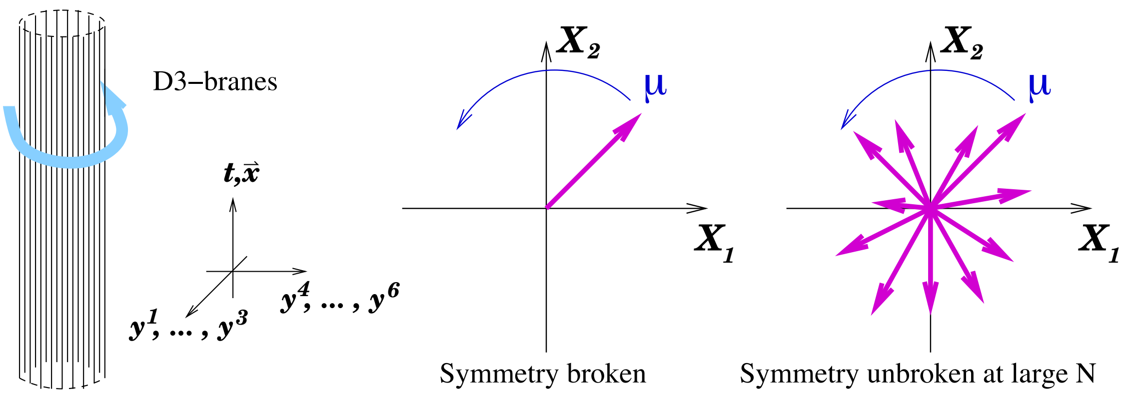
| # | Active boson | $q_a$ | $q_b$ | $q_c$ | $q_d$ | $m_a$ | $m_b$ | $m_c$ | $m_d$ | 1Q-4d | 2Q-4d | 3Q-4d |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $Z_1$ | 3 | −1 | 1 | 1 | −3 | 1 | 1 | 1 | $\mathrm{Y}^{2\mathrm{A}}$ | $\mathrm{N}^{2\mathrm{D}}$ | $\mathrm{N}^{3\mathrm{I}}$ |
| 2 | $Z_1$ | 3 | 1 | −1 | 1 | −3 | 1 | 1 | 1 | $\mathrm{Y}^{2\mathrm{A}}$ | $\mathrm{N}^{2\mathrm{E}}$ | $\mathrm{N}^{3\mathrm{I}}$ |
| 3 | $Z_1$ | 3 | 1 | 1 | −1 | −3 | 1 | 1 | 1 | $\mathrm{Y}^{2\mathrm{A}}$ | $\mathrm{N}^{2\mathrm{E}}$ | $\mathrm{N}^{3\mathrm{I}}$ |
| 4 | $Z_2$ | −1 | 3 | 1 | 1 | 1 | −3 | 1 | 1 | $\mathrm{N}^{2\mathrm{B}}$ | $\mathrm{N}^{2\mathrm{D}}$ | $\mathrm{Y}^{3\mathrm{J}}$ |
| 5 | $Z_2$ | 1 | 3 | −1 | 1 | 1 | −3 | 1 | 1 | $\mathrm{N}^{2\mathrm{C}}$ | $\mathrm{N}^{2\mathrm{E}}$ | $\mathrm{Y}^{3\mathrm{K}}$ |
| 6 | $Z_2$ | 1 | 3 | 1 | −1 | 1 | −3 | 1 | 1 | $\mathrm{N}^{2\mathrm{C}}$ | $\mathrm{N}^{2\mathrm{E}}$ | $\mathrm{Y}^{3\mathrm{K}}$ |
| 7 | $Z_3$ | −1 | 1 | 3 | 1 | 1 | 1 | −3 | 1 | $\mathrm{N}^{2\mathrm{B}}$ | $\mathrm{Y}^{2\mathrm{F}}$ | $\mathrm{Y}^{3\mathrm{J}}$ |
| 8 | $Z_3$ | 1 | −1 | 3 | 1 | 1 | 1 | −3 | 1 | $\mathrm{N}^{2\mathrm{C}}$ | $\mathrm{Y}^{2\mathrm{F}}$ | $\mathrm{Y}^{3\mathrm{K}}$ |
| 9 | $Z_3$ | 1 | 1 | 3 | −1 | 1 | 1 | −3 | 1 | $\mathrm{N}^{2\mathrm{C}}$ | $\mathrm{Y}^{2\mathrm{G}}$ | $\mathrm{Y}^{3\mathrm{K}}$ |
| 10 | $Z_4$ | −1 | 1 | 1 | 3 | 1 | 1 | 1 | −3 | $\mathrm{N}^{2\mathrm{B}}$ | $\mathrm{Y}^{2\mathrm{F}}$ | $\mathrm{Y}^{3\mathrm{J}}$ |
| 11 | $Z_4$ | 1 | −1 | 1 | 3 | 1 | 1 | 1 | −3 | $\mathrm{N}^{2\mathrm{C}}$ | $\mathrm{Y}^{2\mathrm{F}}$ | $\mathrm{Y}^{3\mathrm{K}}$ |
| 12 | $Z_4$ | 1 | 1 | −1 | 3 | 1 | 1 | 1 | −3 | $\mathrm{N}^{2\mathrm{C}}$ | $\mathrm{Y}^{2\mathrm{G}}$ | $\mathrm{Y}^{3\mathrm{K}}$ |
| 13 | $Z_1$ | 3 | −1 | −1 | −1 | −3 | 1 | 1 | 1 | $\mathrm{Y}^{2\mathrm{A}}$ | $\mathrm{N}^{2\mathrm{H}}$ | $\mathrm{N}^{3\mathrm{L}}$ |
| 14 | $Z_2$ | −1 | 3 | −1 | −1 | 1 | −3 | 1 | 1 | $\mathrm{N}^{2\mathrm{B}}$ | $\mathrm{N}^{2\mathrm{H}}$ | $\mathrm{Y}^{3\mathrm{M}}$ |
| 15 | $Z_3$ | −1 | −1 | 3 | −1 | 1 | 1 | −3 | 1 | $\mathrm{N}^{2\mathrm{B}}$ | $\mathrm{Y}^{2\mathrm{G}}$ | $\mathrm{Y}^{3\mathrm{M}}$ |
| 16 | $Z_4$ | −1 | −1 | −1 | 3 | 1 | 1 | 1 | −3 | $\mathrm{N}^{2\mathrm{B}}$ | $\mathrm{Y}^{2\mathrm{G}}$ | $\mathrm{Y}^{3\mathrm{M}}$ |

But boson rule works in every case: non-zero bosons are $Z_1$ for 1Q-4d; $Z_3$ and $Z_4$ for 2Q-4d; and $Z_2$, $Z_3$, $Z_4$ for 3Q-4d.

Suggested interpretation: The singularity in $\langle \mathcal{O}_\chi \mathcal{O}_\chi^\dagger \rangle$ is due to a Fermi surface of a colored fermion, co-existing with a scalar condensate which (at large $N$) leaves the $U(1)$ symmetry unbroken.



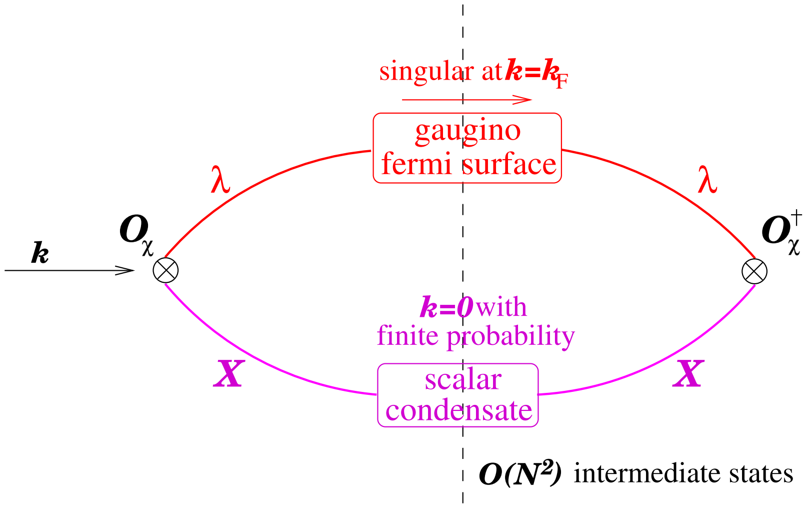
Easiest for me to think about the case of $\mathcal{N} = 4$ SYM in $d = 4$. Large $N$ allows $U(1)$ to remain unbroken even with non-zero scalar condensate:




A common worry is that scalar condensate can run away along flat directions. But perhaps this is not relevant at large $N$. Here's why:

- Only a subleading fraction of directions satisfy $[X^I, X^J] = 0$.
- Cases considered are finitely far from SUSY limit, so it's probably more representative to think of non-commuting directions.
- In non-commuting directions, condensate is limited by $V \sim g^2 \operatorname{tr}[X^I, X^J]^2$.

So—plausibly—the singularity at $k = k_F$, with residue $\sim N^2$ in $AdS_5$ calculations, owes to diagrams in $\mathcal{N} = 4$ SYM roughly like this:

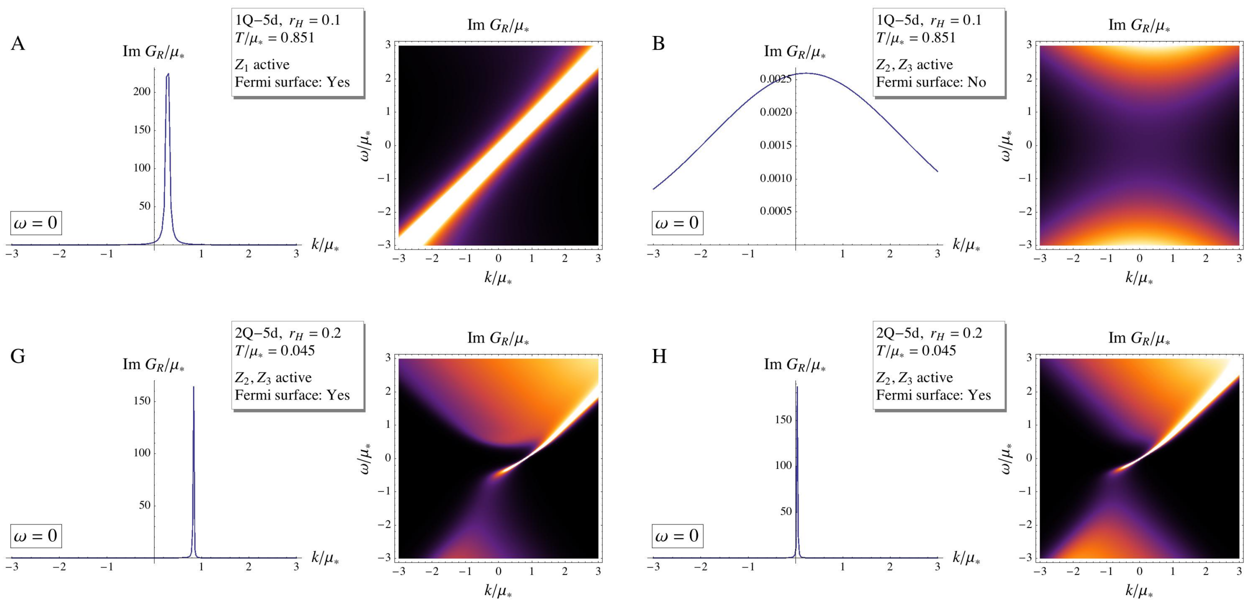



This account contrasts strongly with earlier works claiming that the Fermi surfaces are best understood in terms of color singlet fermions *in the gauge theory* [Huijse-Sachdev '11], and if colored fermions have Fermi surfaces, they are hidden from supergravity calculations.



A closer look at examples shows that $k_F$ is often significantly smaller than the natural scale

$$\mu_* = \sqrt{T^2 + \mu_1^2 + \mu_2^2} \quad \text{(5-d)} \qquad \mu_* = \sqrt{T^2 + \textstyle\sum_j \mu_j^2} \quad \text{(4-d)}\,. \tag{19}$$



There are two unrelated reasons for this:

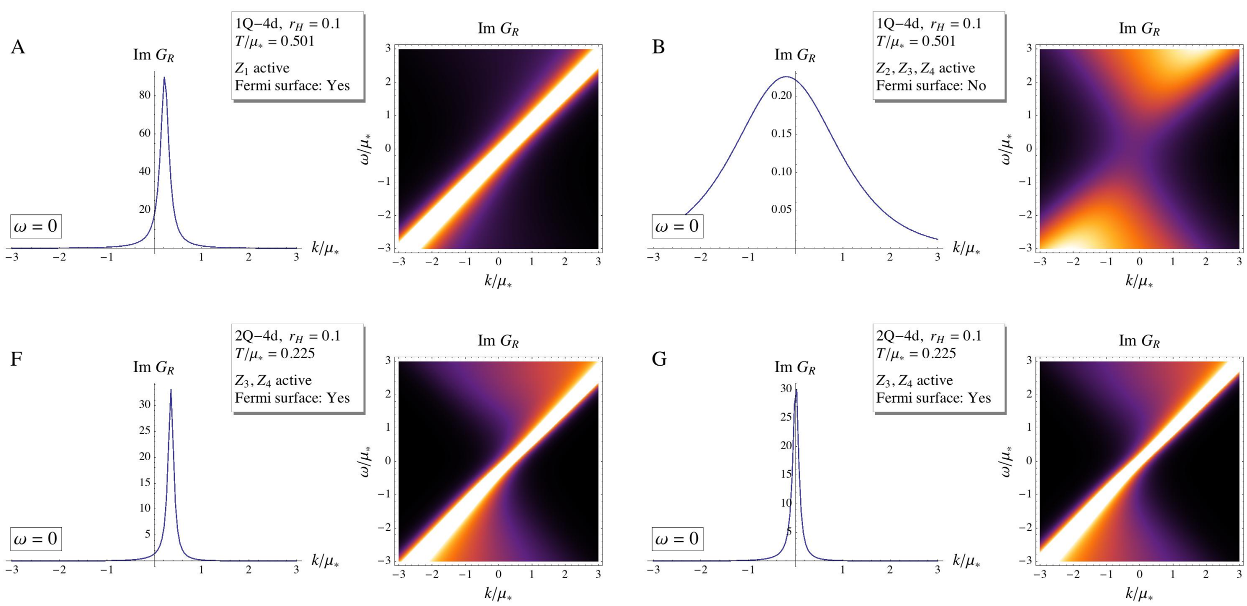
1. $\mu_1 \ll \mu_*$ for the 1Q-5d (Case A), so we naturally have small Fermi surfaces.
2. Case G involves the gaugino $\lambda_1^{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}$, which carries charge under $U(1)$ of the 2Q-BH background, whereas Case H involves the gaugino $\lambda_3^{\left(-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)}$, which is *neutral* under this $U(1)$.

Viewing #1 as trivial, we suggest the following

> "Fermion Rule:" The value of $k_F$ is suppressed, though it may not vanish, when $\lambda$ is neutral under the $U(1)$ charge of the black hole.



A detailed look at 4-d cases provide supports the boson rule and gives some additional evidence in favor of the fermion rule.




- Chemical potential $\mu_a$ is small for case A.
- $k_F$ is larger for case F (charged $\lambda$) than for case G (neutral $\lambda$).

# 8. Summary

- Black holes derived from spinning branes generically have finite entropy at zero temperature, but when one of the spins vanishes, so does the extremal entropy.
- Field theory understanding of holographic Fermi surfaces is probably easier without extremal entropy complicating the story.
- We added slight non-extremality to avoid singular supergravity backgrounds.
- Holographic Fermi surfaces appear or don't appear in correlators of $\mathcal{O}_\chi = \mathrm{tr}\,\lambda Z$ precisely if $Z$ has an expectation value.
- Probably what's going on is that we're seeing a Fermi surface of the color-charged fermions $\lambda$, not some composite color-singlet created by $\mathcal{O}_\chi$.
- Neutral fermions have smaller Fermi surfaces, though their $k_F$ may not be exactly $0$.

