JGRG23

Proceedings of
the 23rd Workshop on General Relativity
and Gravitation in Japan

5—8 November 2013

50th Anniversary Auditorium, Hirosaki University

Aomori, Japan

Volume 1

Workshop Information
Oral Presentations: First Day

Contents

Preface

There has been a significant progress in astrophysical and cosmological observations in recent years. Cosmology has entered an era of precision science. Astrophysical black holes have been observed in many frequency bands with better resolutions and sensitivities. Gamma-ray burst observations have brought a new puzzle into relativistic astrophysics. And, gravitational wave interferometers are now opening a new window for astrophysics and fundamental physics. On the theoretical side, motivated by unified theories of fundamental interactions, especially string theory, many efforts have been made for studies of physics in five (or higher) dimensional spacetimes, and there is a growing interest in experimental verifications of extra-dimension models. There have been also interesting developments in various other areas such as alternative theories of gravity, quantum gravity, and spacetime singularities. The main purpose of this workshop is to overview these recent developments and new directions in research on gravitation, cosmology, and relativistic astrophysics. The topics may include quantum gravity, string cosmology, inflationary cosmology, the generation and evolution of density fluctuations, observational cosmology, gravitational lensing, black holes, gamma-ray bursts, sources of gravitational radiation, gravitational wave experiments, modified gravity models and so on.

This workshop is supported by

- Graduate School of Science and Technology, Hirosaki University
- JSPS Grant-in-Aid for Scientific Research (A) 21244033
- Grant-in-Aid for Scientific Research on Innovative Areas No.24103006
- Aomori Prefecture (via Hirosaki Tourism and Convention Bureau)

We would like to thank all the participants and the above organizations for their kindly help of JGRG23.

December 20, 2013
Hideki Asada
(on behalf of the JGRG23 LOC)

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Tomohiro Harada (Rikkyo University)
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Local Organizing Committee

Hideki Asada (Hirosaki; Chair)
Masumi Kasai (Hirosaki)
Yuuiti Sendouda (Hirosaki)
Ryuichi Takahashi (Hirosaki)

Presentation Award

The JGRG presentation award program was established at the occasion of JGRG22 in 2012. This year, we are pleased to announce the following five winners of the Outstanding Presentation Award for their excellent presentations at JGRG23. The winners were selected by the selection committee consisting of the JGRG23 SOC based on ballots of the participants.

Ryo Namba (Kavli IPMU, University of Tokyo)
"Gauge-flation confronted with CMB observations"

Akira Oka (University of Tokyo)
"Cosmological Upper-Bound for f(R) Gravity through Redshift-Space Distortion"

Masato Nozawa (KEK)
"Supersymmetric Plebanski-Demianski solution"

Hiroyuki Nakano (Yukawa Institute for Theoretical Physics, Kyoto University)
"Spin-Regge-Wheeler-Zerilli formalism and gravitational waves"

Sakine Nishi (Rikkyo University)
"Cosmological matching conditions in Horndeski's theory"

Oral Presentations: First Day

Tuesday 5 November

9:00 Reception desk opens

9:30 Hideki Asada (Hirosaki University)
Opening address
[*]

Morning 1 [Chair: Takahiro Tanaka]

9:35 Tsutomu Kobayashi (Rikkyo University) [Invited]
"Horndeski's theory: a unified description of modified gravity"
[JGRG23(2013)110501]

10:25-45 Break

Morning 2 [Chair: Shinji Mukohyama]

10:45 Rampei Kimura (RESCEU, University of Tokyo)
"Derivative interactions in nonlinear massive gravity"
[JGRG23(2013)110502]

11:05 Chunshan Lin (Kavli IPMU)
"Massive graviton on a spatial condensation web"
[JGRG23(2013)110503]

11:25 Yasuho Yamashita (YITP, Kyoto University)
"Higher dimensional gravity and bigravity"
[JGRG23(2013)110504]

11:45 Yingli Zhang (Yukawa Institute for Theoretical Physics)
"Coleman-deLuccia instantons in nonlinear massive gravity"
[JGRG23(2013)110505]

12:05 Ivan Dario Arraut (Osaka University & KEK)
"Massive Gravity, Black Hole solutions and Relevant scales."
[JGRG23(2013)110506]

12:25-14:00 Lunch

Afternoon 1 [Chair: Yasusada Nambu]

14:00 Shi Pi (APCTP)
"Impact of heavy fields on power spectrum and bispectrum of the curvature perturbation"
[JGRG23(2013)110507]

14:20 Xian Gao (Tokyo Institute of Technology)
"Features in the curvature power spectrum after a sudden turn of the inflationary trajectory"
[JGRG23(2013)110508]

14:40 Toshifumi Noumi (RIKEN)
"Primordial spectra from sudden turning trajectory"
[JGRG23(2013)110509]

15:00 Ryo Saito (Yukawa Institute for Theoretical Physics, Kyoto University)
"Excitation of a heavy scalar field: Turn in the inflaton trajectory"
[JGRG23(2013)110510]

15:20-40 Break

Afternoon 2 [Chair: Hideo Kodama]

15:40 Tomotake Matsumura (KEK) [Invited]
"LiteBIRD, Lite (Light) satellite for the studies of B-mode polarization and inflation from cosmic background radiation detection"
[JGRG23(2013)110511]

16:30 Ryo Namba (Kavli IPMU, University of Tokyo)
"Gauge-flation confronted with CMB observations"
[JGRG23(2013)110512]

16:50 Daisuke Yamauchi (RESCEU, University of Tokyo)
"CMB ISW-lensing bispectrum from cosmic strings"
[JGRG23(2013)110513]

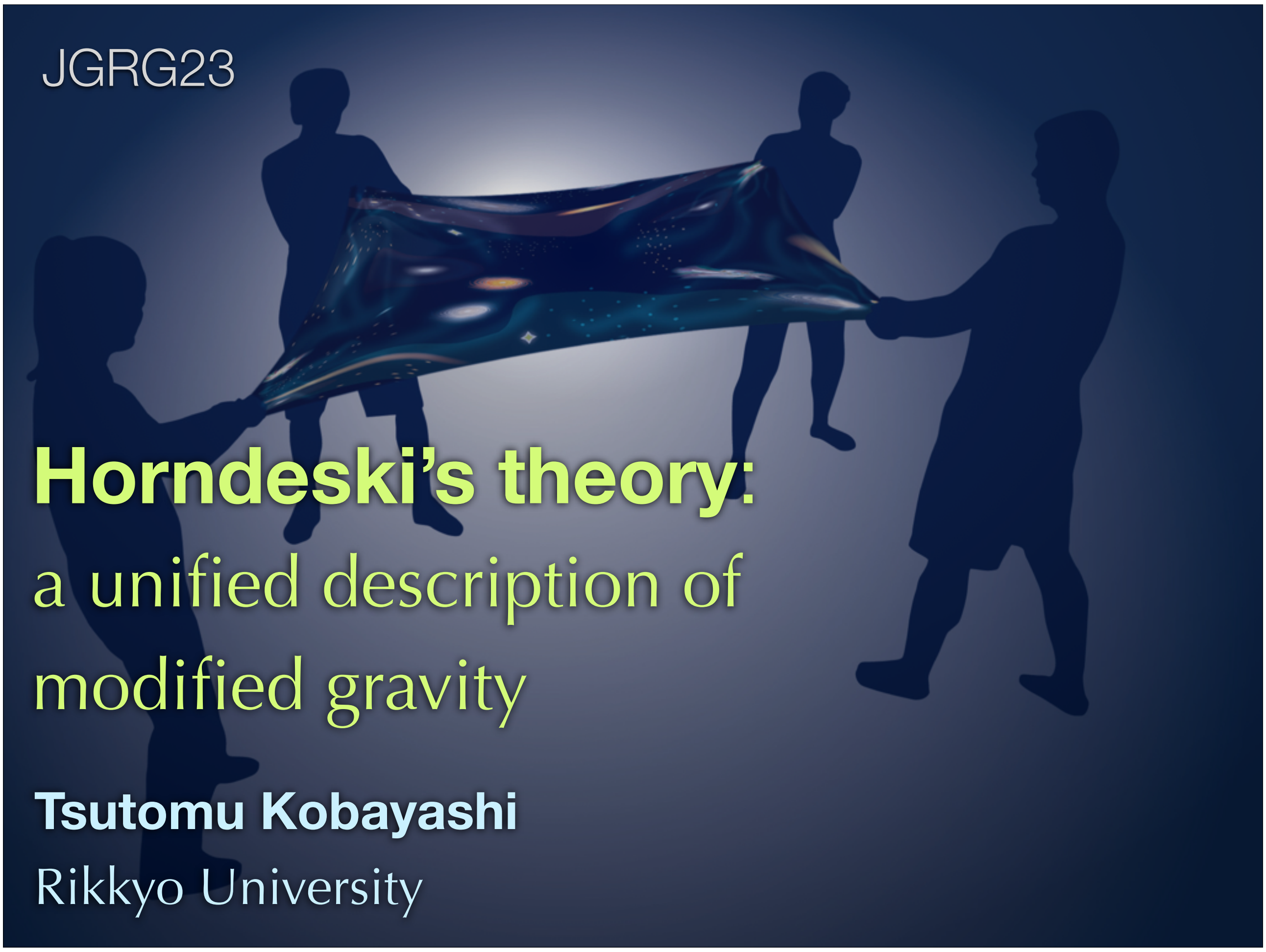
Afternoon 3 [Chair: Yuuiti Sendouda]

17:10-18:12 Poster short presentations

“Horndeski’s theory: a unified description of modified gravity”

by Tsutomu Kobayashi (invited)

[JGRG23(2013)110501]



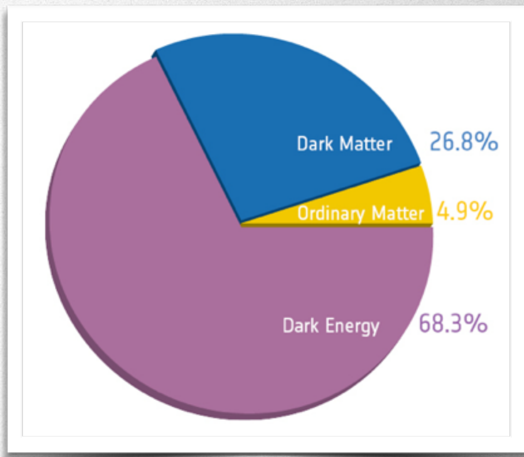
JGRG23

Horndeski's theory:
a unified description of modified gravity

Tsutomu Kobayashi
Rikkyo University

Why modified gravity?

- Cosmic acceleration
  – Our understanding of the Universe is *incomplete*
- Need better understanding of gravity – dark energy or modified gravity?
- Precision cosmology era
  – cosmological tests of gravity



Planck

Modifying GR

- General relativity: massless spin-2
- Modified gravity: new d.o.f. (scalars, vectors)
  - *f*(*R*), DGP, galileons, TeVeS, massive gravity, …
- Let's consider scalar-tensor theories
  - Gravity mediated by $g_{\mu\nu}$ and ϕ
  - Most typical
- Aim: **theoretical framework to describe all scalar-tensor theories in a unified manner**

Talk plan

- Introduction & Motivations
- From galileons to Horndeski
  – Introducing the most general scalar-tensor theory with second-order EOMs
- Screened modified gravity from Horndeski's theory
  – How to evade small-scale tests
- Some other topics
  – Inflation, Multi-field extension, …
- Summary

From galileons to Horndeski

Nicolis, Rattazzi, Trincherini (2009)

Galileon in flat space

Starting point

$$\mathcal{L} = c_1\phi + c_2 X - c_3 X\partial^2\phi + c_4 X\left[(\partial^2\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right]$$
$$-\frac{c_5}{3}X\left[(\partial^2\phi)^3 - 3\partial^2\phi(\partial_\mu\partial_\nu\phi)^2 + 2(\partial_\mu\partial_\nu\phi)^3\right]$$

where $X := -\frac{1}{2}(\partial\phi)^2$

Unique scalar-field theory in 4D Minkowski having

- Galilean shift symmetry: $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$
- 2nd-order field equation

Deffayet, Esposito-Farese, Vikman (2009)

Covariant galileon

Step 2: Covariantize flat-space galileon

$$\eta_{\mu\nu} \to g_{\mu\nu}, \ \partial_\mu \to \nabla_\mu$$

$$\mathcal{L} = c_1\phi + c_2 X - c_3 X\Box\phi + \frac{c_4}{2}X^2 R + c_4 X\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + c_5 X^2 G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{c_5}{3}X\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

- 2nd-order field equations both for ϕ and $g_{\mu\nu}$
- Nonminimal coupling to gravity is necessary
- Forget about symmetry…

Deffayet, Gao, Steer, Zahariade (2011)

Generalized galileon

Step 3: Promote X, X^2 to arbitrary functions of ϕ, X

$$\mathcal{L} = c_1\phi + c_2 X - c_3 X\Box\phi + \frac{c_4}{2}X^2 R + c_4 X\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + c_5 X^2 G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{c_5}{3}X\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

⇓

$$\mathcal{L} = G_2(X,\phi) - G_3(X,\phi)\Box\phi + G_4(X,\phi)R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(X,\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$\longrightarrow$ **Second-order field equations**

Horndeski's theory

$$\begin{aligned}\mathcal{L}_H \;=\;& \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\Big[\kappa_1 \nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi \\ &+\kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\Big] \\ &+\delta^{\alpha\beta}_{\mu\nu}\Big[(F+2W)R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\Big] \\ &-6\left(F_\phi + 2W_\phi - X\kappa_8\right)\Box\phi + \kappa_9\end{aligned}$$

Mathematically rigorous proof that this is ***the most general scalar-tensor theory with second-order field equations in 4D***

Horndeski (1974); Rediscovered by Charmousis et al. (2011)

Horndeski's theory

$$\begin{aligned}\mathcal{L}_H \;=\;& \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\Big[\kappa_1 \nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi \\ &+\kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\Big] \\ &+\delta^{\alpha\beta}_{\mu\nu}\Big[(F+2W)R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\Big] \\ &-6\left(F_\phi + 2W_\phi - X\kappa_8\right)\Box\phi + \kappa_9\end{aligned}$$

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Horndeski (1974); Rediscovered by Charmousis et al. (2011)

Generalized galileon = Modern rephrasing of Horndeski's theory

The dictionary — *TK, Yamaguchi, Yokoyama (2011)*

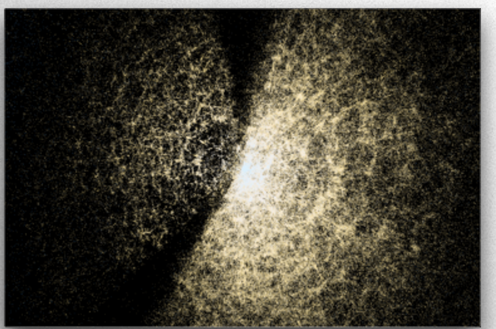
$$\begin{aligned} G_2 &= \kappa_9 + 4X\int^X \mathrm{d}X'\left(\kappa_{8\phi} - 2\kappa_{3\phi\phi}\right), \\ G_3 &= 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X \mathrm{d}X'(\kappa_8 - 2\kappa_{3\phi}), \\ G_4 &= 2F - 4X\kappa_3, \\ G_5 &= -4\kappa_1 \end{aligned}$$

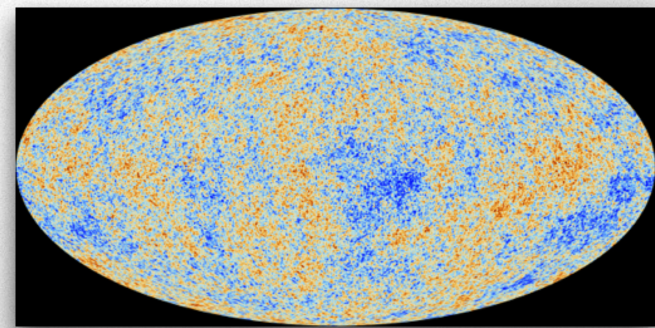
Cosmological tests of gravity

Large-scale structure tests
– power spectra, weak lensing, ISW, …

Modified evolution of density perturbations can be studied in a unified manner using Horndeski's theory

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + a^2(1-2\Psi)\mathrm{d}\mathbf{x}^2, \quad \delta = \delta\rho/\bar{\rho}, \quad \phi = \bar{\phi} + \delta\phi$$





De Felice, TK, Tsujikawa (2011)

Density perturbations

Evolution of density perturbation in any modified gravity w/ ϕ

GR	Scalar-tensor theories
$\ddot{\delta}_k + 2H\delta_k + \frac{k^2}{a^2}\Phi_k = 0$ (Minimally coupled matter)	
$\frac{k^2}{a^2}\Phi_k = -4\pi G\rho\delta_k$	$\frac{k^2}{a^2}\Phi_k = -4\pi G_{\mathrm{eff}}(t,k)\rho\delta_k$
$\Psi_k = \Phi_k$	$\Psi_k = \eta(t,k)\Phi_k$

The most general formulas: $G_{\mathrm{eff}}(t,k) = \cdots, \quad \eta(t,k) = \cdots$

Screened modified gravity from Horndeski's theory

Why screening mechanism?

- ϕ participates in gravity dynamics: $\Phi \neq \Psi$ (in Joran frame)
- Solar-system tests: $|\Psi/\Phi - 1| < \mathcal{O}(10^{-4})$

⇨ ϕ must be screened in the vicinity of sources

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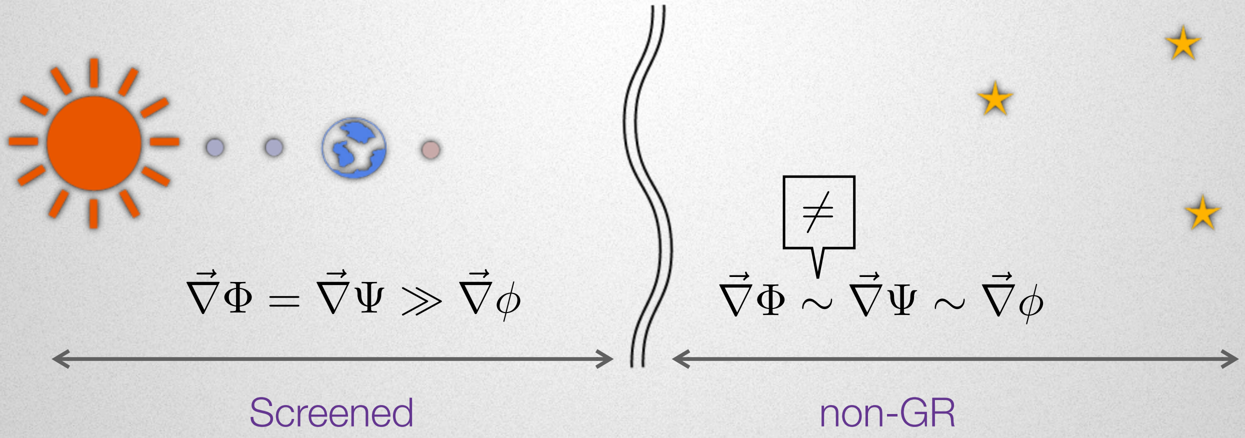

Key idea

Scalar π coupled to matter with the same strength as gravity:

$$\mathcal{L} = -\frac{1}{16\pi G}(\partial\pi)^2 + \pi T^\mu_\mu \quad \text{(Einstein frame)}$$

⇨ $\Box\pi = 8\pi G\rho$

⇨ $\vec{\nabla}\pi \sim \vec{\nabla}\Phi$

($\pi \cdots$ dimensionless)

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$$\Longrightarrow \quad \Box\pi = 8\pi G\rho$$

$$\Longrightarrow \quad \vec{\nabla}\pi \sim \vec{\nabla}\Phi$$

($\pi \cdots$ dimensionless)

Small-scale tests can be evaded if

ϕ is effectively weakly coupled to matter in the vicinity of the source (in Einstein frame)

Key idea

Introduce derivative self-interaction term:

$$\mathcal{L} = -\frac{1}{16\pi G}\left[(\partial\pi)^2 + \frac{r_c^2}{2}(\partial\pi)^2\Box\pi\right] + \pi T^\mu_\mu$$

$$\Longrightarrow \quad \Box\pi \sim 8\pi G\rho$$

$$\Longrightarrow \quad \mathcal{L}_{\rm kin} \sim -\frac{1 + 4\pi r_c^2 G\rho}{16\pi G}(\partial\pi)^2$$

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⇒ $\Box\pi \sim 8\pi G\rho$

⇒ $\mathcal{L}_{\rm kin} \sim -\frac{1+4\pi r_c^2 G\rho}{16\pi G}(\partial\pi)^2$ $\qquad G_{\rm eff} \ll G$

Vainshtein mechanism *Vainshtein (1972)*

Small-scale tests can be evaded if

ϕ **is effectively weakly coupled to matter in the vicinity of the source** (in Einstein frame)

An illustrative example

$$\mathcal{L} = -\frac{1}{16\pi G}\left[(\partial\pi)^2 + \frac{r_c^2}{2}(\partial\pi)^2\Box\pi\right] + \pi T_\mu^\mu$$

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Look for spherically symmetric solution:

$$\partial_r\left[r^2\partial_r\pi + r_c^2 r(\partial_r\pi)^2\right] = 8\pi G\rho r^2$$

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Algebraic equation for $\partial_r\pi/r$

$$\Rightarrow \quad \left(\frac{\partial_r\pi}{r}\right) + r_c^2\left(\frac{\partial_r\pi}{r}\right)^2 = \frac{r_g}{r^3}$$

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$$\Rightarrow \frac{\partial_r\pi}{r} = \frac{1}{2r_c^2}\left(-1+\sqrt{1+\frac{4r_g r_c^2}{r^3}}\right)$$

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Vainshtein radius $r_V := \left(r_g r_c^2\right)^{1/3}$

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$$\frac{\partial_r\pi}{r} = \frac{1}{2r_c^2}\left(-1 + \sqrt{1 + \frac{4r_g r_c^2}{r^3}}\right)$$

- $\partial_r\pi \sim \frac{r_g}{r^2} \sim \partial_r\Phi$ for $r \gg r_V$
- $\partial_r\pi \sim \left(\frac{r}{r_V}\right)^{3/2}\frac{r_g}{r^2} \ll \partial_r\Phi$ for $r \ll r_V$

Vainshtein radius $r_V := (r_g r_c^2)^{1/3}$

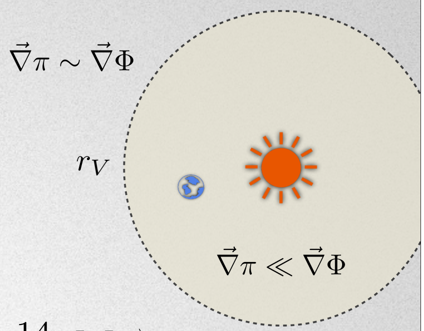
Screened inside Vainshtein radius!

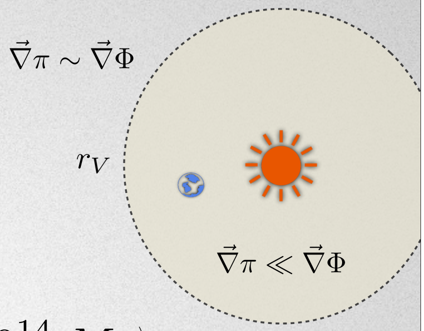
Estimate of Vainshtein radius:

Suppose $r_c = 3$ Gpc

✓ $r_V \sim 100$ pc for the Sun ($M = M_\odot$)

✓ $r_V \sim 1$ Mpc for a galaxy cluster ($M = 10^{14}\ M_\odot$)





Estimate of Vainshtein radius:

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Lessons from this example:

- Naively, scalar-tensor theory predicts $\partial\pi \sim \partial\Phi$
- However, non-linear derivative interaction can be large, leading to self-screening in the vicinity of source

$$r_c^2 \partial\partial\pi\partial\pi \sim \partial\pi \ll \partial\Phi \quad \text{(even if } \Phi \ll 1\text{)}$$

Vainshtein from Horndeski

2 derivatives acting on ϕ , central for implementing Vainshtein mech.

$$\begin{aligned}\mathcal{L} \;=\; & G_2(X,\phi) - G_3(X,\phi)\Box\phi + G_4(X,\phi)R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(X,\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

The most general theory exhibiting Vainshtein screening mechanism can be derived from Horndeski's Lagrangian

Narikawa, TK, Yamauchi, Saito (2013); Koyama, Niz, Tasinato (2013)

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2 derivatives acting on ϕ, central for implementing Vainshtein mech.

$$\mathcal{L} = G_2(X,\phi) - G_3(X,\phi)\Box\phi + G_4(X,\phi)R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
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Narikawa, TK, Yamauchi, Saito (2013); Koyama, Niz, Tasinato (2013)

Cosmological background … see *Kimura, TK, Yamamoto (2012)*

Taylor expand

$$g_{\mu\nu} = \eta_{\mu\nu} + M_{\rm Pl}^{-1}h_{\mu\nu}, \quad \phi = \underline{\underline{\phi_0}} + \pi \qquad h_{\mu\nu}, \pi = (\text{mass})$$

const.

⇨ Effective action for $h_{\mu\nu}$ and π

$M_{\rm Pl}^{-1}h_{\mu\nu} = \mathcal{O}(\epsilon)$

$M_{\rm Pl}^{-1}\pi \lesssim \mathcal{O}(\epsilon)$

Vainshtein from Horndeski

2 derivatives acting on ϕ, central for implementing Vainshtein mech.

$$\mathcal{L} = G_2(X,\phi) - G_3(X,\phi)\Box\phi + G_4(X,\phi)R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+G_5(X,\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

The most general theory exhibiting Vainshtein screening mechanism can be derived from Horndeski's Lagrangian

Narikawa, TK, Yamauchi, Saito (2013); Koyama, Niz, Tasinato (2013)

Cosmological background … see *Kimura, TK, Yamamoto (2012)*

Taylor expand

$$g_{\mu\nu} = \eta_{\mu\nu} + M_{\rm Pl}^{-1}h_{\mu\nu}, \quad \phi = \underline{\underline{\phi_0}} + \pi \qquad h_{\mu\nu}, \pi = (\text{mass})$$

const.

⇨ Effective action for $h_{\mu\nu}$ and π

$M_{\rm Pl}^{-1}h_{\mu\nu} = \mathcal{O}(\epsilon)$

$M_{\rm Pl}^{-1}\pi \lesssim \mathcal{O}(\epsilon)$

Expansion scheme

$$\mathcal{L}_{\rm eff} \supset (\partial h)^2,\ (\partial\pi)^2,\ \partial h\partial\pi,$$
$$\{(\partial h)^2, (\partial\pi)^2, \partial h\partial\pi\} \times (\partial^2\pi)^n,$$
$$(\partial\pi)^4(\partial^2\pi)^n,\ (\partial h)^2(\partial\pi)^2(\partial^2\pi)^n, \cdots$$

2 derivatives acting on π, cubic or higher, but can be as large as quadratic terms

Smaller than other terms

Ignore mass term: $G_2(\phi, X) \supset G_{2\phi\phi}\pi^2$

Example:

$$G_3(\phi, X)\Box\phi \supset -G_{3\phi}(\partial\pi)^2 - \frac{1}{2}G_{3X}(\partial\pi)^2\Box\pi - G_{3\phi}M_{\rm Pl}^{-1}h^{\mu\nu}\pi\partial_\mu\partial_\nu\pi + \cdots$$

Koyama, Niz, Tasinato (2013)

Effective theory for Vainshtein mech.

$$\begin{aligned}\mathcal{L}_{\rm eff} = & -\frac{1}{2}h^{\mu\nu}\delta G_{\mu\nu} \\ & +\eta\mathcal{L}_2^{\rm gal} + \frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal} \\ & -\xi h^{\mu\nu}X_{\mu\nu}^{(1)} - \frac{\alpha}{\Lambda^3}h^{\mu\nu}X_{\mu\nu}^{(2)} + \frac{\beta}{2\Lambda^6}h^{\mu\nu}X_{\mu\nu}^{(3)} \\ & +\frac{1}{2M_{\rm Pl}}h^{\mu\nu}T_{\mu\nu}\end{aligned}$$

$\eta, \mu, \cdots$: dimensionless coefficients

Λ : mass scale (defined in the next slide)

Koyama, Niz, Tasinato (2013)

Effective theory for Vainshtein mech.

$$
\begin{aligned}
\mathcal{L}_{\rm eff} \;=\; & -\frac{1}{2}h^{\mu\nu}\delta G_{\mu\nu} \\
& +\eta\mathcal{L}_2^{\rm gal} + \frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal} \\
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\end{aligned}
$$

→ Jordan frame

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Koyama, Niz, Tasinato (2013)

Effective theory for Vainshtein mech.

Galileon terms

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$\longrightarrow$ Jordan frame

Nonminimal coupling between metric and scalar

$X^{(i)}_{\mu\nu}$: tensors constructed from $\partial_\mu\partial_\nu\pi$

$\eta, \mu, \cdots$: dimensionless coefficients

Λ : mass scale (defined in the next slide)

Some definitions

Parameters of effective theory:

$$G_4 = \frac{M_{\rm Pl}^2}{2}$$
$$G_{4\phi} = M_{\rm Pl}\xi$$
$$G_{2X} - 2G_{3\phi} = \eta$$
$$-G_{3X} + 3G_{4\phi X} = \frac{\mu}{\Lambda^3}$$

$$G_{4X} - G_{5\phi} = \frac{M_{\rm Pl}}{\Lambda^3}\alpha,$$
$$G_{4XX} - \frac{2}{3}G_{5\phi X} = \frac{\nu}{\Lambda^6},$$
$$G_{5X} = -\frac{3M_{\rm Pl}}{\Lambda^6}\beta$$
$$G_{5XX} = -\frac{3\varpi}{\Lambda^9}$$

$$\mathcal{L}_2^{\rm gal} = -\frac{1}{2}(\partial\pi)^2, \quad \mathcal{L}_3^{\rm gal} = -\frac{1}{2}(\partial\pi)^2\partial^2\pi, \quad \mathcal{L}_4^{\rm gal} = \cdots,$$
$$X^{(1)}_{\mu\nu} := \eta_{\mu\nu}\partial^2\pi - \partial_\mu\partial_\nu\pi, \quad X^{(2)}_{\mu\nu} = \cdots$$

DGP, galileons, and massive gravity can be reproduced by choosing appropriately $\xi, \eta, \mu, \cdots$

Frames

de Rham, Gabadadze, Heisenberg, Pirtskhalava (2011)

$$\mathcal{L}_{\rm eff} = -\frac{1}{2}h^{\mu\nu}\delta G_{\mu\nu} + \eta\mathcal{L}_2^{\rm gal} + \frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal}$$
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$X_{\mu\nu}^{(n)} \sim (\partial\partial\pi)^n$

– Mixing of π and $h_{\mu\nu}$

Frames

de Rham, Gabadadze, Heisenberg, Pirtskhalava (2011)

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Disformal transformation:

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} + 2\xi\pi\eta_{\mu\nu} - \frac{2\alpha}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi$$

Frames

de Rham, Gabadadze, Heisenberg, Pirtskhalava (2011)

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$$+\frac{1}{2M_{\rm Pl}}\tilde{h}^{\mu\nu}T_{\mu\nu} - \frac{2\xi}{M_{\rm Pl}}\pi T^{\mu}_{\mu} + \frac{2\alpha}{M_{\rm Pl}\Lambda^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}$$

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de Rham, Gabadadze, Heisenberg, Pirtskhalava (2011)

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Disformal transformation:

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} + 2\xi\pi\eta_{\mu\nu} - \frac{2\alpha}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi$$

Cannot be removed by field redefinition

$$\mathcal{L}_{\rm eff} = -\frac{1}{2}\tilde{h}^{\mu\nu}\widetilde{\delta G}_{\mu\nu} + \eta_{\rm new}\mathcal{L}_2^{\rm gal} + \frac{\mu_{\rm new}}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu_{\rm new}}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi_{\rm new}}{\Lambda^9}\mathcal{L}_5^{\rm gal}$$
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Nonminimal coupling to matter

Narikawa, TK, Yamauchi, Saito (2013)

Spherically symmetric solutions

In terms of $x(r) := \frac{1}{\Lambda^3}\frac{\pi'}{r}$, $A(r) := \frac{1}{M_{\rm Pl}\Lambda^3}\frac{M(r)}{8\pi r^3}$ (enclosed mass)

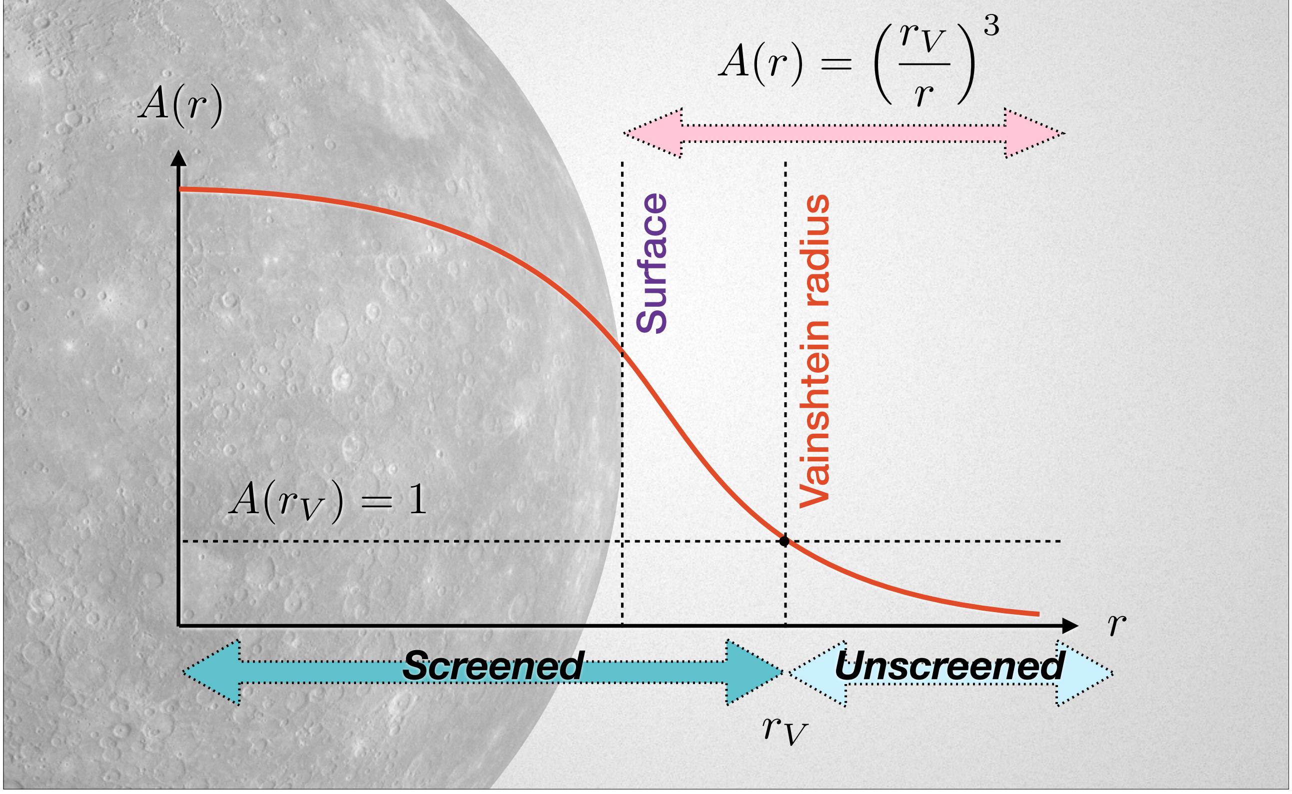
EOMs (w/non-relativistic source) can be written as

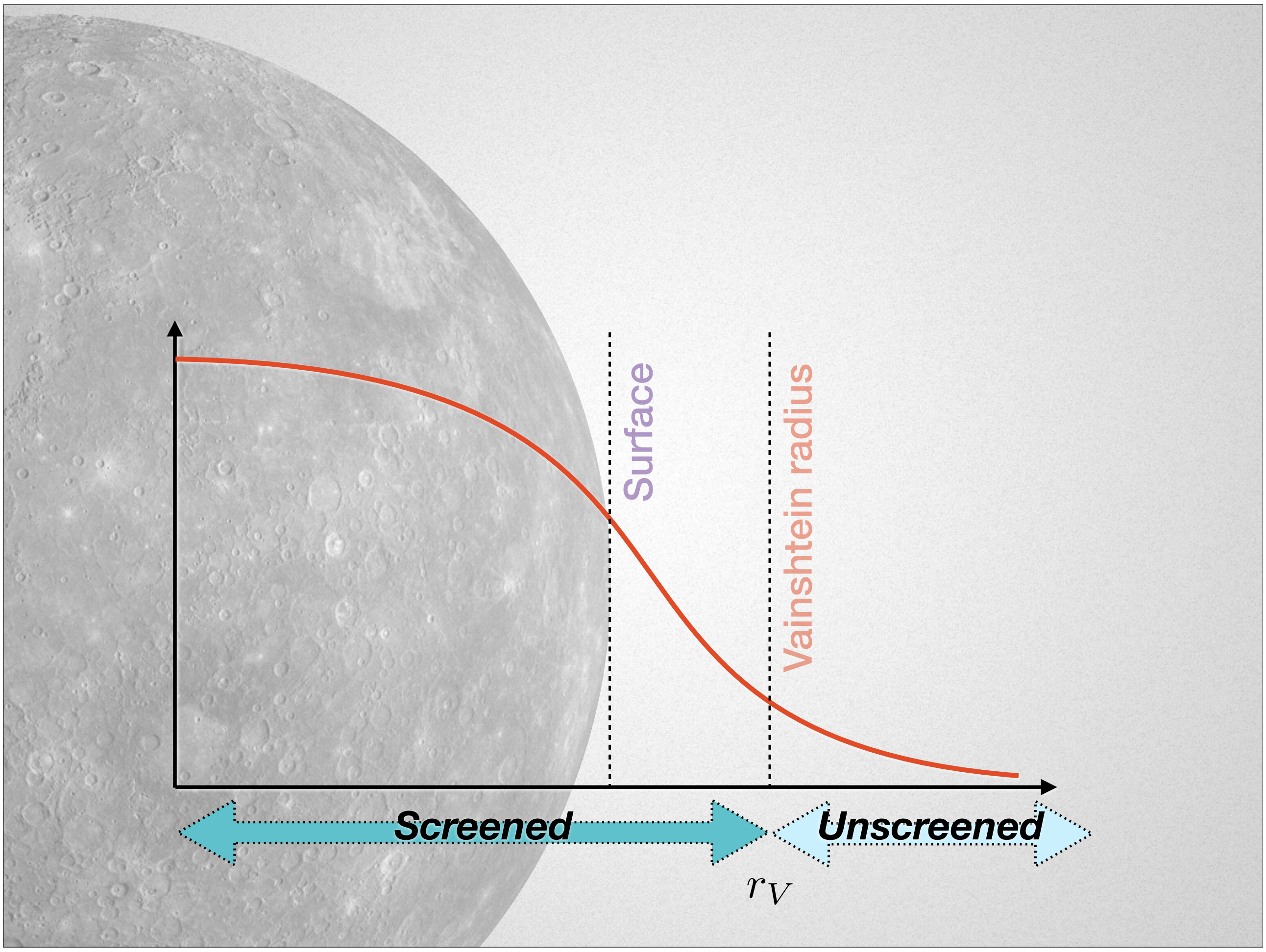
$$\begin{cases} \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x + [\mu + 6\alpha\xi - 3\beta A(r)]\, x^2 + \left(\nu + 2\alpha^2 + 4\beta\xi\right) x^3 - 3\beta^2 x^5 = 0 \\ \frac{1}{\Lambda^3}\frac{\Phi'}{r} = -\xi x + \beta x^3 + A(r) \\ \frac{1}{\Lambda^3}\frac{\Psi'}{r} = \xi x + \alpha x^2 + \beta x^3 + A(r) \end{cases}$$

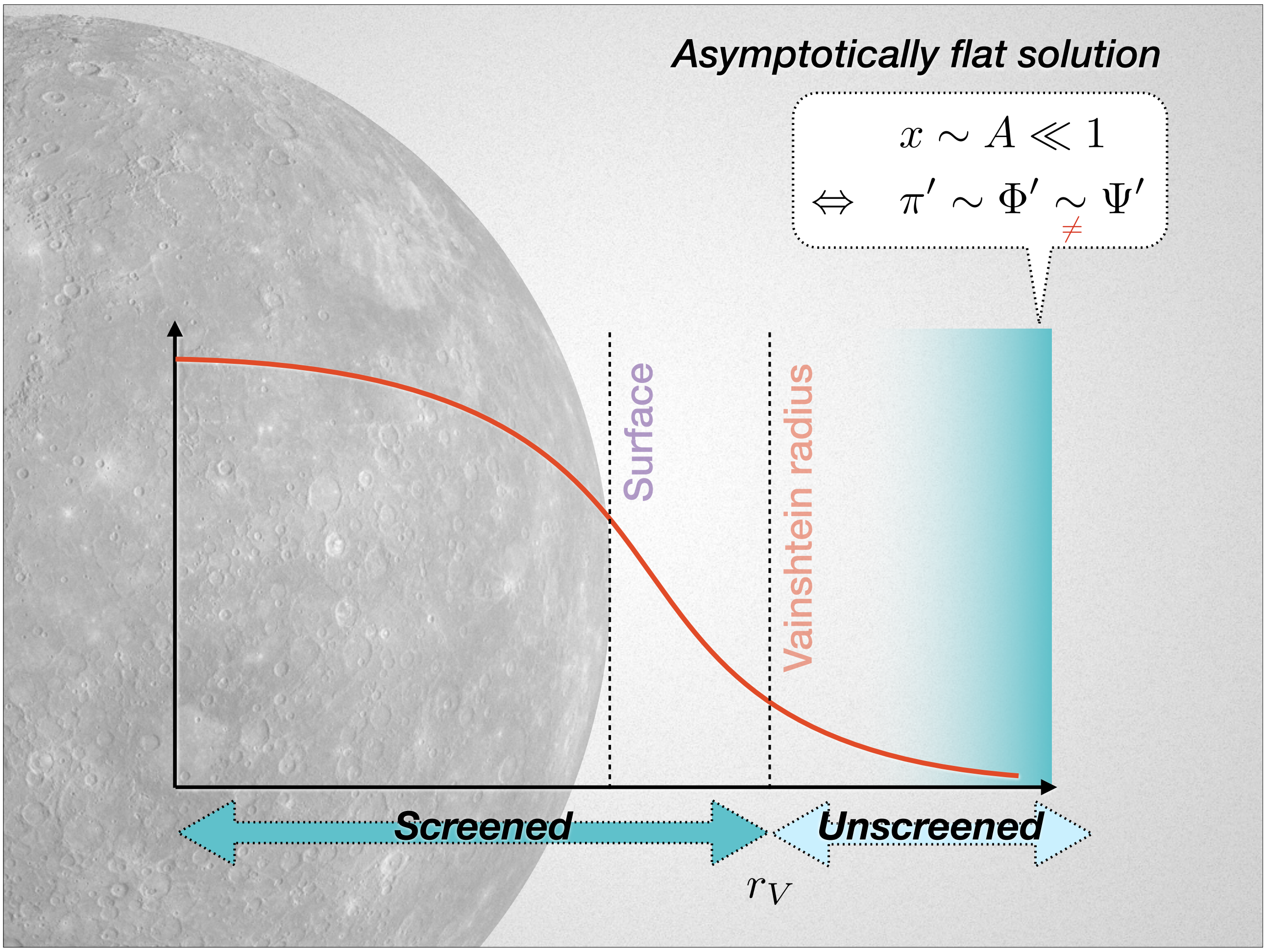
Solve **quintic algebraic equation**

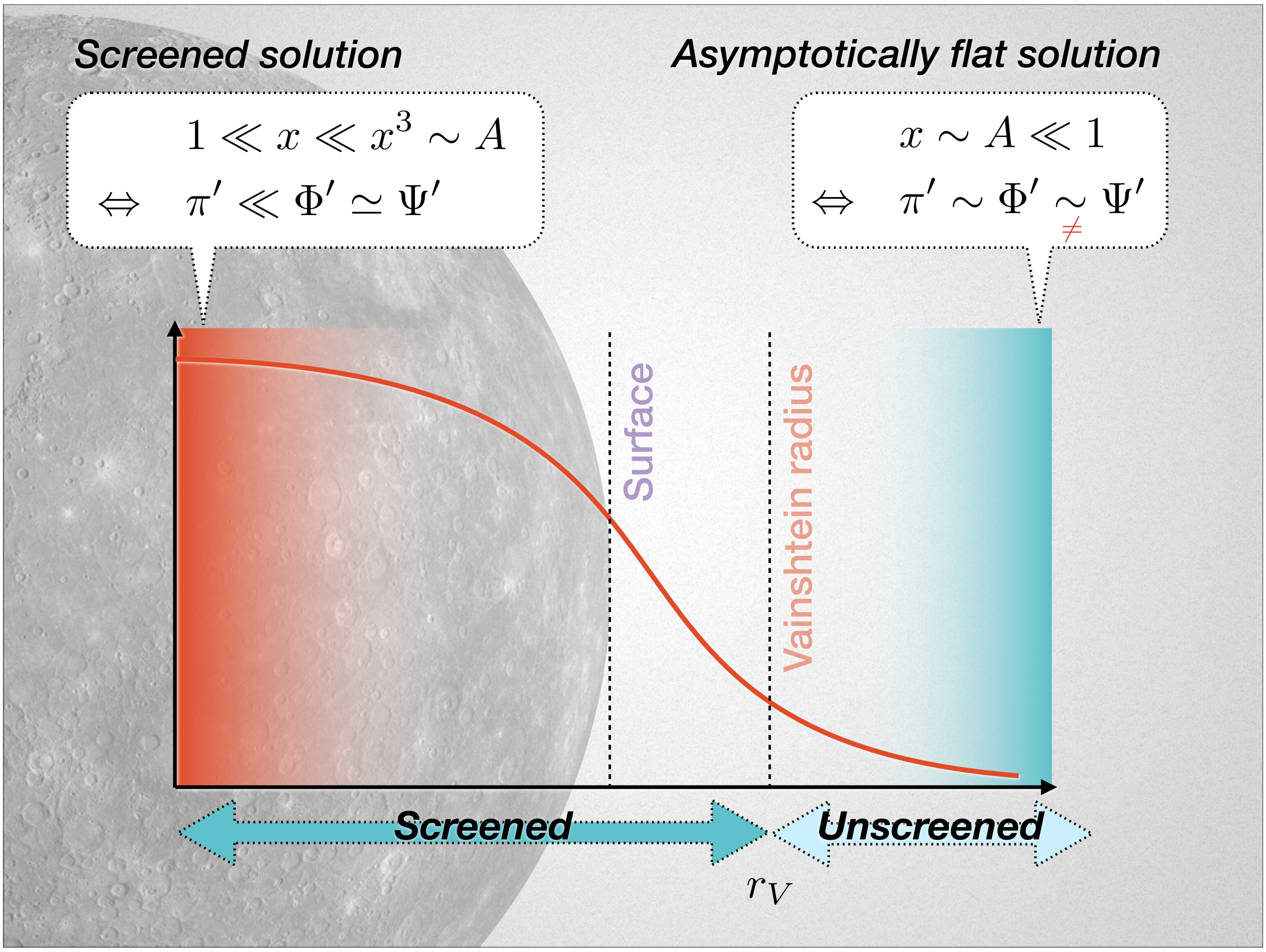
$$\longrightarrow \quad x = x[A(r)] = x(r) \quad \longrightarrow \quad \Phi(r), \Psi(r)$$

Density profile $\rho(r) \longrightarrow A(r) \propto M(r)/r^3$




Surface
Vainshtein radius
Screened
Unscreened
r_V


Asymptotically flat solution
$x \sim A \ll 1$
$\Leftrightarrow \quad \pi' \sim \Phi' \sim \Psi'$
$\neq$
Surface
Vainshtein radius
Screened
Unscreened
r_V


Screened solution
Asymptotically flat solution
$1 \ll x \ll x^3 \sim A$
$\Leftrightarrow \quad \pi' \ll \Phi' \simeq \Psi'$
$x \sim A \ll 1$
$\Leftrightarrow \quad \pi' \sim \Phi' \underset{\neq}{\sim} \Psi'$
Surface
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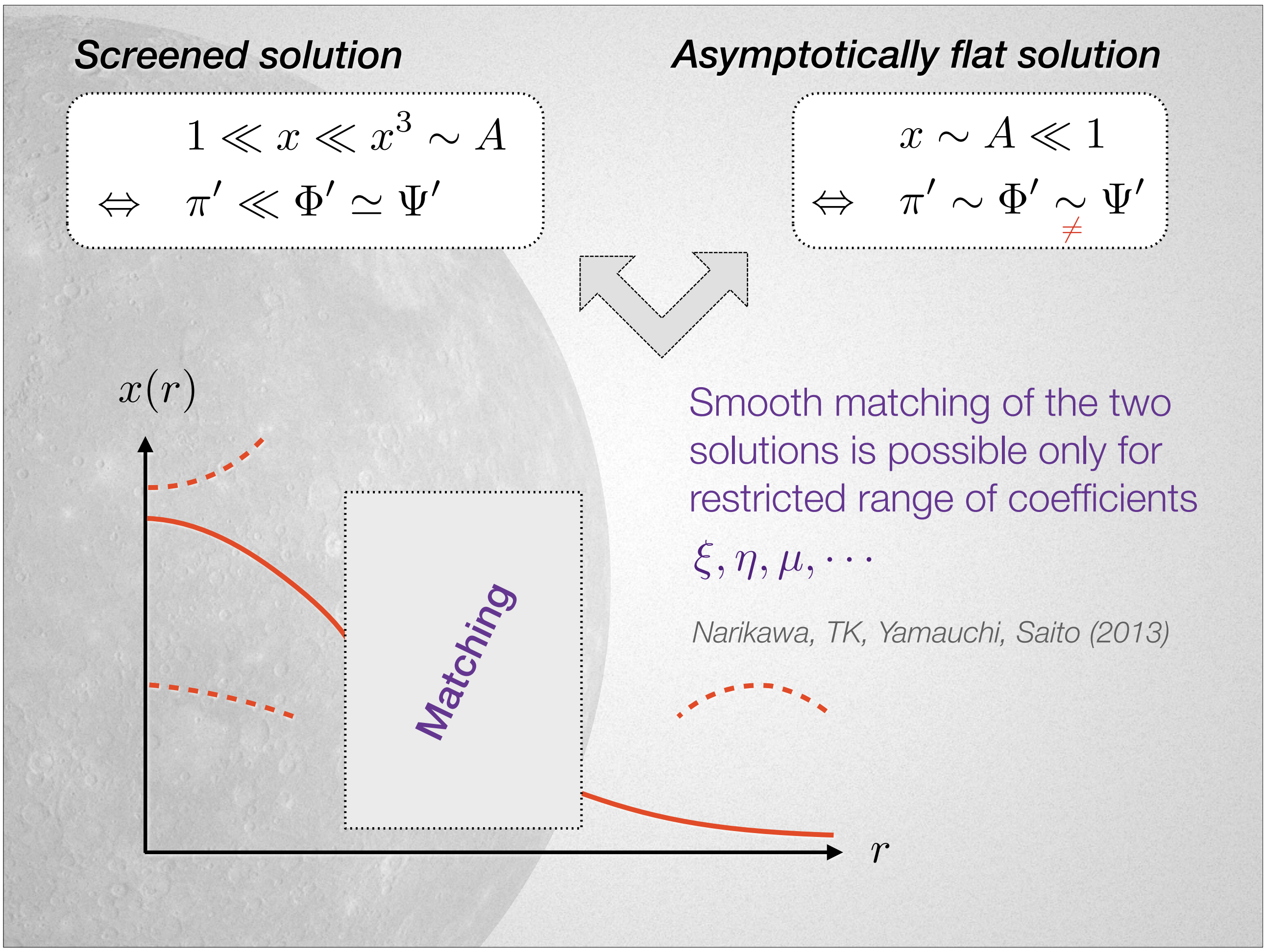
Screened
Unscreened
r_V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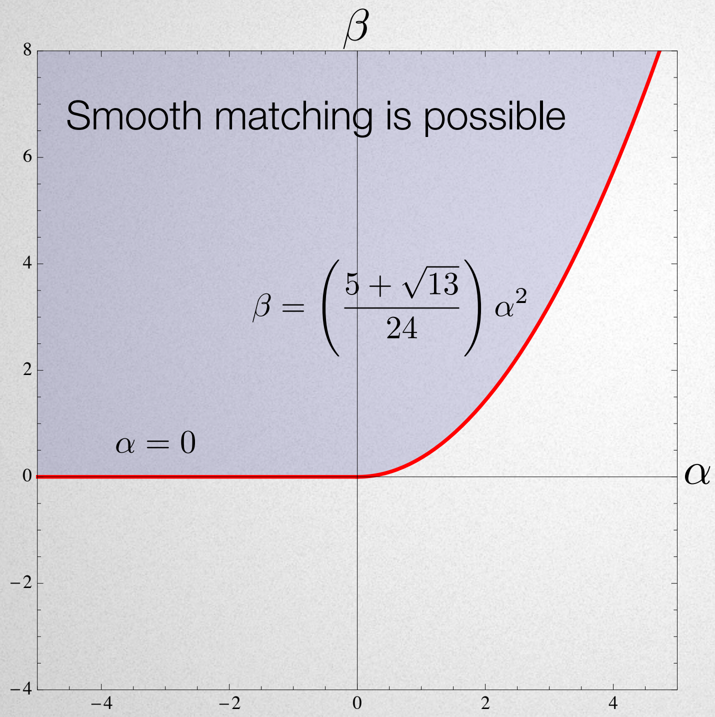
$x(r)$
Matching
r
Smooth matching of the two solutions is possible only for restricted range of coefficients
$\xi, \eta, \mu, \cdots$
Narikawa, TK, Yamauchi, Saito (2013)

The case of massive gravity



Decoupling limit of massive gravity

de Rham, Gabadadze, Tolley (2011)

= 2-parameter subclass of Horndeski's theory

$$\xi = 1,\ \alpha \neq 0,\ \beta \neq 0,$$
$$\text{others} = 0$$

Generic analysis [*Narikawa et al. (2013)*] correctly reproduces previous results [*Sjors, Mortsell (2011); Sbisa et al. (2012)*].

Stability of screened solution

$$\pi \to \pi(r) + \delta\pi(t, r, \theta, \varphi),\quad \Phi \to \Phi + \delta\Phi,\quad \Psi \to \Psi + \delta\Psi$$

$$\Rightarrow\quad \mathcal{L} = \frac{1}{2}\left[\mathcal{K}_t(r)(\partial_t\delta\pi)^2 - \mathcal{K}_r(r)(\partial_r\delta\pi)^2 - \mathcal{K}_\Omega(r)(\partial_\Omega\delta\pi)^2\right]$$

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- (surface) $\ll r \ll r_V$, $\beta \neq 0$ $\Rightarrow$ $\mathcal{K}_r\mathcal{K}_\Omega < 0$

Nonminimal coupling, $\beta\tilde{h}^{\mu\nu}X^{(3)}_{\mu\nu}$, which cannot be removed by disformal transformation, is prohibited by stability

Koyama, Niz, Tasinato (2013)

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Koyama, Niz, Tasinato (2013)

- $\mathcal{L}_{\rm eff} \supset +\alpha\partial_\mu\pi\partial_\nu\pi T^{\mu\nu} \sim +\alpha\rho(\partial_t\delta\pi)^2$

$\alpha \geq 0$ is required for avoiding ghost

Berezhiani, Chkareuli, Gabadadze (2013)

Stability of screened solution

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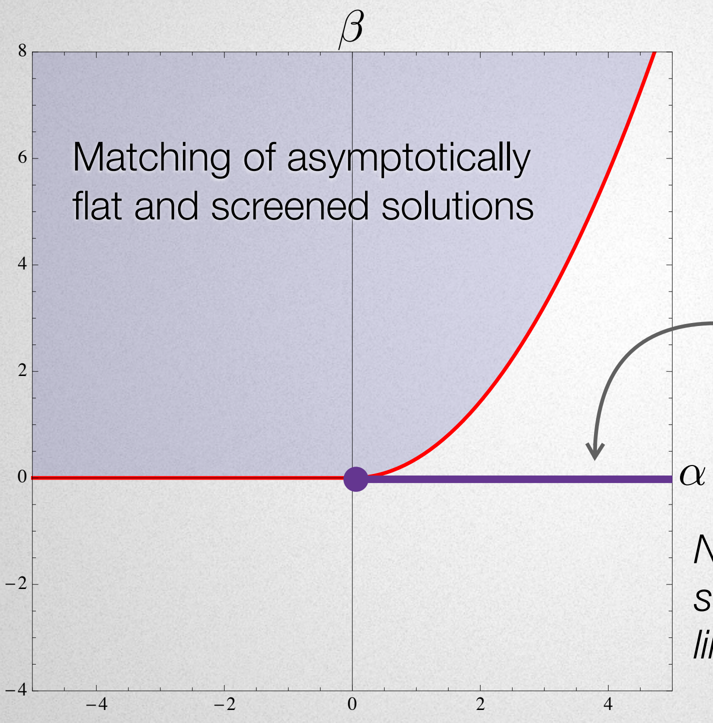
Taylor coefficients are strongly constrained by stability

The case of massive gravity



Stability

No stable, asymptotically flat, screened solution in decoupling limit of massive gravity

Some other topics

Application to inflation

- Horndeski's theory can be used as well to describe all single-field inflation models – **Generalized G-inflation**

  TK, Yamaguchi, Yokoyama (2011)

- **Higgs inflation**
  – Consistent with observations if nonminimal coupling or nonstandard kinetic term is introduced
  Cervantes-Cota, Dehnen (1995); Bezrukov, Shaposhnikov (2008); ……
  – All of those models can be studied in a unified manner using Horndeski's theory *Kamada et al. (2012)*

  – Higgs G(alileon)-inflation *Kamada et al. (2011, 2013)*

Talk by Taro Kunimitsu on Thursday

Nicolis et al. (2009); Creminelli et al. (2012); Hinterbichler et al. (2013)

Stable violation of NEC

- Einstein gravity + "$\mathcal{L}_\phi = P(\phi, X)$"
  – **Null energy condition** (NEC)

$$-2M_{\rm Pl}^2 \dot{H} = \rho + P > 0$$

  is forced by stability

- Galileon terms stabilize fluctuations around NEC violating background

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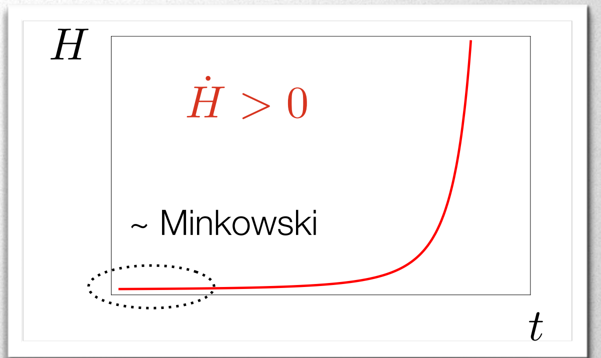
"Galilean genesis"

Starting the Universe from Minkowski?