# “De Sitter Vacua from a D-term Generated Racetrack Uplift”

Yoske Sumitomo

[JGRG24(2014)111007]

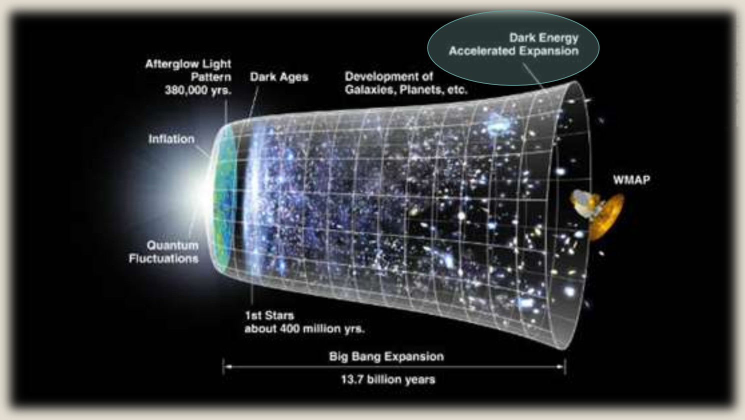
# DE SITTER VACUA FROM A D-TERM GENERATED RACETRACK UPLIFT

Yoske Sumitomo
KEK Theory Center, Japan

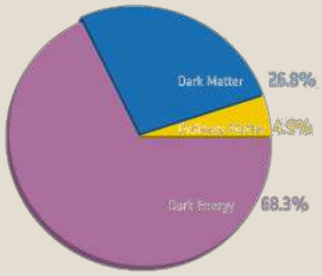
M. Rummel, YS, arXiv:1407.7580

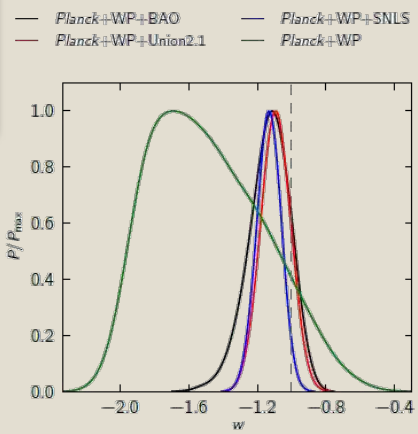
## Dark Energy

Dominant source for late time expansion




After Planck



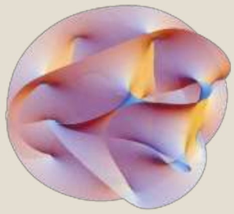

Planck+WMAP+BAO

$w = \frac{p}{\rho} = -1.13^{+0.24}_{-0.25}$ (95% CL)

agrees with
the positive cosmological constant
(de-Sitter vacuum)

# Type IIB models

A Calabi-Yau Space:



Type IIB on CY has no-scale structure

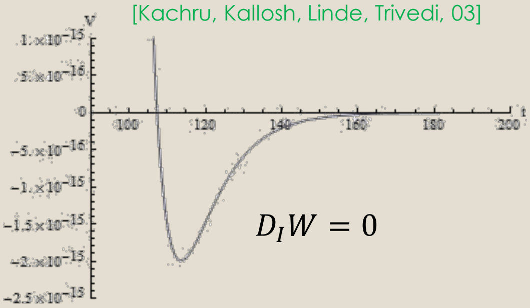
generating hierarchy: $V = V_{\text{Flux}} + V_{\text{NP}} + V_{\alpha'} + \cdots$

$V_{\text{Flux}}$: $\mathcal{O}(\mathcal{V}^{-2})$ complex

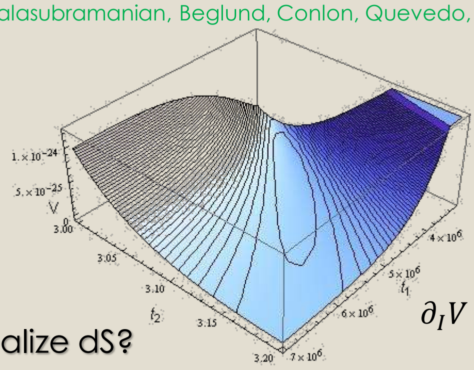
$V_{\text{NP}} + V_{\alpha'}$: $\mathcal{O}(\ll \mathcal{V}^{-2})$ Kähler

Complex structure moduli + dilaton can be integrated out.

E.g. KKLT [Kachru, Kallosh, Linde, Trivedi, 03]




Large Volume Scenario (LVS) [Balasubramanian, Beglund, Conlon, Quevedo, 05]




Both minima stay at AdS. How to realize dS?

# Need for uplift

An uplift to survive

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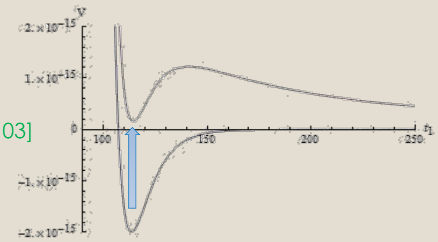
# Some uplift models

- Explicit SUSY breaking

  Probe approximation

  $V = V_{SUGRA} + V_{D3-\overline{D3}}$ [Kachru, Pearson, Verlinde, 01], [KKLT, 03]

  $V_{D3-\overline{D3}} = 2T_3 \int d^4x \sqrt{-g_4} \sim \mathcal{O}(\mathcal{V}^{-4/3})$



  Backreaction of $\overline{D3}$? → a singularity exists, but finite action

  Safe or not? [DeWolfe, Kachru, Mulligan, 08], [McGuirk, Shiu, YS, 09], [Klevanov] [Bena, Giecold, Grana, Halmagyi, Kuperstein, Massai, 09-14], [Dymarsky, 11], [Bena, Blaback, Danielsson, Junghans, Kuperstein, Schmidt, Van Riet, Wrase, Zagermann, 11-14], [Bena, Buchel, Dias, 12], [Kutasov, Wissanji, 12], [Dymarsky, Massai, 13],...

- Non-zero minimum of flux potential [Saltman, Silverstein, 04]

  $V = V_{\rm NP} + V_{\alpha'} + V_{\rm Flux}$ $\qquad V_{\rm Flux} = e^K |D_{S,U_i} W_0|^2 \sim \mathcal{O}(\mathcal{V}^{-2})$

  tuned to balance with $V_{\rm NP} + V_{\alpha'}$ (generically $\ll V_{\rm Flux}$)

See also [Danielsson, Dibitetto, 13], [Blaback, Roest, Zavala, 13], [Kallosh, Linde, Vercnocke, Wrase, 14]

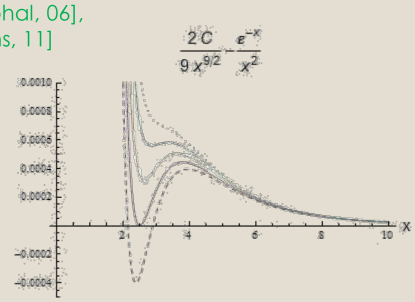
# Some uplift models

- Kähler Uplift [Balasubramanian, Berglund, 04], [Westphal, 06], [Rummel, Westphal, 11], [de Alwis, Givens, 11]

  Focus on other region

  $K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right), \quad W = W_0 + A_1 e^{-a_1 T_1} + \cdots$

  → $V \sim -\frac{W_0 a_1^3 A_1}{2\,\gamma_1^2}\left(\frac{2C}{9x_1^{9/2}} - \frac{e^{-x_1}}{x_1^2}\right)$



  Concern: upper bound on volume, $x_1 = a_1\, Re\, T_1 \lesssim 3.11$

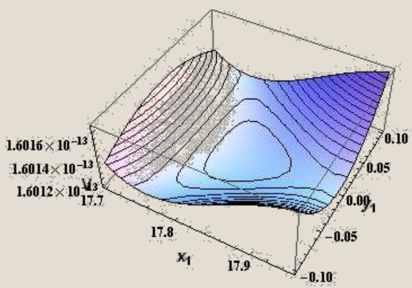
  ↓

- Racetrack Kähler Uplift

  $W = W_0 + A_1 e^{-a_1 T_1} + B_1 e^{-b_1 T_1}$

  → opens up a new region of solutions

  No upper bound [YS, Tye, Wong, 13]



# Some uplift models

- D-term uplift [Burgess, Kallosh, Quevedo, 03], [Cremades, Garcia del Moral, Quevedo, 07], [Krippendorf, Quevedo, 09]

  A magnetic flux on the D7-brane induces D-term:

  $$V_D = \frac{1}{\mathrm{Re}(f_D)}\left(\sum c_{Dj}\hat{K}_j\varphi_j - \xi_D\right)^2 \qquad \xi_D = \frac{1}{4\pi\mathcal{V}}\int J\wedge D_D\wedge\mathcal{F}_D$$

  For D7 wrapping large volume (and $\langle\varphi\rangle \neq 0$)

  ➡ $V_{up} \sim \mathcal{O}(\mathcal{V}^{-8/3})$ tuned by mild warping or something else to balance with $V_{LVS} \propto \mathcal{V}^{-3}$

- Dilaton-dependent non-perturbative effects [Cicoli, Maharana, Quevedo, Burgess, 12]

  $$K \ni -\ln(S+\bar{S}) + \alpha\frac{\rho^2}{\mathcal{V}} \qquad W \ni B\, e^{-b(S+h\,\rho)}$$

  ➡ $V_{up} \propto h^2\frac{e^{-2b\langle s\rangle}}{\mathcal{V}}$ while $V_{LVS} \propto \mathcal{V}^{-3}$

  Dilaton value should be chosen accordingly.

# D-TERM GENERATED RACETRACK UPLIFT

M. Rummel, YS, arXiv:1407.7580

# D-term generated racetrack uplift

Three-Kähler Swiss-Cheese (simplified)

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right), \qquad \mathcal{V} = (T_a + \bar{T}_a)^{3/2} - (T_b + \bar{T}_b)^{3/2} - (T_c + \bar{T}_c)^{3/2}$$

$$W = W_0 + A_2 e^{-a_2 T_b} + A_3 e^{-a_3(T_b + T_c)}$$

LVS region

$$\hat{V} \equiv \frac{V_F}{W_0^2} \sim \frac{3\xi}{4\mathcal{V}^3} + \mathcal{O}\left(\frac{e^{-a_i\tau_i}}{\mathcal{V}^2}\right) + \mathcal{O}\left(\frac{e^{-2a_i\tau_i}}{\mathcal{V}}\right) \sim \mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right)$$

Redefine $T_s = (T_b + T_c)/2\,,\ Z = (T_b - T_c)/2$

Suppose D-term condition fixes $Z = 0$ where $c_i = A_i/W_0$

$$\hat{V} \sim \frac{3\xi}{4\mathcal{V}} + \frac{4c_2 x_s}{\mathcal{V}^2} e^{-x_s} + \frac{2\sqrt{2}c_2^2\sqrt{x_s}}{3\mathcal{V}} e^{-2x_s} + \frac{4\beta c_3 x_s}{\mathcal{V}^2} e^{-\beta x_s} + \cdots$$

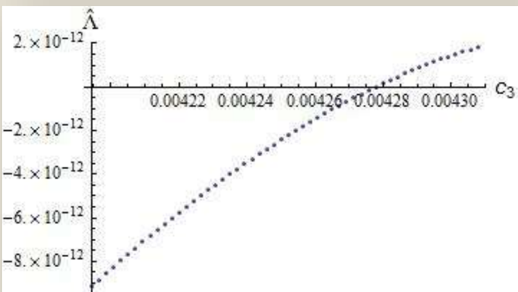
for $\mathcal{V},\ x_s = a_2\tau_s$ $\beta = 2a_3/a_2$ and other parameters redefined

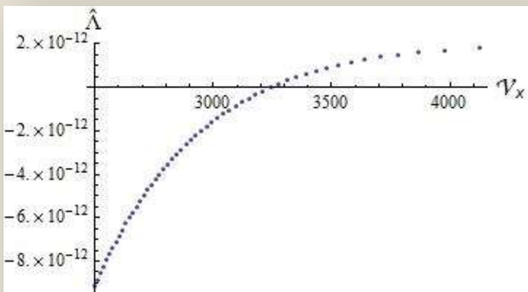
# D-term generated racetrack uplift

Illustration of the uplift $x_s = a_2\tau_s$

$$\hat{V} \sim \frac{3\xi}{4\mathcal{V}} + \frac{4c_2 x_s}{\mathcal{V}^2} e^{-x_s} + \frac{2\sqrt{2}c_2^2\sqrt{x_s}}{3\mathcal{V}} e^{-2x_s} + \frac{4\beta c_3 x_s}{\mathcal{V}^2} e^{-\beta x_s} + \cdots$$

When $c_2 = -0.01,\ \xi = 5,\ \beta = 5/6$, and increase $c_3$




All vacua have positive-definite Hessian, and stable.

Minkowski point: $c_3 \sim 4.28 \times 10^{-3}, \mathcal{V} \sim 3240, x_s \sim 3.07$, (thus $c_3^2$ negligible)

Analytically, $\beta < 1, c_3 > 0$ required at Minkowski

# D-term condition: Z stabilization

Magnetized D7-branes wrapping the Calabi-Yau four-cycle

$$\Rightarrow \quad V_D = \frac{1}{\mathrm{Re}(f_D)}\left(\sum c_{Dj}\widehat{K}_j\varphi_j - \xi_D\right)^2 \qquad \xi_D = \frac{1}{4\pi\mathcal{V}}\int J \wedge D_D \wedge \mathcal{F}_D$$

A choice of anomalous U(1) fluxes on D7 $\mathcal{F}_D$ would give

$$V_D \propto \frac{1}{\mathrm{Re}(f_D)}\frac{1}{\mathcal{V}^2}(\sqrt{\tau_b} - \sqrt{\tau_c})^2$$ mass term for $\mathrm{Re}\,Z = \mathrm{Re}\,(T_b - T_c)/2$

D-term potential stabilizes corresponding moduli at high scale:

$$V_D \gg V_{F,LVS} \sim \mathcal{O}(\mathcal{V}^{-3})$$

Also, the imaginary part of $Z$ is eaten by a massive anomalous U(1) gauge boson at the string scale $\mathcal{O}(\mathcal{V}^{-1/2})$ (Stuckelberg mechanism).

Hence, safely realizing the proposed uplift mechanism in type IIB.

# Summary & Discussion

- We explored an uplift mechanism achieved in F-term with multi-Kähler moduli structure.
- With the help of D-term constraint, the heavy moduli Z is integrated out.
- Proposed potential is different from simple racetrack in F-term.
- Resulting uplift term becomes $\frac{e^{-\beta x_s}}{\mathcal{V}^2}$, and hence no other suppressions are needed.
- Proposed uplift mechanism works in the presence of additional moduli, and is realizable in many compactifications.

“Electric field quench in AdS/CFT”

Shunichiro Kinoshita

[JGRG24(2014)111008]

The 24th workshop on General Relativity and Gravitation @ IPMU 2014/11/10

# Electric field quench in AdS/CFT

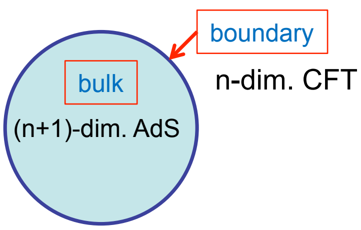
Shunichiro Kinoshita
(Osaka City University Advanced Mathematical Institute)
K. Hashimoto (Osaka, RIKEN), K. Murata (Keio),
T. Oka (Tokyo)

Based on JHEP 09(2014)126 (arXiv:1407.0798)

# AdS/CFT correspondence

- A duality relating a classical gravity in (n+1)-dim. anti-de Sitter (AdS) space and a strongly correlated conformal field theory (CFT) in n-dim.
  - Holography, the gauge/gravity duality

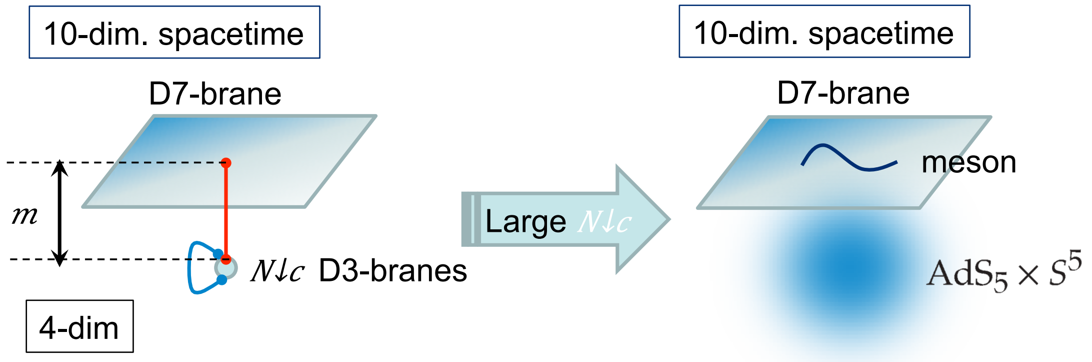
Maldacena (1998)




- The classical dynamics of gravity corresponds to the quantum physics of strongly correlated gauge theory
  - General relativity could describe strongly corrected quantum systems (with finite temperature), which are too difficult to solve.
  - QCD, Quark-gluon plasma(QGP), condensed matter physics, …

# Holographic QCD constructed by D3/D7

Karch, Katz (2002), Grana, Polchinski (2002), Bertolini *et al*. (2002)




N=4 super Yang-Mills + N=2 quark multiplet

A probe D7-brane is embedded in $AdS_5 \times S^5$

$$ds^2 = r^2[-dt^2 + d\vec{x}_3^2] + \frac{1}{r^2}[d\rho^2 + \rho^2 d\Omega_3^2 + dw^2 + d\bar{w}^2]$$

DBI action: $S_{D7} = -\mu_7 \int d^8y \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})}$

Embedding function
$w(\rho) = m\,(\text{const.})$

quark mass

string between D3-branes and D7-brane ⇔ "quark"
fluctuations of D7-brane ⇔ "meson"

# Confinement/deconfinement in the meson sector

Mateos, Myers, Thomson (2006, 2007)

- If one includes electric field or finite temperature in the system, the phase transition occurs

| Gravity side | Gauge theory side |
|---|---|
| Black hole in the bulk<br>or<br>Gauge field on the brane | Finite temperature in the gluon sector<br>or<br>Finite electric field |
| The brane is bending | Expectation values change |
| $w(\rho) \sim m + \frac{c}{\rho^2} + \cdots,$ | $\langle\psi\bar{\psi}\rangle \propto c$ : quark condensate |
| $a_x(\rho) \sim -E_x t + \frac{j_x}{2\rho^2} + \cdots,$ | $\langle\psi\gamma_\mu\bar{\psi}\rangle \propto j_\mu$ : electric current |
| The brane intersects a horizon or not | Deconfinement or confinement |
| The fluctuations dissipate or are confined | Mesons are unstable or stable |

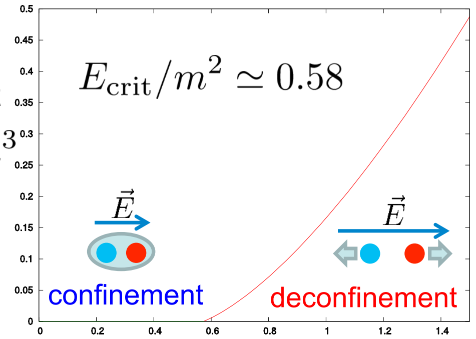
# Electric field case

Karch, O'Bannon (2007)
Erdmenger, Meyer, Shock (2007)
Albash, Filev, Johnson, Kundu (2007)

- Schwinger effect

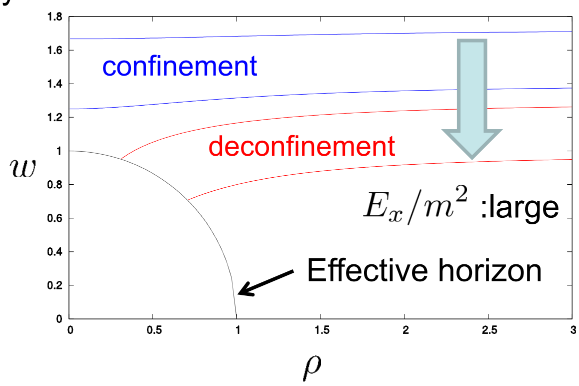
Electric field vs current in the boundary

current $\langle j_x \rangle / m^3$




Electric field $E_x/m^2$

Profile of the brane in the bulk




$$ds^2 = r^2[-dt^2 + d\vec{x}_3^2] + \frac{1}{r^2}[d\rho^2 + \rho^2 d\Omega_3^2 + dw^2 + d\bar{w}^2]$$

Beyond the critical electric field, an effective horizon emerges on the brane
The electric current becomes non-zero value = Schwinger effect

# Our setup: time-dependent electric field

Hashimoto, SK, Murata, Oka JHEP 09 (2014) 126

- Bulk spacetime
  - $AdS_5 \times S^5$

$$ds^2 = \frac{1}{z^2}[-dV^2 - 2dVdz + dx^2 + d\vec{x}_2^2] + d\phi^2 + \cos^2\phi d\Omega_3^2 + \sin^2\phi d\psi^2$$

- D7-brane

The brane is symmetric in $x_3$, $\Omega_3$ -directions

  - Embedding function :

$$V = V(u,v), \quad z = Z(u,v), \quad \phi = \Phi(u,v), \quad \psi = 0$$

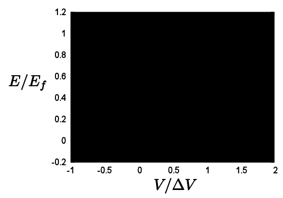
  - Gauge field :

$$2\pi\alpha' A_a dy^a = a_x(u,v)dx$$

Boundary conditions of $a_x$ at the AdS boundary = electric field in the boundary theory

$$a_x(Z=0,V) \equiv -\int^V dV' E(V')$$

$$E(V) = \begin{cases} 0 & (V<0) \\ E_f[V - \frac{\Delta V}{2\pi}\sin(2\pi V/\Delta V)]/\Delta V & (0 \le V \le \Delta V) \\ E_f & (V > \Delta V) \end{cases}$$

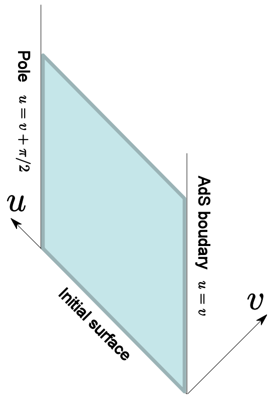


# Equations of motion of the brane

- DBI action

$$S[V,Z,\Phi,a_x] = -\int d^8\sigma \sqrt{-\det(h_{ab}+f_{ab})} \propto \int dudv \frac{\cos^2\Phi}{Z^3}\sqrt{\xi},$$

$$\xi \equiv (h_{uv} + Z^2\partial_u a_x \partial_v a_x)^2 - (h_{uu} + Z^2(\partial_u a_x)^2)(h_{vv} + Z^2(\partial_v a_x)^2)$$




$$h_{uv} = -Z^{-2}(V_{,u}V_{,v} + V_{,u}Z_{,v} + Z_{,u}Z_{,v}) + \Phi_{,u}\Phi_{,v},$$

$$h_{uu} = -Z^{-2}V_{,u}(V_{,u} + 2Z_{,u}) + \Phi_{,u}^2, \quad h_{vv} = -Z^{-2}V_{,v}(V_{,v} + 2Z_{,v}) + \Phi_{,v}^2$$

Coordinate conditions:

$$C_1 \equiv h_{uu} + Z^2(\partial_u a_x)^2 = 0,$$

$$C_2 \equiv h_{vv} + Z^2(\partial_v a_x)^2 = 0$$

$$\partial_u\partial_v \mathbf{X} + \mathbf{f}(\mathbf{X}, \partial_u\mathbf{X}, \partial_v\mathbf{X}) = 0 \qquad \mathbf{X} = (V, Z, \Phi, a_x)$$

2-dimensional non-linear wave equations on an effective metric

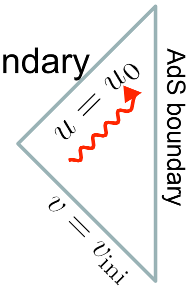
Effective metric: $\gamma_{ab} \equiv h_{ab} + f_{ac}f_{bd}h^{cd}$

# Revisit of deconfinement and thermalization

- If static, both of deconfinement and thermalization are given by the same condition in gravity side: the horizon exists or not
- In time-dependent cases, this definition is not so useful
  - When does the horizon form for temporal observers at the AdS boundary?
  - Since we have no preferred time-slice in gravity, the formation time is ambiguous

# Redshift factor and surface gravity

- Redshift factor
  - The ratio between the energy observed on the AdS boundary and the initial surface (static region)




$$R(u_0) = \frac{k^a \xi_a|_{v=v_{\rm ini}}}{k^a \xi_a|_{v=u_0}} = -\frac{m}{2}\frac{V_{,v}(u_0,u_0)}{\Phi_{,u}(u_0,v_{\rm ini})}$$

- Surface gravity
  - Relation between times to define the Initial state and the final state

$$\kappa(u_0) = \frac{d}{du}\log R(u_0)$$

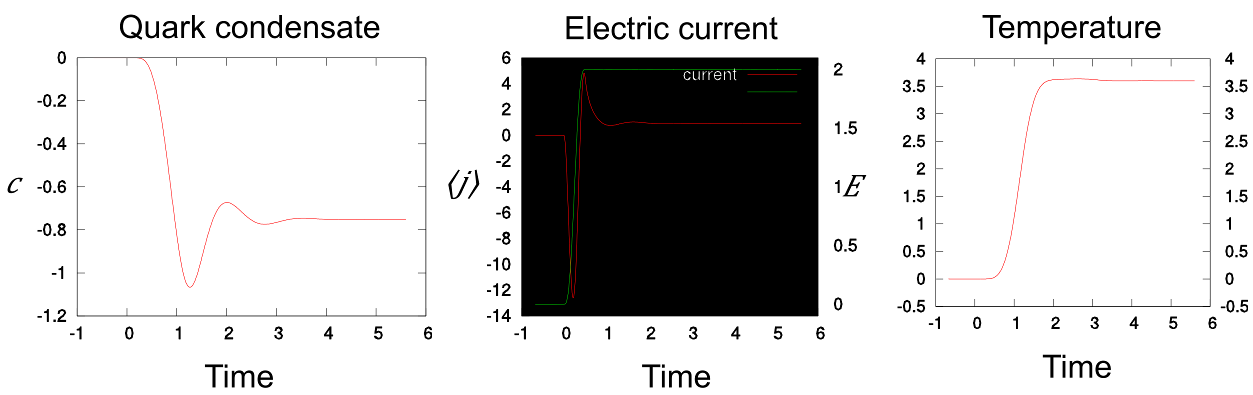
If this quantity is almost constant, it becomes Hawking temperature observed at $u_0$

Redshift becomes too large ⇔ deconfinment
Surface gravity becomes constant ⇔ thermalization

We can define these only from the causal past of temporal observers

# Numerical results super-Schwinger-limit

- Strong electric field ($E_f = 2.0,\ \Delta V = 0.50$)

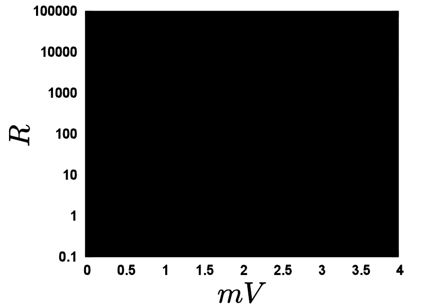



- The meson sector is deconfined by the Schwinger effect
- The system has been relaxed and thermalized
  - The effective horizon emerges on the worldvolume

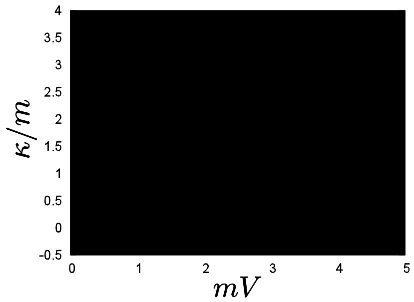
# Thermalization time

- We can estimate thermalization time explicitly

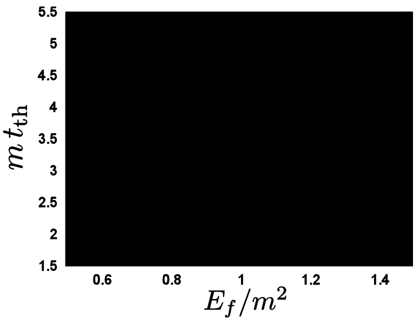
Time evolution of the redshift factor




Time evolution of the surface gravity




thermaliztion time



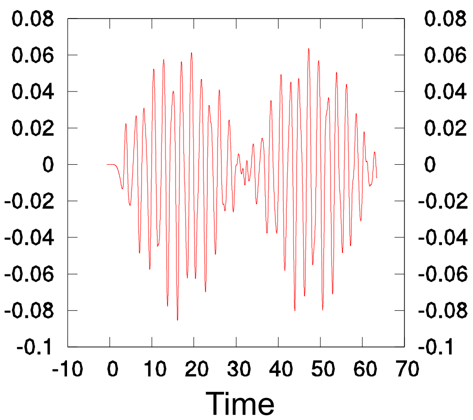

$t_{\rm th} \sim 1/\kappa \sim 1/\sqrt{E_f}$

As the electric field is larger,
it becomes typical thermal scale

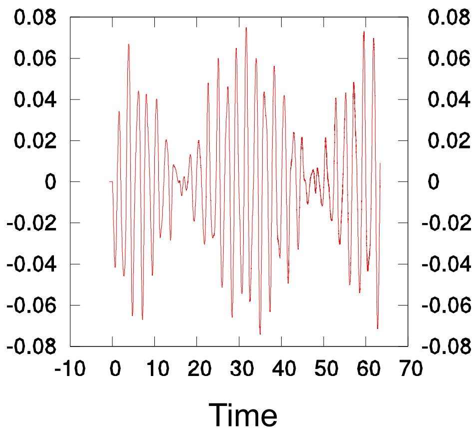
# Numerical results sub-Schwinger-limit

- Weak electric field ($E_f = 0.10$, $\Delta V = 2.0$)

Quark condensate



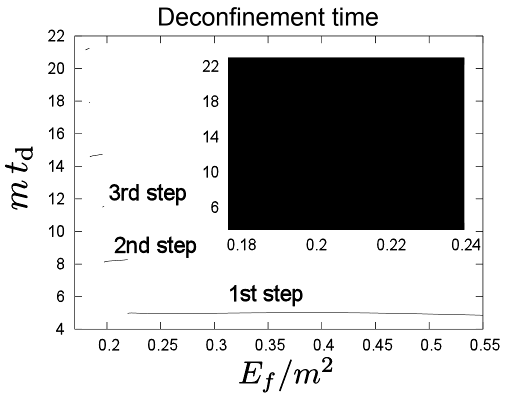

Electric current

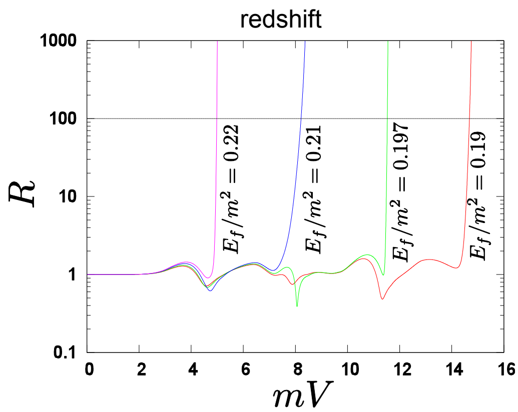



- Normal modes of fluctuations of the brane and the gauge field
  ⇔ discrete spectrum of meson
- “beat” ⇔ meson mixing
  - The Stark effect leads to splitting of degenerate mass spectrum

# Non-equilibrium deconfinement

- We found that sufficiently rapid quench causes the deconfinement transition even below the critical electric field
  - Notice that deconfinement is defined by divergence of redshift factor measured at the AdS boundary

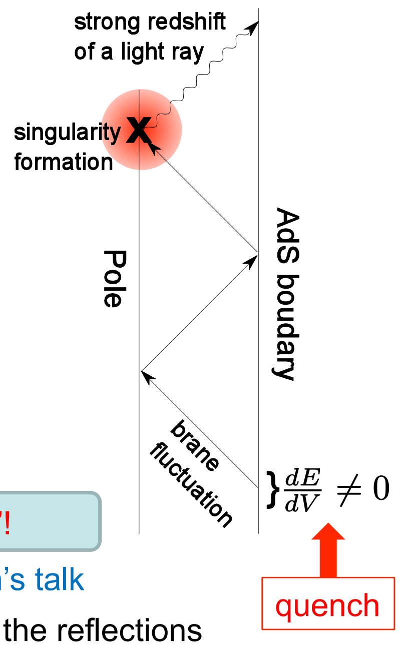



The deconfinement time becomes discrete with respect to the electric field

# "Turbulence" on the brane?

- What is happening on the brane?
  - The fluctuation caused by the quench at the boundary is amplified during coming and going on the brane
  - After reflecting several times, a strongly red-shifted region emerges at the center and then a naked-singularity will form on the brane




It seems to be similar to "AdS turbulent instability"!