Galilean genesis

- Example: $\mathcal{L}_\phi = a_1 e^{2\lambda\phi} X + a_2 X^2 + a_3 X\Box\phi$ *Nicolis et al. (2009); Creminelli et al. (2012)*

Genesis solution – $e^{\lambda\phi} \propto \frac{1}{(-t)}, \quad H \propto \frac{1}{(-t)^3}, \quad a = 1 + \frac{\text{const}}{(-t)^2} \quad (-\infty < t < 0)$

⇨ $\dot{H} > 0,$ Minkowski in asymptotic past, stable

- More complicated DBI-type Lagrangian *Hinterbichler et al. (2013)*

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- All those models admitting genesis solution can be described by *TK, Nishi, Tanahashi, Yamaguchi, in preparation*

$$G_2 = e^{4\lambda\phi} g_2(Y), \quad G_3 = e^{2\lambda\phi} g_3(Y), \quad G_4 = \frac{M_{\rm Pl}^2}{2} + e^{2\lambda\phi} g_4(Y),$$
$$G_5 = e^{-2\lambda\phi} g_5(Y), \quad \text{where } Y = e^{-2\lambda\phi} X$$

- Matching to standard cosmology, perturbations, …

See poster by Sakine Nishi (P20)

Galilean genesis

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⇨ $\dot{H} > 0$, Minkowski in asymptotic past, stable

- More complicated DBI-type Lagrangian — *Hinterbichler et al. (2013)*
- All those models admitting genesis solution can be described by — *TK, Nishi, Tanahashi, Yamaguchi, in preparation*

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- Matching to standard cosmology, perturbations, …

See poster by Sakine Nishi (P20)

Galilean genesis

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Multi-field extension?

Take the same route… — *Deffayet et al. (2010); Padilla et al. (2010)*

✓ **Multi-galileons** in flat space

$$\mathcal{L} = \sum_{I,J,K,\cdots=1}^{N} \left(a_{IJ} X^{IJ} + b_{IJK} X^{IJ} \partial^2 \phi^K + \cdots \right)$$

where $X^{IJ} := -\frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi^J$

Galilean shift symmetry: $\partial_\mu \phi^I \to \partial_\mu \phi^I + b_\mu^I$

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⇩ *Promote X^{IJ} to functions of ϕ^I and X^{IJ}*

✓ **Generalized** multi-galileons

Padilla, Sibanesan (2013)

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More general than $\mathcal{L} = P(X^{IJ}, \phi^K)$

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Generalized multi-galileons

$$\begin{aligned}\mathcal{L} \;=\; & G_2(X^{IJ},\phi^K) - G_{3L}(X^{IJ},\phi^K)\Box\phi^L + G_4(X^{IJ},\phi^K)R \\ & + G_{4,\langle IJ\rangle}\left(\Box\phi^I\Box\phi^J - \nabla_\mu\nabla_\nu\phi^I\nabla^\mu\nabla^\nu\phi^J\right) + G_{5L}(X^{IJ},\phi^K)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi^L \\ & - \frac{1}{6}G_{5I,\langle JK\rangle}\left[\Box\phi^I\Box\phi^J\Box\phi^K - 3\Box\phi^{(I}\nabla_\mu\nabla_\nu\phi^J\nabla^\mu\nabla^\nu\phi^{K)} + 2\nabla_\mu\nabla_\nu\phi^I\nabla^\nu\nabla^\lambda\phi^J\nabla_\lambda\nabla^\mu\phi^K\right]\end{aligned}$$

Symmetrized derivative:

$$G_{,\langle IJ\rangle} := \frac{1}{2}\left(\frac{\partial G}{\partial X^{IJ}} + \frac{\partial G}{\partial X^{JI}}\right)$$

2nd-order equations are maintained by imposing that

$$G_{3IJK} := G_{3I,\langle JK\rangle}, \quad G_{4IJKL} := G_{4,\langle IJ\rangle,\langle KL\rangle},$$
$$G_{5IJK} := G_{5I,\langle JK\rangle}, \quad G_{5IJKLM} := G_{5IJK,\langle LM\rangle},$$

are symmetric in all of indices *I, J, K, …*

Is this the most general multi-scalar-tensor theory with 2nd-order field equations?

– Padilla and Sibanesan conjectured YES

TK, Tanahashi, Yamaguchi (2013)

This is NOT the most general

Counterexample: **generalization of multi-field DBI**

Renaux-Petel et al. (2011)

$$\mathcal{L} = \sqrt{-\gamma}\left(-\lambda + \frac{M^2}{2} R[\gamma]\right) \qquad \gamma_{\mu\nu} = g_{\mu\nu} + f\delta_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J$$

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Usual multi-DBI

!

$$\mathcal{L} \supset \delta_{IJ}\delta_{KL}L^{\mu\alpha\nu\beta}\partial_\mu\phi^I\partial_\nu\phi^J\partial_\alpha\phi^K\partial_\beta\phi^L$$

$L^{\mu\alpha\nu\beta} := R^{\mu\alpha\nu\beta} + \cdots$: Double-dual Riemann

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✓ 2nd-order field equations

✓ Nonzero provided that spacetime is curved and there are multiple fields

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- ✓ 2nd-order field equations
- ✓ Nonzero provided that spacetime is curved and there are multiple fields
- ✓ **Not included in generalized multi-galileons** (even after integration by parts)

Summary

- **Horndeski's theory** – *the most general scalar-tensor theory with second-order field equations* – is a very useful framework for studying modified gravity (and other interesting aspects of cosmology)

- **Vainshtein mechanism** – *screening fifth force*

- The most general *multi*-scalar-tensor theory – incomplete; need more systematic way

  Ongoing project w/ Gao, Ohashi, Tanahashi, Yamaguchi

Thank you

Examples

- Nonminimal coupling (traditional scalar-tensor theory, *f*(*R*), …)

$$G_4(\phi, X) = f(\phi) \quad \longrightarrow \quad f(\phi)R$$

- DGP effective theory (= cubic galileon)

$$G_3(\phi, X) = X \quad \longrightarrow \quad X\Box\phi$$

- Decoupling limit of massive gravity

$$G_4 = \frac{M_{\rm Pl}^2}{2} + M_{\rm Pl}\phi + \frac{M_{\rm Pl}}{\Lambda^3}\alpha X, \quad G_5 = -3\frac{M_{\rm Pl}}{\Lambda^6}\beta X$$

- Nonminimal coupling to Gauss-Bonnet

$$\xi(\phi)\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)$$

$$G_2 = 8\xi^{(4)}X^2(3-\ln X), \quad G_3 = 4\xi^{(3)}X(7-3\ln X),$$
$$G_4 = 4\xi^{(2)}X(2-\ln X), \quad G_5 = -4\xi^{(1)}\ln X, \quad \xi^{(n)} := \frac{\partial^n\xi}{\partial\phi^n}$$

"Derivative interactions in nonlinear massive gravity"

by Rampei Kimura

[JGRG23(2013)110502]

Derivative interactions in nonlinear massive gravity

Rampei Kimura
RESCEU, University of Tokyo

JGRG 2013 @ Hirosaki University

Based on
RK, Daisuke Yamauchi
Phys. Rev. D 88, 084025 (2013) [arXiv:308.0523]

Contents of this talk

1. *Fierz-Pauli and dRGT massive gravity*
2. *Derivative interaction in Fierz-Pauli massive gravity*
3. *Nonlinear derivative interactions*
4. *Summary*

Motivation

Can we construct healthy massive gravity?

"*Linear*" massive gravity

- *Fierz-Pauli massive gravity* (Fierz, Pauli, 1939)

$$S = M_{\rm Pl}^2 \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{4} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \right]$$

Linearized Einstein-Hilbert term

Only allowed mass term which does not have ghost at linear order

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = -\frac{1}{2}(\Box h_{\mu\nu} - \partial_\mu \partial_\alpha h^\alpha_\nu - \partial_\nu \partial_\alpha h^\alpha_\mu + \partial_\mu \partial_\nu h^\alpha_\alpha - \eta_{\mu\nu} \Box h^\alpha_\alpha + \eta_{\mu\nu} \partial_\alpha \partial_\beta h^\alpha_\beta)$$

(1) *Linear theory*
(2) *Lorentz invariant theory*
(3) *No ghost at linear order (5 DOF)*
(4) *Simple nonlinear extension contains ghost at nonlinear level*
(Boulware-Deser ghost, 6th DOF) (Boulware, Deser, 1971)

"*Nonlinear*" massive gravity

- *de Rham-Gabadadze-Tolley massive gravity* (de Rham, Gabadadze, Tolley, 2011)

$$S_{MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right] + S_m[g_{\mu\nu},\psi]$$

$$\mathcal{U}_2 = \varepsilon_{\mu\alpha\rho\sigma}\varepsilon^{\nu\beta\rho\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta$$
$$\mathcal{U}_3 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\rho}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta$$
$$\mathcal{U}_4 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta\mathcal{K}^\rho{}_\sigma$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{\delta^\mu{}_\nu - H^\mu{}_\nu}$$
$$= \delta^\mu{}_\nu - \sqrt{\eta_{ab}g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b}$$

ϕ^a is called Stuckelberg field, which restores general covariance

(1) *Nonlinear theory*
(2) *Lorentz invariant theory*
(3) *No ghost at full order (5 DOF, No BD ghost)* (Hassan, Rosen, 2011)
(4) *Unique theory of massive spin-2 field as an extension of general relativity*

Derivative interaction

- *Fierz-Pauli mass term*

$$\mathcal{U}_{\rm FP} = \varepsilon^{\mu\alpha\rho\sigma}\varepsilon^{\nu\beta}{}_{\rho\sigma}h_{\mu\nu}h_{\alpha\beta}$$

→ *h_{00} becomes a Lagrange multiplier*

- *Derivative interaction in Fierz-Pauli theory* (Kurt Hinterbichler, 2013)

$$\mathcal{L}_{2,3} \sim M_{\rm Pl}^2\,\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\delta}\partial_\mu\partial_\alpha\, h_{\nu\beta}\, h_{\rho\gamma}\, h_{\sigma\delta}$$

Levi-Civita structure ensures that the Lagrangian is linear in h_{00}

→ *h_{00} becomes a Lagrange multiplier, which kills BD ghost*

Our work : Is there any consistent "nonlinear" derivative interactions in de Rham-Gabadadze-Tolley massive gravity??

$$S_{MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right] + S_{int} + S_m[g_{\mu\nu},\psi],$$

Decoupling limit

de Rham, Gabadadze (2010)

- *Decoupling limit : Easy to capture high energy behavior within Compton wavelength of massive graviton*

$$H_{\mu\nu} \to \frac{h_{\mu\nu}}{M_{\rm Pl}} + 2\frac{\partial_\mu\partial_\nu\pi}{M_{\rm Pl}m^2} - \frac{\partial_\mu\partial_\alpha\pi\partial_\mu\partial^\alpha\pi}{M_{\rm Pl}^2 m^4}$$

π is the scalar mode of massive graviton

$$M_{\rm Pl}\to\infty, \qquad m\to 0, \qquad \Lambda_3 = (M_{\rm Pl}m^2)^{1/3} = \text{fixed}, \qquad \frac{T_{\mu\nu}}{M_{\rm Pl}} = \text{fixed}$$

- *dRGT Lagrangian in the decoupling limit*

$$\mathcal{L}_{\rm DL} = -\frac{1}{4}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + \frac{1}{M_{\rm Pl}}h^{\mu\nu}T_{\mu\nu}$$

Standard gravity part

$$-h^{\mu\nu}\left[\frac{1}{4}\varepsilon_\mu{}^{\rho\gamma\alpha}\varepsilon_{\nu\rho\gamma}{}^{\beta}\Pi_{\alpha\beta} + \frac{3\alpha_3+4}{16\Lambda_3^3}\varepsilon_\mu{}^{\gamma\alpha\rho}\varepsilon_{\nu\gamma}{}^{\beta\sigma}\Pi_{\alpha\beta}\Pi_{\rho\sigma} + \frac{\alpha_3+4\alpha_4}{16\Lambda_3^6}\varepsilon_\mu{}^{\alpha\gamma\rho}\varepsilon_\nu{}^{\beta\delta\sigma}\Pi_{\alpha\beta}\Pi_{\gamma\delta}\Pi_{\rho\sigma}\right]$$

Galileon type interactions

- *2nd order differential EOM (NO BD ghost)*
- *Cutoff energy scale is Λ_3*

$$\varepsilon\varepsilon\Pi \equiv \varepsilon^{\mu\alpha\beta\gamma}\varepsilon^{\nu}{}_{\alpha\beta\gamma}\partial_\mu\partial_\nu\pi$$

$$\varepsilon_\mu\varepsilon_\nu\Pi \equiv \varepsilon_\mu{}^{\alpha\gamma\delta}\varepsilon_\nu{}^{\beta}{}_{\gamma\delta}\partial_\alpha\partial_\beta\pi$$

$$\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$$

Guidelines for construction of Lagrangian

- *Candidates for derivative interactions by using the Riemann tensor*

$$\mathcal{L}_{int} \supset M_{\rm Pl}^2\sqrt{-g}HR,\ M_{\rm Pl}^2\sqrt{-g}H^2R,\ M_{\rm Pl}^2\sqrt{-g}H^3R,\ \cdots$$

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

- *Guidelines*

(1) Linearization of $h_{\mu\nu}$ reproduces Fierz-Pauli theory

- *Lorentz invariance*
- *Free of Boulware-Deser ghost at linear level*

(2) Cut off energy scale is Λ_3

- *All nonlinear terms below Λ_3 have to be eliminated*

(3) Free of Boulware-Deser ghost

Derivative interactions and its energy scales

- *General form of Lagrangian*

$$\mathcal{L}_{int} \supset M_{\rm Pl}^2\sqrt{-g}HR,\ M_{\rm Pl}^2\sqrt{-g}H^2R,\ M_{\rm Pl}^2\sqrt{-g}H^3R,\ \cdots$$

- *The Lagrangian in the decoupling limit can be schematically written as*

$$\mathcal{L}_{int} \sim \Lambda_\lambda^{2-n_h-3n_\pi} h^{n_h-1}\partial^2 h\,(\partial^2\pi)^{n_\pi}$$

	n_h =1	n_h =2
n_π =1	∞	Λ_3
n_π =2	Λ_5	Λ_3
n_π =3	Λ_4	Λ_3
n_π =4	$\Lambda_{11/3}$	Λ_3
...	...	...
n_π =n	$\Lambda_{(3n-1)/(n-1)}$	Λ_3

These has to be eliminated

$$\Lambda_\lambda = (M_p m^{\lambda-1})^{1/\lambda}$$

$$\lambda = \frac{n_h + 3n_\pi - 2}{n_h + n_\pi - 2}$$

HR order term

- *General Lagrangian of HR order*

$$\mathcal{L}_{int,1} = M_{\rm Pl}^2\sqrt{-g}H_{\mu\nu}(R^{\mu\nu} + d\,Rg^{\mu\nu})$$

Linearizing $h_{\mu\nu}$ gives the same order of the linearized Einstein-Hilbert

$$\mathcal{L}_{int,1}^{(2)} \propto M_{\rm Pl}^2\left[\sqrt{-g}R\right]_{h^2}$$

$$\rightarrow \quad d = -1/2$$

- *In terms of Levi-Civet symbol,*

$$\mathcal{L}_{int,1} = M_{\rm Pl}^2\sqrt{-g}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}R_{\mu\alpha\nu\beta}\,H_{\rho\gamma}$$

The Lagrangian satisfies requirement (1) : Fierz-Pauli theory at linear theory

HR order term in the decoupling limit

- *The lowest order term in the decoupling limit*

$$\mathcal{L}_{int,1}\Big|_{\partial^2 h\,\partial^2\pi} = -\frac{2}{m^2}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu\partial_\alpha\, h_{\nu\beta}\,\partial_\rho\partial_\gamma\pi$$
$$= -\frac{2}{m^2}\partial_\gamma(\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu\partial_\alpha\, h_{\nu\beta}\,\partial_\rho\pi)$$

Total derivative

- *The next order term in the decoupling limit*

$$\mathcal{L}_{int,1}\Big|_{\partial^2 h\,(\partial^2\pi)^2} = \frac{1}{\Lambda_5^5}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu\partial_\alpha\, h_{\nu\beta}\,\partial_\rho\partial_a\pi\partial^a\partial_\gamma\pi$$

This is not zero or total derivative

→ *The counter part of this term can eliminate this term*

$$\mathcal{L}_{int,1,2} = \frac{1}{4}M_{\rm Pl}^2\sqrt{-g}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}R_{\mu\alpha\nu\beta}\,H_{\rho a}H^a{}_\gamma$$

Λ_5 term is eliminated !

HR order term in the decoupling limit

- *HR order Lagrangian*

$$\mathcal{L}_{int,1} = M_{\rm Pl}^2\sqrt{-g}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}R_{\mu\alpha\nu\beta}\,H_{\rho\gamma}$$
$$\sim \Lambda_\lambda^{2-n_h-3n_\pi}h^{n_h-1}\partial^2 h\,(\partial^2\pi)^{n_\pi}\Big|_{\rm DL}$$

	$n_h = 1$
$n_\pi = 1$	∞
$n_\pi = 2$	Λ_5
$n_\pi = 3$	Λ_4
$n_\pi = 4$	$\Lambda_{11/3}$
...	...
$n_\pi = n$	$\Lambda_{(3n-1)/(n-1)}$

Automatically total derivative ($n_\pi = 1$)

These terms can be always eliminated by adding higher order terms ($n_\pi = 2, \ldots, n$)

HR order term

- *The total Lagrangians including counter terms is given by*

$$\mathcal{L}_{int,1} = M_{\rm Pl}^2 \sqrt{-g}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma} R_{\mu\alpha\nu\beta}$$
$$\times \left(H_{\rho\gamma} + \frac{1}{4} H_{\rho a} H^{a}{}_{\gamma} + \frac{1}{8} H_{\rho a} H^{a}{}_{b} H^{b}{}_{\gamma} + \frac{5}{64} H_{\rho a} H^{a}{}_{b} H^{b}{}_{c} H^{c}{}_{\gamma} + \cdots \right)$$

$$= 2\,\mathcal{K}_{\rho\gamma}$$

$$\mathcal{K}^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \sqrt{\delta^{\mu}{}_{\nu} - H^{\mu}{}_{\nu}} = -\sum_{n=1}^{\infty} \bar{d}_n (H^n)^{\mu}_{\nu},$$

- *The final Lagrangian of HR order term*

$$\mathcal{L}_{int,1} = M_{\rm Pl}^2 \sqrt{-g}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma} R_{\mu\alpha\nu\beta}\,\mathcal{K}_{\rho\gamma}$$

The Lagrangian satisfies requirements (2) : Λ_3 theory in the decoupling limit

H²R order term

- *Starting point of the Lagrangian is*

$$\mathcal{L}_{int,2} = M_{\rm Pl}^2 \sqrt{-g}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\delta} R_{\mu\alpha\nu\beta}\, H_{\rho\gamma}\, H_{\sigma\delta}$$

This is the only combination that the lowest order Λ_5 term becomes a total derivative

- *With the same method of the previous case, we get the resumed Lagrangian of H^2R order term*

$$\mathcal{L}_{int,2} = M_{\rm Pl}^2 \sqrt{-g}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\delta} R_{\mu\alpha\nu\beta}\,\mathcal{K}_{\rho\gamma}\,\mathcal{K}_{\sigma\delta}$$

- *H^3R or higher order terms??*

In four dimension, there is no total derivative combination of the lowest order term in the decoupling limit

General derivative interaction Lagrangians

- *In 4 dimension, the derivative interaction for massive graviton is*

$$\mathcal{L}_{int} = M_{\rm Pl}^2 \sqrt{-g}\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} R_{\mu\alpha\nu\beta} \left(\alpha\, g_{\rho\gamma}\, \mathcal{K}_{\sigma\delta} + \beta\, \mathcal{K}_{\rho\gamma}\, \mathcal{K}_{\sigma\delta}\right)$$

α and β are parameters

- *We can also construct derivative interactions in arbitrary dimensions D*

$$\mathcal{L}_{int}^{(D,d,m)} = M_{\rm Pl}^{D-2} m^{2-d} \sqrt{-g}\, \varepsilon^{\mu_1\mu_2\cdots\mu_D} \varepsilon^{\nu_1\nu_2\cdots\nu_D} R_{\mu_1\nu_1\mu_2\nu_2} \cdots R_{\mu_{d-1}\nu_{d-1}\mu_d\nu_d} \times g_{\mu_{d+1}\nu_{d+1}} \cdots g_{\mu_m\nu_m} \mathcal{K}_{\mu_{m+1}\nu_{m+1}} \cdots \mathcal{K}_{\mu_D\nu_D}$$

d is even number

$2 \leq d \leq m \leq D-1$

These Lagrangians satisfy the requirements (1) and (2)

Boulware-Deser ghost??

- *We constructed the general nonlinear derivative interactions, but we still need to check the requirement (3) : the existence of BD ghost*
  - *Λ_3 theory in the decoupling limit*

$$\mathcal{L}_{\rm DL} \sim \boxed{\frac{1}{\Lambda_3^3}\pi\left[R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right]_{h^2}} + \boxed{\frac{1}{\Lambda_3^{3n_\pi}}\mathcal{O}[h\partial^2 h\,(\partial^2\pi)^{n_\pi}]}$$

EOM is 2nd order differential equation

These terms yield 4th order differential Eq for h and π

There are extra degrees of freedom, which leads to ghost...

Ghost appears at Λ_3

Other derivative interactions (in progress)

- *In 4 dimension, we found other Λ_3 derivative interactions without the Riemann tensor*

$$\mathcal{L}'_{int,1} = M_{\rm Pl}^2 \sqrt{-g}\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma}\, \nabla_\alpha \mathcal{K}_{\nu\beta}\, \nabla_\mu \mathcal{K}_{\rho\gamma},$$
$$\mathcal{L}'_{int,2} = M_{\rm Pl}^2 \sqrt{-g}\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta}\, \nabla_\alpha \mathcal{K}_{\nu\beta}\, \nabla_\mu \mathcal{K}_{\rho\gamma}\, F_{\delta\sigma}(H_{\alpha\beta})$$

- *Λ_3 theory in the decoupling limit*

$$\mathcal{L}_{\rm DL} \sim \frac{1}{\Lambda_3^3}\, \pi \left[R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right]_{h^2} + \frac{1}{\Lambda_3^{3n_\pi}} \mathcal{O}[h\partial^2 h\, (\partial^2\pi)^{n_\pi}]$$

EOM is 2nd order differential equation (coming from $L'_{int,2}$)

These terms yield 4th order differential Eq for h and π (coming from $L'_{int,1}$ and $L'_{int,2}$)

We cannot kill higher derivative terms in EOM even if we combine all four derivative interaction terms…

Summary

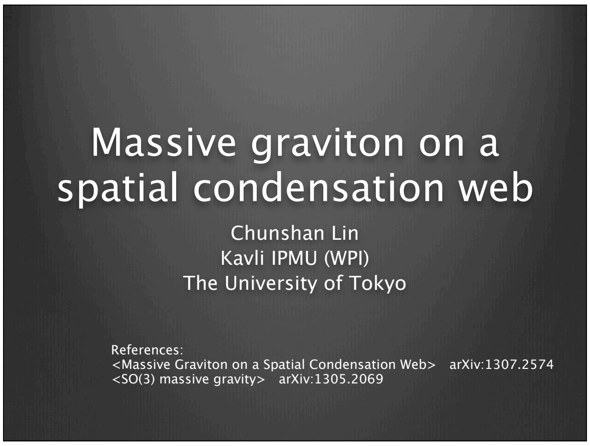
- *We found the most general derivative interactions in dRGT massive gravity*
  - *The energy scales below Λ_3 can be eliminated by adding counter terms*
  - *The Lagrangians can be resumed by using K tensor*
  - *The most general derivative interactions in dRGT theory contain four interactions*
  - *Nonlinear terms contribute at Λ_3*
- *Appropriate DOF?*
  - *4th order differential EOM of the scalar and tensor mode in the decoupling limit*
  - *Ghost appears at Λ_3 in dRGT theory + derivative interactions*

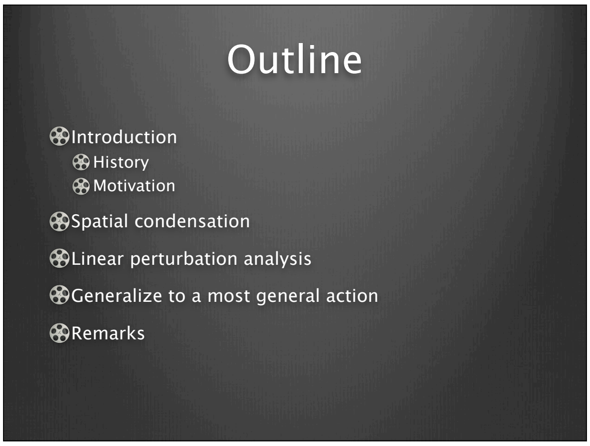
The mass scale of the derivative interactions should be $M < M_{pl}$

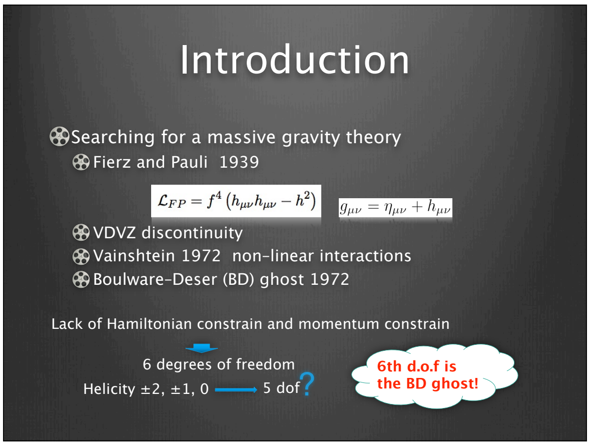
“Massive graviton on a spatial condensation web”

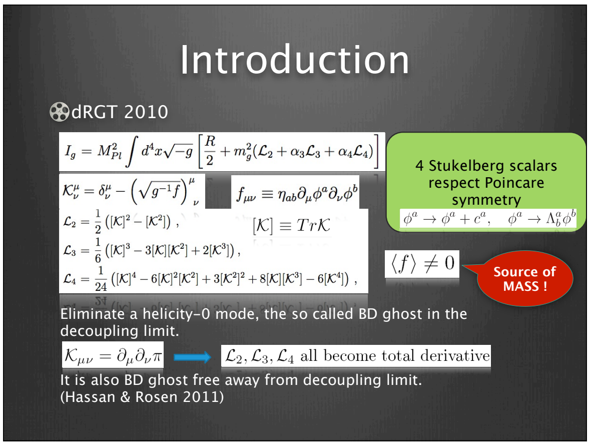
by Chunshan Lin

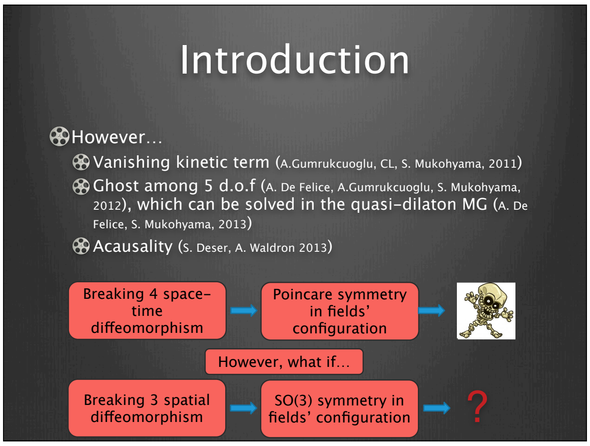
[JGRG23(2013)110503]

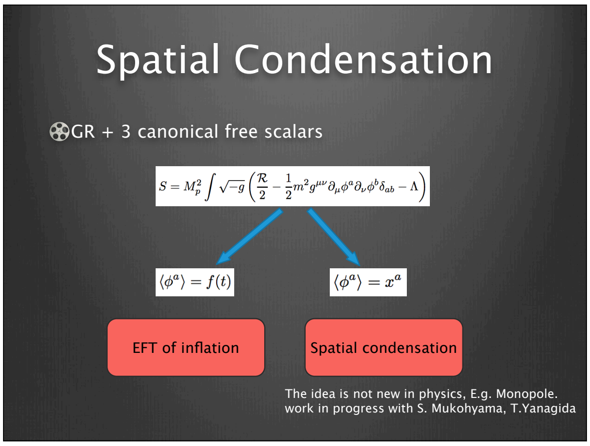

Massive graviton on a spatial condensation web
Chunshan Lin
Kavli IPMU (WPI)
The University of Tokyo
References:
<Massive Graviton on a Spatial Condensation Web> arXiv:1307.2574
<SO(3) massive gravity> arXiv:1305.2069

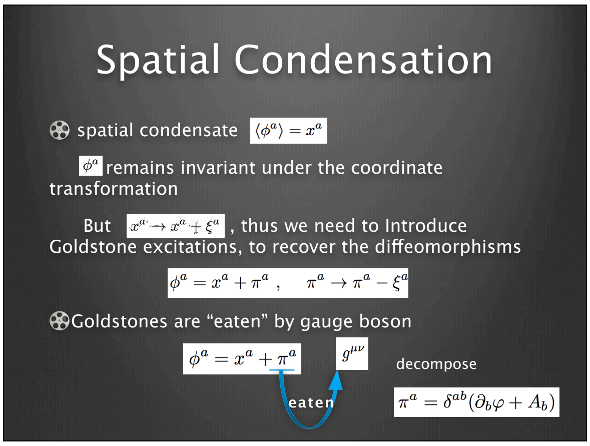

Outline
Introduction
History
Motivation
Spatial condensation
Linear perturbation analysis
Generalize to a most general action
Remarks


Introduction
Searching for a massive gravity theory
Fierz and Pauli 1939
$\mathcal{L}_{FP} = f^4 \left(h_{\mu\nu} h_{\mu\nu} - h^2 \right)$
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
VDVZ discontinuity
Vainshtein 1972 non-linear interactions
Boulware-Deser (BD) ghost 1972
Lack of Hamiltonian constrain and momentum constrain
6 degrees of freedom
Helicity ±2, ±1, 0 → 5 dof?
6th d.o.f is the BD ghost!

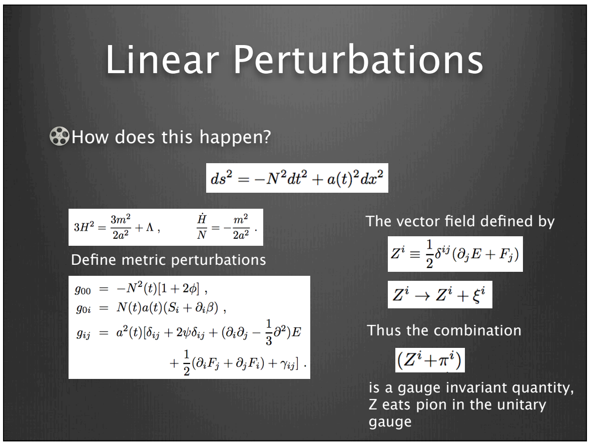

Introduction
dRGT 2010
$I_g = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right]$
$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} f} \right)^\mu_\nu$
$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$
$\mathcal{L}_2 = \frac{1}{2} \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right),$
$[\mathcal{K}] \equiv Tr\mathcal{K}$
$\mathcal{L}_3 = \frac{1}{6} \left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \right),$
$\mathcal{L}_4 = \frac{1}{24} \left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4] \right),$
4 Stukelberg scalars respect Poincare symmetry
$\phi^a \to \phi^a + c^a, \quad \phi^a \to \Lambda^a_b \phi^b$
$\langle f \rangle \neq 0$
Source of MASS !
Eliminate a helicity-0 mode, the so called BD ghost in the decoupling limit.
$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi$ → $\mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4$ all become total derivative
It is also BD ghost free away from decoupling limit.
(Hassan & Rosen 2011)


Introduction
However…
Vanishing kinetic term (A.Gumrukcuoglu, CL, S. Mukohyama, 2011)
Ghost among 5 d.o.f (A. De Felice, A.Gumrukcuoglu, S. Mukohyama, 2012), which can be solved in the quasi-dilaton MG (A. De Felice, S. Mukohyama, 2013)
Acausality (S. Deser, A. Waldron 2013)
Breaking 4 space-time diffeomorphism
Poincare symmetry in fields' configuration
However, what if…
Breaking 3 spatial diffeomorphism
SO(3) symmetry in fields' configuration
?

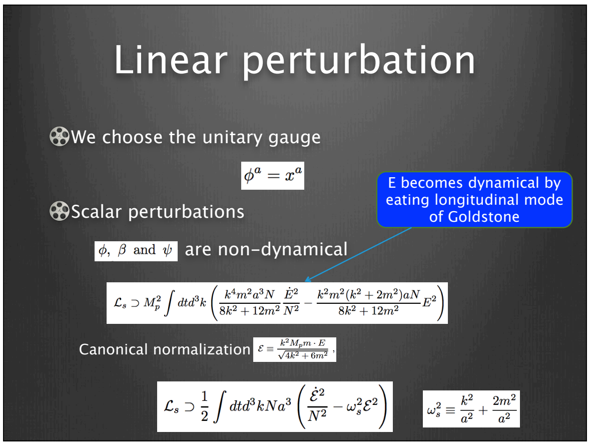

Spatial Condensation
GR + 3 canonical free scalars
$S = M_p^2 \int \sqrt{-g}\left(\frac{\mathcal{R}}{2} - \frac{1}{2}m^2 g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b\delta_{ab} - \Lambda\right)$
$\langle\phi^a\rangle = f(t)$
$\langle\phi^a\rangle = x^a$
EFT of inflation
Spatial condensation
The idea is not new in physics, E.g. Monopole.
work in progress with S. Mukohyama, T.Yanagida

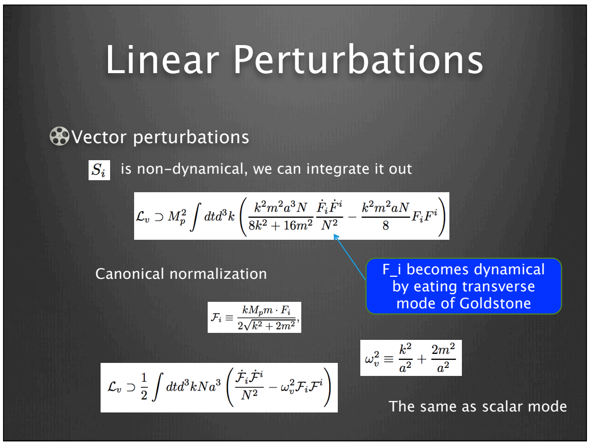


Spatial Condensation
spatial condensate $\langle \phi^a \rangle = x^a$
ϕ^a remains invariant under the coordinate transformation
But $x^a \to x^a + \xi^a$, thus we need to Introduce Goldstone excitations, to recover the diffeomorphisms
$\phi^a = x^a + \pi^a$, $\pi^a \to \pi^a - \xi^a$
Goldstones are "eaten" by gauge boson
$\phi^a = x^a + \pi^a$
$g^{\mu\nu}$
eaten
decompose
$\pi^a = \delta^{ab}(\partial_b \varphi + A_b)$



Linear Perturbations
How does this happen?
$ds^2 = -N^2 dt^2 + a(t)^2 dx^2$
$3H^2 = \frac{3m^2}{2a^2} + \Lambda$, $\frac{\dot{H}}{N} = -\frac{m^2}{2a^2}$.
Define metric perturbations
$g_{00} = -N^2(t)[1 + 2\phi]$,
$g_{0i} = N(t)a(t)(S_i + \partial_i \beta)$,
$g_{ij} = a^2(t)[\delta_{ij} + 2\psi\delta_{ij} + (\partial_i\partial_j - \frac{1}{3}\partial^2)E + \frac{1}{2}(\partial_i F_j + \partial_j F_i) + \gamma_{ij}]$.
The vector field defined by
$Z^i \equiv \frac{1}{2}\delta^{ij}(\partial_j E + F_j)$
$Z^i \to Z^i + \xi^i$
Thus the combination
$(Z^i + \pi^i)$
is a gauge invariant quantity, Z eats pion in the unitary gauge


Linear perturbation
We choose the unitary gauge
$\phi^a = x^a$
Scalar perturbations
E becomes dynamical by eating longitudinal mode of Goldstone
$\phi,\ \beta$ and ψ are non-dynamical
$\mathcal{L}_s \supset M_p^2 \int dt d^3k \left(\frac{k^4 m^2 a^3 N}{8k^2 + 12m^2} \frac{\dot{E}^2}{N^2} - \frac{k^2 m^2 (k^2 + 2m^2) a N}{8k^2 + 12m^2} E^2 \right)$
Canonical normalization $\mathcal{E} \equiv \frac{k^2 M_p m \cdot E}{\sqrt{4k^2 + 6m^2}}$,
$\mathcal{L}_s \supset \frac{1}{2} \int dt d^3k N a^3 \left(\frac{\dot{\mathcal{E}}^2}{N^2} - \omega_s^2 \mathcal{E}^2 \right)$
$\omega_s^2 \equiv \frac{k^2}{a^2} + \frac{2m^2}{a^2}$


Linear Perturbations
Vector perturbations
S_i is non-dynamical, we can integrate it out
$\mathcal{L}_v \supset M_p^2 \int dt d^3k \left(\frac{k^2 m^2 a^3 N}{8k^2 + 16m^2} \frac{\dot{F}_i \dot{F}^i}{N^2} - \frac{k^2 m^2 a N}{8} F_i F^i \right)$
F_i becomes dynamical by eating transverse mode of Goldstone
Canonical normalization
$\mathcal{F}_i \equiv \frac{k M_p m \cdot F_i}{2\sqrt{k^2 + 2m^2}}$,
$\omega_v^2 \equiv \frac{k^2}{a^2} + \frac{2m^2}{a^2}$
$\mathcal{L}_v \supset \frac{1}{2} \int dt d^3k N a^3 \left(\frac{\dot{\mathcal{F}}_i \dot{\mathcal{F}}^i}{N^2} - \omega_v^2 \mathcal{F}_i \mathcal{F}^i \right)$
The same as scalar mode

Linear Perturbations

- Tensor Perturbations

$$\mathcal{L}_T \supset M_p^2 \int dt d^3k \left[\frac{a^3}{4N}\dot{\gamma}_{ij}\dot{\gamma}^{ij} - \frac{(k^2+2m^2)aN}{4}\gamma_{ij}\gamma^{ij}\right]$$

Canonical normalization

$$\tilde{\gamma}_{ij} \equiv \frac{M_p}{2}\gamma_{ij},$$

$$\mathcal{L}_T \supset \frac{1}{2}\int dt d^3k N a^3 \left(\frac{\dot{\tilde{\gamma}}_{ij}\dot{\tilde{\gamma}}^{ij}}{N^2} - \omega_T^2 \tilde{\gamma}_{ij}\tilde{\gamma}^{ij}\right)$$

$$\omega_T^2 \equiv \frac{k^2}{a^2} + \frac{2m^2}{a^2}$$

Surprisingly! All 5 degrees have exactly the same dispersion relations!

Some remarks

- EFT is valid up to quantum gravity scale, say, Planck scale
- No vDVZ discontinuity, we recover GR at the massless limit work in progress (C. Lin and R. Kimura)...
- Apply to inflation, we expect...
  - IR safe inflation
  - Graviton mass suppressed large scale primordial perturbation
    It may be relative to another large scale anomaly:
    the fluctuations at multiple moment l <40 are smaller than expected by 5% to 10%.
  - B mode polarizations of CMB differs from GR

Generalization

- Taking SO(3) residual symmetry as building principle,

$$S = M_p^2 \int \sqrt{-g}\left[\frac{\mathcal{R}}{2} - m^2 \mathcal{U}\left(g^{\mu\nu}, f_{\mu\nu}\right)\right].$$

$$f_{\mu\nu} \equiv \partial_\mu \phi^a \partial_\nu \phi^b \delta_{ab}$$

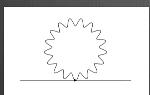
We are also free to add the derivative coupling term, e.g.

$$\text{Horndeski term } G^{\mu\nu} f_{\mu\nu},$$

Thank You!

IR safe inflation

- Review the IR divergence issue in GR



$$\mathcal{H}_I \supset \gamma_{ij}^2 \left(\partial_k \delta\sigma\right)^2$$

$$\delta g_{ij} = a(t)^2 \gamma_{ij}$$

Quadratic action for tensor modes

$$\mathcal{L}_T = \frac{M_p^2}{8} \int dt d^x a^3 \left[\dot{\gamma}_{ij}^2 - a^{-2} \partial_l \gamma_{ij} \partial_l \gamma_{ij}\right]$$

Equation of motion

$$\gamma_{ij}'' + 2aH\gamma_{ij}' - \frac{\nabla^2}{a^2}\gamma_{ij} = 0$$

quatization

$$\gamma_{ij}(x) = \sum_{s=\pm} \int d^3k \left[a(\mathbf{k}) \mathbf{e_{ij}}(\mathbf{k}, \mathbf{s}) \gamma_{\mathbf{k},\mathbf{s}} \mathbf{e}^{\mathbf{ik \cdot x}} + \mathbf{h.c.}\right]$$

Polarization tensor

$$e_{ij}(k,s) e^{ij}(k,s') = \delta_{ss'}$$

IR safe inflation

- Choose the Bounch-David vacuum, we get solution

$$\gamma_{\pm,k} = \frac{H}{(2\pi)^{3/2}\sqrt{k^3}}(1+ik\eta)e^{-ik\eta}$$

Power spectum

$$P_{GW}(k) = 2|\gamma_k(\eta)|^2 = \frac{2H^2}{(2\pi)^3 k^3}\left[1+\mathcal{O}(k^2\eta^2)\right]$$

Loop obtained from contracting two γs

$$\begin{aligned}\langle\zeta(x)\zeta(x)\rangle_{1loop} &\propto \int d^3k P_{GW}(k)\\ &\propto \int \frac{dk}{k}\,.\end{aligned}$$

Diverges at UV and IR

IR safe inflation

- Inflation on a spatial condensation web

$$S = \int\sqrt{-g}\left(\frac{M_p^2}{2}\mathcal{R} - M_p^2 m^2\sum_{a=1}^{3}\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma)\right),$$

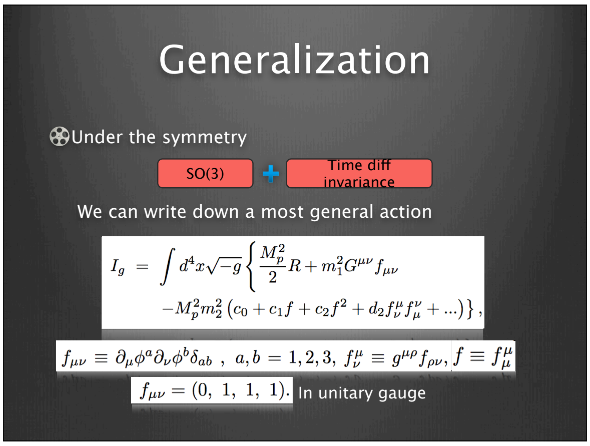
- Spatial condensation a^-2, diluted away rapidly
- Curvature perturbation is still scale invariant
- Tensor modes get a mass

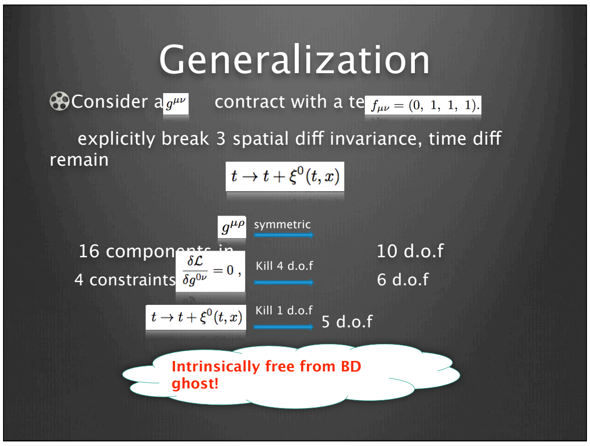
$$\gamma_{\pm,k} = \frac{H}{(2\pi)^{3/2}\sqrt{\tilde{k}^3}}(1+i\tilde{k}\eta)e^{-i\tilde{k}\eta}$$

$$\tilde{k} \equiv \sqrt{k^2+2m^2}\,.$$

$$\begin{aligned}\langle\zeta(x)\zeta(x)\rangle_{1loop} &\propto \int_0^{a(t)H(t)} d^3k P_{GW}(\tilde{k})\\ &\propto Ht + \log\left(H/m\right),\end{aligned}$$

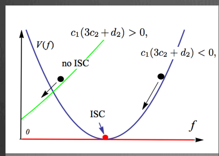
Converges at IR


Generalization
Under the symmetry
SO(3) + Time diff invariance
We can write down a most general action
$I_g = \int d^4x\sqrt{-g}\left\{\frac{M_p^2}{2}R + m_1^2 G^{\mu\nu} f_{\mu\nu} - M_p^2 m_2^2 \left(c_0 + c_1 f + c_2 f^2 + d_2 f^\mu_\nu f^\nu_\mu + \ldots\right)\right\},$
$f_{\mu\nu} \equiv \partial_\mu\phi^a\partial_\nu\phi^b\delta_{ab}\ ,\ a,b = 1,2,3,\ f^\mu_\nu \equiv g^{\mu\rho}f_{\rho\nu},\ f \equiv f^\mu_\mu$
$f_{\mu\nu} = (0,\ 1,\ 1,\ 1).$ In unitary gauge


Generalization
Consider a $g^{\mu\nu}$ contract with a te $f_{\mu\nu} = (0,\ 1,\ 1,\ 1).$
explicitly break 3 spatial diff invariance, time diff remain
$t \to t + \xi^0(t,x)$
$g^{\mu\rho}$ symmetric
16 components in
10 d.o.f
4 constraints $\frac{\delta\mathcal{L}}{\delta g^{0\nu}} = 0\,,$ Kill 4 d.o.f
6 d.o.f
$t \to t + \xi^0(t,x)$ Kill 1 d.o.f
5 d.o.f
Intrinsically free from BD ghost!

ISC

- Infinitely strong coupling issue



In the case of $c_1(3c_2+d_2)<0,$ By fine tuning the cosmological constant, cancel out the effective energy density and pressure of the mass term, one can get an Einstein static universe. However, scalar mode and vector mode have vanishing kinetic term. Without a mass gap, it is infinitely strong coupling.

But, we don't care...

- Einstein static universe is unstable by itself anyway;
- We never live in a static cosmic background;
- $c_1(3c_2+d_2)>0,$ is required to exclude the Einstein static solution.

Cosmology

- We take FRW ansatz

$$ds^2=-N^2dt^2+a(t)^2dx^2\ ,$$

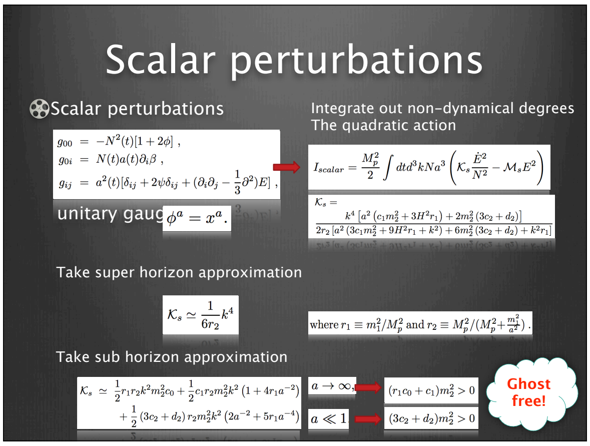
We have such two background Einstein equations

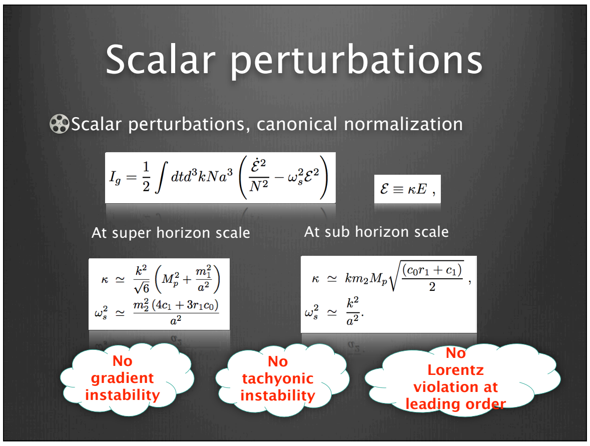
$$3H^2 = r_2 m_2^2\left[c_0+3c_1a^{-2}+3(3c_2+d_2)a^{-4}\right],$$

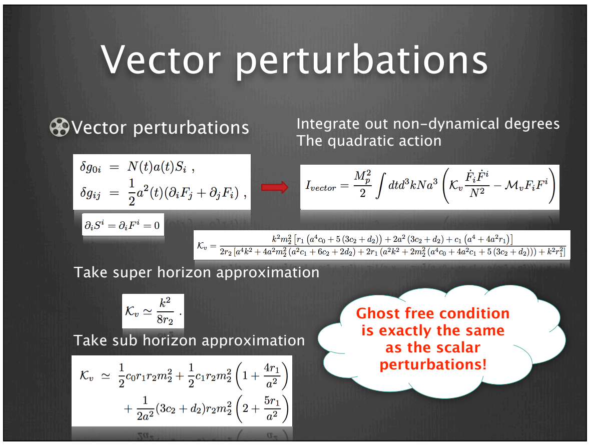
$$\frac{\dot{H}}{N} = r_2^2 m_2^2\left[\frac{r_1c_0-3c_1}{3a^2}-\frac{(2+r_1a^{-2})(3c_2+d_2)}{a^4}\right]$$

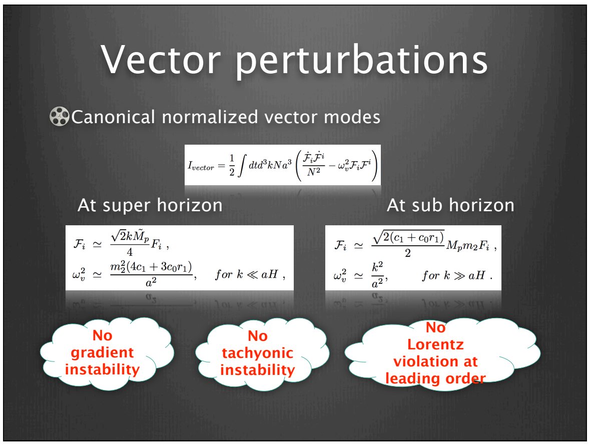
where $r_1\equiv m_1^2/M_p^2$ and $r_2\equiv M_p^2/(M_p^2+\frac{m_1^2}{a^2})$.

Noting that since the the SO(3) symmetry in the fields' configuration, the constraint equations of 3 Stuckelberg scalars are trivially satisfied!


Scalar perturbations
Scalar perturbations
$g_{00} = -N^2(t)[1+2\phi]$, $g_{0i} = N(t)a(t)\partial_i\beta$, $g_{ij} = a^2(t)[\delta_{ij} + 2\psi\delta_{ij} + (\partial_i\partial_j - \frac{1}{3}\partial^2)E]$,
unitary gauge $\phi^a = x^a$.
Integrate out non-dynamical degrees
The quadratic action
$I_{scalar} = \frac{M_p^2}{2}\int dtd^3kNa^3\left(\mathcal{K}_s\frac{\dot{E}^2}{N^2} - \mathcal{M}_sE^2\right)$
$\mathcal{K}_s = \frac{k^4\left[a^2\left(c_1m_2^2 + 3H^2r_1\right) + 2m_2^2\left(3c_2+d_2\right)\right]}{2r_2\left[a^2\left(3c_1m_2^2 + 9H^2r_1 + k^2\right) + 6m_2^2\left(3c_2+d_2\right) + k^2r_1\right]}$
Take super horizon approximation
$\mathcal{K}_s \simeq \frac{1}{6r_2}k^4$
where $r_1 \equiv m_1^2/M_p^2$ and $r_2 \equiv M_p^2/(M_p^2 + \frac{m_1^2}{a^2})$.
Take sub horizon approximation
$\mathcal{K}_s \simeq \frac{1}{2}r_1r_2k^2m_2^2c_0 + \frac{1}{2}c_1r_2m_2^2k^2\left(1+4r_1a^{-2}\right) + \frac{1}{2}\left(3c_2+d_2\right)r_2m_2^2k^2\left(2a^{-2}+5r_1a^{-4}\right)$
$a \to \infty$, $(r_1c_0 + c_1)m_2^2 > 0$
$a \ll 1$ $(3c_2+d_2)m_2^2 > 0$
Ghost free!


Scalar perturbations
Scalar perturbations, canonical normalization
$I_g = \frac{1}{2}\int dtd^3kNa^3\left(\frac{\dot{\mathcal{E}}^2}{N^2} - \omega_s^2\mathcal{E}^2\right)$
$\mathcal{E} \equiv \kappa E$,
At super horizon scale
$\kappa \simeq \frac{k^2}{\sqrt{6}}\left(M_p^2 + \frac{m_1^2}{a^2}\right)$
$\omega_s^2 \simeq \frac{m_2^2\left(4c_1 + 3r_1c_0\right)}{a^2}$
At sub horizon scale
$\kappa \simeq km_2M_p\sqrt{\frac{(c_0r_1+c_1)}{2}}$,
$\omega_s^2 \simeq \frac{k^2}{a^2}$.
No gradient instability
No tachyonic instability
No Lorentz violation at leading order


Vector perturbations
Vector perturbations
Integrate out non-dynamical degrees
The quadratic action
$\delta g_{0i} = N(t)a(t)S_i ,$
$\delta g_{ij} = \frac{1}{2}a^2(t)(\partial_i F_j + \partial_j F_i) ,$
$\partial_i S^i = \partial_i F^i = 0$
$I_{vector} = \frac{M_p^2}{2}\int dtd^3kNa^3\left(\mathcal{K}_v\frac{\dot{F}_i\dot{F}^i}{N^2} - \mathcal{M}_v F_i F^i\right)$
$\mathcal{K}_v = \frac{k^2 m_2^2\left[r_1\left(a^4c_0 + 5\left(3c_2 + d_2\right)\right) + 2a^2\left(3c_2 + d_2\right) + c_1\left(a^4 + 4a^2 r_1\right)\right]}{2r_2\left[a^4k^2 + 4a^2m_2^2\left(a^2c_1 + 6c_2 + 2d_2\right) + 2r_1\left(a^2k^2 + 2m_2^2\left(a^4c_0 + 4a^2c_1 + 5\left(3c_2 + d_2\right)\right)\right) + k^2r_1^2\right]}$
Take super horizon approximation
$\mathcal{K}_v \simeq \frac{k^2}{8r_2} .$
Take sub horizon approximation
$\mathcal{K}_v \simeq \frac{1}{2}c_0 r_1 r_2 m_2^2 + \frac{1}{2}c_1 r_2 m_2^2\left(1 + \frac{4r_1}{a^2}\right) + \frac{1}{2a^2}(3c_2 + d_2) r_2 m_2^2\left(2 + \frac{5r_1}{a^2}\right)$
Ghost free condition is exactly the same as the scalar perturbations!