Murata-kun's talk

Discreteness of the deconfinement time = number of the reflections
Deconfine (divergence of the redshift factor) = singularity formation

# Summary

- We have studied response of the strongly coupled gauge theory against an electric field quench, by using the AdS/CFT correspondense
  - We have numerically solved dynamics of the probe D7-brane under time-dependent boundary conditons
- We have proposed a new definition of deconfinement and thermalization in gravity side
  - Non-equilibrium decomfinement below the Schwinger limit

- A probe-brane version of turbulent instability?
- Applying AC electric field

“Turbulent meson condensation in quark deconfinement”

Keiju Murata

[JGRG24(2014)111009]

# Turbulent meson condensation in quark deconfinement

Keio University, Japan

Keiju Murata

with K.Hashimoto, S.Kinoshita, T.Oka

- K,Hashimoto,S.Kinoshita,KM,T.Oka, "Electric Field Quench in AdS/CFT", arXiv:1407.0798, accepted in JHEP.
- K,Hashimoto,S.Kinoshita,KM,T.Oka, "Turbulent meson condensation in quark deconfinement", arXiv:1408.6293

# Non-equilibrium process in AdS/CFT

- N=4 SYM
- QCD
- Condensed matter physics
- etc…

AdS/CFT ⟷ Gravity theories

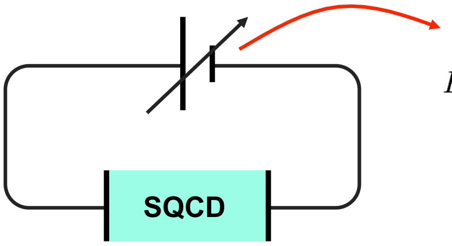
**Non-equilibrium**

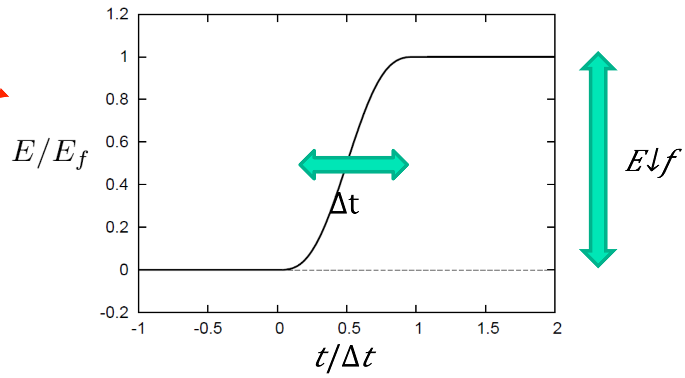
**First-principles calculation is not tractable.**

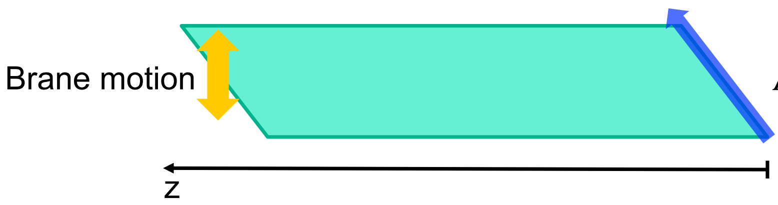
**Tractable.**
At least, there is no problem in the formulation. (Cauchy problem)

**The AdS/CFT gives one of the hopeful approaches to study the non-equilibrium process in strongly coupled systems.**

# Electric field quench in N=2 supersymmetric QCD



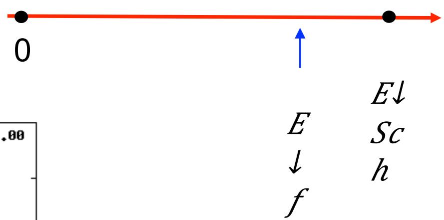



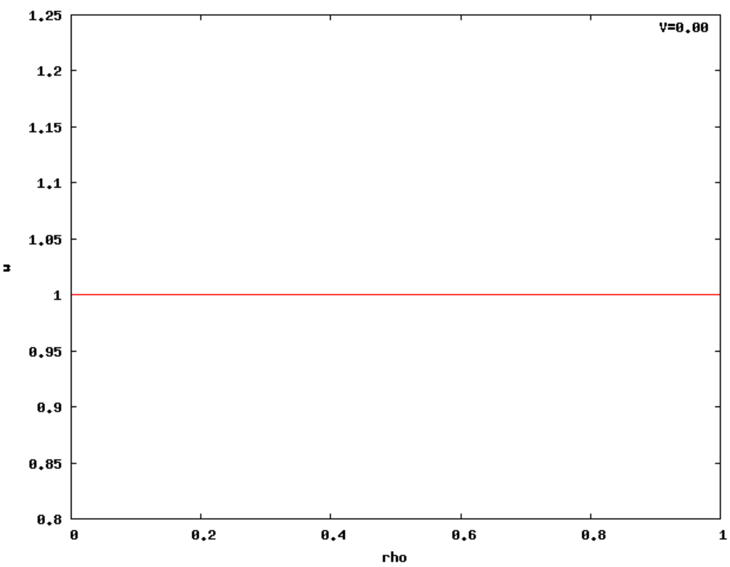

Boundary condition:

$F_{tx}|_{z=0} = -E(t)$

# Dynamics of the D7-brane (subcritical case)

$E_f/m^2 = 0.19,\ m\Delta t = 2$



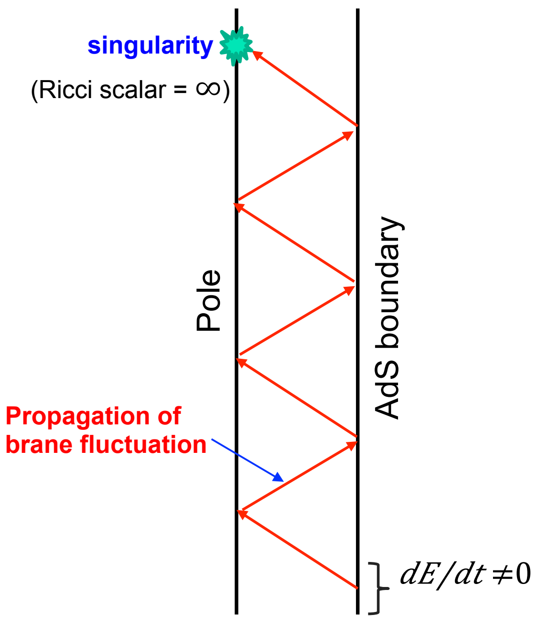




At t ~12, we can see a singular behavior.

Our numerical calculation has crushed there.

What's happened?

# Singularity formation




**D-brane version of the weakly turbulent instability.** Bizon&Rostworowski, 11

A nakid singularity appears on the D-brane.

There is a strong redshift near the singularity.

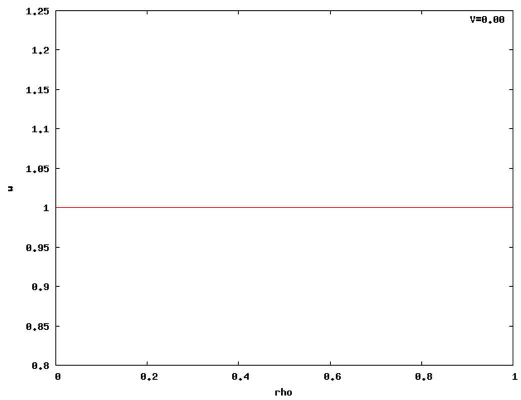
Deconfinement of mesons.

**In this talk, we study the detail of the "turbulence" of the D7-brane and gives a field theory interpretation.**

# Mode decomposition of the non-linear solution

$$\delta W(t,z)=W(t,z)-m$$

Non-linear solution      Background solution




We decompose the non-linear solution into eigen functions in linear theory.

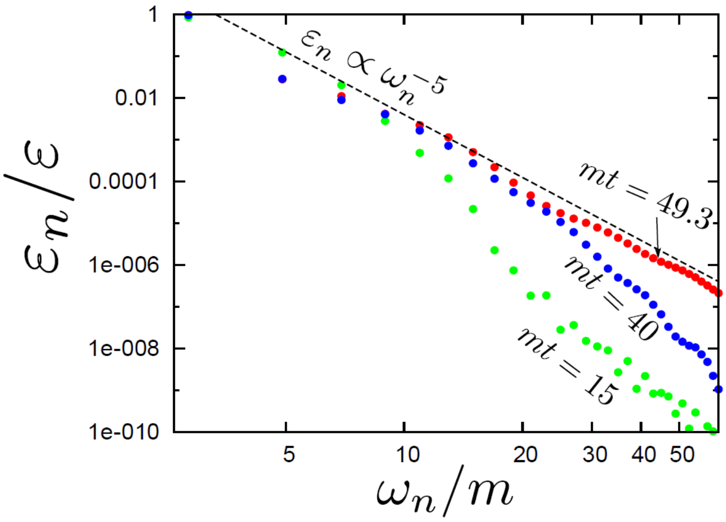
$$\delta W(t,z)=\sum_n c_n(t) e_n(z)$$

Energy contribution from n-th mode

$$\epsilon_n(t)=1/2\,(c_n^2+\omega_n^2 c_n^2)$$

$$\omega_n^2=4(n+1)(n+2)$$

# Energy transfer from large to small scale




**There is an energy flow from large scale to small scale.**

Weak turbulence

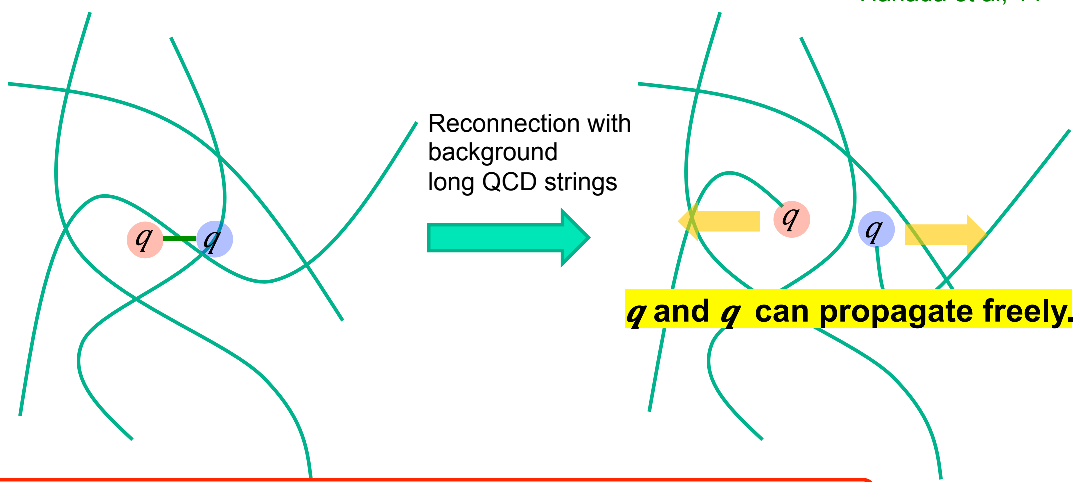
**Many heavy mesons are produced just before the deconfinement.**

Why are many heavy mesons produced before the deconfinement?

# Deconfinement occurs due to heavy quark condensate

Polyakov, 78
Pisarski&Alvaretz, 82
Patel, 84
Lucini et al, 05
Hanada et al, 14

Heavy mesons = long QCD strings

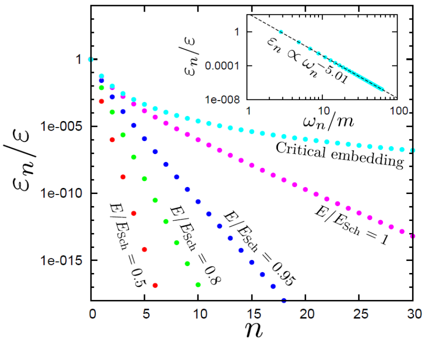



Heavy meson condensation → Quark deconfinement

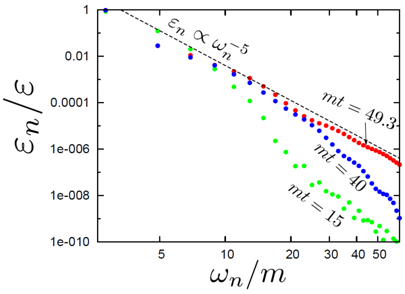
**Our AdS/CFT calculation supports this idea.**

# Kolmogorov-like law just before the deconfinement

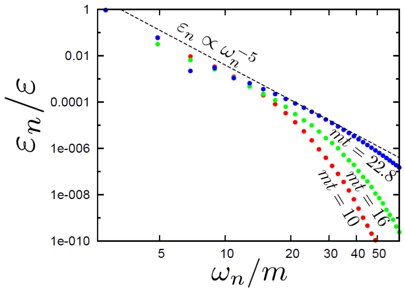
Quasi-static analysis for Electric field quench




Electric field quench




Mass quench




The spectrum seems to approach power law: $\varepsilon_n \propto \omega_n^{-5}$

Universal for the deconfinement transition in N=2 SQCD?

# Summary

We found a “weakly turbulence” in the D3/D7 system.

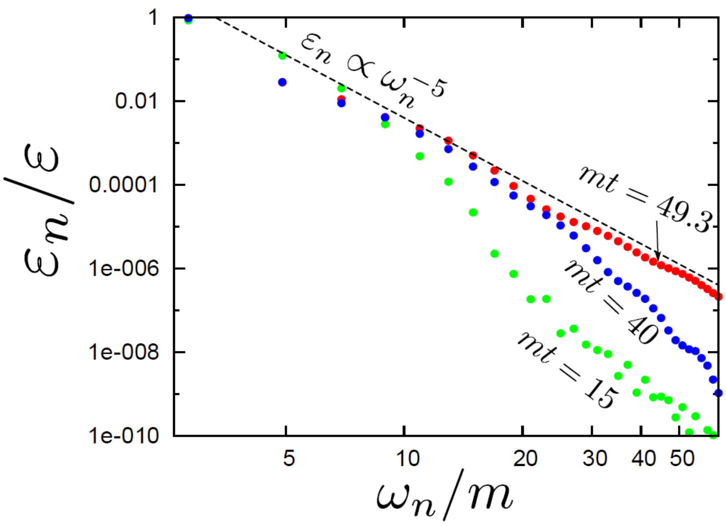
There is a energy flow from large to small scale.

This can be regarded as production of many heavy mesons in SQCD.

Just before the deconfinement spectrum becomes $\varepsilon_n \propto \omega_n^{-5}$ .

Universal for the deconfinement transition in N=2 SQCD?

# Energy transfer from large to small scale




We decomposed a non-linear solution by eigenfunctions for linear perturbations.

**There is an energy flow from large scale to small scale.**

**Many heavy mesons are produced just before the deconfinement.**

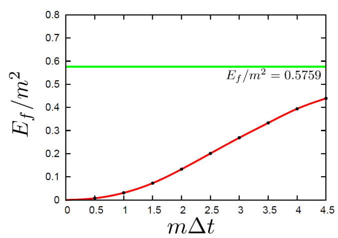
Why are many heavy mesons produced before the deconfinement?

# Summary

We found a "weakly turbulence" in the D3/D7 system.

There is a energy flow from large to small scale.

The turbulence does not occur for arbitrary small perturbation.



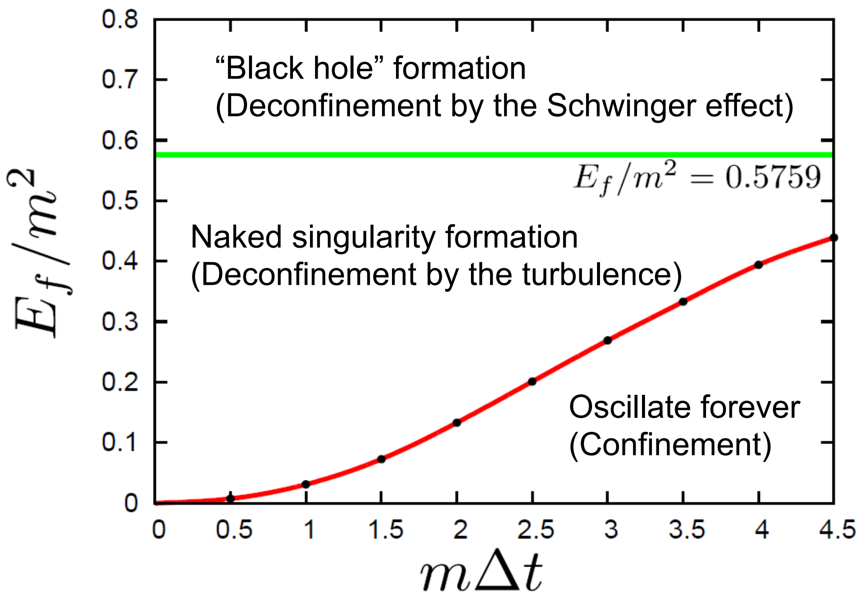

Electric field is not essential for turbulence.

Giving a finite perturbation is important.

Just before the deconfinement spectrum becomes $\varepsilon_n \propto \omega_n^{-5}$ .

Universal for the deconfinement transition in N=2 SQCD?

# Dynamical phase diagram

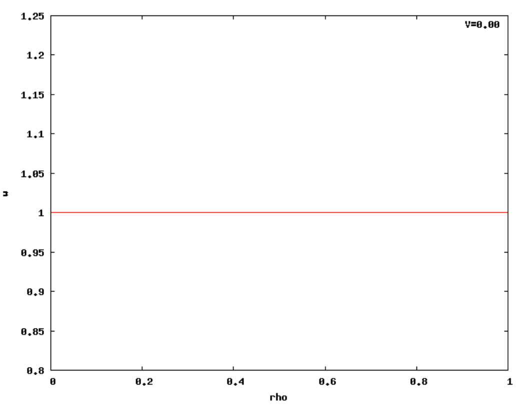



# Mode decomposition of the non-linear solution

$\delta W(t,z)=W(t,z)-m$

Non-linear solution

Background solution




We decompose the non-linear solution into eigen functions in linear theory.

$\delta W(t,z)=\sum_n c_n(t) e_n(z)$

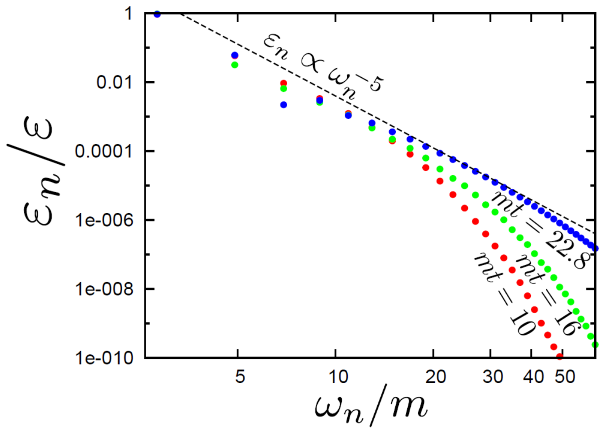
Energy contribution from n-th mode

$\varepsilon_n(t)=1/2\,(c_n^2+\omega_n^2 c_n^2)$

$\omega_n^2=4(n+1)(n+2)$

# Weak turbulence by mass quench

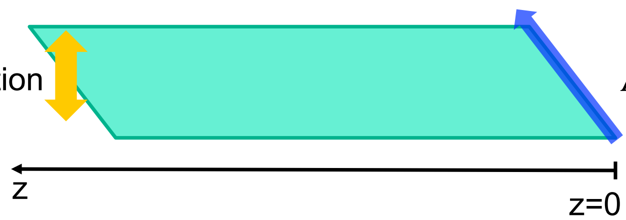
We found the weakly turbulence for the quark mass quench. $m(t)$




Electric field on the D7-brane in not essential for the turbulence.

(Giving a finite perturbation is important.)