Vector perturbations
Canonical normalized vector modes
$I_{vector} = \frac{1}{2}\int dtd^3kNa^3\left(\frac{\dot{\mathcal{F}}_i\dot{\mathcal{F}}^i}{N^2} - \omega_v^2\mathcal{F}_i\mathcal{F}^i\right)$
At super horizon
$\mathcal{F}_i \simeq \frac{\sqrt{2}k\tilde{M}_p}{4}F_i ,$
$\omega_v^2 \simeq \frac{m_2^2(4c_1 + 3c_0 r_1)}{a^2}, \quad for\ k \ll aH ,$
At sub horizon
$\mathcal{F}_i \simeq \frac{\sqrt{2(c_1 + c_0 r_1)}}{2}M_p m_2 F_i ,$
$\omega_v^2 \simeq \frac{k^2}{a^2}, \quad for\ k \gg aH .$
No gradient instability
No tachyonic instability
No Lorentz violation at leading order

Tensor perturbations

- Tensor perturbations

$$\delta g_{ij} = a(t)^2 \gamma_{ij} \,, \qquad \partial_i \gamma^{ij} = \gamma^i_i = 0 \,.$$

The quadratic action

$$I_{tensor} = \frac{M_p^2}{4} \int dt d^3k N a^3 \left(\mathcal{K}_T \frac{\dot{\gamma}_{ij}^2}{N^2} - \mathcal{M}_T^2 \gamma_{ij}^2 \right)$$

$$\mathcal{K}_T = \frac{1}{r_2} \,, \qquad \mathcal{M}_T^2 = \frac{k^2}{a^2} \cdot \left(1 + \frac{3r_1}{a^2}\right) + M_{GW}^2 \,.$$

Canonical normalization

$$\tilde{\gamma}_{ij} \equiv \sqrt{\frac{2}{r_2}} \cdot \gamma_{ij} \,.$$

$$I_{tensor} = \frac{M_p^2}{8} \int dt d^3k N a^3 \left[\frac{\dot{\tilde{\gamma}}_{ij}^2}{N^2} - \left(\frac{c_s^2 k^2}{a^2} + \tilde{M}_{GW}^2 \right) \tilde{\gamma}_{ij}^2 \right] ,$$

$$c_s^2 \equiv \frac{M_p^2 + \frac{3m_1^2}{a^2}}{M_p^2 + \frac{m_1^2}{a^2}} \simeq 1 \,, \qquad \tilde{M}_{GW}^2 \simeq \frac{m_2^2(4c_1 + 3c_0 r_1)}{a^2} \,.$$

Gravitation waves receive a mass!

UV cutoff

- Decoupling limit

$$m_2 \to 0, \; M_p \to \infty, \quad keeping \; (M_p m_2) \; fixed$$

The action for helicity 0 mode (roughly and schematically)

$$I_\varphi = M_p^2 m_2^2 \int \left(k^2 \dot{\varphi}^2 - k^4 \varphi^2 - k^6 \varphi^3 - k^8 \varphi^8 - ... \right) ,$$

Canonical normalization

$$\varphi^c \equiv (M_p m_2 k) \varphi \,,$$

$$I_\varphi = \int \dot{\varphi}^{c2} - k^2 \varphi^{c2} - \frac{k^3 \varphi^{c3}}{M_p m_2} - \frac{k^4 \varphi^{c4}}{M_p^2 m_2^2} - ...$$

EFT approach breaks down at energy scale

$$\Lambda_2 = \sqrt{M_p m_2},$$

Questions

- There are lots of questions we can ask
  - Massive graviton couples to matter;
  - Do we have black hole solutions?
  - The feature on the tensor mode;
  - The observational effect due to the scalar and vector modes?
  - Do we have a more general action?
  - Does it affect the structure formation
  - ……

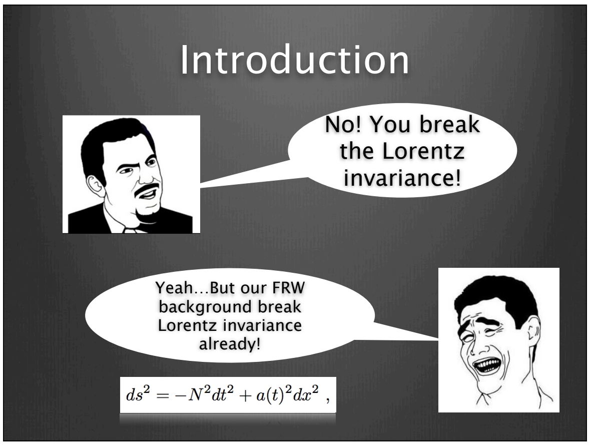
Conclusion

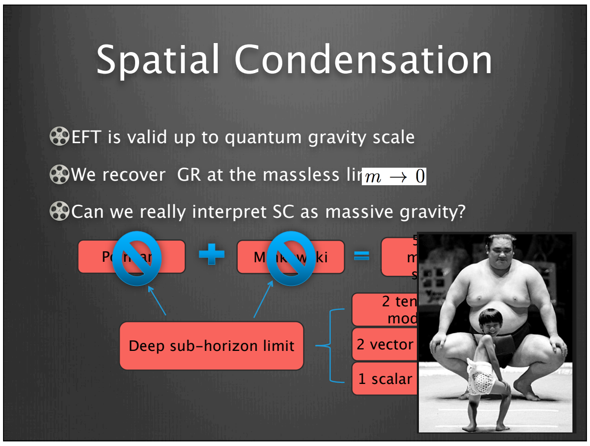
- Spatial condensation

$$\langle \phi^a \rangle = x^a$$

- It solves the IR divergence problem of inflationary correlation function
- Most general action

$$I_g = \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} R + m_1^2 G^{\mu\nu} f_{\mu\nu} - M_p^2 m_2^2 \left(c_0 + c_1 f + c_2 f^2 + d_2 f^\mu_\nu f^\nu_\mu + ... \right) \right\},$$


Introduction
No! You break the Lorentz invariance!
Yeah…But our FRW background break Lorentz invariance already!
$ds^2 = -N^2 dt^2 + a(t)^2 dx^2$,


Spatial Condensation
EFT is valid up to quantum gravity scale
We recover GR at the massless lim $m \to 0$
Can we really interpret SC as massive gravity?
Deep sub-horizon limit
2 vector
1 scalar

“Higher dimensional gravity and bigravity”

by Yasuho Yamashita

[JGRG23(2013)110504]

Higher dimensional gravity and bigravity

YITP, Kyoto University
Yasuho Yamashita
in collaboration with Takahiro Tanaka

13年11月5日火曜日

Introduction

13年11月5日火曜日

de Rham-Gabadadze-Tolley bigravity

dRGT bigravity model :

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}}{2}(R + V(g,\tilde{g})) + L_m\right] + \frac{\chi M_{pl}}{2}\int d^4x\sqrt{-\tilde{g}}\tilde{R}$$

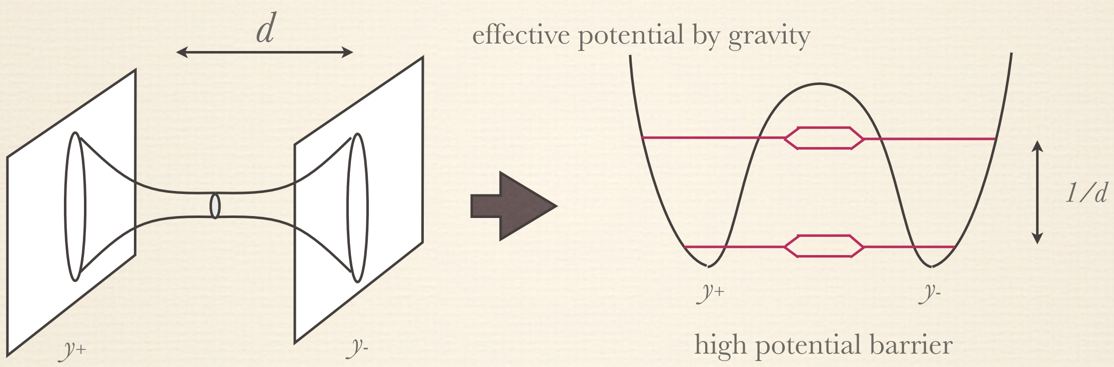
$$V = m^2\sum_{n=0}^{4} c_n\epsilon^{\mu_1\cdots\mu_n}_{\nu_1\cdots\nu_n}K^{\nu_1}_{\mu_1}\cdots K^{\nu_n}_{\mu_n}\ ,\ K^{\mu}_{\nu} = (\sqrt{g^{-1}\tilde{g}})^{\mu}_{\nu}$$

no ghost condition determines the form of interaction
... technical and artificial

We want to embed this model to higher dimensional gravity.

How ?

4-dim mass spectrum ~ eigenvalue problem in quantum mechanics



high potential barrier
→ nearly degenerate two small mass

However high potential barrier = thin throat ...unstable

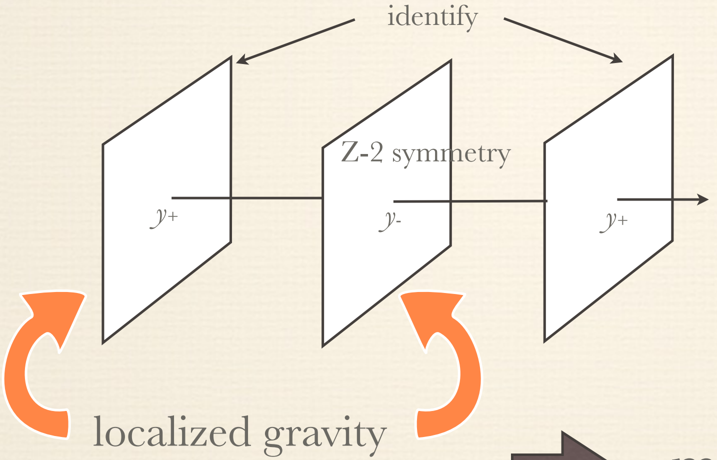
$\kappa_4^2\int d^4x\sqrt{-g^{(4)}}R^{(4)}$ can take its place → **DGP model**

2-brane Dvali-Gabadadze-Poratti model

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}R + \sum_{\sigma=\pm}\int d^4x\sqrt{-g_\sigma^{(4)}}\left(\frac{1}{2\kappa_{4(\sigma)}^2}R_\sigma^{(4)} + L_m\right)$$

parameter

$$r_c^{(\pm)} = \frac{\kappa^2}{2\kappa_{4(\pm)}^2}$$



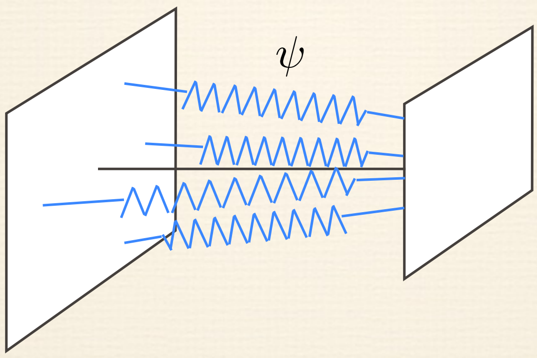
→ mass hierarchy

stabilization mechanism (Goldberger & Wise)

Radion = brane separation

→ scalar d.o.f in addition to two gravitons

...We need to remove this to reproduce bigravity



$$S_s = \int d^5x\sqrt{-g}\left(-\frac{1}{2}g^{ab}\psi_{,a}\psi, b - V_B(\psi) - \sum_{\sigma=\pm}V_{(\sigma)}(\psi)\delta(y-y_\sigma)\right)$$

mass spectrum

graviton's mass spectrum

massless mode always exists

the lowest massive mode

junction condition:

$$K^{(\pm)}_{\mu\nu} = r_c^{(\pm)} \left(G^{\pm(4)}_{\mu\nu} - \frac{1}{3} G^{\pm(4)} g_{\mu\nu} \right)$$

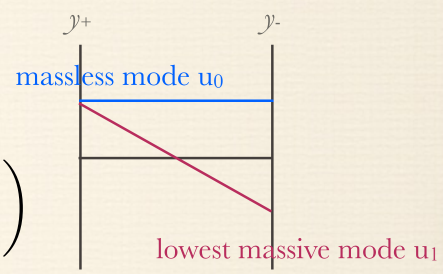
$$\updownarrow$$

$$g_{\mu\nu}/d \simeq r_c \Box^{(4)} g_{\mu\nu} = r_c m_1^2 g_{\mu\nu}$$

$$\Rightarrow \quad m_1^2 \simeq \frac{1}{r_c d} \ll \frac{1}{d^2}$$

hierarchy

eigenfunctions



mass spectrum (scalar mode)

stabilization mechanism → **There is no zero mode!**

If stabilization is weak; $\left|\frac{\mathcal{H}'}{\mathcal{H}^2}\right| \ll 1 \quad \rightarrow \quad \mu^2 \simeq \frac{2\sum_\sigma \frac{\sigma\mathcal{H}a^{-2}}{1-\sigma 2 r_c^{(\sigma)}\mathcal{H}_\sigma}}{\int_{y_+}^{y_-}\frac{\mathcal{H}^2}{a^4(-\mathcal{H}')}}$

$\mathcal{H}$: 5-d curvature scale

stronger stabilization → **large μ^2**

$1 \mp 2r_c^{(\pm)}\mathcal{H}_\pm < 0$ make μ^2 negative : ghost !!

→ corresponds to the **self accelerating branch**

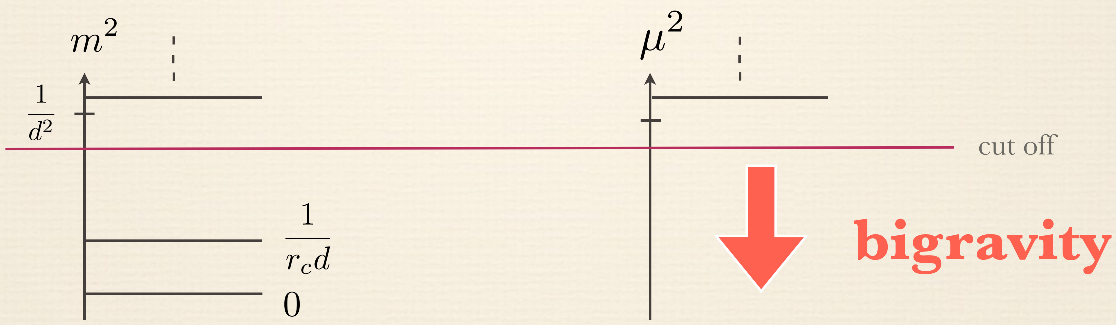
: K.Izumi et. al. (2007)

bigravity as the effective theory

For small d, large r_c and strong stabilization,

TT mode :
one 0 mode
one massive mode

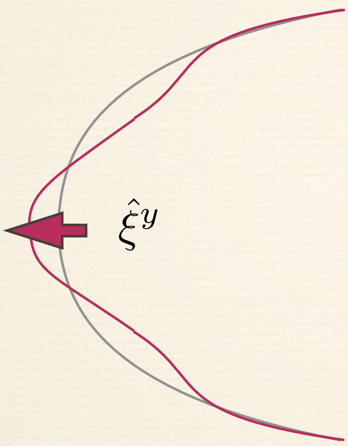
scalar mode :
no mode



Correspondence between DGP 2-brane model and dRGT bigravity

metrics on branes

location of the brane is also perturbed: $y = y_\pm + \hat{\xi}^y_\pm$



$\hat{\xi}^y \leftrightarrow \phi$ *and* T

metrics on branes

$$h_{\mu\nu}(y_\pm) + \nabla_\mu \hat{\xi}_\nu + \nabla_\nu \hat{\xi}_\mu = h^{(TT)}_{\mu\nu}(y_\pm) - \tilde{\gamma}_{\mu\nu}\left(\phi(y_\pm) + 2\mathcal{H}_\pm \hat{\xi}^y_\pm\right)$$

DGP model and dRGT bigravity

	DGP	dRGT
variables	$h_{\mu\nu}(y_\pm) = h^{(0)}_{\mu\nu} u_0(y_\pm) + h^{(1)}_{\mu\nu} u_1(y_\pm)$	$h_{\mu\nu}$, $\tilde{h}_{\mu\nu}$
parameters	$r_c^{(\pm)},\ d \to m_1^2,\ u_0(y_\pm),\ u_1(y_\pm)$	$M_{pl},\ \chi,\ m^2,\ c_n$ $\to m_{eff},\ \chi\omega^2$

ω : scale of $\tilde{h}_{\mu\nu}$ compared with $h_{\mu\nu}$

$$m_1^2 \leftrightarrow m_{eff}^2 \qquad r_c^{(+)}/r_c^{(-)} \leftrightarrow \chi\omega^2$$

DGP model can be shown to be identical to dRGT bigravity in linear regime.

ghost in DGP model

$$2\left(\sum_i \frac{u_i^2(y_+)}{m_i^2 - 2H^2}\right) + \frac{1}{H_+^2(2r_c\mathcal{H}_+ - 1)}\left(\frac{2\kappa^2}{3H_+^2(2r_c\mathcal{H}_+ - 1)}\left(\sum_i \frac{v_i^2(y_+)}{\mu_i^2 + 4H^2}\right) + \mathcal{H}_+\right) = 0$$

diverges as $m^2 \to 2H^2$: Higuchi bound

diverges as $\mu^2 \to 4H^2$
: critical mass that scalar ghost appears

H : 4-d comoving curvature scale

$2r_c\mathcal{H}_+ - 1 > 0$: self-accelerating branch

$\mu_i^2 + 4H^2 \to \mp\epsilon$ means $m_i^2 - 2H^2 \to \pm\epsilon$

➡ **ghost never disappears** : K.Izumi et. al. (2007)

$2r_c\mathcal{H}_+ - 1 < 0$: normal branch

The same identity prohibits m_i^2 & μ_i^2 from crossing their critical masses

➡ **no ghost**

Higuchi ghost in dRGT bigravity

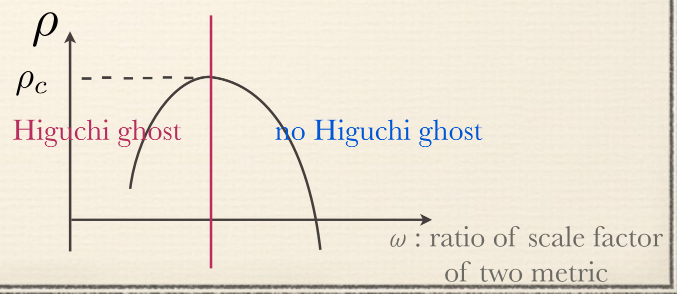
choose the branch connected to the vacuum flat spacetime
with positive graviton mass

⇩ adding cosmological const. little by little

Corresponding deSitter solution exists with no Higuchi ghost

⇩

At the critical energy density,



summary

- We can obtain ghost-free bigravity from DGP 2-brane model.
- This bigravity is confirmed to be identical to dRGT model at least in linear regime.
- In each model, the possible way of ghost appearance at high energies seems to be different.

  ... Truncation of the scalar mode by hand can explain the difference.

basic equations

scalar mode

$$\left[\hat{L}^{(\phi)} + \frac{\Box^{(4)}}{\psi'^2}\right]\phi = \sum_{\sigma=\pm}\left(\sigma\frac{4a^2\kappa^2}{3}(1-\sigma 2r_cH_\pm)^{-1}(Z_{(\sigma)} \pm r_c\phi) - \frac{2\epsilon^{(\sigma)}}{\psi'^2}\Box^{(4)}\phi\right)\delta(y-y_\sigma)$$

$$a_\pm^{-2}\Box^{(4)}Z_{(\pm)} = \pm\frac{\kappa^2}{6}T^{(\pm)} \qquad Z_{(\pm)} = (1\mp 2r_cH_\pm)\underline{\hat{\xi}^y_\pm} \mp r_c\phi(y_\pm)$$

brane bending

$$\epsilon^{(\pm)} \equiv \frac{2}{V''^{(\pm)} \mp 2\psi''/\psi'} \quad \to 0$$

TT mode

$$\left[\hat{L}^{(TT)} + a^{-2}\Box^{(4)}\right]h^{(TT)}_{\mu\nu} = \sum_{\sigma=\pm}\left(-2\kappa^2\Sigma^{(\sigma)}_{\mu\nu} - 2r_ca_\sigma^{-2}\Box^{(4)}h^{(TT)}_{\mu\nu}\right)\delta(y-y_\sigma)$$

$$\Sigma^{(\pm)}_{\mu\nu} \equiv \left(T^{(\pm)}_{\mu\nu} - \frac{1}{4}T^{(\pm)}\gamma^{\pm}_{\mu\nu}\right) \pm \frac{2}{\kappa^2}\left(\nabla_\mu\nabla_\nu - \frac{1}{4}\eta_{\mu\nu}\Box^{(4)}\right)Z_{(\pm)}$$

perturbation

background : vacuum deSitter brane with small H

metric $\quad ds^2 = dy^2 + a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu$

scalar field $\quad \psi(y)$

perturbation

Newton gauge

$$h_{yy} = 2\phi\ ,\ \ h_{y\mu} = 0\ ,\ \ h_{\mu\nu} = h^{(TT)}_{\mu\nu} - \phi\gamma_{\mu\nu}$$

$$\delta\psi = \frac{3}{2\kappa^2\psi'}[\partial_y + 2H]\phi \qquad \gamma_{\mu\nu} = a^2(y)\eta_{\mu\nu}$$

TT mode solution

$$\left[\hat{L}^{(TT)} + a^{-2}(\Box^{(4)} - 2H^2)\right] h^{(TT)}_{\mu\nu} = \sum_{\sigma=\pm}\left(-2\kappa^2\Sigma^{(\sigma)}_{\mu\nu} - 2r_c a_\sigma^{-2}(\Box^{(4)} - 2H^2)h^{(TT)}_{\mu\nu}\right)\delta(y - y_\sigma)$$

↕ m_i^2

source

mode expansion $h^{(TT)}_{\mu\nu} = \sum_i h^{(i)}_{\mu\nu}(x^\rho)u_i(y)$

$$\left[\frac{m_i^2}{a^2}\left(1 + 2r_c\sum_{\sigma=\pm}\delta(y - y(\sigma))\right)\right]u_i = -\hat{L}^{(TT)}u_i(y)$$

solution with source

$$h^{(TT)}_{\mu\nu}(y) = -2\kappa^2\sum_i\frac{u_i(y_+)u_i(y)}{\Box^{(4)} - 2H^2 - m_i^2}\Sigma_{\mu\nu}$$

scalar mode solution

μ_i^2 ↕

$$\left[\hat{L}^{(\phi)} + \frac{(\Box^{(4)} + 4H^2)}{\psi'^2}\right]\phi = \sum_{\sigma=\pm}\sigma\frac{4a^2\kappa^2}{3(1 - \sigma 2r_c\mathcal{H}_\sigma)}(Z_{(\sigma)} + \sigma r_c\phi)\delta(y - y_\sigma)$$

source term

mode expansion $\phi = \sum_i \phi^{(i)}(x^\rho)v_i(y)$

$$\frac{\mu_i^2 + 4H^2}{\psi'^2}v_i(y) = \left[-\hat{L}^{(\phi)} + \sum_{\sigma=\pm}\frac{4r_c\kappa^2a^2}{3(1 - \sigma 2r_c\mathcal{H}_\pm)}\delta(y - y(\sigma))\right]v_i(y)$$

solution with source

$$\phi(y) = \frac{4\kappa^2a_+^2}{3(1 - 2r_c\mathcal{H}_+)}\sum_i\frac{v_i(y_+)v_i(y)}{\Box^{(4)} - \mu_i^2}Z$$

mass spectrum (TT mode)

nondimensionalization $Y = y/d$

$$\frac{1}{a^2}\partial_Y a^4 \partial_Y \frac{1}{a^2} u_i = -\frac{(m_i d)^2}{a^2} u_i \qquad \pm\left(\partial_Y - 2\frac{\partial_Y a}{a}\right) u_i = -\frac{r_c d m_i^2}{a^2} u_i$$

zero mode :

md<<1 massive mode

if $r_c >> d$, r.h.s. of j.c. can contribute to 0-th order eq.

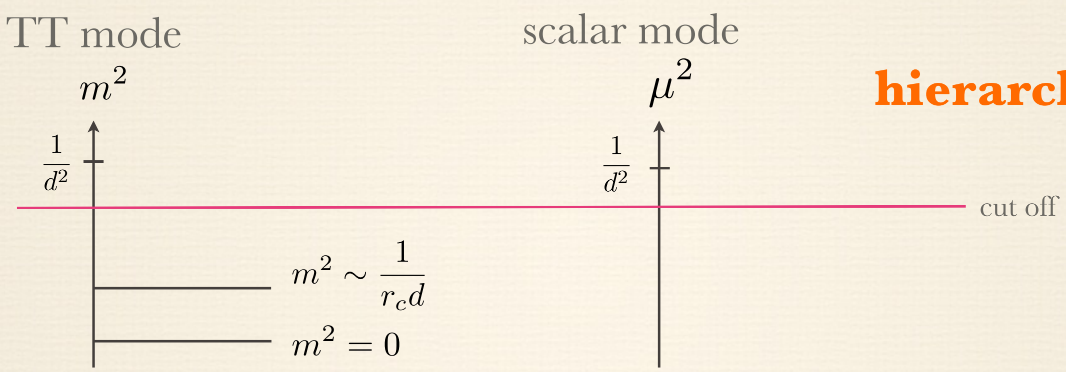
Higuchi ghost in dRGT bigravity

In dRGT model, equation for the de Sitter solution insists

$$\frac{\kappa_4^2}{m^2}\rho_m = \frac{c_1}{\chi\omega} + \left(\frac{6c_2}{\chi} - c_0\right) + \left(\frac{18c_3}{\chi} - 3c_1\right)\omega + \left(\frac{24c_4}{\chi} - 6c_2\right)\omega^2 - 6c_3\omega^3 \equiv f(\omega)$$

$$\tilde{m}^2 - 2H^2 = -\frac{m^2\omega}{3} f'(\omega)$$

summary



hierarchy

1 massless graviton

$$\Box h^{(0)}_{\mu\nu} = -2\kappa^2 \left(T^{(+)}_{\mu\nu} - \frac{1}{2}\tilde{\gamma}_{\mu\nu} T^{(+)} \right)$$

1 massive graviton

$$(\Box - m^2) h^{(m)}_{\mu\nu} = -2\kappa^2 \left(T^{(+)}_{\mu\nu} - \frac{1}{3}\tilde{\gamma}_{\mu\nu} T^{(+)} \right)$$

⟺ **dRGT bigravity**

"Coleman-deLuccia instantons in nonlinear massive gravity"

by Yingli Zhang

[JGRG23(2013)110505]

JGRG23

Coleman-de Luccia instantons in nonlinear Massive Gravity

Ying-li Zhang
YITP , Kyoto University

5, November, 2013

Based on:

YZ, Ryo Saito and Misao Sasaki, JCAP 02(2013)029 [1210.6224]
Misao Sasaki, Dong-han Yeom and YZ, CQG 30(2013)232001[1307.5948]
Ryo Saito, Misao Sasaki, Dong-han Yeom and YZ, in preparison

Outline

1. Motivation
2. Setup of model
3. Coleman-de Luccia solutions
4. Conclusion and Future Prospects

Massive Gravity theory

General Relativity (GR): $S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}R\,,$

In 3+1 dim, for symmetric tensor $g_{\mu\nu}$, the propagating degrees of freedom (dof) can be counted as:

$$6 \; -4 \; = 2$$

Lagrangian multiplier

Helicity ± 2

Such situation changes in the Massive Gravity Theory.

In Massive Gravity (MG), the mass of graviton is non-vanishing, which breaks the gauge invariance

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}[R(g) - m^2 V(g)]$$

$$\supset -\frac{m^2}{16\pi G}\int d^4x \sqrt{\gamma} N V(\gamma, N, N^i)$$

Generally speaking, the dof is

$$6 \; -0 = 6$$

No Lagrangian multiplier…

Helicity $\pm 2, \;\; \pm 1, \;\; 0,$

(Boulware & Deser '72)

Recently, a non-linear construction of massive gravity theory (dRGT) is proposed, where the BD ghost is removed by specially designed non-linear terms, so that the lapse function N becomes a Lagrangian Multiplier, which removes the ghost degree of freedom.

Non-linear Massive Gravity (dRGT)

C. de Rham, G. Gabadadze, Phys. Rev. D 82, 044020 (2010);
C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett 106, 231101 (2011);
S. F. Hassan and R. A. Rosen, JHEP 1107, 009 (2011)

$$S_{MG} = \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right],$$

where $[\mathcal{K}] = tr\,(K_\mu^\nu)$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right),$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right),$$

$$\mathcal{K}_\nu^\mu \equiv \delta_\nu^\mu - \sqrt{g^{\mu\sigma} G_{ab}(\phi) \partial_\nu \phi^a \partial_\sigma \phi^b}.$$

fiducial metric

Stuckelberg field

7

Self-accelerating solution is found in context of non-linear massive gravity, where two branches exist with effective cosmological constant consists of a contribution from mass of graviton. A. E. Gumrukcuoglu et. al. JCAP 106, 231101(2011);

$$\Lambda_{\pm} = -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

There seems to be some hope to explain the current acceleration, but…

Very small m_g^2 from observation ⟶ Cosmological constant problem

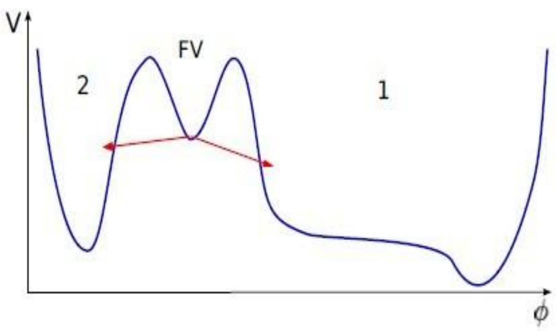
8

A possible resolution: Landscape of Vacua

S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)

• the field can (and will) tunnel from a metastable minimum to a lower one;

• this process is driven by instanton.



S. Coleman and F. de Luccia, Phys.Rev. D21, 3305, (1980)

As a first step, we study the stability of a vacuum in the context of non-linear Massive Gravity with constant graviton mass

Moreover, studies on Hartle-Hawking no-boundary proposal make the inflationary scenario exponentially probable. Misao Sasaki, Dong-han Yeom and YZ, CQG 30(2013)232001

2. Setup of model

$$S = S_{MG} + S_m,$$

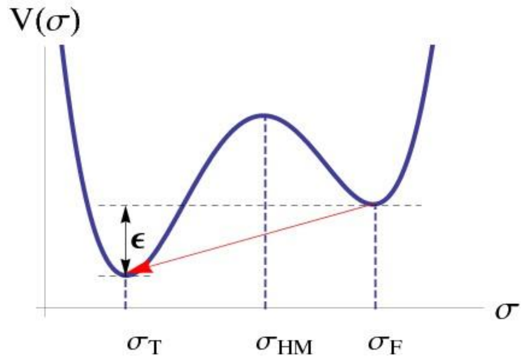
$$S_m \equiv -\int d^4x\,\sqrt{-g}\left[\frac{1}{2}(\partial\sigma)^2 + V(\sigma)\right],$$

- potential $V(\sigma)$

local minima: σ_F

global minima: σ_T

local max: $\sigma_{\rm HM}$



- tunneling probability per unit time per unit volume

$$\Gamma/V = Ce^{-B},$$

$$B = S_E[g_{\mu\nu,B}, \phi_B] - S_E[g_{\mu\nu,F}, \phi_F],$$

↑ bounce solution ↑ 'false vacuum'

Lowest action ↓

usually, bounce solutions are explored by assuming an O(4) symmetry

➢ spacetime metric: Euclidean

$$g_{\mu\nu}dx^\mu dx^\nu = N(\xi)^2 d\xi^2 + a(\xi)^2\Omega_{ij}dx^i dx^j,$$

$$\Omega_{ij} \equiv \delta_{ij} + \frac{K\delta_{il}\delta_{jm}x^l x^m}{1 - K\delta_{lm}x^l x^m}, \quad K > 0$$

Note: the fiducial metric may not respect the symmetry

- fiducial metric: deSitter

$$G_{ab}(\phi)d\phi^a d\phi^b \equiv -(d\phi^0)^2 + b(\phi^0)^2\Omega_{ij}d\phi^i d\phi^j,$$

$$b(\phi^0) \equiv F^{-1}\sqrt{K}\cosh(F\phi^0).$$

↑ fiducial Hubble parameter

$\longrightarrow$ the O(4)-symmetric solutions are obtained by setting

$$\phi^0 = f(\xi), \quad \phi^i = x^i.$$

Inserting these ansatz into the action, we obtain the constraint equation by varying with respect with f

$$(i\dot{a} + Nb_{,f})\left[\left(3-\frac{2b}{a}\right)+\alpha_3\left(1-\frac{b}{a}\right)\left(3-\frac{b}{a}\right)+\alpha_4\left(1-\frac{b}{a}\right)^2\right]=0,$$

$$\dot{a} \equiv \frac{da}{d\xi} \qquad b_{,f} \equiv \frac{db}{df} = \sqrt{K}\sinh(Ff)$$

$\rightarrow$

- Branch I $\quad Nb_{,f} = -i\dot{a},\quad$ Not considered below
- Branch II $\quad \left(3-\frac{2b}{a}\right)+\alpha_3\left(1-\frac{b}{a}\right)\left(3-\frac{b}{a}\right)+\alpha_4\left(1-\frac{b}{a}\right)^2=0.$

$\rightarrow$

$$b = X_\pm a, \qquad X_\pm \equiv \frac{1+2\alpha_3+\alpha_4 \pm \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}.$$

Friedmann equation & EOM for tunneling field

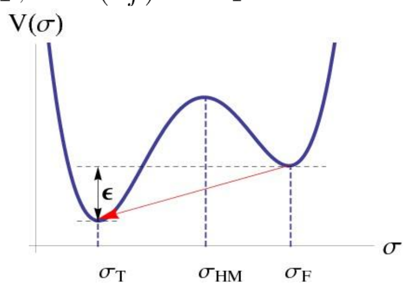
$$\begin{cases} \frac{3}{a^2}\left(\frac{da}{d\tau}\right)^2 - \frac{3K}{a^2} = \frac{1}{2}\left(\frac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm, \\ \frac{d^2\sigma}{d\tau^2} + \frac{3}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0 \end{cases}$$

where $\mathrm{d}\tau \equiv N\mathrm{d}\xi,$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

3. Coleman-de Luccia(CDL) solutions

- CDL solutions can be found when $\sigma(0)=\sigma_\mathrm{T}, \quad \sigma(\tau_f)=\sigma_\mathrm{F}$



$$a(\tau)\begin{cases} = a_\mathrm{T}(\tau) \equiv H_\mathrm{T}^{-1}\sqrt{K}\cos\left(H_\mathrm{T}\tau\right), & \tau<\tau_0 \\ = a_\mathrm{F}(\tau) \equiv H_\mathrm{F}^{-1}\sqrt{K}\cos\left(H_\mathrm{F}\tau+\theta_\mathrm{F}\right), & \tau>\tau_0 \end{cases}$$

$$b(\tau) = X_\pm a(\tau) \implies -\left(f'(\tau)\right)^2 = \begin{cases} X_\pm^2\frac{K-(a_\mathrm{T}H_\mathrm{T})^2}{K-(a_\mathrm{T}FX_\pm)^2}, & \tau<\tau_0 \\ X_\pm^2\frac{K-(a_\mathrm{T}H_\mathrm{F})^2}{K-(a_\mathrm{F}FX_\pm)^2}, & \tau>\tau_0 \end{cases}$$

- difference from GR in action is the mass term

$$S^{\mathrm{mass}} \equiv -m_g^2\int d^4x_E\,\sqrt{\Omega}\left(\mathcal{L}_{2E}+\alpha_3\mathcal{L}_{3E}+\alpha_4\mathcal{L}_{4E}\right)$$
$$= 2\pi^2K^{-\frac{3}{2}}m_g^2Y_\pm\int \mathrm{d}\tau\; a^3(\tau)\sqrt{-(f')^2},$$

$$Y_\pm \equiv 3(1-X_\pm)+3\alpha_3(1-X_\pm)^2+\alpha_4(1-X_\pm)^3,$$

15

• thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall} \,,$$

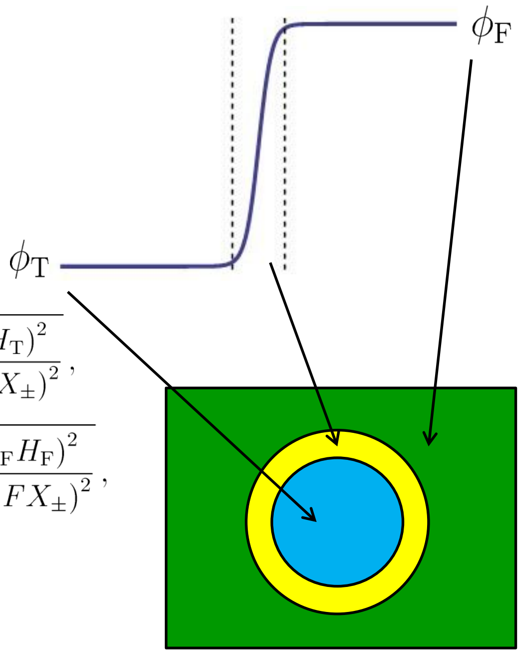
$$\begin{cases} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}|_{\tau<\tau_0} \,, \\ B_{\rm outside} \equiv S_{\rm outside} - S_{\rm F}|_{\tau>\tau_0} \,, \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}|_{\tau=\tau_0} \,, \end{cases}$$

$$S_{\rm inside} = m_g^2 Y_\pm X_\pm \int {\rm d}^3x\sqrt{\Omega} \int_{-\pi/(2H_{\rm T})}^{\tau_0(1-\delta)} {\rm d}\tau\; a_{\rm T}^3 \sqrt{\frac{K-(a_{\rm T}H_{\rm T})^2}{K-(a_{\rm T}FX_\pm)^2}} \,,$$

$$S_{\rm outside} = m_g^2 Y_\pm X_\pm \int {\rm d}^3x\sqrt{\Omega} \int_{\tau_0(1+\delta)}^{\pi/(2H_{\rm F})} {\rm d}\tau\; a_{\rm F}^3 \sqrt{\frac{K-(a_{\rm F}H_{\rm F})^2}{K-(a_{\rm F}FX_\pm)^2}} \,,$$

$$S_{\rm wall} = m_g^2 Y_\pm \int {\rm d}^3x\sqrt{\Omega} \int_{\tau_0(1-\delta)}^{\tau_0(1+\delta)} {\rm d}\tau\; a^3(\tau)\sqrt{-(f')^2}$$

where $\delta \ll 1$



16

• thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall} \,,$$

$$\begin{cases} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}|_{\tau<\tau_0} \,, \\ B_{\rm outside} \equiv \cancel{S_{\rm outside} - S_{\rm F}}|_{\tau>\tau_0} \,, \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}|_{\tau=\tau_0} \,, \end{cases}$$

17

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\text{inside}} + B_{\text{outside}} + B_{\text{wall}} \,,$$

$$\begin{cases} B_{\text{inside}} \equiv S_{\text{inside}} - S_{\text{F}}\big|_{\tau<\tau_0} \,, \\ B_{\text{outside}} \equiv S_{\text{outside}} - S_{\text{F}}\big|_{\tau>\tau_0} \,, \\ B_{\text{wall}} \equiv S_{\text{wall}} - S_{\text{F}}\big|_{\tau=\tau_0} \,, \end{cases}$$

$$\frac{3}{a^2}\left(\frac{da}{d\tau}\right)^2 - \frac{3K}{a^2} = \frac{1}{2}\left(\frac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm ,$$

$$\downarrow$$

$$a' = \sqrt{K + \frac{a^2}{3}\left[\frac{\sigma'^2}{2} - V(\sigma) - \Lambda_\pm\right]}$$

$$\downarrow$$

$$\int_0^{\tau_0(1-\delta)} \mathrm{d}\tau \;=\; \int_0^{a_0} \left(\frac{da}{d\tau}\right)^{-1} \mathrm{d}a$$

$$\downarrow$$

$$B_{\text{inside}} = 2\pi^2 K^{-\frac{3}{2}} m_g^2 Y_\pm X_\pm \int_0^{a_0} a^3 \mathrm{d}a \left\{ \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\text{T}}/3}} \sqrt{\frac{K - (aH_{\text{T}})^2}{K - (aFX_\pm)^2}} - \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\text{F}}/3}} \sqrt{\frac{K - (aH_{\text{F}})^2}{K - (aFX_\pm)^2}} \right\}$$

$$= \mathcal{O}(\epsilon)$$

18

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\text{inside}} + B_{\text{outside}} + B_{\text{wall}} \,,$$

$$\begin{cases} B_{\text{inside}} \equiv S_{\text{inside}} - S_{\text{F}}\big|_{\tau<\tau_0} \,, \\ B_{\text{outside}} \equiv S_{\text{outside}} - S_{\text{F}}\big|_{\tau>\tau_0} \,, \\ B_{\text{wall}} \equiv S_{\text{wall}} - S_{\text{F}}\big|_{\tau=\tau_0} \,, \end{cases}$$

$$\frac{3}{a^2}\left(\frac{da}{d\tau}\right)^2 - \frac{3K}{a^2} = \frac{1}{2}\left(\frac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm ,$$

$$\downarrow$$

$$a' = \sqrt{K + \frac{a^2}{3}\left[\frac{\sigma'^2}{2} - V(\sigma) - \Lambda_\pm\right]}$$

$$\downarrow$$

$$\int_0^{\tau_0(1-\delta)} \mathrm{d}\tau \;=\; \int_0^{a_0} \left(\frac{da}{d\tau}\right)^{-1} \mathrm{d}a$$

$$\downarrow$$

$$B_{\text{inside}} = 2\pi^2 K^{-\frac{3}{2}} m_g^2 Y_\pm X_\pm \int_0^{a_0} a^3 \mathrm{d}a \left\{ \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\text{T}}/3}} \sqrt{\frac{K - (aH_{\text{T}})^2}{K - (aFX_\pm)^2}} - \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\text{F}}/3}} \sqrt{\frac{K - (aH_{\text{F}})^2}{K - (aFX_\pm)^2}} \right\}$$

$$= \mathcal{O}(\epsilon)$$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall}\,,$$

$$\begin{cases} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}|_{\tau<\tau_0}\,, \\ B_{\rm outside} \equiv S_{\rm outside} - S_{\rm F}|_{\tau>\tau_0}\,, \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}|_{\tau=\tau_0}\,, \end{cases}$$

$$\frac{d^2\sigma}{d\tau^2} + \frac{3}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0$$

$$\downarrow \quad \frac{1}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} \ll 1$$

$$\sigma' \simeq \sqrt{2\big[V(\sigma) - V(\sigma_{\rm T})\big]}$$

$$\downarrow \quad {\rm d}\tau = \left(\frac{{\rm d}\sigma}{{\rm d}\tau}\right)^{-1}{\rm d}\sigma$$

$$B_{\rm wall} \simeq 2\pi^2 K^{-\frac{3}{2}} a_0^3 m_g^2 Y_\pm \int_{\sigma_{\rm T}}^{\sigma_{\rm F}} \frac{{\rm d}\sigma}{\sqrt{2\big[V(\sigma) - V(\sigma_{\rm T})\big]}}\left[\sqrt{-(f')^2}\Big|_{\tau<\tau_0} - \sqrt{-(f')^2}\Big|_{\tau>\tau_0}\right]$$

$$= \mathcal{O}(\epsilon)$$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall}\,,$$

$$\begin{cases} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}|_{\tau<\tau_0}\,, \\ B_{\rm outside} \equiv S_{\rm outside} - S_{\rm F}|_{\tau>\tau_0}\,, \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}|_{\tau=\tau_0}\,, \end{cases}$$

$$\frac{d^2\sigma}{d\tau^2} + \frac{3}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0$$

$$\downarrow \quad \frac{1}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} \ll 1$$

$$\sigma' \simeq \sqrt{2\big[V(\sigma) - V(\sigma_{\rm T})\big]}$$

No difference from GR ?

$$\downarrow \quad {\rm d}\tau = \left(\frac{{\rm d}\sigma}{{\rm d}\tau}\right)^{-1}{\rm d}\sigma$$

$$B_{\rm wall} \simeq 2\pi^2 K^{-\frac{3}{2}} a_0^3 m_g^2 Y_\pm \int_{\sigma_{\rm T}}^{\sigma_{\rm F}} \frac{{\rm d}\sigma}{\sqrt{2\big[V(\sigma) - V(\sigma_{\rm T})\big]}}\left[\sqrt{-(f')^2}\Big|_{\tau<\tau_0} - \sqrt{-(f')^2}\Big|_{\tau>\tau_0}\right]$$

$$= \mathcal{O}(\epsilon)$$

- CDL as perturbations around Hawking-Moss (HM) solutions

Expand the potential $V(\sigma)$ around $\sigma = \sigma_{\rm HM}$ as follows:

$$V(\sigma) = V(\sigma_{\rm HM}) - \frac{M^2}{2}(\sigma_{\rm HM} - \sigma)^2 + \frac{m}{3}(\sigma_{\rm HM} - \sigma)^3 + \frac{\nu}{4}(\sigma_{\rm HM} - \sigma)^4 + \cdots,$$

near the HM limit where $M^2 \equiv 4H_{\rm HM}^2(1+\chi^2)$ with $\chi^2 \ll 1$,
the regular solutions are perturbatively found to be

$$a(\tau) = \tilde{H}_{\rm HM}^{-1}\cos\left(\tilde{H}_{\rm HM}\tau\right)\left[1 + \frac{\varepsilon_M^2 H_{\rm HM}^2}{8}\cos^2\left(\tilde{H}_{\rm HM}\tau\right)\right] + \mathcal{O}(\varepsilon_M^3)$$

$$\sigma(\tau) = \sigma_{\rm HM} + \varepsilon_M H_{\rm HM}\sin\left(\tilde{H}_{\rm HM}\tau\right) + \frac{\varepsilon_M^2 m}{12}\left[1 - 2\sin^2\left(\tilde{H}_{\rm HM}\tau\right)\right]$$

$$-\varepsilon_M^3 H_{\rm HM}\sin\left(\tilde{H}_{\rm HM}\tau\right)\left[\frac{3H_{\rm HM}^2 - 4\mu}{56}\cos^2\left(\tilde{H}_{\rm HM}\tau\right) - \frac{m^2}{36H_{\rm HM}^2}\sin^2\left(\tilde{H}_{\rm HM}\tau\right)\right] + \mathcal{O}(\varepsilon_M^4)$$

$\tilde{H}_{\rm HM} \equiv H_{\rm HM}(1 + H_{\rm HM}^2\varepsilon_M^2/24)$

$\mu \equiv \nu + m^2/18H_{\rm HM}^2$

$\varepsilon_M^2 \equiv 84\chi^2/(16H_{\rm HM}^2 + 9\mu)$

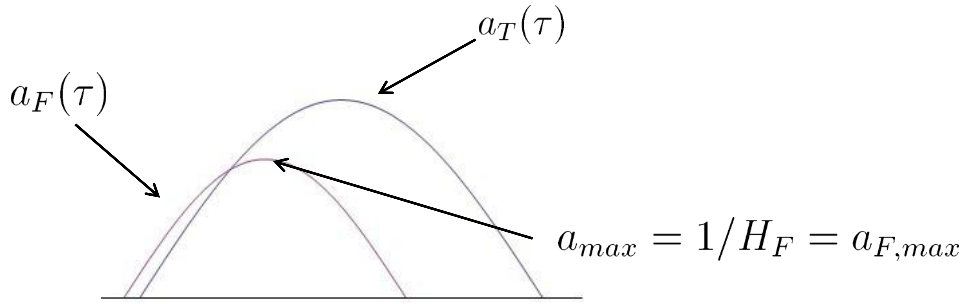
$Y_\pm \equiv 3(1 - X_\pm) + 3\alpha_3(1 - X_\pm)^2 + \alpha_4(1 - X_\pm)^3,$

$$\delta^{(2)}S = \frac{\pi^2 m_g^2 X_\pm Y_\pm H_{\rm HM}^2 \varepsilon_M^2}{2\tilde{H}_{\rm HM}^4\sqrt{1 - \tilde{\alpha}^2}}$$

Hence, if $Y_\pm > 0$, HM dominates over CDL, vise versa.

In GR, perturbations in action vanish until $\mathcal{O}(\varepsilon_M^4)$, and CDL always dominate over HM.

Reconsideration of thin-wall result



$$b(\tau) \equiv F^{-1}\sqrt{K}\cosh\left(Ff(\tau)\right) = X_{\pm}a(\tau) \longrightarrow -(f')^2 = \frac{X_{\pm}^2 (a')^2}{K-(FX_{\pm}a)^2}$$

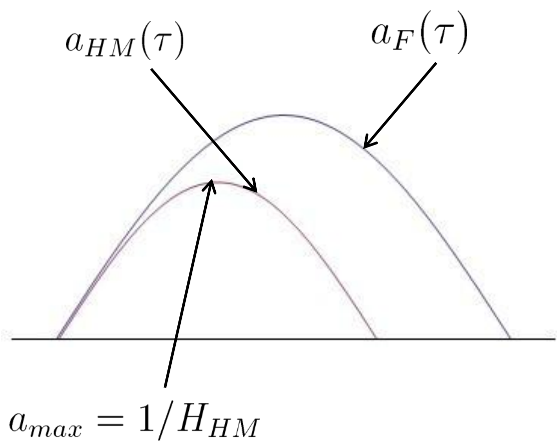
$$S^{\rm mass} = 4\pi^2 K^{-\frac{3}{2}} m_g^2 X_{\pm} Y_{\pm} \int_0^{a_{\rm max}} \frac{a^3 {\rm d}a}{\sqrt{K-(FX_{\pm}a)^2}}$$

$$= -\frac{4\pi^2 K^{-\frac{3}{2}} m_g^2 X_{\pm} Y_{\pm}}{3(FX_{\pm})^4}\left[\sqrt{K-(FX_{\pm}a)^2}\left(2K+(FX_{\pm}a)^2\right)\right]_0^{a_{\rm max}}$$

$$B^{\rm mass}_{\rm thin-wall} \equiv S^{\rm mass} - S^{\rm mass}_{\rm F} \propto \left[\sqrt{K-(FX_{\pm}a)^2}\left(2K+(FX_{\pm}a)^2\right)\right]_{a_{\rm F,max}}^{a_{\rm max}} = 0\,,$$

This explains the reason why no contribution in thin-wall limit. However, in HM case, $a_{\rm max} = a_{\rm HM,max} \equiv H_{\rm HM}^{-1}$

$$B^{\rm mass}_{\rm HM} = -\frac{4\pi^2 K^{-\frac{3}{2}} m_g^2 X_{\pm} Y_{\pm}}{3(FX_{\pm})^4}\left[\sqrt{K-(FX_{\pm}a)^2}\left(2K+(FX_{\pm}a)^2\right)\right]_{H_{\rm F}^{-1}}^{H_{\rm HM}^{-1}} \neq 0$$



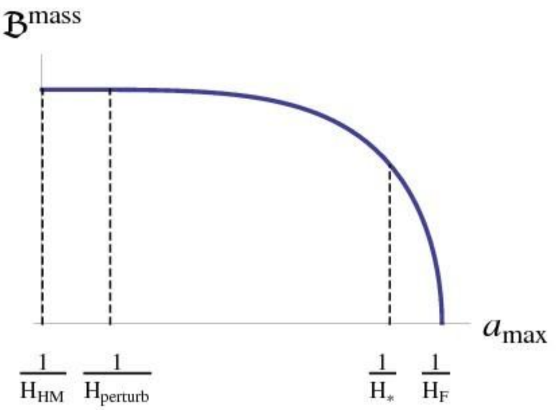
25

Defining

$$\mathfrak{B}^{\rm mass} \equiv -\frac{3(FX_\pm)^4 B^{\rm mass}}{4\pi^2 m_g^2 X_\pm Y_\pm} = \left[\sqrt{1-(FX_\pm a)^2}\left(2+(FX_\pm a)^2\right)\right]_{H_{\rm F}^{-1}}^{a_{\rm max}} ,$$

$$\Delta\Gamma \equiv \frac{\Gamma_{\rm MG}}{\Gamma_{\rm GR}} \simeq \exp\left(\frac{4\pi^2 m_g^2 Y_\pm \mathfrak{B}^{\rm mass}}{3F^4 X_\pm^3}\right) .$$

- HM solution gives largest correction term where a_{max} is smallest;
- when a_{max} increases, correction shrinks gradually;
- at thin-wall limit, the behavior of CDL solution is the same as GR.



26

Under the thin-wall approximation, one can compare the probability of CDL process to HM process as follows

$$\ln\left(\frac{P_{\rm CDL}}{P_{\rm HM}}\right) \approx 4\pi^2\left(\frac{16}{\Sigma^2} - \frac{m_g^2 Y_\pm \mathfrak{B}^{\rm mass}(a_{\rm max}=H_{\rm HM}^{-1})}{3F^4 X_\pm^3}\right) .$$

$$\Sigma \equiv \int_{\sigma_{\rm T}}^{\sigma_{\rm F}} d\sigma \sqrt{2[V(\sigma)-V(\sigma_{\rm T})]}$$

In GR, $m_g = 0$, CDL process dominates over HM one.

However, provided that parameters and their combinations are of order unity, if $m_g^2 > \mathcal{O}\left(F^4\Sigma^{-2}\right)$ HM process dominates over CDL.

Implications?

Summary and future work

- We constructed a model in which the tunneling field minimally couples to the non-linear massive gravity;
- corrections to CDL tunneling changes monotonically when one goes beyond thin-wall approximation until HM case;
- under the thin-wall approximation, the HM process may dominate over CDL one, it is interesting to investigate its implications;
- it would be a further work to generalize our analysis to extended massive gravity theories, e.g. mass-varying theory, quasi-dilaton massive gravity, SO(3) massive gravity…

"Massive Gravity, Black Hole solutions and Relevant scales."

by Ivan Dario Arraut

[JGRG23(2013)110506]

On the consistency of the Black Hole solutions inside the dRGT non-linear massive gravity

Ivan Arraut, in collaboration with
Hideo Kodama
Osaka University and KEK Theory Center
(Tsukuba, Ibaraki).
Paper in preparation.

Motivation

- 1). Recently, dRGT found a ghost-free version of non-linear massive gravity at all orders. However, some other pathologies might exist.
- 2). Recently, it was found that inside the bigravity formalism, the Gregory_laflamme instability is reproduced, except in the Partially massless regime.
- 3). Although there are some previous works on Black Holes stabilities in massive gravity, nobody has derived general expressions inside the dRGT formalism. That´s what we did.

Formulation of the dRGT massive gravity

- The action is given by:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}(R + m^2 U(g,\phi))$$

With the effective potential on two free parameters:

$$U(g,\phi) = U_2 + \alpha_3 U_3 + \alpha_4 U_4$$

Our notation:

$$U_2 = Q^2 - Q_2$$

$$U_3 = Q^3 - 3QQ_2 + 2Q_3$$

$$U_4 = Q^4 - 6Q^2 Q_2 + 8QQ_3 + 3Q_2^2 - 6Q_4$$

$$Q = Q_1 \qquad Q_n = Tr(Q^n)^\mu{}_\nu$$

$$Q^\mu{}_\nu = \delta^\mu{}_\nu - M^\mu{}_\nu$$

$$(M^2)^\mu{}_\nu = g^{\mu\alpha} f_{\alpha\nu}$$

$$f_{\mu\nu} = \eta_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$$

The field equations can be computed as:

$$G_{\mu\nu} = -m^2 X_{\mu\nu}$$

$$X_{\mu\nu} = \frac{\delta U}{\delta g^{\mu\nu}} - \frac{1}{2} U g_{\mu\nu}$$

The other field equation is obtained from the Bianchi identity and corresponds to the dynamics of the Stückelberg fields. That equation is satisfied for a family of solutions with one free parameter.

The Schwarzschild-de Sitter solution

- If we want to reproduce the SdS solutions inside the dRGT formalism, the following condition must be satisfied:

$$m^2 X_{\mu\nu} = \Lambda g_{\mu\nu}$$

$$g^{\mu\alpha} X_{\alpha\nu} = -Q - \frac{1}{2}(Q^2 - Q_2) + (1+Q)Q^\mu{}_\nu - (Q^2)^\mu{}_\nu + \frac{\alpha_3}{2}\{3(Q_2 - Q^2) - Q^3 + 3QQ_2$$
$$-2Q_3 + 3(2Q + Q^2 - Q_2)Q^\mu{}_\nu - 6(1+Q)(Q^2)^\mu{}_\nu + 6(Q^3)^\mu{}_\nu\} + \alpha_4\{-2Q^3 + 6QQ_2 - 4Q_3$$
$$+6(Q^2 - Q_2)Q^\mu{}_\nu - 12Q(Q^2)^\mu{}_\nu + 12(Q^3)^\mu{}_\nu\}($$

$$X^\mu{}_\nu = \chi_0 + \chi_1 Q^\mu{}_\nu + \chi_2 (Q^2)^\mu{}_\nu + \chi_3 (Q^3)^\mu{}_\nu$$

We show that if the theory satisfies the condition:

$$12\alpha_4 = 1 + 3\alpha_3 + 9\alpha_3^2$$

Then any metric form:

$$ds^2 = g_{tt}dt^2 + 2g_{tr}dtdr + g_{rr}dr^2 + r^2 S_0^2 d\Omega_2^2$$

Is a solution with:

$$S_0 = \frac{3\alpha_3 + 1}{3\alpha_3 + 2}$$

$S_0 \neq 1$ Independent of the value taken by the parameter α_3

In the unitary gauge, for the Stuckelberg fields defined by:

$$\phi^0 = t, \quad \phi^i = x^i = r\Omega^i \ (i = 1, 2, 3),$$
$$f_{\mu\nu}dx^\mu dx^\nu = \eta_{ab}d\phi^a d\phi^b = -dt^2 + dr^2 + r^2 d\Omega_2^2.$$

The solution corresponds to:

$$\Lambda = m^2 \frac{1 - S_0}{S_0} = \frac{m^2}{3\alpha_3 + 1}.$$

For any metric in the unitary gauge, we have:

$$(M^2) = (g^* f_*) = \begin{pmatrix} -g^{tt} & g^{tr} & 0 & 0 \\ -g^{tr} & g^{rr} & 0 & 0 \\ 0 & 0 & \frac{1}{S^2} & 0 \\ 0 & 0 & 0 & \frac{1}{S^2} \end{pmatrix}$$

The root square of this matrix is defined by:

$M^\mu{}_\nu = (1-Q)^\mu{}_\nu$ With:

$$Q^\mu{}_\nu = \begin{pmatrix} a & c & 0 & 0 \\ -c & b & 0 & 0 \\ 0 & 0 & 1-\frac{1}{S} & 0 \\ 0 & 0 & 0 & 1-\frac{1}{S} \end{pmatrix}$$

Black Hole solutions:

Here we consider the Black Hole solutions:

$$ds^2 = -\mu^2 f(Sr)dt^2 - 2\mu h'(r) f(Sr) dt dr + \frac{S^2 - (h'(r))^2 f^2(Sr)}{f(Sr)} dr^2 + S^2 r^2 d\Omega_2^2$$

It is possible to demonstrate that the following combination:

$$c^2 + (1-a)(1-b) = \frac{1}{\mu S}$$

Is an invariant under coordinate transformations. In fact, it is just the determinant of the matrix

$$M^\mu{}_\nu = (1-Q)^\mu{}_\nu$$

Perturbation analysis

We will use the gauge invariant formulation, assuming a metric. (**Kodama, Ishibashi and Seto, PRD, 62,064022**):

$$ds^2 = g_{MN}dz^M dz^N = g_{ab}(y)dy^a dy^b + r^2(y)d\sigma_n^2$$

g_{ab} ⇨ Is a 2-dimensional Lorentzian metric.

And: ⇨ $d\sigma_n^2 = \gamma_{ij}dx^i dx^j$

Is the metric of constant sectional curvature K on a bi-dimensional subspace. The internal metric is given by:

$$\hat{R}_{ij} = (n-1)K\gamma_{ij}$$

More details about this approach can be found on the papers of Kodama and Ishibashi.

We use the Harmonic expansion and define the following set of gauge invariant quantities:

$$F_{ai}^{(1)} = rF_a Y_i \qquad \tau_{ai}^{(1)} = r\tau_a Y_i \qquad \tau_{ij}^{(1)} = r^2\tau_T Y_{ij}$$

For vector type perturbations.

And:

$$F^{(0)}_{ab} = F_{ab}Y \qquad F^{(0)} = 2r^2 FY$$
$$\Sigma^{(0)}_{ab} = \Sigma_{ab}Y \qquad \Sigma^{(1)}_{ai} = r\Sigma_a Y_i$$
$$\Sigma^{(0)} = r^2\Sigma Y \qquad \Pi^{(0)}_{ij} = r^2\tau_T Y_{ij}$$

For scalar type perturbations.

In order to use the standard formulation for perturbations, we re-scale the distance and time as follows:

$$f_{\mu\nu}dx^\mu dx^\nu = -\frac{1}{\mu^2}dt^2 + \frac{dr^2}{S_0^2} + \frac{r^2}{S_0^2}d\Omega_2^2$$

$$g_{\mu\nu} = -f(r)(dt + h'(r)dr)^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2$$
$$= g_{ab}dy^a dy^b + r^2 d\Omega_2^2$$

If we take into account that the background metric and the corresponding matrix $M = g^* f_*$ are direct sum of two dimensional submatrices:

$$g = g_{(1)}(t,r) \oplus g_{(2)}(\theta,\phi)$$

$$M = M_{(1)} \oplus M_{(2)}$$

Then we can see that the perturbations will decouple in the same way, except for the case of perturbation of the traces: δQ_n

With the redefinitions:

$$\alpha = 1 + 3\alpha_3 \qquad \beta = 3(\alpha_3 + 4\alpha_4)$$

We concentrate on the family of solutions satisfying the conditions: $\beta = \alpha^2$

Gabadadze and collegagues PRD 85, 044024

From the perturbation of the matrix $X^{\mu}{}_{\nu}$

We obtain the following results:

$$\delta X^i{}_j = \omega(r)(H_L \delta^i{}_j - H_T Y^i{}_j)$$

$$\omega(r) = \frac{1+\alpha}{\alpha}\{\beta(c^2+ab)+\alpha(a+b)+1\}$$

$$\delta X^a{}_b = 0$$

$$\delta X^a{}_i = 0$$

Vector perturbations:

$$h_{ab} = 0 \quad h_{ai} = r f_a Y_i \quad h_{ij} = 2r^2 H_T Y_{ij}$$

(Harmonic expansions)

And the Harmonic expansions for the energy-momentum tensor are:

$$\kappa^2 \tau^{\mu}{}_{\nu} := \kappa^2 \delta T^{\mu}{}_{\nu} = -m^2 \delta X^{\mu}{}_{\nu}$$

$$\tau^a{}_b = 0 \quad \tau^a{}_i = r\tau^a Y_i \quad \tau^i{}_j = \tau_T Y^i{}_j$$

From the previous calculations, we get:

$$\tau^a = 0$$

$$\kappa^2 \tau_T = m^2 \omega(r) H_T$$

These source terms have to satisfy the conservation equation:

$$D_a(r^3\tau^a) + \frac{(l+2)(l-1)}{2[l(l+1)-1]^{1/2}} r^2 \tau_T = 0 \quad \rightarrow (l-1)\omega(r) H_T = 0$$

With $K = 1$ Then: $H_T = 0$

For $l \geq 2.$

In this case, the perturbations are just identical to the Einstein's case.

The exceptional mode l=1:

For this mode, H_T does not exist and as a consequence F_a is not gauge invariant anymore. Its gauge transformation is:

$$\delta y^a = 0,\ \delta z^A = LV^A \Rightarrow \delta F_a = -r D_a L$$

In general, f_a in the Einstein case, is a linear combination of the gauge modes given above and the standard rotational perturbation corresponding to the metric component of the Kerr metric:

$$f_a = -r D_a L - \frac{2aM}{r} \delta^t_a$$

Scalar perturbations

The metric perturbation harmonic expansion is:

$$h_{ab} = f_{ab} Y \quad h_{ai} = r f_a Y_i \quad h_{ij} = 2r^2 \left(H_L \gamma_{ij} Y + H_T Y_{ij} \right)$$

And the source perturbations:

$$\delta T_{ab} = \tau_{ab} Y \quad \delta T^a{}_i = r \tau^a Y_i \quad \delta T^i{}_j = \delta P \delta^i{}_j Y + \tau_T Y^i{}_j$$

From the previous results:

$$\delta X_{ab} = \delta g_{ac} X^c{}_b + g_{ac} \delta X^c{}_b = \frac{\Lambda}{m^2} f_{ab} Y$$

And then:

$$\kappa^2 \tau_{ab} = -\Lambda f_{ab}$$

$$\kappa^2 \tau^a = 0$$

$$\kappa^2 \delta P = -m^2 \omega(r) H_L$$

$$\kappa^2 \tau_T = m^2 \omega(r) H_T$$

⇒

The gauge invariant quantities are:

$$\Sigma_{ab} = \tau_{ab} - 2\Lambda D_{(a} X_{b)} = -\Lambda F_{ab}$$

$$\Sigma_a = \tau_a = 0$$

$$\kappa^2 \Sigma_L = -m^2 \omega H_L$$

Similar analysis for this case can be performed as before, finding that there is no instability.

Gauge invariant formulation of the dRGT theory:

If we define the perturbation:

$$\sigma^\alpha = \delta\phi^\alpha$$

Its gauge transformation under coordinate transformations $\delta_g x^\mu = \zeta^\mu$ is:

$$\delta_g \sigma^\alpha = -\ell_\zeta \phi^\alpha = -\zeta^\mu \partial_\mu \phi^\alpha$$

For the Stückelberg fields in the unitary gauge, the gauge transformation becomes:

$$\delta_g \sigma^t = -\frac{T^t}{\mu} \qquad \delta_g \sigma^r = -\frac{T^r}{S_0} \qquad \delta_g \sigma_T = -\frac{L}{S_0}$$

With: $\sigma^A = \sigma_T Y^A$

For vector perturbations:

$$\sigma^a = 0 \qquad \sigma^A = \sigma_T V^A$$

And we can construct the following gauge invariant:

$$\hat{\sigma} = \sigma_T + \frac{1}{kS_0} H_T$$

For generic modes, the source terms can be expressed in terms of this gauge invariant as:

$$\tau^a = 0 \qquad \kappa^2 \tau_T = m^2 \omega(r) k S_0 \hat{\sigma}_T$$

Scalar perturbations:

$$\Sigma_{ab} = -\Lambda F_{ab} \qquad \kappa^2 \Sigma_a = 0 \qquad \kappa^2 \tau_T = m^2 \omega(r) k S_0 \hat{\sigma}_T$$

$$\kappa^2 \Sigma_L = m^2 \omega(r) \left(\frac{kS_0}{2} \hat{\sigma}_T + \frac{S_0}{r} D_a r \hat{\sigma}^a - F \right)$$

These source terms are written in terms of gauge the invariants:

$$\hat{\sigma}^t = \sigma^t + \frac{X^t}{\mu}$$

$$\hat{\sigma}^r = \sigma^r + \frac{X^r}{S_0}$$

$$\hat{\sigma}_T = \sigma_T + \frac{1}{kS_0} H_T$$

Then if we want to recover the standard results, some constraints on the dynamics of the Stückelberg fields must be imposed.

Conclusions

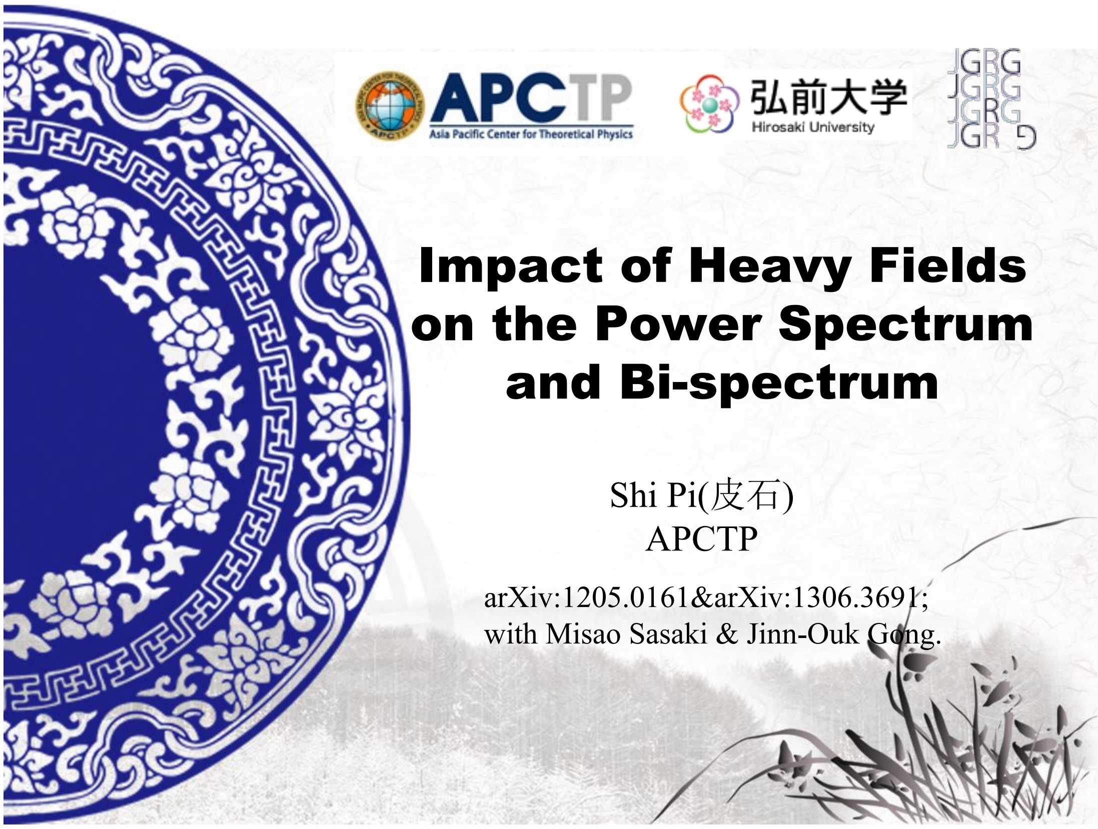
- 1). We have derived general expressions for the Black Hole perturbations inside the dRGT formalism.
- 2). When we allow the Stückelberg fields to be dynamical, some special constraints have to be imposed in order to keep the tehory inside the standard behavior of GR.

This is the JGRG23

“Impact of heavy fields on power spectrum and bispectrum of the curvature perturbation”

by Shi Pi

[JGRG23(2013)110507]


APCTP
Asia Pacific Center for Theoretical Physics
弘前大学
Hirosaki University
Impact of Heavy Fields on the Power Spectrum and Bi-spectrum
Shi Pi(皮石)
APCTP
arXiv:1205.0161&arXiv:1306.3691;
with Misao Sasaki & Jinn-Ouk Gong.


Introduction
Effective Field Approach
In-in formalism Approach
Summary

Inflation

The essense of inflation:

- ❖ An accelerated expansion.
- ❖ Lasts for 60 efoldings.
- ❖ Quantum fluctuations.

Inflation zoology

- ❖ Early Inflations: Starobinsky model. Single field inflation (canonical/non-canonical, potentials...).
- ❖ Multi-field Inflation. (light, heavy, ...)
- ❖ New Physics Stuff. (Sugra, modified gravity, string landscape,...)

Single Field

Single field inflation models with non-canonical kinetic. (Armendariz-Picon et al, hep-th/9904075)

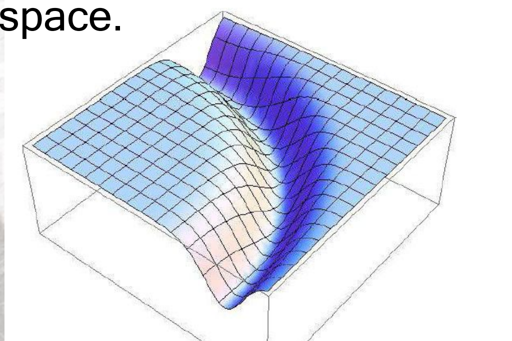
$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{pl}^2 R + 2P(X,\phi)\right]$$

This includes some typical models:

- ❖ Slow-roll inflation (Linde 1982).
- ❖ Kinetic driven k-inflation.
- ❖ DBI inflation (Silverstein et al, hep-th/0310221).

Multi-field

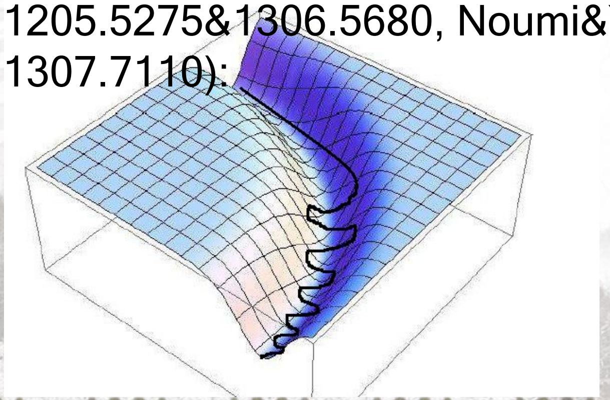
Multi-field inflation models with canonical kinetic. It can be described by a turning motion in a moduli space.



$$M_{\text{eff}}^2 = V'' + 3\dot{\theta}_0^2$$

Sudden Turn

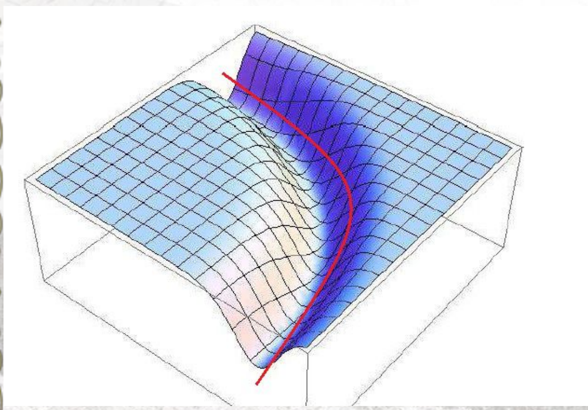
Sudden turn case
(Shiu&Xu 1108.0981, Gao&Langlois 1205.5275&1306.5680, Noumi&Yamaguchi 1307.7110):



$\dot{\theta} >> H$

Soft Turn

Soft turn case.



$\dot{\theta} << H$

Soft turn classify

Slow-roll multi-field inflation can be categorized by

❖ $\dot{\theta} \ll H,\ M_{\text{eff}} \ll H$

2-field inflation with small couplings. Gordon et al astro-ph/0009131.

❖ $\dot{\theta} \ll H,\ M_{\text{eff}} \sim H$

Original quasi-single field inflation. Chen&Wang 0909.0496.

❖ $\dot{\theta} \ll H,\ M_{\text{eff}} \gg H$

Effective field theory after integrating heavy fields out. Tolley 0910.1853. Achucarro 1010.3693.

Power Spectrum

The power spectrum:

❖ $\dot{\theta} \ll H,\ M_{\text{eff}} \ll H$

Enhanced spectrum for curv.pert. Spectrum of entropy pert. Gordon et al astro-ph/0107089.

❖ $\dot{\theta} \ll H,\ M_{\text{eff}} \sim H$

Small correction to the single-field result. Chen&Wang 0911.3380.

❖ $\dot{\theta} \ll H,\ M_{\text{eff}} \gg H$

Small correction prop to M^{-2}. Achucarro et al 1010.3693. Chen 1205.0160. SP 1205.0161.

Non-Gaussianity

The non-Gaussianity of the corresonding models:

- $\dot{\theta} \ll H,\, M_{\rm eff} \ll H$
  Local shape. Maybe suppressed by slow-roll parameters . Vernizzi&Wands astro-ph/0603799.
- $\dot{\theta} \ll H,\, M_{\rm eff} \sim H$
  Trasition from local to equilateral. Chen&Wang 0911.3380. Noumi&Yamaguchi 1211.1624.
- $\dot{\theta} \ll H,\, M_{\rm eff} \gg H$
  Equilateral. Prop to $1/M^6$. Gong, SP & Sasaki 1306.3691.

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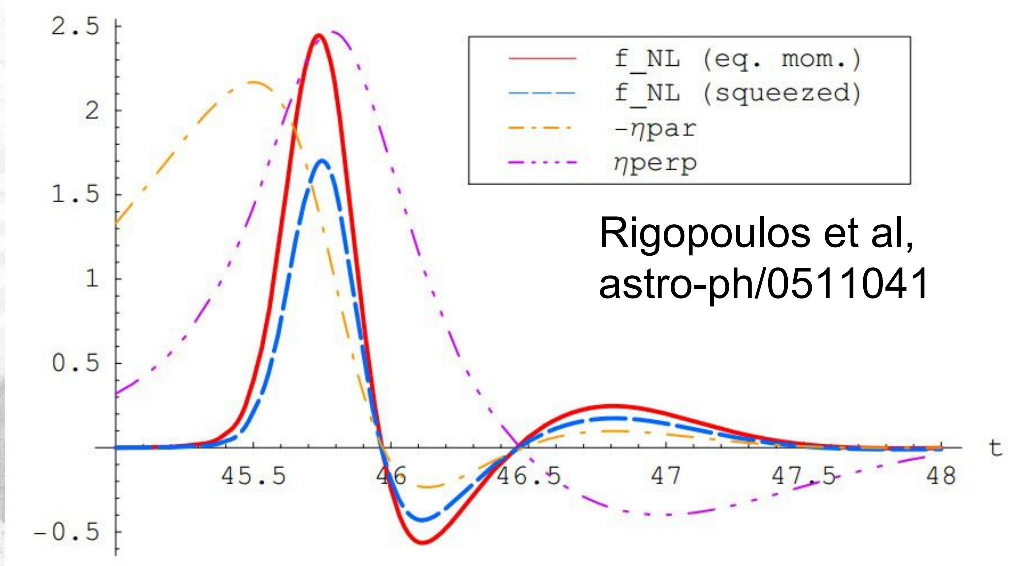
  Equilateral. Prop to $1/M^6$. Gong, Pi & Sasaki 1306.3691.

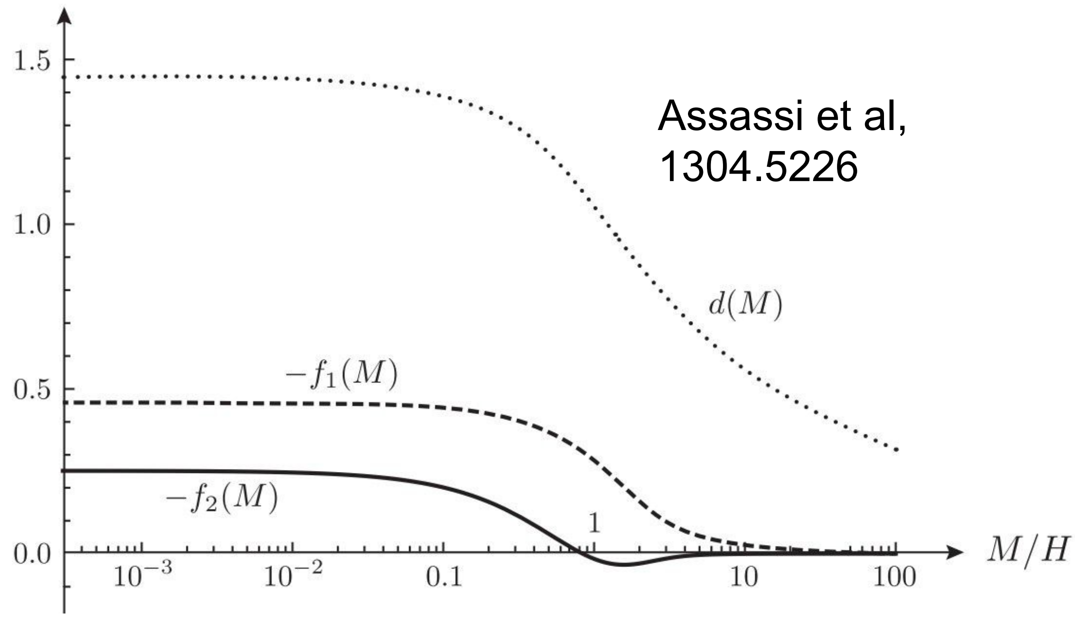
nG with light field

$\dot{\theta} \ll H,\ M_{\rm eff} \ll H$



nG with light field

$\dot{\theta} \ll H,\ M_{\rm eff} \sim H$



Introduction

Effective Action of Inflation

In-in formalism

Summary

QSF Inflation

Quasi-single field inflation can
- ❖ be solved analytically (in principle);
- ❖ mimic the trasition from multi-field to single-field;
- ❖ be embedded into complicated field configurations;
- ❖ reveal the essential of non-canonical kinetic terms and non-linear interactions.

Action

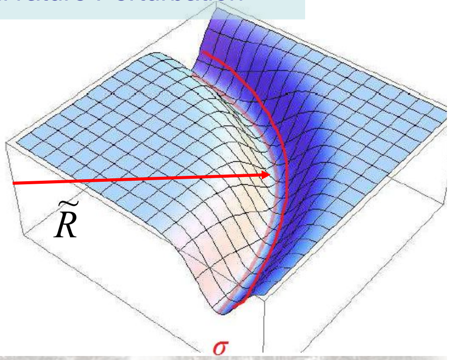
Curvature Perturbation

$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\left(\tilde{R}+\sigma\right)^2(\partial_\mu\theta)^2 - \frac{1}{2}(\partial_\mu\sigma)^2 - V_{\rm sr}(\theta) - V(\sigma)\right]$$

Isocurvature Perturbation

Variables:

- ❖ σ is the radial field.
- ❖ θ is the angular field.
- ❖ $V_{\rm sr}$ is the slow-roll potential along the trajectory.
- ❖ V is the heavy potential perpendicular to it.



EOM

Equation of motions:

$$3m_{\rm Pl}^2H^2 = \frac{1}{2}R^2\dot{\theta}_0^2 + V_{\rm sr} + V\,,$$

$$-2m_{\rm Pl}^2\dot{H} = R^2\dot{\theta}_0^2\,,$$

$$0 = R^2\ddot{\theta}_0 + 3R^2H\dot{\theta}_0 + V_{\rm sr}'\,,$$

$$0 = V' - R\dot{\theta}_0^2\,.$$

Slow-roll Parameters

Slow-roll parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{R^2\dot{\theta}_0^2}{2m_{\rm Pl}^2 H^2} \approx \frac{m_{\rm Pl}^2}{2}\left(\frac{V'_{\rm sr}}{RV_{\rm sr}}\right)^2 ,$$

$$\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \approx -2m_{\rm Pl}^2\frac{V''_{\rm sr}}{R^2V_{\rm sr}} + 2m_{\rm Pl}^2\left(\frac{V'_{\rm sr}}{RV_{\rm sr}}\right)^2 .$$

Introduction

Perturbation theory:

$$\theta(t,\boldsymbol{x}) = \theta_0(t) + \delta\theta(t,\boldsymbol{x}) ,$$

$$\sigma(t,\boldsymbol{x}) = \sigma_0 + \delta\sigma(t,\boldsymbol{x}) .$$

Perturb the potential:

$$V(\sigma) = V(\sigma_0) + V'(\sigma_0)\delta\sigma + \frac{1}{2}V''(\sigma_0)\delta\sigma^2 + \frac{1}{6}V'''(\sigma_0)\delta\sigma^3 + \cdots .$$

Potential Series

When will the expansion be valid:

$$V''(\sigma_0) > \frac{1}{3}V'''(\sigma_0) > \frac{1}{4}V''''(\sigma_0) > \ldots$$

Perturbative Action

$$S[\delta\theta,\delta\sigma] = \int dt d^3x a^3 \left[\frac{1}{2}R^2\dot{\delta\theta}^2 - \frac{R^2}{2a^2}(\nabla\delta\theta)^2 + \frac{1}{2}\dot{\delta\sigma}^2 - \frac{1}{2a^2}(\nabla\delta\sigma)^2 - \frac{1}{2}m_{\text{eff}}^2\delta\sigma^2 + 2R\dot{\theta}_0\delta\dot{\theta}\delta\sigma + R\delta\sigma\dot{\delta\theta}^2 + \dot{\theta}_0\delta\dot{\theta}\delta\sigma^2 - \frac{R}{a^2}\delta\sigma\left(\nabla\delta\theta\right)^2 - \frac{1}{6}V'''(\sigma_0)\delta\sigma^3 + \cdots\right]$$

2nd order coupling: $+ 2R\dot{\theta}_0\delta\dot{\theta}\delta\sigma$

3rd order coupling suppressed by slow-roll: $+R\delta\sigma\dot{\delta\theta}^2 + \dot{\theta}_0\delta\dot{\theta}\delta\sigma^2 - \frac{R}{a^2}\delta\sigma\left(\nabla\delta\theta\right)^2$

Interactions from heavy fields: $-\frac{1}{6}V'''(\sigma_0)\delta\sigma^3$

Constraint for Pert.

Equation of motion for heavy perturbation field $\delta\sigma$

$$\ldots\sigma + \ldots H\delta\ldots - \left(\frac{\nabla^2}{a^2} - m_{\text{eff}}^2\right)\delta\sigma + \frac{V'''}{2}\delta\sigma^2 = 2R\dot{\theta}_0\dot{\delta\theta}$$

The solution for $\delta\sigma$ is

$$\delta\sigma = \frac{2R\dot{\theta}_0}{m_{\text{eff}}^2}\dot{\delta\theta} + \left(\frac{R}{m_{\text{eff}}^2 c_s^2} - \frac{2R^2\dot{\theta}_0^2}{m_{\text{eff}}^2}\frac{V'''}{m_{\text{eff}}^4}\right)\dot{\delta\theta}^2 + \cdots$$

EFT of single field

$$\frac{1}{c_s^2} \equiv 1 + \frac{4\dot{\theta}^2}{m_{\text{eff}}^2}$$

$$S_{\text{eff}}[\delta\theta] = \int dt d^3x a^3 \left[\frac{1}{2}R^2\dot{\delta\theta}^2\left(1 + 4\frac{\dot{\theta}_0^2}{m_{\text{eff}}^2}\right) - \frac{R^2}{2a^2}(\nabla\delta\theta)^2 \right.$$
$$\left. + \left(\frac{2R^2\dot{\theta}_0^2}{m_{\text{eff}}^2} + \frac{4R^2\dot{\theta}_0^3}{m_{\text{eff}}^4} - \frac{4R^3\dot{\theta}_0^3}{3m_{\text{eff}}^6}V'''\right)\dot{\delta\theta}^3 - \frac{2R^2\dot{\theta}_0}{a^2 m_{\text{eff}}^2}\dot{\delta\theta}\,(\nabla\delta\theta)^2\right]$$

$$\mathcal{R} = -\frac{H}{\dot{\theta}_0}\delta\theta$$

Power Spectrum

Power spectrum of the curvature perturbation $R\theta$. (Garriga&Mukhanov hep-th/9904176)

$$\mathcal{P}_{\mathcal{R}} = \frac{H^2}{8\pi^2 m_{\rm Pl}^2 \epsilon c_s}$$

where c_s is the effective speed of sound

$$\frac{1}{c_s^2} \equiv 1 + \frac{4\dot{\theta}^2}{m_{\rm eff}^2}$$

non-Gaussianity

Calculate the dominant non-Gaussianity is similar to that of the general single-field inflation (Chen, Huang, Kachru, Shiu hep-th/0605045)

$$\langle \mathcal{R}(\boldsymbol{p}_1)\mathcal{R}(\boldsymbol{p}_2)\mathcal{R}(\boldsymbol{p}_3)\rangle = (2\pi)^7 \delta^{(3)}(\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3)\mathcal{P}_{\mathcal{R}}^2 \frac{S_\lambda}{(p_1 p_2 p_3)^2}$$

$$S_\lambda = -\frac{6\dot{\theta}_0^2}{m_{\rm eff}^2}\left[1 - \frac{2\dot{\theta}_0^2}{m_{\rm eff}^2} c_s^2 \left(1 + \frac{RV'''}{3m_{\rm eff}^2}\right)\right] \frac{p_1 p_2 p_3}{(p_1 + p_2 + p_3)^3}$$

Non-linear parameter

$$\frac{1}{c_s^2} = 1 + \frac{4\dot{\theta}^2}{m_{\rm eff}^2}$$

The equilateral non-linear parameter $f_{\rm NL}$ is

$$-\frac{20}{81}\left(\frac{\dot{\theta}_0}{m_{\rm eff}}\right)^2 + \frac{40}{81}c_s^2\left(\frac{\dot{\theta}_0}{m_{\rm eff}}\right)^4 + \frac{40}{243}\frac{RV'''}{m_{\rm eff}^2}c_s^2\left(\frac{\dot{\theta}_0}{m_{\rm eff}}\right)^4$$

nG from linear couplings

nG from non-linear couplings: Heavy field self-interaction

Comparison

We only consider the leading order nG:

❖ Which one can dominate the nG?

V'''-term is possible to dominant.

❖ Can it be large?

Only the V'''-term is possible to be large.

Reason to dominate

Recall the condition for potential series:

$$\frac{V'''\delta\sigma}{3V''} < 1$$

Here $\delta\sigma$ is solved by integrated e.o.m. $\delta\sigma = \frac{2R\dot{\theta}_0}{m_{\rm eff}^2}\dot{\delta\theta}$

And $(\delta\theta)^{\cdot}$ can be estimated by the power spectrum of the curvature perturbation.
All together can give an estimation of the upper limit for V'''.

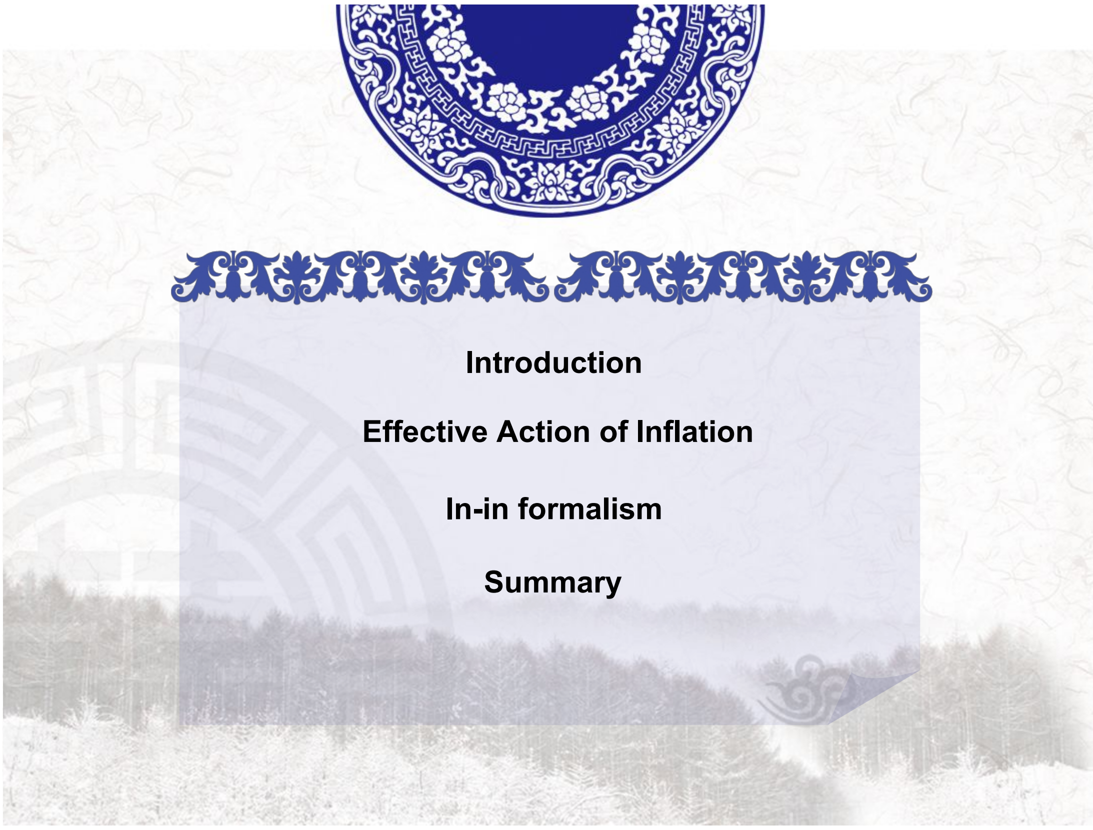
Upper Limit

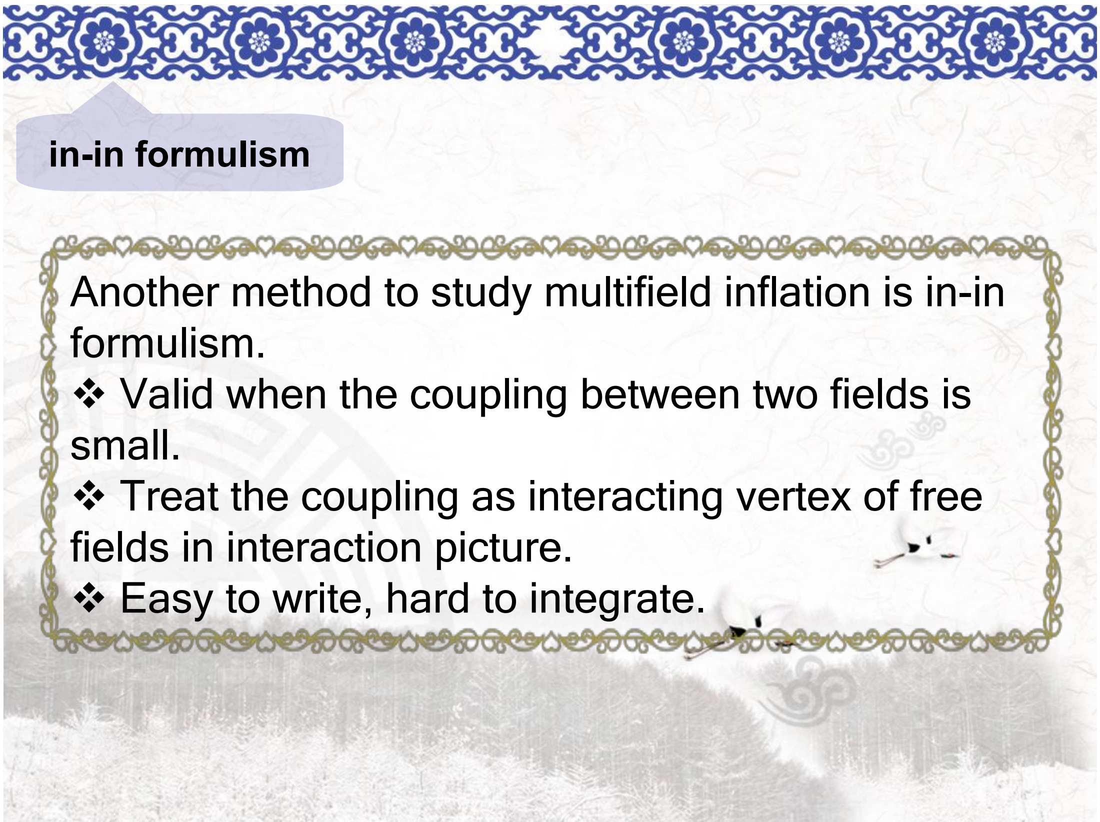
A large prefactor of order 10^6

Use this upper limit for V''' we can have the upper limit for V'''-term in $f_{\rm NL}$.

$$V'''\text{-term} < \frac{1}{\eta P_R^{1/2}}\left(\frac{\dot{\theta}}{m_{eff}}\right)^2$$

Linear nG


Introduction
Effective Action of Inflation
In-in formalism
Summary


in-in formulism
Another method to study multifield inflation is in-in formulism.
❖ Valid when the coupling between two fields is small.
❖ Treat the coupling as interacting vertex of free fields in interaction picture.
❖ Easy to write, hard to integrate.

Program

Program to calculate in in-in formalism

- ❖ Begin with the perturbative action.
- ❖ Define canonical conjugate momenta for each field.
- ❖ Define Hamiltonian.
- ❖ Divided into free part and interaction part.
- ❖ Replacing canonical conjugate momenta with the one in the interaction picture, i.e. defined by free Hamiltonian rather than full Hamiltonian.
- ❖ Cancel conjugate momenta with field velocity.

Hamiltonian

$$\mathcal{H}_0 = a^3\left[\frac{1}{2}R^2\dot{\delta\theta_I}^2 + \frac{R^2}{2a^2}(\nabla\delta\theta_I)^2 + \frac{1}{2}\dot{\delta\sigma_I}^2 + \frac{1}{2a^2}(\nabla\delta\sigma_I)^2 + \frac{1}{2}M_{\rm eff}^2\delta\sigma_I^2\right] ,$$

$$\mathcal{H}_2^I = -2R\dot\theta_0 a^3\delta\sigma_I\dot{\delta\theta}_I ,$$

$$\mathcal{H}_3^I = -a^3R\delta\sigma_I\dot{\delta\theta_I}^2 - a^3\dot\theta_0\dot{\delta\theta}_I\delta\sigma_I^2 + aR\delta\sigma_I\left(\nabla\delta\theta_I\right)^2 + \frac{a^3}{6}V'''\delta\sigma_I^3 ,$$

$$M_{\rm eff}^2 = V'' + 3\dot\theta_0^2 .$$

The condition to do so is to keep the interaction Hamiltonian smaller than the free Hamiltonian, i.e.

$$\dot\theta_0 < H$$

Split the free field

We split the Hamiltonian from free part and interaction part. The former one can be solved by invoking "free field" which satisfied

$$\delta\theta_I(\boldsymbol{p}) = u_p a_{\boldsymbol{p}} + u_p^* a_{-\boldsymbol{p}}^\dagger ,$$

$$\delta\sigma_I(\boldsymbol{p}) = v_p b_{\boldsymbol{p}} + v_p^* b_{-\boldsymbol{p}}^\dagger ,$$

Mode Function

Light field (curv. pert.)
Hankel equation of order 3/2

Annihilation and creation operators obey the ordinary commutation relations, whereas the mode functions are governed by the eoms:

$$\frac{d^2 u_p}{d\tau^2} - \frac{2}{\tau}\frac{du_p}{d\tau} + p^2 u_p = 0 ,$$

$$\frac{d^2 v_p}{d\tau^2} - \frac{2}{\tau}\frac{dv_p}{d\tau} + \left(p^2 + \frac{M_{\rm eff}^2}{H^2\tau^2}\right) v_p = 0 ,$$

Heavy field: Hankel equation of order $\nu = \sqrt{9/4 - m^2/H^2}$

Moltivation

In our work we forcus on the case when M_{eff} is very large. This is because
1, We can have an analytical result.
2, It is also the case when EFT is valid.
3, A large mass hierachy in the early universe is interesting.
4, Large equilateral nG is still possible on *Planck* data.

Free Field Solution $\mu = \sqrt{M_{\text{eff}}^2/H^2 - 9/4}$

We can solve the eoms in the large mass case.

$$u_p = \frac{H}{R\sqrt{2p^3}}(1 + ip\tau)e^{-ip\tau} \, ,$$

$$v_p = -ie^{-\frac{\pi}{2}\mu + i\frac{\pi}{4}} \frac{\sqrt{\pi}}{2} H(-\tau)^{3/2} H_{i\mu}^{(1)}(-p\tau)$$

Then the interactions can be treated as perturbations to this free propagating plane waves.

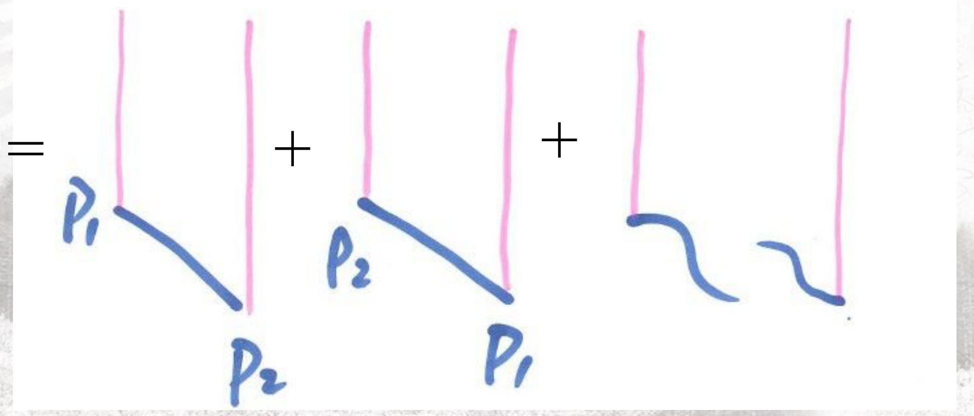
Correlation Function

We first write down the 2-point function

$$
\begin{aligned}
&\langle \delta\theta^2 \rangle \\
\equiv\ & \langle 0 | \left[\bar{T} \exp\left(i \int_{t_0}^{t} dt' H_I(t') \right) \right] \delta\theta_I^2(t) \left[T \exp\left(-i \int_{t_0}^{t} dt' H_I(t') \right) \right] | 0 \rangle \\
\sim\ & \mathcal{P_R}^{(0)} + \delta \mathcal{P_R} \\
=\ & \frac{H^4}{4\pi^2 R^2 \dot{\theta}^2} \left[1 + \frac{\delta \mathcal{P_R}}{\mathcal{P_R}^{(0)}} \right] .
\end{aligned}
$$

Two point function:

$\langle \zeta^2 \rangle \sim \langle \delta\theta^2 \rangle$

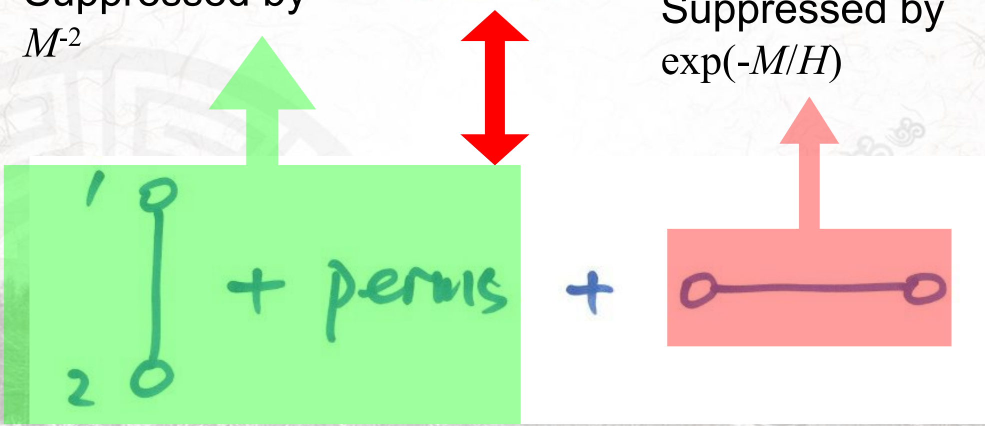


Diagrammatica

$$\langle \delta\theta^2 \rangle$$

Suppressed by M^{-2}

Suppressed by $\exp(-M/H)$



Power Spectrum

The power spectrum for curvature perturbation is (Chen&Wang 1205.0160, SP&Sasaki 1205.0161)

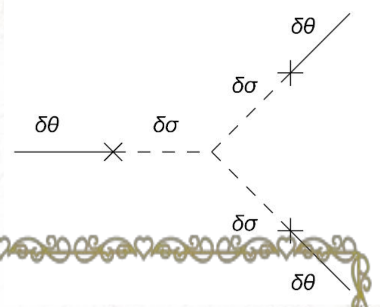
$$\mathcal{P}_\mathcal{R} \approx \mathcal{P}_\mathcal{R}^{(0)} \left[1 + 2\frac{H^2}{M_{\rm eff}^2} \left(\frac{\dot\theta}{H} \right)^2 \right]$$

After introducing the sound speed

$$\frac{1}{c_s^2} \equiv 1 + \frac{4\dot\theta^2}{m_{\rm eff}^2}$$

We see that this result is consistent with EFT.

Correlation Function

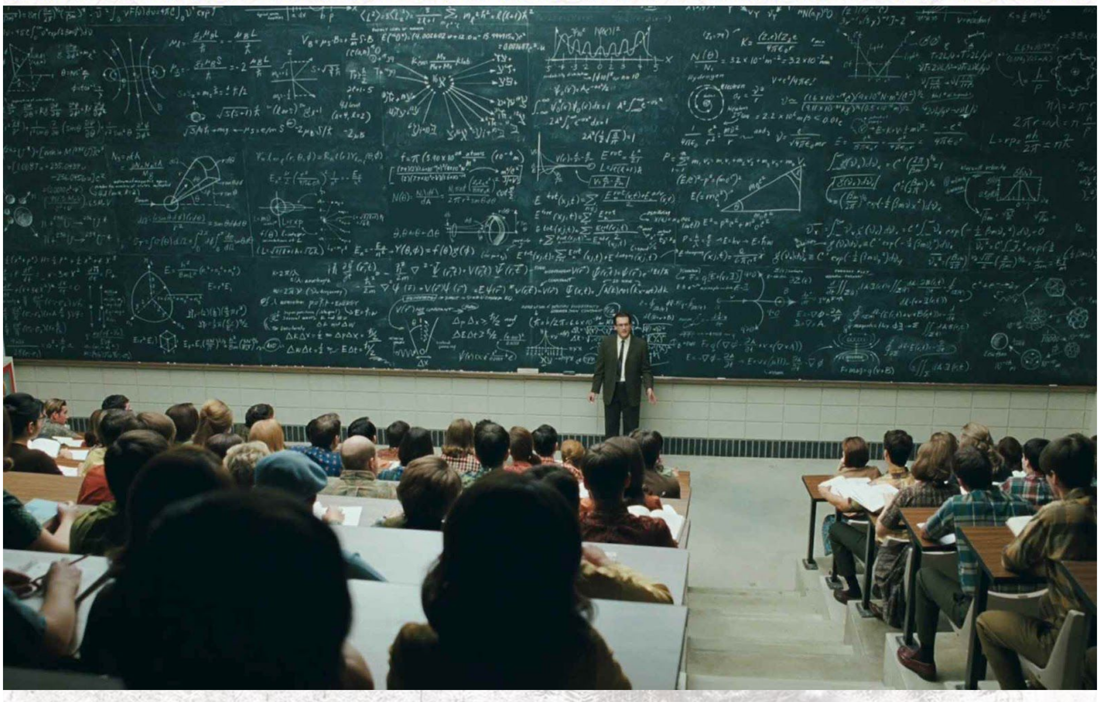


The 3-point function

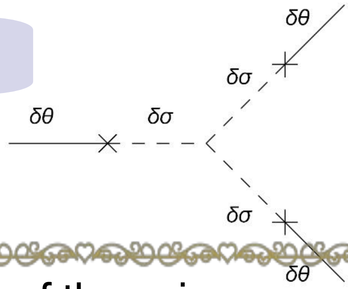
$$\langle \mathcal{R}^3 \rangle = -\left(\frac{H}{\dot{\theta}_0}\right)^3 \langle \delta\theta^3 \rangle$$

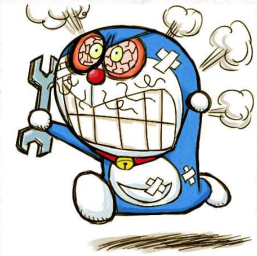
$$\langle \delta\theta^3 \rangle = \left\langle 0 \left| \left[\bar{T} \exp\left(i \int_{t_0}^{t} dt' H_I(t') \right) \right] \delta\theta_I^3 \left[T \exp\left(-i \int_{t_0}^{t} dt' H_I(t') \right) \right] \right| 0 \right\rangle$$

After Wick contraction we have 60 terms (10 different terms+50 permutations of 3 momenta).