# D7-brane with time-dependent electric field




Boundary condition:

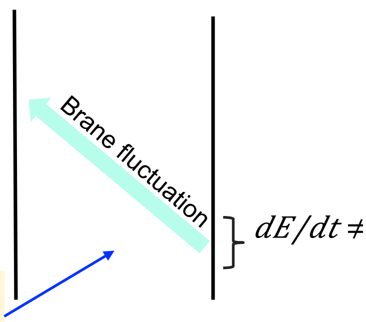
$F_{tx}|_{z=0} = -E(t)$

**Time-dependent electric field induces the brane motion.**

Numerical caluculation ( (1+1)-dim PDE)

We take pure AdS spacetime as background. (Zero temperature gluon plasma.)

$$ds^2 = \frac{1}{z^2}\left[-dt^2 + dz^2 + dx^2\right] + d\phi^2 + \cos^2\Phi\, d\Omega_3^2 + \sin^2\Phi\, d\psi^2$$




Before the quench, the brane is static.

# Hint from condensed matter physics.

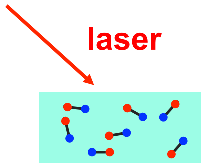
Yoshida&Asano,11
Mott,61,68
Zimmermann et al,78

By the injection of the laser, many exitons are excited in a Mott insulator.
**(exiton = bound state of electron and hole)**

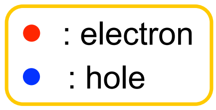
If we consider a electron in the crowd of exitons, its Coulomb force is screened.

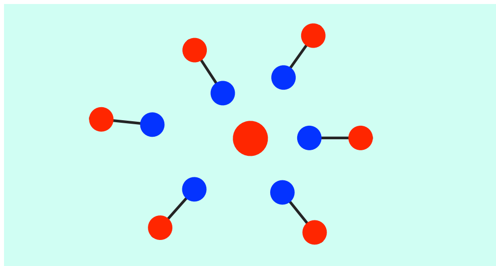
Since the binding force becomes small, exitons deconfine.

exiton-Mott transition



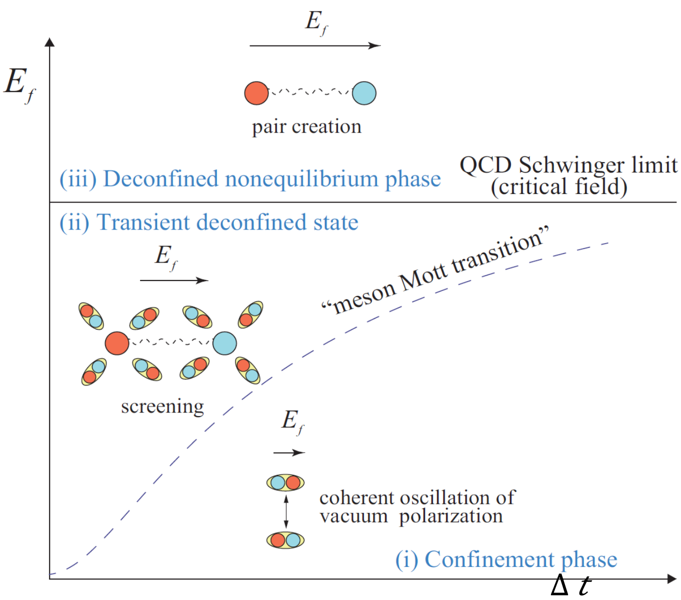

**Mott insulator**



Similar mechanism may be working in SQCD.

# Meson-Mott transition

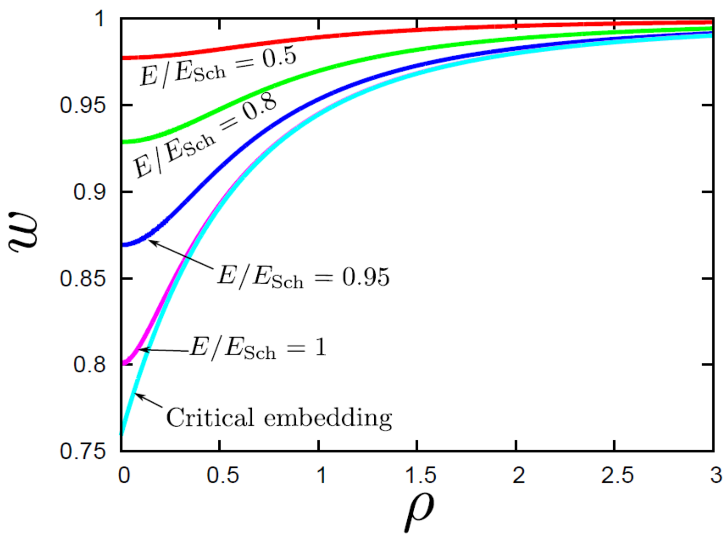


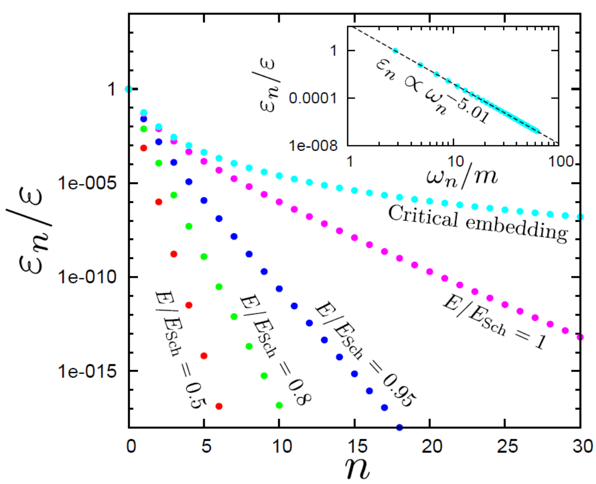

The strong interaction is screened by the crowd of mesons.

Deconfinement

**meson-Mott transition**

# Quasi-static analysis



“Brane-Antibrane and Closed Superstrings at Finite Temperature in the Framework of Thermo Field Dynamics”

Kenji Hotta

[JGRG24(2014)111010]

# Brane-Antibrane and Closed Superstrings at Finite Temperature in the Framework of Thermo Field Dynamics

arXiv:1411.xxxx

$+\alpha$

Hokkaido Univ.

Kenji Hotta

# 1. Introduction

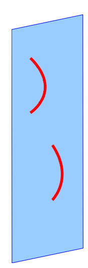
- Superstring

  1-dim. extended object in 10-dim. spacetime (bulk)


- D$p$ -brane

  p-dim. extended object in 10-dim. spacetime

  A hypersurface the ends of open strings can attach to



## Hagedorn Temperature (type II)

maximum temperature for perturbative strings

A single energetic string captures most of the energy.

$$d_n \sim e^{2\pi\sqrt{2n}}$$
$$\Omega(E) \sim e^{\beta_H E}$$
$$Z(\beta) = \int_0^\infty dE\ \Omega(E)\ e^{-\beta E}$$

$$\beta_H \equiv \frac{1}{T_H} = 2\pi\sqrt{2\alpha'}$$

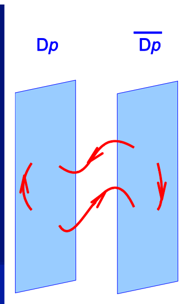
$Z(\beta) \to \infty$ for $\beta < \beta_H$

## Dp-$\overline{\mathrm{D}p}$ pairs (type II)

unstable at zero temperature

open string tachyon → tachyon potential

Sen's conjecture potential height=brane tension




## Brane-antibrane Pair Creation Transition Hotta

finite temperature system of Dp-$\overline{\mathrm{D}p}$ pairs

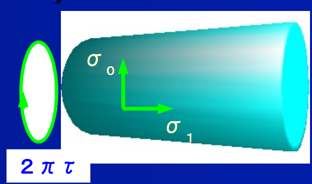
↓ based on Matsubara formalism and on BSFT

1-loop (cylinder world sheet)

Conformal invariance is broken by the boundary terms.

ambiguity in choosing the Weyl factors

→ cylinder boundary action Andreev-Oft




finite temperature effective potential

↓

D9-$\overline{\mathrm{D}9}$ pairs become stable near the Hagedorn temperature.

## ■ Thermo Field Dynamics (TFD) Takahashi-Umezawa

statistical average

$$\langle A \rangle = Z^{-1}(\beta) \sum_n \langle n|\hat{A}|n\rangle e^{-\beta E_n}$$

We can represent it as

$$\langle A \rangle = \left\langle 0(\beta) \middle| \hat{A} \middle| 0(\beta) \right\rangle$$

by introducing a fictitious copy of the system.

$$|0(\beta)\rangle = Z^{-\frac{1}{2}}(\beta) \sum_n e^{-\frac{\beta E_n}{2}} |n, \tilde{n}\rangle$$ thermal vacuum state

$$|n, \tilde{n}\rangle = |n\rangle \otimes |\tilde{n}\rangle$$

The fictitious state is interpreted as `hole' state.

We cannot represent it as

$$|0(\beta)\rangle = \sum_n |n\rangle f_n(\beta)$$

for ordinary number $f_n(\beta)$, since

$$f_n^*(\beta) f_m(\beta) = Z^{-1}(\beta) e^{-\beta E_n} \delta_{nm}$$

cannot be satisfied.

Hawking-Unruh effect can be described by TFD.
It is expected that TFD is available to non-equilibrium system.

(real time formalism)

TFD has been applied to string theory

| | |
|---|---|
| string field theory | Leblanc |
| D-brane | Vancea, Cantcheff, etc. |
| closed bosonic string | Abdalla-Gadelha-Nedel |
| AdS background | Grada-Vancea, etc. |
| pp-wave background | Nedel-Abdalla-Gadelha, etc. |

At the lowest order we do not use one-loop amplitude.

→ There is no problem of the choice of Weyl factors.

finite temperature system of D*p*-D*p* and closed superstring based on TFD?

# Contents

1. Introduction ✓
2. Brane-antibrane Pair in TFD
3. Closed Superstring in TFD
4. Application to Cosmology
5. Conclusion and Discussion

# 2. Brane-antibrane Pair in TFD

- Light-Cone Momentum

We consider a single first quantized string.

light-cone momentum

$$\begin{aligned} p^+ &= p^0 + p^1 \\ p^- &= p^0 - p^1 \end{aligned}$$

$$p^0 = \frac{1}{2}(p^+ + p^-)$$

$$p^+ p^- - |\boldsymbol{p}|^2 - M^2 = 0$$

$$p^- = \frac{|\boldsymbol{p}|^2 + M^2}{p^+}$$

partition function for a single string

$$\begin{aligned} Z_1(\beta) &= \mathrm{Tr}\ \exp\left(-\beta p^0\right) = \mathrm{Tr}\ \exp\left[-\frac{1}{2}\,\beta(p^+ + p^-)\right] \\ &= \mathrm{Tr}\ \exp\left[-\frac{1}{2}\,\beta\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2}{p^+}\right)\right] \end{aligned}$$

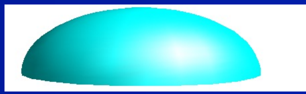
- **BSFT** (Boundary String Field Theory) (BV formalism)

  solution of classical master eq. (superstring)

$$S_{eff} = Z$$

$S_{eff}$ : effective action $Z$ : 2-dim. partition function

$$S_2 = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\ \partial_a X_\mu \partial^a X^\mu + \int_{\partial\Sigma} d\tau |T|^2 + \cdots$$

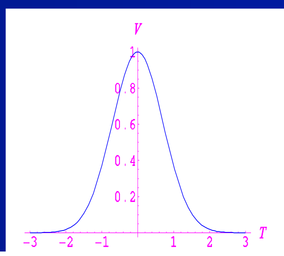


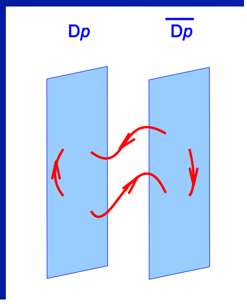
- **Disk** (tree level tachyon potential)

$$V(T) = 2\tau_p v_p \exp(-8|T|^2), \qquad \tau_p = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}} g_s}$$

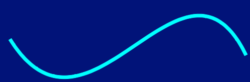
$T$: complex scalar field $\tau_p$ : tension of D*p*-brane

$g_s = e^\phi$ : coupling of strings $v_p$ : *p*-dim. volume




- **Mass Spectrum**



We consider an open string

on <u>a Brane-antibrane pair.</u>

mass spectrum

$$M_{NS}{}^2 = \frac{1}{\alpha'}\left(N_B + N_{NS} + 2|T|^2 - \frac{1}{2}\right) \quad \text{space time boson}$$

$$M_R{}^2 = \frac{1}{\alpha'}\left(N_B + N_R + 2|T|^2\right) \quad \text{space time fermion}$$

number ops.

$$N_B = \sum_{l=1}^{\infty}\sum_{I=1}^{8} \alpha_{-l}^I \alpha_l^I \quad \text{oscillation mode of world sheet boson}$$

$$N_{NS} = \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=1}^{8} r b_{-r}^I b_r^I \quad \text{oscillation mode of world sheet fermion (NS b. c)}$$

$$N_R = \sum_{m=1}^{\infty}\sum_{I=1}^{8} m d_{-m}^I d_m^I \quad \text{oscillation mode of world sheet fermion (R b. c)}$$

We will show only the NS mode case.

## Bogoliubov Transformation

generator of Bogoliubov tr.

$$\mathcal{G}_{1NS} = \mathcal{G}_B + \mathcal{G}_{NS}$$

$$\mathcal{G}_B = i\sum_{l=1}^{\infty}\frac{1}{l}\,\theta_l\left(\alpha_{-l}\cdot\tilde{\alpha}_{-l} - \tilde{\alpha}_l\cdot\alpha_l\right)$$

$$\mathcal{G}_{NS} = i\sum_{r=\frac{1}{2}}^{\infty}\theta_r\left(b_{-r}\cdot\tilde{b}_{-r} - \tilde{b}_r\cdot b_r\right)$$

$$\tanh\theta_l = \exp\left(-\frac{\beta l}{4\alpha' p^+}\right)$$

$$\tan\theta_r = \exp\left(-\frac{\beta r}{4\alpha' p^+}\right)$$

## Thermal Vacuum State

thermal vacuum state for a single string

$$\begin{aligned}|0_{1NS}(\theta)\rangle &\equiv e^{-iG_{1NS}}|0\rangle\rangle\left|p^+\right\rangle|\boldsymbol{p}\rangle\\ &= \prod_{l=1}^{\infty}\left\{\left(\frac{1}{\cosh(\theta_l)}\right)^8\exp\left[\frac{1}{l}\tanh(\theta_l)\alpha_{-l}\cdot\tilde{\alpha}_{-l}\right]\right\}\\ &\quad\times\prod_{r=\frac{1}{2}}^{\infty}\left\{(\cos(\theta_r))^8\exp\left[\tan(\theta_r)b_{-r}\cdot\tilde{b}_{-r}\right]\right\}|0\rangle\rangle\left|p^+\right\rangle|\boldsymbol{p}\rangle\end{aligned}$$

$$\alpha_l|0\rangle\rangle = b_r|0\rangle\rangle = \tilde{\alpha}_l|0\rangle\rangle = \tilde{b}_r|0\rangle\rangle = 0 \qquad \text{for positive } l, r$$

## Free Energy for a Single String

$$F_{1NS}(\theta) = \left\langle 0_{1NS}(\theta)\right|\left(H_{1NS} - \frac{1}{\beta}K_{1NS}\right)\left|0_{1NS}(\theta)\right\rangle$$

Hamiltonian

$$H_{1NS} = \frac{1}{2}\left(p^+ + \frac{|\boldsymbol{p}|^2 + M_{NS}^2}{p^+}\right)$$

entropy

$$\begin{aligned}K_{1NS} &= -\sum_{l=1}^{\infty}\frac{1}{l}\left\{\alpha_{-l}\cdot\alpha_l\ln\sinh^2\theta_l - \alpha_l\cdot\alpha_{-l}\ln\cosh^2\theta_l\right\}\\ &\quad-\sum_{r=\frac{1}{2}}^{\infty}\left\{b_{-r}\cdot b_r\ln\sin^2\theta_r + b_r\cdot b_{-r}\ln\cos^2\theta_r\right\}\end{aligned}$$

$$\begin{aligned}F_{1NS}(\beta) &= \frac{1}{2}\left(p^+ + \frac{|\boldsymbol{p}|^2}{p^+}\right) + \frac{|T|^2}{\alpha' p^+} + \frac{8}{\beta}\sum_{l=1}^{\infty}\ln\left[1 - \exp\left(-\frac{\beta l}{2\alpha' p^+}\right)\right]\\ &\quad - \frac{1}{4\alpha' p^+} - \frac{8}{\beta}\sum_{r=\frac{1}{2}}^{\infty}\ln\left[1 + \exp\left(-\frac{\beta r}{2\alpha' p^+}\right)\right]\end{aligned}$$

This is not useful for analysis of thermodynamical system of strings.

free energy for a single string

→ partition function for a single string → free energy for multiple strings (string gas)

## Partition Function for a Single String

$$Z_{1NS}(\beta) = \frac{v_p}{(2\pi)^p} \int_0^\infty dp^+ \int_{-\infty}^{\infty} d^p p \ \exp\left(-\beta F_{1NS}\right)$$

$$\tau \equiv \frac{2\pi\beta}{\beta_H{}^2 p^+} = \frac{\beta}{4\pi\alpha' p^+}, \quad \beta_H \ = \ 2\pi\sqrt{2\alpha'}$$

$$Z_{1NS}(\beta) \ = \ \frac{16\pi^4 \beta v_p}{\beta_H{}^{p+1}} \int_0^\infty \frac{d\tau}{\tau} \ \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2} \left\{\frac{\vartheta_3(0|i\tau)}{\vartheta_1{}'(0|i\tau)}\right\}^4 \exp\left(-\frac{\pi\beta^2}{\beta_H{}^2\tau}\right)$$

## Free Energy for Multiple Strings

Free energy for multiple strings can be obtained from the following eq.

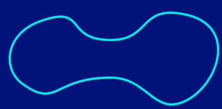
$$F(\beta) = -\sum_{w=1}^{\infty} \frac{1}{\beta w}\left\{Z_{1NS}(\beta w) - (-1)^w Z_{1R}(\beta w)\right\}$$

$$F(\beta) \ = \ -\frac{16\pi^4 v_p}{\beta_H{}^{p+1}} \int_0^\infty \frac{d\tau}{\tau} \ \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2\tau}$$
$$\times \left[\left(\frac{\vartheta_3(0|i\tau)}{\vartheta_1{}'(0|i\tau)}\right)^4 \left\{\vartheta_3\left(0\left|\frac{i\beta^2}{\beta_H{}^2\tau}\right.\right) - 1\right\} - \left(\frac{\vartheta_2(0|i\tau)}{\vartheta_1{}'(0|i\tau)}\right)^4 \left\{\vartheta_4\left(0\left|\frac{i\beta^2}{\beta_H{}^2\tau}\right.\right) - 1\right\}\right]$$

This equals to the free energy based on Matsubara formalism.

This implies that our choice of Weyl factors in the case of Matsubara formalism is quite natural.

# 3. Closed Superstring in TFD



## First Quantized String

mass spectrum

$$M_{NSNS}{}^2 \ = \ \frac{2}{\alpha'}\left(N_B + N_{NS} + \overline{N}_B + \overline{N}_{NS} - 1\right)$$
$$M_{RR}{}^2 \ = \ \frac{2}{\alpha'}\left(N_B + N_R + \overline{N}_B + \overline{N}_R\right)$$

space time boson

$$M_{NSR}{}^2 \ = \ \frac{2}{\alpha'}\left(N_B + N_{NS} + \overline{N}_B + \overline{N}_R - \frac{1}{2}\right)$$
$$M_{RNS}{}^2 \ = \ \frac{2}{\alpha'}\left(N_B + N_R + \overline{N}_B + \overline{N}_{NS} - \frac{1}{2}\right)$$

space time fermion

The calculation is more complicated than the brane-antibrane case.
(GSO projection, level-matching condition)

We can compute free energy for a single string based on TFD.

We can reproduce the free energy for multiple strings based on Matsubara formalism.

## Second Quantized String

Closed superstring field theory is not well-established.
However, we are considering ideal gas of string.
In this case, we can treat closed strings as a collection of bosons and fermions.

generator of Bogoliubov tr.

$$G_{NSNS} = i\sum_{\alpha} \theta_{NSNS,\alpha}\left(A^\dagger_{NSNS,\alpha}\tilde{A}^\dagger_{NSNS,\alpha} - \tilde{A}_{NSNS,\alpha}A_{NSNS,\alpha}\right)$$

$$\alpha = \{p^+, \boldsymbol{p}, N_B, N_{NS}, \overline{N}_B, \overline{N}_{NS}\}$$

thermal vacuum state for multiple strings

$$\begin{aligned}
|0_{NSNS}(\theta)\rangle\rangle &\equiv \mathcal{P}Pe^{-iG_{NSNS}}|0\rangle\rangle \\
&= \mathcal{P}P\exp\left[\sum_{\alpha}\theta_{NSNS,\alpha}\left(A^\dagger_{NSNS,\alpha}\tilde{A}^\dagger_{NSNS,\alpha} - \tilde{A}_{NSNS,\alpha}A_{NSNS,\alpha}\right)\right]|0\rangle\rangle \\
&= \prod_{\alpha}\mathcal{P}_{NSNS,\alpha}P_{NSNS,\alpha} \\
&\qquad \left\{\frac{1}{\cosh(\theta_{NSNS,\alpha})}\exp\left[\tanh(\theta_{NSNS,\alpha})A^\dagger_{NSNS,\alpha}\tilde{A}^\dagger_{NSNS,\alpha}\right]\right\}|0\rangle\rangle
\end{aligned}$$

## Free Energy for Multiple NS-NS String

$$F_{NSNS}(\theta) = \left\langle 0_{NSNS}(\theta)\middle|\left(H_{NSNS} - \frac{1}{\beta}K_{NSNS}\right)\middle|0_{NSNS}(\theta)\right\rangle$$

Hamiltonian

$$H_{NSNS} = \frac{1}{2}\sum_{\alpha}\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2_{NSNS}}{p^+}\right)A^\dagger_{NSNS,\alpha}A_{NSNS,\alpha}$$

entropy

$$\begin{aligned}
K_{NSNS} &= -\sum_{\alpha}\left(A^\dagger_{NSNS,\alpha}A_{NSNS,\alpha}\ln\sinh^2\theta_{NSNS,\alpha}\right. \\
&\qquad \left. -A_{NSNS,\alpha}A^\dagger_{NSNS,\alpha}\ln\cosh^2\theta_{NSNS,\alpha}\right)
\end{aligned}$$

level-matching condition

$$\mathcal{P}_{NSNS,\alpha} = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \ \exp\left[2\pi i\tau_1\left(N_B + N_{NS} - \overline{N}_B - \overline{N}_{NS}\right)\right]$$

GSO projection

$$P_{NSNS,\alpha} = \frac{1}{4}\left(1 + G_{n_r}\right)\left(1 + \overline{G}_{\overline{n}_r}\right)$$

$$G_{n_r} = -(-1)^{\sum_{r=\frac{1}{2}}^{\infty} n_r}$$

## Free Energy for Multiple Strings

Summing over the free energy for all sectors, we obtain

$$F(\beta) = F_{NSNS}(\beta) + F_{RR}(\beta) + F_{NSR}(\beta) + F_{RNS}(\beta)$$

$$\begin{aligned} F(\beta) = & -\frac{8(2\pi)^8 v_9}{{\beta_H}^{10}} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \int_0^\infty d\tau_2 \frac{1}{\tau_2^6} \frac{1}{|\vartheta_1{}'(0|\tau)|^8} \\ & \times \Bigg[ \left\{ \left(\vartheta_3^4 - \vartheta_4^4\right)\left(\bar\vartheta_3^4 - \bar\vartheta_4^4\right) + \vartheta_2^4 \bar\vartheta_2^4 \right\}(0|\tau) \sum_{w=1}^{\infty} \exp\left(-\frac{2\pi w^2 \beta^2}{{\beta_H}^2 \tau_2}\right) \\ & - \left\{ \left(\vartheta_3^4 - \vartheta_4^4\right)\bar\vartheta_2^4 + \vartheta_2^4 \left(\bar\vartheta_3^4 - \bar\vartheta_4^4\right) \right\}(0|\tau) \sum_{w=1}^{\infty} (-1)^w \exp\left(-\frac{2\pi w^2 \beta^2}{{\beta_H}^2 \tau_2}\right) \Bigg] \end{aligned}$$

This equals to the free energy
based on Matsubara formalism.

# 4. Application to Cosmology

## Brane World Cosmology

If the universe is sufficiently hot,
the D9-$\overline{\text{D9}}$ pairs are stable.

All the lower-dim. D-branes in type IIB string theory are realized as topological defects through tachyon condensation from D9-$\overline{\text{D9}}$ pairs.

↓

Various kinds of branes may form
through tachyon condensation.

`Brane World Formation Scenario'

cf) homogeneous and isotropic tachyon condensation
Hotta 2006

# 5. Conclusion and Discussion

- Brane-antibrane in TFD

  We computed thermal vacuum state and partition function for a single string on a Brane-antibrane pair based on TFD.

  The free energy for multiple strings agrees with that based on the Matsubara formalism.

  There are no problem of the choice of the Weyl factors.

- Closed Superstring Gas in TFD

  We computed thermal vacuum state and free energy for multiple closed superstrings based on TFD.

  The free energy for multiple strings agrees with that based on the Matsubara formalism.

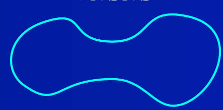
- String Field Theory

  We need to use second quantized string field theory in order to obtain the thermal vacuum state for multiple open strings.

- D-brane boundary state of closed string  cf) Cantcheff

  The thermal vacuum state is reminiscent of the D-brane boundary state of a closed string.

$$|B9_{mat},\eta\rangle_{NSNS} = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\,\alpha_{-n}\cdot\tilde{\alpha}_{-n} + i\eta\sum_{u>0}\psi_{-u}\cdot\tilde{\psi}_{-u}\right]|B9_{mat},\eta\rangle^{(0)}_{NSNS}$$



- Hawking-Unruh Effect

  closed strings in curved spacetime

  Unruh Effect in bosonic open string theory  Hata-Oda-Yahikozawa

  black hole firewall  Almheiri-Marolf-Polchinski-Sully

  Planck solid model  Hotta

“An upper bound on the number of Killing-Yano tensors”

Tsuyoshi Houri

[JGRG24(2014)111011]

JGRG24 @ IPMU, Tokyo, 10 November 2014

# An upper bound on the number of Killing-Yano tensors

Tsuyoshi Houri
(Kobe University, Japan)

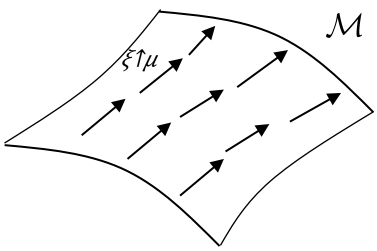
with Yukinori Yasui (Osaka City U., Japan)

Ref. arXiv:1410.1023[gr-qc], to appear in CQG

# Spacetime symmetry

- Killing vector fields:

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$




- Killing-Yano tensors:

A generalisation of Killing vector fields

$$\nabla_\mu \xi_{\nu_1 \nu_2 \dots \nu_n} + \nabla_{\nu_1} \xi_{\mu \nu_2 \dots \nu_n} = 0$$

$$\xi_{[\mu_1 \mu_2 \dots \mu_n]} = \xi_{\mu_1 \mu_2 \dots \mu_n}$$

# Why Killing-Yano tensors?

- Separability

  Hamilton-Jacobi equations for geodesics,
  Klein-Gordon and Dirac equations

- Exact solutions

  Stationary, axially symmetric black holes with
  spherical horizon topology

# The purpose of this talk

To show a simple method for finding Killing-Yano tensors for a given metric.

*including Killing vector fields

# The method

1. To compute an upper bound on the number of KY tensors.
2. To get an ansatz for solving the KY equations.

Any metric in Any coordinates,
Any dimensions, Any rank

# THE METHOD

# Killing vector fields

## Killing equation

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$

- $\nabla_\mu \xi_\nu = L_{\mu\nu}$ ,   $L_{\mu\nu} = \nabla_{[\mu} \xi_{\nu]}$
- $\nabla_\mu L_{\nu\rho} = -R_{\nu\rho\mu}{}^{\sigma} \xi_\sigma$


- $\nabla_\mu \xi_\nu = L_{\mu\nu}$ ,   $L_{\mu\nu} = \nabla_{[\mu} \xi_{\nu]}$
- $\nabla_\mu L_{\nu\rho} = -R_{\nu\rho\mu}{}^{\sigma} \xi_\sigma$


- Killing connection

$$D_\mu \xi_A \equiv \nabla_\mu \begin{pmatrix} \xi_\nu \\ L_{\nu\rho} \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -R_{\nu\rho\mu}{}^{\sigma} & 0 \end{pmatrix} \begin{pmatrix} \xi_\sigma \\ L_{\mu\nu} \end{pmatrix}$$

- $\xi_A = (\xi_\mu, L_{\mu\nu})$ : a section of $E^1 \equiv \Lambda^1(M) \oplus \Lambda^2(M)$
- $D_\mu$ : a connection on $E^1$

$$D_\mu \xi_A = 0$$

# The key

**Killing vector fields $\Leftrightarrow$ Parallel sections of $E^1$**

- The number of parallel sections of $E^1$ is bound by the rank of $E^1$, which is given by

$$N = \binom{n}{1} + \binom{n}{2} = n(n+1)/2 \quad .$$

Hence, the maximum number of Killing vector fields is given by $n(n+1)/2$.

# Curvature conditions

$$D_\mu \xi_A = 0$$

$$R_{\mu\nu A}{}^{B}\, \xi_B \equiv (D_\mu D_\nu - D_\nu D_\mu)\xi_A = 0$$

- The number of the solutions provides an upper bound on the number of Killing vector fields.
- The solutions themselves can be used as an ansatz for solving Killing equation.

# Killing-Yano tensors of rank p

- Killing connection [Semmelmann 2002]

**Rank-p KY tensors** $\Leftrightarrow$ **Parallel sections of** $E^p=\Lambda^p(M)\oplus\Lambda^{p+1}(M)$

$$D_\mu \xi_A=0 \qquad \xi_A=(\xi_{\mu_1\ldots\mu_p},\ L_{\mu_1\ldots\mu_{p+1}})$$

- The maximal number

$$N=\binom{n}{p}+\binom{n}{p+1}=\binom{n+1}{p+1}$$

- Curvature conditions [TH-Yasui 2014]

$$R_{\mu\nu A}{}^{B}\,\xi_B\equiv(D_\mu D_\nu-D_\nu D_\mu)\xi_A=0$$

# Curvature conditions on Killing-Yano tensors

[TH-Yasui 2014]

- $\mathcal{R}(X,Y):\Gamma(E^p)\to\Gamma(E^p)$, $\qquad E^p=\Lambda^p(M)\oplus\Lambda^{p+1}(M)$

$$\mathcal{R}(X,Y)=\begin{pmatrix} N_{11}(X,Y) & 0 \\ N_{21}(X,Y) & N_{22}(X,Y)\end{pmatrix}$$

- $N_{11}(X,Y):\Lambda^p(M)\to\Lambda^p(M)$

  $N_{11}(X,Y)=R(X,Y)+\frac{1}{p}\left(i(X)\wedge R^+(Y)-i(Y)\wedge R^+(X)\right)$

- $N_{21}(X,Y):\Lambda^p(M)\to\Lambda^{p+1}(M)$

  $N_{21}(X,Y)=-\frac{p+1}{p}\left((\nabla_X R)^+(Y)-(\nabla_Y R)^+(X)\right)$

- $N_{22}(X,Y):\Lambda^{p+1}(M)\to\Lambda^{p+1}(M)$

  $N_{22}(X,Y)=R(X,Y)+\frac{1}{p}\left(R^+(X)(i(Y))-R^+(Y)(i(X))\right)$

# RESULTS

## Our package of *Mathematica*

INPUT:
- Metric data

FUNCTIONS:
- Compute the Killing curvature
- Solve the curvature conditions

*available at the URL: http://www.research.kobe-u.ac.jp/fsci-pacos/KY_upperbound/

# Symmetry of Kerr spacetime

Kerr metric

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta\, d\phi)^2 + \frac{\sin^2\theta}{\Sigma}(a\,dt - (r^2+a^2)d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma\, d\theta^2$$

$$\Delta = r^2 - 2Mr + a^2\,, \quad \Sigma = r^2 + a^2\cos^2\theta$$

- Two Killing vector fields: $\partial/\partial t$ and $\partial/\partial\phi$
- One rank-2 Killing-Yano tensor:

$$f = a\cos\theta\, dr \wedge (dt - a\sin^2\theta\, d\phi) + r\sin\theta\, d\theta \wedge (a\,dt - (r^2+a^2)d\phi)$$

Our result:

Kerr metric admits exactly two Killing vector fields, one rank-2 and no rank-3 KY tensors.

# The number of rank-p KY tensors

| 4D metrics | $p=1$ | $p=2$ | $p=3$ |
|---|---|---|---|
| Maximally symmetric | 10 | 10 | 5 |
| Plebanski-Demianski | 2 | 0 | 0 |
| Kerr | 2 | 1 | 0 |
| Schwazschild | 4 | 1 | 0 |
| FLRW | 6 | 4 | 1 |
| Self-dual Taub-NUT | 4 | 4 | 0 |
| Eguchi-Hanson | 4 | 3 | 0 |

# The number of rank-p KY tensors

| 5D metrics | $p=1$ | $p=2$ | $p=3$ | $p=4$ |
|---|---|---|---|---|
| Maximally symmetric | 15 | 20 | 15 | 6 |
| Myers-Perry | 3 | 0 | 1 | 0 |
| Emparan-Reall | 3 | 0 | 0 | 0 |
| Kerr string | 3 | 1 | 0 | 1 |

# SUMMARY & FUTURE WORKS

# Summary

- Killing connection

**Rank-p KY tensors** $\Leftrightarrow$ **Parallel sections of** $E^p = \Lambda^p(M) \oplus \Lambda^{p+1}(M)$

$$D_\mu \xi_A = 0 \qquad \xi_A = (\xi_{\mu_1 \ldots \mu_p}, L_{\mu_1 \ldots \mu_{p+1}})$$

- The maximal number

$$N = \binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1}$$

- Curvature conditions

$$R_{\mu\nu A}{}^{B} \xi_B \equiv (D_\mu D_\nu - D_\nu D_\mu)\xi_A = 0$$

# Future works

| Vector fields | Killing | Conformal Killing |
|---|---|---|
| symmetric | Killing-Stackel<br>**Stackel 1895** | Conformal Killing-Stackel |
| anti-symmetric | Killing-Yano<br>**Yano 1952** | Conformal Killing-Yano<br>**Tachibana 1969, Kashiwada 1968** |

“Invariant quantities in the scalar-tensor theories of gravitation”

Laur Jarv

[JGRG24(2014)111012]

The 24th Workshop on General Relativity and Gravitation (JGRG24)
Kavli IPMU, University of Tokyo, 10-14 November 2014

# Invariant quantities in scalar-tensor theories of gravitation

**Laur Järv**
University of Tartu, Estonia

LJ, Piret Kuusk, Margus Saal, Ott Vilson
arXiv:1411.1947

## Motivation

The action of scalar-tensor gravity (STG) is invariant under

- conformal rescaling of the metric $g_{\mu\nu} = e^{2\bar{\gamma}(\bar{\Phi})}\bar{g}_{\mu\nu}$ ,
- reparametrization of the scalar field $\Phi = \bar{f}(\bar{\Phi})$.