A Typical Term

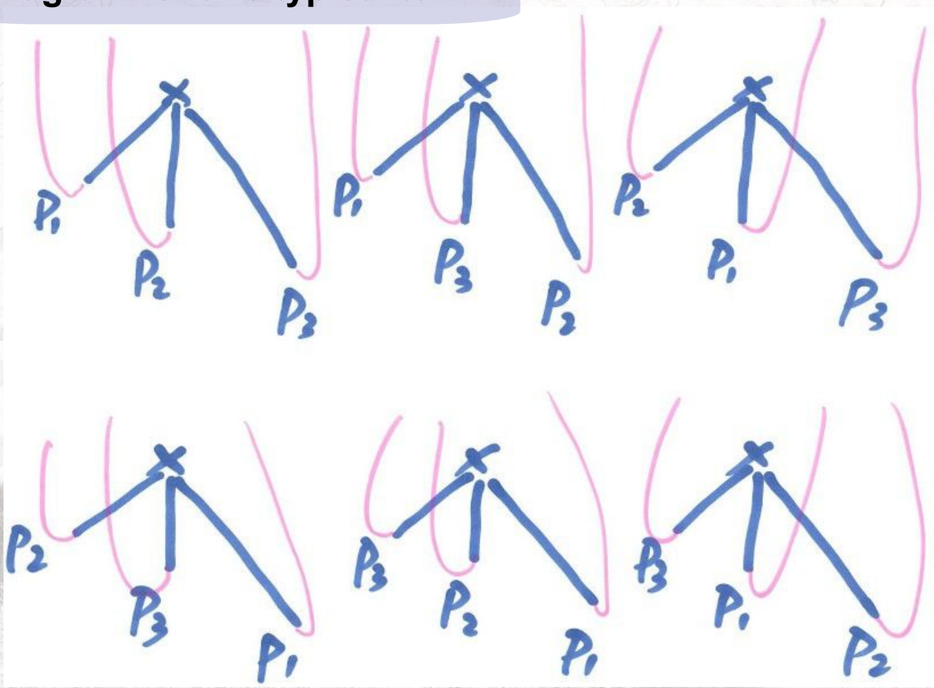


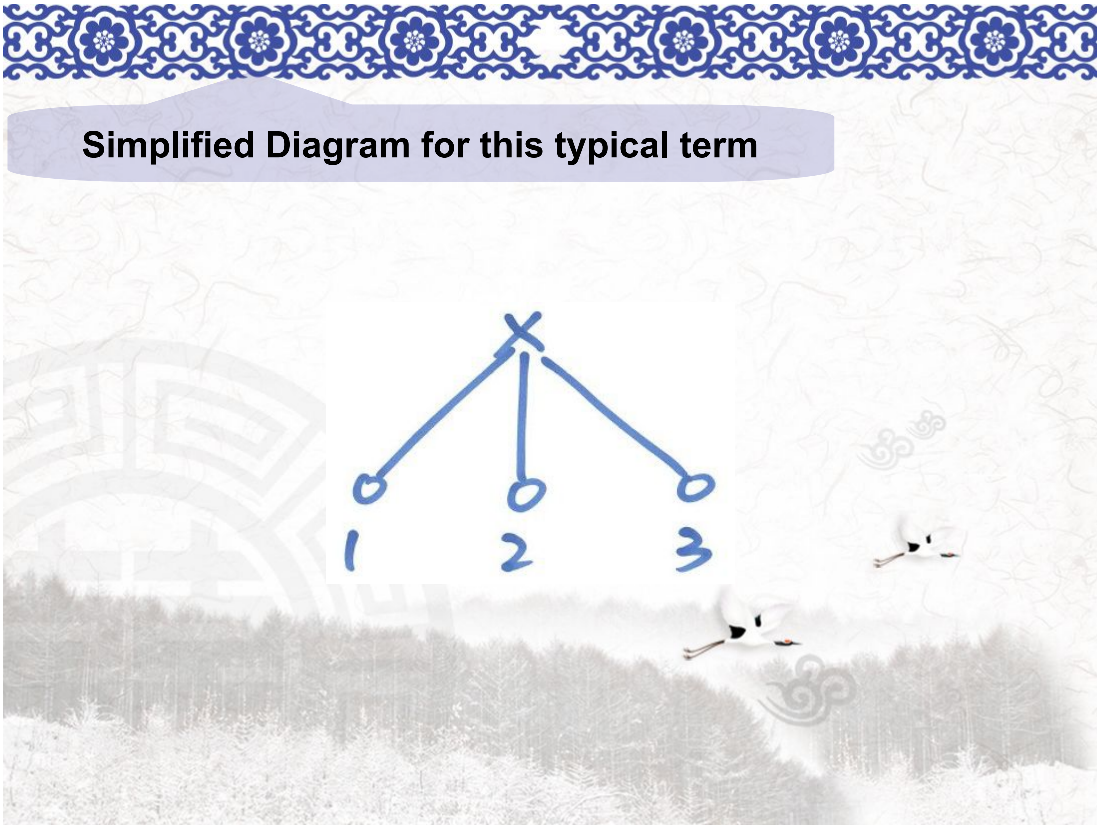


A typical term of these is

$$\langle \delta\theta^3 \rangle = 16 R^3 \dot{\theta}_0^3 V''' u_{p1} u_{p2} u_{p3}(0)$$
$$\mathrm{Re}\left[\int_{-\infty}^{0} d\tau\ a^4 v_{p1} v_{p2} v_{p3}(\tau) \int_{-\infty}^{\tau} d\tau_1\ a^3 v_{p1}^* u_{p1}'^*(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2\ a^3 v_{p2}^* u_{p2}'^*(\tau_2) \int_{-\infty}^{\tau_2} d\tau_3\ a^3 v_{p3}^* u_{p3}'^*(\tau_3) \right]$$

Diagram for this typical term




Simplified Diagram for this typical term
1
2
3

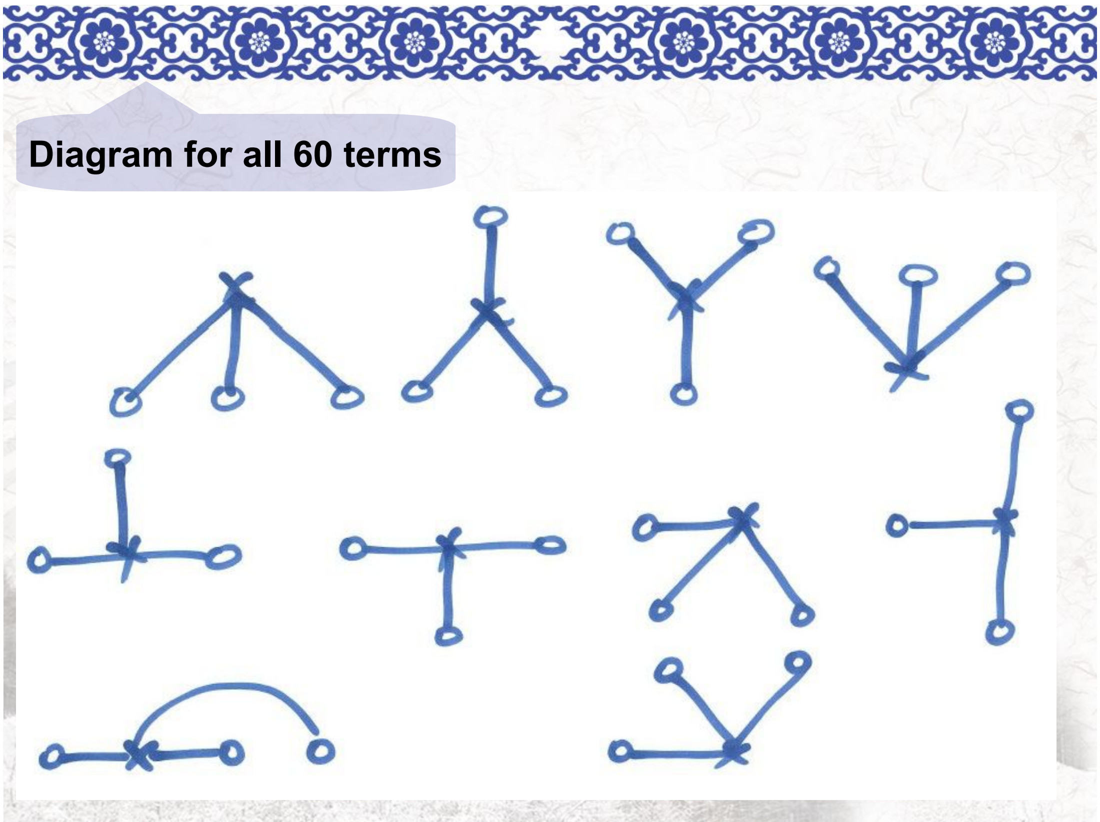
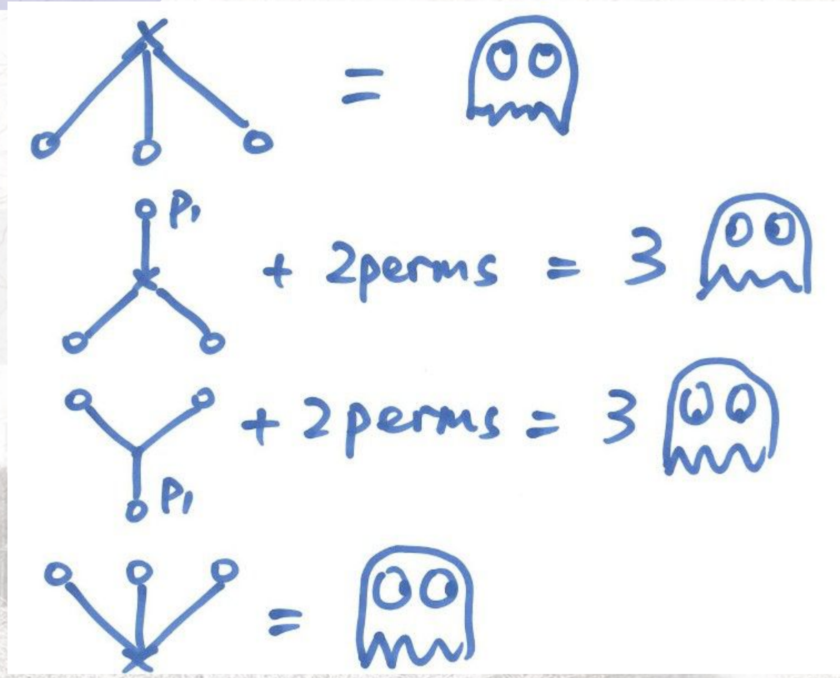


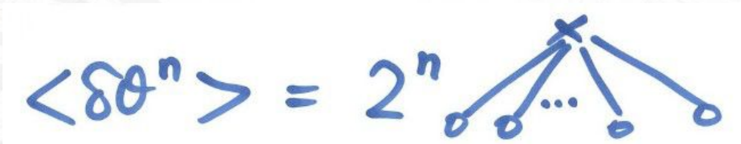
Diagram for all 60 terms

Symmetry



Approximation

These four "ghosts" are different essentially. But they can be the same in the limit when $M_{\text{eff}} \to \infty$. In general, we have



nG Diagram

And for the 3-point function we have

$$\langle \mathcal{R}(\mathbf{p}_1)\mathcal{R}(\mathbf{p}_2)\mathcal{R}(\mathbf{p}_3)\rangle = (2\pi)^7 \delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)\mathcal{P}_{\mathcal{R}}^2 \frac{S_{\delta\sigma^3}(p_1, p_2, p_3)}{p_1^2 p_2^2 p_3^2},$$

$$S_{\delta\sigma^3}(p_1, p_2, p_3) = \frac{4\pi^6 R\dot{\theta}_0^4 c_s^2}{\mu^6 H^6} V''' \frac{p_1 p_2 p_3}{(p_1 + p_2 + p_3)^3},$$

$$f_{\rm NL}^{\delta\sigma^3} = \frac{40}{243}\frac{R\dot{\theta}_0^4 c_s^2}{H^6 \mu^6} V'''.$$

Introduction

Effective Action of Inflation

In-in formalism

Summary

Summary

Our model: Two-field, canonical kinetic, power-law potential, weak coupled, adiabatic turn, massless+very massive.

Our goal:

❖ Correction to the power spectrum of curvature perturbation originating from the rotation in field space.

❖ Correction to the non-Gaussianity due to the heavy-field interactions (by both EFT approach and in-in formalism)

Summary

Our result: It may dominant the nG.

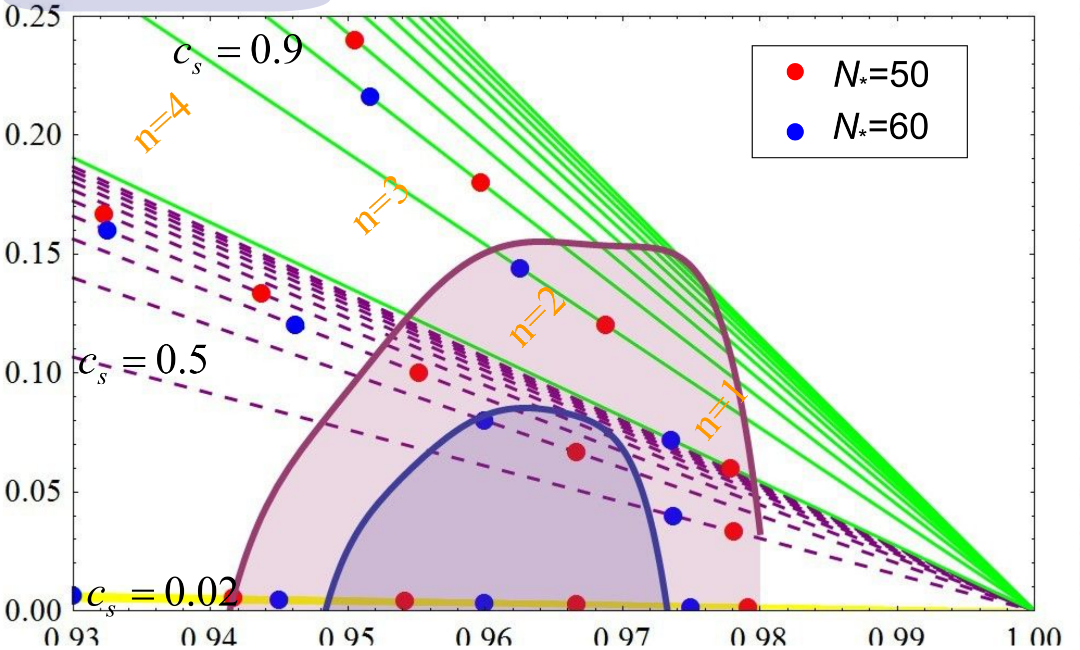
❖ Correction to power spectrum

$$\frac{\Delta P}{P} \propto \left(\frac{\dot{\theta}}{M} \right)^2$$

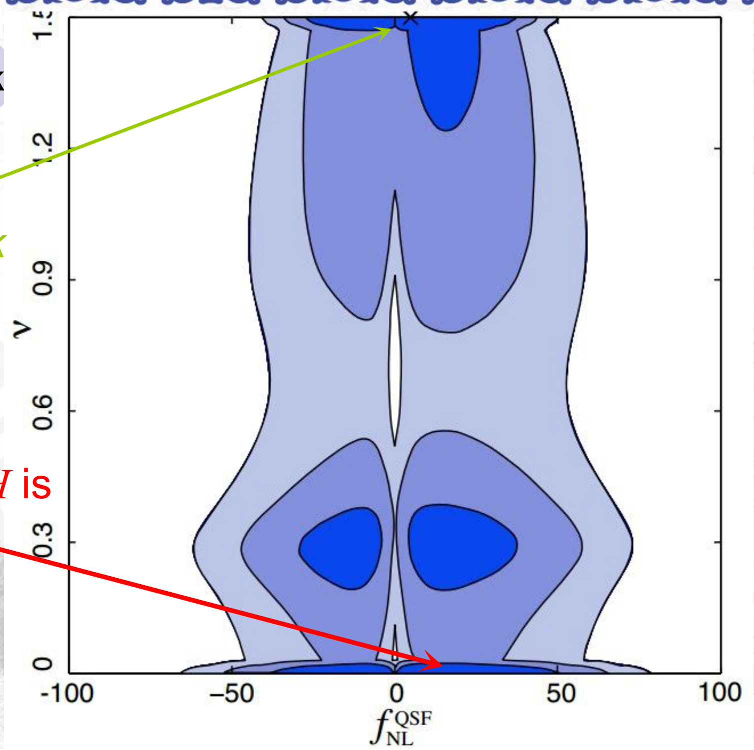
❖Correction to non-Gaussianity (equilateral, can be large)

$$\Delta f_{NL} \propto \left(\frac{\dot{\theta}^4}{M^6} \right) V'''$$

Compare to Planck



Compare to Planck




Thank you!
謝謝！
ありがと！

“Features in the curvature power spectrum after a sudden turn of the inflationary trajectory”

by Xian Gao

[JGRG23(2013)110508]

Features in the curvature power spectrum after a sudden turn of the inflationary trajectory

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5 November, 2013
The 23rd Workshop on General Relativity and Gravitation in Japan
Hirosaki University

Based on works with ***David Langlois*** and ***Shuntaro Mizuno***
JCAP 10 (2012) 040 [arXiv:1205.5275]
JCAP 10 (2013) 023 [arXiv:1306.5680]

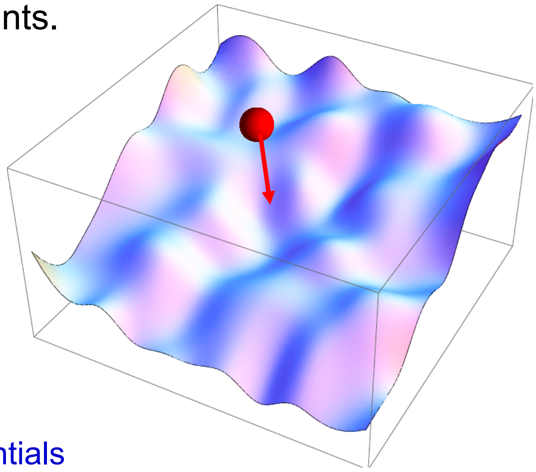
Single field inflation

- The latest observations on CMB are compatible with statistically Gaussian primordial perturbation, which has a nearly flat spectrum with negligible running spectral tilt.
- In particular, the data are compatible with the adiabaticity at 95% CL, which implies there is no evidence for the isocurvature modes and there is only one relevant degree of freedom responsible to the primordial perturbations.
- Beyond the single-field?
  - Theoretical motivation
  - Observational hints: asymmetries, oscillatory features, etc.

Massive fields

Can massive (*M* >= *H*) fields be allowed and play some role in multi-field models?

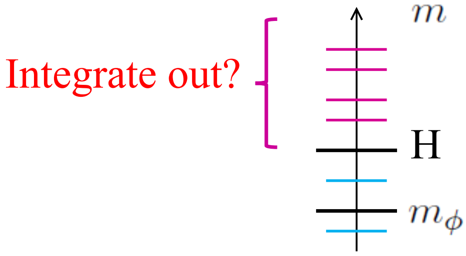
- As long as there is a **light (flat) direction** in the multi-field potential, inflation occurs, while other directions may be heavy.
- Perturbations probe the whole potential landscape, not only the light direction.
- Massive modes may have some imprints.



A landscape of potentials

Heavy modes?

- Naively, an effective theory for the light mode(s) is expected.



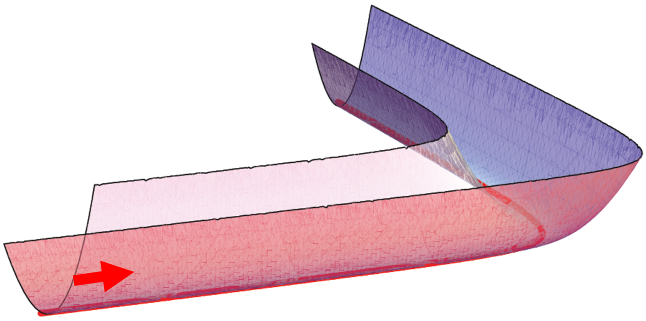
- If there is a bending trajectory:
  - The trajectory generally deviates from the light direction.
  - The **adiabatic** mode can become temporarily **heavy**.
  - The effective single-field description may break down.

- Recent progress: *Tolley & Wyman `09. Cremonini, Lalak & Turzynski '10, Achucarro, Gong, Hardeman, Palma, Patil `10, Shiu & Xu `11, Watson et al '12. Chen & Wang `12, Gong, **Pi** & Sasaki '12, '13, **Noumi**, Yamaguchi & Yokoyama '12, '13, **Saito**, Nakashima, Takamizu,Yokoyama, '12, '13. …*

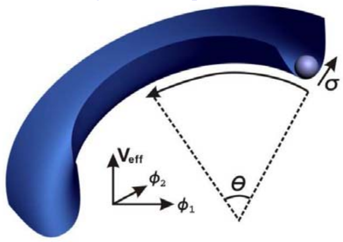
Heavy modes at work: Turning trajectory

Multi-field effects manifest themselves only when the background trajectory is **bending**.

We will concentrate on a **single turning process**, by requiring (the minimal deviation from the standard scenario):
1) the turning process occurs in a **finite** time interval
2) the potential trough is asymptotically **straight** before and after the turn.

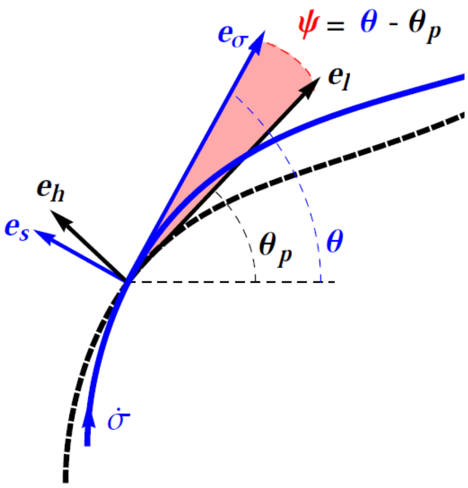


Different from "**constant** turn" in QSI [*Chen & Wang '09, '12*]



[Gong, Pi, Sasaki '13, Noumi, Yamaguchi, Yokoyama '12]

Turning trajectory: a two-field example



The background trajectory is characterized by: $\{\dot\sigma, \psi\}$

- **Velocity**: $\ddot\sigma + 3H\dot\sigma + V_{,\sigma} = 0$
- **Direction**: **A simple approximate equation of motion for ψ ($|\psi|$<<1):**

$$\ddot\psi + 3H\dot\psi + m_h^2\psi \simeq -\ddot\theta_p - 3H\dot\theta_p$$

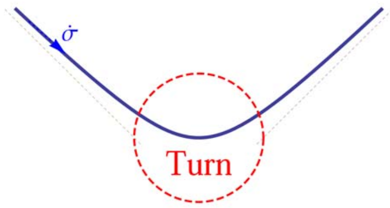
$$\boldsymbol{M} = \mathrm{diag}\{m_l^2, m_h^2\}, \qquad m_h \gtrsim H \gg M_l$$

- In general, the trajectory (adiabatic direction) tends to deviate from the light direction, with turning light direction $\boldsymbol{\theta_p}$ serves as a driving force;
- ψ behaves as a damped oscillator with frequency controlled by $\boldsymbol{m_h}$;

A Gaussian toy model

A toy Gaussian ansatz:

$$\dot{\theta}_p(t) = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2}$$

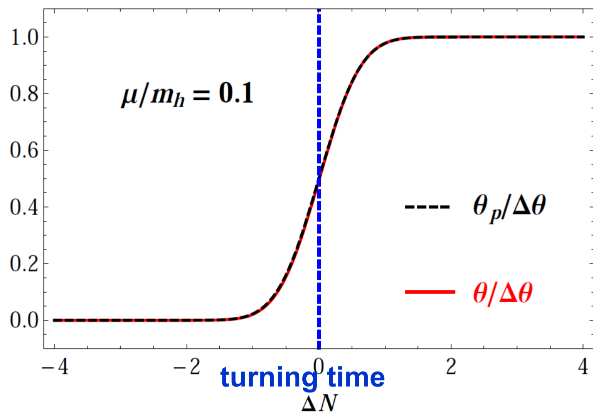


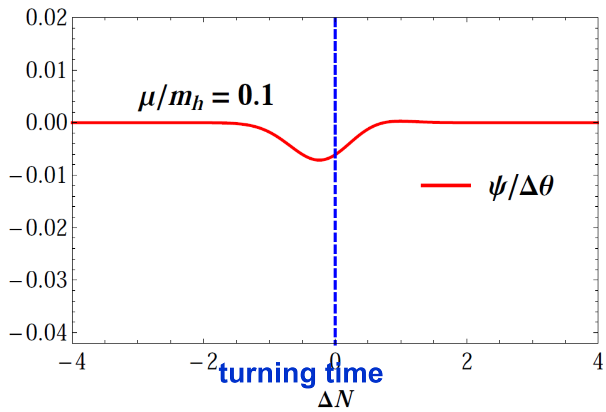
"Energy scale" of the turn: ***μ* = 1/Δt >> *H***

The qualitative behaviors of the trajectory and the perturbations are sensitive to the **ratio: μ/m_h** .

Limit 1: Soft turn ($\mu \ll m_h$)

$$\psi(t) \approx \frac{\Delta\theta}{\sqrt{2\pi}} \left(\frac{\mu}{m_h}\right)^2 e^{-\frac{1}{2}\mu^2 t^2} \left(\mu t - 3\frac{H}{\mu}\right)$$





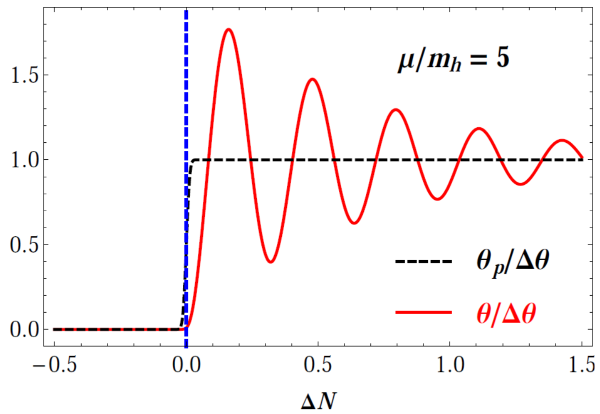
Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

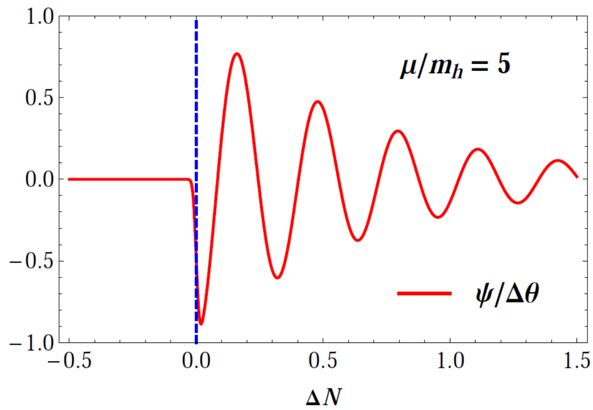
Evolution of $\psi = \theta - \theta_p$
(angle between trajectory & light direction)

→ tiny deviation

Limit 2: Sharp turn (*μ*>≈*m_h*)

$$\psi(t) \approx -\frac{\Delta\theta}{2} e^{-\frac{m_h^2}{2\mu^2}} \mathrm{erfc}\left(-\frac{\mu t}{\sqrt{2}}\right) e^{-\frac{3}{2}Ht} \cos(m_h t - \text{phase})$$





Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

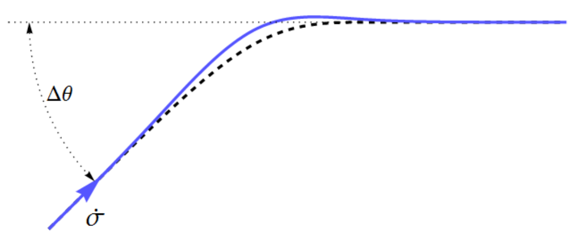
Evolution of $\psi=\theta-\theta_p$
(angle between trajectory & light direction)

→ large deviation with oscillation

Evolution of the trajectory

Soft turn

- Just around the turning point, the trajectory deviates slightly from the light direction of the potential due to the centrifugal force.
- After the turn, the trajectory soon relaxes and re-coincides with the light direction.
- There is no explicit oscillation of the trajectory.
- The adiabatic/entropic modes are approximately the light/heavy modes.



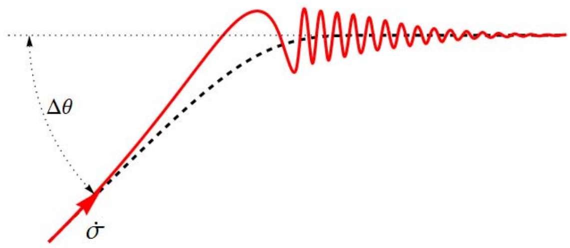
Evolution of the trajectory

Sharp turn

• Soon *after* the sharp turn, the trajectory starts to oscillate, with considerable amplitude.

• The adiabatic/entropic modes get rapidly mixed with light/heavy modes.

• The adibatic (curvature) mode has not necessarily to be light, which can be temporarily due to the oscillation.
[*Achucarro, Gong, Hardeman, Palma, Patil, '10. Shiu & Xu, '11, Chen, '11, '12,* ***Gao****, Langlois, Mizuno, '12, '13*]



Oscillatory background during a sharp turn

When the turn is **sharp**, the oscillating trajectory will induce oscillatory parts in background quantities (*a*, *H* etc).

Deviation from the smooth value: $a = \bar{a} + \Delta a, \quad H = \bar{H} + \Delta H, \quad \epsilon = \bar{\epsilon} + \Delta\epsilon$

An equation of motion for $\Delta\epsilon$

$$\frac{d^2\Delta\epsilon}{dt^2} + 3\bar{H}\frac{d\Delta\epsilon}{dt} - 12\bar{\epsilon}\bar{H}^2\Delta\epsilon = 2\bar{\epsilon}\left[\left(\dot{\theta}_p + \dot{\psi}\right)^2 - \hat{m}_h^2\sin^2\psi\right]$$

Infinitely sharp turn limit (μ→∞):

$$\dot{\theta}_p = \Delta\theta\frac{\mu}{\sqrt{2\pi}}e^{-\frac{1}{2}\mu^2t^2} \xrightarrow{\mu\to\infty} \dot{\theta}_p = \Delta\theta\delta(t)$$

$$\psi(t) \approx -\Theta(t)\,\Delta\theta e^{-\frac{3}{2}\bar{H}t}\cos(\hat{m}_h t)$$

$$\Delta\epsilon \approx \frac{\Theta(t)}{2}\bar{\epsilon}(\Delta\theta)^2 e^{-3\bar{H}t}\cos(2\hat{m}_h t) + \text{non-osci}$$

Two effects on the perturbations

Deviation from the single-field slow-roll (SFSL):

$$\begin{aligned}\mathcal{L} &= \mathcal{L}\left(\theta_p, a\right) \\ &= \mathcal{L}\left(\theta_p, \bar{a} + \Delta a\right) \\ &= \mathcal{L}_0\left(0, \bar{a}\right) + \mathcal{L}_{\mathrm{I}}^{(\mathrm{turn})}\left(\theta_p, \bar{a}\right) + \mathcal{L}_{\mathrm{I}}^{(\mathrm{resonance})}\left(0, \Delta a\right)\end{aligned}$$

- **"Free" part** (SFSL limit):

$$\mathcal{L}_0^{\mathrm{l,h}} = \frac{1}{2}\left[u_{l,h}'^2 - \left(\partial u_{l,h}\right)^2 - \left(\bar{a}^2 m_{l,h}^2 - \bar{a}^2 \bar{H}^2\left(2 - \bar{\epsilon}\right)\right) u_{l,h}^2\right]$$

$\bar{H}$ and $\bar{\epsilon}$ are evaluated by $\bar{a}$.

- **"Interaction" part** (deviation from SFSL):

Effects 1: turning light direction (potential trough)

$$\mathcal{L}_{\mathrm{I}}^{(\mathrm{turn})} = \frac{1}{2}{\theta_p}'^2 u_l^2 + \frac{1}{2}{\theta_p}'^2 u_h^2 + 2{\theta_p}' u_l u_h' + {\theta_p}'' u_l u_h$$

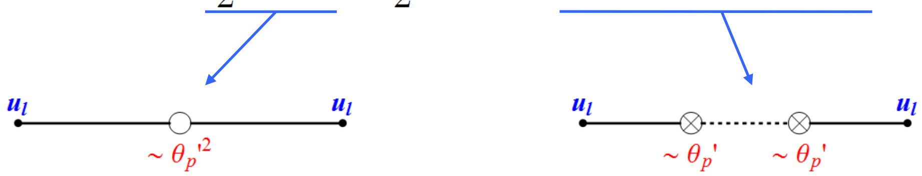
Effects 2: oscillatory background

$$\mathcal{L}_{\mathrm{I}}^{(\mathrm{resonance})} = -\frac{1}{2}\left[\left(\Delta a\right)^2 m_{l,h}^2 - \Delta\left(a^2 H^2\left(2 - \epsilon\right)\right)\right] u_{l,h}^2$$

Effect 1: turn

Two-point interactions:

$$\mathcal{L}_{\mathrm{I}}^{(\mathrm{turn})} = \frac{1}{2}{\theta_p}'^2 u_l^2 + \frac{1}{2}{\theta_p}'^2 u_h^2 + 2{\theta_p}' u_l u_h' + {\theta_p}'' u_l u_h$$



Effective theory:

$$v'' + \left(c_s^2 k^2 + a^2 m_{\mathrm{eff}}^2 - \frac{a''}{a}\right) v = 0 \qquad \text{with} \quad v \equiv u_l / c_s$$

$$c_s^2 \approx 1 - \frac{4{\theta_p}'^2}{a^2 m_h^2} + \mathcal{O}\left(m_h^{-4}\right),$$

$$m_{\mathrm{eff}}^2 \approx -\frac{{\theta_p}'^2}{a^2} + \frac{1}{a^4 m_h^2}\left(4a^2 H^2 \theta_p'^2 + 4\theta_p'^4 + 12 a H \theta_p' \theta_p'' - 3\theta_p''^2 - 2\theta_p' \theta_p'''\right) + \mathcal{O}\left(m_h^{-4}\right).$$

Gaussian ansatz: $\dot{\theta}_p = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2}$

Effect 1: turn

Correction to the spectrum (when EFT is valid):

$$\left(\frac{\Delta P}{P}\right)_{\rm turn} \approx \left(\frac{\Delta P}{P}\right)_0 + \left(\frac{\Delta P}{P}\right)_1 + \mathcal{O}\left(m_h^{-4}\right)$$

where

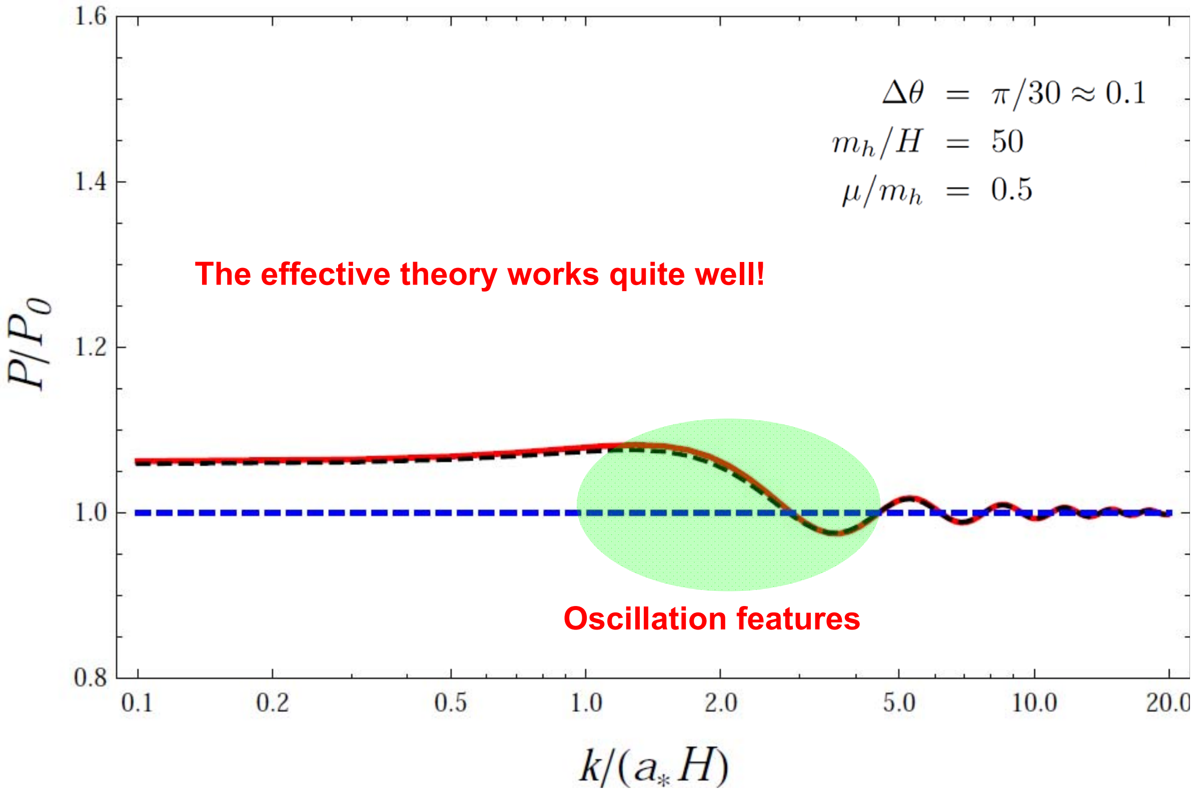
$$\left(\frac{\Delta P}{P}\right)_0 = \frac{(\Delta\theta)^2}{\sqrt{\pi}\, x_*^3}\frac{\mu}{H}\left(x_*\sin x_* + \cos x_*\right)\left(\sin x_* - x_*\cos x_*\right),$$

$$\left(\frac{\Delta P}{P}\right)_1 = \frac{(\Delta\theta)^2}{2\sqrt{\pi}\, x_*^3}\frac{\mu}{H}\left(\frac{\mu}{m_h}\right)^2\left(x_*\sin x_* + \cos x_*\right)\left(\sin x_* - x_*\cos x_*\right).$$

with $x_* \equiv \dfrac{k}{a_* H}$

→ **There are oscillatory features periodic in *k*.**

Effect 1: turn



Effect 2: Resonance

For the light mode:

$$\begin{aligned}\mathcal{L}_{\mathrm{I}}^{(\mathrm{resonance})} &= -\frac{1}{2}\left[(\Delta a)^2 m_l^2 - \Delta\left(a^2 H^2 (2-\epsilon)\right)\right] u_l^2 \\ &\simeq \frac{1}{2}\Delta\left(a^2 H^2 (2-\epsilon)\right) u_l^2 \\ &\simeq -\frac{1}{2}\bar{a}^2 \bar{H}^2 (\Delta\epsilon)_{\mathrm{osci}} u_l^2\end{aligned}$$

In the infinitely sharp turn limit, we have solved:

$$\Delta\epsilon \approx \frac{\Theta(t)}{2}\bar{\epsilon}(\Delta\theta)^2 e^{-3\bar{H}t}\cos\left(2\hat{m}_h t\right) + \text{non-osci}$$

An oscillation in background periodic in cosmic time *t* will induce resonance effect, which is **periodic in (ln *k*)**, in the spectrum of perturbation.
[Chen '11, '12]

Effect 2: Resonance

Contribution to the spectrum of the light mode:

$$\begin{aligned}\left(\frac{\Delta P}{P}\right)_{\mathrm{res}} \approx\ & \Theta\left(\frac{k}{a_* m_h} - 1\right)\frac{\sqrt{\pi}}{4}\bar{\epsilon}\,(\Delta\theta)^2\left(\frac{\bar{H}}{m_h}\right)^{\frac{3}{2}} \\ & \times\left(\frac{a_* m_h}{k}\right)^3 \cos\left[2\frac{m_h}{\bar{H}}\ln\left(\frac{k}{a_* m_h}\right) + 2\frac{m_h}{\bar{H}} - \frac{\pi}{4}\right].\end{aligned}$$

- The oscillation is periodic in **ln *k***, with frequency $2m_h/\bar{H} \gg 1$.
- The resonance features manifest themselves only on very small length scales: $k > a_* m_h \gg a_* \bar{H}$
- The amplitude is rather small: $\bar{\epsilon}\,(\Delta\theta)^2\left(\frac{\bar{H}}{m_h}\right)^{\frac{3}{2}} \ll 1$
- The amplitude is even suppressed on small scales: $\sim 1/k^3$

→ **The resonance feature is subdominant with respect to the oscillatory feature caused by the bending potential valley (light direction).**

Main message from this talk

- Heavy field(s) may play some role in the early Universe.
- Effective single-field description may not be valid.
- Sharp turn may produce oscillatory features in the spectra of light mode(s).

Thank you for your attention!

“Primordial spectra from sudden turning trajectory”

by Toshifumi Noumi

[JGRG23(2013)110509]

Primordial spectra from sudden turning trajectory

Toshifumi Noumi

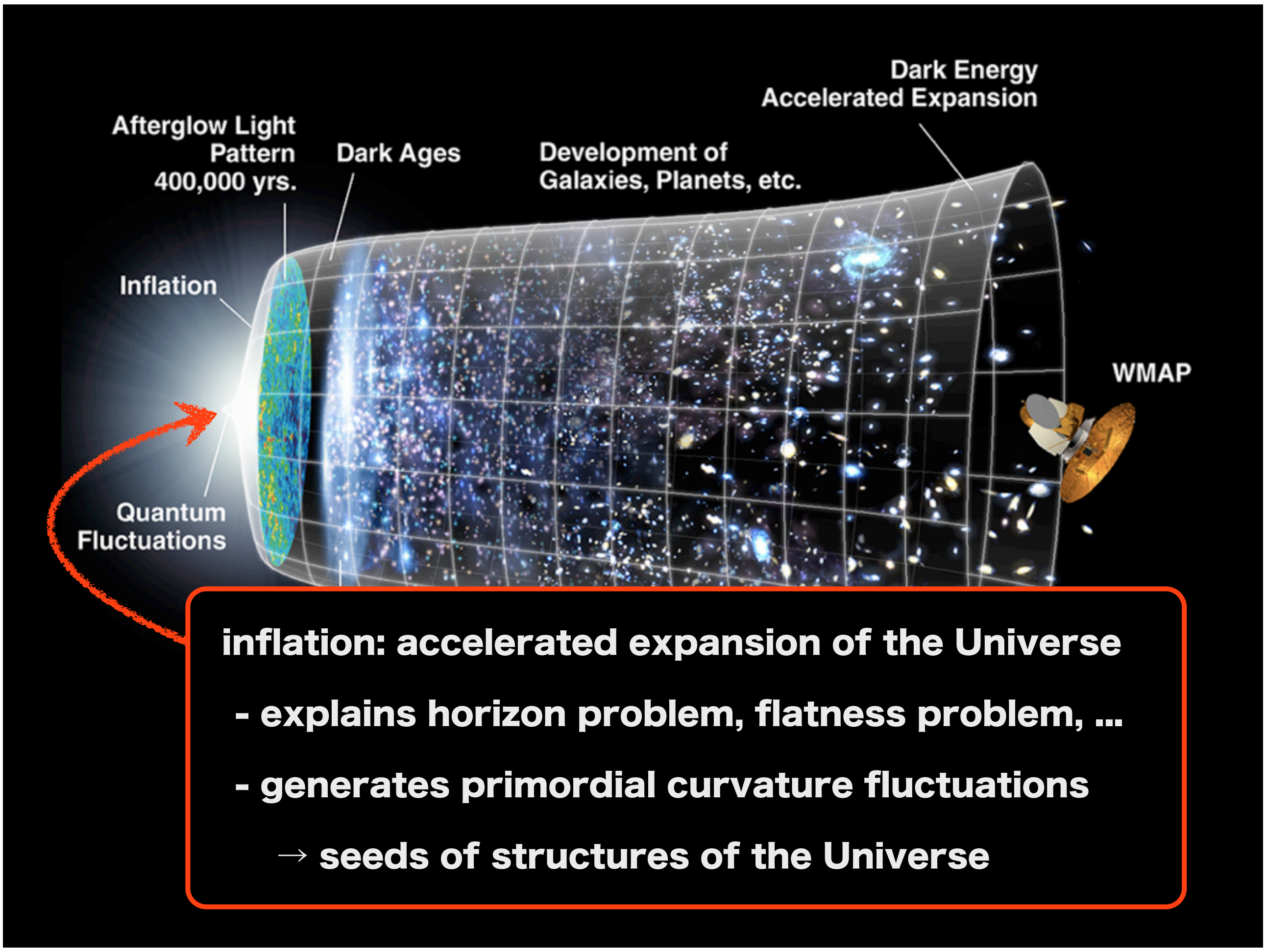
(Math Phys Lab, RIKEN)

mainly based on arXiv:1307.7110 with Masahide Yamaguchi

also JEHP06(2013)051 with M.Yamaguchi and D.Yokoyama

JRGR23 @Hirosaki University, November 5th 2013

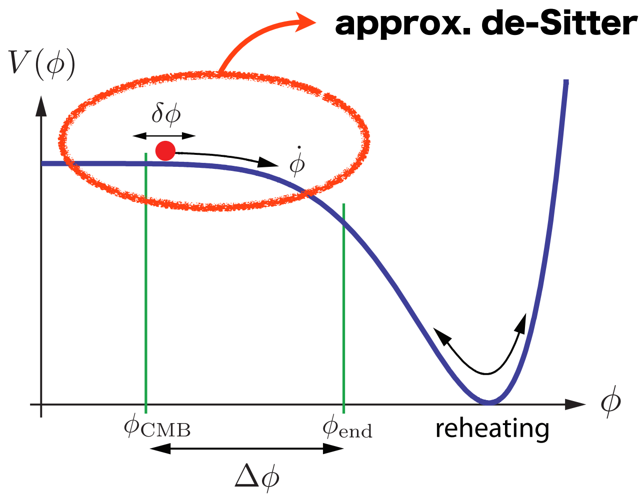
introduction



single-field slow-roll inflation

- introduce an inflaton field:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$$



single-field slow-roll inflation

- introduce an inflaton field:

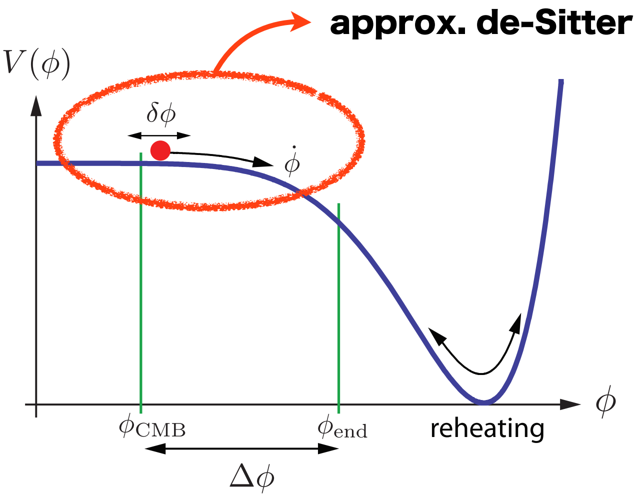
$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

- FRW spacetime

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

- Hubble parameter: $H(t) = \frac{\dot{a}}{a}$

(horizon problem
  observation



$$\longrightarrow \quad \ln\left[\frac{a(t_f)}{a(t_i)}\right] \gtrsim 60 \quad \epsilon = -\frac{\dot{H}}{H^2} \ll 1 \quad \eta = \frac{\dot{\epsilon}}{\epsilon H} \ll 1$$

This simplest model well explains the current observational data!

as a probe of high energy physics?

possibly as a deviation from single-field slow-roll inflation

models based on high energy theory have been also discussed
(ex. supergravity, superstring theory, ...)

one generic feature of such high energy based models:

massive scalar fields other than inflaton

- supergravity: generically $m_{\rm scalar} \sim H$
- extra dimensions: Kaluza-Klein modes
- superstring theory: moduli of compactification

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one generic feature of such high energy based models:

massive scalar fields other than inflaton

- supergravity: generically $m_{\rm scalar} \sim H$
- extra dimensions: Kaluza-Klein modes
- superstring theory: moduli of compactification

can be used as a probe of high energy physics!?
can affect primordial curvature perturbations!?

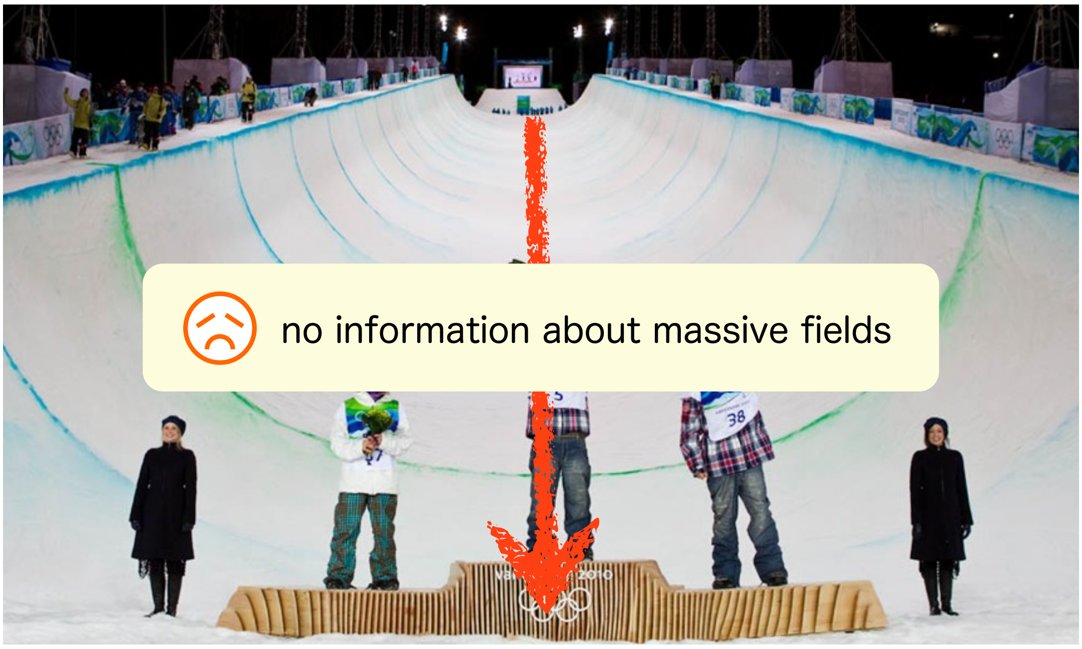
recent works in this direction:
ex. Chen, Shiu-Xu, Achucarro et al, Gao et al, Saito et al, Shi-Sasaki

when heavy fields become relevant?

suppose that the potential has a massive direction
in addition to the slow-roll direction

if you roll along the bottom of potential...

- don’t feel the massive potential
- single field approximation works well



if you roll along the bottom of potential...

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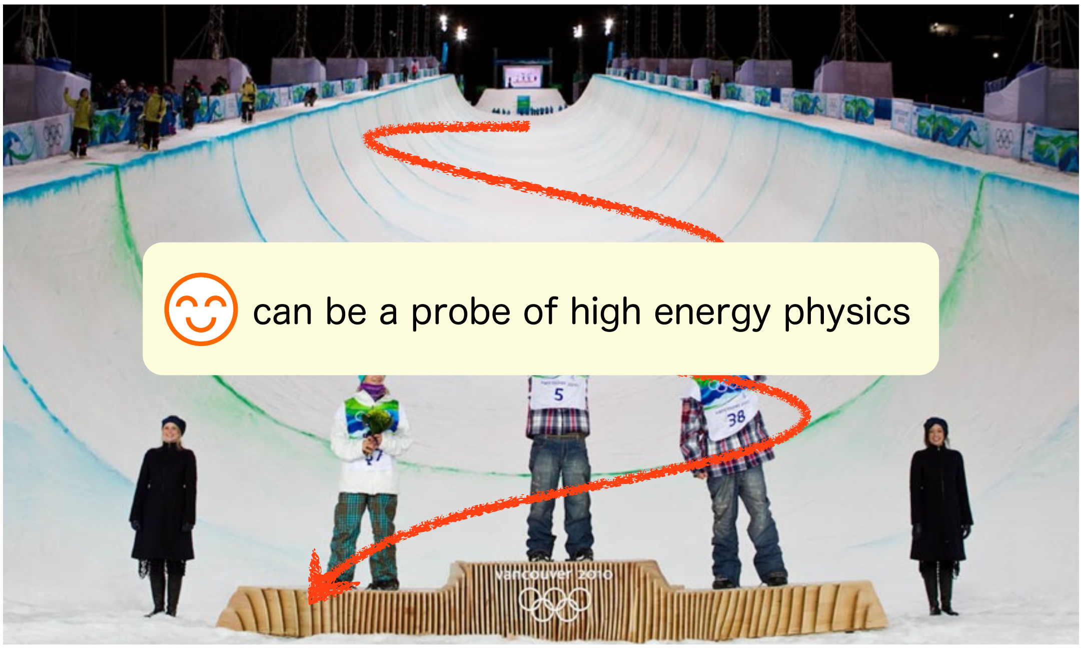
if you turn and climb the potential...

- massive potential becomes relevant to your dynamics
- deviation from single-field slow-roll inflation

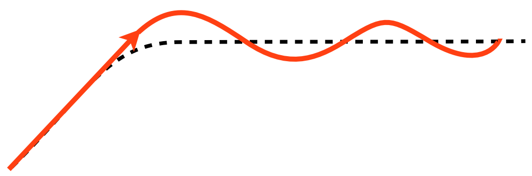


if you turn and climb the potential...

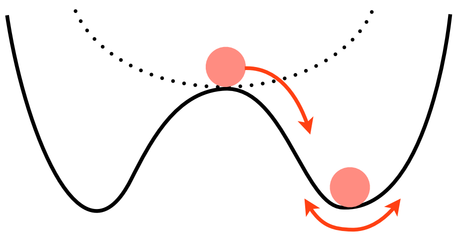
- massive potential becomes relevant to your dynamics
- deviation from single-field slow-roll inflation

possible scenarios for heavy field oscillations:

1. turning potential (cf. talks by Xian Gao and Ryo Saito)



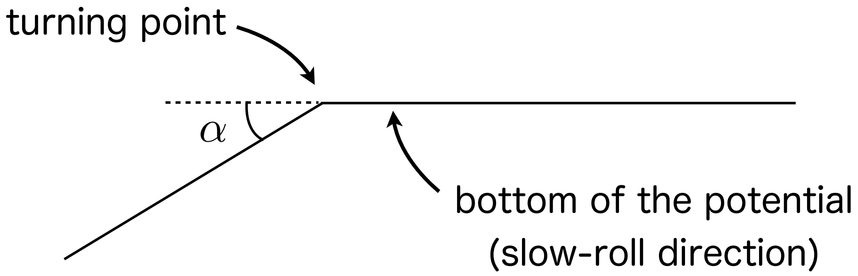
2. phase transition (of massive direction)



it would be meaningful to discuss
effects of such oscillations on primordial curvature perturbations

set up

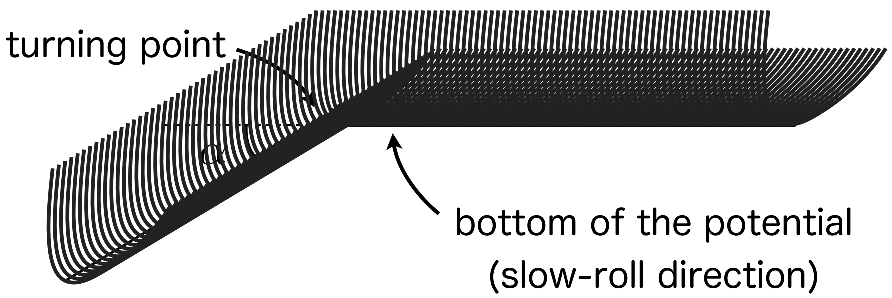
set up



- sudden turning potential:

- canonical kinetic terms

set up



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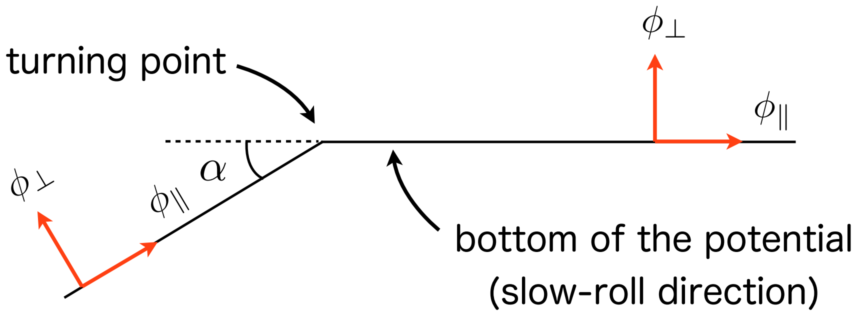
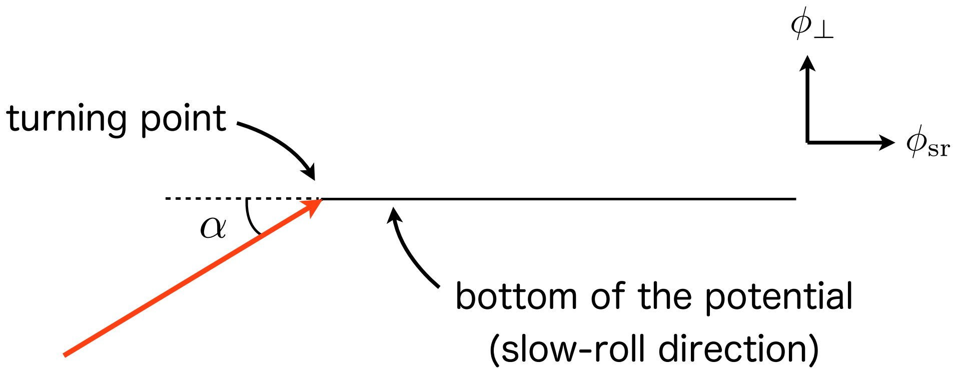


- separable sudden turning potential:

$$V(\phi_i) = V_{\rm sr}(\phi_\parallel) + \frac{m^2}{2}\phi_\perp^2 + \frac{\lambda}{4!}\phi_\perp^4$$

- canonical kinetic terms

set up

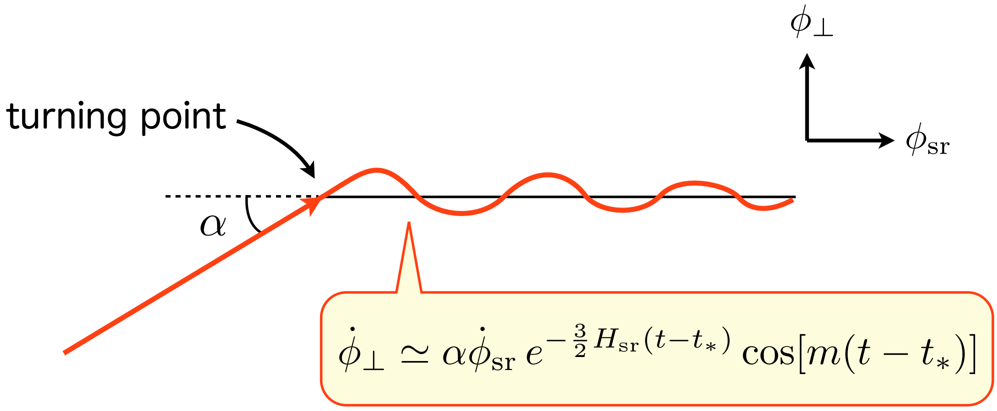


before the turn:

background trajectory is along slow-roll direction

$\rightarrow$ single-field slow-roll inflation

set up



\# before the turn:

background trajectory is along slow-roll direction

$\rightarrow$ single-field slow-roll inflation

\# after the turn:

heavy field oscillations (deviation from single field)

how heavy field oscillations affect inflation?

two effects of heavy field oscillations:

1. deformations of Hubble parameter
2. conversion interactions

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1. deformations of Hubble parameter
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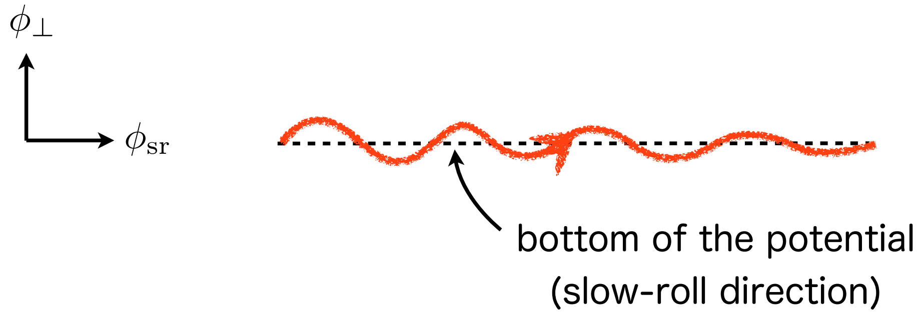
Deformations of Hubble parameter



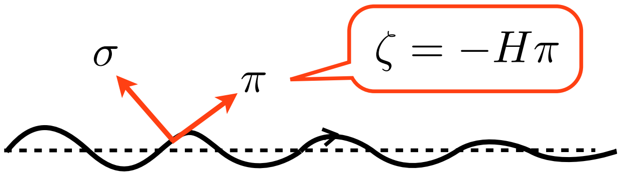
if background trajectory oscillates...

Friedman equation: $-2M_{\rm Pl}^2\dot{H} = \dot{\phi}_{\rm sr}^2 + \dot{\phi}_\perp^2$ ← oscillating

- deformed Hubble parameter $\dot{H} = \dot{H}_{\rm sr} + \delta\dot{H}$ ← oscillating

Deformations of Hubble parameter

※ Hubble deformation affects adiabatic perturbations



※ we take kinetic basis
(cf. potential basis in Gao et al.)