Aspects of the conformal frame issue:

- Physical : which frame is observed, or rescaling of units?
- Mathematical : classical equations equivalent, but what about cosmological perturbations, quantum corrections?

A possible interpretation (e.g. Kamenshchik, Steinwachs 1408.5769)

- Changing conformal frame and parametrization ⇔ a change of coordinates in some abstract generalized field space
- Discrepancies ⇔ theory has not been formulated in a covariant way with respect to that abstract space

A possible way to proceed

- Introduce conformally invariant variables (Catena et al astro-ph/0604492; Postma, Volponi 1407.6874)

## Outline

Scalar-tensor gravity action (no dervivative couplings)

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{A}(\Phi)R - \mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - 2\ell^{-2}\mathcal{V}(\Phi)\right\} + S_{matter}\left[e^{2\alpha(\Phi)}g_{\mu\nu}, \chi\right] . \quad (1)$$

What we do (arXiv:1411.1947):

- Introduce quantities invariant under conformal rescaling and scalar field redefinition,
- Write the field equations and action in terms of these invariants,
- Show how the observables are expressed in terms of invariants, e.g.
  - effective gravitational constant and PPN parameters,
  - fix point properties and periods of oscillation of the scalar field cosmological solutions.
- The scalar field value itself has no physical meaning (in a generic parametrization), only the values of invariant combinations are observable.



## Scalar-tensor gravity (STG)

STG action in a general form, one scalar field $\Phi$, no derivative couplings,

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{A}(\Phi)R - \mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - 2\ell^{-2}\mathcal{V}(\Phi)\right\} + S_{matter}\left[e^{2\alpha(\Phi)}g_{\mu\nu}, \chi\right] . \quad (2)$$

- Four arbitrary functions $\mathcal{A}(\Phi)$, $\mathcal{B}(\Phi)$, $\mathcal{V}(\Phi)$, $e^{2\alpha(\Phi)}$.
- Two dimensionful constants $\kappa^2$, $\ell$ to make $\Phi$ dimensionless.
- Reasonable to assume

$$0 < \mathcal{A} < \infty\,, \qquad 0 < 2\bar{\mathcal{A}}\bar{\mathcal{B}} + 3\left(\bar{\mathcal{A}}'\right)^2\,, \quad (3)$$

$$0 \le \mathcal{V} < \infty\,, \qquad -\infty < \alpha < \infty\,. \quad (4)$$

# Parametrizations of scalar-tensor gravity (STG)

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{A}(\Phi)R - \mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - 2\ell^{-2}\mathcal{V}(\Phi)\right\} + S_m\left[e^{2\alpha(\Phi)}g_{\mu\nu},\chi\right]. \quad (5)$$

By conformal rescaling and scalar field redefinition can fix two functions to get different parametrizations, e.g.

- Jordan frame Brans-Dicke-Bergmann-Wagoner (JF BDBW)

$$\mathcal{A} = \Psi, \qquad \mathcal{B} = \frac{\omega(\Psi)}{\Psi}, \qquad \mathcal{V} = \mathcal{V}(\Psi), \qquad \alpha = 0, \qquad (6)$$

- Jordan frame Boisseau, Esposito-Farèse, Polarski and Starobinsky (JF BEPS)

$$\mathcal{A} = F(\phi), \qquad \mathcal{B} = 1, \qquad \mathcal{V} = \mathcal{V}(\Psi), \qquad \alpha = 0, \qquad (7)$$

- Einstein frame canonical parametrization (EF canonical)

$$\mathcal{A} = 1, \qquad \mathcal{B} = 2, \qquad \mathcal{V} = \mathcal{V}(\varphi), \qquad \alpha = \alpha(\varphi) \qquad (8)$$



# Transformation rules

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{A}(\Phi)R - \mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - 2\ell^{-2}\mathcal{V}(\Phi)\right\} + S_m\left[e^{2\alpha(\Phi)}g_{\mu\nu},\chi\right]. \quad (9)$$

- Under conformal rescaling and scalar field reparametrization

$$g_{\mu\nu} = e^{2\bar{\gamma}(\bar{\Phi})}\bar{g}_{\mu\nu}, \qquad \Phi = \bar{f}(\bar{\Phi}), \qquad (10)$$

  the functions transform as

$$\begin{aligned}
\bar{\mathcal{A}}(\bar{\Phi}) &= e^{2\bar{\gamma}(\bar{\Phi})}\mathcal{A}\left(\bar{f}(\bar{\Phi})\right), \\
\bar{\mathcal{B}}(\bar{\Phi}) &= e^{2\bar{\gamma}(\bar{\Phi})}\left(\left(\bar{f}'\right)^2\mathcal{B}\left(\bar{f}(\bar{\Phi})\right) - 6\left(\bar{\gamma}'\right)^2\mathcal{A}\left(\bar{f}(\bar{\Phi})\right) - 6\bar{\gamma}'\bar{f}'\mathcal{A}'\right), \\
\bar{\mathcal{V}}(\bar{\Phi}) &= e^{4\bar{\gamma}(\bar{\Phi})}\mathcal{V}\left(\bar{f}(\bar{\Phi})\right), \\
\bar{\alpha}(\bar{\Phi}) &= \alpha\left(\bar{f}(\bar{\Phi})\right) + \bar{\gamma}(\bar{\Phi}). \qquad (11)
\end{aligned}$$

- Use these rules to find combinations which remain invariant.

## Basic invariants

Three basic independent quantitites, invariant under rescaling and reparametrization:

$$\mathcal{I}_1(\Phi) \equiv \frac{e^{2\alpha(\Phi)}}{\mathcal{A}(\Phi)}, \tag{12}$$

$$\mathcal{I}_2(\Phi) \equiv \frac{\mathcal{V}(\Phi)}{(\mathcal{A}(\Phi))^2}, \tag{13}$$

$$\mathcal{I}_3(\Phi) \equiv \pm\int\left(\frac{2\bar{\mathcal{A}}\bar{\mathcal{B}}+3\left(\bar{\mathcal{A}}'\right)^2}{4\bar{\mathcal{A}}^2}\right)^{\frac{1}{2}} d\Phi\,. \tag{14}$$

- $\mathcal{I}_1(\Phi) \not\equiv \mathit{const}$ means nonminimal coupling
- $\mathcal{I}_2(\Phi) \not\equiv 0$ means nonvanishing potential
- $\left(\mathcal{I}_3'(\Phi)\right)^2 = \frac{2\omega(\Psi)+3}{4\Psi^2}$ frequently appears in formulas



## More invariants

- Can define infinitely many more invariants using

$$\mathcal{I}_i \equiv \mathfrak{f}(\mathcal{I}_j)\,, \qquad \mathcal{I}_m \equiv \frac{\mathcal{I}_k'}{\mathcal{I}_l'}\,, \qquad \mathcal{I}_r \equiv \int \mathcal{I}_n \mathcal{I}_p' d\Phi\,. \tag{15}$$

- For example

$$\mathcal{I}_4 \equiv \frac{\mathcal{I}_2}{\mathcal{I}_1^2} = \frac{\mathcal{V}}{e^{4\alpha}}, \tag{16}$$

$$\mathcal{I}_5 \equiv \left(\frac{\mathcal{I}_1'}{2\mathcal{I}_1\mathcal{I}_3'}\right)^2 = \frac{(2\alpha'\mathcal{A}-\mathcal{A}')^2}{2\mathcal{A}\mathcal{B}+3\left(\mathcal{A}'\right)^2}\,. \tag{17}$$

- Can also introduce an additional invariant object

$$\hat{g}_{\mu\nu} \equiv \mathcal{A}(\Phi) g_{\mu\nu}\,, \tag{18}$$

## Field equations and action in terms of invariants

Using $\hat{g}_{\mu\nu} \equiv \mathcal{A}(\Phi)g_{\mu\nu}$ can express

- Field equations

$$\hat{G}_{\mu\nu} + \hat{g}_{\mu\nu}\hat{g}^{\rho\sigma}\hat{\nabla}_\rho\mathcal{I}_3\hat{\nabla}_\sigma\mathcal{I}_3 - 2\hat{\nabla}_\mu\mathcal{I}_3\hat{\nabla}_\nu\mathcal{I}_3 + \ell^{-2}\hat{g}_{\mu\nu}\mathcal{I}_2 - \kappa^2\hat{T}_{\mu\nu} = 0\,, \tag{19}$$

$$\hat{\Box}\mathcal{I}_3 - \frac{1}{2\ell^2}\frac{d\mathcal{I}_2}{d\mathcal{I}_3} + \frac{\kappa^2}{4}\frac{d\ln\mathcal{I}_1}{d\mathcal{I}_3}\hat{T} = 0\,. \tag{20}$$

- Action

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\hat{g}}\left\{\hat{R} - 2\hat{g}^{\mu\nu}\hat{\nabla}_\mu\mathcal{I}_3\hat{\nabla}_\nu\mathcal{I}_3 - 2\ell^{-2}\mathcal{I}_2\right\} + S_m\left[\mathcal{I}_1\hat{g}_{\mu\nu}, \chi\right]\,. \tag{21}$$

taking $\mathcal{I}_1(\mathcal{I}_3)$ and $\mathcal{I}_2(\mathcal{I}_3)$.



## PPN parameters in terms of invariants

Translate results from JF BDBW parametrization for general $\omega(\Psi)$, $V(\Psi)$ (Hohmann, LJ, Kuusk, Randla 1309.0031) into invariants

$$G_{\text{eff}} = \mathcal{I}_1\left(1 + \mathcal{I}_5 e^{-m_\Phi r}\right)\,, \tag{22}$$

$$\gamma - 1 = -\frac{2e^{-m_\Phi r}}{G_{\text{eff}}}\mathcal{I}_1\mathcal{I}_5\,, \tag{23}$$

$$\beta - 1 = \frac{1}{2}\frac{\mathcal{I}_1^3\mathcal{I}_5}{G_{\text{eff}}^2}\frac{\mathcal{I}_5'}{\mathcal{I}_1'}e^{-2m_\Phi r} - \frac{m_\Phi r}{G_{\text{eff}}^2}\mathcal{I}_1^2\mathcal{I}_5\,\beta(r)\,, \tag{24}$$

$$m_\Phi = \frac{1}{\ell}\sqrt{\frac{\mathcal{I}_2''}{2\mathcal{I}_1\left(\mathcal{I}_3'\right)^2}}\,, \tag{25}$$

with conditions for asymptotic Minkowski background: $\mathcal{I}_2 = 0$, $\frac{\mathcal{I}_2'}{\mathcal{I}_1'} = 0$.

- PPN parameters manifestly invariant
- matches the calculation in EF canonical parametrization (Schärer et al 1410.7914)

# Scalar field fixed point in FLRW cosmology without matter

Flat FLRW cosmology without matter, scalar field equation
Hubble parameter substituted in ($\varepsilon = \pm 1$ expanding / contracting)

$$\frac{d^2}{d\hat{t}^2}\mathcal{I}_3 = -\varepsilon\sqrt{3\left(\frac{d}{d\hat{t}}\mathcal{I}_3\right)^2 + \frac{3}{\ell^2}\mathcal{I}_2}\,\frac{d}{d\hat{t}}\mathcal{I}_3 - \frac{1}{2\ell^2}\frac{d\mathcal{I}_2}{d\mathcal{I}_3}\,, \tag{26}$$

- Condition for a fixed point ( $\frac{d}{d\hat{t}}\mathcal{I}_3\big|_{\Phi_0} = 0$ and $\frac{d^2}{d\hat{t}^2}\mathcal{I}_3\Big|_{\Phi_0} = 0$) at $\Phi_0$:

$$\frac{\mathcal{I}_2'}{\mathcal{I}_3'}\bigg|_{\Phi_0} = 0\,. \tag{27}$$

- Linearize the equation around the fixed point, solve to get

$$\mathcal{I}_3(\hat{t}) = M_1 e^{\lambda_+^\varepsilon \hat{t}} + M_2 e^{\lambda_-^\varepsilon \hat{t}}\,, \tag{28}$$

where eigenvalues are

$$\lambda_\pm^\varepsilon = \frac{1}{2\ell}\left[-\varepsilon\sqrt{3\mathcal{I}_2} \pm \sqrt{3\mathcal{I}_2 - 2\frac{d^2\mathcal{I}_2}{d\mathcal{I}_3^2}}\,\right]_{\Phi_0}\,. \tag{29}$$



# Scalar field fixed point in FLRW cosmology without matter

- Fixed point condition $\frac{\mathcal{I}_2'}{\mathcal{I}_3'}\Big|_{\Phi_0} = 0$ can be satisfied in two ways

$$\Phi_\bullet \;:\; \mathcal{I}_2'|_{\Phi_\bullet} = 0\,, \quad \frac{1}{\mathcal{I}_3'}\Big|_{\Phi_\bullet} \neq 0\,, \qquad \Psi V' - 2V = 0$$

$$\Phi_\star \;:\; \frac{1}{\mathcal{I}_3'}\Big|_{\Phi_\star} = 0\,, \qquad \frac{1}{\omega} = 0\,.$$

- Taylor expand to express the solution in terms of the scalar field

$$\Phi(\hat{t}) - \Phi_0 = \pm\frac{1}{\mathcal{I}_3'}\bigg|_{\Phi_0}\mathcal{I}_3(\hat{t}) + \frac{1}{4}\left(\frac{1}{(\mathcal{I}_3')^2}\right)'\bigg|_{\Phi_0}\cdot\mathcal{I}_3^2(\hat{t})\,. \tag{30}$$

- For $\Phi_\bullet$ the solution is linear, but for $\Phi_\star$ nonlinear,

$$\Phi(\hat{t}) - \Phi_\star \approx \frac{1}{4}\left(\frac{1}{(\mathcal{I}_3')^2}\right)'\bigg|_{\Phi_\star}\left(M_1 e^{\lambda_+^\varepsilon \hat{t}} + M_2 e^{\lambda_-^\varepsilon \hat{t}}\right)^2\,. \tag{31}$$

- $\Phi_\bullet$ : JF BDBW Faraoni et al gr-qc/0605050v, EF Leon 0812.1013,
$\Phi_\star$ : JF BDBW LJ, Kuusk, Saal 1003.1686, 1006.1246.

## Summary and outlook

LJ, Piret Kuusk, Margus Saal, Ott Vilson 1411.1947

We studied general scalar-tensor gravity (without derivative couplings)

- Constructed quantites that are invariant under conformal rescaling and scalar field redefinition,
- Formulated the theory in terms of these invariant variables,
- Showed how observables like PPN parameters and qualitative features of scalar field cosmological solutions (convergence properties, periods of oscillation) are given in terms of the invariants,
- Explained a particular case where there is correspondence between the Einstein frame linear and Jordan frame nonlinear approximate solutions of the scalar field.

Outlook

- study cosmological perturbations and quantum corrections?
- generalize for theories with derivative couplings and disformal invariance (Horndeski and beyond)?

“Nonlinear mode-coupling of large-scale structure: validity of perturbation theory calculation”

by Atsushi Taruya

[JGRG24(2014)111013]

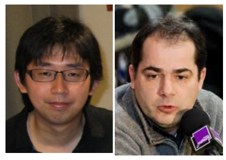
10 Nov. 2014
JGRG24 @ Kavli IPMU

# Nonlinear mode-coupling of large-scale structure :

## validity of perturbation theory calculation

Atsushi TARUYA (YITP)

In collaboration with
Takahiro NISHIMICHI、 Francis BERNARDEAU
(Institut d'Astrophysique de Paris)



# What we did

In the context of cosmological large-scale structure formation,

we characterize the nonlinear response of power spectrum to a small variation in linear counter part from N-body simulations:

**Mode coupling kernel**

$$K(k,q) = q\,\frac{\delta P_{\rm nl}(k)}{\delta P_0(q)}$$

Comparing it with perturbation theory (PT), we found

- ✓ Kernel is generically suppressed at UV domain, in contrast to PT
- ✓ Discrepancy with PT prediction appears even at low-k, where PT works very well

→ helps us to improve theoretical treatment of LSS

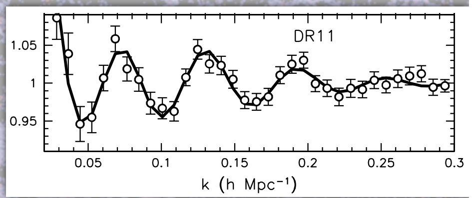
# Large-scale structure (LSS)

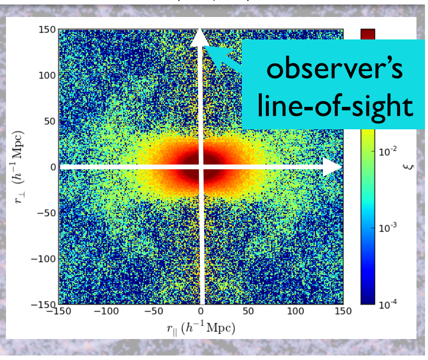
Spatial inhomogeneity of <u>mass distribution</u> at 1~10^3 Mpc

dark matter + baryon (galaxies)

- Precision cosmological probe
  - ✓ baryon acoustic oscillation (**BAO**): cosmic expansion
  - ✓ redshift-space distortions (**RSD**): growth of structure

  imprinted on power spectrum & correlation function




- Accurate theoretical template is needed

# Perturbation theory of LSS

A role of ***nonlinear gravity*** is crucial in characterizing LSS

- needs to be properly incorporated into theoretical template
- For large scales of our interest, nonlinearity is weak

**Perturbation theory (PT) approach**

Basic eqs.

$$\frac{\partial \delta}{\partial t}+\frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right]=0$$

$$\frac{\partial \vec{v}}{\partial t}+\frac{\dot{a}}{a}\vec{v}+\frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v}=-\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi=4\pi G\,\bar{\rho}_{\rm m}\,\delta$$

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Makino, Sasaki & Suto ('92), ...

Large-scale structure
= pressureless & irrotational fluid

Single-stream approximation of collision less Boltzmann eq.

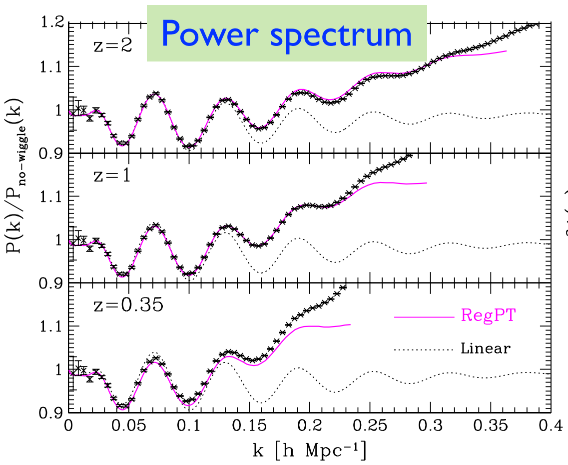
Standard PT

Regarding linear fluctuation $|\delta_0| \ll 1$ as the small expansion parameter : $\delta=\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\cdots$

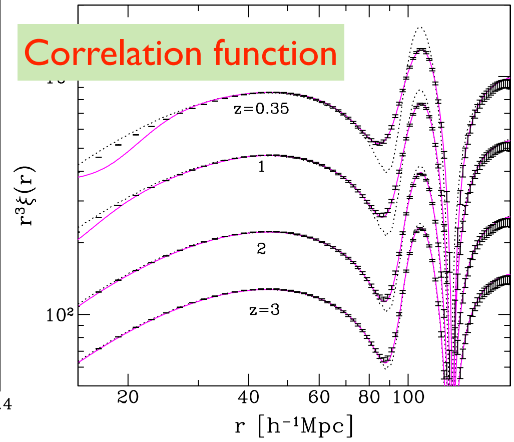
# Standard PT & improved PT

Standard PT is, however, a poor convergence expansion
(e.g., positivity of higher-order corrections is not guaranteed)

⟶ re-organizing PT expansion (RPT, RegPT, iPT, ...)




AT, Bernardeau, Nishimichi & Codis ('12)

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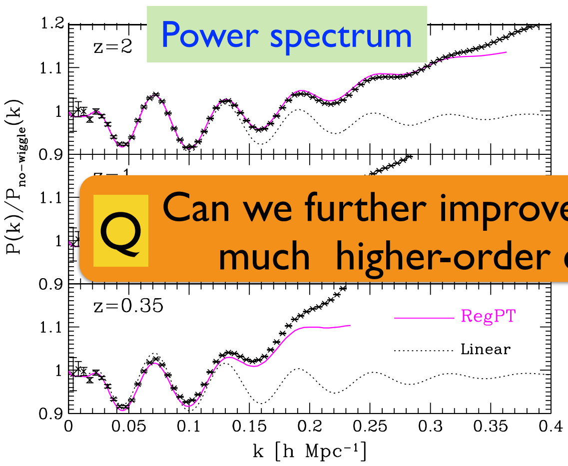


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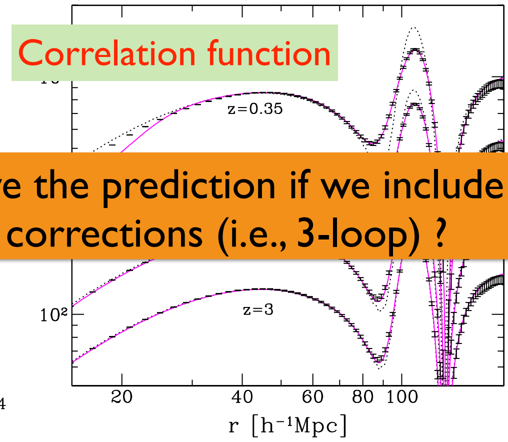
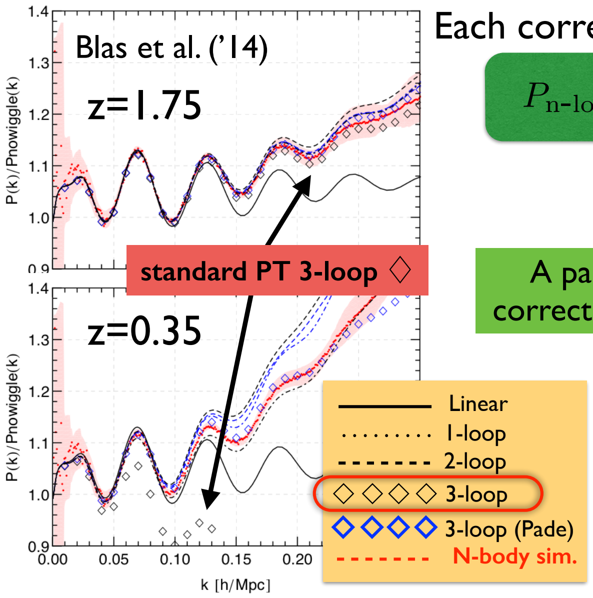



**Q** Can we further improve the prediction if we include much higher-order corrections (i.e., 3-loop) ?

# Curse of UV divergence

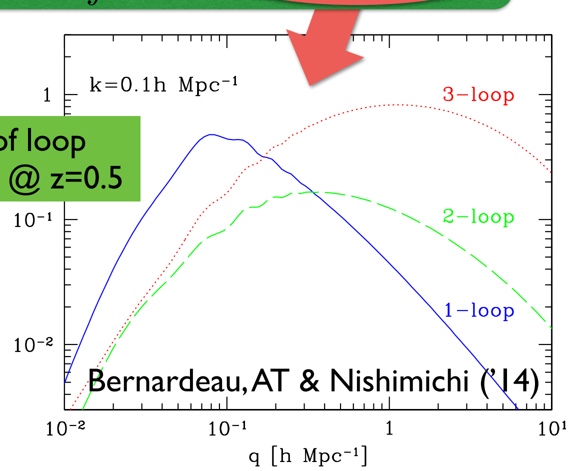
standard PT $P(k) = P_{\text{lin}}(k) + P_{\text{1-loop}}(k) + P_{\text{2-loop}}(k) + \cdots$




Each correction involves mode-coupling integral:

$$P_{\text{n-loop}}(k) \propto \int d\ln q \, K_{\text{n-loop}}(k,q)\, P_0(q)$$

A part of loop correction @ z=0.5




# Curse of UV divergence

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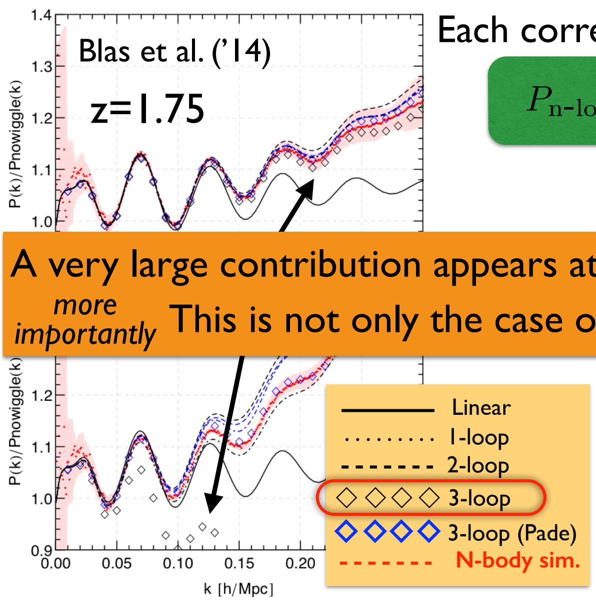


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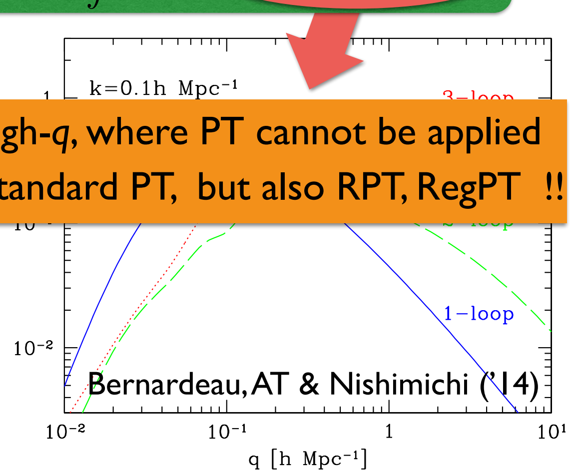



A very large contribution appears at high-*q*, where PT cannot be applied

*more importantly* This is not only the case of standard PT, but also RPT, RegPT !!

# Question

Bad UV behaviors in mode-coupling kernel may indicate the break down of PT

⟶ A simple fluid treatment cannot describe small-scale physics ?
(e.g., halo formation or virialization)

⟶ Need effective field theory treatment ?

Baumann et al. ('12), Carrasco, Herzberg & Senatore ('12), Carrasco et al. ('13ab), Porto, Senatore & Zaldarriaga ('14), …

**Q** How does the mode-coupling structure look like in reality ?

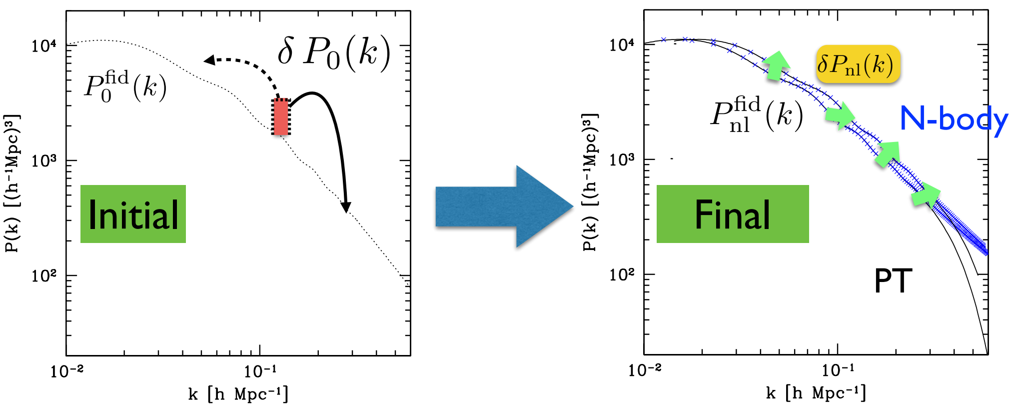
We quantitatively measure the *mode-coupling kernel* from N-body simulations, and compare it with PT calculation

# Measurement of kernel

mode-coupling kernel which we can measure

$$\delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$$

➡ How the small disturbance added in <u>initial power spectrum</u> can contribute to each Fourier mode in <u>final power spectrum</u>



# Measurement of kernel

mode-coupling kernel which we can measure

$$\delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$$

How the small disturbance added in <u>initial power spectrum</u> can contribute to each Fourier mode in <u>final power spectrum</u>

Alternative definition $K(k,q) = q\,\dfrac{\delta P_{\rm nl}(k)}{\delta P_0(q)}$

*(discretized) estimator:*

$$\widehat{K}(k_i,q_j)\,P_0(q_j) \equiv \frac{P^{+}_{\rm nl}(k_i) - P^{-}_{\rm nl}(k_i)}{\Delta \ln P_0\, \Delta \ln q}$$

$\Delta \ln q = \ln q_{j+1} - \ln q_j$

| name | box | particles | start-$z$ | bins | runs | total |
|---|---|---|---|---|---|---|
| L9-N9 | 512 | $512^3$ | 31 | 15 | 4 | 120 |
| L9-N8 | 512 | $256^3$ | 15 | 13 | 4 | 104 |
| L10-N9 | 1024 | $512^3$ | 31 | 15 | 1 | 30 |

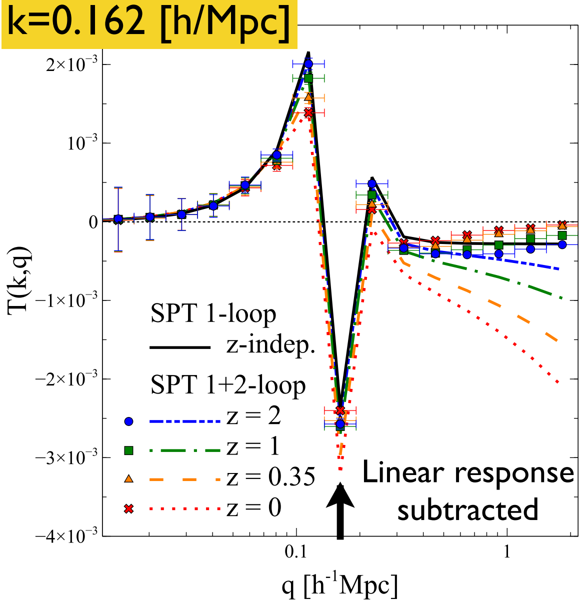
Run many simulations…

by T.Nishimishi

# Result

Normalized kernel $\quad T(k,q) = K(k,q)/P^{\rm lin}(k)$

k=0.162 [h/Mpc]




Black solid : Standard PT 1-loop (z-indept.)

Blue, Green, Orange, Red : 2-loop

q<k : reproduce simulation well

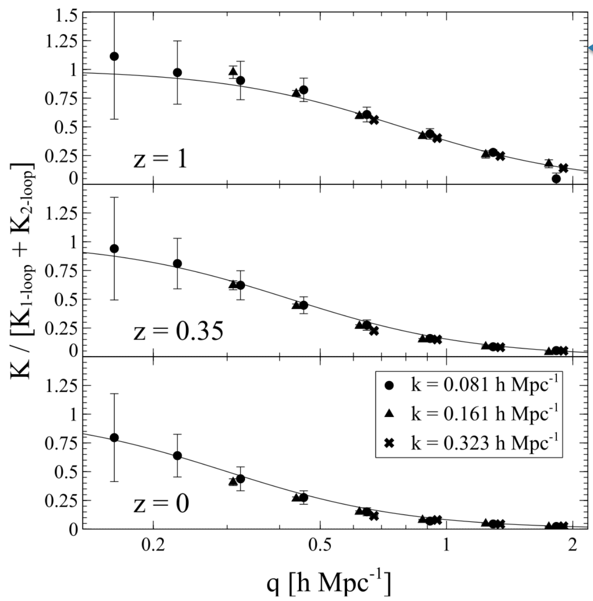
q>k : discrepancy is manifest (particularly large at low-z)

That is,

UV contribution is suppressed

# Characterizing UV suppression

UV suppression is seen at various k & q




ratio of measured kernel to PT prediction

Fitting formula

$q_0(z) = 0.3/D_+^2(z)\ [h\,\mathrm{Mpc}^{-1}]$

$$K_{\rm eff}(k,q) = \left[K^{\text{1-loop}}(k,q) + K^{\text{1-loop}}(k,q)\right]\frac{1}{1+(q/q_0)^2}$$

$K^{\text{1-loop}}$, $K^{\text{1-loop}}$ Standard PT kernel

# Summary & discussion

Measurement of mode-coupling kernel of large-scale structure (LSS) : $K(k,q) = q\,\dfrac{\delta P_{\rm nl}(k)}{\delta P_0(q)}$

*Unlike the standard PT results,*

- There appears UV suppression in N-body simulation at k<<q
- Discrepancy can be seen even at low-k, where standard PT can reproduce the N-body result quite well

**Physical origin** A connection with small-scale physics (formation and merging processes of dark matter halos)

**Implication** Check the validity and limitation for EFTofLSS

⟶ A step toward an improved prescription of LSS