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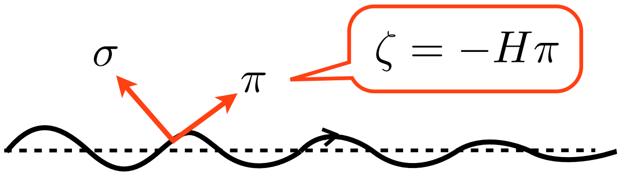

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kinetic term of adiabatic mode is modified:

$$S = \int dt d^3x\, a^3(-M_{\rm Pl}^2 \dot{H}) \left[\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right]$$

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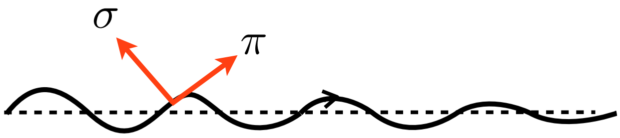

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$$\dot{H} = \dot{H}_{\rm sr} + \delta\dot{H}$$

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Deformations of Hubble parameter

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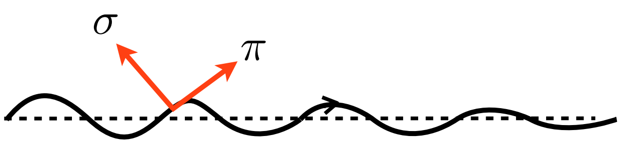

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$$+ \int dt d^3x\, a^3(-M_{\rm Pl}^2 \delta\dot{H}) \left[\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right]$$

deviation from single-field slow-roll inflation

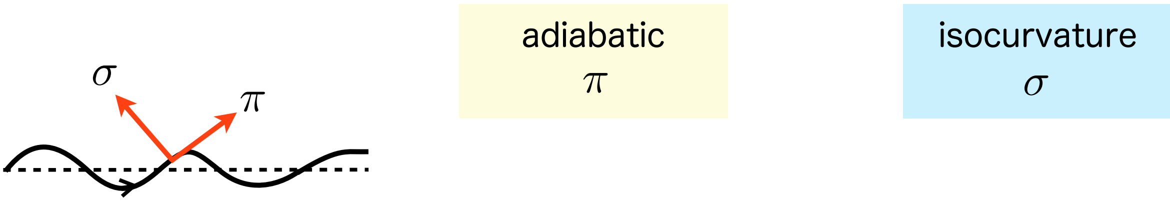
→ can be seen as an oscillating π - π interaction

two effects of heavy field oscillations:

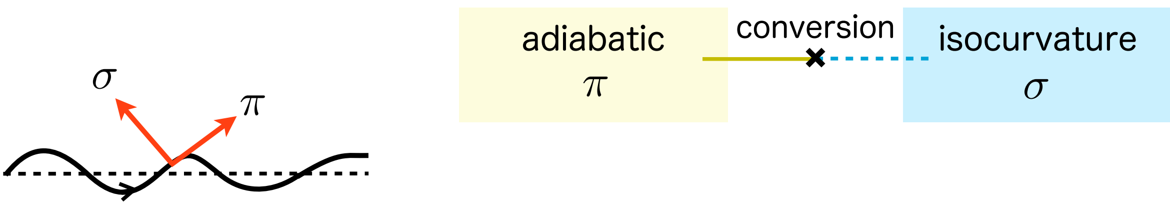
1. deformations of Hubble parameter

2. conversion interactions

conversion interaction



conversion interaction



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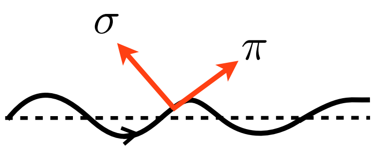

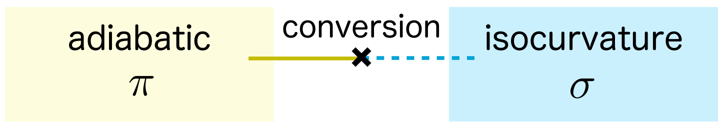

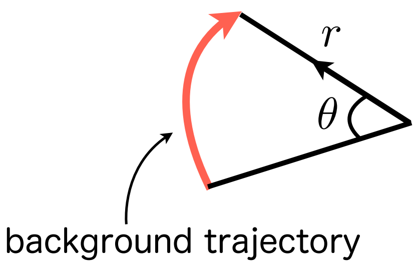

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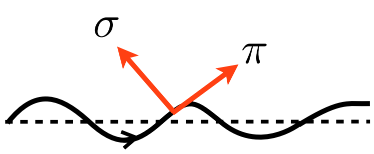
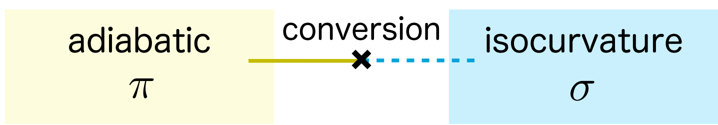
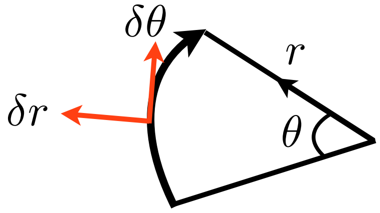


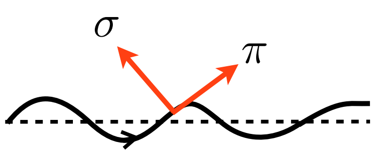


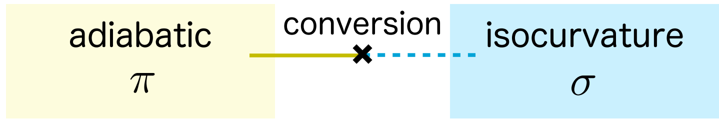


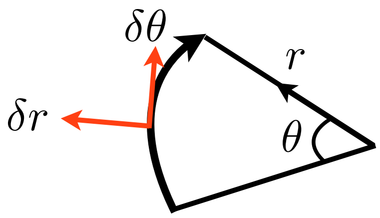
kinetic term : $r^2 \partial_\mu \theta \partial^\mu \theta$

$$r = \bar{r} + \delta r, \quad \theta = \bar{\theta} + \delta\theta$$

conversion interaction



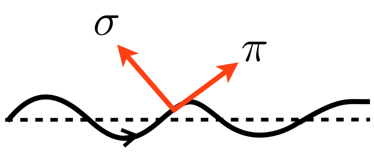


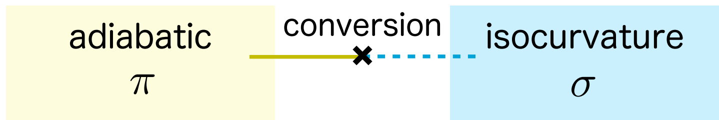


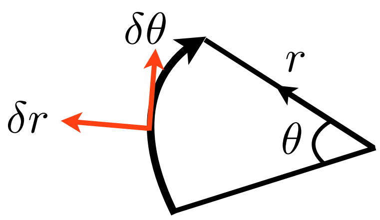
kinetic term : $r^2 \partial_\mu \theta \partial^\mu \theta \ni (\bar{r}\dot{\bar{\theta}})\, \delta r\, \dot{\delta\theta}$

$$r = \bar{r} + \delta r, \;\; \theta = \bar{\theta} + \delta\theta$$

conversion interaction







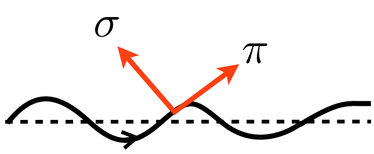
kinetic term : $r^2 \partial_\mu \theta \partial^\mu \theta \ni (\bar{r}\dot{\bar{\theta}})\, \delta r\, \dot{\delta\theta}$

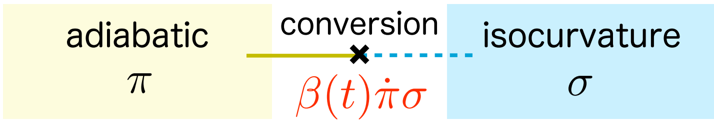
$$r = \bar{r} + \delta r, \;\; \theta = \bar{\theta} + \delta\theta$$

cf. centrifugal force $\ddot{\delta r} \sim r\dot{\theta}^2$

coupling $\propto$ angular velocity $\dot{\bar{\theta}}$

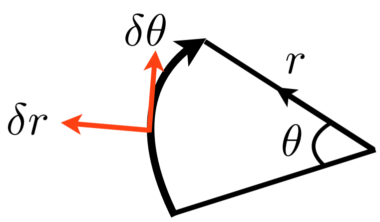
conversion interaction





conversion interaction

with oscillating coupling β



kinetic term : $r^2\partial_\mu\theta\partial^\mu\theta \ni (\bar{r}\dot{\bar{\theta}})\,\delta r\,\delta\dot{\theta}$

$$r = \bar{r} + \delta r, \;\; \theta = \bar{\theta} + \delta\theta$$

cf. centrifugal force $\delta\ddot{r} \sim r\dot{\theta}^2$

coupling $\propto$ angular velocity $\dot{\bar{\theta}}$

two effects of heavy field oscillations:

① Hubble deformation → π - π interaction

$$\int dtd^3x\, a^3(-M_{\rm Pl}^2\delta\dot{H})\left[\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right]$$

② π - σ conversion interaction

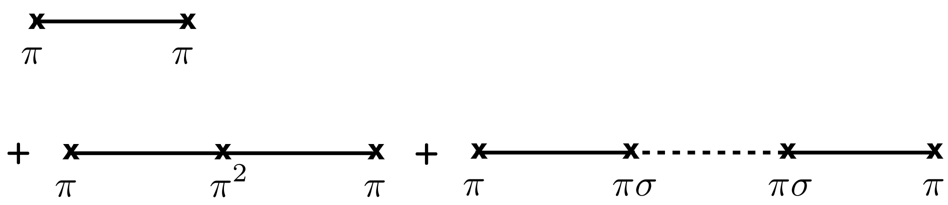
$$\int dtd^3x\, a^3\beta(t)\dot{\pi}\sigma$$

※ $\delta\dot{H}(t)$ and $\beta(t)$ are oscillating

effects on primordial power spectrum

effects on primordial power spectrum

$\langle \pi_{\mathbf{k}}\, \pi_{\mathbf{k}'} \rangle =$

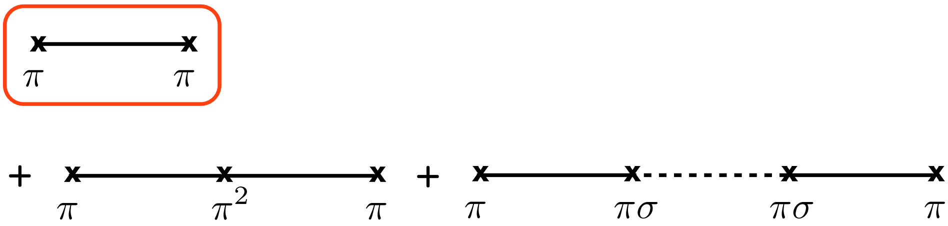


power spectrum: $\mathcal{P}_\zeta(k) = \dfrac{H_{\rm sr}^2}{8\pi^2 M_{\rm sr}^2 \epsilon_{\rm sr}} \left(1 + \mathcal{C}_{\delta H} + \mathcal{C}_{\rm conv}\right)$

$$H^2 \langle \pi_{\mathbf{k}}\, \pi_{\mathbf{k}'} \rangle = \langle \zeta_{\mathbf{k}}\, \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k)$$

effects on primordial power spectrum

$\langle \pi_{\mathbf{k}} \, \pi_{\mathbf{k}'} \rangle =$



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single field slow-roll

→ almost scale-invariant PS

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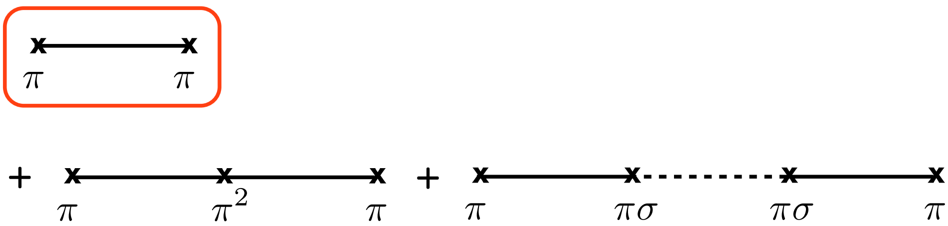

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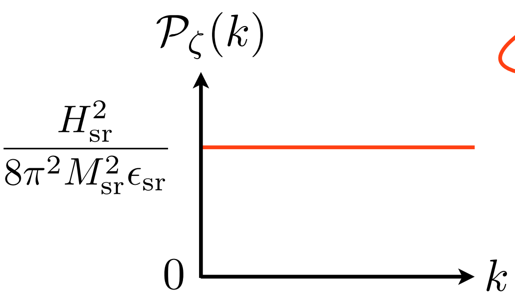

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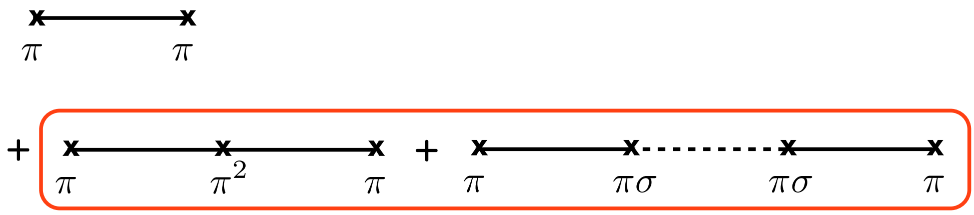

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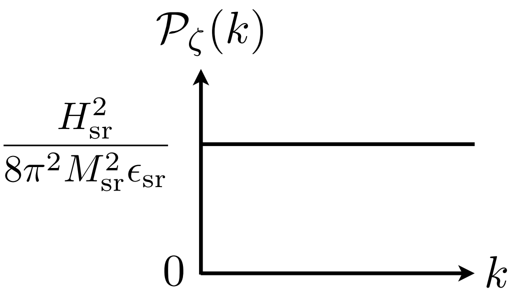
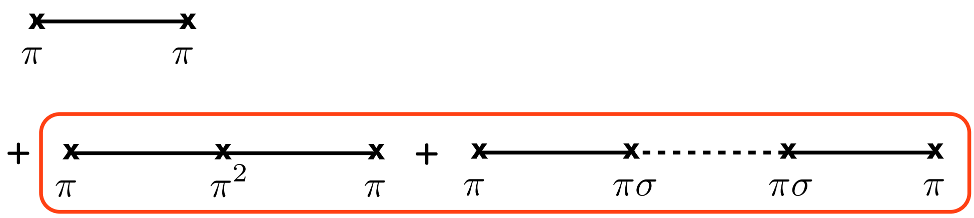
deviations



effects on primordial power spectrum

$\langle \pi_{\mathbf{k}} \, \pi_{\mathbf{k}'} \rangle =$



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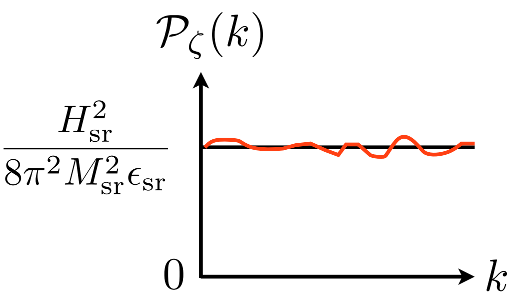
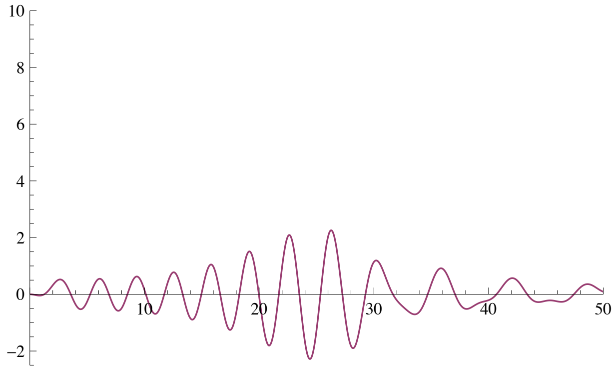
deviations



effects on primordial power spectrum

$\dfrac{\mathcal{C}_{\delta H}(k)}{\alpha^2}$ for $m = 20H_{\rm sr}$

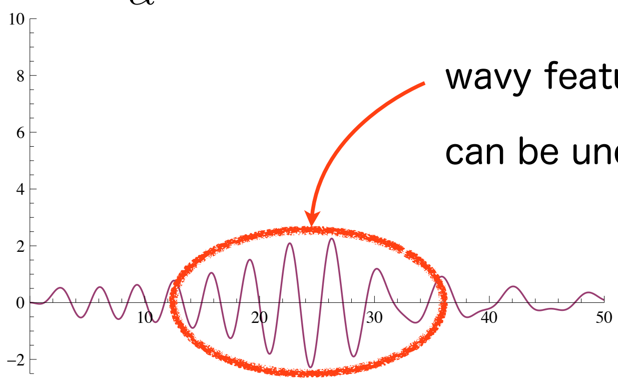


$\dfrac{k}{k_*}$

k_* : scale of the turn

effects on primordial power spectrum

$\dfrac{\mathcal{C}_{\delta H}(k)}{\alpha^2}$ for $m = 20H_{\rm sr}$



$\dfrac{k}{k_*}$

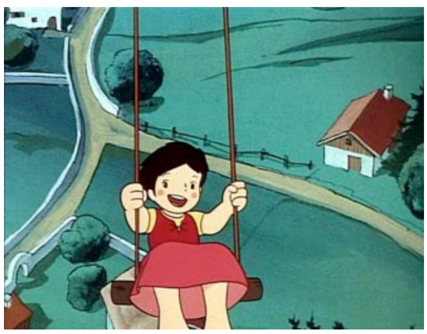
wavy features around $k \sim 20k_*$:

can be understood as resonances

[Chen, Saito et al]

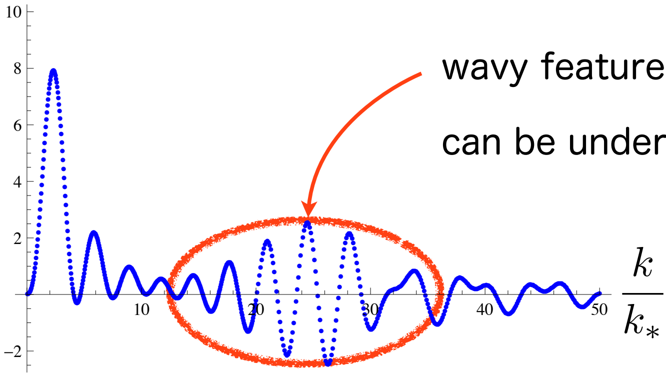
k_* : scale of the turn

cf. swing



effects on primordial power spectrum

$$\frac{\mathcal{C}_{\rm conv}(k)}{\alpha^2}$$ for $m = 20H_{\rm sr}$



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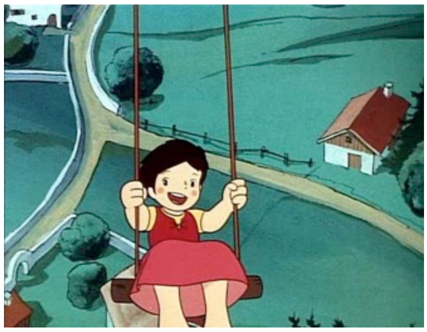
k_* : scale of the turn

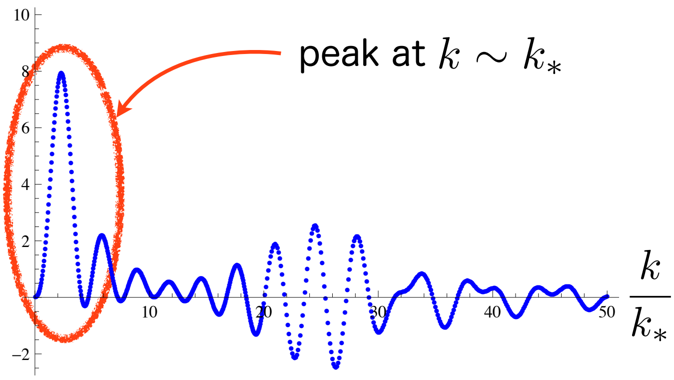
cf. swing



effects on primordial power spectrum

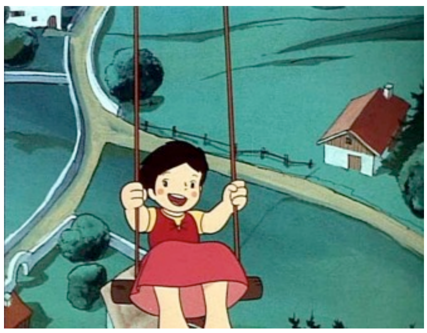
$$\frac{\mathcal{C}_{\rm conv}(k)}{\alpha^2}$$ for $m = 20H_{\rm sr}$

peak at $k \sim k_*$



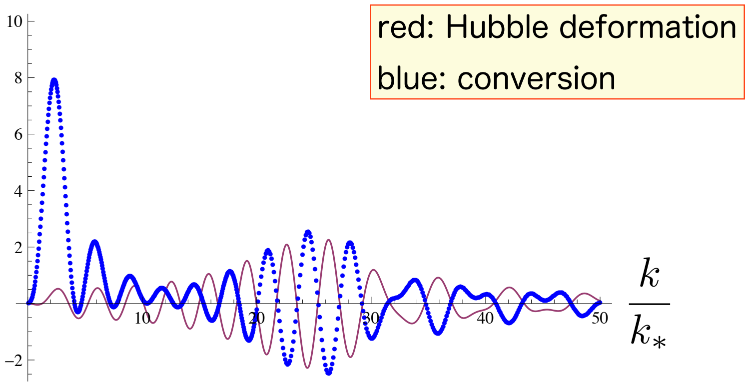
k_* : scale of the turn

cf. swing



effects on primordial power spectrum

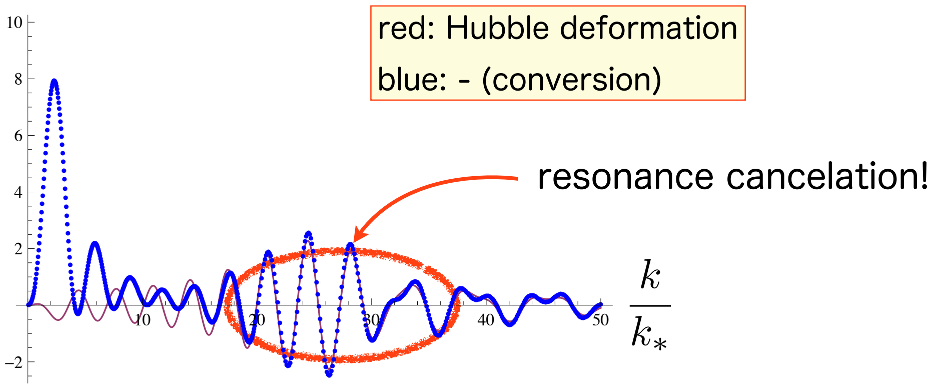
total deviation $\mathcal{C}_{\delta H}(k) + \mathcal{C}_{\rm conv}(k)$



- peak at the turn ~ $\alpha^2 \frac{m}{H_{\rm sr}}$
- resonance from each contribution ~ $\alpha^2 \left(\frac{m}{H_{\rm sr}}\right)^{1/2}$

effects on primordial power spectrum

total deviation $\mathcal{C}_{\delta H}(k) + \mathcal{C}_{\rm conv}(k)$



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effects on primordial power spectrum

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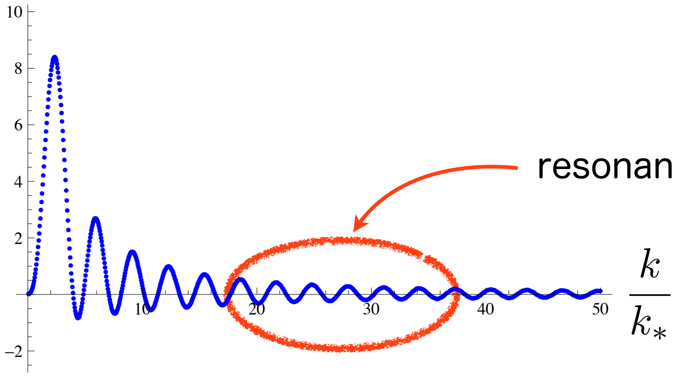


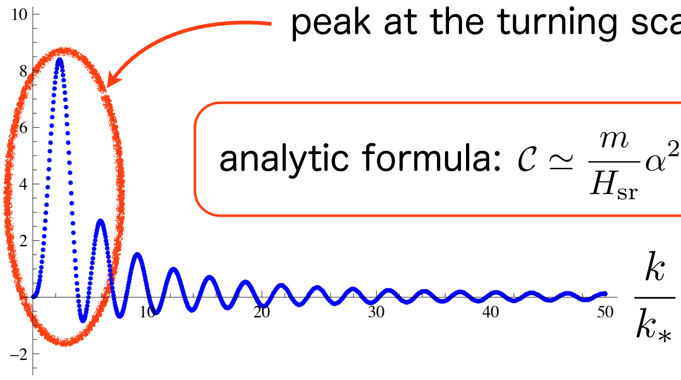
resonance cancelation!

- peak at the turn ~ $\alpha^2 \frac{m}{H_{\rm sr}}$
- resonance from each contribution ~ $\alpha^2 \left(\frac{m}{H_{\rm sr}}\right)^{1/2}$
- resonance cancellation between the two effects

effects on primordial power spectrum

total deviation $\mathcal{C}_{\delta H}(k) + \mathcal{C}_{\rm conv}(k)$

peak at the turning scale cf. Gao et al '13



analytic formula: $\mathcal{C} \simeq \frac{m}{H_{\rm sr}} \alpha^2 \frac{(\sin x_* - x_* \cos x_*)^2}{x_*^3}$

- peak at the turn ~ $\alpha^2 \frac{m}{H_{\rm sr}}$
- resonance from each contribution ~ $\alpha^2 \left(\frac{m}{H_{\rm sr}}\right)^{1/2}$
- resonance cancellation between the two effects

effects on primordial power spectrum

why resonances cancel each other out?

effects on primordial power spectrum

why resonances cancel each other out?

- Hubble deformation effects $M_{\rm Pl}^2 \delta\dot{H}\pi^2 \sim \dot{\phi}_\perp^2 \pi^2$

  ※ $\delta\dot{H}$ originates from velocity $\dot{\phi}_\perp$

effects on primordial power spectrum

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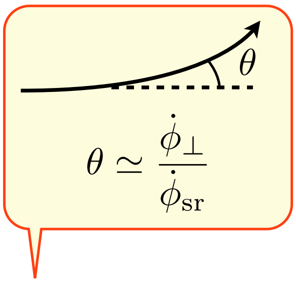

- conversion interactions $\beta \dot{\pi} \sigma \sim \ddot{\phi}_\perp \pi \sigma$
  - ※ conversion originates from angular velocity $\frac{\ddot{\phi}_\perp}{\dot{\phi}_{\rm sr}}$

effects on primordial power spectrum

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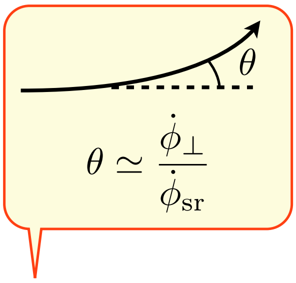

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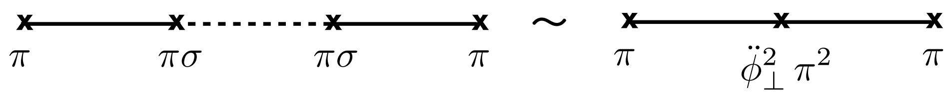
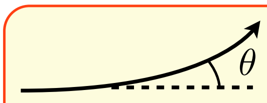
$\rightarrow$



effects on primordial power spectrum

why resonances cancel each other out?



$\theta \simeq \dfrac{\dot\phi_\perp}{\dot\phi_{\rm sr}}$

- Hubble deformation effects $M_{\rm Pl}^2 \delta \dot H \pi^2 \sim \dot\phi_\perp^2 \pi^2$

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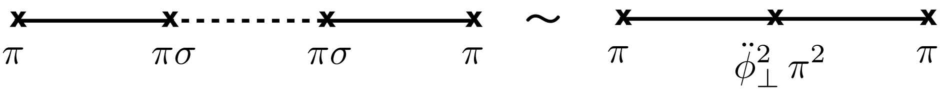
$\rightarrow$ 

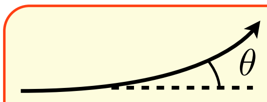
- couplings of the two interactions have opposite phases

  $\dot\phi_\perp^2 \sim \cos^2 mt \;\rightarrow\; \ddot\phi_\perp^2 \sim \sin^2 mt$

  $\rightarrow$ negative correlation between the two resonances

effects on primordial power spectrum

why resonances cancel each other out?



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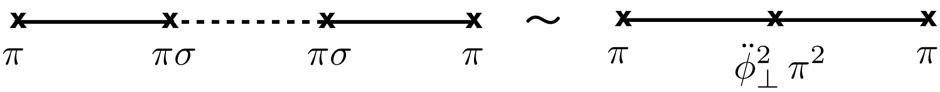
$\rightarrow$ 

- couplings of the two interactions have opposite phases

  $\dot\phi_\perp^2 \sim \cos^2 mt \;\rightarrow\; \ddot\phi_\perp^2 \sim$ s

  canonical kinetic terms

  $\rightarrow$ negative correlation between the two resonances

  $\cos^2 mt + \sin^2 mt = 1$: no oscillations $\rightarrow$ no resonances

Summary and prospects

Summary and prospects

effects of heavy field oscillations on primordial spectra

are discussed as a possible probe of high energy physics

- two effects
  - deformation of Hubble parameter
  - conversion between adiabatic and isocurvature perturbations
- two scales
  - resonance features around mass scale of heavy fields
  - peak at the turning scale
- resonance cancellation in models with canonical kinetic term
- bispectra are also discussed in our paper

Summary and prospects

prospects:

1. primordial spectra for more general models
   - phase trans., two-field open inflation, derivative interaction,...
   - resonance cancellation occurs or not??
   - what is the most robust signal? peak or resonance??
2. detectability
   - peak (spike) in the primordial spectrum
   - oscillating CMB power spectrum??
   - constraints from primordial black holes??

Thank you!

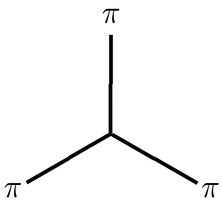
primordial bispectrum

primordial bispectra

scalar three-point functions:

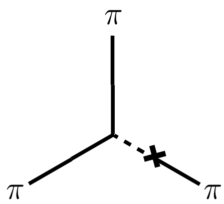
$\langle \pi_{\mathbf{k_1}} \pi_{\mathbf{k_2}} \pi_{\mathbf{k_3}} \rangle$

=



Hubble deformation

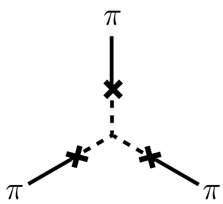
+



conversion

$\mathcal{O}(\alpha^2)$

+

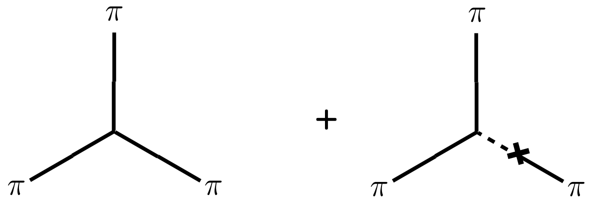


conversion

$\mathcal{O}(\tilde{\lambda}\alpha^2)$ ($\tilde{\lambda} \sim \sigma$ cubic interaction)

primordial bispectra

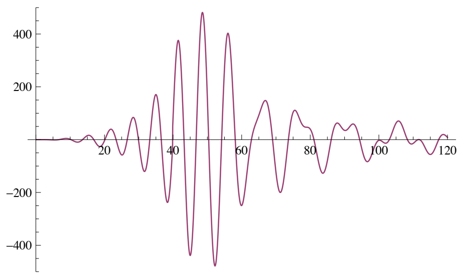
※ shape function: $S(k_1, k_2, k_3) \sim \dfrac{(\text{3-pt})}{(\text{2-pt})^2}$

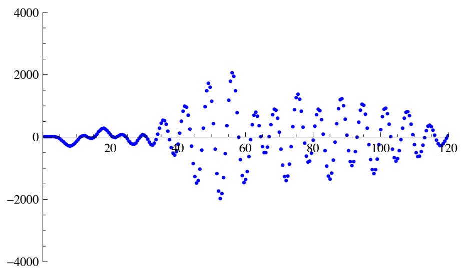


$\mathcal{O}(\alpha^2)$

Hubble deformation   conversion

scale-dependence for equilateral configurations $k_1 = k_2 = k_3$



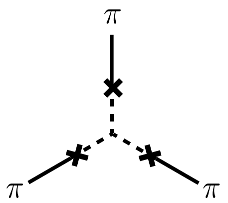


$\dfrac{\sum_i k_i}{k_*}$

- resonances at the mass scale
- not so large non-Gaussianities $f_{NL} \sim \alpha^2 \left(\dfrac{m}{H_{\rm sr}}\right)^{5/2} \times \mathcal{O}(1)$

primordial bispectra

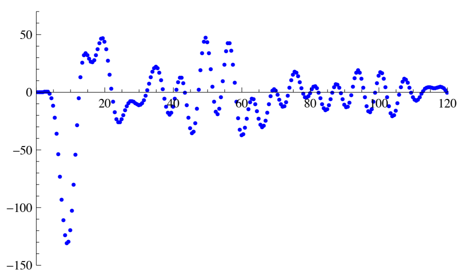
※ shape function: $S(k_1, k_2, k_3) \sim \dfrac{(\text{3-pt})}{(\text{2-pt})^2}$



$\mathcal{O}(\tilde{\lambda}\alpha^2)$

conversion

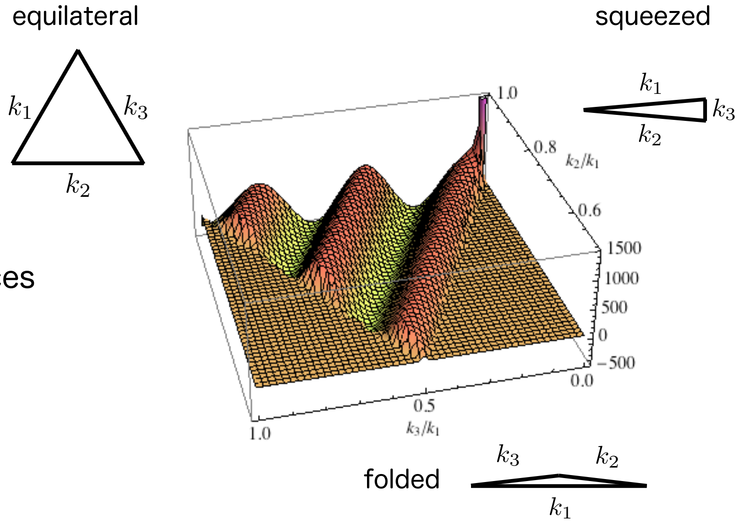
scale-dependence for equilateral configurations $k_1 = k_2 = k_3$



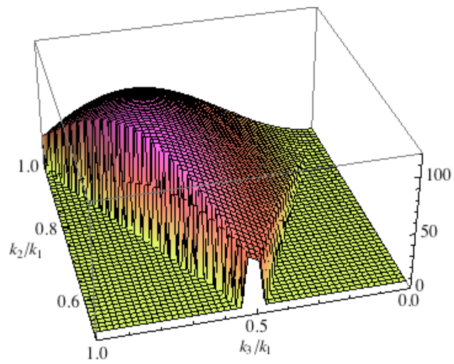
$\dfrac{\sum_i k_i}{k_*}$

- peak at the turning scale
- not so large non-Gaussianities $f_{NL} \sim \tilde{\lambda}\alpha^2 \left(\dfrac{m}{H_{\rm sr}}\right)^{3} \times \mathcal{O}(0.1)$

shape around resonances



shape around the peak



"Excitation of a heavy scalar field: Turn in the inflaton trajectory"

by Ryo Saito

[JGRG23(2013)110510]

2013/11/4 JGRG23 / Hirosaki University

Excitation of a heavy scalar field: Turn in the inflaton trajectory

Ryo Saito (YITP), Shuntaro Mizuno (APC)

RS & S. Mizuno, in preparation

Introduction

Inflation – Accelerated expansion in the very early universe

- solves unnatural points in the standard Big Bang theory.
- provides the seed of the structures in the universe,
  primordial density/curvature fluctuations,

from microscopic quantum fluctuations.

They are supposed to be governed by short-distance physics.

Cosmological observations could provide a window into physics beyond the reach of terrestrial experiments.

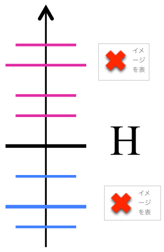
"Heavy" scalar fields

In general, there are a number of scalar fields in a model of inflation.

The primordial fluctuations are originated from fluctuations in light (m<H) scalar fields (Inflatons),

while fluctuations in heavy (M>H) scalar fields are decayed away.

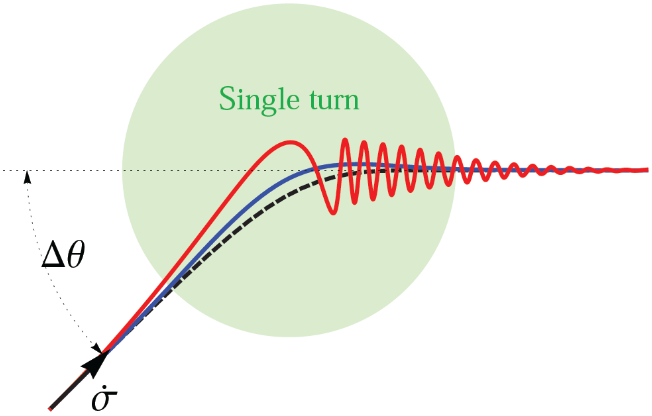
Is there any chance to probe heavy scalar fields?



Heavy scalar fields can produce fine features in the primordial spectra when **the inflaton trajectory is curved**.

[Chen & Wang 09, Tolley & Wyman 09, Achucarro+ 10, Shiu & Xu 11, Chen 11, Pi & Sasaki 12, RS+ 12, 13, Sespedes+ 12, Gao+ 12, Noumi+ 12, 13, Burgess+ 13,.....]

Modeling a turning trajectory



Variation of the angle:
$\Delta\theta$

Duration of the turn:
$\mu \sim (d\theta/dt)/\Delta\theta \sim 1/\Delta t$

[Gao, Langlois, & Mizuno 12]

- Two-field system with a single light/heavy field.
- During a turn, the heavy scalar field is excited from its minimum:

For soft turn (μ<M), it smoothly relaxes to the minimum.

For sharp turn (μ>M), the trajectory oscillates around the minimum.

Fine features in the primordial spectra

Two features induced by the turn:

- Large mixing between the light and heavy modes

[Chen & Wang 09, Tolley & Wyman 09, Achucarro+ 10, 11, 12, Shiu & Xu 11, Pi & Sasaki 12, Sespedes+ 12, Gao+ 12, Noumi+ 12, 13, Burgess+ 13]

For $c_s \sim 1$

$$\frac{\Delta \mathcal{P}_\zeta}{\mathcal{P}_\zeta} \sim \begin{cases} \Delta\theta^2 \left(\frac{\mu}{H}\right) & \text{(For soft turn: } \mu < M) \\ \Delta\theta^2 \left(\frac{M}{H}\right) & \text{(For sharp turn: } \mu > M) \end{cases}$$

at $k/a_{\rm turn} \sim H$

- Resonance between the excited oscillation and the fluctuations

[Chen 11, RS+ 12, 13, Gao+ 12, Noumi+ 12, 13]

For $c_s \sim 1$

Energy fraction of the excited heavy scalar field.

$$\frac{\Delta \mathcal{P}_\zeta}{\mathcal{P}_\zeta} \sim (1-c_s^2)\left(\frac{\epsilon_\chi}{\epsilon}\right)\left(\frac{M}{H}\right)^{1/2} \quad \text{(For sharp turn: } \mu > M)$$

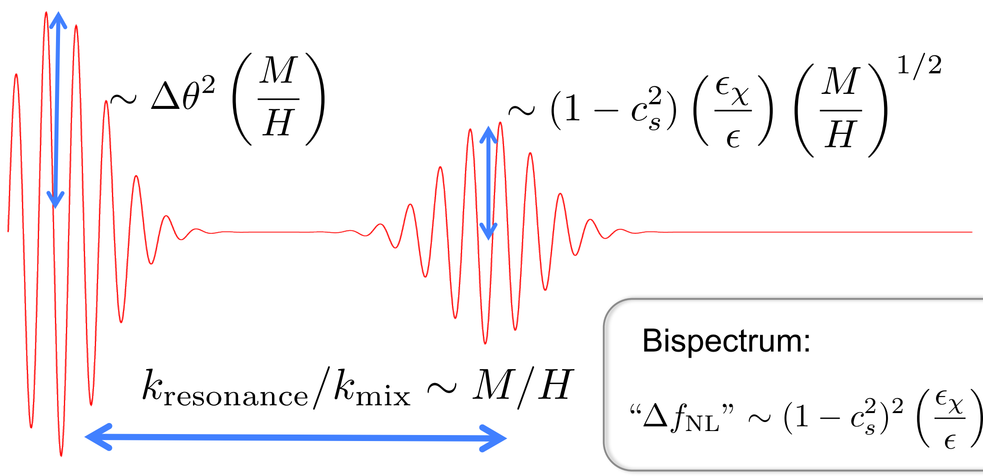
at $k/a_{\rm turn} \sim M$

Fine features in the primordial spectra

For sharp turn,

two correlating features are expected to appear in the power spectrum.

(Features are also expected to be induced in the bispectrum.)



Bispectrum:

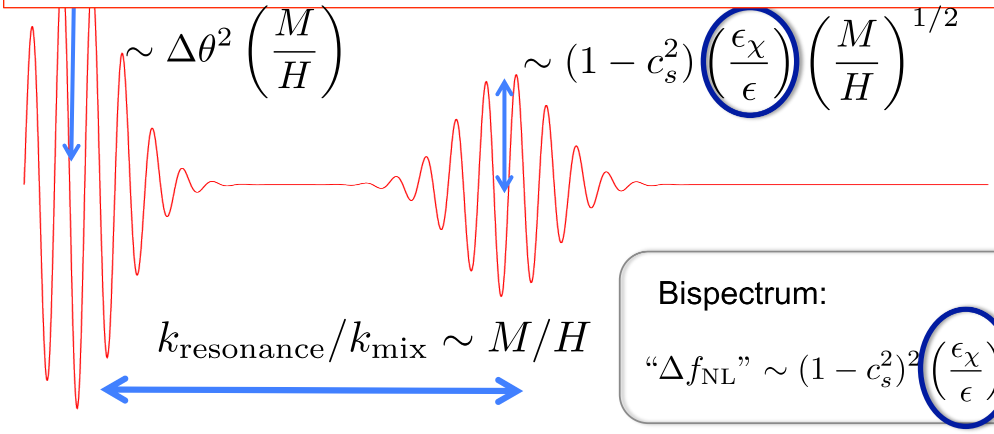
$$\text{“}\Delta f_{\rm NL}\text{”} \sim (1-c_s^2)^2\left(\frac{\epsilon_\chi}{\epsilon}\right)\left(\frac{M}{H}\right)^{5/2}$$

We can obtain an evidence for heavy scalar fields if correlating features are detected in the primordial spectra (power spectrum/bispectrum).

Fine features in the primordial spectra

For sharp turn,
two correlating features are expected to appear in the power spectrum.
(Features are also expected to be induced in the bispectrum.)

Which parameters determine the efficiency of the excitation (incl. Cs) ?



Bispectrum:

$$\text{“}\Delta f_{\rm NL}\text{”} \sim (1-c_s^2)^2\left(\frac{\epsilon_\chi}{\epsilon}\right)\left(\frac{M}{H}\right)^{5/2}$$

We can obtain an evidence for heavy scalar fields if correlating features are detected in the primordial spectra (power spectrum/bispectrum).

Background dynamics

Action (DBI action):

$$P(X^{IJ},\phi^I) = -\frac{1}{f(\phi^I)}\left(\sqrt{\mathcal{D}}-1\right) - V\left(\phi^I\right)\ ,\ \mathcal{D} \equiv \det\left(\delta^I_J - f\partial_\mu\phi^I\partial^\mu\phi_J\right)$$

Derivative couplings
⇒ Cs < 1

Evolution equation:

$$\ddot{\phi}^I + \left(3H - \frac{\dot{c}_s}{c_s}\right)\dot{\phi}^I + c_s V_{,I} = 0\,,\quad c_s \equiv \sqrt{1 - f\dot{\sigma}^2}$$

Speed of sound → Reduction of the friction + Flattening of the potential

More efficient excitation?

Background dynamics

Evolution equation:

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Speed of sound → Reduction of the friction + Flattening of the potential

More efficient excitation?

Numerical estimation

Condition to excite the heavy scalar field (sharp turn):

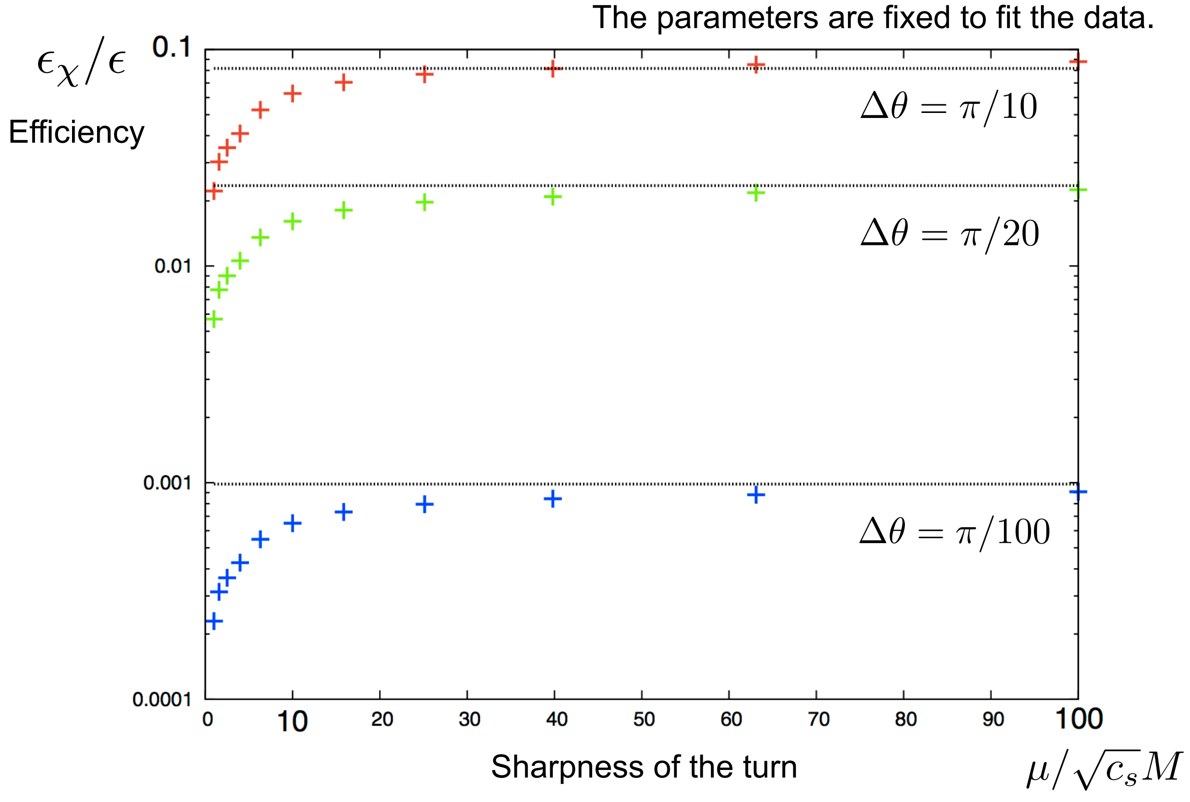
$$\mu > \sqrt{c_s} M$$
$$(\sim 1/\Delta t_{\rm turn})$$

(cf. Gao, Langlois, & Mizuno 12)

The flattening of the potential is more important effect.
(The variation in the speed of sound during the turn can be neglected.)

Efficiency of the excitation

The parameters are fixed to fit the data.



Efficiency of the excitation

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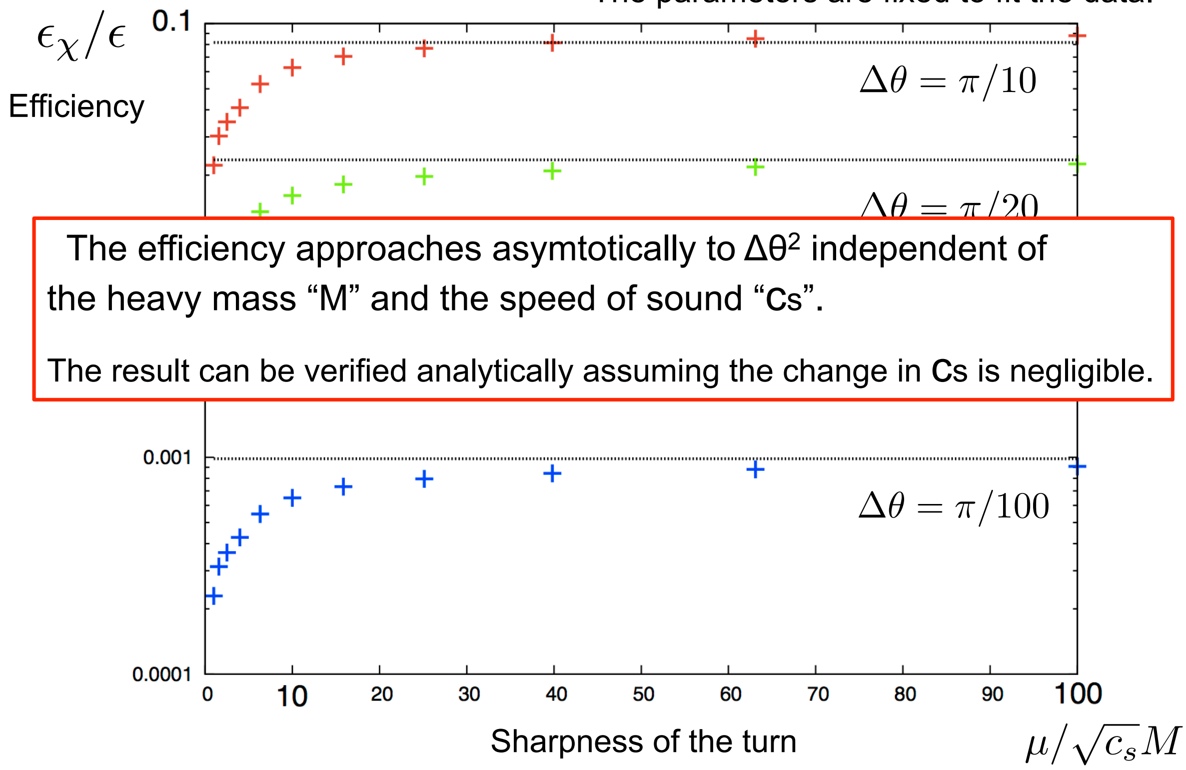


The efficiency approaches asymtotically to Δθ² independent of the heavy mass "M" and the speed of sound "Cs".

The result can be verified analytically assuming the change in Cs is negligible.

Relation between two features

For $c_s \sim 1$ and $\mu \gg \sqrt{c_s}M$

- From the mixing [Gao, Langlois, & Mizuno 12, Noumi & Yamaguchi 13]

$$\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta} \sim (\Delta\theta)^2 \left(\frac{M}{H}\right)$$

- From the resonance

$k_{\mathrm{resonance}}/k_{\mathrm{mix}} \sim M/H$

$c_s = -r/8n_t$ or $\sim -(f_{\mathrm{NL}}^{\mathrm{equil}})^{-1/2}$

$\epsilon_\chi/\epsilon \sim (\Delta\theta)^2$

$$\frac{\Delta\mathcal{P}_{\zeta,\mathrm{resonance}}}{\mathcal{P}_\zeta} \sim (1-c_s^2)\left(\frac{M}{H}\right)^{-1/2}\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta}$$ Suppressed

$$\text{“}\Delta f_{\mathrm{NL,resonance}}\text{”} \sim (1-c_s^2)^2\left(\frac{M}{H}\right)^{3/2}\frac{\Delta\mathcal{P}_{\zeta,\mathrm{mix}}}{\mathcal{P}_\zeta}$$ Enhanced

The features from resonance appear in the bispectrum.

Summary

- Features in the primordial spectra could be a probe of short-distance Physics behind inflation.

- Correlating features induced by a sharp turn in the inflaton trajectory
  ⇒ Large signal in the bispectrum

Simultaneous detection of the features from the mixing and the resonance can strengthen the evidence for heavy DoF during inflation.
↑ We need to analyze the bispectrum taking into account the scale dependence.

- Need to check

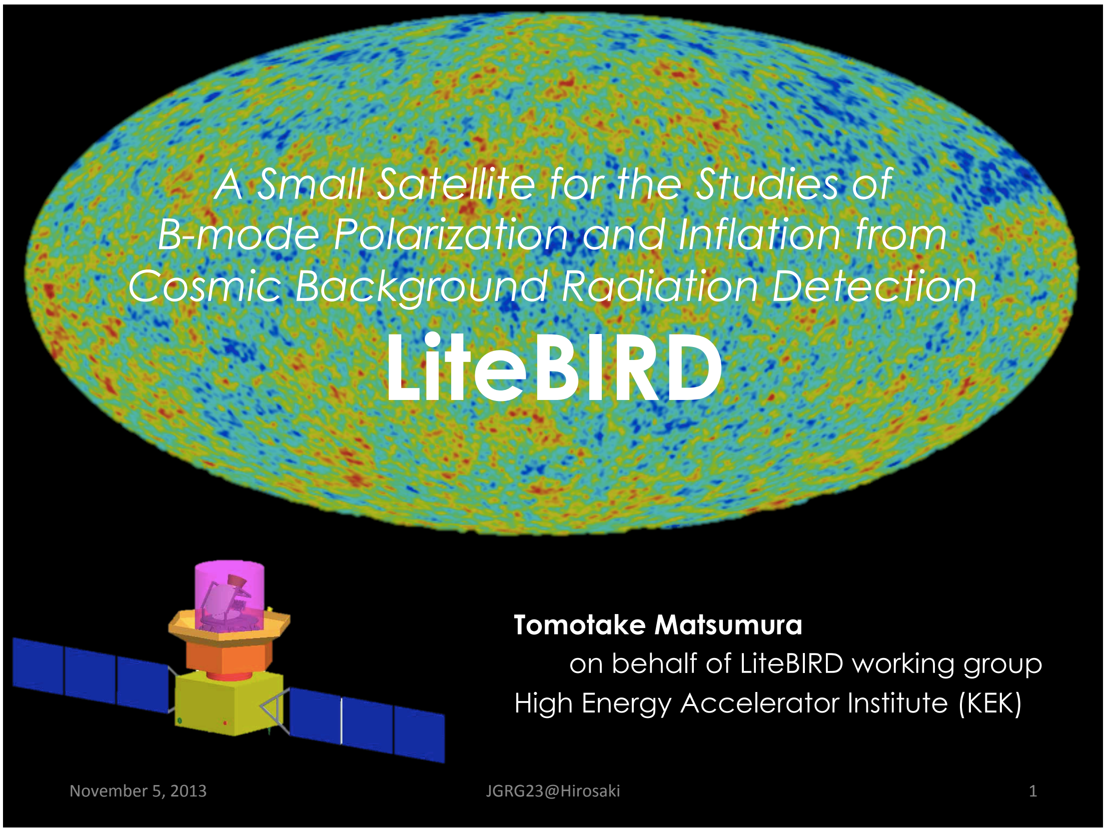
  Features for a small speed of sound $c_s = \mathcal{O}(0.1)$

  • Kinematic basis vs. Mass basis, Mass matrix,…
  (Large mixing through the derivative couplings)
  • Large equilateral bispectrum and folded bispectrum (from non-BD components)

“LiteBIRD, Lite (Light) satellite for the studies of B-mode polarization and inflation from cosmic background radiation detection”

by Tomotake Matsumura (invited)

[JGRG23(2013)110511]


A Small Satellite for the Studies of
B-mode Polarization and Inflation from
Cosmic Background Radiation Detection
LiteBIRD
Tomotake Matsumura
on behalf of LiteBIRD working group
High Energy Accelerator Institute (KEK)
November 5, 2013
JGRG23@Hirosaki
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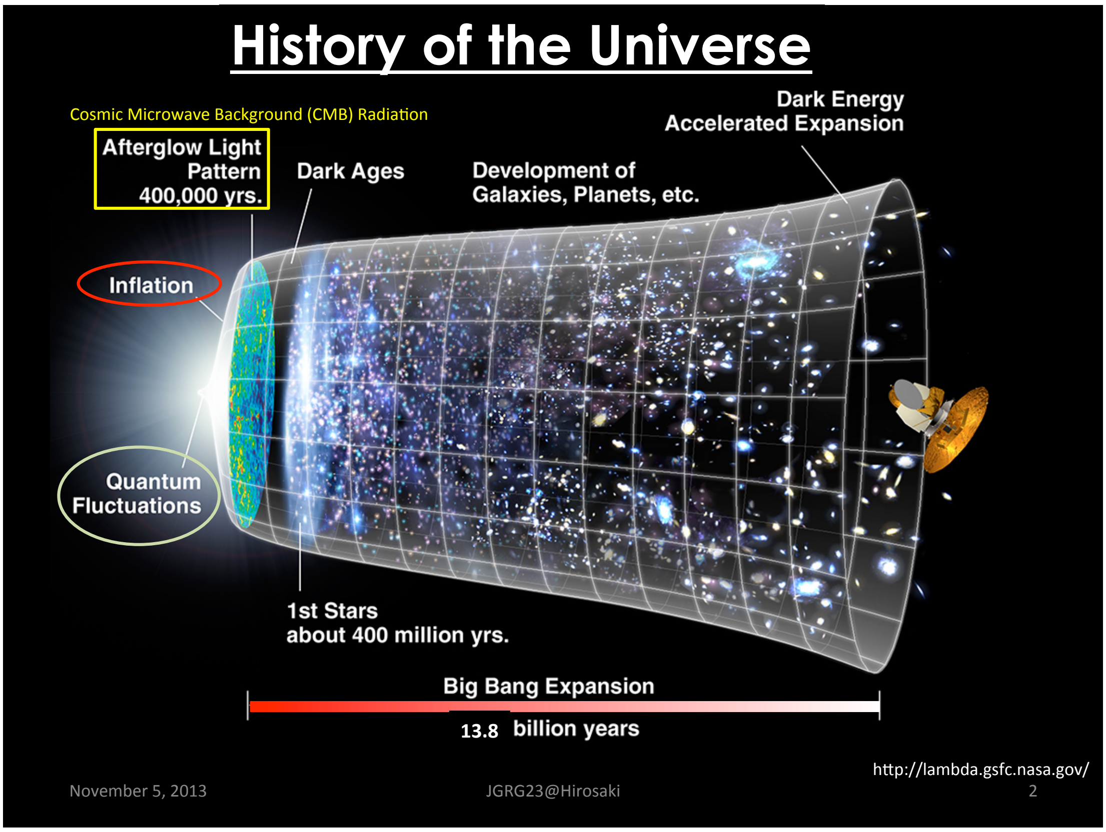
History of the Universe
Cosmic Microwave Background (CMB) Radiation
Afterglow Light Pattern 400,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Dark Energy Accelerated Expansion
Inflation
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.8 billion years
http://lambda.gsfc.nasa.gov/
November 5, 2013
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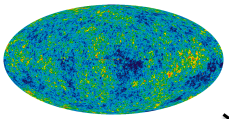
2

6 parameters to describe the Universe

Initial condition
A_s, n_s

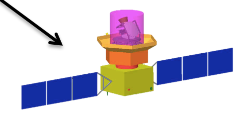
Propagation of plasma
$\Omega_b h^2$: Baryon
$\Omega_c h^2$: Cold dark matter



h: **Hubble parameter**

Reionization
τ

observer



Parameter	*Planck* (CMB+lensing) Best fit	*Planck* (CMB+lensing) 68 % limits	*Planck*+WP+highL+BAO Best fit	*Planck*+WP+highL+BAO 68 % limits
$\Omega_b h^2$	0.022242	0.02217 ± 0.00033	0.022161	0.02214 ± 0.00024
$\Omega_c h^2$	0.11805	0.1186 ± 0.0031	0.11889	0.1187 ± 0.0017
$100\theta_{MC}$	1.04150	1.04141 ± 0.00067	1.04148	1.04147 ± 0.00056
τ	0.0949	0.089 ± 0.032	0.0952	0.092 ± 0.013
n_s	0.9675	0.9635 ± 0.0094	0.9611	0.9608 ± 0.0054
$\ln(10^{10} A_s)$	3.098	3.085 ± 0.057	3.0973	3.091 ± 0.025
Ω_Λ	0.6964	0.693 ± 0.019	0.6914	0.692 ± 0.010
σ_8	0.8285	0.823 ± 0.018	0.8288	0.826 ± 0.012
z_{re}	11.45	$10.8^{+3.1}_{-2.5}$	11.52	11.3 ± 1.1
H_0	68.14	67.9 ± 1.5	67.77	67.80 ± 0.77
Age/Gyr	13.784	13.796 ± 0.058	13.7965	13.798 ± 0.037
$100\theta_*$	1.04164	1.04156 ± 0.00066	1.04163	1.04162 ± 0.00056
r_{drag}	147.74	147.70 ± 0.63	147.611	147.68 ± 0.45
$r_{drag}/D_V(0.57)$	0.07207	0.0719 ± 0.0011		

From Planck 2013 results. I. Overview of products and scientific results

Beyond the standard

- **Particle physicists might think** From H. Murayama, arXiv:0704.2276
  - Non-baryonic dark matter
  - Dark energy
  - Neutrino mass
  - Nearly scale-invariant, Gaussian, and apparently acausal density perturbations
  - Baryon asymmetry
  - …
- **Cosmologists would think**
  - Origin of the structure
  - Flatness problem
  - Horizon problem
  - Monopole problem
  - …

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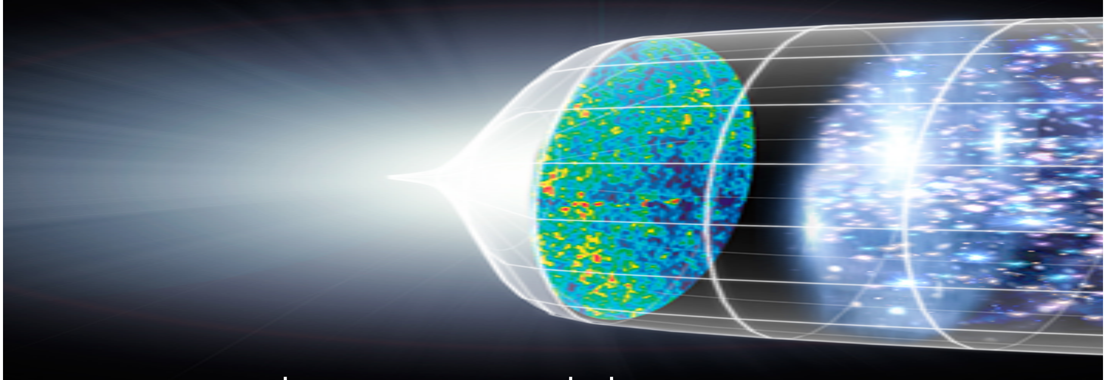
Use CMB to probe these!

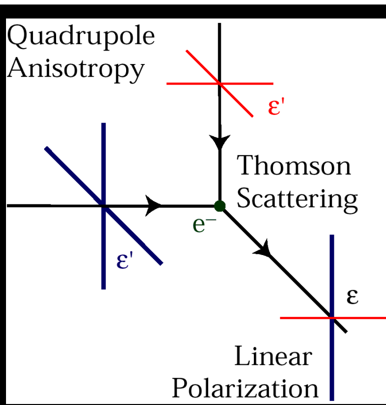
Use CMB to probe beyond



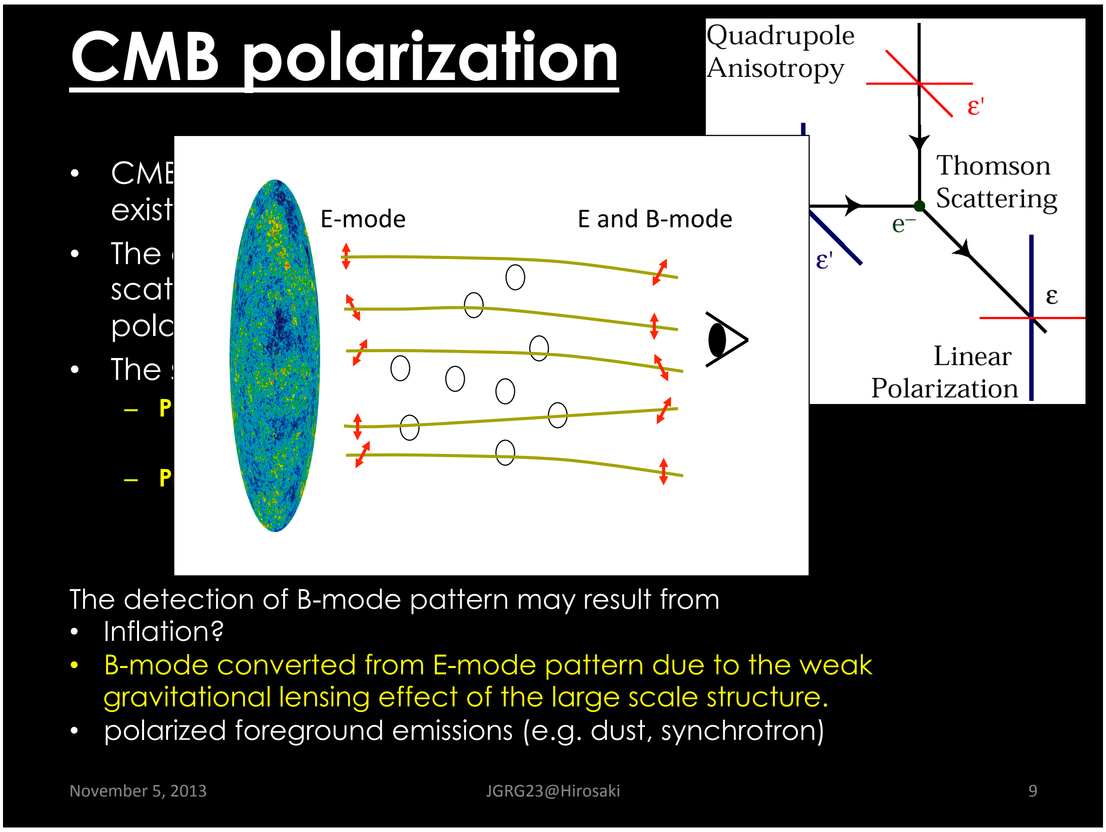
Probing the inflationary paradigm
Search for the imprinted B-mode polarization to look back in time beyond the last scattering surface.

Probing matter distribution
Search for B-mode polarization that is converted from E-mode polarization due to the weak gravitational lensing effect.

CMB polarization



- CMB has polarization regardless of the existence of inflation.
- The quadrupole pattern at the last scattering surface generate the linearly polarized light.
- The source of quadrupole pattern,
  - **Primordial density perturbations**
    - → **E-mode**
  - **Primordial gravitational wave via inflation**
    - → **E-mode and B-mode**


CMB polarization
Quadrupole Anisotropy
ε'
Thomson Scattering
e⁻
ε
Linear Polarization
E-mode
E and B-mode
The detection of B-mode pattern may result from
• Inflation?
• B-mode converted from E-mode pattern due to the weak gravitational lensing effect of the large scale structure.
• polarized foreground emissions (e.g. dust, synchrotron)
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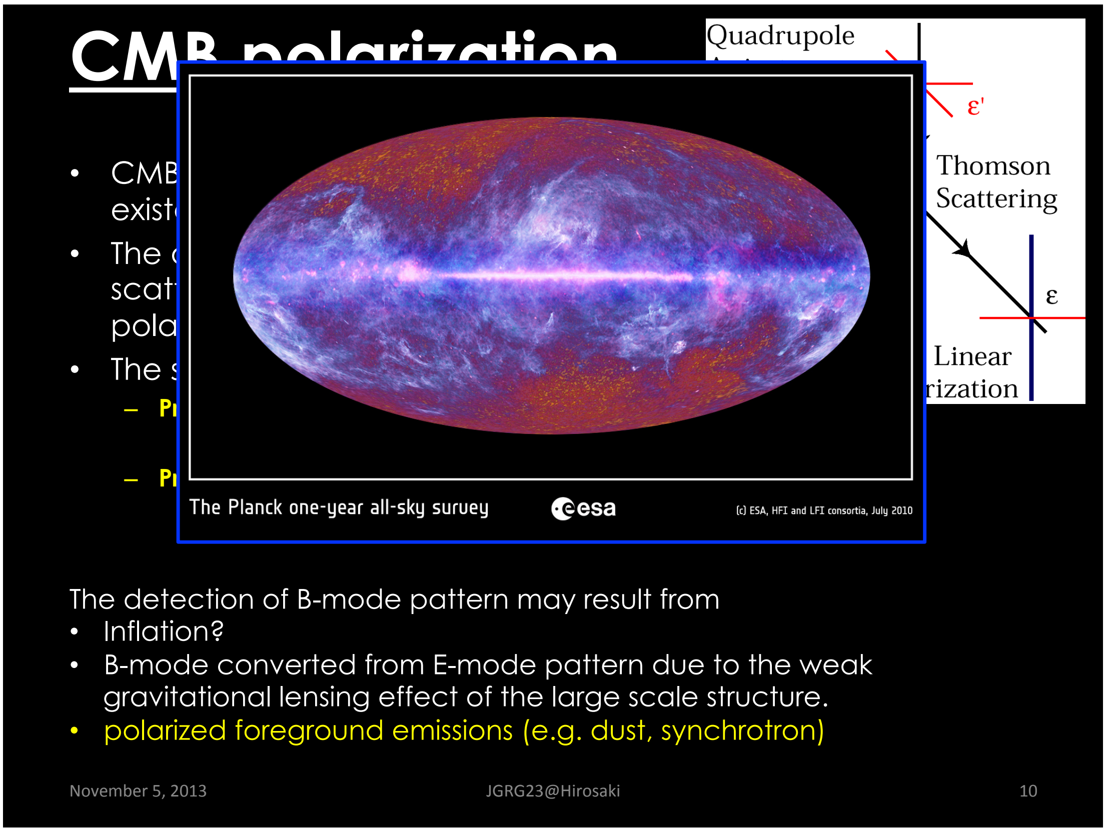
9


Quadrupole
ε'
Thomson Scattering
ε
Linear
The Planck one-year all-sky survey
esa
(c) ESA, HFI and LFI consortia, July 2010
The detection of B-mode pattern may result from
• Inflation?
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November 5, 2013
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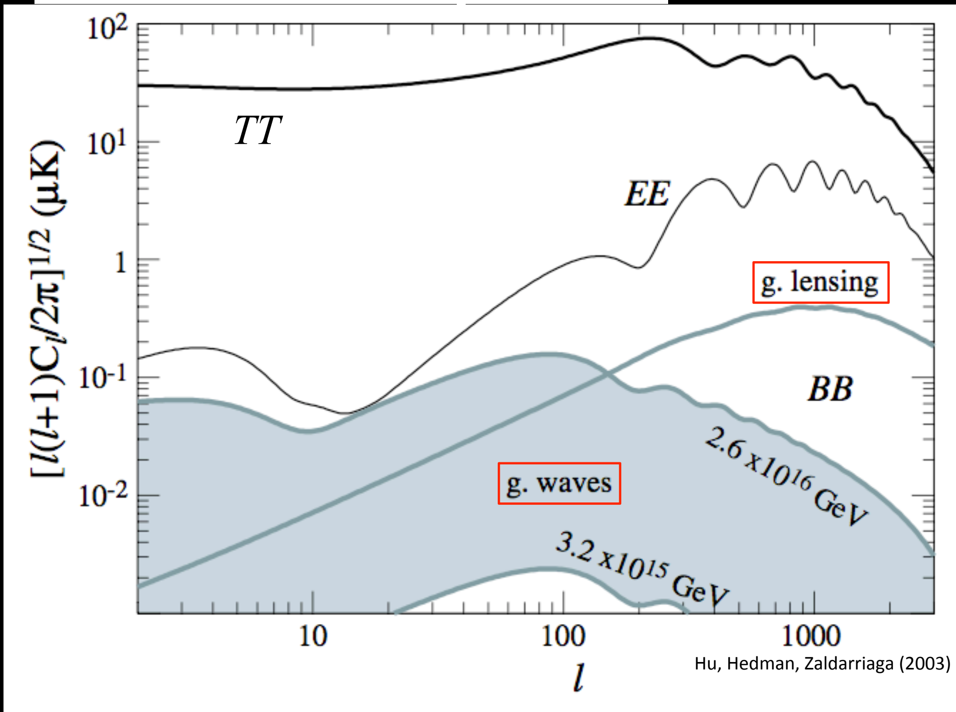
10

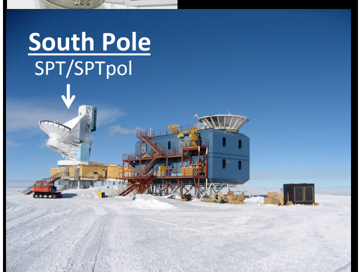
CMB Power spectra

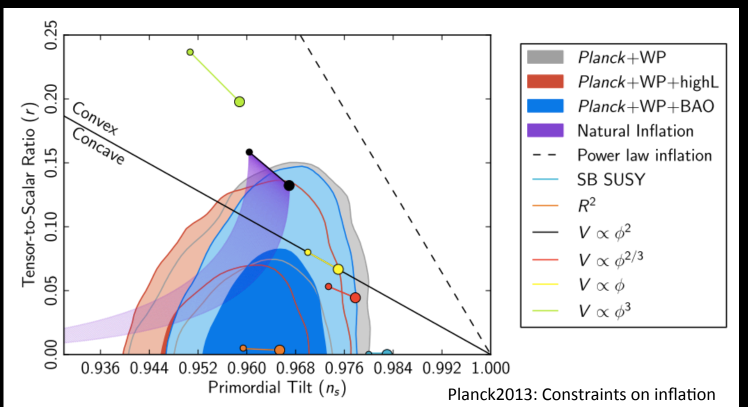


Hu, Hedman, Zaldarriaga (2003)

Experimental limit to inflation models

Planck at L2





Planck2013: Constraints on inflation

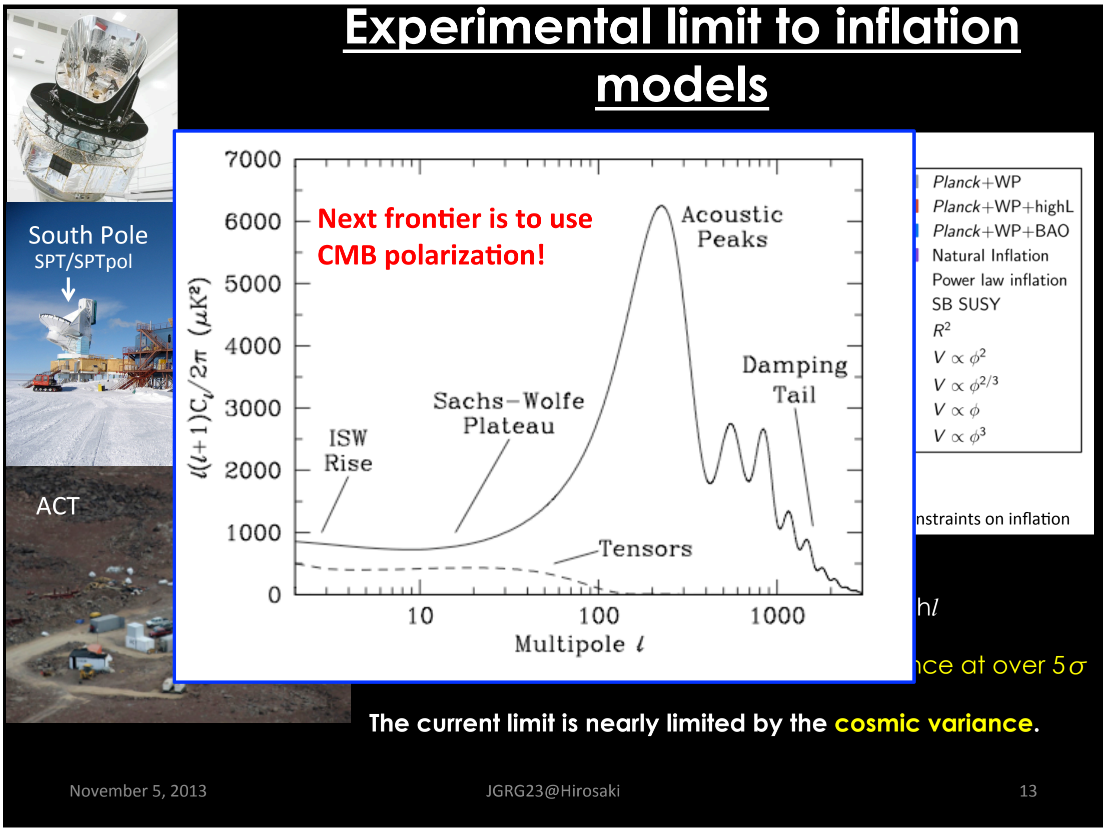
Current best limit on *r* from Planck+WMAP+high*l*

***r* <0.11** (95%C.L.)

n_s **= 0.9548±0.0073**, ruling out the scale invariance at over 5σ

The observational results already started to constraining the inflation models.

The current limit is nearly limited by the cosmic variance.


Experimental limit to inflation models
South Pole
SPT/SPTpol
ACT
7000
6000
5000
4000
3000
2000
1000
0
Next frontier is to use CMB polarization!
Acoustic Peaks
Damping Tail
Sachs–Wolfe Plateau
ISW Rise
Tensors
10
100
1000
Multipole ℓ
ℓ(ℓ+1)C_ℓ/2π (μK²)
Planck+WP
Planck+WP+highL
Planck+WP+BAO
Natural Inflation
Power law inflation
SB SUSY
R²
V ∝ φ²
V ∝ φ^{2/3}
V ∝ φ
V ∝ φ³
nstraints on inflation
hl
nce at over 5σ
The current limit is nearly limited by the cosmic variance.
November 5, 2013
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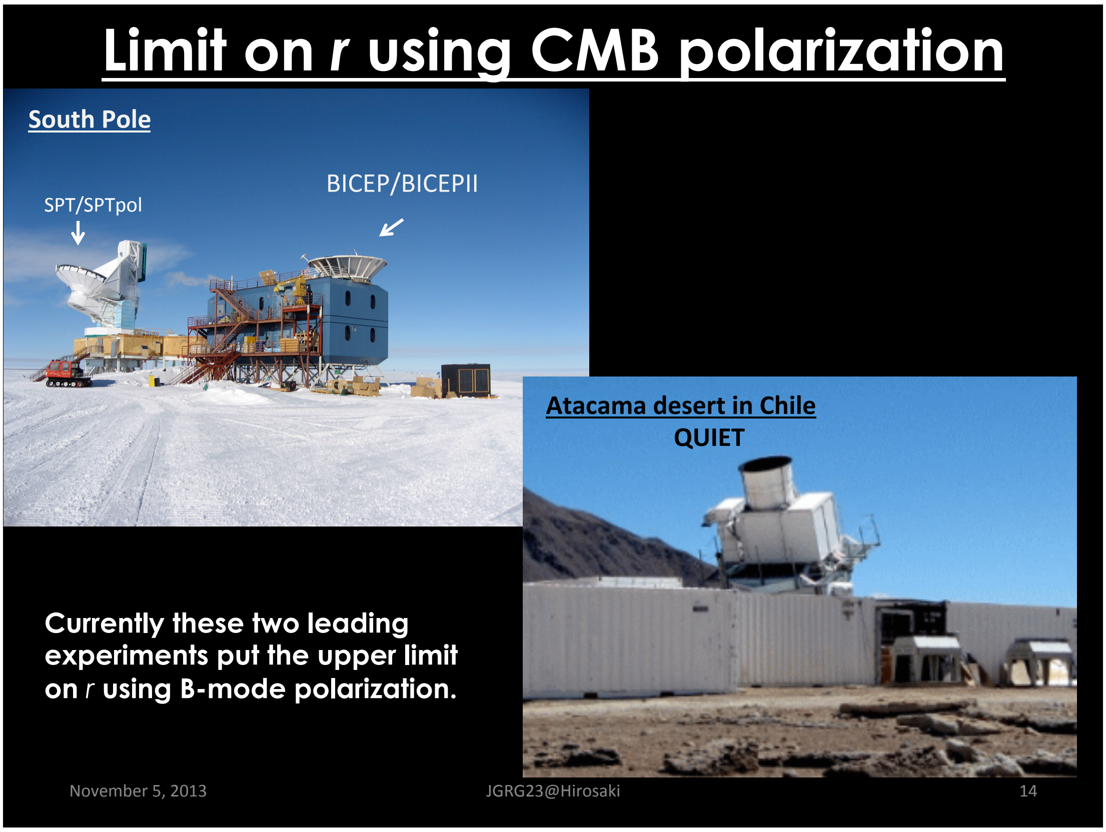
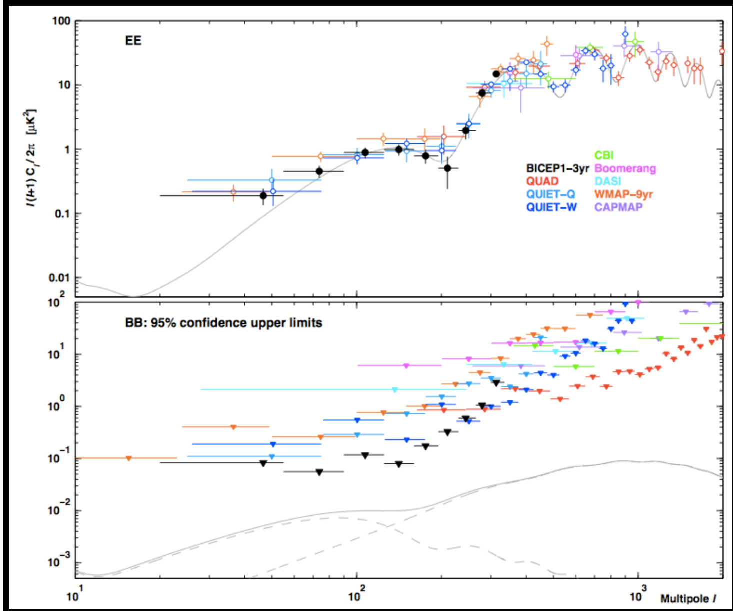
13


Limit on r using CMB polarization
South Pole
SPT/SPTpol
BICEP/BICEPII
Atacama desert in Chile
QUIET
Currently these two leading experiments put the upper limit on r using B-mode polarization.
November 5, 2013
JGRG23@Hirosaki
14

Current limit on *r*



Current best limit on *r*
Planck+WMAP+high*l*
r <0.11 (95%C.L.)

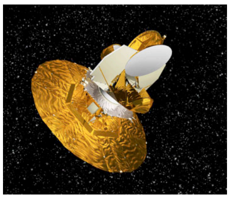
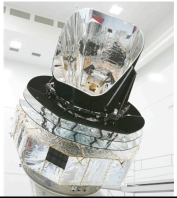
Current best limit from BB power spectrum
BICEP-I three year data, *r* <0.70. Barkats et al. (2013)

Very big community wide efforts to probe this deeper.

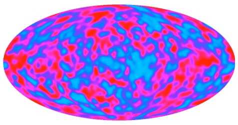
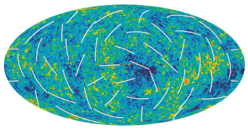
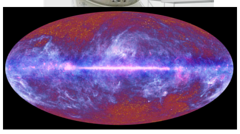
BICEP-I three year data Barkats et al. (2013)

CMB satellite and next generation satellite proposals

CMB satellites

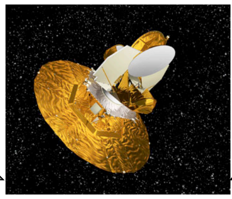




Next generation CMB satellite

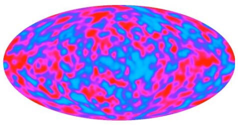
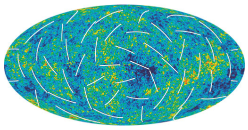
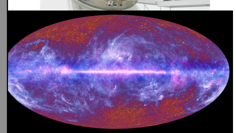




COBE (1989)	**WMAP (2001)**	**Planck (2009)**
Band		
32–90GHz	23–94GHz	30–857GHz（353GHz）
Detectors		
6 radiometers	20 radiometers	11 radiometers + 52 bolometers
Operation temperature		
300/140 K	90 K	100 mK
Angular Resolutino		
~7°	~0.22°	~0.1°
Orbit		
Sun Synch	L2	L2

CMB satellites

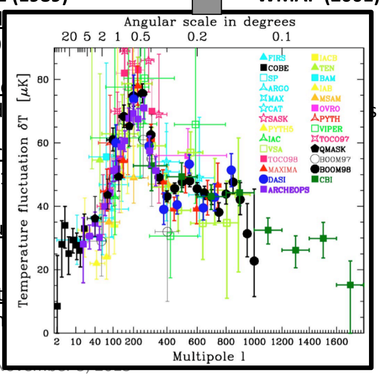



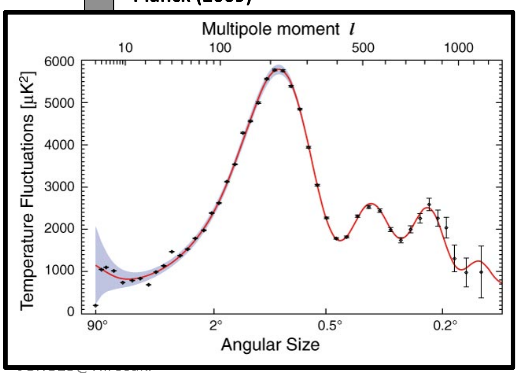

Next generation CMB satellite

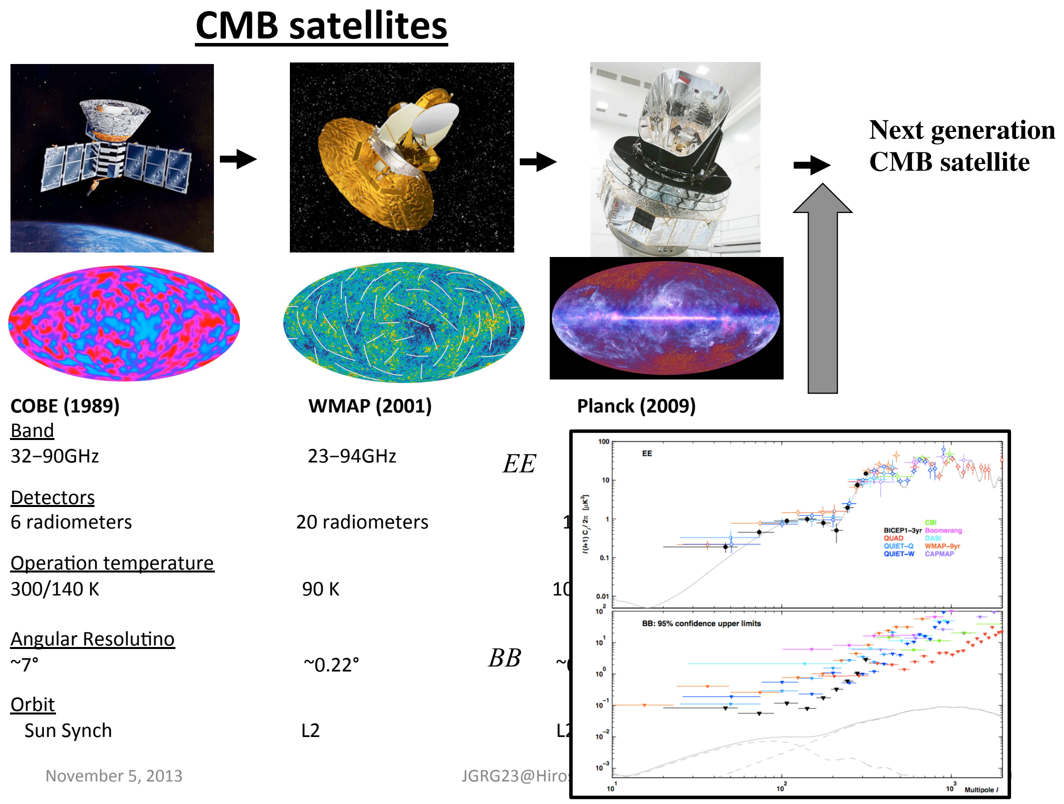




COBE (1989) **WMAP (2001)** **Planck (2009)**

Band
32–9
Dete
6 rad
Oper
300/
Angu
~7°
Orbit
Sun






CMB satellites
Next generation
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Sun Synch
L2
EE
BB
BB: 95% confidence upper limits
BICEP1–3yr
QUAD
QUIET–Q
QUIET–W
CBI
Boomerang
DASI
WMAP–9yr
CAPMAP
Multipole l
November 5, 2013
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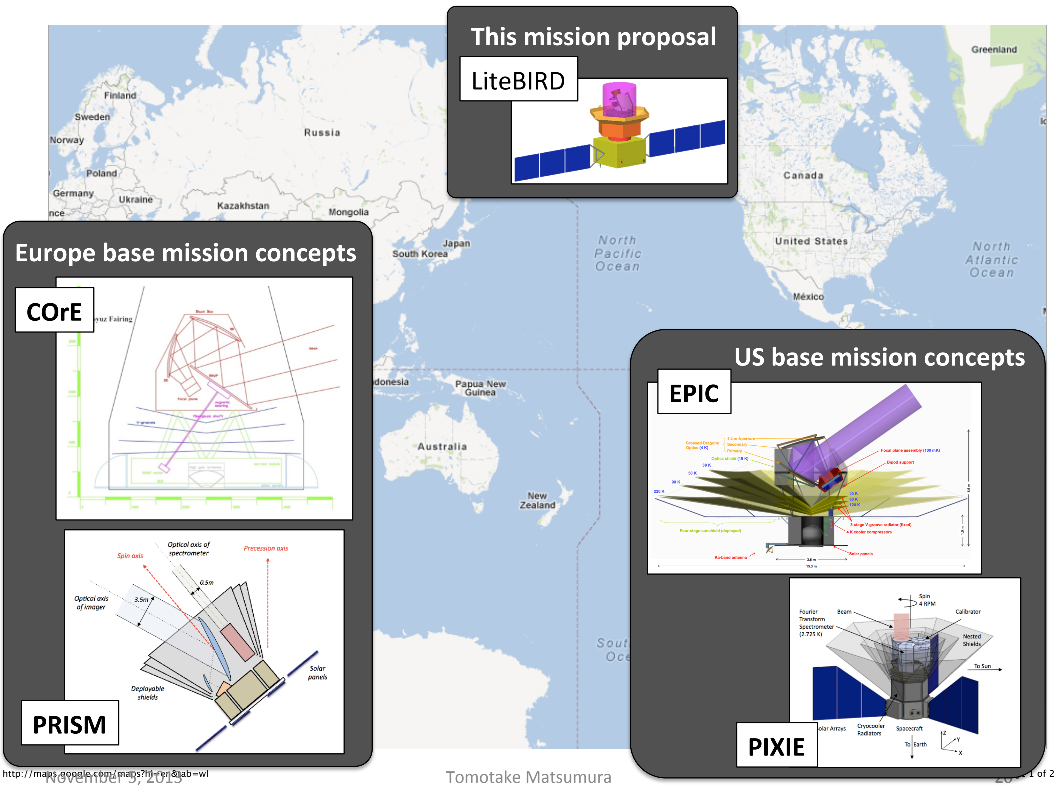
This mission proposal
LiteBIRD
Europe base mission concepts
COrE
PRISM
US base mission concepts
EPIC
PIXIE
Finland
Sweden
Norway
Poland
Germany
Ukraine
Russia
Kazakhstan
Mongolia
Japan
South Korea
North Pacific Ocean
Canada
United States
México
Greenland
North Atlantic Ocean
Papua New Guinea
Australia
New Zealand
http://maps.google.com/maps?hl=en&tab=wl
November 5, 2013
Tomotake Matsumura

LiteBIRD

LiteBIRD working group

- 69 members (as of Oct. 1, 2013)
- International and interdisciplinary

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Y. Chinone
K. Hattori
M. Hazumi (PI)
M. Hasegawa
Y. Hori
N. Kimura
T. Matsumura
H. Morii
R. Nagata
S. Oguri
N. Sato
T. Suzuki
O. Tajima
T. Tomaru
H. Yamaguchi
M. Yoshida

SOKENDAI
Y. Akiba
Y. Inoue
H. Ishitsuka
H. Watanabe

Osaka U.
S. Takakura

JAXA
H. Fuke
I. Kawano
H. Matsuhara
K. Mitsuda
T. Nishibori
A. Noda
S. Sakai
Y. Sato
K. Shinozaki
H. Sugita
Y. Takei
T. Wada
N. Yamasaki
T. Yoshida
K. Yotsumoto

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H. Ishino
A. Kibayashi
Y. Kibe

National Inst. for Fusion Science
S. Takada

UC Berkeley
A. Ghribi
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A. Lee (US PI)
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J. Borrill

Tsukuba U.
M. Nagai

Kavli IPMU
N. Katayama
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Yokohama NU.
S. Murayama
S. Nakamura
K. Natsume

Osaka Pref. U.
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M. Kozu
H. Ogawa

MPA
E. Komatsu

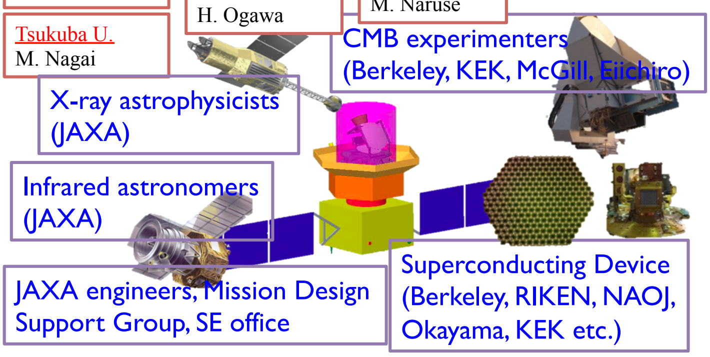
Tohoku U.
M. Hattori
K. Ishidoshiro
K. Morishima

Kinki U.
I. Ohta

Saitama U.
M. Naruse

ATC/NAOJ
K. Karatsu
T. Noguchi
Y. Sekimoto
Y. Uzawa

RIKEN
K. Koga
S. Mima
C. Otani



LiteBIRD mission

- Check simple well-motivated inflationary models
  - ***requirement of the uncertainty on r***

    (**stat.** ⊕ **syst.** ⊕ **foreground** ⊕ **lensing**) **δr < 0.001**

No lose theorem of LiteBIRD

- Many inflationary models predict r>0.01 → >10sigma discovery
- Simple well-motivated inflationary models (single-large-field slow-roll models) have a lower bound on r, r>0.002, from Lyth relation.

$$r = \frac{1}{N^2}\left(\frac{\Delta\phi}{m_{\rm pl}}\right)^2 \approx 2\cdot 10^{-3}\left(\frac{\Delta\phi}{m_{\rm pl}}\right)^2$$

  - no gravitational wave detection at LiteBIRD → exclude well motivated inflationary models (i.e. r<0.002 @ 95% C.L.)
- Early indication from non-space-based projects → power spectra at LiteBIRD !

Design philosophy of LiteBIRD

The science goal of LiteBIRD is to test the well motivated inflationary models (large single field slow roll models) with the sensitivity of δr<0.001.

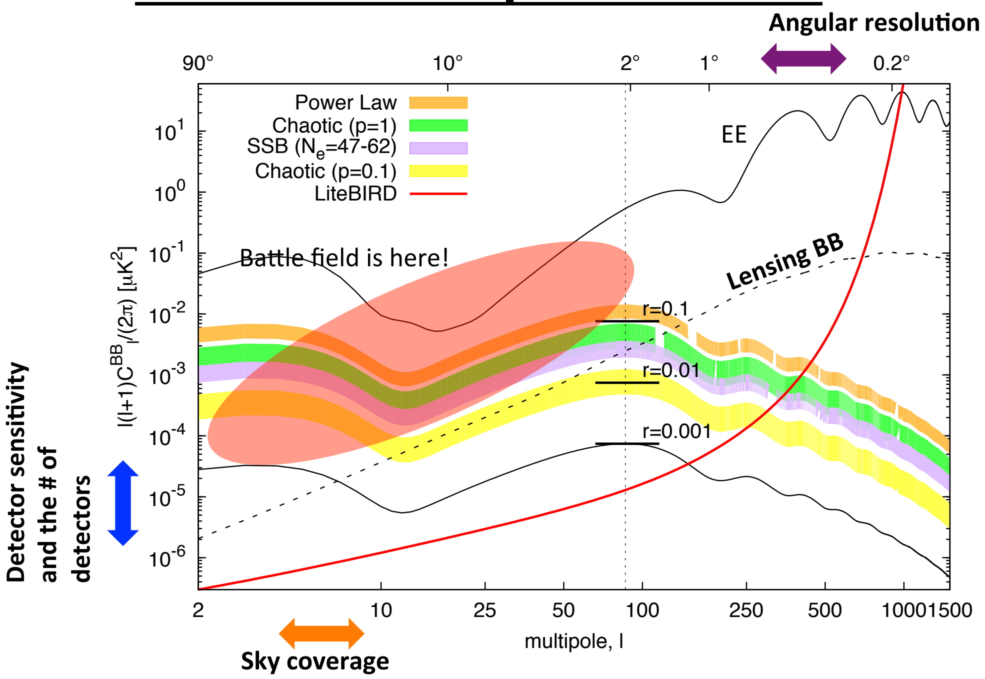
Design philosophy of LiteBIRD

The science goal of LiteBIRD is to test the well motivated inflationary models (large single field slow roll models) with the sensitivity of $\delta r<0.001$.

⬇

What is the instrumental specification in order to achieve this?

Instrumental specifications



Translate to the instrumental specifications

The science goal of LiteBIRD is to test the well motivated inflationary models with the sensitivity of δr<0.001.

Instrumental specifications

- **Frequency coverage 60-270 GHz**
- **Angular resolution: 30 arcmin（@150GHz）**
- **Sensitivity: 2 uK･arcmin**
- **All sky survey**

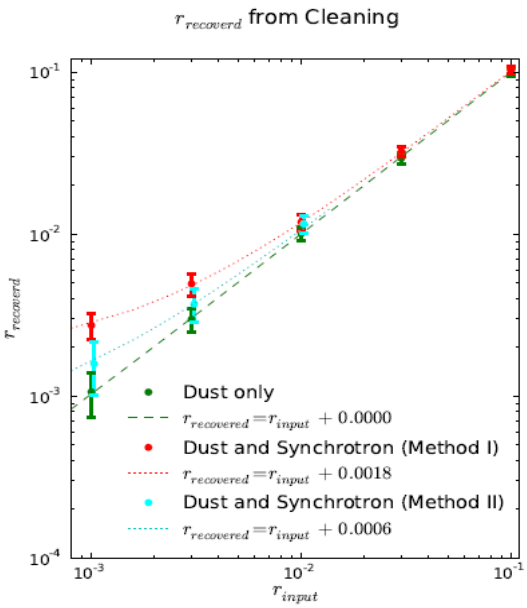
Translate to the instrumental specifications

The science goal of LiteBIRD is to test the well motivated inflationary models with the sensitivity of δr<0.001.

Instrumental specifications

- **Frequency coverage 60-270 GHz**
  - **→ multi-color observation without using external data**
- **Angular resolution: 30 arcmin（@150GHz）**
  - **→ < 1 m telescope**
- **Sensitivity: 2 uK･arcmin**
  - **→ kilo pixel array**
- **All sky survey**
  - **→ spin type scanning strategy**

Foreground removal and observing bands



According to N. Katayama and E. Komatsu, (ApJ 737, 78 (2011), arXiv:1101.5210), the pixel-based polarized foreground removal using template method indicates that we need

→ ≥ 5 bands in 50-270GHz

The method do not assume the spectral shape of the foreground emission.

→ model-independent

The subtraction of the dust and synchrotron emissions with the three bands (60, 100, 240 GHz) was demonstrated with very small bias,

r~0.0006.

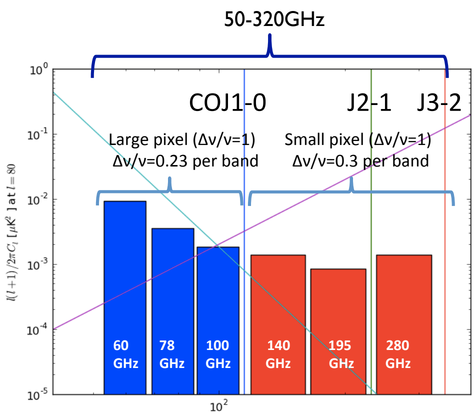
LiteBIRD band selection for multi-chroic pixels

We chose the band locations with the following reasons.

1. **Katayama-Komatsu (2011) suggested the range of frequency from 50-270 GHz based on the template subtraction.**
2. **We want to exclude the CO lines.**
3. **From the practical consideration such as AR coating on a lenslet array, it is reasonable to limit the bandwidth per pixel to Δν/ν~1.**

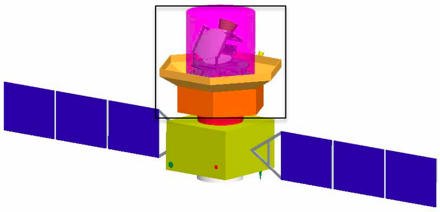
Above three constraints naturally put us to the band locations.

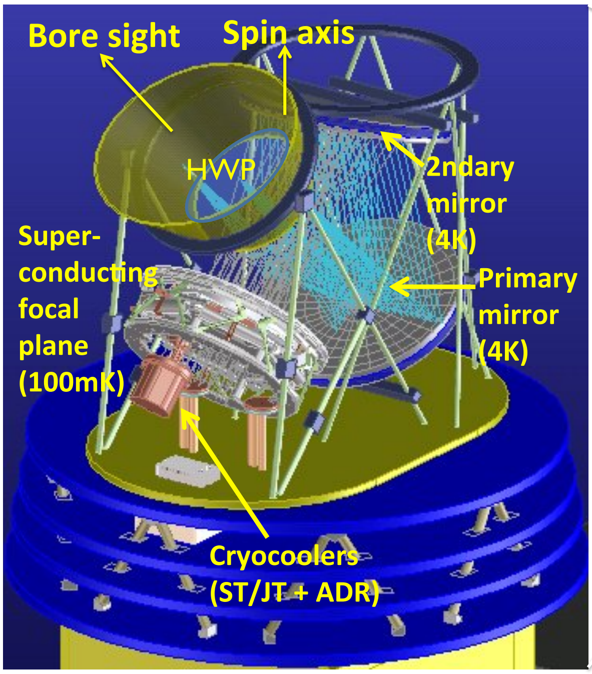
- **Some room for low frequencies.**
- **Option of distributed band centers (more studies needed).**



System overview

- Launch vehicle: H2 or Epsilon
- w/ spin & precession scan strategy
- To be ready for Mission Definition Review in JFY2013
- Target launch year: 2020 (LEO or L2)





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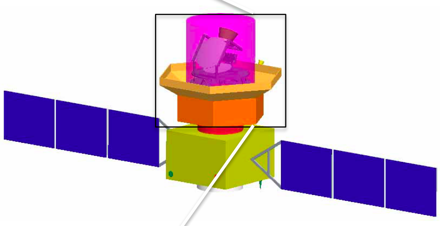

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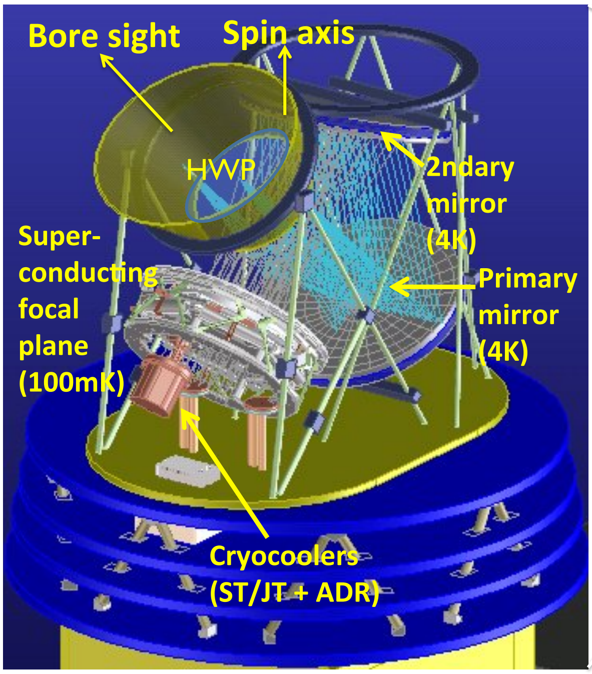
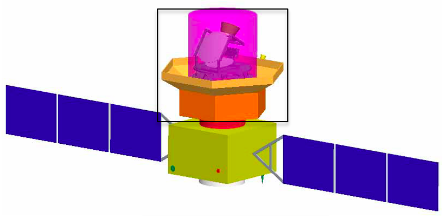
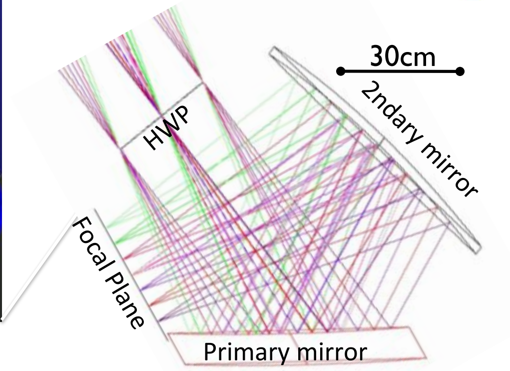


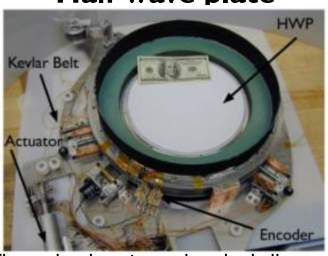




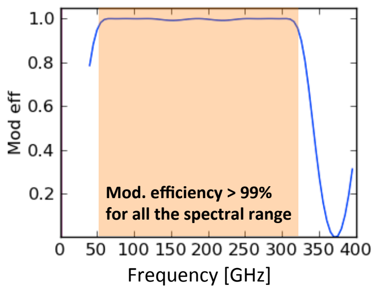
Crossed Dragone Configuration

Optical system

Half-wave plate

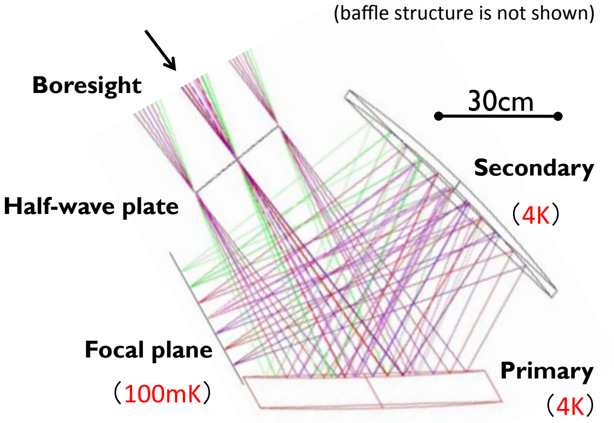


The technology is used at the balloon environment by EBEX.



Cryogenically cooled Cross Dragone telescope

(baffle structure is not shown)

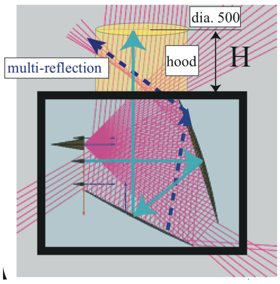


The continuous HWP helps
1) avoid detector $1/f_{knee}$
2) mitigate the differential systematic effects.

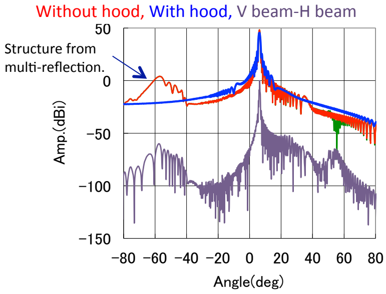
The 7 stacked achromatic HWP plates covers the LiteBIRD bandwidth. The broadband AR coating is required and the simulated based solution exists. It is yet required to be demonstrated experimentally.

Optical system

GRASP simulation for sidelobe evaluation



Without hood, With hood, V beam-H beam



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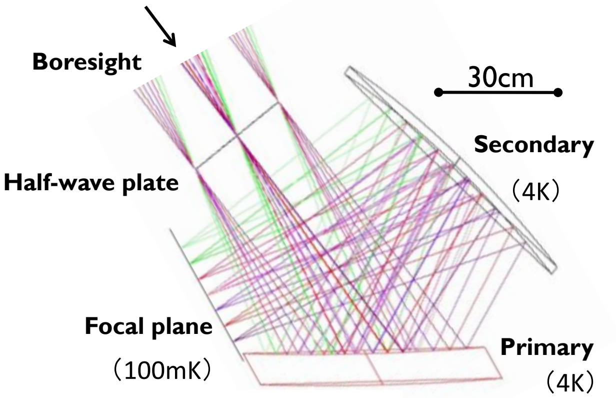


Beam measurements

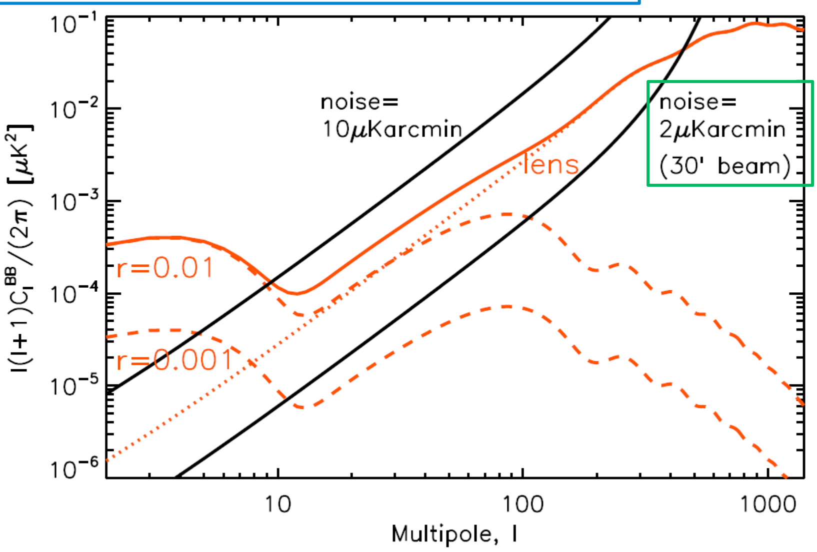
The beam measurement setup is built using the prototype crossed Dragone telescope.

We also consider the option of three mirror Gregorian telescope.

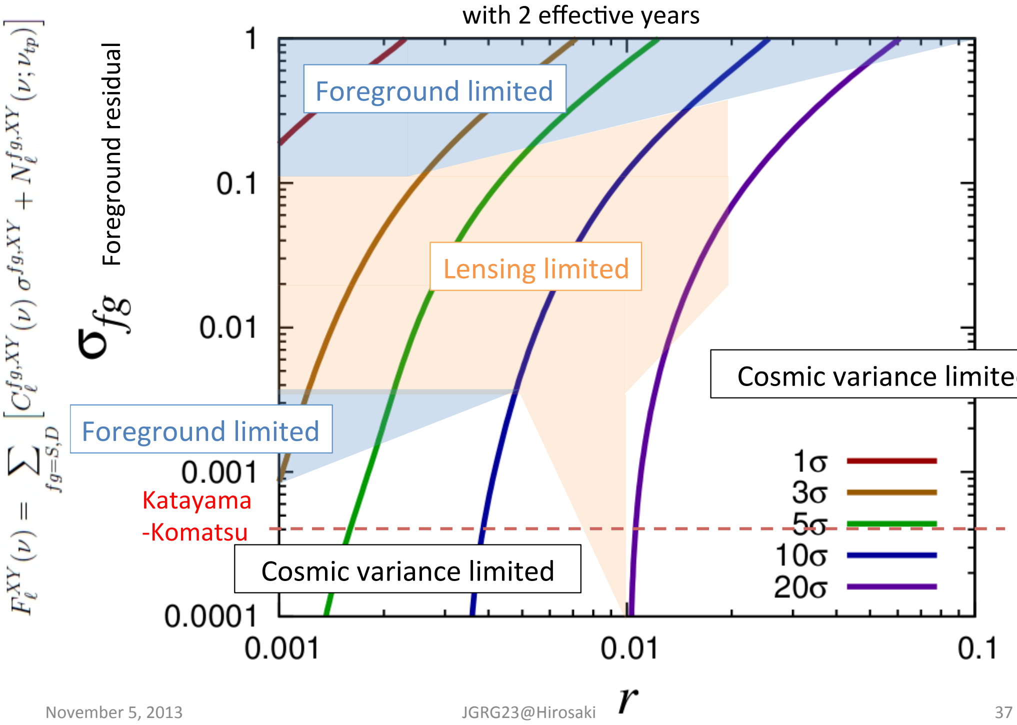
Focal plane requirement

Noise level: goal = 2μK･arcmin
(requirement: < 3μK･arcmin)

To be well below "lensing floor"



Expected sensitivity on *r*



LiteBIRD focal plane design

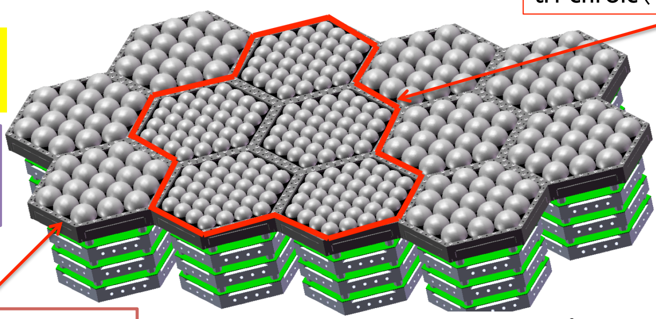
tri-chroic（140/195/280GHz）

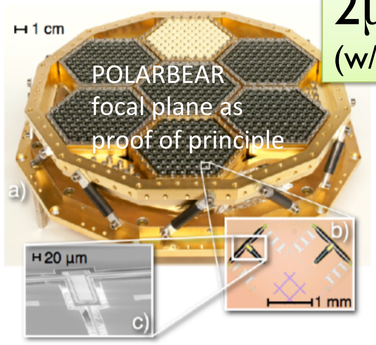
UC Berkeley TES option

2022 TES bolometers

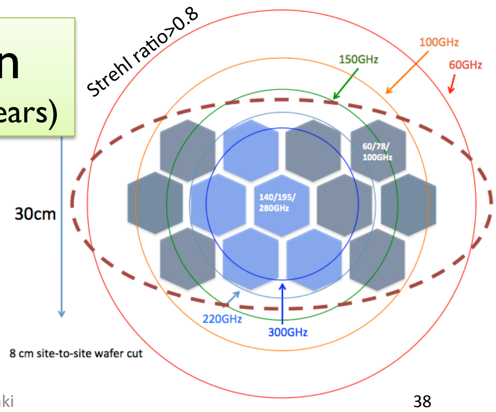
T_{bath} = 100mK

tri-chroic（60/78/100GHz）





2μKarcmin
(w/ 2 effective years)



LiteBIRD focal plane design

UC Berkeley TES option

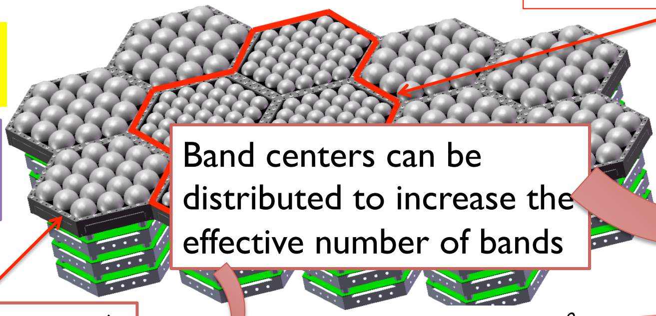
2022 TES bolometers

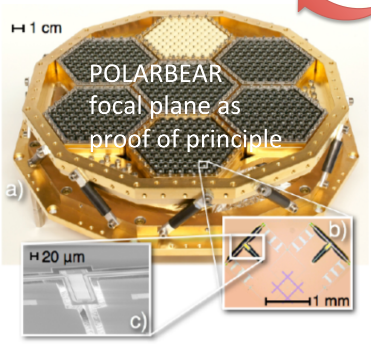
T_{bath} = 100mK

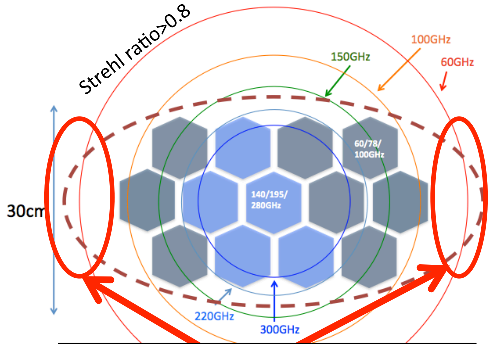
tri-chroic（60/78/100GHz）

tri-chroic（140/195/280GHz）

Band centers can be distributed to increase the effective number of bands







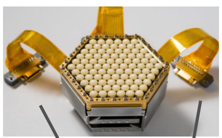
More space to place <60GHz detectors

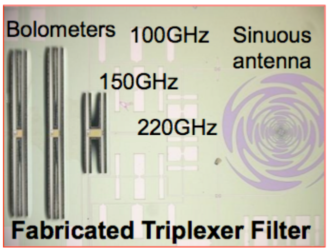
Detector options

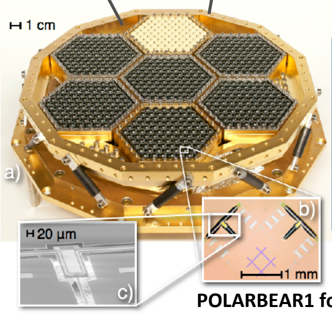
TES option

• A number of ongoing CMB projects employ the TES bolometers, including POLARBEAR-1 & 2, EBEX, SPTpol, and many.

TES from UC Berkeley







POLARBEAR1 focal plane

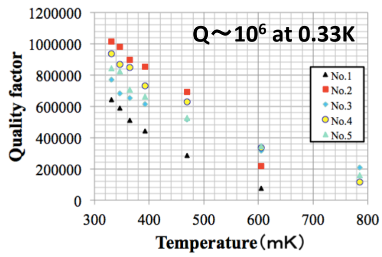
Readout from McGill University

Readout electronics based on SQUID and DfMUX (64 MUX). The required power is 2W/SQUID and the total power is 64W.

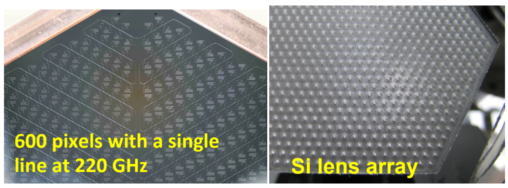
MKID option

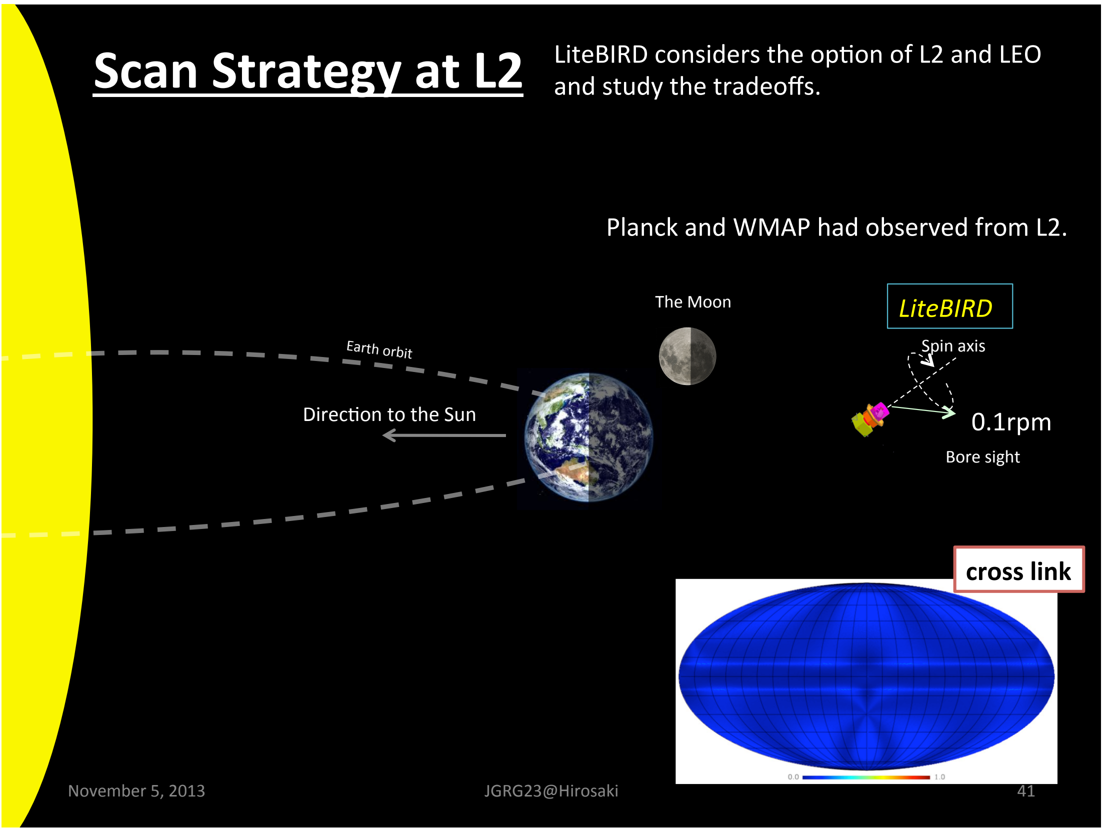
• Large multiplexing factor in the MKID readout
• Large dynamic range
• Fast time constant〜µsec

MKID development in KEK & Okayama Univ.



MKID development in National Astronomical Observatory of Japan





Systematic effect requirements

We set the required level of each systematic effect as 1% of lensing floor in C_l at all l range.

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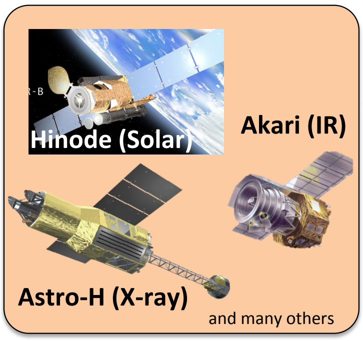
We set the required level of each systematic effect as 1% of lensing floor in C_l at all l range.

Effects	Types	Requirement (bias)	Requirement (random)	Comments	Mitigation
Absolute gain	$E \to B$	Cancel on r	3%	Calibration in every 10 min.	Dipole, planets
Polarization angle	$E \to B$	1 arcmin.	24 arcmin.		
Beam size stability	$E \to B$		O(10%)		Scan strategy
Absolute pointing	$E \to B$	6 arcmin.	25 arcmin.	20degx30deg FOV	Scan strategy
Diff. pointing	$T \to B$	3.5 arcsec.	16 arcsec.		Continuous HWP
Diff. gain	$T \to B$	0.01%	0.3%		Continuous HWP
Diff. beam size	$T \to B$	0.7%	2%		Continuous HWP
Diff. beam ellipticity	$T \to B$	7% @l=2 0.04% @ l=300	2.7 %		Continuous HWP

Satellite BUS system

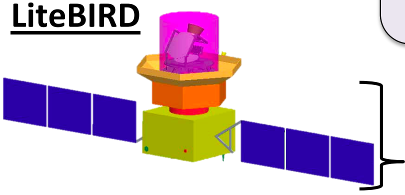


Mitsubishi electronic corp. (MELCO)
NEC
Sumitomo Heavy Industry (SHI)

Systematics study

System requirement to achieve the science goal
- Scan strategy
- Pointing Accuracy
- Spin rate
- System power requirement
- …

LiteBIRD



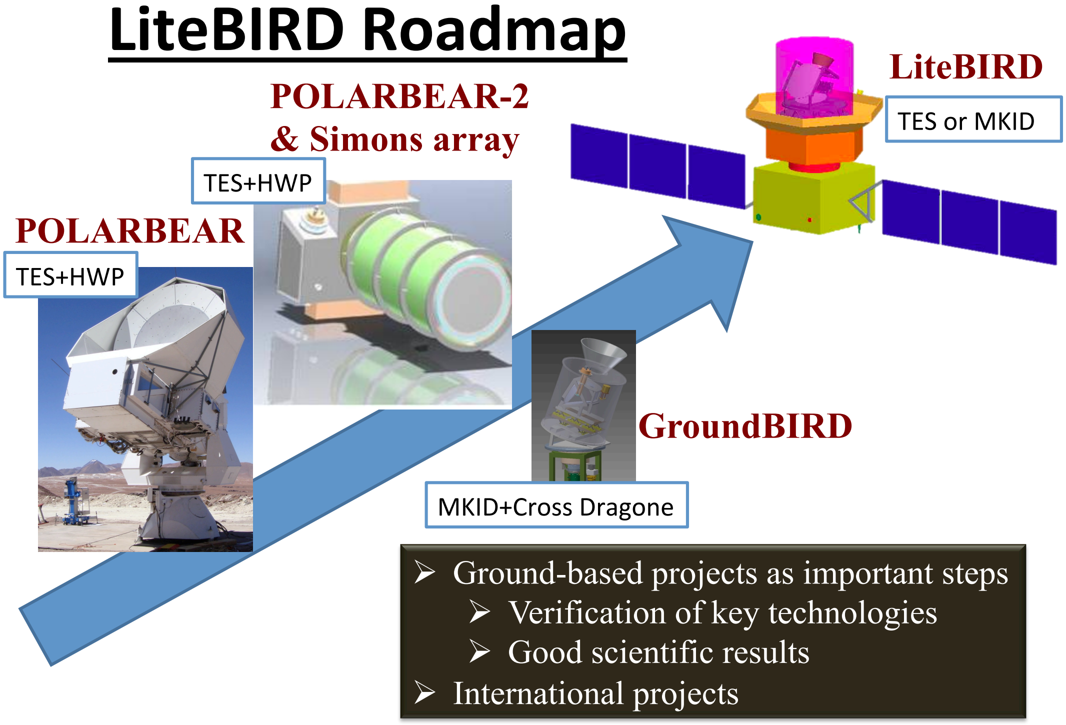
Design of
- Configuration
- Power system
- Thermal architecture
- Attitude control system

Design iteration

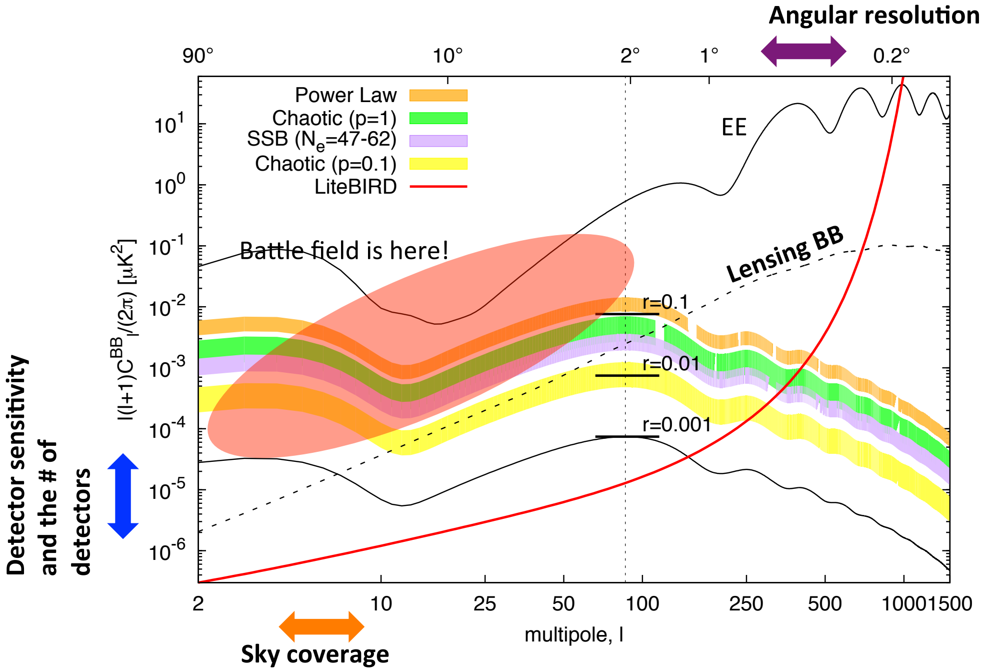
Project status

- Candidate for JAXA's future missions on "fundamental physics"
- Working group authorized by Steering Committee for Space Science (SCSS) of Japan
- One of eight most important future projects by astronomy/ astrophysics division of Science Council of Japan
- Japanese High Energy Physics (HEP) community has also identified CMB polarization measurements and dark energy survey as two important areas of their "cosmic frontier".

LiteBIRD Roadmap



Instrumental specifications



Conclusions

- LiteBIRD is designed to test the well motivated inflationary models with an uncertainty of $\delta r<0.001$ (full success).
- Currently LiteBIRD WG is going through the design iterations to prepare for the mission definition review by the end of this year.
- The R&D for the LiteBIRD technologies are in progress in the ground-based experiments (POLARBEAR, POLARBEAR-2, Simons array, GroundBIRD).

“Gauge-flation confronted with CMB observations”

by Ryo Namba

[JGRG23(2013)110512]

Gauge-flation Confronted with CMB Observations

Ryo Namba

Kavli IPMU

JGRG23 Workshop: November 5, 2013

RN, E. Dimastrogiovanni & M. Peloso, arXiv:1308.1366 (accepted in JCAP)

Introduction

Inflation – A dominant paradigm for the physics in the primordial universe

- Solves the problems in the BB cosmology (horizon, flatness, monopole)
- Consistent with the fluctuations in the CMB and LSS observations

Simplest realization – Scalar-field inflaton φ

- Typically requires a flat potential $V(\varphi)$ $\Leftrightarrow$ UV sensitive
- Flatness is spoiled by radiative corrections and η problems in supergrav.

Shift symmetry to protect the flatness – invariance under $\varphi \to \varphi + \text{const.}$

- Natural inflation Freese, Frieman & Olinto '90
- Observations require axion decay constant $f \gtrsim M_p$ Savage, Freese & Kinney '06
  - $f < M_p$ can be compatible in various mechanisms, e.g. more than one axion
- Symmetry allows interaction with a gauge field $\mathcal{L} \propto \varphi F\tilde{F}$ Anber & Sorbo '10
- New phenomenological predictions
  - Non-Gaussianity, chiral GWs, primordial BHs, . . .

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Chromo-natural inflation – promoted the gauge field to an $SU(2)$

Adshead & Wyman '12

- φ can be integrated out if sitting at its potential minimum
  - Leads to the **Gauge-flation** Sheikh-Jabbari '12

Gauge-flation Maleknejad & Sheikh-Jabbari '11

$$S = \int d^4x \sqrt{-g} \left[\underbrace{\frac{M_p^2}{2} R}_{\text{GR}} \underbrace{- \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}}_{\text{Standard SU(2)}} + \underbrace{\frac{\kappa}{96} \left(F^a_{\mu\nu} \tilde{F}^{a,\mu\nu} \right)^2}_{\text{New term}} \right] + \dots$$

$$\text{Ansatz :} \quad \langle A^a_i \rangle = \hat{\phi}(t)\, \delta^a_i \,, \quad ds^2 = -dt^2 + a^2(t)\, \delta_{ij}\, dx^i\, dx^j$$

- The isotropic FLRW = an attractor of the dynamics of the model
  Maleknejad, Sheikh-Jabbari & Soda '12
- Shares background trajectories with the Chromo-natural inflation
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- The first and only existing stable model with a vector field only
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The **Yang-Mills term** F^2 behaves as the standard radiation

- Massless spin-1 field

The **new term** $(F\tilde{F})^2$ behaves like a cosmological constant

- $F^a_{\mu\nu}\tilde{F}^{a,\mu\nu} = \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}}F^a_{\mu\nu}F^a_{\rho\sigma} \Leftrightarrow$ coupling to gravity only through $\mathrm{Det}\,(g_{\mu\nu})$
  - $T_{\mu\nu}\left[(F\tilde{F})^2\right] \propto g_{\mu\nu}\,(F\tilde{F})^2 \quad\Longleftrightarrow\quad$ cosmological constant

Inflation

Energy density : $\rho = \rho_{YM} + \rho_\kappa$

Pressure : $P = \frac{1}{3}\rho_{YM} - \rho_\kappa$

ρ_{YM}: Yang-Mills F^2, ρ_κ: new $(F\tilde{F})^2$

- $\kappa = 0 \longrightarrow w = 1/3 \longrightarrow$ radiation
- $\rho_\kappa \gg \rho_{YM} \longrightarrow w = -1 \longrightarrow$ **inflation**

$\Longrightarrow$ Slow-roll param. : $\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{2\rho_{YM}}{\rho_{YM}+\rho_\kappa} \ll 1$

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There is only **one free parameter** in the model:

$$\gamma \equiv \frac{g^2 Q^2}{H^2}$$

g: $SU(2)$ coupling

$Q\,\delta^a_i = \langle A^a_i \rangle / a$

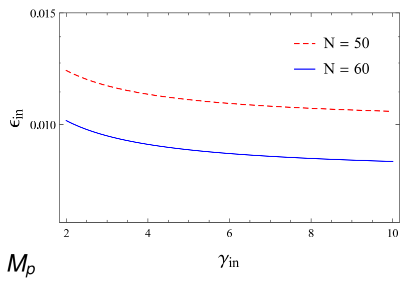
- Other parameters are fixed by
  - Number of e-folds, $N = 50 - 60$
  - Background attractor
  - COBE normalization

$$N_{\rm tot} \simeq \frac{1+\gamma_{\rm in}}{2\,\epsilon_{\rm in}} \ln\left(\frac{1+\gamma_{\rm in}}{\gamma_{\rm in}}\right)$$

- The attractor leads to

$$\epsilon \simeq \frac{Q^2}{M_p^2}(1+\gamma)$$

$$\delta \equiv -\frac{\dot{Q}}{QH} \sim \epsilon^2$$



  - **Small-field** inflation in the sense $Q \ll M_p$
  - $\epsilon \sim \mathcal{O}\left(10^{-2}\right) \sim$ **large-field** value in the single-scalar chaotic inflation
  - Q rolls VERY slowly during inflation

Perturbations

$$\delta A^a_\mu = A^a_\mu - \langle A^a_\mu \rangle \ , \quad \delta g_{\mu\nu} = g_{\mu\nu} - g^{(0)}_{\mu\nu}$$

- (3 × 4 gauge perts.) + (10 metric perts.) = Total 22 d.o.f.

Decomposition

$$\begin{aligned}
\delta A^a_0 &= Y_a + \partial_a Y \\
\delta A^a_i &= \delta Q \, \delta_{ai} + \partial_i \left(M_a + \partial_a M \right) + \epsilon_{iab} \left(U_b + \partial_b U \right) + t_{ia} \\
\delta g_{00} &= 2\phi \\
\delta g_{0i} &= B_i + \partial_i B \\
\delta g_{ij} &= 2\,\psi\,\delta_{ij} + 2\,\partial_i\partial_j E + \partial_i E_j + \partial_j E_i + h_{ij}
\end{aligned}$$

Three decoupled sectors:

1. 8 "Scalar": $\delta Q, \ M, \ Y, \ U, \ \phi, \ B, \ \psi, \ E$
2. 5 "Vector": $M_a, \ Y_a, \ U_a, \ B_i, \ E_i$ $\quad (\partial_a M_a = \partial_a Y_a = \partial_a U_a = \partial_i B_i = \partial_i E_i = 0)$
3. 2 "Tensor": $t_{ia}, \ h_{ij}$ $\quad (\partial_i h_{ij} = \partial_i t_{ia} = \partial_a t_{ia} = h_{ii} = t_{ii} = 0)$

- Turning on the vector vev in general breaks the rotational symmetry
- $SU(2) \cong SO(3)$
- Choice $\langle A^a_i \rangle = \hat{\phi}\, \delta^a_i$ ⇔ Rotational symmetry is preserved by global $SU(2)$
  - Rotation is "canceled" by $SU(2)$ transformation
- The decomposition identifying the $SU(2)$ indices as coordinate ones realizes the decoupling of the 3 sectors in the quadratic-order action.

d.o.f.	Total	SU(2) gauge	GR gauge	Non-dynamical	Physical d.o.f.
Scalar	8	-1	-2	$-(1+2)$	2
Vector	10	-2	-2	$-(2+2)$	2
Tensor	4	0	0	0	4

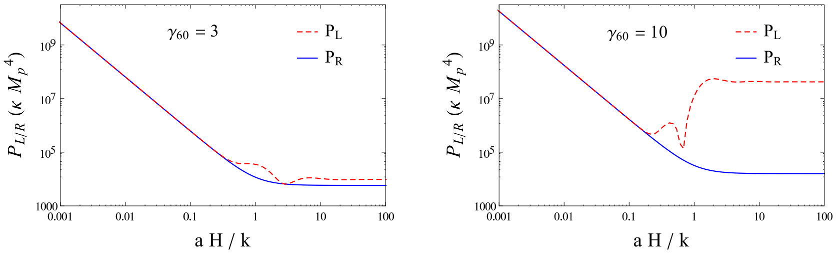
Tensor Modes

(2 tensors) $\times$ (2 d.o.f.) $=$ total 4 d.o.f.

$$\begin{cases} h_{ij} \to h_{L/R} \\ t_{ia} \to t_{L/R} \end{cases} \Longrightarrow \quad h/t \text{ coupled}, \quad L/R \text{ decoupled}$$

- ***CPT* in the tensor sector is broken from** $SU(2)$ (not from $(F\tilde{F})^2$)
- **Tachyonic growth near horizon crossing in** t_L $\Longrightarrow$ **sources** h_L

Gravitational-Wave Power Spectrum



◇ **The larger γ, the larger GW (L mode)**

Scalar Mode

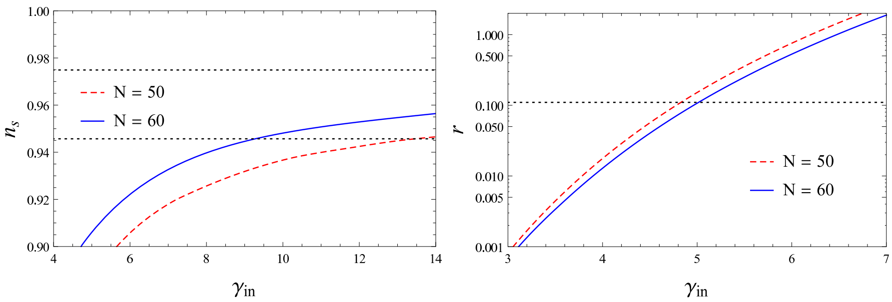
2 **coupled** dynamical d.o.f.

- Coupled system $\Longrightarrow$ Initial quantization in a matrix form
  - 2 initial eigenfrequencies: $\omega_{\rm in} = k, \ \frac{\sqrt{\gamma-2}}{\sqrt{3\gamma}}\, k$
  - **Strong instability** for $\gamma < 2$ $\Longrightarrow$ Theory unstable for $\gamma < 2$

    (coincides with Chromo-natural inflation)

    Missed by Maleknejad & Sheikh-Jabbari '11
- Observable quantity: curvature perturbation $\zeta = -\frac{H}{\dot{\rho}}\delta\rho$
- **Curvature power spectrum** $P_\zeta \cong 2.2 \times 10^{-9} \propto k^{n_s - 1}$
- **The larger γ, the larger n_s**

Phenomenology



Phenomenologically allowed space

$0.9457 < n_s < 0.9749$, $r < 0.11$ at 2σ from Planck

- $N = 50$: $\gamma_{\rm in} \gtrsim 13.5$ for n_s **and** $\gamma \lesssim 4.8$ for r
- $N = 60$: $\gamma_{\rm in} \gtrsim 9.3$ for n_s **and** $\gamma \lesssim 5.0$ for r

$\Longrightarrow$ **NO ALLOWED PARAMETER SPACE**

Conclusions

- The only existing stable inflationary model with a vector field alone
  - Related to Chromo-natural inflation with fewer parameters
  - Does not suffer from the flat-potential issue in scalar-field models
  - No explicit breaking of gauge invariance – no ghosts, stable
  - Interesting in the theoretical perspective
- Phenomenologically, not viable
  - Consistent with the results obtained in the Chromo-natural inflation model
  - “Analogy” between Gauge-flation & Chromo-natural persists in perturbations

◇ Symmetry consideration
  - Rotational symmetry is restored by $SU(2)$ transformation
  - $SU(2)$ spontaneously breaks CPT in the tensor sector with the given background
  - $SU(2)$ can be a subgroup of larger symmetry groups

“CMB ISW-lensing bispectrum from cosmic strings”

by Daisuke Yamauchi

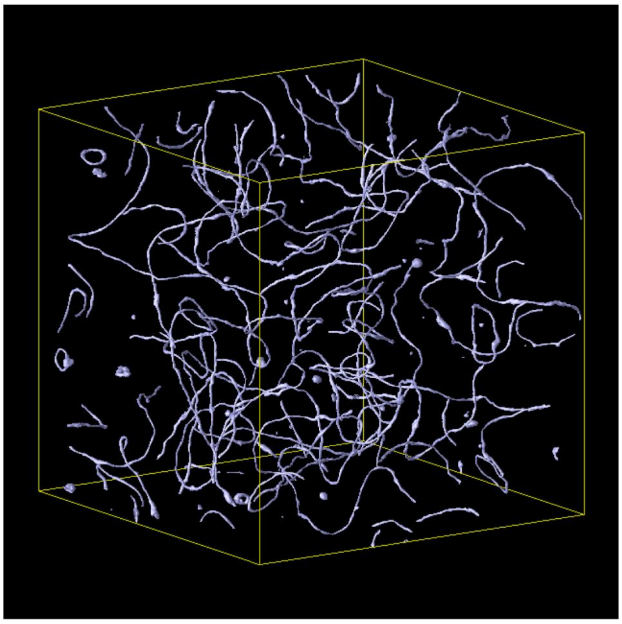
[JGRG23(2013)110513]

CMB ISW-lensing bispectrum from cosmic strings

YAMAUCHI, Daisuke
Research Center for the Early Universe (RESCEU), U. Tokyo

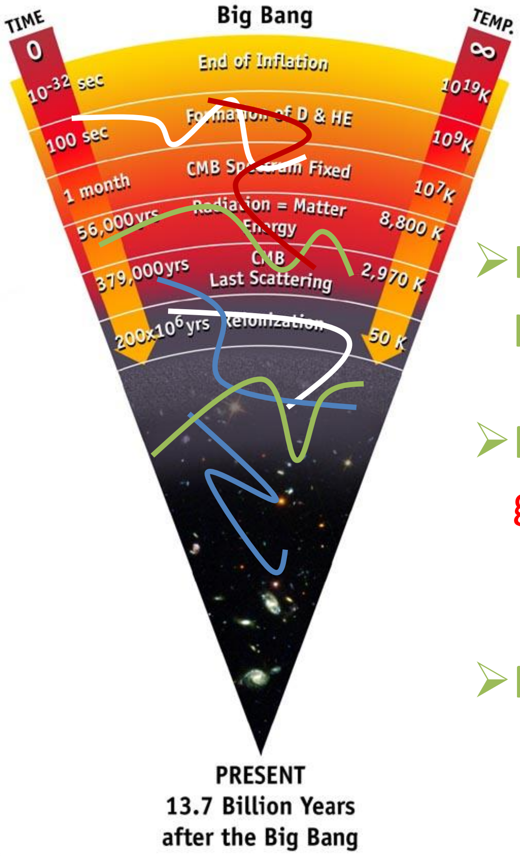
1309.5528 with Sendouda(Hirosaki) and Takahashi(Kumamoto)

Cosmic strings



[Hiramatsu+Sendouda+Takahashi+**DY**+Yoo (2013)]
[see also poster #03 Hiramatsu]

- Line-like topological defects
- generally form during phase transition in the very early universe. [Jeannerot+(2003)]
- could be a probe for the early phases of the universe before the CMB epoch!



Why are cosmic strings still interesting?

- Have a potential to reveal the physics during phase transition
- Possible sources of CMB, GWs, gravitational lensing, 21cm line,…

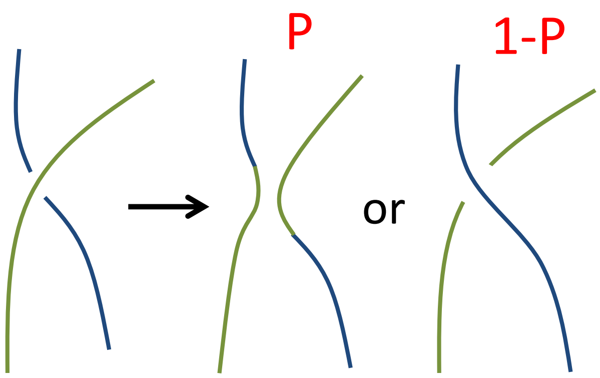
  [→ poster #23 Kitajima]
- Macroscopic objects of superstrings ("*cosmic superstrings*")

Cosmic superstrings

✓ A new type of cosmic strings may be formed at the end of stringy inflation!

[Sarangi+Tye(2002), Jones+(2003), Copeland+(2004)]

✓ Their qualitative properties in the late-time universe should be similar to those of normal cosmic strings, except for the INTERCOMMUTING PROBABILITY "P":



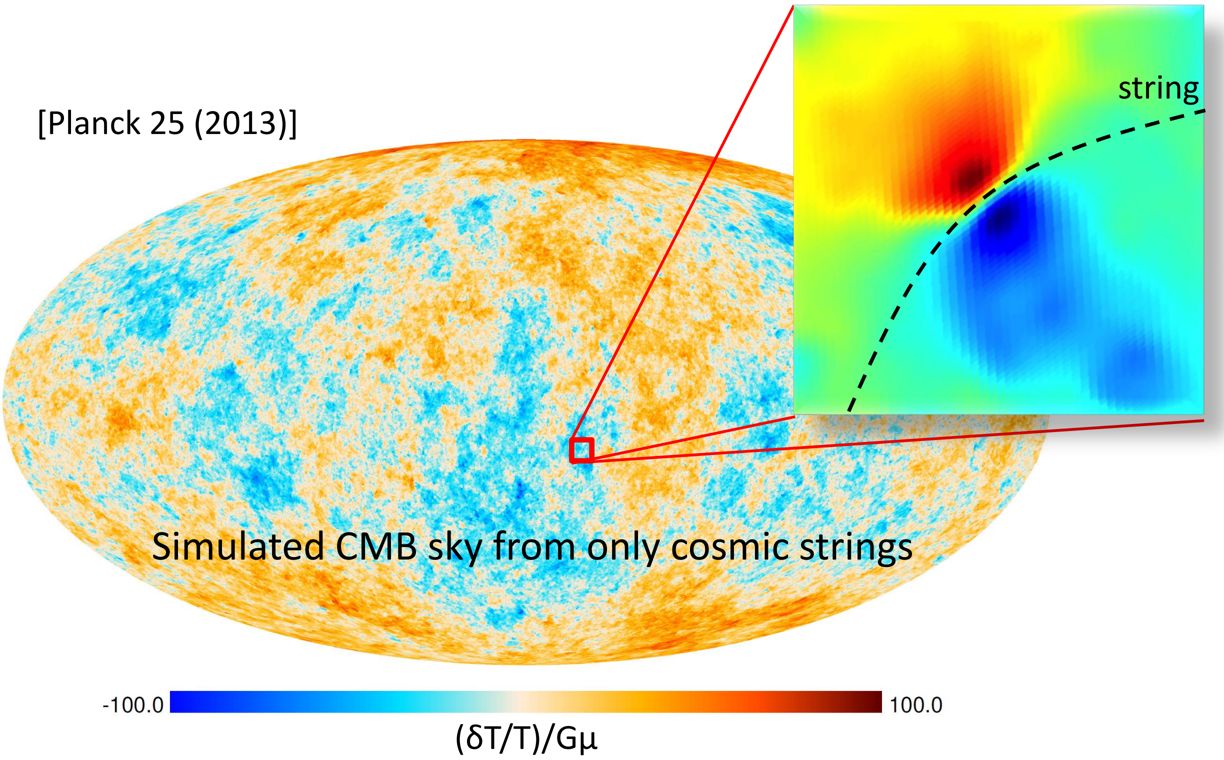
✓Cosmic superstrings

P<<1

✓Normal cosmic strings

P=1

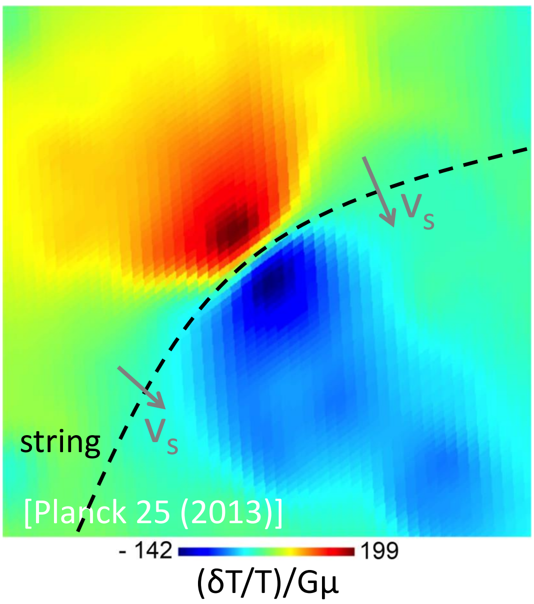
Cosmic strings on the CMB sky



Gott-Kaiser-Stebbins (GKS) effect

[Kaiser+Stebbins(1984), Gott III(1985)]

- ✓ most characteristic post-recombination effect of a cosmic string
- ✓ considered as an integrated Sachs-Wolfe (ISW) effect



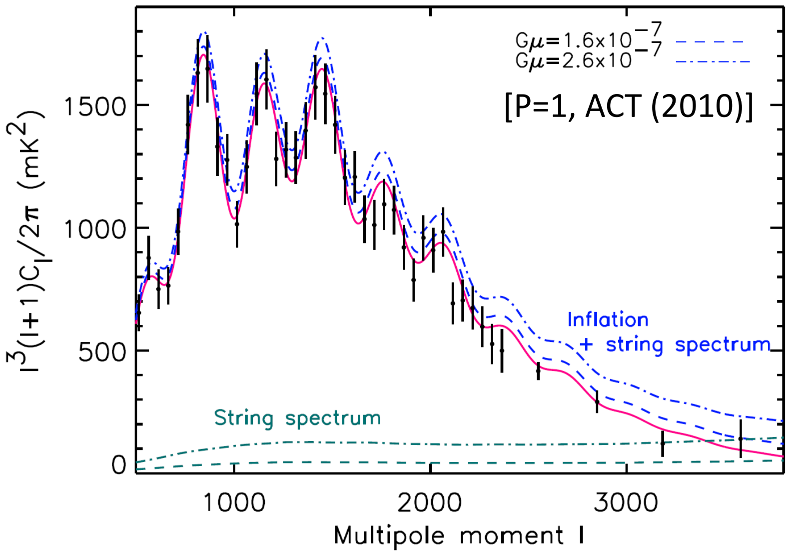
Discontinuities of the CMB temp. fluc. across the strings with the amplitude:

$$\frac{\delta T}{T} \equiv \Theta = 8\pi \frac{v_{\mathrm{s}}}{\sqrt{1-v_{\mathrm{s}}^2}} G\mu$$

(GKS effect with string curvature [**DY**+(2010)])

Current CMB constraint

✓ Cosmic strings would add power to small-scale tail of the CMB temp. power spectrum.



$$G\mu < 1.3 \times 10^{-7}$$

[P=1, Planck 25 (2013)]

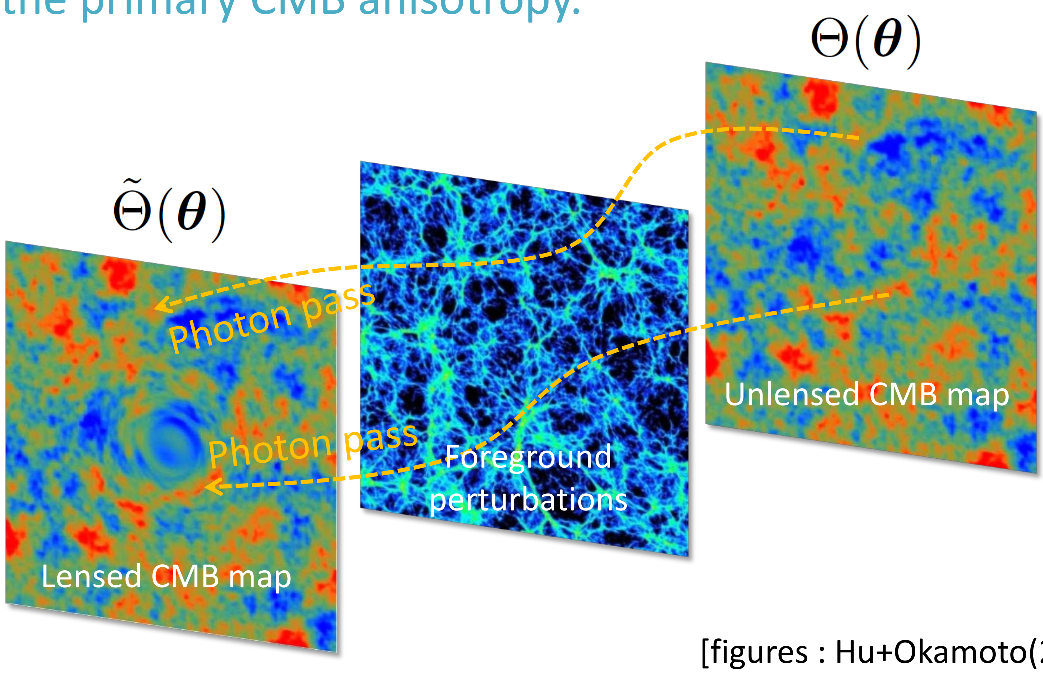
Note : Constraint from CMB lensing curl-mode

$$G\mu P^{-1} \leq 3.4 \times 10^{-5}$$

[Namikawa+**DY**+Taruya (2013)]

CMB lensing

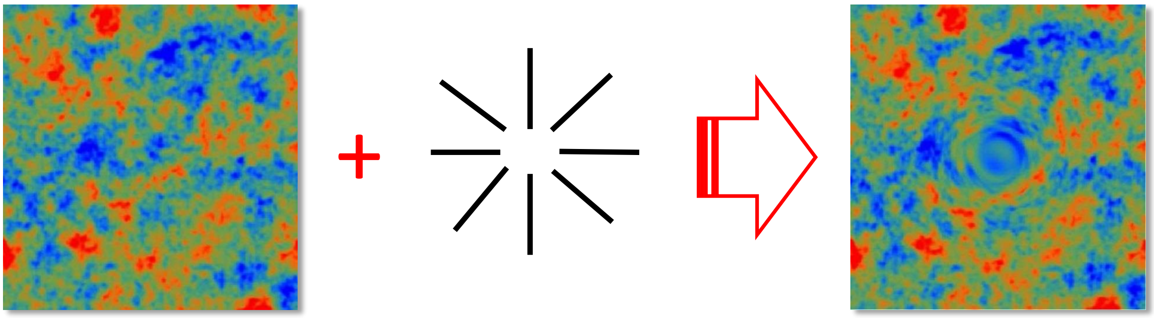
What we observe is a subtly distorted version of the primary CMB anisotropy.



[figures : Hu+Okamoto(2002)]

Lensing potential ϕ

The distortion effect of lensing on the primary CMB is expressed by a remapping with the gradient of the lensing potential ϕ.



Unlensed : $\Theta(\boldsymbol{\theta})$ Deflection field: $\nabla\phi$ Lensed : $\tilde{\Theta}(\boldsymbol{\theta})$

$$\tilde{\Theta}(\boldsymbol{\theta}) = \Theta(\boldsymbol{\theta} + \nabla\phi)$$

ISW-lensing bispectrum

- A lensed fluctuation is a nonlinear function of fields

$$\begin{aligned}\tilde{\Theta}(\boldsymbol{\theta}) &= \Theta(\boldsymbol{\theta} + \nabla\phi) \\ &= \Theta(\boldsymbol{\theta}) + \nabla\phi(\boldsymbol{\theta}) \cdot \nabla\Theta(\boldsymbol{\theta}) + \cdots\end{aligned}$$

Lensing events lead to deviations from Gaussianity

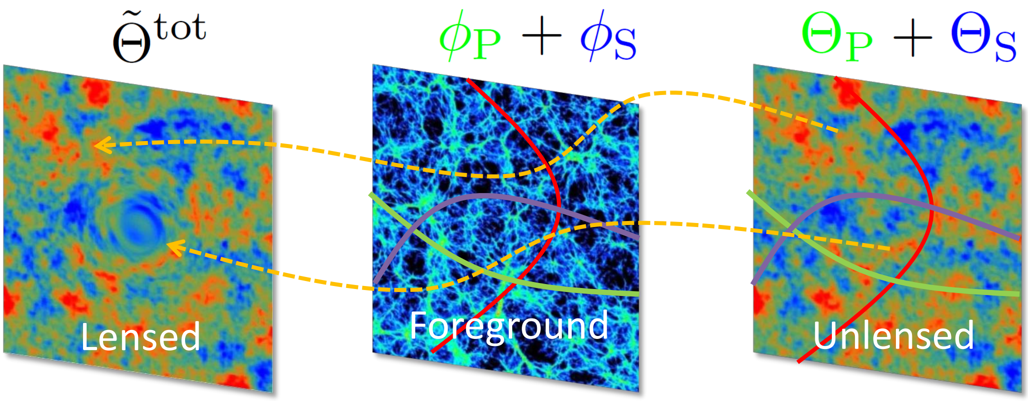
$$B^{\rm lens}(\ell_1, \ell_2, \ell_3) = -\boldsymbol{\ell}_1 \cdot \boldsymbol{\ell}_2 \, C_{\ell_1}^{\Theta\phi} C_{\ell_2}^{\Theta\Theta} + \cdots$$

✓ The cross-correlation due to the late-time evolution induces the *"ISW-lensing" bispectrum*.

CMB lensing from primordial perturbations (P) and cosmic strings (S)

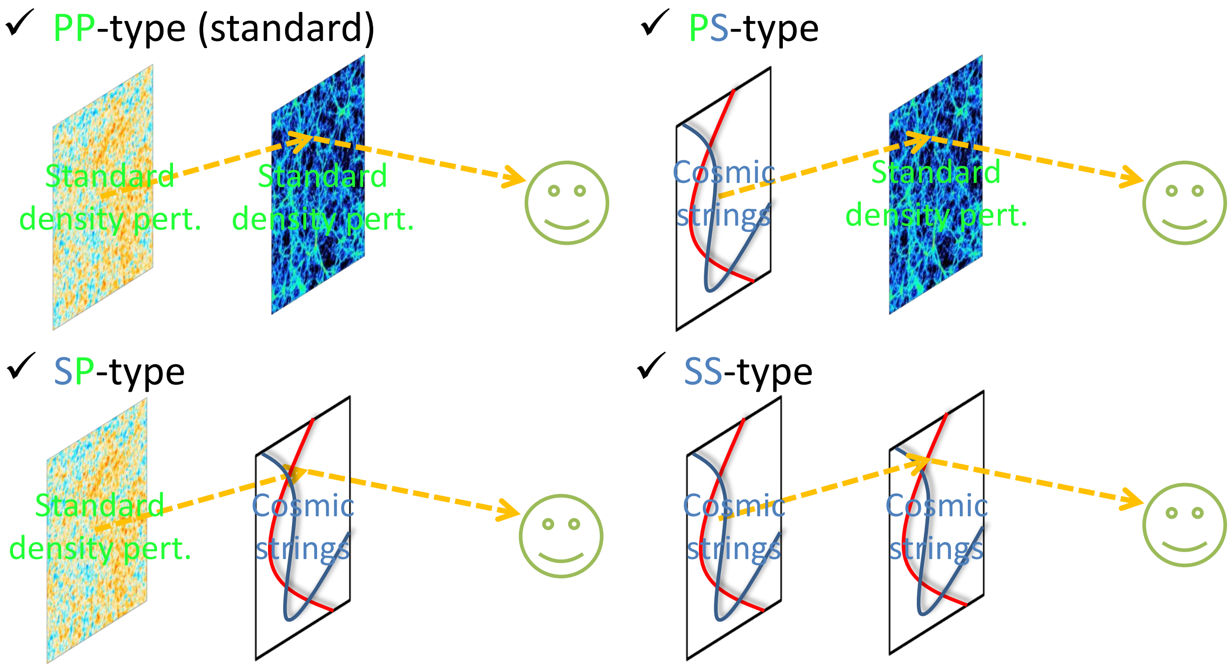
In the case of various independent gravitational sources, the observed CMB anisotropy can be regarded as a superposition of those due to each source.

$$\tilde{\Theta}^{\rm tot}(\boldsymbol{\theta}) = \sum_{\alpha={\rm P,S}} \Theta_\alpha\Big(\boldsymbol{\theta} + \sum_{\beta={\rm P,S}} \nabla\phi_\beta\Big)$$



Various types of CMB lensing

"P" : Primordial density perturbations "S" : Cosmic strings



New !

[DY+Sendouda+Takahashi(2013)]

αβ-type ISW-lensing bispectrum

$$B^{\alpha\beta}(\ell_1,\ell_2,\ell_3) = -\boldsymbol{\ell}_1 \cdot \boldsymbol{\ell}_2 \, C_{\ell_1}^{\Theta_\alpha \phi_\alpha} C_{\ell_2}^{\Theta_\beta \Theta_\beta} + \cdots$$

$B^{\rm tot} = B^{\rm PPP}$: Primordial bispectrum

$+ B^{\rm PP}$: Primordial ISW-lensing [2σ detection, Planck19]

- Cosmic strings

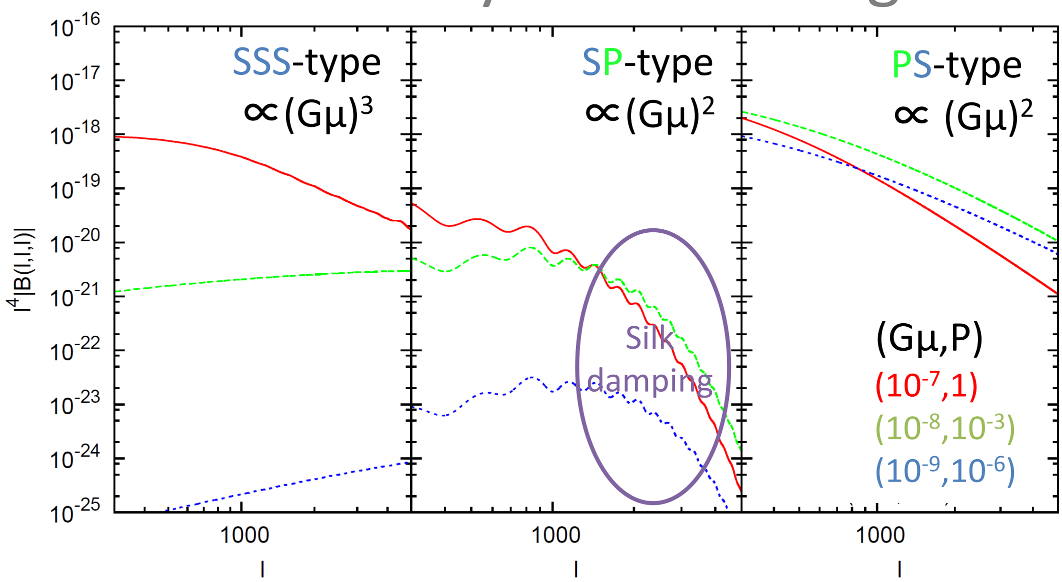
$+ B^{\rm SSS}$: purely due to the GKS effect [Hindmarsh+(2009), Regan+Shellard(2010)]

$+ B^{\rm SP} + B^{\rm PS} + B^{\rm SS}$

: String-induced ISW-lensing

New !

Equilateral-shaped bispectra induced by cosmic strings

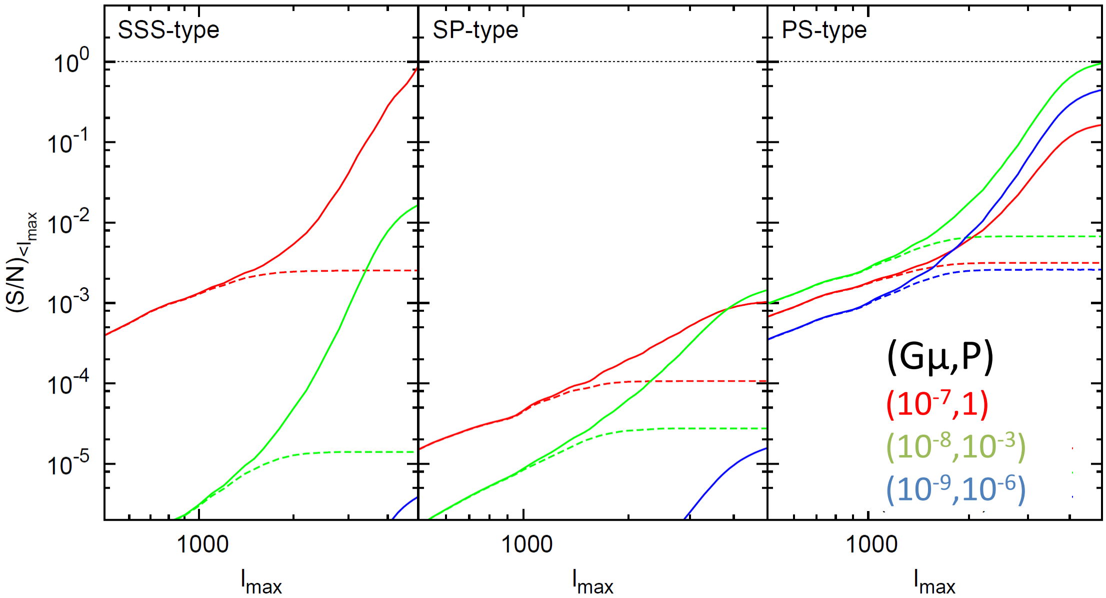


- The standard ISW-L (PP-type) and SP-type bispectra are particularly suppressed due to the Silk damping, so only the SSS- and PS-type bispectra are relevant at small scale.

New !

Cumulative signal-to-noise ratio

Solid : Planck+ACTPol–like noise, dashed : Planck-like noise



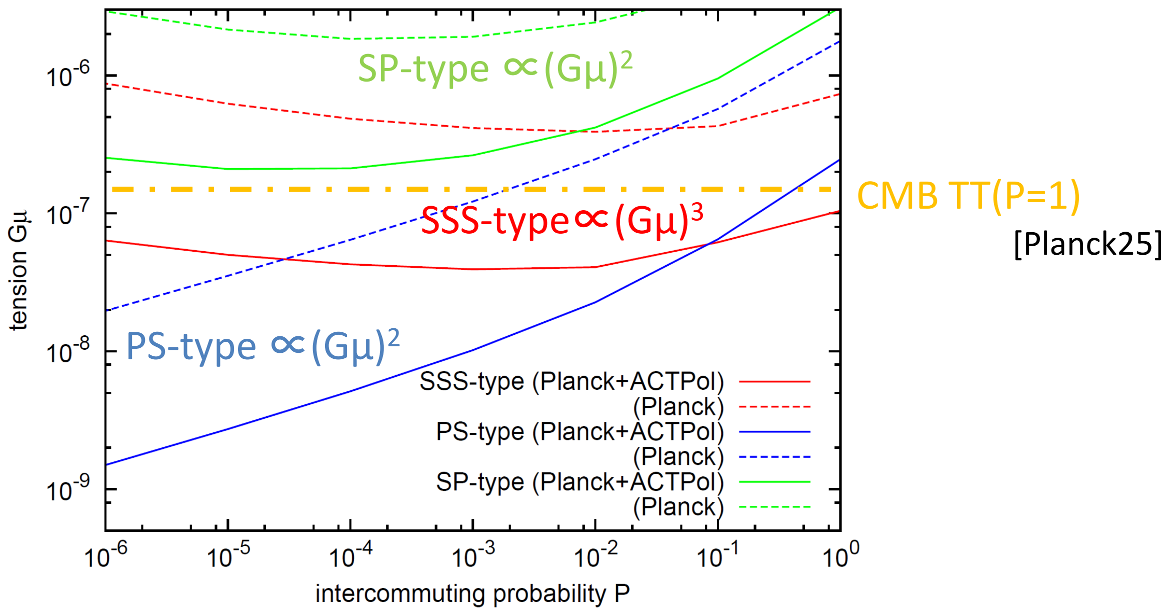
To estimate the feasibility to detect their signals, we quantify (S/N) in the current and future CMB observations. The SP-type is not relevant, as expected.

New !

[**DY**+Sendouda+Takahashi(2013)]

Constraint in Gμ-P plane

Solid : Planck+ACTPol–like noise, dashed : Planck-like noise



For small P, the PS-type ISW-L bispectrum $\propto C_l^{\Theta\rho\phi\rho} C_l^{\Theta s\Theta s} \propto (G\mu)^2$ gives the tighter constraint on Gμ than the SSS-type bispectrum $\propto (G\mu)^3$.

Summary

- A cosmic string segment is expected to cause weak lensing as well as the ISW effect, which are naturally produces the yet another kind of the CMB temp. bispectra, ***string-induced ISW-lensing bispectra (SP-, PS-, SS-type)***.

- The ISW-lensing bispectrum can constrain the string-model parameters even more tightly than the purely GKS-induced bispectrum in the future CMB observations on small scales.